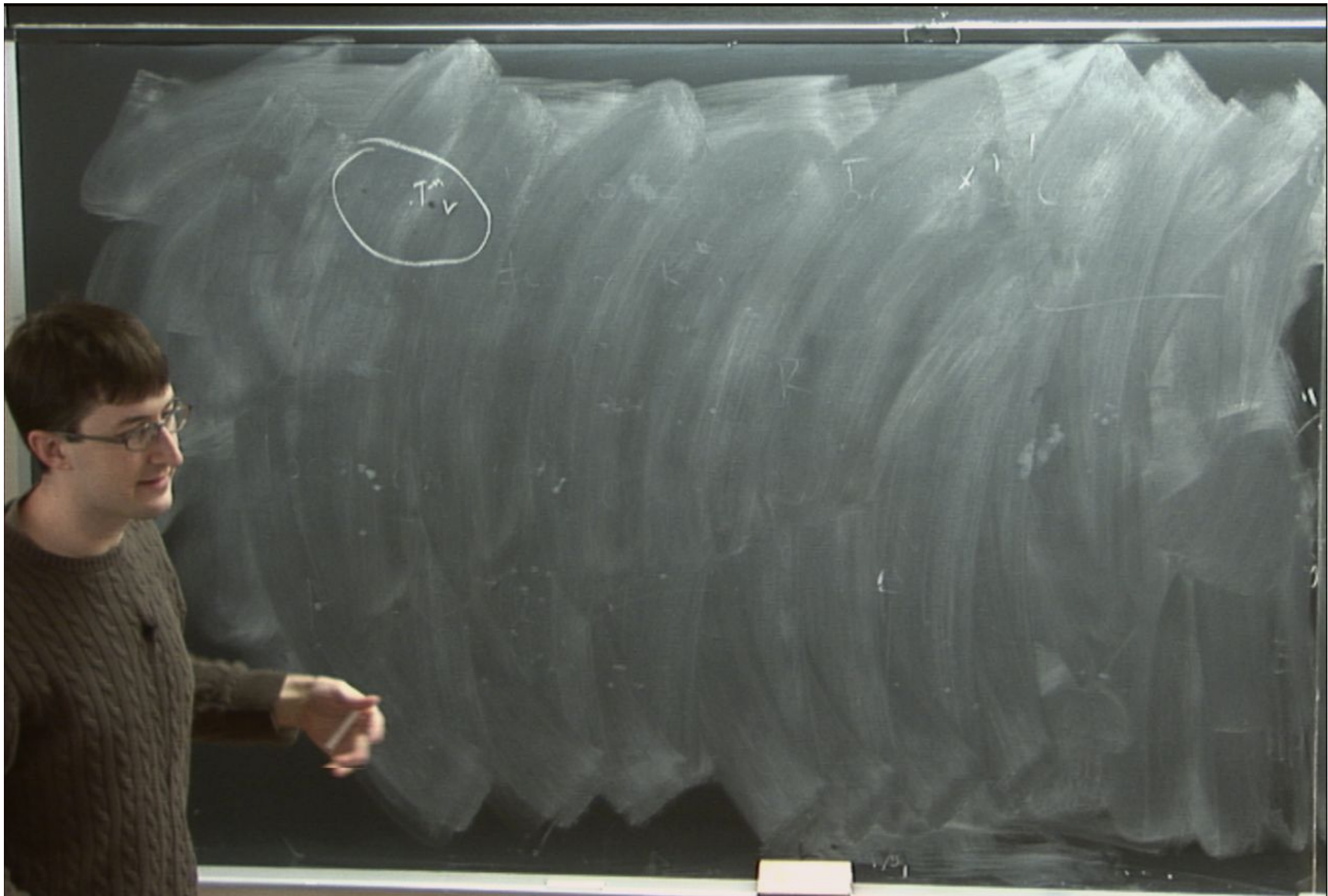


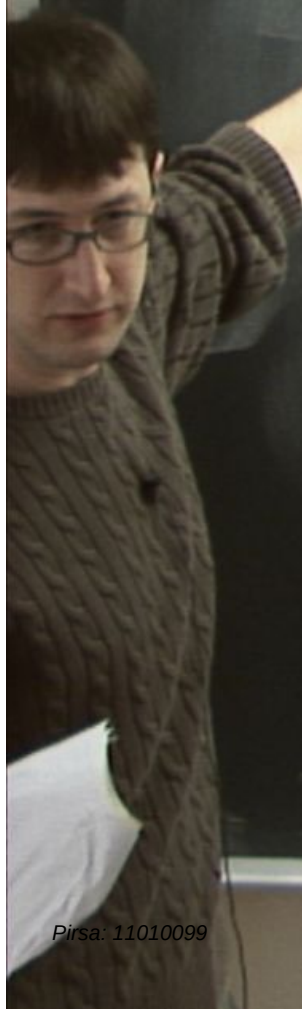
Title: Cosmology Review - Lecture 2

Date: Jan 24, 2011 11:30 AM

URL: <http://pirsa.org/11010099>

Abstract:





