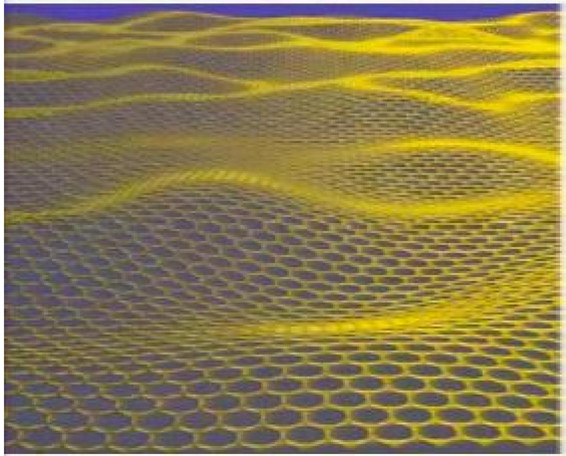
Title: Giant flavor-Hall effect and nonlocal transport in Dirac materials

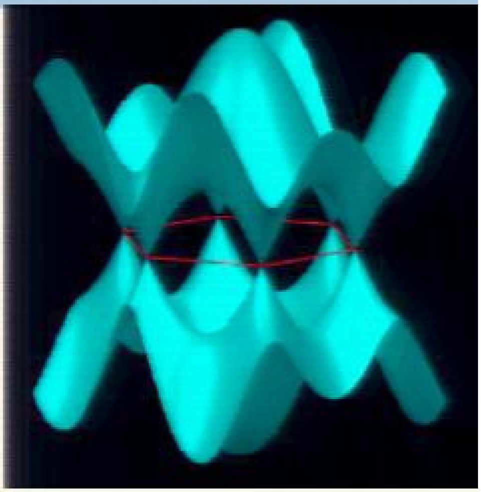
Date: Jan 12, 2011 02:00 PM

URL: http://pirsa.org/11010085

Abstract: Graphene-like materials provide a unique opportunity to explore quantum-relativistic phenomena in a condensed matter laboratory. Interesting phenomena associated with the internal degrees of freedom, spin and valley, including quantum spin-Hall effect, have been theoretically proposed, but could not be observed so far largely due to disorder and density inhonogeneity. We show that weak magnetic field breaks the symmetries that protect flavor (spin, valley) degeneracy, and induces large bulk non-quantized flavor-Hall effect in graphene. The effect occurs due to flavor splitting which generates the imbalance of the Hall resistivities of the two flavor species. At the Dirac point, flavor-Hall effect is greatly enhanced due to anomalous magnetotransport of Dirac-like carriers. The flavor-Hall effect is robust in the presence of disorder and interactions, and can serve as a hallmark of the quasi-relativistic carriers in the system. It manifests itself in large nonlocal transport mediated by long-lived flavor currents, as well as in flavor injection and accumulation experiments. Recent experimental observation of the giant nonlocal transport in graphene following our prediction opens up new opportunities for creating and manipulating flavor currents. Our work also shows that nonlocal electric transport provides a tool to study neutral modes in solid-states systems.

Giant Flavor-Hall effect in Dirac materials



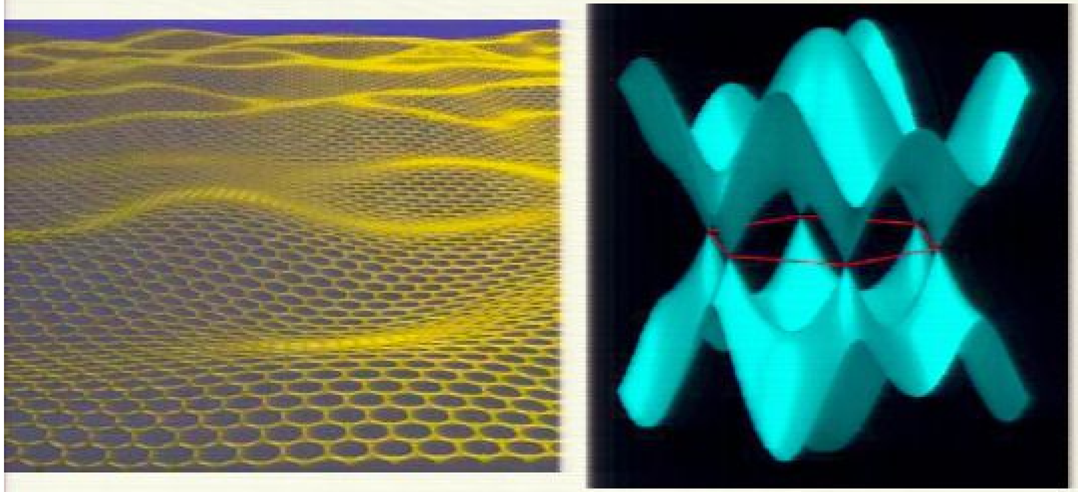


Dima Abanin
Princeton University

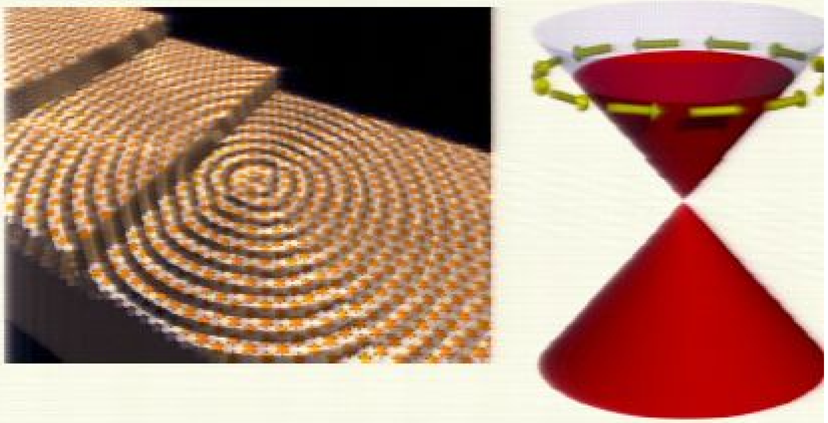
Perimeter Institute
January 12, 2011

Dirac materials

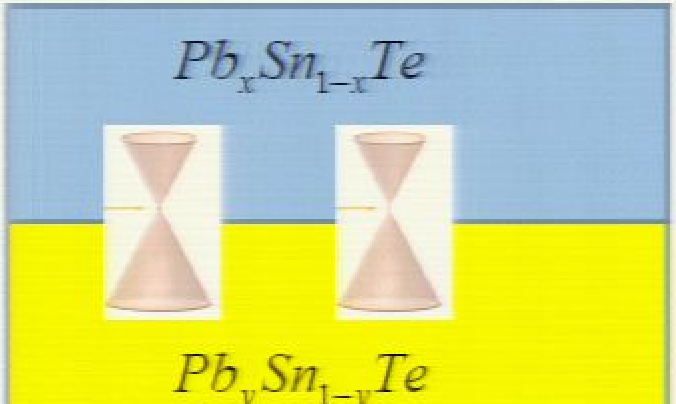
GRAPHENE (2004)



TOPOLOGICAL INSULATORS (2007)



INVERTED CONTACTS (2010)



All objects are three-dimensional

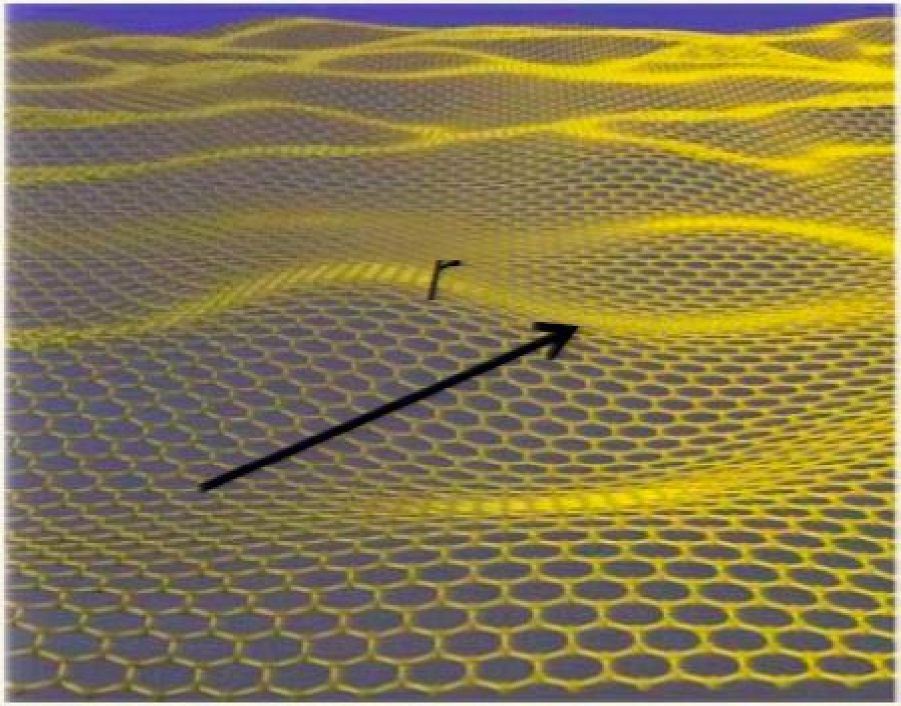
2D crystals cannot exist

-Destroyed by thermal fluctuations

-Displacement

$$< h^2(r) > \propto \ln(r)$$

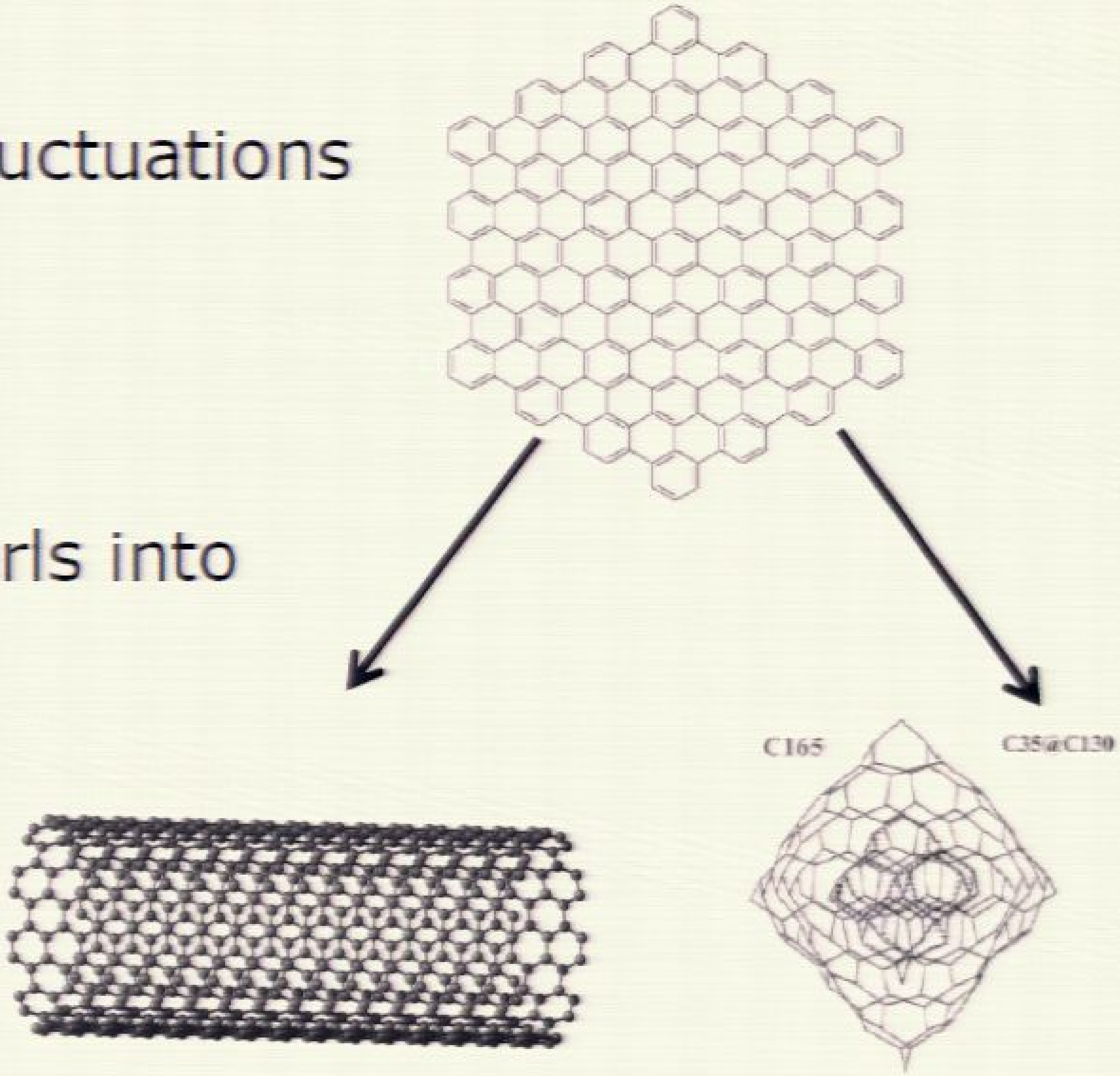
(Mermin-Wagner theorem)



NO bottom-up growth of graphene

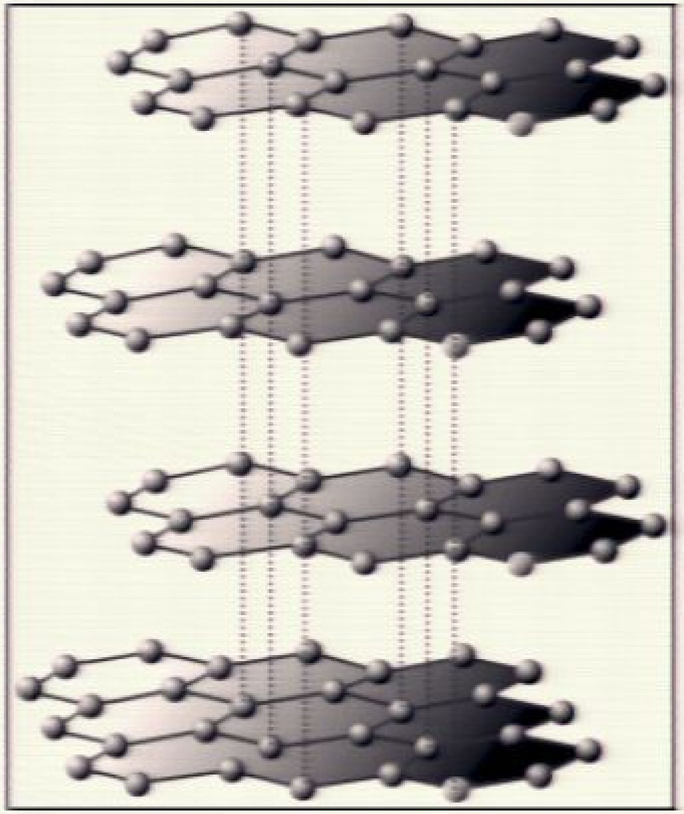
Strong out-of-plane fluctuations

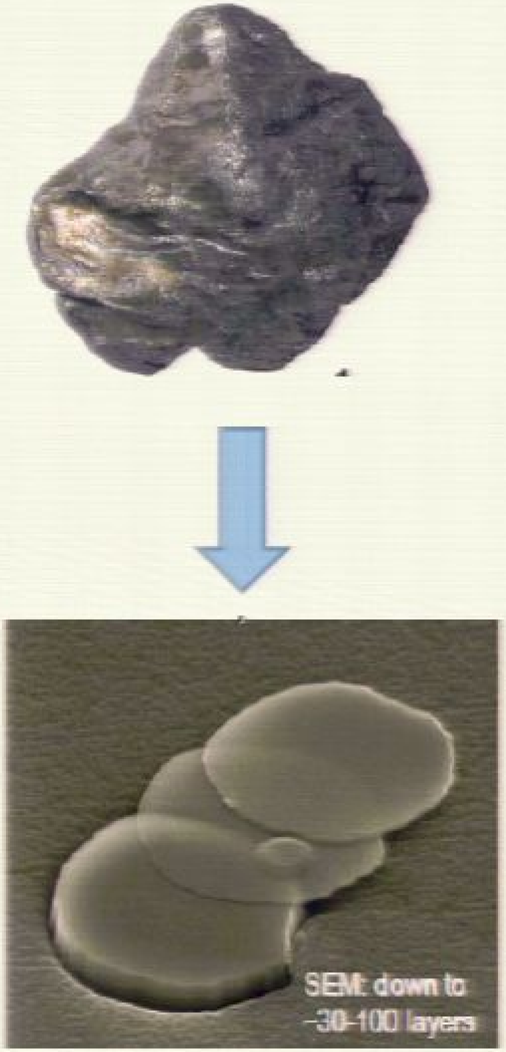
Graphene unstable, curls into nanotubes, fullerenes



Top-down approach

3D graphite is layered; peel off a thin "pancake"



