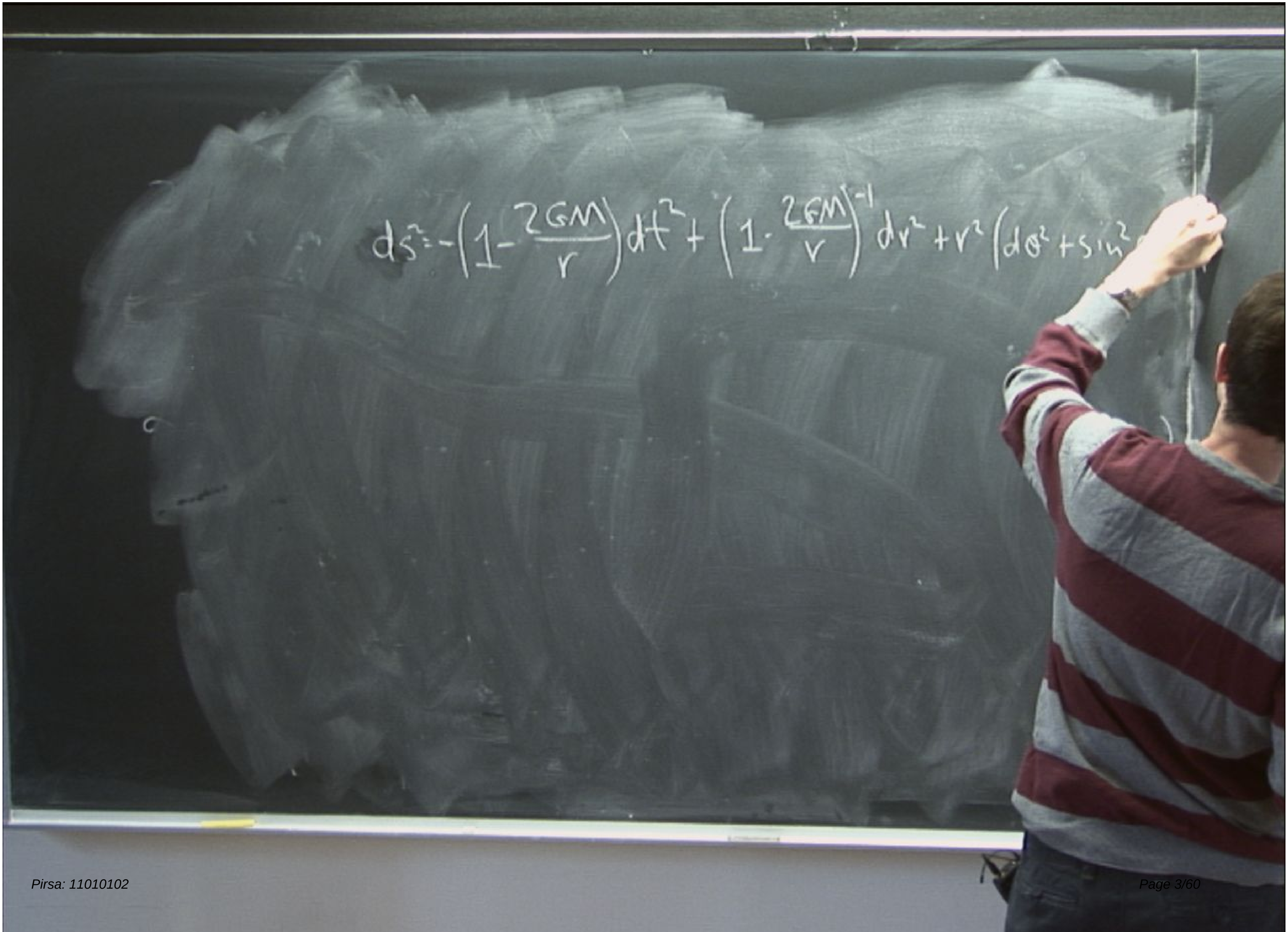
Title: Cosmology Review - Lecture 5

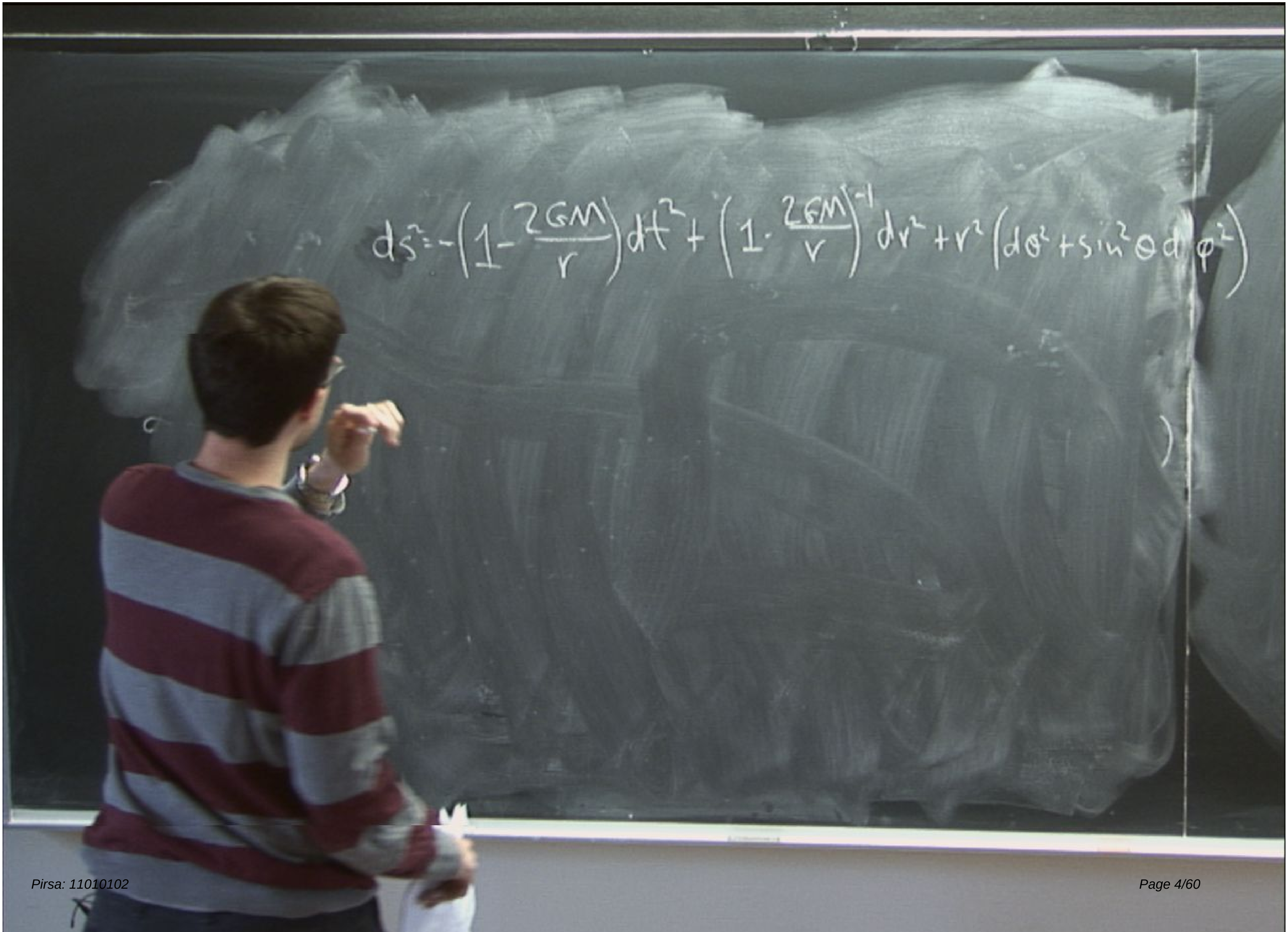
Date: Jan 28, 2011 11:30 AM

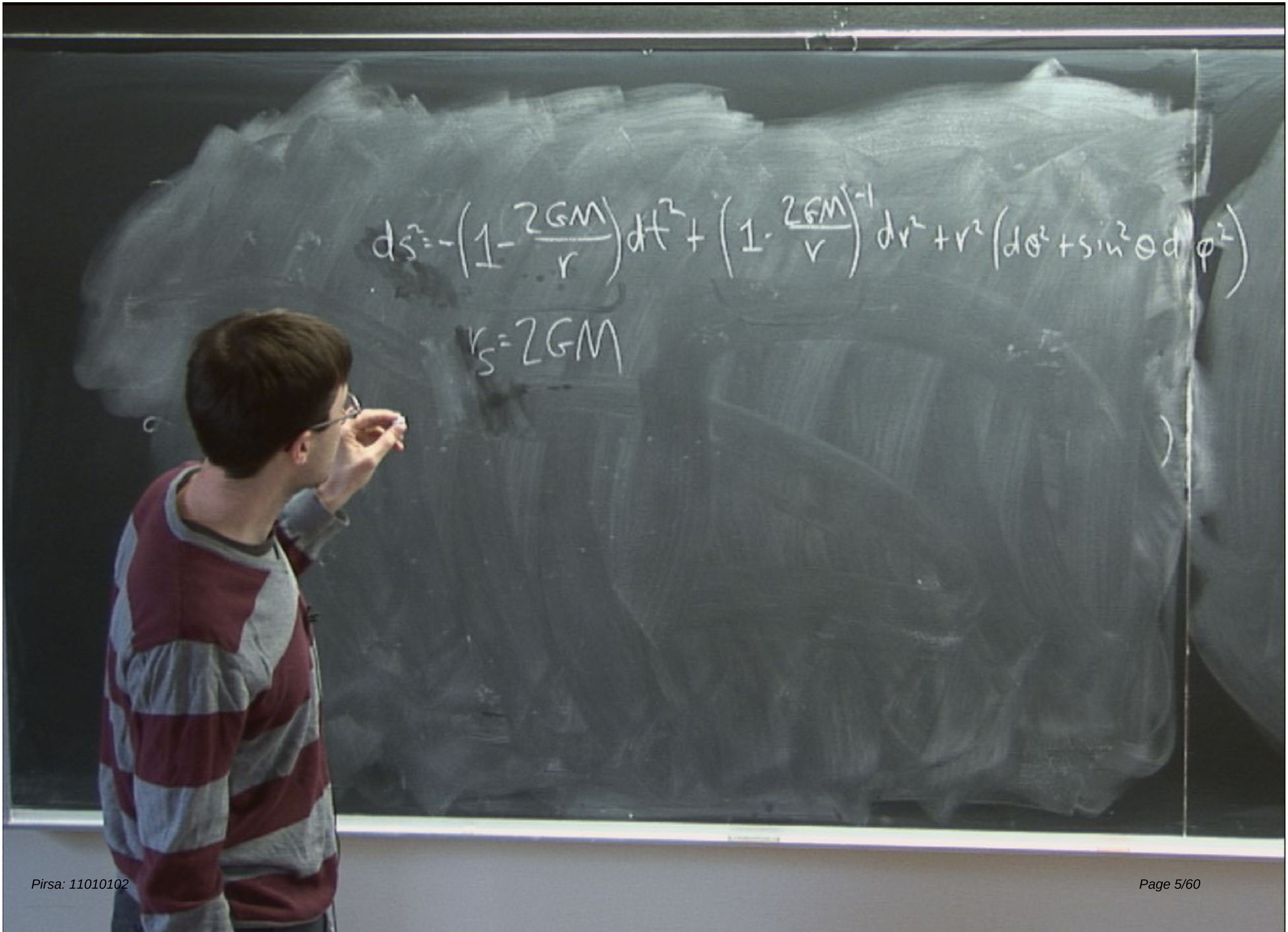
URL: http://pirsa.org/11010102

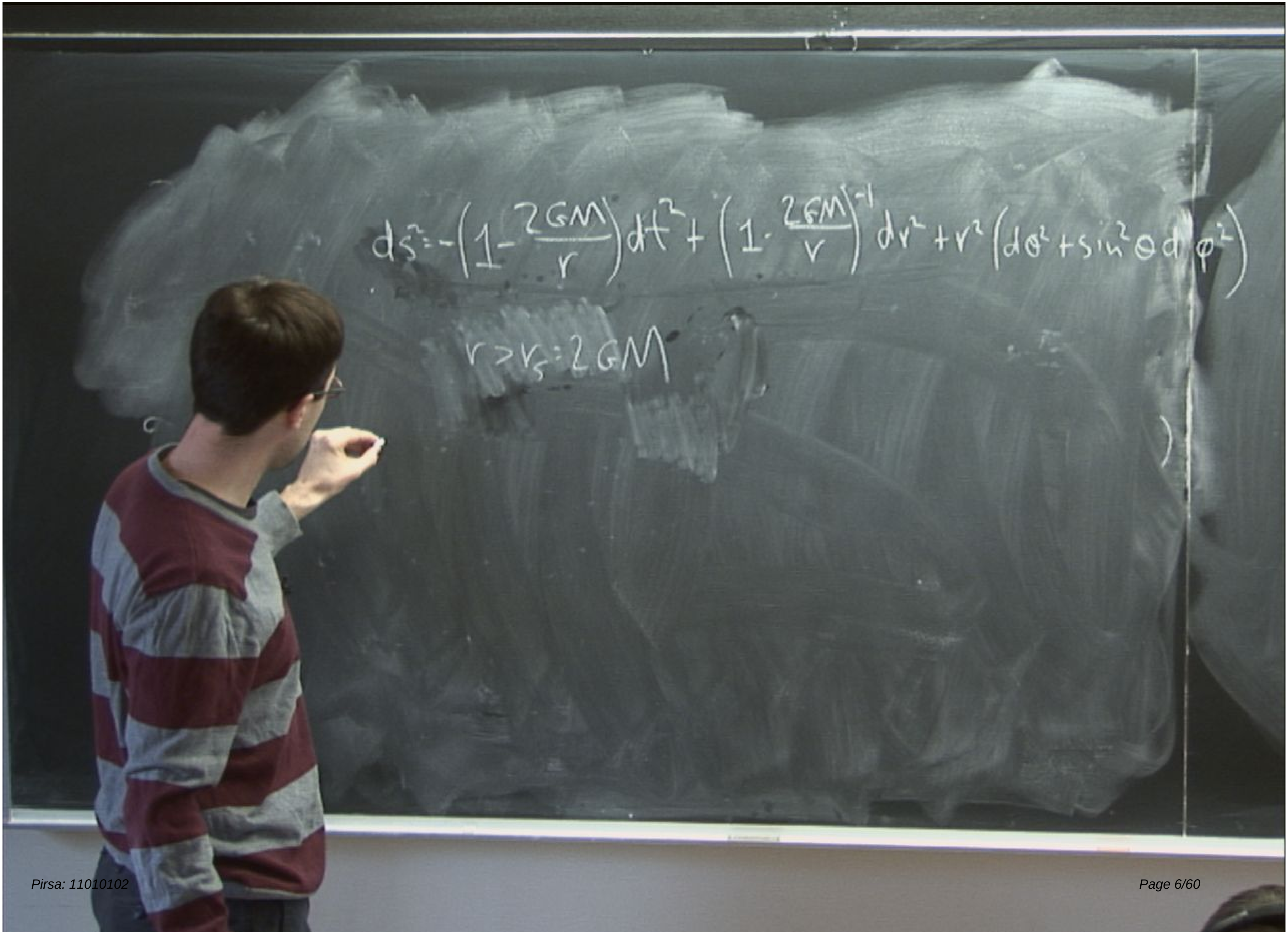
Abstract:

