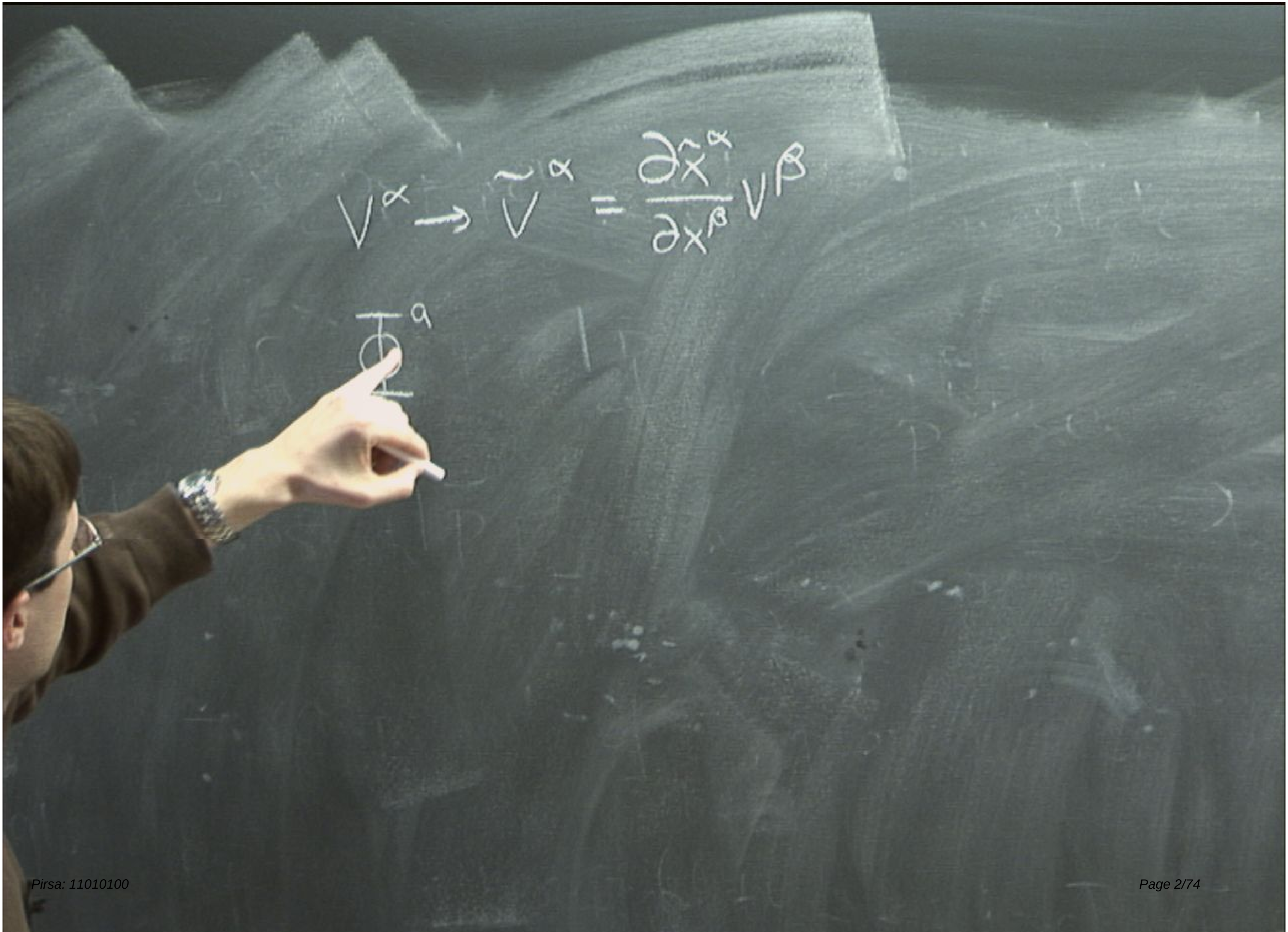
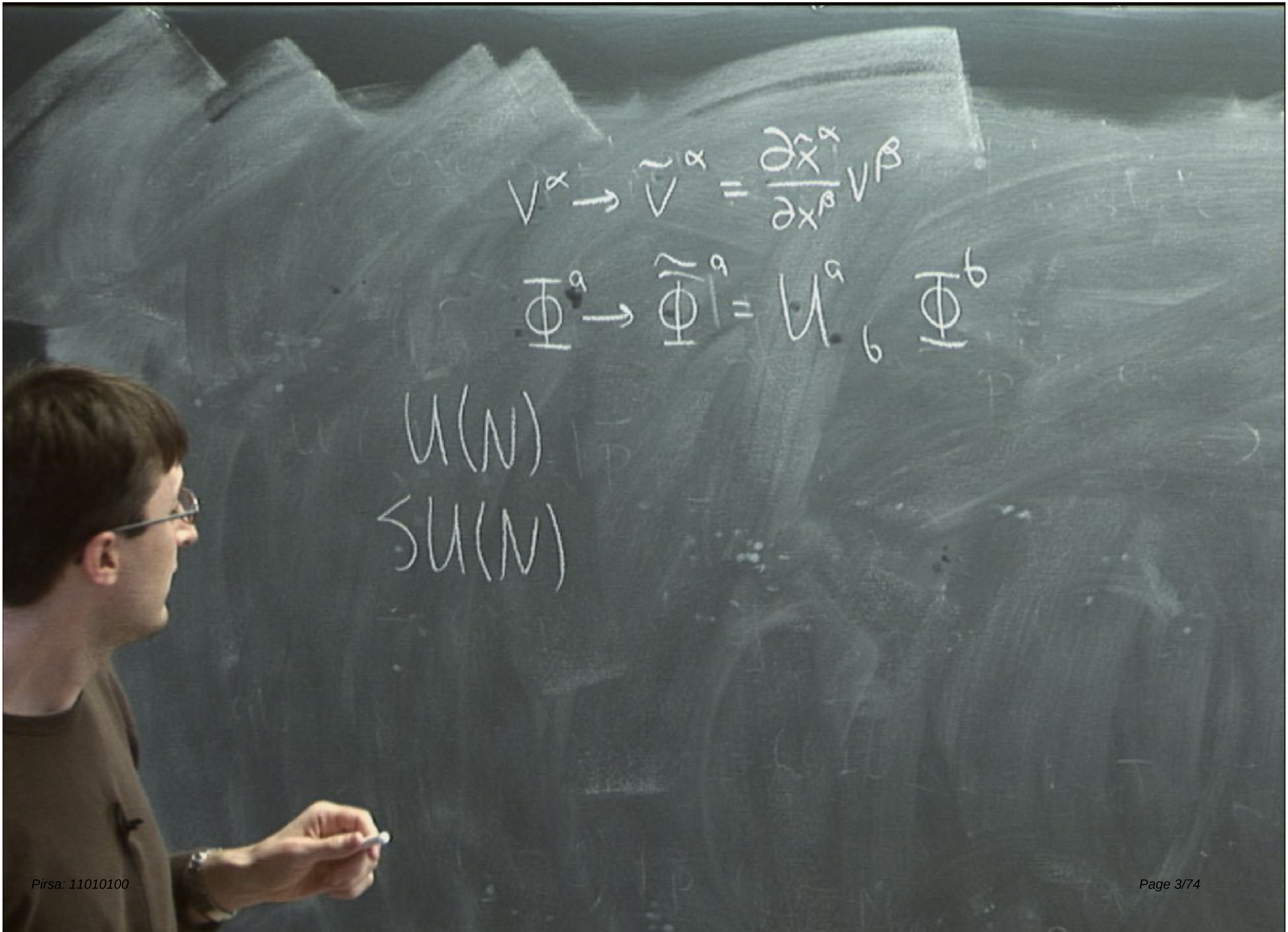
Title: Cosmology Review - Lecture 3

Date: Jan 26, 2011 11:30 AM

URL: http://pirsa.org/11010100

Abstract:


$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta$
Φ^a


$V^\alpha \rightarrow \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta$
$\Phi^a \rightarrow \tilde{\Phi}^a = U^a{}_b \Phi^b$
U(N)
SU(N)

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

$$U(N)$$

$$SU(N)$$

$$V^\alpha \rightarrow \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta$$

$$\Phi^a \rightarrow \tilde{\Phi}^a = U^a{}_b \, \Phi^b$$

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta$$

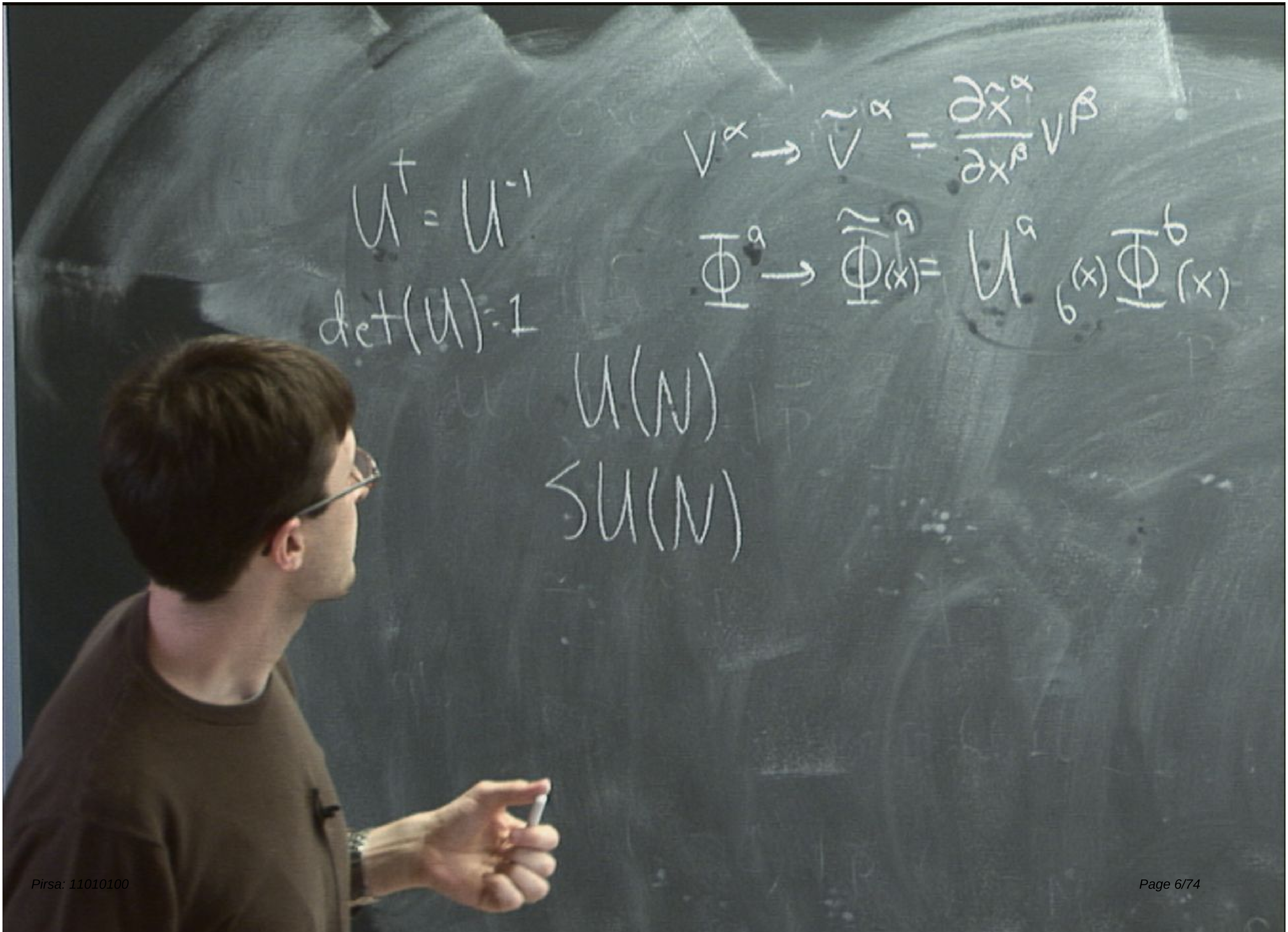
$$U^\dagger = U^{-1}$$

$$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x)$$

$$\det(U) = 1$$

$U(N)$

$SU(N)$


$U^{\dagger} = U^{-1}$
$\det(U) = 1$
$V^{\alpha} \to \tilde{V}^{\alpha} = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta}$
$\Phi^{a} \to \tilde{\Phi}^{a}(x) = U^{a}{}_{b}(x)\, \Phi^{b}(x)$
$U(N)$
$SU(N)$

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \to \nabla_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x) \qquad \partial_\mu \tilde{\Phi}^a = \partial_\mu \left(U^a{}_b \right.$$

$U(N)$

$SU(N)$

$$U^\dagger = U^{-1}$$

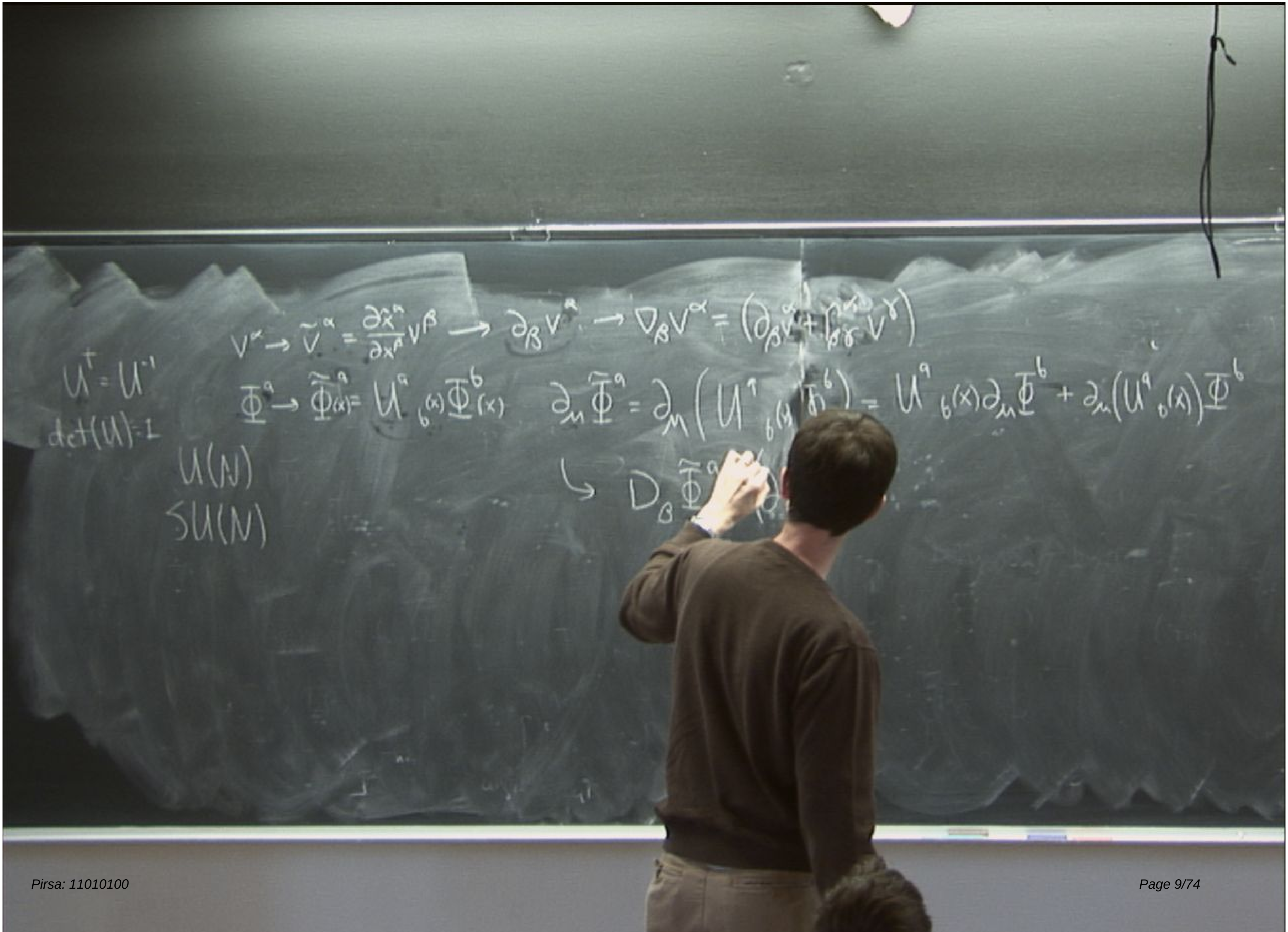
$$\det(U) = 1$$

$U(N)$

$SU(N)$

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \longrightarrow \nabla_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$$

$$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x) \qquad \partial_\mu \tilde{\Phi}^a = \partial_\mu \left(U^a{}_b(x)\, \Phi^b \right)$$


$U^\dagger = U^{-1}$
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$U(N)$
$SU(N)$
$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \to \partial_\beta V^\alpha \to \nabla_\beta V^\alpha = (\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma)$
$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x) \Phi^b(x)$
$\partial_\mu \tilde{\Phi}^a = \partial_\mu (U^a{}_b(x) \Phi^b) = U^a{}_b(x) \partial_\mu \Phi^b + \partial_\mu (U^a{}_b(x)) \Phi^b$
$D_\beta \tilde{\Phi}^a$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$U(N)$

$SU(N)$

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \to \nabla_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$$

$$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\, \Phi^b(x) \qquad \partial_\mu \tilde{\Phi}^a = \partial_\mu \left(U^a{}_b(x)\, \Phi^b\right) = U^a{}_b(x)\, \partial_\mu \Phi^b + \partial_\mu\left(U^a{}_b(x)\right)\Phi^b$$

$$\hookrightarrow D_\beta \tilde{\Phi}^a = \left(\partial_\beta \tilde{\Phi}^a + i g A_\beta \Phi\right.$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \longrightarrow \nabla_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$$

