

Title: Phase Space Gauge Principles and Universal Consequences for Physics and Space-Time

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Abstract: Hints for the possibility of two times emerged in M-theory in 1995. If taken seriously this required new concepts that could solve unitarity (ghost) and causality problems so that physics could be described sensibly in a spacetime with two times. The necessary concept turned out to be a gauge symmetry in phase space. This is an unfamiliar concept, but is one that extends Einstein's approach to the formulation of fundamental equations of physics, by removing the perspective of the observer, not only in position space but more generally in phase space.

This approach led in 1998 to what is now called 2T-physics, which has been formulated so far in classical and quantum mechanics, field theory and partially in string theory. In this lecture I will explain the fundamental aspects of 2T-physics, and will outline the progress from classical mechanics, through the standard model and gravity, all the way to supergravity in d-space plus 2-time dimensions. I will describe how 2T-physics is consistent with 1T-physics in (d-1)-space plus 1-time dimensions, but also how it goes beyond 1T-physics, by making in principle a vast number of verifiable predictions that are systematically missed in 1T-physics, as well as providing new computational tools for physics.

Phase Space Gauge Principles and Universal Consequences for Physics and Space-Time

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January 18, 2011

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- It works for all classical & quantum mechanics, field theory the Standard Model, gravity, all the way to supergravity. String theory is still underdeveloped in this framework.
- Makes in principle a vast number of verifiable predictions that are missed systematically in the 1T formulation of physics at all scales of distance or energy.

Strong hints for 2T-physics came from M-theory (IB -1995):
 Extended SUSY of 11D M-theory is really a SUSY in 12 dimensions

$$\{Q_{32}, Q_{32}\} = (P + Z_{[2]} + Z_{[5]})_{11D} = (T_{[2]} + T_{[6]+})_{12D}$$

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If 2 times are taken seriously there are huge problems,
 how does one remove the ghosts and causality problems?

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e.g. $A_M(X^M)$ has wrong sign kinetic terms,
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e.g. $A_M(X^M) \rightarrow A_\mu(x^\mu)$, etc

What remains?

1T shadows!

with many kinds of 1T spaces

2T-physics developed by finding the
fundamental solution to the
ghost problem & related **causality problem**.

The answer (1998) is a
gauge symmetry in phase space X^M, P_M

This is independent of M-theory, but I think will eventually
 explain (M-)S-theory, with $OSp(1|64)$ SUSY

Like all gauge symmetries, also introduces a
 new universal principle for constructing the
dynamics.

It is compatible with 1T-physics for all known phenomena, furthermore
 goes beyond 1T-physics in revealing additional hidden information.

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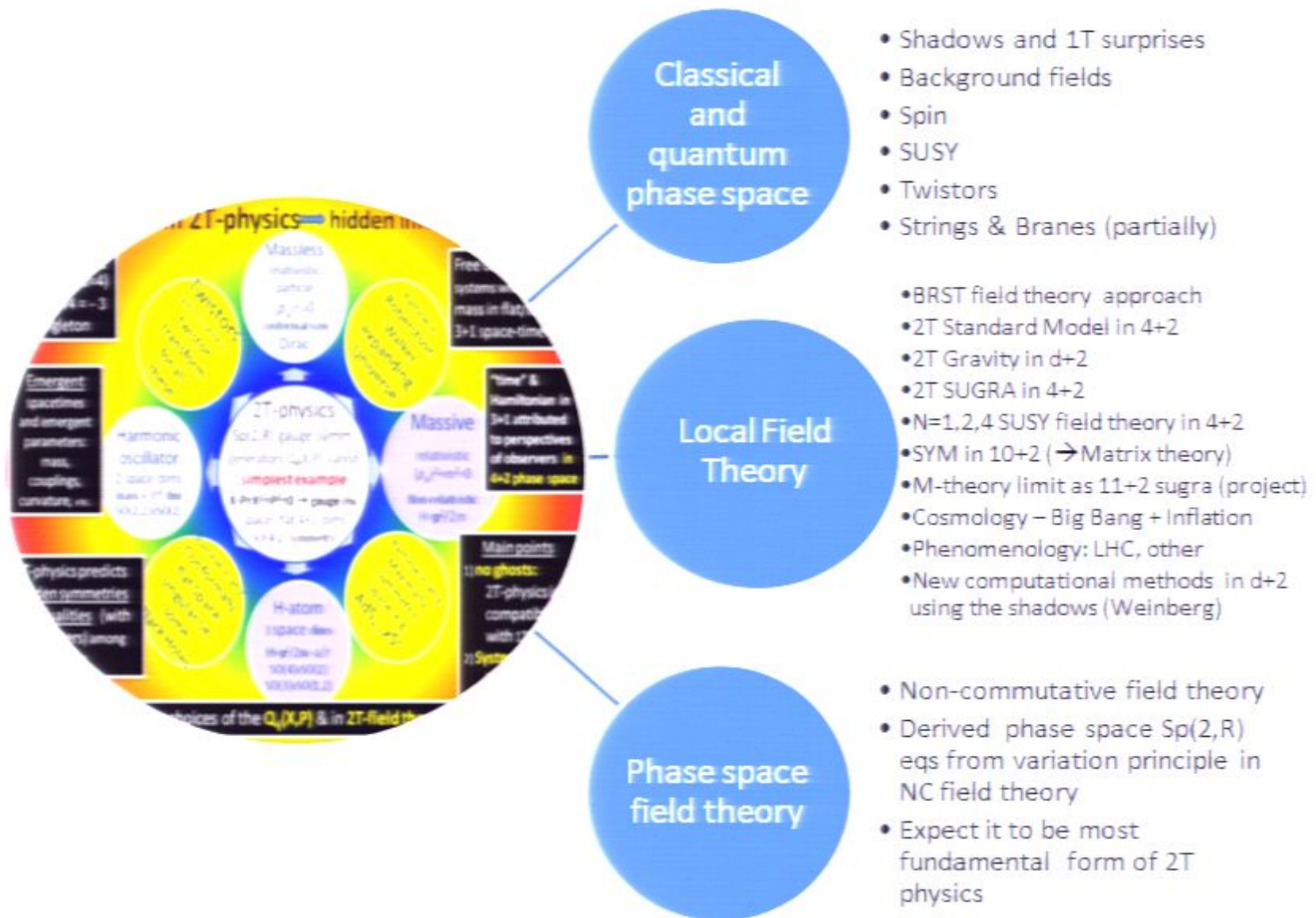
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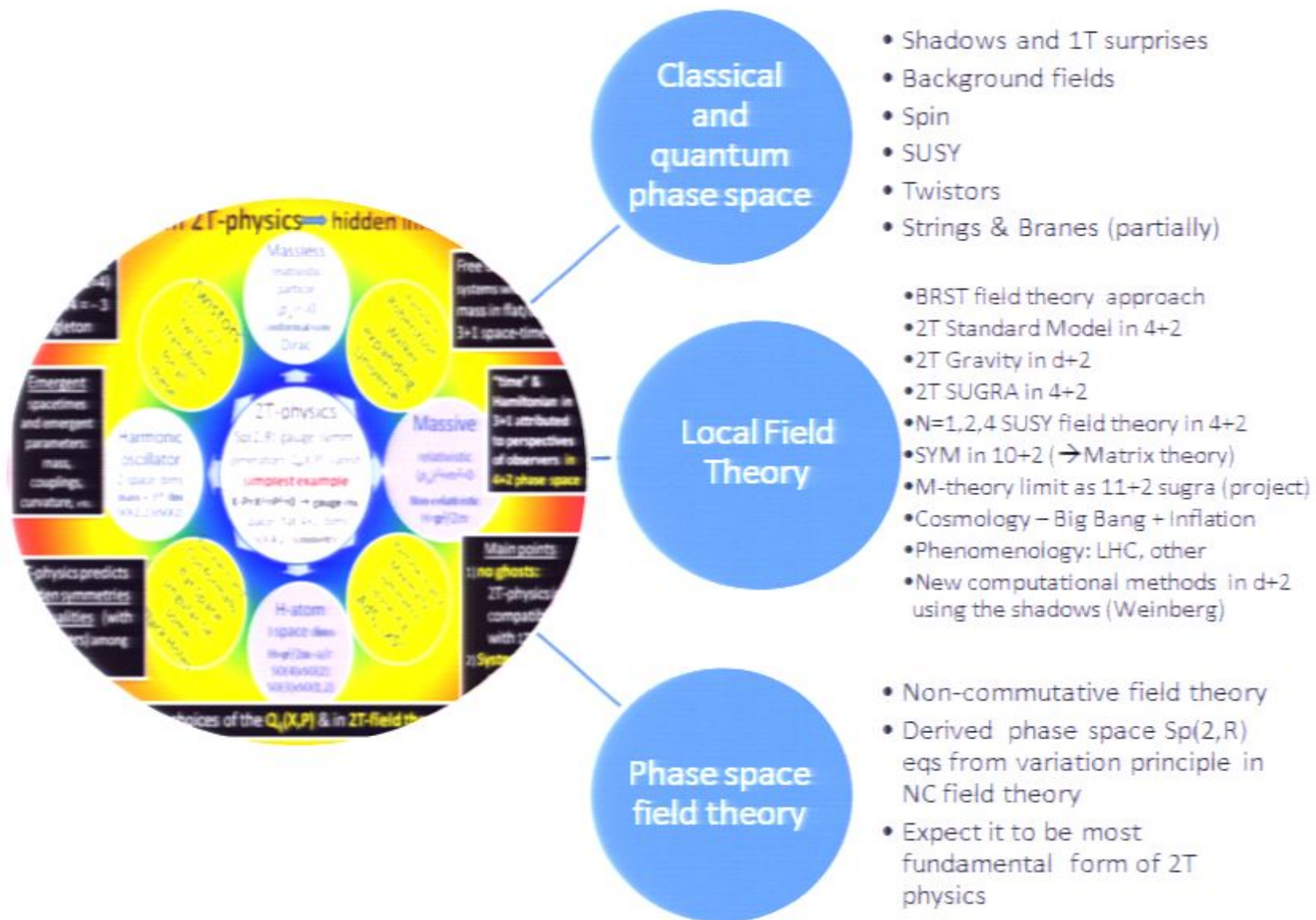
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- **Canonical transformations turned into gauge symmetry.** Start with worldline, then field theory, then ... (not a completed project, but expect ultimately field theory in phase space)
- **2T is not an input**, it is a consequence of gauge symmetry.





Phase space symmetry in particle classical mechanics

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$$L = \partial_\tau X^M P_M - \dots$$

A huge symmetry of the first term in L under GLOBAL canonical transformations.

$$\delta X^M = \frac{\partial \varepsilon(X, P)}{\partial P_M} = \{X^M, \varepsilon\},$$

$$\delta P_M = -\frac{\partial \varepsilon(X, P)}{\partial X^M} = \{P_M, \varepsilon\}$$

$$\delta_\varepsilon \{X^M, P_N\} = 0 \quad \delta_\varepsilon [X^M, P_N] = 0.$$

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These $\varepsilon(X, P)$ contain all gauge transformations
Maxwell, Einstein, Yang-Mills, & MUCH MORE ...

$$L = \partial_\tau X^M P_M - H(X, P)$$

$$\delta_\varepsilon H = \{H, \varepsilon\}$$

$$\varepsilon(X, P) = \Lambda(X) + \varepsilon^M(X) P_M + \varepsilon^{MN}(X) P_M P_N + \dots$$

$$H(X, P) = G^{MN}(X) (P_M + A_M(X)) (P_N + A_N(X)) + \dots$$

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Generalizes with
spin & SUSY, etc.

For non-Abelian YM
use fiber bundles.

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Proposal (1998): To be able to remove ghost degrees of freedom from X, P , promote canonical transformations to a gauge symmetry by localizing on the worldline (every instant of motion)

Gauge Symmetry in Phase Space

$$\varepsilon(X(\tau), P(\tau), \tau) = \sum_a \varepsilon^a(\tau) Q_a(X, P) \quad \{Q_a, Q_b\} = f_{ab}{}^c Q_c$$

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Equation of motion
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 means only
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Analogous to Virasoro
 constraints in string theory.

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Familiar example:
 relativistic particle

$$L = \dot{x}^\mu p_\mu - e(\tau) \frac{p^2}{2}$$

More general, any $Q(X, P)$:
 1-parameter **non-compact** Abelian
 gauge symmetry. **Requires 1T** and
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Sp(2,R)
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Three generators $Q_{ij}(X,P)$: $\{Q_{11}, Q_{22}, Q_{12}=Q_{21}\}$
form Sp(2,R) under Poisson brackets.

$A^{ij}(\tau)$ is the Sp(2,R) gauge potential, $i=1,2$ is label for Sp(2,R) doublet.

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X, P) - H(X, P)$$

Example, flat spacetime:

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In flat case
Global symmetry SO(d,2)
 $L^{MN}=X^M P^N - X^N P^M$, $\{Q_{ij}, L^{MN}\}=0$.
So L^{MN} is gauge invariant.
Could add $H(L^{MN})$, could break
SO(d,2) is OK, still gauge inv

$$X \cdot X = X \cdot P = P \cdot P = 0 \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

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Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint

$$X \cdot X = X \cdot P = P \cdot P = 0 \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M =$	1	$X^{+'}(\tau)$	$\frac{1}{2}x^2$	$X^{-'}(\tau)$	x^μ	$X^\mu(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
	$P^M =$	0	$P^{+'}(\tau)$	$x \cdot p$	$P^{-'}(\tau)$	p^μ	$P^\mu(\tau)$	

$$X \cdot X = X \cdot P = P \cdot P = 0 \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M =$	1	$X^{+'}(\tau)$	$\frac{1}{2}x^2$	$X^{-'}(\tau)$	x^μ	$X^\mu(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
	$P^M =$	0	$P^{+'}(\tau)$	$x \cdot p$	$P^{-'}(\tau)$	p^μ	$P^\mu(\tau)$	
Relativistic massive particle	$X^M =$	$\frac{1+a}{2a}$		$\frac{x^2 a}{1+a}$		x^μ	$\alpha \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
$p^2 + m^2 = 0$	$P^M =$	$\frac{-m^2}{2a x \cdot p}$		$a x \cdot p$		p^μ		

$X \cdot X = X \cdot P = P \cdot P = 0$

$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$

$X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$

Relativistic massless particle $p^2 = 0$	$X^M =$ $P^M =$	<div style="display: flex; flex-direction: column; align-items: center;"> <div>1 $X^{+'}(\tau)$</div> <div>0 $P^{+'}(\tau)$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$\frac{1}{2} x^2$ $X^{-'}(\tau)$</div> <div>$x \cdot p$ $P^{-'}(\tau)$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>x^μ $X^\mu(\tau)$</div> <div>p^μ $P^\mu(\tau)$</div> </div>	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
Relativistic massive particle $p^2 + m^2 = 0$	$X^M =$ $P^M =$	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$\frac{1+a}{2a}$</div> <div>$\frac{-m^2}{2ax \cdot p}$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$\frac{x^2 a}{1+a}$</div> <div>$a \ x \cdot p$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>x^μ</div> <div>p^μ</div> </div>	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M =$ $P^M =$	<div style="display: flex; flex-direction: column; align-items: center;"> <div>t</div> <div>m</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$</div> <div>$H$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$X^0 = \pm \left \mathbf{r} - \frac{t}{m} \mathbf{p} \right$</div> <div>$X^i = \mathbf{r}^i$</div> </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div>$P^0 = 0$</div> <div>$P^i = \mathbf{p}^i$</div> </div>

$X \cdot X = X \cdot P = P \cdot P = 0$

$ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$

$X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$

Relativistic massless particle $p^2 = 0$	$X^M =$	1	$X^{+'}(\tau)$	$\frac{1}{2}x^2$	$X^{-'}(\tau)$	x^μ	$X^\mu(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
	$P^M =$	0	$P^{+'}(\tau)$	$x \cdot p$	$P^{-'}(\tau)$	p^μ	$P^\mu(\tau)$	
Relativistic massive particle $p^2 + m^2 = 0$	$X^M =$	$\frac{1+a}{2a}$		$\frac{x^2 a}{1+a}$		x^μ	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
	$P^M =$	$\frac{-m^2}{2ax \cdot p}$		$a x \cdot p$		p^μ		
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M =$	t		$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$		$X^0 = \pm \left \mathbf{r} - \frac{t}{m} \mathbf{p} \right $	$X^i = \mathbf{r}^i$	
	$P^M =$	m		H		$P^0 = 0$	$P^i = \mathbf{p}^i$	

Mass : modulus in the embedding of d-dimensional phase space into d+2.

$$X \cdot X = X \cdot P = P \cdot P = 0 \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^+ dX^- + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^\pm = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M =$	1	$X^{+}(\tau)$	$\frac{1}{2}x^2$	$X^{-}(\tau)$	x^μ	$X^{\mu}(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
	$P^M =$	0	$P^{+}(\tau)$	$x \cdot p$	$P^{-}(\tau)$	p^μ	$P^{\mu}(\tau)$	
Relativistic massive particle $p^2 + m^2 = 0$	$X^M =$	$\frac{1+a}{2a}$		$\frac{x^2 a}{1+a}$		x^μ	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
	$P^M =$	$\frac{-m^2}{2a x \cdot p}$		$a x \cdot p$		p^μ		
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M =$	t		$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$		$X^0 = \pm \mathbf{r} - \frac{t}{m} \mathbf{p} $	$X^i = \mathbf{r}^i$	
	$P^M =$	m		H		$P^0 = 0$	$P^i = \mathbf{p}^i$	

Mass : modulus in the embedding of d-dimensional phase space into d+2.