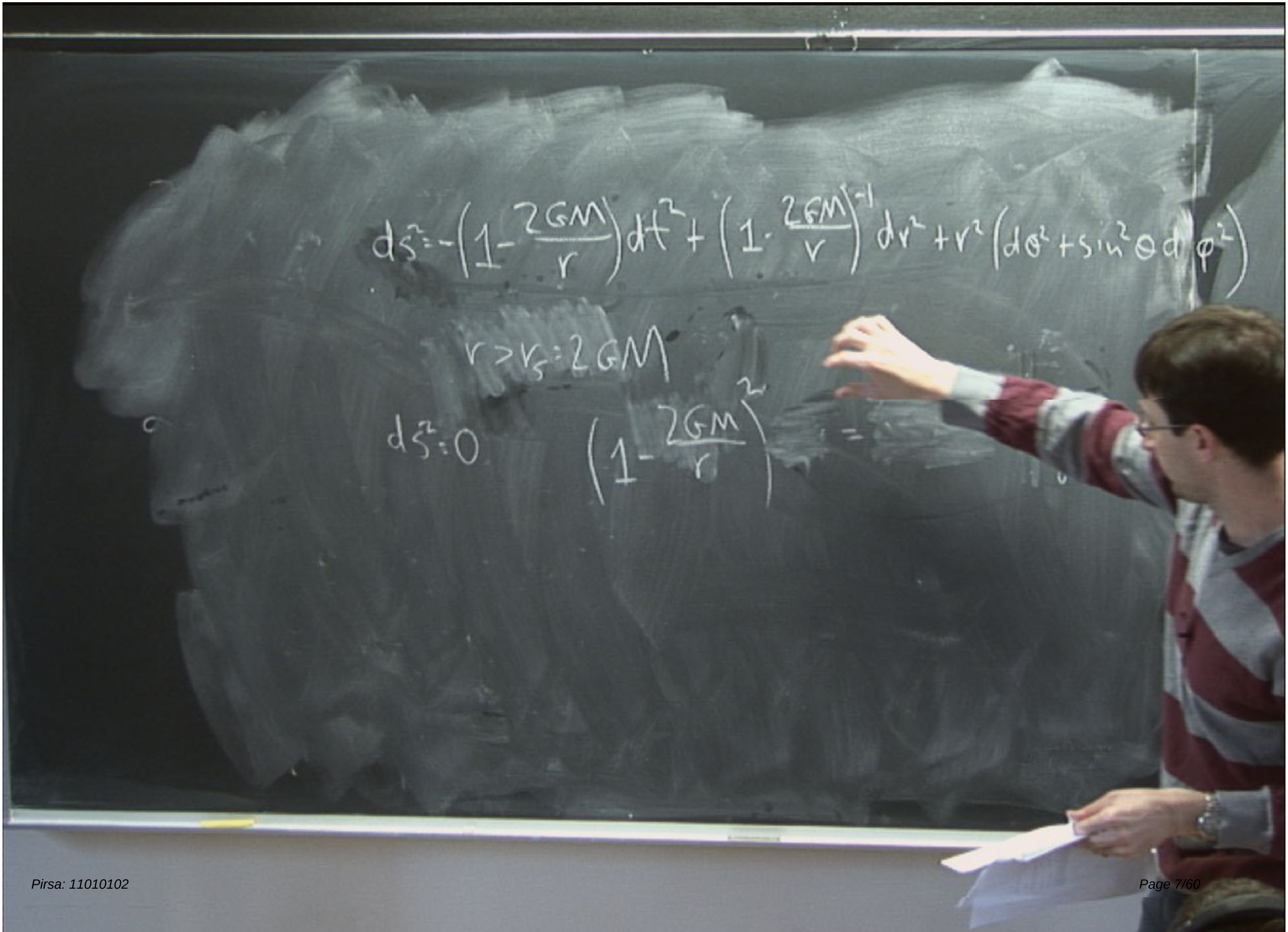
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\right.$


$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$


$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$
$r_s = 2GM$


$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$
$r > r_s = 2GM$


$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$
$r > r_s = 2GM$
$ds^2 = 0$
$\left(1-\frac{2GM}{r}\right)^2$

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

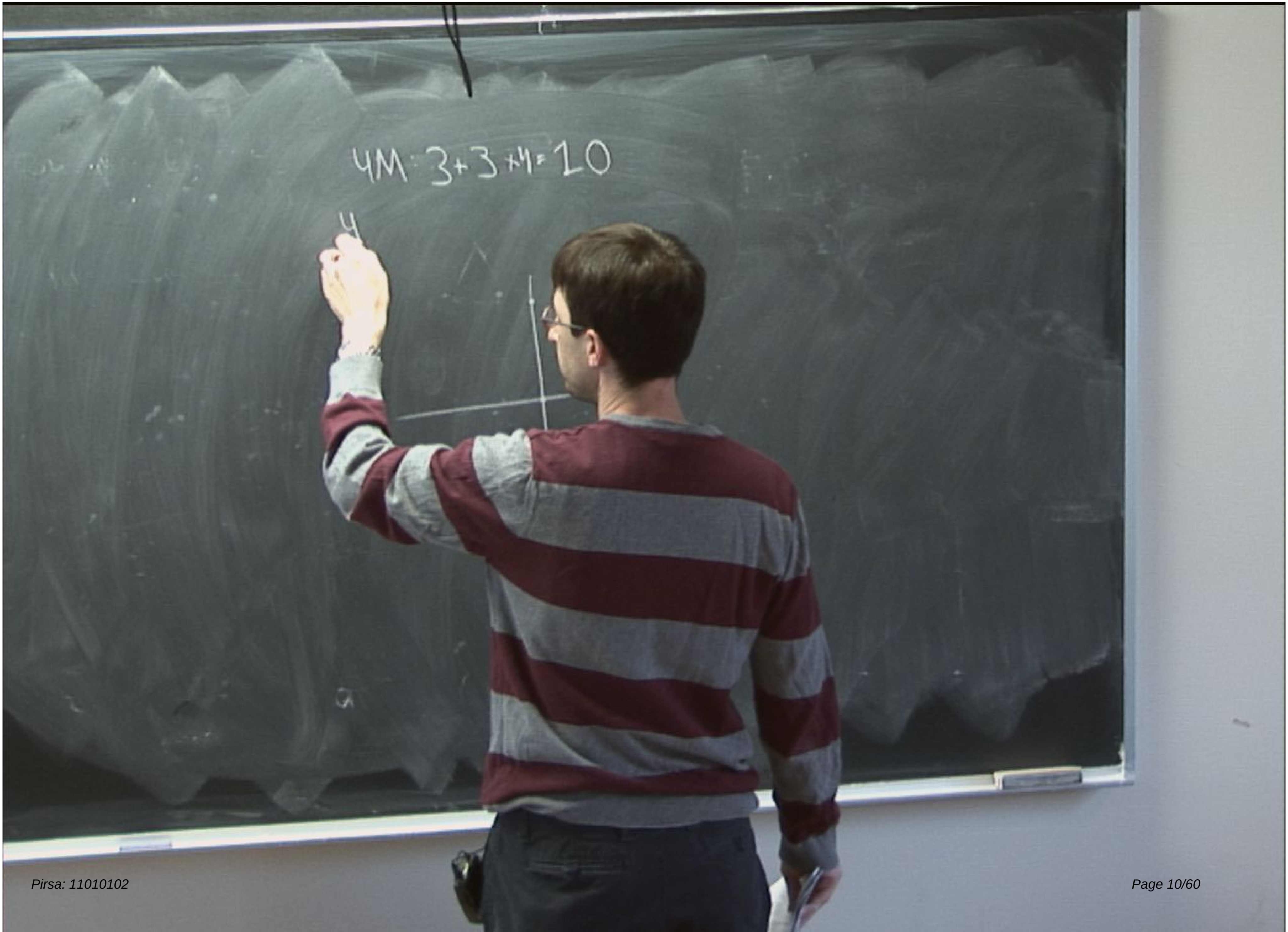
$$r > r_s = 2GM$$

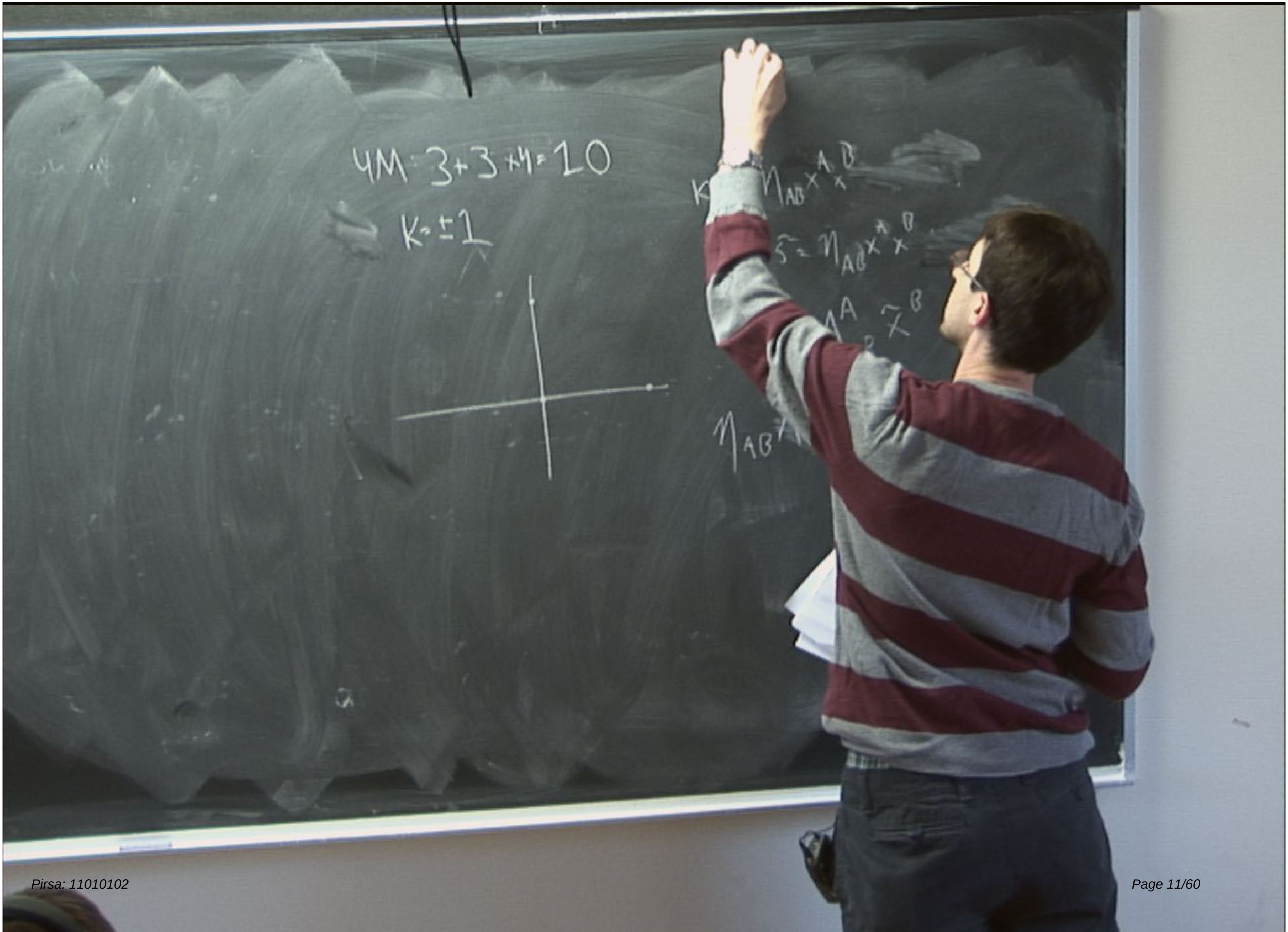
$$ds^2 = 0 \qquad \left(1-\frac{2GM}{r}\right) = \left(\frac{dr}{dt}\right)$$

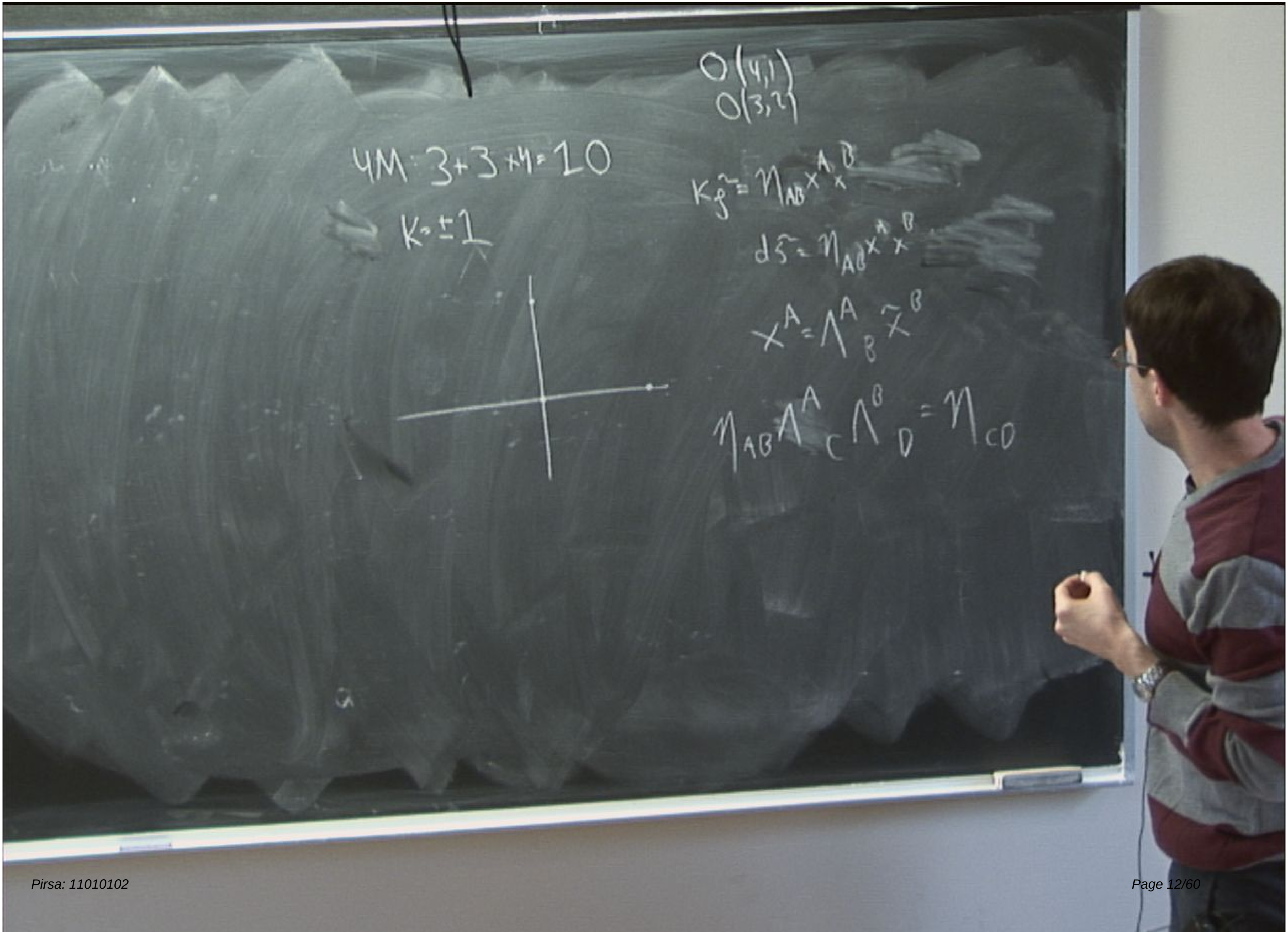
$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$r > r_s = 2GM$$

$$ds^2 = 0 \qquad \left(1-\frac{2GM}{r}\right) = \qquad \frac{dr}{dt}$$


4M = 3+3 ×4 = 10


4M: 3+3×4=10
K=±1


O(4,1)
O(3,2)
4M: 3+3×4=10
K=±1
$K\tilde{g} = \eta_{AB} x^A x^B$
$d\tilde{s} = \eta_{AB} x^A x^B$
$x^A = \Lambda^A{}_B \tilde{x}^B$
$\eta_{AB} \Lambda^A{}_C \Lambda^B{}_D = \eta_{CD}$