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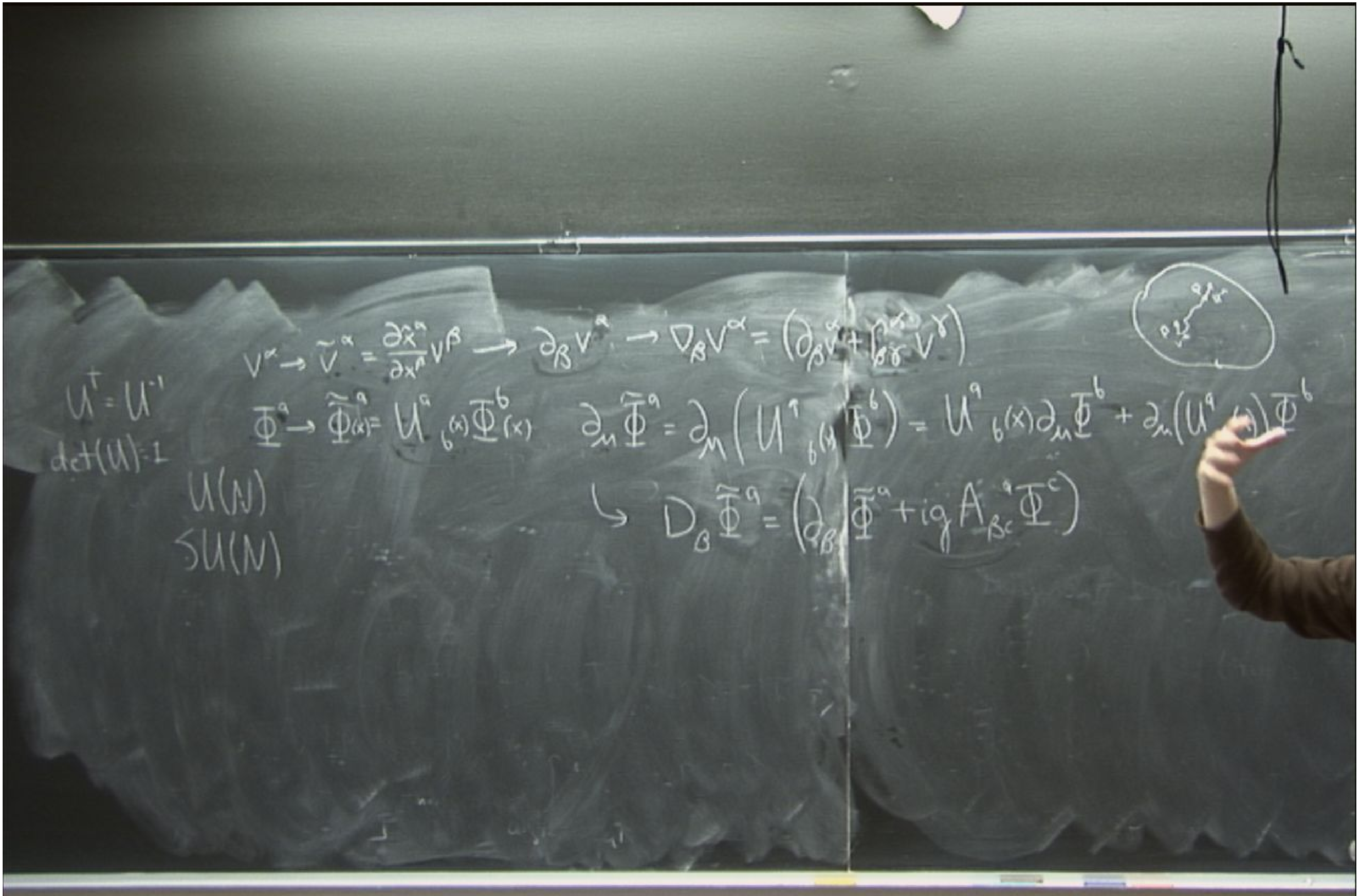
$U(N)$

$SU(N)$

$$\hookrightarrow D_\beta \tilde{\Phi}^a = \left(\partial_\beta \tilde{\Phi}^a + i g A_{\beta c}{}^{a}\, \Phi^c\right)$$


$U^\dagger = U^{-1}$
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U(N)
SU(N)
$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \to \partial_\beta V^\alpha \to D_\beta V^\alpha = (\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma)$
$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\Phi^b(x)$
$\partial_\mu \tilde{\Phi}^a = \partial_\mu (U^a{}_b(x)\Phi^b) = U^a{}_b(x)\partial_\mu \Phi^b + \partial_\mu (U^a{}_b(x))\Phi^b$
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$U^\dagger = U^{-1}$

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U(N)

SU(N)

$$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \to \partial_\beta V^\alpha \to \nabla_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$$

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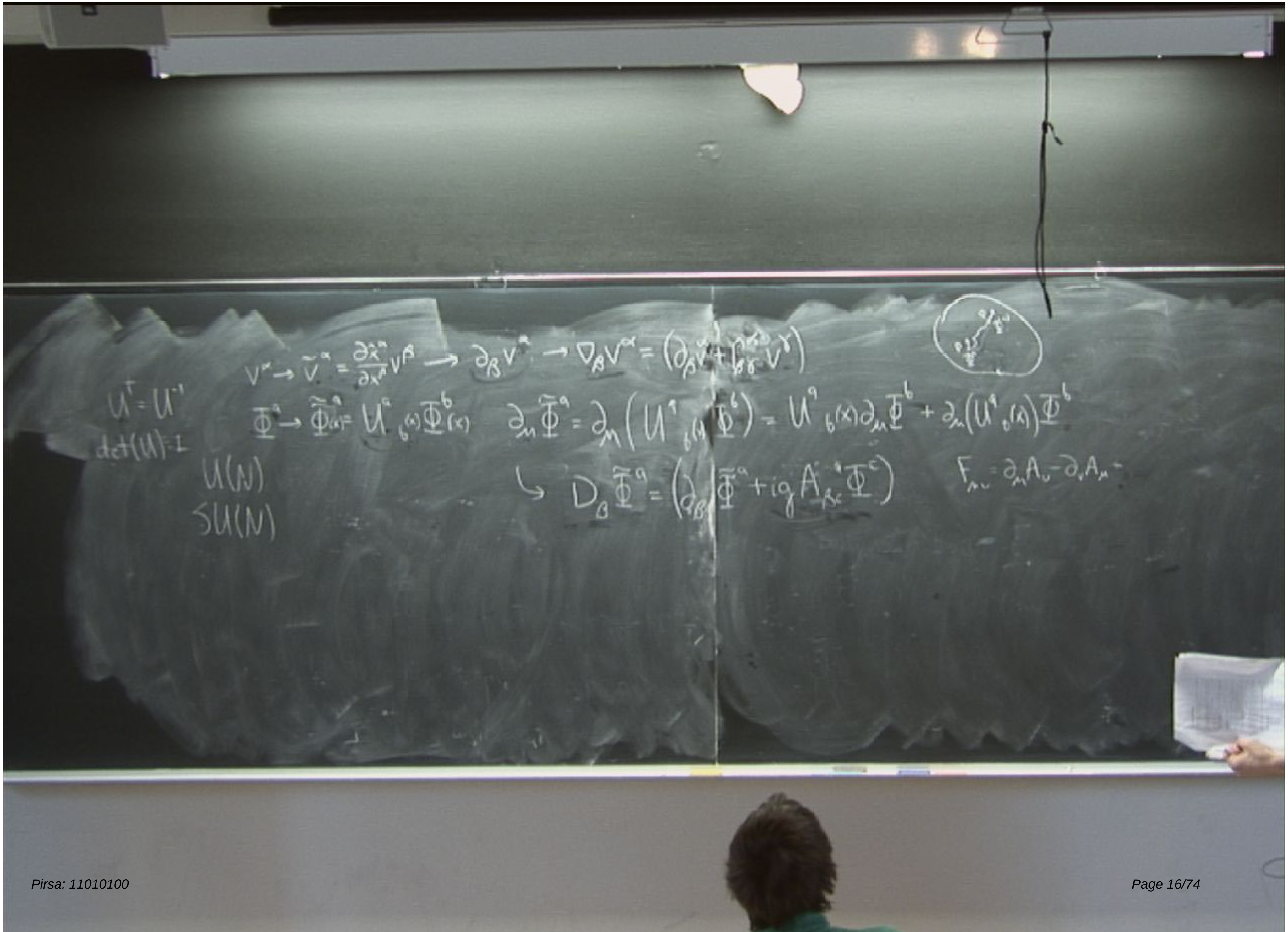
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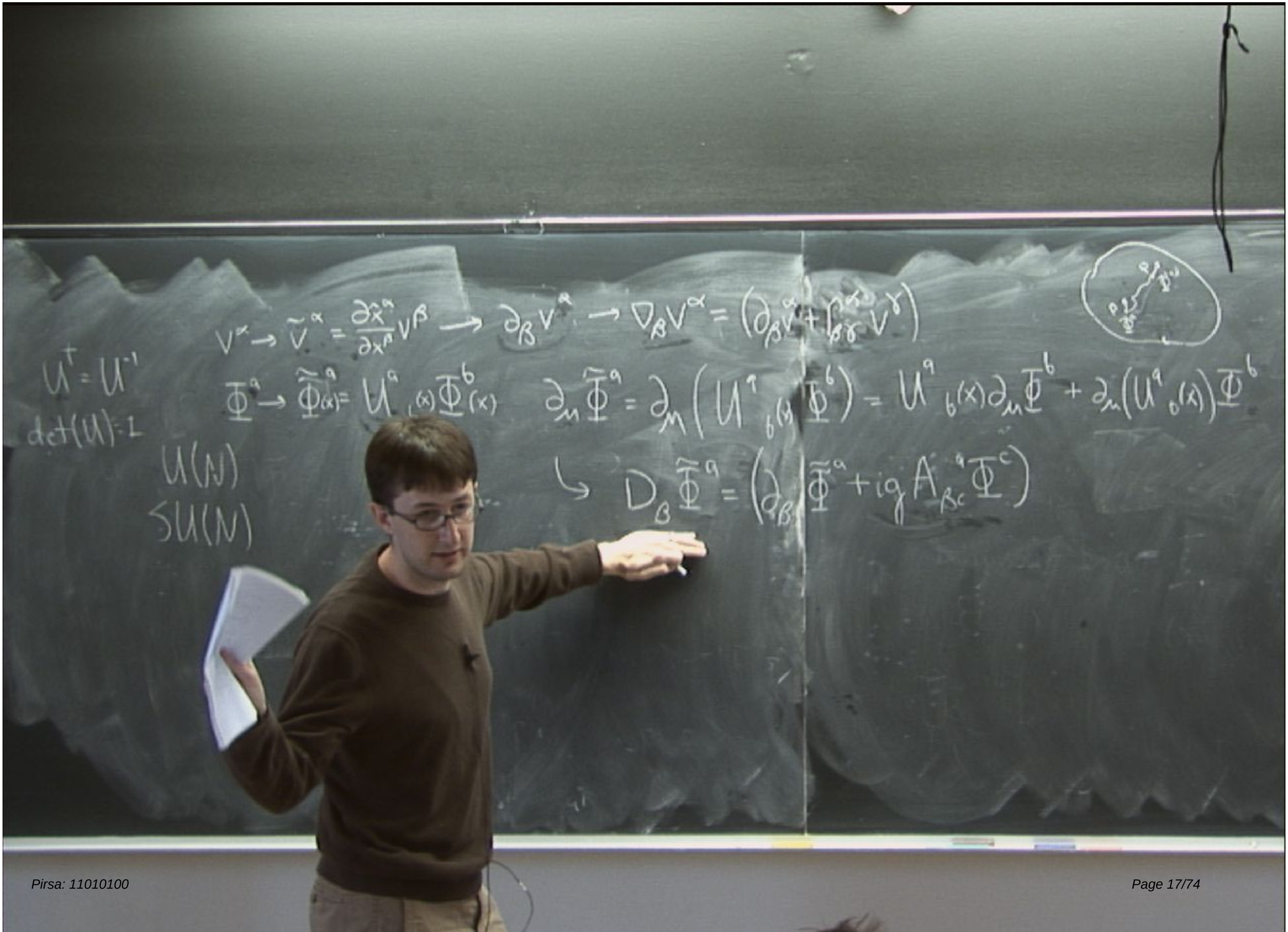
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + i g A_\beta{}^a{}_c \Phi^c)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu -$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SU(N)
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}^a \Phi^c)$

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$SU(N)$

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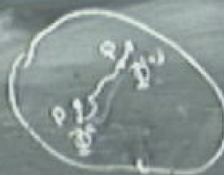
$$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x) \qquad \partial_\mu \tilde{\Phi}^a = \partial_\mu\left(U^a{}_b(x)\,\Phi^b\right) = U^a{}_b(x)\,\partial_\mu \Phi^b + \partial_\mu\left(U^a{}_b(x)\right)\Phi^b$$

$$\hookrightarrow D_\beta \tilde{\Phi}^a = \left(\partial_\beta \tilde{\Phi}^a + i g A_{\beta c}{}^a \Phi^c\right)$$

$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$



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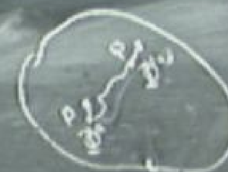

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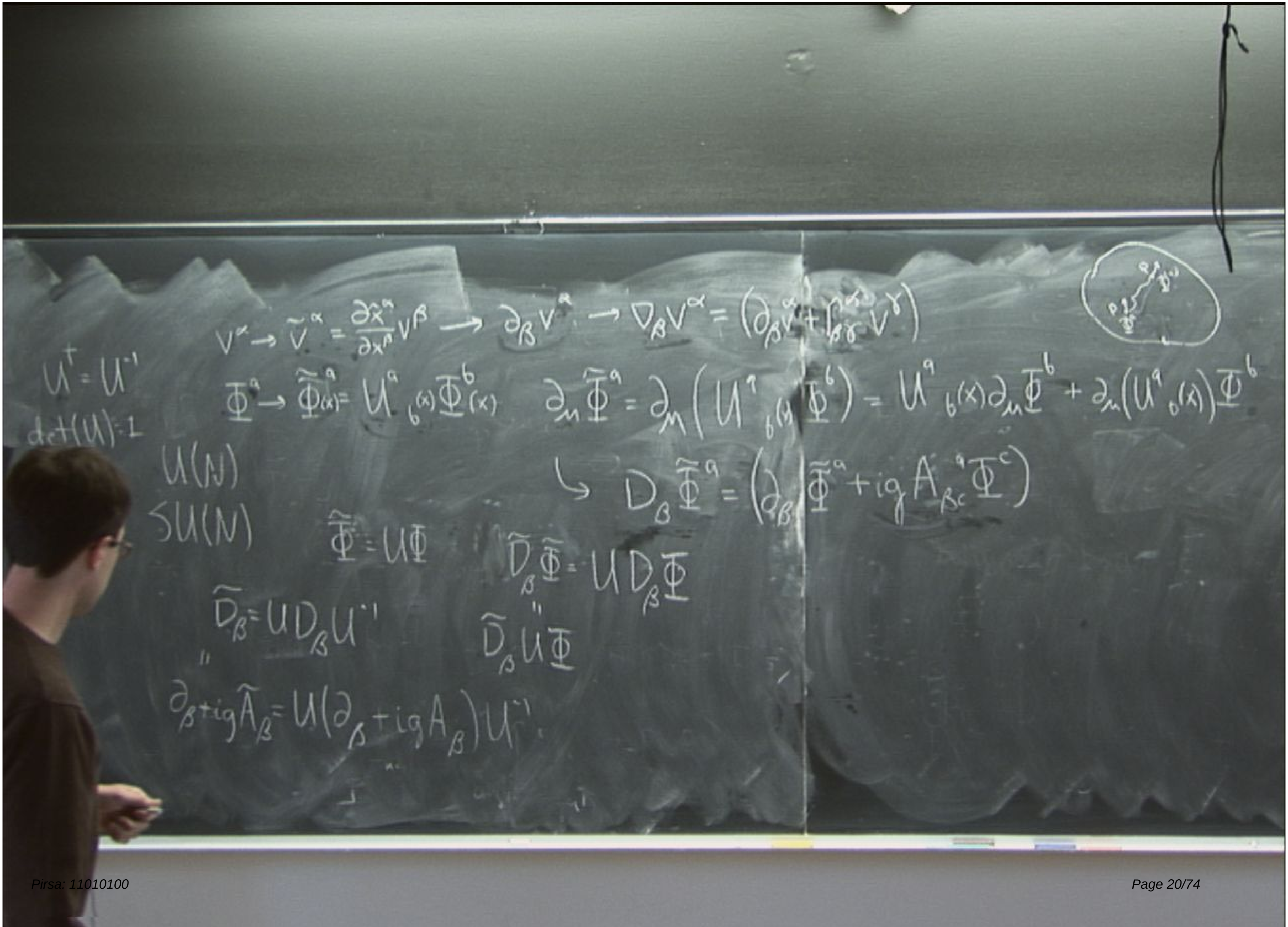
$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

$$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \overset{\shortparallel}{\tilde{D}_\beta U \Phi}$$


$U^\dagger = U^{-1}$
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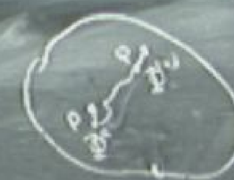
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}^a \Phi^c)$
$\tilde{\Phi} = U\Phi$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta U \Phi$
$\tilde{D}_\beta = U D_\beta U^{-1}$
$\partial_\beta + ig\tilde{A}_\beta = U(\partial_\beta + ig A_\beta) U^{-1}$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \longrightarrow D_\beta V^\alpha = \left(\partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma\right)$



$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\, \Phi^b(x)$ $\quad \partial_\mu \tilde{\Phi}^a = \partial_\mu \left(U^a{}_b(x) \Phi^b \right) = U^a{}_b(x) \partial_\mu \Phi^b + \partial_\mu \left(U^a{}_b(x) \right) \Phi^b$

$U(N)$

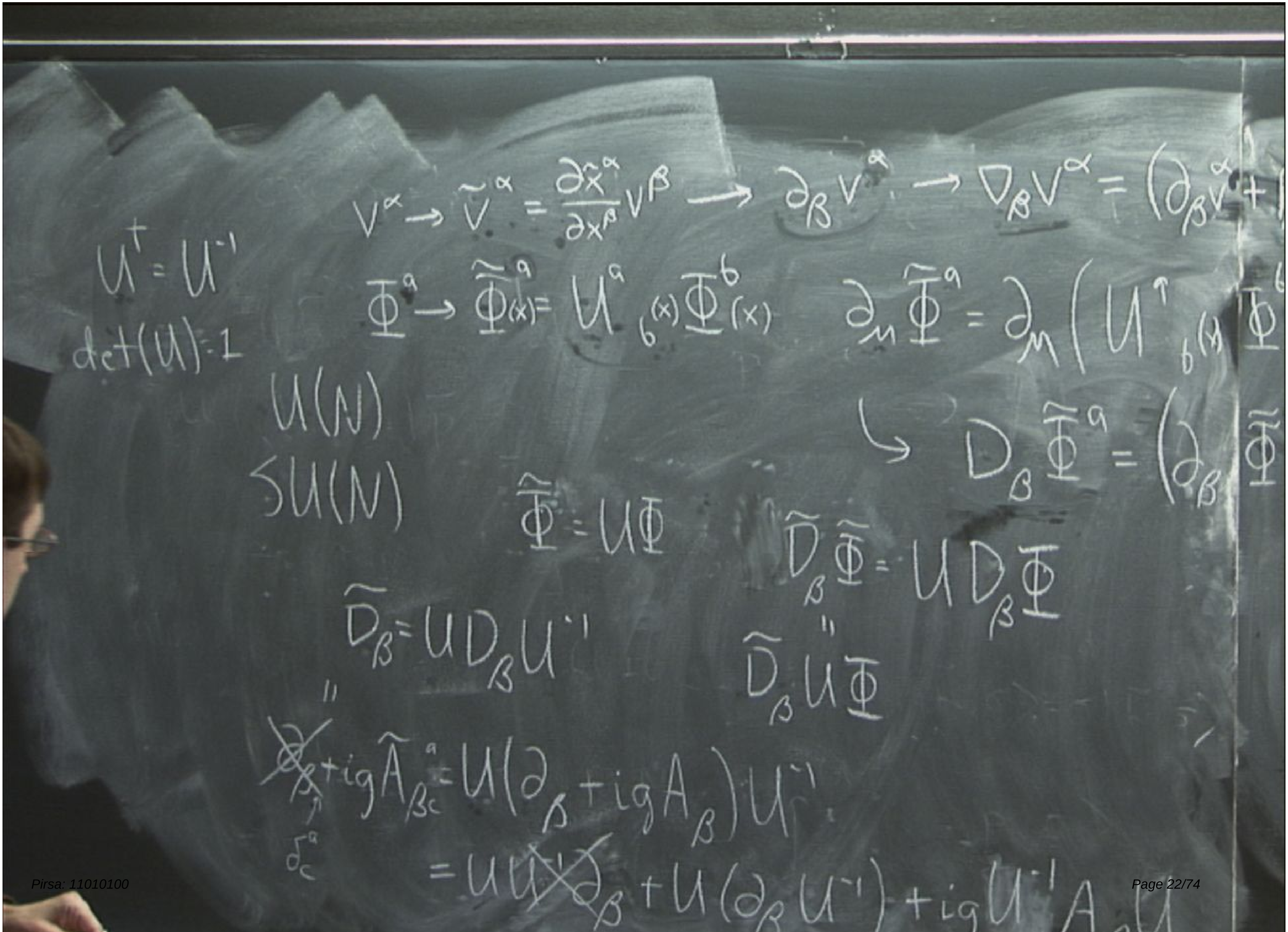
$\hookrightarrow D_\beta \tilde{\Phi}^a = \left(\partial_\beta \tilde{\Phi}^a + i g A_\beta{}^a{}_c \Phi^c \right)$

