Title: Cosmology Review - Lecture 6

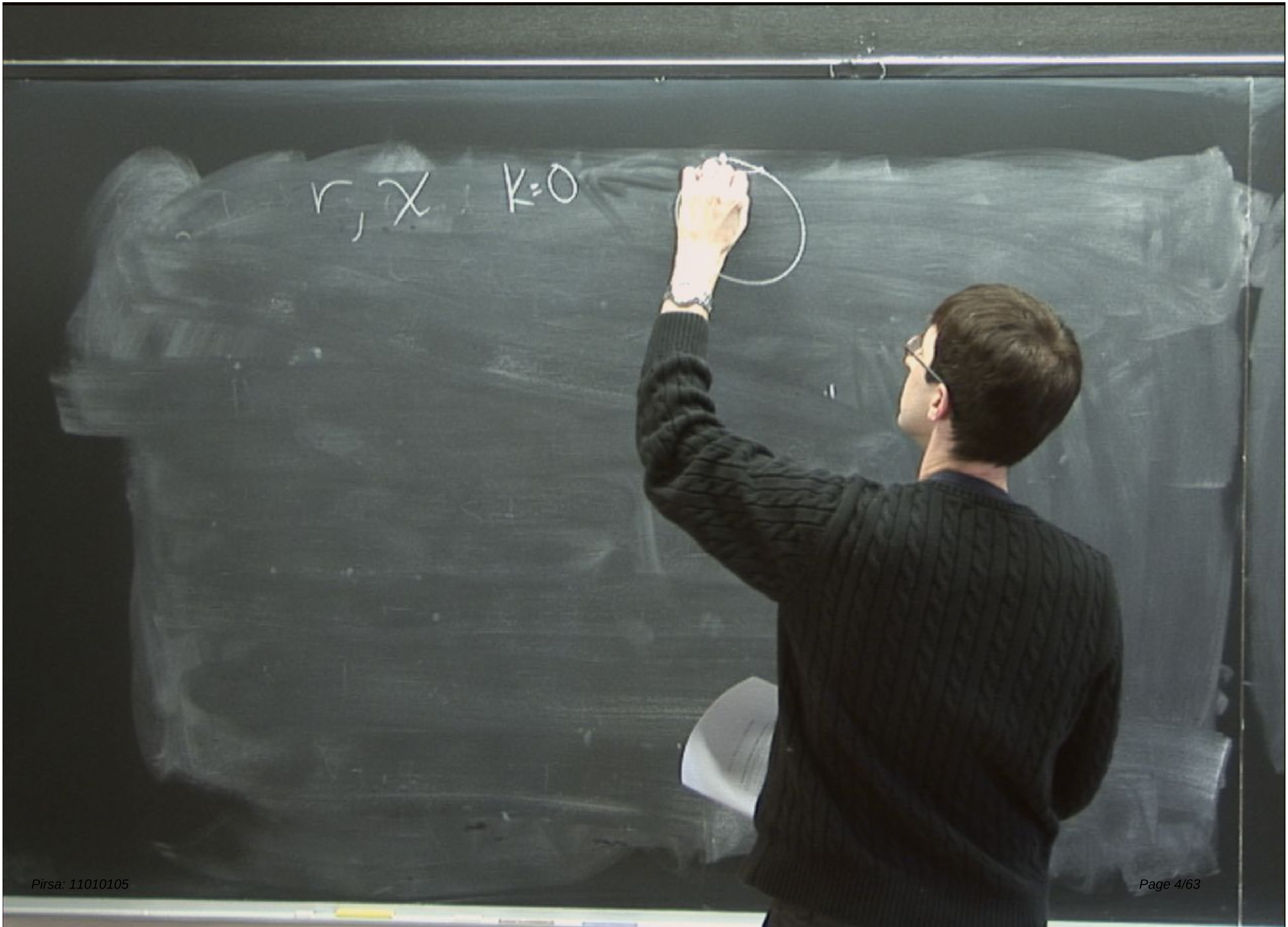
Date: Jan 31, 2011 11:30 AM

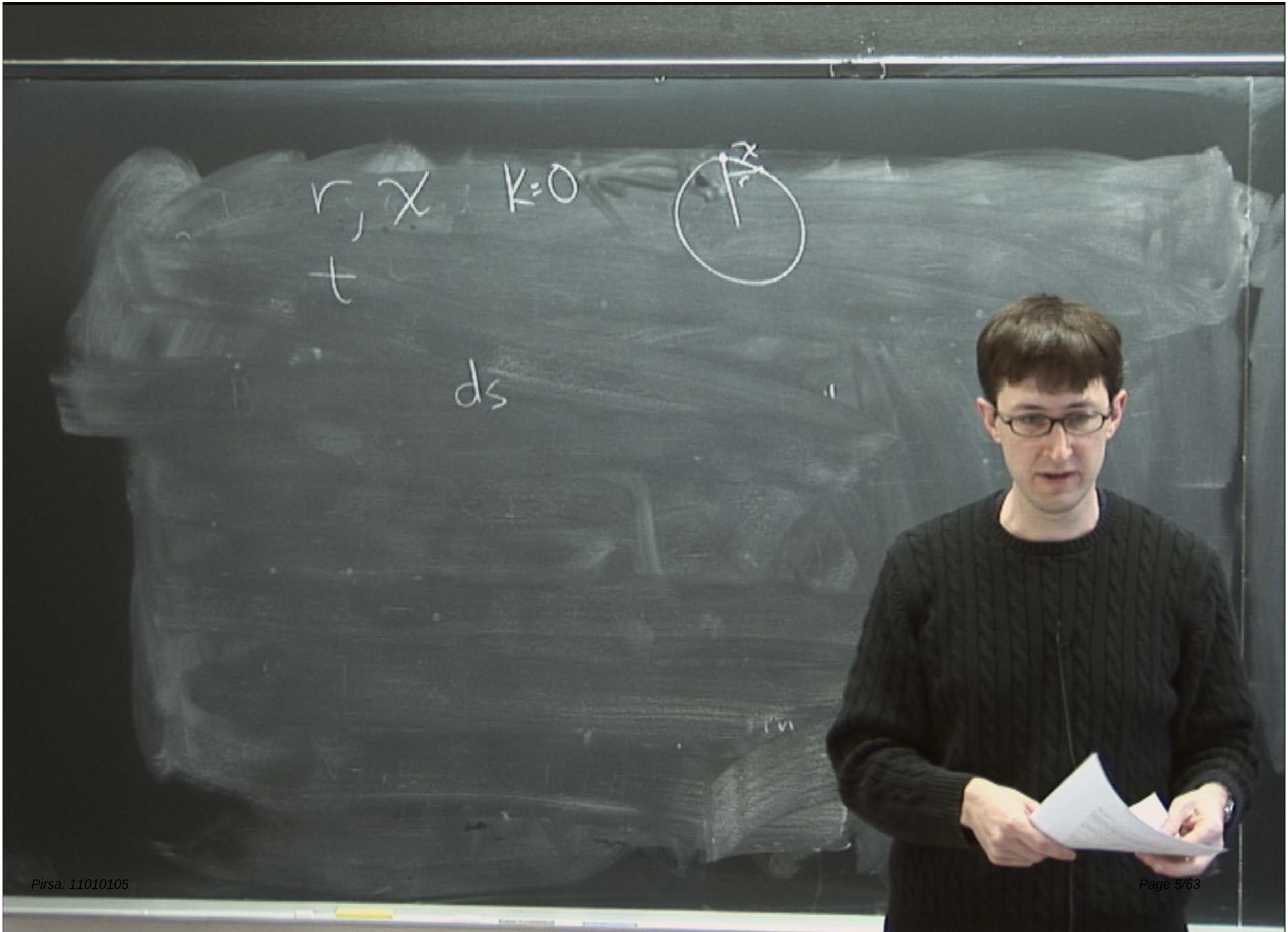
URL: http://pirsa.org/11010105

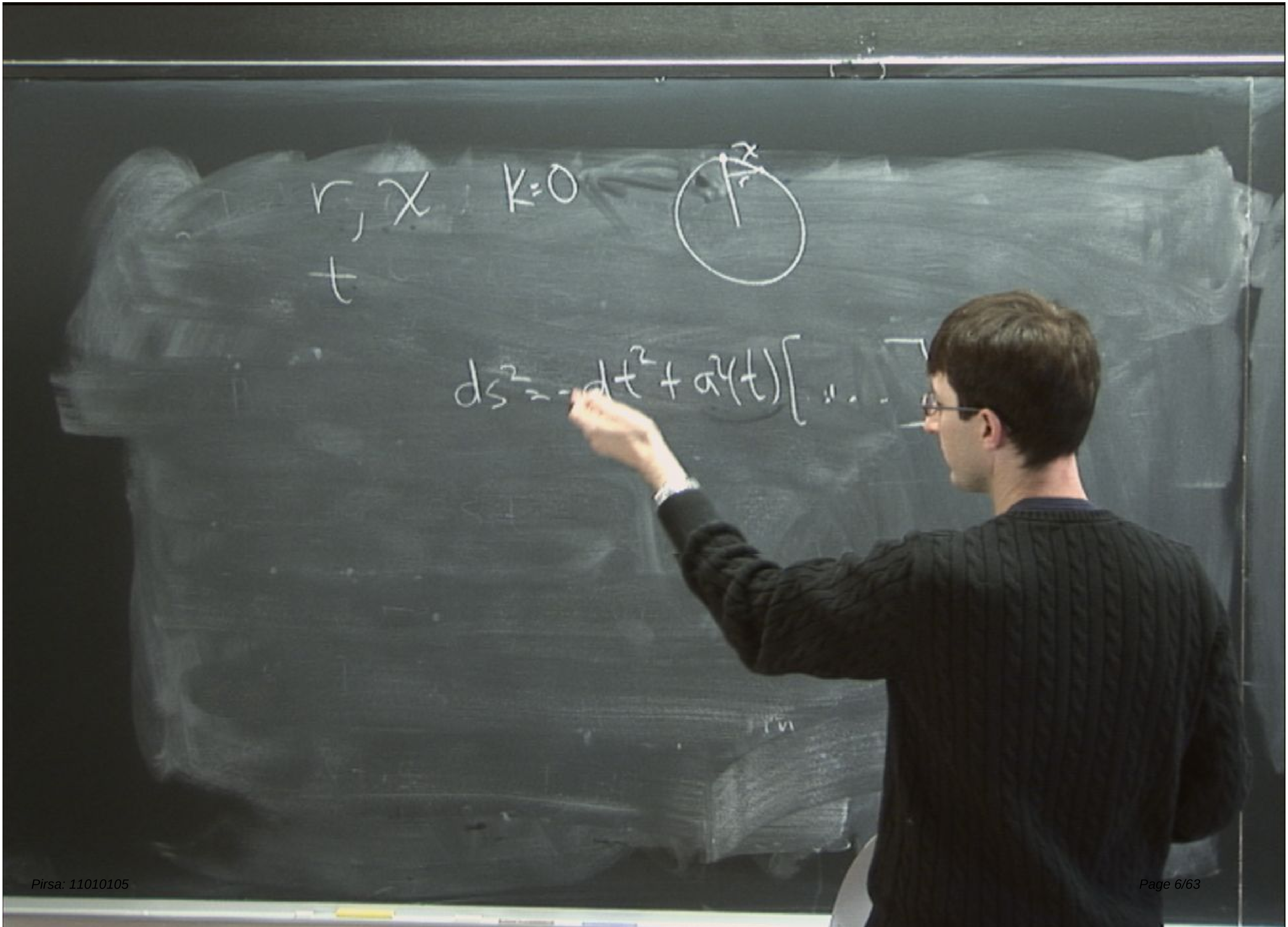
Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


r, χ k=0


r, χ $k=0$


r, χ k=0
t
ds

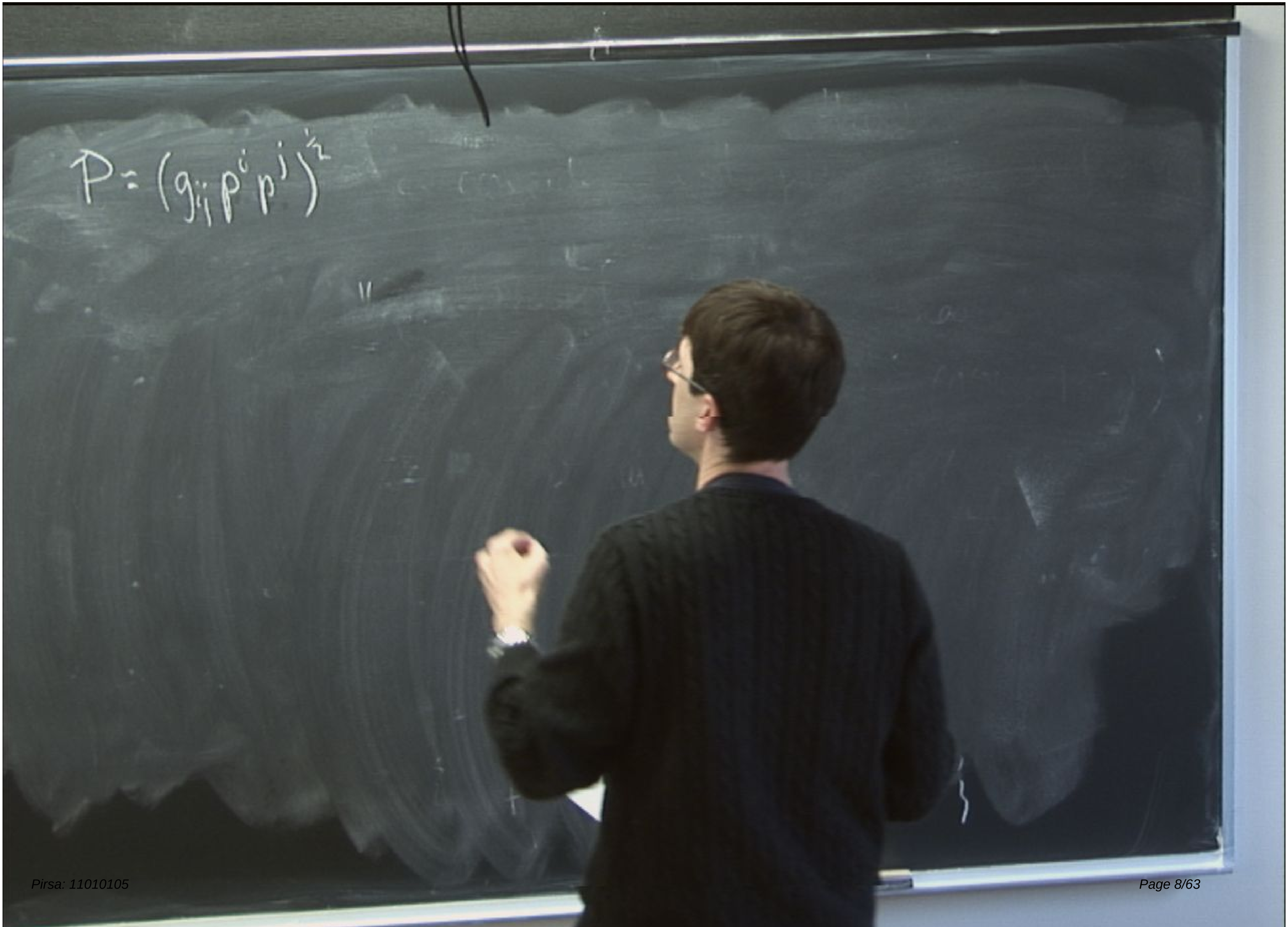

r, χ
k=0
t
ds² = -dt² + a²(t)[...]

