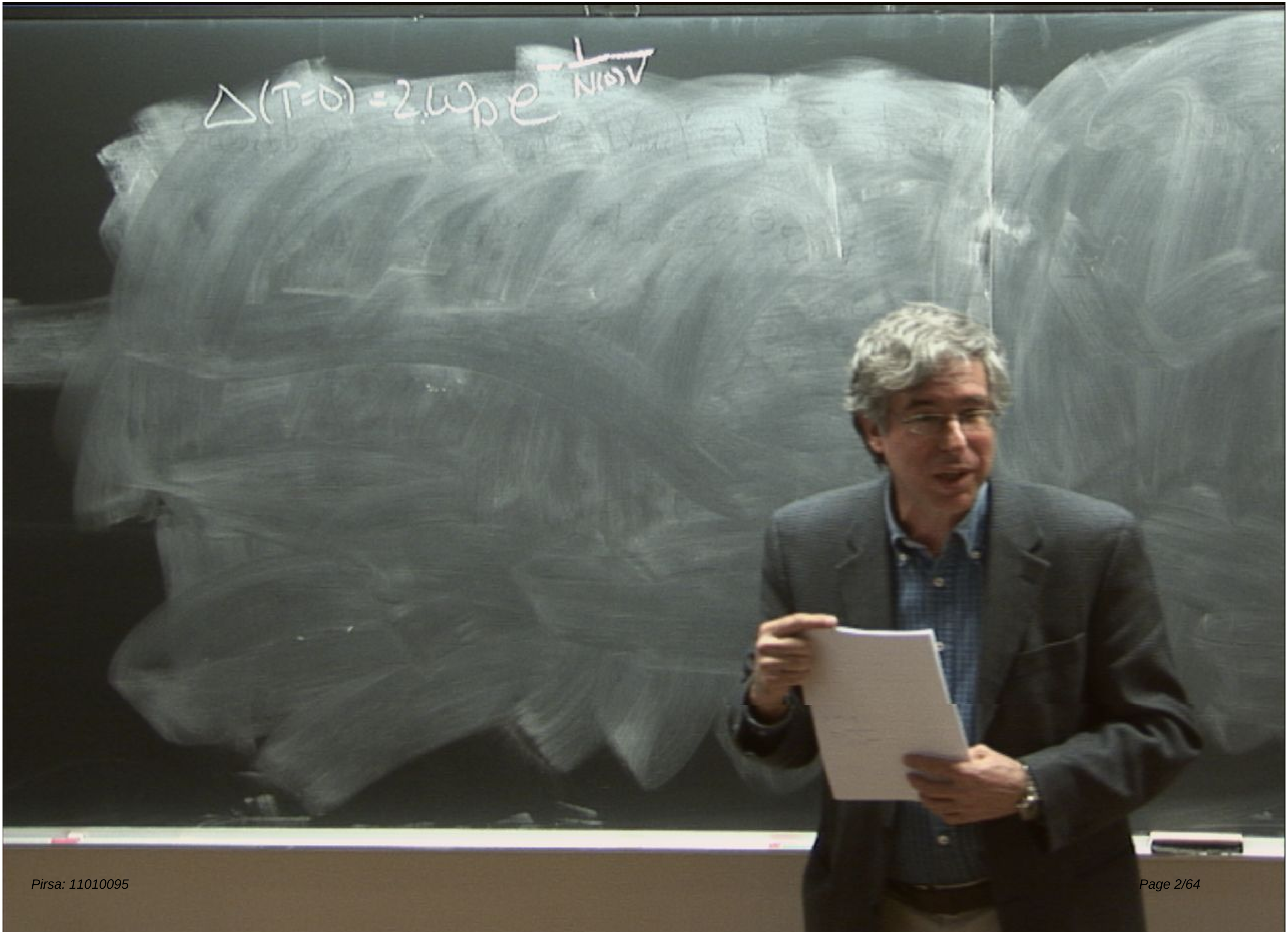
Title: Condensed Matter Review - Lecture 13

Date: Jan 19, 2011 10:15 AM

URL: http://pirsa.org/11010095

Abstract:


$\Delta(T=0) = 2\omega_D e^{-\frac{1}{N(0)V}}$

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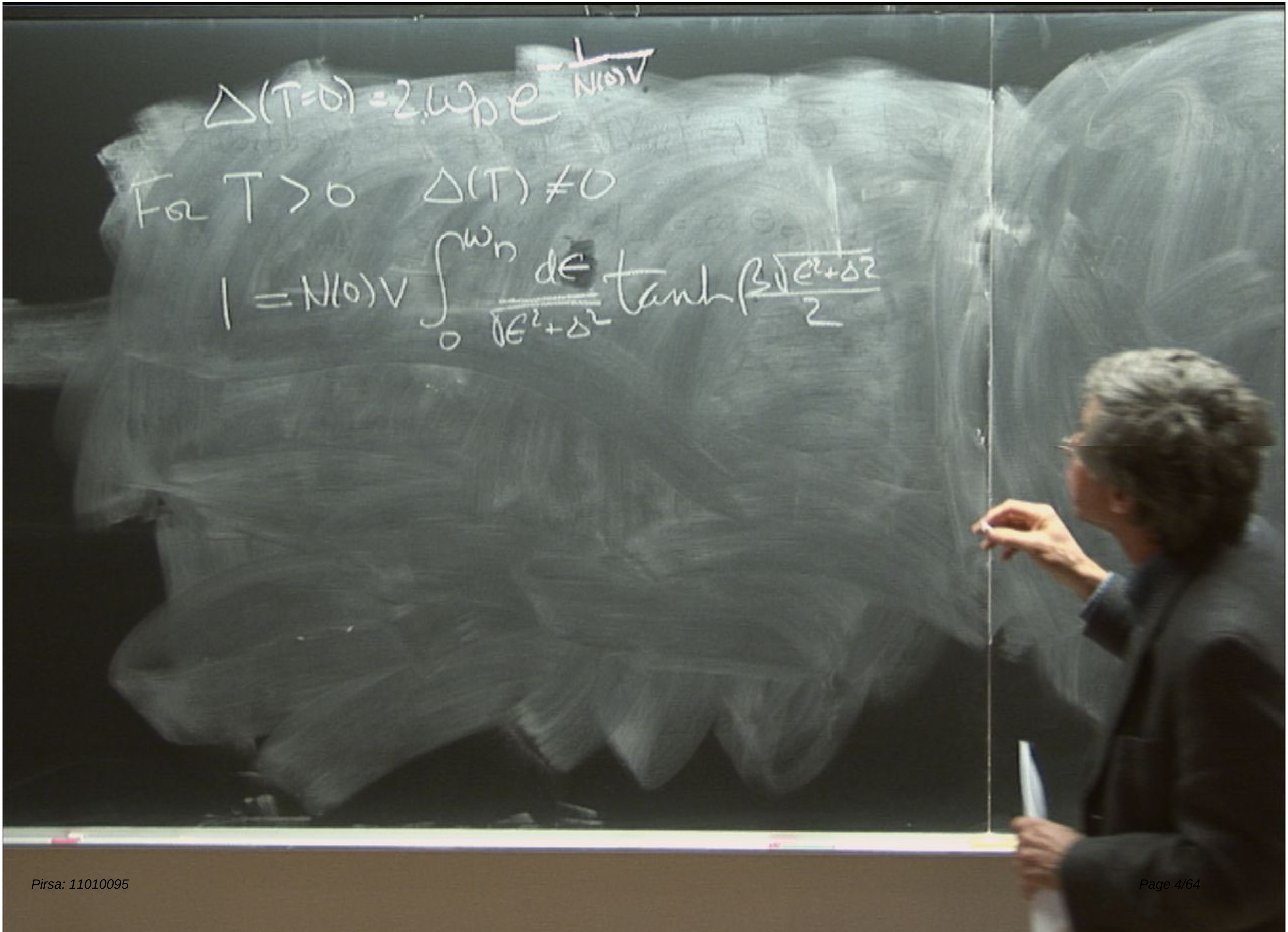
For $T > 0$ $\quad \Delta(T) \neq 0$

$$1 = N(0)V \int_0^{\omega_D} \frac{d\epsilon}{\sqrt{\epsilon^2+\Delta^2}} \tanh \frac{\beta\sqrt{\epsilon^2+\Delta^2}}{2}$$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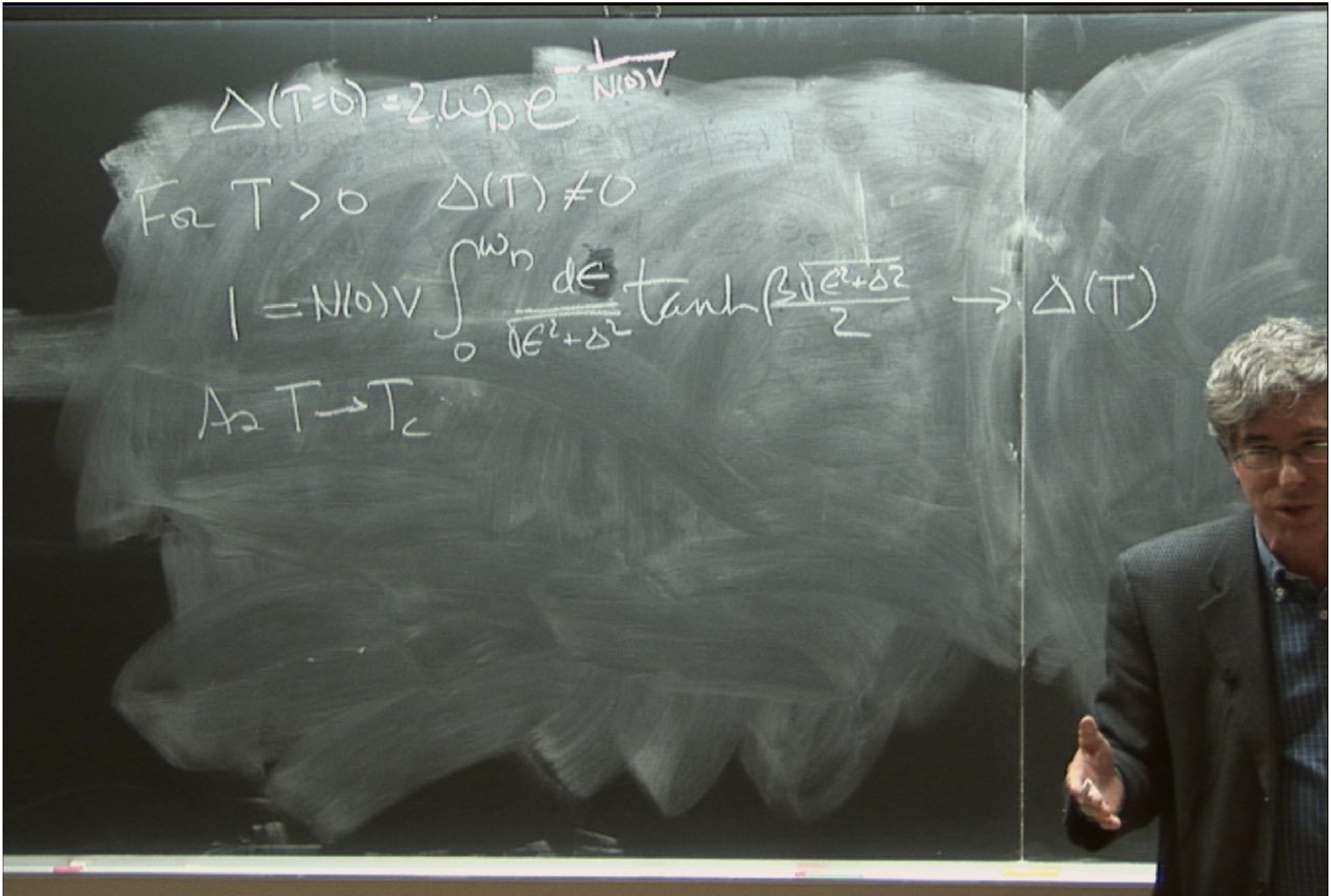
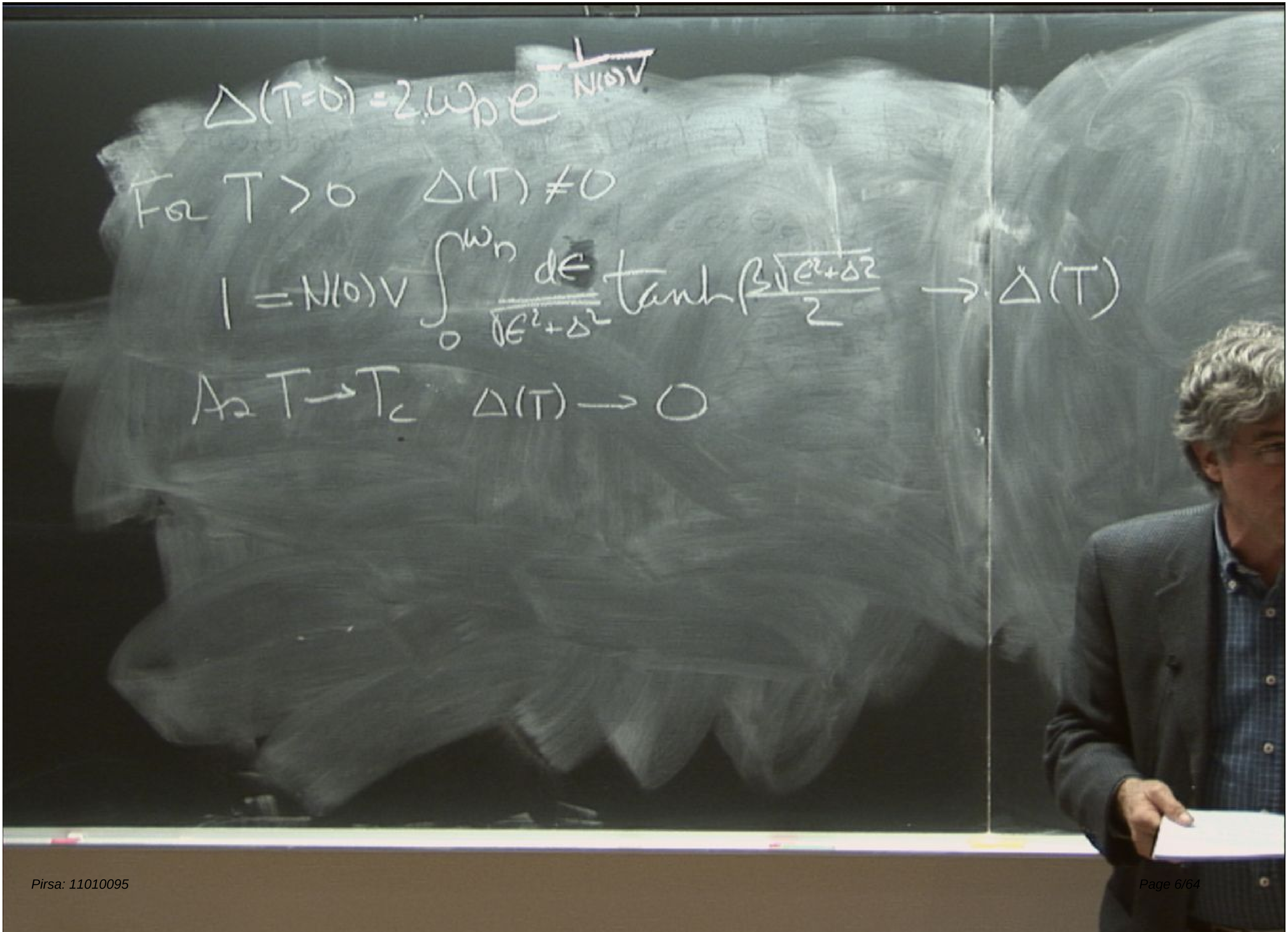

Δ(T=0) = 2ω_D e^(-1/N(0)V)
For T > 0   Δ(T) ≠ 0
1 = N(0)V ∫_0^(ω_D) dε/√(ε²+Δ²) tanh β√(ε²+Δ²)/2 → Δ(T)
As T → T_c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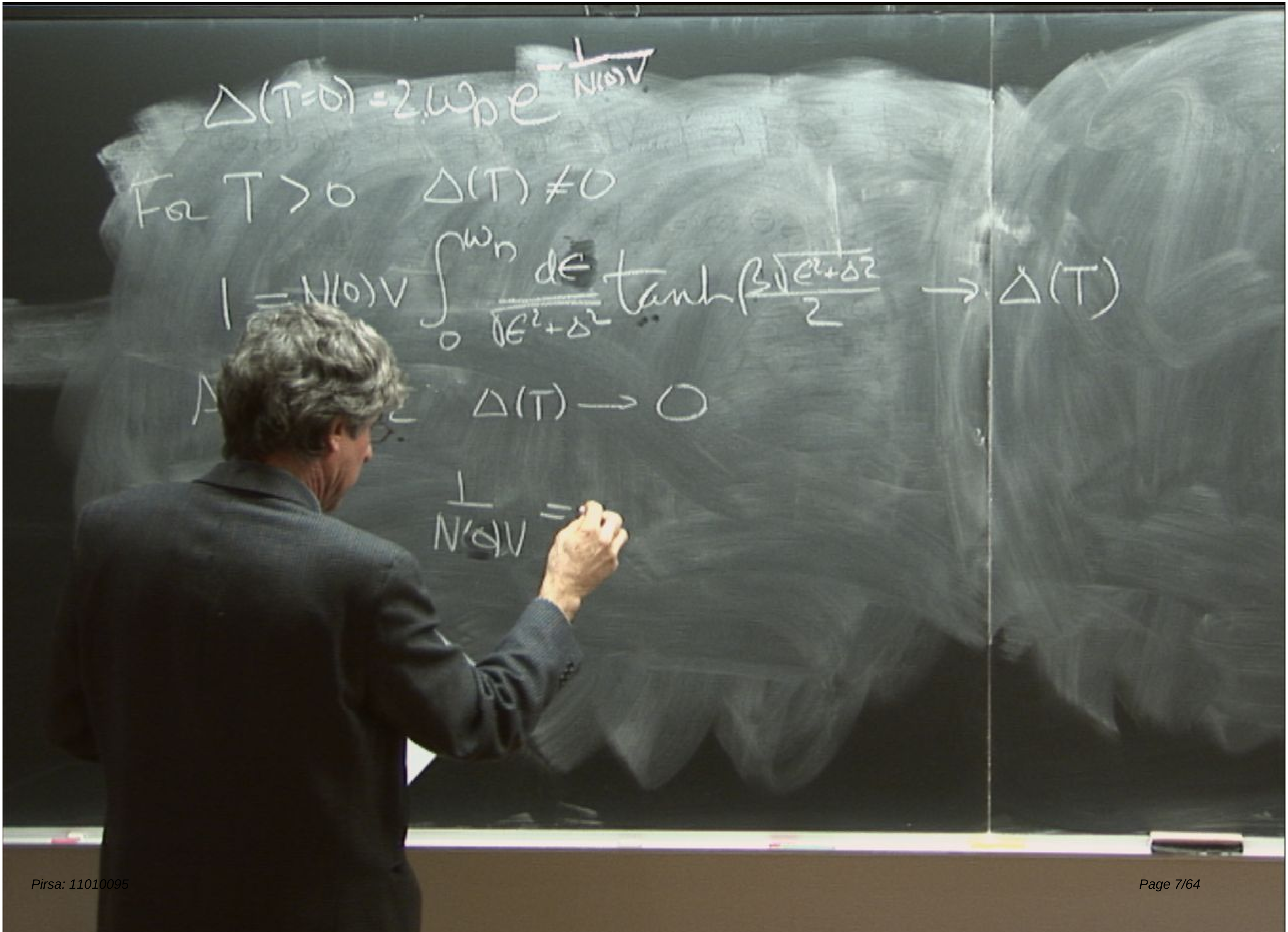
As $T \rightarrow T_c$ $\Delta(T) \rightarrow 0$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$\Delta(T) \rightarrow 0$
$\frac{1}{N(0)V} =$

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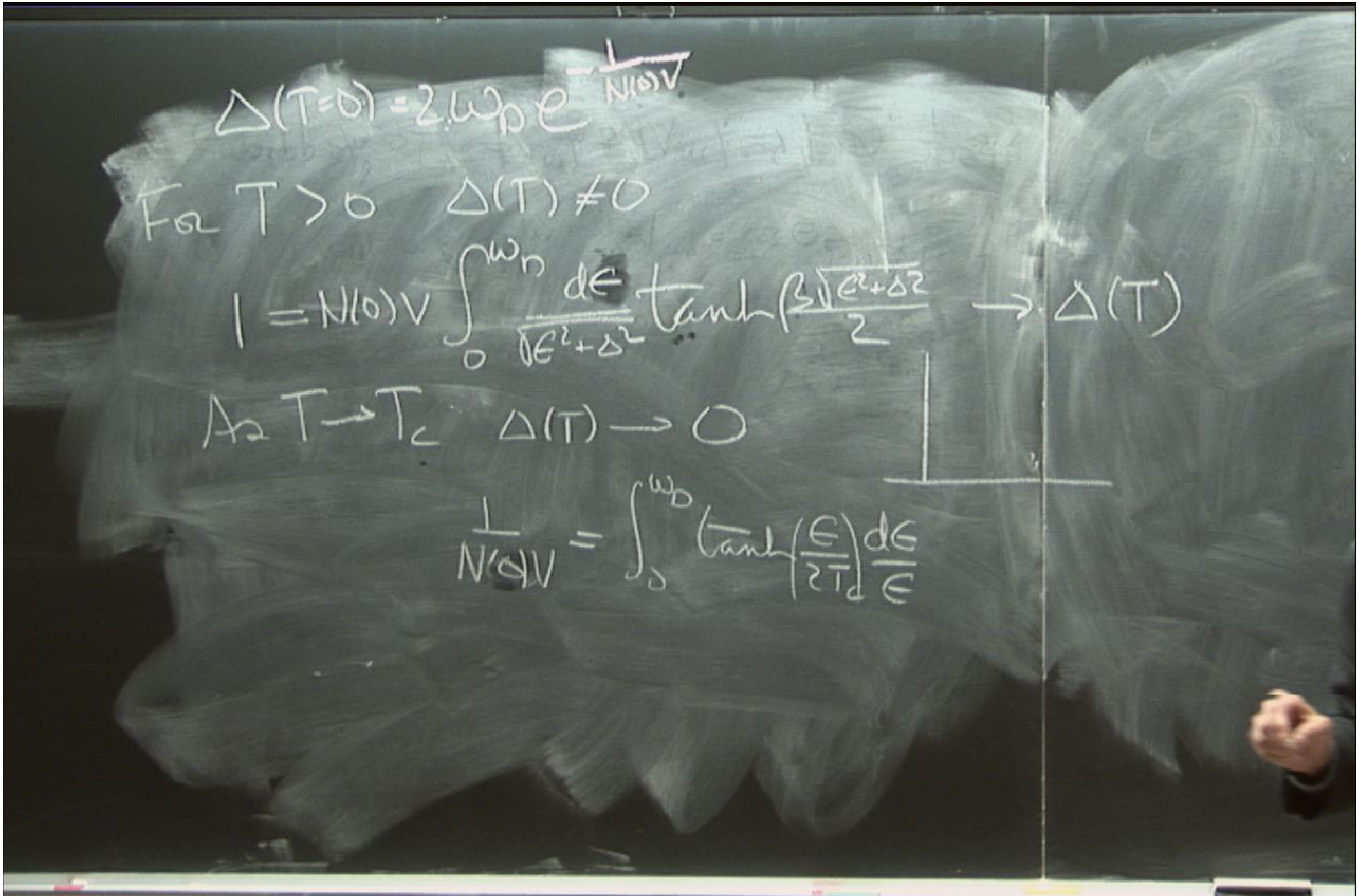
As $T \rightarrow T_c$ $\quad \Delta(T) \rightarrow 0$