$$\begin{matrix} T^{\mu\nu} \\ \gamma^{\alpha} \\ \beta \delta \end{matrix}$$

$$\partial_{\mu}$$

$$\nabla_{\mu} T^{\alpha}_{\beta}$$

$$\begin{matrix} T^{\mu} \\ \nu \\ \rho \\ \sigma \end{matrix}$$

 ∂_{μ}

$$\nabla_{\mu} T^{\alpha}_{\beta}$$

$$R^\alpha_{\beta\gamma\delta} \quad v^\alpha(x)$$

$$(\nabla_\delta \nabla_\gamma - \nabla_\gamma \nabla_\delta) v^\alpha = R^\alpha_{\beta\gamma\delta} v^\beta$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\delta}\nabla_{\gamma}-\nabla_{\gamma}\nabla_{\delta})v^{\alpha}=\underbrace{R^{\alpha}_{\beta\gamma\delta}}_{\text{Riemann tensor}}, v^{\beta}$$



$$R^{\alpha}{}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\delta}\nabla_{\gamma}-\nabla_{\gamma}\nabla_{\delta})v^{\alpha}=R^{\alpha}{}_{\beta\gamma\delta}v^{\beta}$$

$R^\alpha_{\beta\gamma\delta}$ $V^\alpha(x)$

$$(\nabla_\delta \nabla_\gamma - \nabla_\gamma \nabla_\delta) V^\alpha = \underbrace{R^\alpha_{\beta\gamma\delta}}_{\text{Riemann tensor}} V^\beta$$

$$\nabla_\gamma (\nabla_\delta V^\alpha) - \nabla_\delta (\nabla_\gamma V^\alpha)$$

2

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha}=R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\gamma}(\nabla_{\delta}v^{\alpha})+\Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu}-\Gamma^{\alpha}_{\gamma\nu}\nabla_{\delta}v^{\nu}$$

2

$$R^\alpha_{\beta\gamma\delta} \quad v^\alpha(x)$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha_{\beta\gamma\delta} v^\beta$$

$$\nabla_\gamma (\nabla_\delta v^\alpha) - \nabla_\delta (\nabla_\gamma v^\alpha)$$

$$\rightarrow \partial_\delta (\nabla_\gamma v^\alpha) + \Gamma^\alpha_{\delta\nu} \nabla_\gamma v^\nu - \Gamma^\alpha_{\gamma\delta} \nabla_\nu v^\nu$$

2

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\delta}\nabla_{\gamma}-\nabla_{\gamma}\nabla_{\delta})v^{\alpha}=R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha})+\Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu}-\Gamma^{\alpha}_{\gamma\delta}\nabla_{\nu}v^{\nu}-$$

$$\downarrow$$

$$\partial_{\delta}v^{\alpha}+\Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha}=R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha})+\Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu}-\Gamma^{\alpha}_{\gamma\delta}\nabla_{\nu}v^{\nu}-$$

$$\downarrow$$

$$\partial_{\delta}v^{\alpha}+\Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta}=\Gamma^{\alpha}_{\beta\delta,\gamma}-\Gamma^{\alpha}_{\beta\gamma,\delta}+\Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta}-\Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha} = R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha}) - \nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha}) + \Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu} - \Gamma^{\alpha}_{\gamma\delta}\nabla_{\nu}v^{\nu} -$$

$$\downarrow$$

$$\partial_{\delta}v^{\alpha} + \Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta} - \Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha}=R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha})+\Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu}-\Gamma^{\alpha}_{\gamma\delta}\nabla_{\nu}v^{\nu}-$$

$$\downarrow$$

$$\partial_{\delta}v^{\alpha}+\Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta}=\Gamma^{\alpha}_{\beta\delta,\gamma}-\Gamma^{\alpha}_{\beta\gamma,\delta}+\Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta}-\Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R_{\beta\gamma}=R^{\alpha}_{\beta\alpha\gamma}$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha}=R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha}) + \Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu} - \Gamma^{\alpha}_{\gamma\nu}\nabla_{\delta}v^{\nu}$$

$$\downarrow$$

$$\partial_{\delta}v^{\alpha} + \Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta} - \Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R_{\beta\delta} = R^{\alpha}_{\beta\alpha\delta}$$

Riemann

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta}-\nabla_{\delta}\nabla_{\gamma})v^{\alpha}=\underbrace{R^{\alpha}_{\beta\gamma\delta}}_{\text{Riemann tensor}}v^{\beta}$$

$$(\nabla_{\gamma}(\nabla_{\delta}v^{\alpha})-\nabla_{\delta}(\nabla_{\gamma}v^{\alpha}))$$

$$\partial_{\gamma}(\nabla_{\delta}v^{\alpha})+\Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu}-\Gamma^{\alpha}_{\gamma\nu}\nabla_{\delta}v^{\nu}-$$

$$\downarrow$$

$$\partial_{\gamma}v^{\alpha}+\Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta}=\Gamma^{\alpha}_{\beta\delta,\gamma}-\Gamma^{\alpha}_{\beta\gamma,\delta}+\Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta}-\Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R_{\beta\gamma}=R^{\alpha}_{\beta\alpha\gamma}$$