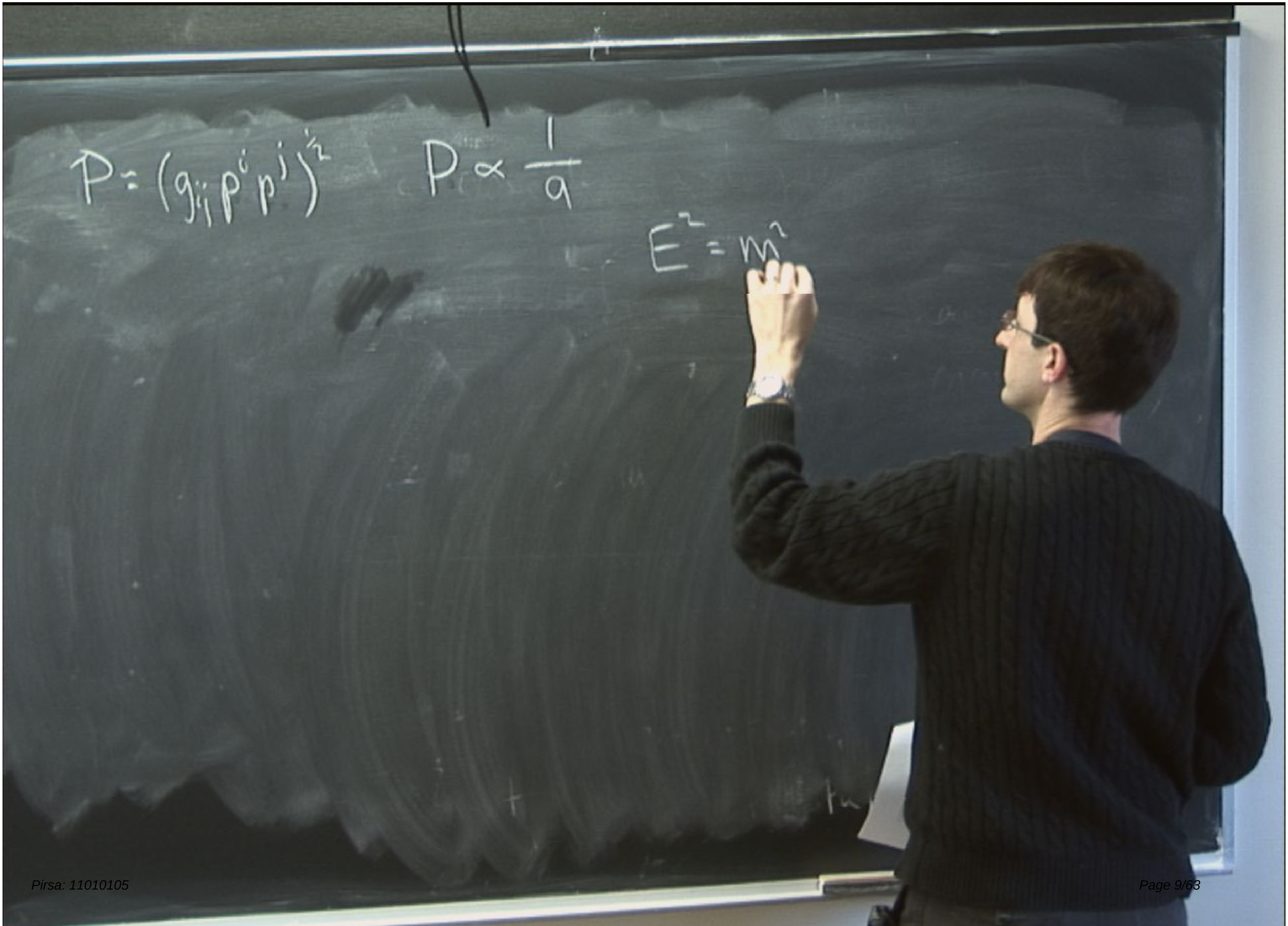
r, χ $\quad K=0$

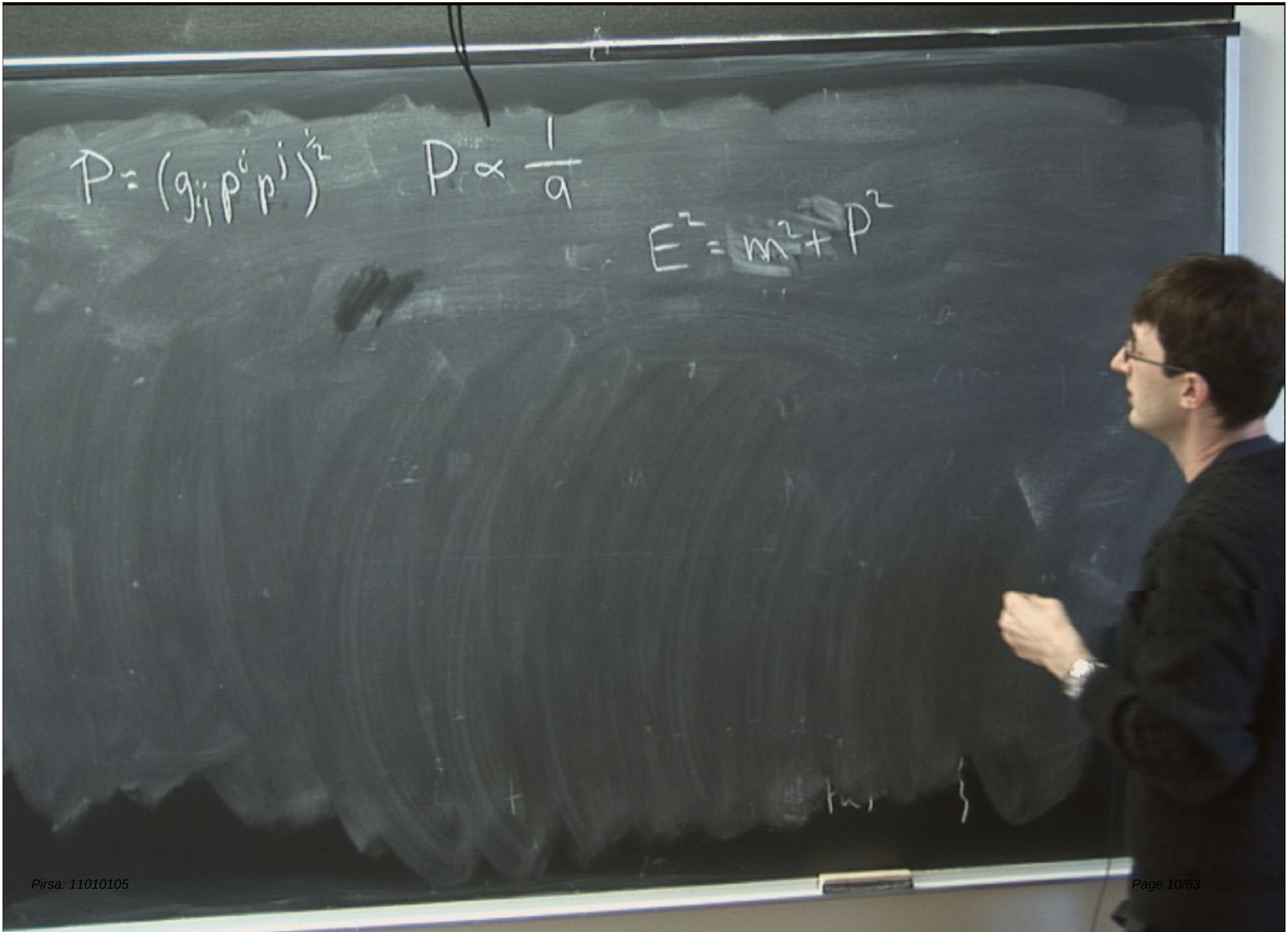
$dt = a\,d\eta$

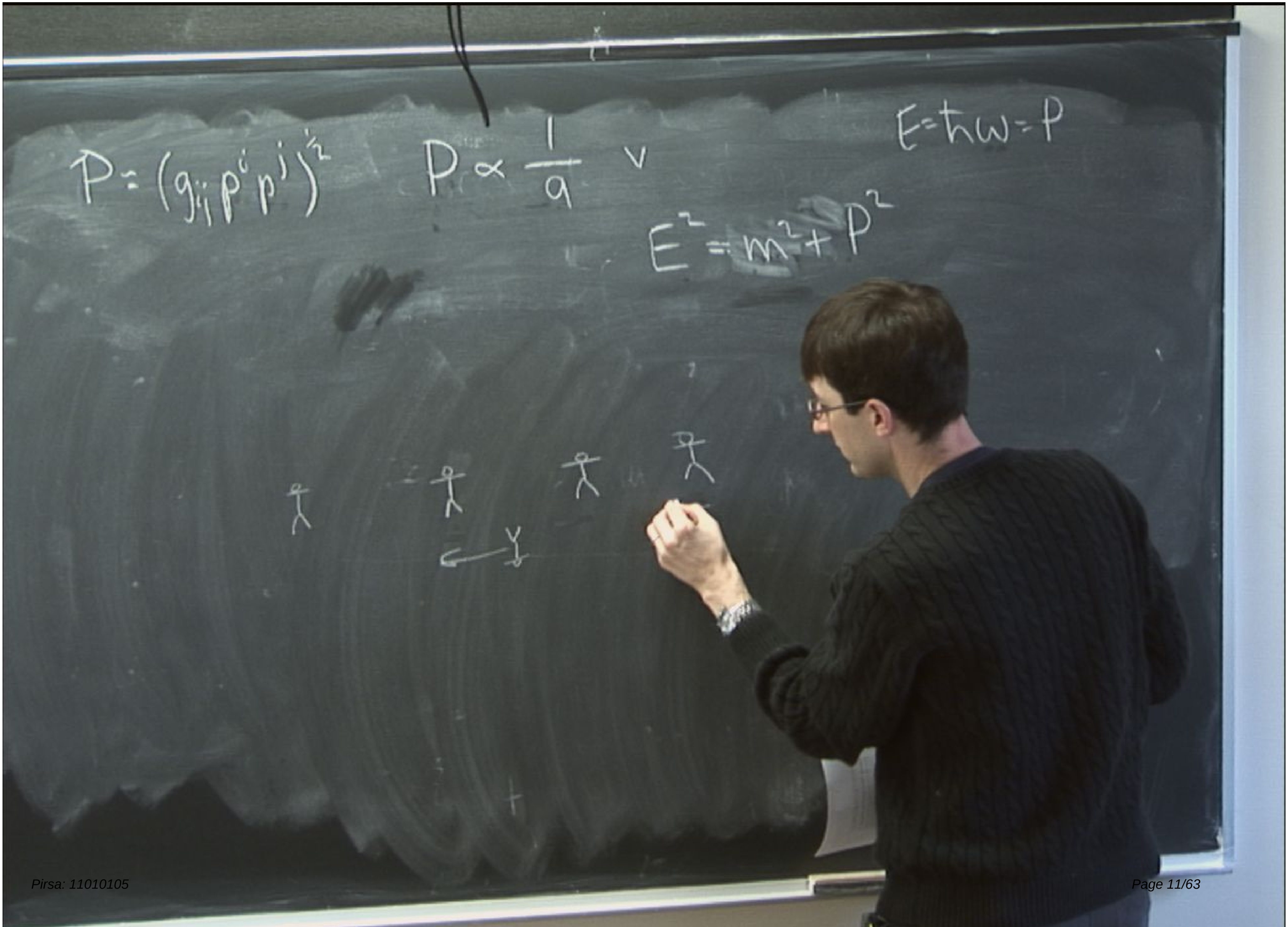
$$ds^2 = -dt^2 + a^2(t)\left[\ \ldots\ \right]$$

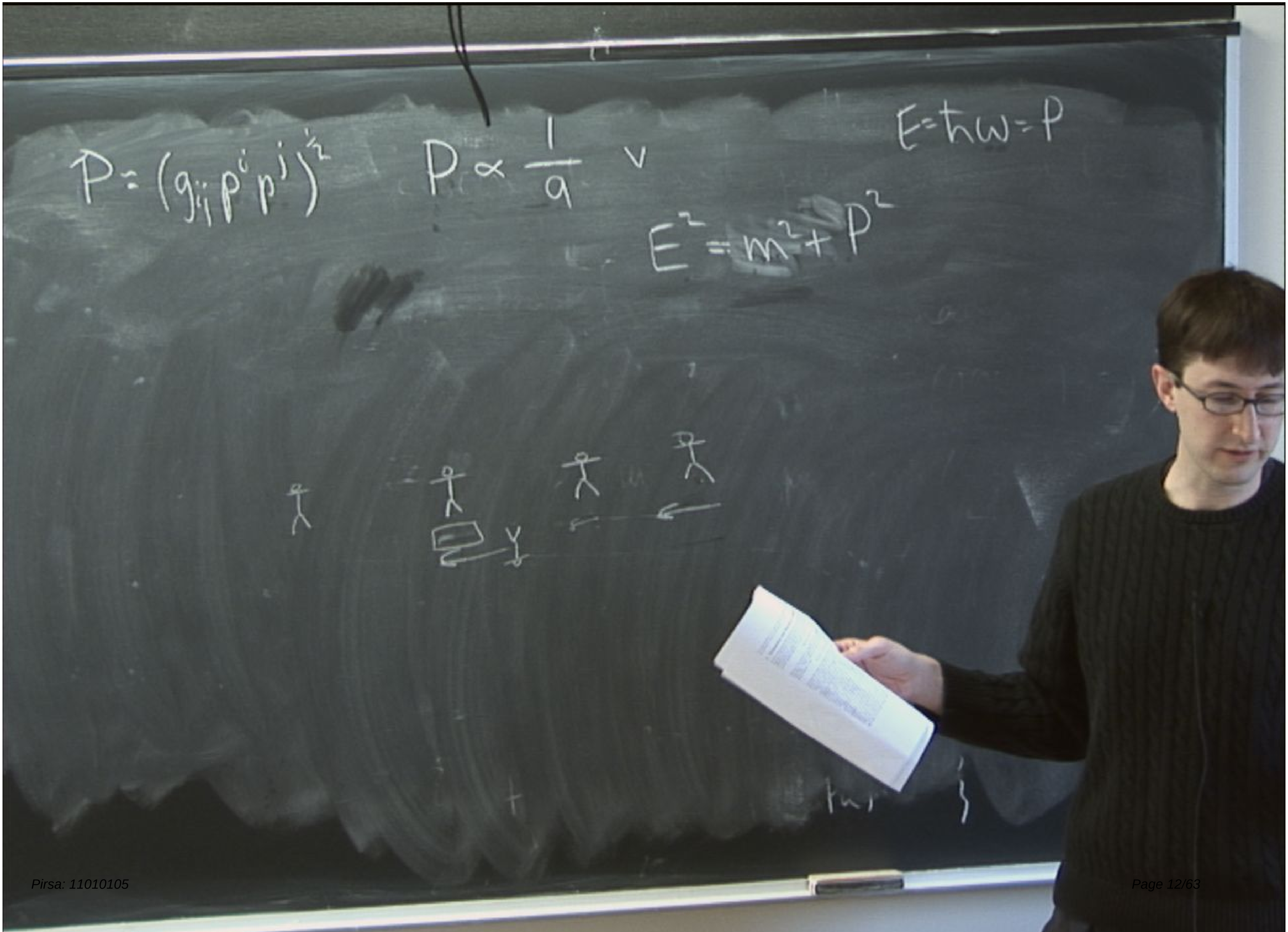
$$= a^2(\eta)\left[-d\eta^2 + \{\ \ldots\ \}\right]$$

