

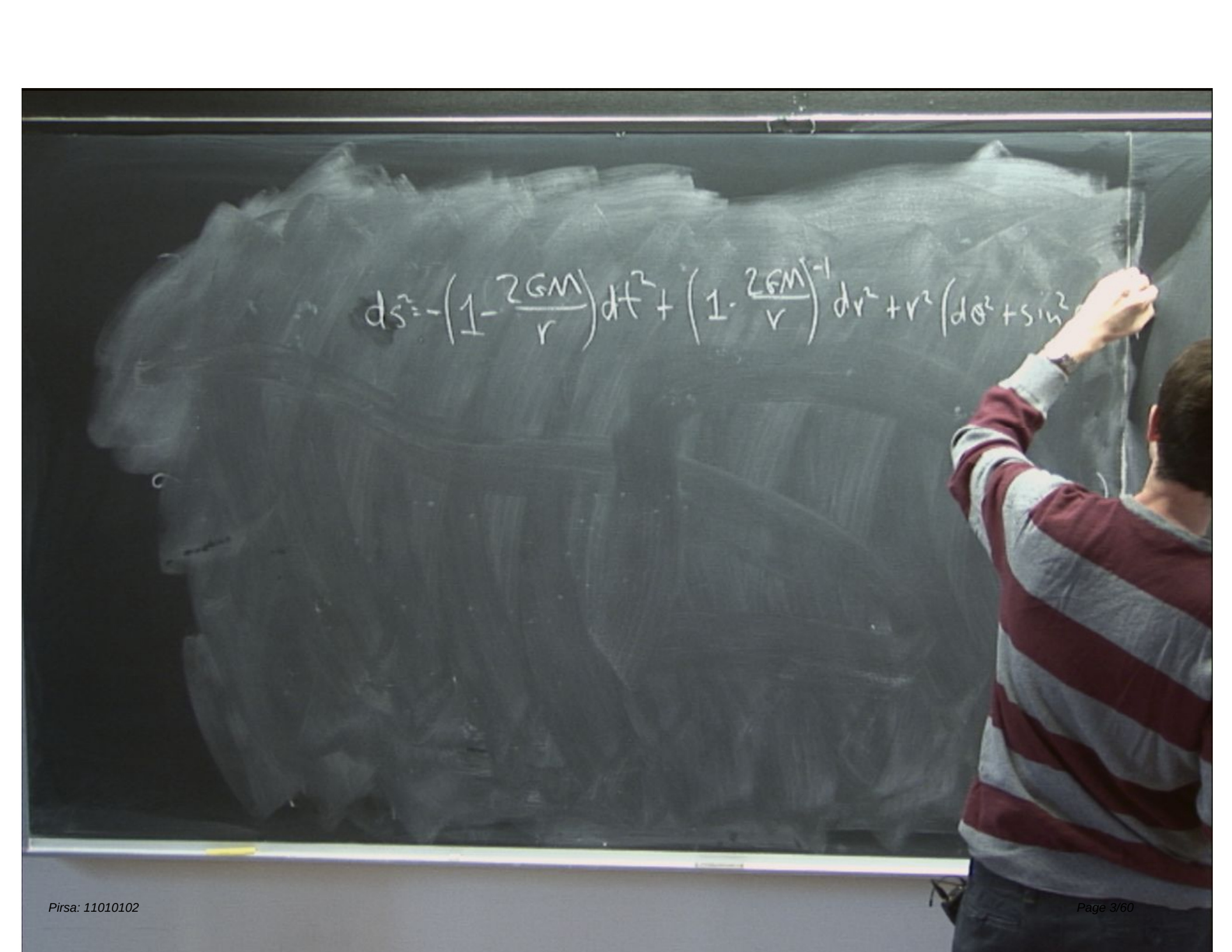
Title: Cosmology Review - Lecture 5

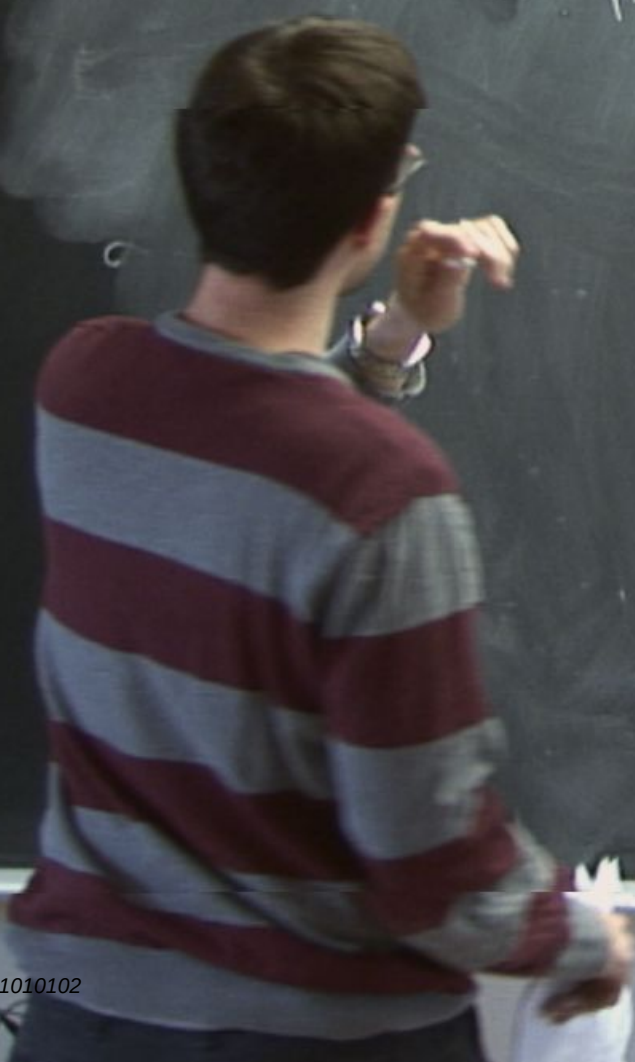
Date: Jan 28, 2011 11:30 AM

URL: <http://pirsa.org/11010102>

Abstract:



A person wearing a red and white striped long-sleeved shirt is standing on the right side of a large chalkboard, writing an equation. The chalkboard is mostly covered in light-colored chalk smudges and erasures. The equation being written is the Schwarzschild metric tensor.
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$



A person with short dark hair, wearing a red and grey horizontally striped sweater, is seen from the back, standing in front of a large chalkboard. They are holding a piece of chalk in their right hand, positioned as if they have just finished writing or are about to write. The chalkboard is filled with a large, somewhat messy white chalk smudge in the background. The equation for the Schwarzschild metric is written in white chalk on the board.

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$r_s = 2GM$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = 0 \quad \left(1 - \frac{2GM}{r}\right)^2 =$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = 0 \quad \left(1 - \frac{2GM}{r}\right) = \left|\frac{dr}{dt}\right|^2$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = 0 \quad \left(1 - \frac{2GM}{r}\right) = \left|\frac{dr}{dt}\right|^2$$