$O(4,1)$
$O(3,2)$

$4M \quad 3+3+4=10$

$K=\pm 1$

$K\ell^2 = \eta_{AB}x^A x^B$

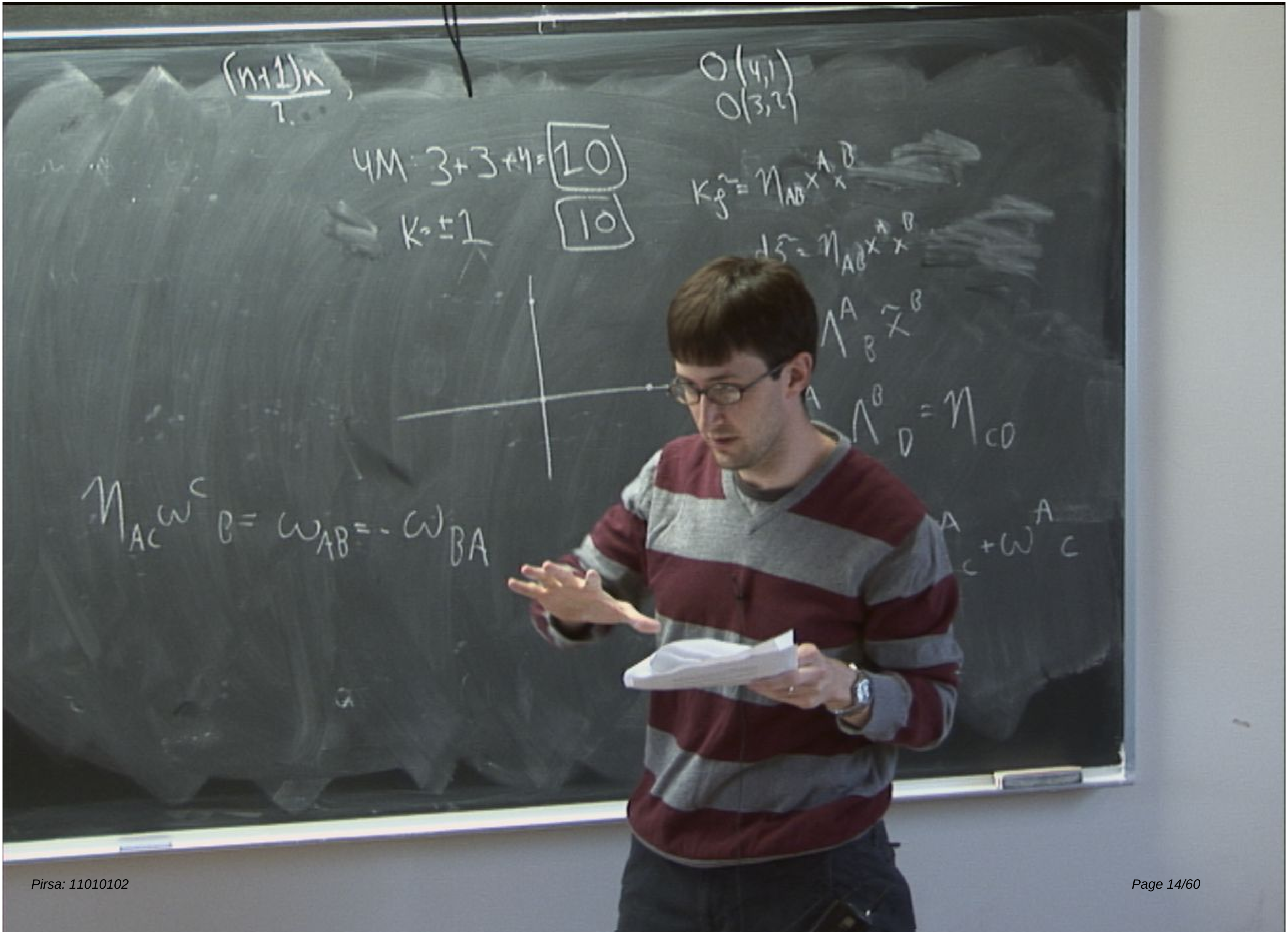
$ds^2 = \eta_{AB}x^A x^B$

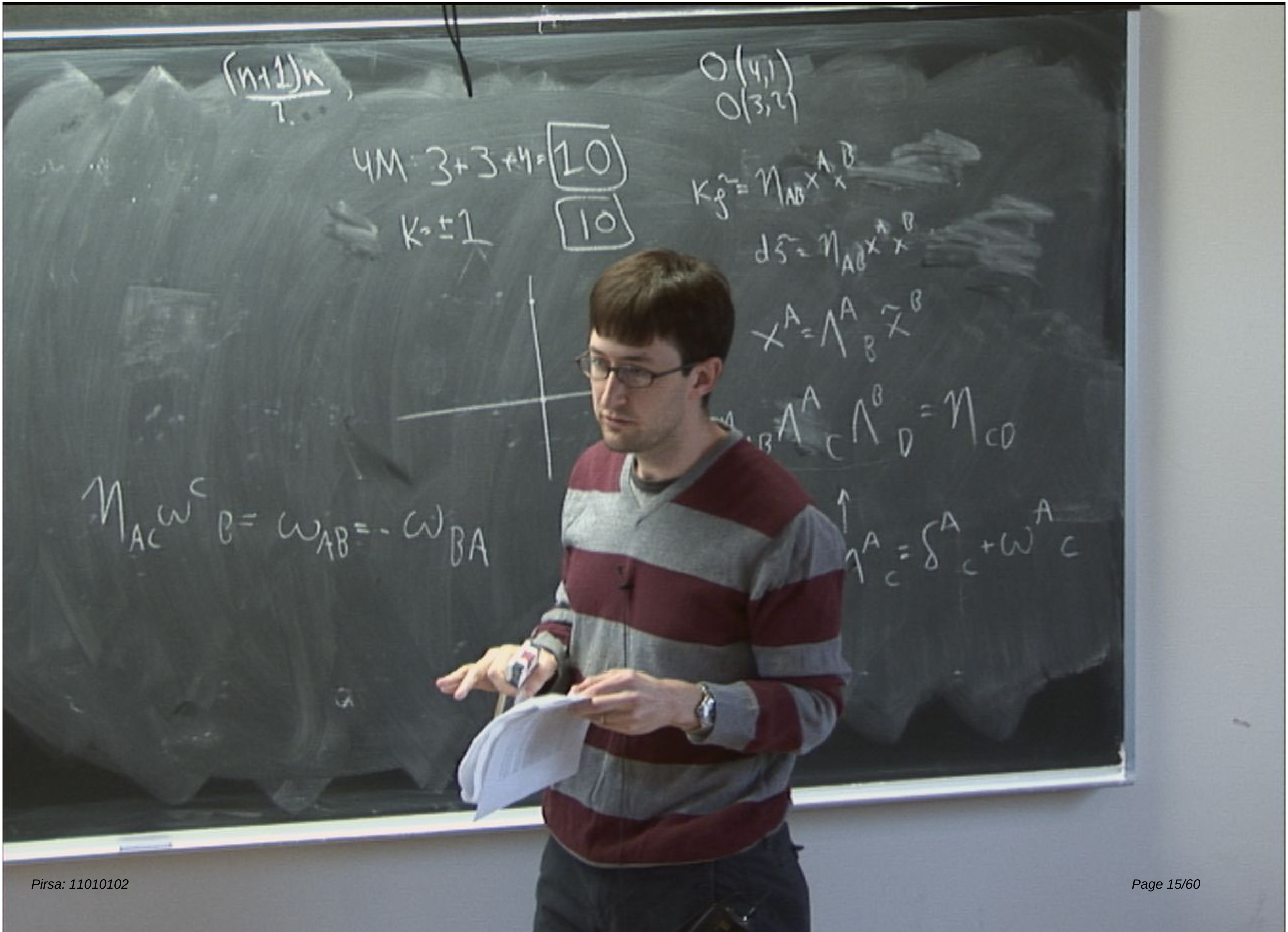
$x^A = \Lambda^A{}_B \tilde{x}^B$

$\eta_{AB}\Lambda^A{}_C\Lambda^B{}_D = \eta_{CD}$

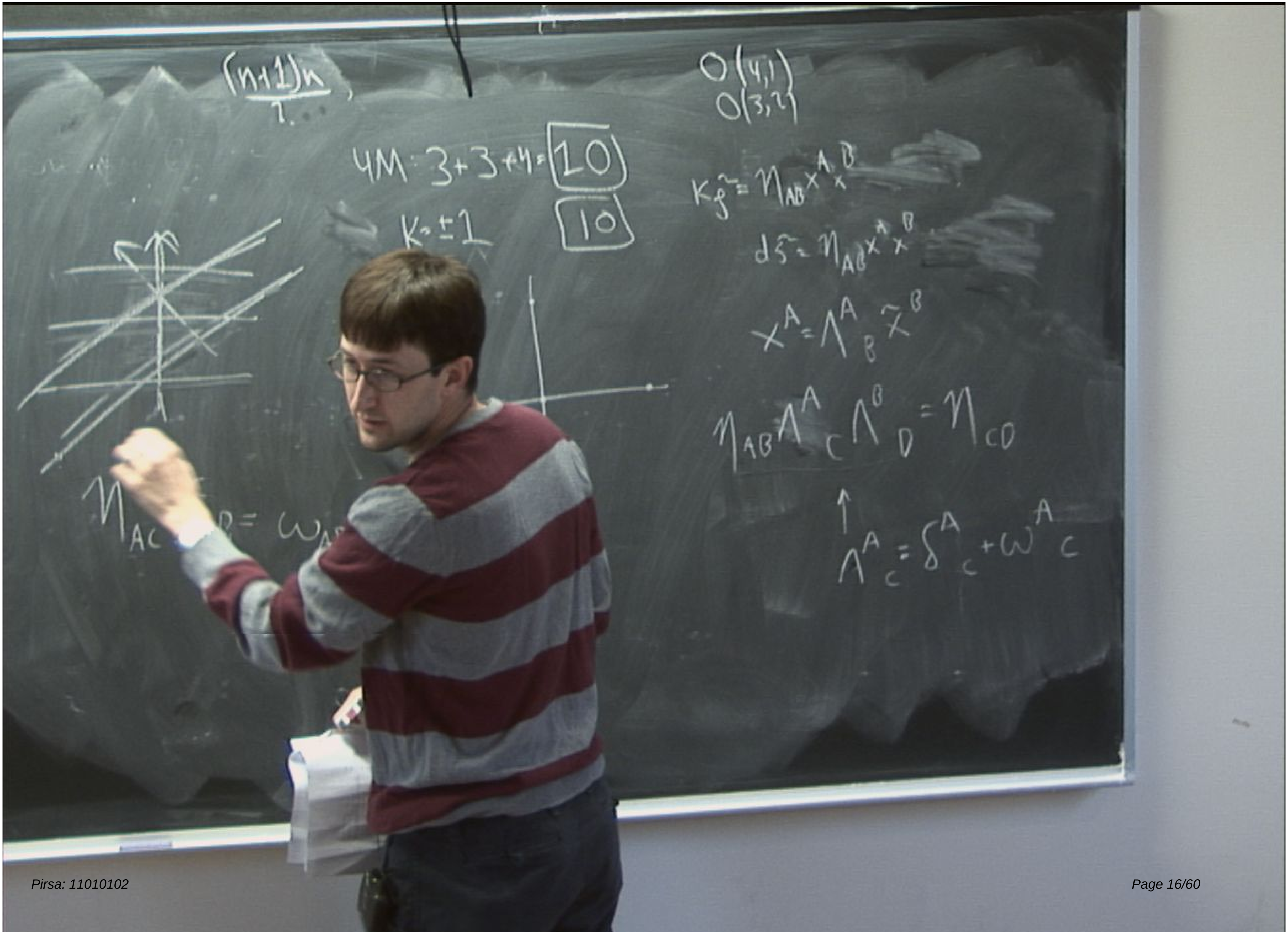
$\Lambda^A{}_C = \delta^A{}_C + \omega^A{}_C$

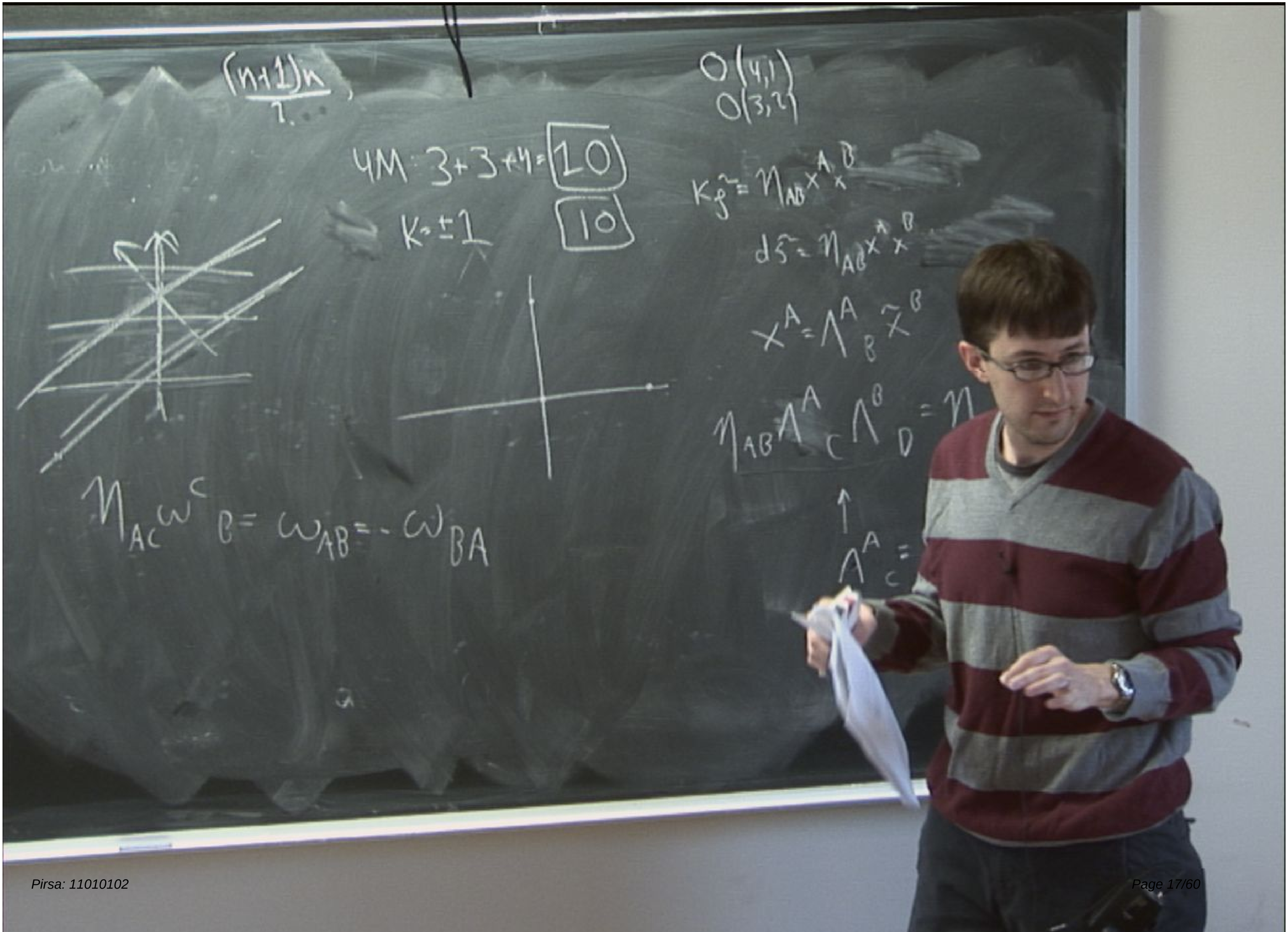
$\eta_{AC}\omega^C{}_B = \omega_{AB} = -\omega_{BA}$