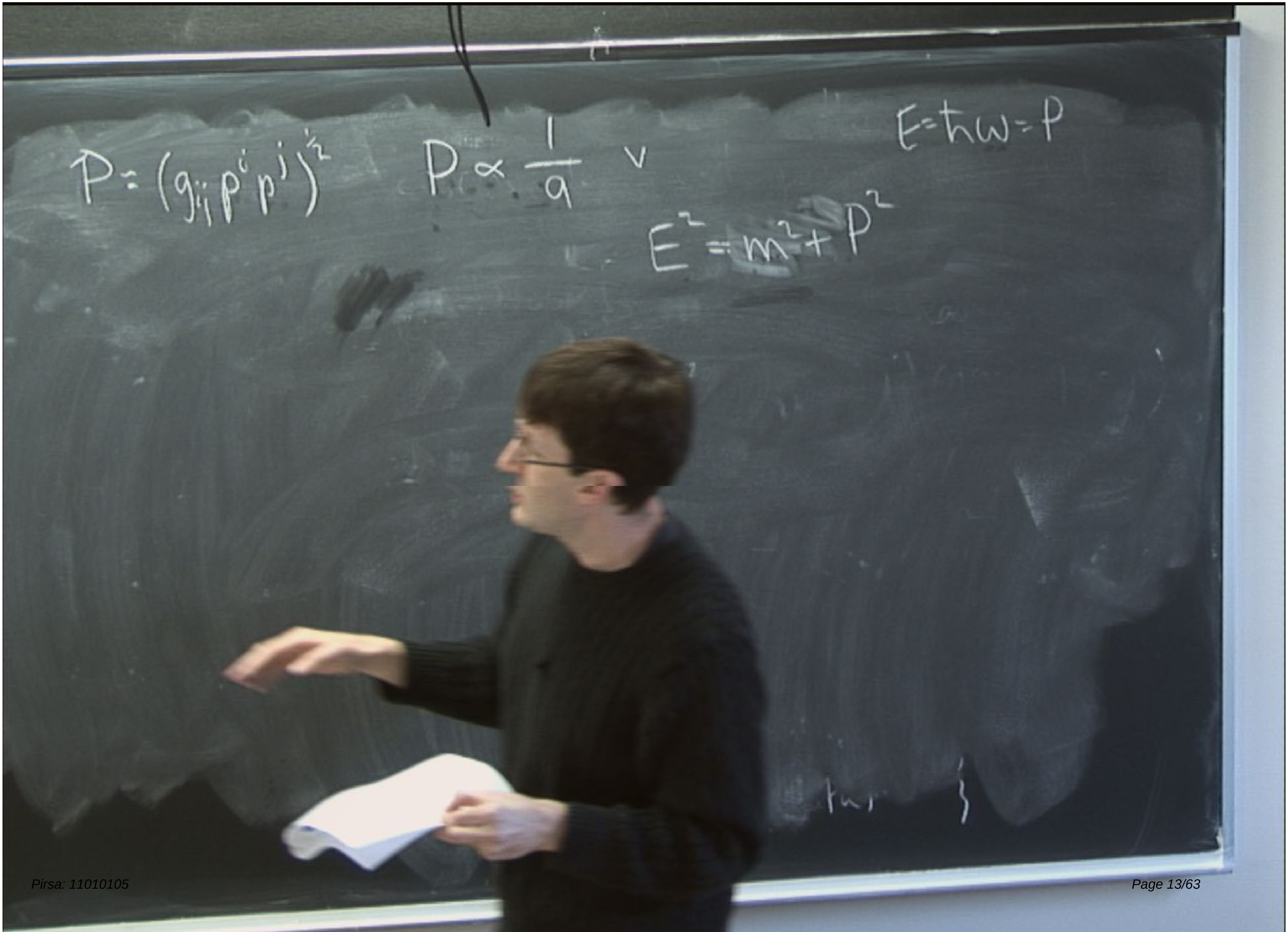

$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$

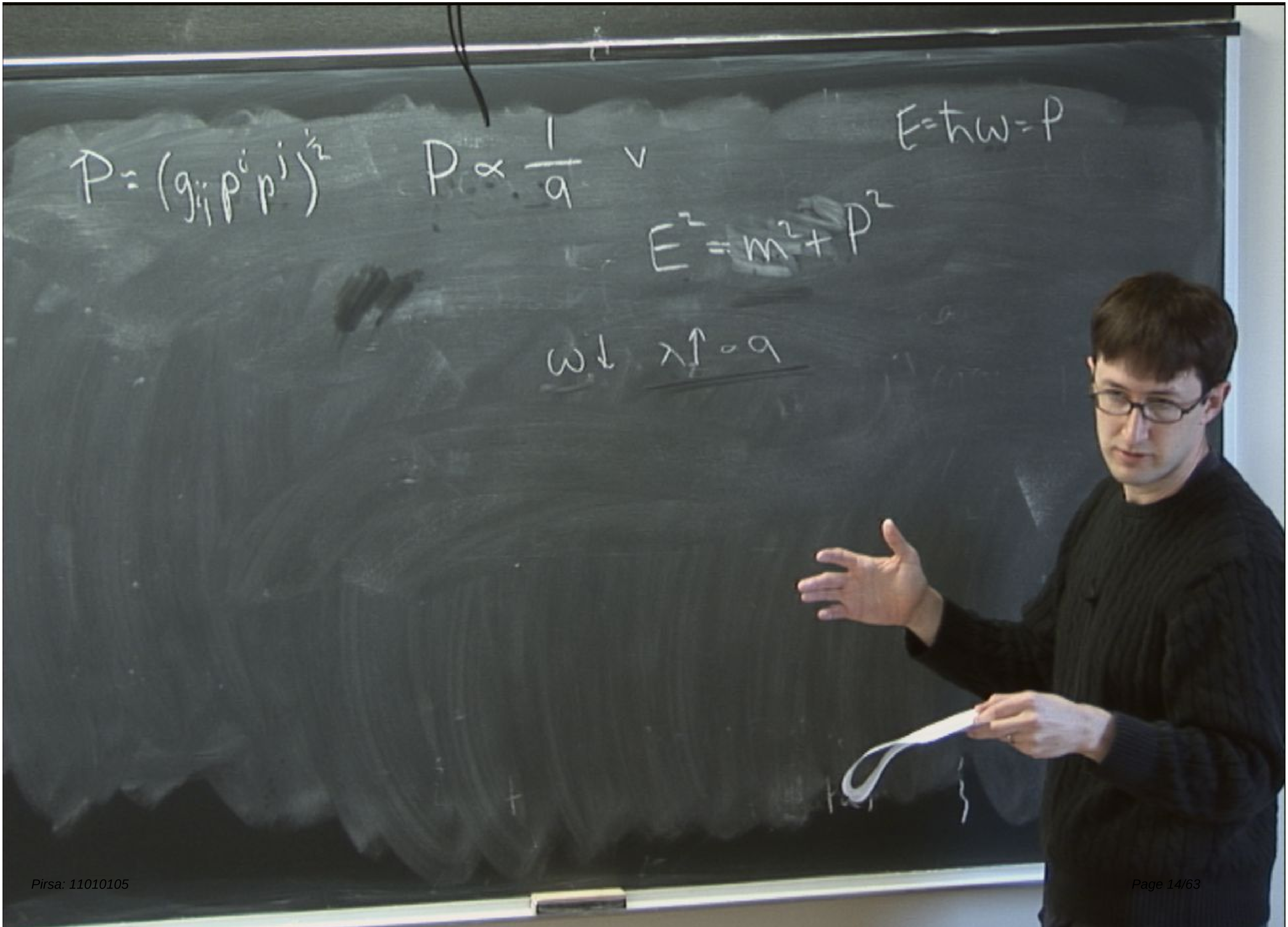

$P = (g_{ij} p^i p^j)^{1/2}$
$P \propto \frac{1}{a}$
$E^2 = m^2$

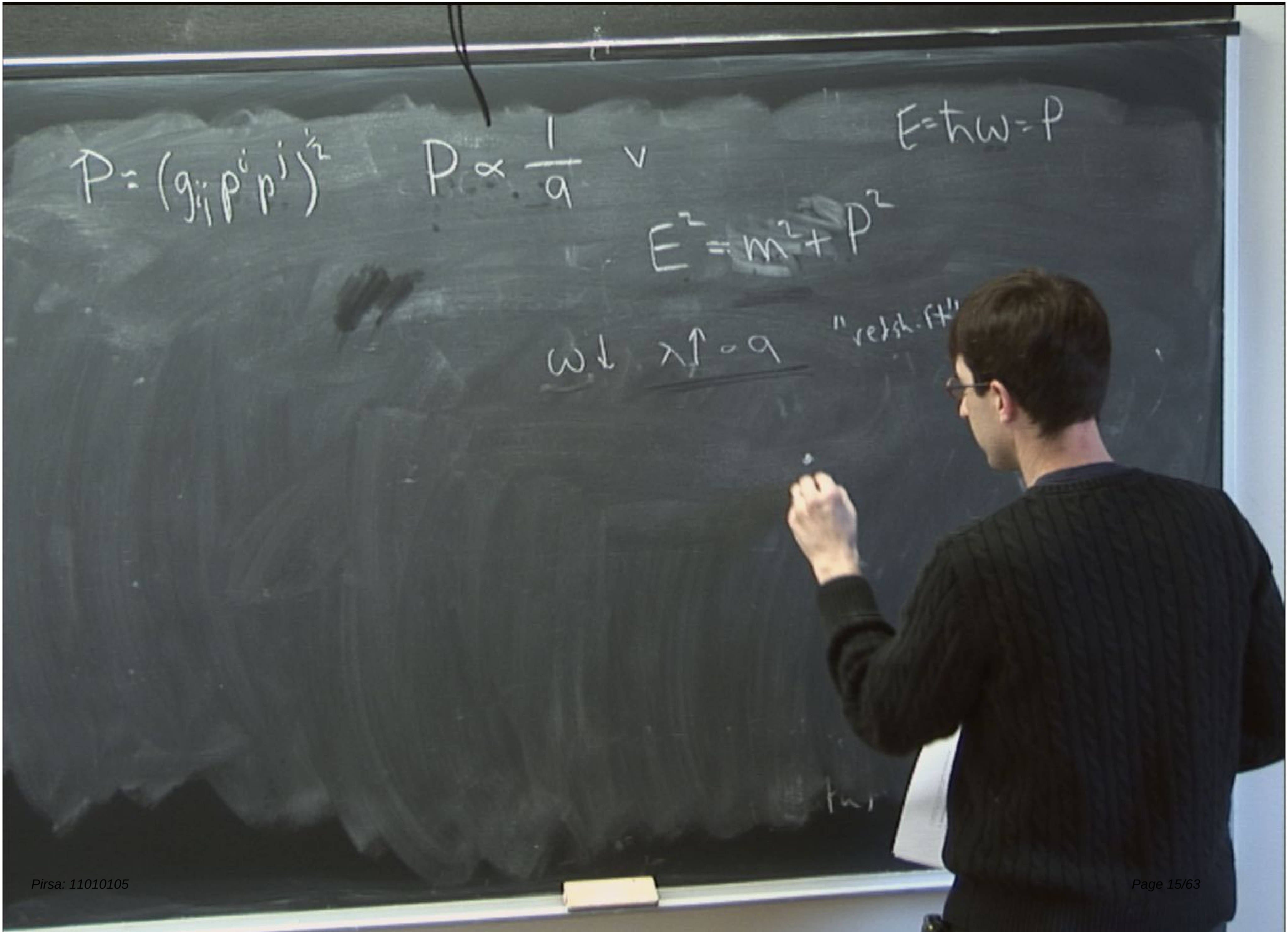

$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
$E^2 = m^2 + P^2$

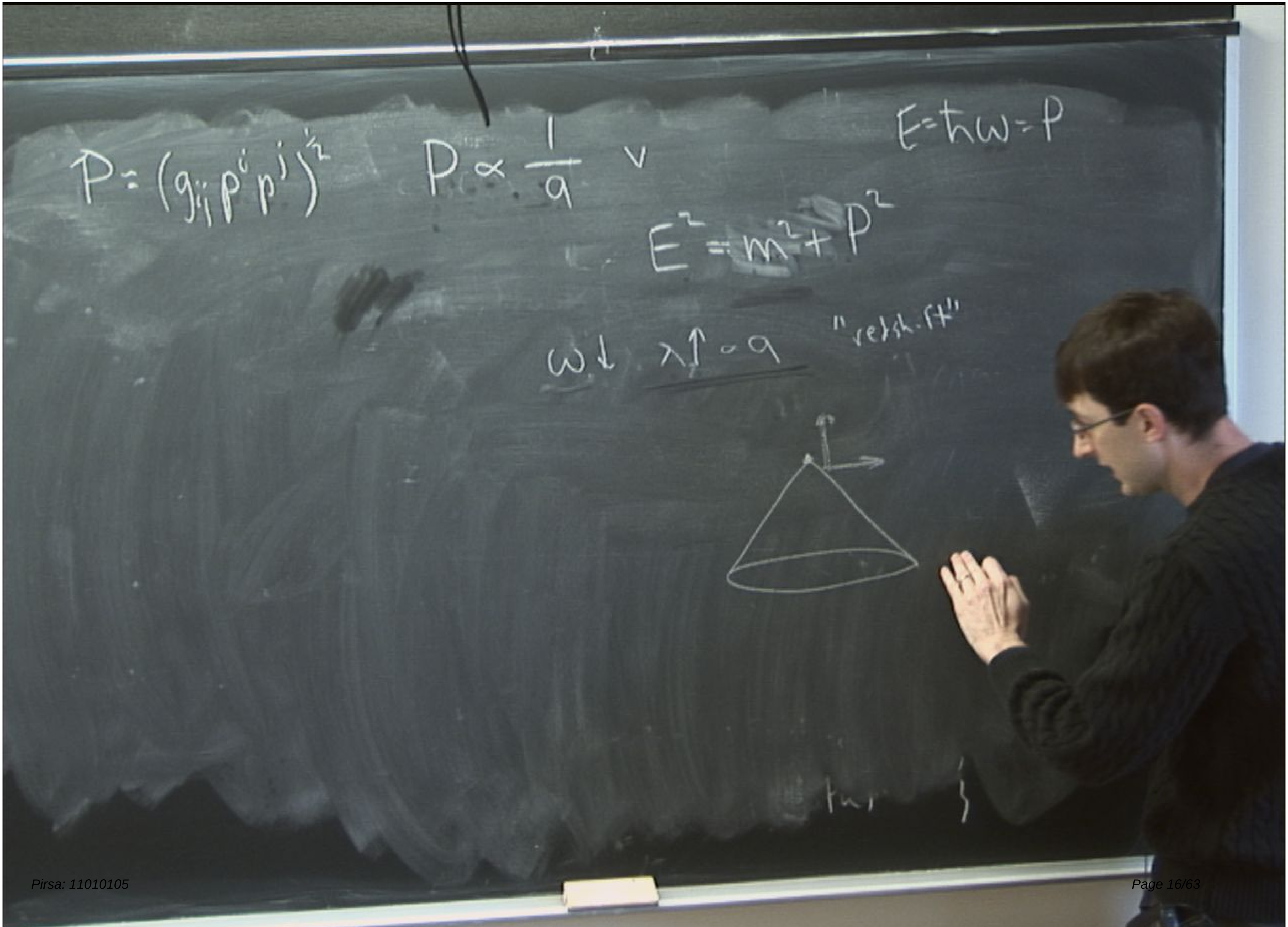

$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$

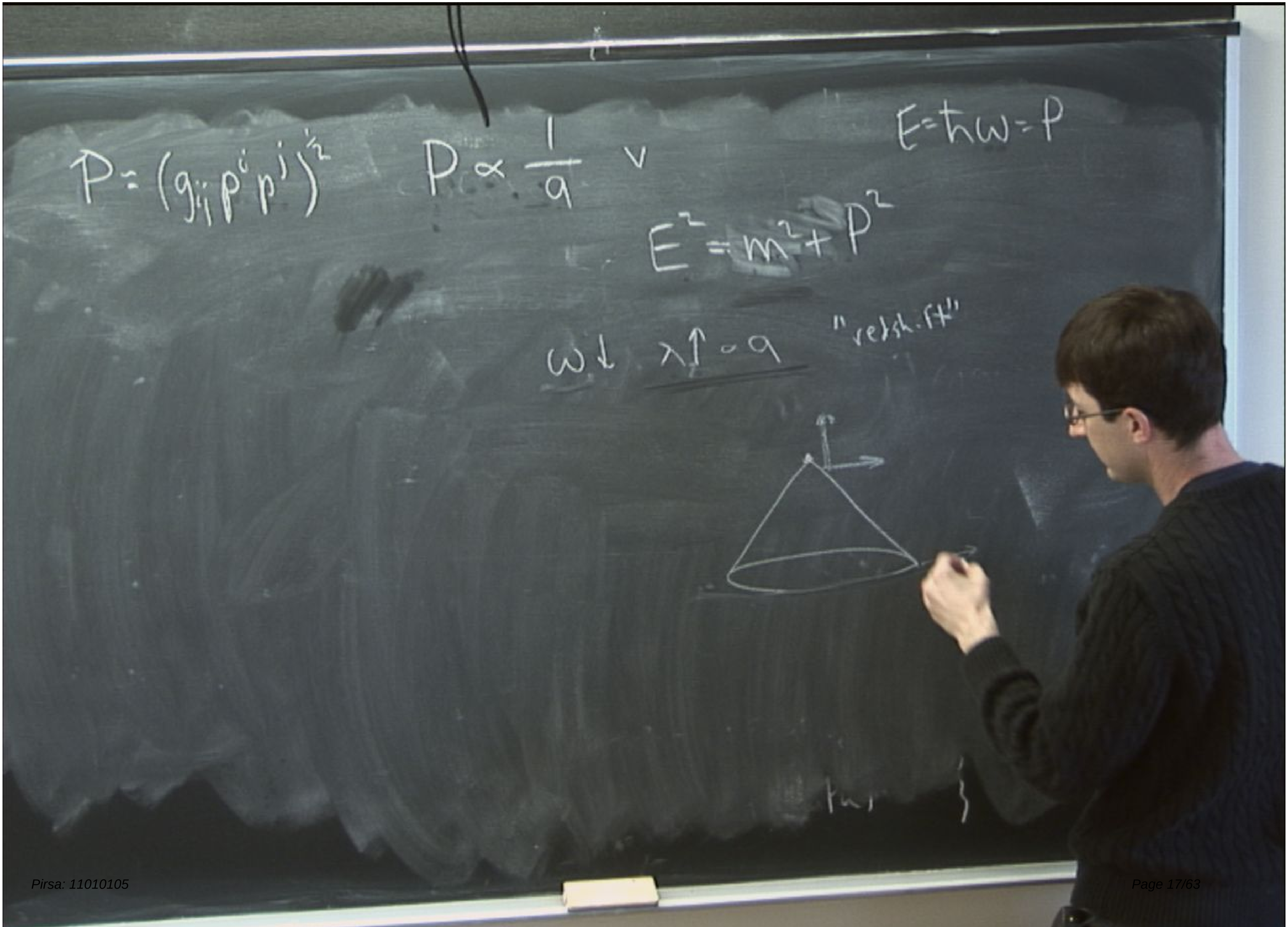

$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = P$
$E^2 = m^2 + p^2$

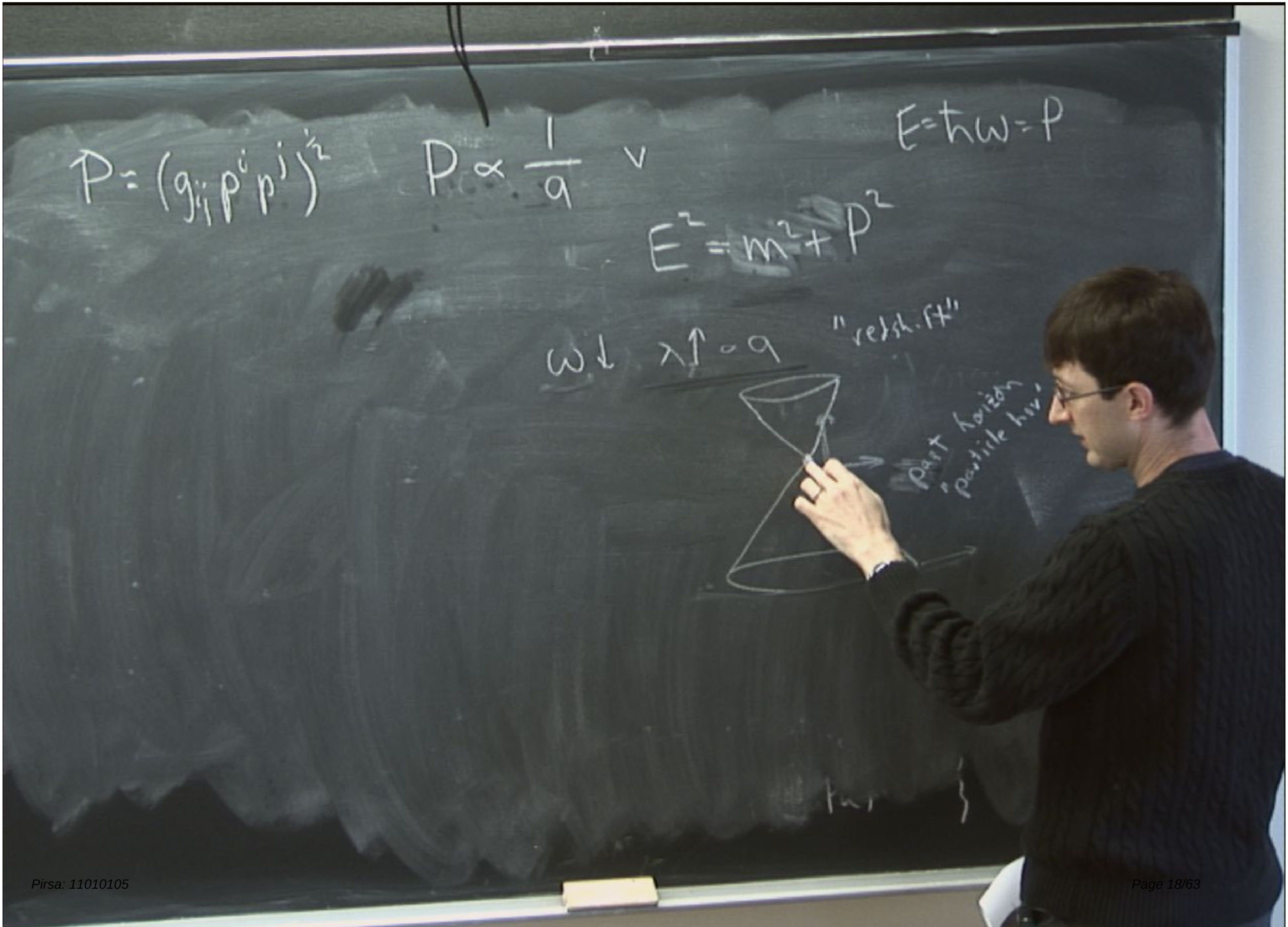

$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$ v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$


$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$ v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$\omega \downarrow$ $\lambda \uparrow \sim a$


$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$\omega\downarrow \quad \lambda\uparrow \propto a$
"redshift"


$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$\omega\downarrow \; \lambda\uparrow \propto a$
"redshift"


$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$ v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$\omega \downarrow \quad \lambda \uparrow \propto a$
"redshift"


$P = (g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$\omega \downarrow \quad \lambda \uparrow \propto a$
"redshift"
past horizon
"particle hor"

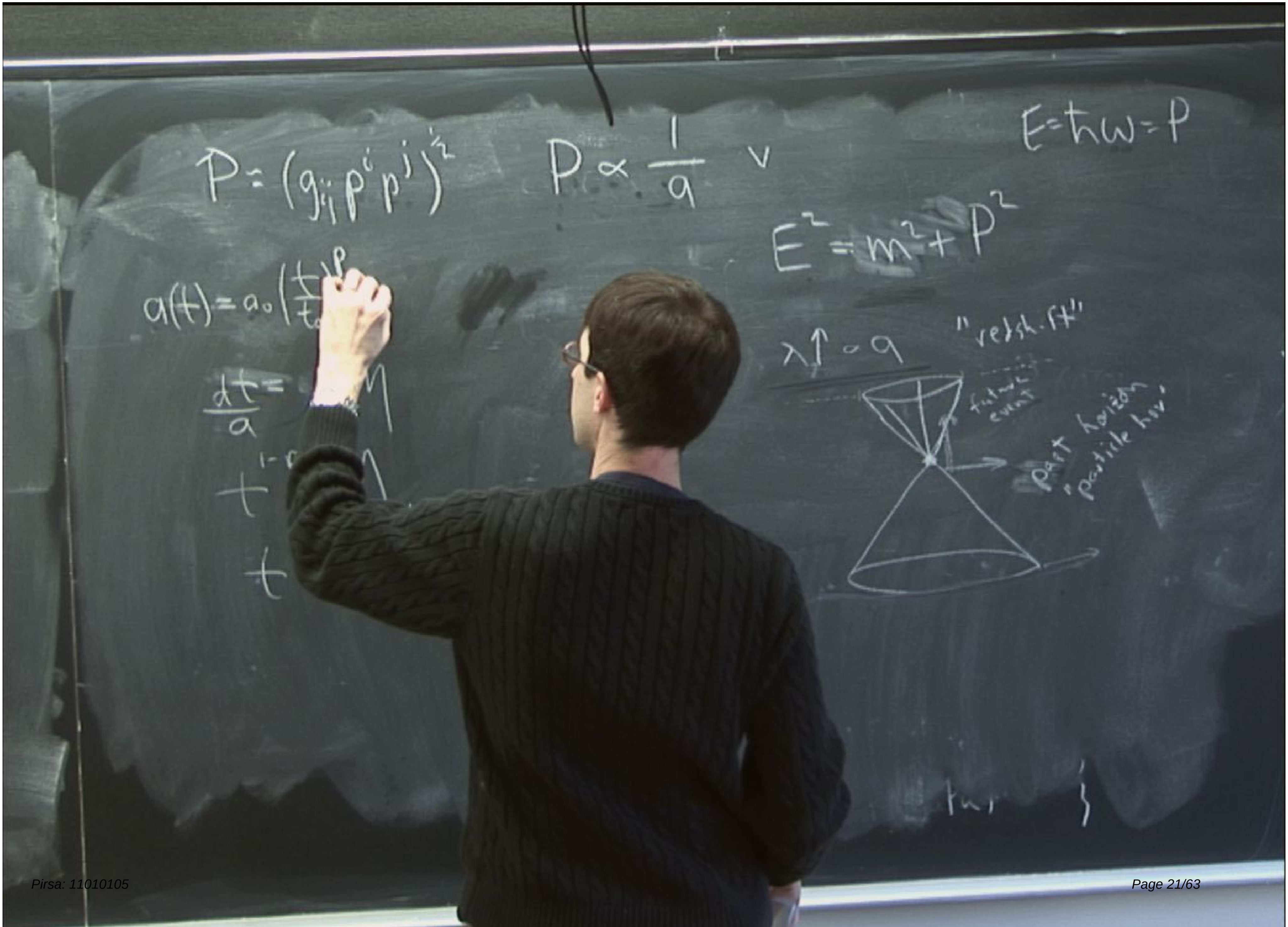
$$E = \hbar\omega = p$$

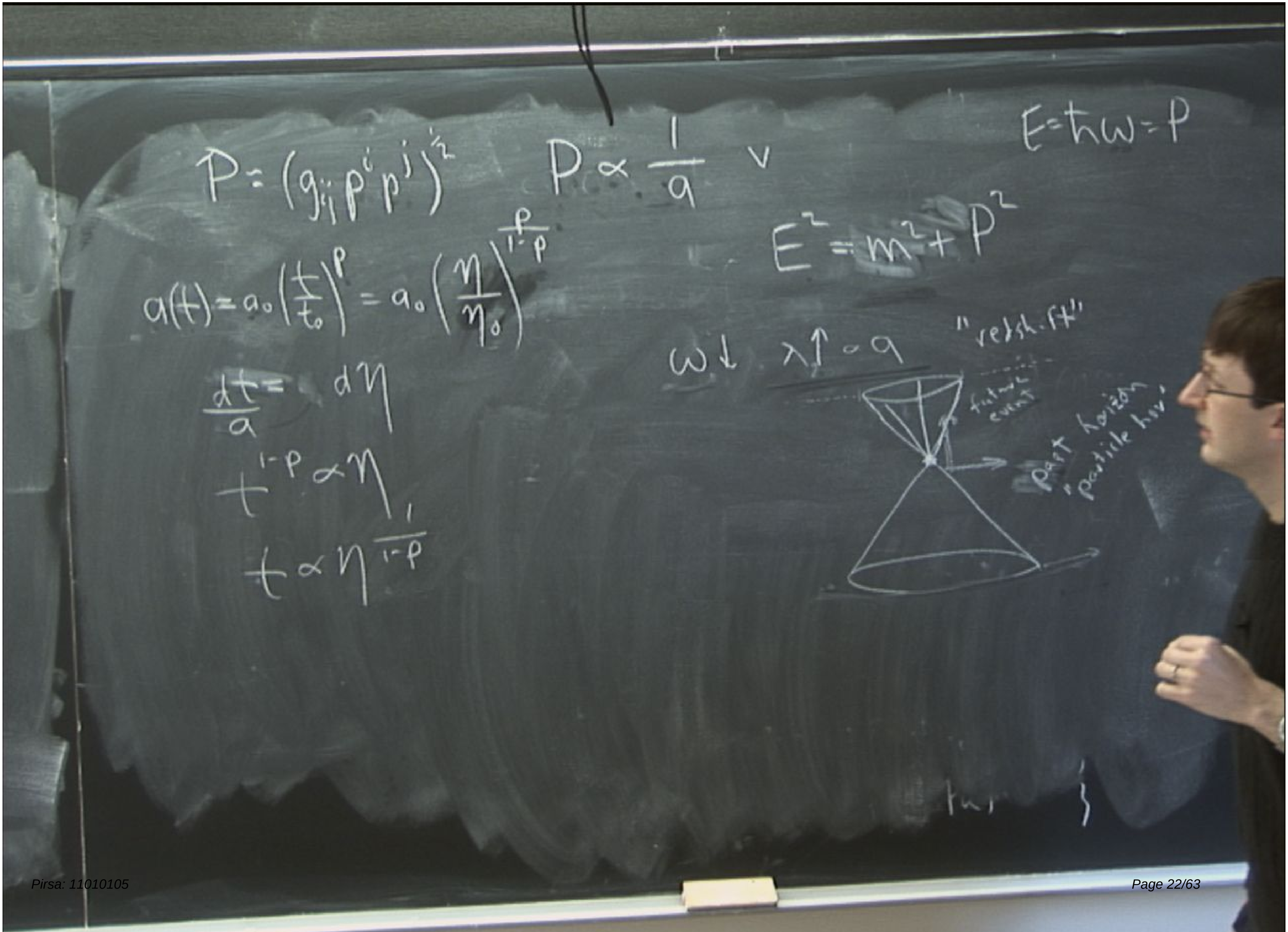
$$p = (g_{ij}\, p^i p^j)^{\frac{1}{2}} \qquad p \propto \frac{1}{a} \quad v$$

$$E^2 = m^2 + p^2$$

$\omega \downarrow \quad \lambda \uparrow \propto a$ "redshift"


P = (g_ij p^i p^j)^½
P ∝ 1/a
E = ħω = P
E² = m² + P²
ω↓ λ↑ ∝ a
"redshift"
future event
past horizon
"particle hor."


P = (g_ij p^i p^j)^1/2
P ∝ 1/a
E = ħω = P
E² = m² + P²
a(t) = a₀(t/t₀)^p
λ↑ ∝ a
"redshift"
future event
past horizon
"particle hor."



$P = (g_{ij} p^i p^j)^{1/2}$
$P \propto \frac{1}{a}$ v
$E = \hbar\omega = P$
$E^2 = m^2 + P^2$
$a(t) = a_0 \left(\frac{t}{t_0}\right)^p = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\omega \downarrow \quad \lambda \uparrow \propto a$
"redshift"
$\frac{dt}{a} = d\eta$
$t^{1-p} \propto \eta$
$t \propto \eta^{\frac{1}{1-p}}$
future event
past horizon
"particle hor."

$K = 0$

$0 < t < \infty$

$$s^2 = -dt^2 + a^2(t)[\ \ ]$$

$$= a^2(\eta)[-d\ \ ]$$

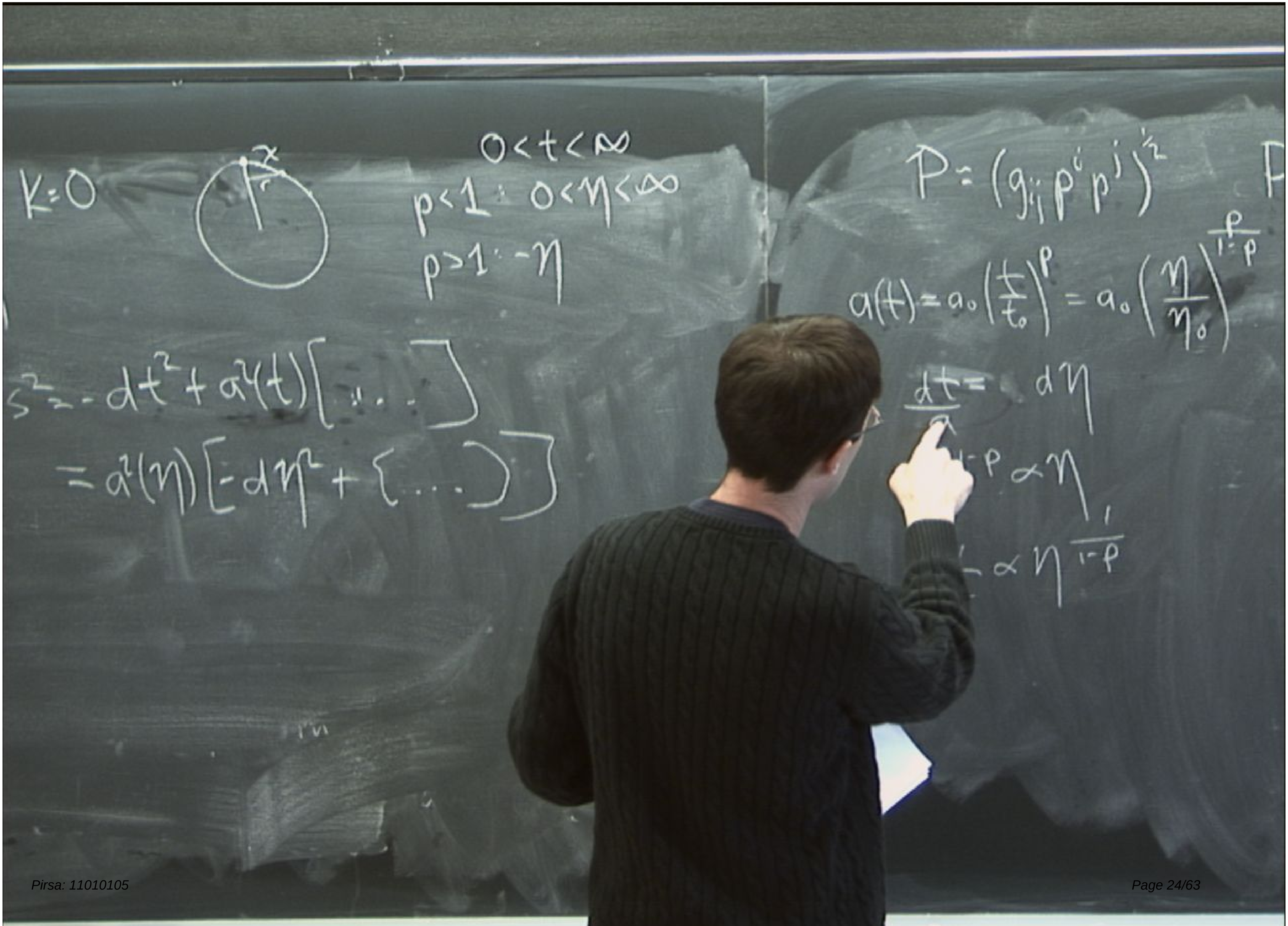
$$P = (g_{ij} p^i p^j)^{\frac{1}{2}} \qquad P$$

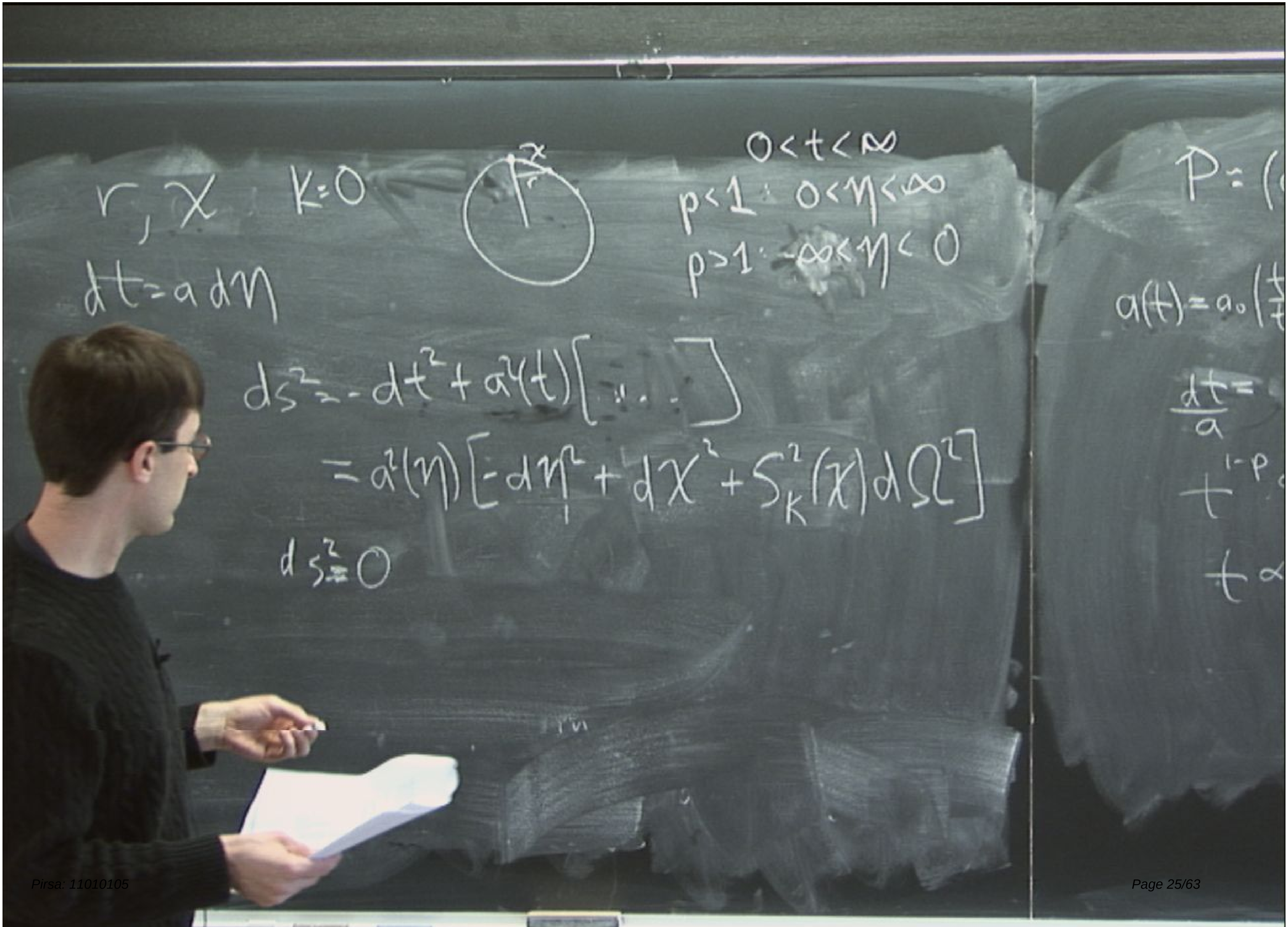
$$a(t) = a_0 \left(\frac{t}{t_0}\right)^p = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$$