$$4M: 3+3+4=10$$

4

$$4M = 3 + 3 + 4 = 10$$

$$K = \pm 1$$



$$K = \eta_{AB} x^A x^B$$

$$\tilde{S} = \eta_{AB} \tilde{x}^A \tilde{x}^B$$

$$\tilde{x}^A = \tilde{\Lambda}^A_B x^B$$

$$\eta_{AB}$$

$$4M: 3+3+4=10$$

$$K = \pm 1$$



$$O(4,1)$$

$$O(3,2)$$

$$K_{\tilde{g}} = \eta_{AB} x^A x^B$$

$$d\tilde{s} = \eta_{AB} x^A x^B$$

$$x^A = \Lambda^A_B \tilde{x}^B$$

$$\eta_{AB} \Lambda^A_C \Lambda^B_D = \eta_{CD}$$

$$O(4,1)$$

$$O(3,2)$$

$$4M: 3+3+4=10$$

$$K = \pm 1$$



$$\eta_{AC} \omega^C_B = \omega_{AB} = -\omega_{BA}$$

$$K_{\tilde{g}} = \eta_{AB} x^A x^B$$

$$d\tilde{s} = \eta_{AB} x^A x^B$$

$$x^A = \Lambda^A_B \tilde{x}^B$$

$$\eta_{AB} \Lambda^A_C \Lambda^B_D = \eta_{CD}$$

$$\uparrow$$

$$\Lambda^A_C = \delta^A_C + \omega^A_C$$

$$\frac{(n+1)n}{2} \dots$$

$$4M: 3+3+4 = \boxed{10}$$

$$K = \pm 1$$

$$\boxed{10}$$

$$O(4,1)$$

$$O(3,2)$$

$$K_g^2 = \eta_{AB} x^A x^B$$

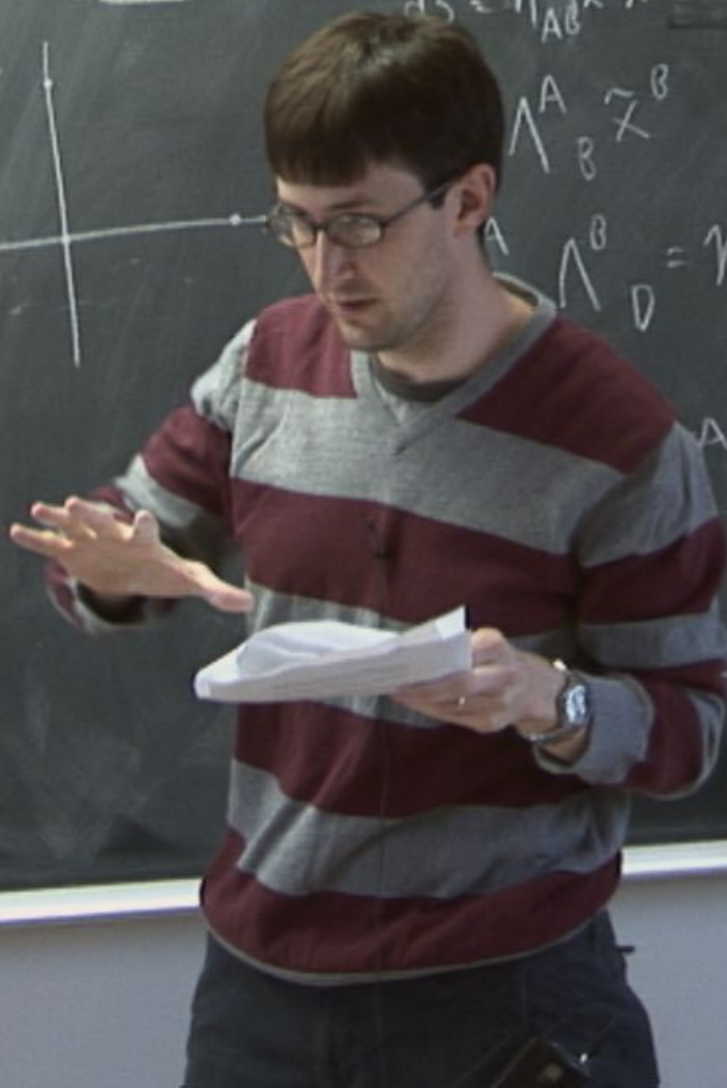
$$dS^2 = \eta_{AB} x^A x^B$$

$$\Lambda^A_B \tilde{x}^B$$

$$\Lambda^B_D = \eta_{CD}$$

$$\eta_{AC} \omega^C_B = \omega_{AB} = -\omega_{BA}$$

$$\omega^A_C + \omega^A_C$$



$$\frac{(n+1)n}{2} \dots$$

$$O(4,1)$$

$$O(3,2)$$

$$4M: 3+3+4 = \boxed{10}$$

$$K = \pm 1$$

$$\boxed{10}$$

$$K_g^2 = \eta_{AB} x^A x^B$$

$$ds^2 = \eta_{AB} x^A x^B$$

$$x^A = \Lambda^A_B \tilde{x}^B$$

$$\Lambda^A_B \Lambda^B_C = \Lambda^A_C$$

$$\Lambda^A_C = \delta^A_C + \omega^A_C$$

$$\eta_{AC} \omega^C_B = \omega_{AB} = -\omega_{BA}$$

$$\frac{(n+1)n}{2} \dots$$

$$4M: 3+3+4 = \boxed{10}$$

$$K = \pm 1$$

$$\boxed{10}$$



$$\mathcal{M}_{AC} = \omega_{AB}$$

$$O(4,1)$$

$$O(3,2)$$

$$K_g^{\sim} = \eta_{AB} x^A x^B$$

$$d\tilde{s} = \eta_{AB} \dot{x}^A \dot{x}^B$$

$$x^A = \Lambda^A_B \tilde{x}^B$$

$$\eta_{AB} \Lambda^A_C \Lambda^B_D = \eta_{CD}$$

$$\uparrow \Lambda^A_C = \delta^A_C + \omega^A_C$$

$$\frac{(n+1)n}{2}$$

$$4M: 3+3+4 = \boxed{10}$$

$$K = \pm 1$$

$$\boxed{10}$$



$$\eta_{AC} \omega^C_B = \omega_{AB} = -\omega_{BA}$$

$$O(4,1)$$

$$O(3,2)$$

$$K_g^2 = \eta_{AB} x^A x^B$$

$$d\tilde{s} = \eta_{AB} \tilde{x}^A \tilde{x}^B$$