$$\frac{1}{N(0)V} = \int_0^{\omega_D} \tanh \frac{\epsilon}{2T_c} \frac{d\epsilon}{\epsilon}$$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As $T \to T_c$ $\Delta(T) \to 0$
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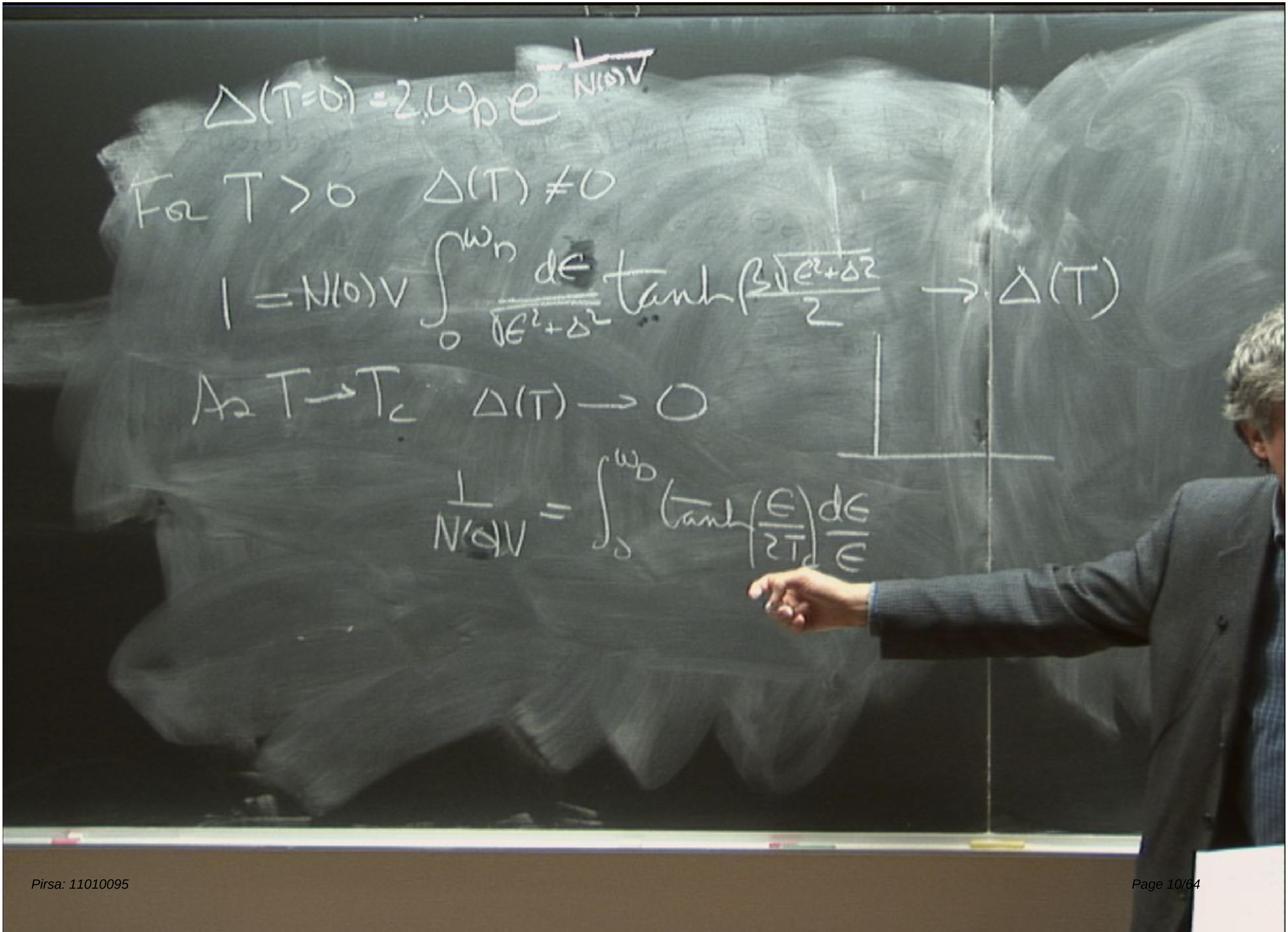
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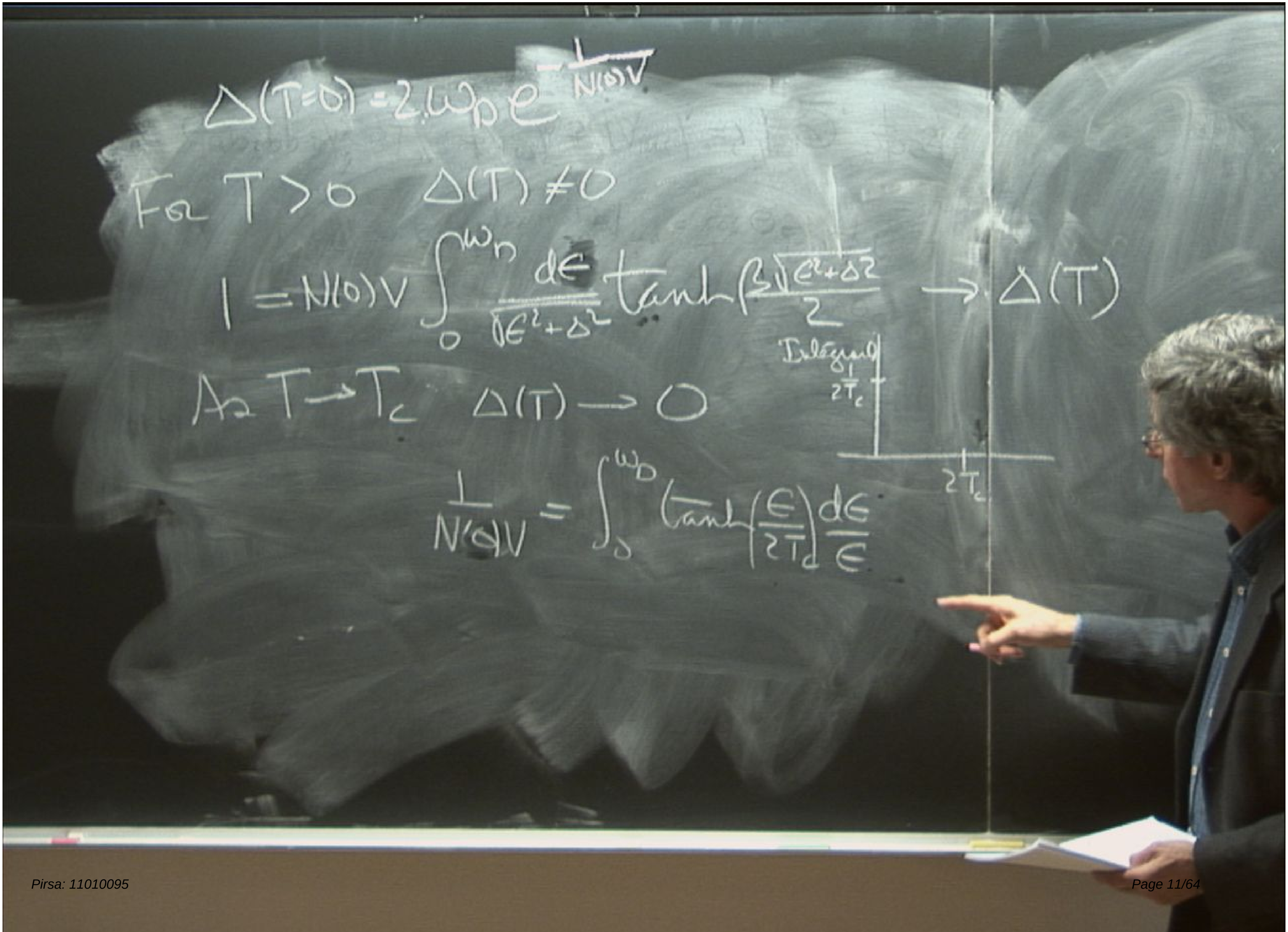

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Integrand
$\frac{1}{2T_c}$
$2T_c$
As $T \rightarrow T_c$ $\Delta(T) \rightarrow 0$
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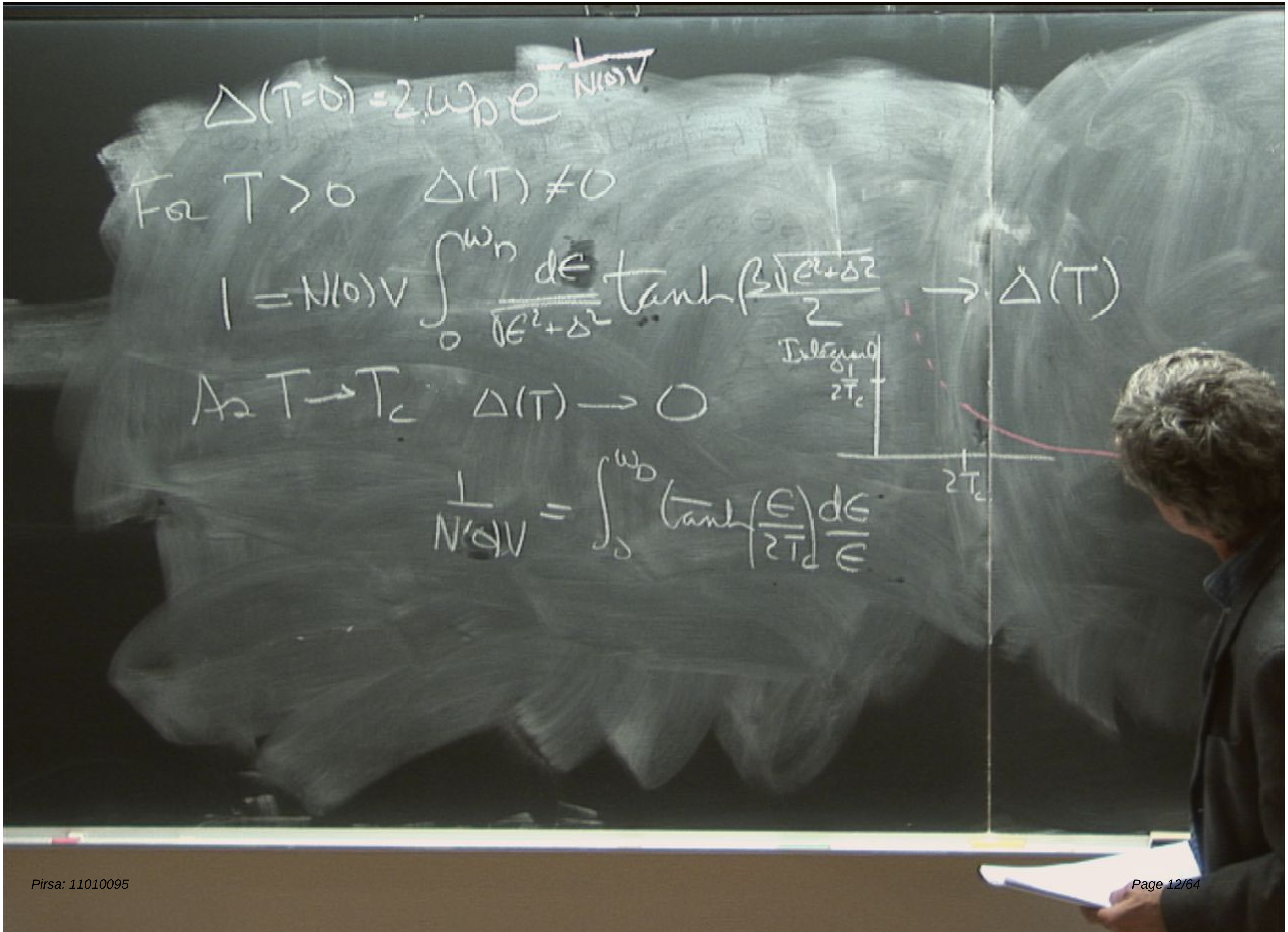
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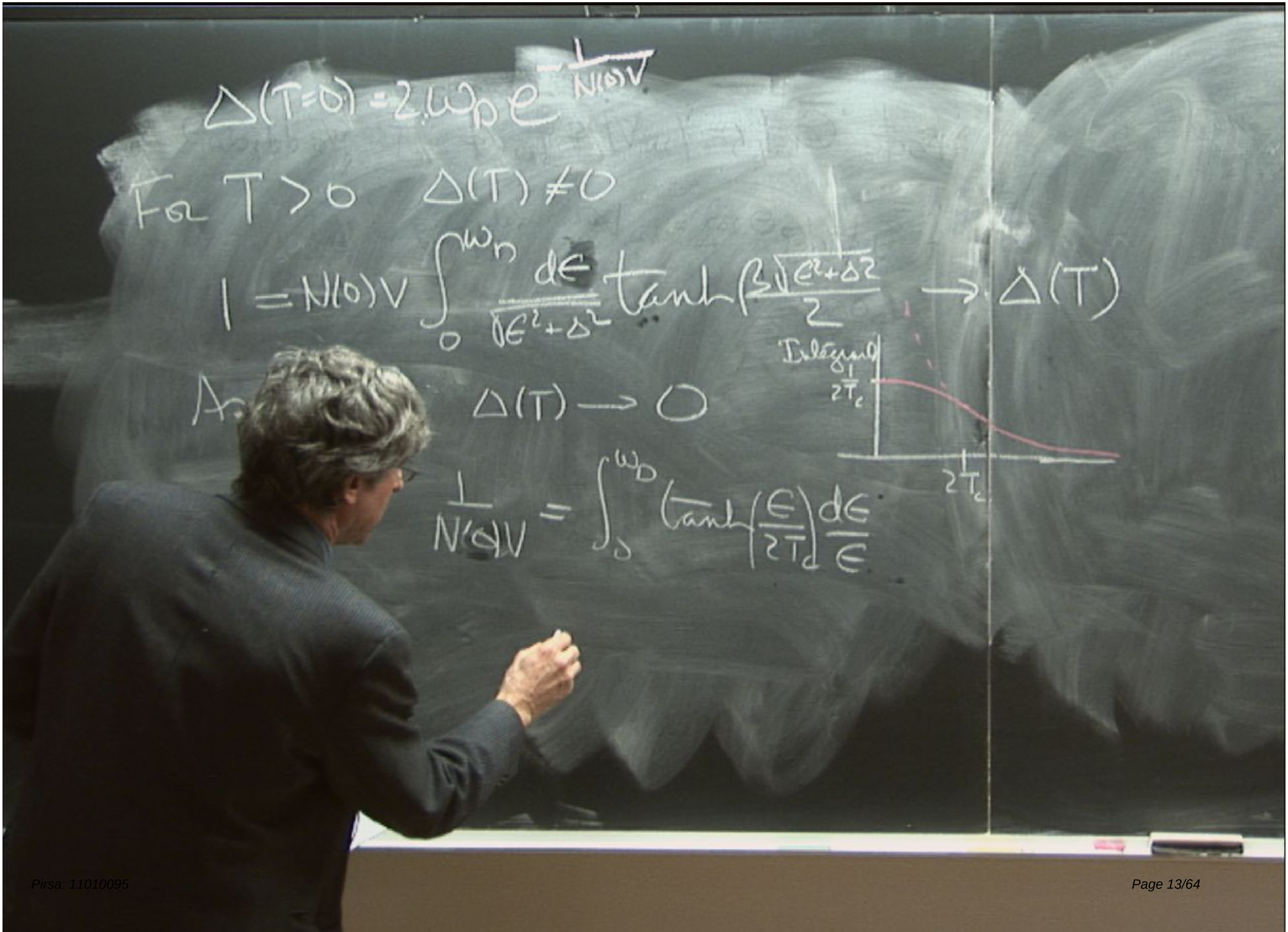
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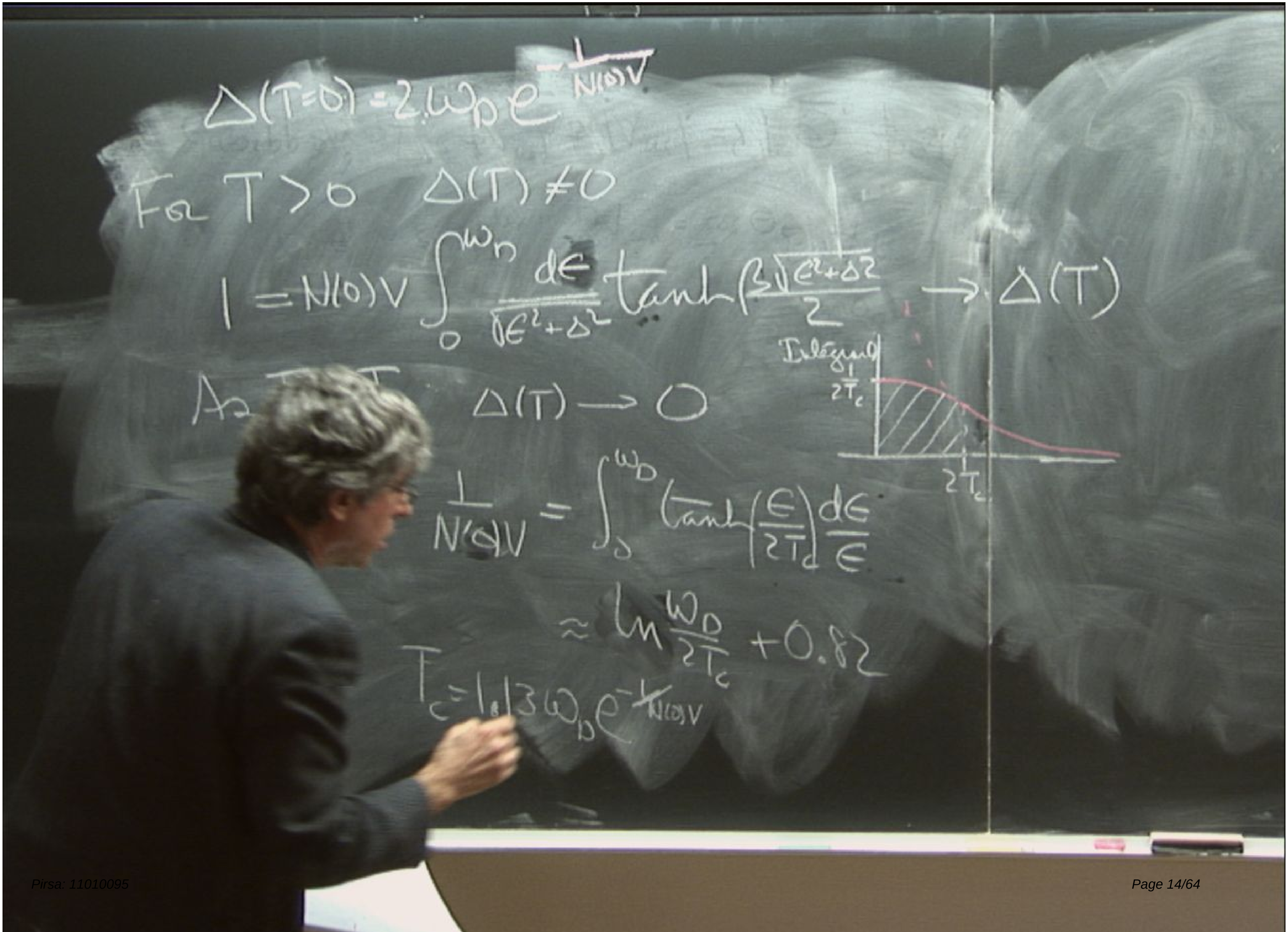
1/N(0)V = ∫_0^{ω_D} tanh(ε/2T_c) dε/ε
Integrand
1/2T_c
2T_c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$\frac{1}{N(0)V} = \int_0^{\omega_D} \tanh\left(\frac{\epsilon}{2T_c}\right)\frac{d\epsilon}{\epsilon}$
$\approx \ln \frac{\omega_D}{2T_c} + 0.82$
$T_c = 1.13\,\omega_D e^{-\frac{1}{N(0)V}}$

$$\Delta(T=0) = 2\omega_D e^{-\frac{1}{N(0)V}}$$

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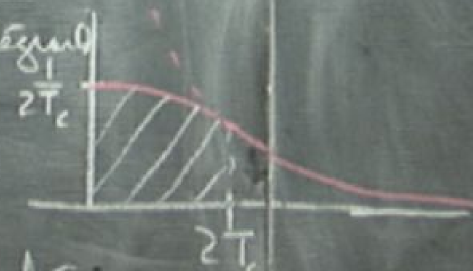
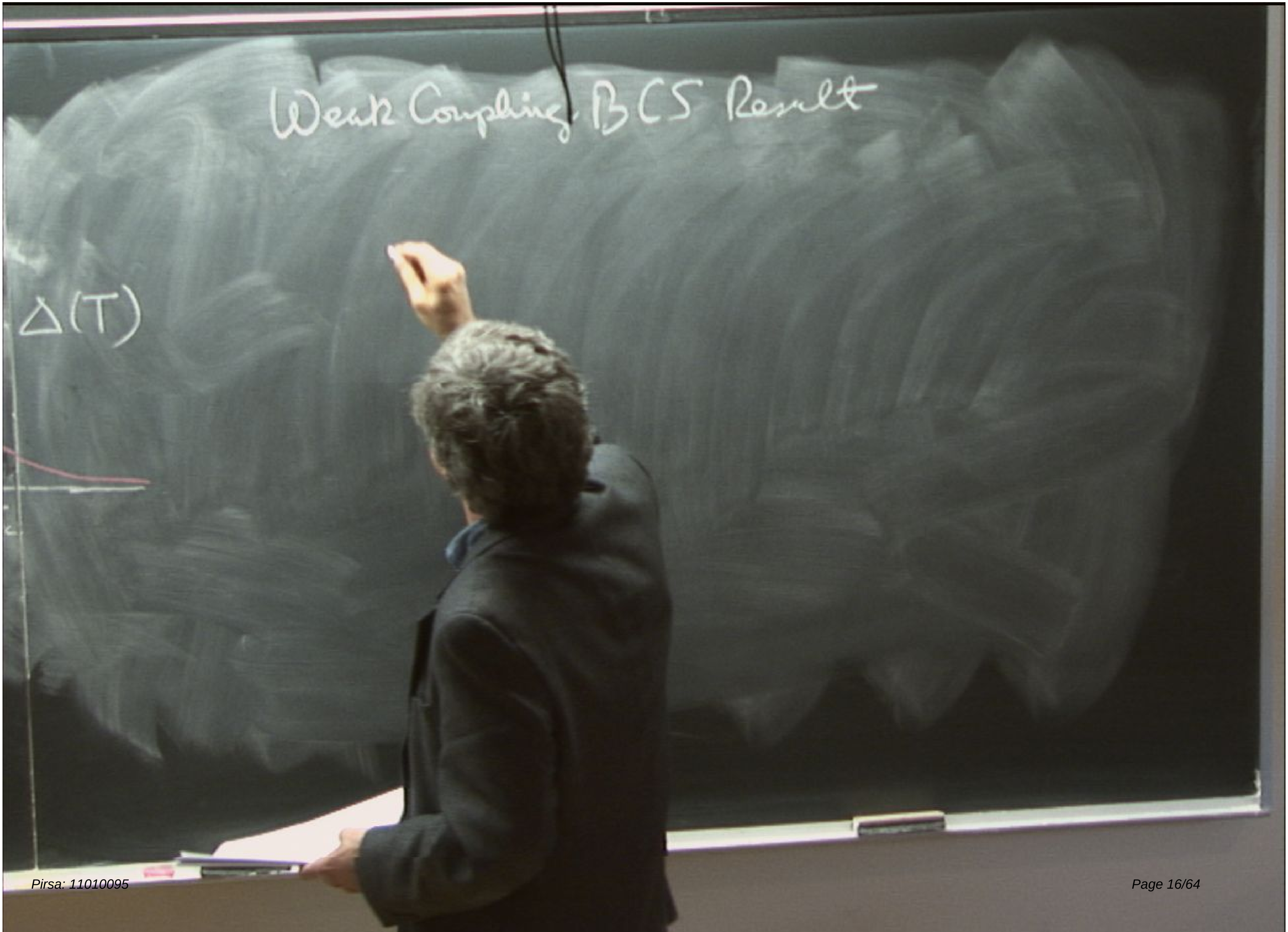
As $T \rightarrow T_c \quad \Delta(T) \rightarrow 0$

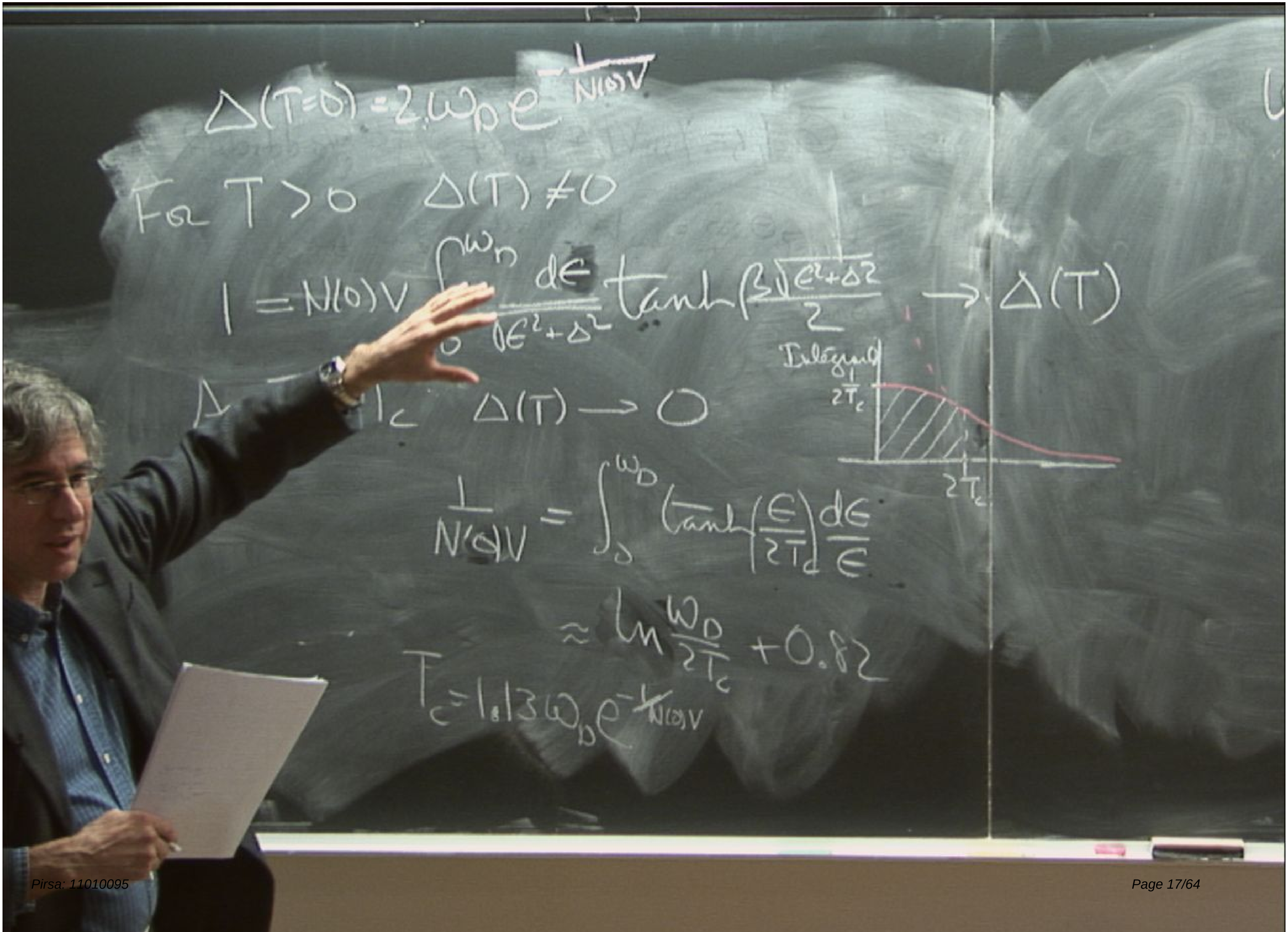


$$\frac{1}{N(0)V} = \int_0^{\omega_D} \tanh\left(\frac{\epsilon}{2T_c}\right)\frac{d\epsilon}{\epsilon}$$

$$\approx \ln \frac{\omega_D}{2T_c} + 0.82$$

$$T_c = 1.13\,\omega_D e^{-1/N(0)V}$$


Weak Coupling BCS Result
Δ(T)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T_c $\Delta(T) \rightarrow 0$
$\frac{1}{N(0)V} = \int_0^{\omega_D} \tanh\left(\frac{\epsilon}{2T_c}\right)\frac{d\epsilon}{\epsilon}$
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$T_c = 1.13\,\omega_D e^{-\frac{1}{N(0)V}}$
Integrand
$\frac{1}{2T_c}$
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$$\Delta(T=0) = 2\,\omega_D\, e^{-\frac{1}{N(0)V}}$$