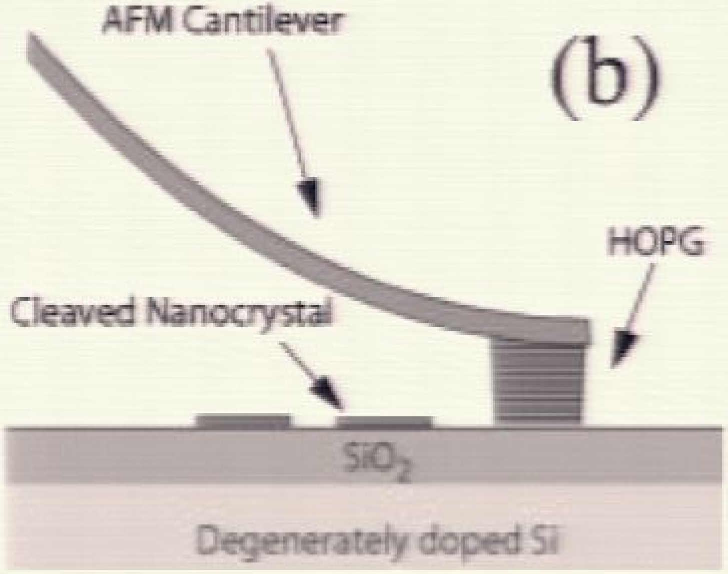


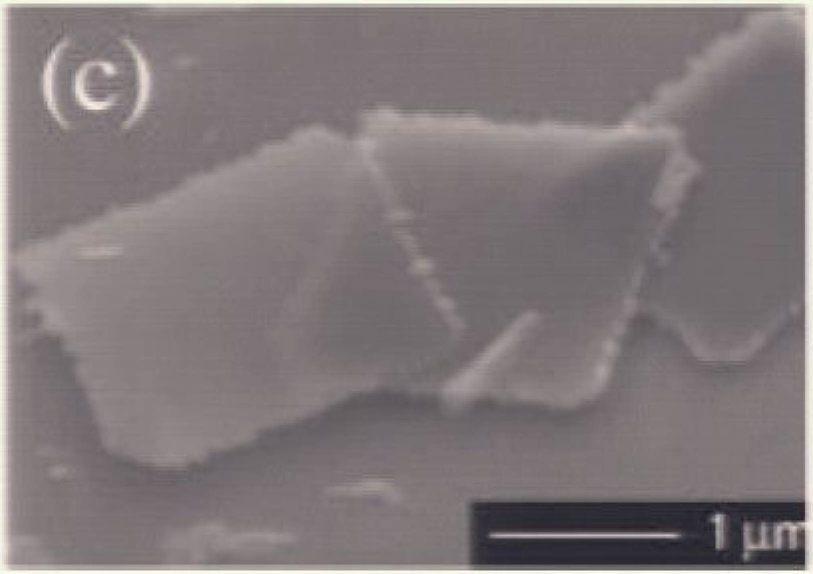
(Kurtz'90,Ebbesen'95,Ohashi'97,
Ruoff'99, Kim'04, McEuen'04)

Top-down approach

Nanopencil (down to 40 layers)

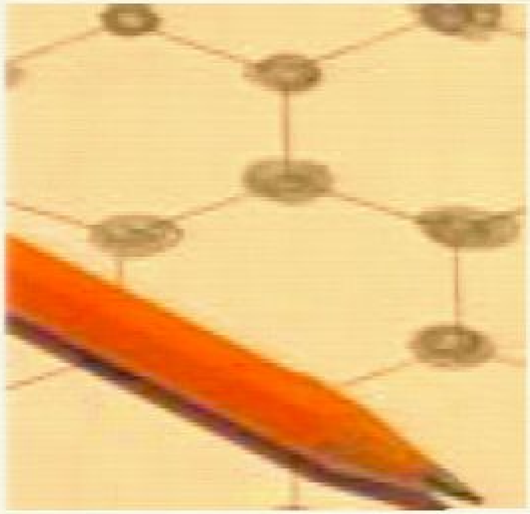
P. Kim'04,McEuen'04





Discovery of monolayer graphene

The trick: write on scotch tape *(Novoselov, Geim et al '04)*



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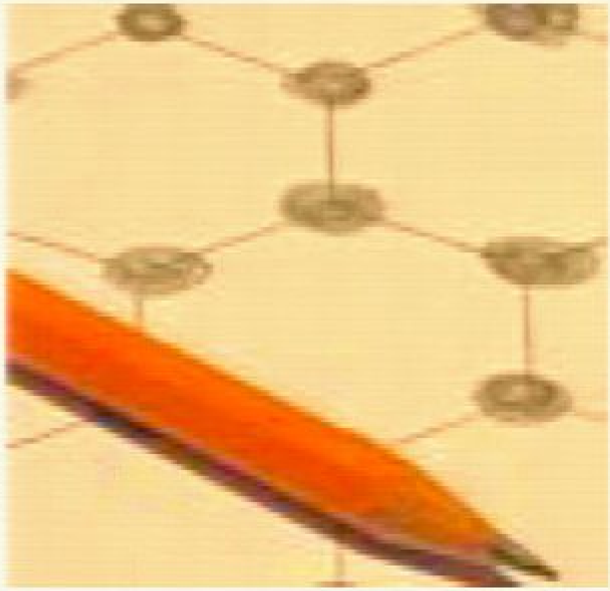
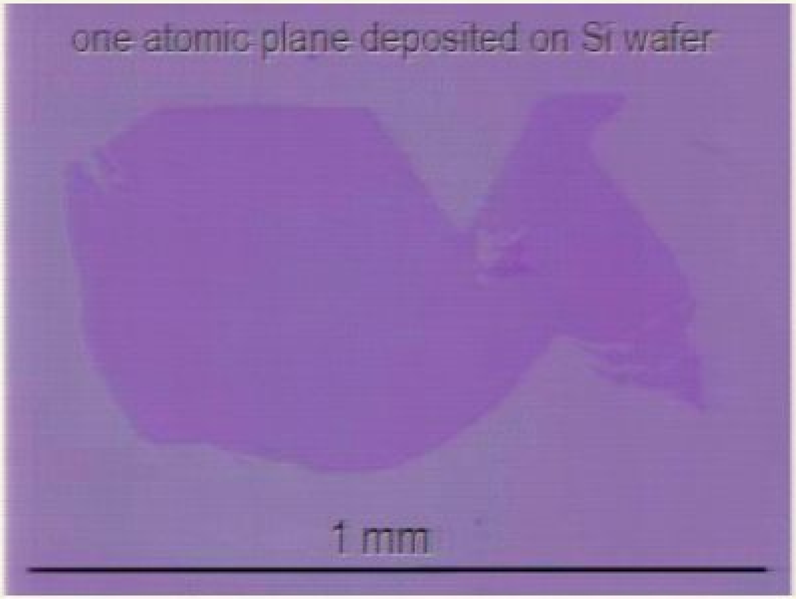
Discovery of monolayer graphene



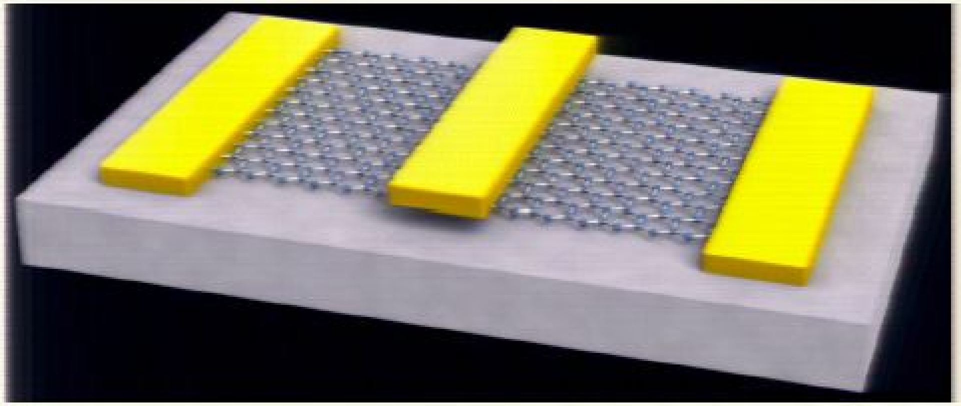


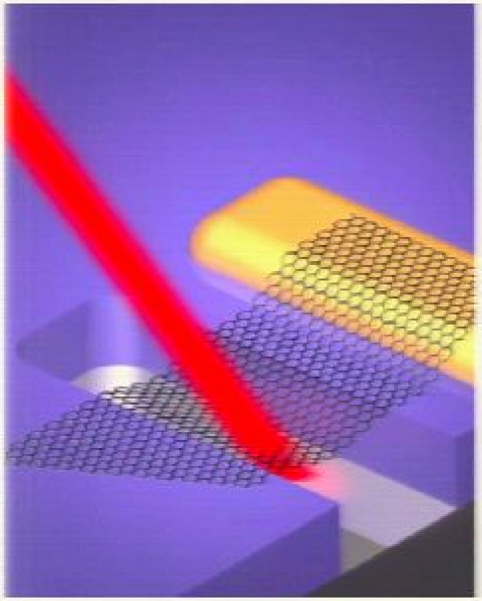
-Many flakes “drawn”

-Certain substrate necessary

-Graphene visible in a microscope

Why is graphene stable?





-Low temperatures limit fluctuations

-Supported by substrate and/or contacts

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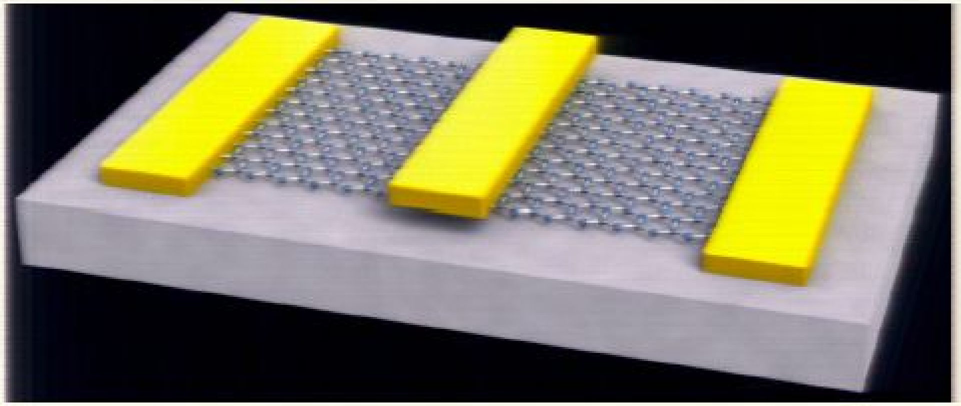

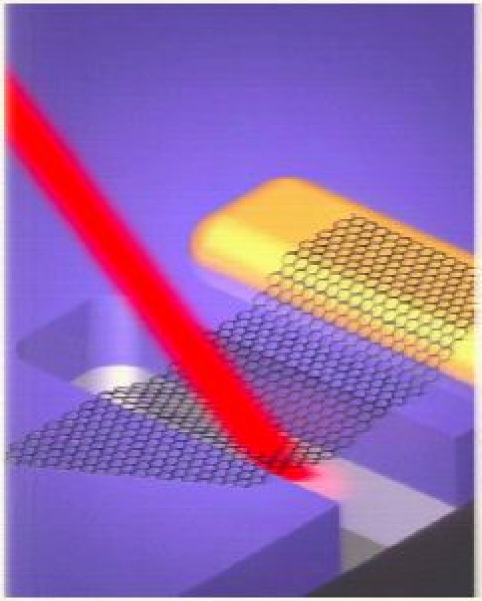

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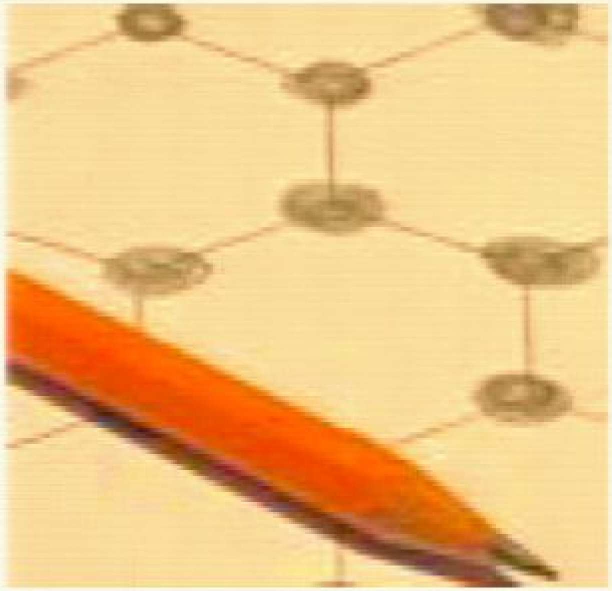
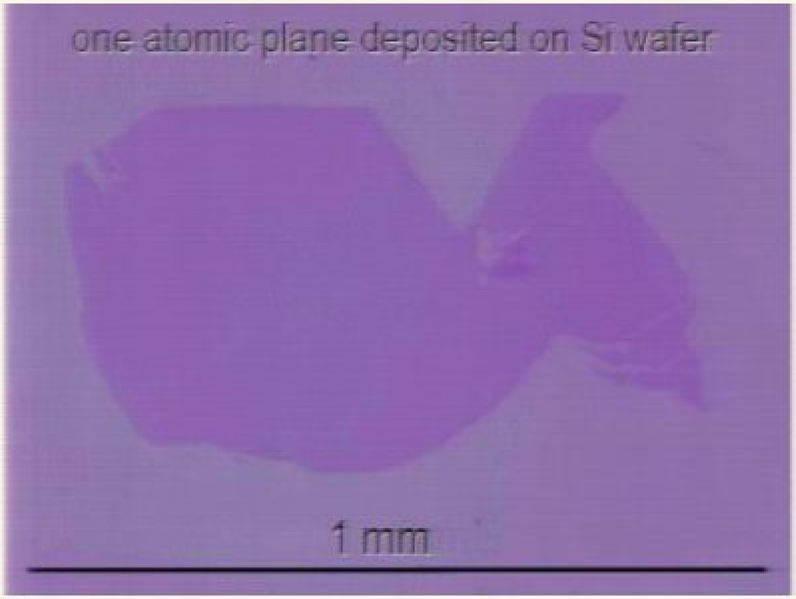
Discovery of monolayer graphene





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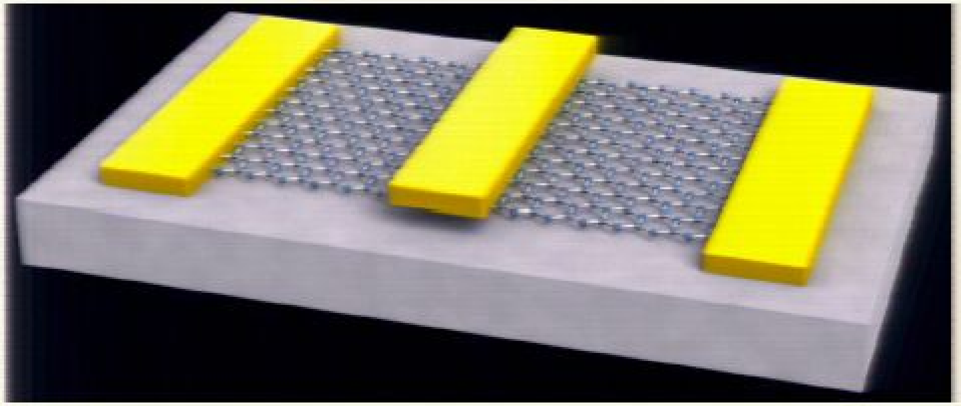
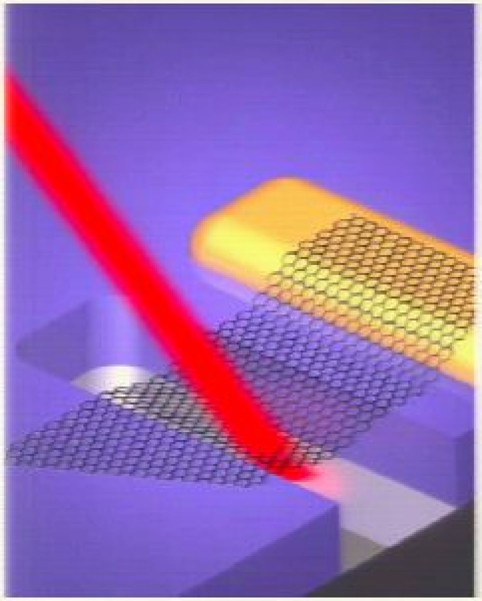




-Low temperatures limit fluctuations

-Supported by substrate and/or contacts

Why is graphene stable?

New type of material:
ATOMICALLY THIN!