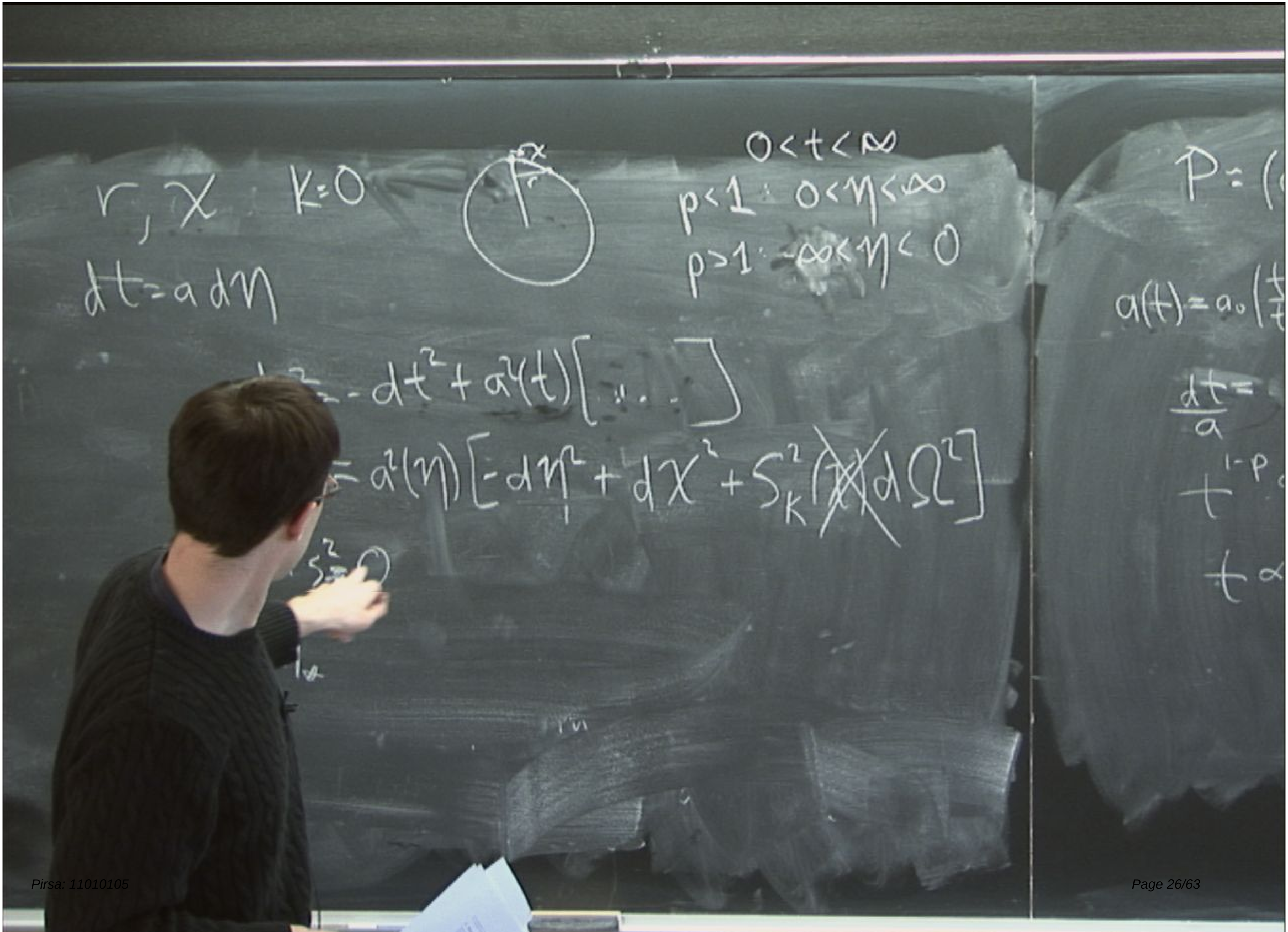
$$\frac{dt}{a} = d\eta$$

$$t^{1-p} \propto \eta$$

$$t \propto \eta^{\frac{1}{1-p}}$$


$K=0$
$0<t<\infty$
$p<1:\ 0<\eta<\infty$
$p>1:\ -\eta$
$ds^2 = -dt^2 + a^2(t)[\ldots]$
$= a^2(\eta)[-d\eta^2 + \{\ldots\}]$
$P = (g_{ij}p^i p^j)^{\frac{1}{2}}$
$a(t) = a_0\left(\frac{t}{t_0}\right)^p = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\frac{dt}{a} = d\eta$
$\alpha\, \eta^{\frac{1}{1-p}}$


r, χ   k=0
dt = a dη
0 < t < ∞
p < 1 : 0 < η < ∞
p > 1 : −∞ < η < 0
ds² = −dt² + a²(t)[...]
= a²(η)[−dη² + dχ² + S_k²(χ)dΩ²]
ds² = 0
P = (
a(t) = a₀(
dt/a =
t^{1−p}
t ∝


r, χ K=0
0<t<∞
p<1 : 0<η<∞
p>1 : -∞<η<0
dt = a dη
- dt² + a²(t)[...]
= a²(η)[-dη² + dχ² + S_k²(χ) dΩ²]
a(t) = a₀(
dt/a =
t ∝

$r, \chi \quad K=0$

$0 < t < \infty$

$p<1: \ 0 < \eta < \infty$

$p>1: \ -\infty < \eta < 0$

$dt = a\, d\eta$

$$ds^2 = -dt^2 + a^2(t)\left[\ \dots\ \right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + \cancel{S_K^2(\chi)\, d\Omega^2}\right]$$

$ds^2 = 0 \quad d\eta = d\chi$

$\eta_* \quad \chi = \eta - \eta_*$

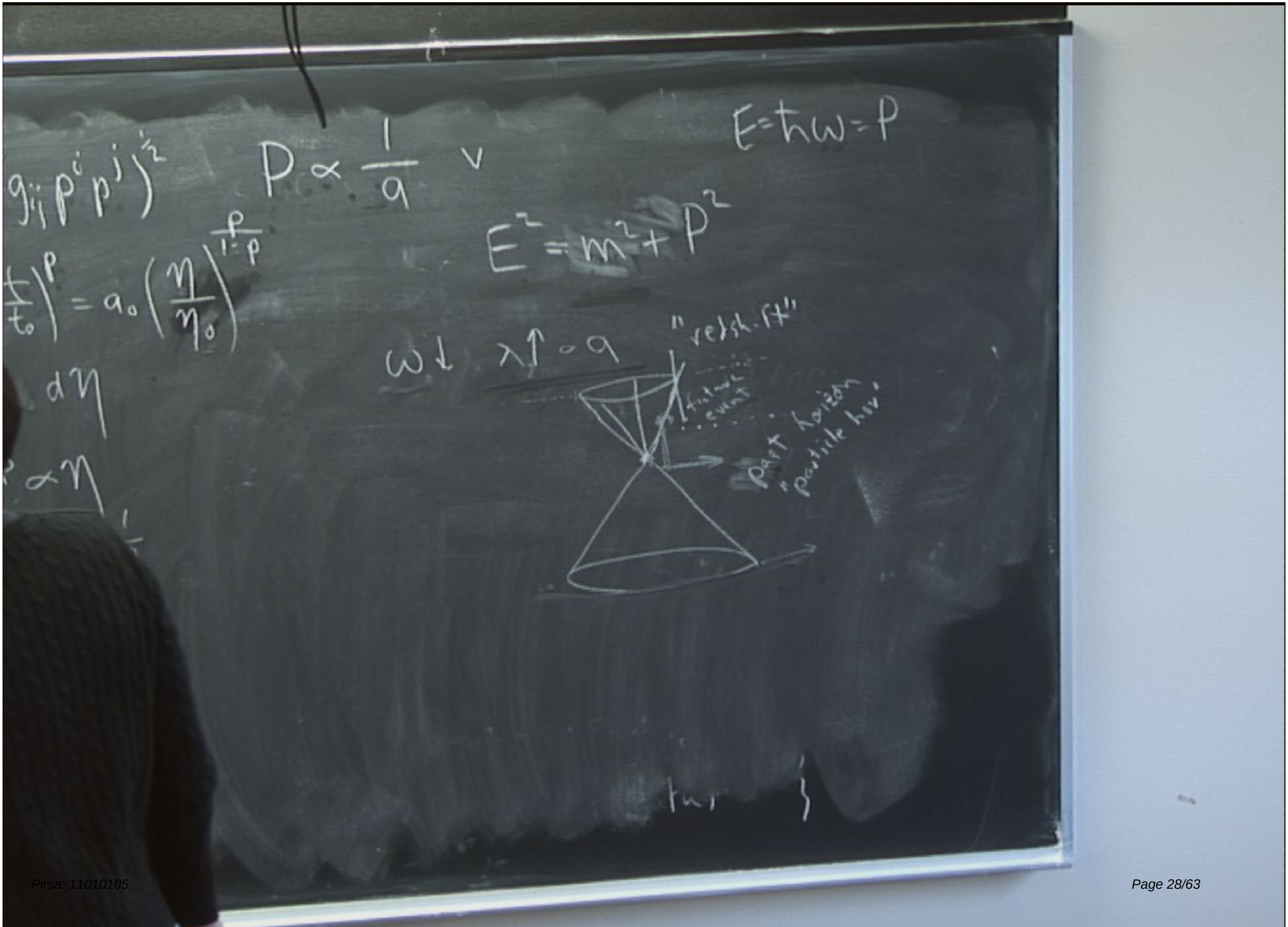
$P = ($

$a(t) = a_0 \left(\frac{t}{t}\right.$

$\frac{dt}{a} =$

t^{1-p}

$t \propto$


E = ħω = p
p ∝ 1/a
E² = m² + p²
ω↓ λ↑ ∝ a
"redshift"
future event
past horizon
"particle hor."
dη

$t = a\, d\eta$

$p > 1: \; -\infty < \eta < 0$

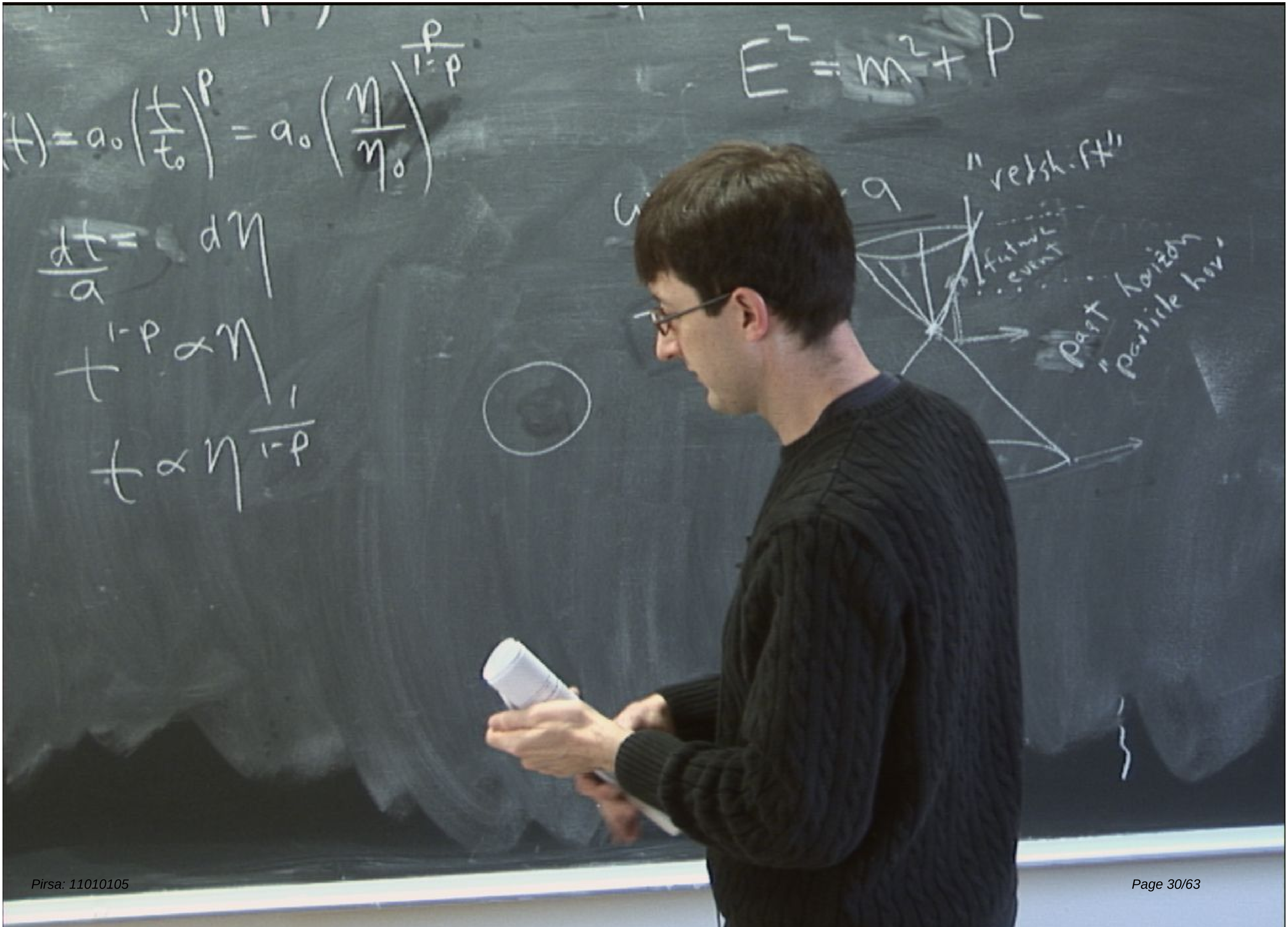
$a(t$

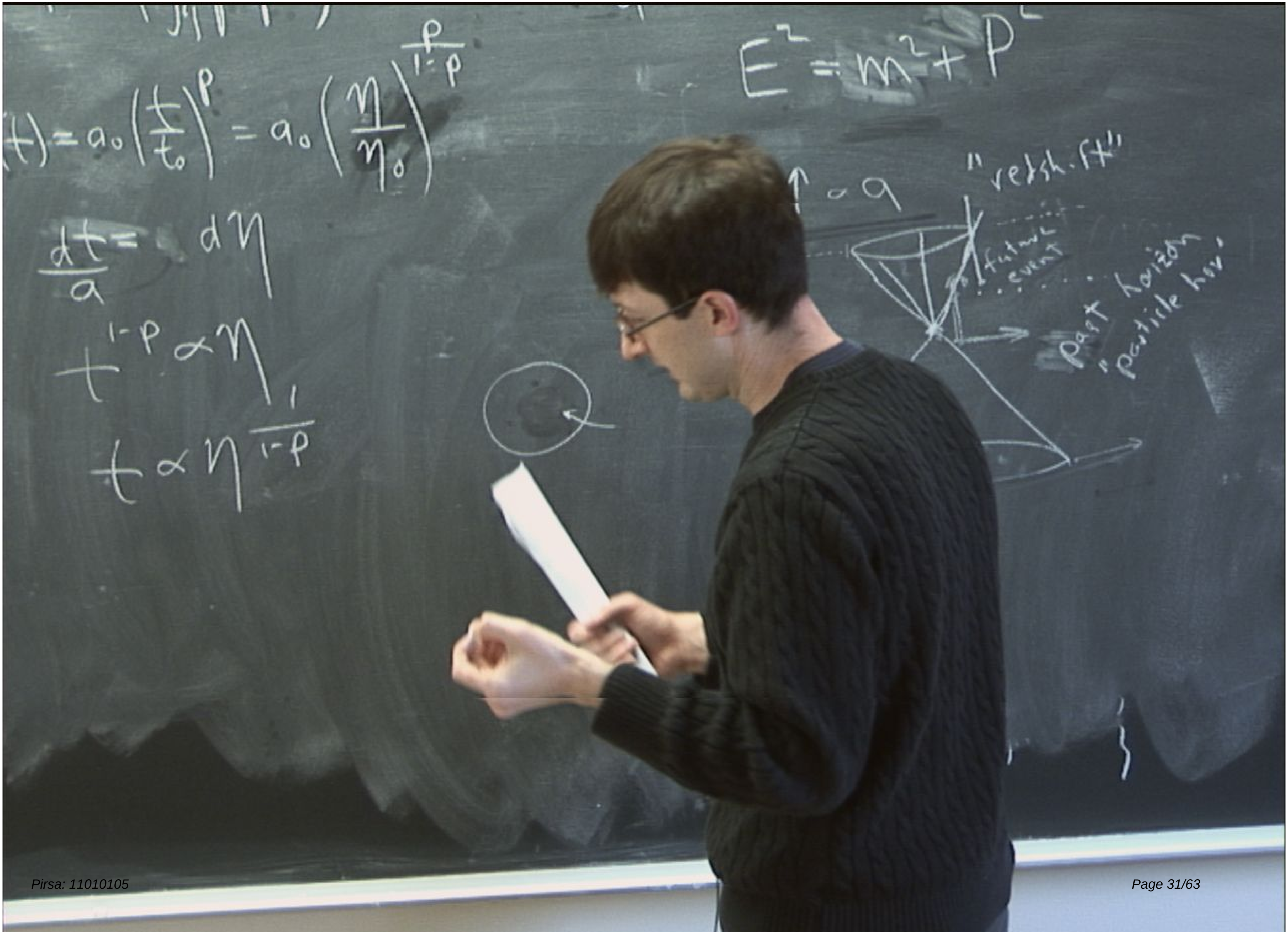
$$ds^2 = -dt^2 + a^2(t)\left[\ldots\right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + \cancel{S_k^2(\chi)}\, d\Omega^2\right]$$

$$ds^2 = 0 \qquad d\eta = d\chi$$

$$\eta_* \qquad \chi = \eta - \eta_* \qquad \chi = -\eta_*$$


$E^2 = m^2 + P^2$
$(t) = a_0\left(\frac{t}{t_0}\right)^p = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\frac{dt}{a} = d\eta$
$t^{1-p} \propto \eta$
$t \propto \eta^{\frac{1}{1-p}}$
"redsh.ft"
future event
past horizon
"particle hor.



$E^2 = m^2 + P^2$
$a(t) = a_0\left(\frac{t}{t_0}\right)^p = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\frac{dt}{a} = d\eta$
$t^{1-p} \propto \eta$
$t \propto \eta^{\frac{1}{1-p}}$
"redsh. ft"
future event
past horizon
"particle hor."