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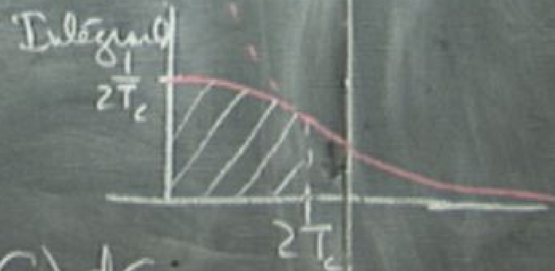

As $T \rightarrow T_c$ $\quad \Delta(T) \rightarrow 0$

$$\frac{1}{N(0)V} = \int_0^{\omega_D} \tanh\left(\frac{\epsilon}{2T_c}\right) \frac{d\epsilon}{\epsilon}$$

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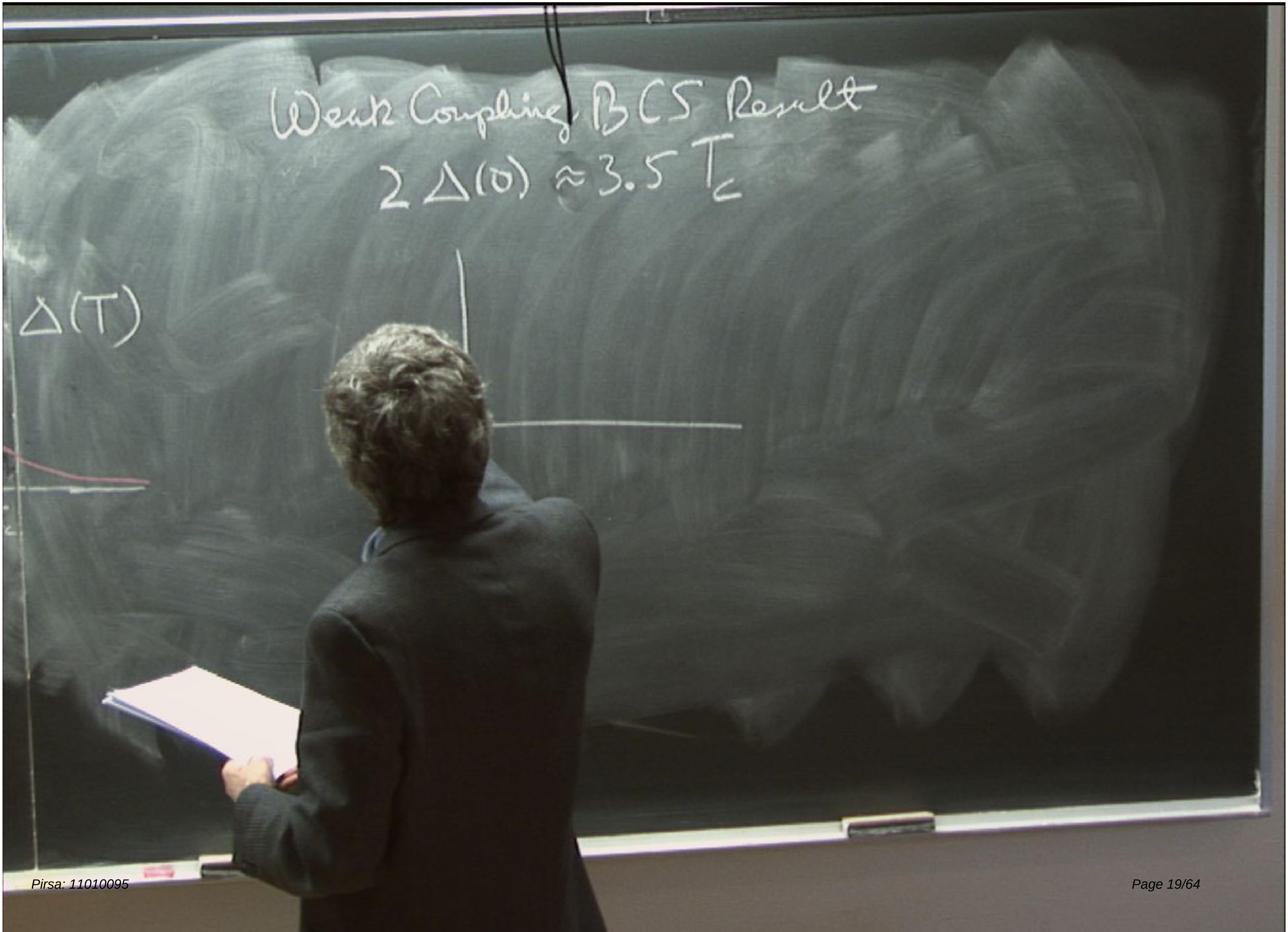
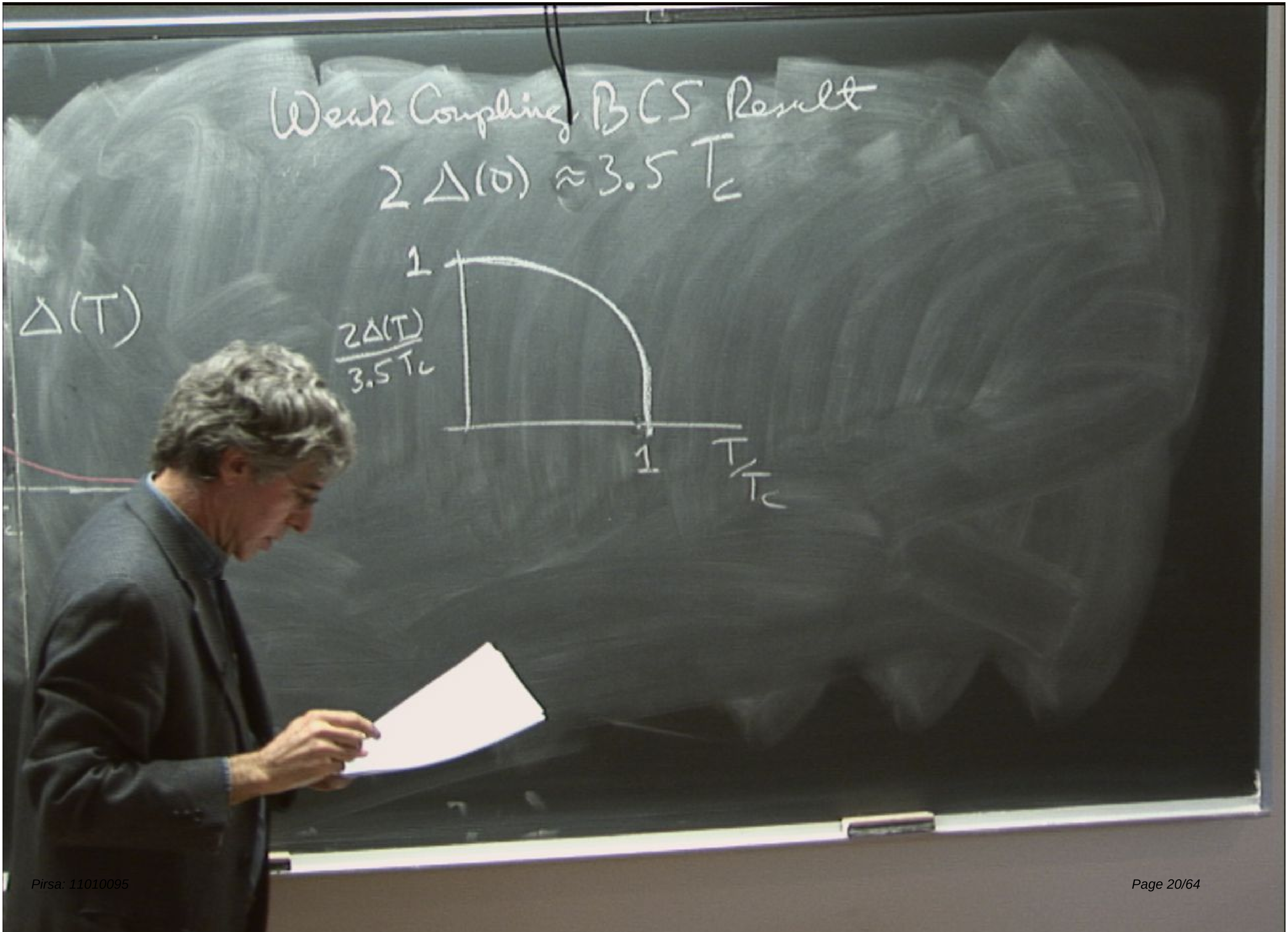
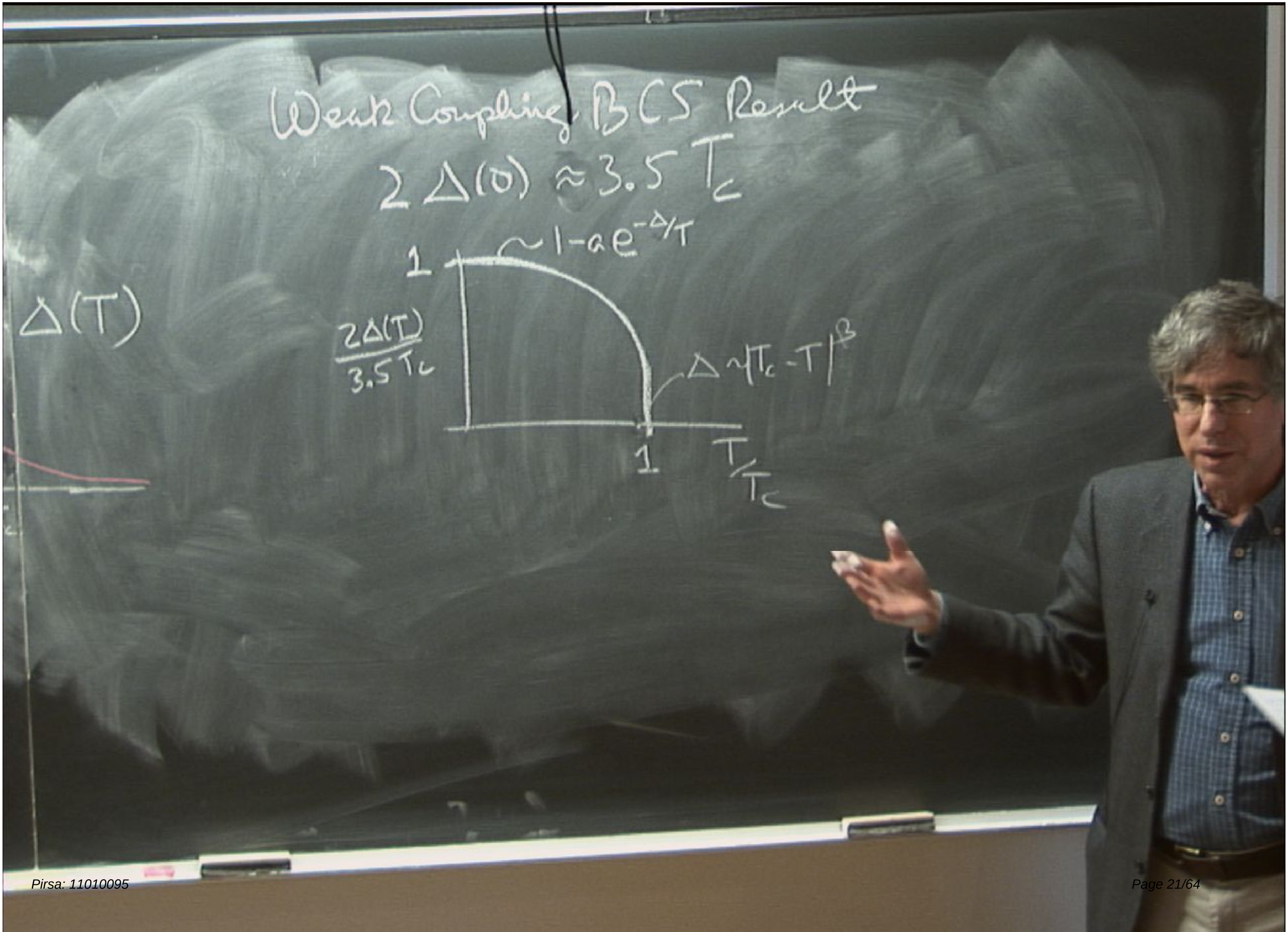
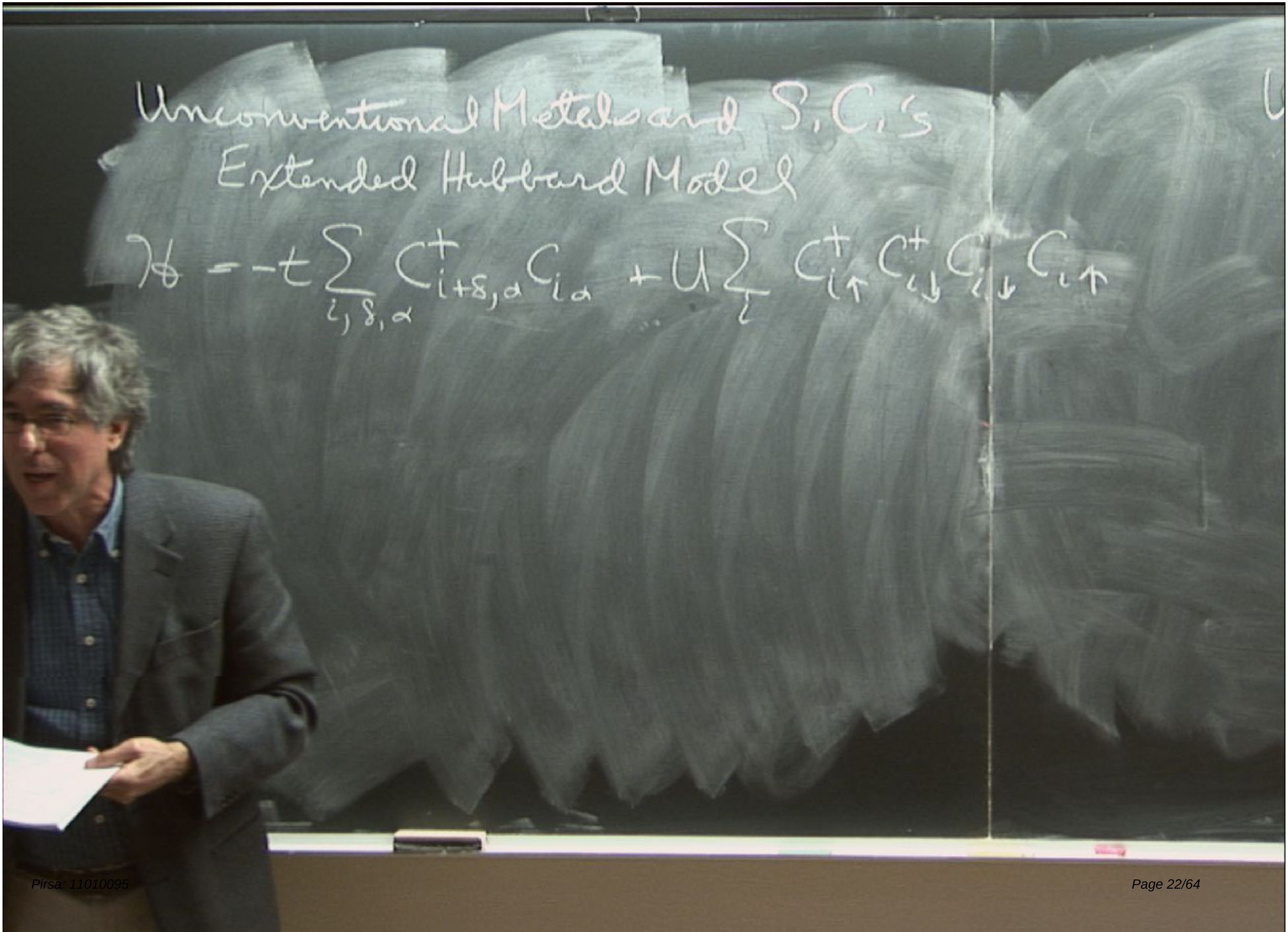
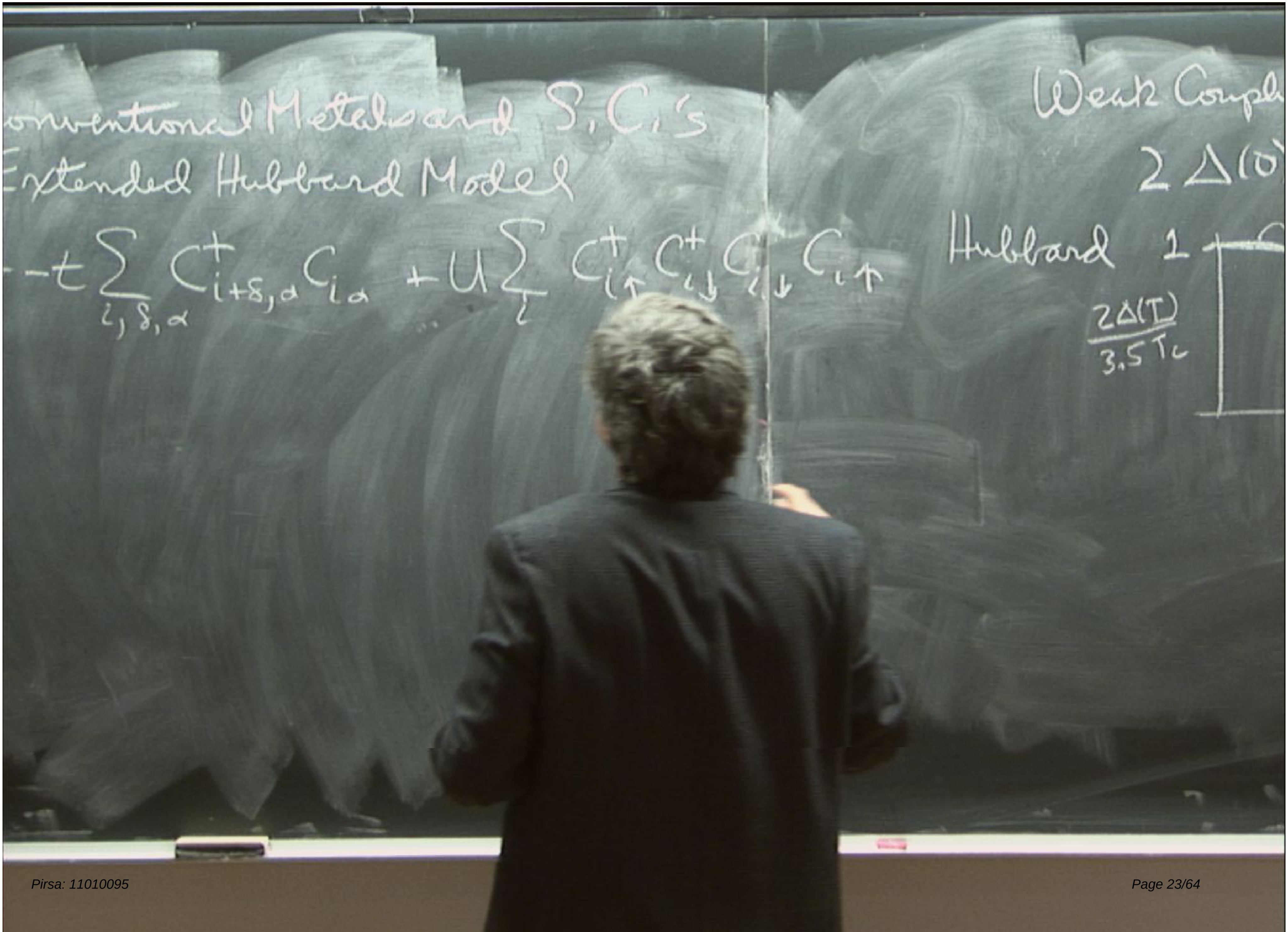

Weak Coupling BCS Result
2 Δ(0) ≈ 3.5 T_c
Δ(T)


Weak Coupling BCS Result
2Δ(0) ≈ 3.5 Tc
1
2Δ(T) / 3.5 Tc
1
T/Tc
Δ(T)


Weak Coupling BCS Result
2Δ(0) ≈ 3.5 T_c
~1-ae^{-Δ/T}
1
2Δ(T) / 3.5 T_c
Δ ~ |T_c - T|^β
1
T/T_c
Δ(T)


