Title: Alpha' - exact Backgrounds for String Cosmology

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Abstract: I will discuss a novel approach to time-dependent background fields of string theory, which allows the identification of configurations which satisfy the conformal invariance conditions to all orders in the Regge slope parameter alpha'. I will present a graviton, dilaton and tachyon configuration which is a resummation of all orders in the Regge slope and is shown to satisfy the conformal invariance conditions order by order, to all orders in alpha', up to field redefinitions which do not change the dependence of the fields on the target space-time coordinates, due to a property termed as "homogeneity" of the Weyl anomaly coefficients. It describes an inflationary universe in the sigma-model frame, while in the Einstein frame it corresponds either to a de Sitter or to a power-law universe, depending on the values of the dilaton and the tachyon amplitudes. Possible ways to connect these different solutions, which may correspond to different epochs of the universe, will also be discussed.

Anna Kostouki,
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Alpha' – exact Backgrounds in (first quantized) String Theory

J. Alexandre, N. Mavromatos: arXiv: 0811.4607 [hep-th]
M. Sakellariadou, F. Yusaf: in preparation

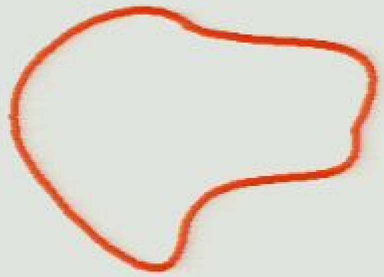
Perimeter Institute for Theoretical Physics, January 26 2011

Outline

- Introduction: String theory & time – dependent background fields
- A (bosonic string) toy – model: Time – dependent graviton, dilaton and tachyon backgrounds, which satisfy conformal invariance conditions to all orders in alpha' due to "homogeneity" properties
- Cosmology: Expanding universe, Hubble rate depending on tachyon and dilaton amplitudes
- Outlook: Further applications of resummation technique: D-particle foam in curved space-time

String Theory & Quantum Gravity

- Closed Bosonic String



D – dimensional "target" space – time

quantize string coordinates, X^{μ}

- Tachyon

lowest state in the spectrum ($M^2 < 0$)

- Graviton, dilaton and antisymmetric tensor

massless states

- Introduce these states as background fields, then study the effective theory

$$g_{\mu\nu}(X) \quad \phi(X) \quad B_{\mu\nu}(X) \quad T(X)$$

String Theory & Quantum Gravity

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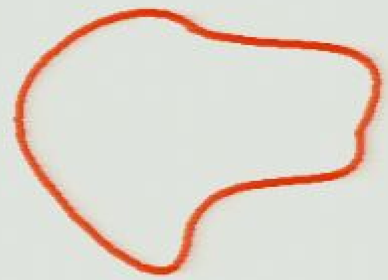

- Tachyon

  lowest state in the spectrum ($M^2 < 0$)

- Graviton, dilaton and antisymmetric tensor

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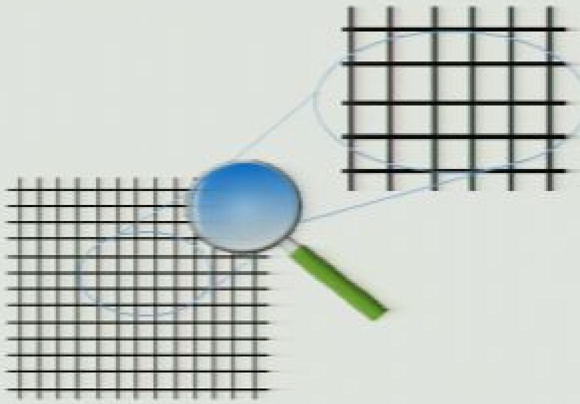
Open strings contain gauge field at massless level $A_\mu(X)$

- Introduce these states as background fields, then study the effective theory

$$g_{\mu\nu}(X) \quad \phi(X) \quad B_{\mu\nu}(X) \quad T(X)$$

World-sheet Conformal Invariance

- Diffeomorphisms and rescalings of the world-sheet coordinates do not affect Physics in the target space.



- Conformal invariance conditions:

$$\boxed{\beta^i(g) = 0}$$

Weyl anomaly coefficients must vanish.

$$g^i = (g_{\mu\nu}, \phi, B_{\mu\nu}, T, \ldots)$$

- Conformal invariance offers a tool for passing from the world-sheet quantum theory to the effective theory for the background fields in the target space.