$$x^A = \Lambda^A_B \tilde{x}^B$$

$$\eta_{AB} \Lambda^A_C \Lambda^B_D = \eta_{CD}$$

$$\Lambda^A_C =$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = 0$$

$$\left(1 - \frac{2GM}{r}\right) = \left|\frac{dr}{dt}\right|^2$$

$$r = 1GM$$

15

$$G = c \cdot 1$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

$$ds^2 = 0$$

$$\left(1 - \frac{2GM}{r}\right) = \left(\frac{dr}{dt}\right)^2$$

$$\frac{dr}{dt}$$

$$r = 1GM$$

$$S \leq M^2$$

$$G = c = 1$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$r > r_s = 2GM$$

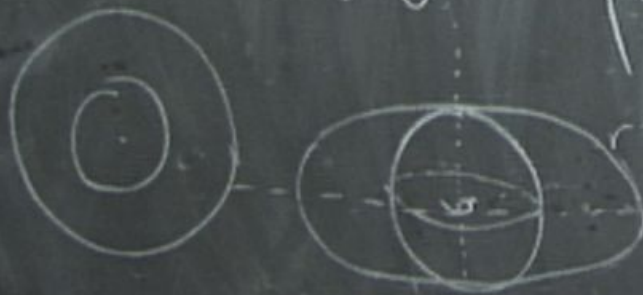
$$ds^2 = 0$$

$$\left(1 - \frac{2GM}{r}\right) = \left(\frac{dr}{dt}\right)^2$$

$$\frac{dr}{dt}$$

$$r = 1GM$$

$$M \leq M_{\text{max}}$$



$$\boxed{n=3}$$

$$ds^2 =$$

coordinates

kinematics

massive

massless

horizon)

$n=3$

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{\rho^2 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

coordinates

kinematics

massive

massless

horizon

dynamics

$n=3$

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

$$\{x^1, x^2, x^3\} = r \{ \sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta \}$$

coordinates

kinematics

massive

massless

horizon

dynamics

$n=3$

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

$$\{x^1, x^2, x^3\} = r \{\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta\}$$

coordinates

kinematics

massive

massless

horizon

dynamics

dr^2

$$\int dx^i dx^j$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \int \tilde{x}^e \quad a(t) = \frac{1}{p} \hat{a}(t)$$

$n=3$

coordinates
kinematics
massive
massless
horizon
dynamics

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i$$

$$\{x^1, x^2, x^3\} = r \{ \sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta \}$$

$$\rightarrow -d^2t^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \int \dot{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$x \quad (k=t+1)$$

$$(k=-1)$$

$$(k=0)$$

$$r = S_k(x)$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$r = S_k(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