Maximally Symmetric Spaces $p^2 - \frac{K (x \cdot p)^2}{1 - K x^2} = 0$	$X^M =$	$1 + \sqrt{1 - K x^2}$	$\frac{x^2/2}{1 + \sqrt{1 - K x^2}}$	x^μ	$g_{\mu\nu} = \eta_{\mu\nu} + \frac{K}{1 - K x^2} x_\mu x_\nu$
	$P^M =$	0	$\frac{\sqrt{1 - K x^2}}{1 + \sqrt{1 - K x^2}} x \cdot p$	$p^\mu - \frac{K x \cdot p x^\mu}{1 + \sqrt{1 - K x^2}}$	

$$X \cdot X = X \cdot P = P \cdot P = 0 \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^+ dX^- + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^\pm = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M =$	1	$X^{+}(\tau)$	$\frac{1}{2}x^2$	$X^{-}(\tau)$	x^μ	$X^{\mu}(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
	$P^M =$	0	$P^{+}(\tau)$	$x \cdot p$	$P^{-}(\tau)$	p^μ	$P^{\mu}(\tau)$	
Relativistic massive particle $p^2 + m^2 = 0$	$X^M =$	$\frac{1+a}{2a}$		$\frac{x^2 a}{1+a}$		x^μ	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
	$P^M =$	$\frac{-m^2}{2ax \cdot p}$		$a x \cdot p$		p^μ		
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M =$	t		$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$		$X^0 = \pm \mathbf{r} - \frac{t}{m}\mathbf{p} $	$X^i = \mathbf{r}^i$	
	$P^M =$	m		H		$P^0 = 0$	$P^i = \mathbf{p}^i$	

Mass : modulus in the embedding of d-dimensional phase space into d+2.

Maximally Symmetric Spaces $p^2 - \frac{K (x \cdot p)^2}{1-Kx^2} = 0$	$X^M =$	$1 + \sqrt{1 - Kx^2}$	$\frac{x^2/2}{1 + \sqrt{1 - Kx^2}}$	x^μ	$g_{\mu\nu} = \eta_{\mu\nu} + \frac{K}{1-Kx^2} x_\mu x_\nu$	curvature
	$P^M =$	0	$\frac{\sqrt{1-Kx^2}}{1 + \sqrt{1-Kx^2}} x \cdot p$	$p^\mu - \frac{K x \cdot p x^\mu}{1 + \sqrt{1-Kx^2}}$		

$X \cdot X = X \cdot P = P \cdot P = 0$

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^+ dX^- + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M = 1$ $P^M = 0$	$X^{+'}(\tau)$ $P^{+'}(\tau)$	$\frac{1}{2}x^2$ $x \cdot p$	$X^{-'}(\tau)$ $P^{-'}(\tau)$	x^μ p^μ	$X^{\mu'}(\tau)$ $P^{\mu'}(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
Relativistic massive particle $p^2 + m^2 = 0$	$X^M = \frac{1+a}{2a}$ $P^M = \frac{-m^2}{2ax \cdot p}$		$\frac{x^2 a}{1+a}$ $a x \cdot p$		x^μ p^μ	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M = t$ $P^M = m$		$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$ H		$X^0 = \pm \mathbf{r} - \frac{t}{m}\mathbf{p} $ $X^i = \mathbf{r}^i$ $P^0 = 0, P^i = \mathbf{p}^i$		

Mass : modulus in the embedding of d-dimensional phase space into d+2.

Maximally Symmetric Spaces $p^2 - \frac{K (x \cdot p)^2}{1-Kx^2} = 0$	$X^M = 1 + \sqrt{1 - Kx^2}$ $P^M = 0$	$\frac{x^2/2}{1 + \sqrt{1 - Kx^2}}$ $\frac{\sqrt{1 - Kx^2}}{1 + \sqrt{1 - Kx^2}} x \cdot p$	x^μ $p^\mu - \frac{K x \cdot p}{1 + \sqrt{1 - Kx^2}} x^\mu$	$g_{\mu\nu} = \eta_{\mu\nu} + \frac{K}{1 - Kx^2} x_\mu x_\nu$ curvature
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Time and Hamiltonian are different in each case \leftrightarrow 1T perspectives in 2T phase space. All can be gauge transformed to each other. There are gauge invariant relations among them. (info absent in 1T-physics)

Any function of $L_{MN} - Y_{MPN}, Y_{NPM}$ is gauge invariant

Gauge choice	M	$0'$ $X^{0'}$	0 X^0	$I = (1', t)$ $X^{1'} X^i$
Robertson-Walker $r < R_0$ (closed universe) $-H^2 + \frac{R_0^2}{a^4(t)}(\mathbf{p}^2 - \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2}) = 0$	$X^M = a(t) \cos(\int^t \frac{dt'}{a(t')})$ $P^M = -H \sin(\int^t \frac{dt'}{a(t')})$	$a(t) \sin(\int^t \frac{dt'}{a(t')})$ $H \cos(\int^t \frac{dt'}{a(t')})$	$X^1 = \mathbf{r}^i a(t) / R_0$ $X^{1'} = \pm a(t) \sqrt{1 - \frac{r^2}{R_0^2}}$ $P^i = \frac{R_0}{a(t)}(\mathbf{p}^i - \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2} \mathbf{r}^i)$ $P^{1'} = \pm \frac{\mathbf{r} \cdot \mathbf{p}}{a(t)} \sqrt{1 - \frac{r^2}{R_0^2}}$	
Robertson-Walker $r > 0$ (open universe) $-H^2 + \frac{R_0^2}{a^4(t)}(\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2}) = 0$	$X^M = a(t) \sinh(\int^t \frac{dt'}{a(t')})$ $P^M = \pm H \cosh(\int^t \frac{dt'}{a(t')})$	$(\pm)' a(t) \sqrt{1 + \frac{r^2}{R_0^2}}$ $(\pm)' \frac{\mathbf{r} \cdot \mathbf{p}}{a(t)} \sqrt{1 + \frac{r^2}{R_0^2}}$	$X^1 = \mathbf{r}^i a(t) / R_0$ $X^{1'} = \pm a(t) \cosh(\int^t \frac{dt'}{a(t')})$ $P^i = \frac{R_0}{a(t)}(\mathbf{p}^i + \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2} \mathbf{r}^i)$ $P^{1'} = H \sinh(\int^t \frac{dt'}{a(t')})$	
Cosmological constant $\Lambda \equiv \frac{3}{R_0^2} > 0$ $-H^2(1 - \frac{r^2}{R_0^2}) + (\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 - r^2}) = 0$	$X^M = \sqrt{R_0^2 - r^2} \sinh \frac{t}{R_0}$ $P^M = \pm \frac{H}{R_0} \sqrt{R_0^2 - r^2} \cosh \frac{t}{R_0}$	R_0 $\frac{R_0 \mathbf{r} \cdot \mathbf{p}}{R_0^2 - r^2}$	$X^1 = \mathbf{r}^i$ $X^{1'} = \pm \sqrt{R_0^2 - r^2} \cosh \frac{t}{R_0}$ $P^i = \mathbf{p}^i + \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2 - r^2} \mathbf{r}^i$ $P^{1'} = \frac{H}{R_0} \sqrt{R_0^2 - r^2} \sinh \frac{t}{R_0}$	
Cosmological constant $\Lambda \equiv -\frac{3}{R_0^2} < 0$ $-H^2(1 + \frac{r^2}{R_0^2}) + (\mathbf{p}^2 - \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 + r^2}) = 0$	$X^M = \sqrt{R_0^2 + r^2} \sin \frac{t}{R_0}$ $P^M = \pm \frac{H}{R_0} \sqrt{R_0^2 + r^2} \cos \frac{t}{R_0}$	$\pm \sqrt{R_0^2 + r^2} \cos \frac{t}{R_0}$ $\frac{H}{R_0} \sqrt{R_0^2 + r^2} \sin \frac{t}{R_0}$	$X^1 = \mathbf{r}^i$ $X^{1'} = R_0$ $P^i = \mathbf{p}^i - \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2 + r^2} \mathbf{r}^i$ $P^{1'} = -\frac{R_0 \mathbf{r} \cdot \mathbf{p}}{R_0^2 + r^2}$	
(d-1)-sphere \times time $-H^2 + (\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 - r^2}) = 0$	$X^M = R_0 \cos \frac{t}{R_0}$ $P^M = -H \sin \frac{t}{R_0}$	$R_0 \sin \frac{t}{R_0}$ $H \cos \frac{t}{R_0}$	$R_0 \hat{n}^I = \frac{X^1 = \mathbf{r}^i}{X^{1'} = \pm \sqrt{R_0^2 - r^2}}$ $P^i = \mathbf{p}^i$ $P^{1'} = \pm \frac{\mathbf{r} \cdot \mathbf{p}}{\sqrt{R_0^2 - r^2}}$	
H-atom, $H < 0$ $H = \frac{\mathbf{p}^2}{2m} - \frac{\alpha}{r}$	$X^M = \frac{r \cos u}{u(t) \equiv \sqrt{\frac{-2mH}{\alpha}}(\mathbf{r} \cdot \mathbf{p} - 2mHt)}$ $P^M = -\frac{m\alpha}{r \sqrt{-2mH}} \sin u$	$r \sin u$ $\frac{m\alpha}{r \sqrt{-2mH}} \cos u$	$X^1 = \mathbf{r}^i - \frac{\alpha}{m\alpha} \mathbf{r} \cdot \mathbf{p} \mathbf{p}^i$ $X^{1'} = -\frac{r}{m\alpha} \sqrt{-2mH} \mathbf{r} \cdot \mathbf{p}$ $P^i = \mathbf{p}^i$ $P^{1'} = \frac{1}{\sqrt{-2mH}} (\frac{m\alpha}{r} - \mathbf{p}^2)$	
H-atom, $H > 0$	$X^M = \frac{r \cosh u}{u(t) \equiv \sqrt{\frac{2mH}{\alpha}}(\mathbf{r} \cdot \mathbf{p} - 2mHt)}$ $P^M = \frac{m\alpha}{r \sqrt{2mH}} \sinh u$	$\frac{r}{m\alpha} \sqrt{2mH} \mathbf{r} \cdot \mathbf{p}$ $\frac{1}{\sqrt{2mH}} (\frac{m\alpha}{r} - \mathbf{p}^2)$	$X^1 = \mathbf{r}^i - \frac{\alpha}{m\alpha} \mathbf{r} \cdot \mathbf{p} \mathbf{p}^i$ $X^{1'} = r \sinh u$ $P^i = \mathbf{p}^i$ $P^{1'} = \frac{m\alpha}{r \sqrt{2mH}} \cosh u$	

An example: Massive relativistic particle gauge

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$$X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'}) \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^M = \left(\frac{1-a}{2a}, \frac{x^2 a}{1+a}, x^\mu \right), \quad a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$$

$$P^M = \left(\frac{-m^2}{2(x \cdot p)a}, (x \cdot p)a, p^\mu \right), \quad P^2 = p^2 + m^2 = 0.$$

Note $a=1$
when $m=0$

embed phase space in 3+1 into
phase space in 4+2

Make 2 gauge choices solve 2
constraints $X^2=X, P=0$

τ reparametrization and one
constraint remains.

$$S = \int d\tau \left(\dot{X}^M P^N - \frac{1}{2} A^{ij} X_i^M X_j^N \right) \eta_{MN} = \int d\tau \left(\dot{x}^\mu p_\mu - \frac{1}{2} A^{22} (p^2 + m^2) \right)$$

An example: Massive relativistic particle gauge

13 23

$$X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'}) \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^M = \left(\frac{1+a}{2a}, \frac{x^2 a}{1+a}, x^\mu \right), \quad a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$$

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Make 2 gauge choices solve 2
constraints $X^2=X, P=0$

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constraint remains.

Gauge
invariants

$$S = \int d\tau \left(\dot{X}^M P^N - \frac{1}{2} A^{ij} X_i^M X_j^N \right) \eta_{MN} = \int d\tau \left(\dot{x}^\mu p_\mu - \frac{1}{2} A^{22} (p^2 + m^2) \right)$$

$$L^{MN} = \varepsilon^{ij} X_i^M X_j^N = X^M P^N - X^N P^M$$



$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu, \quad L^{+'-'} = (x \cdot p) a.$$

$$L^{+' \mu} = \frac{1+a}{2a} p^\mu + \frac{m^2}{2(x \cdot p)a} x^\mu$$

$$L^{-' \mu} = \frac{x^2 a}{1+a} p^\mu - (x \cdot p) a x^\mu$$

$X \cdot X = X \cdot P = P \cdot P = 0$

$$ds^2 = dX^M dX^N \eta_{MN} = -2dX^+ dX^- + dX^\mu dX^\nu \eta_{\mu\nu} \quad X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'})$$

Relativistic massless particle $p^2 = 0$	$X^M = 1$ $P^M = 0$	$X^{+'}(\tau)$ $P^{+'}(\tau)$	$\frac{1}{2}x^2$ $x \cdot p$	$X^{-'}(\tau)$ $P^{-'}(\tau)$	x^μ p^μ	$X^{\mu'}(\tau)$ $P^{\mu'}(\tau)$	Make 2 gauge choices. Solve 2 constraints. Remains 1 gauge choice and 1 constraint
Relativistic massive particle $p^2 + m^2 = 0$	$X^M = \frac{1+a}{2a}$ $P^M = \frac{-m^2}{2ax \cdot p}$		$\frac{x^2 a}{1+a}$ $a x \cdot p$		x^μ p^μ	$a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$	
Non-relativistic massive particle $H - \frac{p^2}{2m} = 0$	$X^M = t$ $P^M = m$		$\frac{\mathbf{r} \cdot \mathbf{p} - tH}{m}$ H		$X^0 = \pm \mathbf{r} - \frac{t}{m}\mathbf{p} $ $X^i = \mathbf{r}^i$ $P^0 = 0, P^i = \mathbf{p}^i$		

Mass : modulus in the embedding of d-dimensional phase space into d+2.

