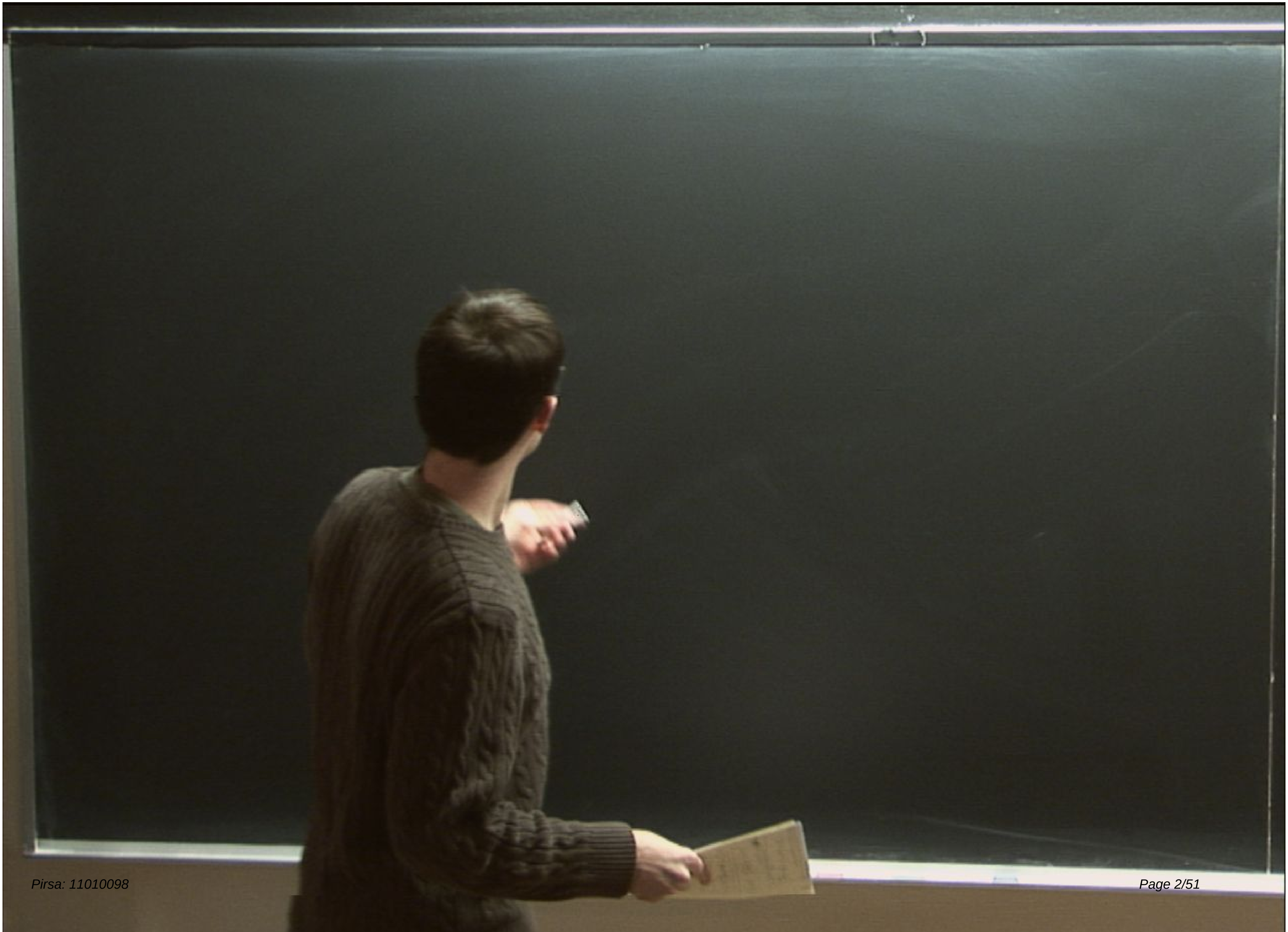
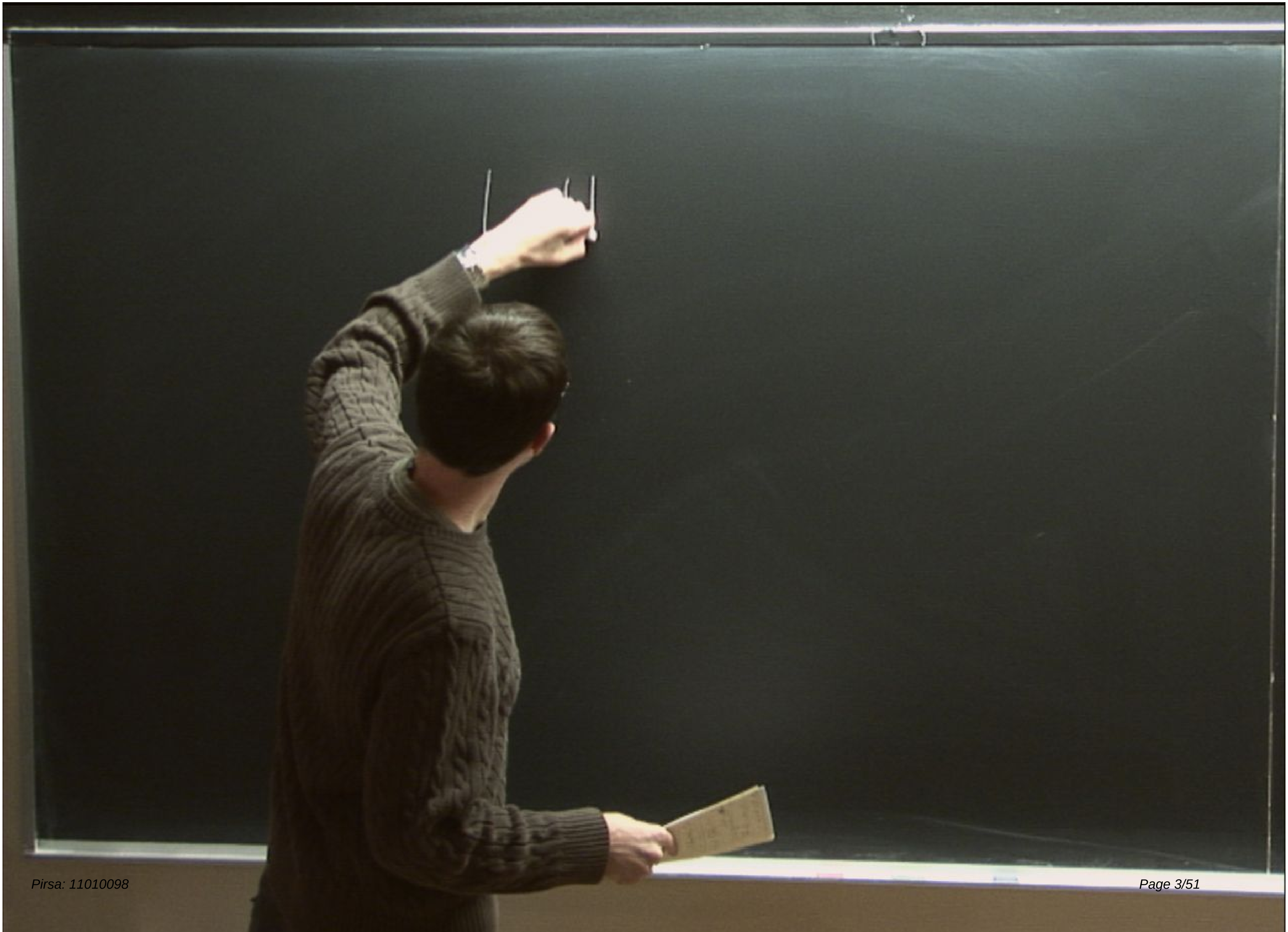
Title: Cosmology Review - Lecture 1