World-sheet Conformal Invariance

$$\beta^g_{\mu\nu} = \beta^B_{\mu\nu} = \beta^\phi = \beta^T = 0$$

Conformal invariance conditions look like equations of motion for the background fields; one has to find background field configurations that satisfy these conditions.

$$\begin{aligned}
\beta^g_{\mu\nu} &= \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \phi - \alpha' \partial_\mu T \partial_\nu T + \mathcal{O}(\alpha'^2) \\
\beta^\phi &= \frac{D-26}{6} - \frac{\alpha'}{2} \nabla^2 \phi + \alpha' \partial^\mu \phi \partial_\mu \phi + \mathcal{O}(\alpha'^2) \\
\beta^T &= -2T - \frac{\alpha'}{2} \nabla^2 T + \alpha' \partial^\mu \phi \partial_\mu T + \mathcal{O}(\alpha'^2) \quad (\, B_{\mu\nu} = 0\,)
\end{aligned}$$

$$\boxed{\text{Regge slope parameter: } \alpha' = (\ell_s)^2}$$

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\end{aligned}$$

Calculations of the Weyl anomaly coefficients are done perturbatively. The parameter in the perturbative expansion is the Regge slope.

Effective Theory

Strings in background fields; CFT on the world-sheet

⇒

Conformal Invariance Conditions

⇓

E.o.m. for the fields in the target space – time

⇐

Effective action in the target space – time

Cosmological Background Fields

- Consider closed bosonic strings in time-dependent backgrounds

$$g_{\mu\nu}(X^0),\ \phi(X^0),\ B_{\mu\nu}(X^0),\ T(X^0)$$

- Weyl anomaly coefficients are also time-dependent only

- Look for configurations that satisfy

$$\beta^g_{\mu\nu}(X^0) = \beta^B_{\mu\nu}(X^0) = \beta^\phi(X^0) = \beta^T(X^0) = 0$$

Cosmological Background Fields

- First example: **Linear Dilaton** *(Myers 1987; Antoniadis et al. 1988)*

$$g_{\mu\nu} = \eta_{\mu\nu} \qquad T = 0$$
$$\phi(X^0) = -Q\, X^0 \qquad B_{\mu\nu} = 0$$

- It satisfies the conformal invariance conditions as long as

$$Q^2 = \frac{D-26}{6\,\alpha'}$$

Cosmological Background Fields

- Target space-time effective theory described by the **sigma-model frame** action

$$S^D_{eff} \propto \int d^D x \sqrt{-g}\, e^{-2\phi} \left[-\frac{2Q^2}{3\,\alpha'} + R + 4\partial_\mu \phi \partial^\mu \phi \right]$$

- **Einstein frame**: $g_{\mu\nu} \rightarrow e^{-\frac{4\phi}{D-2}} g_{\mu\nu} = e^{\frac{4QX^0}{D-2}} \eta_{\mu\nu}$

so that the Einstein – Hilbert term takes the form $\int d^D x \sqrt{-g}\, R$

- Move to FRW coordinates $t = \dfrac{D-2}{2Q} e^{\frac{2QX^0}{D-2}}$
- In FRW form, linearly growing scale factor: $a(t) = t$

Cosmological Background Fields

(Alexandre et al. 2006)

- New Graviton – Dilaton Background Configuration

$$g_{\mu\nu}(X^0) = \frac{A}{(X^0)^2}\eta_{\mu\nu}$$

$$\phi(X^0) = \phi_0 \ln\left(\frac{X^0}{\alpha'}\right)$$

- It has a specific property: it leads to "homogeneous" Weyl anomaly coefficients

Homogeneous β's

- First order graviton – dilaton Weyl anomaly coefficients

$$
\begin{aligned}
\beta^g_{00} &= -\frac{\alpha'}{(X^0)^2}(D-1) + \mathcal{O}(\alpha'^2) \\
\beta^g_{ij} &= \frac{\alpha'}{(X^0)^2}\left[(D-1)+2\phi_0\right]\delta_{ij} + \mathcal{O}(\alpha'^2) \\
\beta^\phi &= \frac{D-26}{6} - \frac{\alpha'}{A}\left[\frac{(D-1)\phi_0}{2}+\phi_0^2\right] + \mathcal{O}(\alpha'^2)
\end{aligned}
$$

- Higher order corrections have the same dependence on X^0

Homogeneous β's

- E.g. the next order correction to the dilaton Weyl anomaly coefficient has the form:

$$\beta^{\phi} = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\phi + \alpha'\partial_\mu\phi\partial^\mu\phi + \frac{\alpha'^2}{6}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \mathcal{O}(\alpha'^3)$$

- For the specific configuration the order (α'^2) correction is a constant

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{(D-1)(2D-3)}{A^2}$$

and similarly for all higher orders....