$n=3$

coordinates
kinematics

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

$$\{x^1, x^2, x^3\} = r \{ \sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta \}$$

$$\rightarrow -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

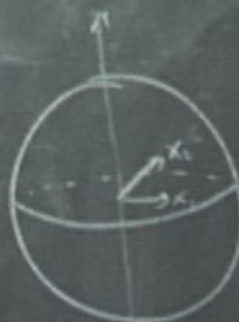
$$\rightarrow -dt^2 + a^2(t) \left[d\chi^2 + S_K^2(\chi) d\Omega^2 \right]$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \hat{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

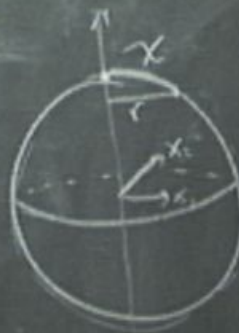


$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \hat{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$



$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$



$$r=1$$

$$X = \frac{\pi}{2}$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = p^e x^e$$

$$a(t) = \frac{1}{p} \hat{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

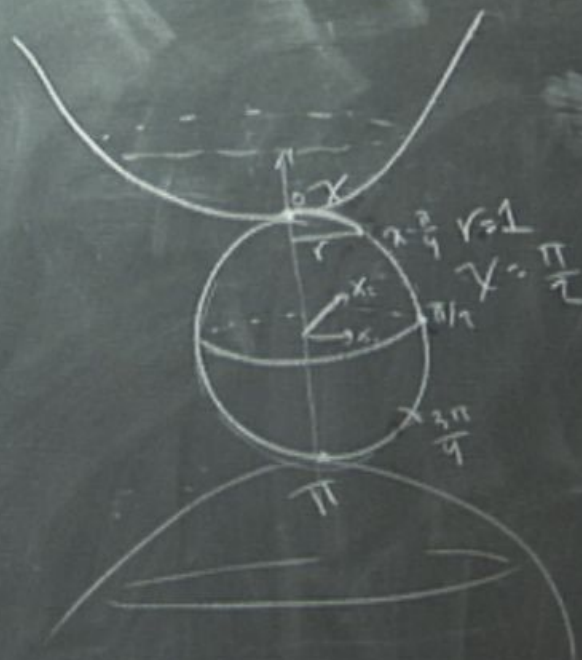


$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \hat{x}^e$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

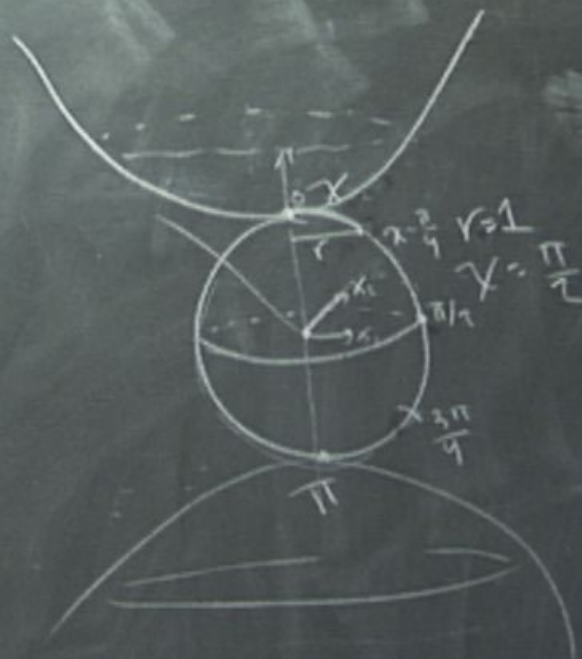


$$dr^2 + r^2 d\Omega^2$$

$$x^e = \int f^p$$

$$a(t) = \frac{1}{p} \tilde{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$