$SU(N)$

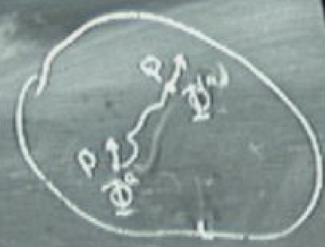
$\tilde{\Phi} = U \Phi$ $\quad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$

$\tilde{D}_\beta = U D_\beta U^{-1}$ $\quad \tilde{D}_\beta U \Phi$

$\partial_\beta + i g \tilde{A}_\beta = U(\partial_\beta + i g A_\beta) U^{-1}$


$U^\dagger = U^{-1}$
$\det(U) = 1$
$U(N)$
$SU(N)$
$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \longrightarrow \nabla_\beta V^\alpha = (\partial_\beta V^\alpha + $
$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\, \Phi^b(x)$
$\partial_\mu \tilde{\Phi}^a = \partial_\mu (U^a{}_b(x)\, \Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$
$\tilde{\Phi} = U\Phi$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta = U D_\beta U^{-1}$
$\tilde{D}_\beta U \Phi$
$\partial_\beta + ig\tilde{A}^a_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$
δ^a_c
$= UU^{-1}\partial_\beta + U(\partial_\beta U^{-1}) + igU^{-1}A_\beta U$

$$\ldots \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma)$$



$$\ldots{}_b(x)\,\Phi^b) = U^a{}_b(x)\,\partial_\mu \Phi^b + \partial_\mu\left(U^a{}_b(x)\right)\Phi^b$$

$$\ldots\tilde{\Phi}^a + ig\,A^a_{\beta c}\,\Phi^c)$$

$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U\left(\partial_\beta U^{-1}\right)$$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$

$$\tilde{\Phi} = U\Phi$$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$U(N)$

$SU(N)$

$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \longrightarrow \partial_\beta V^\alpha \longrightarrow \nabla_\beta V^\alpha = (\partial_\beta V^\alpha + \ldots$

$\Phi^a \to \tilde{\Phi}^a(x) = U^a{}_b(x)\, \Phi^b(x) \qquad \partial_\mu \tilde{\Phi}^a = \partial_\mu \left(U^a{}_b(x) \Phi^b \ldots \right.$

$\hookrightarrow D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi} \ldots$

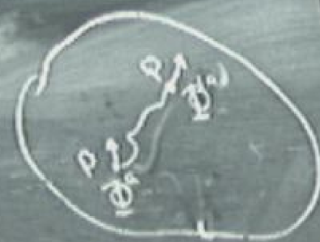
$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$

$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \qquad \tilde{D}_\beta U \Phi$

$\cancel{\partial_\beta} + ig \tilde{A}^a_{\beta c} = U(\partial_\beta + ig A_\beta) U^{-1}$

δ^a_c

$= U \cancel{U^{-1}} \partial_\beta + U(\partial_\beta U^{-1}) + ig U^{-1} A_\beta U$



$$\ldots \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma)$$

$$\ldots{}_b(x)\Phi^b) = U^a{}_b(x)\,\partial_\mu \Phi^b + \partial_\mu\left(U^a{}_b(x)\right)\Phi^b$$

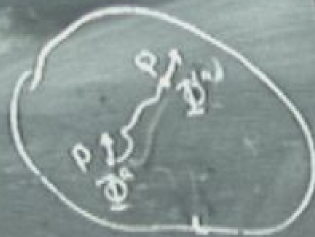
$$\ldots \partial_\beta \ldots {}^a{}_{\beta c}\Phi^c)$$

$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U\left(\partial_\beta U^{-1}\right)$$

$$\tilde{\Phi} = U\Phi$$

$$U\,\partial_\beta U^{-1}\,\Phi$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \ddots \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$



$$\ldots \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma)$$

$$\ldots {}_b(x)\, \Phi^b) = U^a{}_b(x)\, \partial_\mu \Phi^b + \partial_\mu \left(U^a{}_b(x) \right) \Phi^b$$

$$\ldots \partial_\beta \tilde{\Phi}^a + i g A^a_{\beta c} \Phi^c)$$

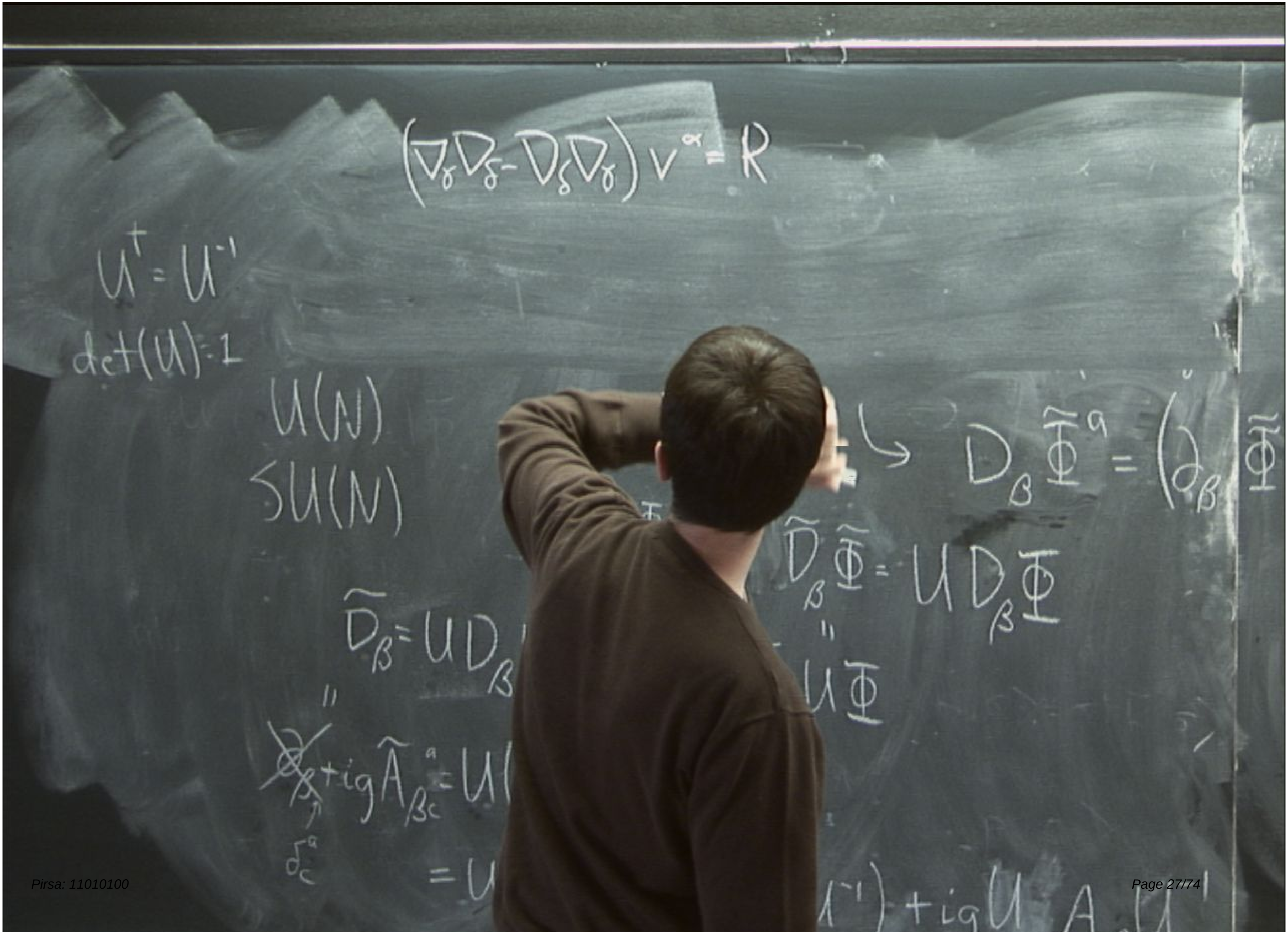
$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U \left(\partial_\beta U^{-1} \right)$$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix}$$

$$\tilde{\Phi} = U \Phi$$

$$U(x)\, \partial_\beta U^{-1}(x)\, \Phi$$

$$= U(x) \left[\partial_\beta U^{-1}(x) \right] \Phi$$


$(\nabla_\gamma D_\delta - D_\delta \nabla_\gamma) v^\alpha = R$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$U(N)$
$SU(N)$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta = U D_\beta$
$U \Phi$
$\cancel{\partial_\beta} + ig \tilde{A}^a_{\beta c} = U($
δ^a_c
$+ ig U A U^{-1}$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

$U(N)$

$SU(N)$

$$\tilde{\Phi} = U\Phi$$

$$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

$$\parallel$$

$$\tilde{D}_\beta U \Phi$$

$$\hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$$

$$\tilde{D}_\beta = U D_\beta U^{-1}$$

$$\parallel$$

$$\cancel{\partial_\beta} + ig\tilde{A}^a_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$$

$$= U\cancel{U^{-1}\partial_\beta} + U(\partial_\beta U^{-1}) + igU A U^{-1}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha{}_{\beta\delta,\gamma} - \Gamma^\alpha{}_{\cdot\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$$

$U^\dagger = U^{-1}$

$\det(U) = 1$

(N)

$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$

$\hookrightarrow D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_\beta{}^a{}_c \tilde{\Phi}^c)$

$\tilde{A}_\beta = U$

$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$

$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix}$

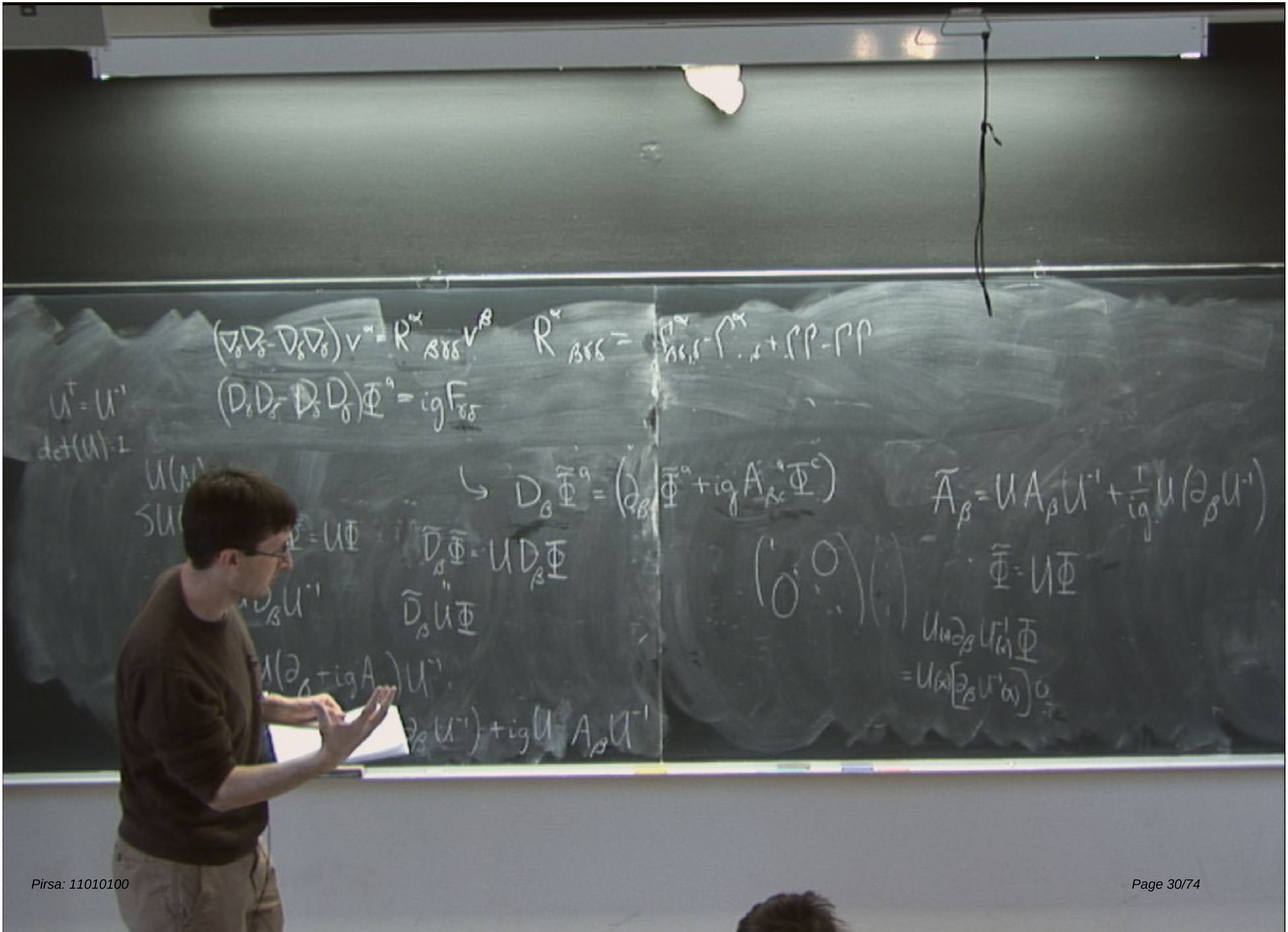
$\tilde{\Phi} =$

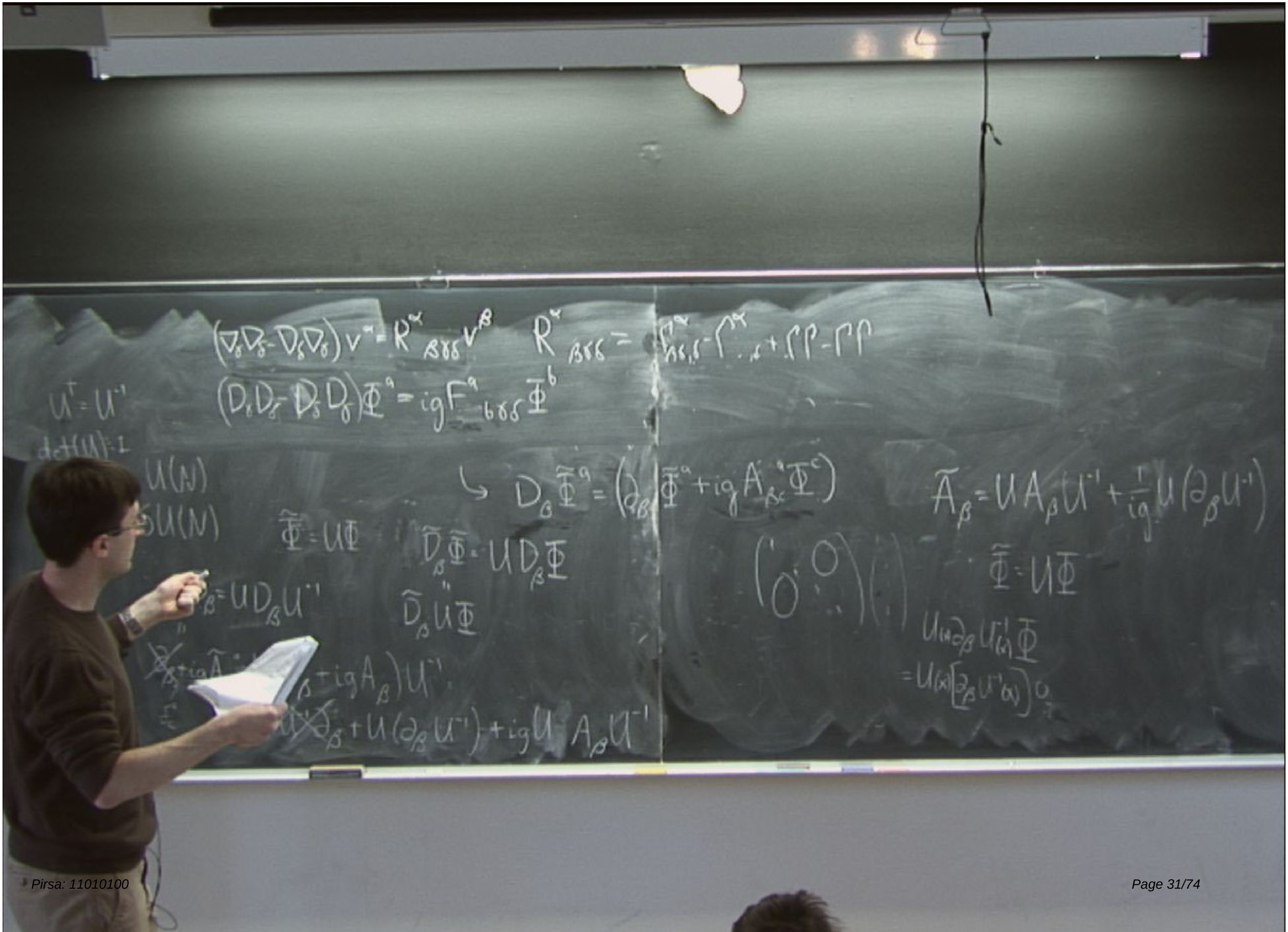
$U(x)\partial_\beta U^{-1}(x)\Phi$

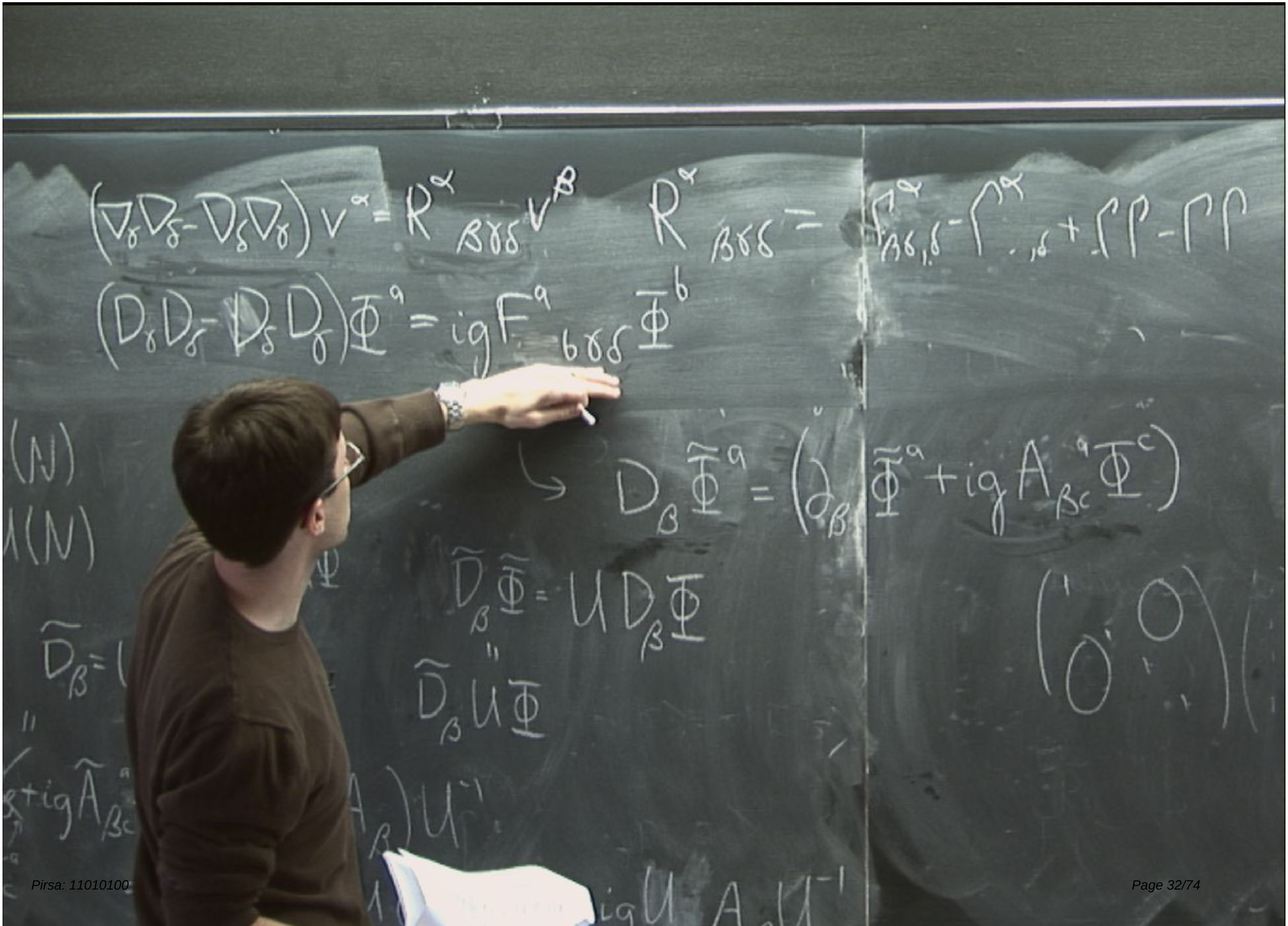
$ig\tilde{A}_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$