Unconventional Metals and S.C.'s
Extended Hubbard Model
$\mathcal{H} = -t \sum_{i,\delta,\alpha} C^{\dagger}_{i+\delta,\alpha} C_{i\alpha} + U \sum_{i} C^{\dagger}_{i\uparrow} C^{\dagger}_{i\downarrow} C_{i\downarrow} C_{i\uparrow}$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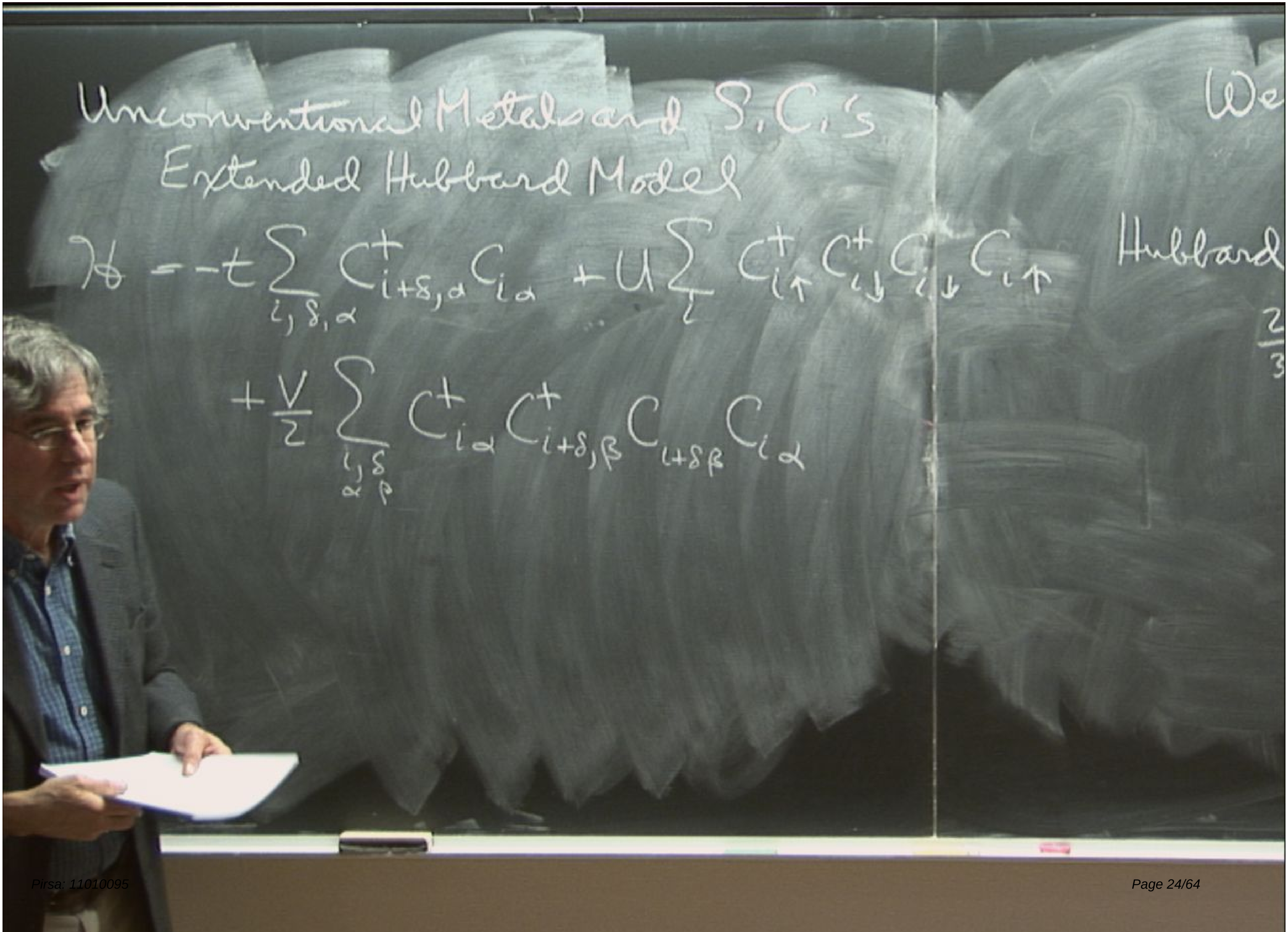
Extended Hubbard Model
-t ∑ C†i+δ,α Ciα + U ∑ C†i↑ C†i↓ Ci↓ Ci↑
i,δ,α
i
Weak Coupl
2Δ(0)
Hubbard 1
2Δ(T)
3.5 Tc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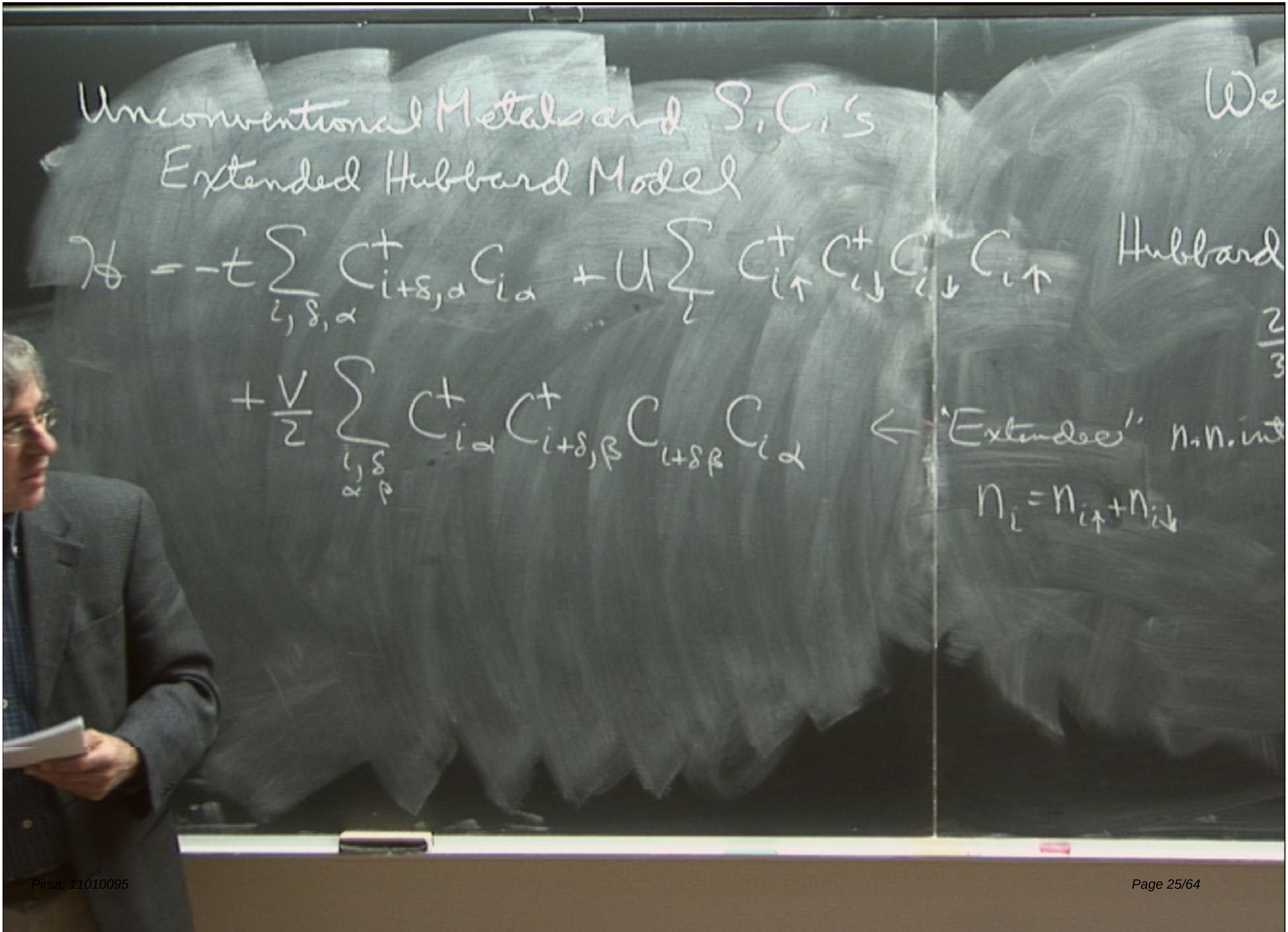
$+ \frac{V}{2} \sum_{\substack{i,\delta \\ \alpha\,\beta}} c^{\dagger}_{i\alpha} c^{\dagger}_{i+\delta,\beta} c_{i+\delta\beta} c_{i\alpha}$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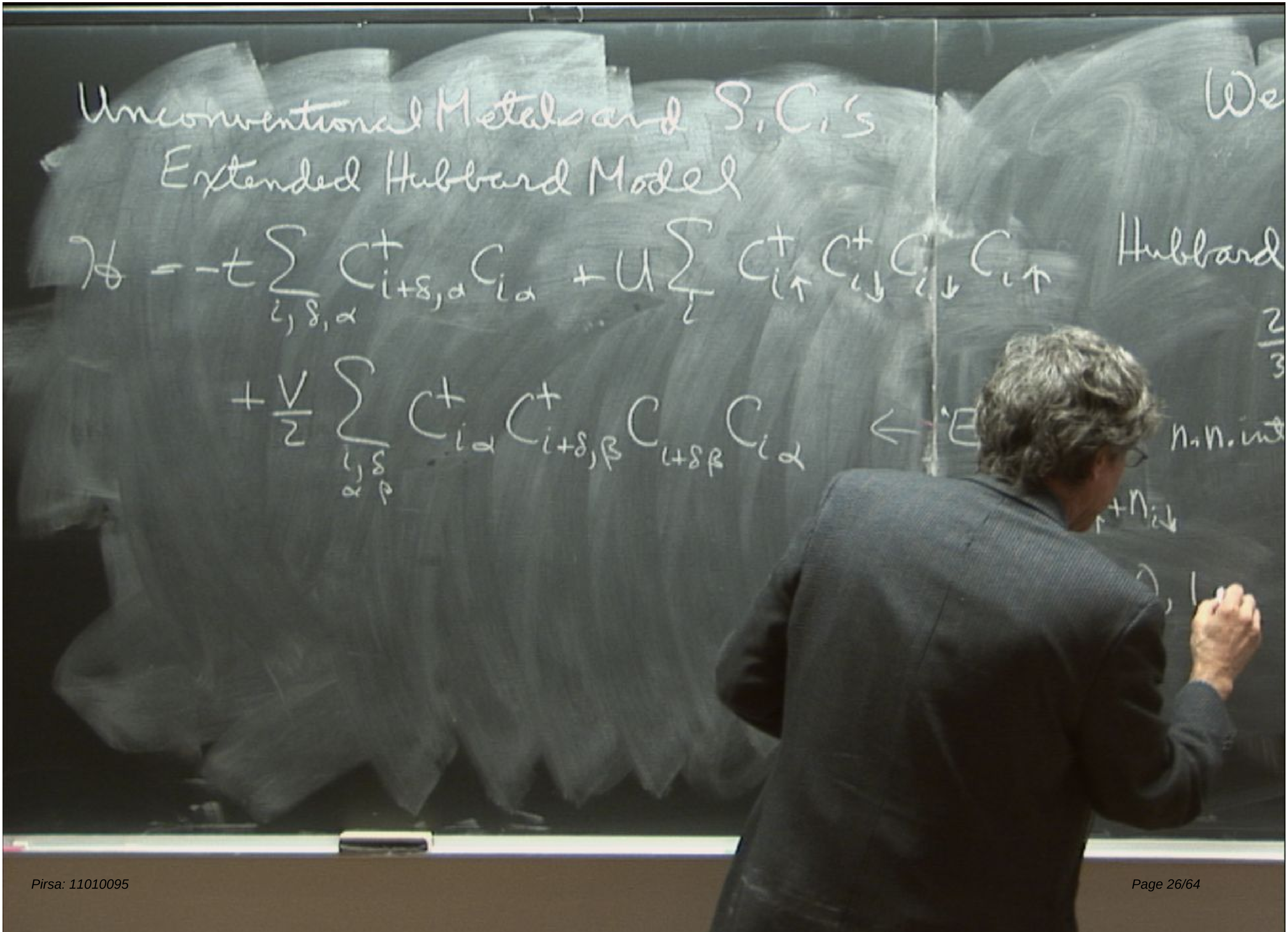
$+ \frac{V}{2} \sum_{i,\delta,\alpha,\beta} C^{+}_{i\alpha} C^{+}_{i+\delta,\beta} C_{i+\delta\beta} C_{i\alpha}$
← "Extended" n.n. int
$n_i = n_{i\uparrow} + n_{i\downarrow}$
Hubbard
We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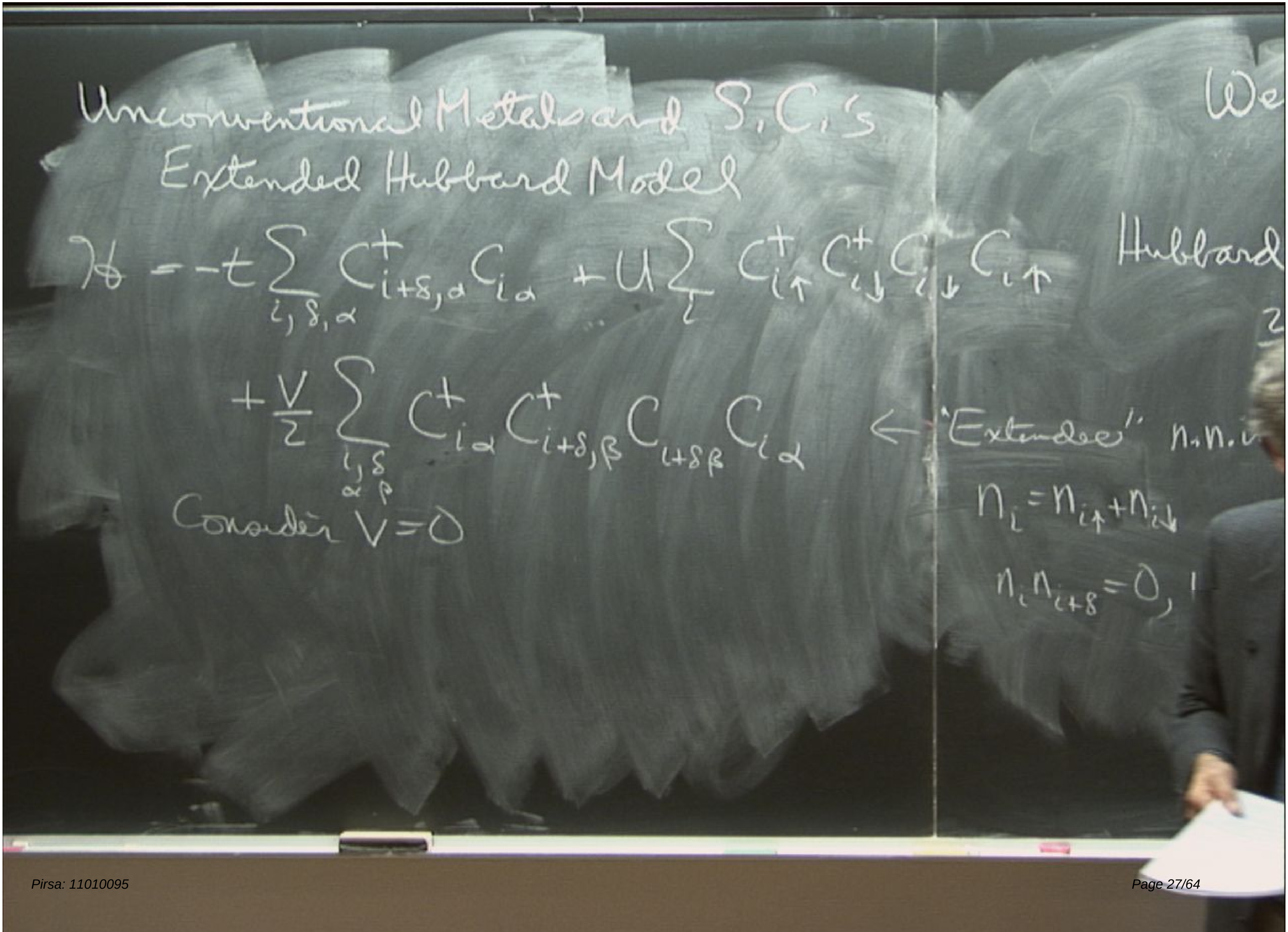
Hubbard
n.n. int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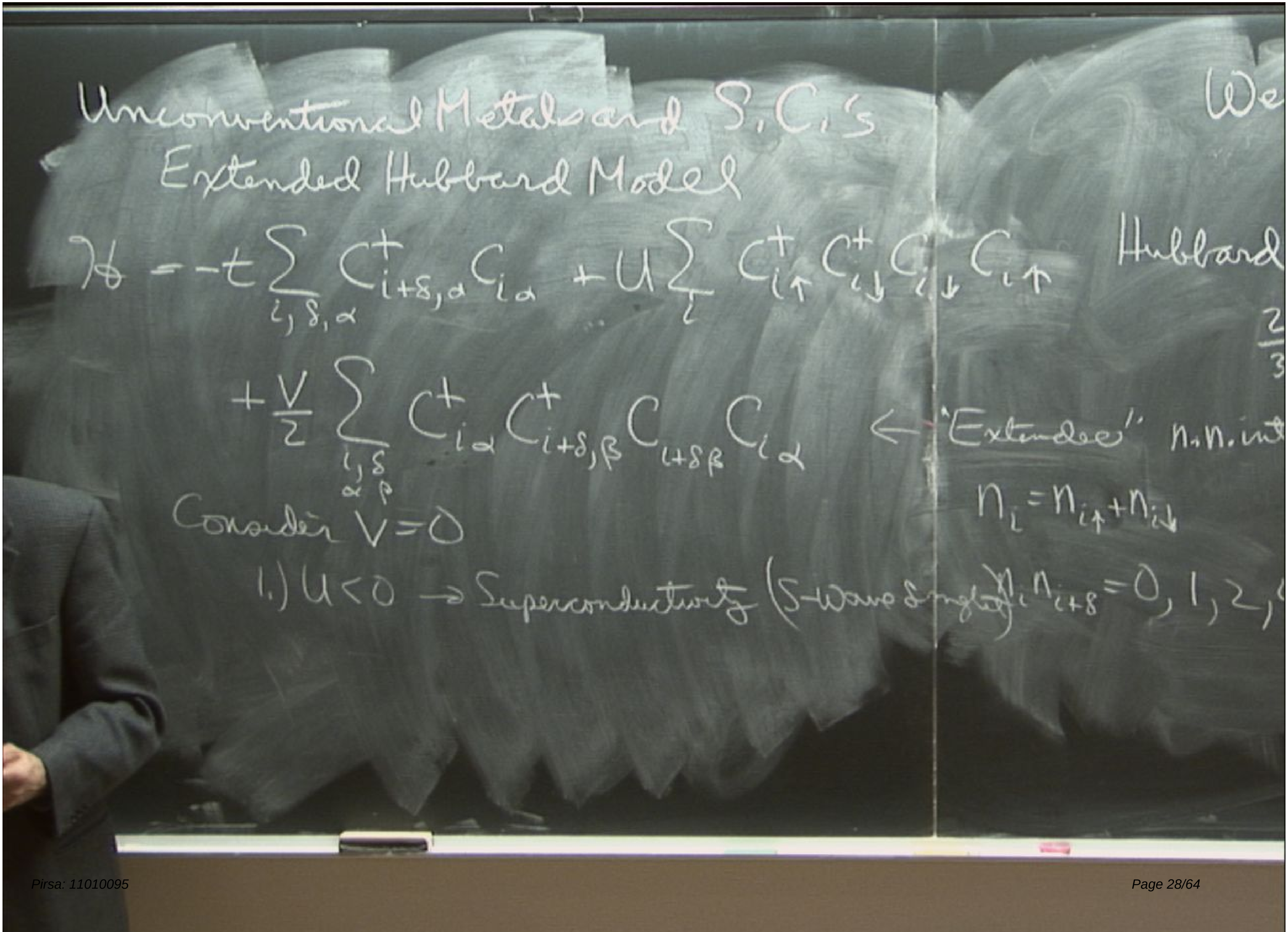
← "Extended" n.n.
Consider $V=0$
We
Hubbard
$n_i = n_{i\uparrow} + n_{i\downarrow}$
$n_i n_{i+\delta} = 0,$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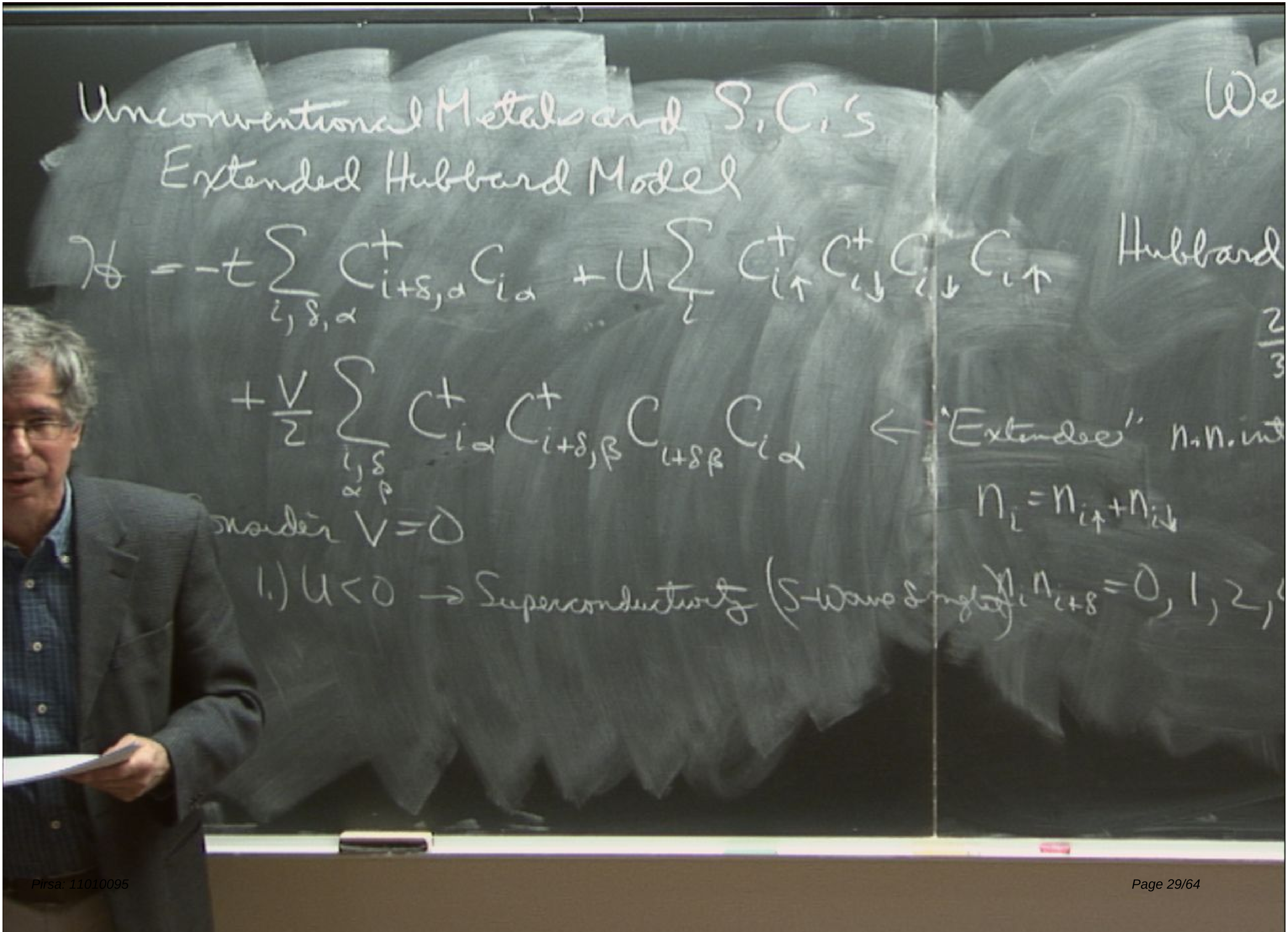
← "Extended" n.n. int
$n_i = n_{i\uparrow} + n_{i\downarrow}$
Consider V=0
1.) U<0 → Superconductivity (S-wave Singlet)
$n_i n_{i+\delta} = 0, 1, 2,$
Hubbard
We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Hubbard
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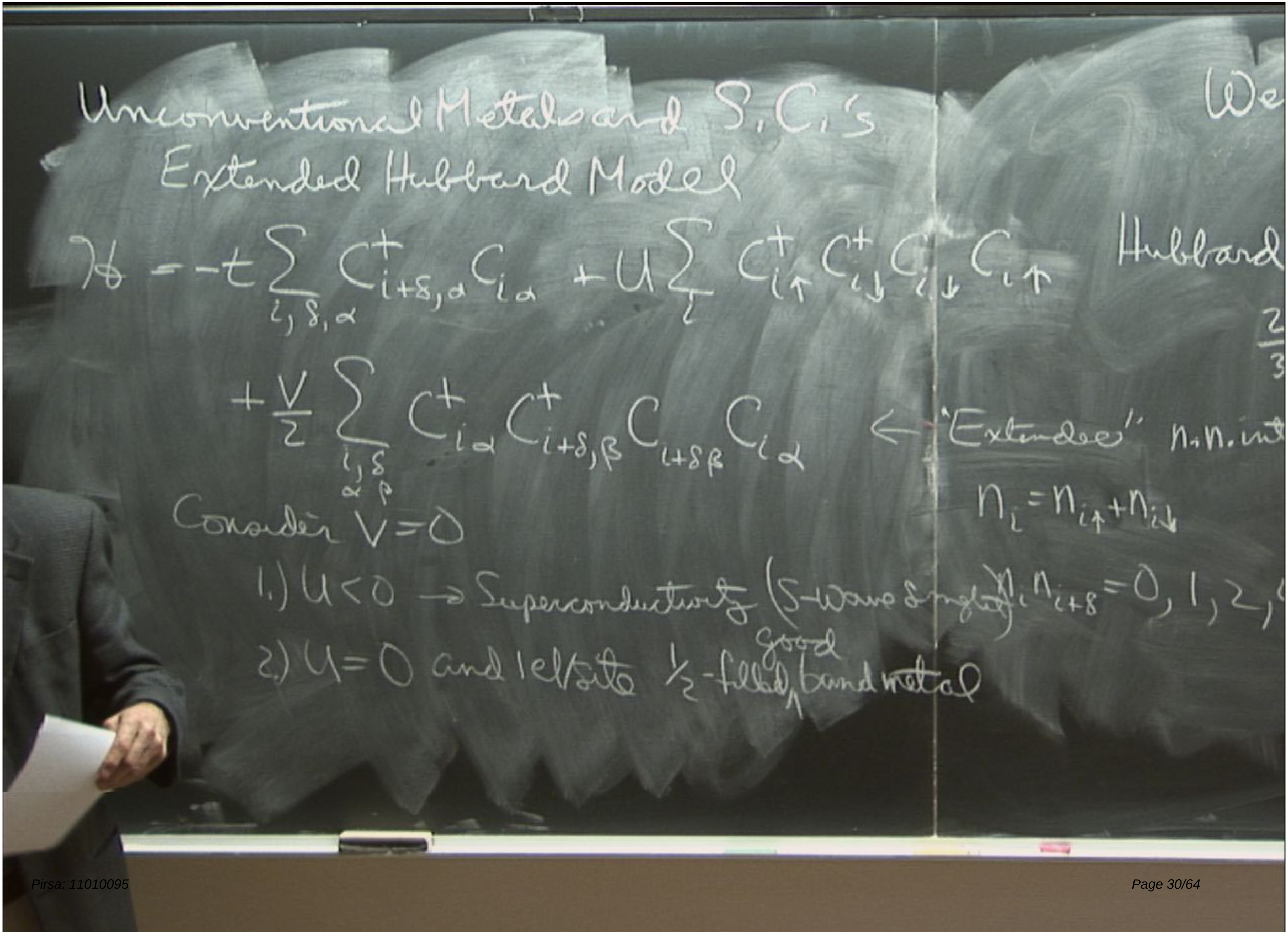
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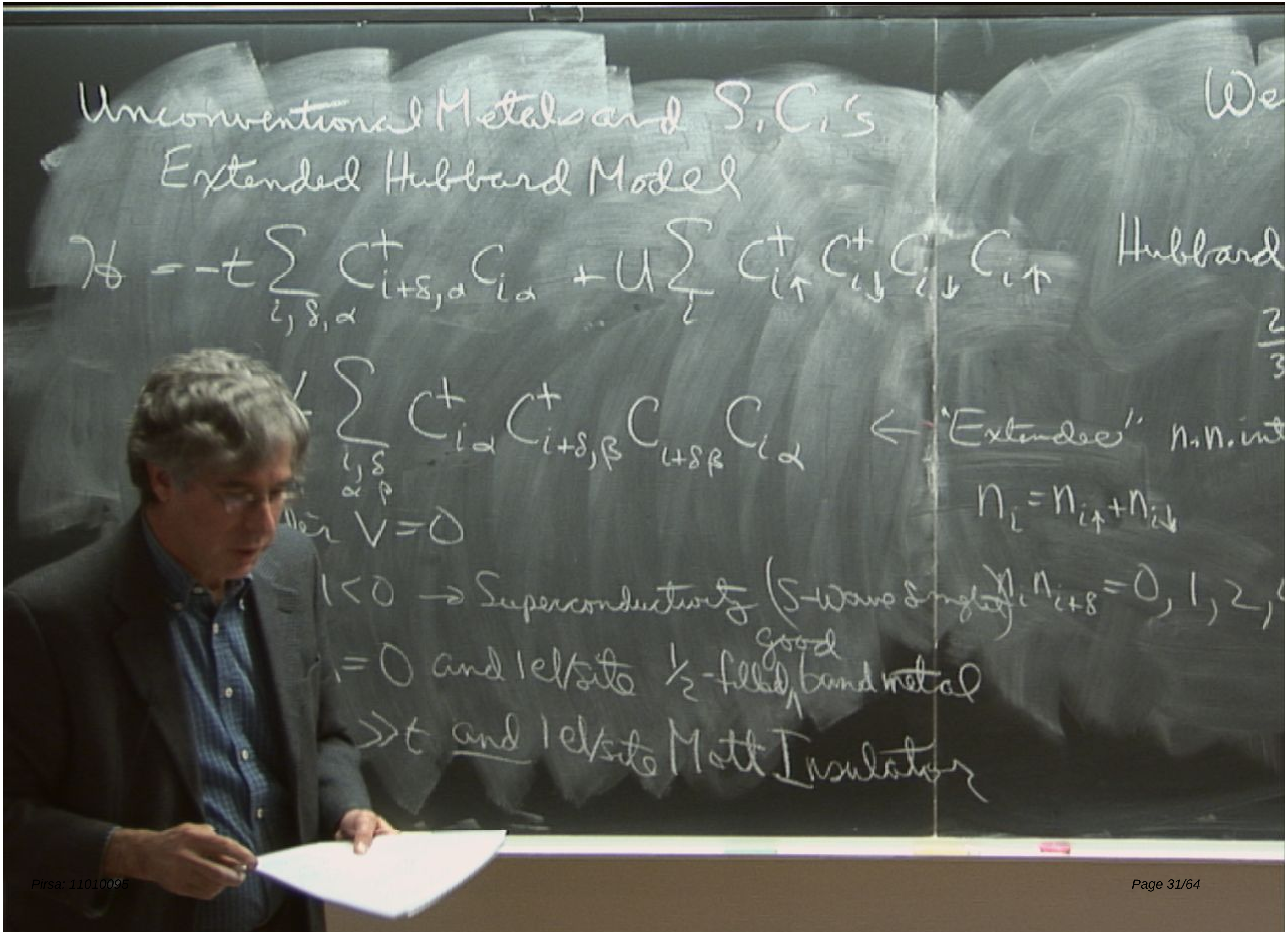
← "Extended" n.n. int
$n_i = n_{i\uparrow} + n_{i\downarrow}$
$n_i n_{i+\delta} = 0, 1, 2,$
Consider $V = 0$
1.) $U < 0$ → Superconductivity (S-wave singlet)
2.) $U = 0$ and 1el/site ½-filled good band metal
Hubbard
We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Extended Hubbard Model
Hubbard
'Extended' n.n. int
V = 0
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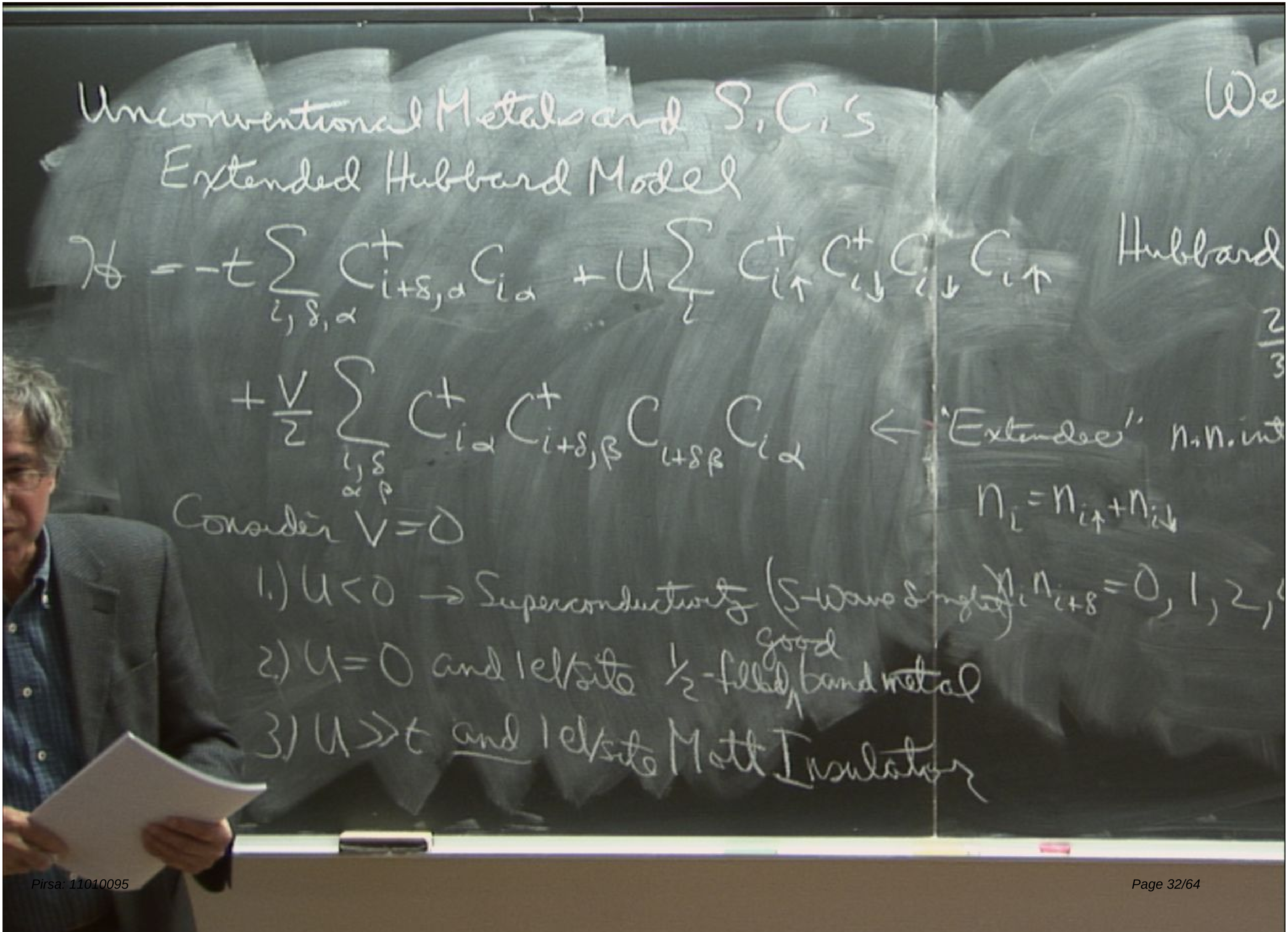
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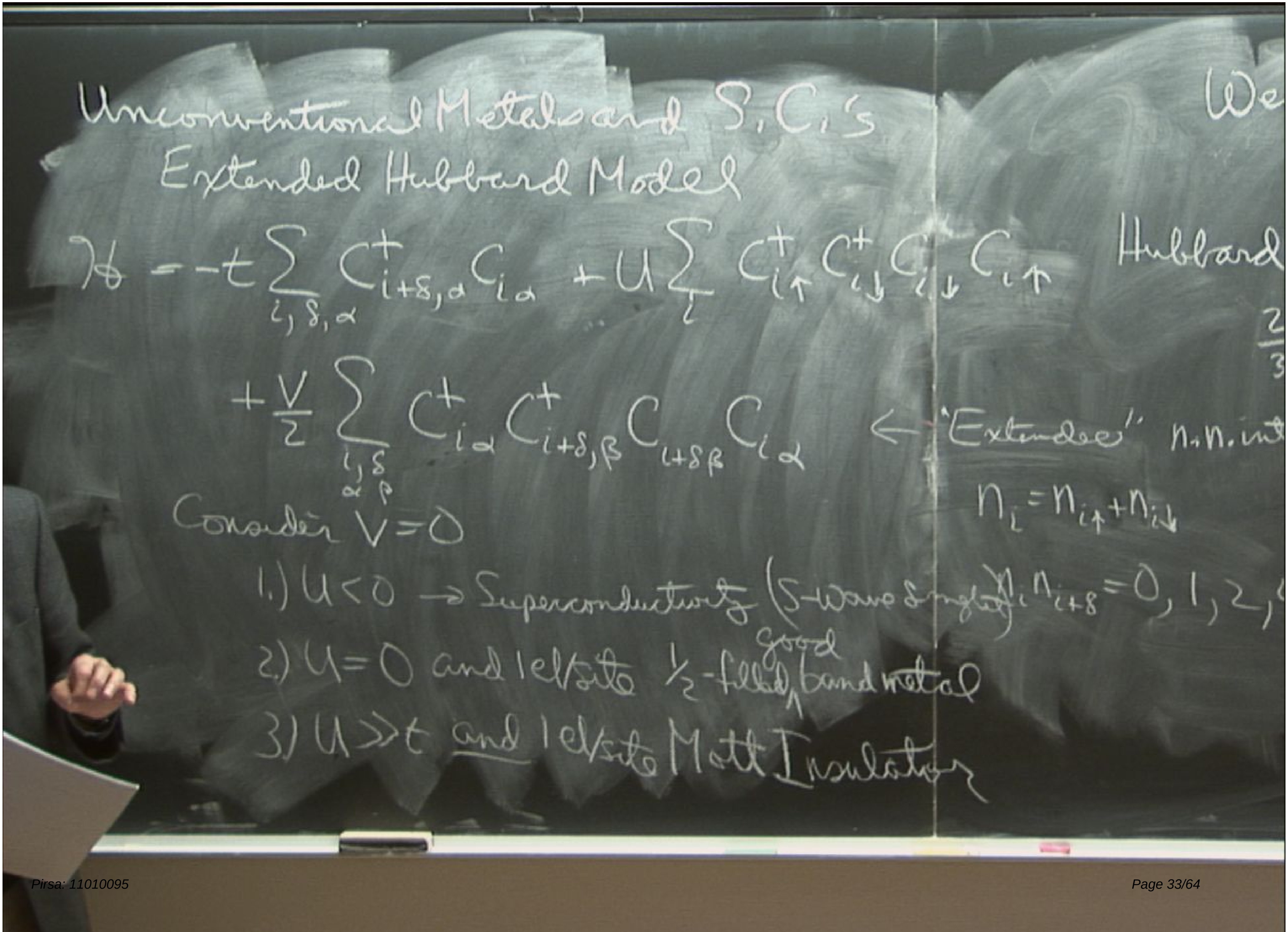
2.) U=0 and 1el/site ½-filled good band metal
3.) U>>t and 1el/site Mott Insulator
We
Hubbard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$+ \frac{V}{2} \sum_{i,\delta,\alpha,\beta} C^{\dagger}_{i\alpha} C^{\dagger}_{i+\delta,\beta} C_{i+\delta\beta} C_{i\alpha}$ ← "Extended" n.n. int
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2.) $U = 0$ and 1el/site ½-filled good band metal
3.) $U \gg t$ and 1el/site Mott Insulator
Hubbard
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Unconventional Metals and S.C.'s