$n=3$

coordinates

kinematics

massive

massless

horizon

dynamics

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

$$\{x^1, x^2, x^3\} = r \{ \sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta \}$$

$$\rightarrow -d^2t^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$\rightarrow -dt^2 + a^2(t) [d\chi^2 + S_K^2(\chi) d\Omega^2]$$

$$dt = a d\eta$$

$n=3$

coordinates

metrics

massive

massless

(horizon)

dynamics

$$ds^2 = -dt^2 + a^2(t) \left[\delta_{ij} + K \frac{\delta_{ik} \delta_{jl} x^k x^l}{1 - K \delta_{ef} x^e x^f} \right] dx^i dx^j$$

$$\{x^1, x^2, x^3\} = r \{ \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta \}$$

$$\rightarrow -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$\rightarrow -dt^2 + a^2(t) \left[d\chi^2 + S_K^2(\chi) d\Omega^2 \right]$$

$$= a^2 \left[-d\eta^2 + d\chi^2 + S_K^2(\chi) d\Omega^2 \right]$$

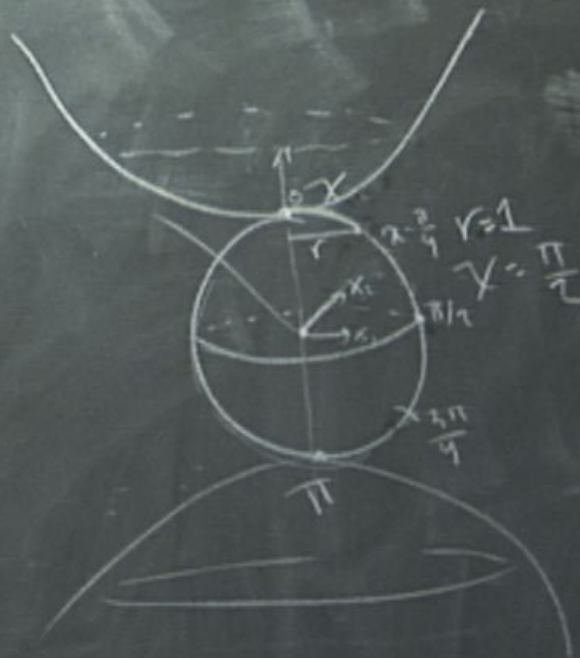
$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin \chi & (K=1) \\ \sinh \chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$$dt = a d\eta$$



$$dr^2 + r^2 d\Omega^2$$

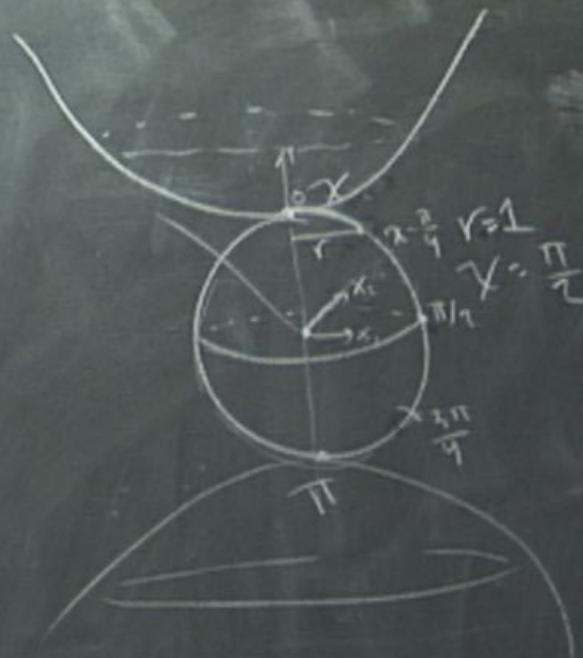
$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$



$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho x^e$$

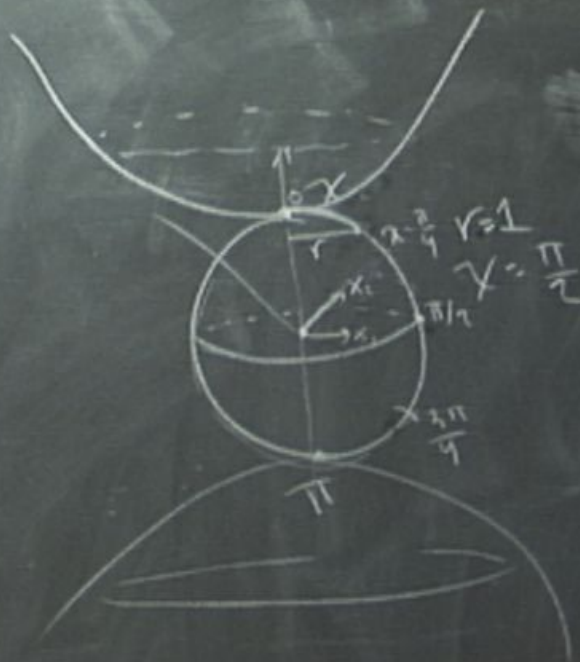
$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$a(t(\eta)) = a(\eta)$$