-Low

-Supported by substrate and/or contacts

GRAPHENE'S SUPERLATIVES

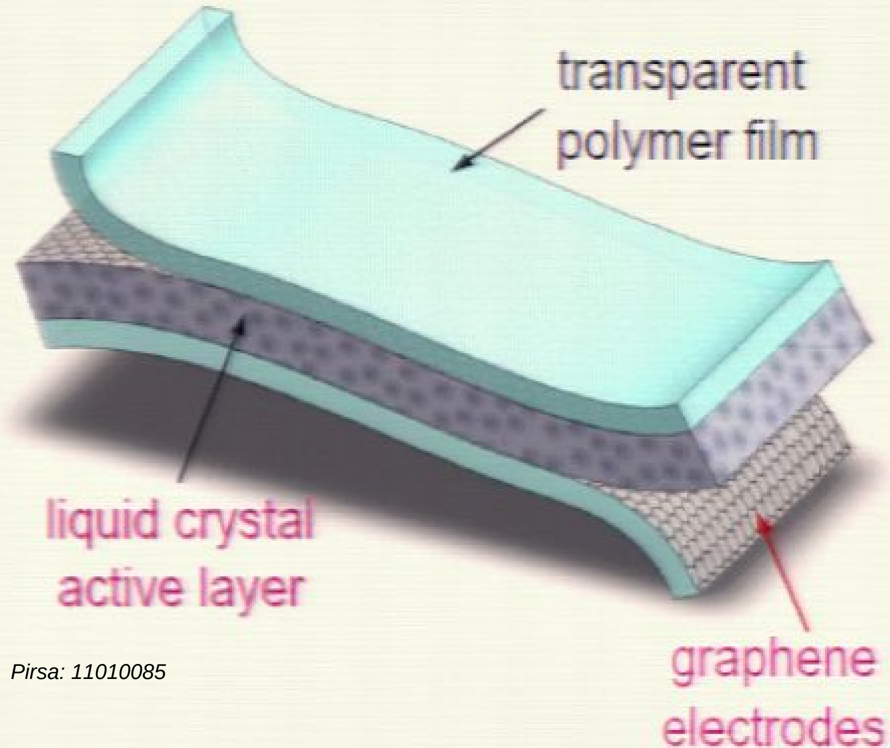
thinnest imaginable material
largest surface area (~2,700 m^2 per gram)
strongest material 'ever measured' (theoretical limit)
stiffest known material (stiffer than diamond)
most stretchable crystal (up to 20% elastically)
record thermal conductivity (outperforming diamond)
highest current density at room T (10^6 times of copper)
completely impermeable (even He atoms cannot squeeze through)
highest intrinsic mobility (100 times more than in Si)
conducts electricity in the limit of no electrons
lightest charge carriers (zero rest mass)
longest mean free path at room T (micron range)

From A. Geim's Nobel lecture '2010

First applications

Bendable graphene-based touch screen (Samsung)

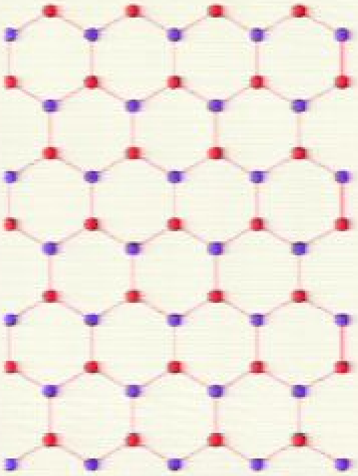
To go on sale 2012

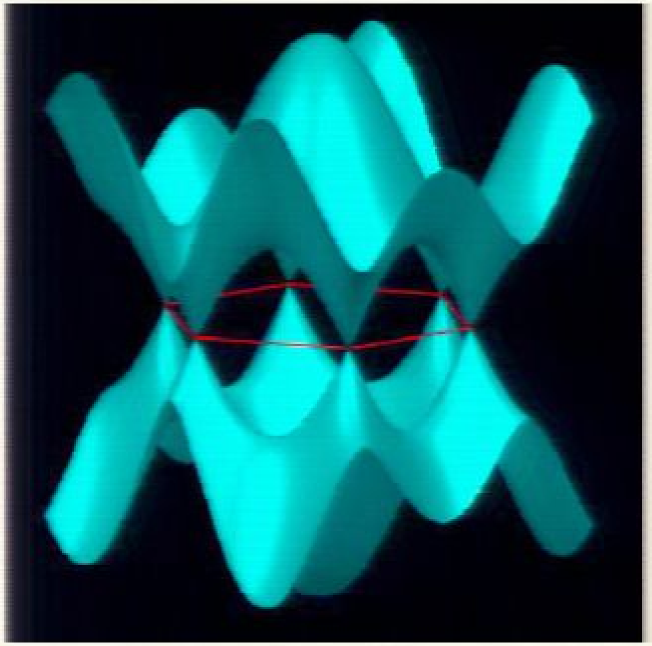


Unique electronic properties

New quantum mechanical phenomena

Graphene

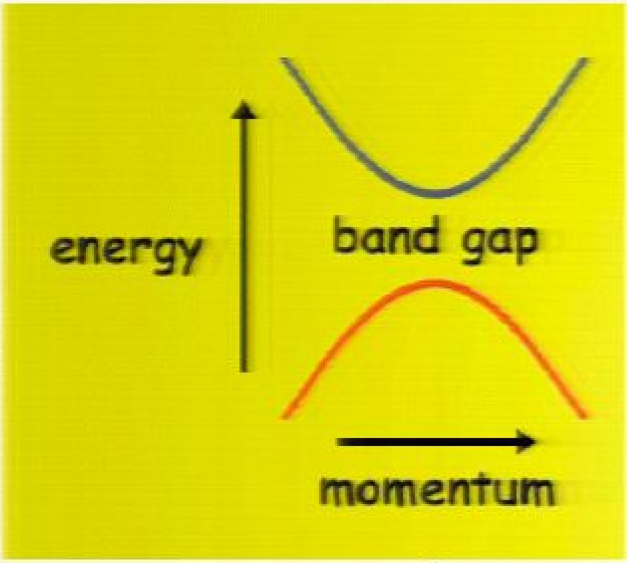




$$\varepsilon = \pm vp$$

Relativistic
quantum mechanics

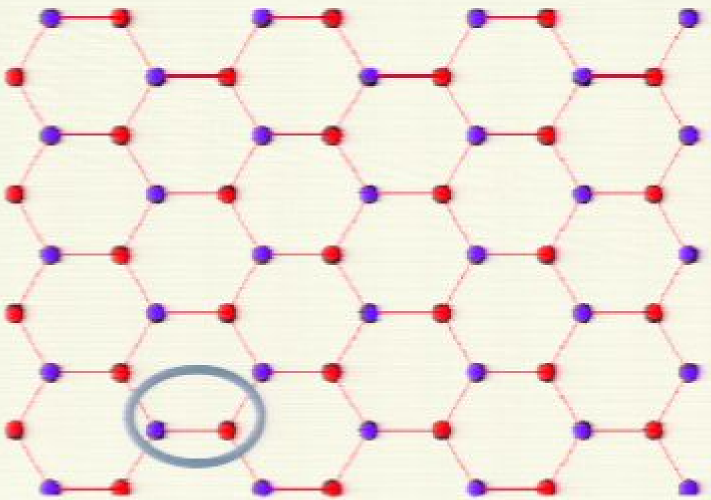
Normal metals



$$\varepsilon = \pm \frac{p^2}{2m_*}$$

Textbook
quantum mechanics

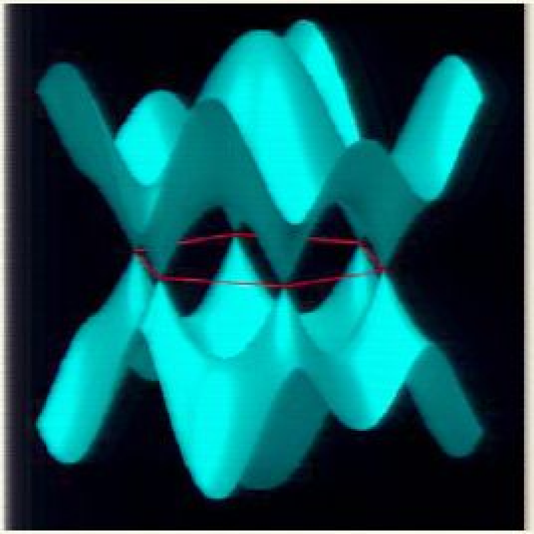
Electronic spectrum



Unit cell

● Sublattice B

● Sublattice A



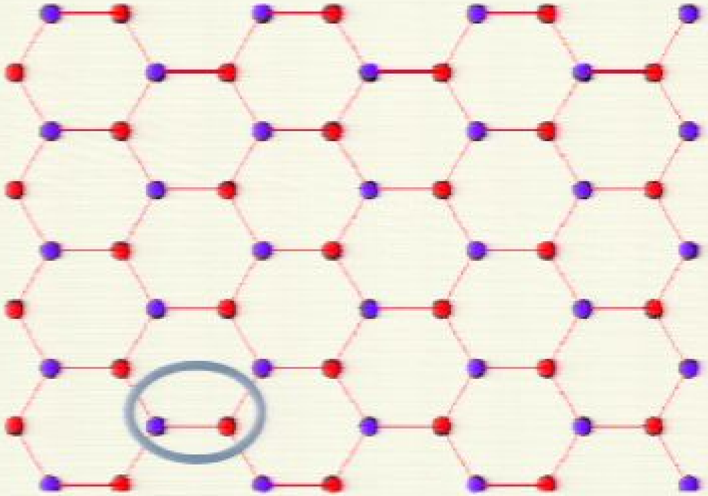
Two valleys

- Diffraction of electrons by a honeycomb lattice
- Two Dirac points (valleys)

$$H_{K(K')} = v_0 \begin{bmatrix} 0 & p_x \pm i p_y \\ p_x \mp i p_y & 0 \end{bmatrix} \qquad v_0 = 10^6 m/s$$

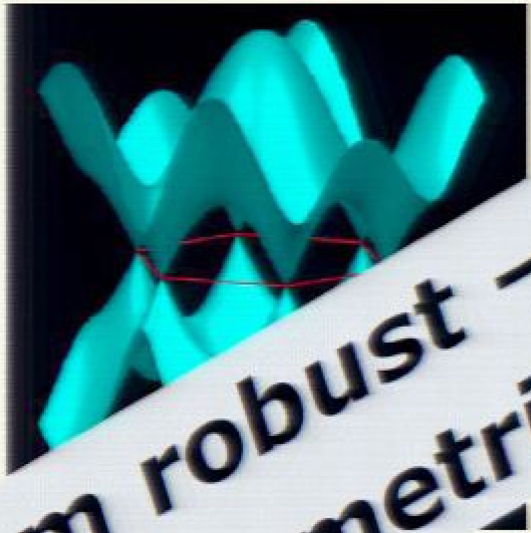
- Two spinor components = sublattice A and B

Electronic spectrum



Unit cell ● Sublat

● S



valleys

Dirac spectrum robust – protected by symmetries!

-Diffraction of … honeycomb lattice

-Two Dir… …eys)

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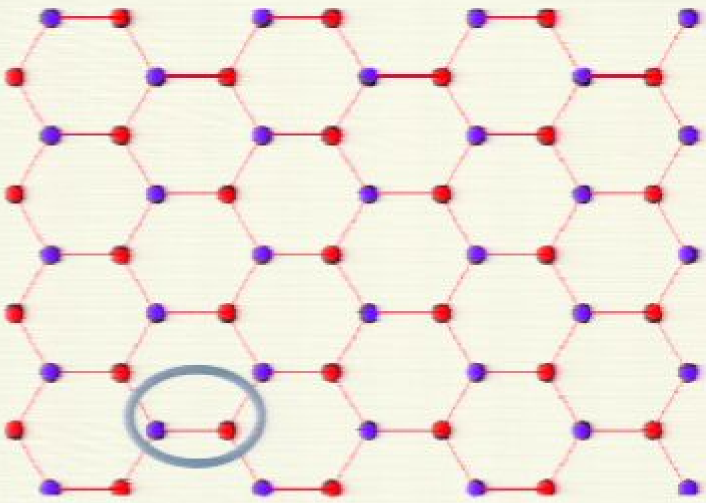

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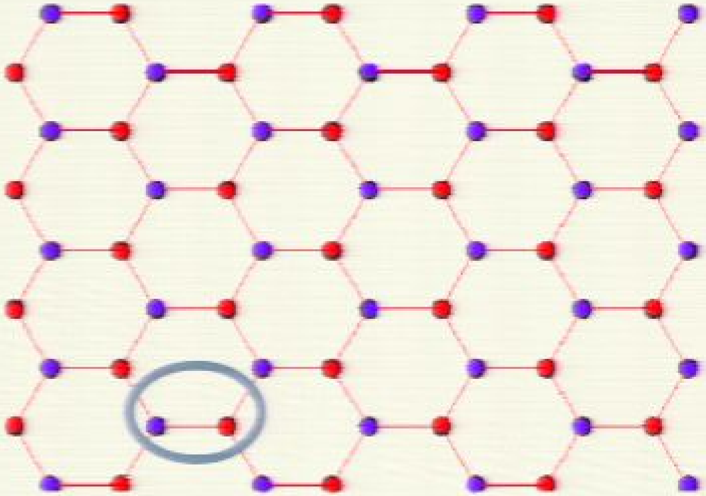

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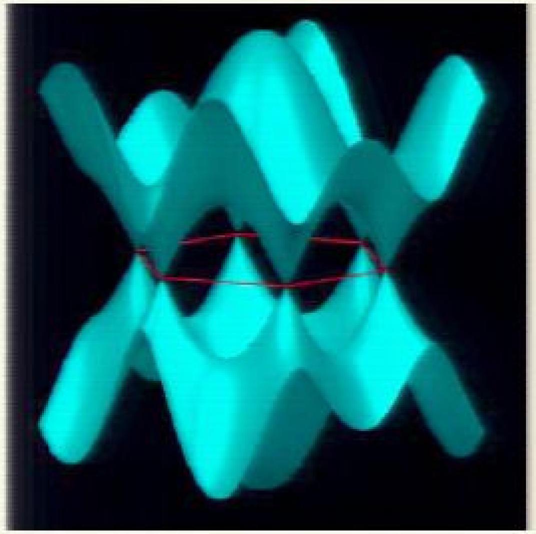
● Sublattice A



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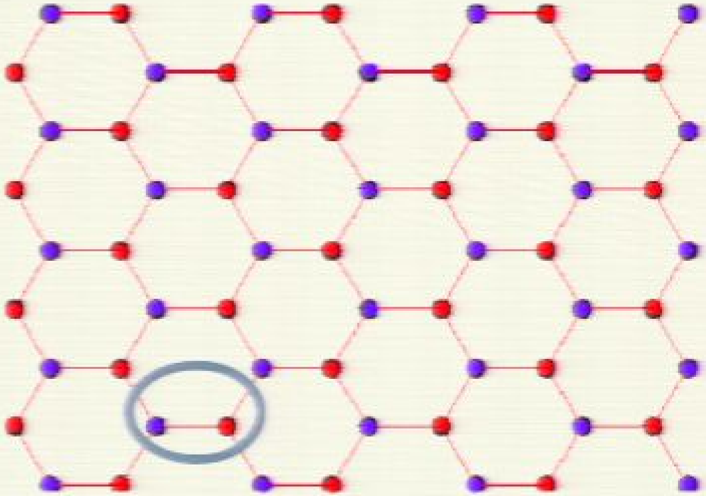

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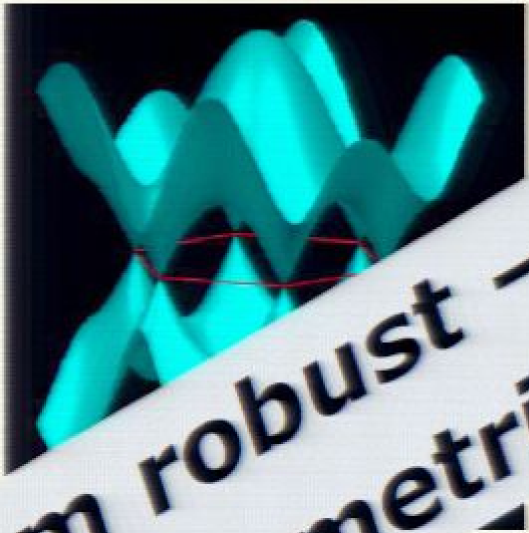

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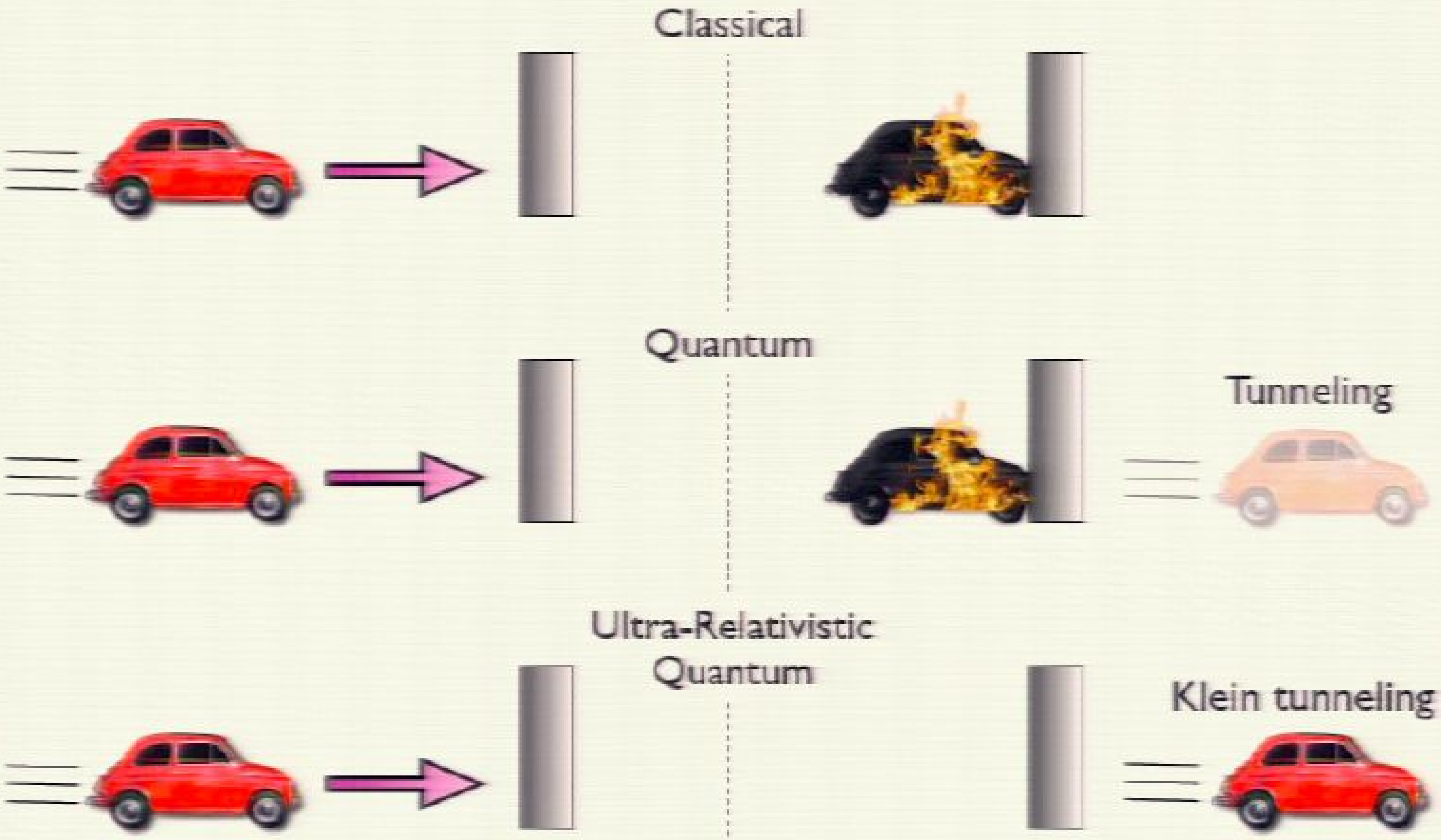
New kind of tunneling