Extended Hubbard Model

$$\mathcal{H} = -t \sum_{i,\delta,\alpha} C^{+}_{i+\delta,\alpha} C_{i\alpha} + U \sum_{i} C^{+}_{i\uparrow} C^{+}_{i\downarrow} C_{i\downarrow} C_{i\uparrow}$$

$$+ \frac{V}{2} \sum_{\substack{i,\delta \\ \alpha\beta}} C^{+}_{i\alpha} C^{+}_{i+\delta,\beta} C_{i+\delta\beta} C_{i\alpha} \quad \leftarrow \text{"Extended" n.n. int}$$

$n_i = n_{i\uparrow} + n_{i\downarrow}$

$n_i n_{i+\delta} = 0, 1, 2, \ldots$

We

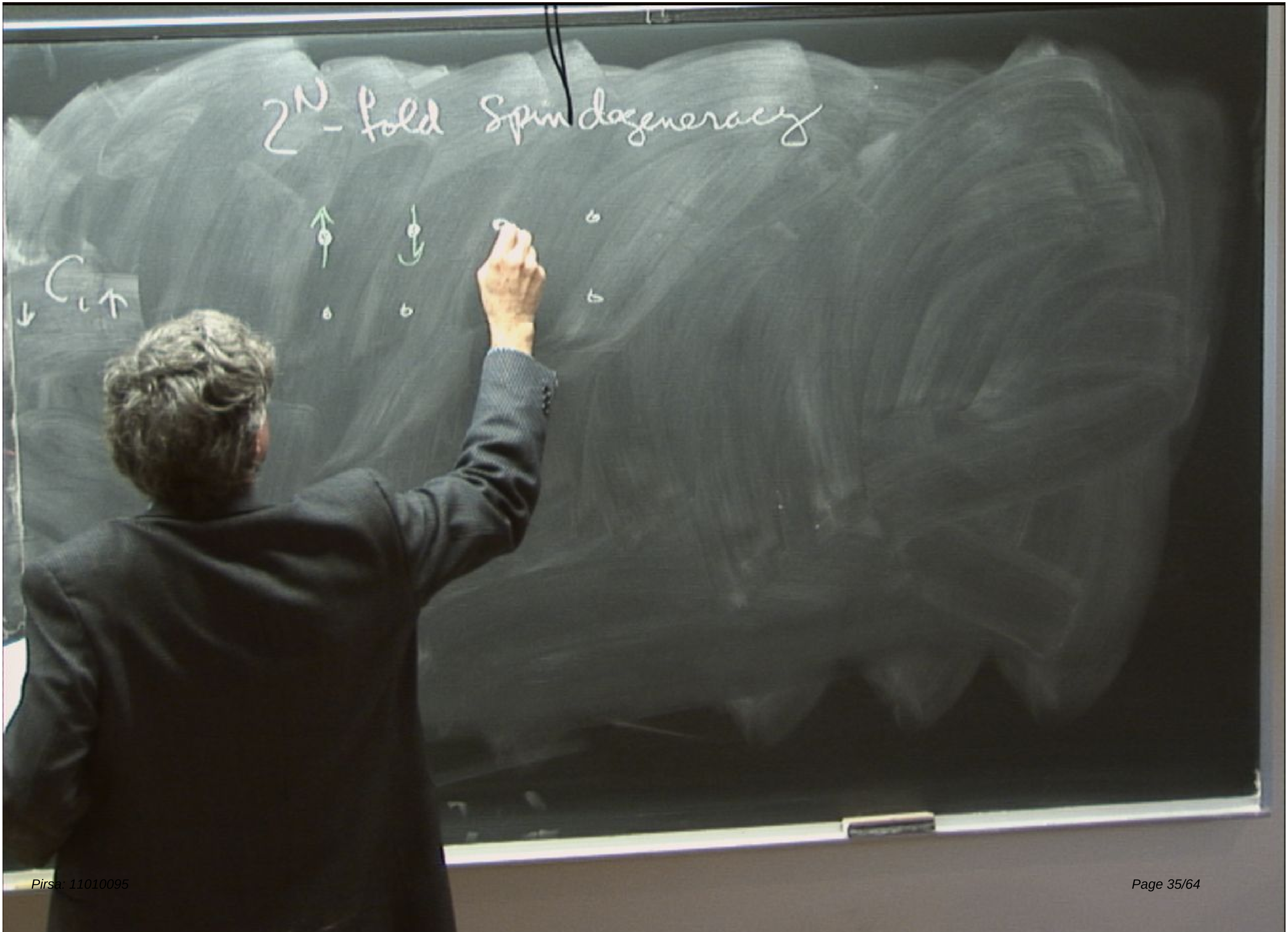
Hubbard

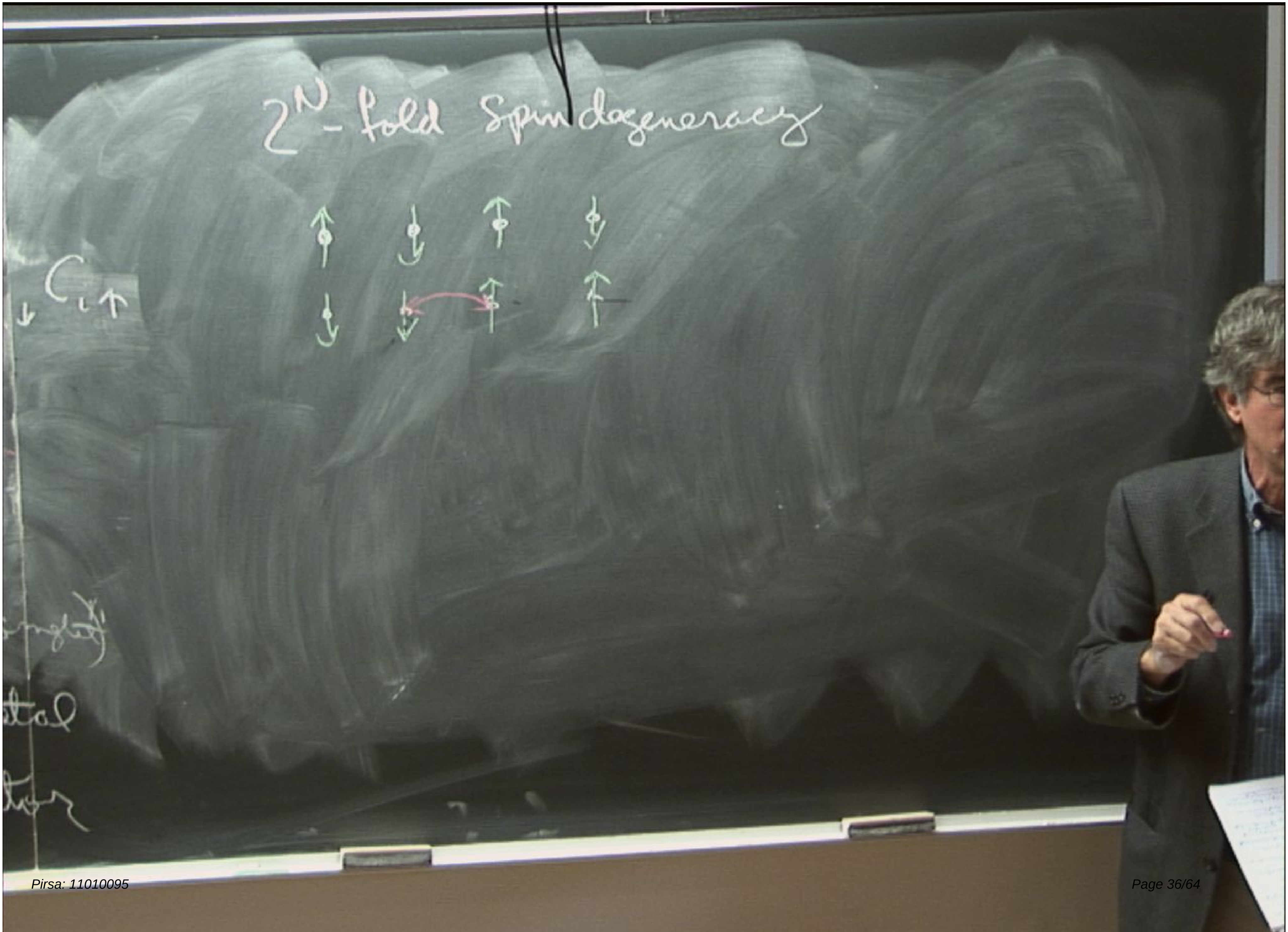
Consider $V = 0$

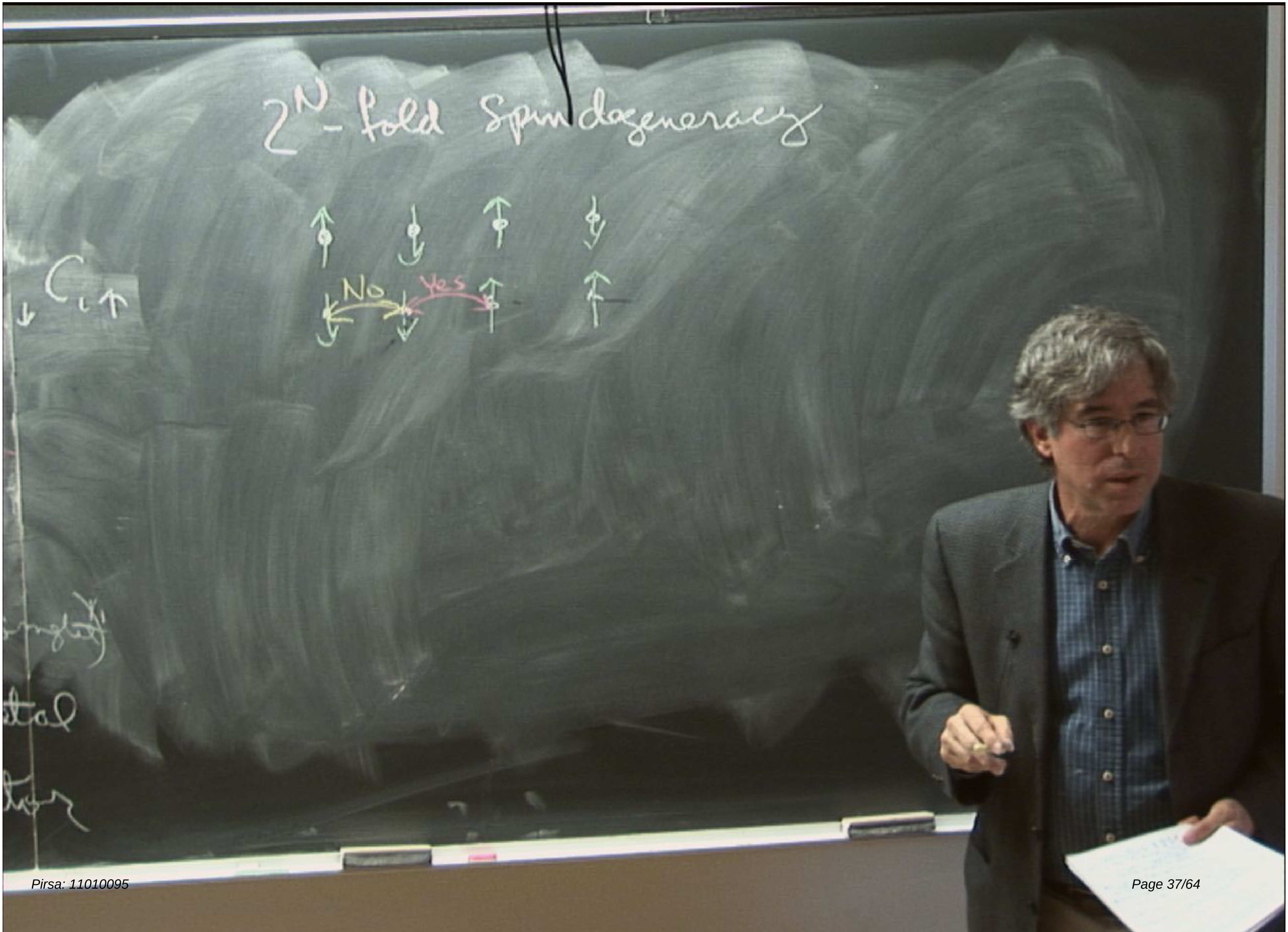
1.) $U < 0 \rightarrow$ Superconductivity (S-wave singlet)

2.) $U = 0$ and 1el/site ½-filled good band metal

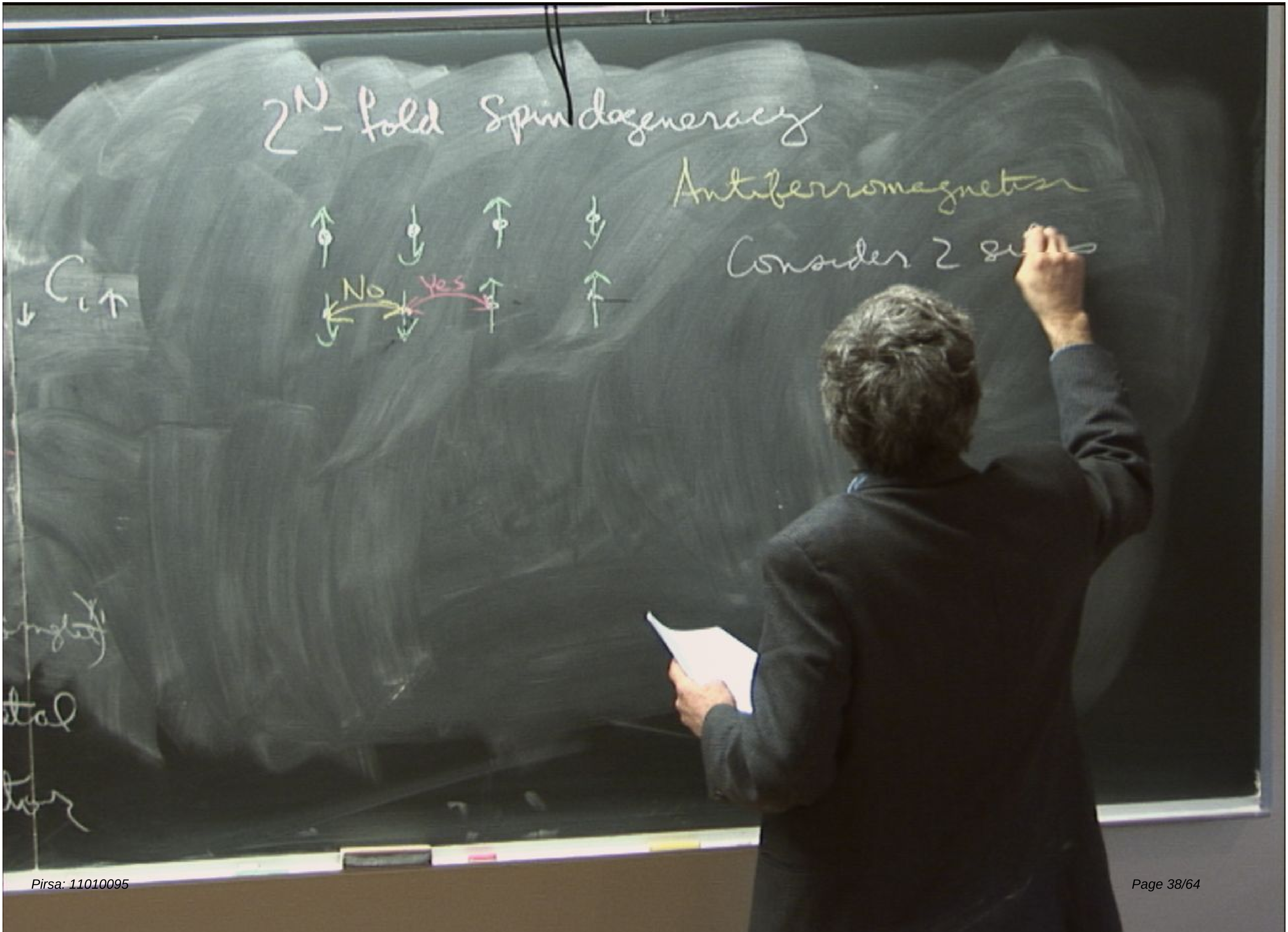
3.) $U \gg t$ and 1el/site Mott Insulator