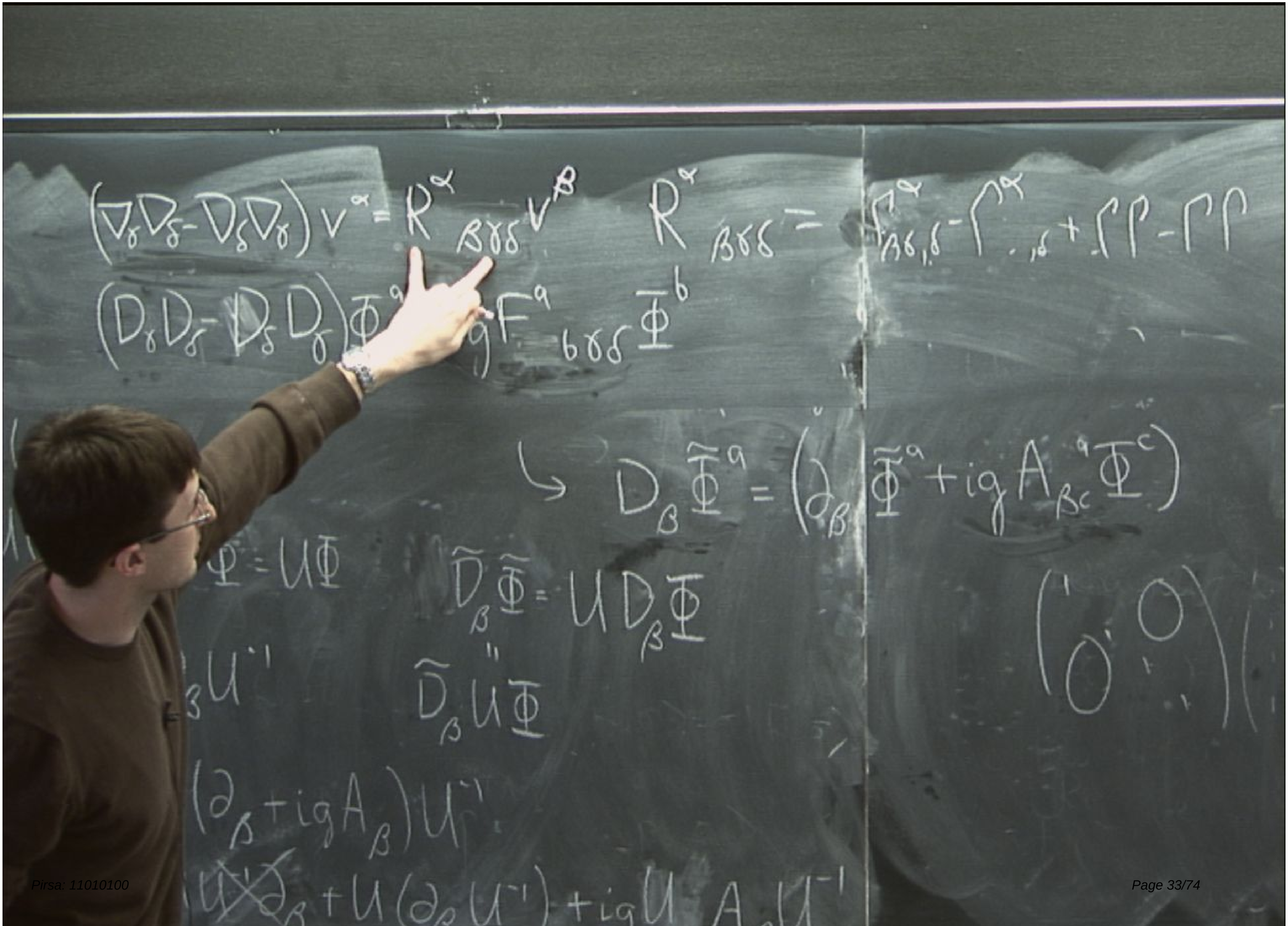
$= U(x)[\partial_\beta U^{-1}(x)]$

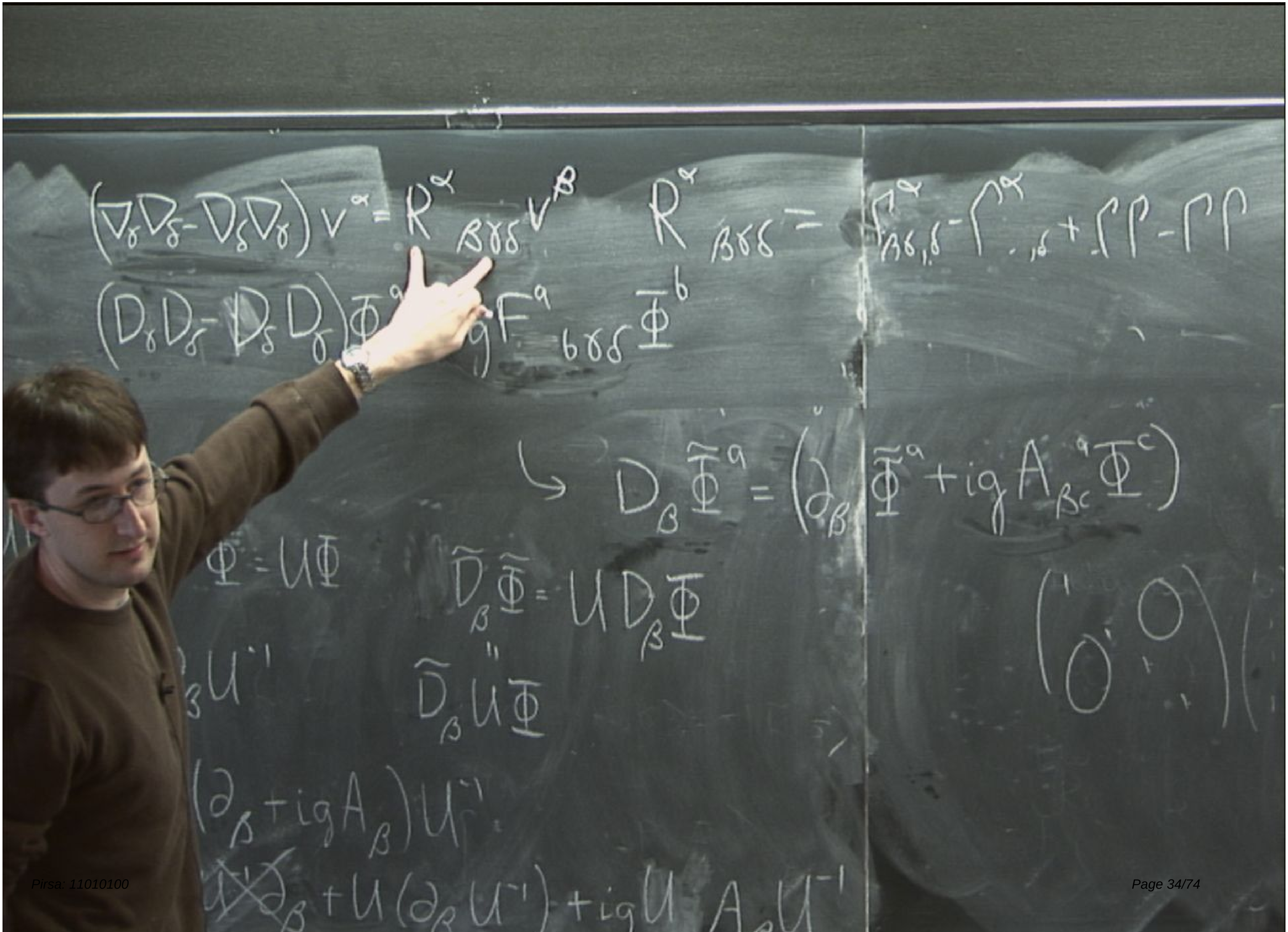
$= U \cancel{U^{-1}} \partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha{}_{\beta\delta,\gamma} - \Gamma^\alpha{}_{\ldots,\delta} + \Gamma\Gamma - \Gamma\Gamma$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F_{\gamma\delta}$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}{}^a \tilde{\Phi}^c)$
$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U (\partial_\beta U^{-1})$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{\Phi} = U \Phi$
$\tilde{D}_\beta U \Phi$
$U(x) \partial_\beta U^{-1}(x) \Phi$
$= U(x) [\partial_\beta U^{-1}(x)] \Phi$
$U(\partial_\beta + ig A_\beta) U^{-1}$
$\partial_\beta U^{-1}) + ig U A_\beta U^{-1}$


$(\nabla_\gamma\nabla_\delta-\nabla_\delta\nabla_\gamma)V^\alpha = R^\alpha{}_{\beta\gamma\delta}V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = igF^a{}_{b\gamma\delta}\Phi^b$
$U^\dagger = U^{-1}$
$U(N)$
$SU(N)$
$\tilde\Phi = U\Phi$
$D_\beta\tilde\Phi^a = (\partial_\beta\Phi^a + igA_{\beta c}{}^a\Phi^c)$
$\tilde A_\beta = UA_\beta U^{-1} + \frac{1}{ig}U(\partial_\beta U^{-1})$
$\tilde D_\beta\tilde\Phi = UD_\beta\Phi$
$\tilde\Phi = U\Phi$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\delta} - \Gamma^\alpha_{\cdot\,,\delta} + \Gamma\Gamma - \Gamma\Gamma$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = igF^a{}_{b\gamma\delta}\Phi^b$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + igA_{\beta c}{}^a \Phi^c)$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta U \Phi$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\cdot\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = g F^a{}_{b\gamma\delta} \Phi^b$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_\beta{}^a{}_c \Phi^c)$
$\tilde{\Phi} = U\Phi$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta U \Phi$
$(\partial_\beta + igA_\beta) U^{-1}$
$\cancel{UU^{-1}}\partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}^a \Phi^c)$
$\tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$
$\tilde{D}_\beta U \Phi$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\delta} - \Gamma^\alpha_{\cdot\,,\delta} + \Gamma\Gamma - \Gamma\Gamma$$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta}\Phi^b$$

$$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}{}^a \Phi^c)$$

$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

$$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$$ig\tilde{A}_{\beta c}{}^a = U(\partial_\beta + igA_\beta)U^{-1}$$

$$= U\cancel{U^{-1}}\partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\cdot\,\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta}\Phi^b \qquad F_{\gamma\delta} =$$

(N)

$$D_\beta \tilde{\Phi}^a = \left(\partial_\beta \tilde{\Phi}^a + ig A^a_{\beta c}\Phi^c\right)$$

(N)

$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

$$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$$

$$+ig\tilde{A}^a_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$$

$$= U\cancel{U^{-1}}\partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\cdot\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$$

$$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b \qquad F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$$

$U(N)$

$SU(N)$

$$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A_\beta{}^a{}_c \tilde{\Phi}^c)$$

$$\tilde{A}_\beta \quad A$$

$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

$$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \ddots \end{pmatrix}$$

$$\tilde{\Phi} = U$$

$$U(x)\partial_\beta U^{-1}(x)\Phi$$

$$\cancel{\partial_\beta} + ig\tilde{A}_{\beta c}{}^a = U(\partial_\beta + igA_\beta)U^{-1}$$

$$\delta^a_c$$

$$= U(x)[\partial_\beta U^{-1}(x)]$$

$$= U\cancel{U^{-1}}\partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\cdot\,\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b \qquad F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$$

(N)

$\hookrightarrow D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}^a + ig A^a_{\beta c} \Phi^c)$

$\tilde{A}_\beta = U A$

$U(N)$

$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$

$\tilde{\Phi} = U$

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$

$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$

$U(x) \partial_\beta U^{-1}(x) \Phi$

$\partial_\beta + ig\tilde{A}_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$

$= U(x)[\partial_\beta U^{-1}(x)]$

$= U\cancel{U^{-1}}\partial_\beta + U(\partial_\beta U^{-1}) + igU A_\beta U^{-1}$

$\Gamma^\alpha_{\beta\gamma,\delta} - \Gamma^\alpha_{\cdot\,\cdot,\gamma} + \Gamma\Gamma - \Gamma\Gamma$

$$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$$

$$\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}^{a} \Phi^c)$$

$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U (\partial_\beta U^{-1})$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$$

$$\tilde{\Phi} = U \Phi$$

$$U(x) \partial_\beta U^{-1}(x) \Phi$$

$$= U(x) [\partial_\beta U^{-1}(x)] \Phi$$

$$\Gamma^{\alpha}_{\beta\gamma,\delta} - \Gamma^{\alpha}_{\cdot\cdot,\gamma} + \Gamma\Gamma - \Gamma\Gamma$$

$$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig\left[A_\gamma, A_\delta\right]$$

$$\partial_\beta \tilde{\Phi}^a + ig\, A_{\beta c}^{\ a}\, \Phi^c)$$

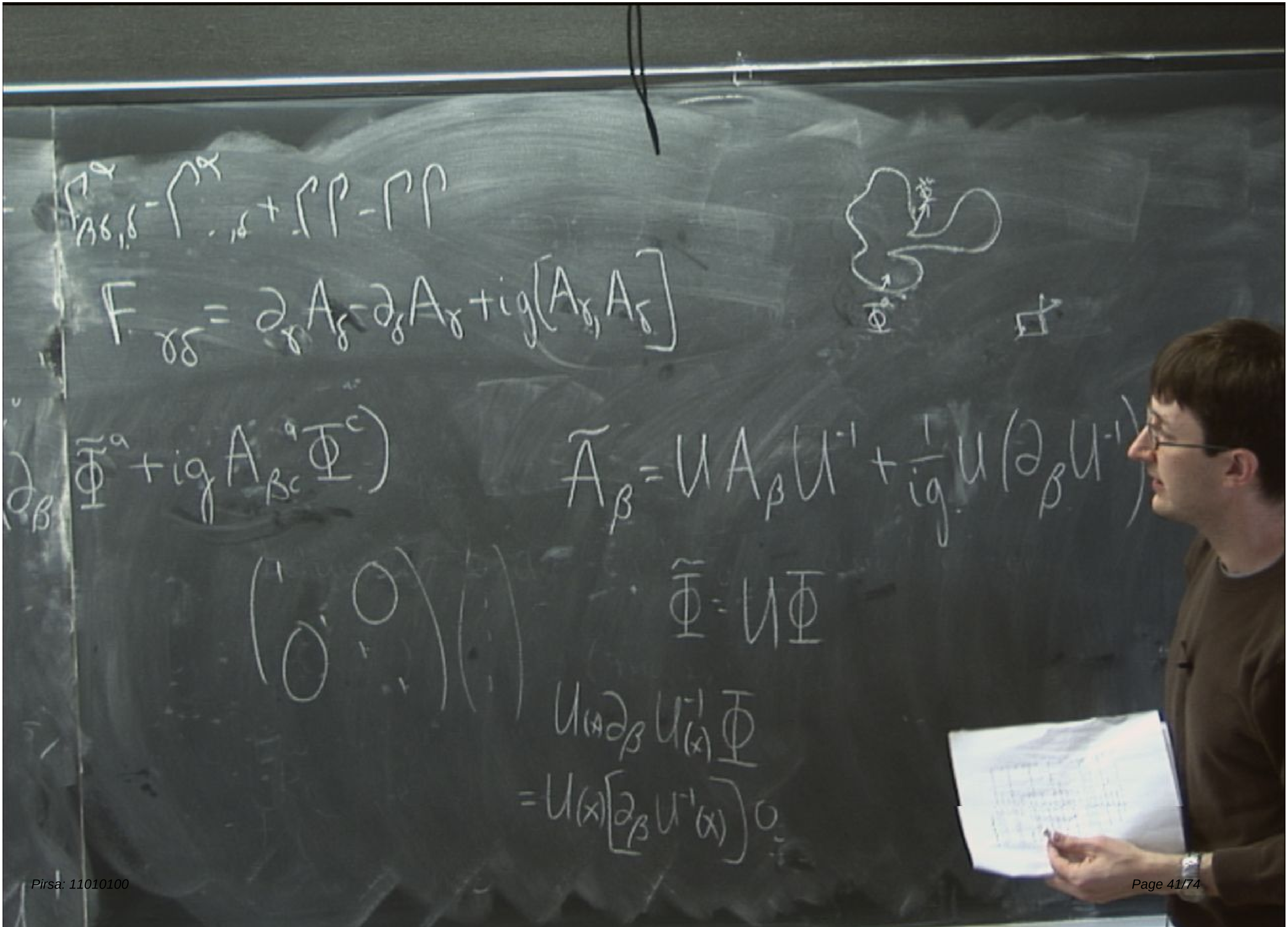
$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U\left(\partial_\beta U^{-1}\right)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \ddots \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$

$$\tilde{\Phi} = U\Phi$$

$$U(x)\,\partial_\beta\, U^{-1}(x)\, \Phi$$

$$= U(x)\left[\partial_\beta U^{-1}(x)\right] 0$$


$\Gamma^{\alpha}_{\beta\gamma,\delta} - \Gamma^{\alpha}_{\cdot\cdot,\gamma} + \Gamma\Gamma - \Gamma\Gamma$
$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$
$\partial_\beta \tilde{\Phi}^a + ig A_{\beta c}^{a} \Phi^c)$
$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U(\partial_\beta U^{-1})$
$\tilde{\Phi} = U\Phi$
$U(x)\partial_\beta U^{-1}_{(x)} \Phi$
$= U(x)[\partial_\beta U^{-1}(x)] \Phi$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} = \Gamma_\beta$$

$$U^\dagger = U^{-1} \qquad (D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$$

$$\det(U) = 1$$

$U(N)$

$SU(N)$

$$\hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$$

$$\tilde{\Phi} = U\Phi \qquad \tilde{D}_\beta \tilde{\Phi} = U D_\beta \Phi$$

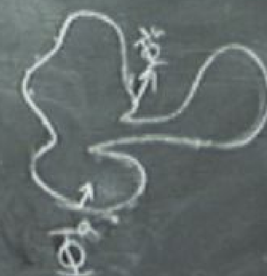
$$\tilde{D}_\beta = U D_\beta U^{-1} \qquad \tilde{D}_\beta U \Phi$$

$$\cancel{\partial_\beta} + ig\tilde{A}^a_{\beta c} = U(\partial_\beta + igA_\beta)U^{-1}$$