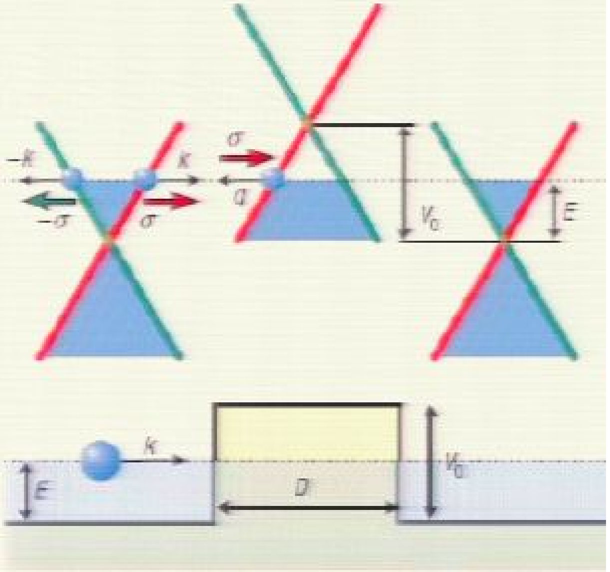


© Mark Rudner

Klein paradox

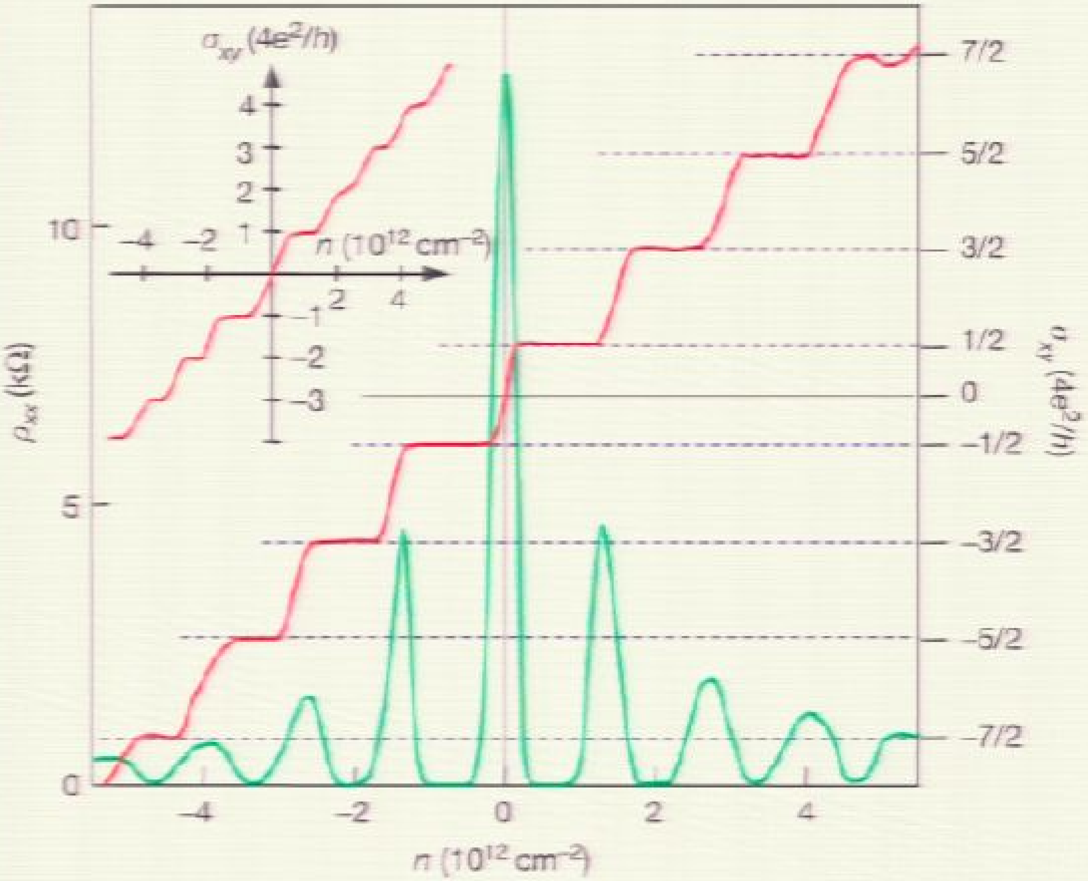
Pseudospin σ aligned with momentum p

$$\psi_p = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\theta_p/2} \\ e^{-i\theta_p/2} \end{bmatrix} \qquad \theta_p = \tan(p_x / p_y)$$



Scalar potential cannot flip pseudospin → NO BACKSCATTERING

Half-integer quantum Hall effect



$$\sigma_{xy} = 4 \times (N + 1/2)\frac{e^2}{h}$$

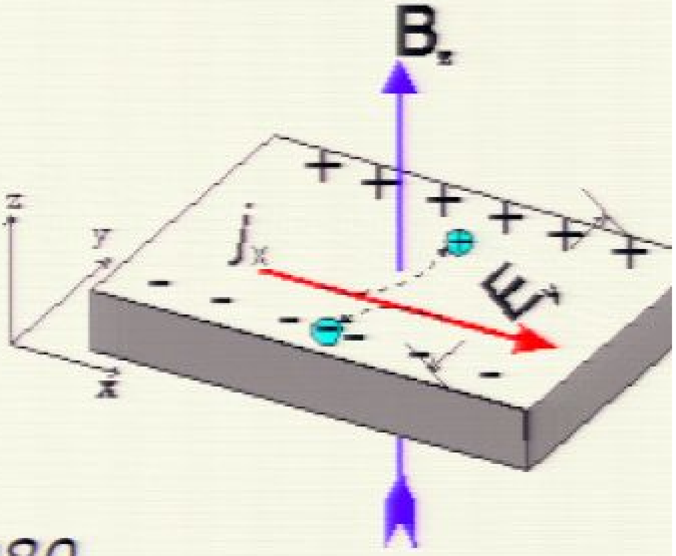
-4=2*2 (valley, spin)

-At room temperature (previously, only at 30K)

Novoselov et al'05;Zhang et al'05

Integer quantum Hall effect

Cclassical Hall effect: *Hall'1891*



$$\rho_{xy} = -\frac{B}{ne}$$

n carrier density (non-universal)

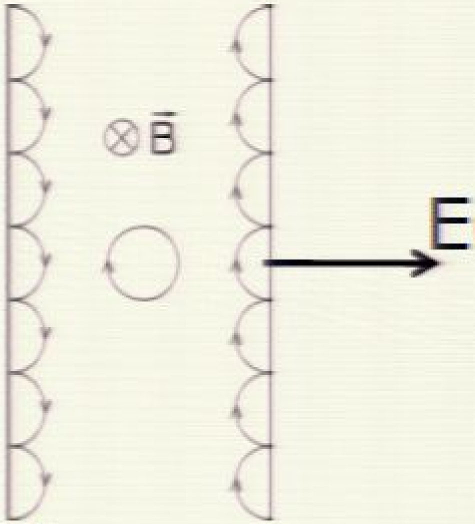
Quantum Hall effect: *Von Klitzing'1980*

$$\rho_{xy} = -\frac{h}{Ne^2}$$

N integer

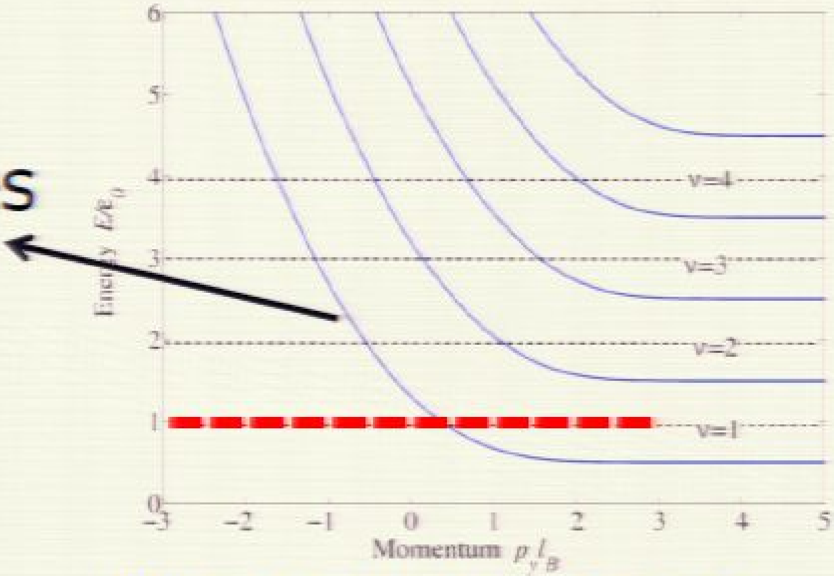
UNIVERSAL

Quantum Hall edge states



Edge states

Position space

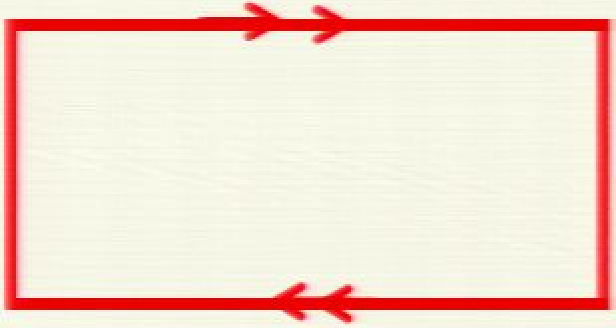


Momentum space

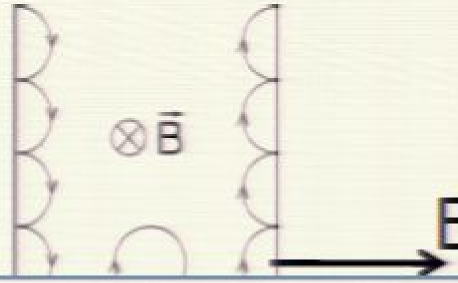
-One chiral edge mode per Landau Level; topological protection

-N edge modes → $\sigma_{xy} = N\frac{e^2}{h}$

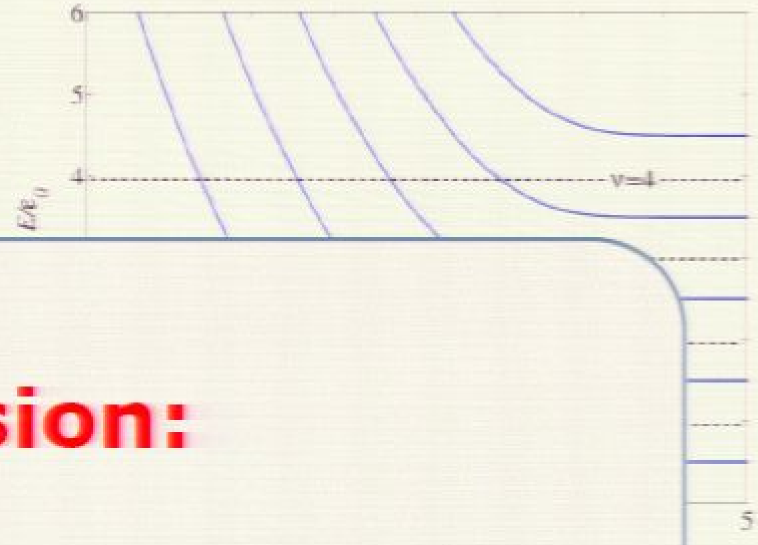
-Explains quantum Hall effect



Quantum Hall edge states



Edge states



-Quantization precision: 1/1000000000

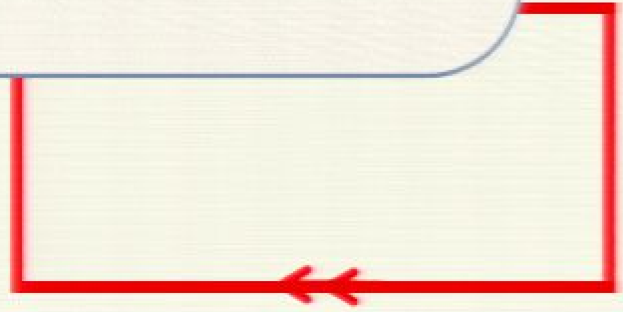
-Used as a standard in metrology

-O

leve

-N edge modes → $\sigma_{xy} = N\frac{e}{h}$

-Explains quantum Hall effect

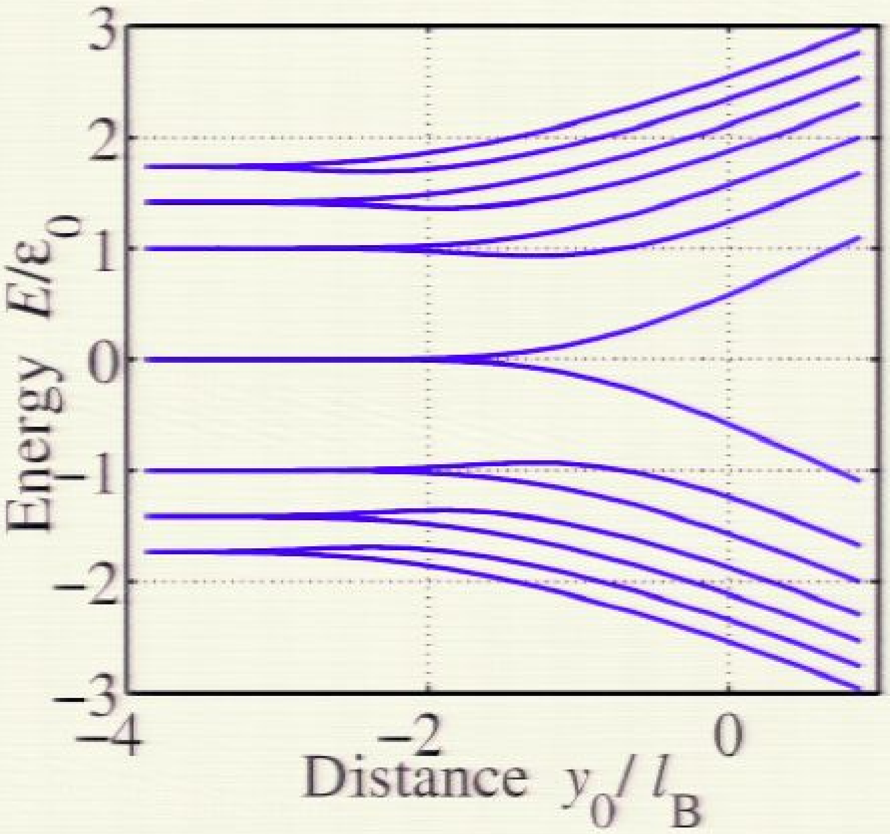


QH edge states in graphene

-Dirac spectrum
-Zeroth Landau level
(Semenoff'83,Jackiw'84)
-Contributes

$$\sigma_{xy} = \pm\frac{1}{2}\frac{e^2}{h}$$

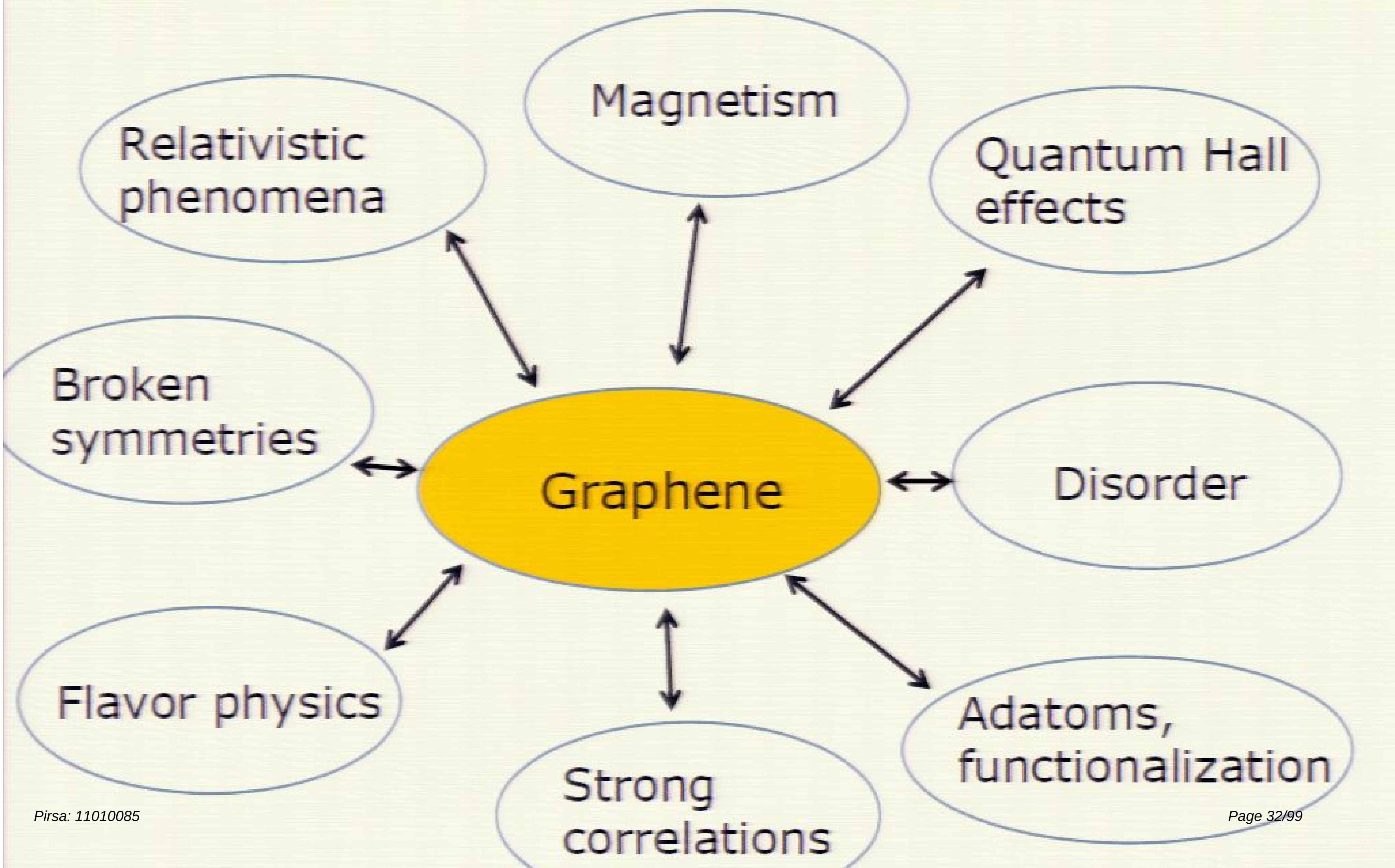
per specie



-Symmetric splitting of 0th Landau level
-Odd number of edge modes
-Explains half-integer QHE