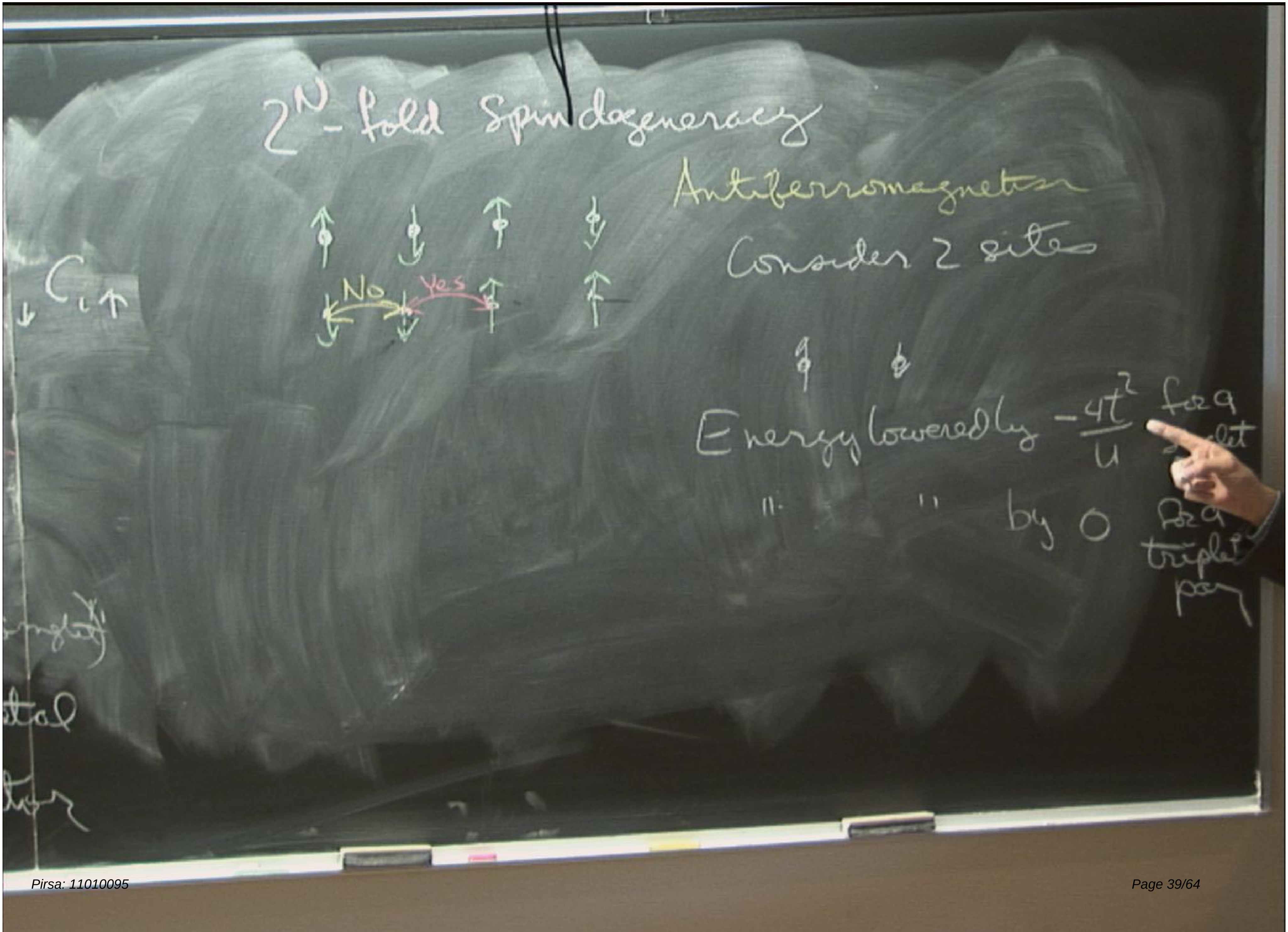

2^N - fold Spin degeneracy
C_i ↑
↓

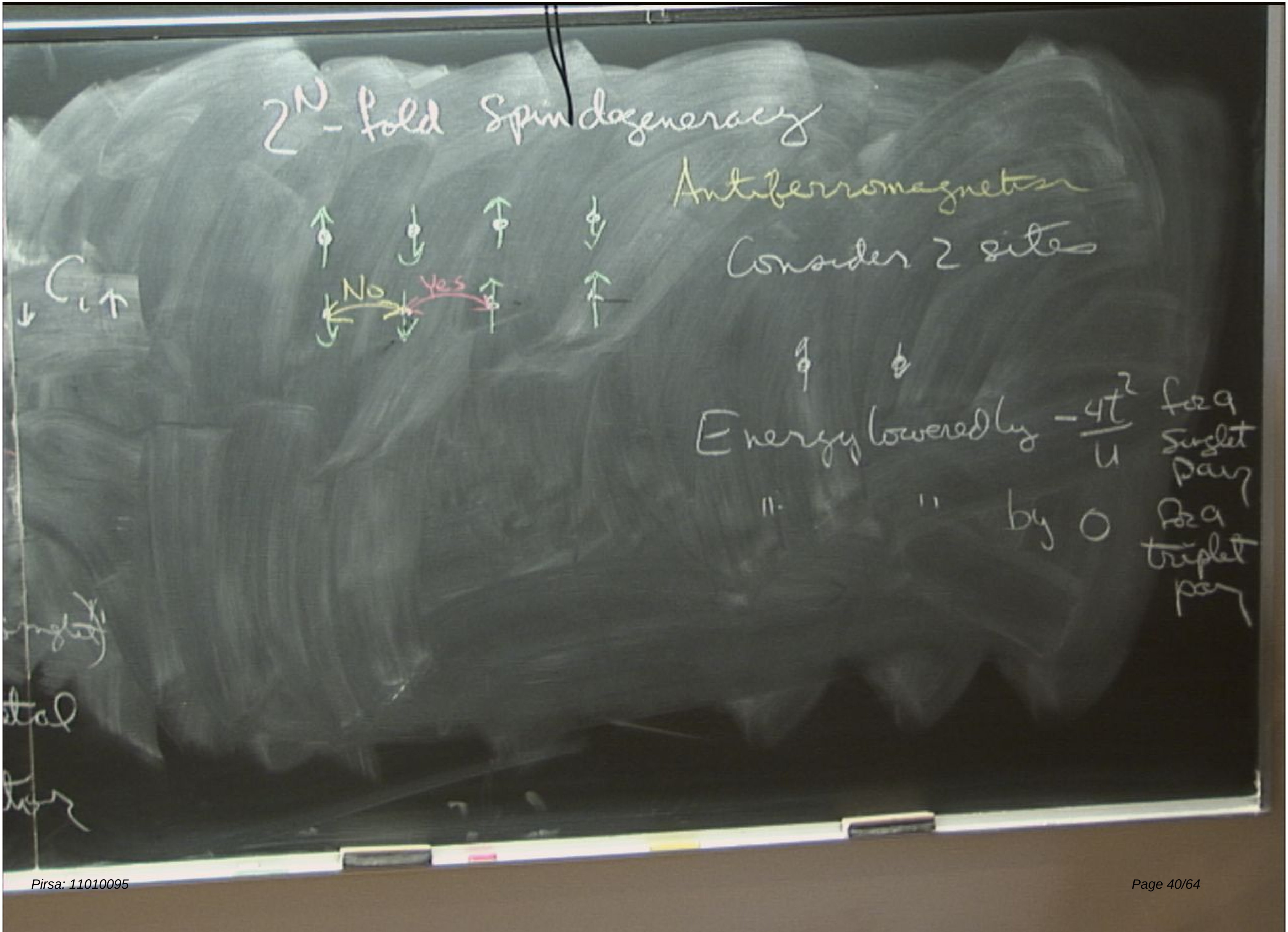

2^N - fold Spin degeneracy



2^N- fold Spin degeneracy
No
Yes


2^N-fold Spin degeneracy
Antiferromagnetism
Consider 2 su
No
Yes


2^N - fold Spin degeneracy
Antiferromagnetism
Consider 2 sites
No
Yes
Energy lowered by -4t^2/U for a singlet
" " by 0 for a triplet pair



2^N - fold Spin degeneracy
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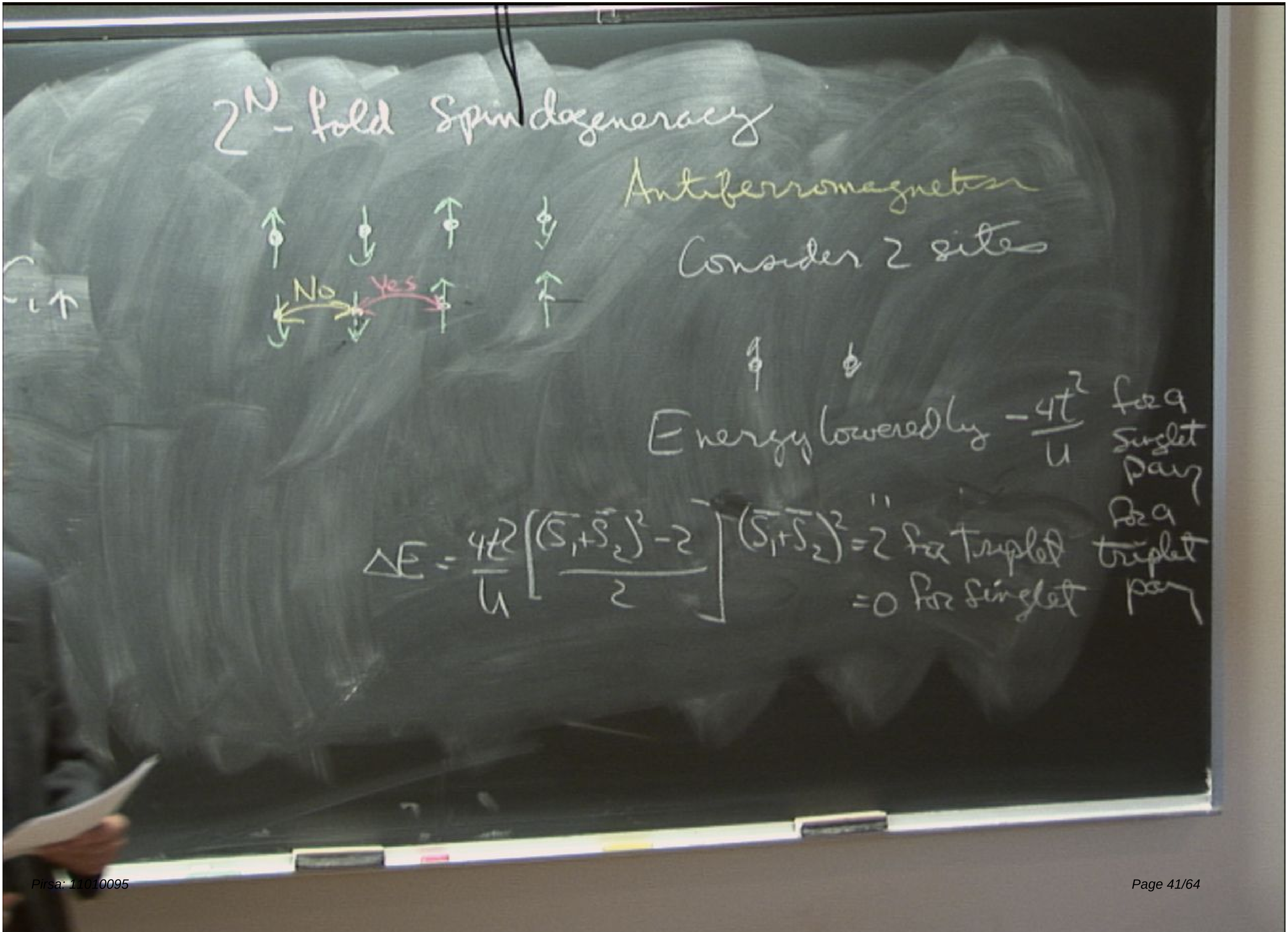
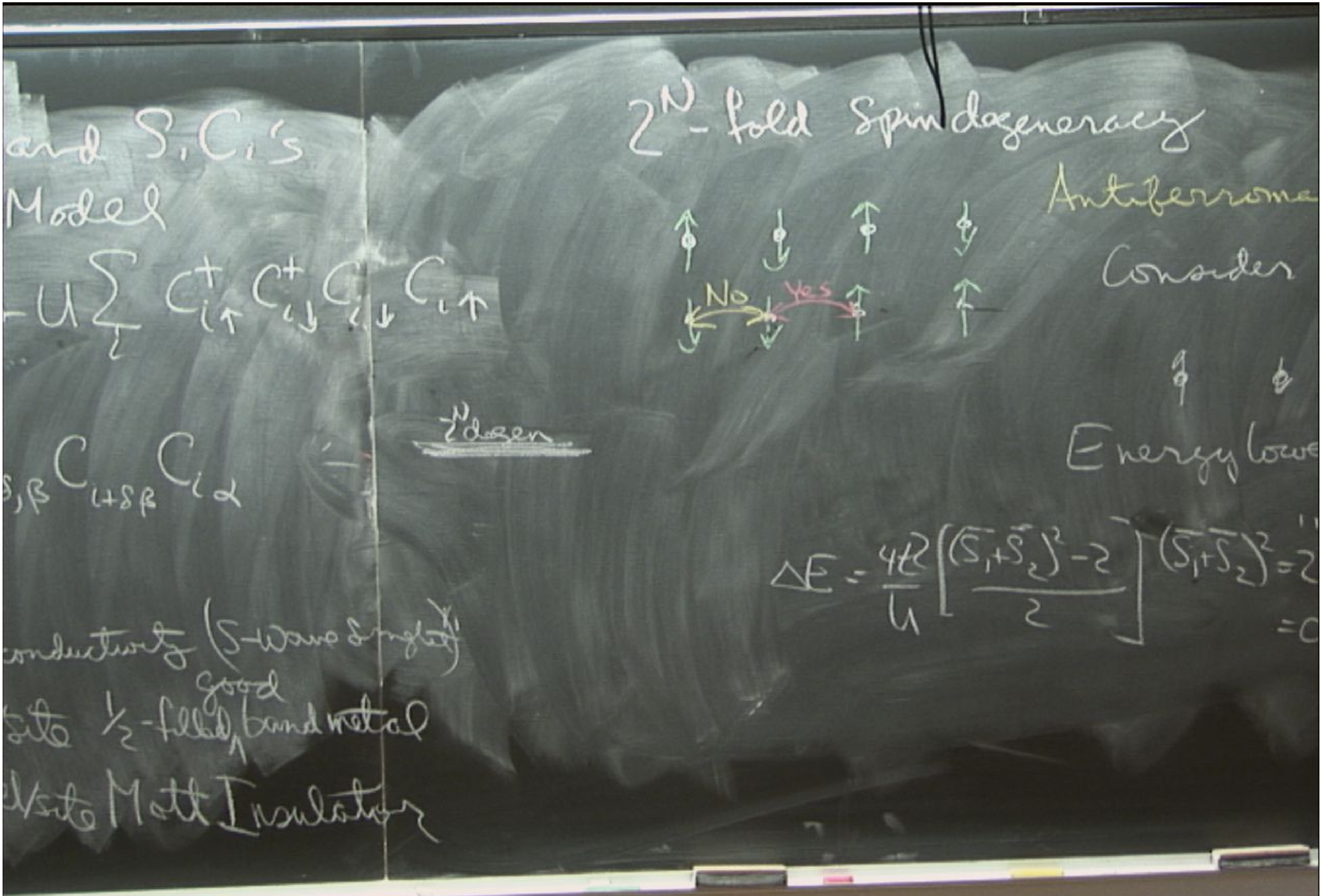
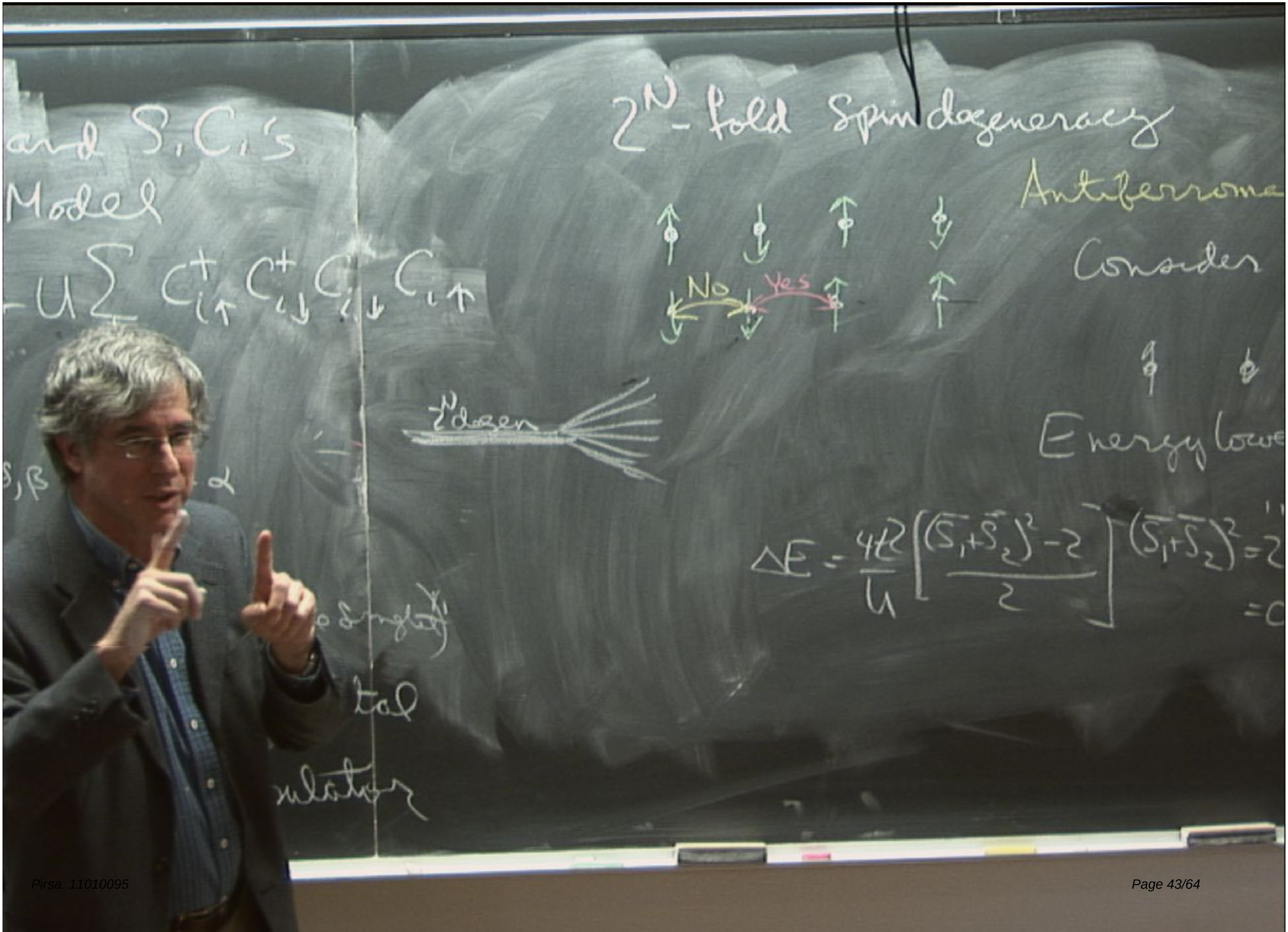
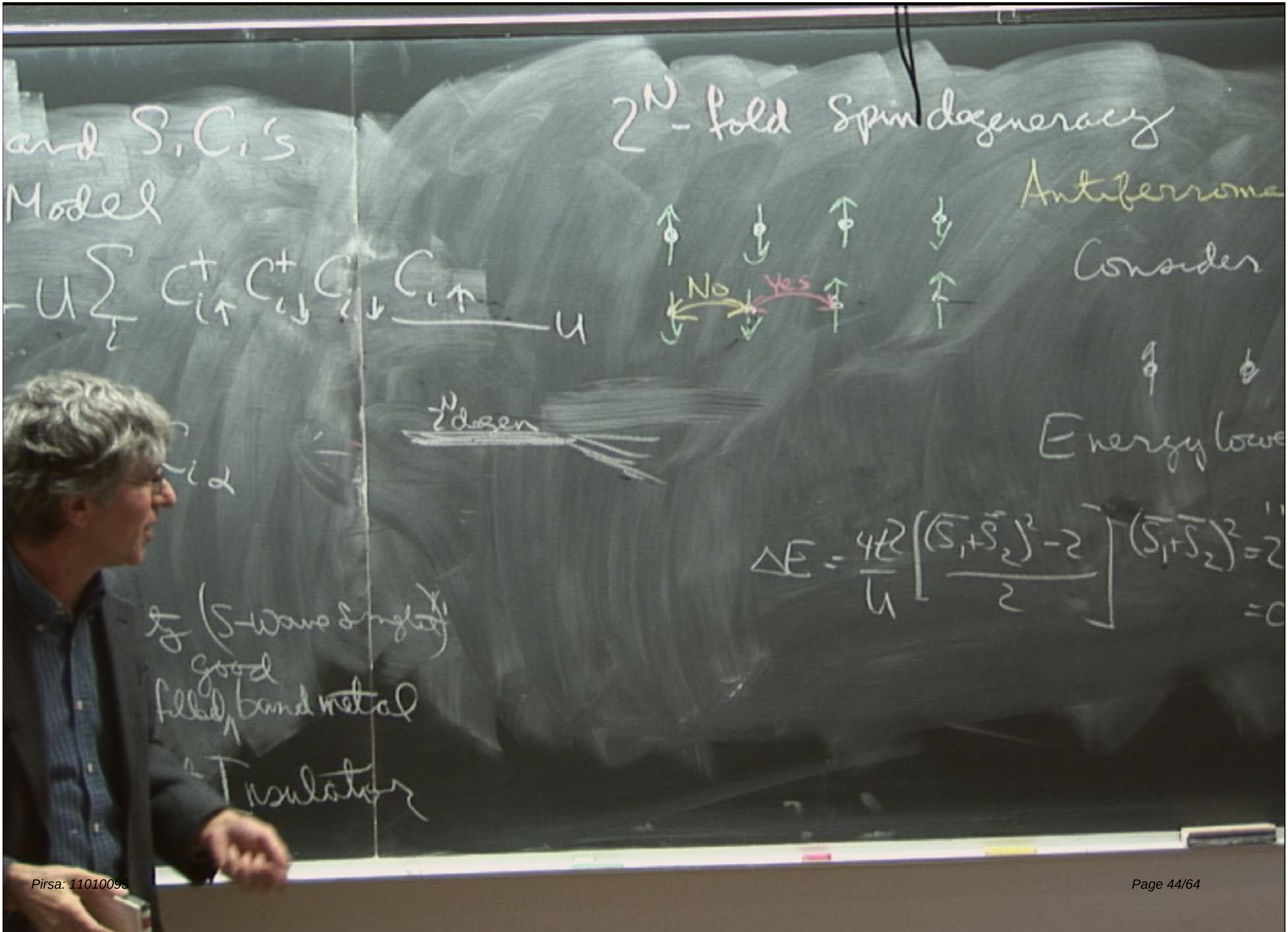
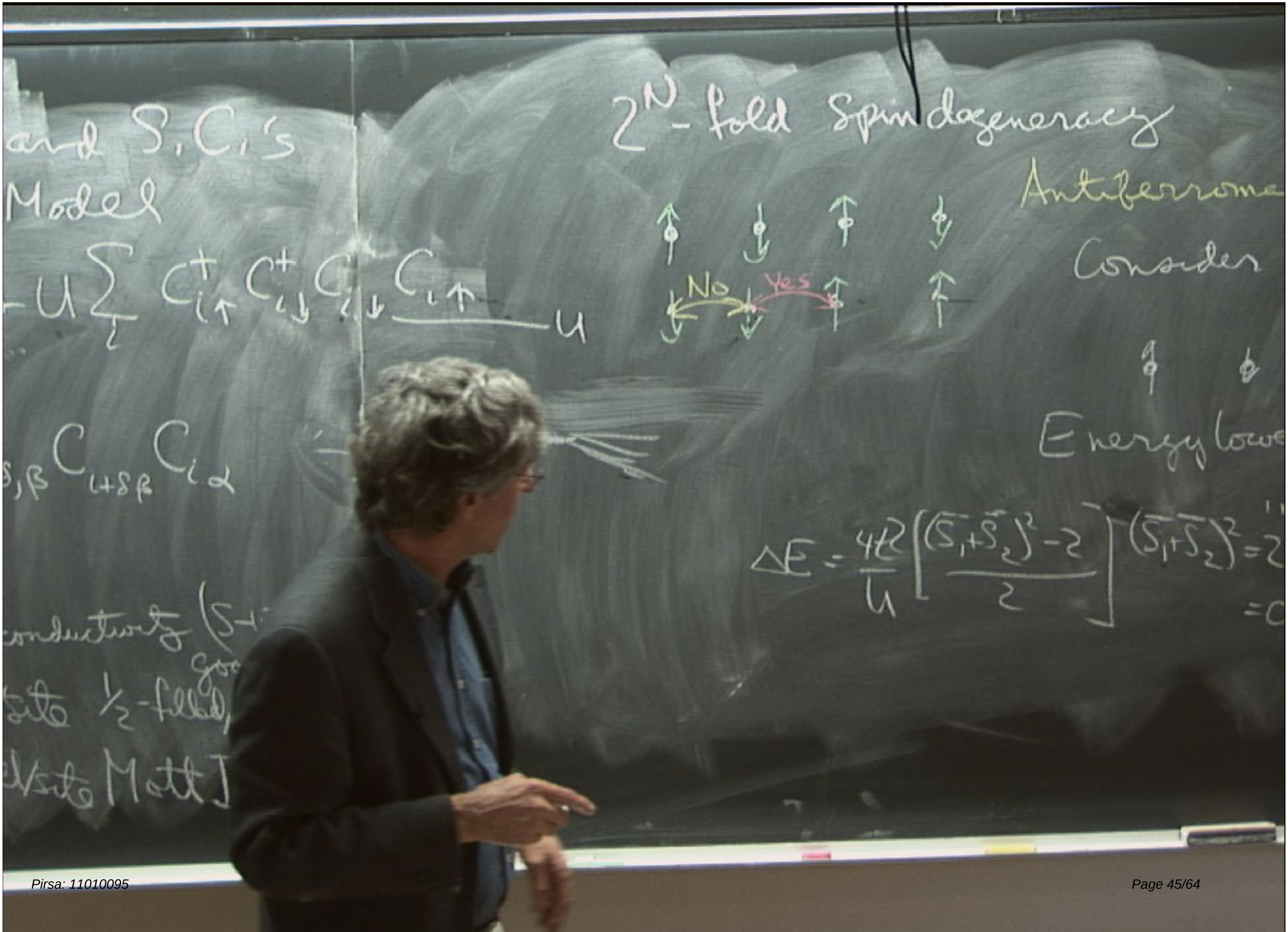