Maximally Symmetric Spaces $p^2 - \frac{K (x \cdot p)^2}{1 - K x^2} = 0$	$X^M = 1 + \sqrt{1 - K x^2}$ $P^M = 0$	$\frac{x^2/2}{1 + \sqrt{1 - K x^2}}$ $\frac{\sqrt{1 - K x^2}}{1 + \sqrt{1 - K x^2}} x \cdot p$	x^μ $p^\mu - \frac{K x \cdot p x^\mu}{1 + \sqrt{1 - K x^2}}$	$g_{\mu\nu} = \eta_{\mu\nu} + \frac{K}{1 - K x^2} x_\mu x_\nu$ curvature
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Time and Hamiltonian are different in each case <-> 1T perspectives in 2T phase space. All can be gauge transformed to each other. There are gauge invariant relations among them. (info absent in 1T-physics)

Any function of $L_{MN} - Y_{MPN}, Y_{NPM}$ is gauge invariant

Gauge choice	M	θ' $X^{0'}$	θ X^0	$I = (I', t)$ $X^{I'} X^I$
Robertson-Walker $r < R_0$ (closed universe) $-H^2 + \frac{R_0^2}{a^2(t)} (\mathbf{p}^2 - \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2}) = 0$	$X^M = a(t) \cos(\int^t \frac{dt'}{a(t')})$ $P^M = -H \sin(\int^t \frac{dt'}{a(t')})$	$a(t) \sin(\int^t \frac{dt'}{a(t')})$ $H \cos(\int^t \frac{dt'}{a(t')})$	$X^I = \mathbf{r}^I a(t) / R_0$ $X^{I'} = \pm a(t) \sqrt{1 - \frac{r^2}{R_0^2}}$ $P^I = \frac{R_0}{a(t)} (\mathbf{p}^I - \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2} \mathbf{r}^I)$ $P^{I'} = \pm \frac{\mathbf{r} \cdot \mathbf{p}}{a(t)} \sqrt{1 - \frac{r^2}{R_0^2}}$	
Robertson-Walker $r > 0$ (open universe) $-H^2 + \frac{R_0^2}{a^2(t)} (\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2}) = 0$	$X^M = a(t) \sinh(\int^t \frac{dt'}{a(t')})$ $P^M = \pm H \cosh(\int^t \frac{dt'}{a(t')})$	$(\pm)' a(t) \sqrt{1 + \frac{r^2}{R_0^2}}$ $(\pm)' \frac{\mathbf{r} \cdot \mathbf{p}}{a(t)} \sqrt{1 + \frac{r^2}{R_0^2}}$	$X^I = \mathbf{r}^I a(t) / R_0$ $X^{I'} = \pm a(t) \cosh(\int^t \frac{dt'}{a(t')})$ $P^I = \frac{R_0}{a(t)} (\mathbf{p}^I + \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2} \mathbf{r}^I)$ $P^{I'} = H \sinh(\int^t \frac{dt'}{a(t')})$	
Cosmological constant $\Lambda \equiv \frac{3}{R_0^2} > 0$ $-H^2(1 - \frac{r^2}{R_0^2}) + (\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 - r^2}) = 0$	$X^M = \sqrt{R_0^2 - r^2} \sinh \frac{t}{R_0}$ $P^M = \pm \frac{H}{R_0} \sqrt{R_0^2 - r^2} \cosh \frac{t}{R_0}$	R_0 $\frac{R_0 \mathbf{r} \cdot \mathbf{p}}{R_0^2 - r^2}$	$X^I = \mathbf{r}^I$ $X^{I'} = \pm \sqrt{R_0^2 - r^2} \cosh \frac{t}{R_0}$ $P^I = \mathbf{p}^I + \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2 - r^2} \mathbf{r}^I$ $P^{I'} = \pm \frac{H}{R_0} \sqrt{R_0^2 - r^2} \sinh \frac{t}{R_0}$	
Cosmological constant $\Lambda \equiv -\frac{3}{R_0^2} < 0$ $-H^2(1 + \frac{r^2}{R_0^2}) + (\mathbf{p}^2 - \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 + r^2}) = 0$	$X^M = \sqrt{R_0^2 + r^2} \sin \frac{t}{R_0}$ $P^M = \pm \frac{H}{R_0} \sqrt{R_0^2 + r^2} \cos \frac{t}{R_0}$	$\pm \sqrt{R_0^2 + r^2} \cos \frac{t}{R_0}$ $\frac{H}{R_0} \sqrt{R_0^2 + r^2} \sin \frac{t}{R_0}$	$X^I = \mathbf{r}^I$ $X^{I'} = R_0$ $P^I = \mathbf{p}^I - \frac{\mathbf{r} \cdot \mathbf{p}}{R_0^2 + r^2} \mathbf{r}^I$ $P^{I'} = -\frac{R_0 \mathbf{r} \cdot \mathbf{p}}{R_0^2 + r^2}$	
(d-1)-sphere \times time $-H^2 + (\mathbf{p}^2 + \frac{(\mathbf{r} \cdot \mathbf{p})^2}{R_0^2 - r^2}) = 0$	$X^M = R_0 \cos \frac{t}{R_0}$ $P^M = -H \sin \frac{t}{R_0}$	$R_0 \sin \frac{t}{R_0}$ $H \cos \frac{t}{R_0}$	$R_0 \hat{n}^I = \frac{X^I = \mathbf{r}^I}{X^{I'} = \pm \sqrt{R_0^2 - r^2}}$ $P^I = \mathbf{p}^I$ $P^{I'} = \pm \frac{\mathbf{r} \cdot \mathbf{p}}{\sqrt{R_0^2 - r^2}}$	
H-atom, $H < 0$ $H = \frac{\mathbf{p}^2}{2m} - \frac{\alpha}{r}$	$X^M = \frac{r \cos u}{u(t) \equiv \sqrt{\frac{-2mH}{m\alpha}} (\mathbf{r} \cdot \mathbf{p} - 2mHt)}$ $P^M = -\frac{m\alpha}{r \sqrt{-2mH}} \sin u$	$r \sin u$ $\frac{m\alpha}{r \sqrt{-2mH}} \cos u$	$X^I = \mathbf{r}^I - \frac{\alpha}{m\alpha} \mathbf{r} \cdot \mathbf{p} \frac{\mathbf{p}^I}{\sqrt{-2mH}}$ $X^{I'} = -\frac{\alpha}{m\alpha} \sqrt{-2mH} \mathbf{r} \cdot \mathbf{p}$ $P^I = \mathbf{p}^I$ $P^{I'} = \frac{1}{\sqrt{-2mH}} (\frac{m\alpha}{r} - \mathbf{p}^2)$	
H-atom, $H > 0$	$X^M = \frac{r \cosh u}{u(t) \equiv \sqrt{\frac{2mH}{m\alpha}} (\mathbf{r} \cdot \mathbf{p} - 2mHt)}$ $P^M = \frac{m\alpha}{r \sqrt{2mH}} \sinh u$	$\frac{\alpha}{m\alpha} \sqrt{2mH} \mathbf{r} \cdot \mathbf{p}$ $\frac{1}{\sqrt{2mH}} (\frac{m\alpha}{r} - \mathbf{p}^2)$	$X^I = \mathbf{r}^I - \frac{\alpha}{m\alpha} \mathbf{r} \cdot \mathbf{p} \frac{\mathbf{p}^I}{\sqrt{2mH}}$ $X^{I'} = r \sinh u$ $P^I = \mathbf{p}^I$ $P^{I'} = \frac{m\alpha}{r \sqrt{2mH}} \cosh u$	

An example: Massive relativistic particle gauge

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$$X^{\pm'} = \frac{1}{\sqrt{2}} (X^{0'} \pm X^{1'}) \quad ds^2 = dX^M dX^N \eta_{MN} = -2dX^{+'} dX^{-'} + dX^\mu dX^\nu \eta_{\mu\nu}$$

$$X^M = \left(\frac{1+a}{2a}, \frac{x^2 a}{1+a}, x^\mu \right), \quad a \equiv \sqrt{1 + \frac{m^2 x^2}{(x \cdot p)^2}}$$

$$P^M = \left(\frac{-m^2}{2(x \cdot p)a}, (x \cdot p)a, p^\mu \right), \quad P^2 = p^2 + m^2 = 0.$$

Note $a=1$
when $m=0$

embed phase space in 3+1 into
phase space in 4+2

Make 2 gauge choices solve 2
constraints $X^2=X, P=0$

τ reparametrization and one
constraint remains.

Gauge
invariants

$$S = \int d\tau \left(\dot{X}^M P^N - \frac{1}{2} A^{ij} X_i^M X_j^N \right) \eta_{MN} = \int d\tau \left(\dot{x}^\mu p_\mu - \frac{1}{2} A^{22} (p^2 + m^2) \right)$$

$$L^{MN} = \varepsilon^{ij} X_i^M X_j^N = X^M P^N - X^N P^M$$

$$\delta X^\mu = \omega_{MN} \{L^{MN}, X^\mu\}, \quad \delta p^\mu = \omega_{MN} \{L^{MN}, p^\mu\},$$



$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu, \quad L^{+'-'} = (x \cdot p) a.$$

$$L^{+' \mu} = \frac{1+a}{2a} p^\mu - \frac{m^2}{2(x \cdot p)a} x^\mu$$

$$L^{-' \mu} = \frac{x^2 a}{1+a} p^\mu - (x \cdot p) a x^\mu$$

$so(d,2)$ is hidden symmetry of the massive
action. Looks like conformal transformations deformed
by mass. The symmetry in 1T theory is a clue of extra
1+1 dimensions, including 2T.

Shadows from 2T-physics → hidden info in 1T-physics¹⁴

Shadows from 2T-physics → hidden info in 1T-physics¹⁴

2T-physics

$Sp(2, R)$ gauge symm.
generators $Q_{ij}(X, P)$ vanish

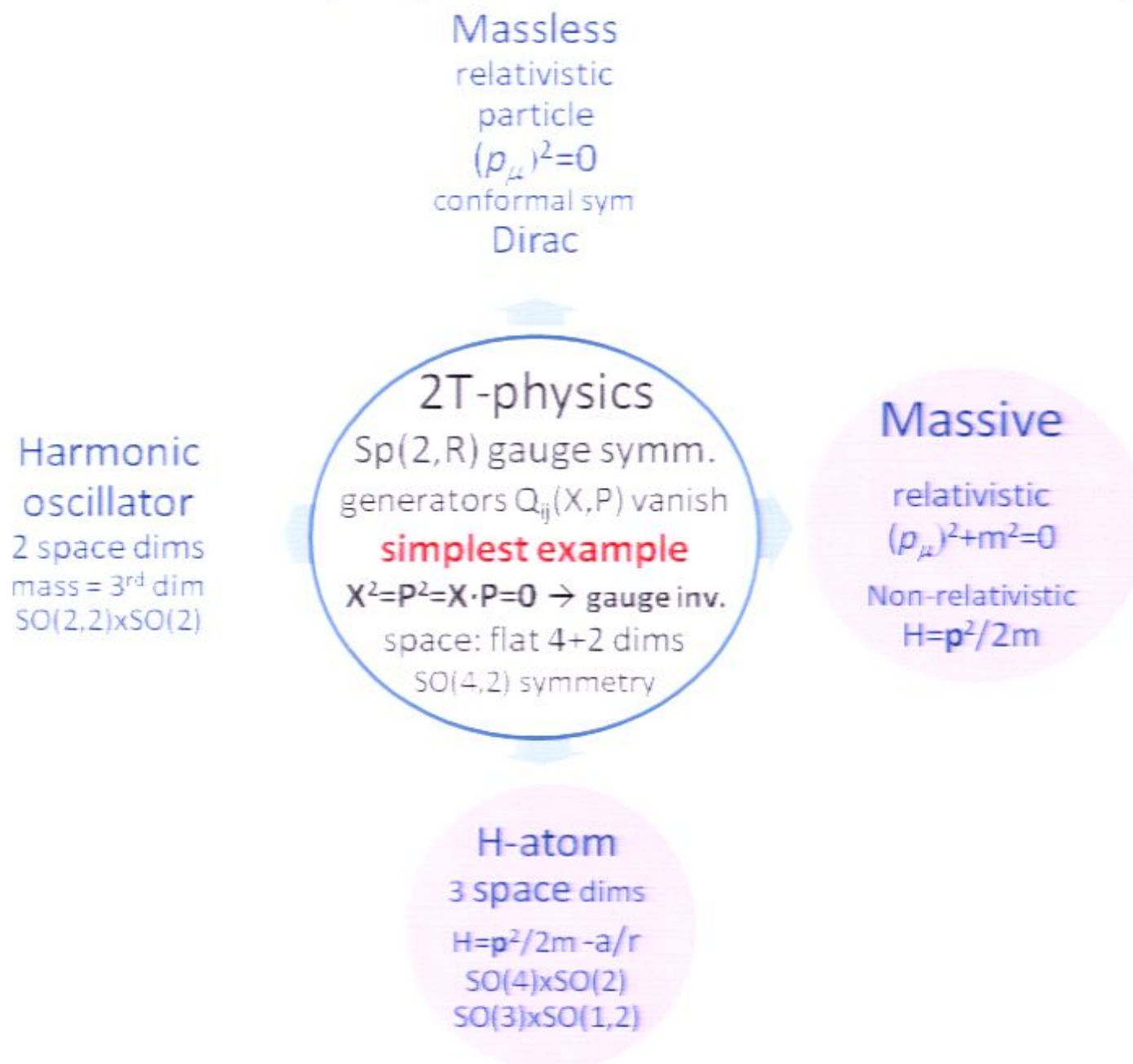
simplest example

$X^2 = P^2 = X \cdot P = 0 \rightarrow$ gauge inv.

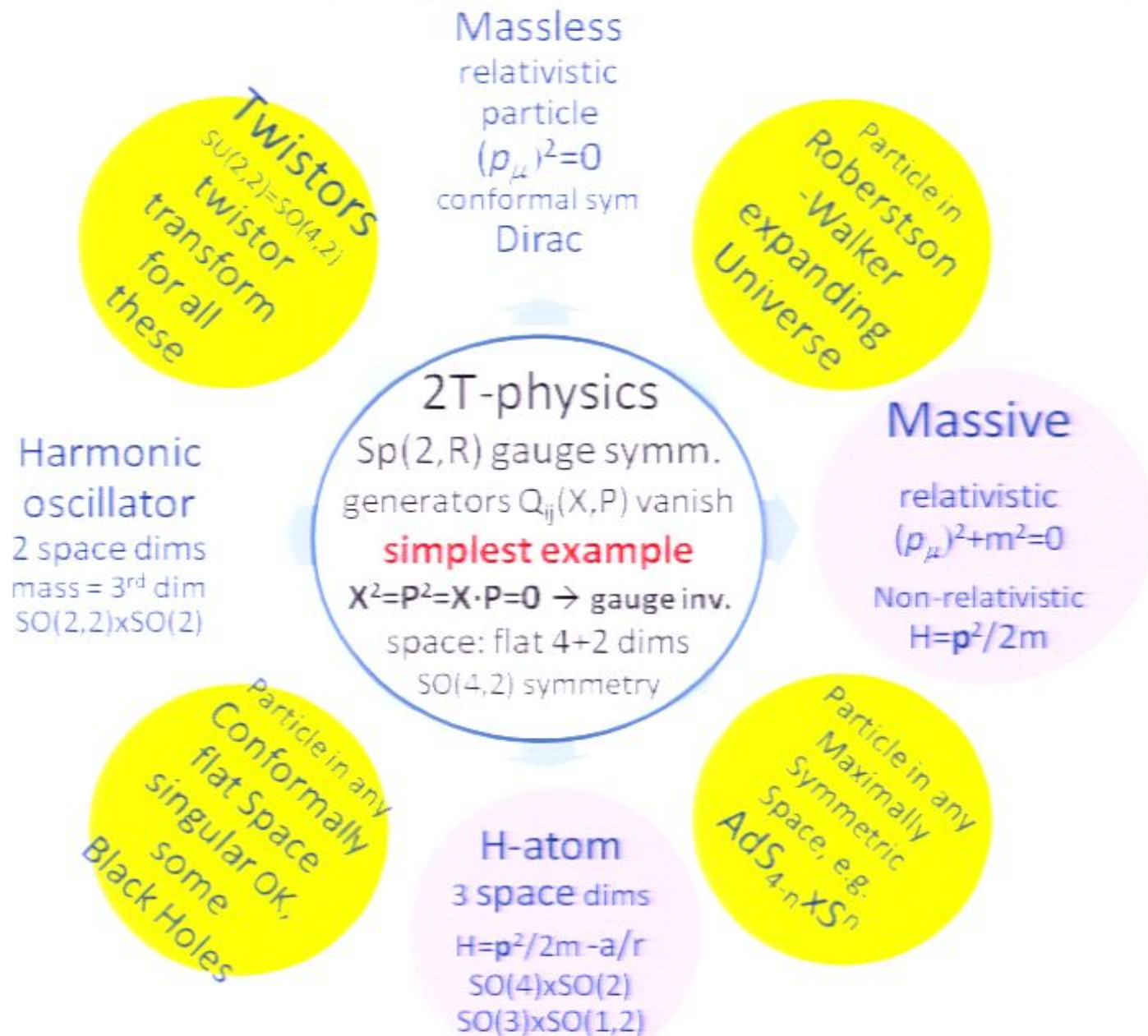
space: flat 4+2 dims

$SO(4, 2)$ symmetry

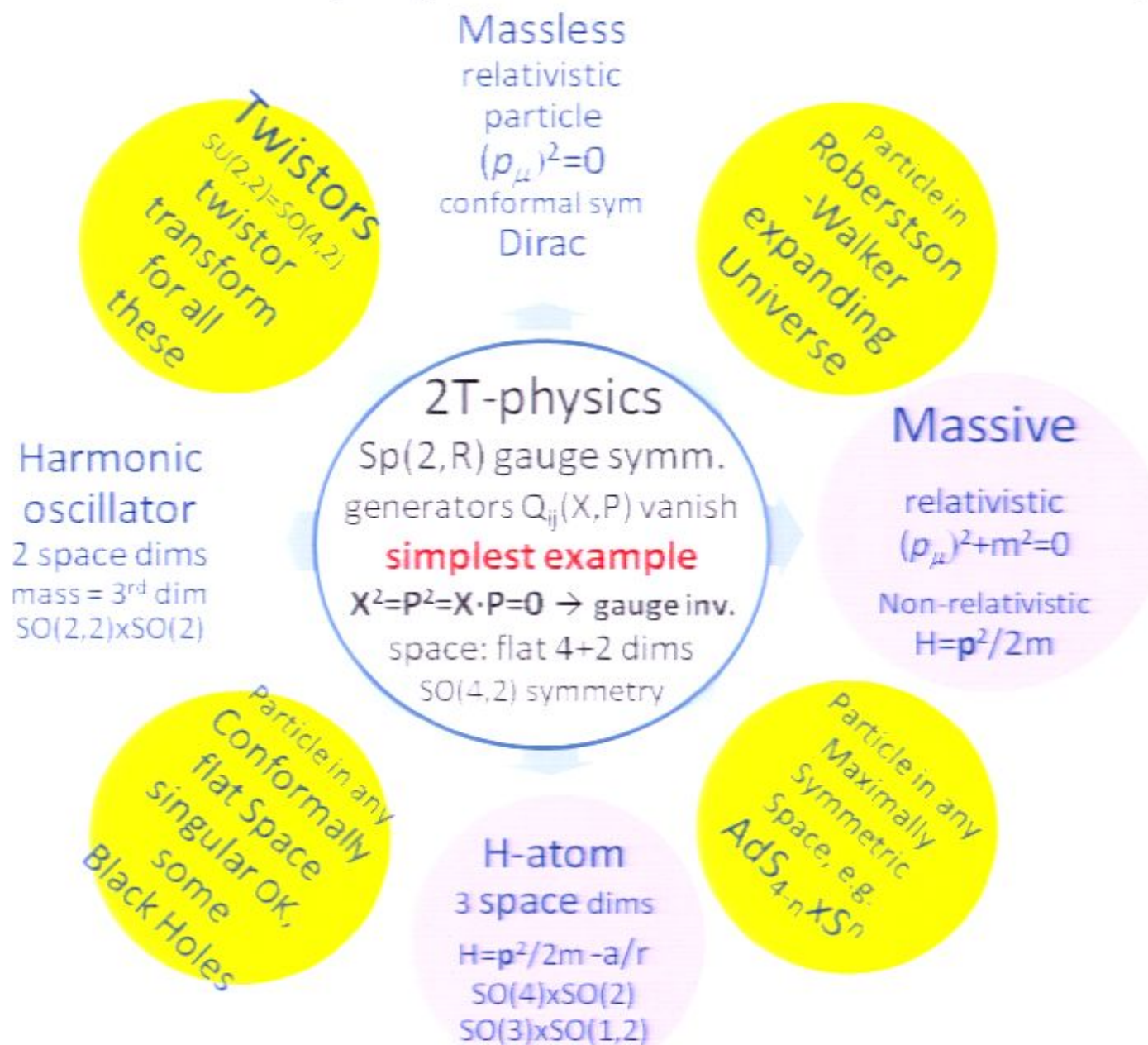
Shadows from 2T-physics → hidden info in 1T-physics¹⁴