DA, Lee, Levitov
PRL'06;
Brey, Fertig

Graphene and condensed matter physics



Relativistic phenomena

Magnetism

Quantum Hall effects

Broken symmetries

Graphene

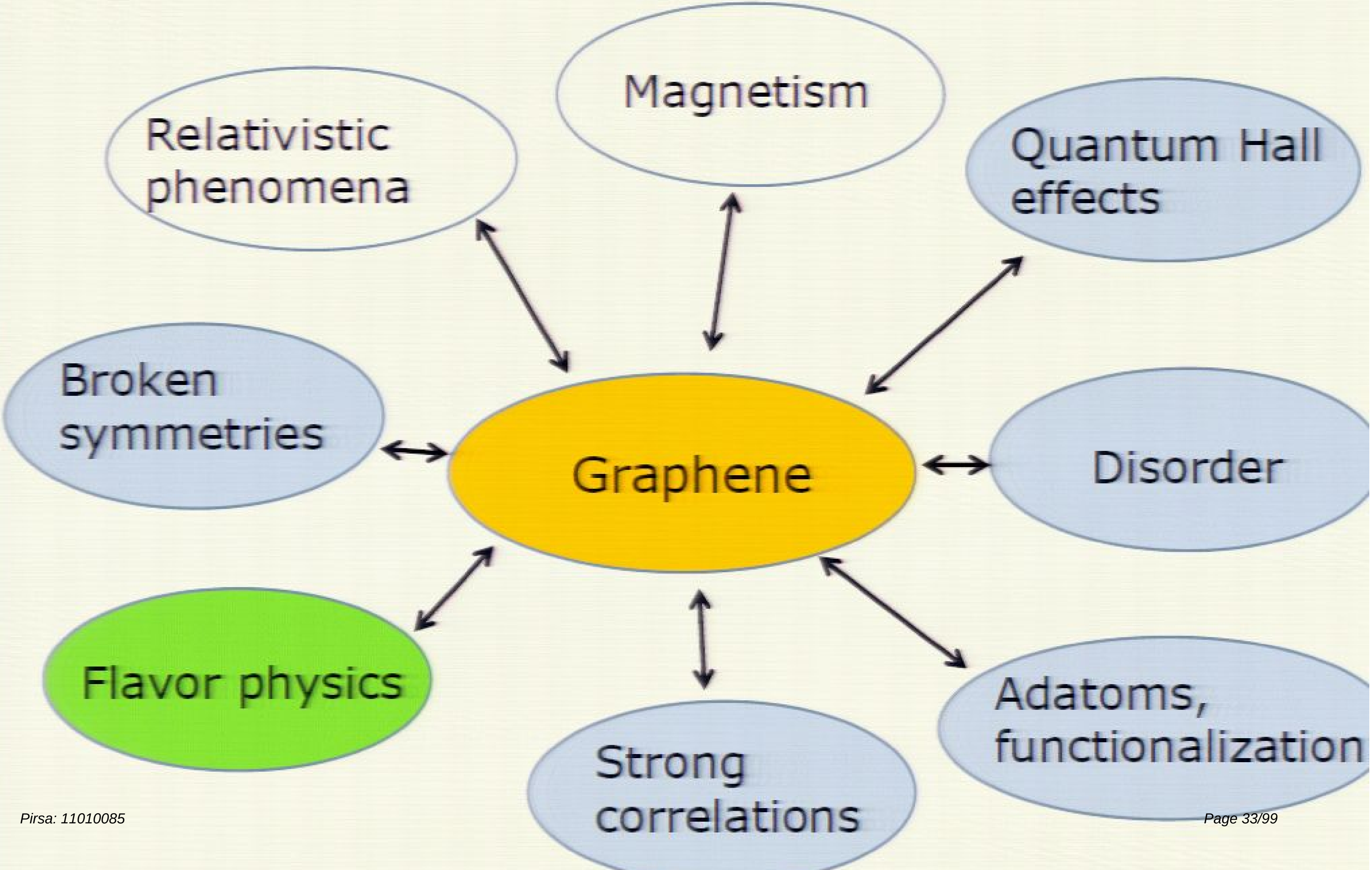
Disorder

Flavor physics

Strong correlations

Adatoms, functionalization

My work on graphene



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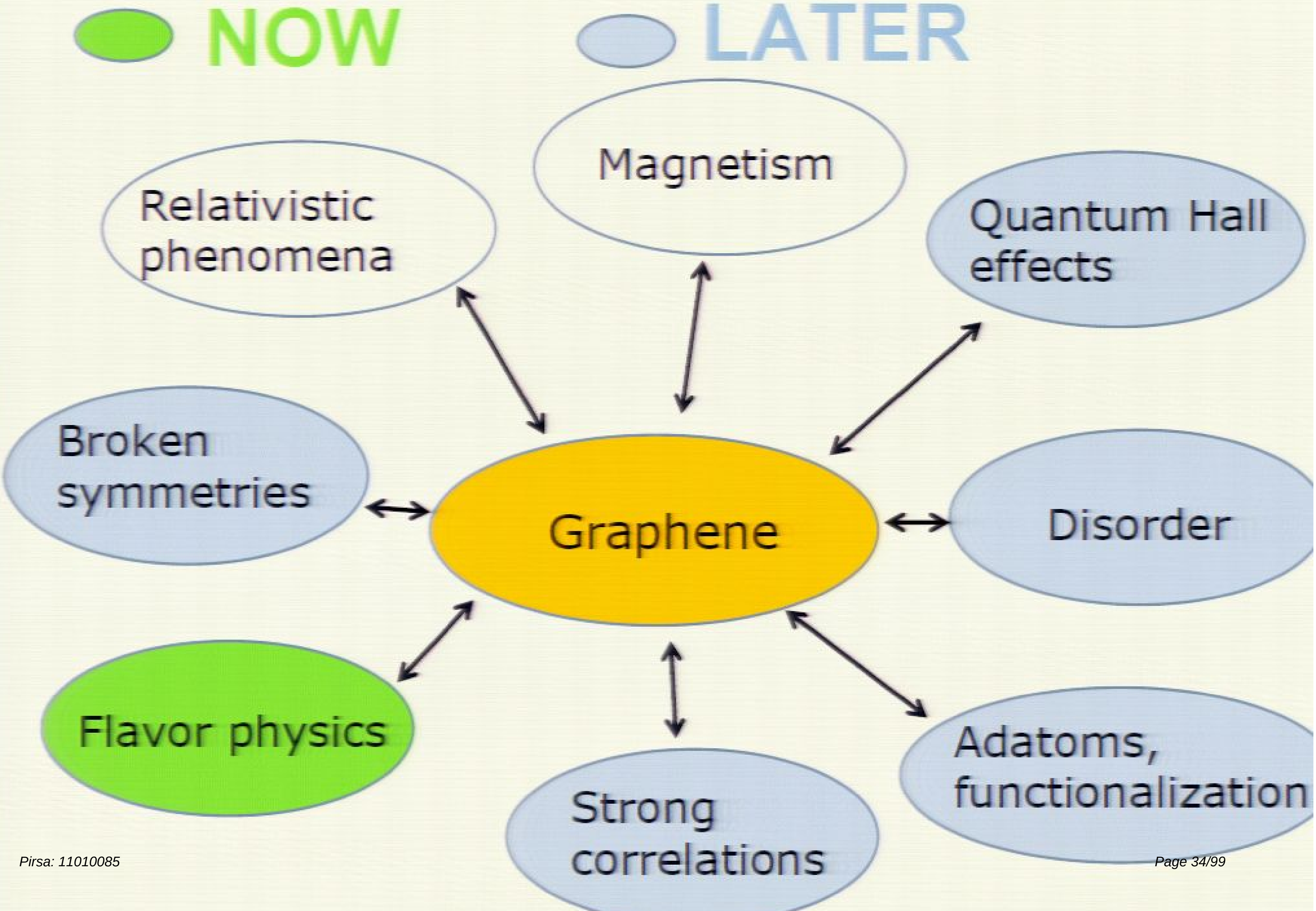


🟢 NOW

🔵 LATER

Magnetism

Relativistic phenomena

Quantum Hall effects

Broken symmetries

Graphene

Disorder

Flavor physics

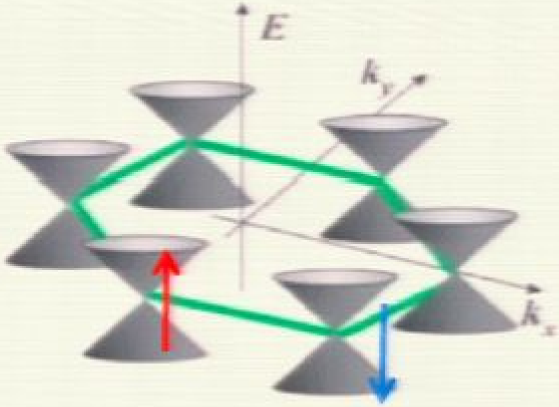
Strong correlations

Adatoms, functionalization

Neutral degrees of freedom

USE SPIN FOR INFORMATION PROCESSING

“Spintronics” -- many groups involved, very active field



GRAPHENE: spin, valley

-slow spin and valley relaxation

-spin qubits, spintronics

TOP. INSULATORS: spin, Majorana fermions

-locking of spin and momentum

Spintronics and valleytronics

nature

LETTERS

Electronic spin transport and spin precession in single graphene layers at room temperature

Nikolaos Tombros[1], Csaba Jozsa[1], Mihaita Popinciuc[2], Harry T. Jonkman[2] & Bart J. van Wees[1]

Ferromagnetic electrodes

BUT: short spin lifetime

nature

LETTERS

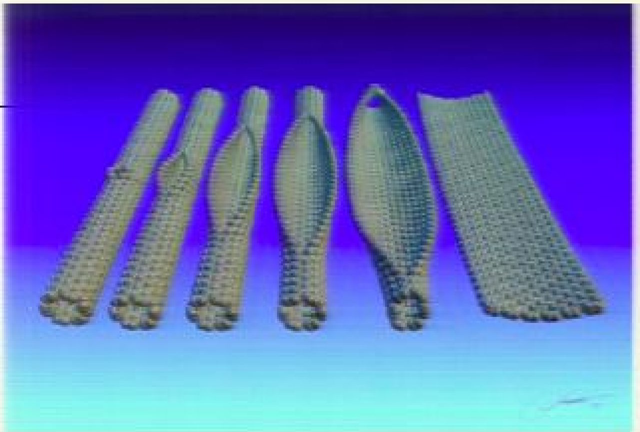
Half-metallic graphene nanoribbons

Young-Woo Son[1,2], Marvin L. Cohen[1,2] & Steven G. Louie[1,2]

LETTERS

Valley filter and valley valve in graphene

A. RYCERZ[1,2], J. TWORZYDŁO[3] AND C. W. J. BEENAKKER[1]*



Nanoribbons with **perfect edge**

BUT: impossible to make (disorder)

Spintronics and valleytronics

nature

LETTERS

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Ferromagnetic electrodes

nature

LETTERS

LETTERS

Valley

A. RYCERZ[1,2], J.

NEEDED:
A NEW WAY TO GENERATE & DETECT NEUTRAL MODES

Nanoribbons with **perfect edge**

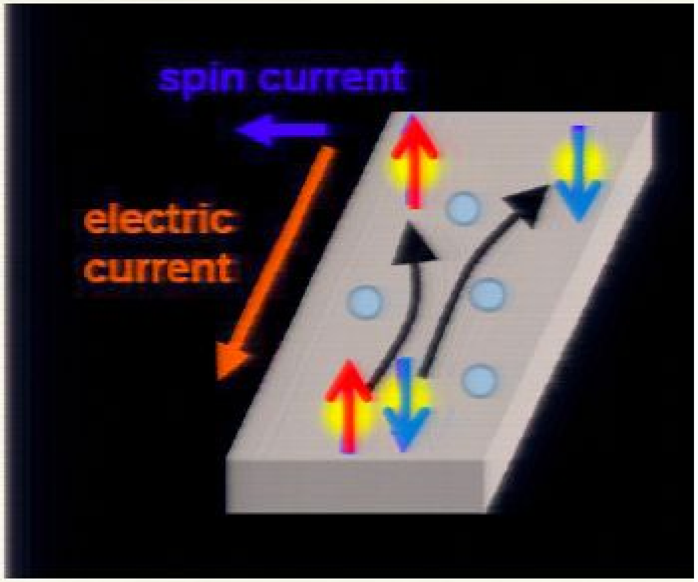
BUT: impossible to make (disorder)

Spin-Hall effect (conventional)

Charge current → spin current

$$j_s^x = \sigma_{SH} E_y$$ Dyakonov, Perel'71; Hirsch '99

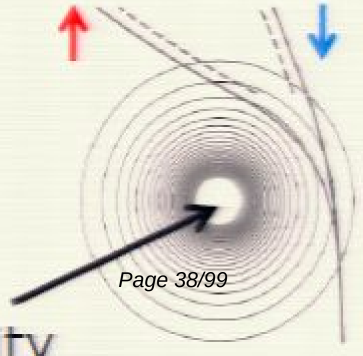
Due to spin-orbital scattering



Dimensionless
Spin-Hall coefficient $\xi = \dfrac{\sigma_{SH}}{\sigma_{xx}}$

Weak effect (SO small) $\xi \sim 0.0005$

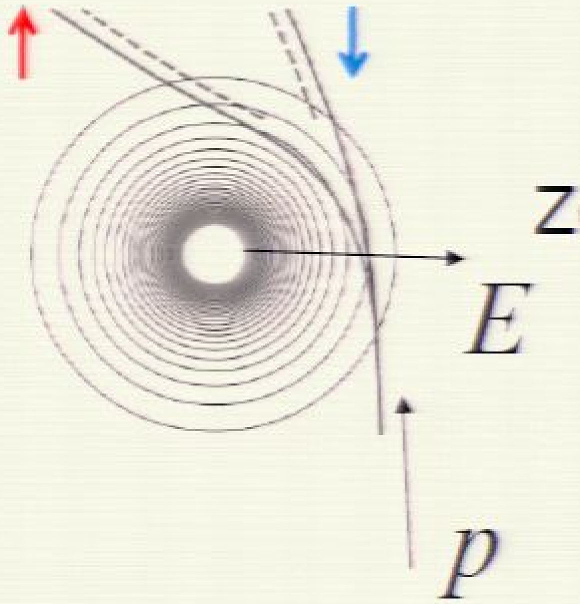
GaAs, Al, InAs, ..



Reminder: spin-orbit interaction

Moving frame, Lorentz transformation

$$B = \frac{[p \times E]}{mc}$$

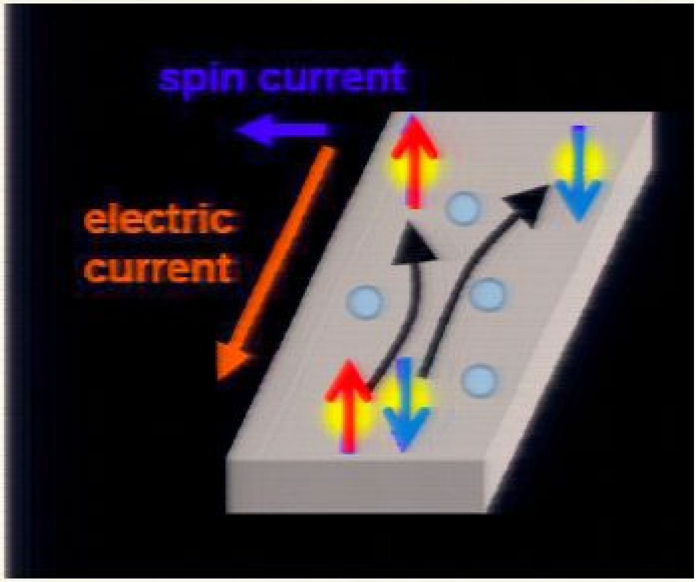


Zeeman interaction:

$$H_{SO} = g\mu_B \sigma B = \lambda \sigma [p \times E], \lambda = \frac{g\mu_B}{mc}$$

Spin-selective scattering

Spin-Hall effect (conventional)



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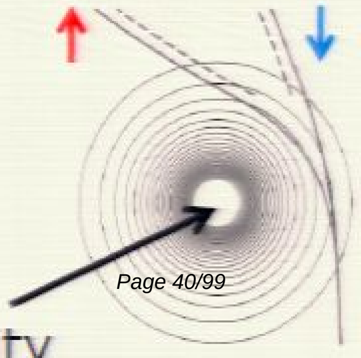

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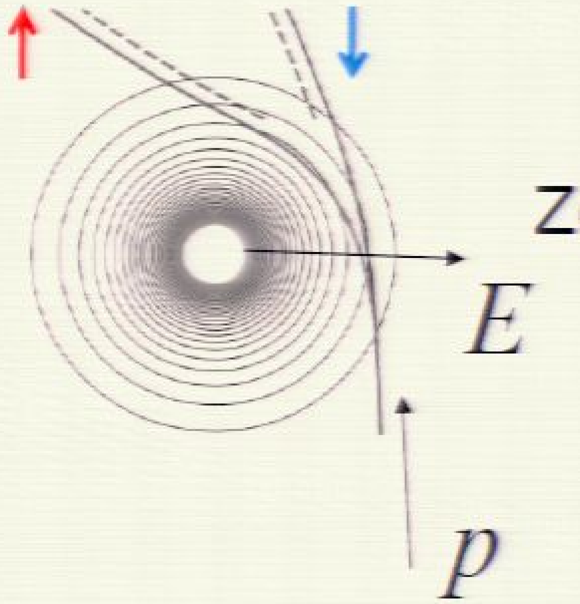


Zeeman interaction:

$$H_{SO} = g\mu_B \sigma B = \lambda \sigma [p \times E], \lambda = \frac{g\mu_B}{mc}$$