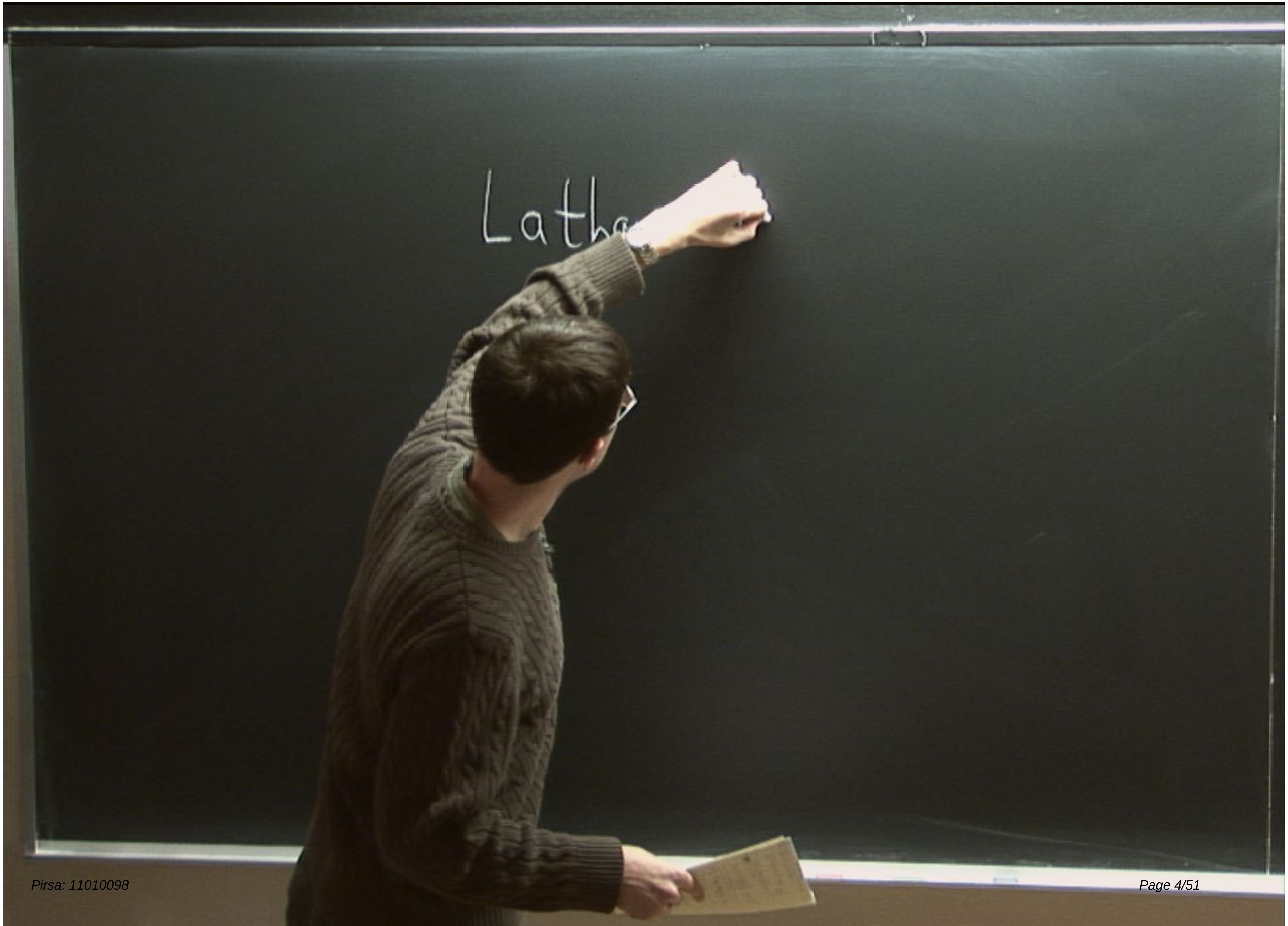
Date: Jan 24, 2011 09:00 AM

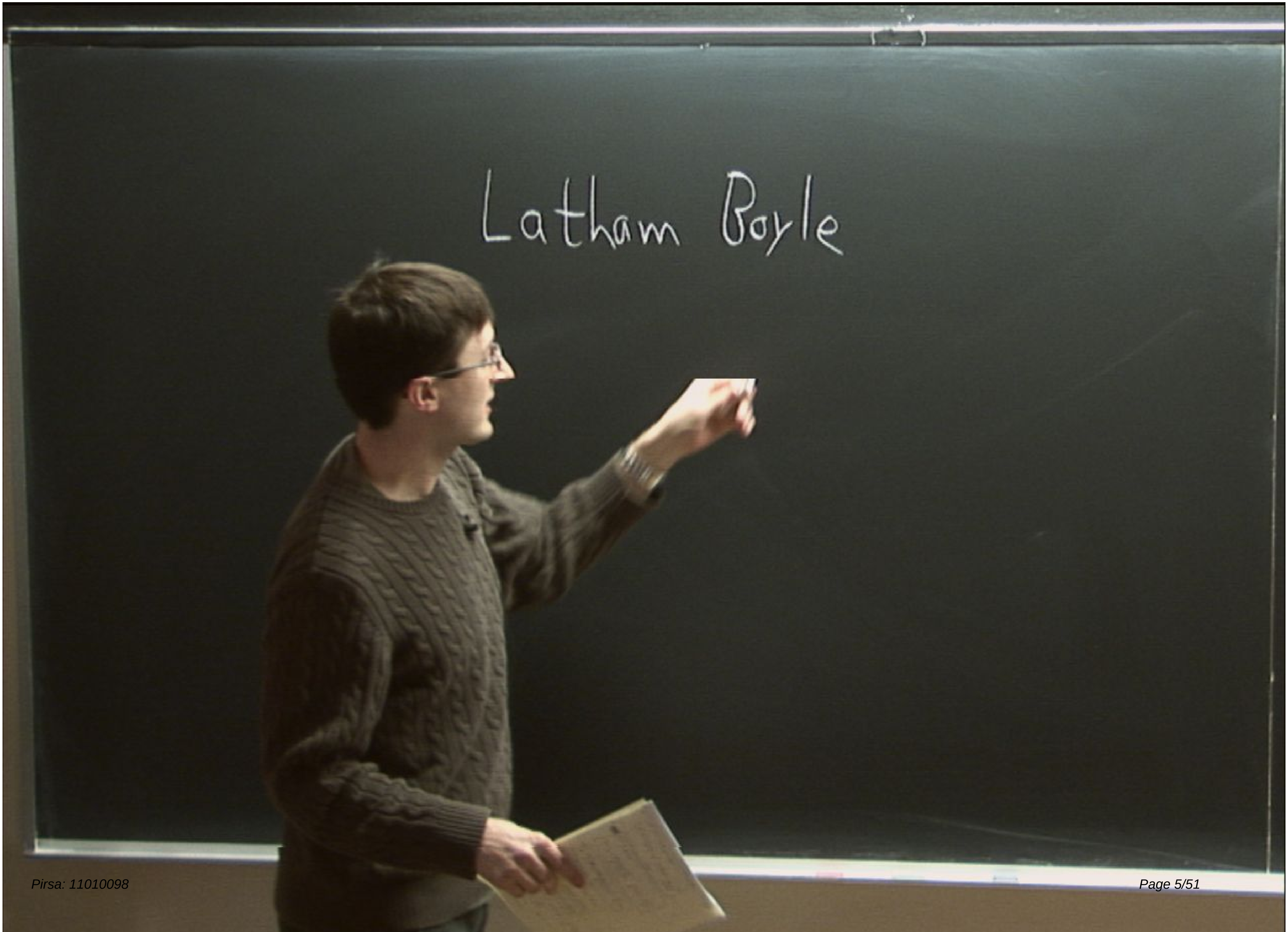
URL: http://pirsa.org/11010098

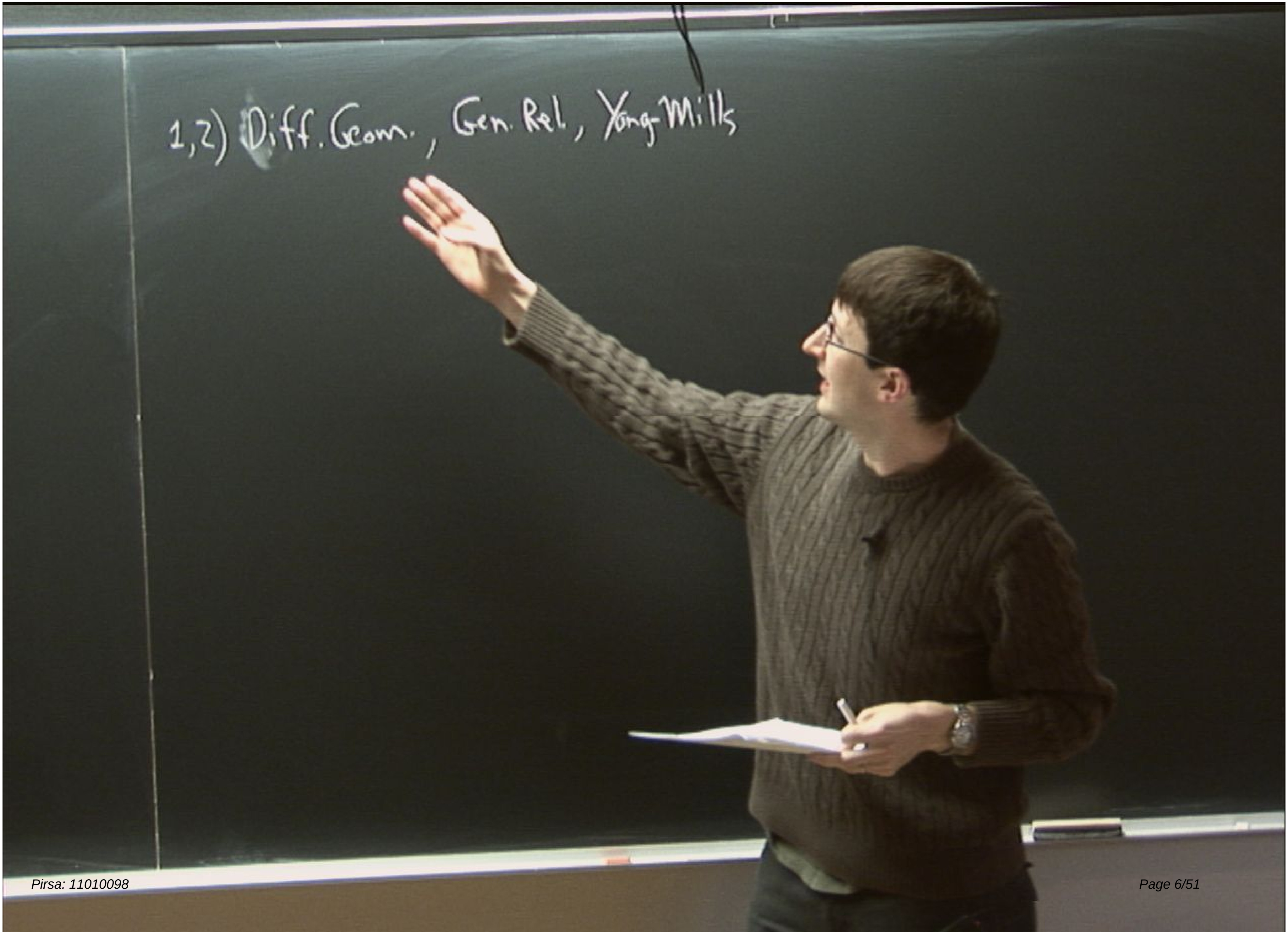
Abstract:

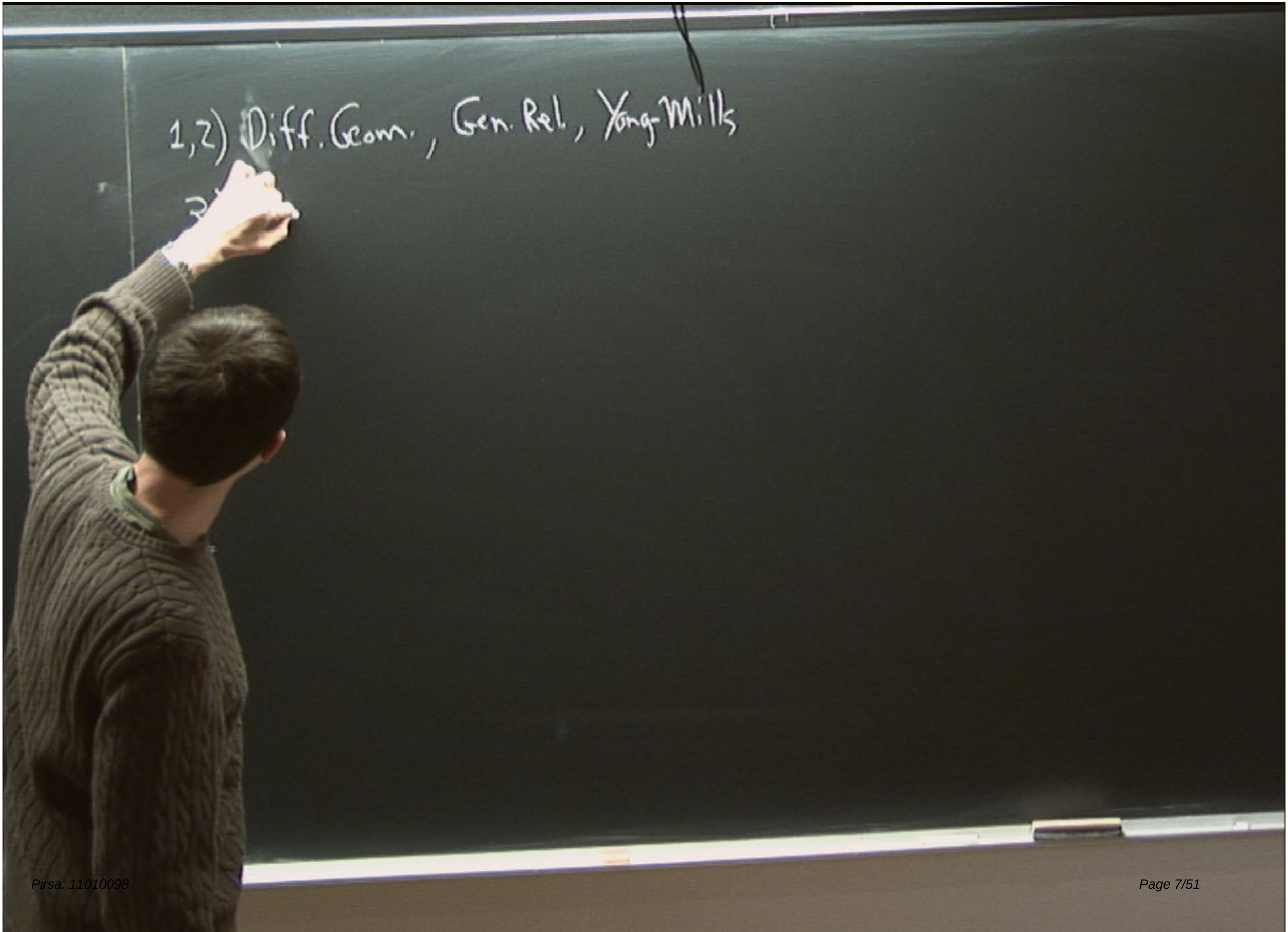







Latham Boyle


1,2) Diff. Geom., Gen. Rel., Yang-Mills


1,2) Diff. Geom. , Gen. Rel. , Yang-Mills

1,2) Diff. Geom., Gen. Rel., Yang-Mills

3) Whirlwind Tour: Maximally symmetric → Minkowski, → de Sitter, → anti de Sitter

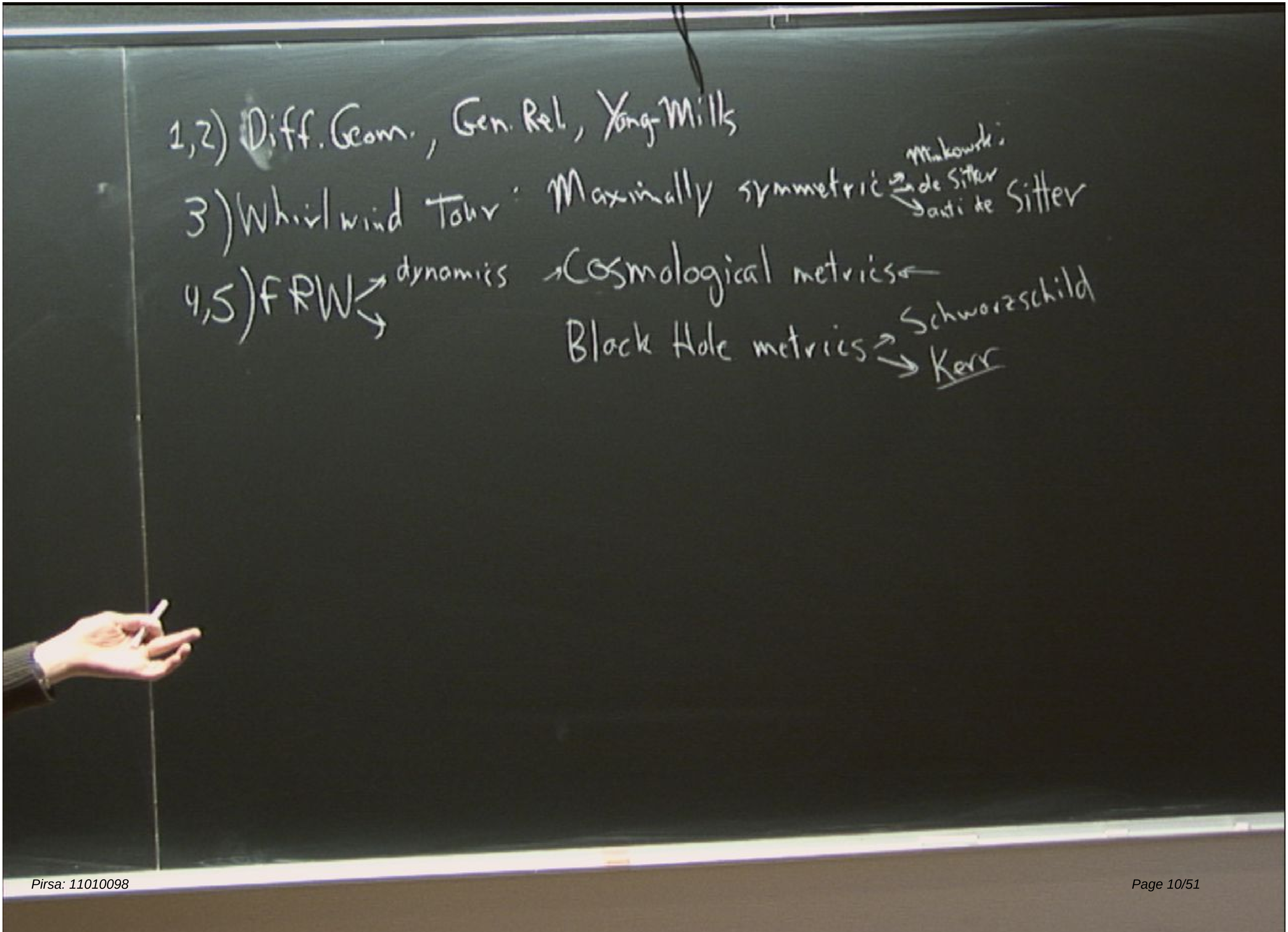
1,2) Diff. Geom., Gen. Rel, Yang-Mills

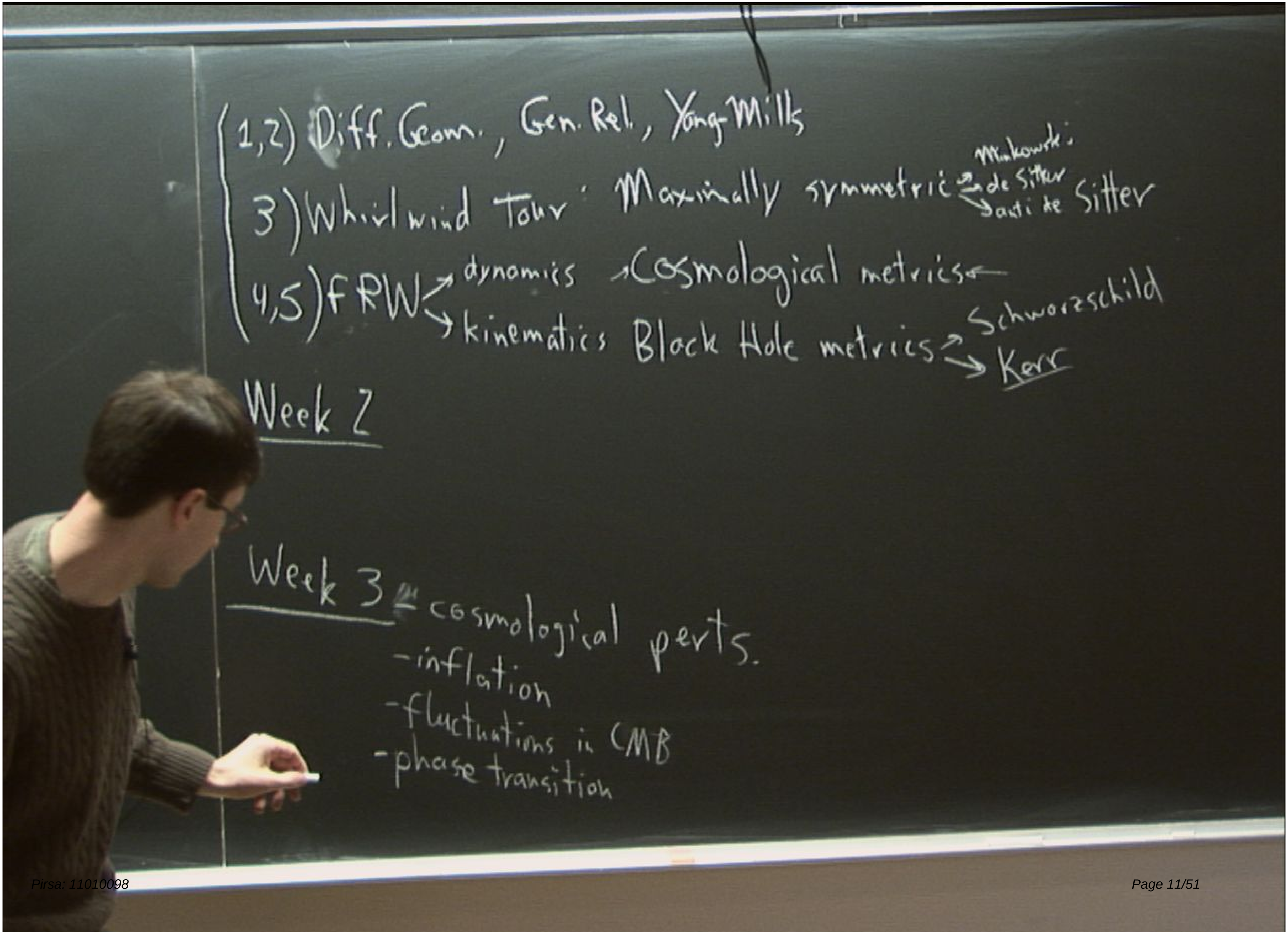
3) Whirlwind Tour: Maximally symmetric → Minkowski → de Sitter → anti de Sitter Sitter