Homogeneous β's

- Similarly, all terms in $\beta^g_{\mu\nu}$ are proportional to $(X^0)^{-2}$

- This property of the configuration are called "homogeneity"

$$\beta^g_{00} = \frac{A}{(X^0)^2} \sum_{k=1}^{\infty} A_k \left(\frac{\alpha'}{A}\right)^k$$

$$\beta^g_{ij} = \delta_{ij} \frac{A}{(X^0)^2} \sum_{k=1}^{\infty} B_k \left(\frac{\alpha'}{A}\right)^k$$

$$\beta^\phi = \sum_{k=0}^{\infty} C_k \left(\frac{\alpha'}{A}\right)^k$$

$A_k,\ B_k,\ C_k$: dimensionless constants

Field Redefinitions

- General field redefinitions leave the S-matrix unaffected and are therefore a symmetry of the theory

$$g^i \to \tilde{g}^i = g^i + \delta g^i$$

- ...but alter the Weyl anomaly coefficients

$$\beta^i(g) \to \tilde{\beta}^i(\tilde{g})$$

$$\tilde{\beta}^i(g) = \beta^i(g) - \delta g^j \frac{\delta \beta^i}{\delta g^j} + \beta^j \frac{\delta(\delta g^i)}{\delta g^j}$$

- For time-dependent backgrounds:

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- For time-dependent backgrounds:

$$(x^0)) = \beta^i(g(x^0)) - \int dy^0\, \delta g^j(y^0)\, \frac{\delta \beta^i(g(x^0))}{\delta g^j(y^0)} + \int dy^0\, \beta^j(g(y^0))\, \frac{\delta(\delta g^i)(x^0)}{\delta g^j(y^0)}$$

Field Redefinitions

- There is an ambiguity in the definition of the Weyl anomaly coefficients starting from 2nd order in alpha'

(Metsaev, Tseytlin, 1987)

$$\beta^i = \sum_{k=1}^{\infty} (\alpha')^k \beta^i_k$$

$$g^i \to \tilde{g}^i = g^i + \sum_{k=1}^{\infty} (\alpha')^k \bar{g}^i_k$$

$$\delta\beta^i \equiv \sum_{k=1}^{\infty} (\alpha')^k \delta\beta^i_k = \sum_{\ell,m=1}^{\infty} (\alpha')^{\ell+m} \left[\beta^j_\ell \frac{\partial(\bar{g}^i_k)}{\partial g^j} - \bar{g}^j_\ell \frac{\partial(\beta^i_k)}{\partial g^j} \right]$$

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$$\delta\beta^i_1 = 0 \qquad \delta\beta^i_2 = \beta^j_1 \frac{\partial(\delta g^i_1)}{\partial g^j} - \delta g^j_1 \frac{\partial(\beta^i_1)}{\partial g^j}$$

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Field Redefinitions

Starting from the first order Weyl anomaly coefficients, it is possible to make a redefinition of the graviton and the dilaton fields that

- does not change their functional dependence on the time coordinate

$$
\begin{aligned}
\tilde{g}_{\mu\nu} &= g_{\mu\nu} + \alpha' g_{\mu\nu}\left(a_1 R + a_2 \partial_\mu\phi\partial^\mu\phi + a_3\nabla^2\phi\right)\\
\tilde{\phi} &= \phi + \alpha'\left(b_1 R + b_2\partial_\mu\phi\partial^\mu\phi + b_3\nabla^2\phi\right)
\end{aligned}
$$

- adds new (second order) terms to the Weyl anomaly coefficients that are homogeneous to the existing ones

Field Redefinitions

$$\beta^g_{00} = \frac{\alpha'}{(X^0)^2}\sum_{k=0}^{1} A_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

$$\beta^g_{ij} = \frac{\alpha'}{(X^0)^2}\delta_{ij}\sum_{k=0}^{1} B_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

$$\beta^\phi = \sum_{k=0}^{2} C_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

Field Redefinitions

- Result of the redefinition:

$$\tilde{\beta}^g_{00} = \frac{\alpha'}{(X^0)^2}\sum_{k=0}^{1}\tilde{A}_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

$$\tilde{\beta}^g_{ij} = \frac{\alpha'}{(X^0)^2}\delta_{ij}\sum_{k=0}^{1}\tilde{B}_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

$$\tilde{\beta}^\phi = \sum_{k=0}^{2}\tilde{C}_k\left(\frac{\alpha'}{A}\right)^k + \mathcal{O}\left((\alpha')^3\right)$$