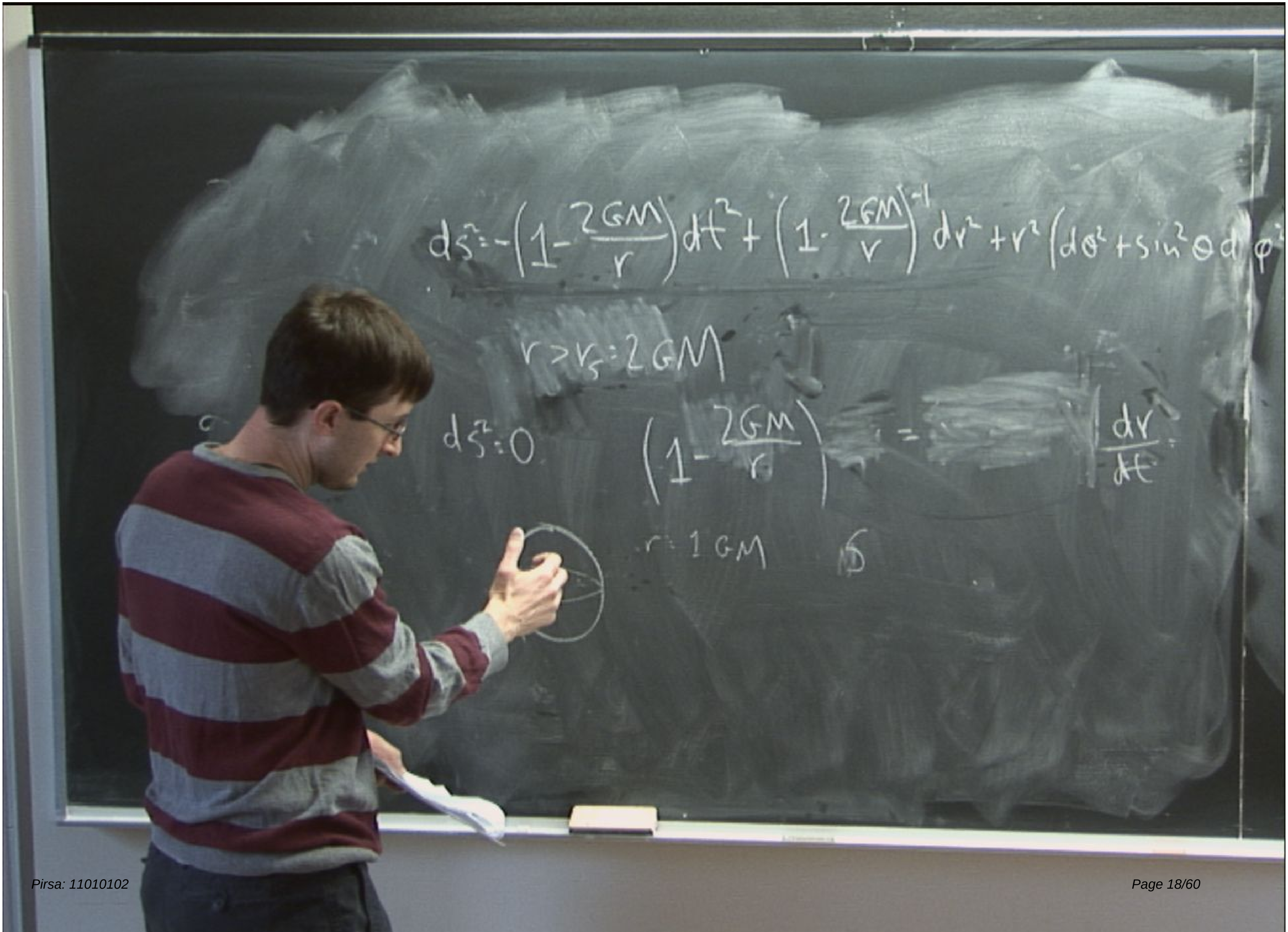

(n+1)n
2
O(4,1)
O(3,2)
4M 3+3+4=10
K=±1
10
BA

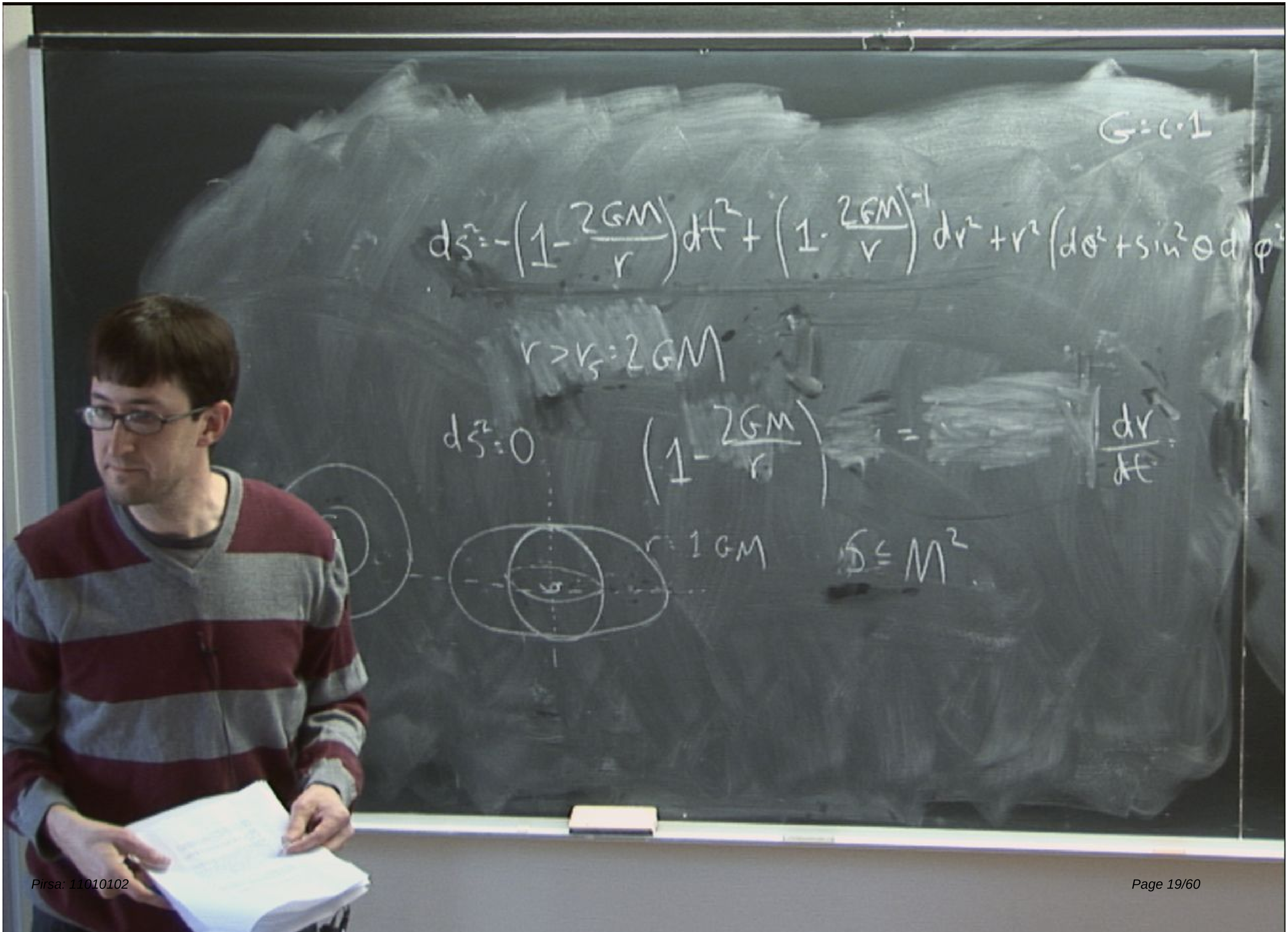


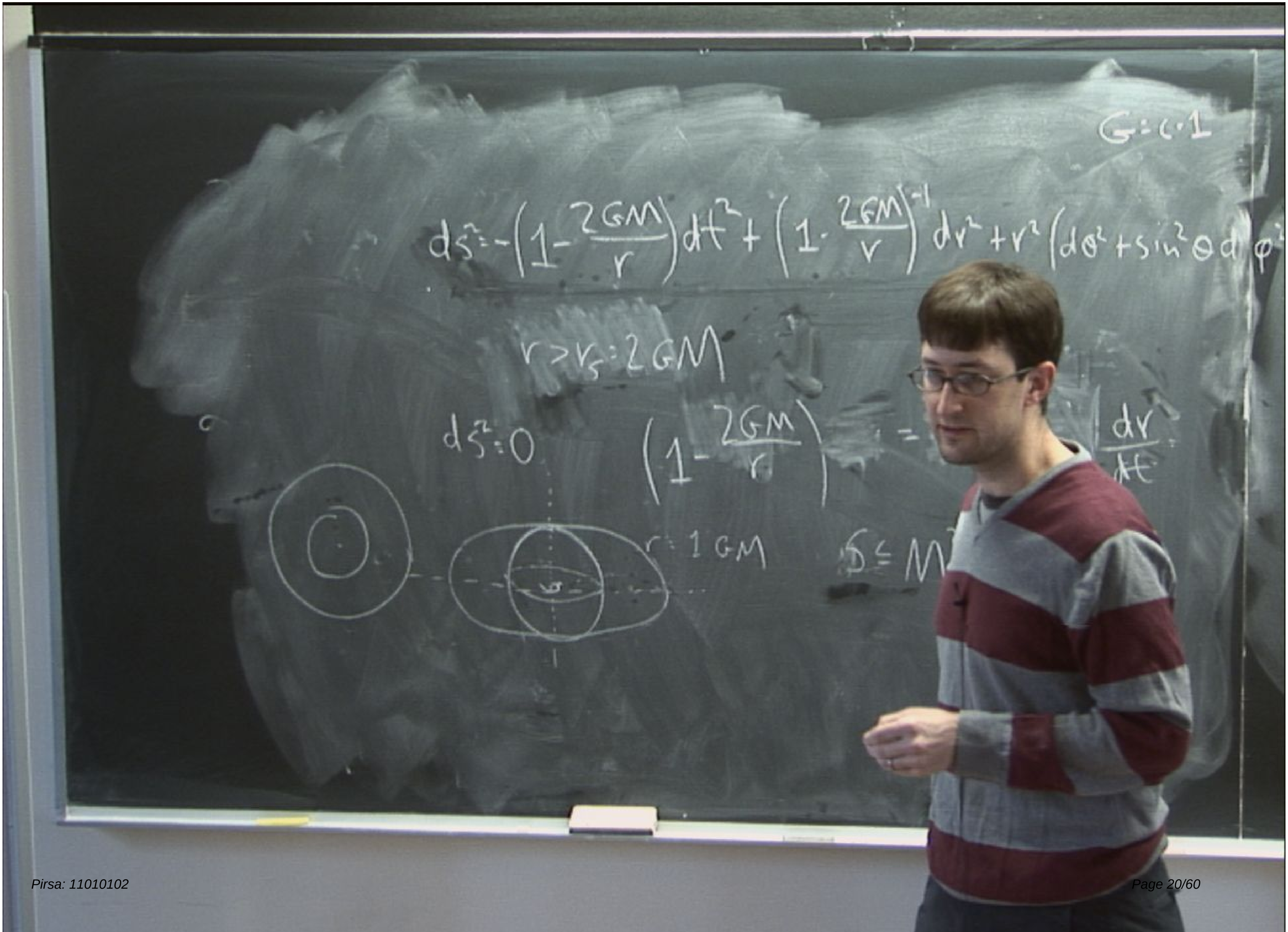
O(4,1)
O(3,2)
4M 3+3+4 = 10
K = ±1
10

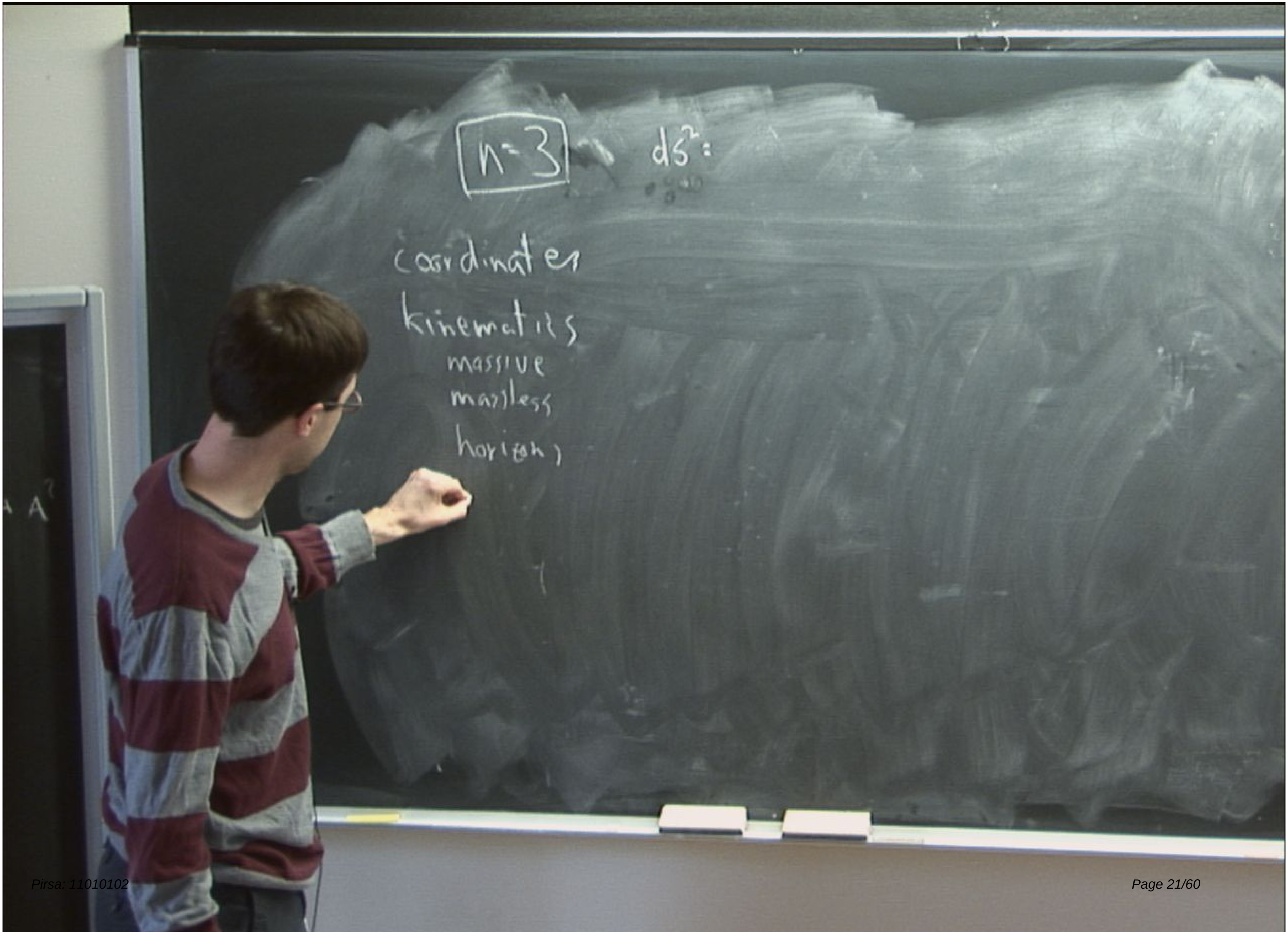

$\frac{(n+1)n}{2}$
$O(4,1)$
$O(3,2)$
4M: 3+3+4= 10
K=±1
10
$x^A = \Lambda^A{}_B \tilde{x}^B$
$\eta_{AB}\Lambda^A{}_C\Lambda^B{}_D = \eta_{CD}$
$\Lambda^A{}_C = \delta^A{}_C + \omega^A{}_C$

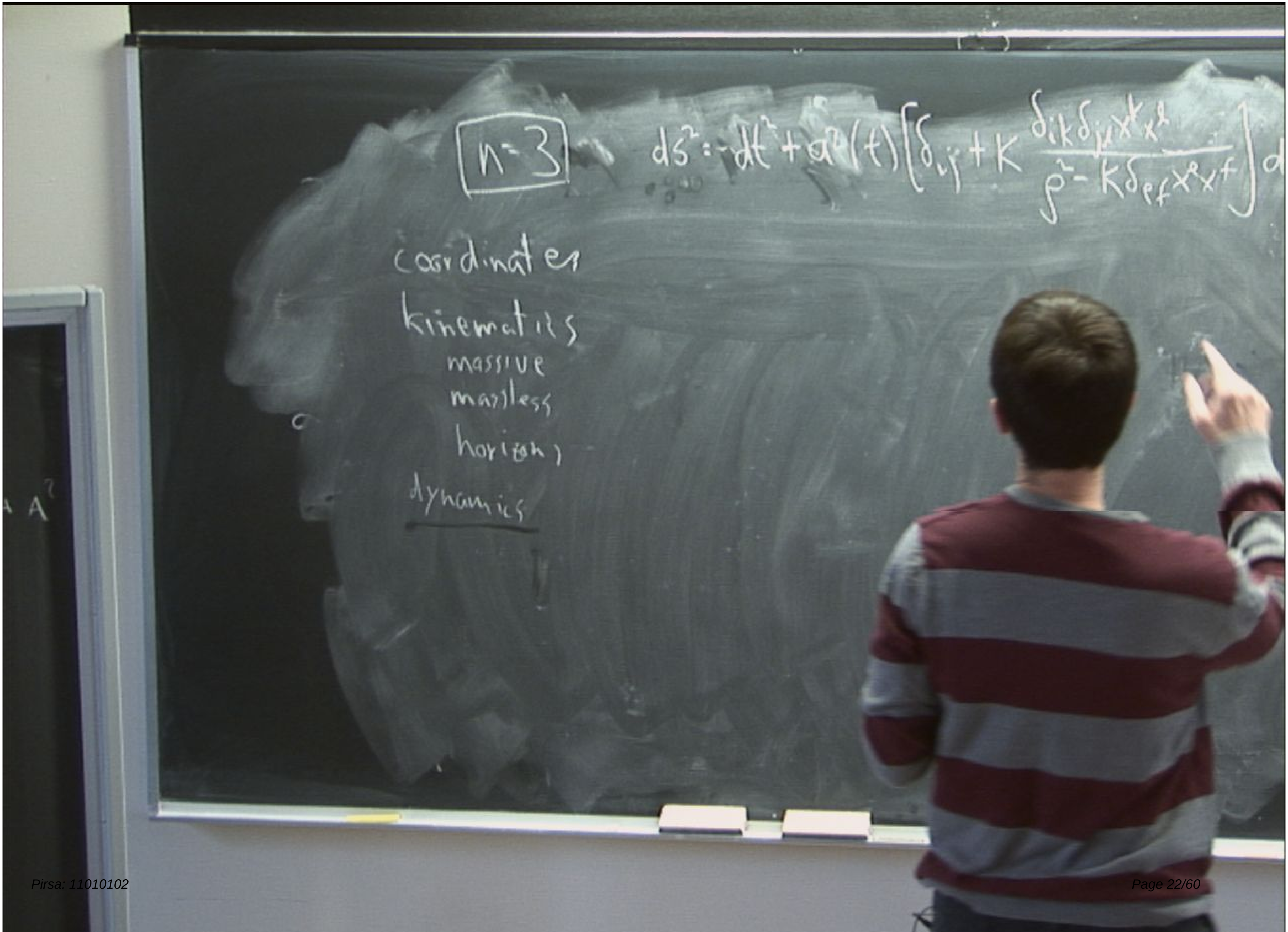

$\frac{(n+1)n}{2}$
$O(4,1)$
$O(3,2)$
4M: 3+3+4 = $\boxed{10}$
$K = \pm 1$
$\boxed{10}$
$K\tilde{g} = \eta_{AB} x^A x^B$
$d\tilde{s} = \eta_{AB} x^A x^B$
$x^A = \Lambda^A{}_B \tilde{x}^B$
$\eta_{AB} \Lambda^A{}_C \Lambda^B{}_D = \eta$
$\Lambda^A{}_C =$
$\eta_{AC} \omega^C{}_B = \omega_{AB} = -\omega_{BA}$


$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$
$r > r_s = 2GM$
$ds^2 = 0$
$\left(1-\frac{2GM}{r}\right)$
$\frac{dr}{dt}$
$r = 1GM$


G = c = 1
ds² = -(1 - 2GM/r) dt² + (1 - 2GM/r)⁻¹ dr² + r² (dθ² + sin²θ dφ²)
r > r_s = 2GM
ds² = 0
(1 - 2GM/r) = dr/dt
r = 1 GM
δ ≤ M²