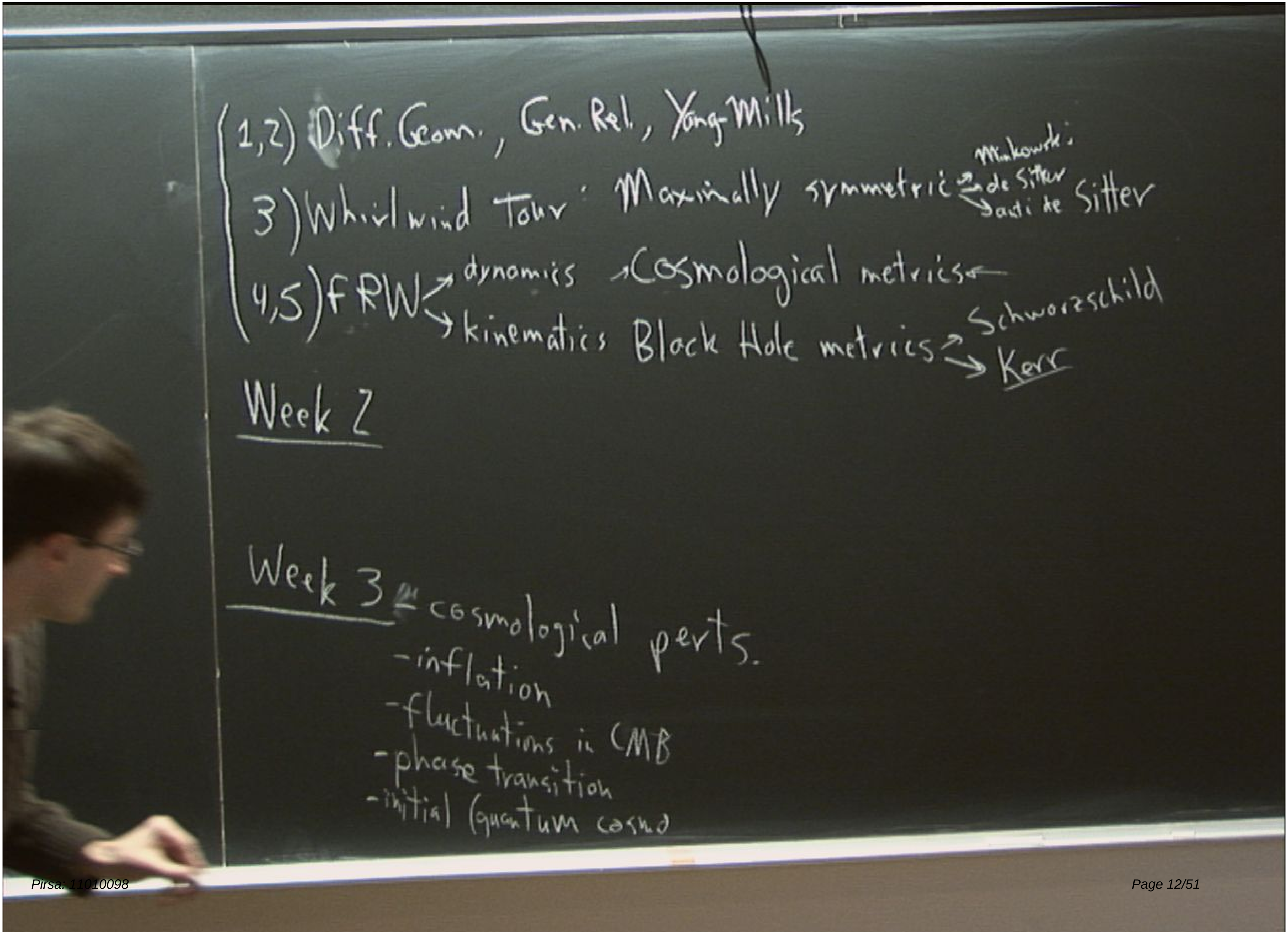
Cosmological metrics ←

4,5)

Black Hole metrics → Schwarzschild → Kerr


1,2) Diff. Geom., Gen. Rel, Yang-Mills
3) Whirlwind Tour: Maximally symmetric
Minkowski
de Sitter
anti de Sitter
4,5) FRW
dynamics
Cosmological metrics
Black Hole metrics
Schwarzschild
Kerr


1,2) Diff. Geom., Gen. Rel., Yang-Mills
3) Whirlwind Tour: Maximally symmetric
Minkowski
de Sitter
anti de Sitter
4,5) FRW
dynamics
kinematics
Cosmological metrics
Black Hole metrics
Schwarzschild
Kerr
Week 2
Week 3
cosmological perts.
-inflation
-fluctuations in CMB
-phase transition


(1,2) Diff. Geom., Gen. Rel., Yang-Mills
3) Whirlwind Tour: Maximally symmetric
Minkowski
de Sitter
anti de Sitter
(4,5) FRW
dynamics
kinematics
Cosmological metrics
Black Hole metrics
Schwarzschild
Kerr
Week 2
Week 3
cosmological perts.
-inflation
-fluctuations in CMB
-phase transition
-initial (quantum cosmo)

1,2) Diff. Geom., Gen. Rel., Yang-Mills

3) Whirlwind Tour: Maximally symmetric → Minkowski, de Sitter, anti de Sitter

4,5) FRW → dynamics, kinematics

Cosmological metrics

Black Hole metrics → Schwarzschild, Kerr

Week 2

Week 3 ← cosmological perts.

- inflation
- fluctuations in CMB
- phase transition
- initial (quantum cosmo)

1,2) Diff. Geom., Gen. Rel., Yang-Mills

3) Whirlwind Tour: Maximally symmetric → Minkowski
→ de Sitter
→ anti de Sitter

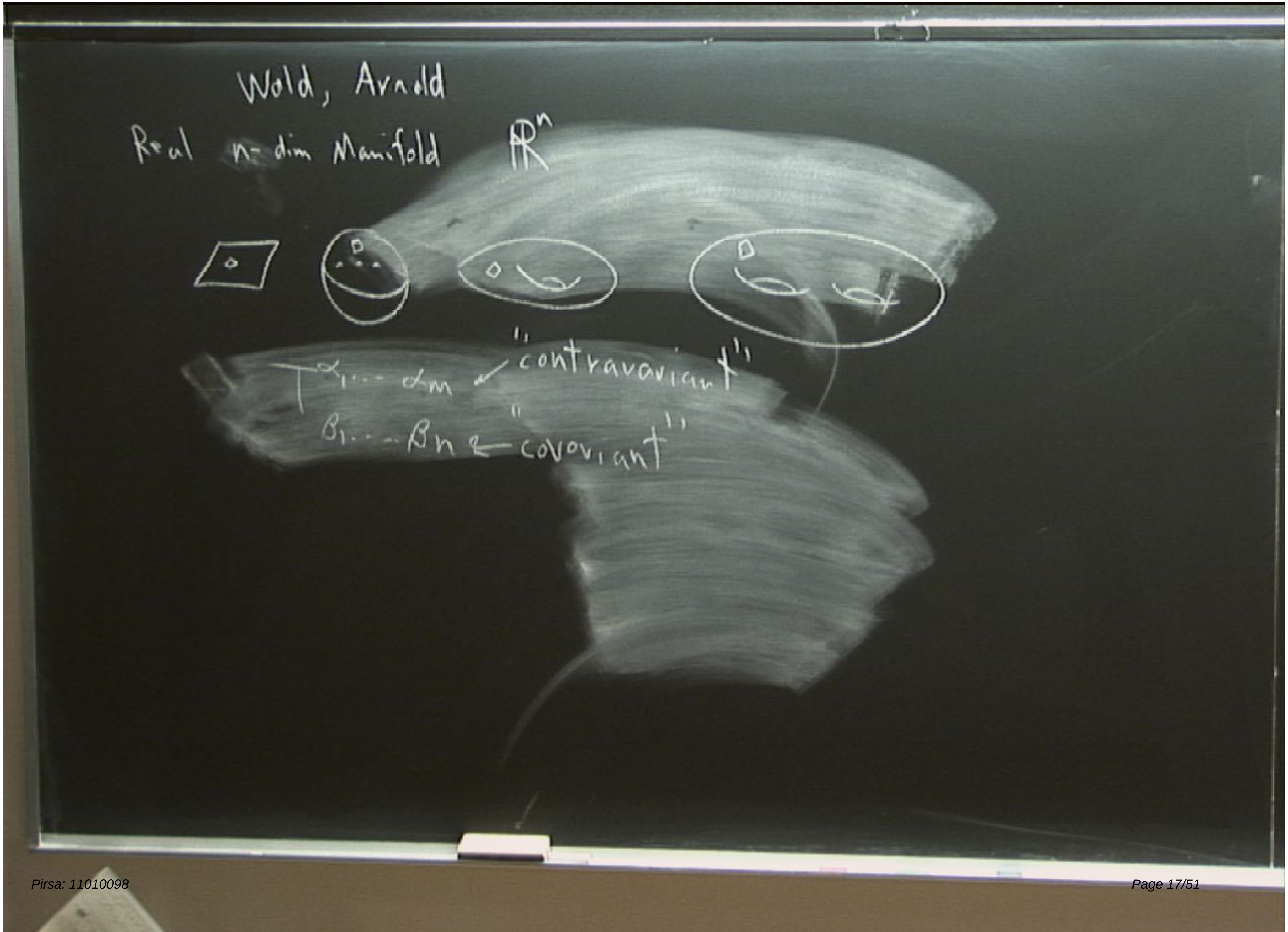
4,5) FRW → dynamics → Cosmological metrics ←
→ kinematics   Black Hole metrics → Schwarzschild
→ Kerr

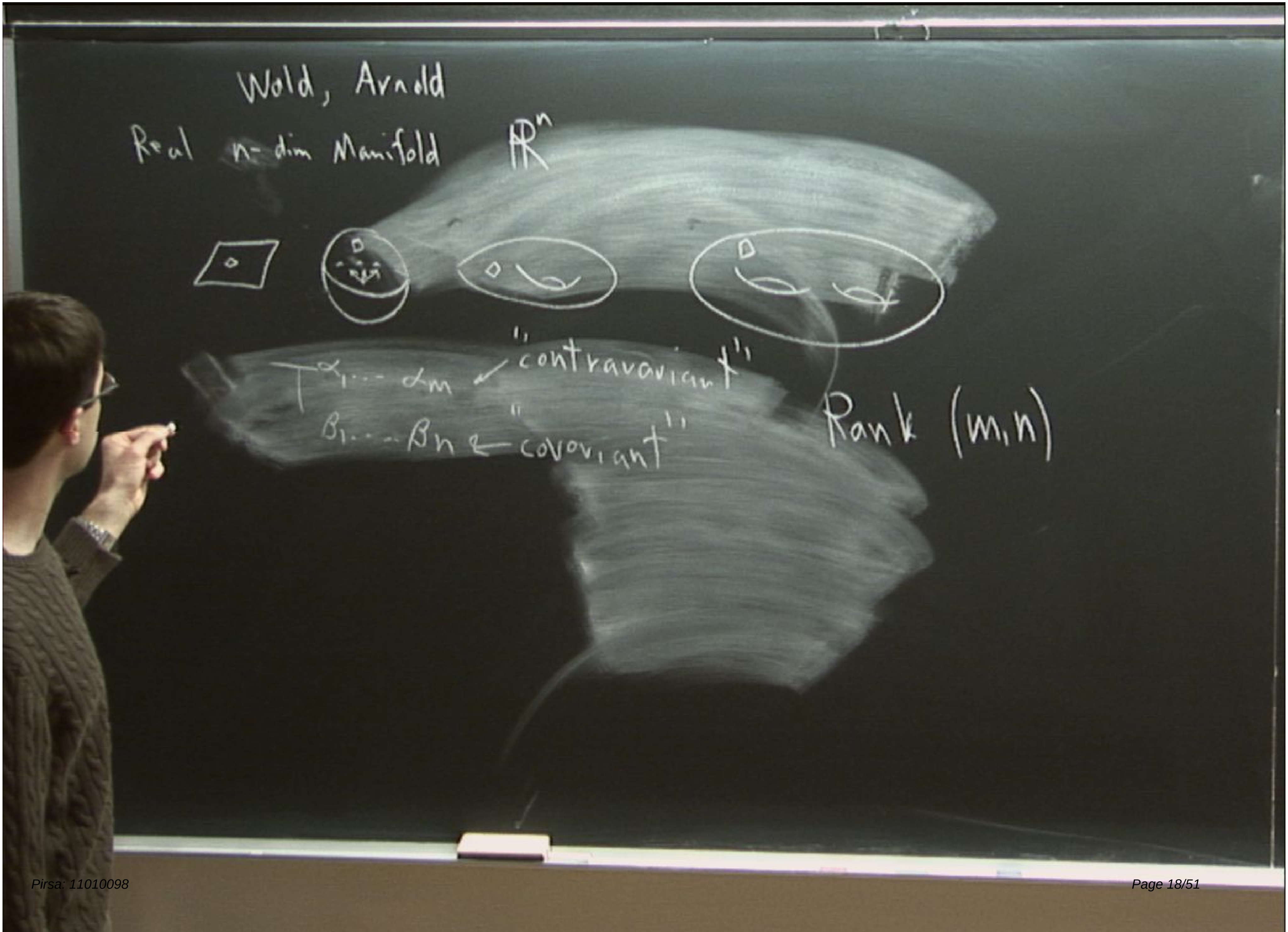
Week 2 → CMB
→ BBN
→ Dark Matter:
→ Dark Energy