- Enough ambiguous constants introduced to set

$$\sum_{k=0}^{1}\tilde{A}_k\left(\frac{\alpha'}{A}\right)^k = \sum_{k=0}^{1}\tilde{B}_k\left(\frac{\alpha'}{A}\right)^k = \sum_{k=0}^{2}\tilde{C}_k\left(\frac{\alpha'}{A}\right)^k = 0$$

Field Redefinitions

- After the field redefinition, the Weyl anomaly coefficients vanish to second order in alpha'.
- The procedure can be repeated order by order to all higher orders in alpha'.
- The proposed configuration satisfies the conformal invariance conditions to an arbitrary order in alpha', up to field redefinitions which do not alter the dependence of the background fields on the time coordinate.
- The proposed configuration should therefore be thought of as a resummation of all orders in alpha' and thus an exact background for perturbative string theory.

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$$\begin{aligned}\tilde{g}_{\mu\nu} &= g_{\mu\nu} + \alpha' g_{\mu\nu}\left(a_1 R + a_2 \partial_\mu \phi \partial^\mu \phi + a_3 \nabla^2 \phi\right) \\ \tilde{\phi} &= \phi + \alpha'\left(b_1 R + b_2 \partial_\mu \phi \partial^\mu \phi + b_3 \nabla^2 \phi\right)\end{aligned}$$

- adds new (second order) terms to the Weyl anomaly coefficients that are homogeneous to the existing ones

Field Redefinitions

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Tachyon Background

- Found that the time-dependent graviton – dilaton configuration

$$g_{\mu\nu}(X^0) = \frac{A}{(X^0)^2}\eta_{\mu\nu} \qquad \phi(X^0) = \phi_0 \ln\left(\frac{X^0}{\alpha'}\right)$$

satisfies conformal invariance conditions to all orders in alpha'

- Add time-dependent tachyon background

$$T(X^0) = \tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

- Weyl anomaly coefficients (almost) have the same homogeneity properties

(Alexandre, A.K., Mavromatos, 2008)

Tachyon Background

- Tachyon Weyl anomaly coefficient has one inhomogeneous term

$$\beta^T = -2T - \underbrace{\frac{\alpha'}{2}\nabla^2 T + \alpha'\partial^\mu\phi\partial_\mu T + \mathcal{O}(\alpha'^2)}_{\text{constants}}$$

$$-2\tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

- but it can also be removed by a field redefinition (that doesn't change the dependence of the tachyon field on the time coordinate)

Tachyon Background

- Field redefinition

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \alpha' g_{\mu\nu} \left(a_1 R + a_2 \partial\phi \cdot \partial\phi + a_3 \partial\phi \cdot \partial T + a_4 \partial T \cdot \partial T + a_5 \nabla^2 \phi + a_6 \nabla^2 T \right)$$

$$\tilde{\phi} = \phi + \alpha' \left(b_1 R + b_2 \partial\phi \cdot \partial\phi + b_3 \partial\phi \cdot \partial T + b_4 \partial T \cdot \partial T + b_5 \nabla^2 \phi + b_6 \nabla^2 T \right)$$

$$\tilde{T} = T \left(1 + \alpha' c R \right) + \alpha' \left(c_1 R + c_2 \partial\phi \cdot \partial\phi + c_3 \partial\phi \cdot \partial T + c_4 \partial T \cdot \partial T + c_5 \nabla^2 \phi + c_6 \nabla^2 T \right)$$

Tachyon Background

- Field redefinition
- Does not alter dependence of fields on the time coordinate, i.e.

$$\tilde{g}_{\mu\nu}(X^0) \propto (X^0)^{-2}\eta_{\mu\nu} \quad \tilde{\phi}(X^0), \tilde{T}(X^0) \propto \ln\left(X^0/\sqrt{\alpha'}\right)$$

- Cancels the inhomogeneous term in β^T as long as

$$c = -\frac{2\,A^2}{(\alpha')^2(D-1)\left[D(D-1)+(D+1)\tau_0^2\right]}$$

- Adds new homogeneous terms of order ${\alpha'}^2$ to all three coefficients
- The new $\mathcal{O}({\alpha'}^2)$ coefficients can be set to zero (there is enough freedom in the parameters of the redefinition)

Tachyon Background

- The configuration

$$g_{\mu\nu}(X^0) = \frac{A}{(X^0)^2}\eta_{\mu\nu}$$

$$\phi(X^0) = \phi_0 \ln\left(\frac{X^0}{\alpha'}\right)$$

$$T(X^0) = \tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

is a consistent closed bosonic string background, satisfying the conformal invariance conditions to an arbitrary order in the Regge slope, up to field redefinitions which do not alter the time-dependence of the fields.