$$R^{\alpha}_{\beta\gamma\delta} \quad v^{\alpha}(x)$$

$$(\nabla_{\gamma}\nabla_{\delta} - \nabla_{\delta}\nabla_{\gamma})v^{\alpha} = R^{\alpha}_{\beta\gamma\delta}v^{\beta}$$

$$\nabla_{\gamma}(\nabla_{\delta}v^{\alpha}) - \nabla_{\delta}(\nabla_{\gamma}v^{\alpha})$$

$$\rightarrow \partial_{\delta}(\nabla_{\gamma}v^{\alpha}) + \Gamma^{\alpha}_{\delta\nu}\nabla_{\gamma}v^{\nu} - \Gamma^{\alpha}_{\gamma\delta}\nabla_{\nu}v^{\nu} -$$

$$\partial_{\delta}v^{\alpha} + \Gamma^{\alpha}_{\delta\nu}v^{\nu}$$

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\gamma\sigma}\Gamma^{\sigma}_{\beta\delta} - \Gamma^{\alpha}_{\delta\sigma}\Gamma^{\sigma}_{\beta\gamma}$$

$$R_{\beta\gamma} = R^{\alpha}_{\beta\alpha\gamma}$$

Riemann

$$\begin{matrix} T^\mu \\ \nu \\ \Gamma^\alpha \\ \beta \end{matrix}$$

$$\partial_\mu \boxed{\nabla_\mu T^\alpha_\beta}$$

$$g_{\mu\nu}(x) u^\mu v^\nu$$

Riemann

$$\Gamma^\alpha_{\beta\gamma} T^\mu_\nu$$

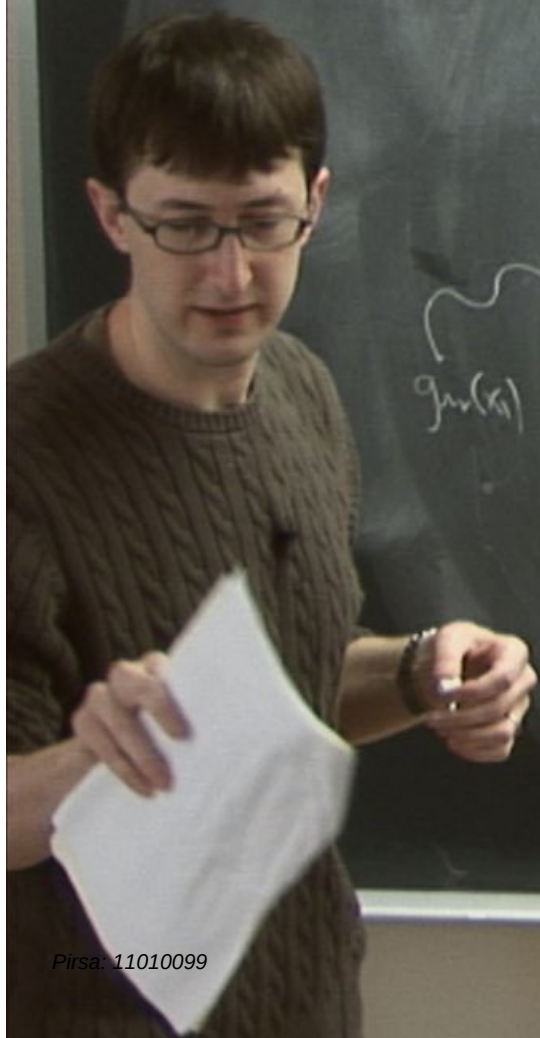
$$\partial_\mu \boxed{\nabla_\mu T^\alpha_\beta}$$

$$g_{\mu\nu}(x) u^\mu u^\nu$$

$$g_{\mu\nu;\alpha}$$

$$\left(\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right)$$

$$\boxed{R_{\mu\nu\alpha\beta}}$$



$$\begin{matrix} T^m \\ \Gamma^\alpha \\ \beta \end{matrix}$$

$$\partial_\mu \boxed{\nabla_\mu T^\alpha_\beta}$$

$$g_{\mu\nu}(x) u^\mu u^\nu$$

$$g_{\mu\nu;\alpha} = 0$$

$$g_{\mu\nu}(x_1) \quad g_{\mu\nu}(x_2)$$



Riemann

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} (g_{\mu\beta;\gamma} + g_{\mu\gamma;\beta} - g_{\beta\gamma;\mu})$$

$$\partial_\mu \left(\nabla_\mu T^\alpha_\beta \right)$$

$$g_{\mu\nu}(x) u^\mu u^\nu$$

$$g_{\mu\nu;\alpha} = 0$$

$$g_{\mu\nu}(x_1) \rightarrow g_{\mu\nu}(x_2)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta} = 0$$

Riemann

$$\begin{matrix} T^{\mu} \\ \nu \\ \rho \sigma \end{matrix}$$

$$\partial_{\mu} \boxed{\nabla_{\mu} T^{\alpha}_{\beta}}$$

$$g_{\mu\nu}(x) u^{\mu} u^{\nu}$$

$$\boxed{g_{\mu\nu;\alpha} = 0}$$

$$g_{\mu\nu}(x_1) \quad g_{\mu\nu}(x_2)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta} = 0$$