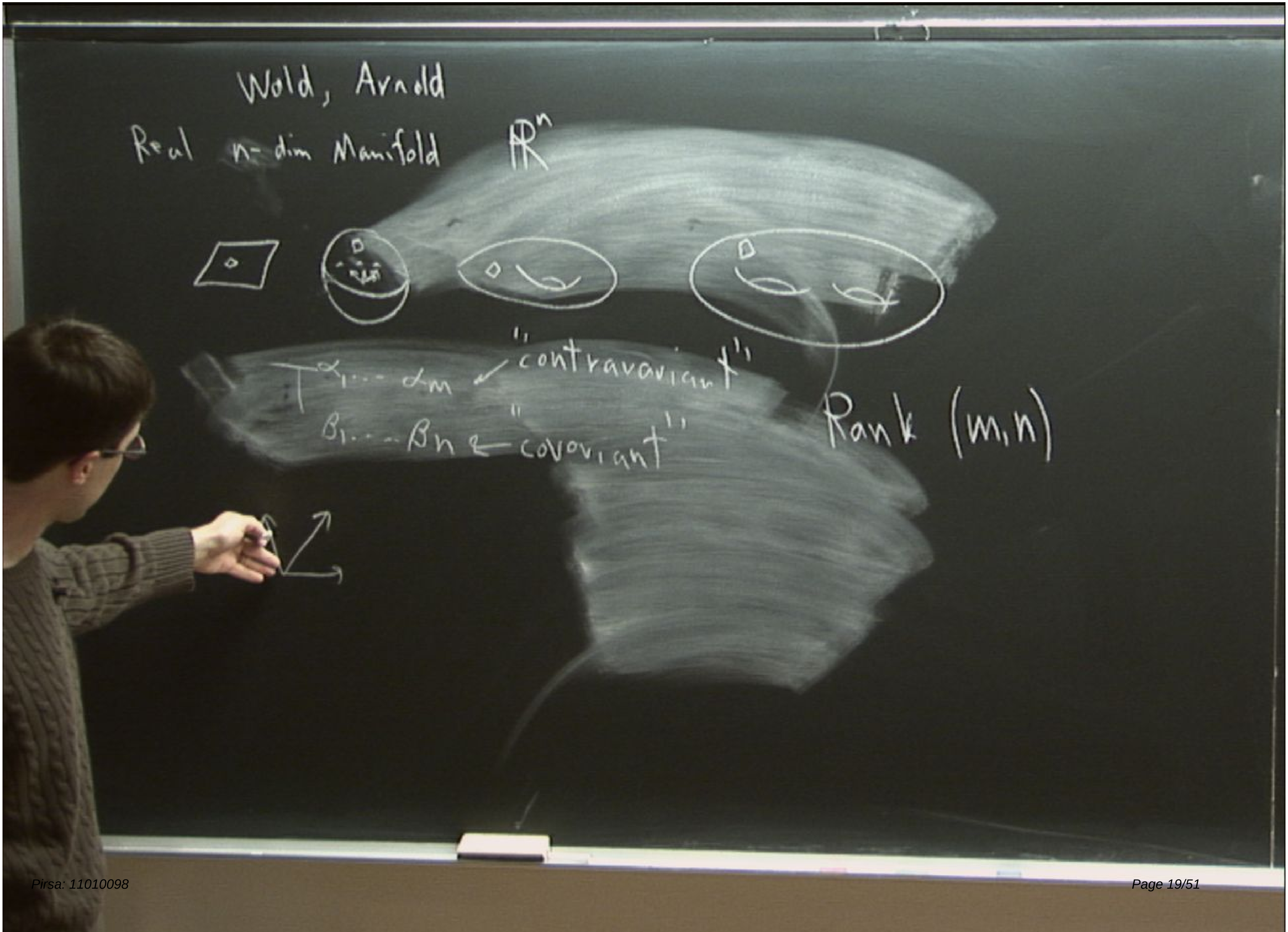
Week 3 → matter/anti-matter → Sakharov conditions, baryogenesis/leptogenesis
- cosmological perts.
- inflation
- fluctuations in CMB
- phase transition
- initial (quantum cosmo)

Real n-dim Manifold $\mathbb{R}^n$

Real n-dim Manifold $\mathbb{R}^n$


Wald, Arnold
Real n-dim Manifold
$\mathbb{R}^n$
$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$
"contravariant"
"covariant"


Wold, Arnold
Real n-dim Manifold
$\mathbb{R}^n$
$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$
"contravariant"
"covariant"
Rank (m,n)


Wold, Arnold
Real n-dim Manifold
$\mathbb{R}^n$
$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$
"contravariant"
"covariant"
Rank (m,n)

Wald, Arnold

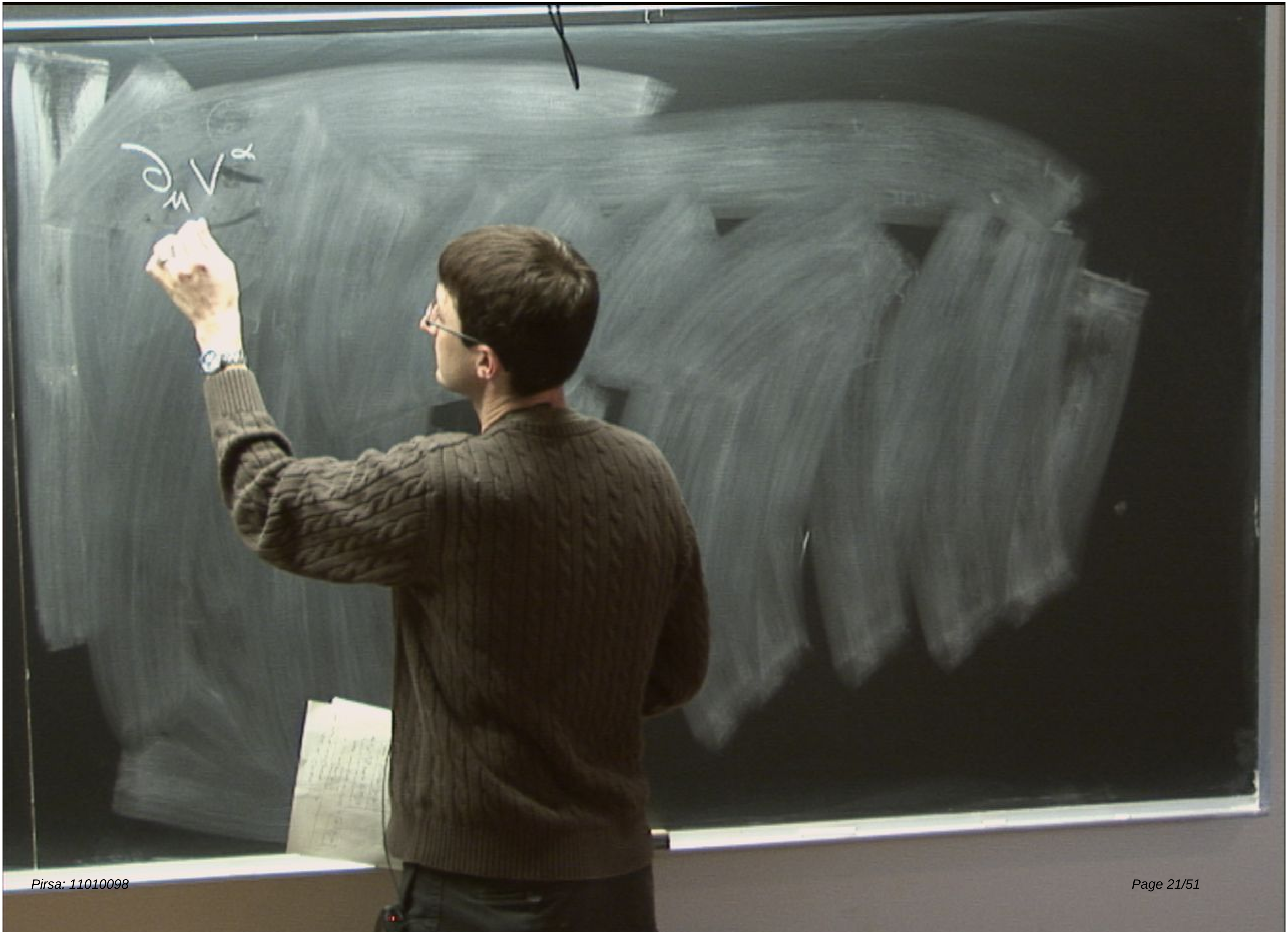
Real n-dim Manifold $\mathbb{R}^n$

$T^{\alpha_1\ldots\alpha_m}$ ← "contravariant"

$x \to \tilde{x}$ $\quad {}_{\beta_1\ldots\beta_n}$ ← "covariant"

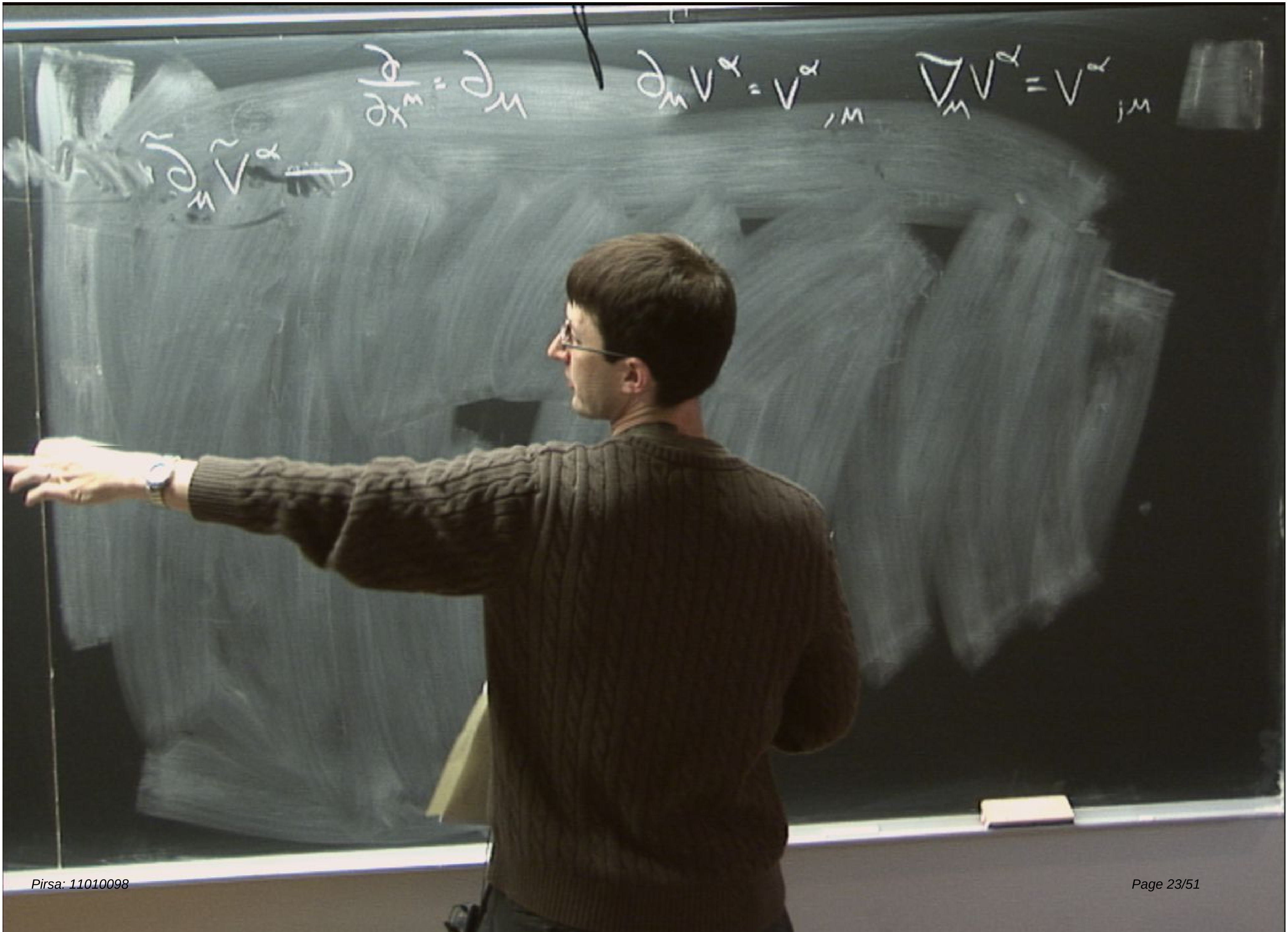
Rank (m,n)

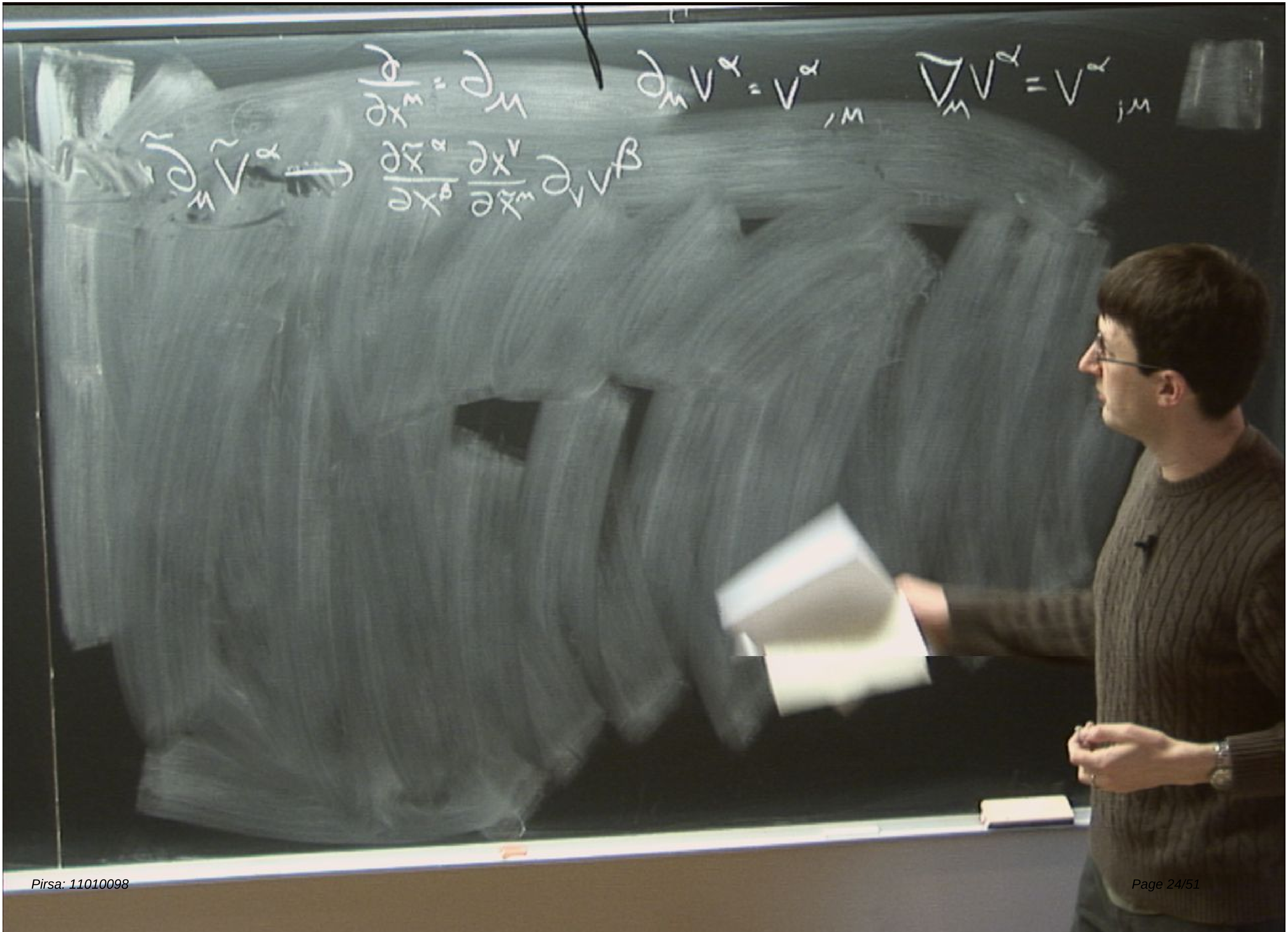
$$\tilde{T}^{\alpha_1\ldots\alpha_m}{}_{\beta_1\ldots\beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\gamma_1}}\cdots\frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\gamma_m}}\frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}}\cdots\frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1\ldots\gamma_m}{}_{\delta_1\ldots\delta_n}$$