Comments on the solutions

- The vacuum configuration found is exact in alpha', but is not a non-perturbative string solution.
- Field redefinitions are well-defined in the context of perturbative string theory.
- Massive degrees of freedom have not been taken into account.

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Effective Theory

Effective target space action from closed string partition function

(Tseytlin, 1987)

$$S_{eff}^{(D)} = \beta^i \frac{\partial}{\partial g^i} \mathcal{Z}[g^i]$$

$$\Downarrow$$

$$S_{eff}^{(D)} = \int d^D x \sqrt{-g}\, e^{-2\phi - T} \left[\frac{1}{2} g^{\mu\nu} \beta^g_{\mu\nu} - 2\beta^\phi - \beta^T \right]$$

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Effective Theory

Ambiguities in the (tachyon part of the) effective action...

(Swanson, 2008)

$$S_{eff}^{(D)} \propto \int d^D x \sqrt{-g}\, e^{-2\phi} \Big\{ f_0(T) + f_1(T)\, R + 4 f_2(T) \partial_\mu \phi \partial^\mu \phi$$
$$- f_3(T) \partial_\mu T \partial^\mu T - f_4(T) \partial_\mu \phi \partial^\mu T + \mathcal{O}(\alpha') \Big\}$$

$f_1(T)$: important for determining the form of the background metric in the Einstein frame

Following Tseytlin's proposal $\quad f_1(T) = e^{-T}$

Sigma-model frame

Solution in the sigma-model frame:

- Inflationary metric $g_{\mu\nu}(\bar{x}^0) = \left(\eta_{00},\, e^{2\,\bar{x}^0/\sqrt{A}}\,\eta_{ij}\right)$

$$\left(\begin{array}{rcl} x^0 & = & \sqrt{A}\,e^{-\bar{x}^0/\sqrt{A}} \\ x^i & = & \bar{x}^i \end{array}\right)$$

(the zero mode of X^0 plays the role of conformal time)

- Dilaton and tachyon: linear in time $\phi,\, T \propto \bar{x}^0$

Einstein frame

Passage to the Einstein frame:

$$g^E_{\mu\nu} = e^{\omega(\phi,T)} g_{\mu\nu}$$

$$\omega(\phi, T) = \frac{-4\,\phi + 2\ln f_1(T)}{D-2}$$

$$S^{(D)}_{eff} = \beta^i \frac{\partial}{\partial g^i} \mathcal{Z}[g^i] \quad \Longrightarrow \quad f_1(T) = e^{-T}$$

$$\Longrightarrow \quad \omega(x^0) = -\frac{2\,(2\,\phi_0 + \tau_0)}{D-2} \ln \frac{x^0}{\sqrt{\alpha'}}$$

Einstein frame

Solution in the Einstein frame:

- If $2\,\phi_0 + \tau_0 = 0$: Inflationary expansion

$$a(t) = \exp\left(\frac{t - t_0}{\sqrt{A}}\right)$$

- If $2\phi_0 + \tau_0 \neq 0$: Power-law expansion

$$a(t) = a_0 \left(\frac{t - t_0}{\sqrt{A}}\right)^{1+\frac{D-2}{2\phi_0+\tau_0}}$$

accelerated if $2\phi_0 + \tau_0 > 0\,;$

Minkowski if $2\phi_0 + \tau_0 = 2 - D$

Comments

- The configuration presented is consistent with the conformal invariance conditions of the world-sheet theory in any number of target space-time dimensions.
- A toy-model for inflation in 4 dimensions, without the need for compactification, has been constructed.
- The vacuum configuration found is exact in alpha', but the model has been formulated within the frame of perturbative string theory (the freedom of performing field redefinitions relies on the S-matrix approach).

Comments

Consistency with Perturbative String Theory

- String coupling has to be small (at late times)

$$g_s = e^{\phi} = \left(\frac{x^0}{\sqrt{\alpha'}}\right)^{\phi_0} \Longrightarrow \begin{cases} \phi_0 > 0 & \text{accelerating solutions} \\ \phi_0 < 0 & \text{decelerating solutions} \end{cases}$$

- No horizons $\Longrightarrow$ eternally accelerating solutions are ruled out

  - Exit from inflation?
  - Incomplete effective action?
  - Off-shell tachyon? *(Alexandre, Mavromatos 2009)*

Exit from Inflation?