2^N-fold Spin degeneracy
Antiferromagnetism
Consider 2 sites
No
Yes
Energy lowered by -4t²/U for a singlet pair
ΔE = 4t²/U [((S₁+S₂)² - 2)/2]
(S₁+S₂)² = 2 for triplet
= 0 for singlet
for a triplet pair

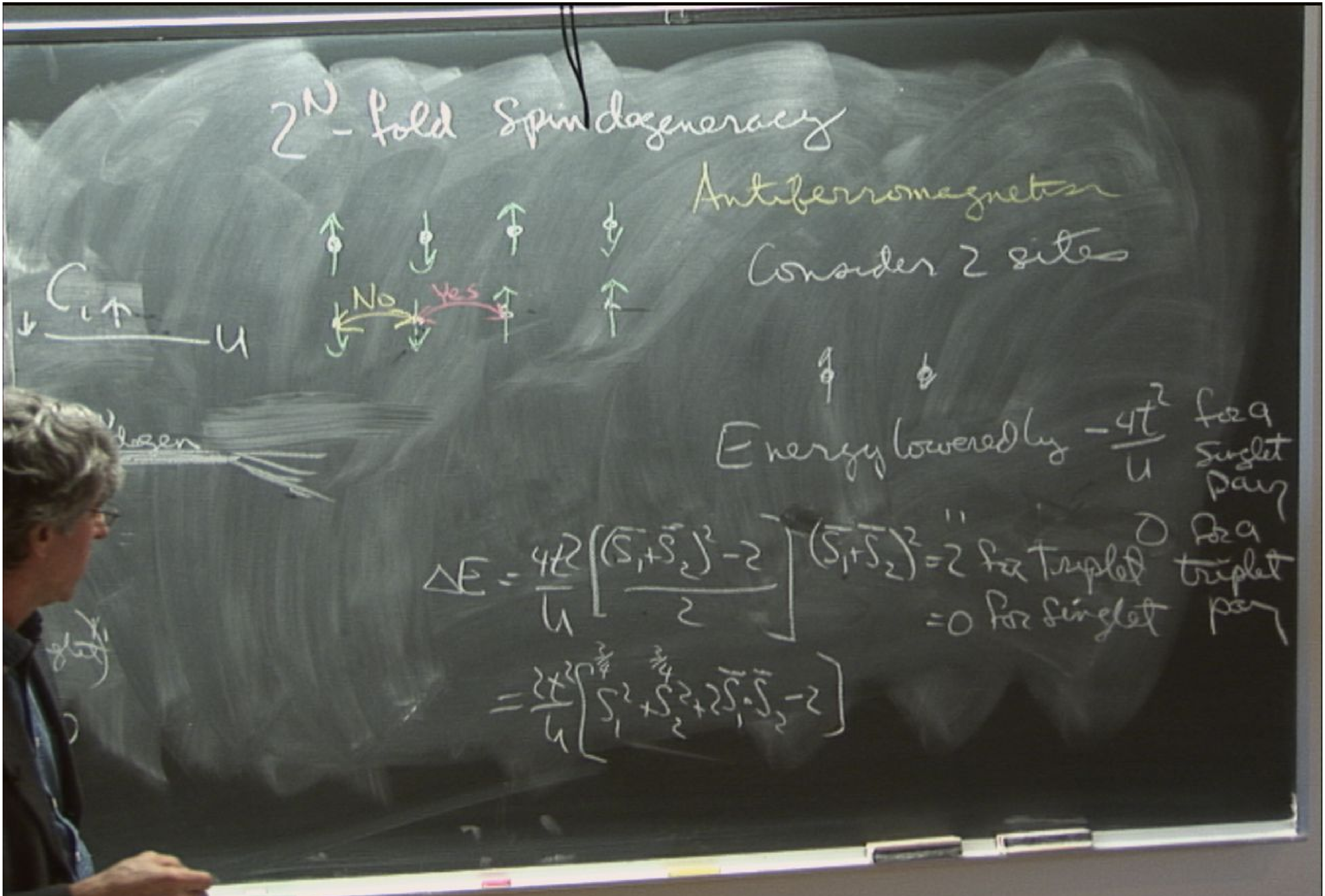

and S, C_i's
Model
-U Σ_i C†_i↑ C†_i↓ C_i↓ C_i↑
C_{i+δβ} C_{iα}
conductivity (S-wave singlet)
good
site ½-filled band metal
site Mott Insulator
2^N - fold Spin degeneracy
Antiferroma
No
Yes
Consider
2^N degen
Energy lowe
ΔE = 4t²/U [((S_1+S_2)² - 2)/2]
(S_1+S_2)² = 2


and S, C_i's
Model
-U Σ C†_{i↑} C†_{i↓} C_{i↓} C_{i↑}
2^N - fold Spin degeneracy
Antiferroma
Consider
No
Yes
2^N degen
Energy lowe
ΔE = -4t²/U [((S_1+S_2)² - 2)/2]
(S_1+S_2)² = 2



and $S_i C_i$'s
Model
$-U\sum_i C^\dagger_{i\uparrow} C^\dagger_{i\downarrow} C_{i\downarrow} C_{i\uparrow}$
2^N-fold Spin degeneracy
Antiferroma
No
Yes
Consider
Energy lowe
$\Delta E = \frac{4t^2}{U}\left[\frac{(\vec{S}_1+\vec{S}_2)^2-2}{2}\right]$
good
filled band metal
Insulator


2N- fold Spin degeneracy
Antiferroma
No
Yes
Consider
Model
Energy lowe
conductivity
Mott


2^N - fold Spin degeneracy
Antiferromagnetism
Consider 2 sites
No
Yes
U
Energy lowered by $-\frac{4t^2}{U}$ for a singlet pair
0 for a triplet pair
$\Delta E = \frac{4t^2}{U}\left[\frac{(\vec{S}_1+\vec{S}_2)^2-2}{2}\right]$
$(\vec{S}_1+\vec{S}_2)^2 = 2$ for triplet
$= 0$ for singlet
$= \frac{2t^2}{U}\left[S_1^2 + S_2^2 + 2\vec{S}_1\cdot\vec{S}_2 - 2\right]$
3/4
3/4

2^N-fold Spin degeneracy

Antiferromagnetism

Consider 2 sites

$c_{i\uparrow}$ ——— U

2^N degen

Energy lowered by $-\frac{4t^2}{U}$ for a singlet pair

" " 0 for a triplet pair

$$\Delta E = \frac{4t^2}{U}\left[\frac{(\vec{S}_1+\vec{S}_2)^2-2}{2}\right]$$

$(\vec{S}_1+\vec{S}_2)^2 = 2$ for Triplet

$= 0$ for singlet

$$= \frac{2t^2}{U}\left[\overset{3/4}{S_1^2} + \overset{3/4}{S_2^2} + 2\vec{S}_1\cdot\vec{S}_2 - 2\right]$$

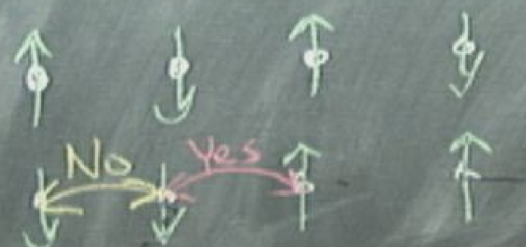
$$= \frac{4t^2}{U}\left[\vec{S}_1\cdot\vec{S}_2 - 1/4\right]$$

2^N-fold spin degeneracy

Antiferromagnetism

Consider 2 sites

$c_{i\uparrow}$ ——— U



2^N degen

Energy lowered by $-\frac{4t^2}{U}$ for a singlet pair

" 0 for a triplet pair

$$\Delta E = \frac{4t^2}{U}\left[\frac{(\vec{S}_1+\vec{S}_2)^2-2}{2}\right]$$

$(\vec{S}_1+\vec{S}_2)^2 = 2$ for triplet

$= 0$ for singlet

$$= \frac{2t^2}{U}\left[S_1^2 + S_2^2 + 2\vec{S}_1\cdot\vec{S}_2 - 2\right] \quad (S_1^2 = S_2^2 = 3/4)$$

$$= \frac{4t^2}{U}\left[\vec{S}_1\cdot\vec{S}_2 - 1/4\right]$$

2^N- fold Spin degeneracy

Antiferromagnetism

Consider 2 sites

$C_{i\uparrow}$ ——— U

No Yes

2^N degen

Energy lowered by $-\frac{4t^2}{U}$ for a singlet pair

" 0 for a triplet pair

$$\Delta E = \frac{4t^2}{U}\left[\frac{(\vec{S}_1+\vec{S}_2)^2 - 2}{2}\right]$$

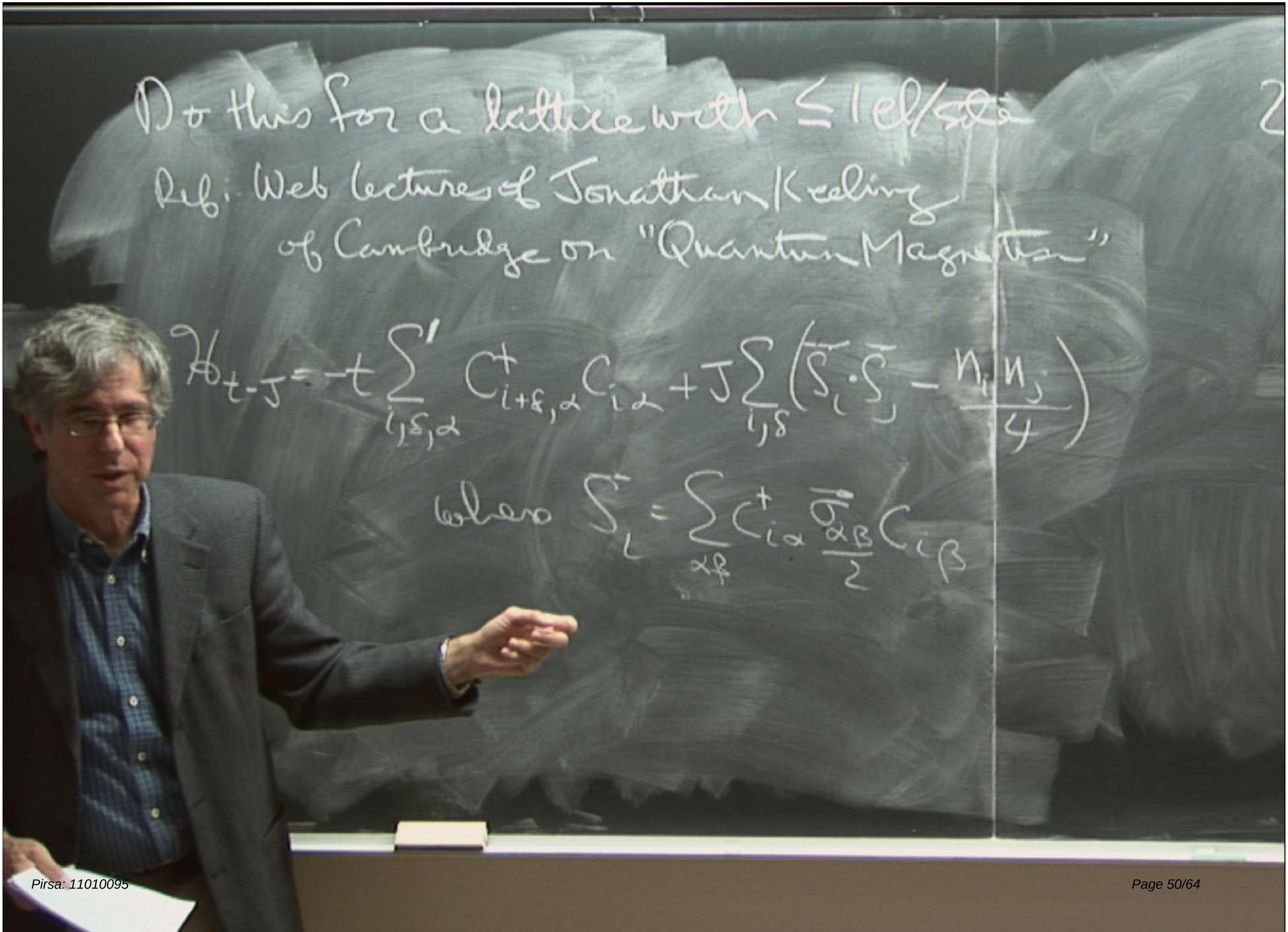
$(\vec{S}_1+\vec{S}_2)^2 = 2$ for Triplet

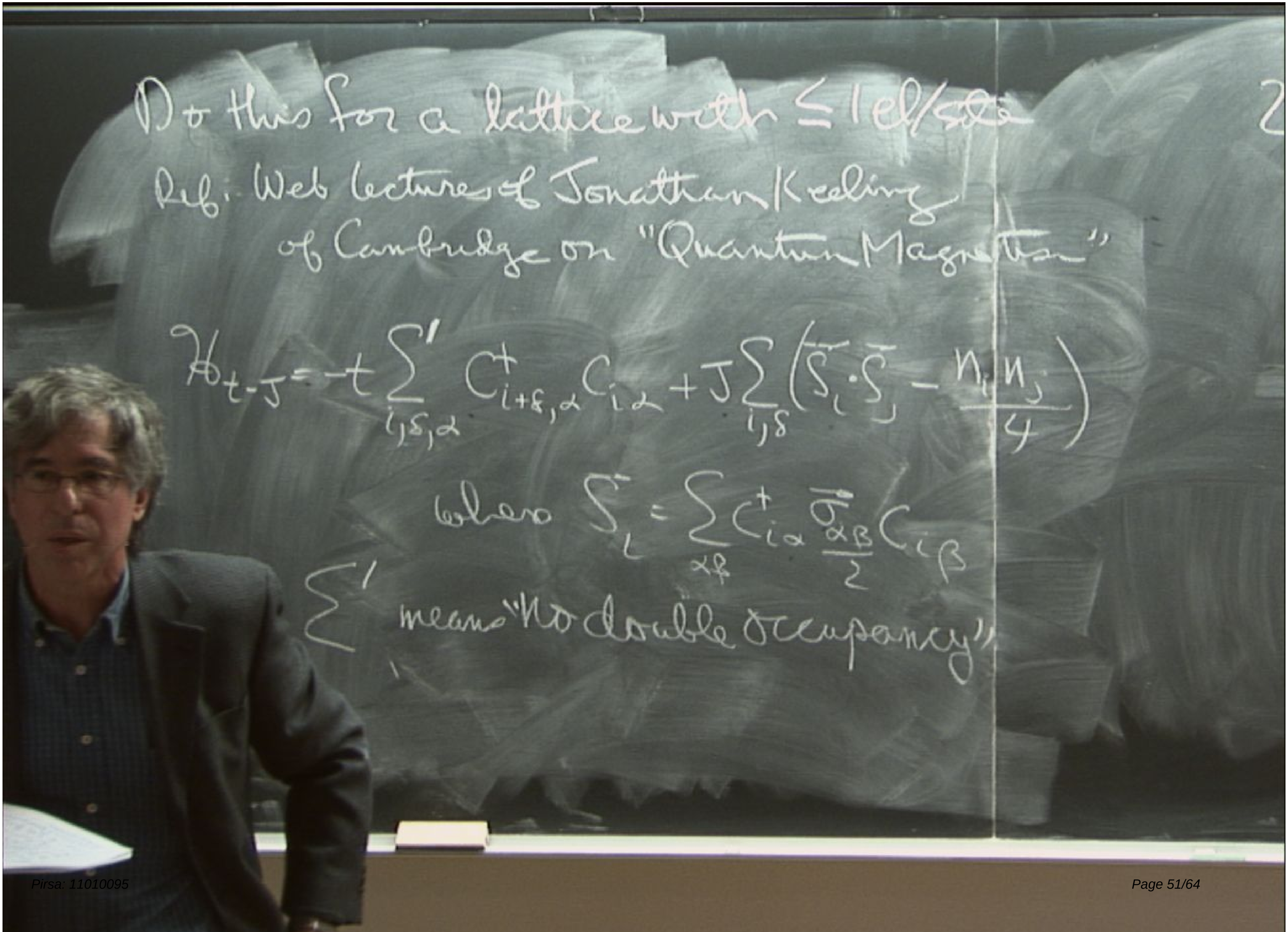
$= 0$ for singlet

$$J = \frac{2t^2}{U}\left[S_1^2 + S_2^2 + 2\vec{S}_1\cdot\vec{S}_2 - 2\right]$$

($S_1^2 = \frac{3}{4}$, $S_2^2 = \frac{3}{4}$)

$$= \frac{4t^2}{U}\left[\vec{S}_1\cdot\vec{S}_2 - \frac{1}{4}\right]$$


Do this for a lattice with ≤ 1 el/site
Ref: Web lectures of Jonathan Keeling
of Cambridge on "Quantum Magnetism"
$\mathcal{H}_{t-J} = -t \sum'_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} + J \sum_{i,\delta} \left(\vec{S}_i \cdot \vec{S}_j - \frac{n_i n_j}{4} \right)$
where $\vec{S}_i = \sum_{\alpha\beta} c^{\dagger}_{i\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$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where $\vec{S}_i = \sum_{\alpha\beta} c^{\dagger}_{i\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$
$\sum'$ means "No double occupancy"

Do this for a lattice with ≤ 1 el/site

Ref: Web lectures of Jonathan Keeling of Cambridge on "Quantum Magnetism"

$$\mathcal{H}_{t-J} = -t \sum'_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} + J \sum_{i,\delta} \left(\vec{S}_i \cdot \vec{S}_{i+\delta} - \frac{n_i n_{i+\delta}}{4} \right), \quad J = \frac{4t^2}{U}$$

where $\vec{S}_i = \sum_{\alpha\beta} c^{\dagger}_{i\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$

$\sum'$ means "No double occupancy"

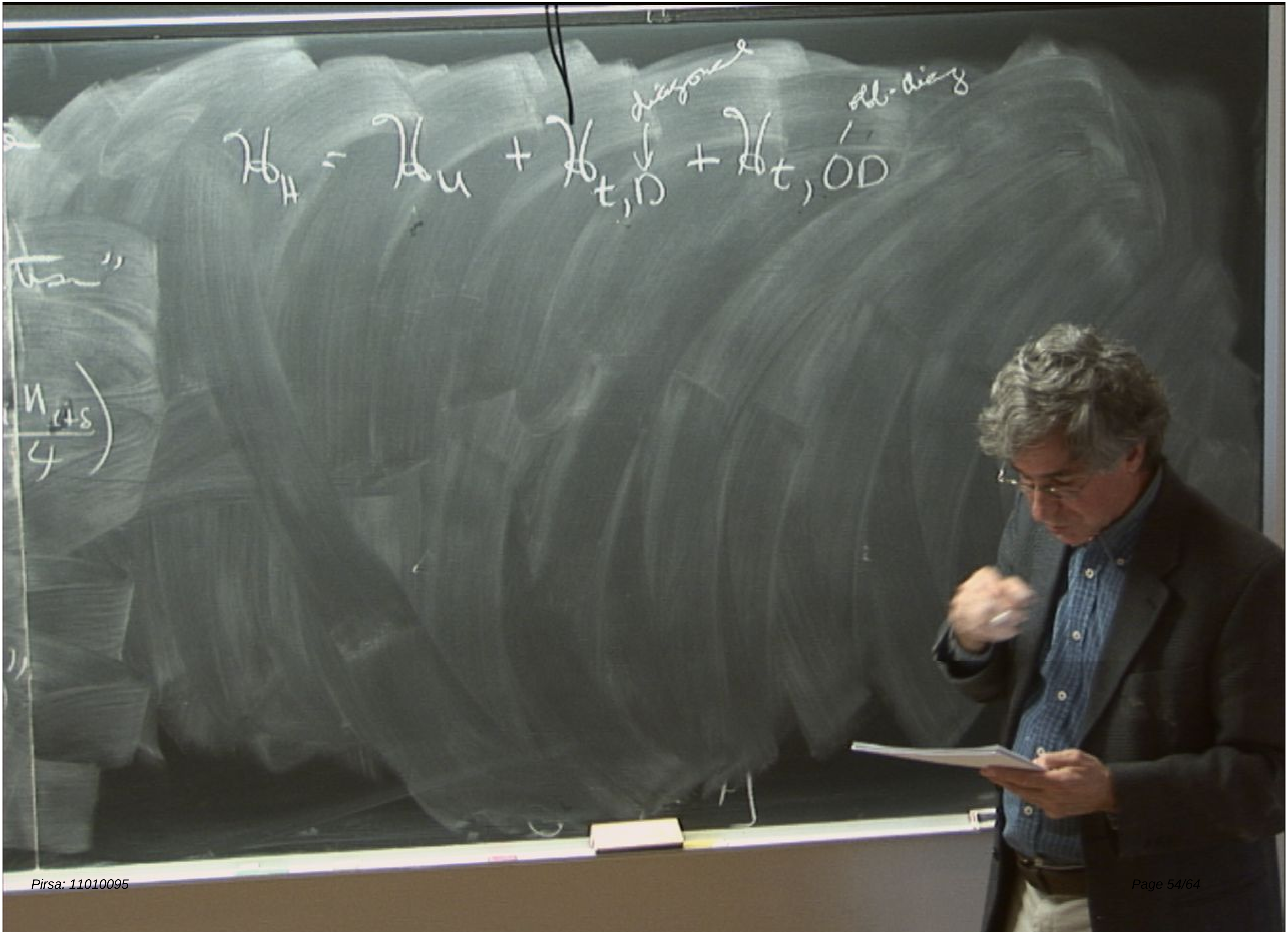
Do this for a lattice with ≤ 1 el/site

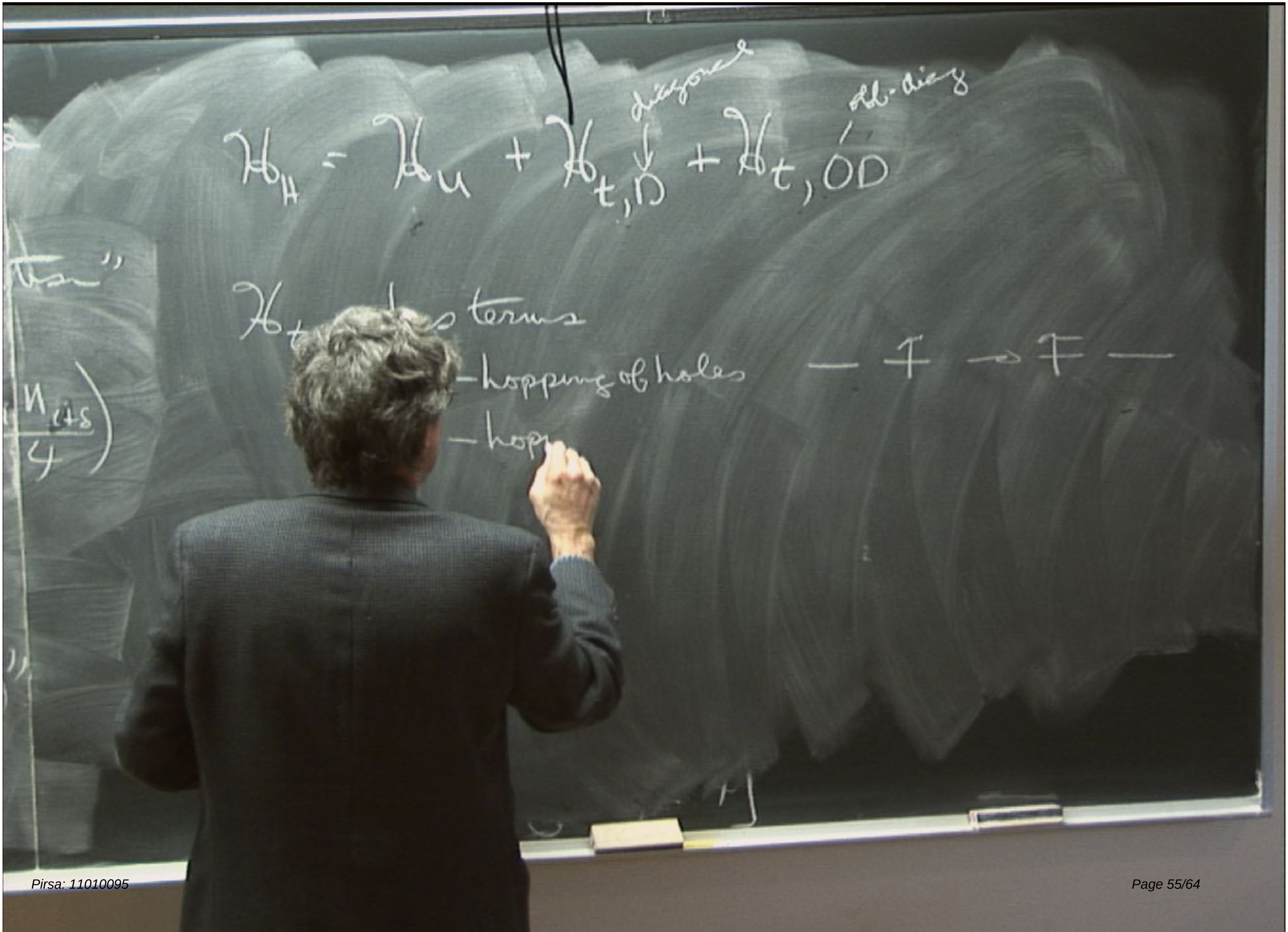
Ref: Web lectures of Jonathan Keeling of Cambridge on "Quantum Magnetism"

$$\mathcal{H}_{t-J} = -t \sum_{i,\delta,\alpha}{}' c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} + \overset{4t^2/U}{J} \sum_{i,\delta} \left(\vec{S}_i \cdot \vec{S}_{i+\delta} - \frac{n_i n_{i+\delta}}{4} \right)$$