Riemann

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta})$$

$$\partial_{\mu} \left(\nabla_{\mu} T^{\alpha}_{\beta} \right)$$

$$g_{\mu\nu}(x) u^{\mu} u^{\nu}$$

$$g_{\mu\nu;\alpha} = 0$$

$$g_{\mu\nu}(x)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta} = 0$$

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta})$$

Riemann

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\delta\gamma,\beta})$$

$$\partial_{\mu} \left(\nabla_{\mu} T^{\alpha}_{\beta} \right)$$

$$g_{\mu\nu}(x) u^{\mu} u^{\nu}$$

$$g_{\mu\nu;\alpha} = 0$$

$$g_{\mu\nu}(x_1) \rightarrow g_{\mu\nu}(x_2)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\delta\gamma;\beta} = 0$$

$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\delta\gamma,\beta})$$

Riemann

$$\begin{matrix} T^{\mu} \\ \nu \\ \rho \\ \sigma \end{matrix}$$

$$\partial_{\mu} \boxed{\nabla_{\mu} T^{\alpha}_{\beta}}$$

$$g_{\mu\nu}(x) u^{\mu} u^{\nu}$$

$$\boxed{g_{\mu\nu;\alpha} = 0}$$

$$g_{\mu\nu}(x_1) \rightarrow g_{\mu\nu}(x_2)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta} = 0$$

$$\overset{\alpha}{\nearrow} \beta\gamma = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta})$$

Levi-Civita

Riemann

$$\Gamma_{\alpha\beta}^{\mu} = \frac{1}{2} g^{\mu\nu} (g_{\alpha\nu;\beta} + g_{\beta\nu;\alpha} - g_{\alpha\beta;\nu})$$

$$\partial_{\mu} \left(\nabla_{\mu} T^{\alpha}_{\beta} \right)$$

$$g_{\mu\nu}(x) u^{\mu} u^{\nu}$$

$$g_{\mu\nu;\alpha} = 0$$

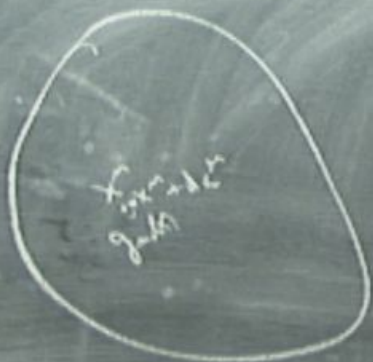
$$g_{\mu\nu}(x_1) \rightarrow g_{\mu\nu}(x_2)$$

$$g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta} = 0$$

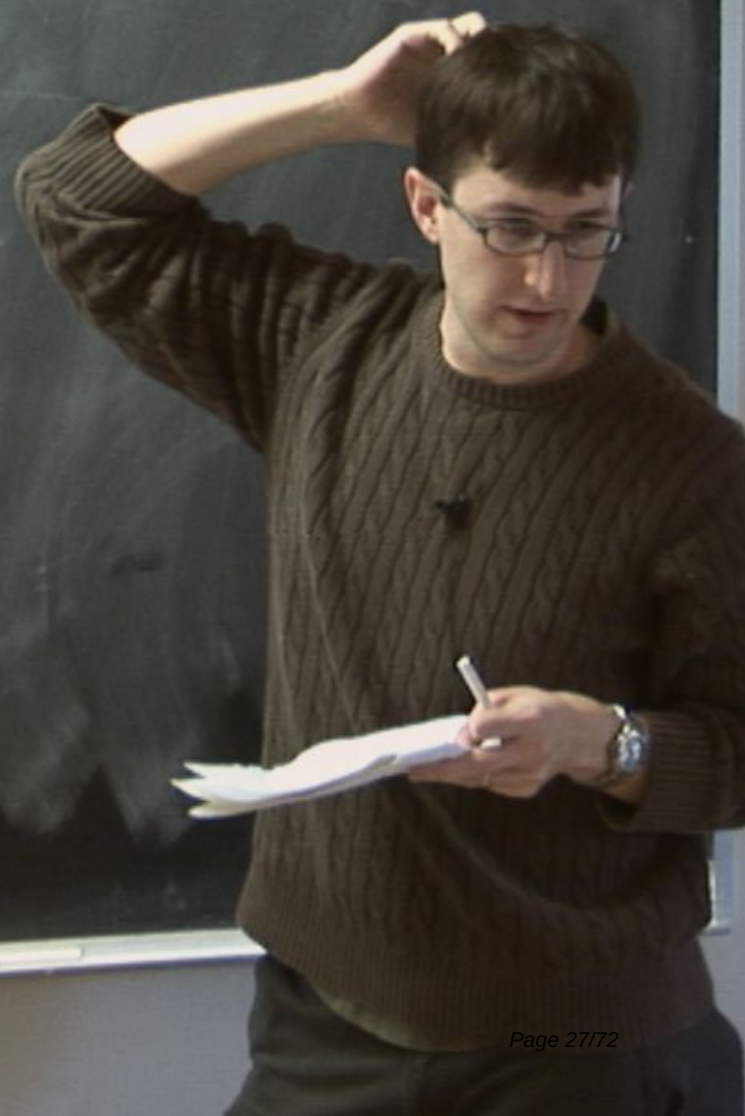
$$\Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta;\gamma} + g_{\gamma\delta;\beta} - g_{\beta\gamma;\delta})$$