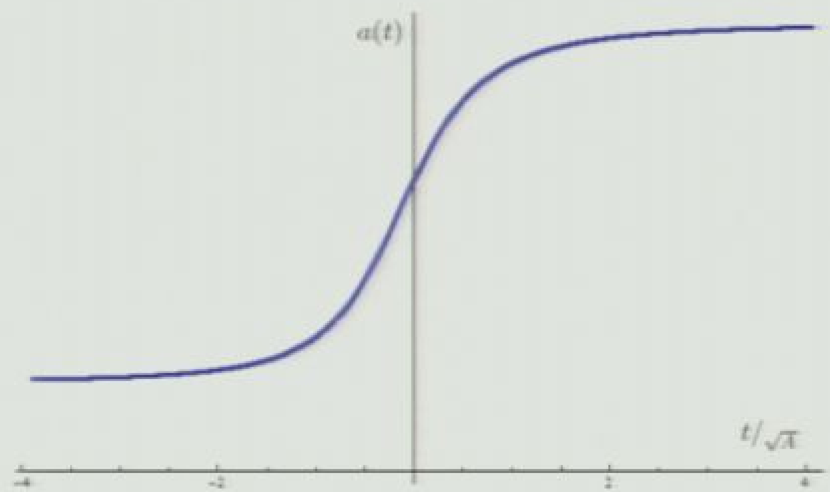
- Inflation takes place in this model as long as the dilaton and tachyon amplitudes satisfy the anti-alignment condition

$$2\,\phi_0 + \tau_0 \;=\; 0$$

- Mechanism changing the value of the dilaton/tachyon amplitude and thus disturbing the anti-alignment condition?
- The universe then enters a phase of power-law expansion

Exit from Inflation?

- The analysis made so far has been relying on $f_1(T) = e^{-T}$
- If this is not correct? Try different choices for $f_1(T)$ ...
- It is possible to obtain solutions in the Einstein frame that interpolate between flat universes with intermediate inflationary phases



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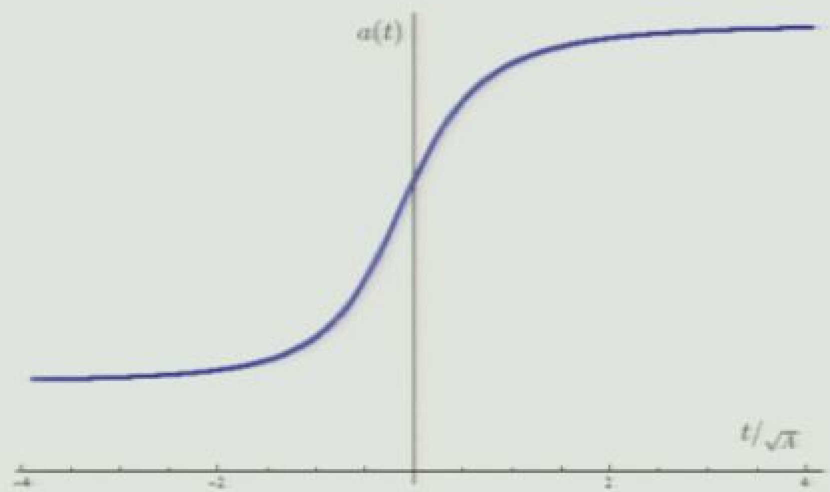

Effective Theory

Effective target space action from closed string partition function

(Tseytlin, 1987)

$$S^{(D)}_{eff} = \beta^i \frac{\partial}{\partial g^i} \mathcal{Z}[g^i]$$

$$\Downarrow$$

$$S^{(D)}_{eff} = \int d^D x \sqrt{-g}\, e^{-2\phi - T} \left[\frac{1}{2} g^{\mu\nu} \beta^g_{\mu\nu} - 2\beta^\phi - \beta^T \right]$$

Tachyon Background

- Field redefinition
- Does not alter dependence of fields on the time coordinate, i.e.

$$\tilde{g}_{\mu\nu}(X^0) \propto (X^0)^{-2}\eta_{\mu\nu} \quad \tilde{\phi}(X^0), \tilde{T}(X^0) \propto \ln\left(X^0/\sqrt{\alpha'}\right)$$