δ^a_c

$$= UU^{-1}\cancel{\partial_\beta} + U(\partial_\beta U^{-1}) + igU A U^{-1}$$

$$\ldots_{\gamma\delta} = \Gamma^{\alpha}_{\beta\gamma,\delta} - \Gamma^{\alpha}_{\cdot\,\cdot,\delta} + \Gamma\Gamma - \Gamma\Gamma$$



$$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$$

$$= \left(\partial_\beta \Phi^a + ig A^{a}_{\beta c} \Phi^c\right)$$

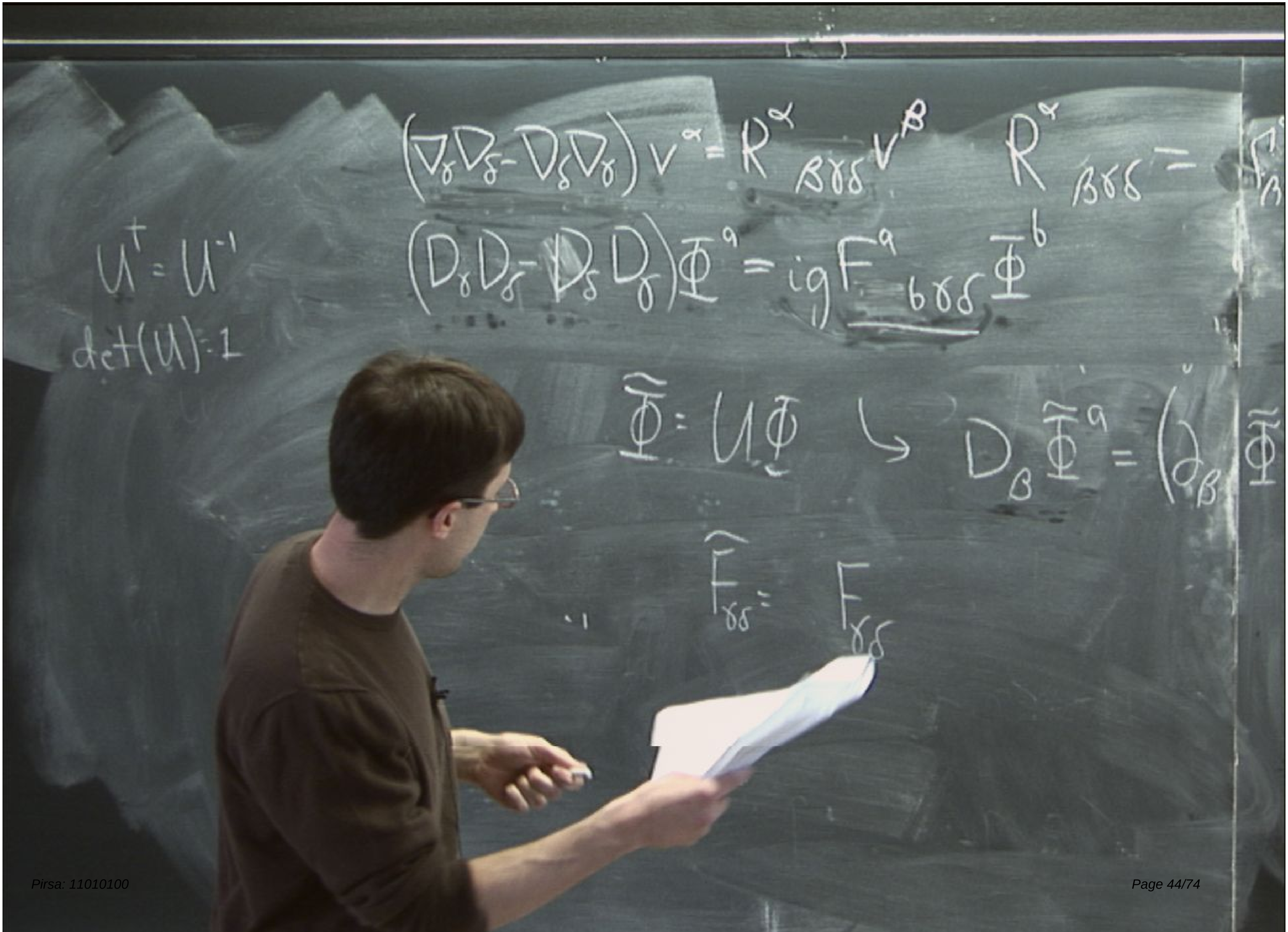
$$\tilde{A}_\beta = U A_\beta U^{-1} + \frac{1}{ig} U(\partial_\beta U^{-1})$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \vdots \end{pmatrix}$$

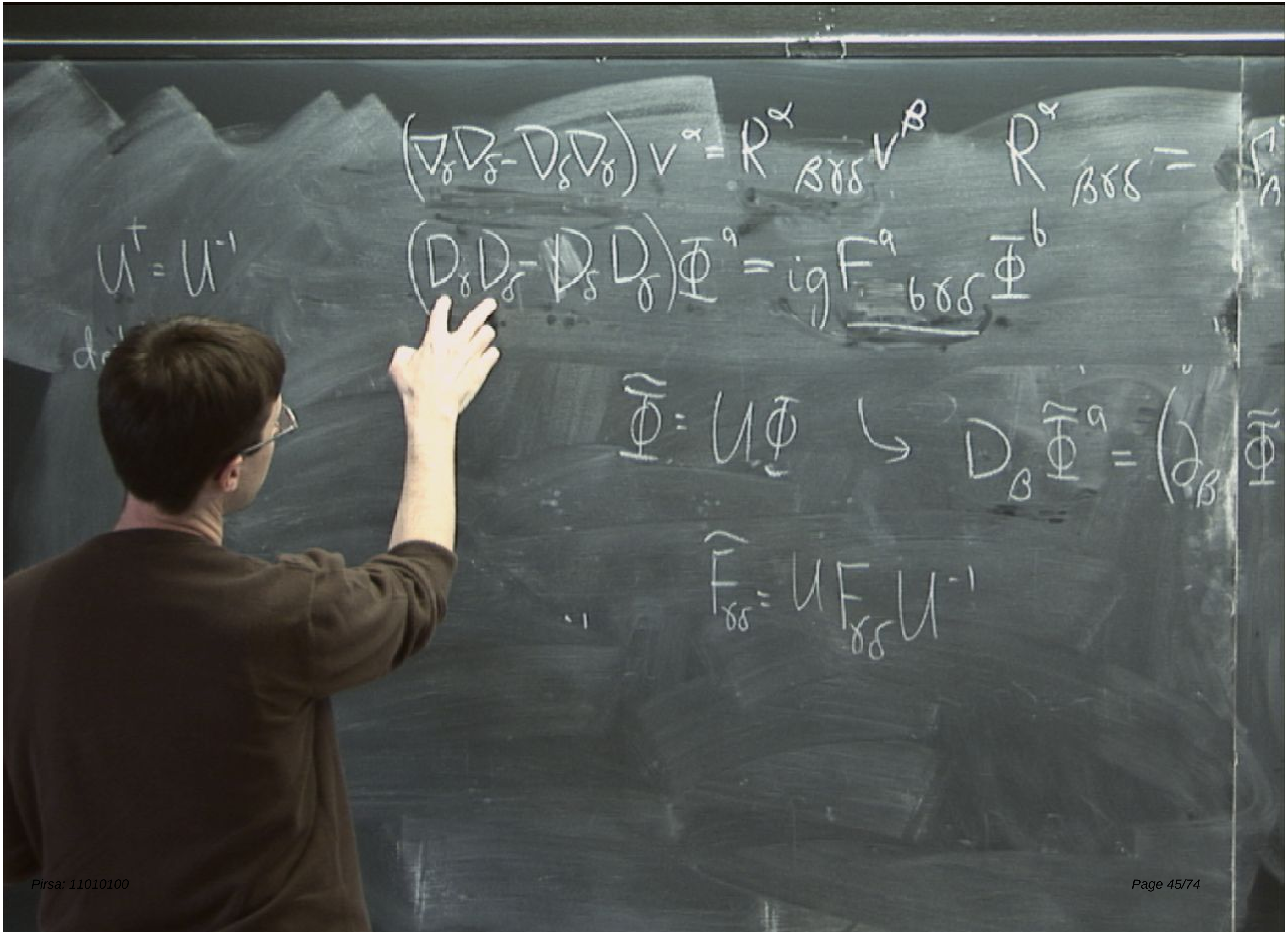
$$\tilde{\Phi} = U\Phi$$

$$U(x)\partial_\beta U^{-1}(x)\Phi$$

$$= U(x)\left[\partial_\beta U^{-1}(x)\right]0$$



$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$
$\tilde{\Phi} = U\Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$
$\tilde{F}_{\gamma\delta} = F_{\gamma\delta}$


$(\nabla_\gamma D_\delta - D_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$
$\tilde{\Phi} = U \Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta \tilde{\Phi}$
$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$$U^\dagger = U^{-1}$$

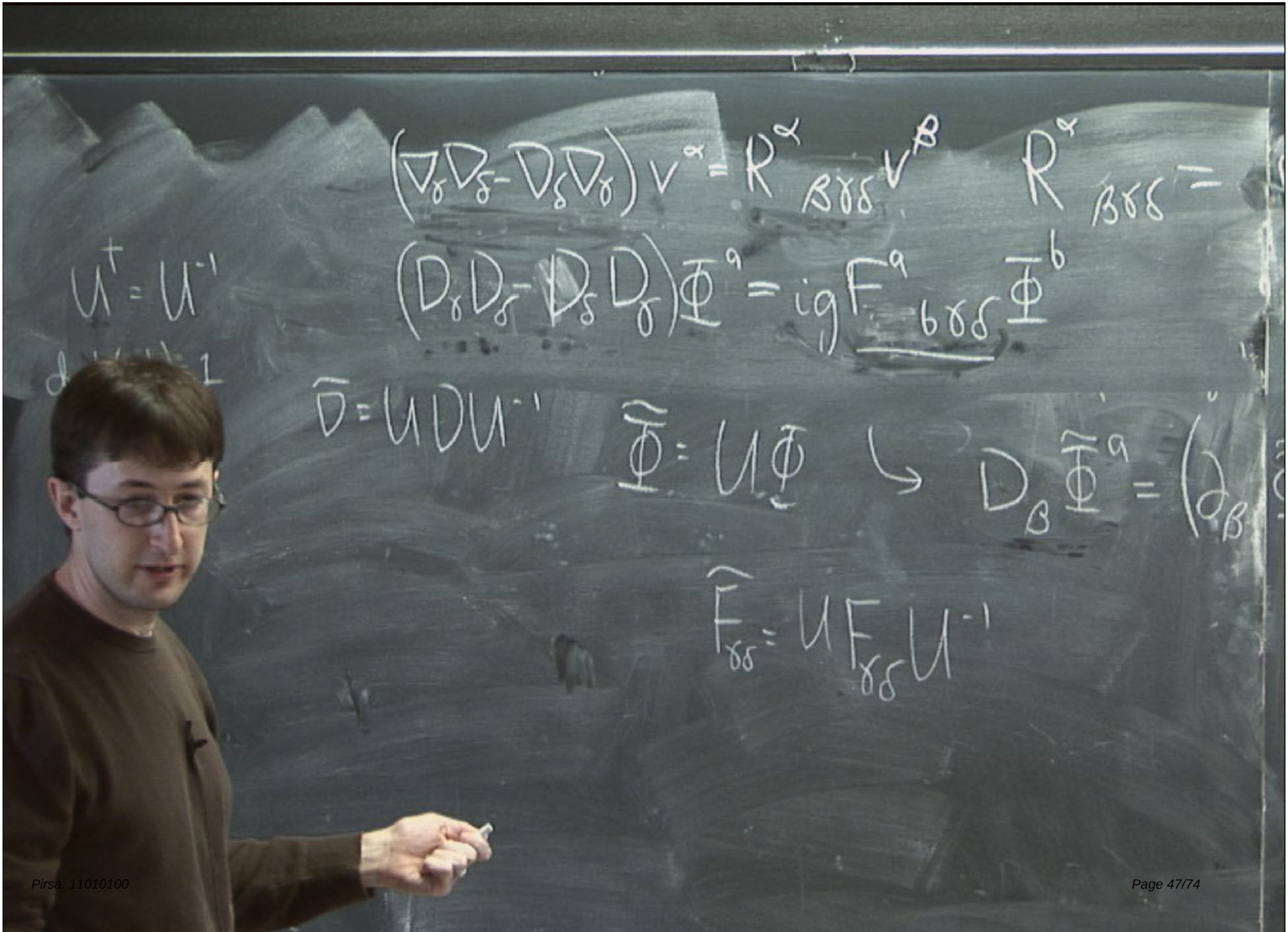
$$\det(U) = 1$$

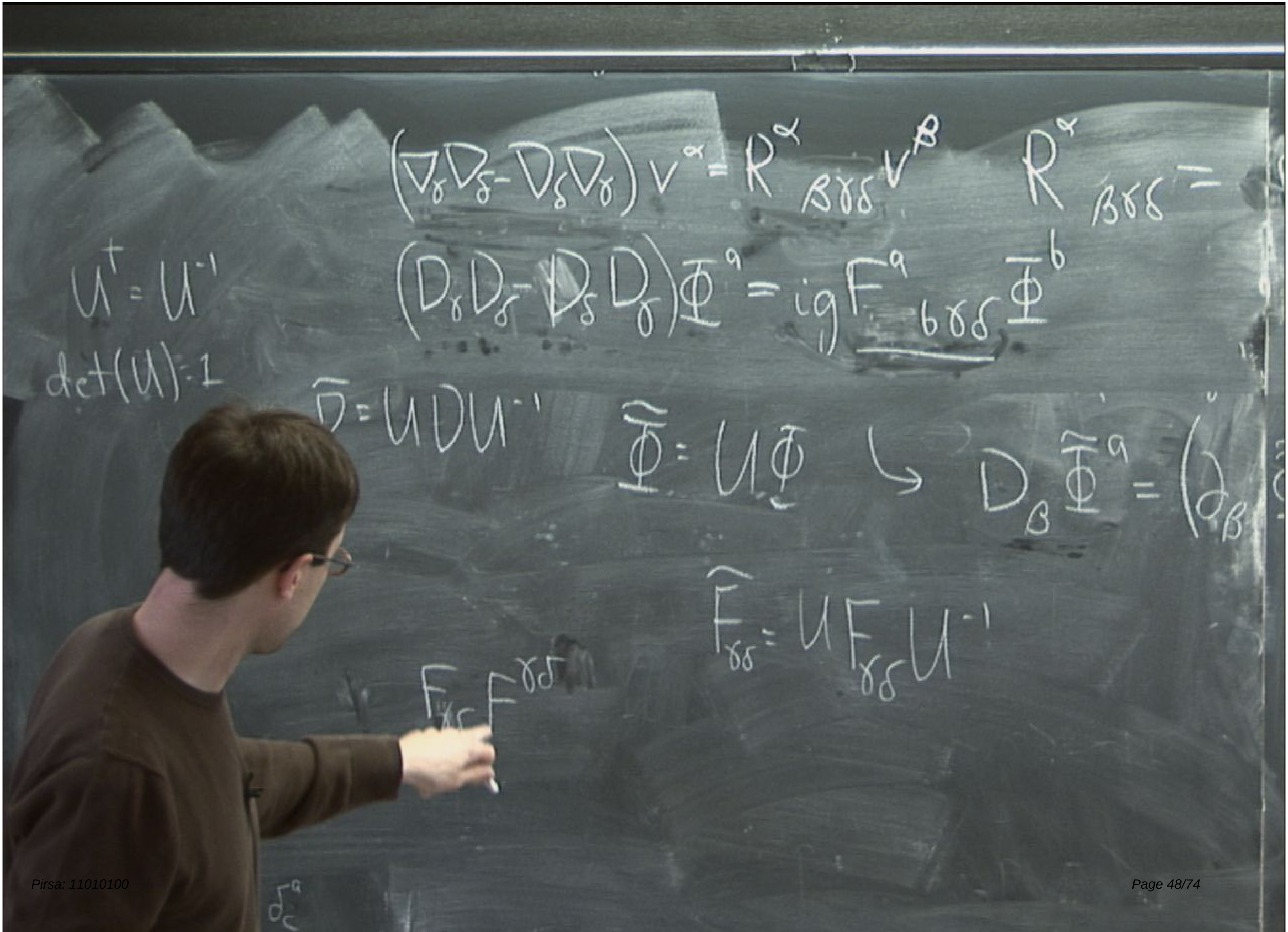
$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = i g F^a{}_{b\gamma\delta}\Phi^b$$

$$\tilde{D} = U D U^{-1}$$

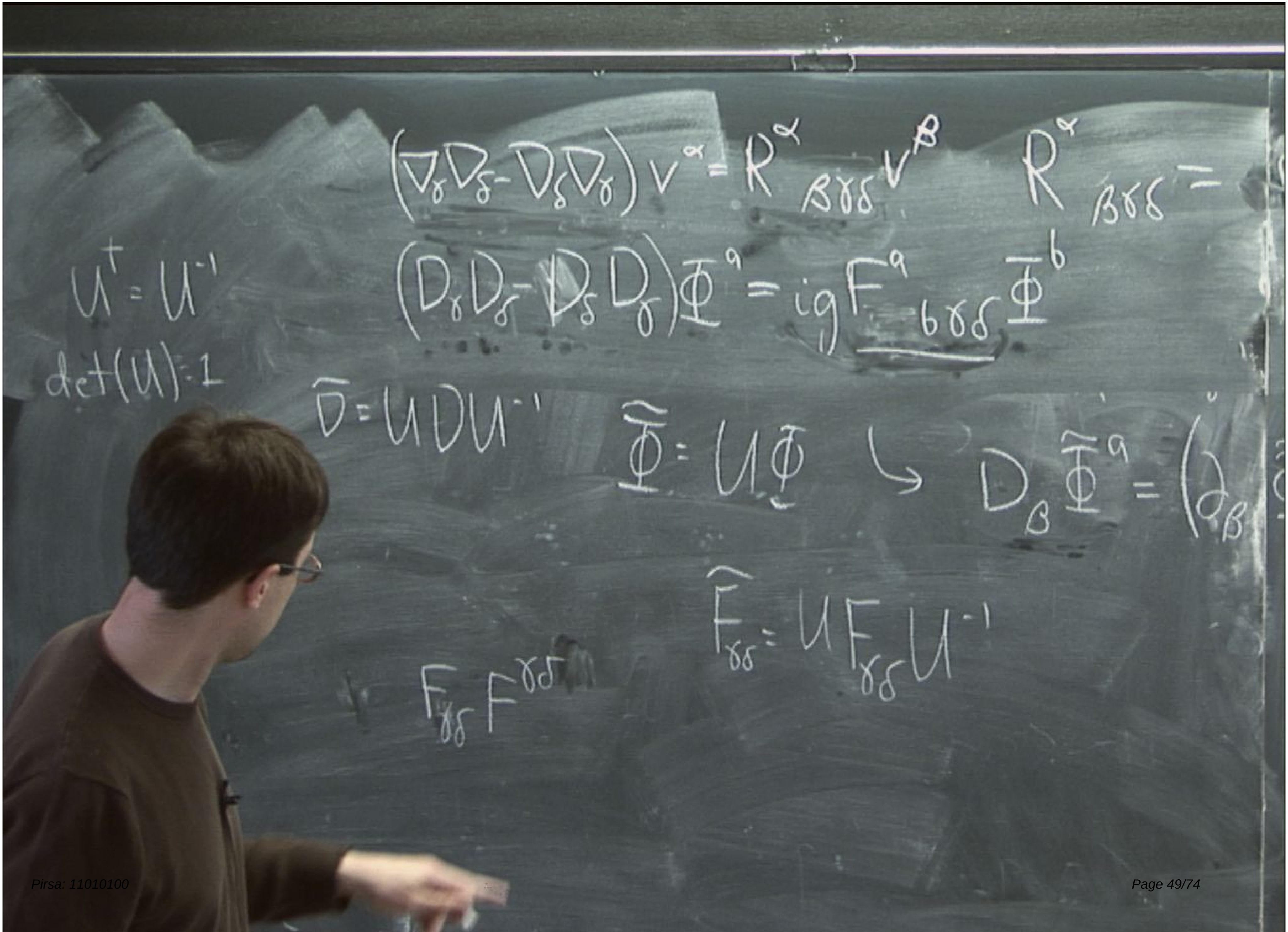
$$\tilde{\Phi} = U \Phi \quad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = i g F^a{}_{b\gamma\delta} \Phi^b$
$\tilde{D} = U D U^{-1}$
$\tilde{\Phi} = U \Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta$
$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$