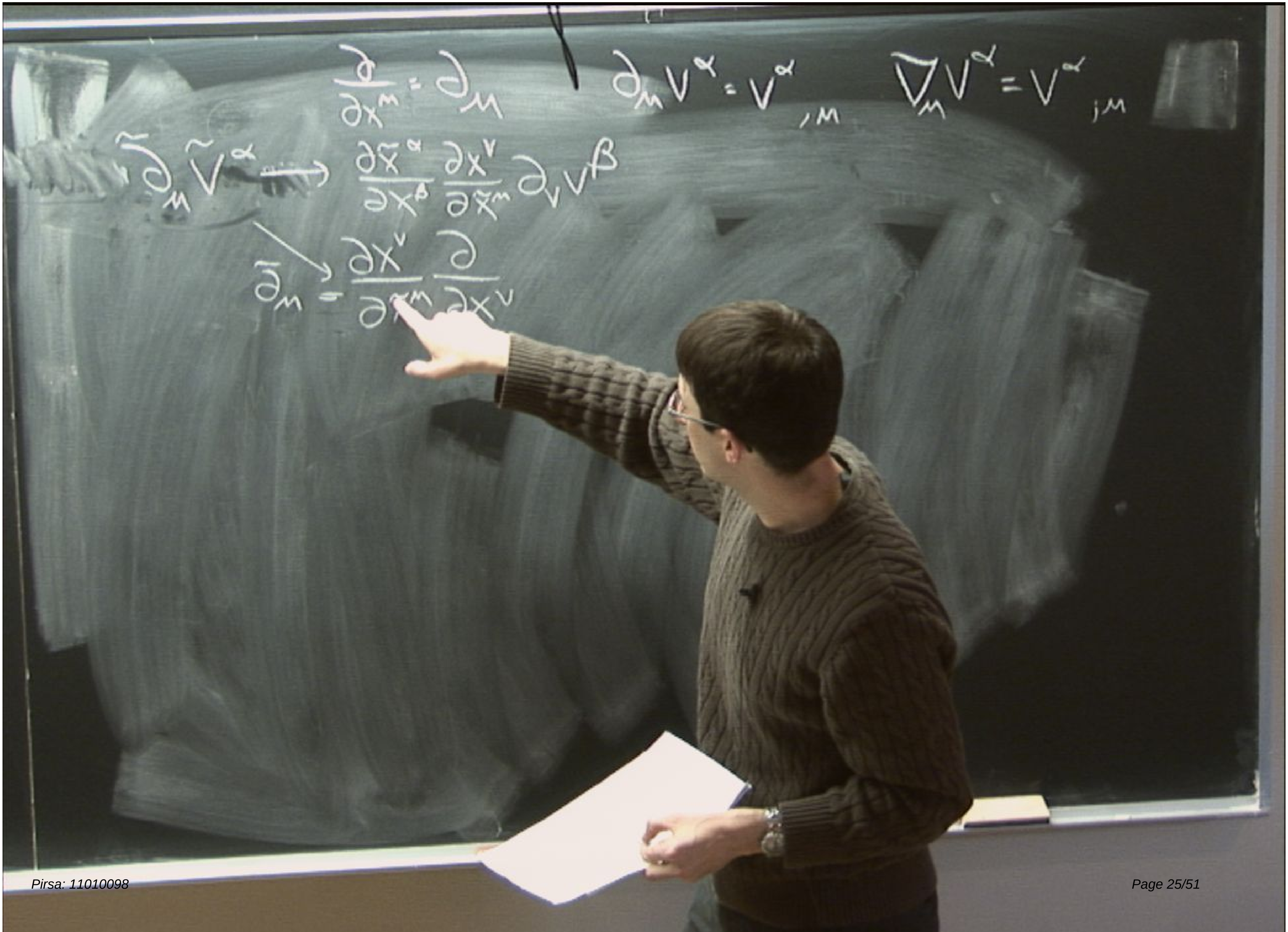

$\partial_\mu V^\alpha$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha = V^\alpha{}_{,\mu} \qquad \nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha =$$


$\frac{\partial}{\partial x^\mu} = \partial_\mu$
$\partial_\mu V^\alpha = V^\alpha{}_{,\mu}$
$\nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$
$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow$


$\frac{\partial}{\partial x^\mu} = \partial_\mu$
$\partial_\mu V^\alpha = V^\alpha{}_{,\mu}$
$\nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$
$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$



$\frac{\partial}{\partial x^\mu} = \partial_\mu$
$\partial_\mu V^\alpha = V^\alpha{}_{,\mu}$
$\nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$
$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$
$\tilde{\partial}_\mu = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu}$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha = V^\alpha{}_{,\mu} \qquad \nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\partial_\nu V^\beta$$

$$\tilde{\partial}_\mu = \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}$$

$f(x(\tilde{x}))$

Rank (m,n)

$$\frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \ldots \gamma_m}{}_{\delta_1 \ldots \delta_n}$$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha = \ldots$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$$

$$\tilde{\partial}_\mu = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu}$$

$$\frac{\partial}{\partial \tilde{x}^m} f(x(\tilde{x}))$$

Rank (m,n)

$$\frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \ldots \gamma_m}{}_{\delta_1 \ldots \delta_n}$$

$$\frac{\partial}{\partial x^m} = \partial_m \qquad \partial_m V^\alpha$$

$$\tilde{\partial}_m \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^m} \partial_\nu V^\beta$$

$$\tilde{\partial}_m = \frac{\partial x^\nu}{\partial \tilde{x}^m} \frac{\partial}{\partial x^\nu}$$

$$\frac{\partial}{\partial \tilde{x}^\mu} f(x(\tilde{x}))$$

$$\frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} f(x)$$

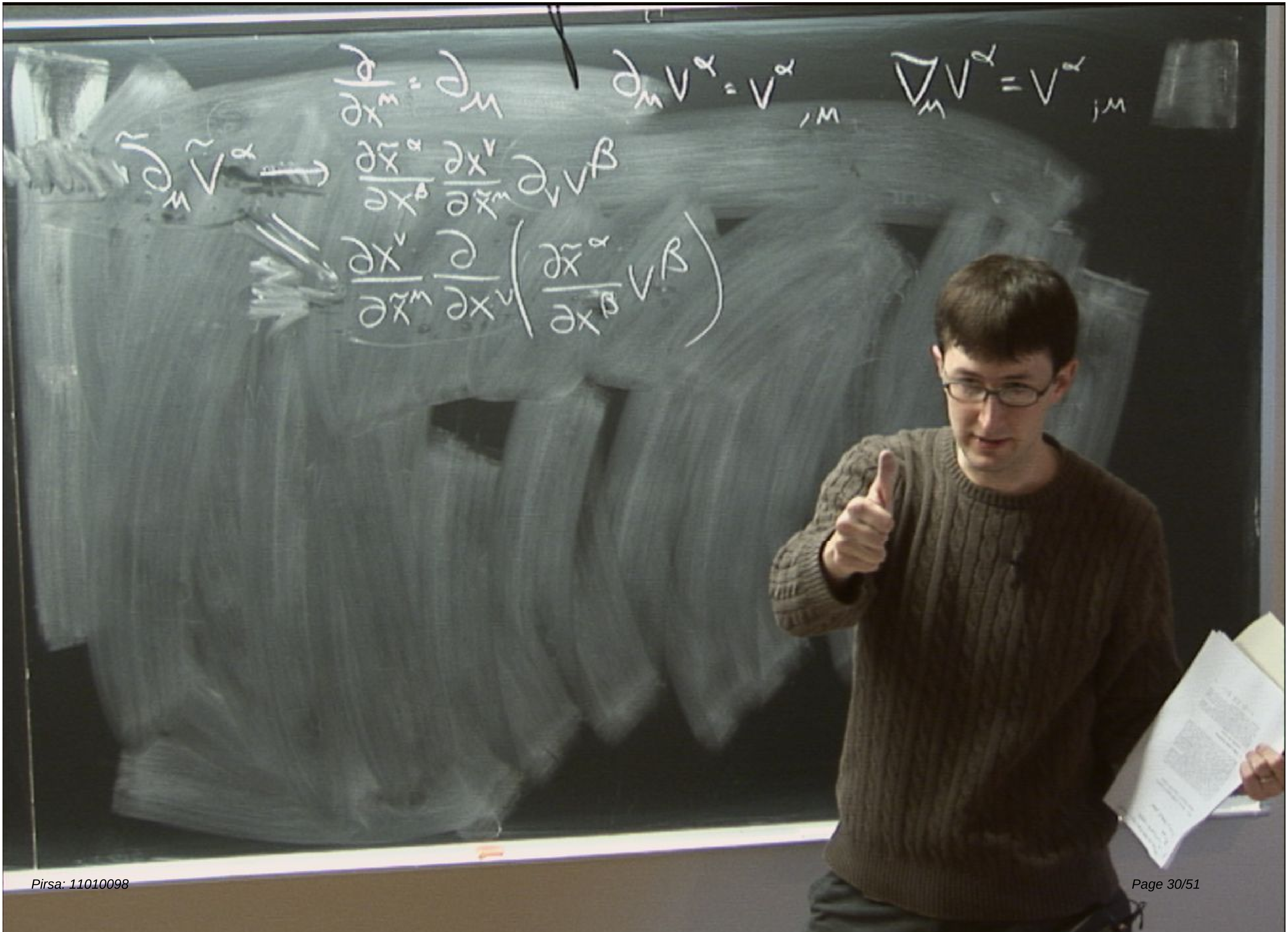
Rank (m,n)

$$\frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \ldots \gamma_m}{}_{\delta_1 \ldots \delta_n}$$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$$

$$\tilde{\partial}_\mu = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu}$$


$\frac{\partial}{\partial x^\mu} = \partial_\mu$
$\partial_\mu V^\alpha = V^\alpha{}_{,\mu}$
$\nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$
$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$
$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right)$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha = V^\alpha{}_{,\mu} \qquad \nabla_\mu V^\alpha = V^\alpha{}_{;\mu}$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\partial_\nu V^\beta$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta}V^\beta\right)$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\left[\frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\partial_\nu V^\beta + \frac{\partial^2 \tilde{x}^\alpha}{\partial x^\beta \partial x^\nu}V^\beta\right]$$

$$\nabla_\mu$$

$$\frac{\partial}{\partial x^\mu} = \partial_\mu \qquad \partial_\mu V^\alpha = V^\alpha{}_{,\mu} \qquad \nabla_\mu V^\alpha = V^\alpha$$

$$\tilde{\partial}_\mu \tilde{V}^\alpha \longrightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu V^\beta$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right)$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left[\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \partial_\nu V^\beta + \frac{\partial^2 \tilde{x}^\alpha}{\partial x^\beta \partial x^\nu} V^\beta \right]$$

$$\underline{\nabla_\mu V^\alpha} = \partial_\mu V^\alpha + \underline{\Gamma^\alpha_{\mu\nu}} V^\nu$$

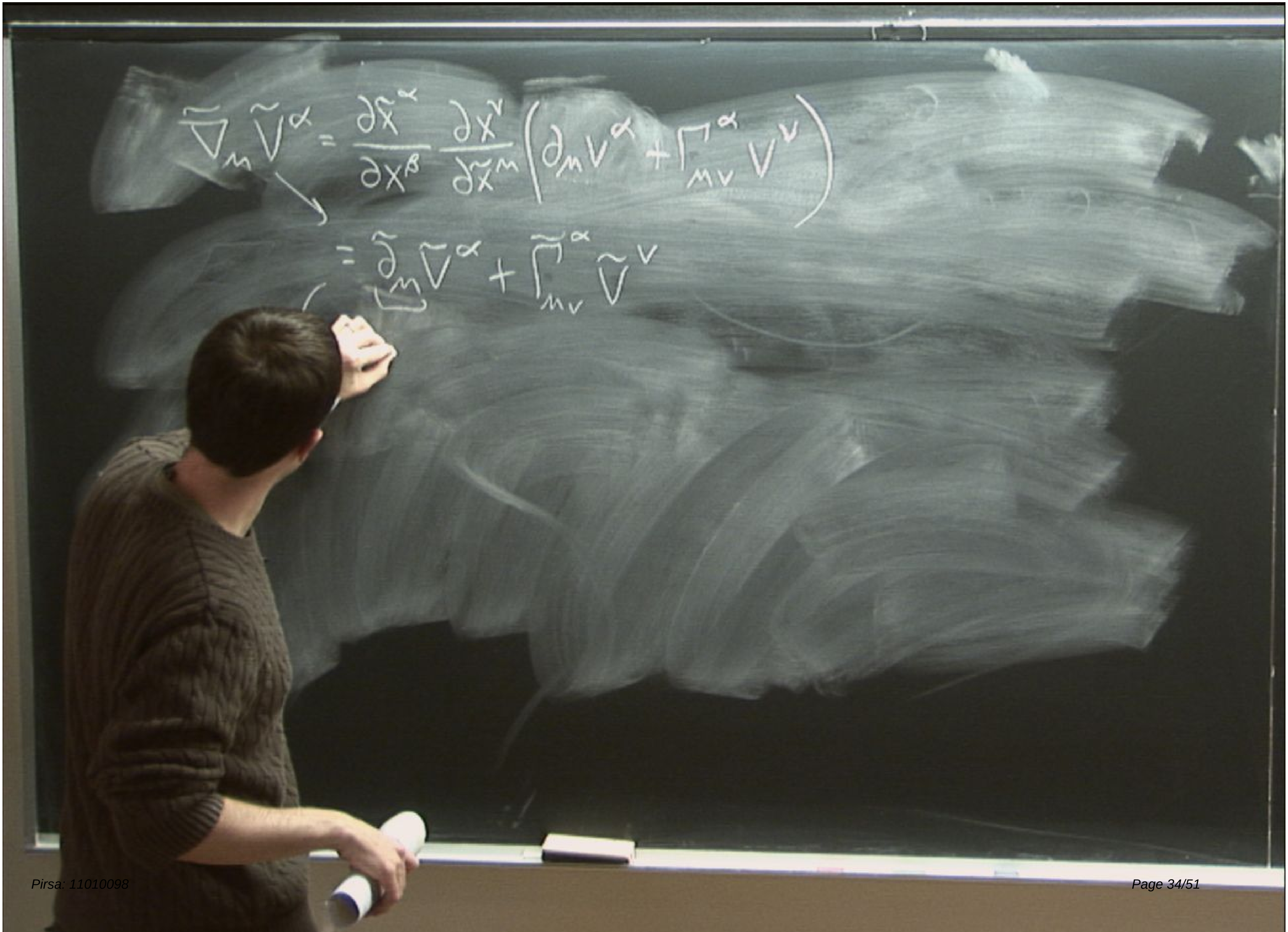
∇

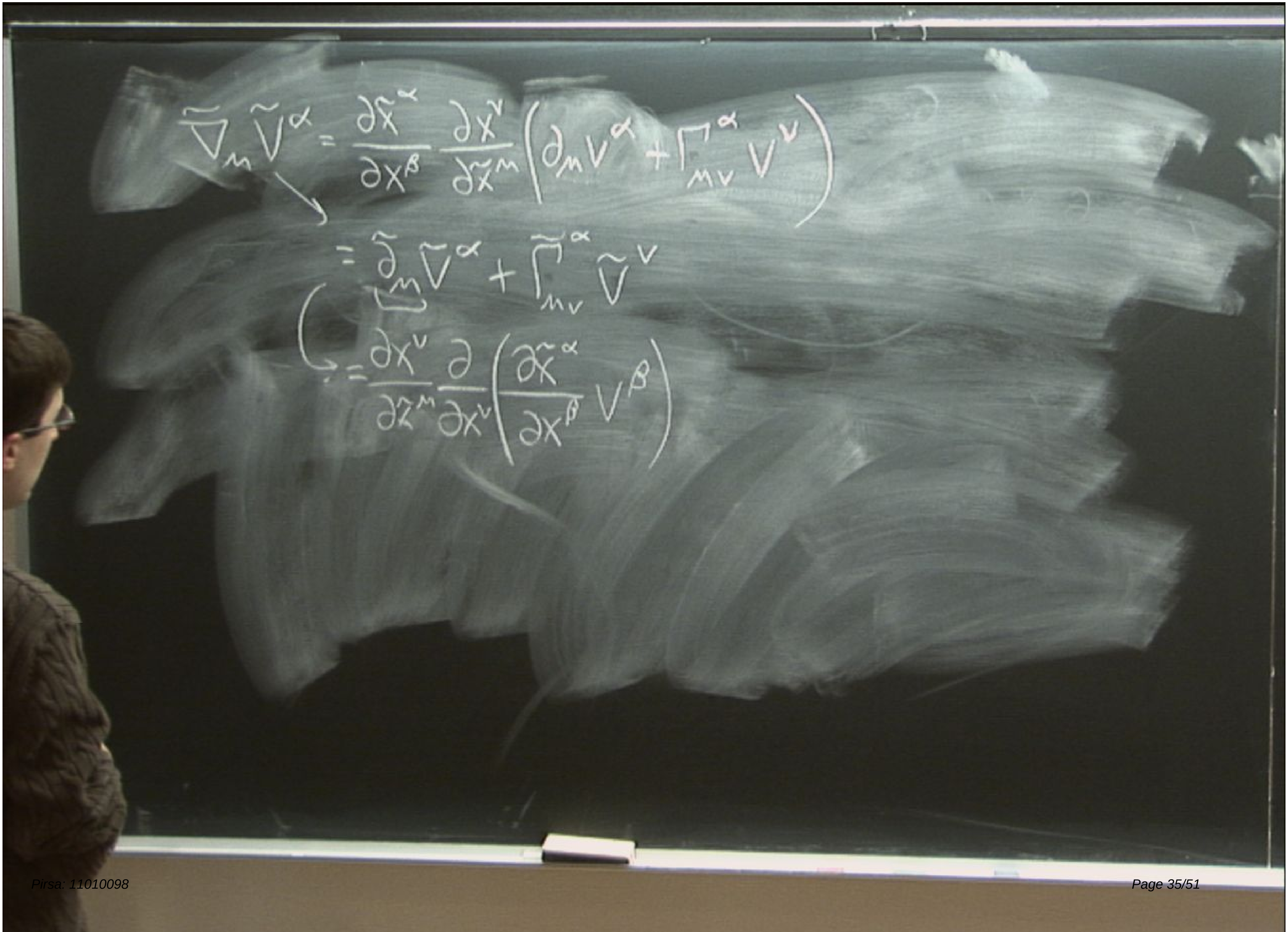
$\frac{\partial}{\partial x^\nu} f(x)$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \partial_\nu V^\beta$$