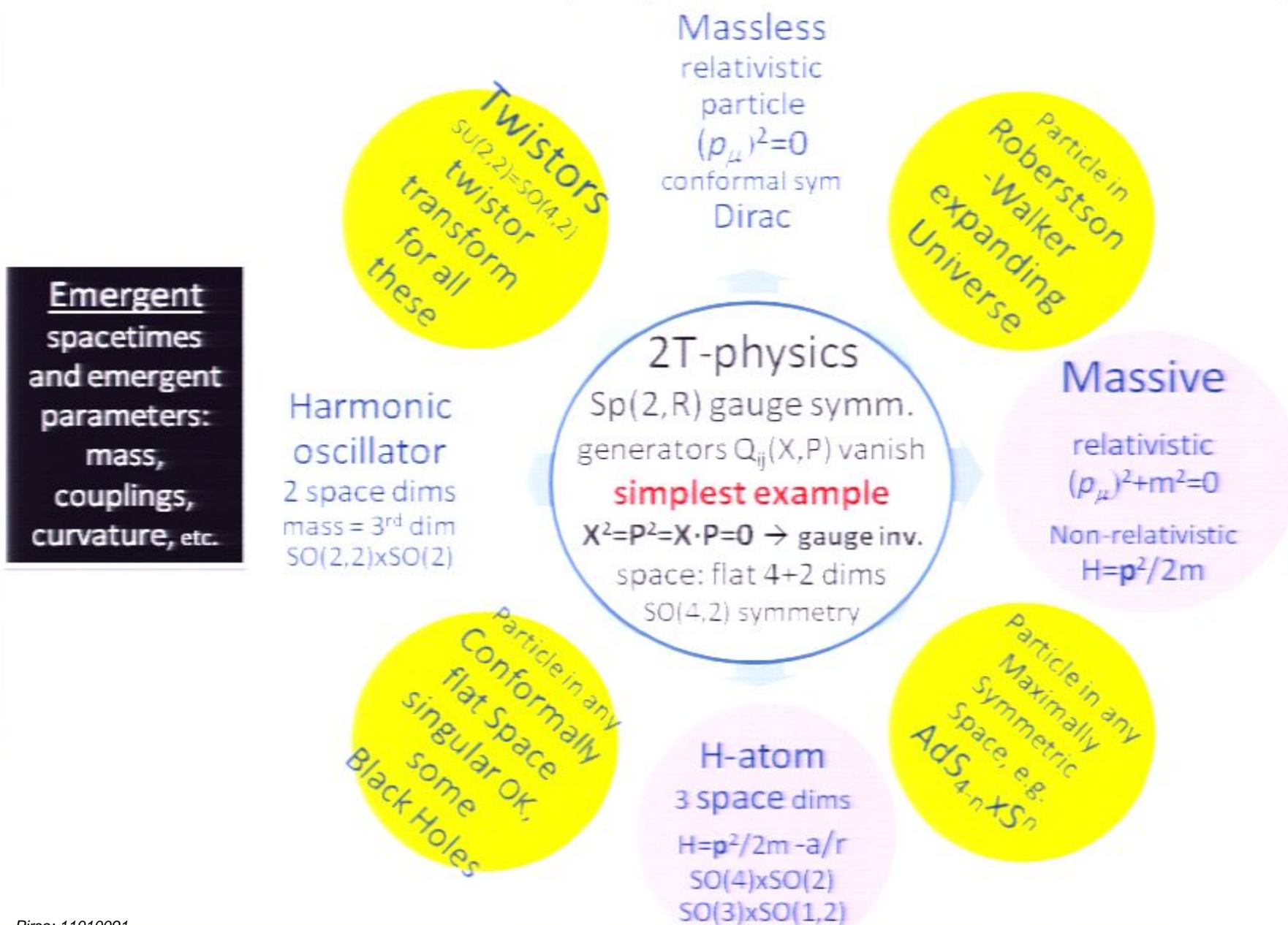
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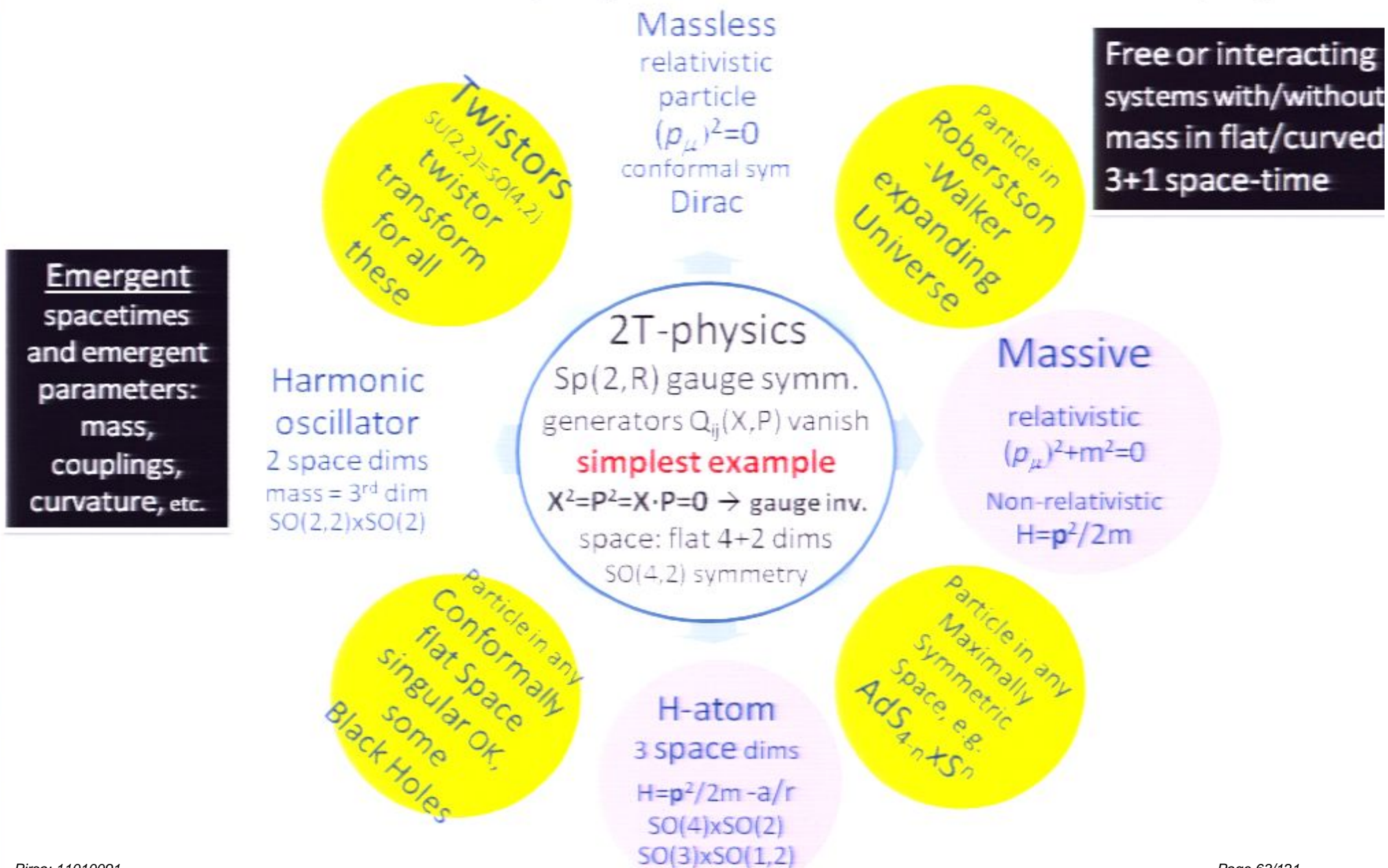
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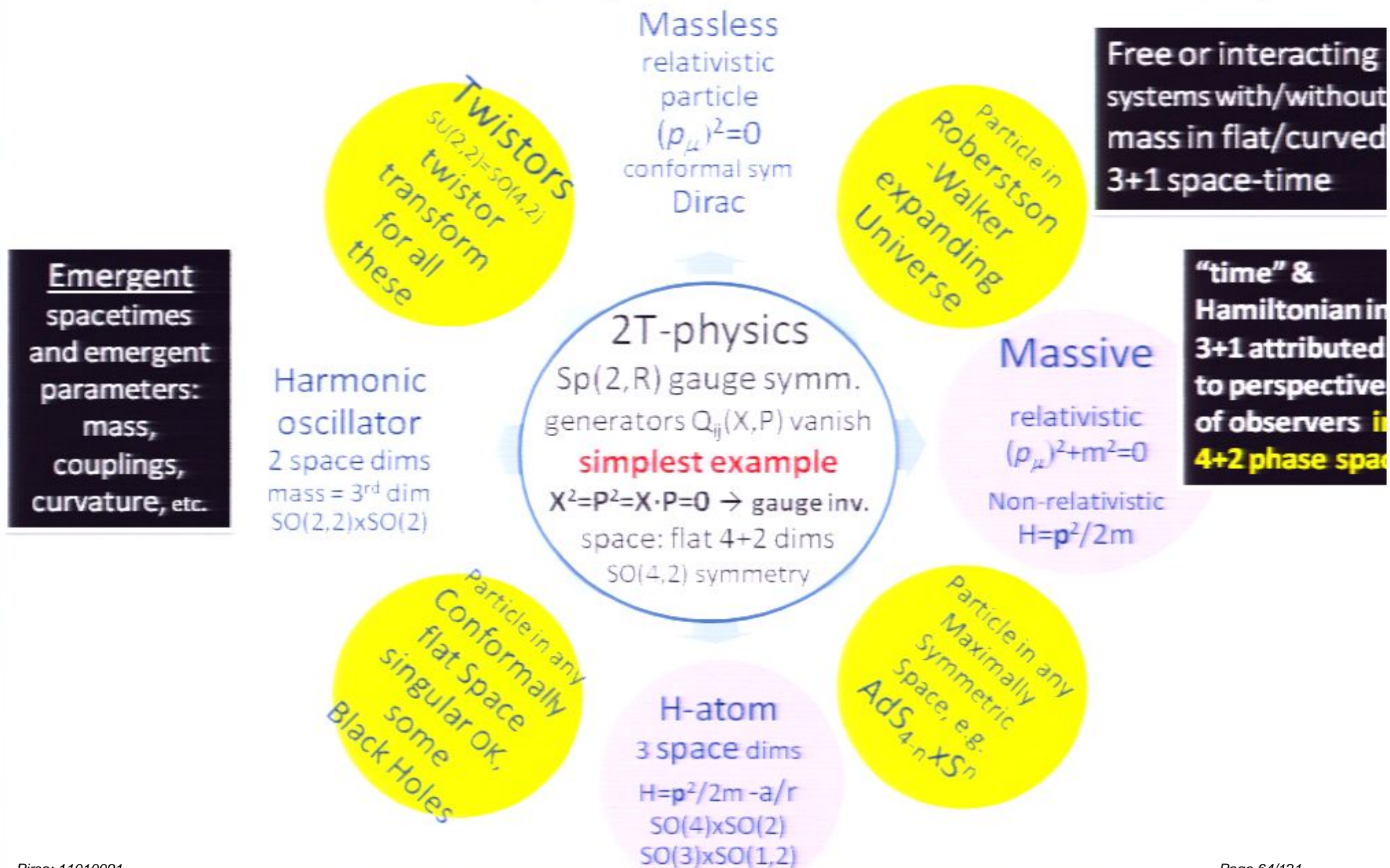
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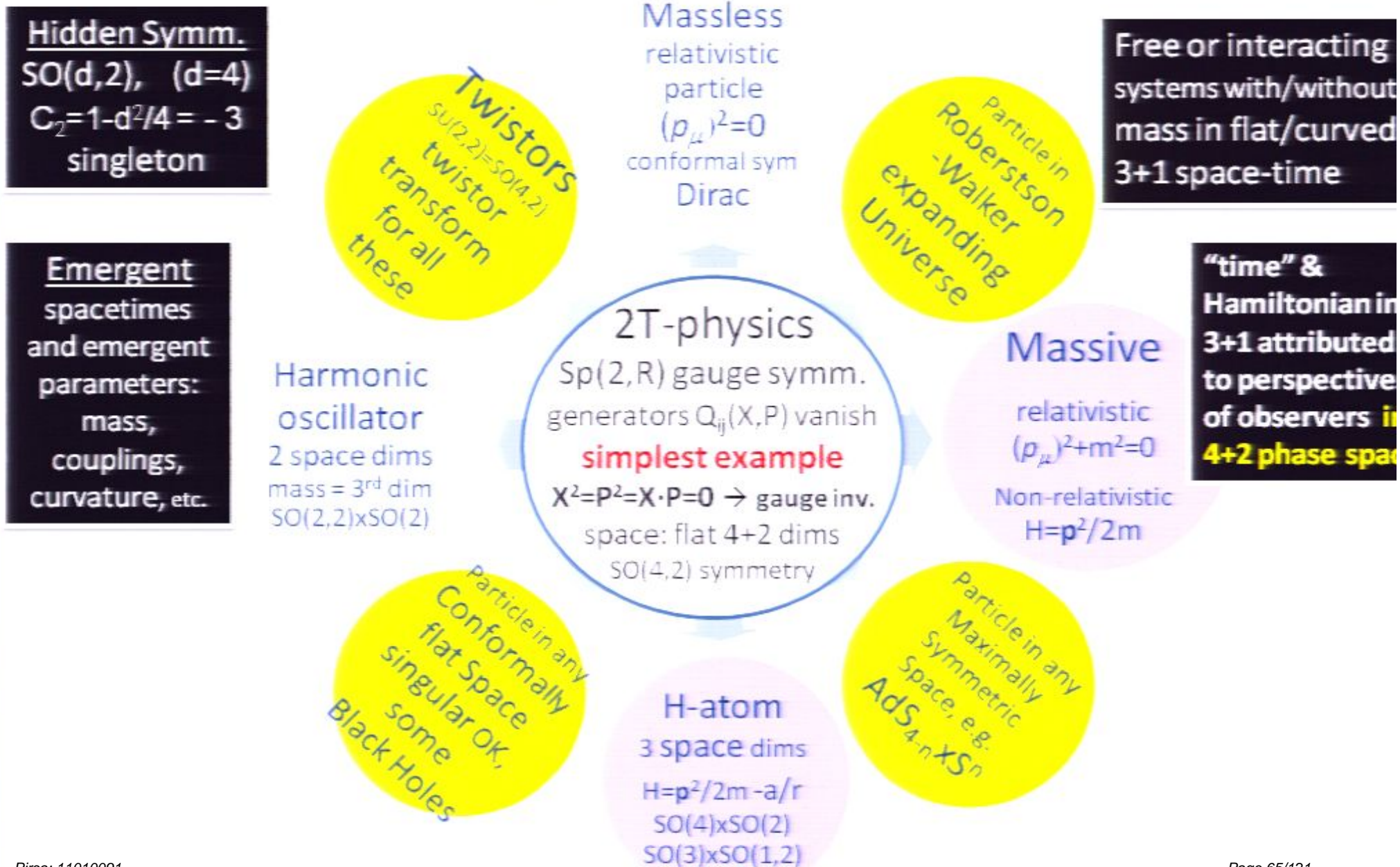
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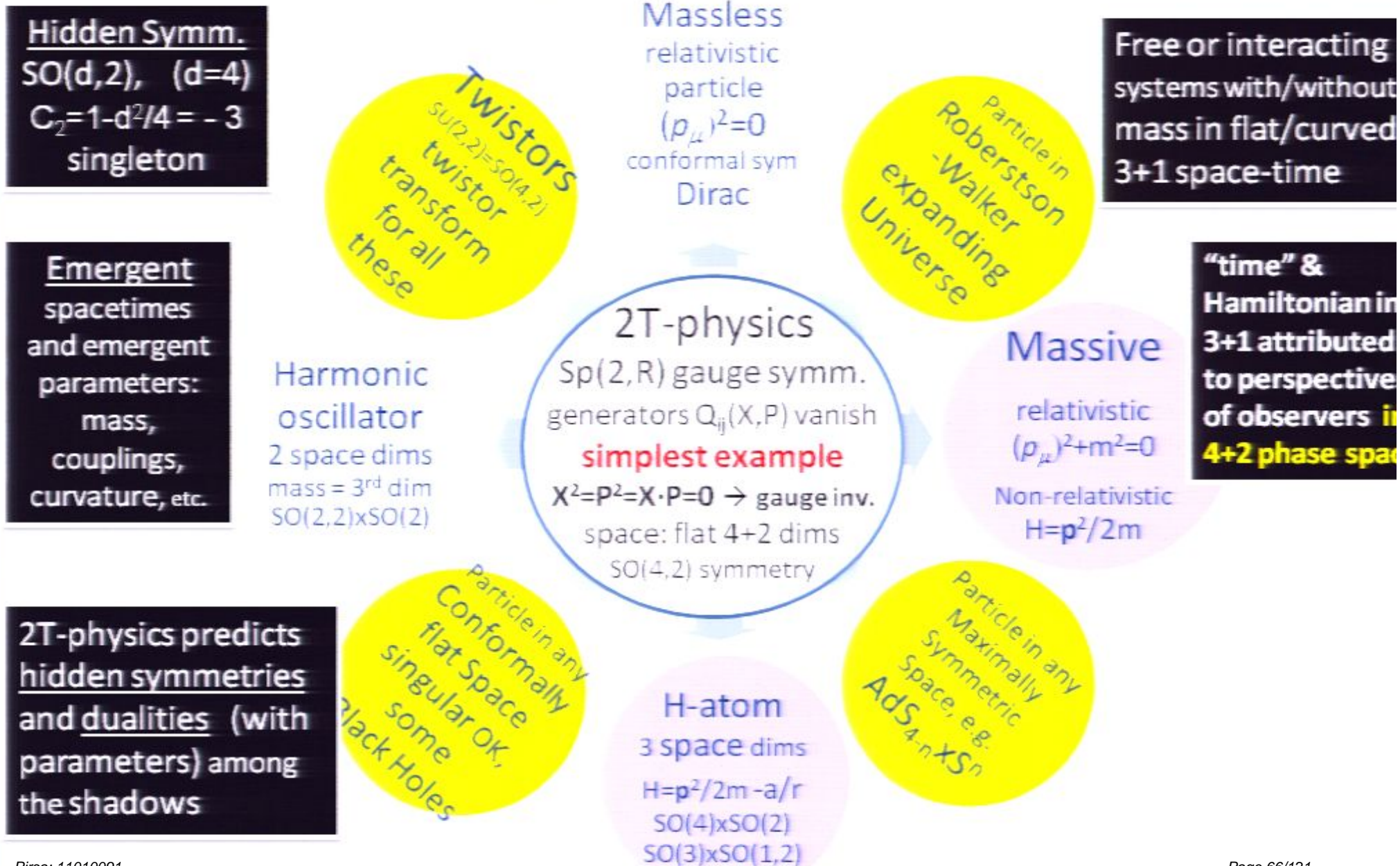
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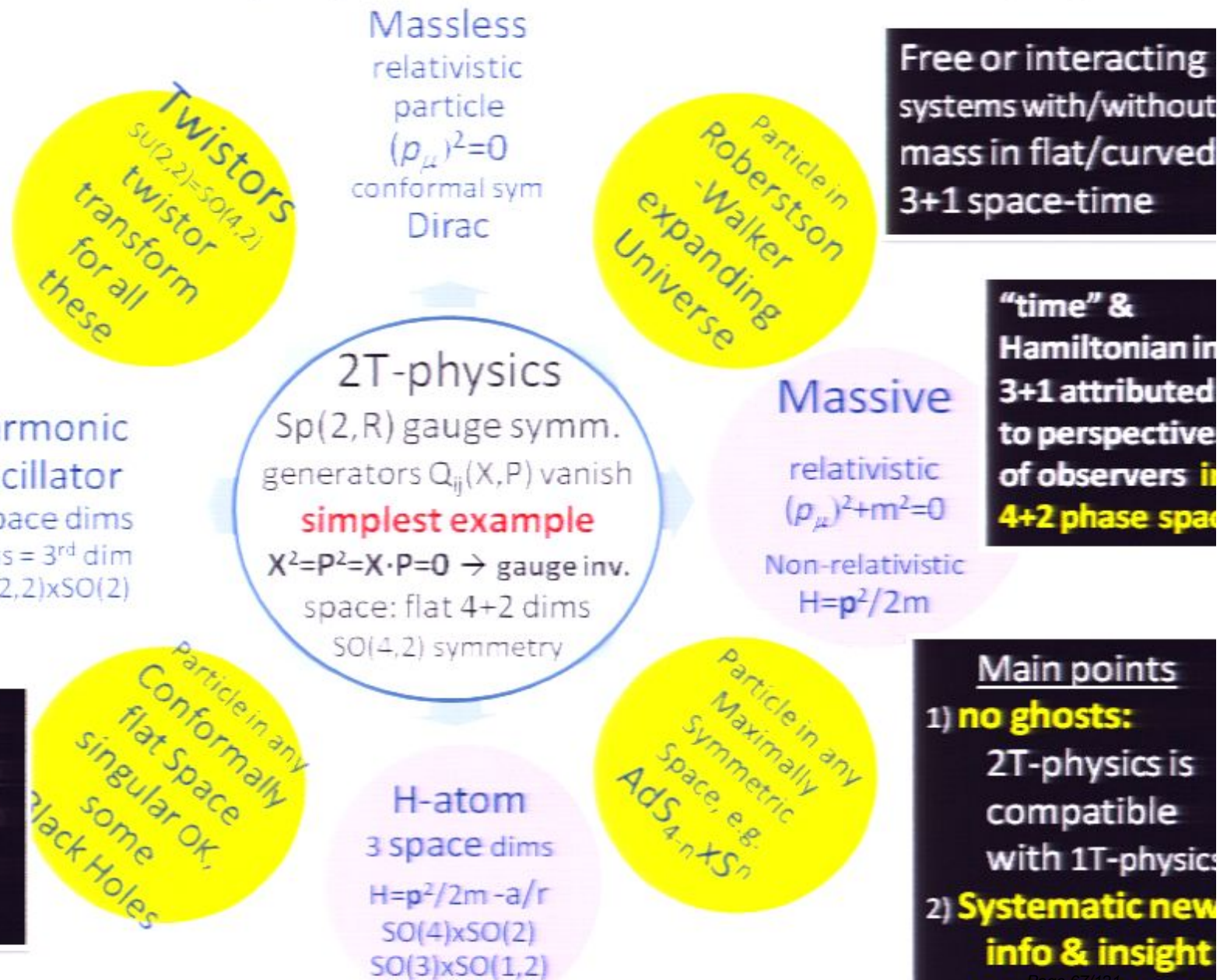