$0 < t < \infty$

$p < 1: \; 0 < \eta < \infty \quad \ddot{a} < 0$

$p > 1: \; -\infty < \eta < 0 \quad \ddot{a} > 0$

$P = (g_{ij} p^i p^j)^{\frac{1}{2}} \quad P$

$a(t) = a_0 \left(\frac{t}{t_0}\right)^p = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$

$= d\eta$

$\propto \eta$

$\eta^{\frac{1}{1-p}}$

$)[\;\ldots\;]$

$\eta^2 + d\chi^2 + S_k^2(\cancel{\chi}) d\Omega^2]$

$d\chi$

$\chi = -\eta_*$

$r, \chi \quad K=0$

$0 < t < \infty$

$p < 1: \; 0 < \eta < \infty$

$p > 1: \; -\infty < \eta < 0$

$dt = a\, d\eta$

$$ds^2 = -dt^2 + a^2(t)[\;\ldots\;]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2(\chi)\ldots\right.$$

$ds^2 = 0 \quad d\eta = -d\chi$

$\eta_* \quad \chi = \eta - \eta_* \quad \chi = -\eta_*$

$r, \chi \quad K=0$

$0 < t < \infty$

$p<1: \; 0<\eta<\infty$

$p>1: \; -\infty<\eta<0$

$dt = a\, d\eta$

$$ds^2 = -dt^2 + a^2(t)\left[\ldots \right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + \ldots d\Omega^2\right]$$

$ds^2 = 0 \quad d\eta = -d\chi$

$\eta_* \quad \chi = \eta - \eta \ldots$

$r, \chi \quad K=0$

$0 < t < \infty$

$p<1: \; 0 < \eta < \infty$

$p>1: \; -\infty < \eta < 0$

$dt = a\, d\eta$

$$ds^2 = -dt^2 + a^2(t)\left[\;\ldots\; \right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2\cancel{(\chi)}\, d\Omega^2\right]$$

$ds^2 = 0 \quad d\eta = -d\chi$

$\eta_* \quad \chi = \eta_* - \eta \quad \chi = -\eta_*$

$r, \chi \quad K=0$

$0 < t < \infty$

$p<1: \; 0 < \eta < \infty$

$p>1: \; -\infty < \eta < 0$

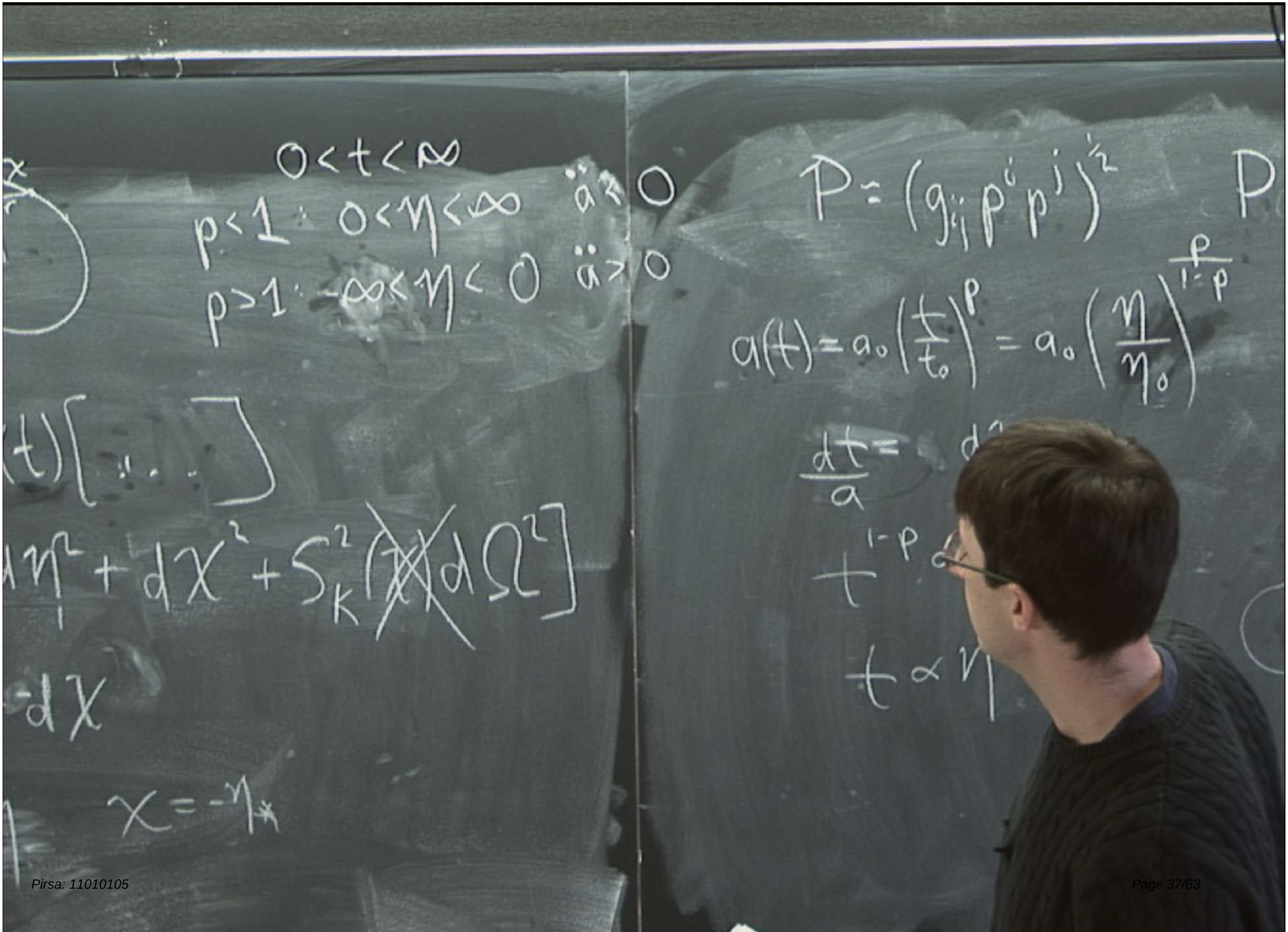
$dt = a\, d\eta$

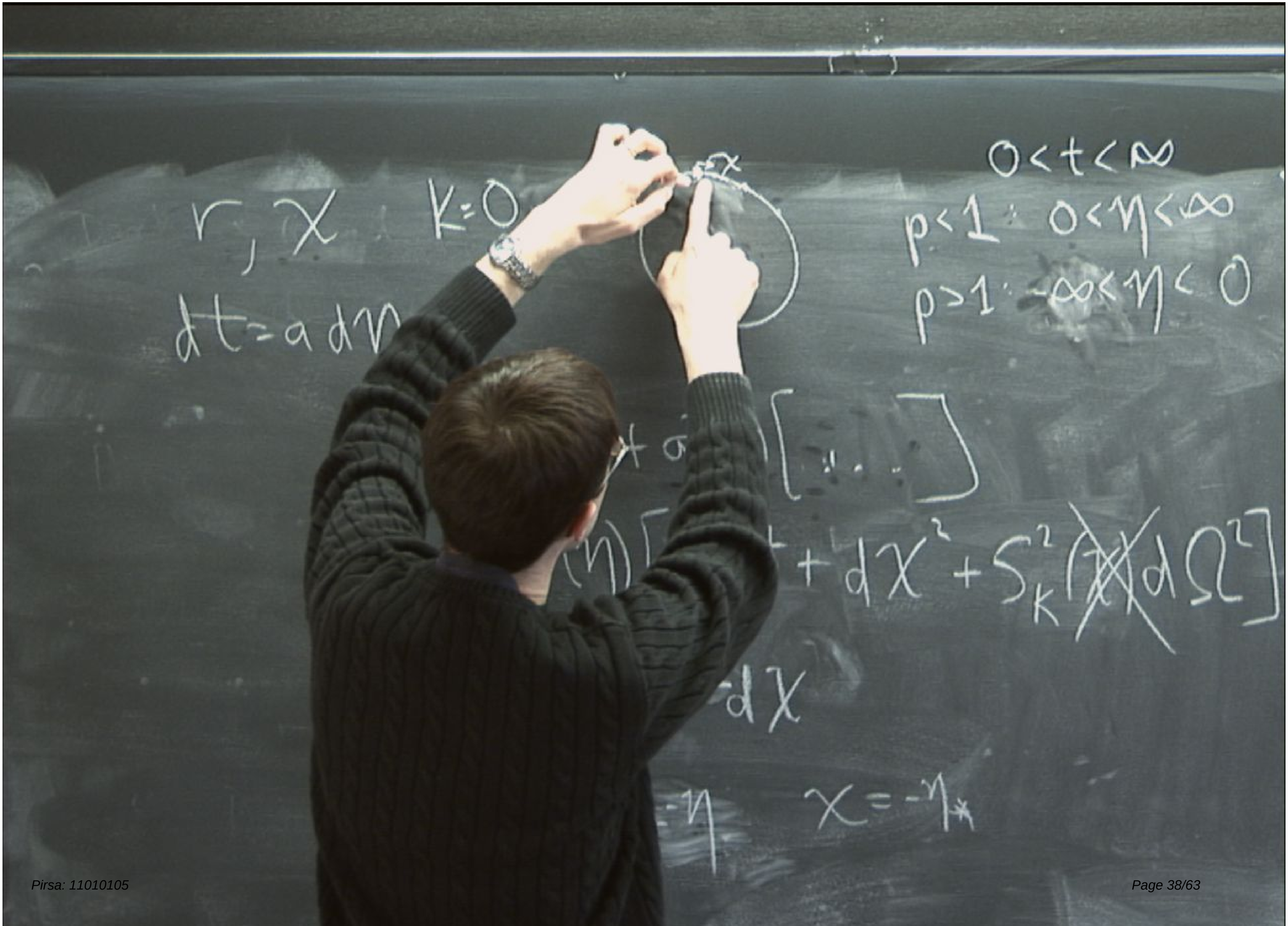
$$ds^2 = -dt^2 + a^2(t)\left[\;\cdots\;\right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2(\cancel{\chi})\, d\Omega^2\right]$$

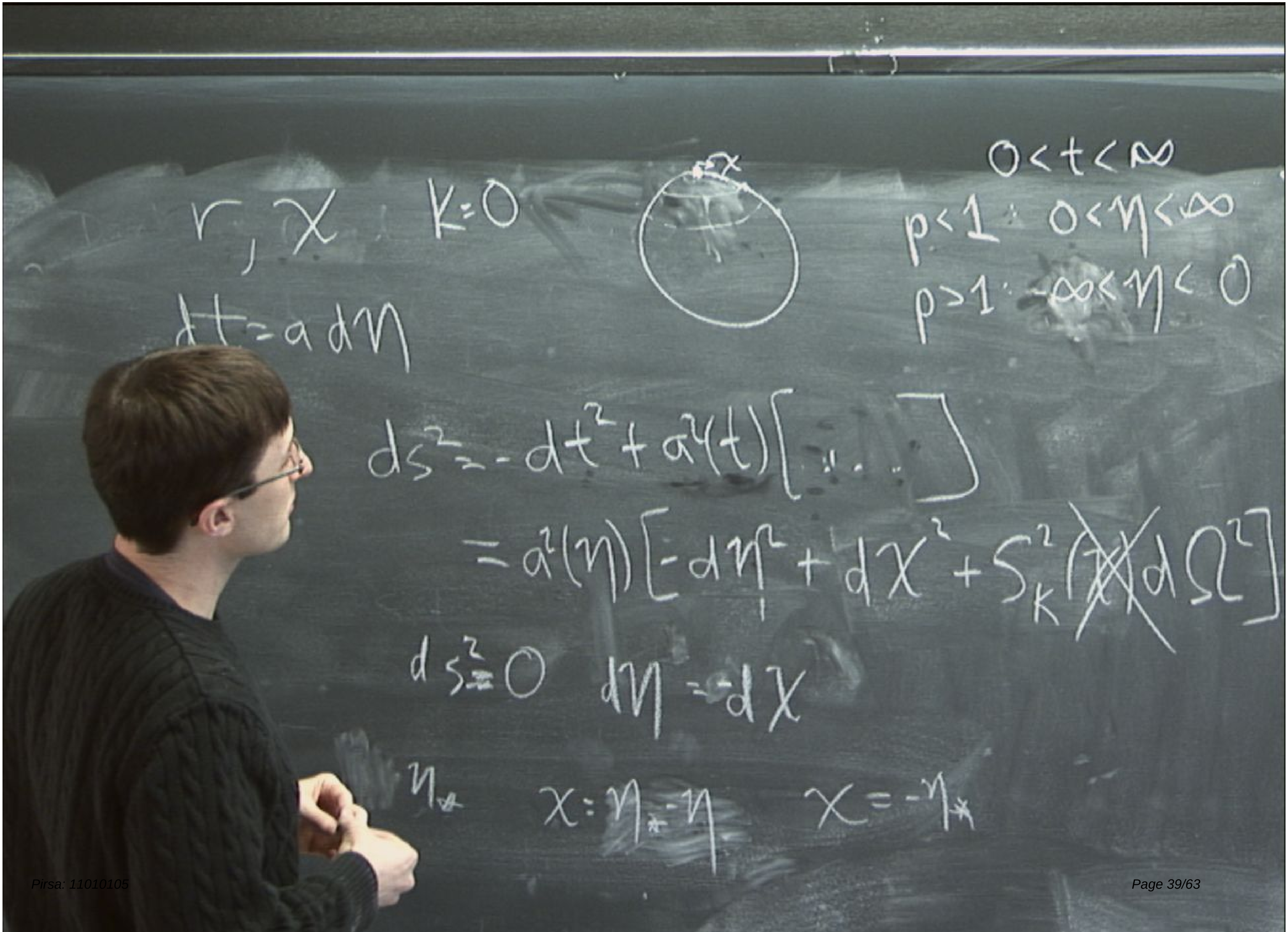
$ds^2 = 0 \quad d\eta = -d\chi$

$\eta_* \quad \chi = \eta_* - \eta \quad \chi = -\eta_*$


$0<t<\infty$
$p<1:\ 0<\eta<\infty \quad \ddot{a}<0$
$p>1:\ -\infty<\eta<0 \quad \ddot{a}>0$
$P=(g_{ij}p^ip^j)^{\frac{1}{2}}$
$a(t)=a_0\left(\frac{t}{t_0}\right)^p=a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\frac{dt}{a}=$
t^{1-p}
$t\propto\eta$
$d\eta^2+d\chi^2+S_k^2(\cancel{\chi})d\Omega^2]$
$d\chi$
$\chi=-\eta_*$



r, χ   K=0
dt = a dη
0<t<∞
p<1 : 0<η<∞
p>1 : -∞<η<0
+ dχ² + S_k² dΩ²]
dχ
χ = -η*


r, χ   K=0
0<t<∞
p<1 : 0<η<∞
p>1 : -∞<η<0
dt = a dη
ds² = -dt² + a²(t)[...]
= a²(η)[-dη² + dχ² + S_K²(χ) dΩ²]
ds² = 0   dη = -dχ
η*   χ = η* - η   χ = -η*

r, χ $\quad K=0$

$0<t<\infty$

$p<1: \ 0<\eta<\infty$

$p>1: \ -\infty<\eta<0$

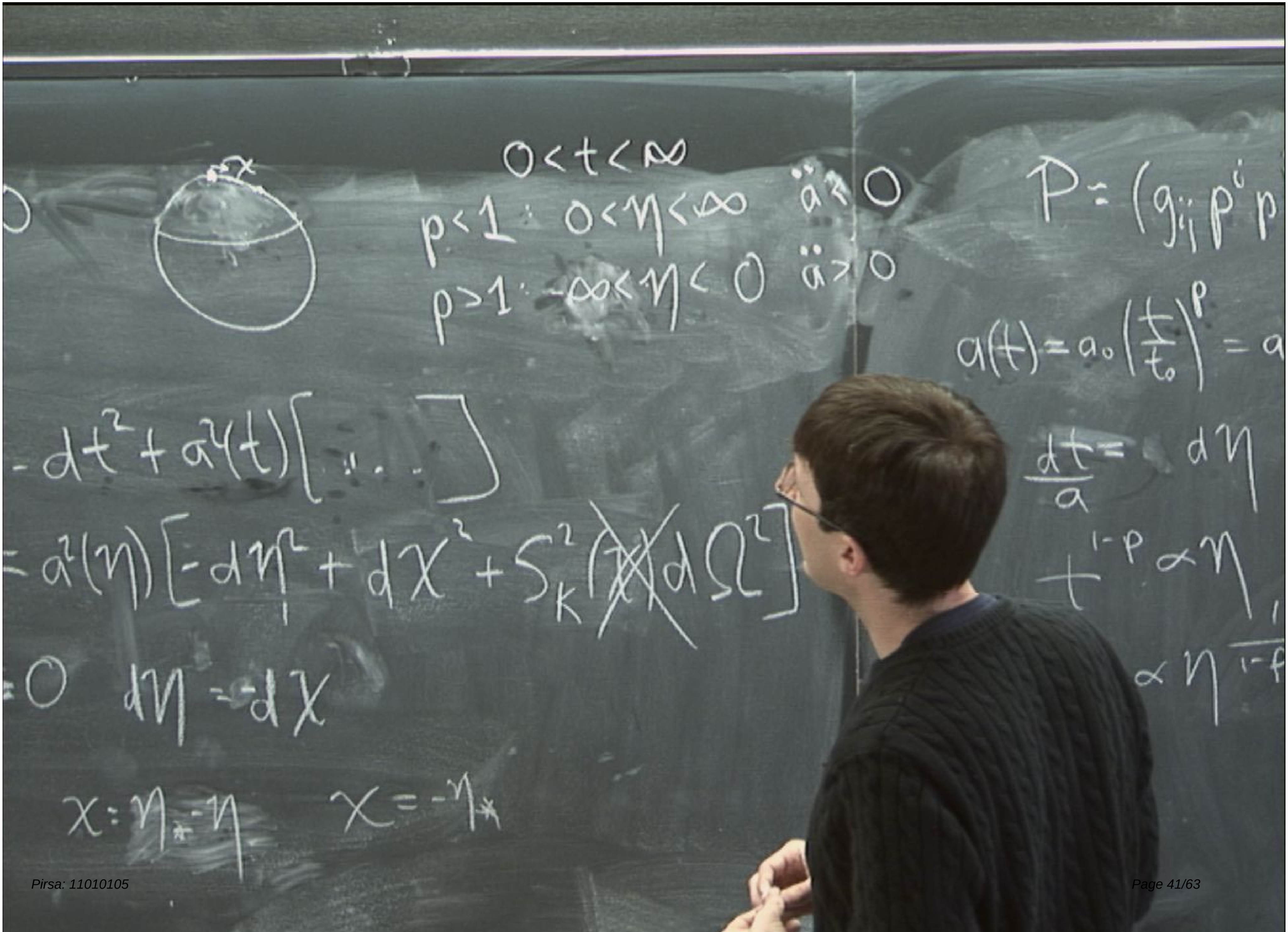
$dt = a\,d\eta$

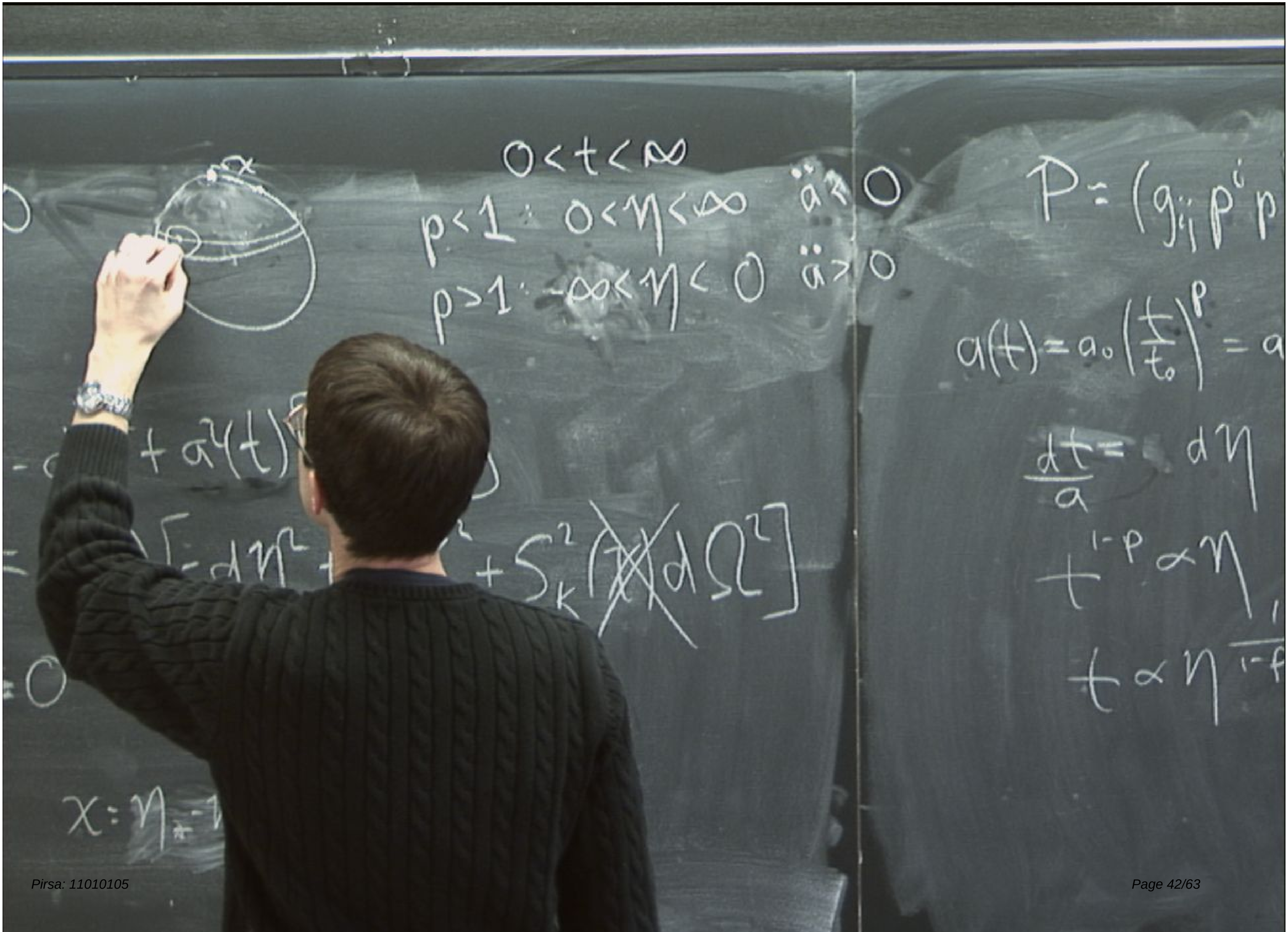
$$ds^2 = -dt^2 + a^2(t)\left[\ \dots\ \right]$$

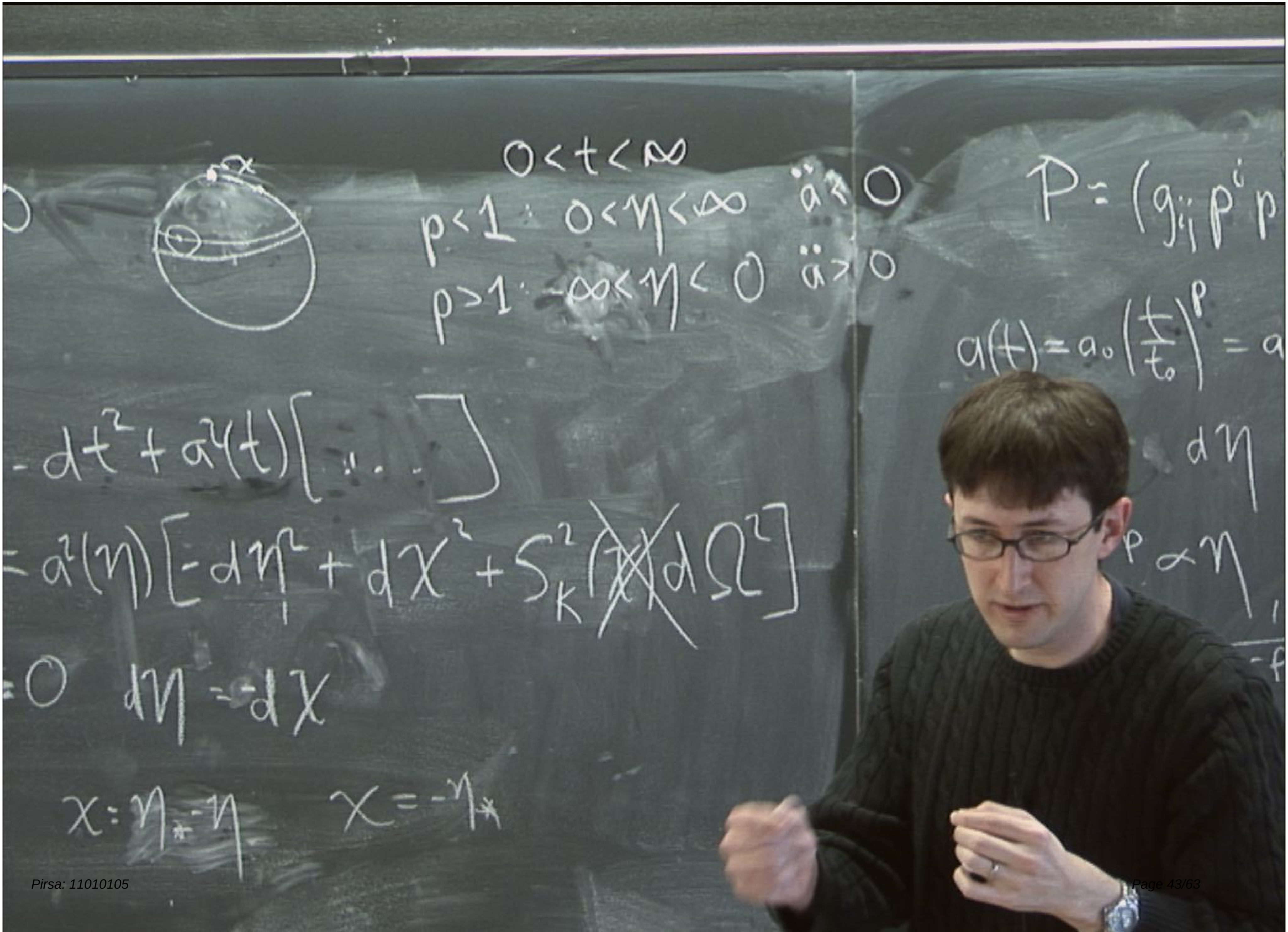
$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2(\cancel{\chi})\,d\Omega\right.$$