$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$
$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$

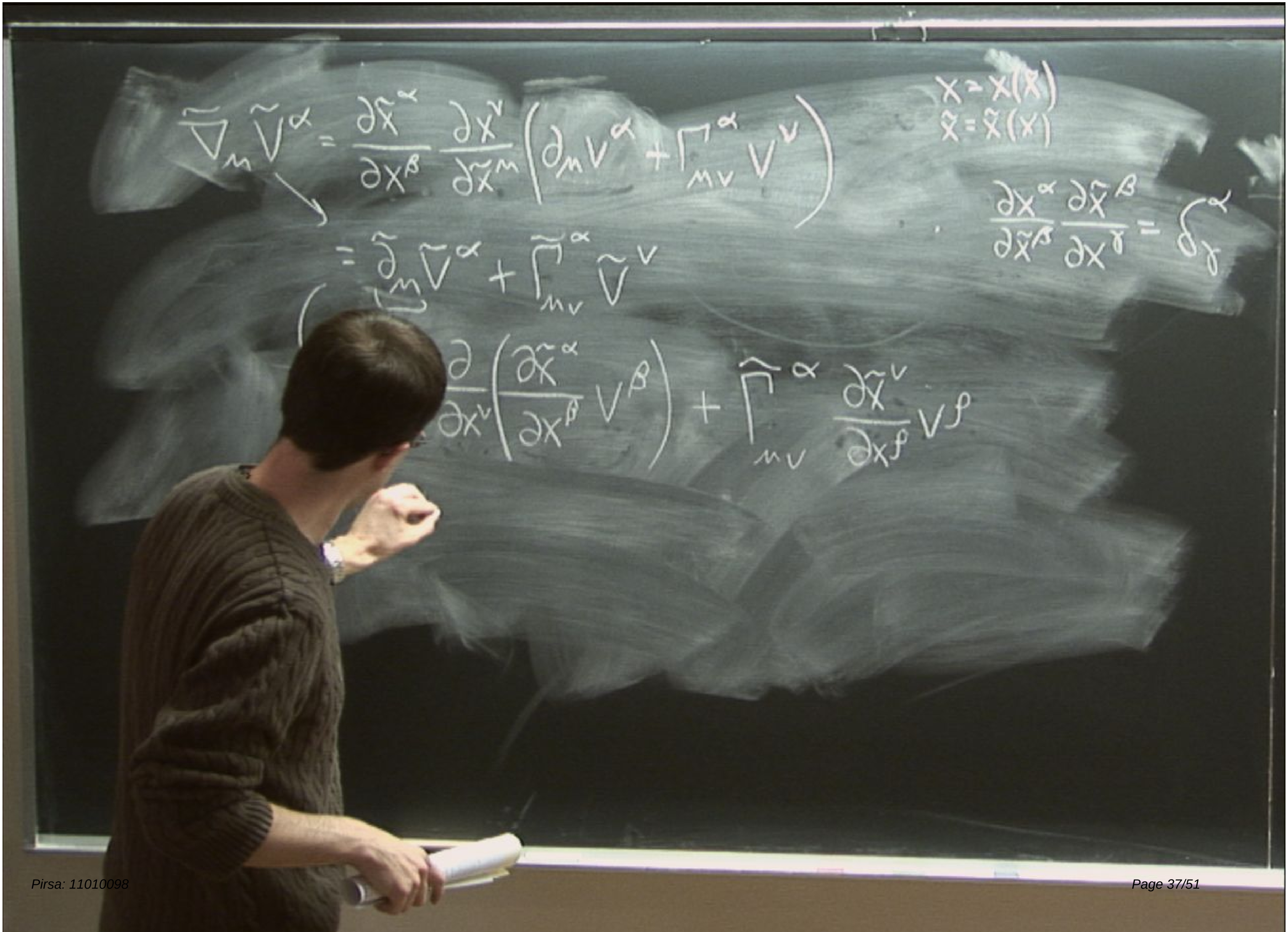

$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$
$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$
$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta\right)$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right) + \tilde{\Gamma}^\alpha_{\mu\nu} \frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$T \quad {}_\gamma = S \quad {}_\gamma$$


$\tilde\nabla_\mu \tilde V^\alpha = \frac{\partial \tilde x^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde x^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$
$= \tilde\partial_\mu \tilde V^\alpha + \tilde\Gamma^\alpha_{\mu\nu}\tilde V^\nu$
$\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde x^\alpha}{\partial x^\beta} V^\beta\right) + \tilde\Gamma^\alpha_{\mu\nu}\frac{\partial \tilde x^\nu}{\partial x^\rho} V^\rho$
$x = x(\tilde x)$
$\tilde x = \tilde x(x)$
$\frac{\partial x^\alpha}{\partial \tilde x^\beta}\frac{\partial \tilde x^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$$

$$x = x(\tilde{x})$$
$$\tilde{x} = \tilde{x}(x)$$

$$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta}\frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta\right) + \tilde{\Gamma}^\alpha_{\mu\nu}\frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial x^\sigma}{\partial \tilde{x}^\beta}\frac{\partial x^\tau}{\partial \tilde{x}^\gamma}\Gamma^\rho_{\sigma\tau} + \frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$$

$$x = x(\tilde{x})$$
$$\tilde{x} = \tilde{x}(x)$$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$$

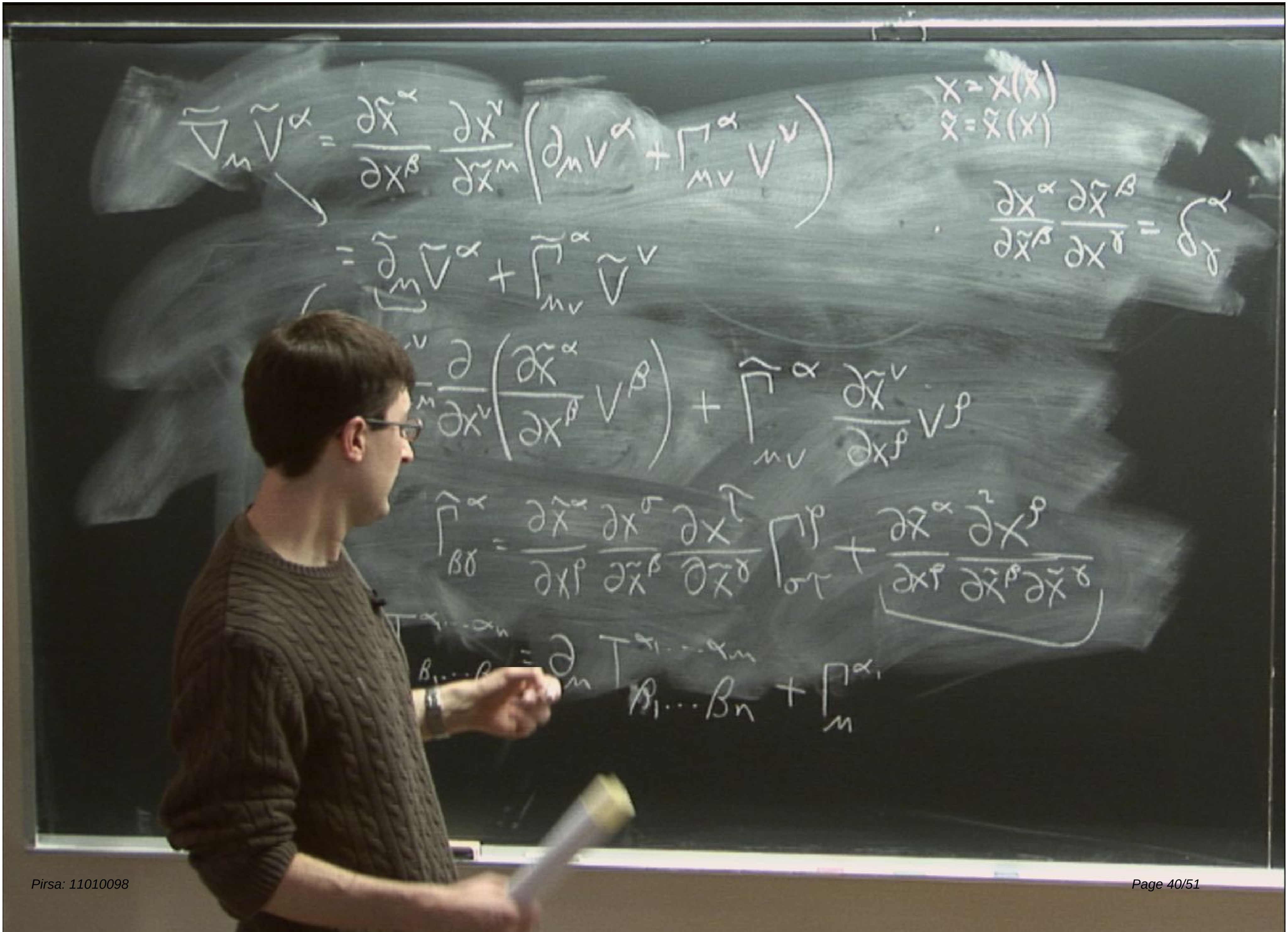
$$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta}\frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$$

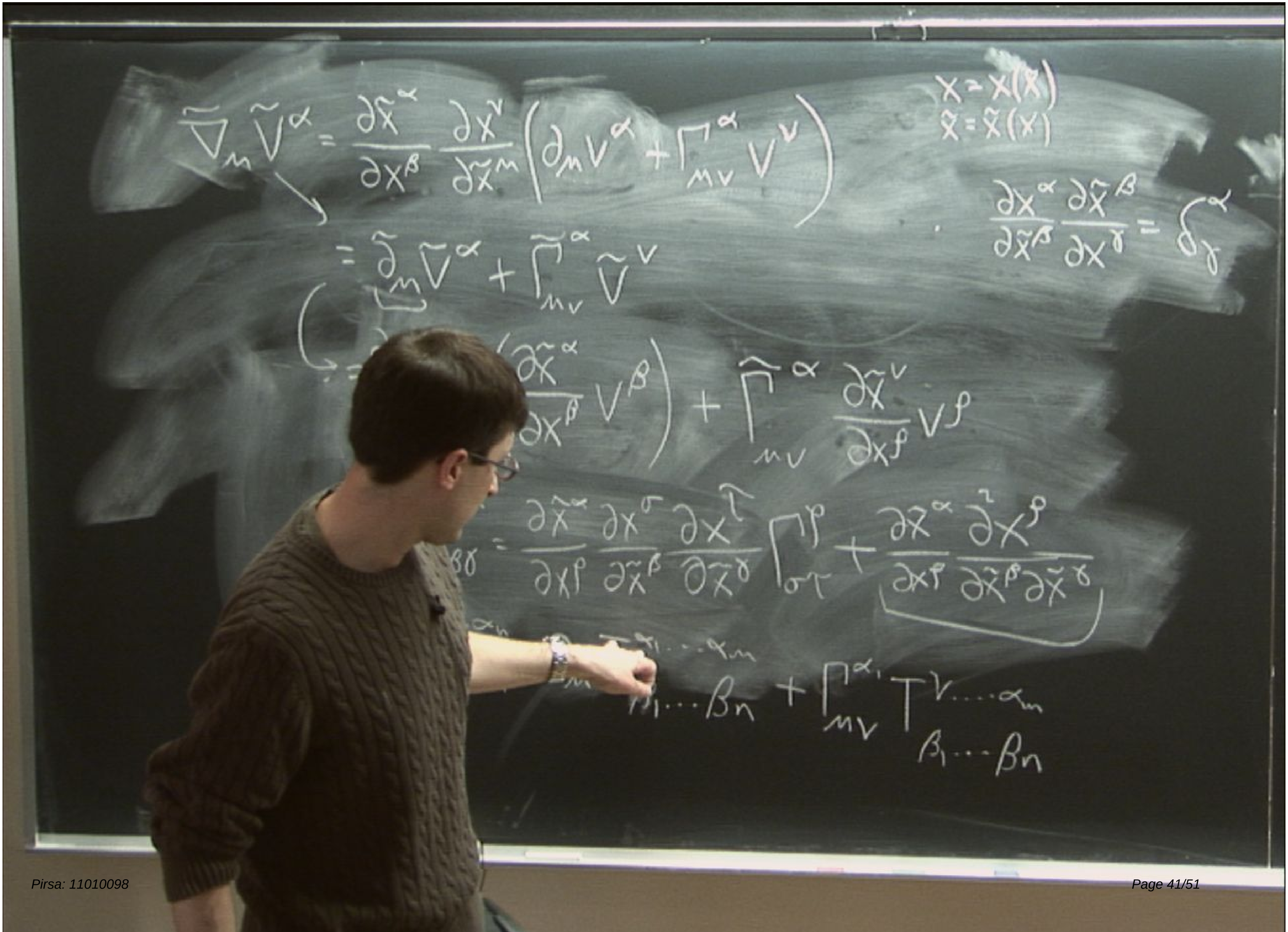
$$= \underbrace{\tilde{\partial}_\mu}\tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu}\tilde{V}^\nu$$