$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = igF^a{}_{b\gamma\delta}\Phi^b$
$\tilde{D} = UDU^{-1}$
$\tilde{\Phi} = U\Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta$
$\tilde{F}_{\gamma\delta} = UF_{\gamma\delta}U^{-1}$
$F_{\gamma\delta}F^{\gamma\delta}$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \quad R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$
$\tilde{D} = U D U^{-1}$
$\tilde{\Phi} = U\Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta$
$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$
$F_{\gamma\delta} F^{\gamma\delta}$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig\underline{F^a{}_{b\gamma\delta}}\,\Phi^b$$

$$\tilde{D} = U D U^{-1} \qquad \tilde{\Phi} = U\Phi \qquad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

$$(F_{\gamma\delta} F^{\gamma\delta})^a{}_b$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta}\Phi^b$$

$$\tilde{D} = U D U^{-1} \qquad \tilde{\Phi} = U\Phi \quad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

$$\tilde{F}_{\gamma\delta}\tilde{F}^{\gamma\delta} = U F_{\gamma\delta} F^{\gamma\delta} U^{-1}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

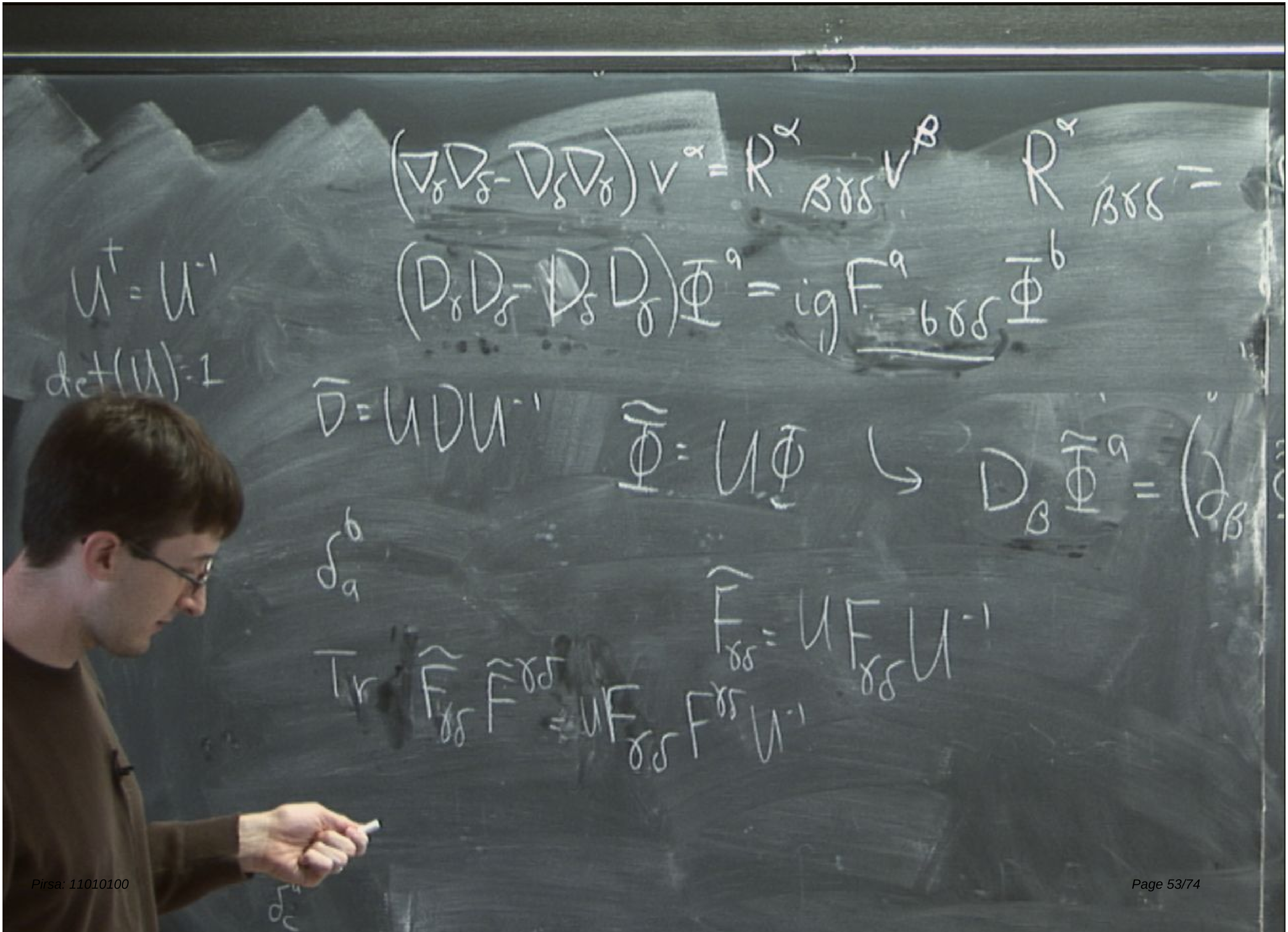
$$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$$

$$\tilde{D} = U D U^{-1} \qquad \tilde{\Phi} = U \Phi \quad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

$$\mathrm{Tr}\ \tilde{F}_{\gamma\delta} \tilde{F}^{\gamma\delta} = U F_{\gamma\delta} F^{\gamma\delta} U^{-1}$$

$$\delta^a_c$$


$(\nabla_\gamma D_\delta - D_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$\tilde{D} = U D U^{-1}$
$\tilde{\Phi} = U \Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta$
δ^b_a
$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$
$\mathrm{Tr}\, \tilde{F}_{\gamma\delta} \tilde{F}^{\gamma\delta} = U F_{\gamma\delta} F^{\gamma\delta} U^{-1}$
δ^a_c

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$$U^\dagger = U^{-1}$$

$$\det(U) = 1$$

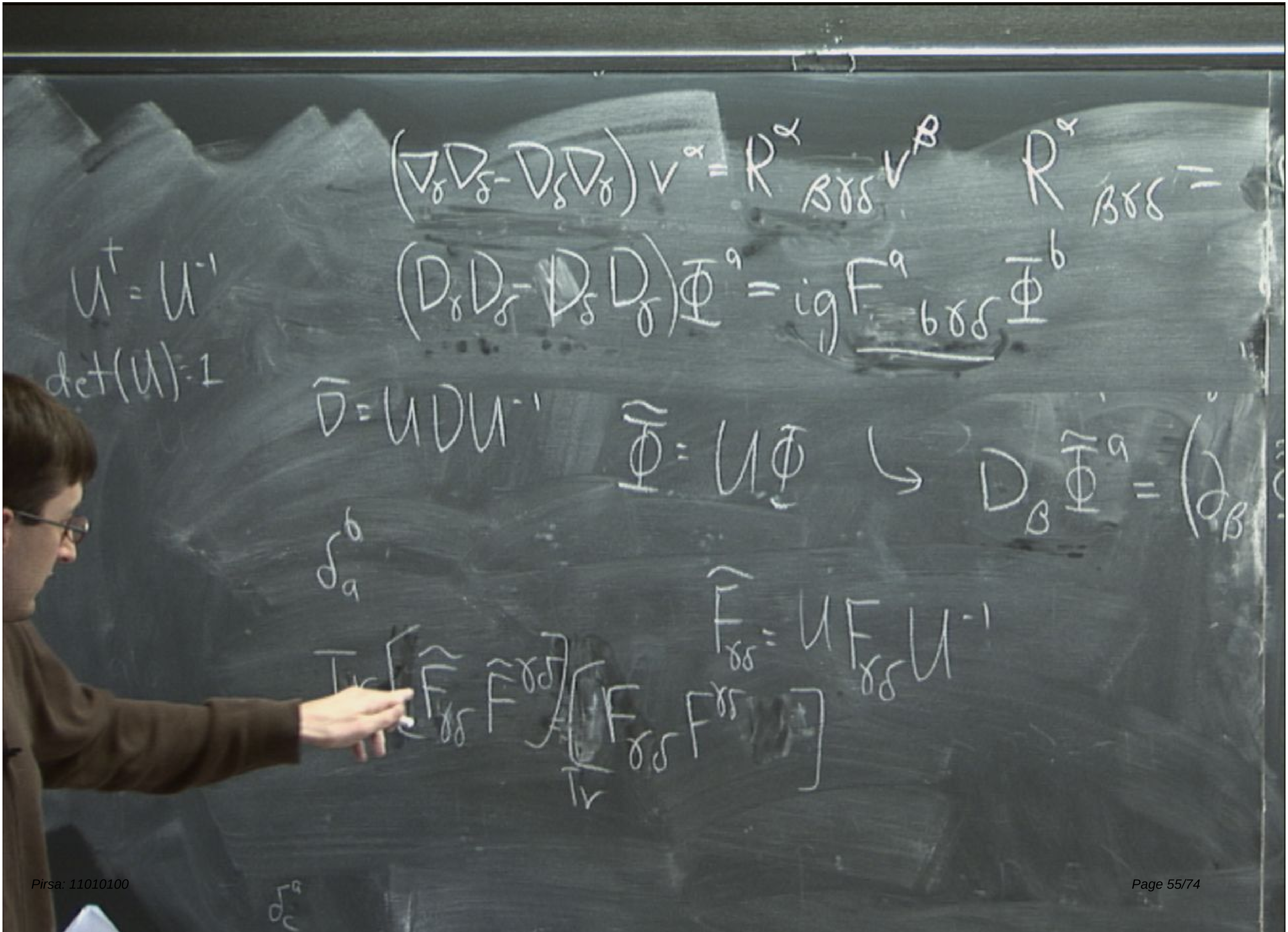
$$(D_\gamma D_\delta - D_\delta D_\gamma) \Phi^a = ig F^a{}_{b\gamma\delta} \Phi^b$$

$$\tilde{D} = U D U^{-1} \qquad \tilde{\Phi} = U \Phi \quad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

$$\delta_a^b$$

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

$$\mathrm{Tr}[\tilde{F}_{\gamma\delta} \tilde{F}^{\gamma\delta}] = \mathrm{Tr}[U F_{\gamma\delta} F^{\gamma\delta} U^{-1}]$$


$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} =$
$U^\dagger = U^{-1}$
$\det(U) = 1$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = i g F^a{}_{b\gamma\delta}\Phi^b$
$\tilde{D} = U D U^{-1}$
$\tilde{\Phi} = U\Phi$
$D_\beta \tilde{\Phi}^a = (\partial_\beta$
δ_a^b
$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$
$\mathrm{Tr}[\tilde{F}_{\gamma\delta} \tilde{F}^{\gamma\delta}] = \mathrm{Tr}[F_{\gamma\delta} F^{\gamma\delta}]$
δ_c^a

$$(\nabla_\gamma D_\delta - D_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig \underline{F^a{}_{b\gamma\delta}}\,\Phi^b$$

$$\tilde{D} = U D U^{-1} \qquad \tilde{\Phi} = U\Phi \quad \hookrightarrow \quad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

δ_a^b

$$\tilde{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

$$\mathrm{Tr}[\tilde{F}_{\gamma\delta}\tilde{F}^{\gamma\delta}] = \mathrm{Tr}[F_{\gamma\delta}F^{\gamma\delta}]$$

δ_c^a

$$(\nabla_\gamma D_\delta - D_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta \qquad R^\alpha{}_{\beta\gamma\delta} =$$

$U^\dagger = U^{-1}$

$\det(U) = 1$

$$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = igF^a{}_{b\gamma\delta}\Phi^b$$

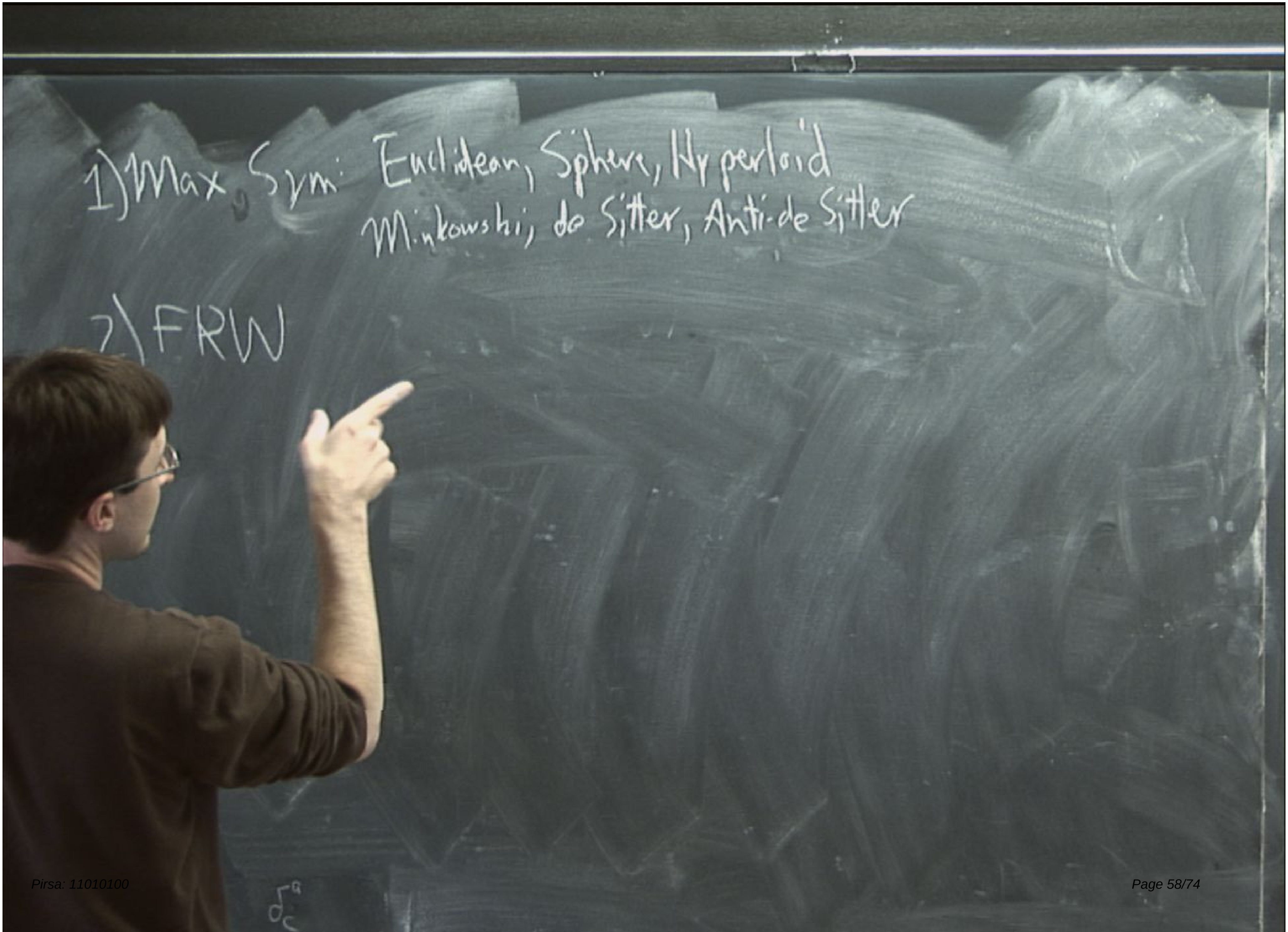
$$\hat{D} = U D U^{-1} \qquad \tilde{\Phi} = U\Phi \qquad D_\beta \tilde{\Phi}^a = (\partial_\beta$$