Shadows from 2T-physics → hidden info in 1T-physics ¹⁴

Hidden Symm.
 $SO(d,2)$, ($d=4$)
 $C_2=1-d^2/4 = -3$
 singleton

Emergent
 spacetimes
 and emergent
 parameters:
 mass,
 couplings,
 curvature, etc.

2T-physics predicts
hidden symmetries
 and dualities (with
 parameters) among
 the shadows



How shadows might be used in
“AdS-CFT” type correspondence

15 03

How shadows might be used in “AdS-CFT” type correspondence

15 03

Shadows from
Flat 4+2, all
have $SO(4,2)$ in
the same repr.



Flat 3+1
Robertson-Walker 3+1
 AdS_4
 $AdS_3 \times S^1$
 $AdS_2 \times S^2$
 $R \times S^3$
deSitter₄
All conf. flat spacetimes 4D
H-atom₃₊₁
Massive particles₃₊₁
Etc.

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The usual case
 AdS_5

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All conf. flat spacetimes 4D
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How shadows might be used in “AdS-CFT” type correspondence

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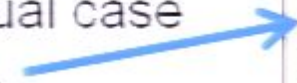


Flat 4+1
Robertson-Walker 4+1
 AdS_5
 $AdS_4 \times S^1$
 $AdS_3 \times S^2$
 $AdS_4 \times S^1$
 $R \times S^4$
 $deSitter_5$
All conf. flat in 5D
 $H\text{-atom}_{4+1}$
 $Massive\ particles_{4+1}$
Etc.

Shadows from Flat 4+2, all have $SO(4,2)$ in the same repr.



The usual case
 AdS_5



Flat 3+1
Robertson-Walker 3+1
 AdS_4
 $AdS_3 \times S^1$
 $AdS_2 \times S^2$
 $R \times S^3$
 $deSitter_4$
All conf. flat spacetimes 4D
 $H\text{-atom}_{3+1}$
 $Massive\ particles_{3+1}$
Etc.

How shadows might be used in “AdS-CFT” type correspondence

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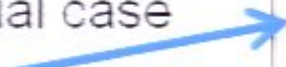
Flat 4+1
Robertson-Walker 4+1
 AdS_5
 $AdS_4 \times S^1$
 $AdS_3 \times S^2$
 $AdS_4 \times S^1$
 $R \times S^4$
deSitter₅
All conf. flat in 5D
H-atom₄₊₁
Massive particles₄₊₁
Etc.

Shadows from Flat 4+2, all have $SO(4,2)$ in the same repr.



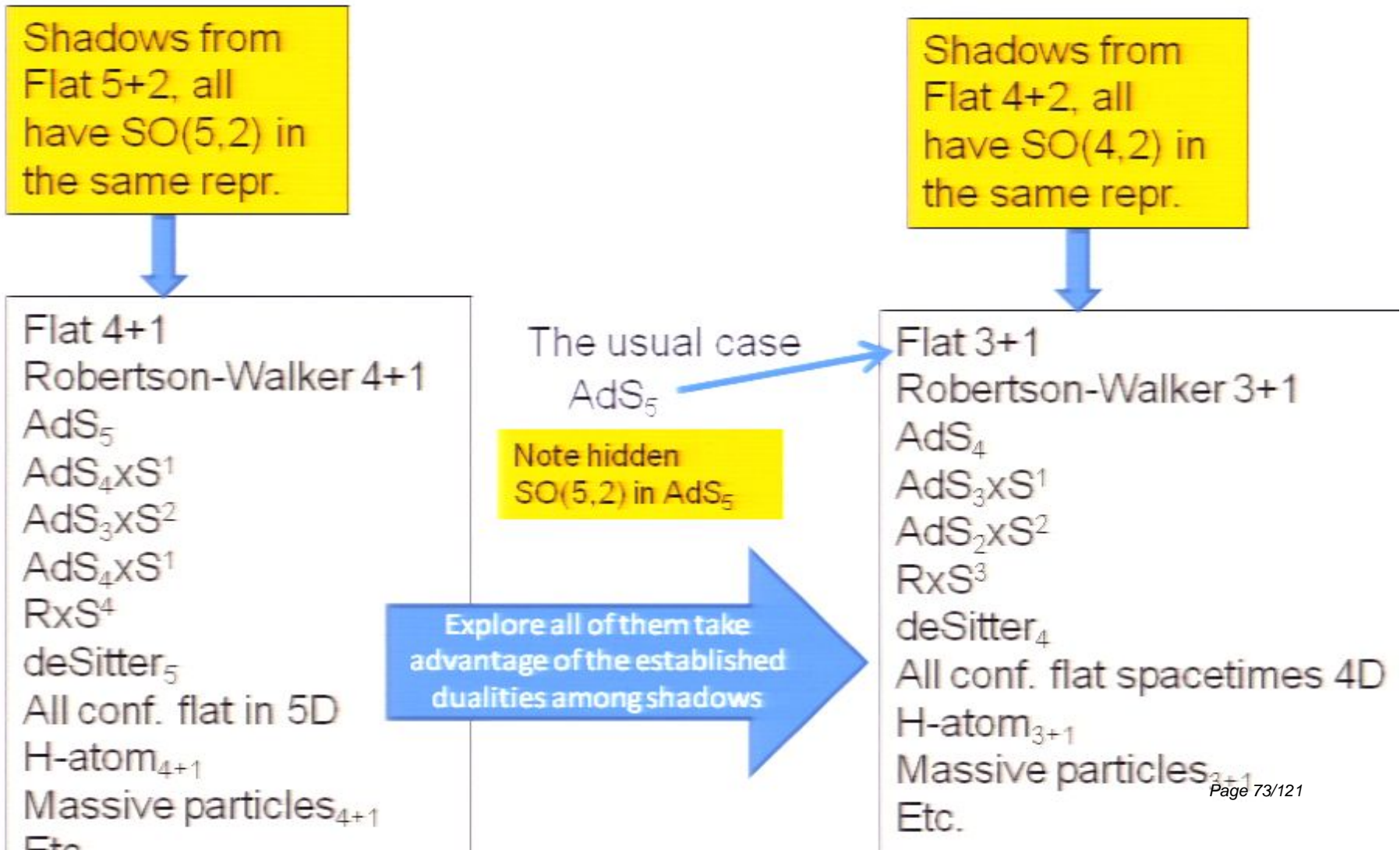
Flat 3+1
Robertson-Walker 3+1
 AdS_4
 $AdS_3 \times S^1$
 $AdS_2 \times S^2$
 $R \times S^3$
deSitter₄
All conf. flat spacetimes 4D
H-atom₃₊₁
Massive particles₃₊₁
Etc.