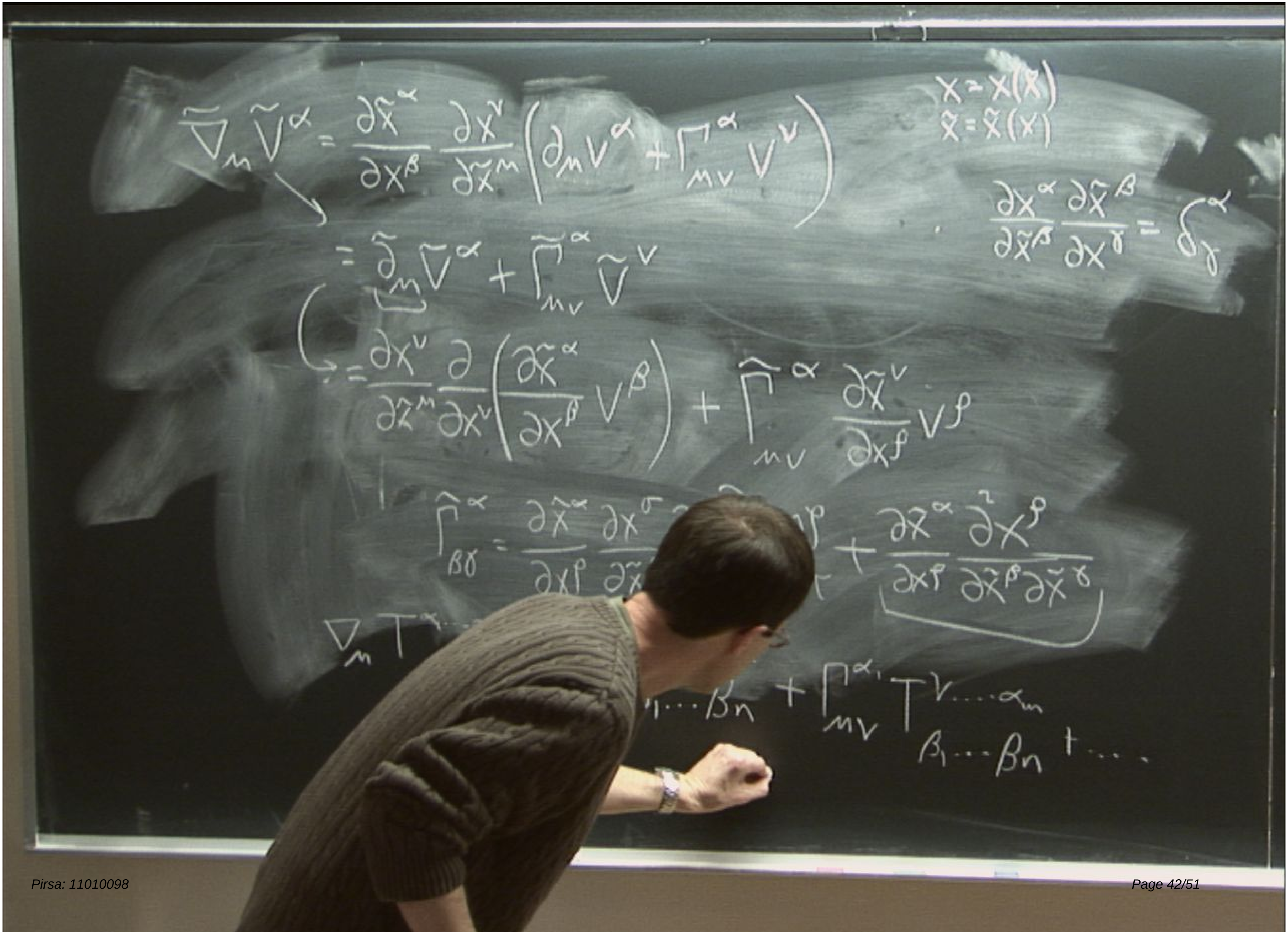
$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta\right) + \tilde{\Gamma}^\alpha_{\mu\nu}\frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial x^\sigma}{\partial \tilde{x}^\beta}\frac{\partial x^\tau}{\partial \tilde{x}^\gamma}\Gamma^\rho_{\sigma\tau} + \underbrace{\frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}}$$

$$\nabla_\mu T^{\alpha_1\cdots\alpha_n}{}_{\beta_1\cdots\beta_n}$$


$\tilde\nabla_\mu \tilde V^\alpha = \frac{\partial \tilde x^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde x^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$
$x = x(\tilde x)$
$\tilde x = \tilde x(x)$
$\frac{\partial x^\alpha}{\partial \tilde x^\beta}\frac{\partial \tilde x^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$
$= \tilde\partial_\mu \tilde V^\alpha + \tilde\Gamma^\alpha_{\mu\nu}\tilde V^\nu$
$\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde x^\alpha}{\partial x^\beta}V^\beta\right) + \tilde\Gamma^\alpha_{\mu\nu}\frac{\partial \tilde x^\nu}{\partial x^\rho}V^\rho$
$\tilde\Gamma^\alpha_{\beta\gamma} = \frac{\partial \tilde x^\alpha}{\partial x^\rho}\frac{\partial x^\sigma}{\partial \tilde x^\beta}\frac{\partial x^\tau}{\partial \tilde x^\gamma}\Gamma^\rho_{\sigma\tau} + \frac{\partial \tilde x^\alpha}{\partial x^\rho}\frac{\partial^2 x^\rho}{\partial \tilde x^\beta \partial \tilde x^\gamma}$
$= \partial_\mu T^{\alpha_1\dots\alpha_m}{}_{\beta_1\dots\beta_n} + \Gamma^{\alpha_1}_{\mu}$


$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$
$x = x(\tilde{x})$
$\tilde{x} = \tilde{x}(x)$
$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$
$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$
$+ \tilde{\Gamma}^\alpha_{\mu\nu} \frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$
$= \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\gamma} \Gamma^\rho_{\sigma\tau} + \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$+ \Gamma^{\alpha_1}_{\mu\nu} T^{\nu \dots \alpha_m}_{\beta_1 \dots \beta_n}$


$x = x(\tilde{x})$
$\tilde{x} = \tilde{x}(x)$
$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$
$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$
$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$
$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right) + \tilde{\Gamma}^\alpha_{\mu\nu} \frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta}$ … $+ \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\nabla_\mu T^{\alpha \dots}$
… $_{\beta_1 \dots \beta_n} + \Gamma^{\alpha_1}_{\mu\nu} T^{\nu \dots \alpha_m}_{\beta_1 \dots \beta_n} + \dots$

$$X = X(\tilde{X})$$
$$\tilde{X} = \tilde{X}(X)$$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\nu V^\beta + \Gamma^\beta_{\nu\gamma} V^\gamma \right)$$

$$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right) + \tilde{\Gamma}^\alpha_{\mu\nu} \frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\gamma} \Gamma^\rho_{\sigma\tau} + \underbrace{\frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}}$$

$$\nabla_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} = \partial_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} + \Gamma^{\alpha_1}_{\mu\nu} T^{\nu \ldots \alpha_m}_{\beta_1 \ldots \beta_n} + \ldots - \Gamma^\nu_{\mu\beta_1} T^{\alpha_1 \ldots \alpha_m}_{\nu \ldots \beta_n}$$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu \right)$$

$$x = x(\tilde{x})$$
$$\tilde{x} = \tilde{x}(x)$$

$$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu} \tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial}{\partial x^\nu} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right) + \hat{\Gamma}^\alpha_{\mu\nu} \frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$\hat{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\gamma} \Gamma^\rho_{\sigma\tau} + \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$$

$$\boxed{\nabla_\mu T^{\alpha_1 \ldots \alpha_m}{}_{\beta_1 \ldots \beta_n}} = \partial_\mu T^{\alpha_1 \ldots \alpha_m}{}_{\beta_1 \ldots \beta_n} + \Gamma^{\alpha_1}_{\mu\nu} T^{\nu \ldots \alpha_m}{}_{\beta_1 \ldots \beta_n} + \ldots - \Gamma^\nu_{\mu\beta_1} T^{\alpha_1 \ldots \alpha_m}{}_{\nu \ldots \beta_n} - \ldots$$

$$x = x(\tilde{x})$$
$$\tilde{x} = \tilde{x}(x)$$

$$\tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}\frac{\partial x^\nu}{\partial \tilde{x}^\mu}\left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu\right)$$

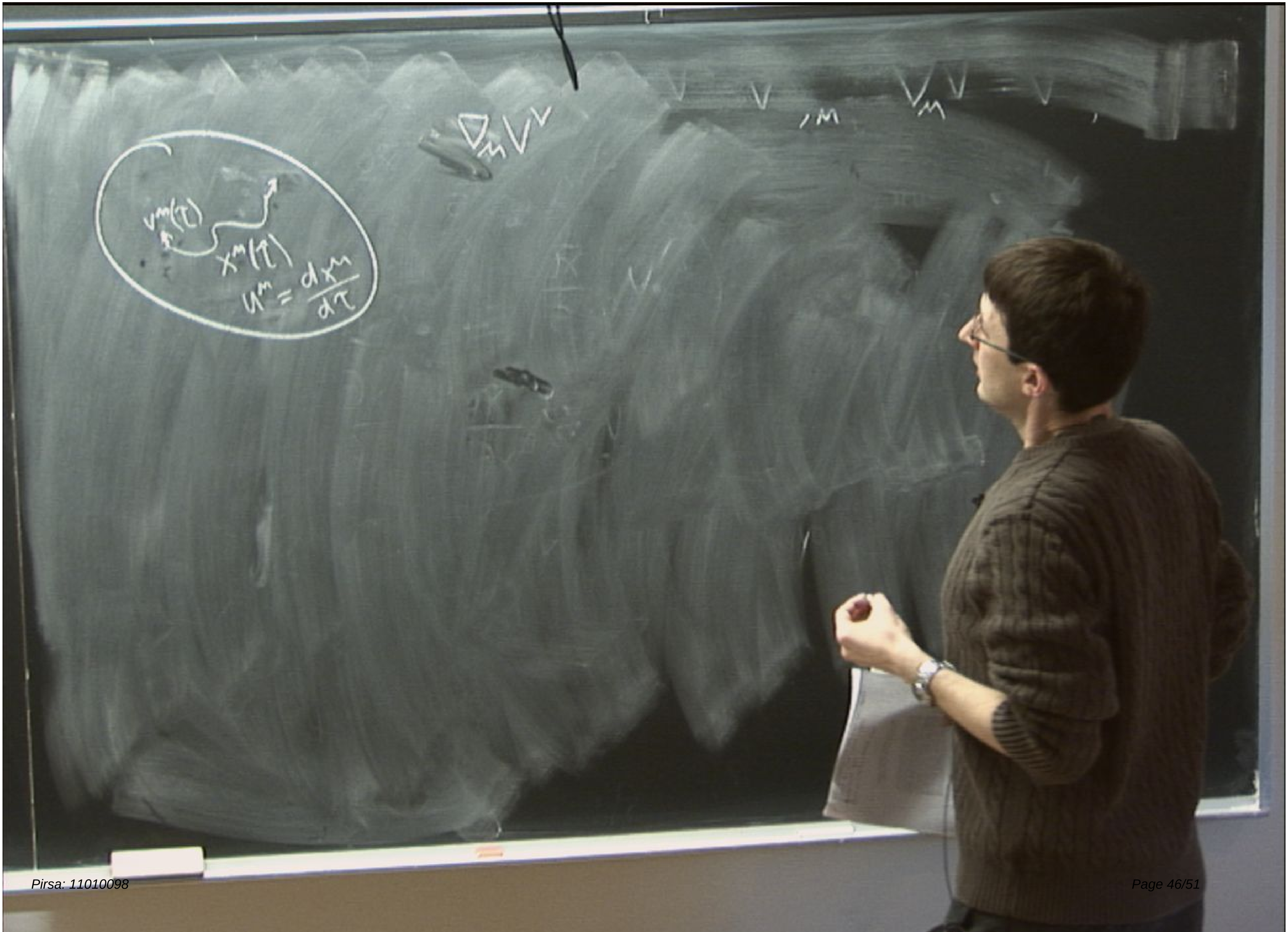
$$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta}\frac{\partial \tilde{x}^\beta}{\partial x^\gamma} = \delta^\alpha_\gamma$$

$$= \tilde{\partial}_\mu \tilde{V}^\alpha + \tilde{\Gamma}^\alpha_{\mu\nu}\tilde{V}^\nu$$

$$= \frac{\partial x^\nu}{\partial \tilde{x}^\mu}\frac{\partial}{\partial x^\nu}\left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta\right) + \tilde{\Gamma}^\alpha_{\mu\nu}\frac{\partial \tilde{x}^\nu}{\partial x^\rho} V^\rho$$

$$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial x^\sigma}{\partial \tilde{x}^\beta}\frac{\partial x^\tau}{\partial \tilde{x}^\gamma}\Gamma^\rho_{\sigma\tau} + \frac{\partial \tilde{x}^\alpha}{\partial x^\rho}\frac{\partial^2 x^\rho}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$$

$$\boxed{\nabla_\mu T^{\alpha_1\ldots\alpha_m}{}_{\beta_1\ldots\beta_n}} = \partial_\mu T^{\alpha_1\ldots\alpha_m}{}_{\beta_1\ldots\beta_n} + \Gamma^{\alpha_1}_{\mu\nu} T^{\nu\ldots\alpha_m}{}_{\beta_1\ldots\beta_n} + \ldots - \Gamma^{\nu}_{\mu\beta_1} T^{\alpha_1\ldots\alpha_m}{}_{\nu\ldots\beta_n} - \ldots$$