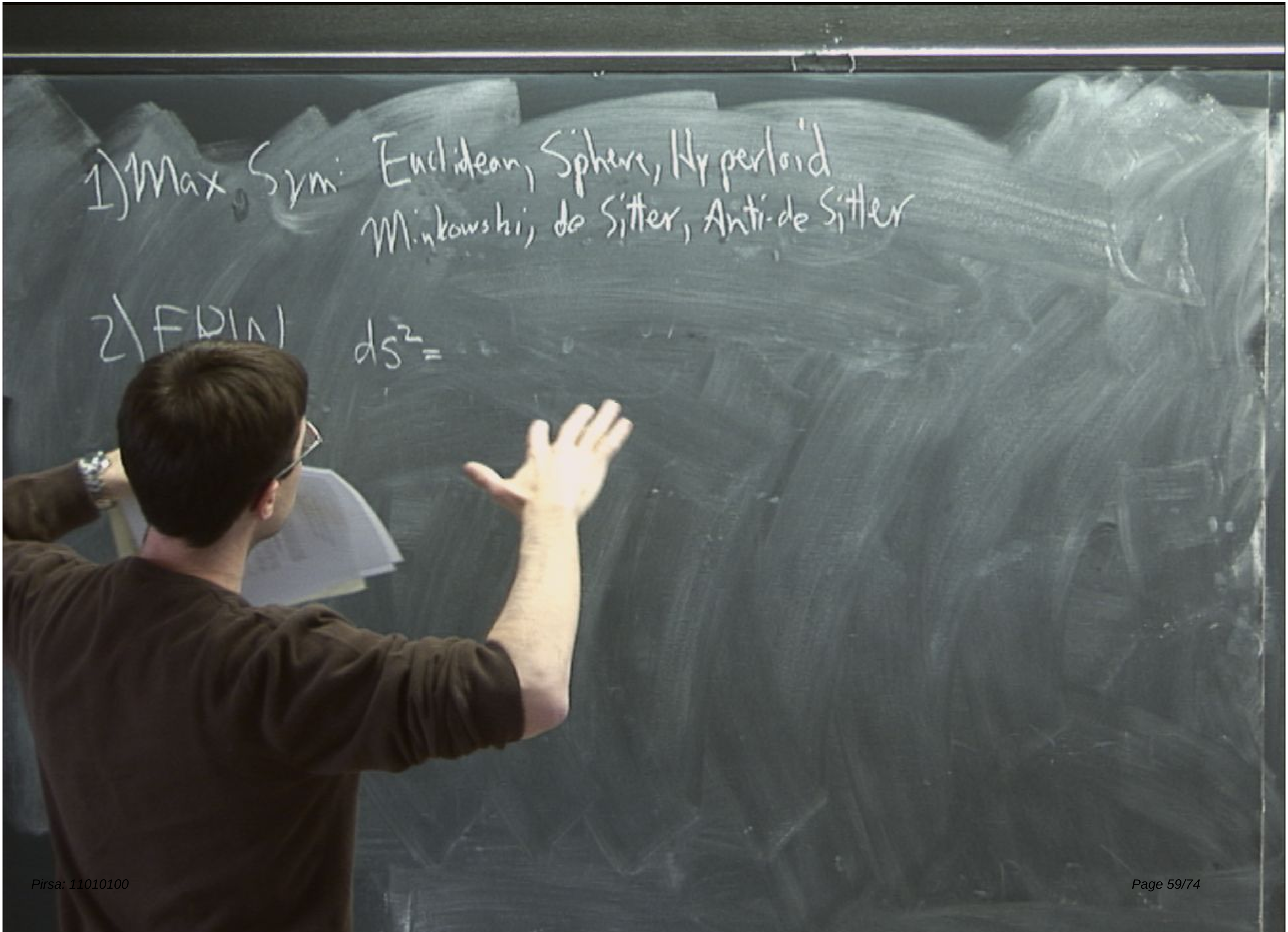
δ_a^b

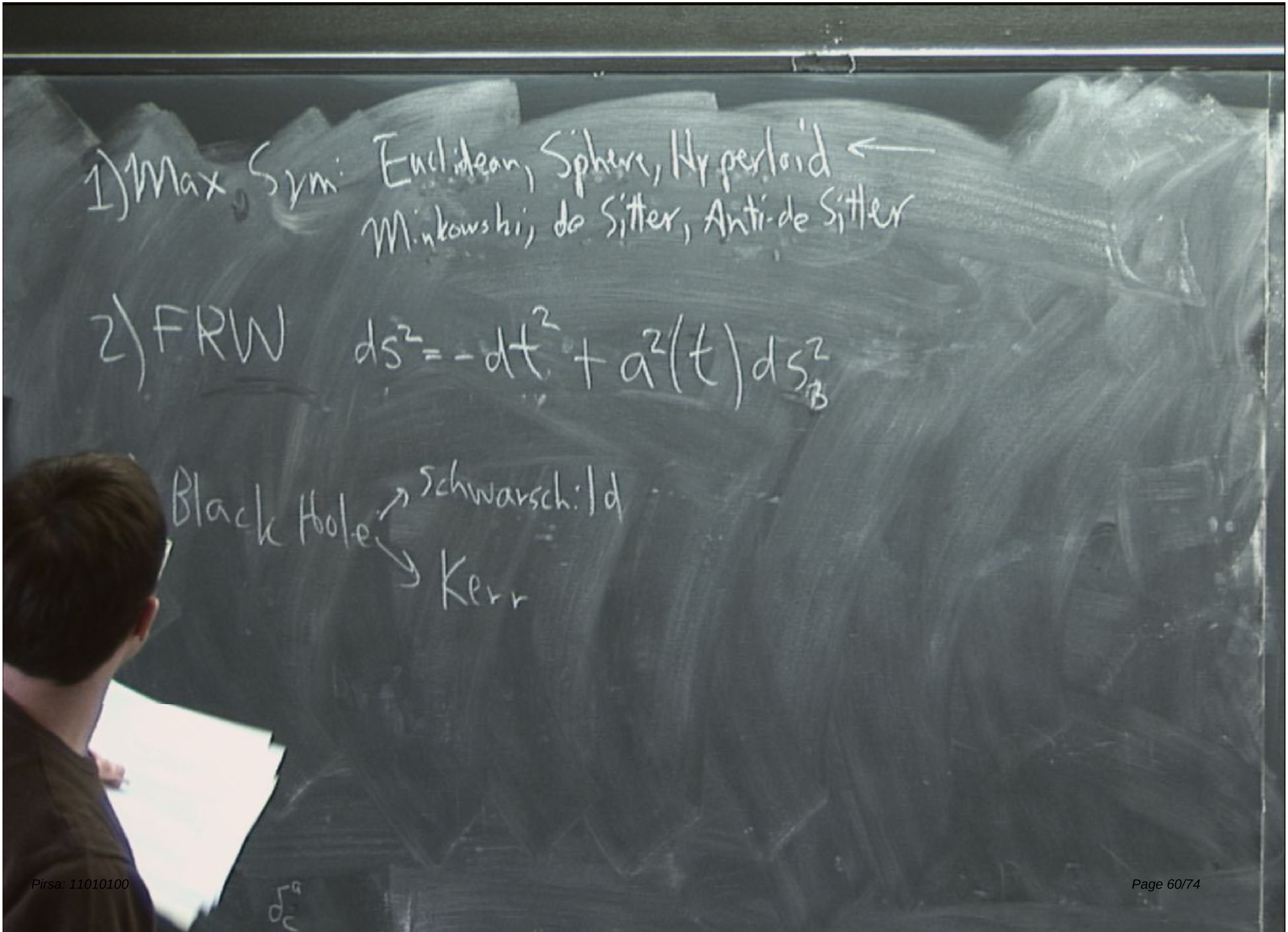
$$\hat{F}_{\gamma\delta} = U F_{\gamma\delta} U^{-1}$$

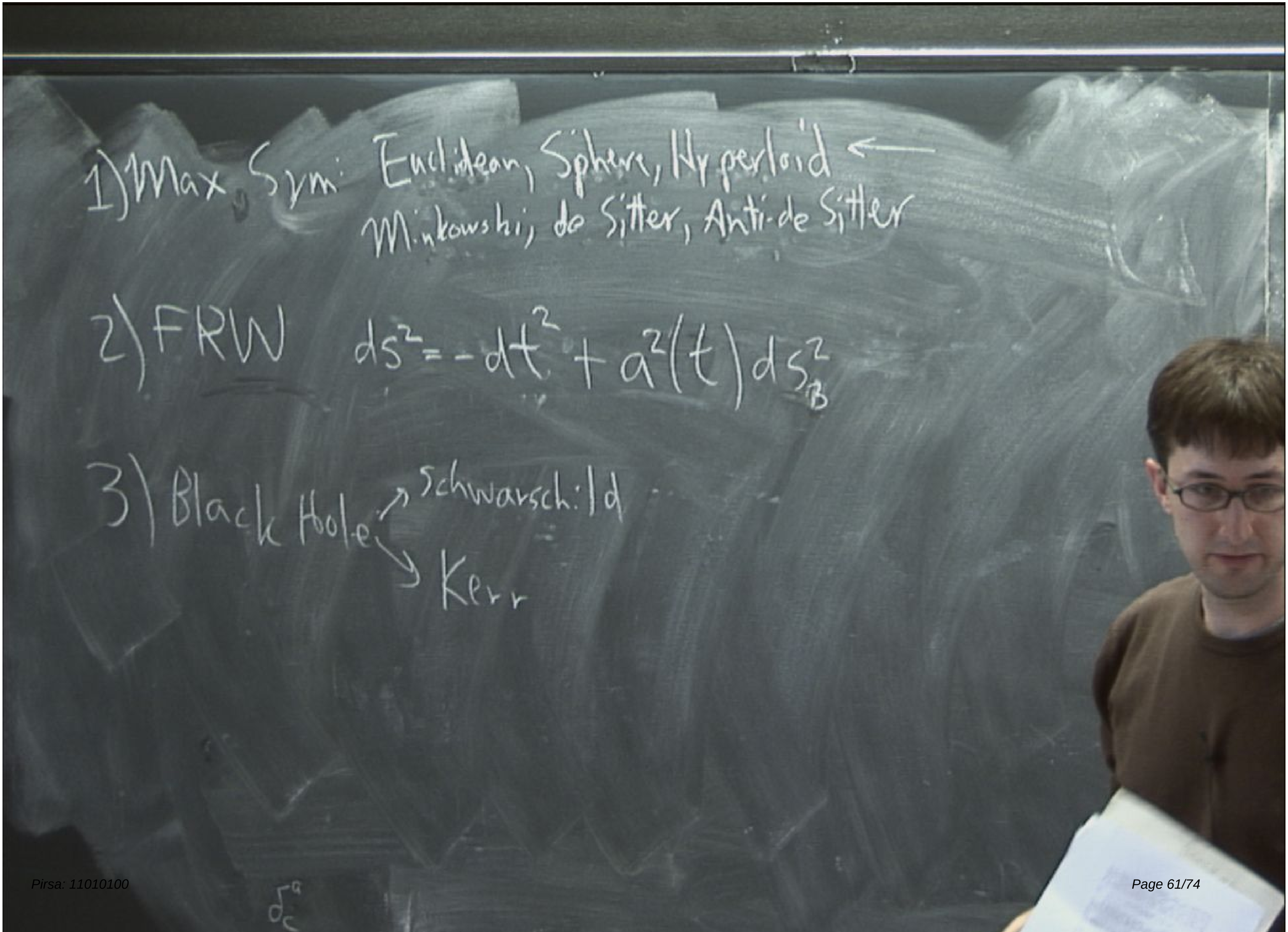
$$\mathrm{Tr}[\hat{F}_{\gamma\delta}\tilde{F}^{\gamma\delta}] = \mathrm{Tr}[F_{\gamma\delta}F^{\gamma\delta}]$$

δ_c^a


1) Max. Sym: Euclidean, Sphere, Hyperboloid
Minkowski, de Sitter, Anti-de Sitter
2) FRW


1) Max. Sym: Euclidean, Sphere, Hyperloid
Minkowski, de Sitter, Anti-de Sitter
2) FRW $ds^2 =$


1) Max Sym: Euclidean, Sphere, Hyperboloid
Minkowski, de Sitter, Anti-de Sitter
2) FRW $ds^2 = -dt^2 + a^2(t)\,ds_3^2$
Black Hole
Schwarzschild
Kerr


1) Max Sym: Euclidean, Sphere, Hyperloid
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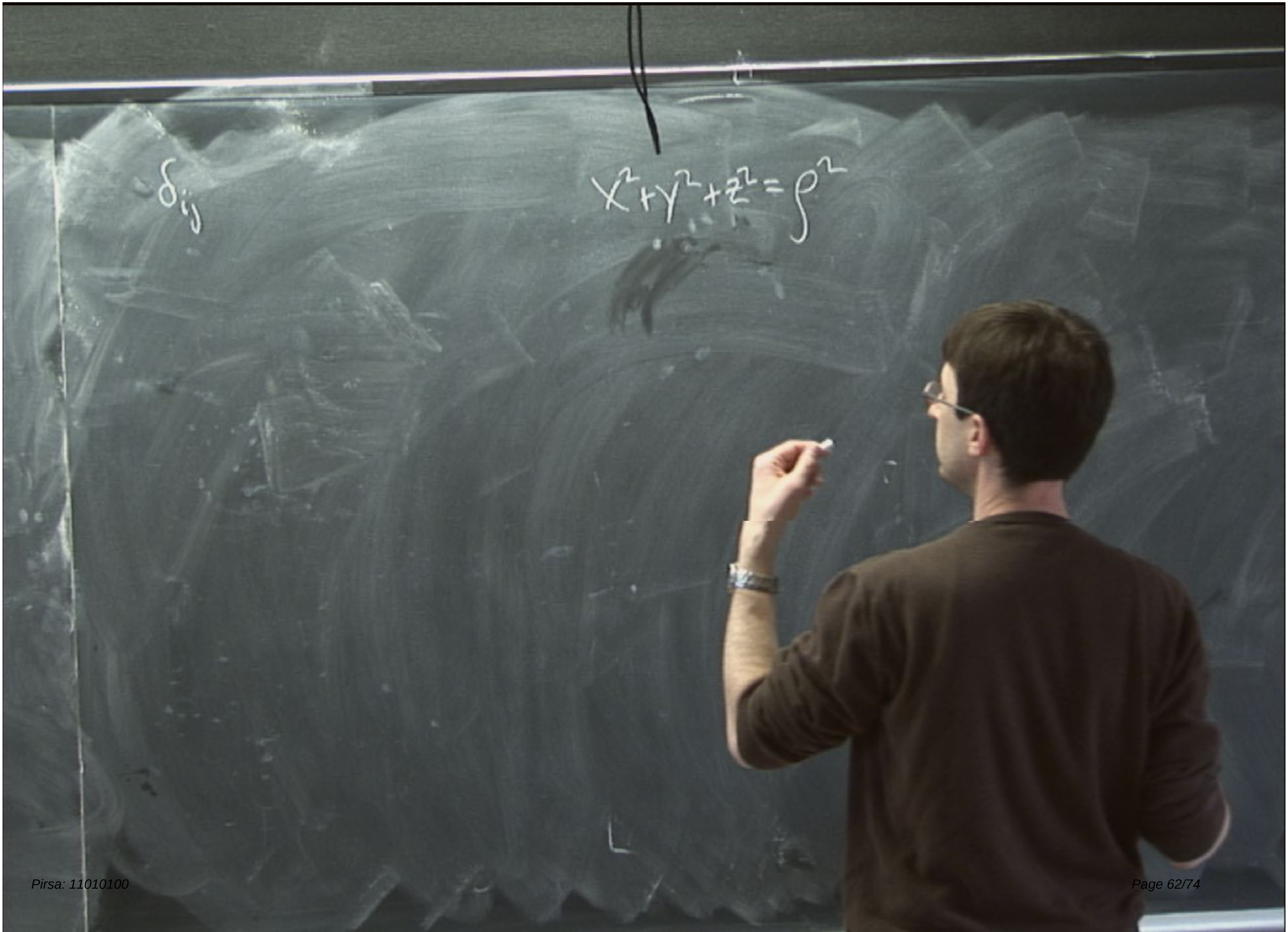

δ_{ij}
$x^2+y^2+z^2=\rho^2$

δ_{ij}

$$x^2+y^2+z^2=\rho^2$$

$$dx^2+dy^2+dz^2=ds^2$$

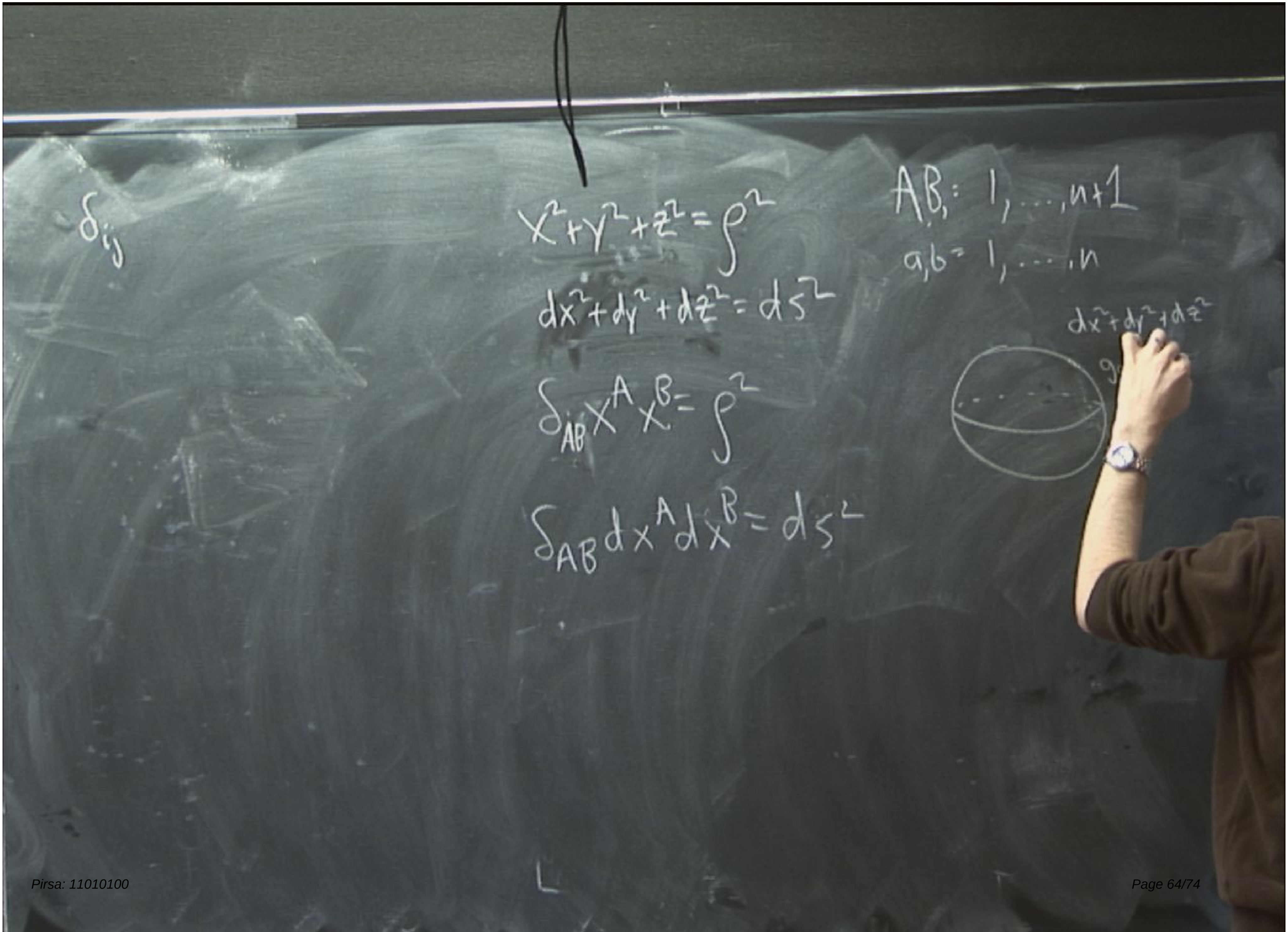
$$\delta_{AB}X^A X^B=\rho^2$$

$$\delta_{AB}dx^A dx^B=ds^2$$

$A,B: 1,\ldots,n+1$

$a,b = 1,\ldots,n$

$dx^2+dy^2+dz^2$


δ_{ij}
$x^2+y^2+z^2=\rho^2$
$dx^2+dy^2+dz^2=ds^2$
$\delta_{AB}X^A X^B=\rho^2$
$\delta_{AB}dx^A dx^B=ds^2$
$A,B = 1,\dots,n+1$
$a,b = 1,\dots,n$
$dx^2+dy^2+dz^2$
g