Levi-Civita

Riemann



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

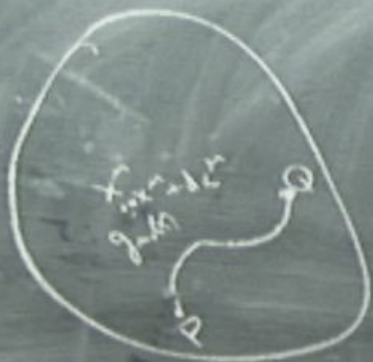




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_R$$

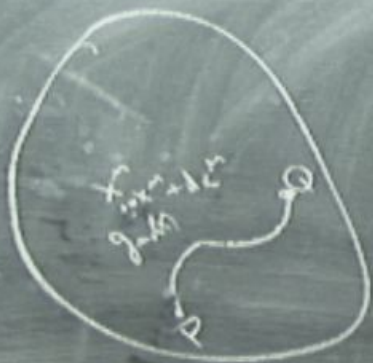




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_R$$

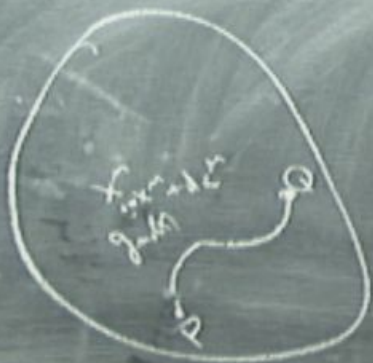




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_R$$

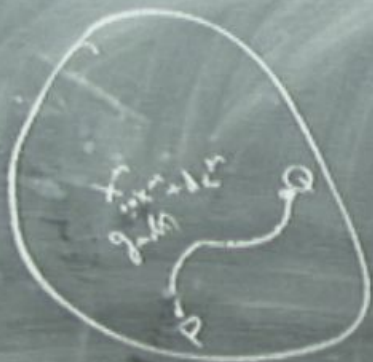




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

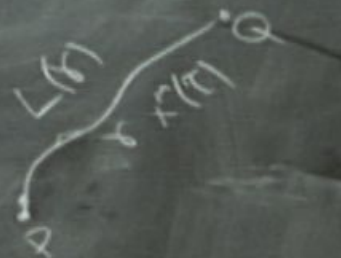
$$\int_P^Q ds = \int_R$$

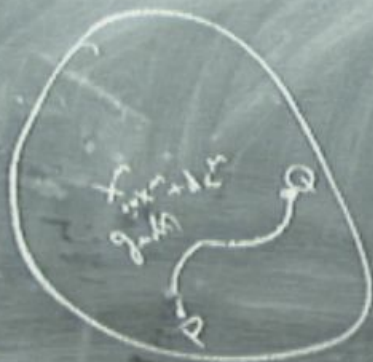




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

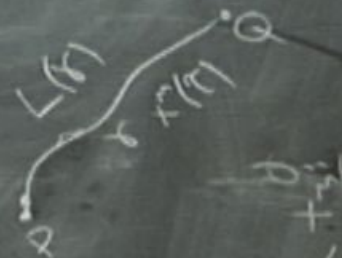
$$\int_P^Q ds = \int_R$$





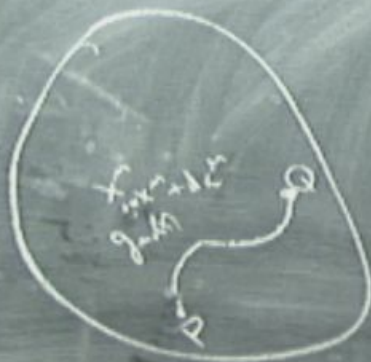
$$ds^2 = g_{\mu\nu}(x) dx^{\mu} dx^{\nu}$$

$$\int_P^Q ds = \int_R$$



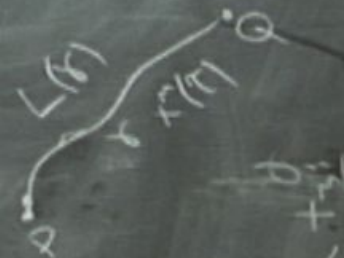
$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}^{\mu}} \right) - \frac{\partial L}{\partial x^{\mu}} = 0$$

So that



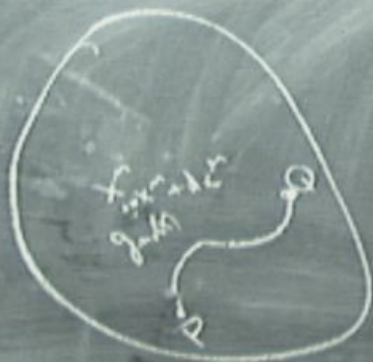
$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$