G = c = 1
ds² = -(1 - 2GM/r) dt² + (1 - 2GM/r)⁻¹ dr² + r² (dθ² + sin²θ dφ²
r > r_s = 2GM
ds² = 0
(1 - 2GM/r)
dr/dt
r = 1 GM


n=3
ds²=
coordinates
kinematics
massive
massless
horizons


n=3
$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{\rho^2 - K\delta_{ef}x^e x^f}\right]d$
coordinates
kinematics
massive
massless
horizons
dynamics

$\boxed{n=3}$

$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1 - K\delta_{ef}x^e x^f}\right]dx^i dx^j$$

coordinates

$$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$$

kinematics

massive

massless

horizons

dynamics

$\boxed{n=3}$

$$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1 - K\delta_{ef}x^e x^f}\right]dx^i dx^j$$

coordinates

$$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$$

kinematics

massive

massless

horizons

dynamics

dr^2

$dr^2 + r^2 d\Omega^2$

$x^e = \rho \tilde{x}^e$ $\quad a(t) = \frac{1}{\rho}\tilde{a}(t)$

$\boxed{n=3}$ $ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1-K\delta_{ef}x^e x^f}\right]dx^i dx^j$

coordinates $\{x^1, x^2, x^3\} = r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$

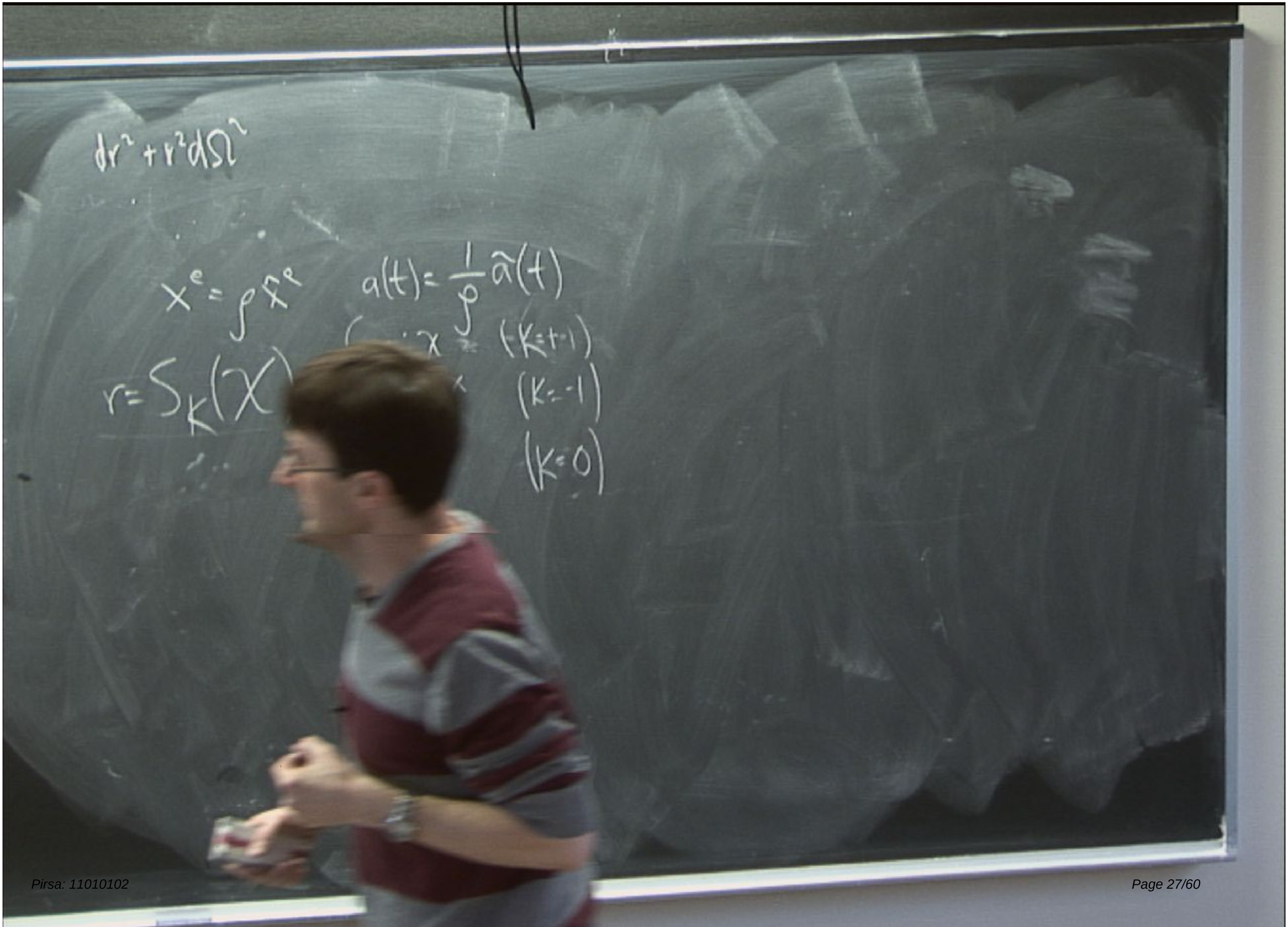
kinematics $\rightarrow -d^2t^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$

massive

massless

horizons

dynamics

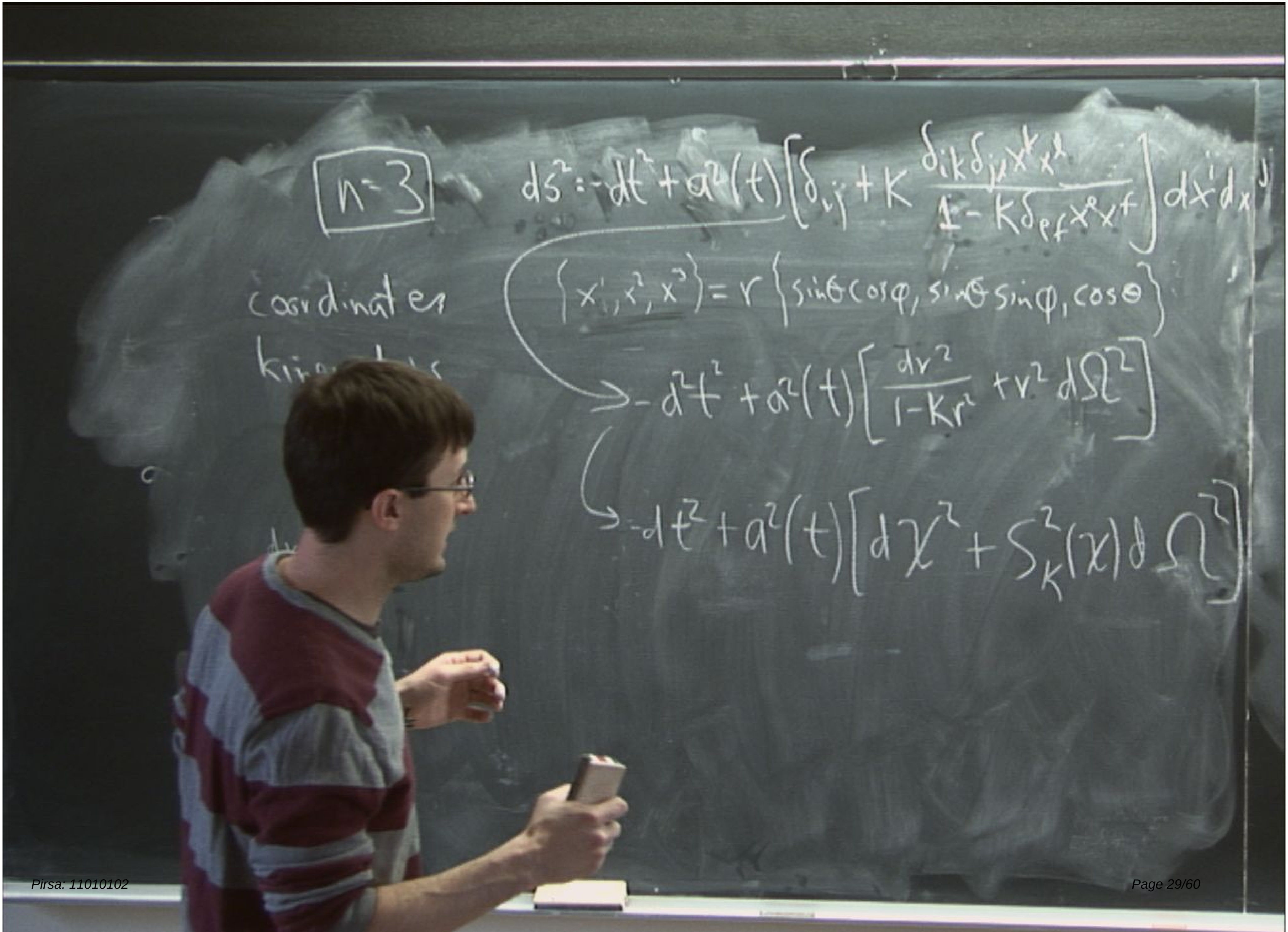


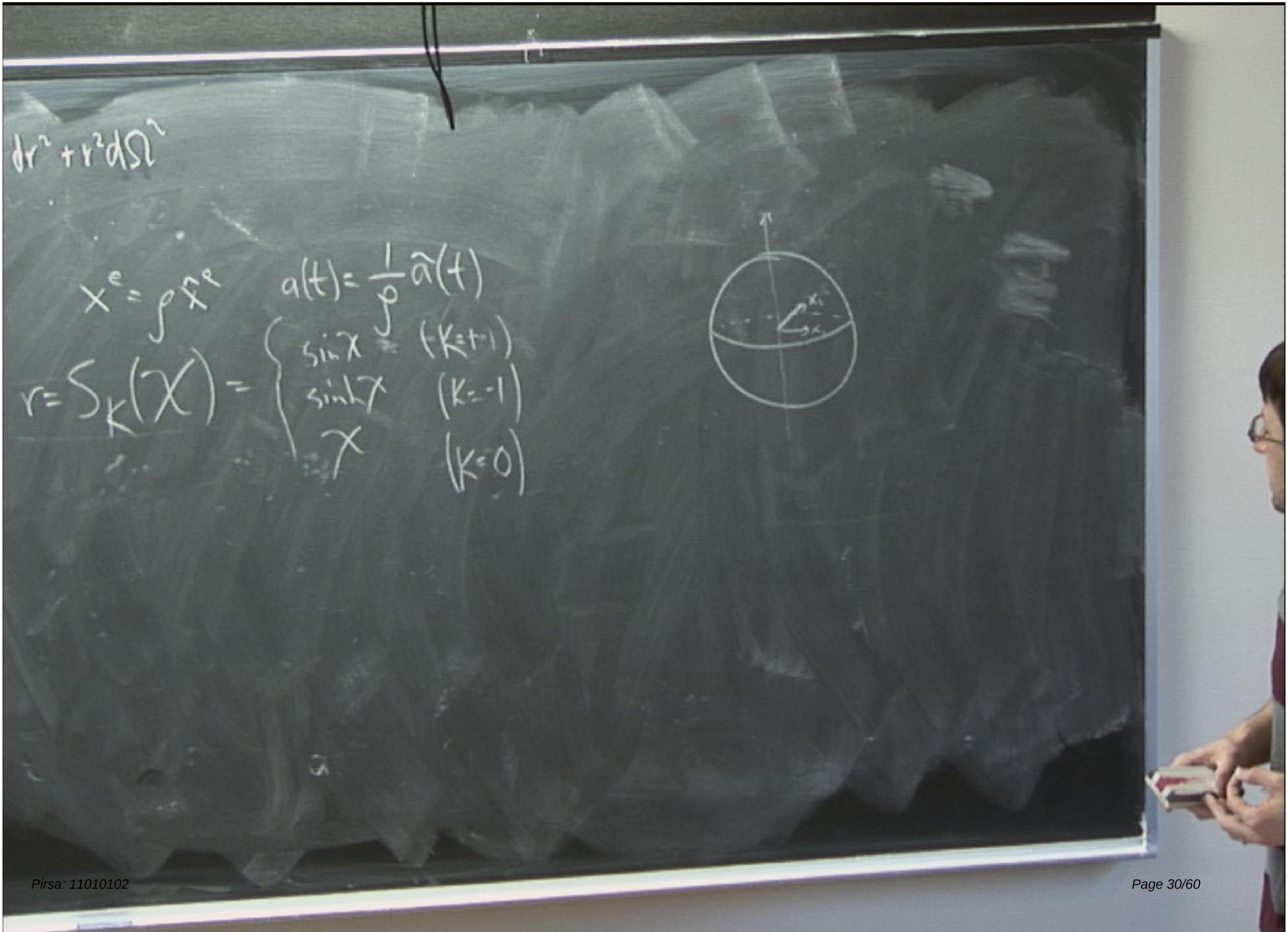
$dr^2 + r^2 d\Omega^2$
$x^e = \rho \hat{x}^e$
$a(t) = \frac{1}{\rho} \tilde{a}(t)$
$(k = +1)$
$r = S_k(\chi)$
$(k = -1)$
$(k = 0)$

$dr^2 + r^2 d\Omega^2$

$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K = +1) \\ \sinh\chi & (K = -1) \\ \chi & (K = 0) \end{cases}$$

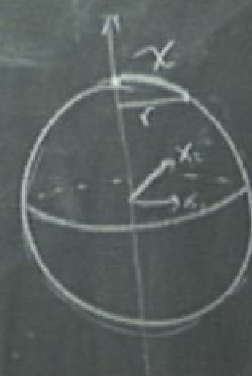

$n=3$
$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1-K\delta_{ef}x^e x^f}\right]dx^i dx^j$
coordinates
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$
$-d^2t^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$-dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$


$dr^2 + r^2 d\Omega^2$
$x^e = \rho \hat{x}^e$
$a(t) = \frac{1}{\rho} \tilde{a}(t)$
$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho\, \hat{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