- Cancels the inhomogeneous term in β^T as long as

$$c = -\frac{2\,A^2}{(\alpha')^2(D-1)\left[D(D-1)+(D+1)\tau_0^2\right]}$$

- Adds new homogeneous terms of order α'^2 to all three coefficients
- The new $\mathcal{O}(\alpha'^2)$ coefficients can be set to zero (there is enough freedom in the parameters of the redefinition)

Tachyon Background

- Tachyon Weyl anomaly coefficient has one inhomogeneous term

$$\beta^T = -2T \underbrace{- \frac{\alpha'}{2}\nabla^2 T + \alpha' \partial^\mu \phi \partial_\mu T + \mathcal{O}(\alpha'^2)}_{\text{constants}}$$

$$- 2\tau_0 \ln\left(\frac{X^0}{\sqrt{\alpha'}}\right)$$

- but it can also be removed by a field redefinition (that doesn't change the dependence of the tachyon field on the time coordinate)

Field Redefinitions

- After the field redefinition, the Weyl anomaly coefficients vanish to second order in alpha'.
- The procedure can be repeated order by order to all higher orders in alpha'.
- The proposed configuration satisfies the conformal invariance conditions to an arbitrary order in alpha', up to field redefinitions which do not alter the dependence of the background fields on the time coordinate.
- The proposed configuration should therefore be thought of as a resummation of all orders in alpha' and thus an exact background for perturbative string theory.

Field Redefinitions

Starting from the first order Weyl anomaly coefficients, it is possible to make a redefinition of the graviton and the dilaton fields that

- does not change their functional dependence on the time coordinate

$$\begin{aligned} \tilde{g}_{\mu\nu} &= g_{\mu\nu} + \alpha' g_{\mu\nu}\left(a_1 R + a_2 \partial_\mu\phi\partial^\mu\phi + a_3\nabla^2\phi\right) \\ \tilde{\phi} &= \phi + \alpha'\left(b_1 R + b_2\partial_\mu\phi\partial^\mu\phi + b_3\nabla^2\phi\right) \end{aligned}$$

- adds new (second order) terms to the Weyl anomaly coefficients that are homogeneous to the existing ones

Field Redefinitions

- There is an ambiguity in the definition of the Weyl anomaly coefficients starting from 2nd order in alpha'

(Metsaev, Tseytlin, 1987)

$$\beta^i = \sum_{k=1}^{\infty} (\alpha')^k \beta^i_k$$

$$g^i \to \tilde{g}^i = g^i + \sum_{k=1}^{\infty} (\alpha')^k \bar{g}^i_k$$

Field Redefinitions

- General field redefinitions leave the S-matrix unaffected and are therefore a symmetry of the theory

$$g^i \to \tilde{g}^i = g^i + \delta g^i$$

- ...but alter the Weyl anomaly coefficients

$$\beta^i(g) \to \tilde{\beta}^i(\tilde{g})$$

$$\tilde{\beta}^i(g) = \beta^i(g) - \delta g^j \frac{\delta \beta^i}{\delta g^j} + \beta^j \frac{\delta(\delta g^i)}{\delta g^j}$$

- For time-dependent backgrounds:

$$(x^0)) = \beta^i(g(x^0)) - \int dy^0 \, \delta g^j(y^0) \, \frac{\delta \beta^i(g(x^0))}{\delta g^j(y^0)} + \int dy^0 \, \beta^j(g(y^0)) \, \frac{\delta(\delta g^i)(x^0)}{\delta g^j(y^0)}$$

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Effective Theory

Ambiguities in the (tachyon part of the) effective action...

(Swanson, 2008)

$$S^{(D)}_{eff} \propto \int d^D x \sqrt{-g}\, e^{-2\phi} \Big\{ f_0(T) + f_1(T)\, R + 4 f_2(T) \partial_\mu \phi \partial^\mu \phi$$
$$- f_3(T) \partial_\mu T \partial^\mu T - f_4(T) \partial_\mu \phi \partial^\mu T + \mathcal{O}(\alpha') \Big\}$$

$f_1(T)$: important for determining the form of the background metric in the Einstein frame

Following Tseytlin's proposal $\quad f_1(T) = e^{-T}$

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$$f_3(T) + \frac{3}{2}\left(\frac{f_1'(T)}{f_1(T)}\right)^2 > 0$$

$$\frac{f_2(T)}{f_1(T)} + \frac{\left(6 f_1'(T) - f_4(T)\right)^2}{4 f_1(T) f_3(T) + 6\left(f_1'(T)\right)^2} < \frac{3}{2}$$