$ds^2 = 0 \quad d\eta = -d\chi$

$\eta_* \quad \chi = \eta_* - \eta \quad \chi = -\eta_*$


0 < t < ∞
p < 1 : 0 < η < ∞   ä < 0
p > 1 : -∞ < η < 0   ä > 0
P = (g_ij p^i p
a(t) = a_0 (t/t_0)^p = a
-dt^2 + a^2(t)[...]
= a^2(η)[-dη^2 + dχ^2 + S_k^2(χ) dΩ^2]
dt/a = dη
t^(1-p) ∝ η
= 0   dη = -dχ
∝ η^(1/(1-p))
χ = η_* - η   χ = -η_*


0<t<∞
p<1 : 0<η<∞ ä<0
p>1 : -∞<η<0 ä>0
a(t) = a₀ (t/t₀)^p = a
dt/a = dη
t^(1-p) ∝ η
t ∝ η


0 < t < ∞
p < 1 : 0 < η < ∞   ä < 0
p > 1 : −∞ < η < 0   ä > 0
P = (g_ij p^i p
a(t) = a_0 (t/t_0)^p = a
− dt^2 + a^2(t)[...]
= a^2(η)[−dη^2 + dχ^2 + S_k^2(χ) dΩ^2]
= 0   dη = −dχ
dη
p ∝ η
χ = η_* − η   χ = −η_*

$$0 < t < \infty$$

$$p < 1: \quad 0 < \eta < \infty \qquad \ddot{a} < 0$$

$$p > 1: \quad -\infty < \eta < 0 \qquad \ddot{a} > 0$$

$$-dt^2 + a^2(t)\left[\ \dots\ \right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + \cancel{S_k^2(\chi)}\, d\Omega^2\right]$$

$$= 0 \qquad d\eta = -d\chi$$

$$\chi = \eta_* - \eta \qquad \chi = -\eta_*$$

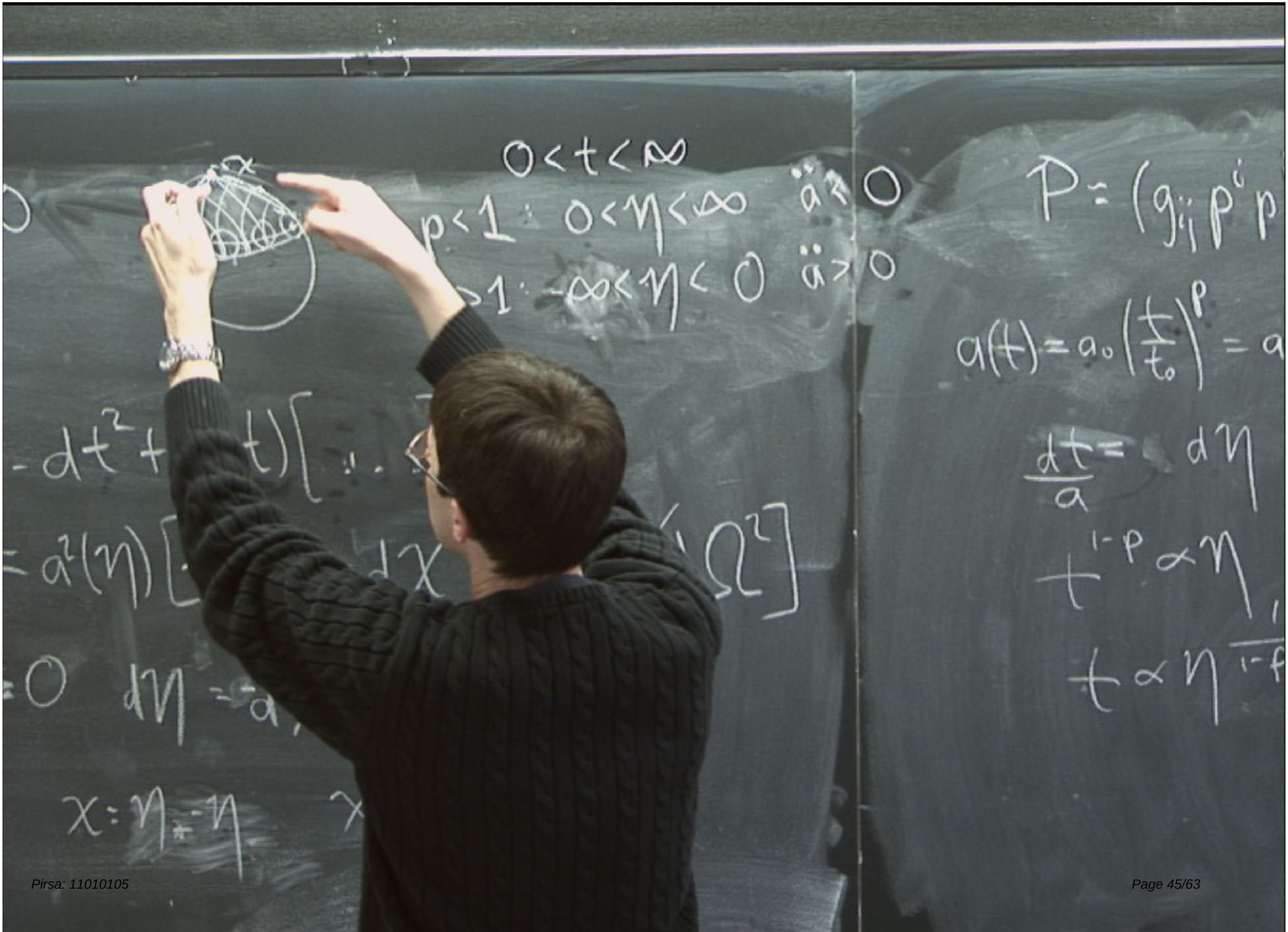
$$P = (g_{ij} p^i p$$

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^p = a$$

$$\frac{dt}{a} = d\eta$$

$$t^{1-p} \propto \eta$$

$$t \propto \eta$$


0<t<∞
p<1 : 0<η<∞
>1 : -∞<η<0 ä>0
a(t)=a₀(t/t₀)^p
dt/a = dη
t^{1-p} ∝ η
x=η₀-η

$$0<t<\infty$$

$$p<1: \quad 0<\eta<\infty \quad \ddot{a}<0$$

$$p>1: \quad -\infty<\eta<0 \quad \ddot{a}>0$$

$$P = (g_{ij} p^i p$$

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^p = a$$

$$-dt^2 + a^2(t)\left[\ldots\right]$$

$$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_k^2(\cancel{\chi})\, d\Omega^2\right]$$

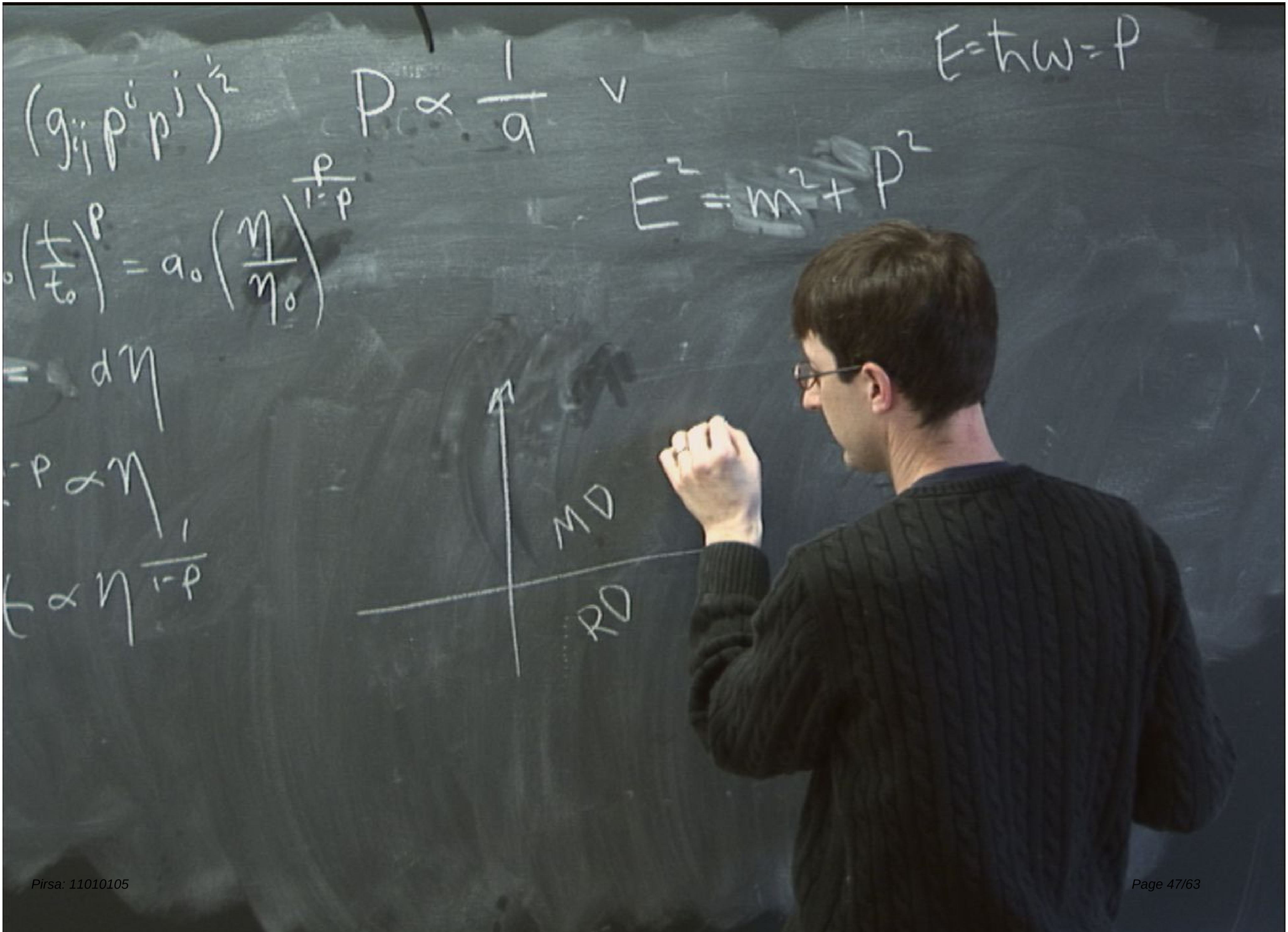
$$\frac{dt}{a} = d\eta$$

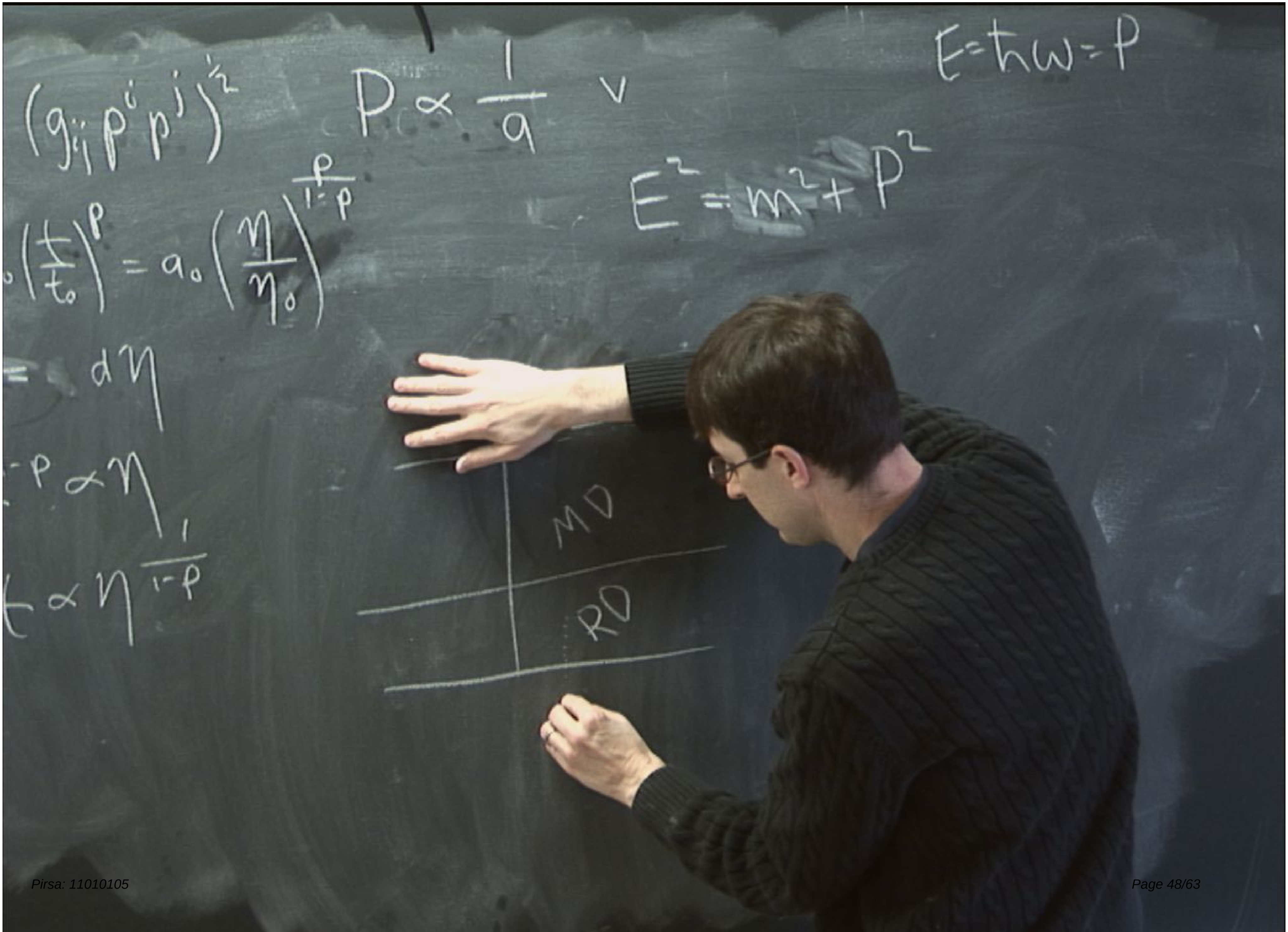
$$t^{1-p} \propto \eta$$

$$t$$

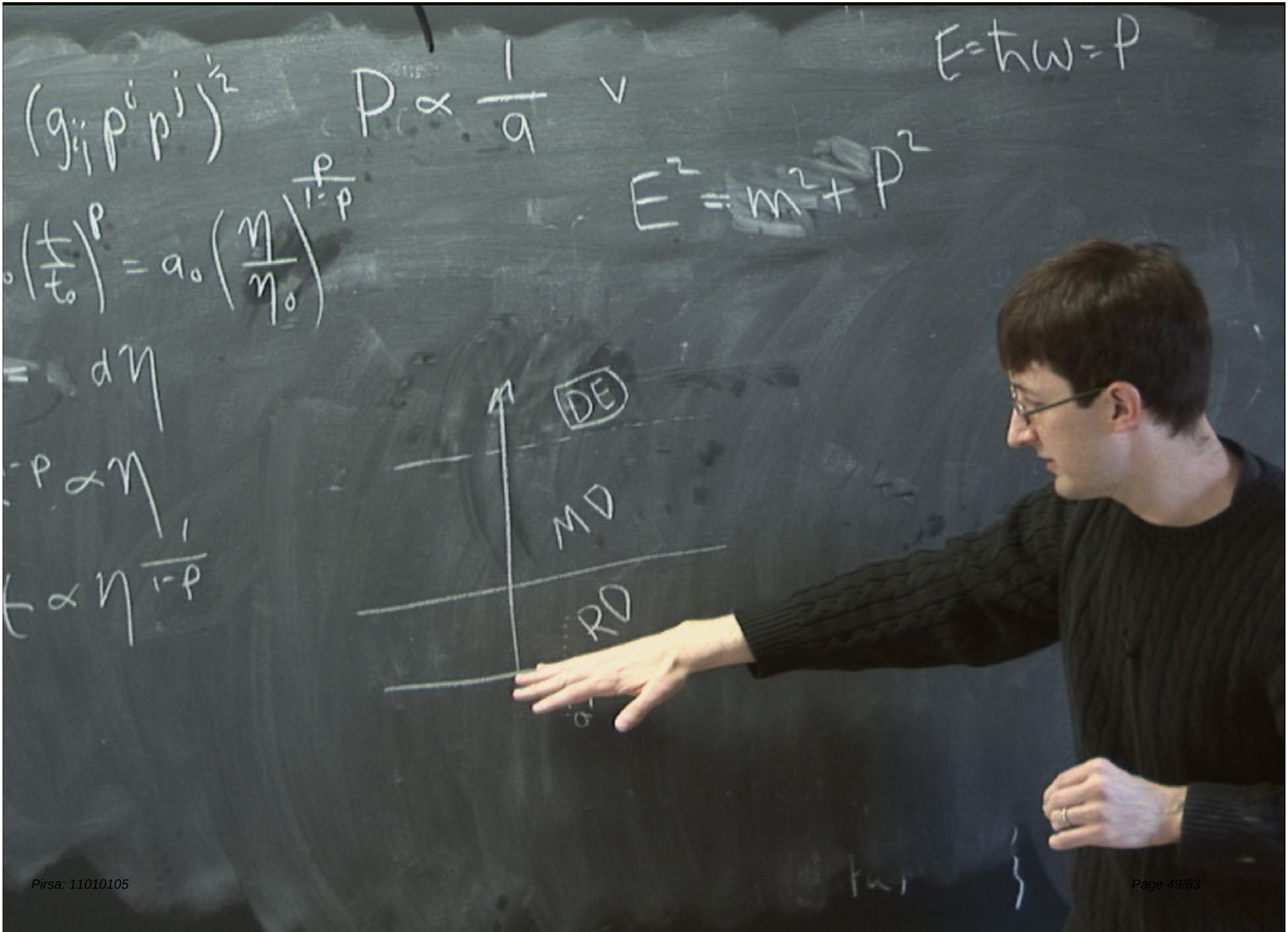
$$=0 \quad d\eta = -d\chi$$

$$\chi = \eta_* - \eta \qquad \chi = -\eta_*$$

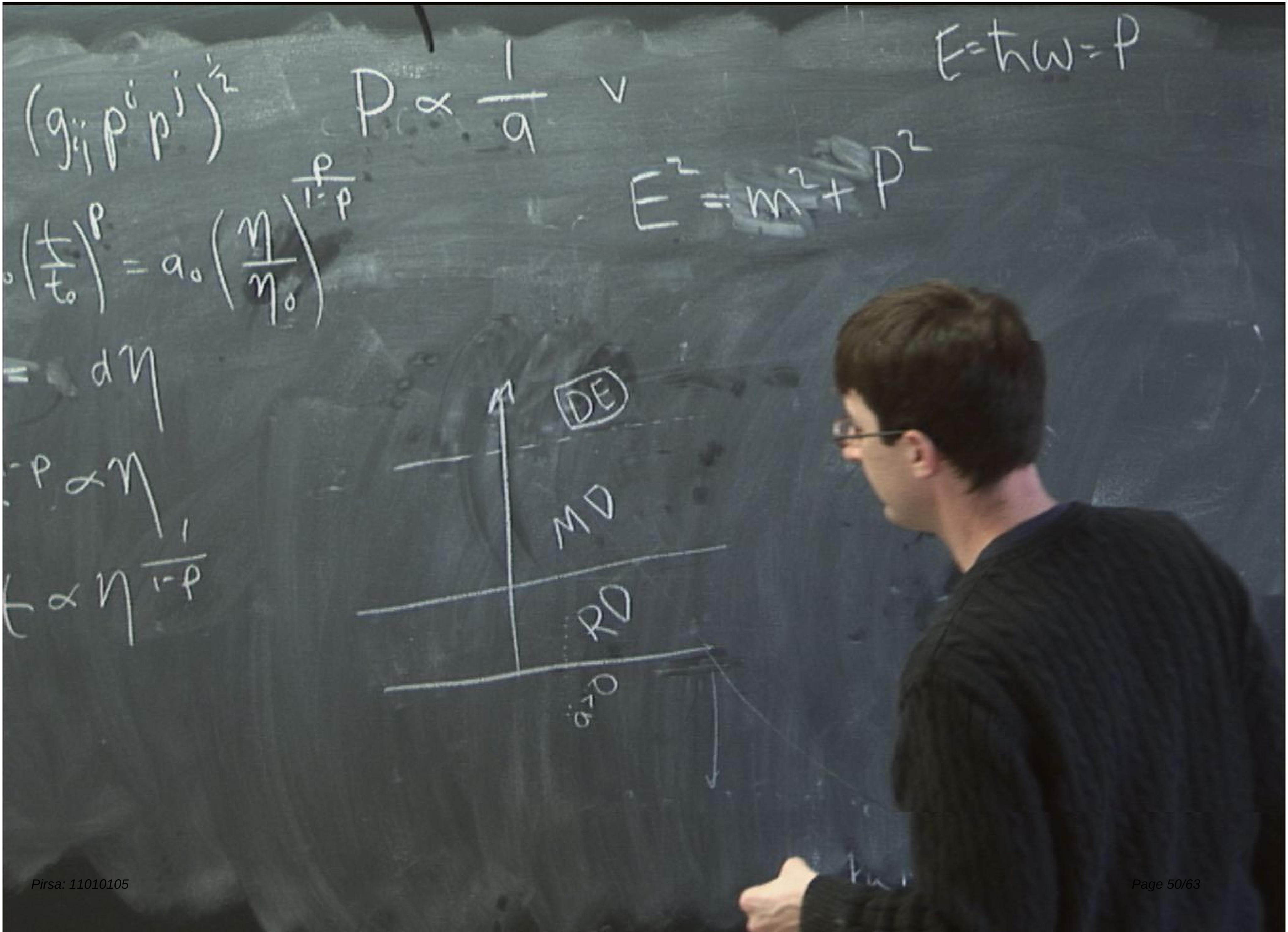

$E = \hbar\omega = p$
$P \propto \frac{1}{a}$
v
$E^2 = m^2 + p^2$
MD
RD



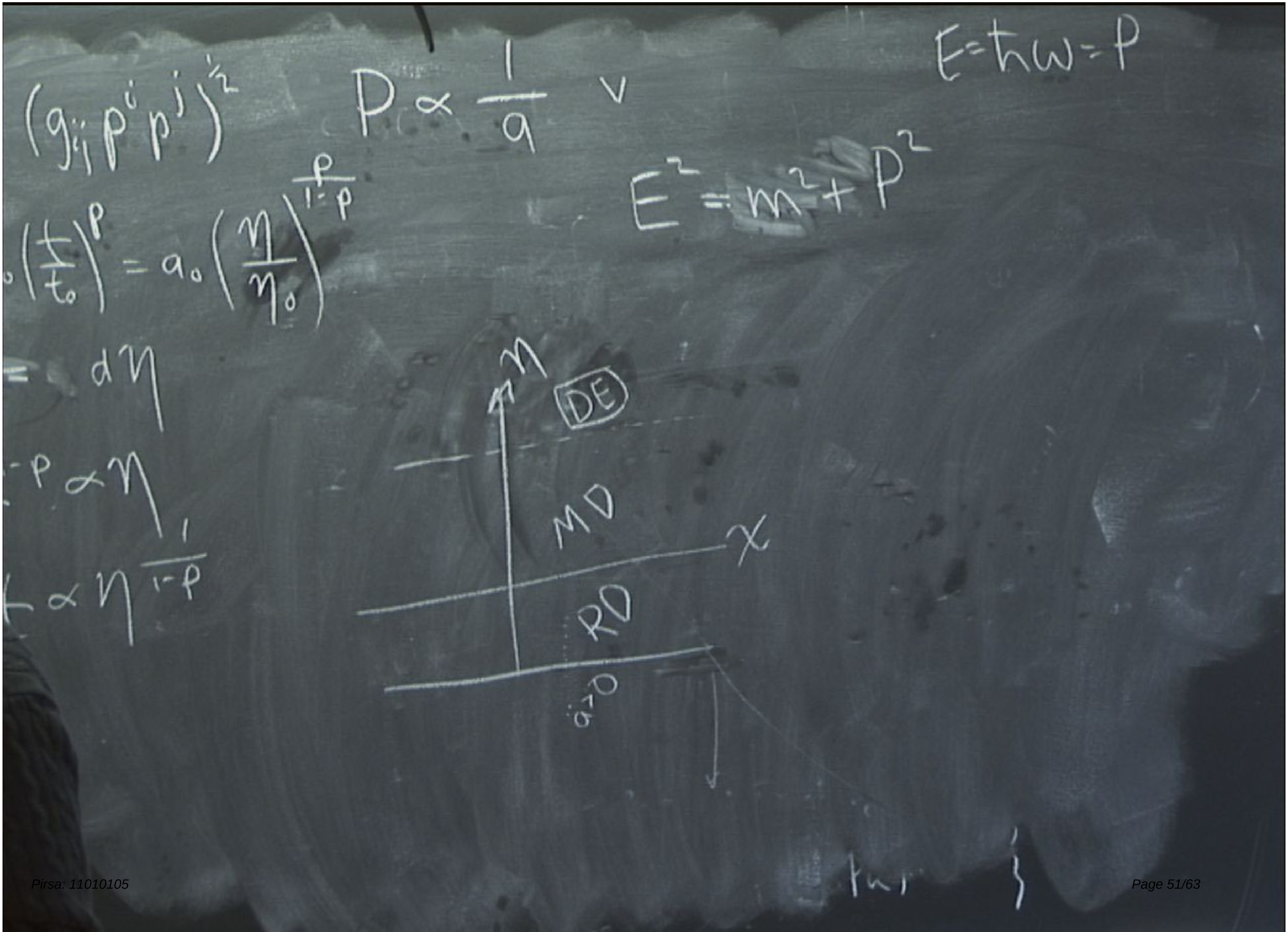
$(g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = p$
$E^2 = m^2 + p^2$
$\left(\frac{t}{t_0}\right)^p = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$d\eta$
$\propto \eta$
$t \propto \eta^{\frac{1}{1-p}}$
MD
RD