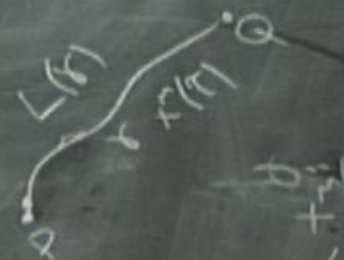
$$\int_P^Q \sqrt{g_{\mu\nu}(x)} dx^\mu dx^\nu$$

$$\int dt L(\dot{x}, x)$$



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$

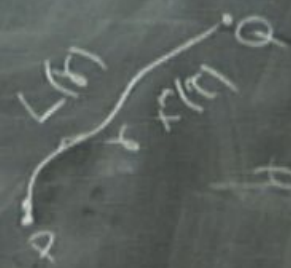


$$S[x] = \int dt L(x, \dot{x}, t)$$



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$



$$S[x] = \int dt L(x, \dot{x}, t)$$

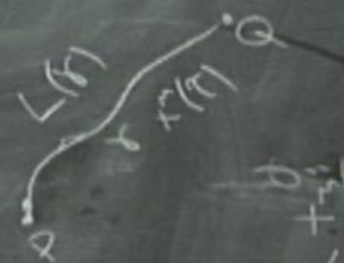
$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}^\mu}$$



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right)$$



$$S[x] = \int dt L(x, \dot{x}, t)$$

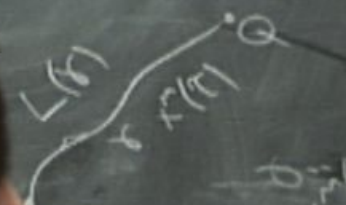
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$ds = \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} dt$$

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right)$$



$$g_{\mu\nu}(x)$$

$$\int dt L(x, \dot{x}, t)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

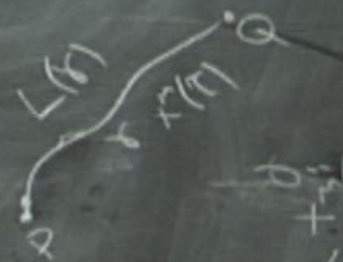




$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right)$$



$$\int dt L(x, \dot{x}, t)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

$$\frac{d}{dt}$$

$$\nabla_\mu U^\alpha = 0$$

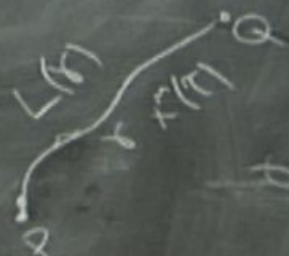
Riemann



$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\int_P^Q ds = \int_P^Q$$

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right)$$



$$\frac{d}{dt} L(x)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$G = c = 1$$

$$x^\mu(\tau) \quad g_{\mu\nu}$$

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

$$\frac{d}{d\tau} p^\mu = m \dot{x}^\mu(\tau)$$

$$u^\mu \nabla_\mu u^\alpha = 0$$

$$(E, \vec{p})$$

Riemann

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$= \int d^4x \underbrace{\mathcal{L}(x)}_{\substack{d^4x' \\ x \sim x'}}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$\uparrow$
 Φ, ψ

$$R = g^{\alpha\beta} R_{\alpha\beta}$$

$$\Gamma^\alpha_{\beta\gamma}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$\Gamma^\alpha_{\beta\gamma}$