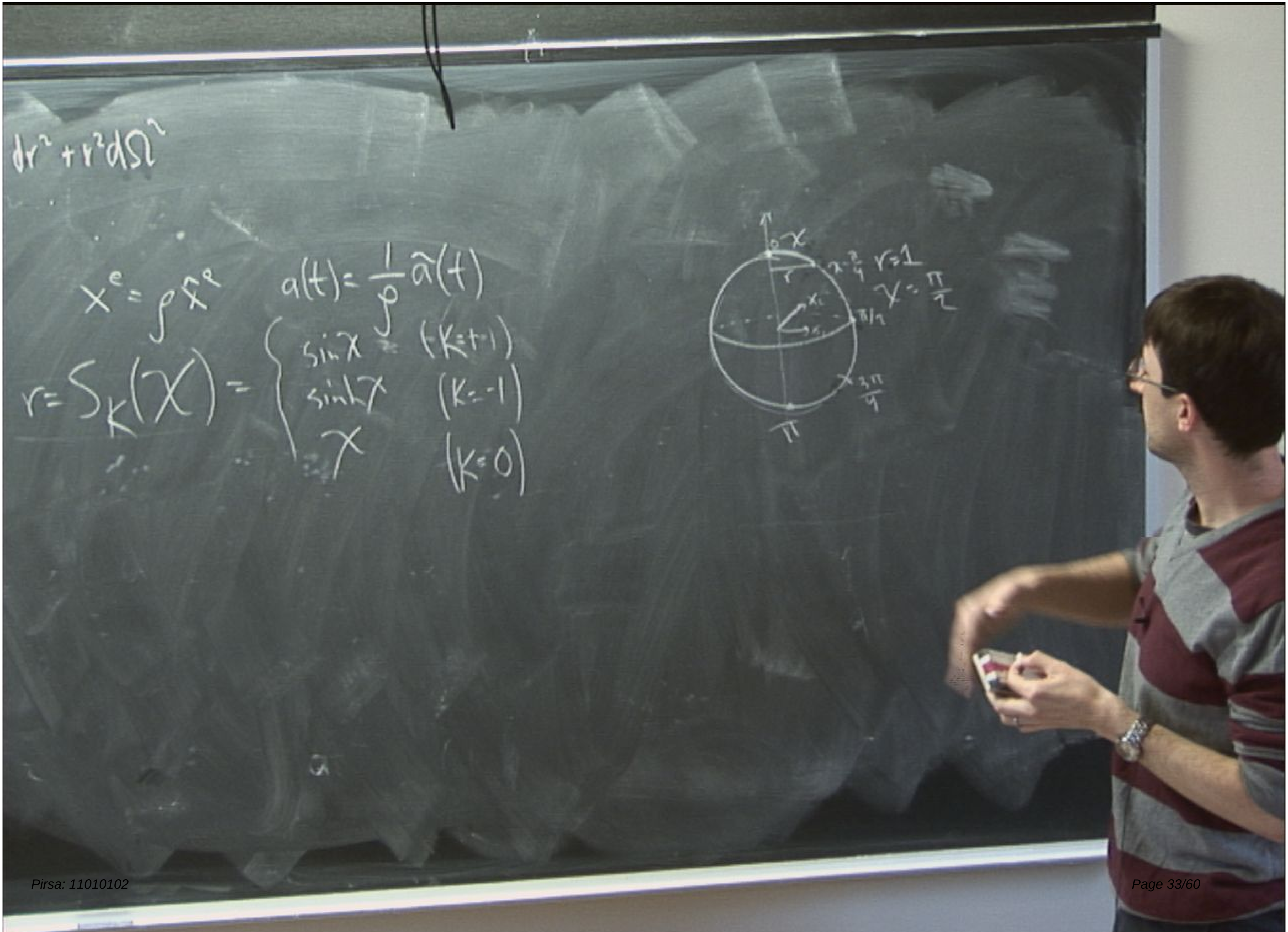


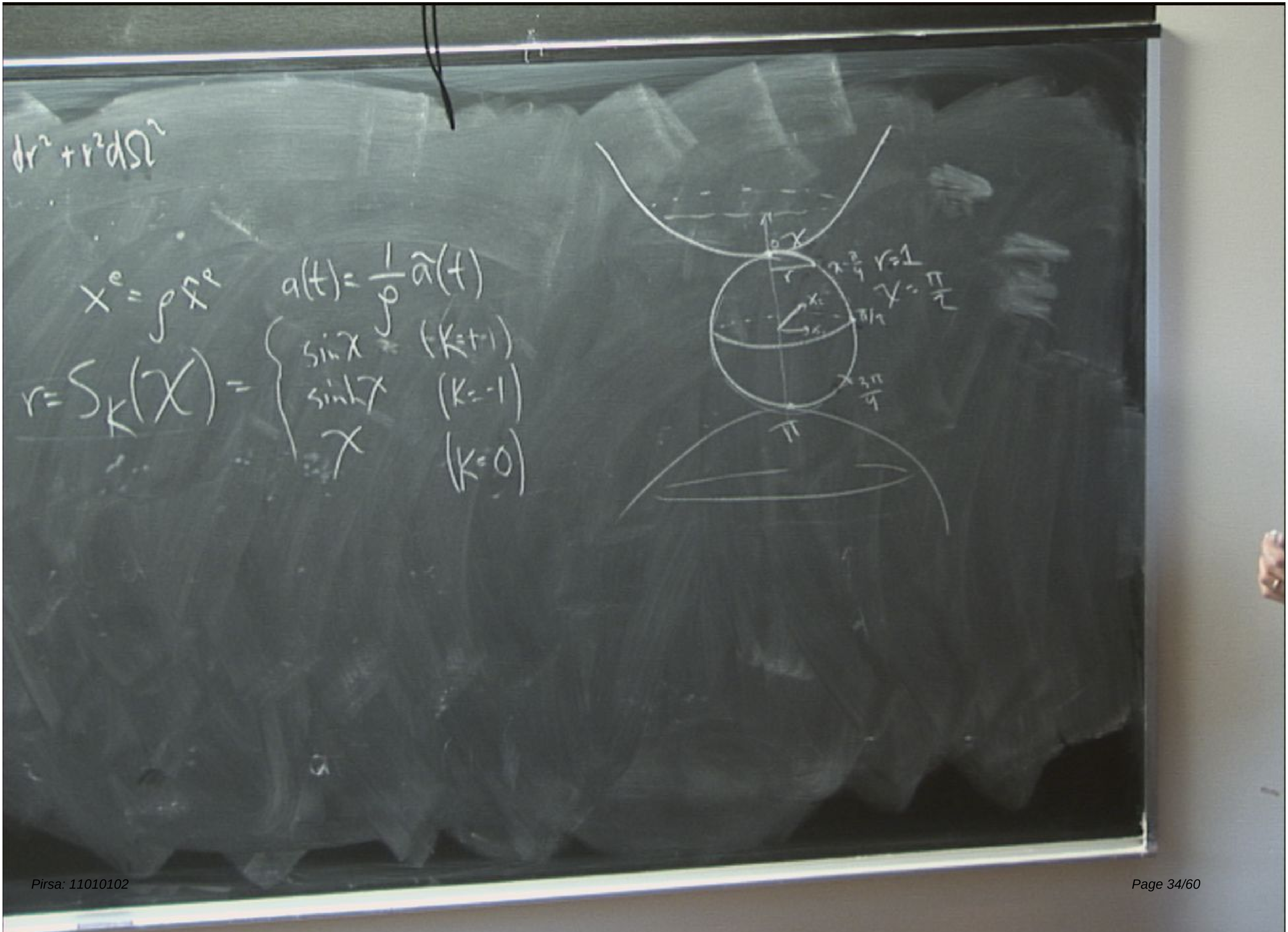
$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \hat{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$r = 1$
$\chi = \frac{\pi}{2}$


$dr^2 + r^2 d\Omega^2$
$x^e = \rho \hat{x}^e$
$a(t) = \frac{1}{\rho}\tilde{a}(t)$
$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$
$r=1$
$\chi = \frac{\pi}{2}$
$\frac{3\pi}{4}$
π

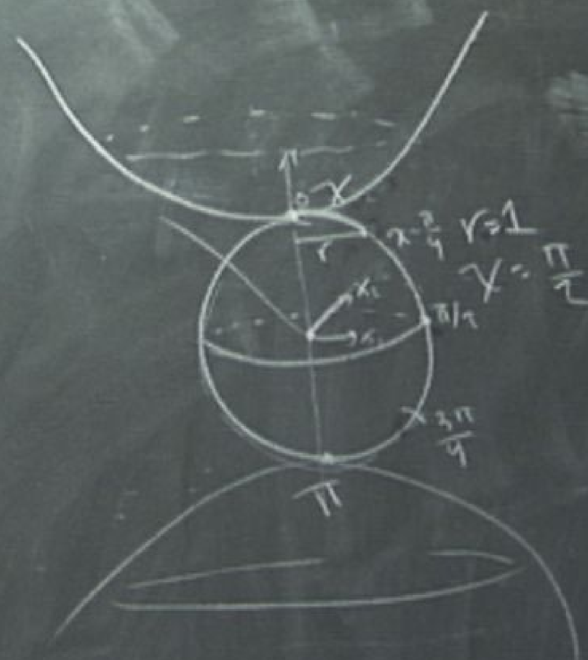


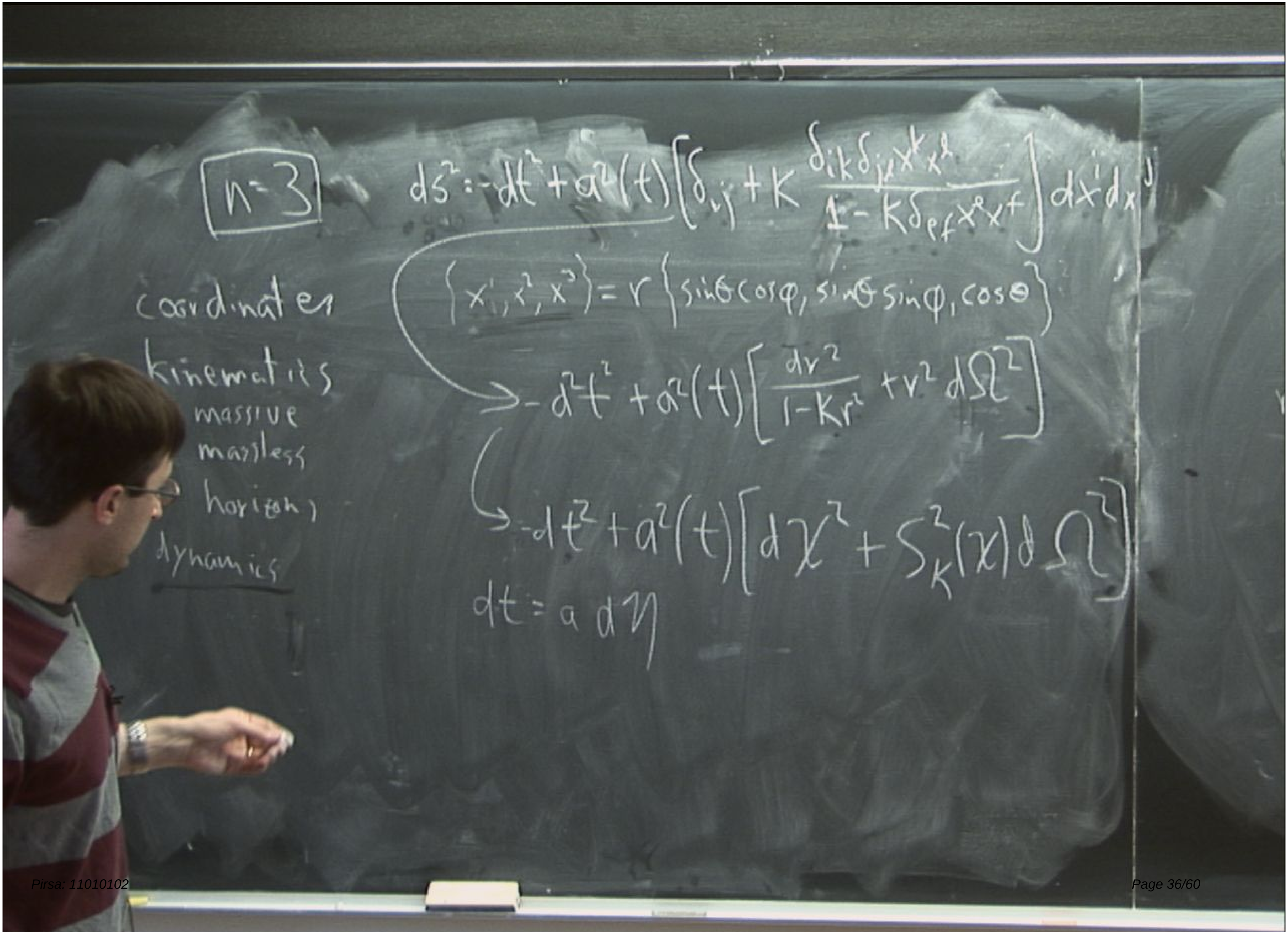
$dr^2 + r^2 d\Omega^2$
$x^e = \rho \hat{x}^e$
$a(t) = \frac{1}{\rho}\tilde{a}(t)$
$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$
$r=1$
$\chi = \frac{\pi}{2}$
$\frac{3\pi}{4}$
π

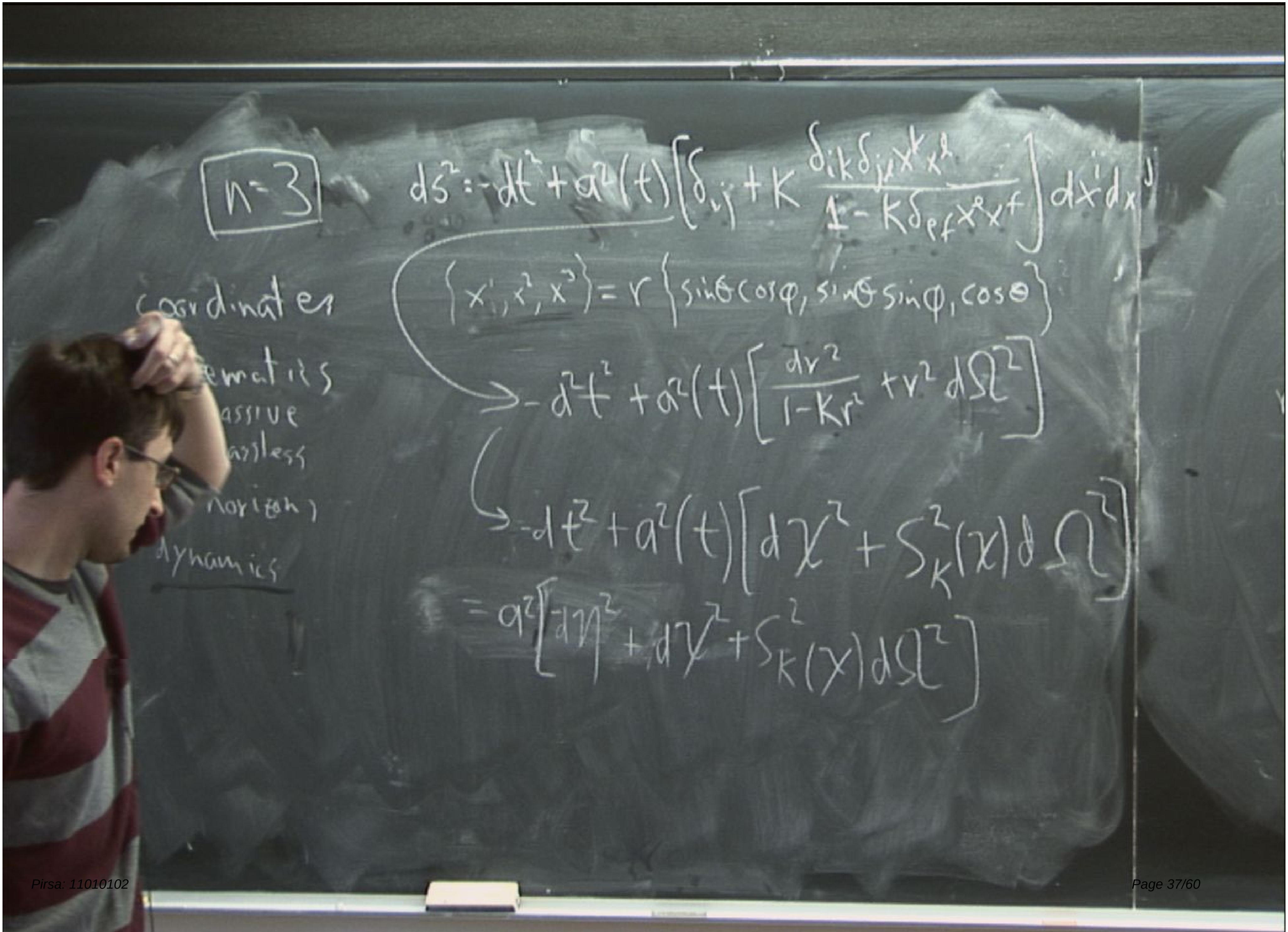
$dr^2 + r^2 d\Omega^2$

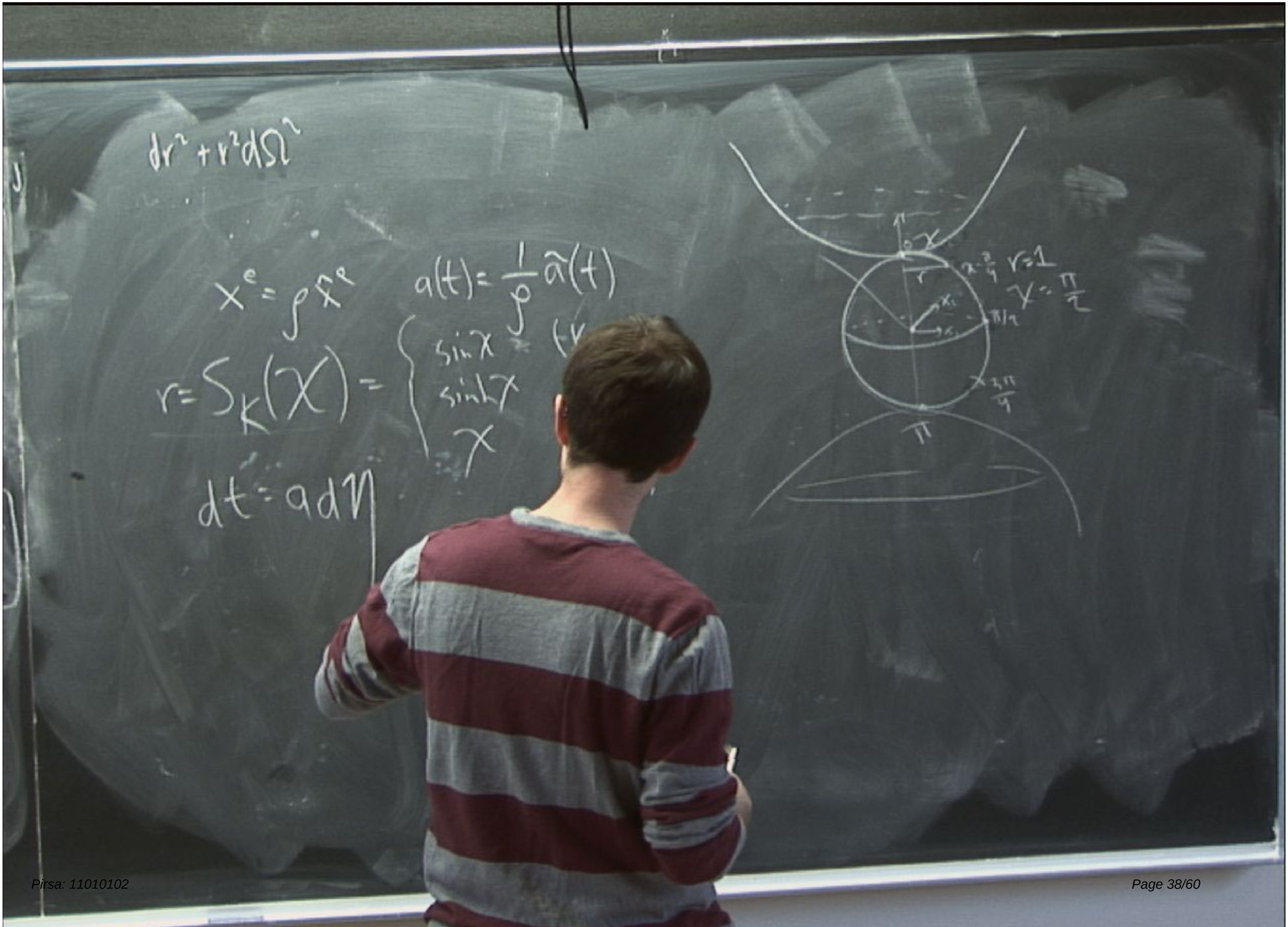
$x^e = \rho \hat{x}^e$ $\qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$