$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u = \frac{dx}{dt}$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$dr^2 + r^2 d\Omega^2$$

$$\dot{x}^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\tau)$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta \, a(\eta)$$

$$dr^2 + r^2 d\Omega^2$$

$$\dot{x}^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\tau)$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_K(X) = \begin{cases} \sin X & (K=+1) \\ \sinh X & (K=-1) \\ X & (K=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta \, a(\eta)$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$U^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{d}{d\lambda} \left(\Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \right) = 0$$

$$\vec{u} = \frac{dx^\mu}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$\lambda = \frac{d\tau}{m}$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^\alpha}{d\lambda} + \Gamma_{\alpha\beta}^\gamma p^\beta p^\gamma = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$a(t) = \frac{1}{\rho} \hat{a}(t)$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \Gamma_{\alpha\beta}^0 p^\alpha p^\beta = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda} \frac{d\lambda}{d\tau}$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$a(t) = \frac{1}{\rho} \tilde{a}(t)$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H g_{ij} p^i p^j$$

$$u^\mu = \frac{dx^\mu}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$dr^2 + r^2 d\Omega^2$$

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$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j}$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

t

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

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$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 =$$

$$u^\mu = \frac{dx^\mu}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$



$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = g_{\mu\nu} p^\mu p^\nu = (p^0)^2 = \frac{g_{ij} p^i p^j}{p^2}$$

$$E^2 = m^2 + \vec{p}^2$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$



$$dr^2 + r^2 d\Omega^2$$



$$x^e = p^e \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = g_{\mu\nu} p^\mu p^\nu = (p^0)^2 = \frac{g_{ij} p^i p^j}{p^2}$$

$$E^2 = m^2 + \vec{p}^2$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$dr^2 + r^2 d\Omega^2$$



$$x^e = p^e \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

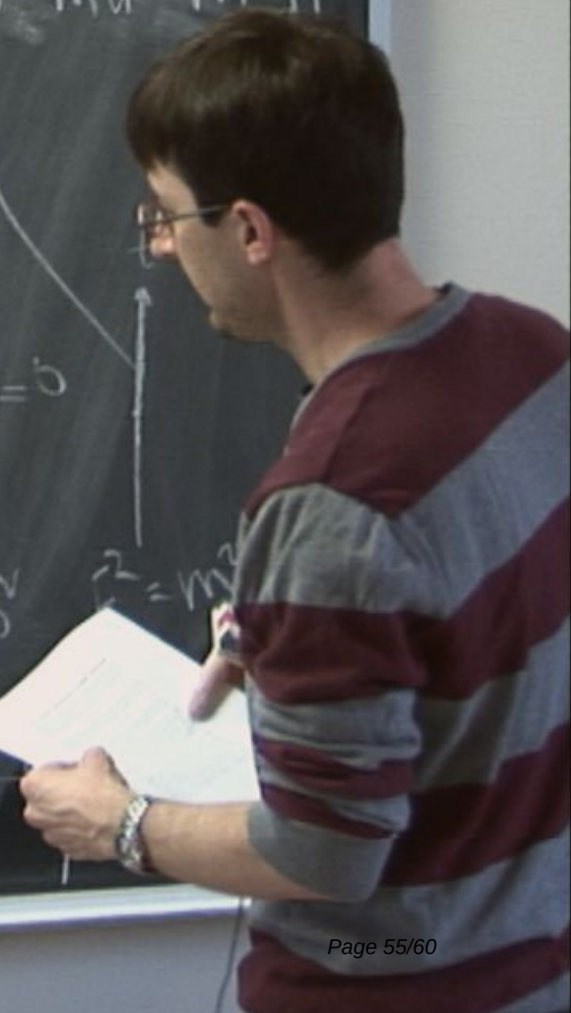
$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = g_{\mu\nu} p^\mu p^\nu = (p^0)^2 = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\lambda} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$