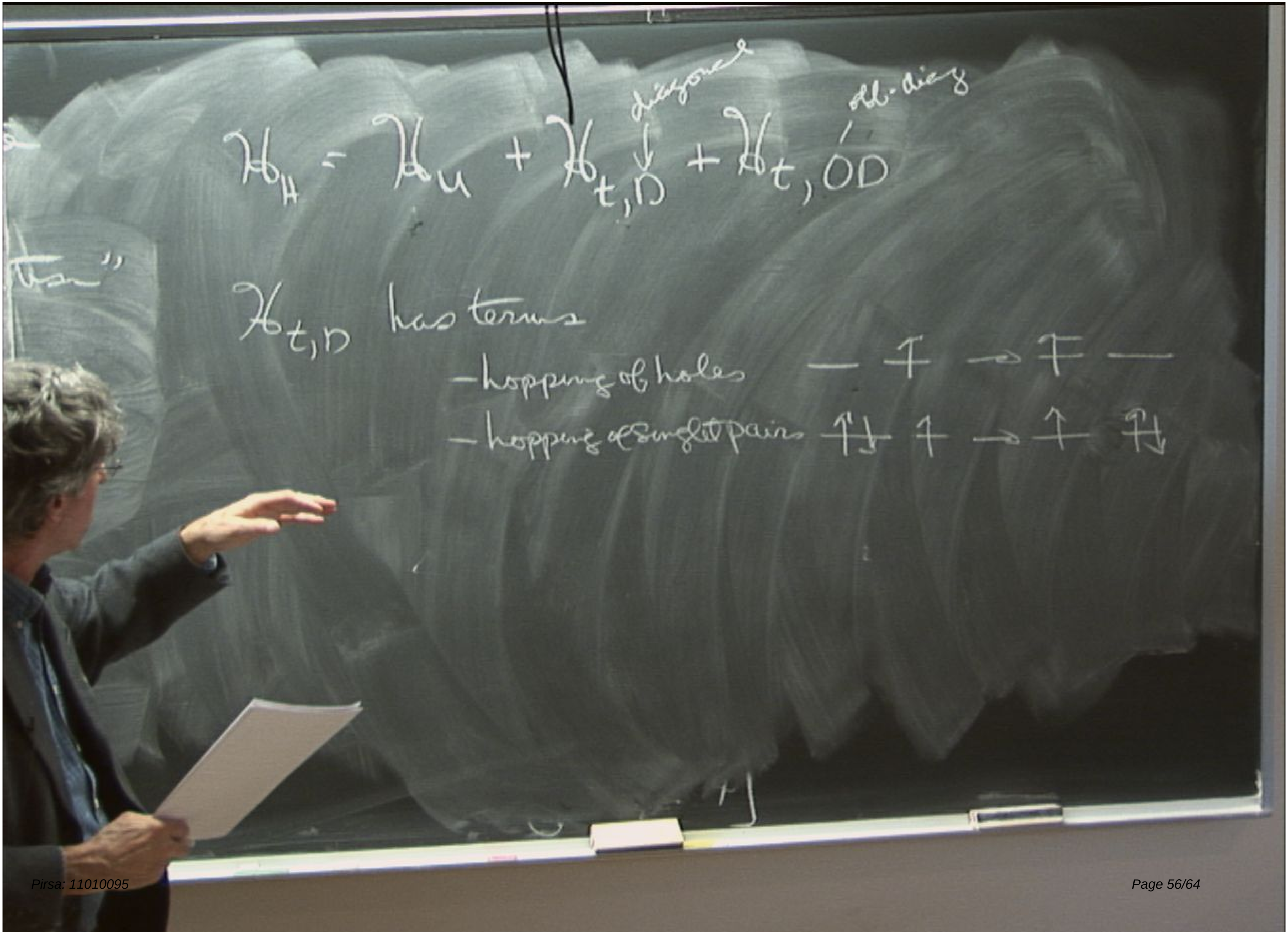
where $\vec{S}_i = \sum_{\alpha\beta} c^{\dagger}_{i\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} c_{i\beta}$

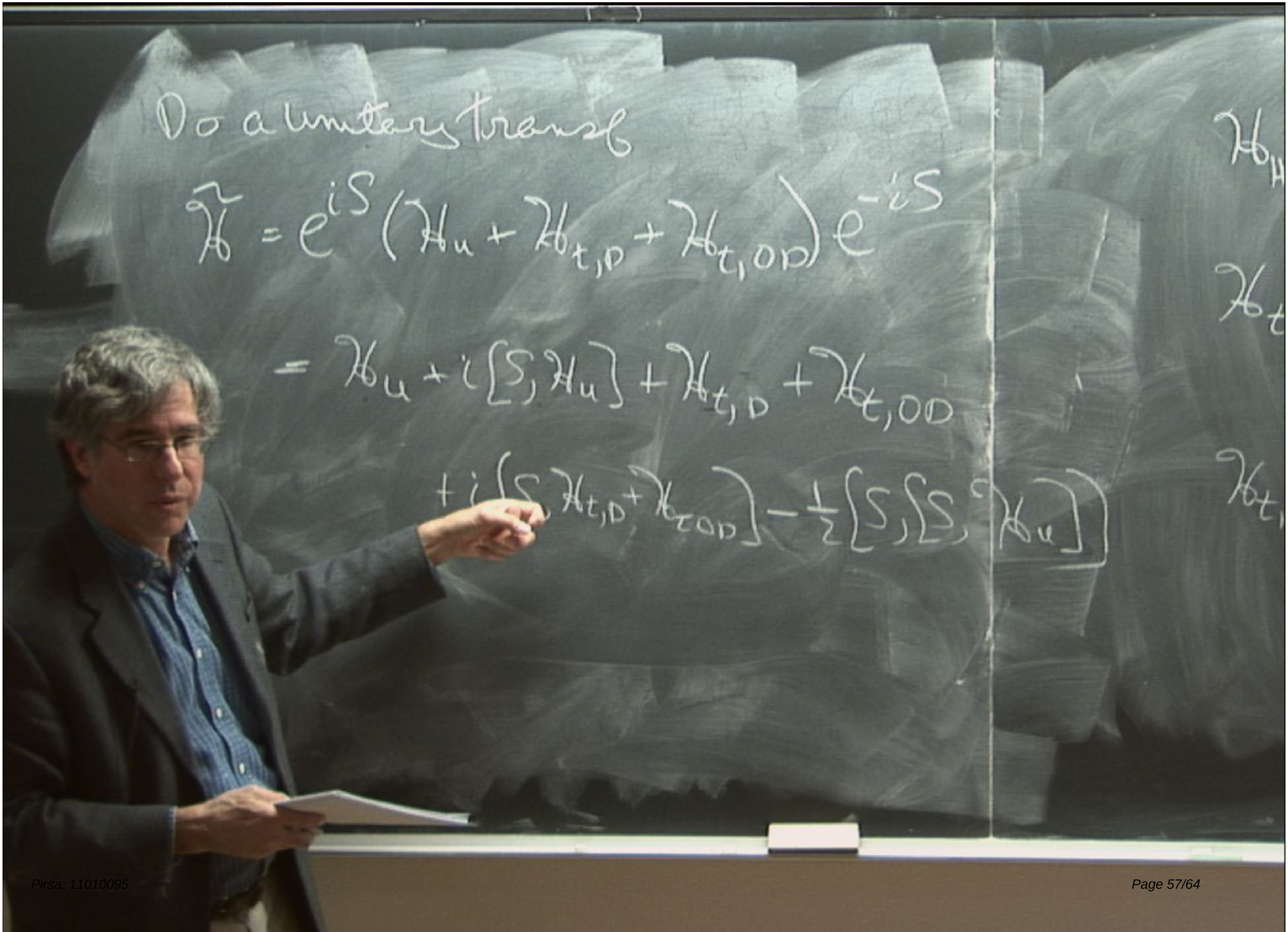
$\sum'$ means "No double occupancy"

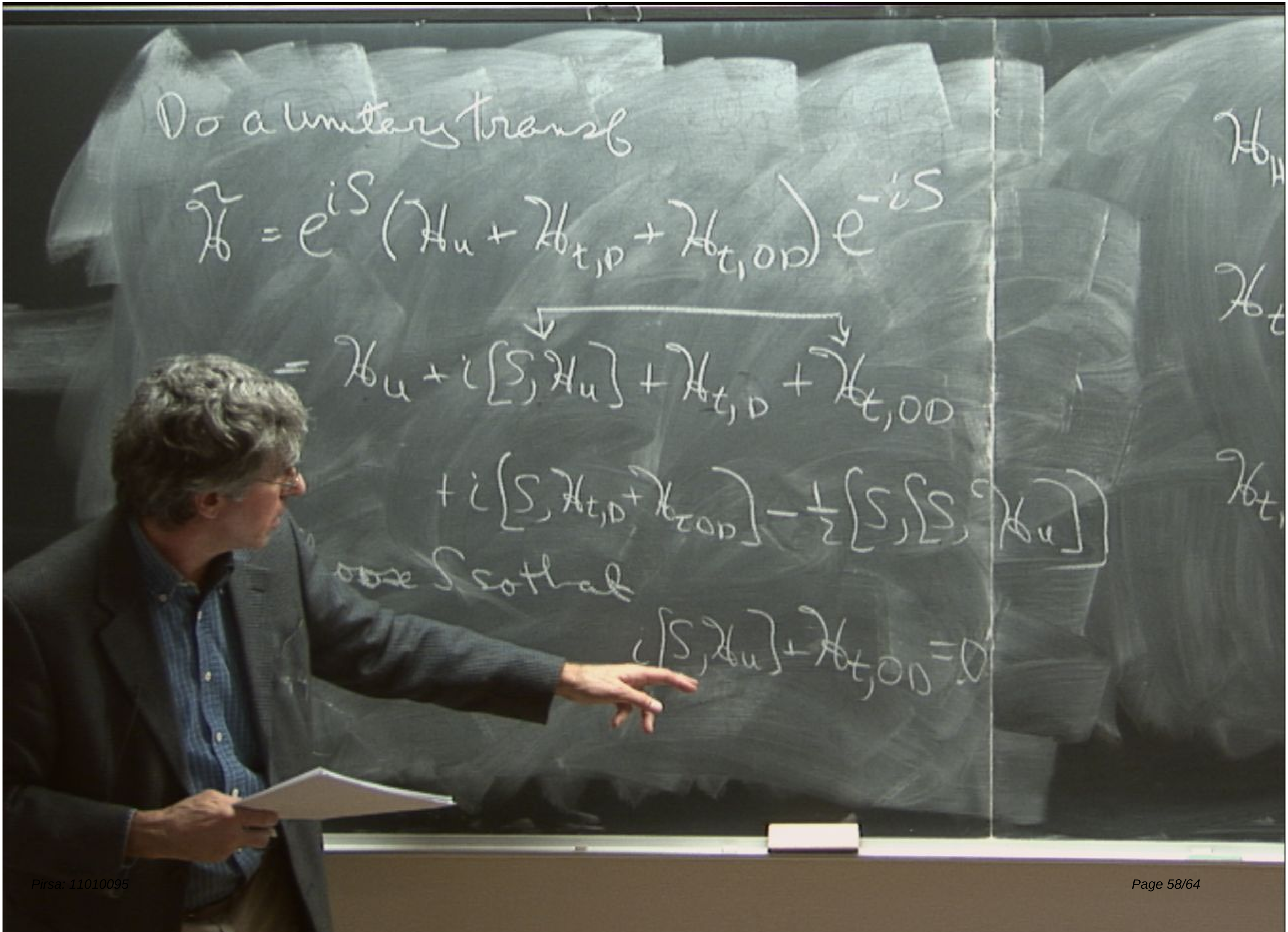

diagonal
off-diag
H_H = H_U + H_t,D + H_t,OD



$\mathcal{H}_H = \mathcal{H}_U + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$
terms
hopping of holes


diagonal
off-diag
$\mathcal{H}_H = \mathcal{H}_U + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$
$\mathcal{H}_{t,D}$ has terms
– hopping of holes
– hopping of singlet pairs


Do a unitary transf
$\tilde{\mathcal{H}} = e^{iS}(\mathcal{H}_u + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD})e^{-iS}$
$= \mathcal{H}_u + i[S, \mathcal{H}_u] + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$
$+ i[S, \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}] - \frac{1}{2}[S,[S,\mathcal{H}_u]]$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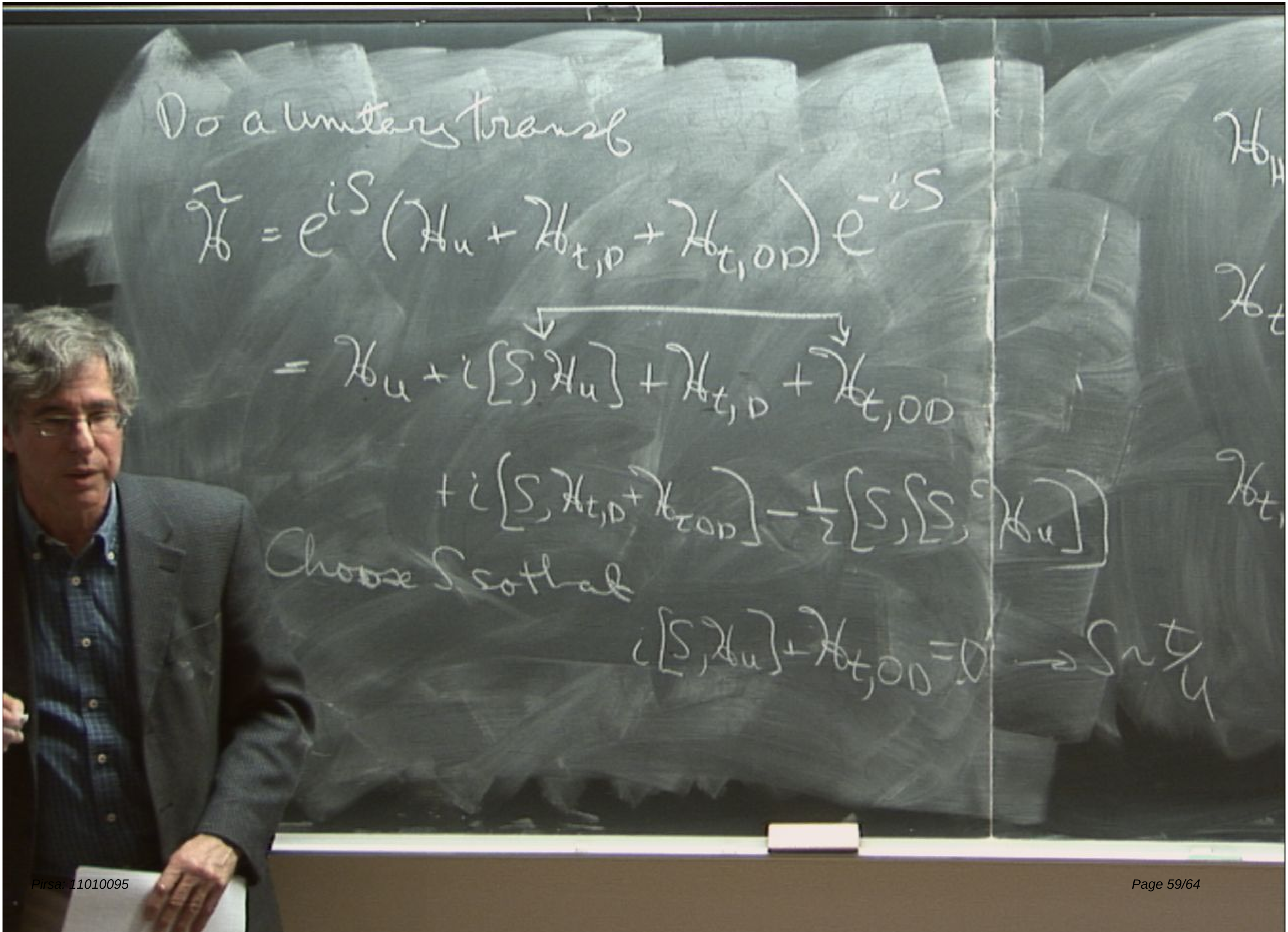
$= \mathcal{H}_u + i[S, \mathcal{H}_u] + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$
$+ i[S, \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}] - \frac{1}{2}[S,[S,\mathcal{H}_u]]$
S so that
$i[S, \mathcal{H}_u] + \mathcal{H}_{t,OD} = 0$


Do a unitary transf
$\tilde{\mathcal{H}} = e^{iS}(\mathcal{H}_u + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD})e^{-iS}$
$= \mathcal{H}_u + i[S,\mathcal{H}_u] + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$
$+ i[S,\mathcal{H}_{t,D} + \mathcal{H}_{t,OD}] - \frac{1}{2}[S,[S,\mathcal{H}_u]]$
Choose S so that
$i[S,\mathcal{H}_u] + \mathcal{H}_{t,OD} = 0 \rightarrow S \sim t/u$

Do a unitary transf

$$\tilde{\mathcal{H}} = e^{iS}\left(\mathcal{H}_u + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}\right) e^{-iS}$$

$$= \mathcal{H}_u + i[S,\mathcal{H}_u] + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$$

$$+ i[S,\mathcal{H}_{t,D} + \mathcal{H}_{t,OD}] - \frac{1}{2}[S,[S,\mathcal{H}_u]]$$

Choose S so that

$$i[S,\mathcal{H}_u] + \mathcal{H}_{t,OD} = 0 \rightarrow S \sim t/U$$

$\mathcal{H}_H$

$\mathcal{H}_t$

$\mathcal{H}_{t,}$

Do a unitary transf

$$\tilde{\mathcal{H}} = e^{iS}(\mathcal{H}_u + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD})e^{-iS}$$

$$= \mathcal{H}_u + i[S,\mathcal{H}_u] + \mathcal{H}_{t,D} + \mathcal{H}_{t,OD}$$

$$+ i[S,\mathcal{H}_{t,D} + \mathcal{H}_{t,OD}] - \frac{1}{2}[S,[S,\mathcal{H}_u]]$$

Choose S so that

$$i[S,\mathcal{H}_u] + \mathcal{H}_{t,OD} = 0 \rightarrow S \sim t/U$$

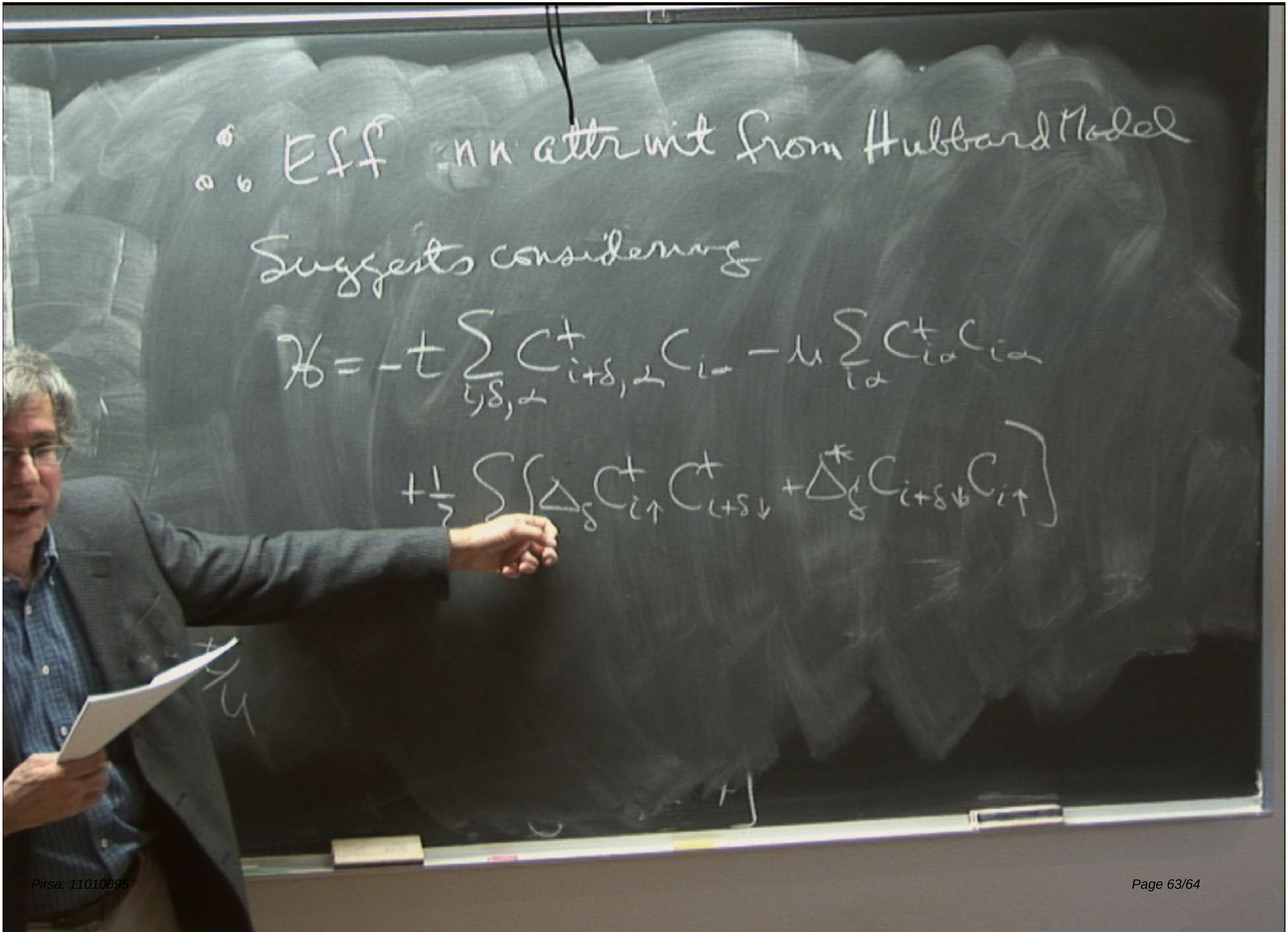
$\mathcal{H}_{\mu}$

∴ Eff nn attr int from Hubbard Model

Suggests considering

$$\mathcal{H} = -t \sum_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i\alpha} c^{\dagger}_{i\alpha} c_{i\alpha}$$

$$+ \frac{1}{2} \sum_{i,\delta} \left[\Delta_{\delta} c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_{\delta} c_{i+\delta\downarrow} c_{i\uparrow} \right]$$


∴ Eff nn attr int from Hubbard Model
Suggests considering
$\mathcal{H} = -t \sum_{i,\delta,\alpha} c^{+}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i\alpha} c^{+}_{i\alpha} c_{i\alpha}$
$+ \frac{1}{2} \sum [\Delta_\delta c^{+}_{i\uparrow} c^{+}_{i+\delta\downarrow} + \Delta^{*}_\delta c_{i+\delta\downarrow} c_{i\uparrow}]$

∴ Eff nn attr int from Hubbard Model

Suggests considering

$$\mathcal{H} = -t \sum_{i,\delta,\alpha} c^{\dagger}_{i+\delta,\alpha} c_{i\alpha} - \mu \sum_{i\alpha} c^{\dagger}_{i\alpha} c_{i\alpha}$$

$$+ \frac{1}{2} \sum_{i,\delta} \left[\Delta_{\delta} c^{\dagger}_{i\uparrow} c^{\dagger}_{i+\delta\downarrow} + \Delta^{*}_{\delta} c_{i+\delta\downarrow} c_{i\uparrow} \right]$$

$\mathcal{H}_U$

$\to \sim t/U$