The usual case
 AdS_5

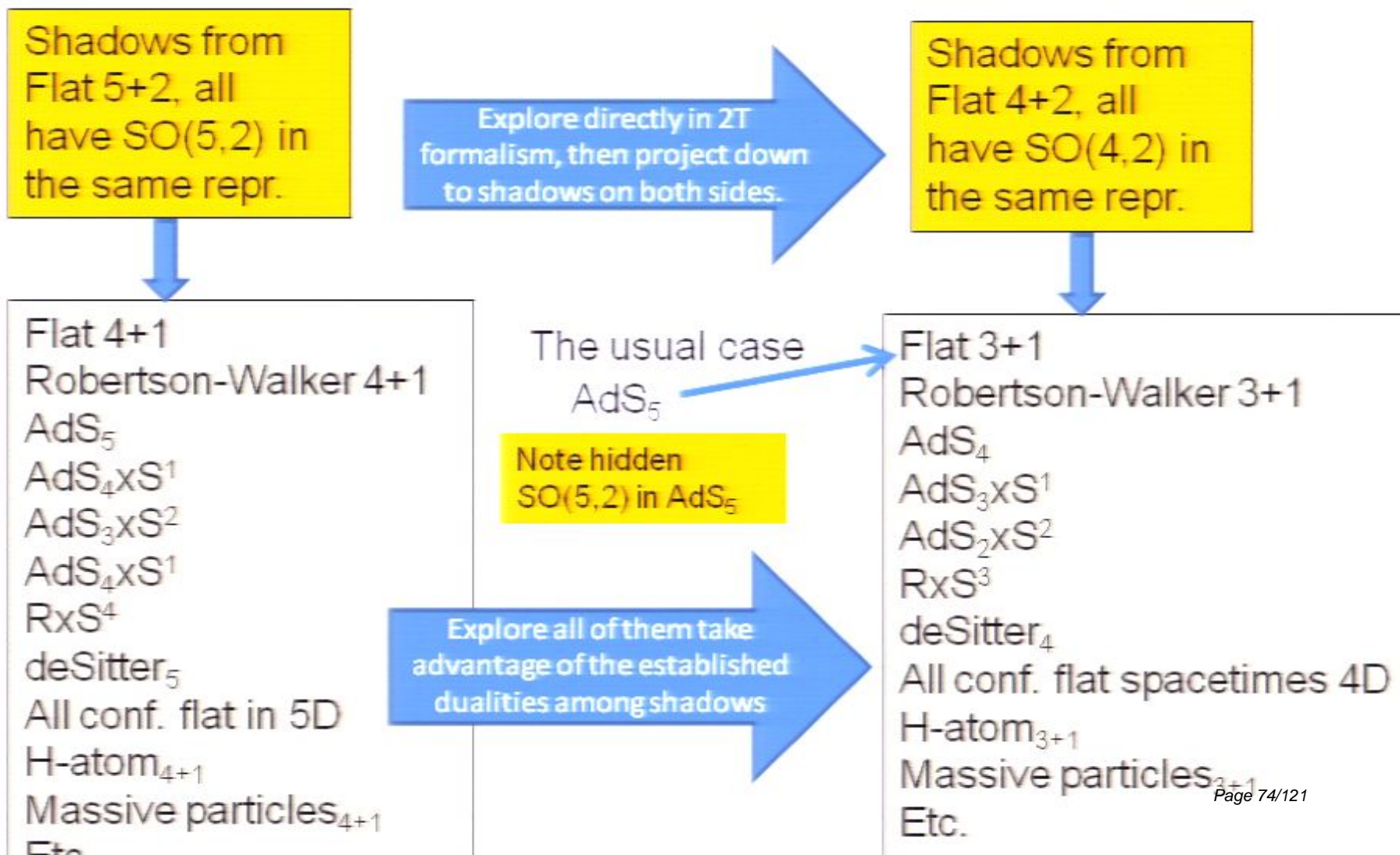


Note hidden
 $SO(5,2)$ in AdS_5

How shadows might be used in “AdS-CFT” type correspondence



How shadows might be used in “AdS-CFT” type correspondence



2T Field Theory, including interactions.

- Physical states $Q_{ij}(X, \partial)\Phi(X)=0$ -- field eoms.
Similar to Virasoro constraints in string theory.
What is the action that generates these as eoms through the variational principle?

Gravity as background in 2T-physics

0804.1585 [hep-th]

$$S = \int d\tau (\partial_\tau X^M P_M(\tau) - \frac{1}{2} A^{\mu\nu}(\tau) Q_{\mu\nu}(X(\tau), P(\tau)))$$

$$Q_{11} = W(X), \quad Q_{12} = V^M(X) P_M, \quad Q_{22} = G^{MN}(X) P_M P_N$$

Compare to
flat case

$$Q_{11}^{flat} = X \cdot X, \quad Q_{12}^{flat} = X \cdot P, \quad Q_{22}^{flat} = P \cdot P,$$

$$W_{flat}(X) = X \cdot X, \quad V_{flat}^M(X) = X^M, \quad G_{flat}^{MN}(X) = \eta^{MN}$$

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$$\{Q_{11}, Q_{22}\} = 4Q_{12} - V^M = \frac{1}{2} G^{MN} \partial_N W, \quad \{A, B\} \equiv \frac{\partial A}{\partial X^M} \frac{\partial B}{\partial P_M} + \frac{\partial A}{\partial P_M} \frac{\partial B}{\partial X^M}$$

$$\{Q_{11}, Q_{12}\} = 2Q_{11} - V^M \partial_M W = 2W,$$

$$\{Q_{22}, Q_{12}\} = -2Q_{22} - \mathcal{L}_V G^{MN} = -2G^{MN}$$

Sp(2,R) algebra puts
kinematic constraints on
background geometry

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Sp(2,R) algebra puts
kinematic constraints on
background geometry

$$\begin{aligned} -2G^{MN} &= V^K \partial_K G^{MN} - \partial_K V^M G^{KN} - \partial_K V^N G^{MK} \\ &= -\nabla^M V^N - \nabla^N V^M \equiv \mathcal{L}_V G^{MN} \end{aligned}$$

$$G_{MN} = \nabla_M V_N = \frac{1}{2} \nabla_M \partial_N W$$

Solve kinematics, and
impose $Q_{11}=Q_{12}=0$:

Get all shadows, e.g.

$$S = \int d\tau (\partial_\tau x^\mu p_\mu(\tau) - \frac{1}{2} A^{22}(\tau) g^{\mu\nu}(x(\tau)) p_\mu(\tau) p_\nu(\tau))$$

Rules for 2T field theory, spins=0, $\frac{1}{2}$, 1
Impose $Sp(2, R)$ singlet condition in flat space !!

18 23

Rules for 2T field theory, spins=0, $\frac{1}{2}$, 1
Impose $Sp(2, R)$ singlet condition in flat space !!

Use BRST approach for $Sp(2, R)$. Like string field theory:

I.B.+Kuo, hep-th/0605267
I.B. hep-th/060645

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 I.B. hep-th/060645

Flat
space

$$S_{kin} = \int d^{d+2}X \delta(X^2) \left[\begin{array}{l} \frac{1}{2} \bar{\Phi} D^2 \Phi + \frac{i}{2} \bar{\Psi} X \mathcal{D} \Psi + h.c. \\ -\frac{1}{4} F_{MN} F^{MN} \Omega^{\frac{2(d-4)}{d-2}} \end{array} \right], \quad \Omega \text{ is dilaton}$$

Rules for 2T field theory, spins=0, 1/2, 1

Impose Sp(2,R) singlet condition in flat space !!

18/28

Use BRST approach for Sp(2,R). Like string field theory: I.B.+Kuo, hep-th/0605267
I.B. hep-th/060645

Flat
space

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There is explicit X^M , no translation invariance, only **SO(d,2) spacetime invariance**. This SO(d,2) becomes conformal symmetry in the "conformal shadow", but a hidden SO(d,2) symmetry in other shadows.

$$S_{yukawa} = \int d^{d+2}X \delta(X^2) \Omega^{-\frac{d-4}{d-2}} \left[\bar{\Psi}_L X \Psi_R \right] \Phi + h.c., \quad \Psi_{LR} \text{ spinors of SO}(d,2)$$

Double the size
spinor as SO(d-1,1)
+ Fermionic gauge sym

$$S_{scalars} = \int d^{d+2}X \delta(X^2) V(\Omega, \Phi), \quad V(\Omega, H) = \Omega^{\frac{2d}{d-2}} V\left(1, \frac{\Phi}{\Omega}\right)$$

Homogeneous $V(\Omega, \Phi)$
Only dimensionless
couplings among scalars

$$S_{anomalies} \sim \int d^{d+2}X \delta(X^2) \varepsilon^{M_1 M_2 M_3 \dots M_{d+2}} (X_{M_1} \hat{c}_{M_2} \ln \Omega) (A_{M_3 \dots M_{d+2}})$$

$$X = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}, \quad \Omega = \frac{1}{2} (x^2 + y^2 + z^2 + w^2)$$

Two sets of eqs:
 $P + \bar{P} = 0$

Dimensionless eqs:
 $X = 0, \quad X \cdot P + \bar{X} \cdot \bar{P} = 0$

Minimizing the action gives two
equations to get all 3 Sp(2,R)
constraints for each field,
including interaction !!

Gravity & SM in 2T-physics Field Theory

Gauge symmetry and consistency with $Sp(2, \mathbb{R})$ lead to a unique gravity action in $d+2$ dims, with **no parameters at all**.

Gravity & SM in 2T-physics Field Theory

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Pure gravity has three fields:

$G_{MN}(X)$, metric

$\Omega(X)$, dilaton

$W(X)$, appears in $\delta(W)$, and ...

$$S^0 = \gamma \int d^{d+2}X \sqrt{G} \left\{ \begin{array}{l} \delta(W) \left[\Omega^2 R(G) + \frac{1}{2a} \partial\Omega \cdot \partial\Omega - V(\Omega) \right] \\ + \delta'(W) \left[\Omega^2 (1 - \nabla^2 W) + \partial W \cdot \partial\Omega^2 \right] \end{array} \right\} \quad \begin{array}{l} \text{No scale,} \\ a = \frac{(d-2)}{8(d-1)}. \end{array}$$

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The equations of motion reproduce the $Sp(2,R)$ constraints, called **kinematic equations**, (proportional to $\delta'(W)$, $\delta''(W)$), and also the **dynamical equations** (proportional to $\delta(W)$).

Progress in 2T-physics

20/21

Progress in 2T-physics

- **Local $\text{Sp}(2, \mathbb{R})$: A general principle in Class. & Quant. Mech.**
A principle for a higher unification and deeper insight into physics & space-time
Reveals more physics phenomena that are systematically missed in 1T-physics.

Progress in 2T-physics

29/23

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A principle for a higher unification and deeper insight into physics & space-time
Reveals more physics phenomena that are systematically missed in 1T-physics.

- **Principles of 2T field theory in $d+2$ dimensions**

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IB+Y.C.Kuo 0605267, IB 0606045, 0610187, 0804.1585; IB+S.H.Chen 0811.2510

➔ Phenomenological applications: Cosmology 1004.0752, LHC 0606045, 0610187, and in progress
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no scale models
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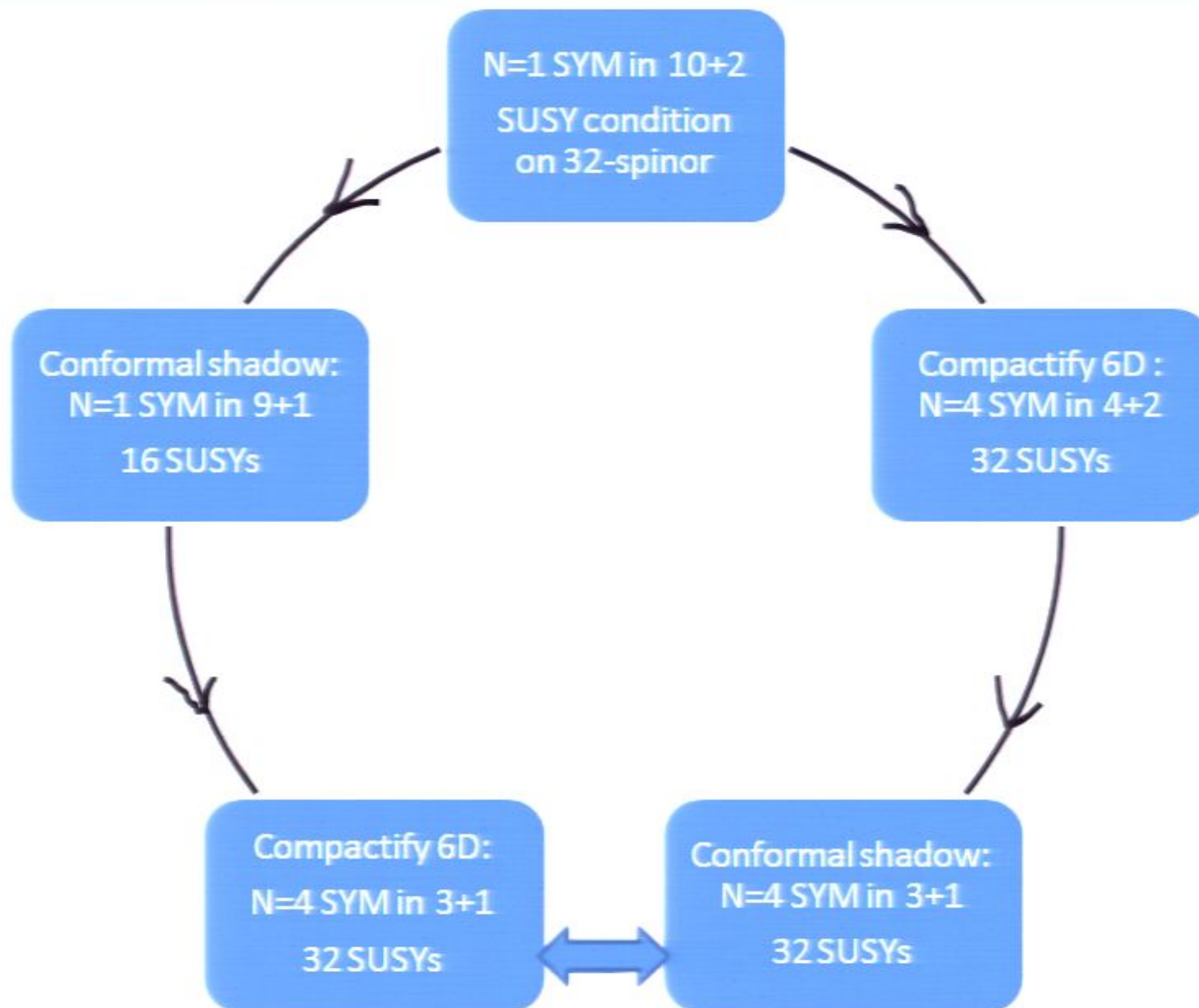
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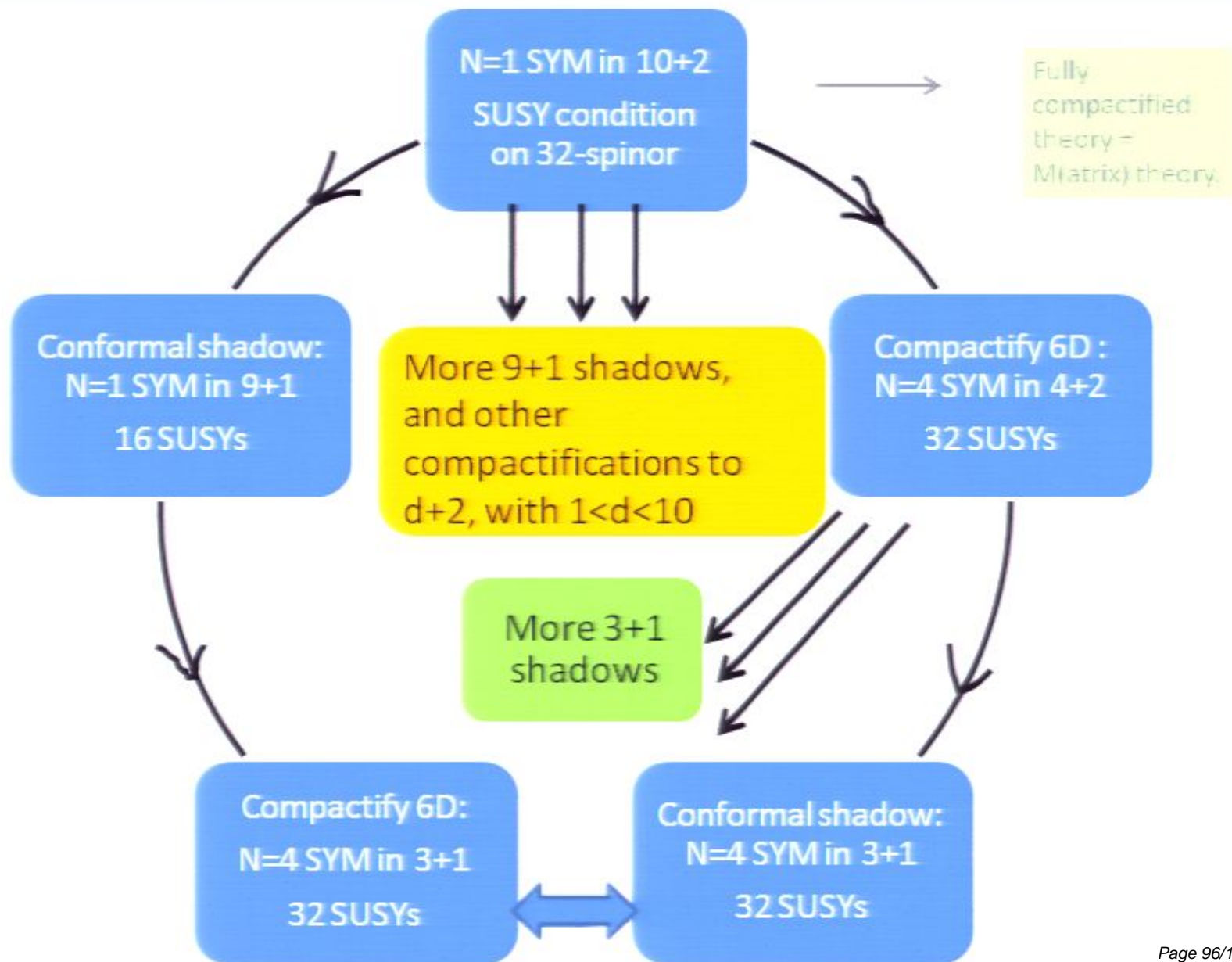
- **A more fundamental approach – field theory in phase space (full Q,P symm)**

IB + Deliduman 0103042, IB + S.J.Rey 0104135, IB 0106013, + under development.