Spin-selective scattering

Exception: quantum spin Hall effect

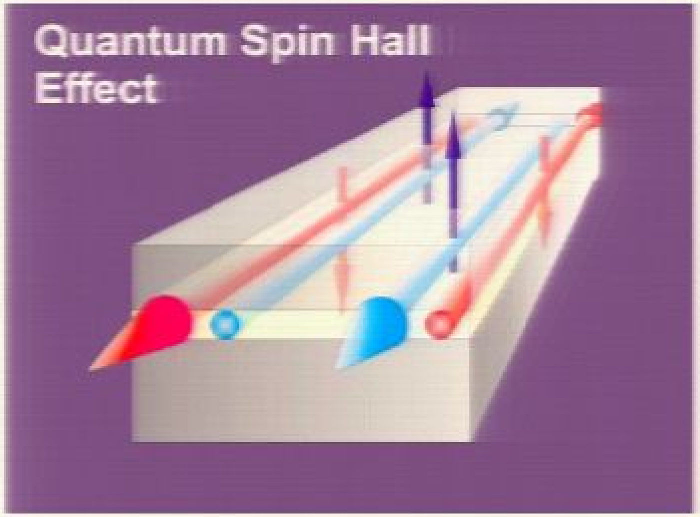
Kane, Mele '05

-Topologically non-trivial gap in the bulk

-Spin-filtered edge states

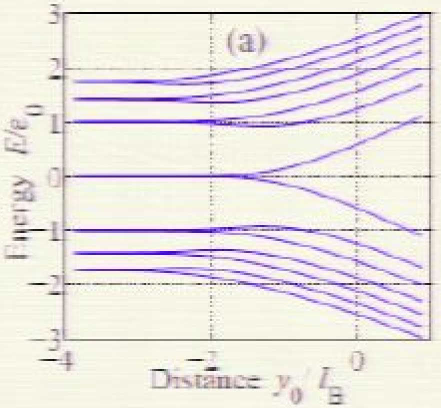
-Quantized $\sigma_{SH} = \frac{2e^2}{h}$ $\xi = 1$

-Observed in HgTe at T<1K



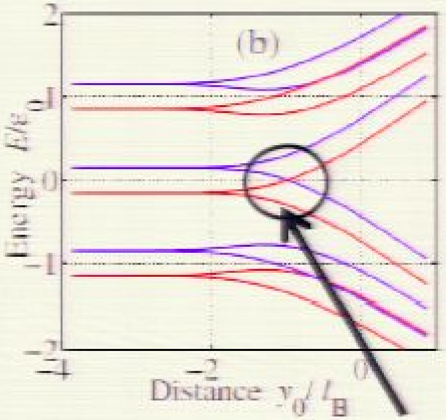
Bernevig et al '06, Konig et al'07

Quantum spin-Hall effect in graphene



DA,Lee,Levitov,PRL'06,SSC'07
Brey,Fertig, PRB'06

Spin splitting



DA,Lee,Levitov, PRL'06, DA, Novoselov,Lee, Geim,Levitov, PRL'07

spin-filtered edge states

-Quantizing B-field

-Originates from unusual Landau levels

-Expected up to T~100K

-Not yet observed (clean edge needed)

This work: giant SHE w/o spin-orbit

-Giant, even at room *T* and low *B*

$$\xi \approx 0.1$$

-Stems from Dirac spectrum

-Naturally generalized for valley

This work: giant SHE w/o spin-orbit

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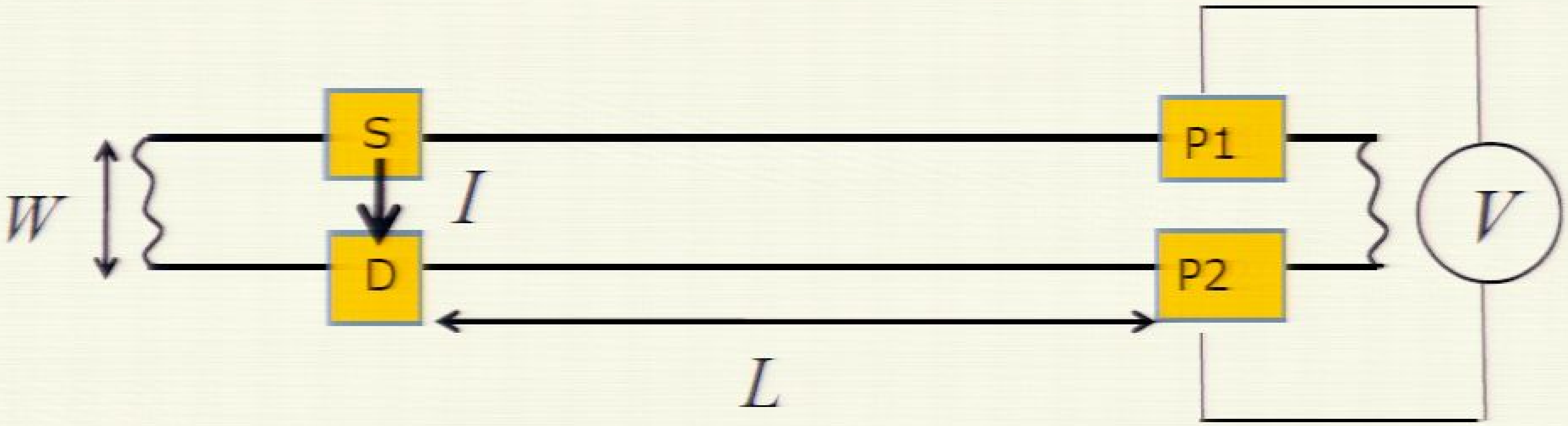
-Stems from Dirac spectrum

-Natu

NUMBER IMPORTANT!

Giant magneto-resistance (MR)	"Colossal" MR
dR/R=0.1	dR/R=0.8
Buckley prize'09	Nobel prize'07
(Moodera,Miyazaki, Mezervey,Tedrow)	*(Grunberg,Fert'88)*

How to detect SHE?



-Nonlocal transresistance $R_{nl} = V / I$

-Charge contribution (no neutral modes)

How to detect SHE?

-Nonlocal transresistance $R \quad = V / I$

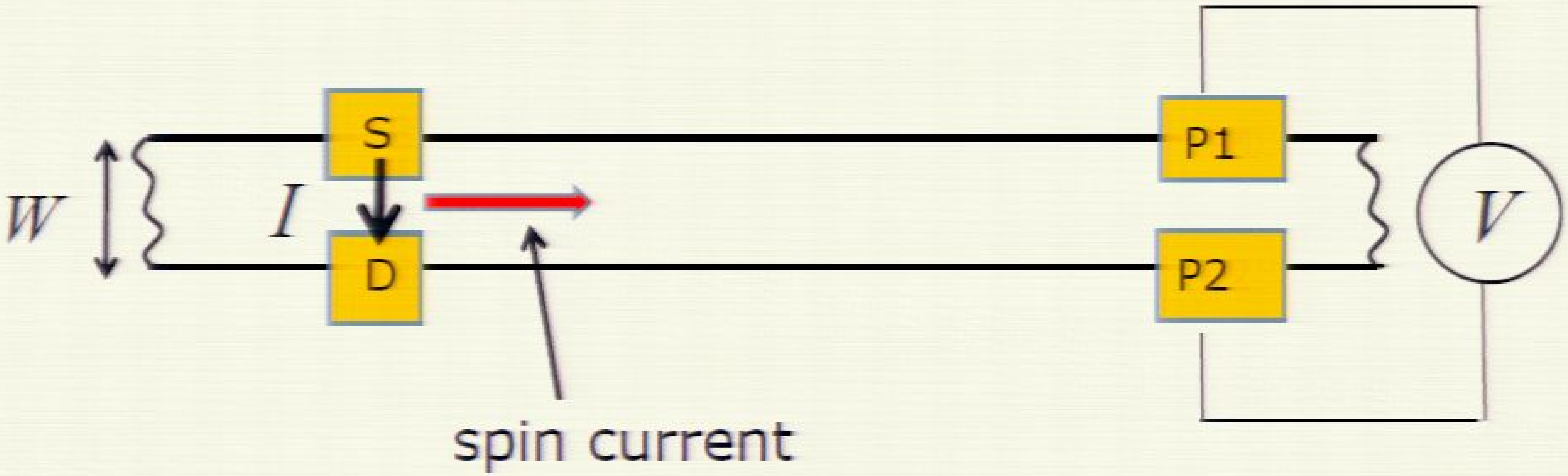
A:

$$R_{nl} = \rho e^{-\pi L/W} \approx 10^{-7}\,\Omega$$

NEGLIGIBLE!

-Charge contribution (n s)

How to detect SHE?



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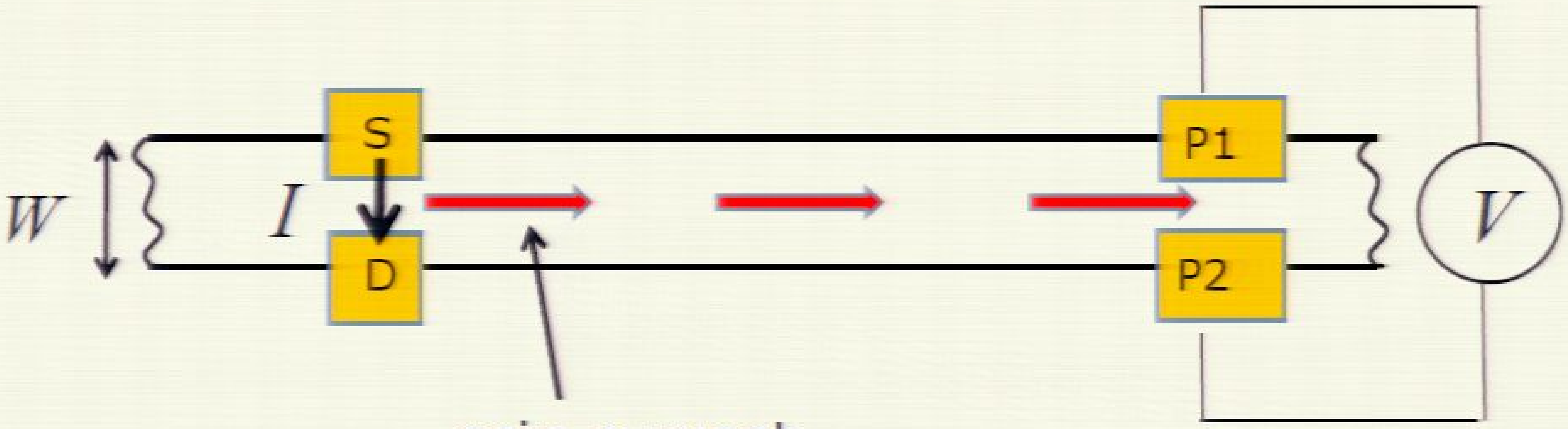
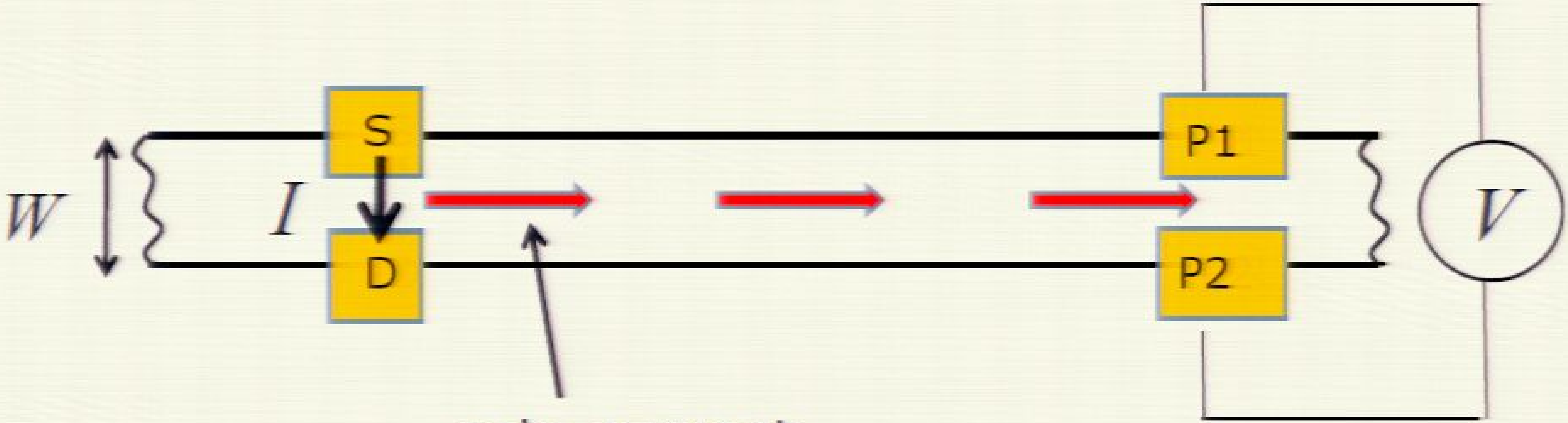


spin current

Current → spin current→ Propagates far away → Nonlocal Voltage

DA, Shytov, Levitov, Halperin PRB '09

How to detect SHE?



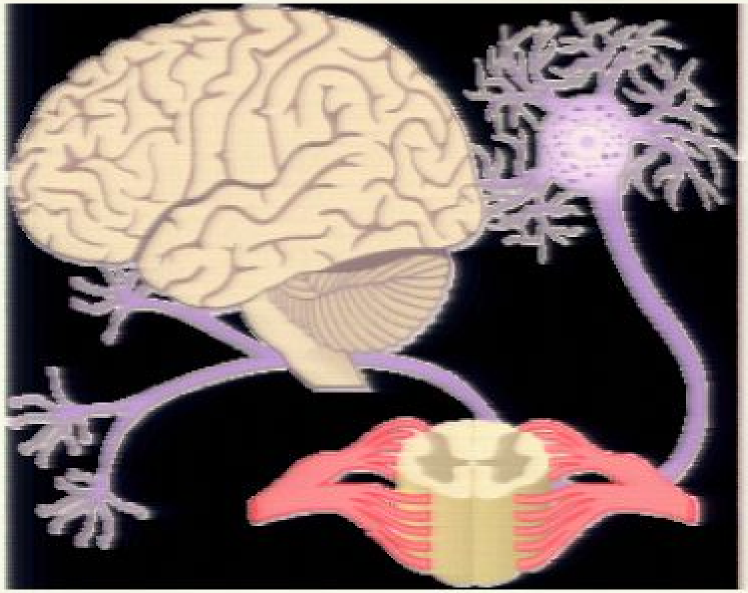
spin current

Current → spin current→ Propagates far away → Nonlocal Voltage

Message: SHE manifest itself in nonlocal electric transport

DA, Shytov, Levitov, Halperin PRB '09

Neurons and nonlocality

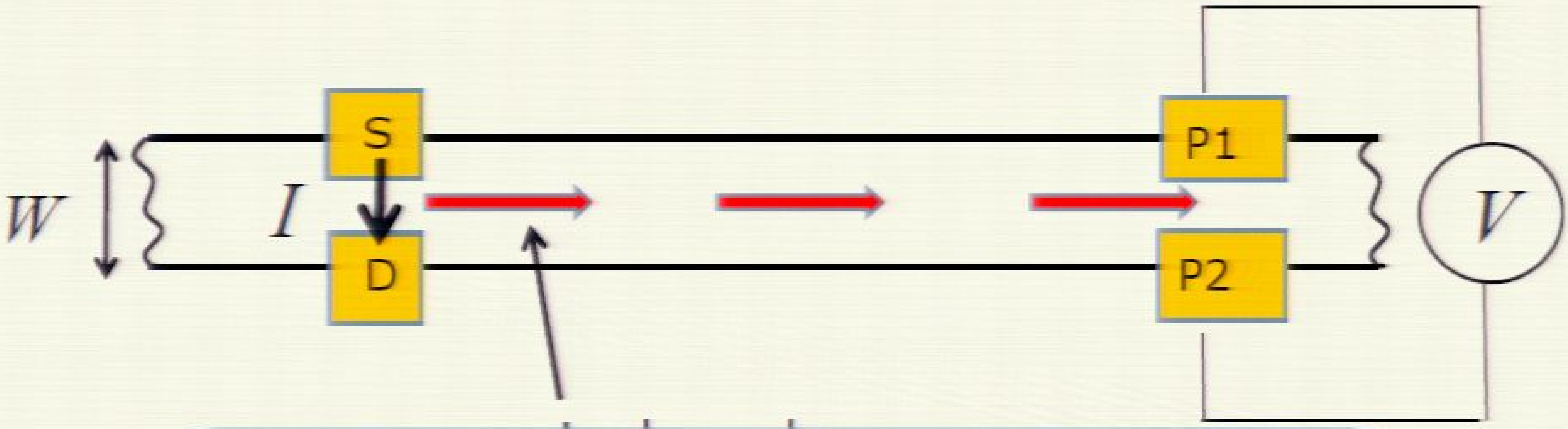


Neuron=spin mode

Monitor=electric current

Brain=Voltmeter

How to detect SHE?