∇μ Vν
vm(τ)
xm(τ)
um = dxm/dτ

$v^{\mu}(\tau)$

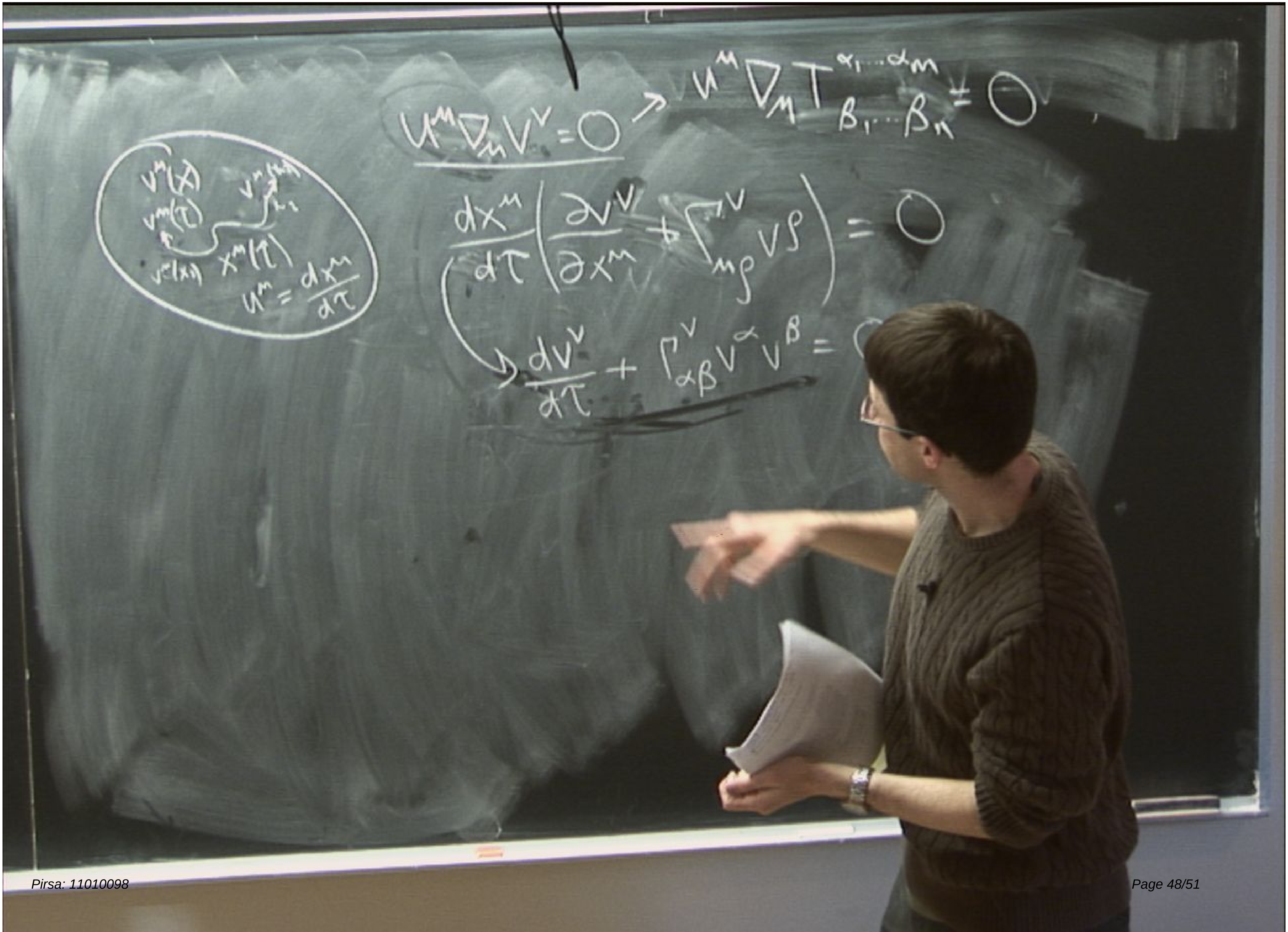
$x^{\mu}(\tau)$

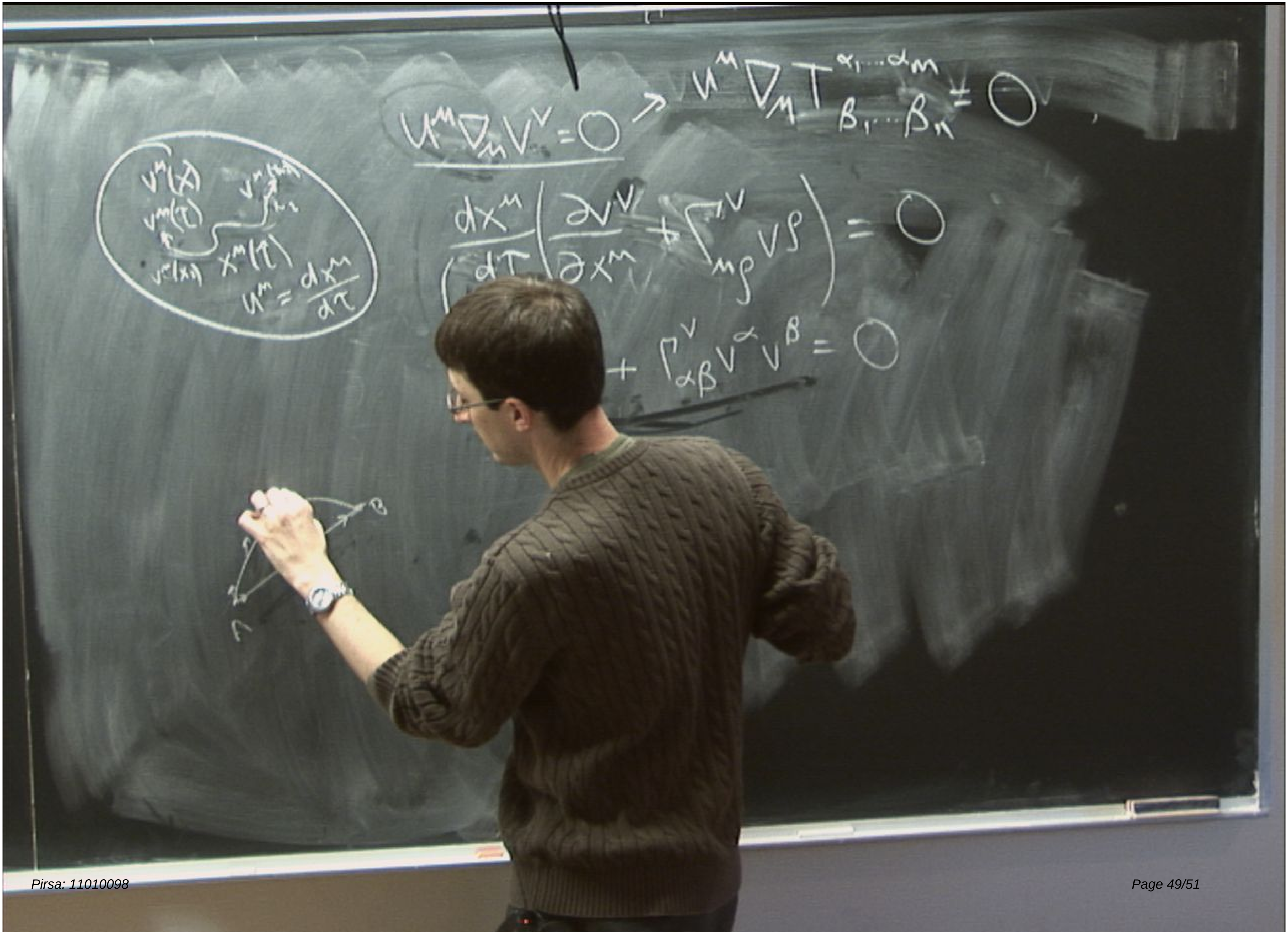
$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

$$u^{\mu}\nabla_{\mu}V^{\nu} = 0 \rightarrow u^{\mu}\nabla_{\mu}T^{\alpha_1\ldots\alpha_m}{}_{\beta_1\ldots\beta_n} = 0$$

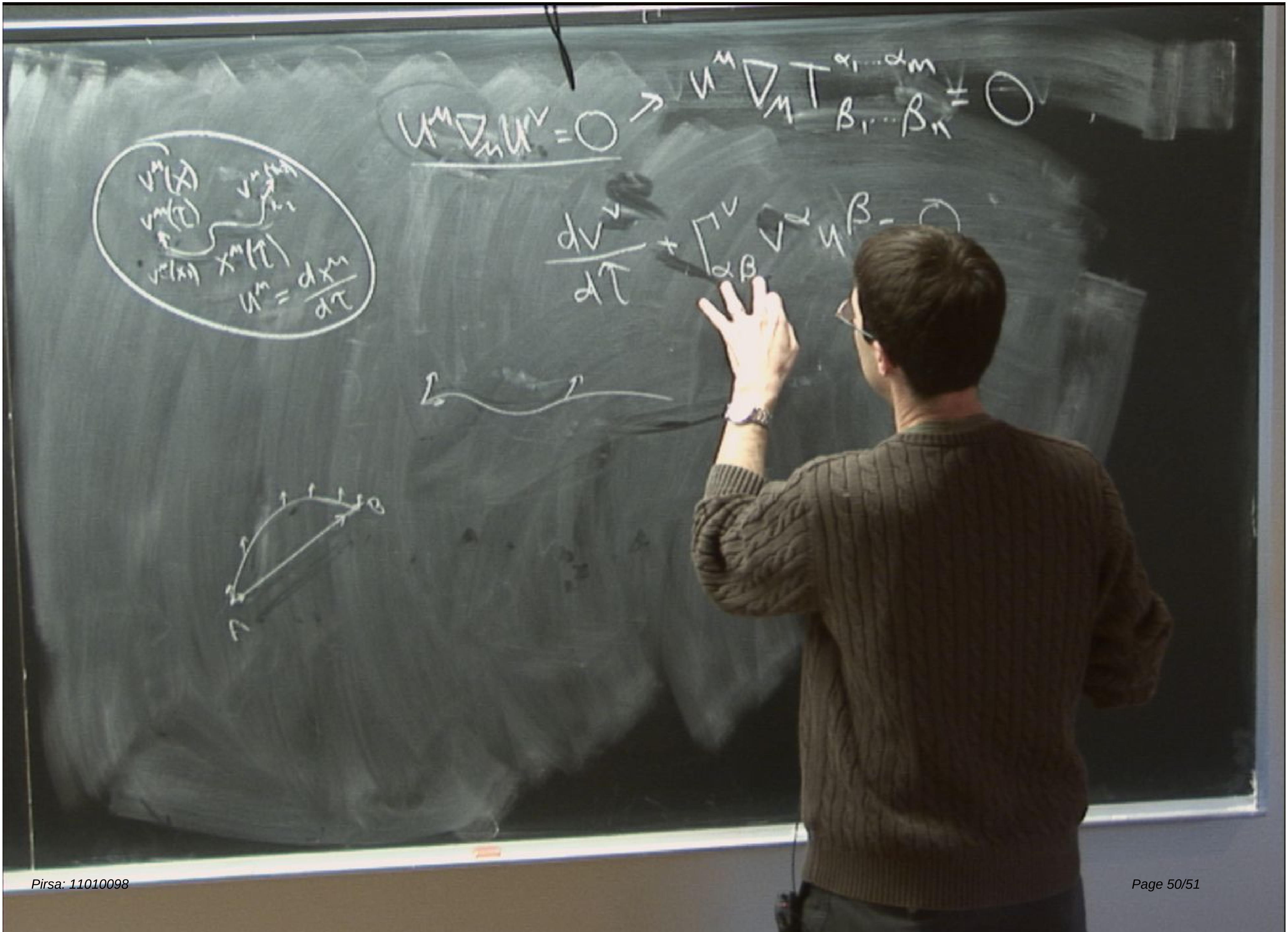
$$\frac{dx^{\mu}}{d\tau}\left(\frac{\partial V^{\nu}}{\partial x^{\mu}} + \Gamma^{\nu}_{\mu\rho}V^{\rho}\right) = 0$$

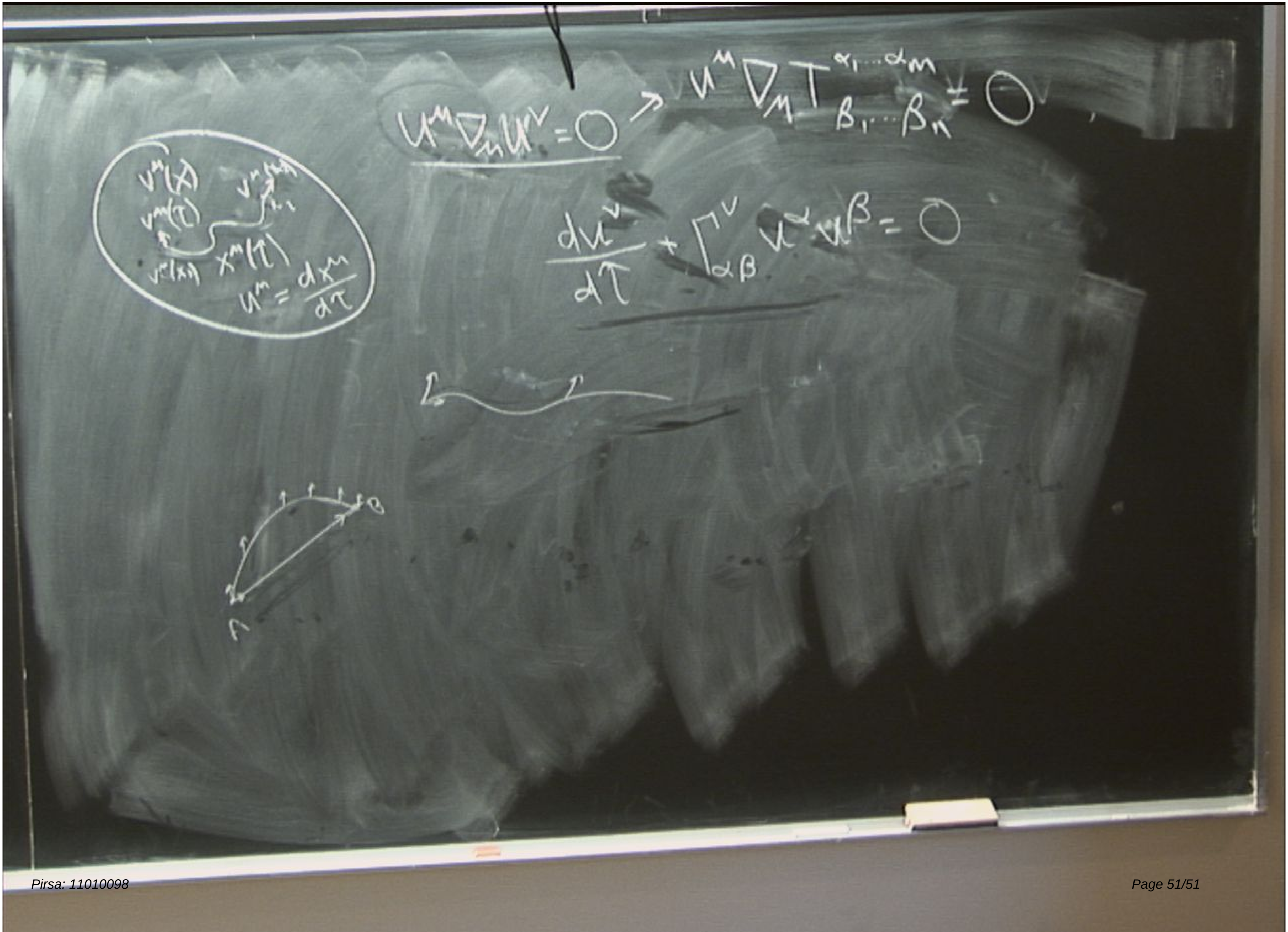
$$\frac{dV^{\nu}}{d\tau} + \Gamma^{\nu}_{\alpha\beta}V^{\alpha}V^{\beta} = 0$$


$U^\mu \nabla_\mu V^\nu = 0 \rightarrow U^\mu \nabla_\mu T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} = 0$
$V^\mu(x)$
$V^\mu(\tau)$
$x^\mu(\tau)$
$U^\mu = \frac{dx^\mu}{d\tau}$
$\frac{dx^\mu}{d\tau}\left(\frac{\partial V^\nu}{\partial x^\mu} + \Gamma^\nu_{\mu\rho} V^\rho\right) = 0$
$\frac{dV^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta} V^\alpha V^\beta = 0$



$U^\mu \nabla_\mu V^\nu = 0 \rightarrow U^\mu \nabla_\mu T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} = 0$
$V^\mu(\lambda)$
$V^\mu(\tau)$
$X^\mu(\tau)$
$U^\mu = \frac{dx^\mu}{d\tau}$
$\frac{dx^\mu}{d\tau}\left(\frac{\partial V^\nu}{\partial x^\mu} + \Gamma^\nu_{\mu\rho} V^\rho\right) = 0$
$+ \Gamma^\nu_{\alpha\beta} V^\alpha V^\beta = 0$
B
A


$u^\mu \nabla_\mu u^\nu = 0$
$u^\mu \nabla_\mu T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n} = 0$
$\frac{dv^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta} v^\alpha u^\beta = 0$
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$


$u^\mu \nabla_\mu u^\nu = 0$
$u^\mu \nabla_\mu T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} \neq 0$
$\frac{du^\nu}{d\tau} + \Gamma^\nu_{\alpha\beta} u^\alpha u^\beta = 0$
$v^\mu(\lambda)$
$v^\mu(\tau)$
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$
A
B