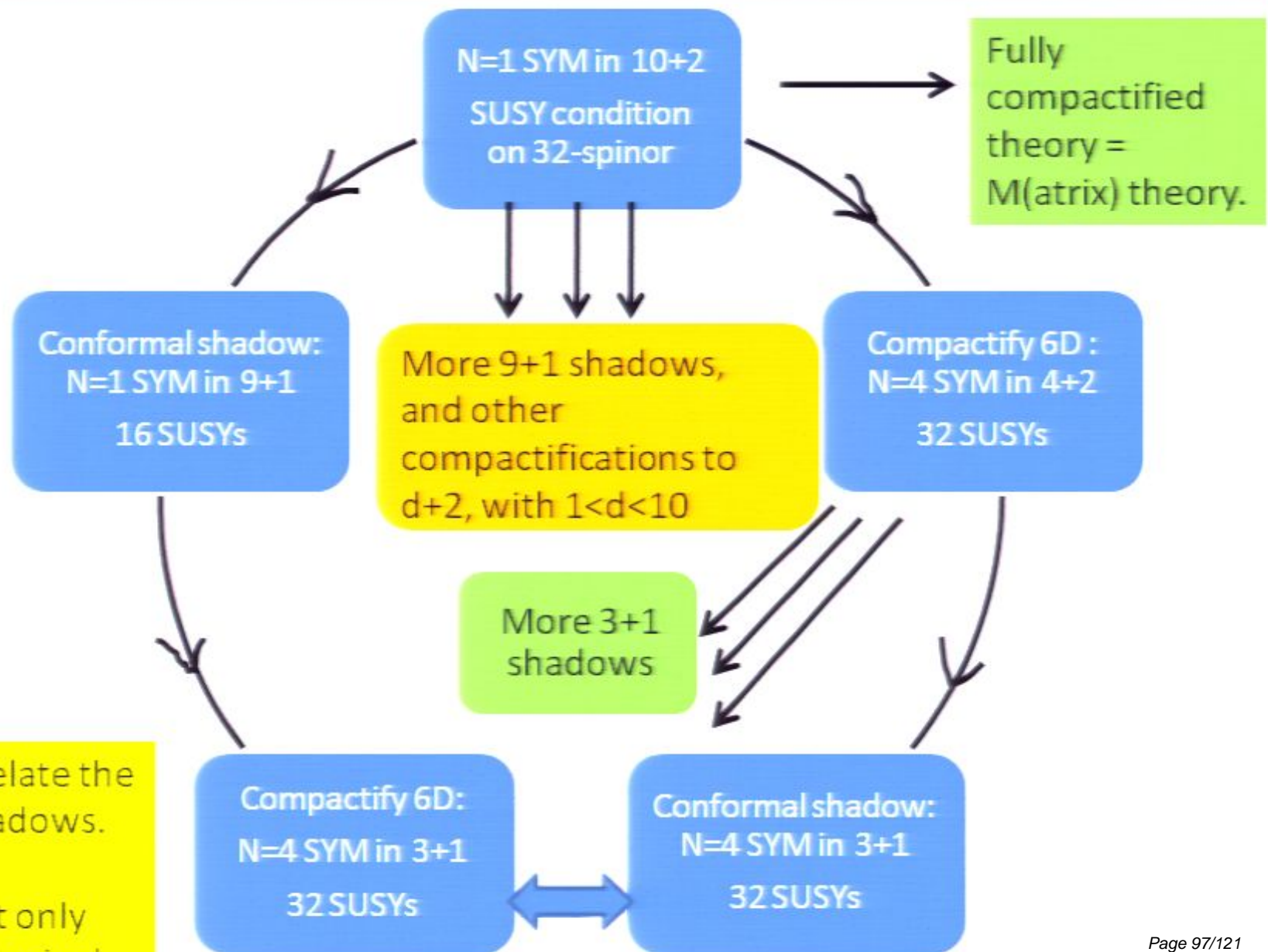
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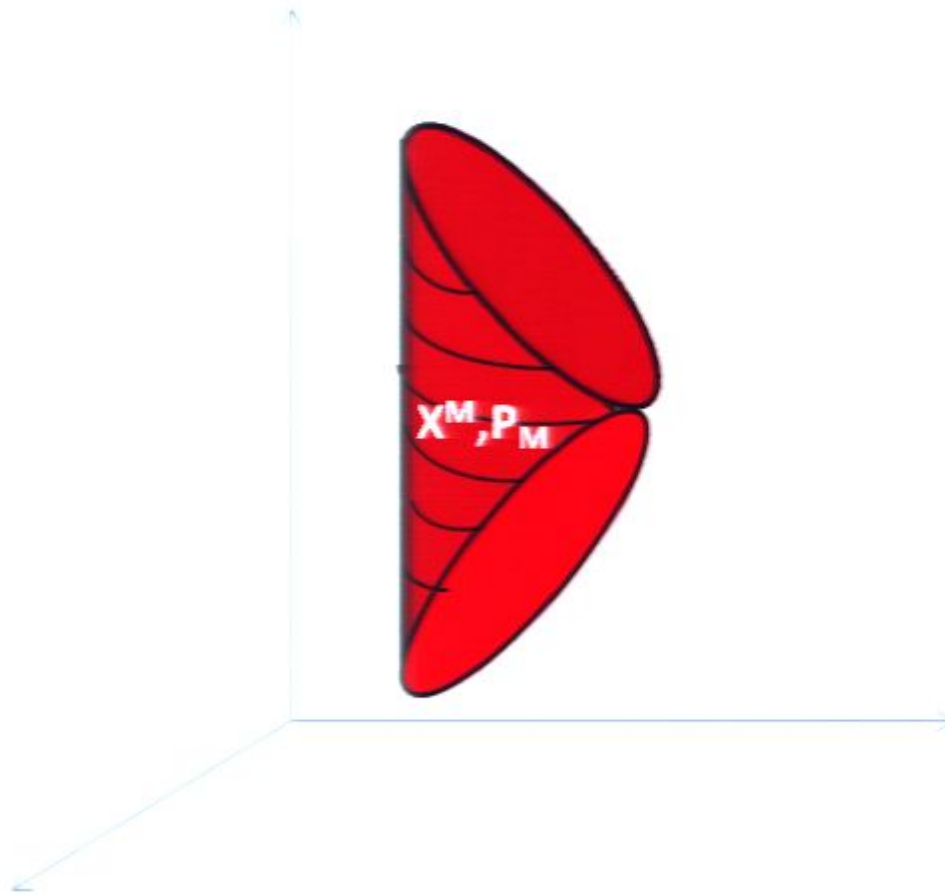


2T-Physics as a completion and unifying framework for 1T-physics

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The relation between 2T-physics and 1T-physics described by an analogy :

Consider object in the room \approx
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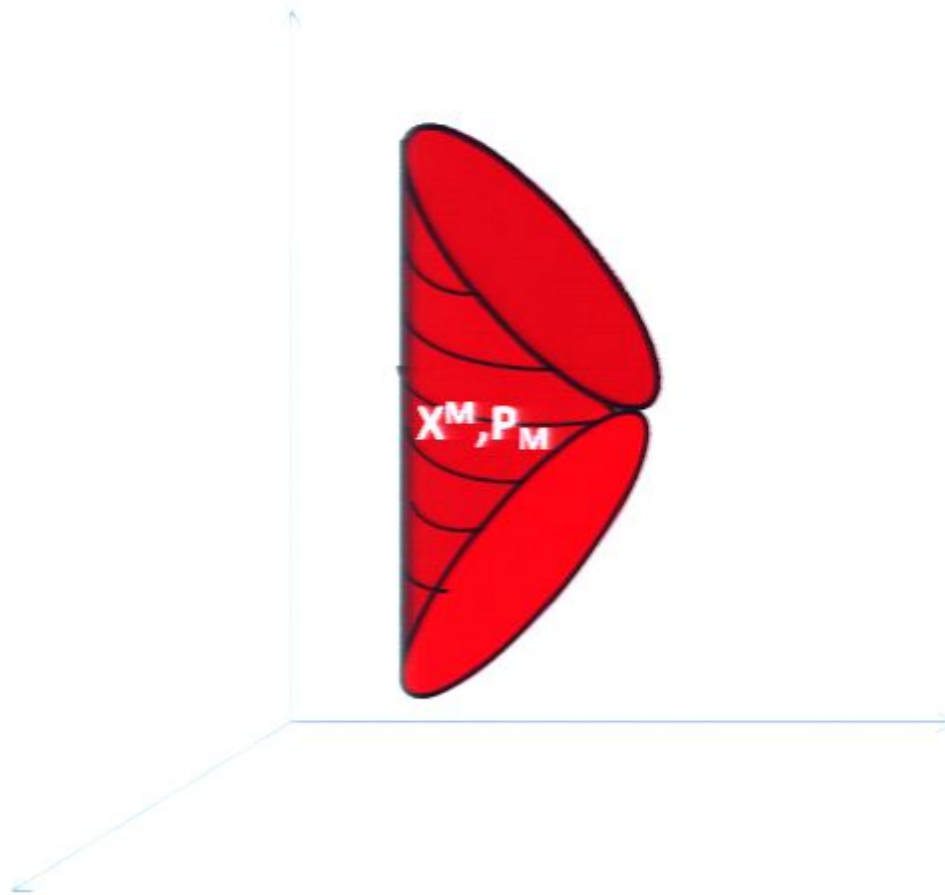


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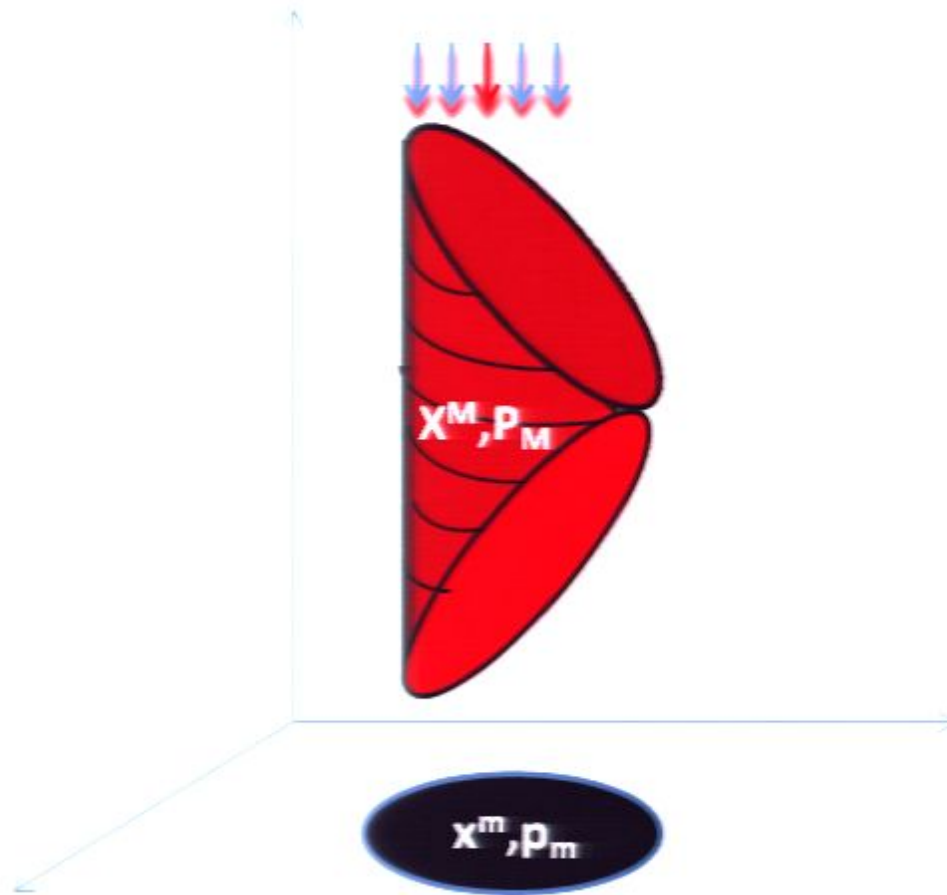


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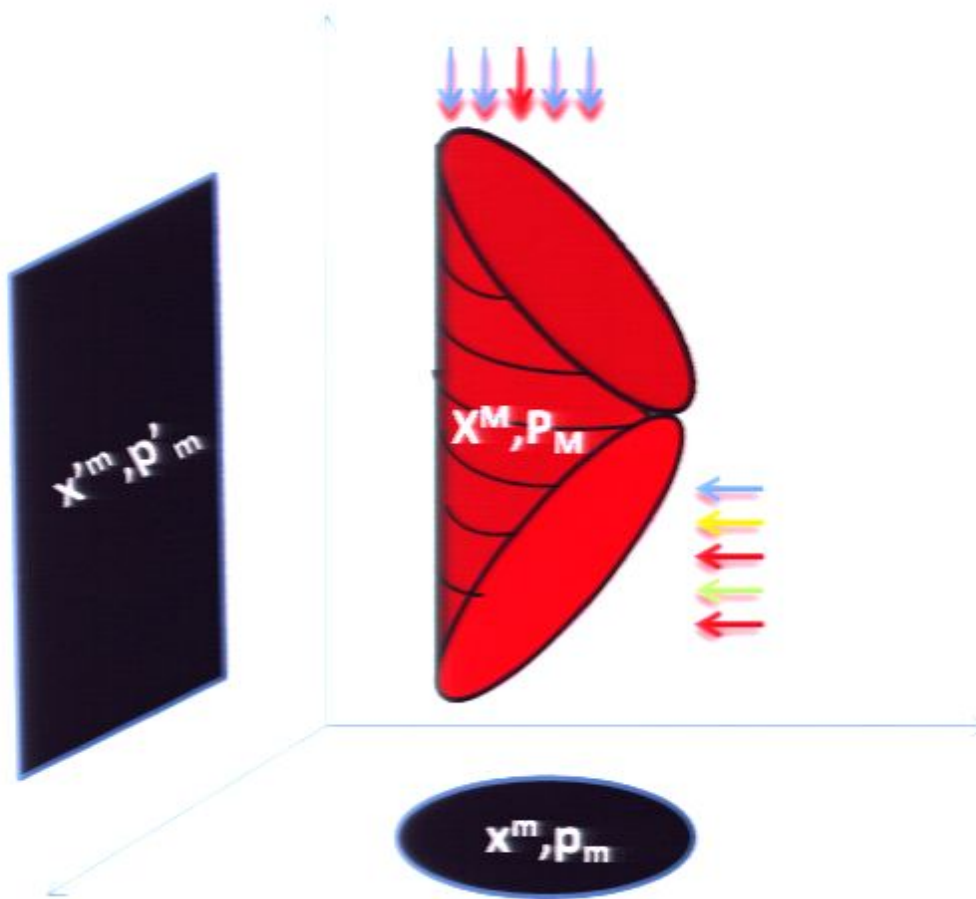