$\boxed{n=3}$
$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1-K\delta_{ef}x^e x^f}\right]dx^i dx^j$
coordinates
kinematics
massive
massless
horizons
dynamics
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$
$-dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$-dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$
$dt = a\, d\eta$


$\boxed{n=3}$
$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{\delta_{ik}\delta_{jl}x^k x^l}{1-K\delta_{ef}x^e x^f}\right]dx^i dx^j$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$
$-d^2t^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$-dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$
$= a^2\left[-d\eta^2 + d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$
coordinates
ematics
assive
assless
horizons
dynamics



dr² + r²dΩ²
x^e = ρ x̃^e
a(t) = 1/ρ ã(t)
r = S_K(χ) =
sin χ
sinh χ
χ
dt = a dη
r=1
χ = π/2
3π/4
π

$$dr^2 + r^2 d\Omega^2$$

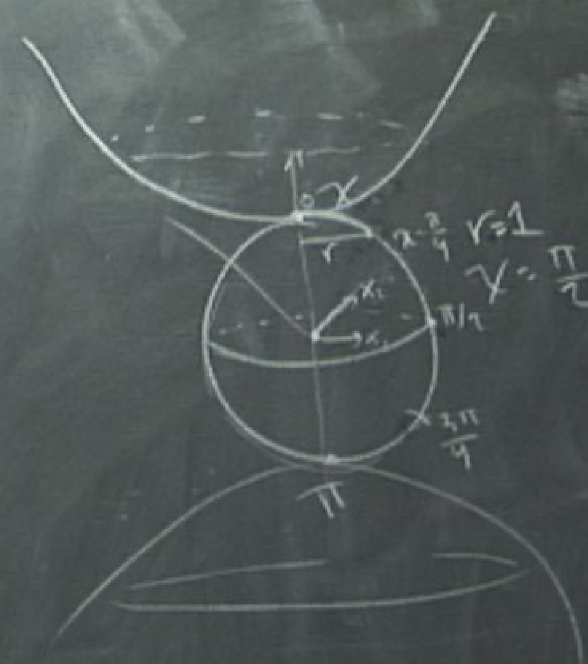
$$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

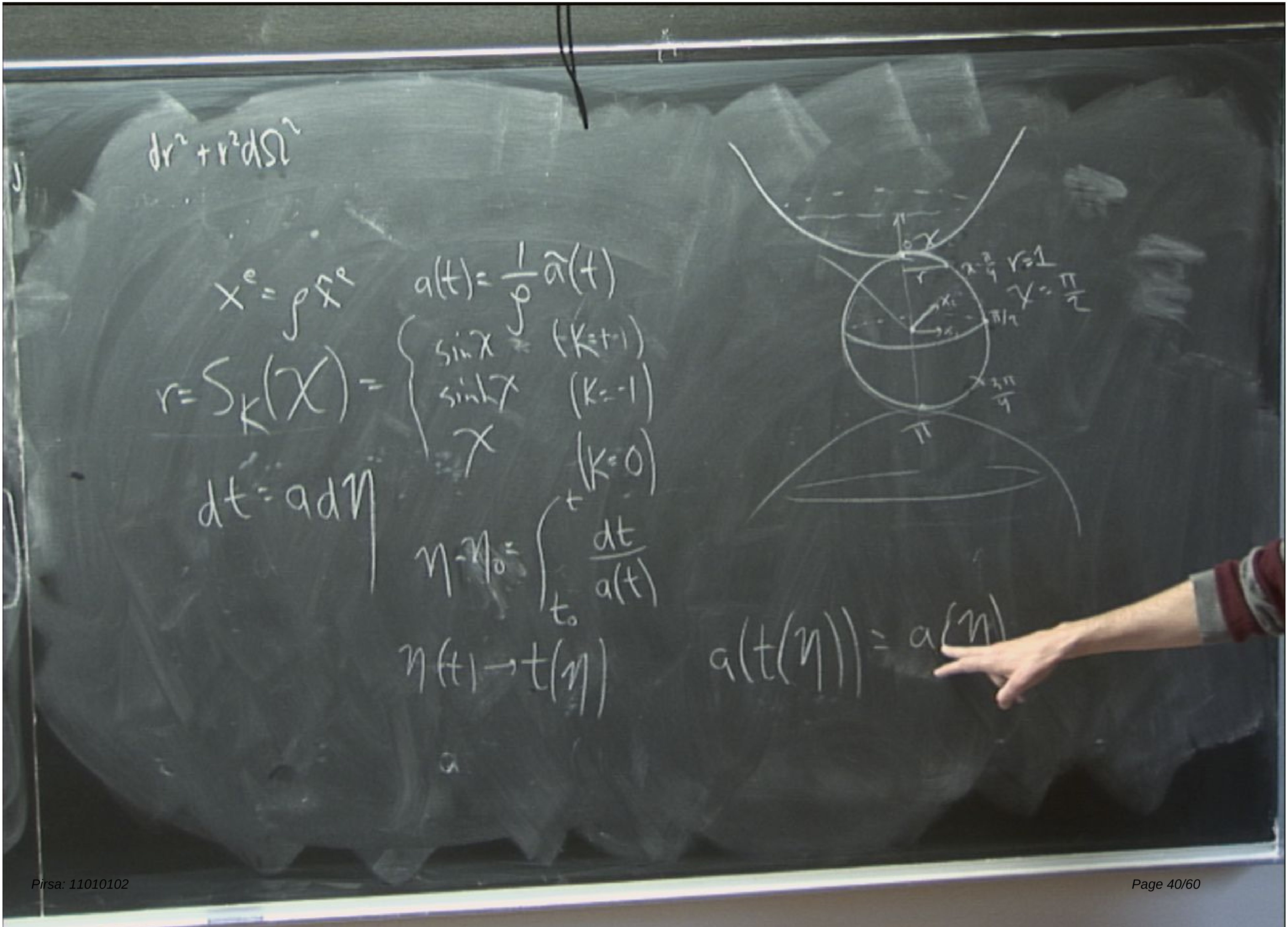
$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$$dt = a\,d\eta$$

$$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$$

$$\eta(t) \to t(\eta)$$




$dr^2 + r^2 d\Omega^2$
$x^e = \rho \tilde{x}^e$
$a(t) = \frac{1}{\rho} \tilde{a}(t)$
$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$
$dt = a\, d\eta$
$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$
$\eta(t) \to t(\eta)$
$a(t(\eta)) = a(\eta)$
χ
r
$\chi = \frac{\pi}{4}$ $r = 1$
$\chi = \frac{\pi}{2}$
$\pi/4$
$\frac{3\pi}{4}$
π

$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u = \frac{dx^\alpha}{\;}$$

$$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$$dt = a\, d\eta$$

$$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$$

$$\eta(t) \to t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$$

$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\tau)$

$dr^2 + r^2 d\Omega^2$

$u^\mu \nabla_\mu u^\alpha = 0$

$x^e = \rho \tilde{x}^e \quad a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$dt = a\,d\eta$

$$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$$

$\eta(t) \rightarrow t(\eta)$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$$

$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\tau)$$

$$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K = +1) \\ \sinh\chi & (K = -1) \\ \chi & (K = 0) \end{cases}$$

$$dt = a\, d\eta$$

$$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$$

$$\eta(t) \to t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$$

$$dr^2 + r^2 d\Omega^2$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$$

$$p^\mu = \frac{dx^\mu}{d\lambda}$$

$$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K = +1) \\ \sinh\chi & (K = -1) \\ \chi & (K = 0) \end{cases}$$

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$$\eta(t) \to t(\eta)$$

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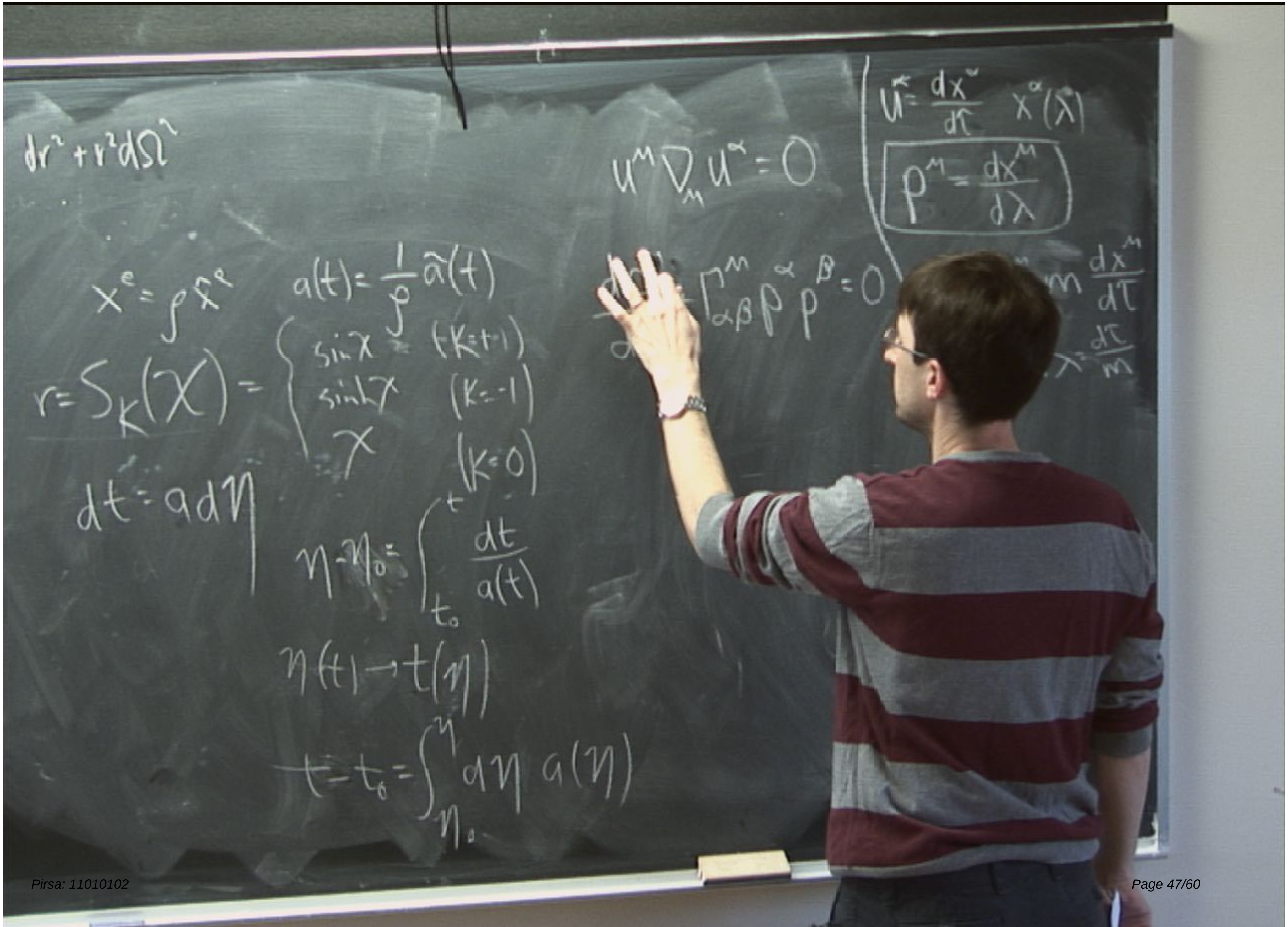
$dr^2 + r^2 d\Omega^2$
$x^e = \rho \hat{x}^e$
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$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$
$dt = a\,d\eta$
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$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$
$u^\mu \nabla_\mu u^\alpha = 0$
$u^\alpha = \frac{dx^\alpha}{d\tau}$ $x^\alpha(\lambda)$
$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$
$\Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$
$m \frac{dx^\mu}{d\tau}$
$\lambda = \frac{d\tau}{m}$

$dr^2 + r^2 d\Omega^2$

$x^e = \rho\, \tilde{x}^e$

$a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

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$$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$$

$u^\mu \nabla_\mu u^\alpha = 0$

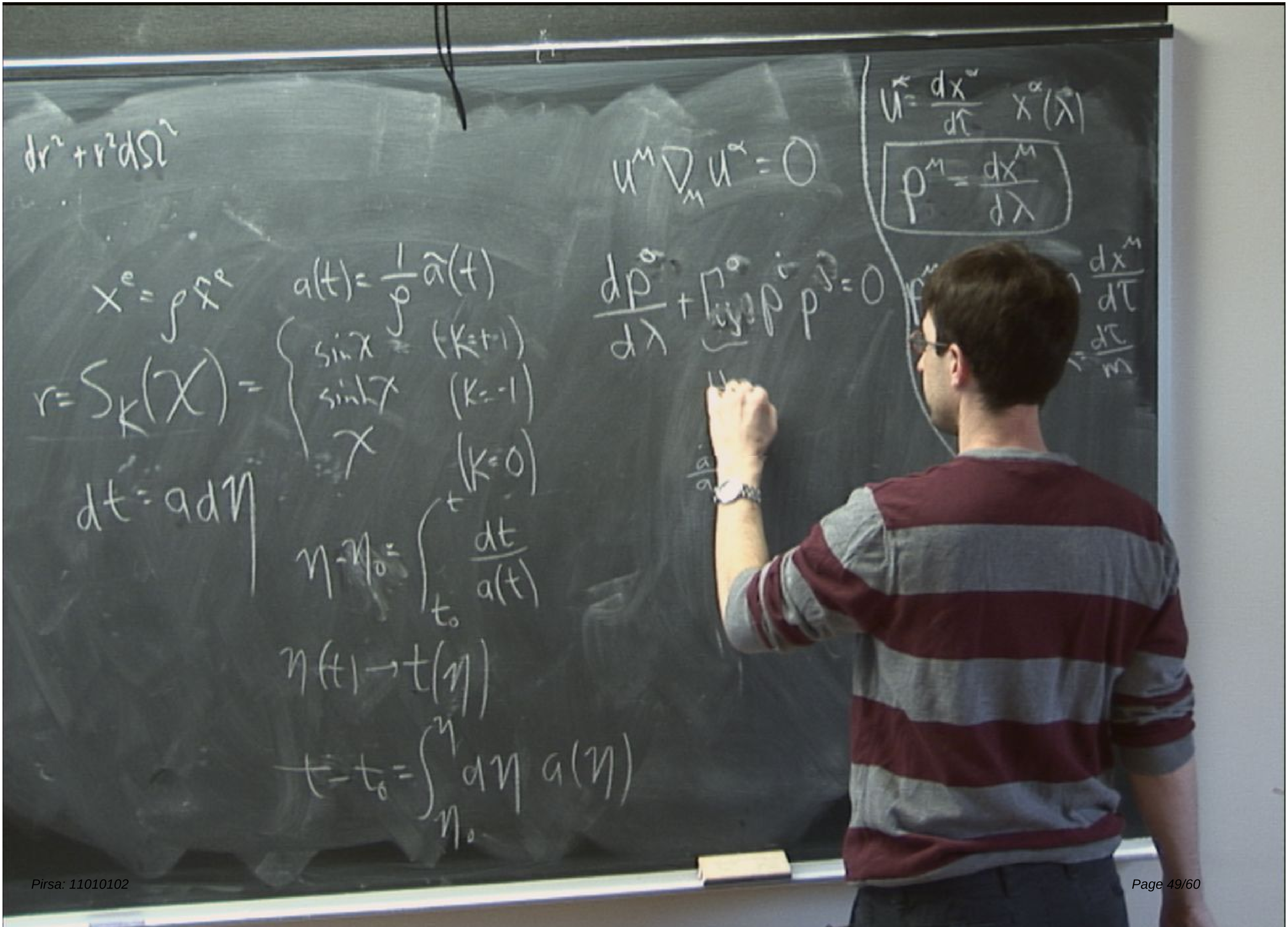
$$\frac{dp^\sigma}{d\lambda} + \Gamma^\sigma_{\alpha\beta} p^\alpha p^\beta = 0$$

$u^\alpha = \frac{dx^\alpha}{d\tau}$ $\quad x^\alpha(\lambda)$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$

$d\lambda = \frac{d\tau}{m}$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$\eta(t) \rightarrow t(\eta)$
$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$
$u^\mu \nabla_\mu u^\alpha = 0$
$\frac{dp^0}{d\lambda} + \Gamma^0_{ij} p^i p^j = 0$
$u^\alpha = \frac{dx^\alpha}{d\tau}$ $x^\alpha(\lambda)$
$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$
$\frac{dx^\mu}{d\tau}$
$\frac{d\tau}{m}$

$$dr^2 + r^2 d\Omega^2$$

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$$u^\mu \nabla_\mu u^\alpha = 0$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \qquad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{H g_{ij}} p^i p^j = 0 \qquad p^\mu = m u^\mu = m\frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

$$\frac{\dot{a}}{a}$$

$$\frac{dp^0}{d\lambda} + H g_{ij} p^i p^j$$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