$R = g^{\alpha\beta} R_{\alpha\beta}$

Φ, ψ

$$g^{\alpha\beta}$$

$$g_{\alpha\beta}$$

$$V^\alpha g_{\alpha\beta} = V_\beta$$

$$g^{\alpha\beta}$$

$$g_{\alpha\beta}$$

$$V^\alpha g_{\alpha\beta} = V_\beta$$

$$V_\alpha g^{\alpha\beta} = V^\beta$$

$$T^{\alpha\beta} \delta_{\alpha\beta} = V^\delta$$

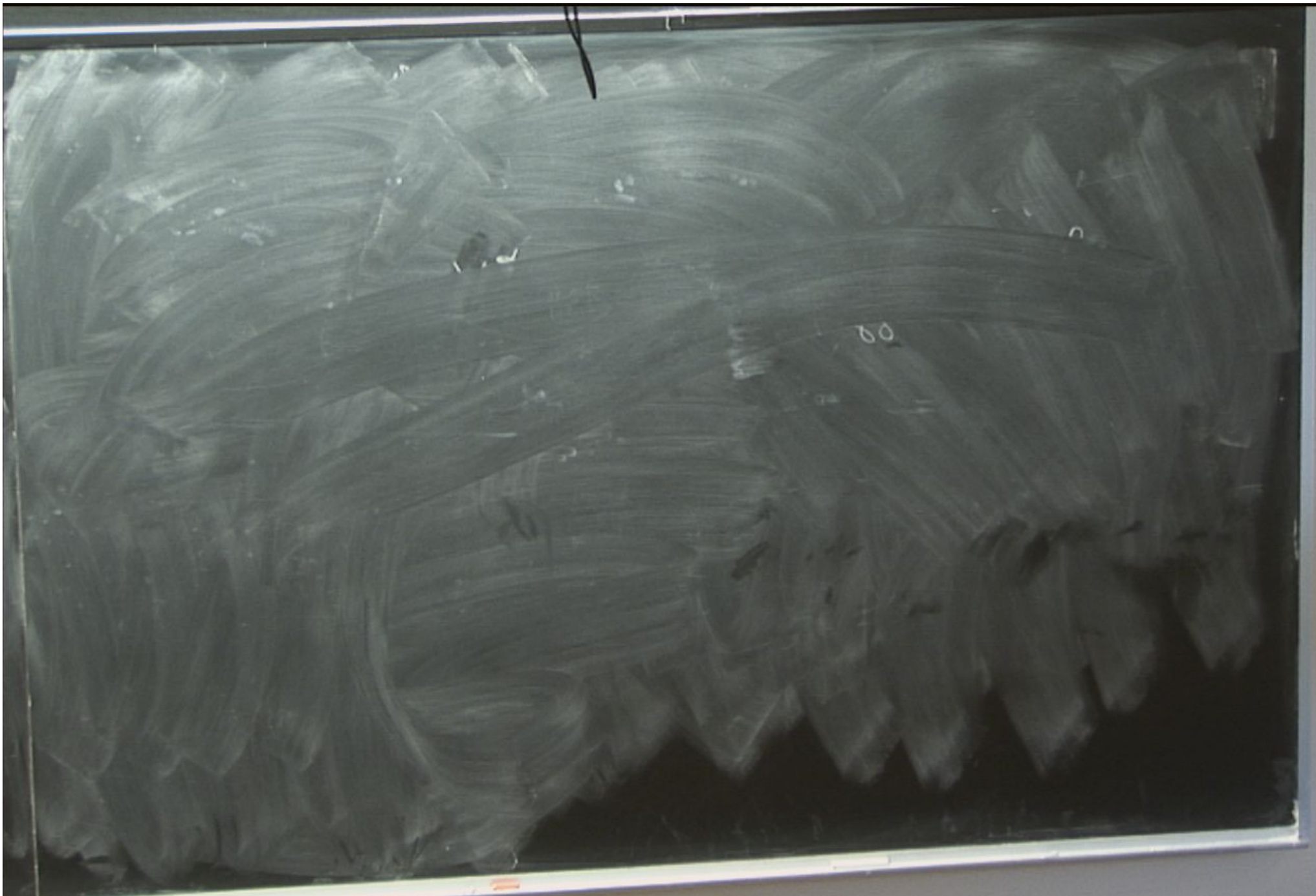
$$g^{\alpha\beta}$$

$$g_{\alpha\beta}$$

$$V^\alpha g_{\alpha\beta} = V_\beta$$

$$V_\alpha g^{\alpha\beta} = V^\beta$$

$$\delta^{\alpha\beta} V_\beta = V^\alpha$$



$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$\uparrow$
 $\Phi, \bar{\psi}$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\Gamma^\alpha_{\mu\nu} \quad \delta g_{\mu\nu} \quad \delta S = 0 \quad \Phi, \bar{\Psi}$$

$$R = g^{\alpha\beta} R_{\alpha\beta}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\Gamma^\alpha_{\beta\gamma} \quad \delta g_{\mu\nu} \quad \delta S = 0 \quad \Phi, \bar{\Phi} \quad \text{Einstein}$$

$$R = g^{\alpha\beta} R_{\alpha\beta} \quad \delta \Phi \quad \delta S$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\Gamma^\alpha_{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\psi}$$

Einstein Eqs

$$R = g^{\alpha\beta} R_{\alpha\beta}$$

$$\Gamma_{\beta\delta}^{\alpha} = \frac{1}{2} g^{\alpha\epsilon} (g_{\beta\delta,\epsilon} + g_{\epsilon\delta,\beta} - g_{\beta\epsilon,\delta})$$

$$\Gamma_{\beta\delta}^{\alpha} = \frac{1}{2} g^{\alpha\epsilon} (g_{\beta\delta,\epsilon} + g_{\epsilon\delta,\beta} - g_{\beta\epsilon,\delta})$$

$$R_{\beta\delta\epsilon}$$

00

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$\delta g_{\mu\nu}$

$\delta S = 0$

$\Phi, \bar{\psi}$

Einstein Eqs

$$R = g^{\alpha\beta} R_{\alpha\beta}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\psi}$$

Einstein Eqs

$$R = g^{\alpha\beta} R_{\alpha\beta}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$\Gamma^\alpha_{\beta\gamma}$