$E = \hbar\omega = p$
$(g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E^2 = m^2 + p^2$
DE
MD
RD



E=ħω=p
(g_ij p^i p^j)^1/2
P ∝ 1/a
E^2 = m^2 + p^2
DE
MD
RD



$(g_{ij} p^i p^j)^{\frac{1}{2}}$
$P \propto \frac{1}{a}$
v
$E = \hbar\omega = p$
$E^2 = m^2 + p^2$
$(\frac{t}{t_0})^p = a_0 (\frac{\eta}{\eta_0})^{\frac{p}{1-p}}$
$d\eta$
$\propto \eta$
$\propto \eta^{\frac{1}{1-p}}$
η
DE
MD
RD
x
$a \to 0$

$(g_{ij} p^i p^j)^{\frac{1}{2}}$ $\quad P \propto \frac{1}{a}$ $\quad$ v $\qquad E = \hbar\omega = P$

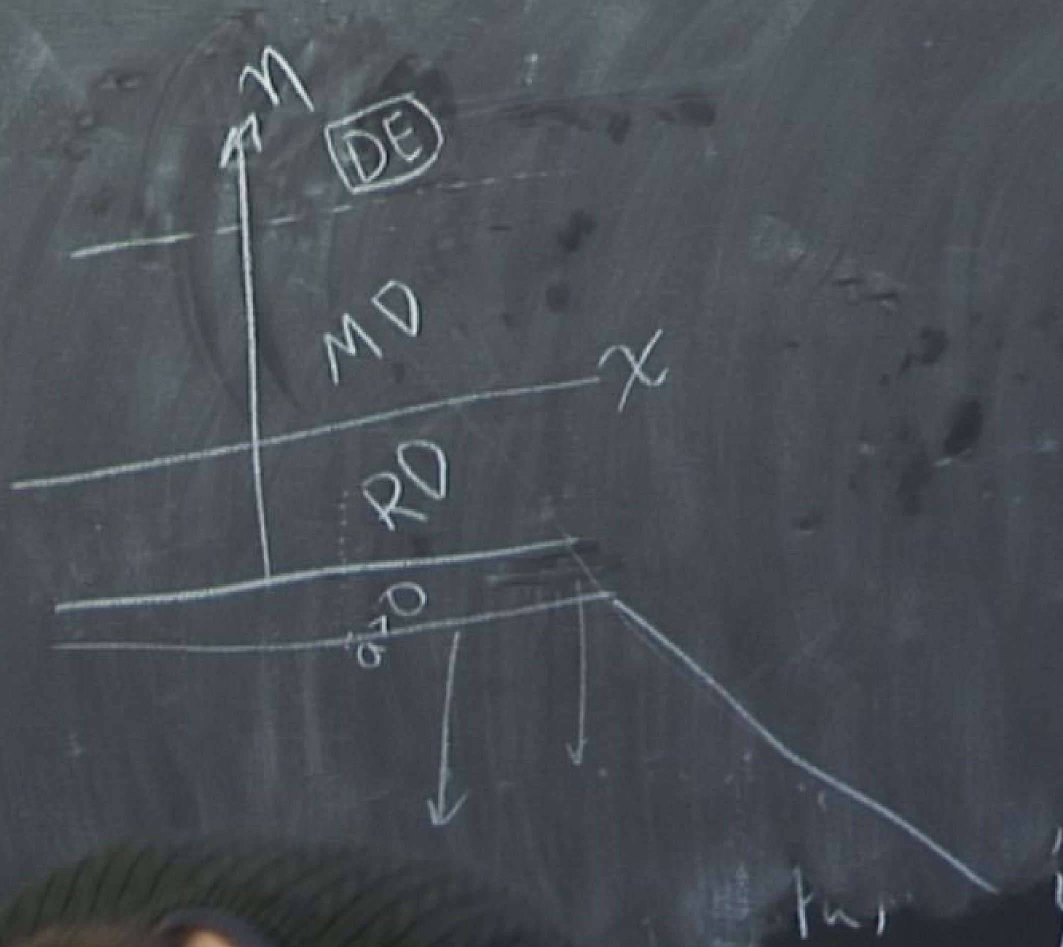
$E^2 = m^2 + P^2$

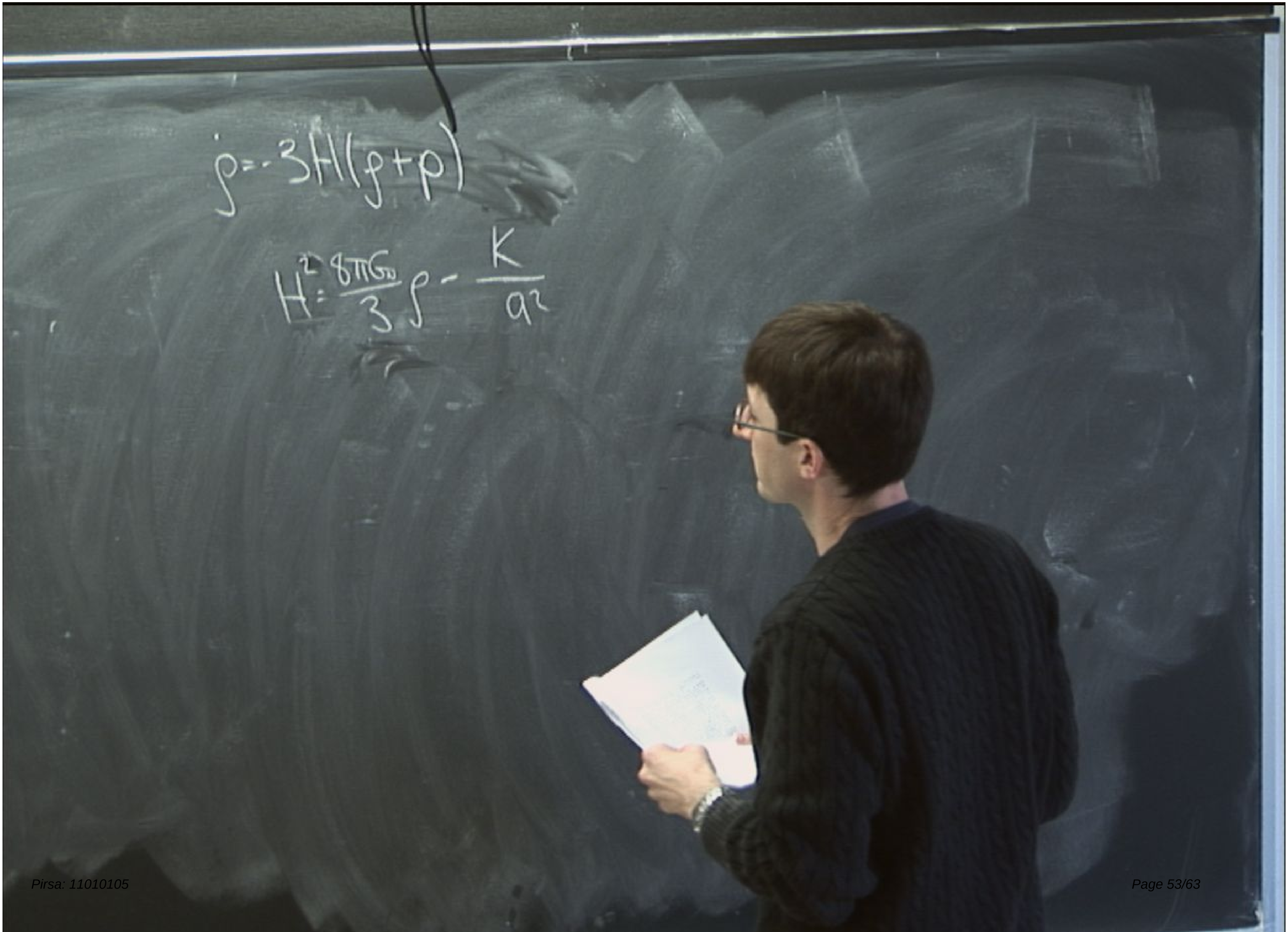
$\left(\frac{t}{t_0}\right)^p = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$

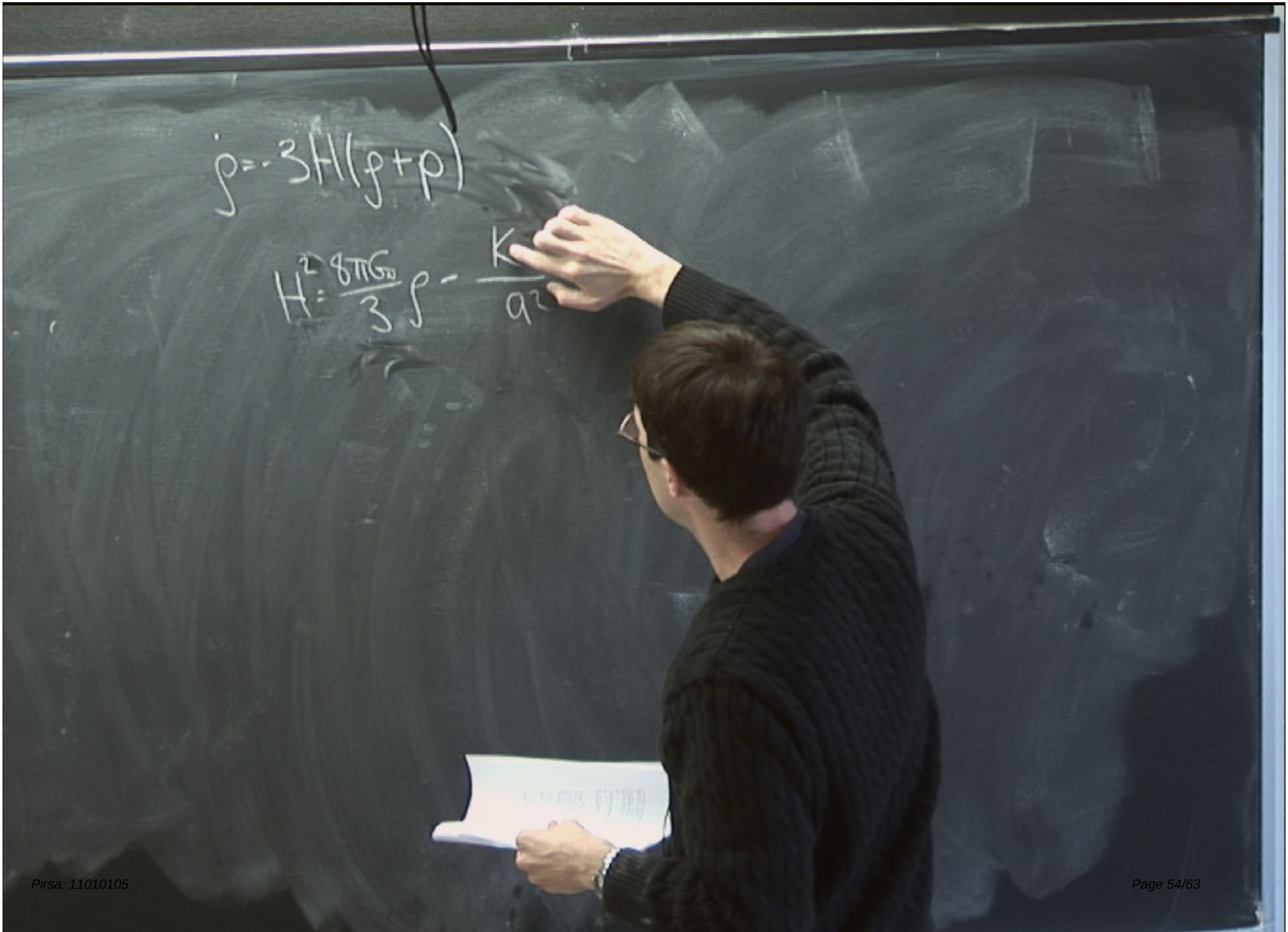
$= d\eta$

${}^{1-p} \propto \eta$

$t \propto \eta^{\frac{1}{1-p}}$




$\dot{\rho} = -3H(\rho + p)$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$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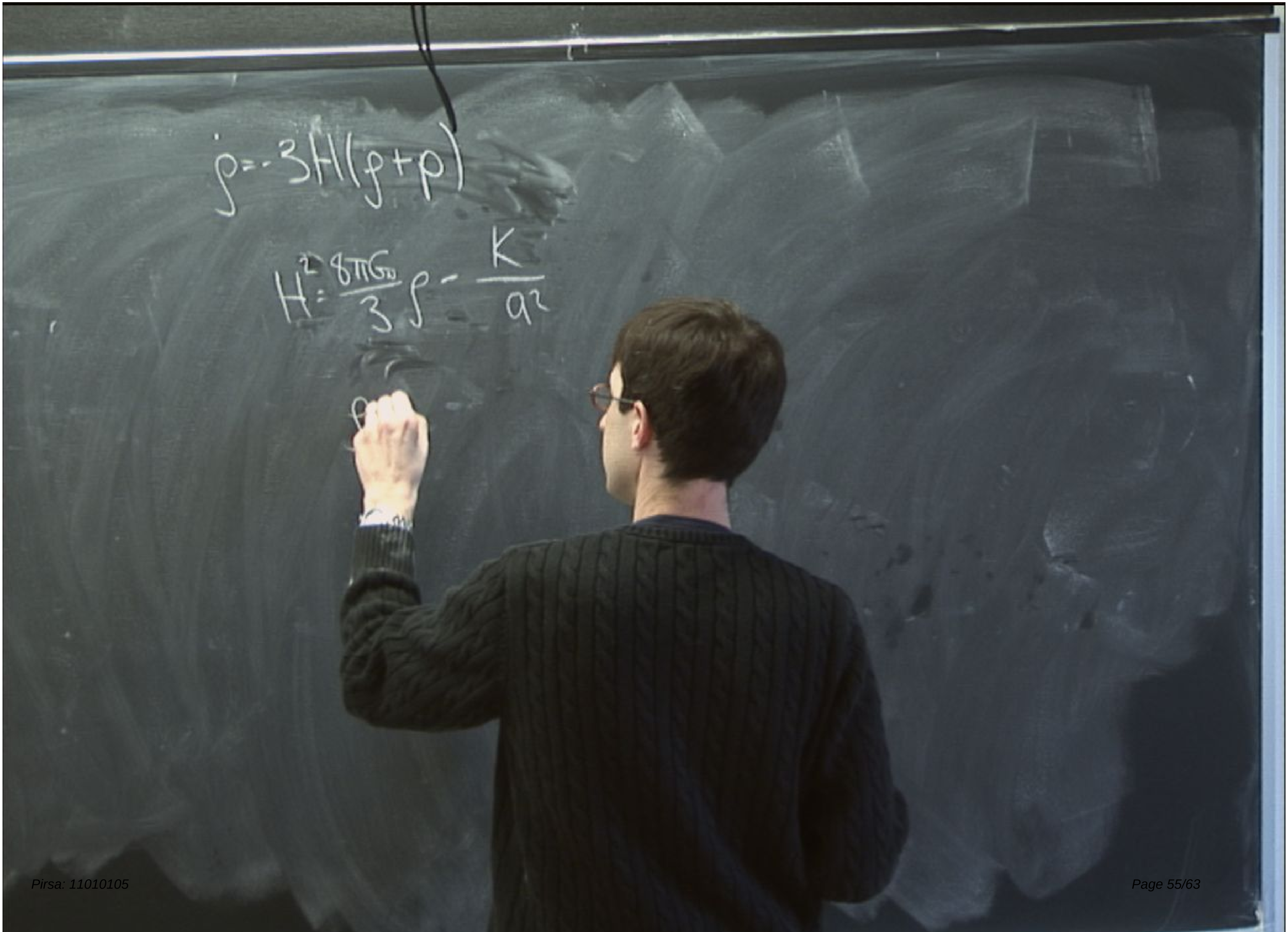
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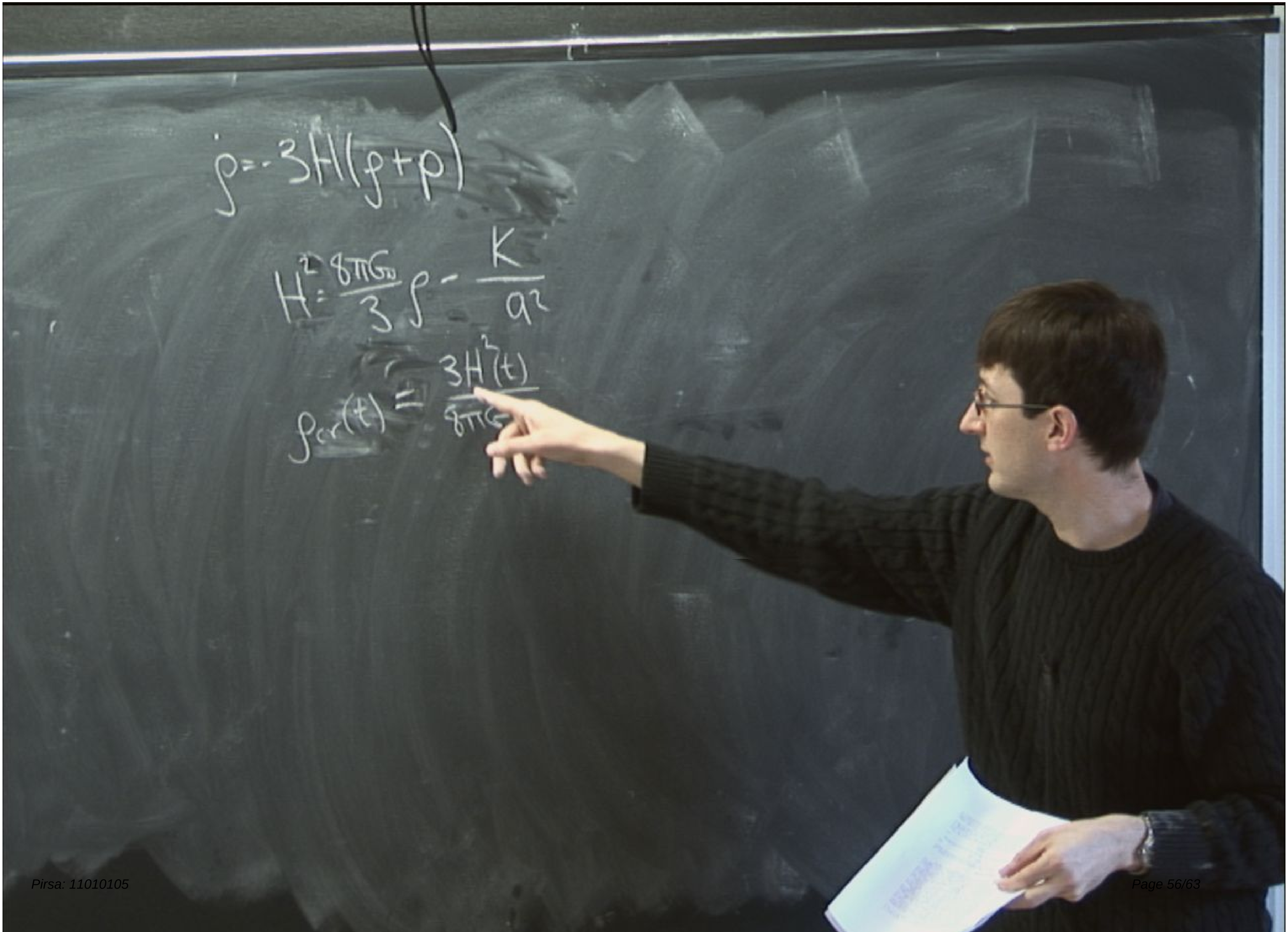
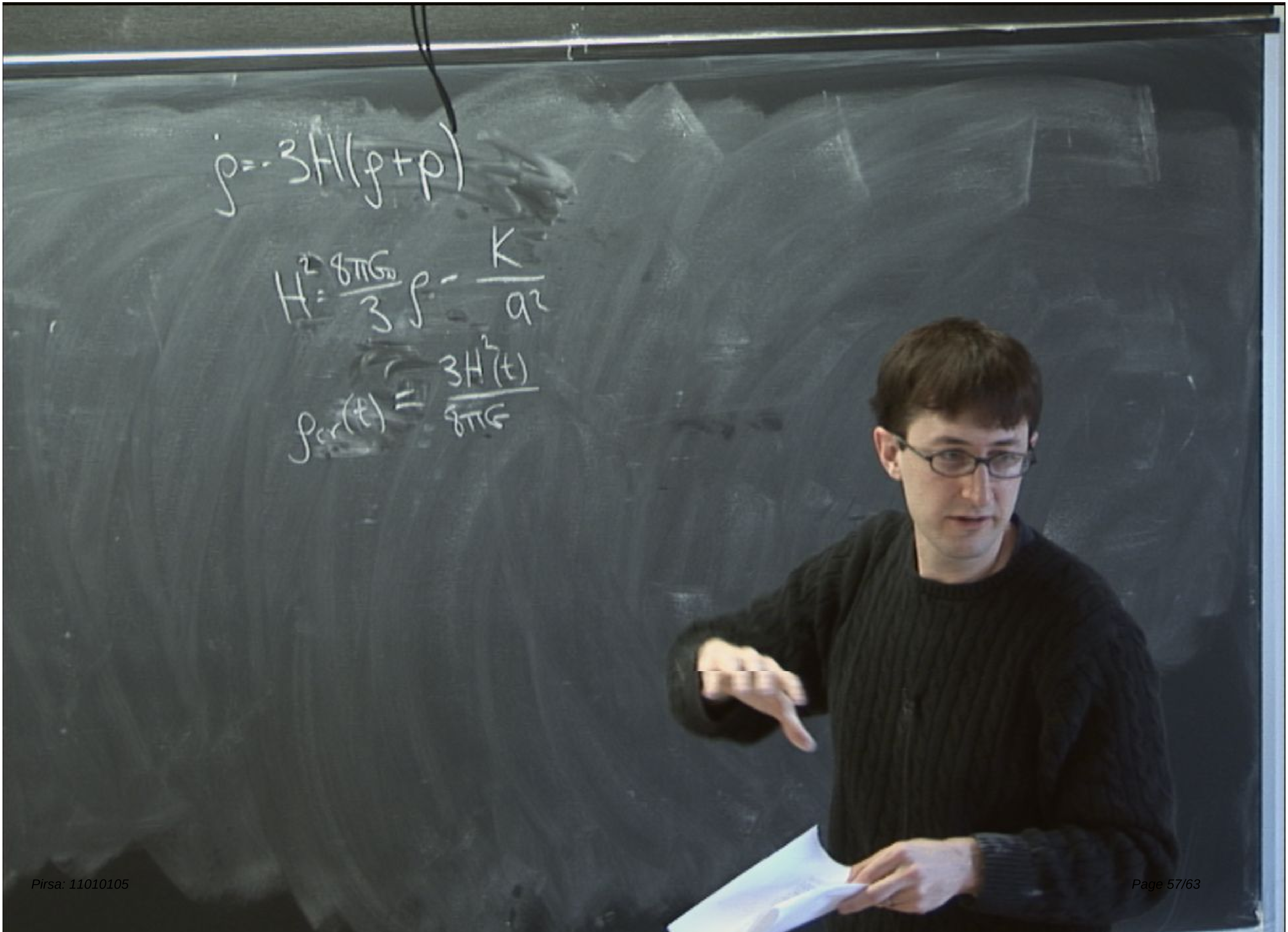