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$$x^e = p^e \tilde{x}^e$$

$$a(t) = \frac{1}{p} \tilde{a}(t)$$

$$\tilde{x}^e = \begin{cases} \sin x & (k=+1) \\ \sinh x & (k=-1) \\ x & (k=0) \end{cases}$$

$a d\eta$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^m \nabla_m u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = g_{\mu\nu} p^\mu p^\nu = (p^0)^2 = \frac{g_{ij} p^i p^j}{p^2}$$

$$E^2 = m^2 + \vec{p}^2$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^m = \frac{dx^m}{d\lambda}}$$

$$p^m = m u^m = m \frac{dx^m}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

t

$$\frac{dP}{d\lambda} = - \frac{d\eta}{d\lambda}$$

$$x^e = \rho \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma_{ij}^0}_{H g_{ij}} p^i p^j = 0$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = g_{\mu\nu} p^\mu p^\nu = (p^0)^2 = \frac{g_{ij} p^i p^j}{p^2}$$

$$E^2 = m^2 + \vec{p}^2$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

t

$$\frac{dP}{d\eta} = -\frac{dg}{d\eta}$$

$$\ln P = -\ln a + C$$

$$P \propto \frac{1}{a}$$

$$u^m \nabla_m u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$x^e = p \tilde{x}^e$$

$$a(t) = \frac{1}{p} \tilde{a}(t)$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$\frac{dp^0}{d\lambda} + \Gamma_{ij}^0 p^i p^j = 0$$

$$H g_{ij}$$

$$\frac{dp^0}{d\lambda} + H \left[g_{ij} p^i p^j \right]$$

$$p^m = m u^m = m \frac{dx^m}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$p_1^2 + p_2^2$$

$$\frac{dP}{d\eta} = -\frac{d\eta}{d\eta}$$

$$\ln P = -\ln a + C$$

$$\boxed{P \propto \frac{1}{a}}$$

$$E = P = \hbar \omega$$

$$U^{\mu} \nabla_{\mu} U^{\alpha} = 0$$

$$U^{\alpha} = \frac{dx^{\alpha}}{d\tau} \quad x^{\alpha}(\lambda)$$

$$\boxed{p^{\mu} = \frac{dx^{\mu}}{d\lambda}}$$

$$x^e = p^e \tilde{x}^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=+1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$a(t) = \frac{1}{p} \hat{a}(t)$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$\frac{dp^0}{d\lambda} + \Gamma_{ij}^0 p^i p^j = 0$$

$$\frac{\dot{a}}{a} = H$$

$$\frac{dp^0}{d\lambda} = \dots$$

$$m^2 = \dots$$

$$p^{\mu} = m \frac{dx^{\mu}}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$\frac{dP}{d\eta} = -\frac{da}{a}$$

$$\ln P = -\ln a + C$$

$$\boxed{P \propto \frac{1}{a}}$$

$$E = P = \hbar \omega$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\boxed{u^\mu = \frac{dx^\mu}{d\tau} \quad x^\alpha(\lambda)}$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$x^e = p x^e$$

$$r = S_k(X) = \begin{cases} \sin X & (k=1) \\ \sinh X & (k=-1) \\ X & (k=0) \end{cases}$$

$$dt = a d\eta$$

$$a(t) = \frac{1}{p} \hat{a}(t)$$

$$\eta - \eta_0 = \int_{t_0}^t \frac{dt}{a(t)}$$

$$\eta(t) \rightarrow t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} a(\eta) d\eta$$

$$\frac{dp^0}{d\lambda} + \Gamma_{\mu\nu}^0 p^\mu p^\nu = 0$$

$$\frac{dp^0}{d\lambda} + \frac{H}{k} p^0 = 0$$

$$m^2 = \dots$$

$$m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$