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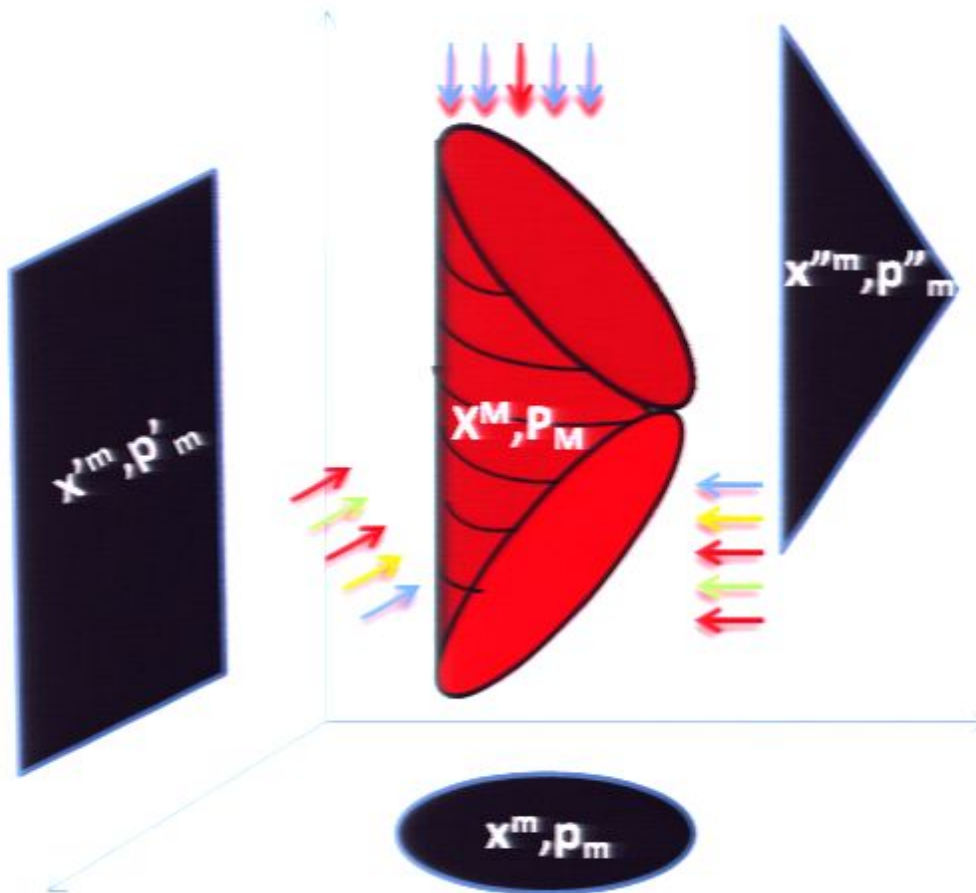
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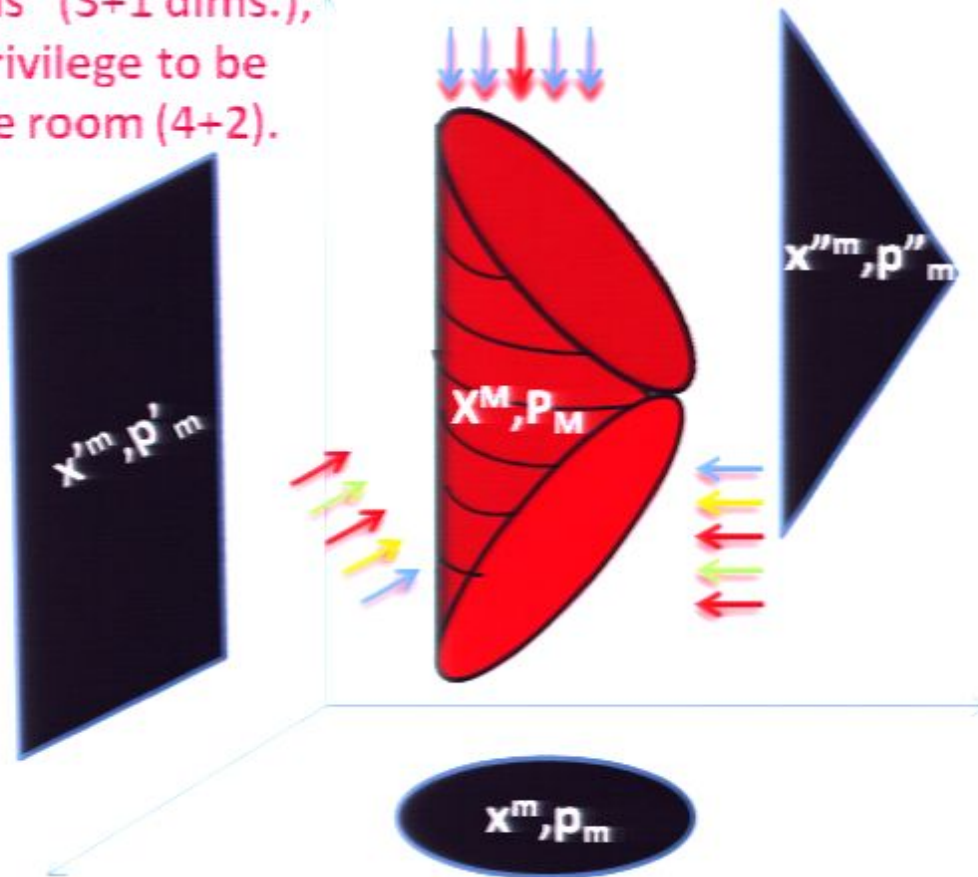
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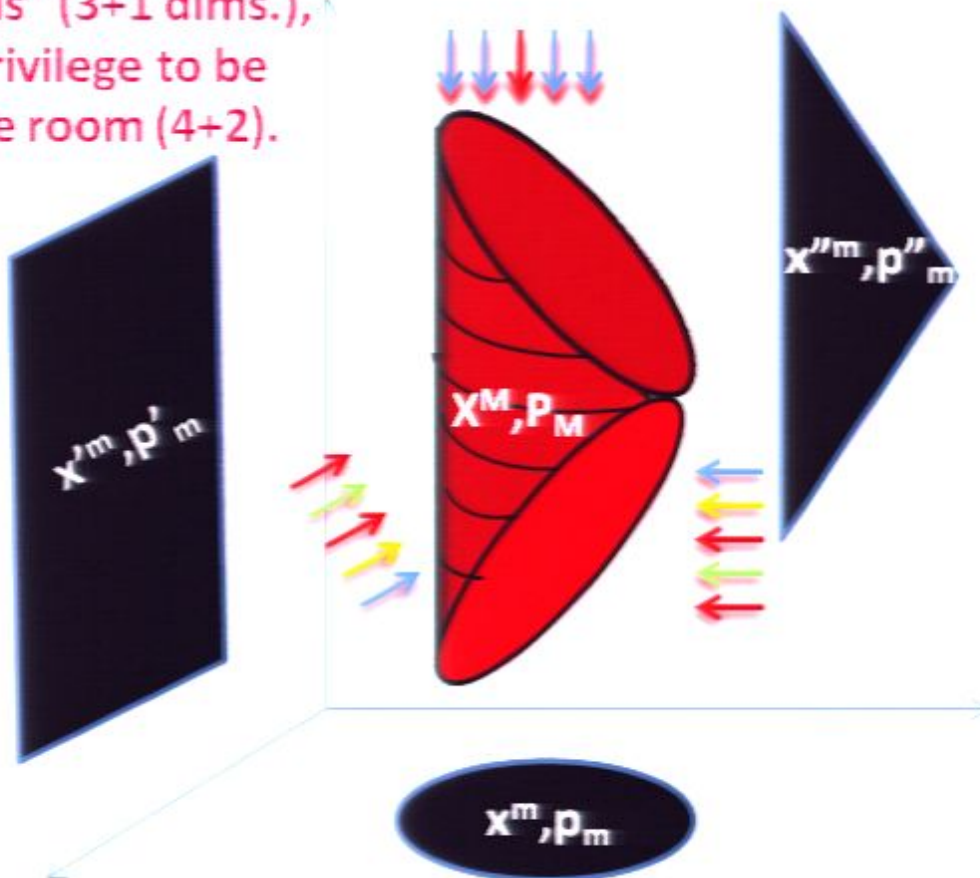
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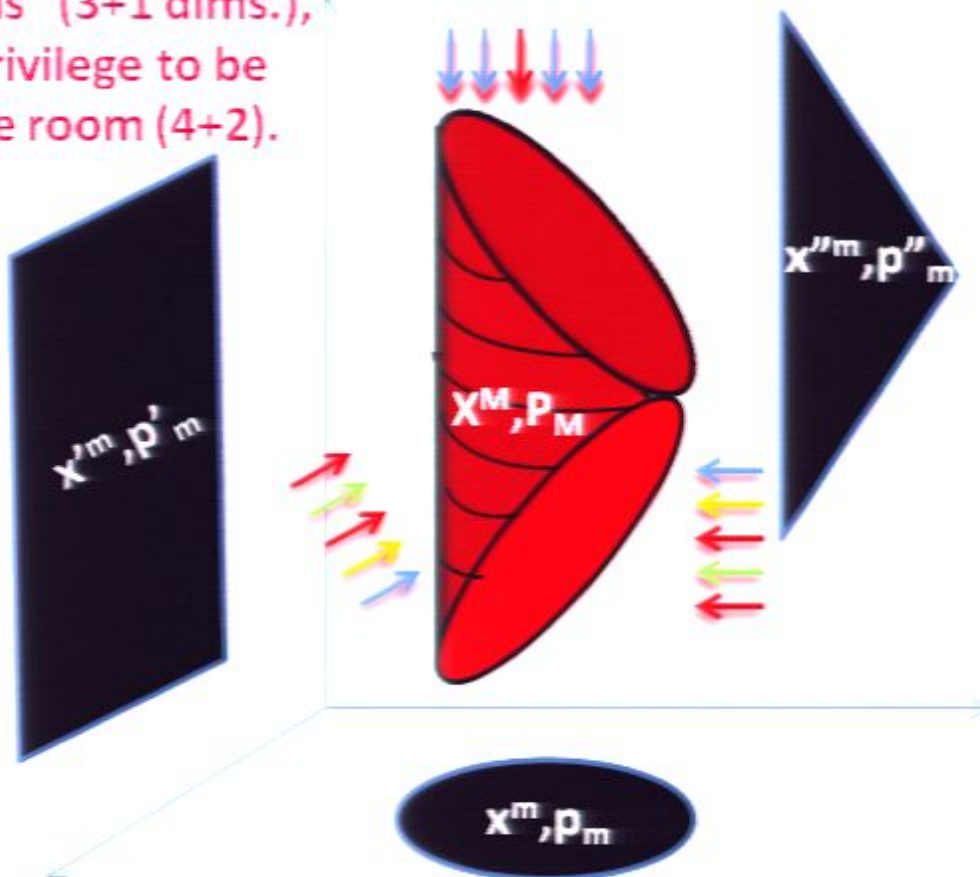
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ONE 2T system \rightarrow ^{holographic} MANY 1T systems

Predict many relations among the
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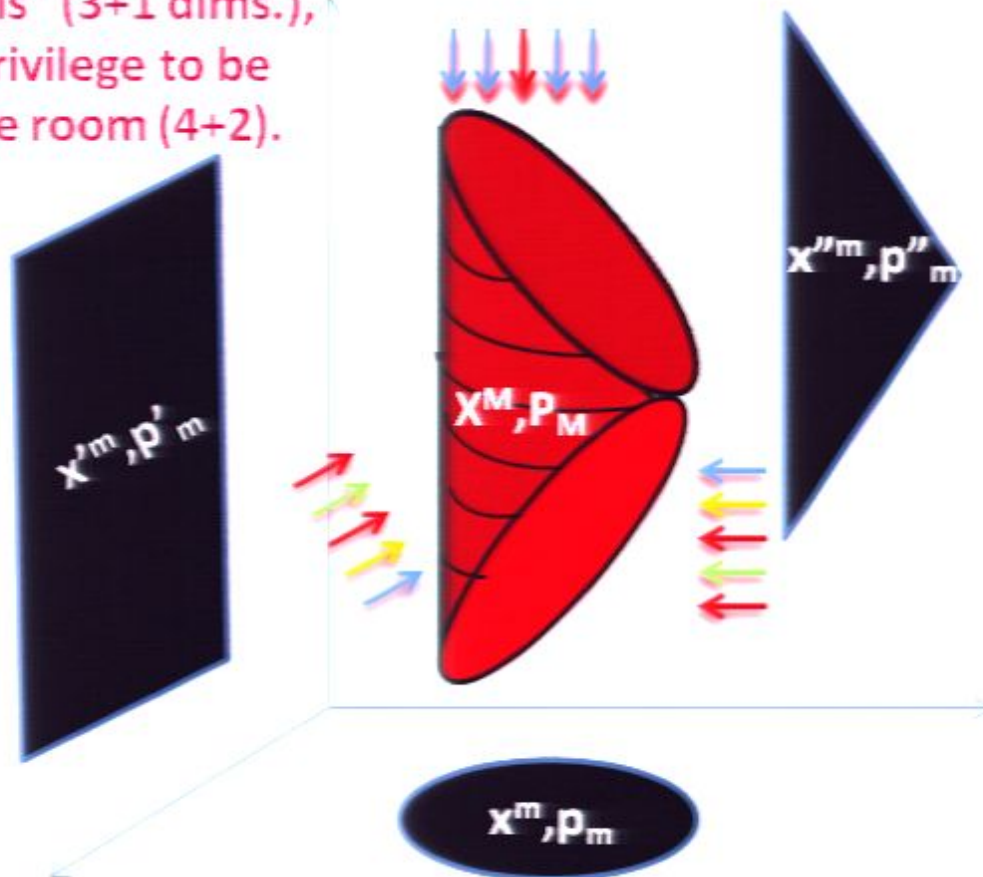
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Contains systematically missed information in 1T-physics approach.

This info related to higher spacetime:
Instead of interpreting the shadows as different dynamical systems (1T), must recognize they are perspectives in higher spacetime. Then, we can indirectly “see” the extra 1+1 dims.

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- 1) 1T-physics is incomplete !!!
- 2) 2T-physics makes new testable predictions, and provides new computational tools

Do you need 2T? YES!

11 12

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"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can't ignore chromium."

Do you need 2T? YES!

Examples and
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2T-physics is a new direction in higher dimensional unification.

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Expect more predictions at every scale of distance or energy, and more powerful computational tools in future research ...

A quotation from Gell-Mann

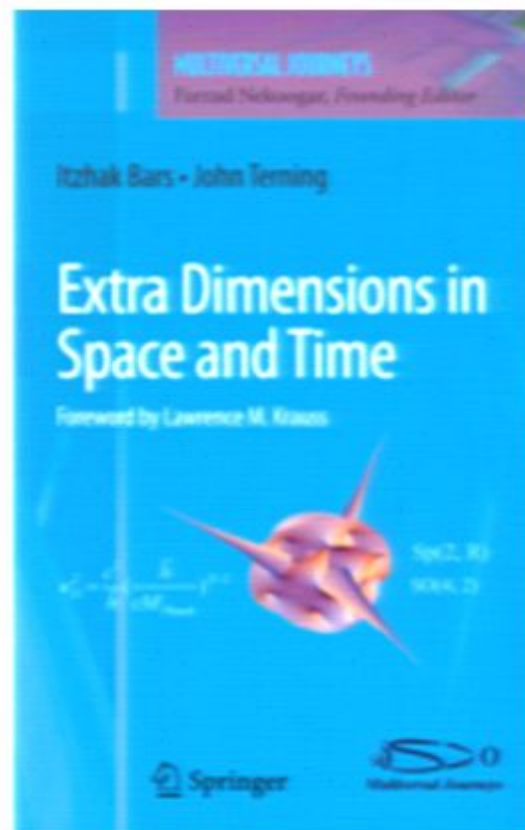
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Anything which is
not forbidden is
compulsory!

Where to find more information on 2T-physics

For concepts and technical guidance on over 50 papers
My recent talk: **arXiv:1004.0688**

A book at an elementary level for science enthusiasts (Springer 2009):



By
Itzhak Bars
and
John Terning

It can be downloaded at your
university if your library has a
contract with Springer
(e.g. here at PI)
DOI: 10.1007/978-0-387-77638-5

$$\begin{aligned} \mathcal{L}^{MN} : & \quad \mathcal{L}^{\mu\nu} \\ & \quad \mathcal{L}^{+\mu} \\ & \quad \mathcal{L}^{-\mu} \\ & \quad \mathcal{L}^{+-} \end{aligned}$$

$$L^{MN} : \begin{matrix} L^{\mu\nu} \\ L^{+\mu} \\ L^{-\mu} \\ L^{+-} \end{matrix}$$

$$|c_2, \dots, L^{++}\rangle$$

$$L^{MN}: \begin{matrix} L^{\mu\nu} \\ L^{+n} \\ L^{-n} \\ L^{+-} \end{matrix}$$

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$$p^\mu \rightarrow p^\mu + \epsilon^\mu$$

$$(L^{+n})^2 = 0$$

$$|c_2 \dots \underbrace{j(L^{+n})}_l \rangle$$

$$\frac{SO(2) \times SO(2)}{SO(4)} \rightarrow SO(4)$$

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$$|c_2 \dots \underbrace{(\textcircled{m})}_{(L^{(1)})} ; \underbrace{(\textcircled{n})}_{(L^{(2)})} \rangle$$

$\frac{SO(8) \times SO(1,2)}{SO(9)}$

$H = -\frac{1}{(\alpha')^2}$

$$Z^{MN} : \begin{matrix} L^{\mu\nu} \\ L^{+n} \\ L^{-n} \\ L^{+-} \end{matrix}$$

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$$(L^{+n})^2 = 0$$

$$U = \langle L^{+n} | \dots \rangle$$

$$|c_2 \dots \underbrace{(L^{+n})}_{\text{sort}(1) \times \text{sort}(2)} \dots \rangle$$

$$\frac{\text{sort}(1) \times \text{sort}(2)}{10k}$$

$$H = \dots$$

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$$\frac{\text{sort}(1) \times \text{sort}(2)}{r_{0k}} \rightarrow \text{sort}(1) \rightarrow \underbrace{L^{00}}_{\text{sort}}$$

$$H = \frac{1}{(\dots)}$$