Originally predicted for GaAs;

Small $R_{nl} \propto \xi^2 \rho_{xx} < 10^{-6}\Omega$

DA, Shytov, Levitov, Halperin PRB '09

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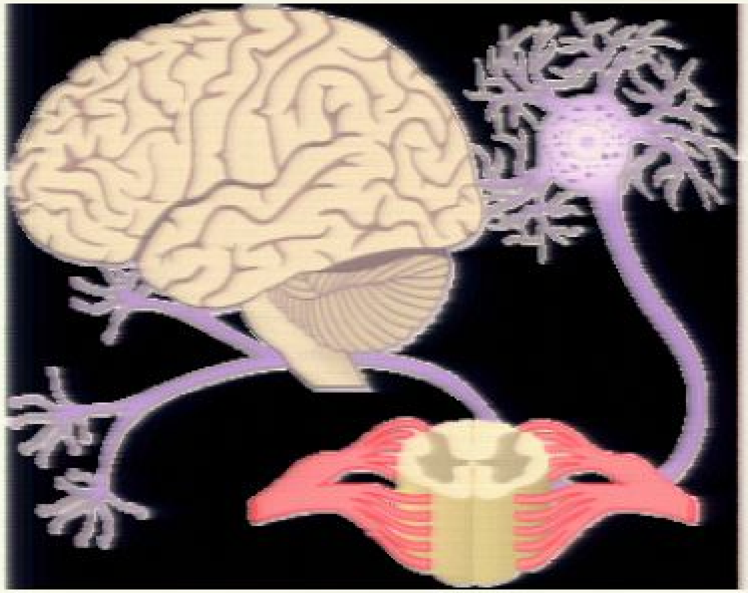

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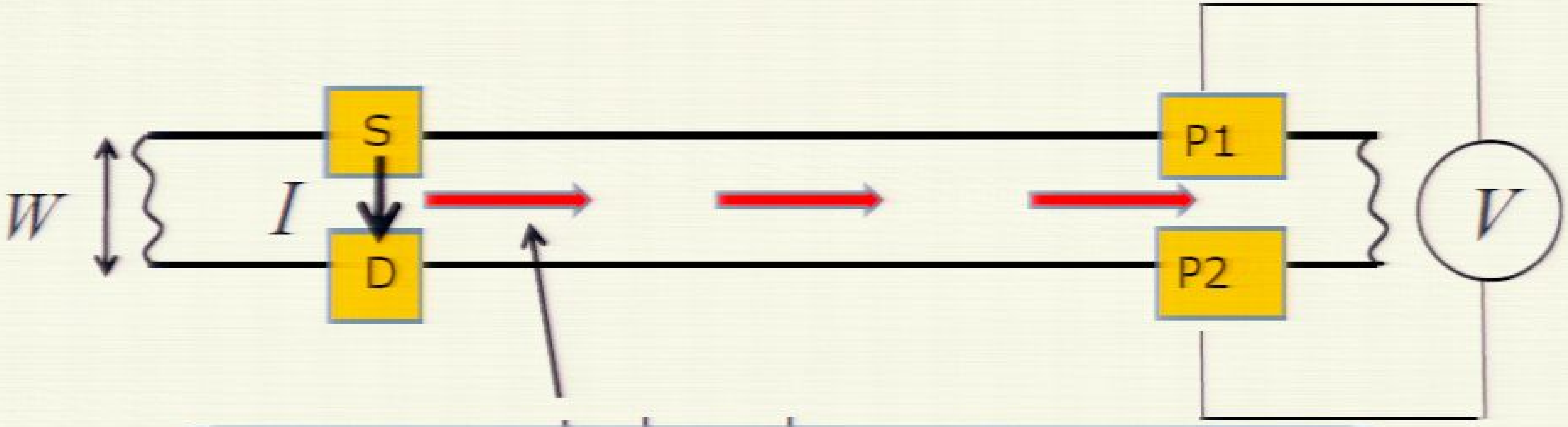

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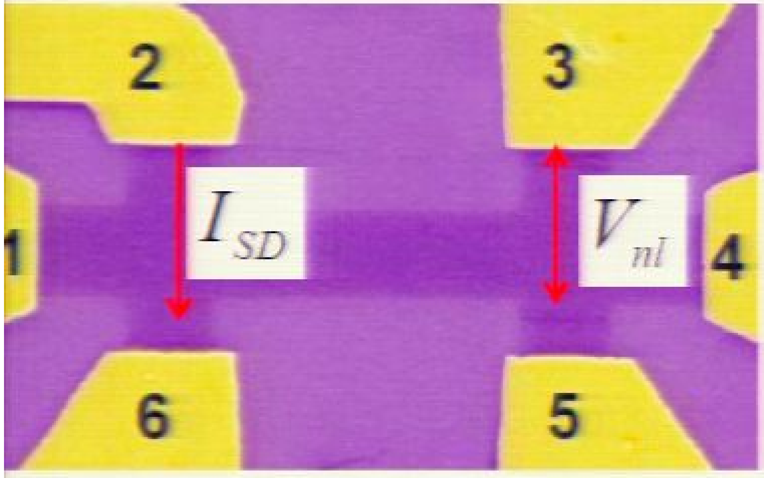
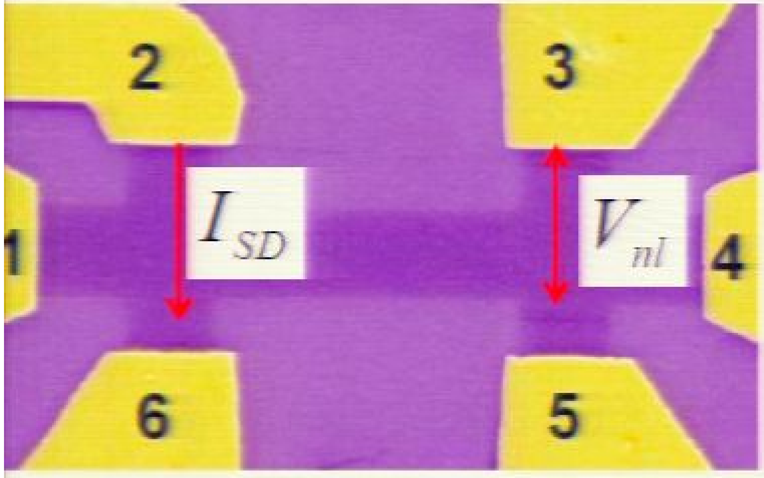
Search for neutral modes in graphene



$$R_{nl} = \frac{V_{35}}{I_{26}}$$

$R_{nl} = 0 @ B = 0$ where it is only charge

Search for neutral modes in graphene



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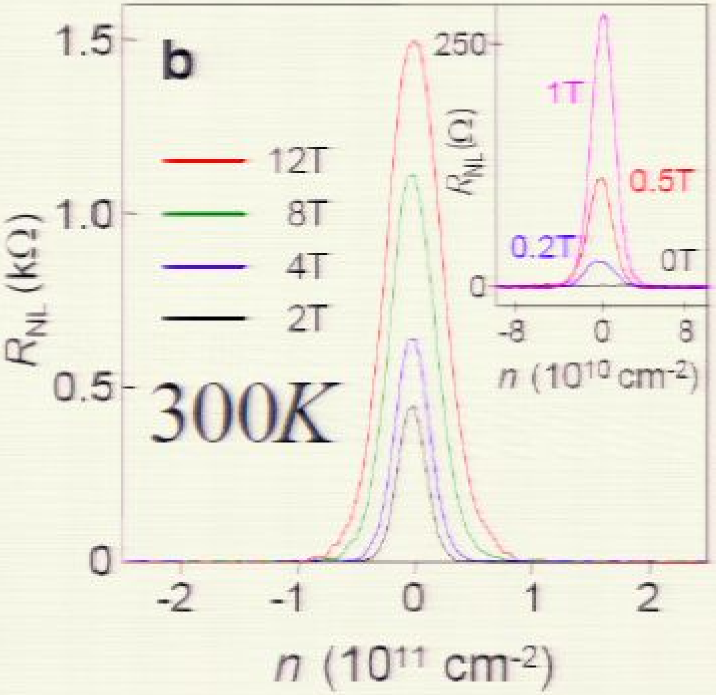
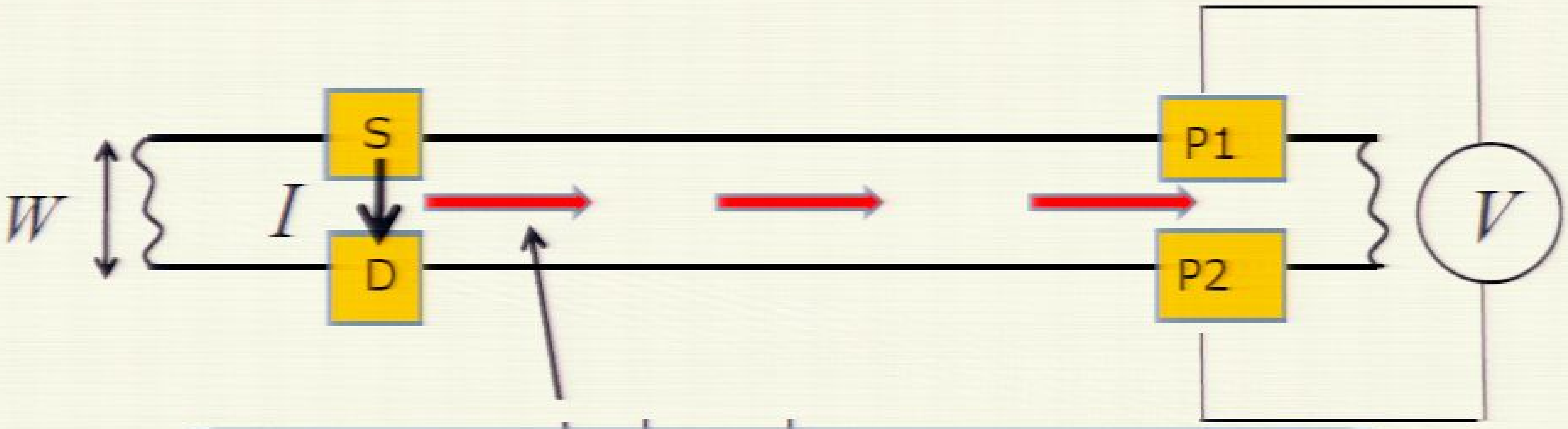


$R_{nl} = 0 @ B = 0$ where it is only charge

Giant nonlocality in a broad range of B, T

How to detect SHE?



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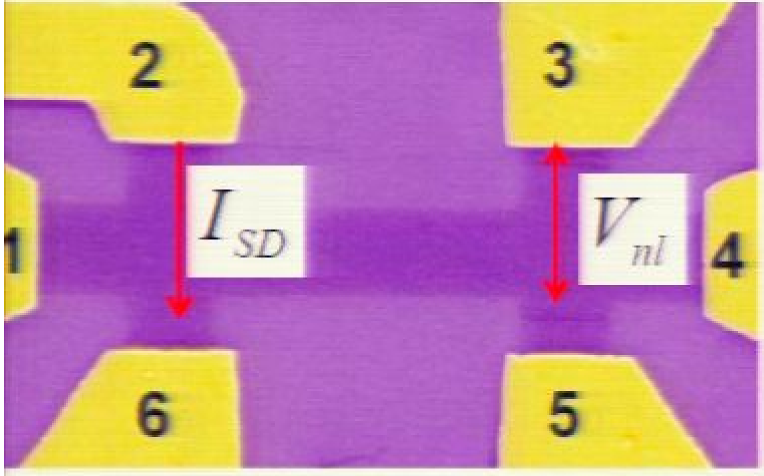

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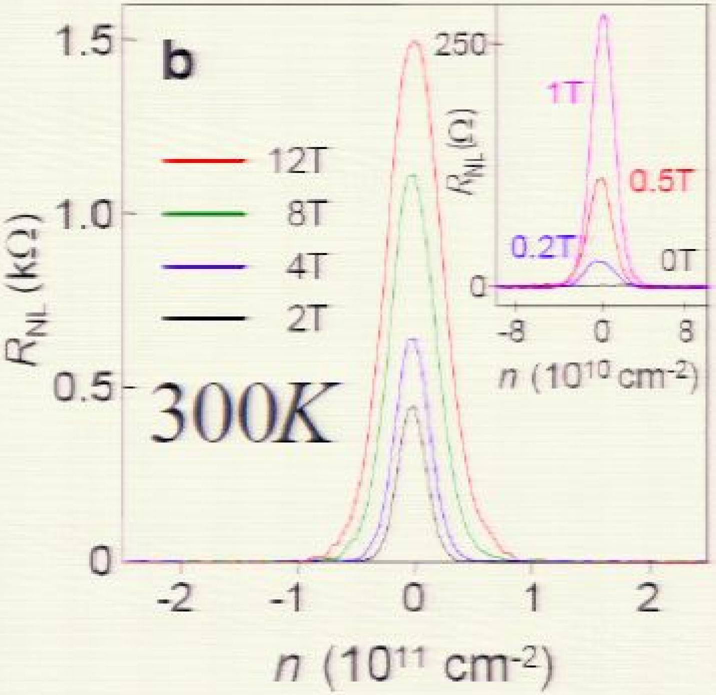
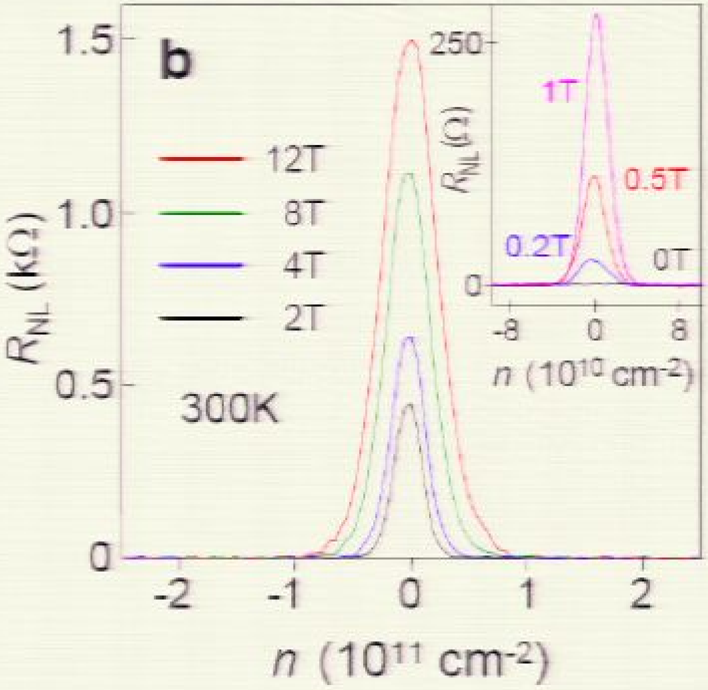


$R_{nl} = 0 @ B = 0$ where it is only charge

Giant nonlocality in a broad range of B, T

Features

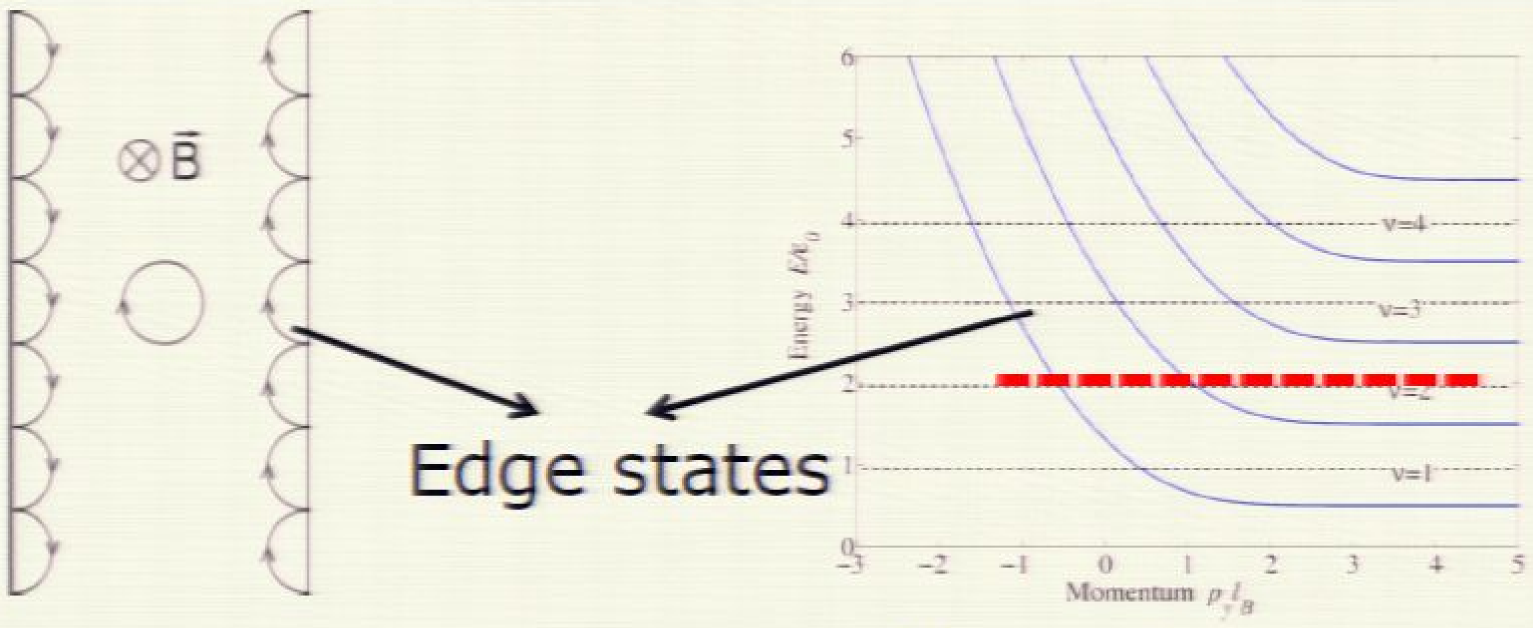
- -Peaked at the Dirac point
- -Grows with B, 1/T
- -Largest in clean samples



Precedents

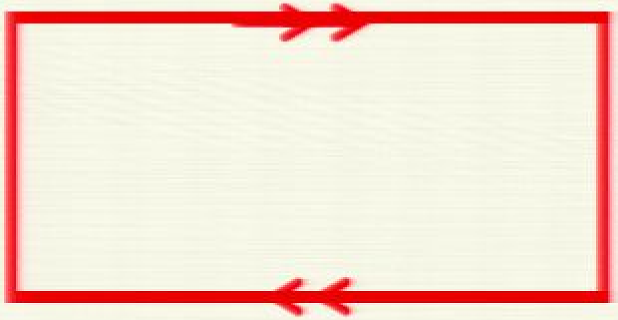
Nonlocality from edge transport
in quantum Hall effect regime