$\dot{\rho} = -3H(\rho + p)$
$H^2 = \frac{8\pi G_0}{3}\rho - \frac{K}{a^2}$
$\rho_{cr}(t) = \frac{3H^2(t)}{8\pi G}$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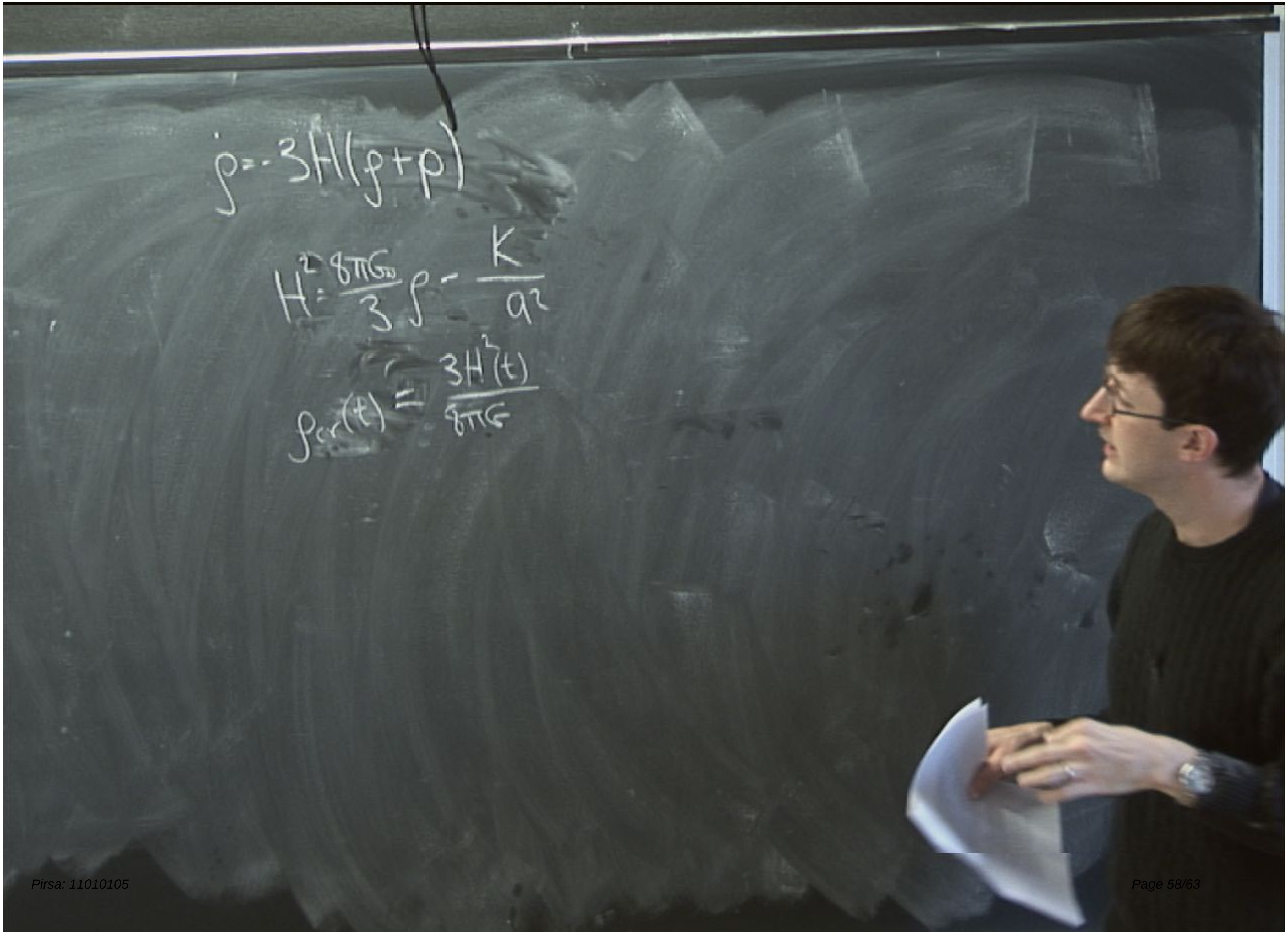
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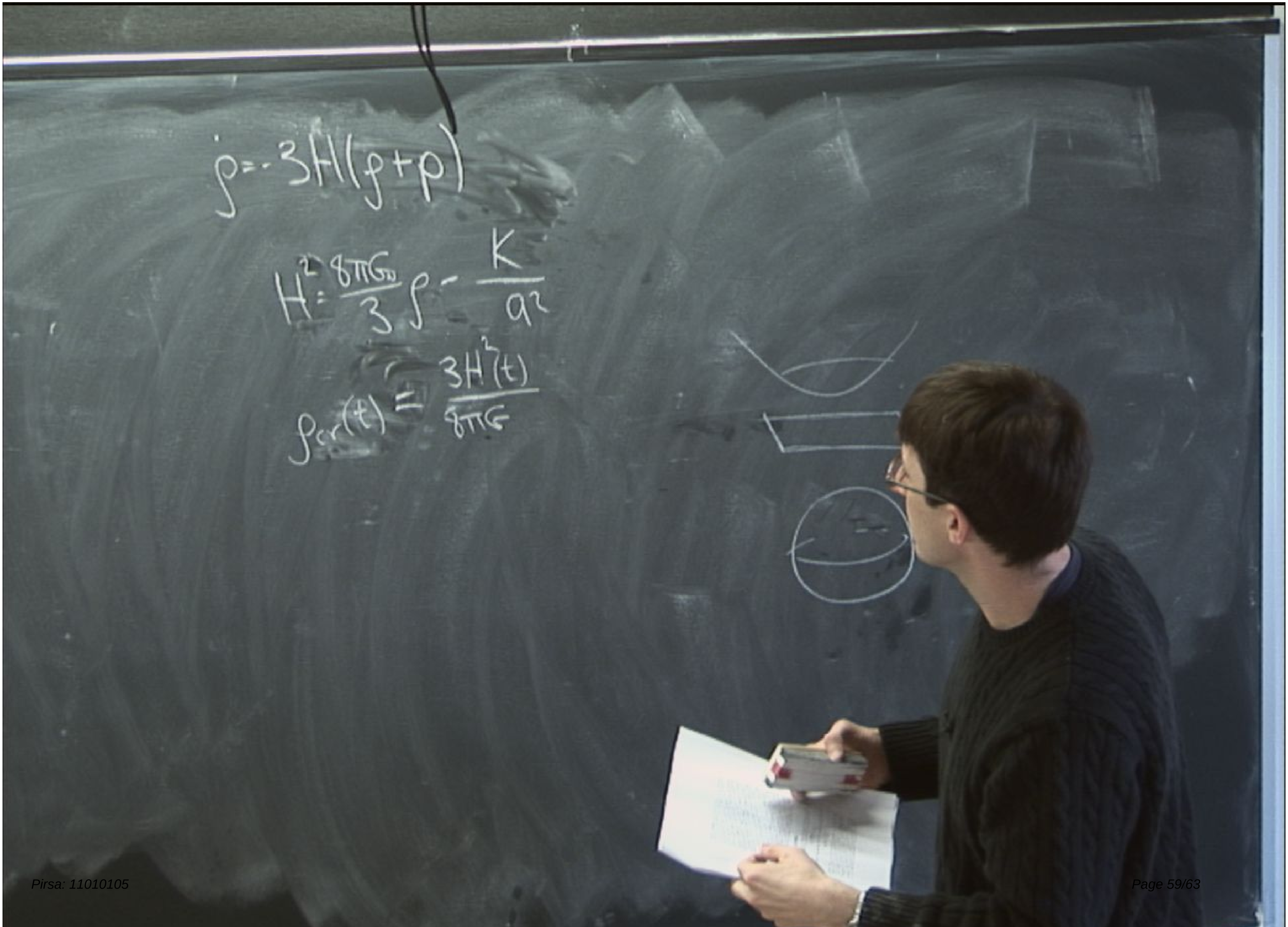
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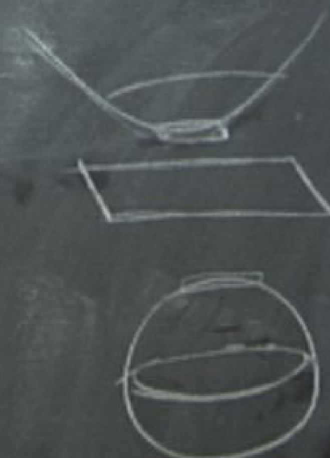
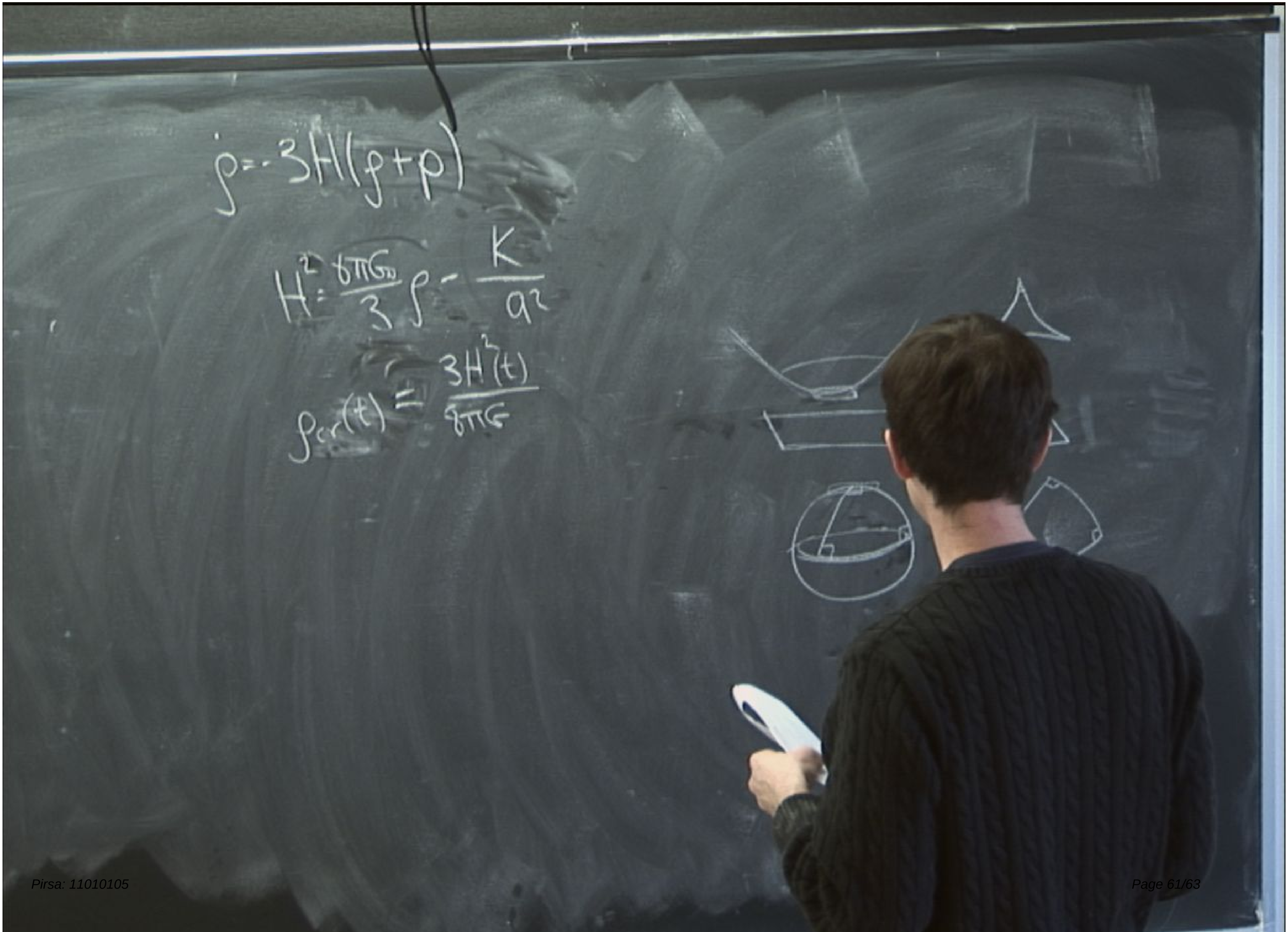
$$\dot{\rho} = -3H(\rho + p)$$

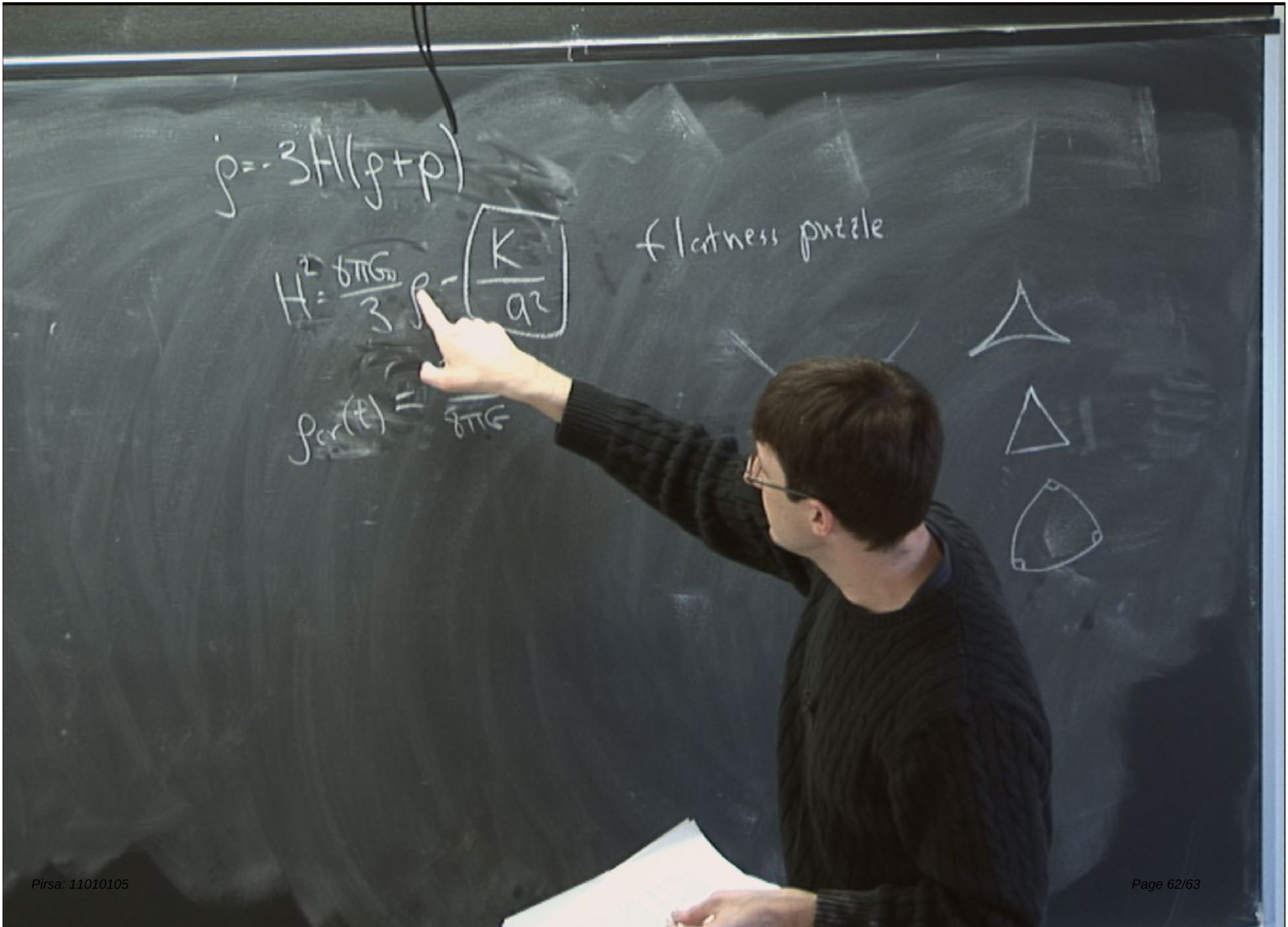
$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

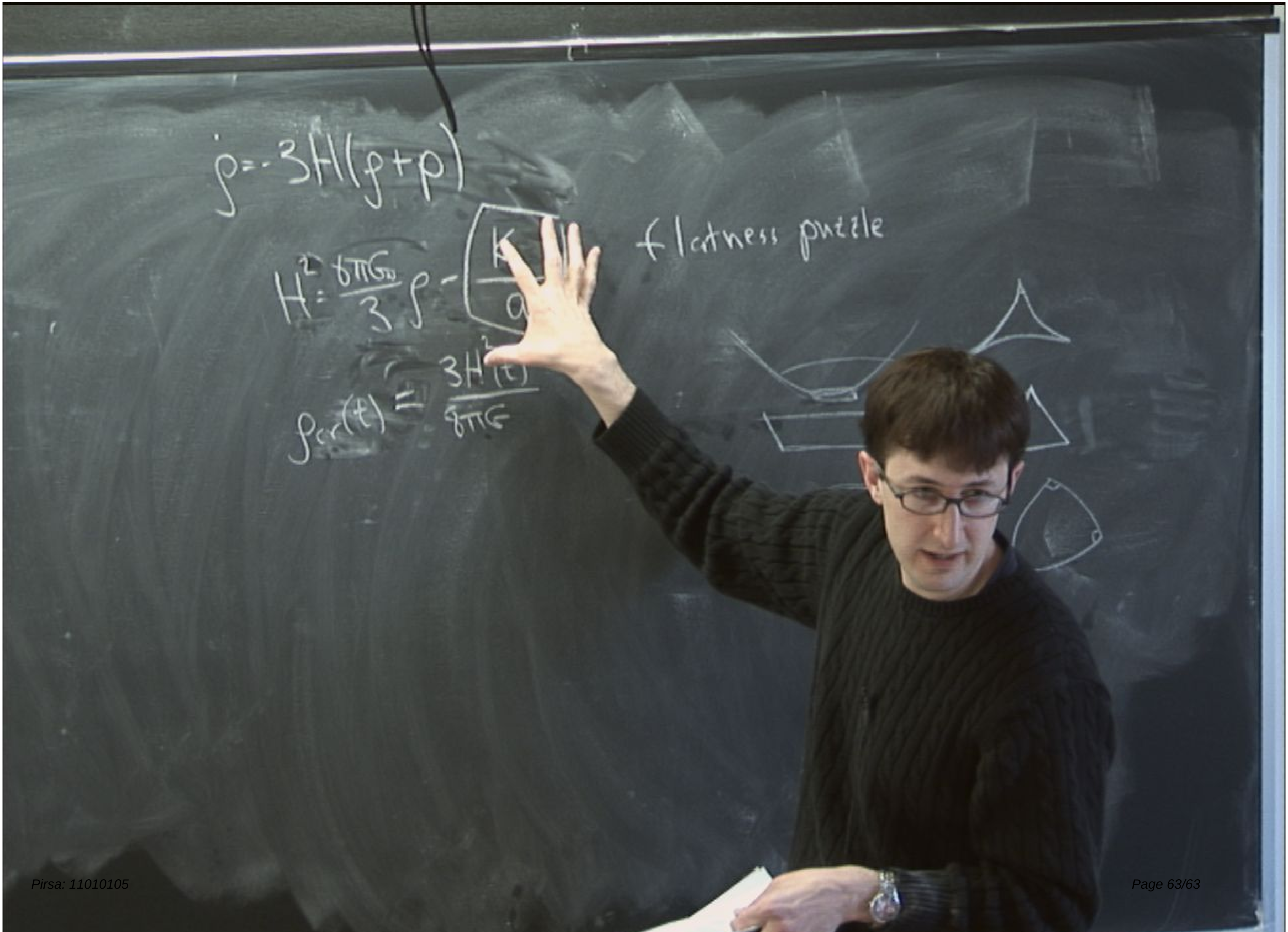
$$\rho_{cr}(t) = \frac{3H^2(t)}{8\pi G}$$





$\dot{\rho} = -3H(\rho + p)$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
$\rho_{cr}(t) = \frac{3H^2(t)}{8\pi G}$


ρ̇ = -3H(ρ+p)
H² = 8πG₀/3 ρ - K/a²
flatness puzzle
ρcr(t) = ... 8πG


$\dot{\rho} = -3H(\rho + p)$
$H^2 = \frac{8\pi G_N}{3}\rho - \boxed{\frac{K}{a}}$
flatness puzzle
$\rho_{cr}(t) = \frac{3H^2(t)}{8\pi G}$