δ_{ij}

$$x^2 + y^2 + z^2 = \rho^2$$

$$dx^2 + dy^2 + dz^2 = ds^2$$

$$\delta_{AB} X^A X^B = \rho^2$$

$$\delta_{AB} dx^A dx^B = ds^2$$

$A, B = 1, \ldots, n+1$

$a, b = 1, \ldots, n$

$dx^2 + dy^2 + dz^2$

$g_{ab} dx^a dx^b$

δ_{ij}

$$x^2+y^2+z^2=\rho^2$$

$$dx^2+dy^2+dz^2=ds^2$$

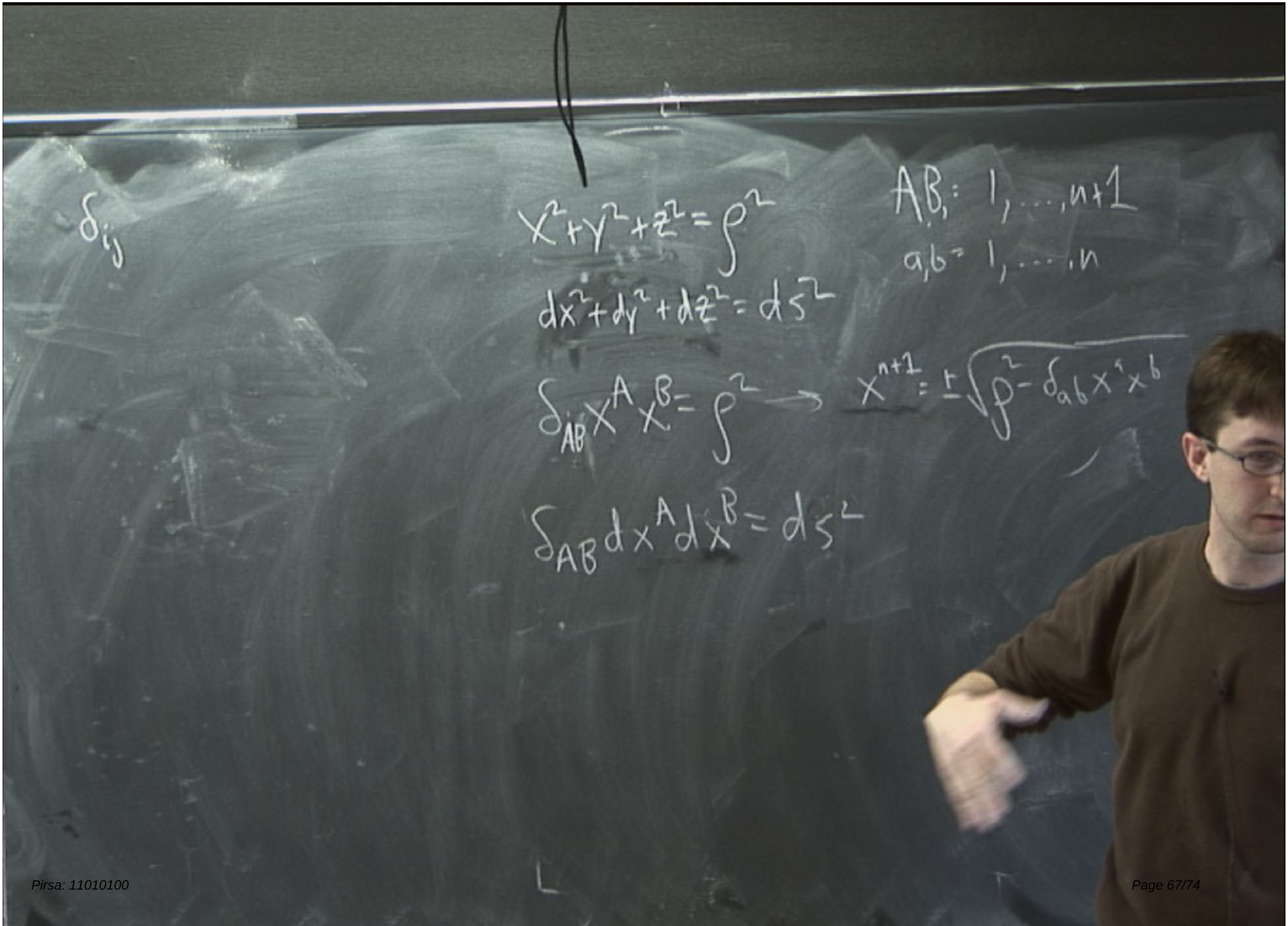
$$\delta_{AB}x^Ax^B=\rho^2$$

$$\delta_{AB}dx^Adx^B=ds^2$$

$A,B: 1,\dots,n+1$

$a,b = 1,\dots,n$

$$dx^2+dy^2+dz^2 = \boxed{g_{ab}}dx^a dx^b$$


δ_{ij}
$x^2+y^2+z^2=\rho^2$
$dx^2+dy^2+dz^2=ds^2$
$\delta_{AB}x^Ax^B=\rho^2 \rightarrow x^{n+1}=\pm\sqrt{\rho^2-\delta_{ab}x^ax^b}$
$\delta_{AB}dx^Adx^B=ds^2$
$A,B:\ 1,\dots,n+1$
$a,b=1,\dots,n$

$$x^2+y^2+z^2=\rho^2$$

$A,B = 1,\dots,n+1$

$a,b = 1,\dots,n$

$$dx^2+dy^2+dz^2=ds^2$$

$$\delta_{AB}X^A X^B=\rho^2 \longrightarrow X^{n+1}=\pm\sqrt{\rho^2-\delta_{ab}x^a x^b}$$

$$\delta_{AB}\,dx^A dx^B=ds^2$$

$$ds^2=\left[\delta_{ij}+\frac{\delta_{ik}\delta_{jl}x^k x^l}{\rho^2-\delta_{kl}x^k x^l}\right]dx^i dx^j$$

δ_{ij}

$X^2+Y^2+Z^2$

dx^2+dy^2+

$a^2(t)\,ds_B^2$

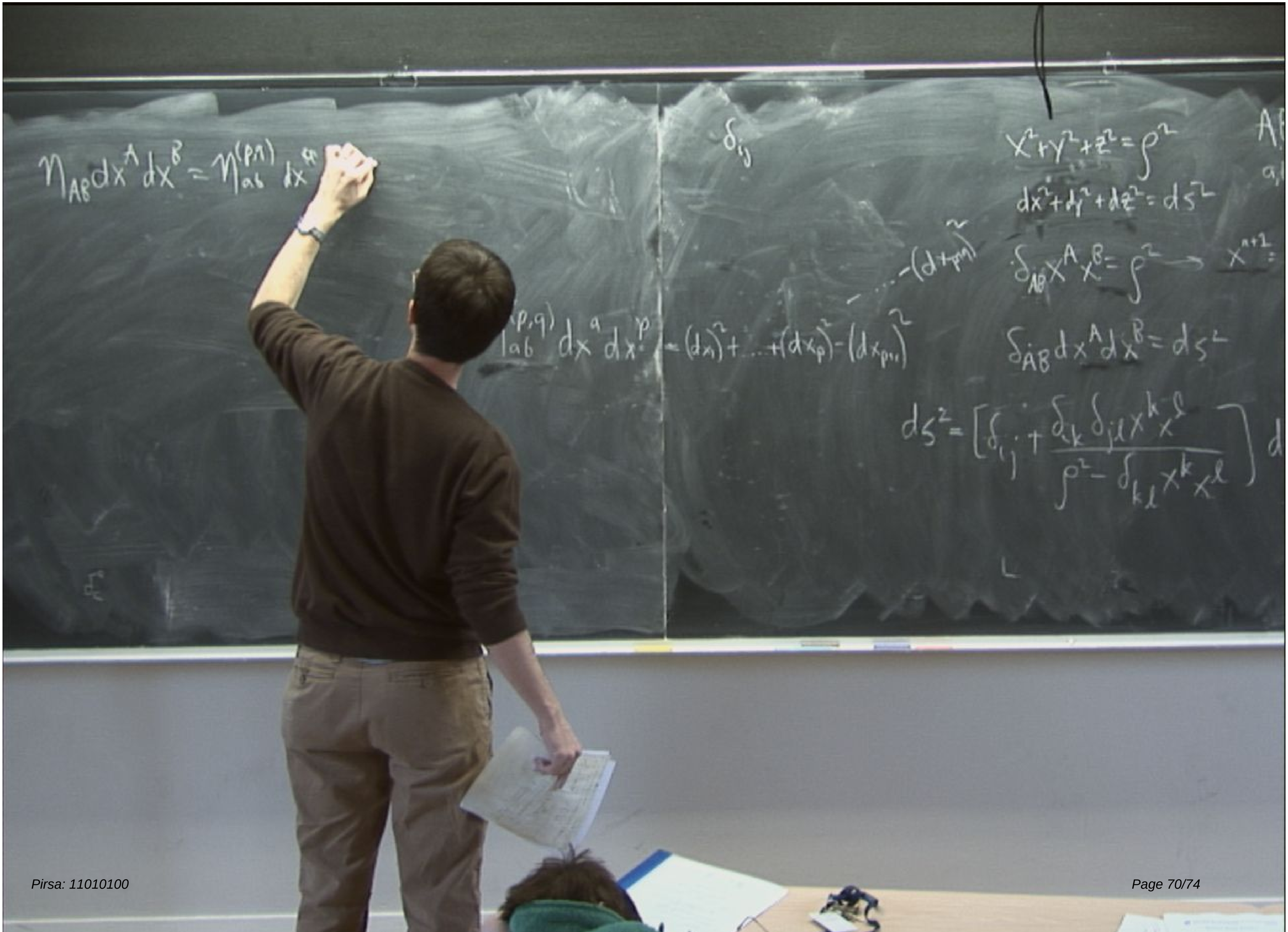
$\delta_{AB}X^AX^B=$

$-(d+p+q)^2$

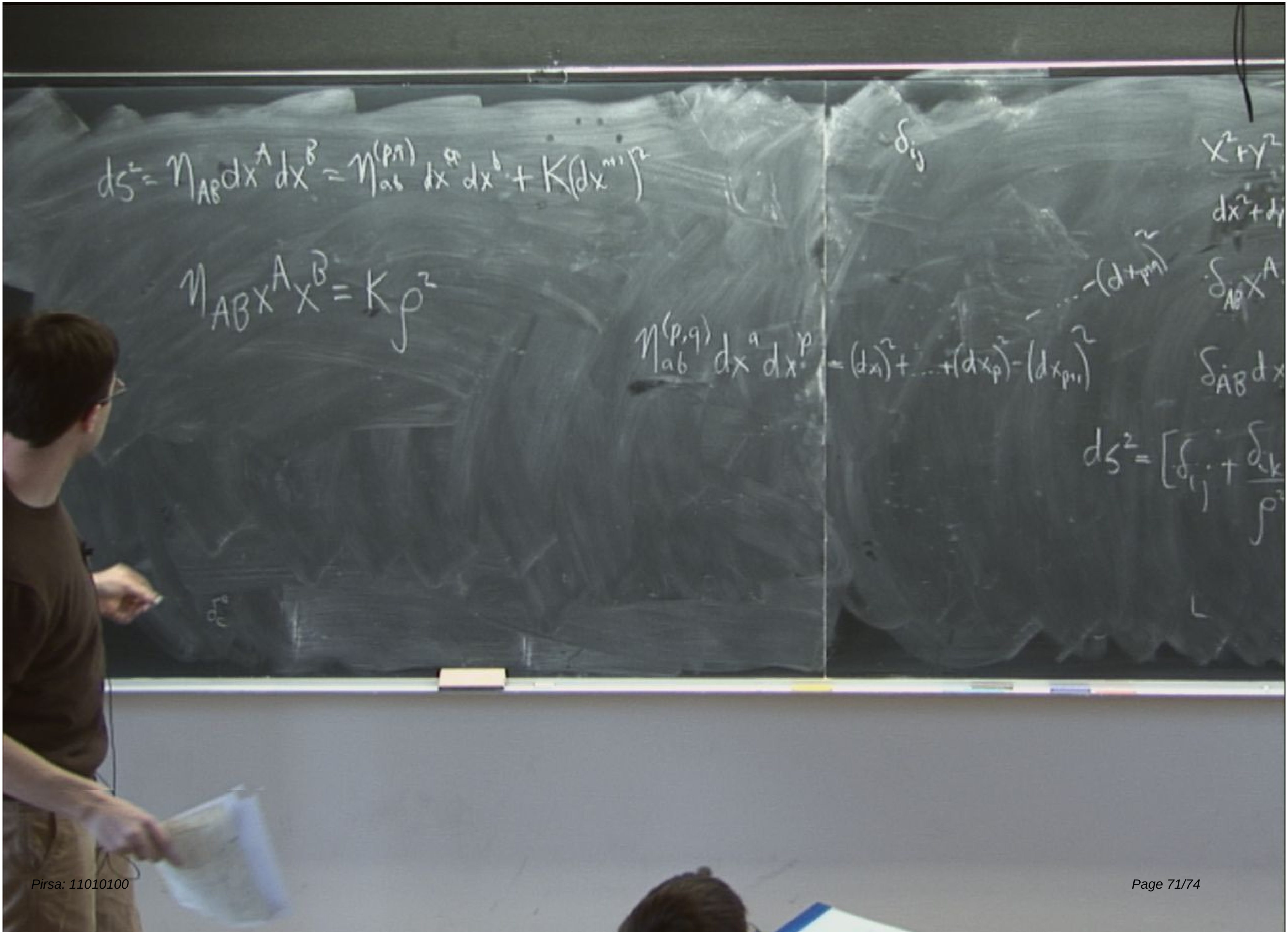
$\eta^{(p,q)}_{ab}\,dx^a\,dx^b = (dx_1)^2+\dots+(dx_p)^2-(dx_{p+1})^2$

$\delta_{AB}\,dx^A$

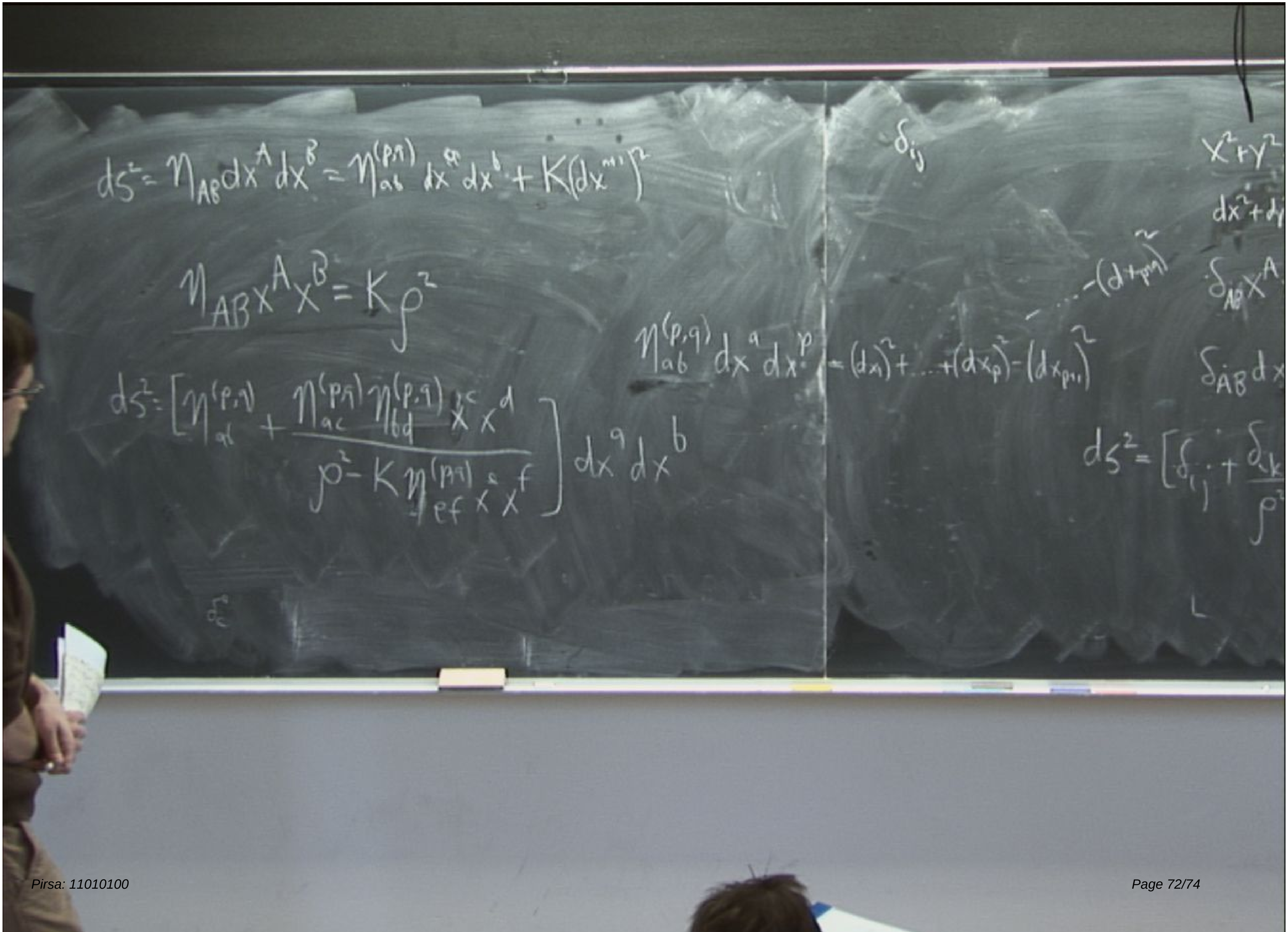
$ds^2=\left[\delta_{ij}+\frac{\delta_{ik}\delta}{\rho^2-}\right.$



$\eta_{AB} dx^A dx^B = \eta^{(p,q)}_{ab} dx^a$
δ_{ij}
$x^2+y^2+z^2=\rho^2$
$dx^2+dy^2+dz^2=ds^2$
$\delta_{AB} x^A x^B = \rho^2 \to x^{n+1} =$
$\delta_{AB} dx^A dx^B = ds^2$
$ds^2 = \left[\delta_{ij} + \frac{\delta_{ik}\delta_{jl} x^k x^l}{\rho^2 - \delta_{kl} x^k x^l}\right]$



$ds^2 = \eta_{AB} dx^A dx^B = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\eta_{AB} x^A x^B = K\rho^2$
$\eta^{(p,q)}_{ab} dx^a dx^b = (dx_1)^2 + \dots + (dx_p)^2 - (dx_{p+1})^2 \dots - (dx_{p+q})^2$
δ_{ij}
x^2+y^2
dx^2+dy
$\delta_{AB} x^A$
$\delta_{AB} dx$
$ds^2 = [\delta_{ij} + \frac{\delta_{ik}}{\rho^2}$



$ds^2 = \eta_{AB} dx^A dx^B = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{m+1})^2$
$\eta_{AB} x^A x^B = K \rho^2$
$ds^2 = \left[\eta^{(p,q)}_{ab} + \frac{\eta^{(p,q)}_{ac} \eta^{(p,q)}_{bd} x^c x^d}{\rho^2 - K \eta^{(p,q)}_{ef} x^e x^f} \right] dx^a dx^b$
$\eta^{(p,q)}_{ab} dx^a dx^b = (dx_1)^2 + \dots + (dx_p)^2 - (dx_{p+1})^2 \dots - (dx_{p+q})^2$
δ_{ij}
$x^2 + y^2$
$\delta_{AB} x^A$
$\delta_{AB} dx$
$ds^2 = [\delta_{ij} +$

δ_{ij}

	$K=+1$	$K=-1$	$K=0$
$(n,0)$	n-sphere	n-hyperbolid	Euclidean
$(n\text{-}1,1)$	n-d de Sitter	anti-de Sitter	Mink.

$$x^p \;=\; (dx_1)^2 + \ldots + (dx_p)^2 - (dx_{p+1})^2$$

$K=+1,\ K$

δ_{ij}

	$K=+1$	$K=-1$	$K=0$
$(n,0)$	n-sphere	n-hyperbolic	Euclidean
$(n-1,1)$	n-d de Sitter	anti-de Sitter	Mink.

$$x^p \quad = (dx_1)^2 + \ldots + (dx_p)^2 - (dx_{p+1})^2$$

$K=+1,\ K$

$$R^{\alpha}{}_{\beta\gamma\delta} = K \frac{(n-1)}{\rho^2} [$$