Open String Gauge Backgrounds

- Study time-dependent gauge field backgrounds, as a different application of the technique of resummation in alpha'
- The effective action for the gauge field is known to all orders in alpha' and in the field strength, but not to all orders in derivatives of the field strength.
- DBI action

$$S_{DBI} = -T_p \int d^p x \, e^{-\phi} \sqrt{-\det\left(g_{\mu\nu} - 2\pi\alpha' F_{\mu\nu}\right)}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Open String Gauge Backgrounds

- The e.o.m. for the gauge field is homogeneous for a time-dependent configuration of the form

$$A^i \propto u^i X^0$$

- along with the graviton – dilaton configuration

$$g_{\mu\nu}(X^0) = \frac{A}{(X^0)^2}\eta_{\mu\nu}$$

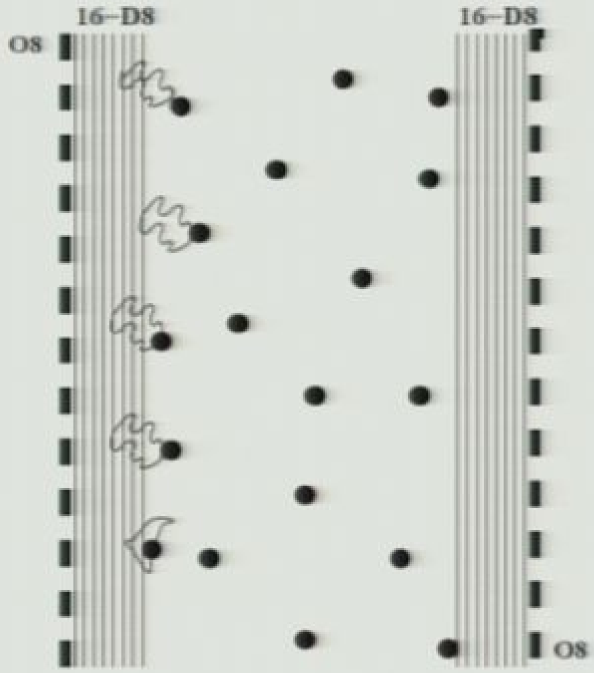
$$\phi(X^0) = \phi_0 \ln\left(\frac{X^0}{\alpha'}\right)$$

- => such backgrounds are exact solutions to the world-sheet conformal invariance conditions

D-particle Foam in Curved Space-time

- Time-dependent backgrounds of this type appear in D-particle foam models

Ellis, Mavromatos, Nanopoulos, 1999
Gravanis, Mavromatos, 2002
Mavromatos, Papavassiliou, 2003



- Two stacks of 16 D8-branes and one O8-plane
- One extra bulk dimension, filled with D-particles
- D-particles interact with photons on the branes and gravitons in the bulk

D-particle Foam in Curved Space-time

- Consider $D3$-brane in bulk space-time filled with D-particles
- Bulk graviton and dilaton configurations:

$$g_{\mu\nu}(X^0) = \frac{A}{(X^0)^2}\eta_{\mu\nu}$$

$$\phi(X^0) = \phi_0 \ln\left(\frac{X^0}{\alpha'}\right)$$

- The presence of the D-particle foam can be effectively described by a time-dependent gauge field
- Study recoil operator, take Liouville-dressed version...

$$u^i = \gamma v^i$$

$$\frac{u_i\, A}{4\pi\alpha'}\int_\Sigma d^2\xi\, \frac{\epsilon^{ab}}{(X^0)^2}\partial_a X^0 \partial_b X^i \quad \Longrightarrow \quad A^i(x^0) = \frac{u^i\, x^0}{2\pi\alpha'}$$

D-particle Foam in Curved Space-time

- Such time-dependent backgrounds are conformal (up to field redefinitions...)
- They are conformal even if higher-derivative corrections in the field strength are included in the effective action / equations of motion (the homogeneity properties remain valid)
- Role in cosmology? (enhanced growth of large scale structure? *Dodelson, Liguori 2006*)

Summary

- Time-dependent backgrounds for perturbative string theory have been studied.
- Certain non-trivial configurations have been shown to guarantee conformal invariance of the theory, to all orders in the Regge slope, due to their "homogeneity" properties.
- In the case of a graviton – dilaton – tachyon background these configurations correspond to a de Sitter universe.
- The method can be applied to other models relevant to string cosmology; a *D*-particle foam model is under investigation.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