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$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{Hg_{ij}} p^i p^j = 0$$

$$\frac{\dot a}{a}$$

$$\frac{dp^0}{d\lambda} + H\, \boxed{g_{ij} p^i p^j}$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \qquad x^\alpha(\lambda)$$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

t

$dr^2 + r^2 d\Omega^2$

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$u^\mu \nabla_\mu u^\alpha = 0$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{Hg_{ij}} p^i p^j = 0$$

$\frac{\dot{a}}{a}$

p^2

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$m^2 =$

$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$

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t

$dr^2 + r^2 d\Omega^2$

$x^e = \rho \hat{x}^e$

$a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$r = S_K(\chi) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

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$u^\mu \nabla_\mu u^\alpha = 0$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{H g_{ij}} p^i p^j = 0$$

$\frac{\dot a}{a}$

p^2

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$m^2 = -g_{\mu\nu} p^\mu p^\nu$

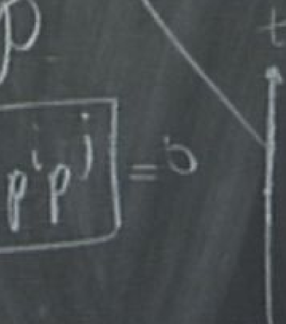
$= (p^0)^2 - \underbrace{g_{ij} p^i p^j}_{p^2}$

$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$

$$\boxed{p^\mu = \frac{dx^\mu}{d\lambda}}$$

$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$

$d\lambda = \frac{d\tau}{m}$



$E^2 = m^2 + \vec{p}^{\,2}$

$$dr^2 + r^2 d\Omega^2$$

$$x^e = \rho\, \tilde{x}^e \qquad a(t) = \frac{1}{\rho}\tilde{a}(t)$$

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$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{H g_{ij}} p^i p^j = 0$$

$$H = \frac{\dot{a}}{a}$$

$$p^2$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = -g_{\mu\nu} p^\mu p^\nu$$

$$= (p^0)^2 - \underbrace{g_{ij} p^i p^j}_{p^2}$$

$$E^2 = m^2 + \vec{p}^{\,2}$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} \qquad x^\alpha(\lambda)$$

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$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{Hg_{ij}} p^i p^j = 0$$

$$p^\mu = m u^\mu = m \frac{dx^\mu}{d\tau}$$

$$\frac{\dot{a}}{a} \qquad p^2$$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij} p^i p^j} = 0$$

$$m^2 = -g_{\mu\nu} p^\mu p^\nu$$

$$= (p^0)^2 -$$

$$p^2 = m^2$$

$H\,p^2$

$x^e = \rho\, \hat{x}^e$

$a(t) = \frac{1}{\rho}\tilde{a}(t)$

$$(\ldots) = \begin{cases} \sin\chi & (K=+1) \\ \sinh\chi & (K=-1) \\ \chi & (K=0) \end{cases}$$

$a\,d\eta$

$$\eta - \eta_0 = \int_{t_0}^{t} \frac{dt}{a(t)}$$

$$\eta(t) \to t(\eta)$$

$$t - t_0 = \int_{\eta_0}^{\eta} d\eta\, a(\eta)$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{Hg_{ij}} p^i p^j = 0$$

p^2

$\frac{\dot{a}}{a}$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij}p^ip^j} = 0$$

$$m^2 = -g_{\mu\nu}p^\mu p^\nu = (p^0)^2 - \underbrace{g_{ij}p^ip^j}_{p^2}$$

$u^\alpha = \frac{dx^\alpha}{d\tau} \quad x^\alpha(\lambda)$

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$$p^\mu = m u^\mu = m\frac{dx^\mu}{d\tau}$$

$$d\lambda = \frac{d\tau}{m}$$

t

$$E^2 = m^2 + \vec{p}^{\,2}$$

$\frac{dP}{\ } = - \frac{dq}{q}$

$x^e = \rho\, \tilde{x}^e$

$a(t) = \frac{1}{\rho}\,\tilde{a}(t)$

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$$\frac{dp^0}{d\lambda} + \underbrace{\Gamma^0_{ij}}_{Hg_{ij}}\, p^i p^j = 0$$

$\frac{\dot a}{a}$ $\quad p^2$

$$\frac{dp^0}{d\lambda} + H \boxed{g_{ij}\,p^i p^j} = 0$$

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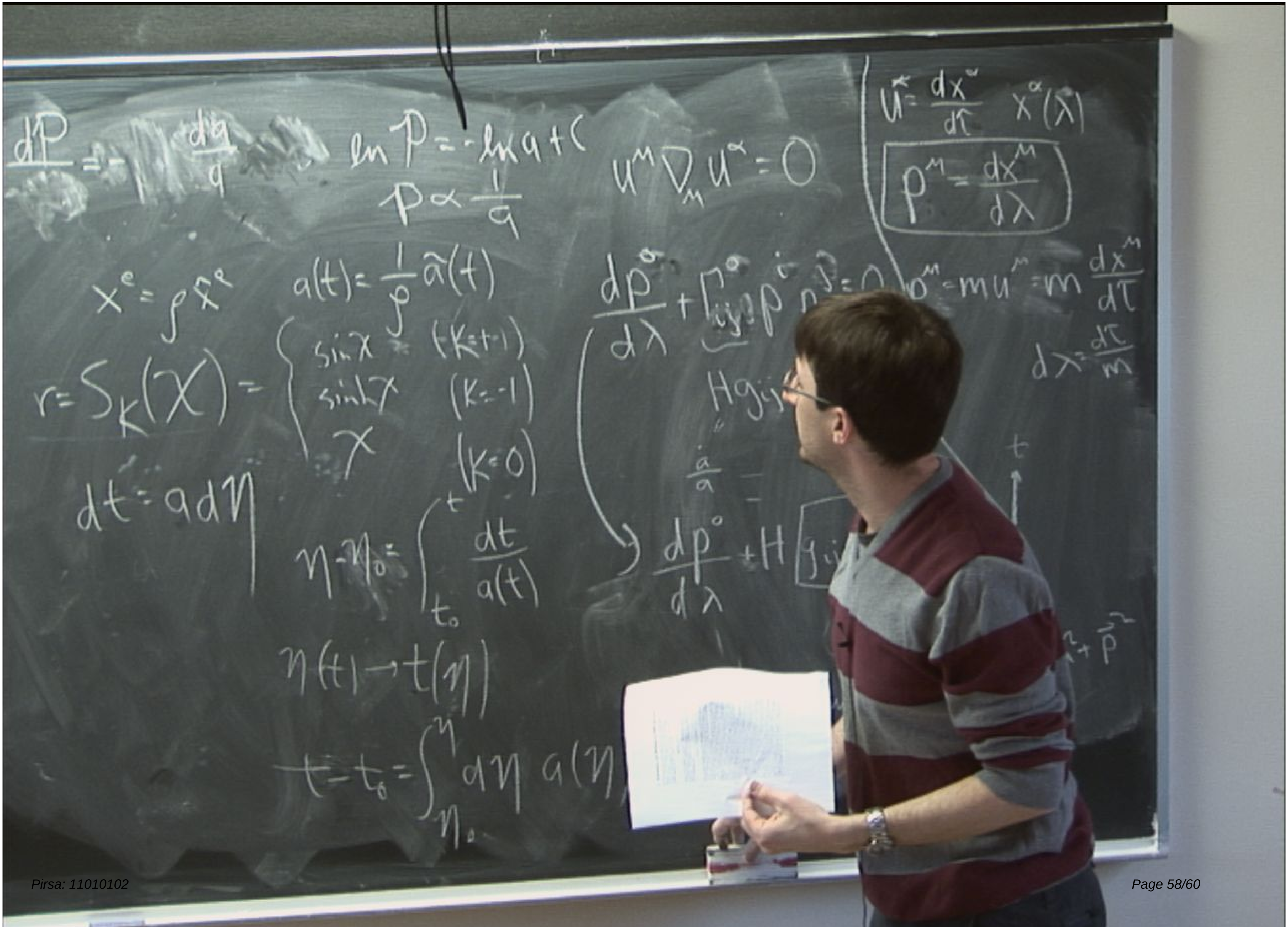
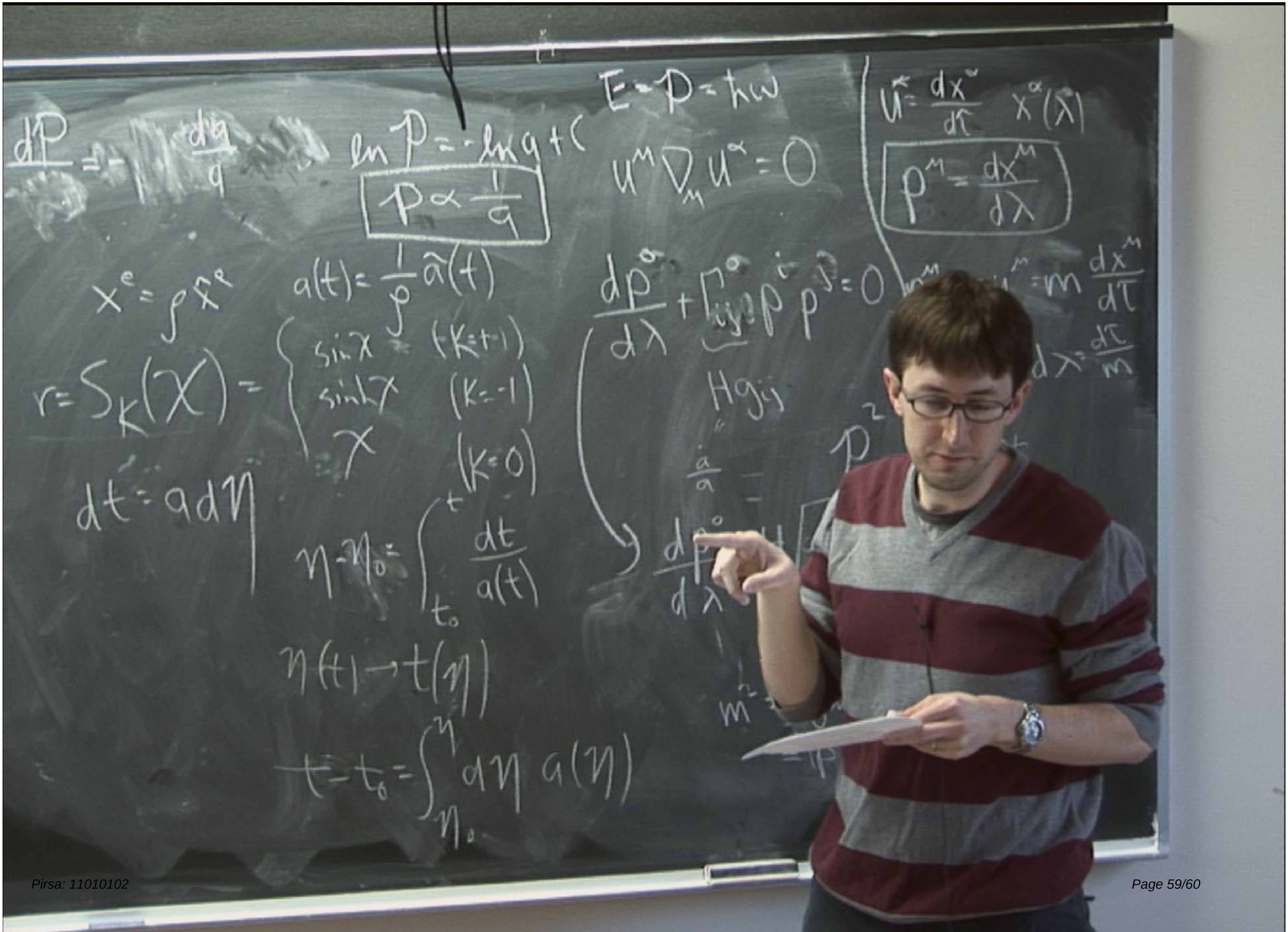
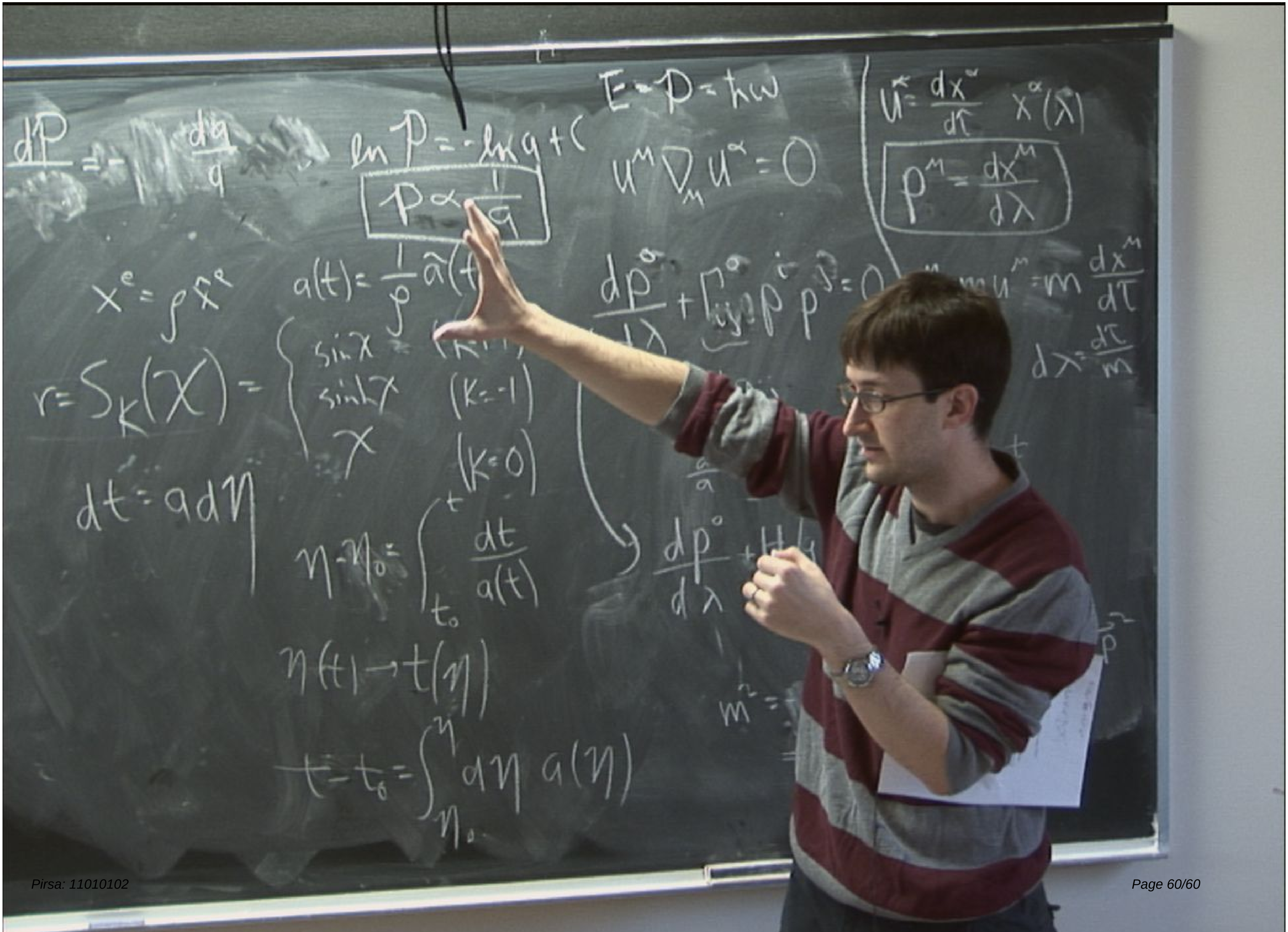

ln P = - ln a + C
P ∝ 1/a
u^μ ∇_μ u^α = 0
u^α = dx^α/dτ   x^α(λ)
p^μ = dx^μ/dλ
x^e = ρ x̂^e
a(t) = 1/ρ ã(t)
r = S_K(χ) =
sin χ (K = +1)
sinh χ (K = -1)
χ (K = 0)
dt = a dη
η - η_0 = ∫_{t_0}^{t} dt/a(t)
η(t) → t(η)
t - t_0 = ∫_{η_0}^{η} dη a(η)
dp^0/dλ + Γ^0_{ij} p^i p^j = 0
H g_{ij}
p^μ = m u^μ = m dx^μ/dτ
dλ = dτ/m
dp^0/dλ + H g_{ij}


E = p = ħω
ln p = -ln a + C
p ∝ 1/a
u^μ ∇_μ u^α = 0
u^α = dx^α/dτ   x^α(λ)
p^μ = dx^μ/dλ
a(t) = 1/ρ ã(t)
x^e = ρ x̃^e
r = S_k(χ) = sin χ (k=+1), sinh χ (k=-1), χ (k=0)
dt = a dη
η - η_0 = ∫_{t_0}^{t} dt/a(t)
η(t) → t(η)
t - t_0 = ∫_{η_0}^{η} dη a(η)
dp^0/dλ + Γ^0_{ij} p^i p^j = 0
Hg_{ij}
dλ = dτ/m



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