$g^{\mu\nu}$

$\delta g_{\mu\nu}$

$\delta S = 0$

$\Phi, \bar{\psi}$

Einstein Eqs

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta \Gamma^\alpha_{\mu\beta}$$

$$g^{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\Phi}$$

$$\delta S = 0$$

$$r$$

Einstein Eqs

$$\Gamma_{\beta\delta}^{\alpha} = \frac{1}{2} g^{\alpha\epsilon} (g_{\beta\delta,\epsilon} + g_{\epsilon\delta,\beta} - g_{\beta\epsilon,\delta})$$

$$R_{\beta\delta\epsilon}$$

00

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$g^{\mu\nu}$

$\delta g_{\mu\nu}$

$\delta S = 0$

$\Phi, \bar{\Phi}$

Einstein Eqs

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

var
88 $g^{\mu\nu}$

$\delta g_{\mu\nu}$

$\delta S = 0$

$\Phi, \bar{\Phi}$

Einstein Eqs

$$\left. \frac{2\Lambda}{5N} + L_m \right\}$$

$$\delta g_{\mu\nu} \quad \delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} + g^{\alpha\beta} \delta g_{\beta\gamma} = 0$$

$$\left. \begin{aligned} \frac{2\Lambda}{5N} + L_m \end{aligned} \right\}$$

$$\delta g$$

$$\delta g_{\mu\nu} \quad \delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} = -g^{\alpha\beta} \delta g_{\beta\gamma}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta}}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta \Gamma^\alpha_{\beta\gamma}$$

$$g^{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\Phi}$$

Einstein Eqs

$$\int d^4x$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta}}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta \Gamma^\alpha_{\beta\gamma}$$

$$g^{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\Phi}$$

Einstein Eqs

$$g = \det(g_{\mu\nu})$$

$$\left. \frac{2\Lambda}{5N} + L_m \right\}$$

$$\delta g_{\mu\nu}$$

$$\delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} = -g^{\alpha\beta} \delta g_{\beta\gamma}$$

$$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta} \quad \text{--- Jacobi Form}$$

$$\delta \sqrt{-g} = \frac{1}{2} (-g)^{-1/2} (-\delta g) = -\frac{1}{2} \sqrt{-g} g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta \Gamma^\alpha_{\beta\gamma} \quad g^{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

Einstein Eqs

$$\delta S = \int d^4x \left[\delta(\sqrt{-g}) \left(\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right) + \sqrt{-g} \delta \left(\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right) \right]$$

$$R_{\alpha\beta}$$

$$\frac{\partial \mathcal{L}_m}{\partial g^{\alpha\beta}}$$

$$\frac{\partial \mathcal{L}_m}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta}$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$$

$$\delta g^{\alpha\beta} \quad g^{\mu\nu}$$

$$\delta g_{\mu\nu}$$

$$\delta S = 0$$

$$\Phi, \bar{\Phi}$$

Einstein Eqs

$$\delta S = \int d^4x \left[\delta(\sqrt{-g}) \left\{ \dots \right\} + \sqrt{-g} \delta g^{\alpha\beta} \frac{R_{\alpha\beta}}{16\pi G_N} + \sqrt{-g} \frac{\partial \mathcal{L}_m}{\partial g^{\alpha\beta}} \delta g^{\alpha\beta} \right]$$

$$g = \det(g_{\mu\nu})$$

$$\left. \frac{2\Lambda}{5N} + L_m \right\}$$

$$\delta g_{\mu\nu}$$

$$\delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} = -g^{\alpha\beta} \delta g_{\beta\gamma}$$

$$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta} \quad \text{--- Jacobi Form.}$$

$$\delta \sqrt{-g} = \frac{1}{2} (-g)^{-1/2} (-\delta g) = -\frac{1}{2} \sqrt{-g} g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$\left. \delta g^{\alpha\beta} \right\}$$

$$g = \det(g_{\mu\nu})$$

$$\left. \frac{2\Lambda}{5N} + L_m \right\}$$

$$\delta g_{\mu\nu}$$

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$$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta} \quad \text{--- Jacobi Form}$$

$$\delta \sqrt{-g} = \frac{1}{2} (-g)^{-1/2} (-\delta g) = -\frac{1}{2} \sqrt{-g} g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \quad \leftarrow$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$g = \det(g_{\mu\nu})$$

$$\left. \frac{2\Lambda}{5N} + L_{\text{m}} \right\}$$

$$\delta g_{\mu\nu}$$

$$\delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} = -g^{\alpha\beta} \delta g_{\beta\gamma}$$

$$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta} \quad \text{--- Jacobi Form}$$

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$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu}} = 8\pi G_N T_{\mu\nu} \quad \leftarrow$$

$$\left. \delta g^{\alpha\beta} \right\}$$

$$R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R$$

$$g = \det(g_{\mu\nu})$$

$$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi)$$

$$\left. \frac{2\Lambda}{5N} + \mathcal{L}_m \right\}$$

$$\delta g_{\mu\nu}$$

$$\delta g^{\mu\nu}$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha_\gamma$$

$$\delta g^{\alpha\beta} g_{\beta\gamma} = -g^{\alpha\beta} \delta g_{\beta\gamma}$$

$$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta} \quad \text{--- Jacobi Form}$$

$$\delta \sqrt{-g} = \frac{1}{2} (-g)^{-1/2} (-\delta g) = -\frac{1}{2} \sqrt{-g} g^{\alpha\beta} \delta g_{\alpha\beta}$$

$$\left[G_{\mu\nu} + \Lambda g_{\mu\nu} \right] = 8\pi G_N T_{\mu\nu} \leftarrow g_{\mu\nu} \mathcal{L}_m - 2 \frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$$

$$R_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R$$

$$\left. \delta g^{\alpha\beta} \right\}$$