Recall quantum Hall edge states



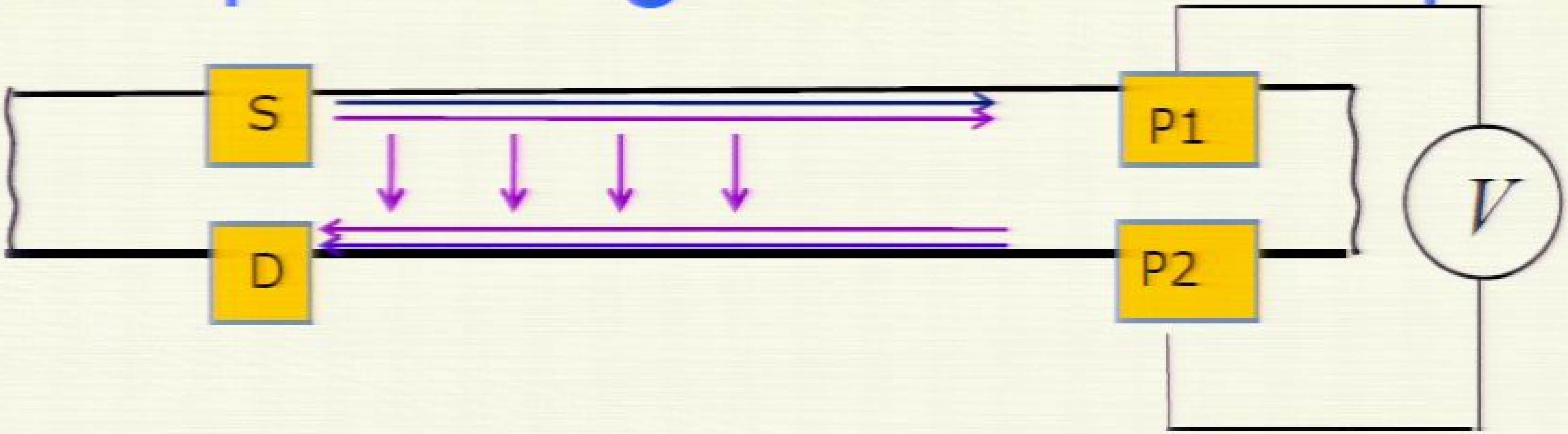
Position space

Momentum space



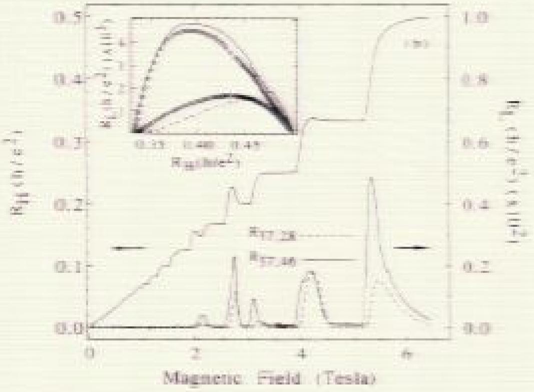
Unidirectional (chiral) edge modes in QHE; One per Landau level

Nonlocal transport from coupled edge states transport



-Edge states or bulk alone → no nonlocality

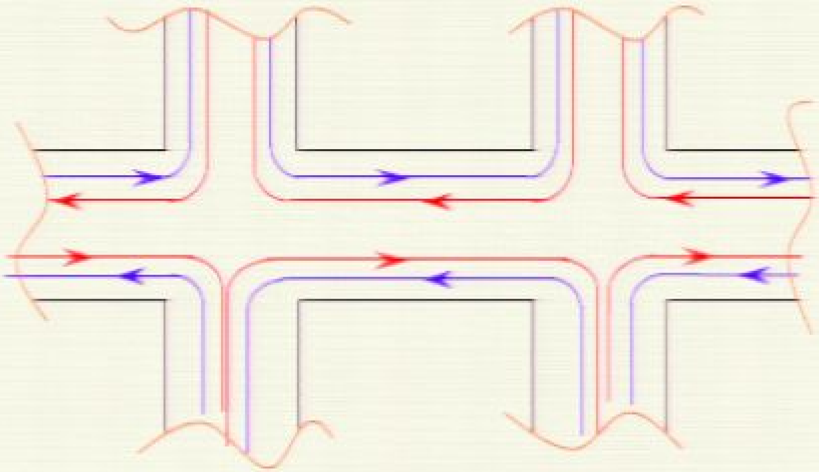
-2+ edge states+bulk →nonlocality



Nonlocal transport from QSHE

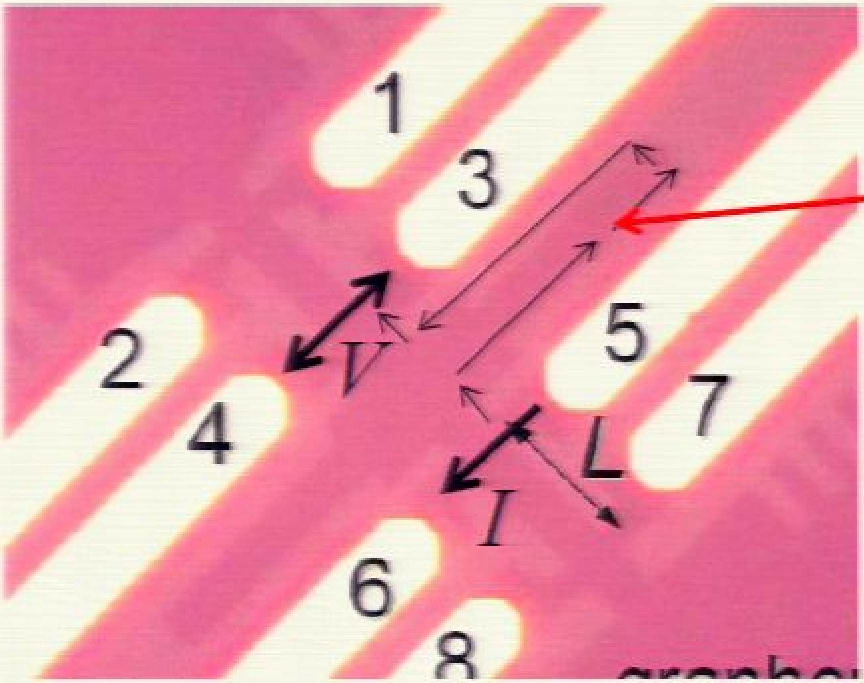
Counter-propagating edge states

Landauer-type 1D transport, nonlocality



DA, Lee, Levitov '06

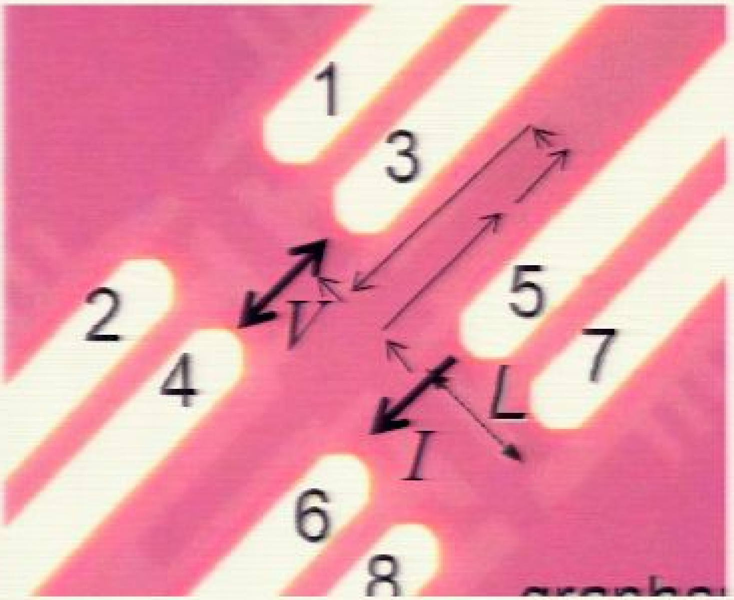
Ruling out the edge state scenario

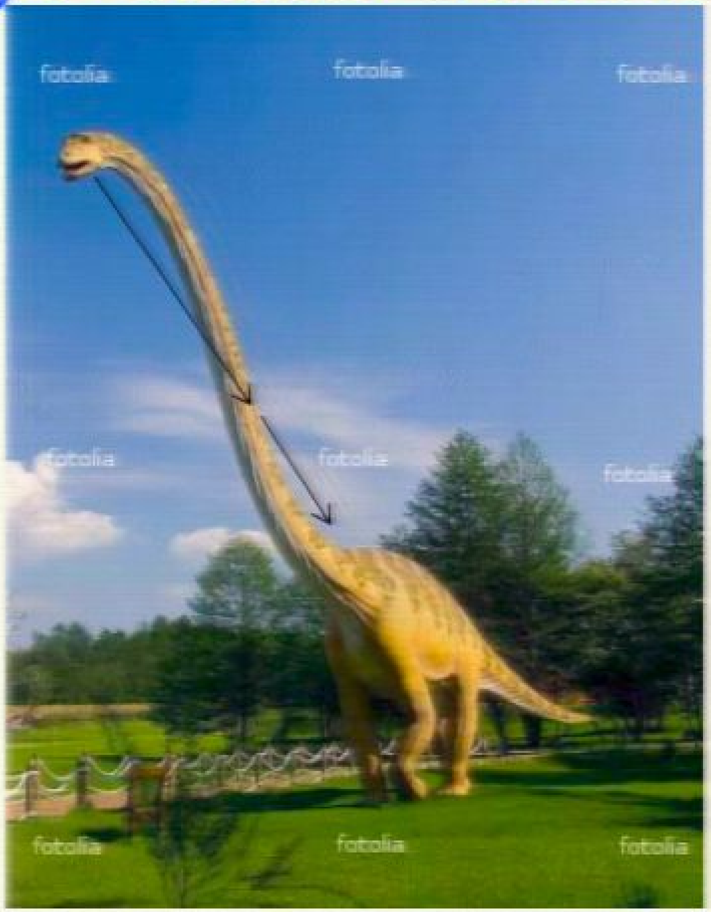


Sample with a belly, extra edge

$$R_{nl} = \frac{V_{34}}{I_{56}}$$

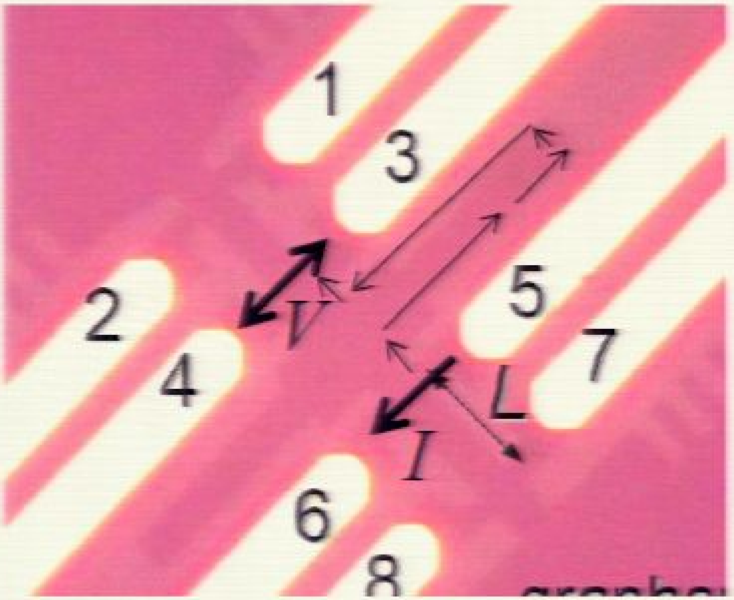
Ruling out the edge state scenario

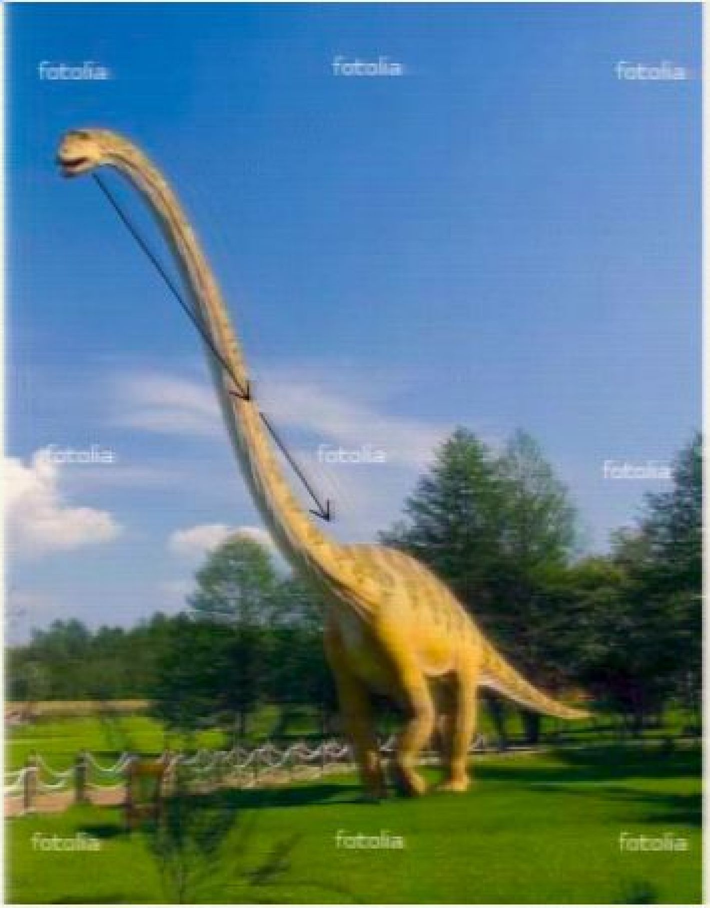




Edge current would be extinct as dinosaurs

Ruling out the edge state scenario



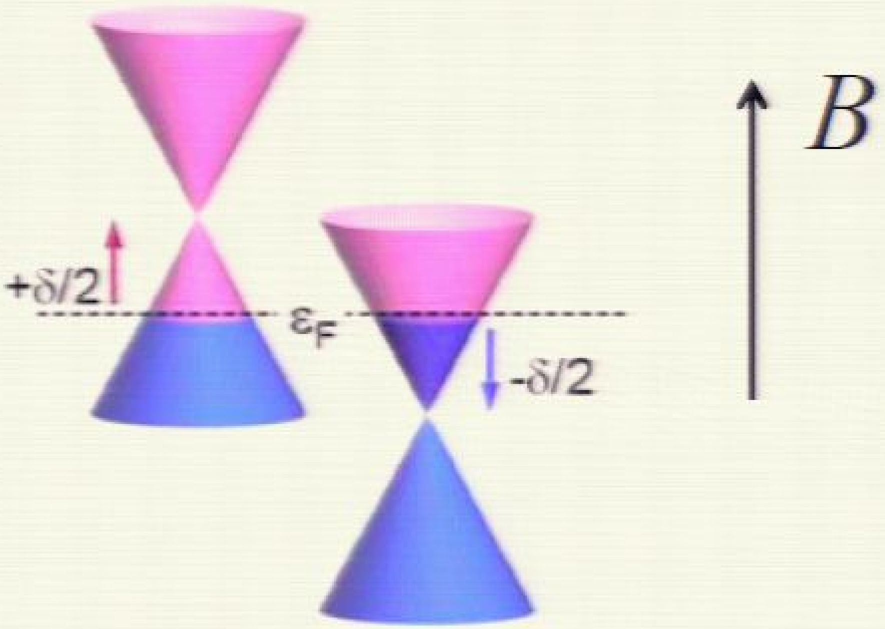


Edge current would be extinct as dinosaurs

BUT: nonlocality is not affected by long edge

Thus, we attribute the nonlocality to the spin-Hall effect

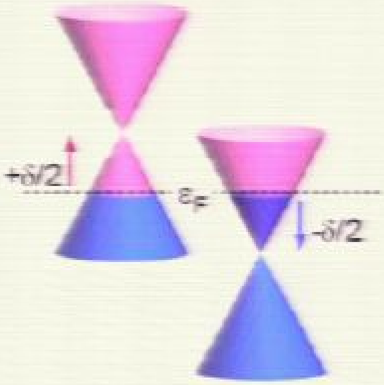
Key idea

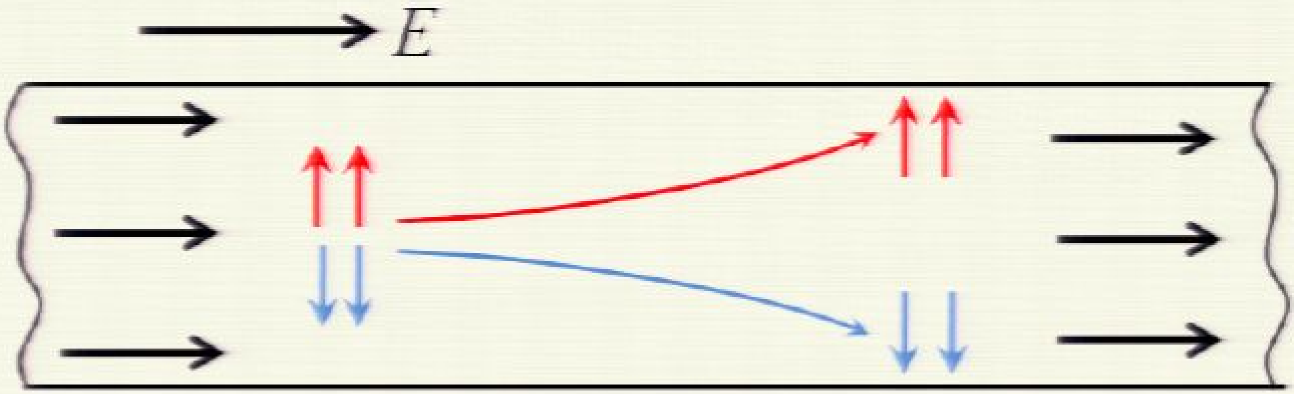


Zeeman-split bands $\varepsilon_{\uparrow(\downarrow)}(k) = vk \pm \delta / 2$

Finite density of electrons and holes

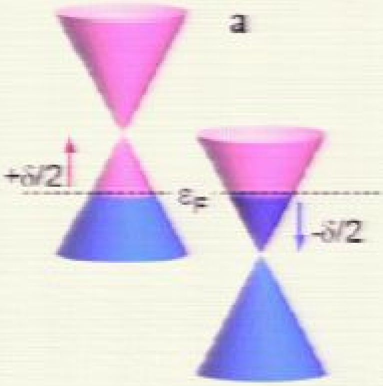
Spin-Hall effect without spin-orbit





Opposite Lorentz force on the up-spin and down-spin

Spin current in the transverse direction



Is it a sizable effect?

Connect to Hall resistivity

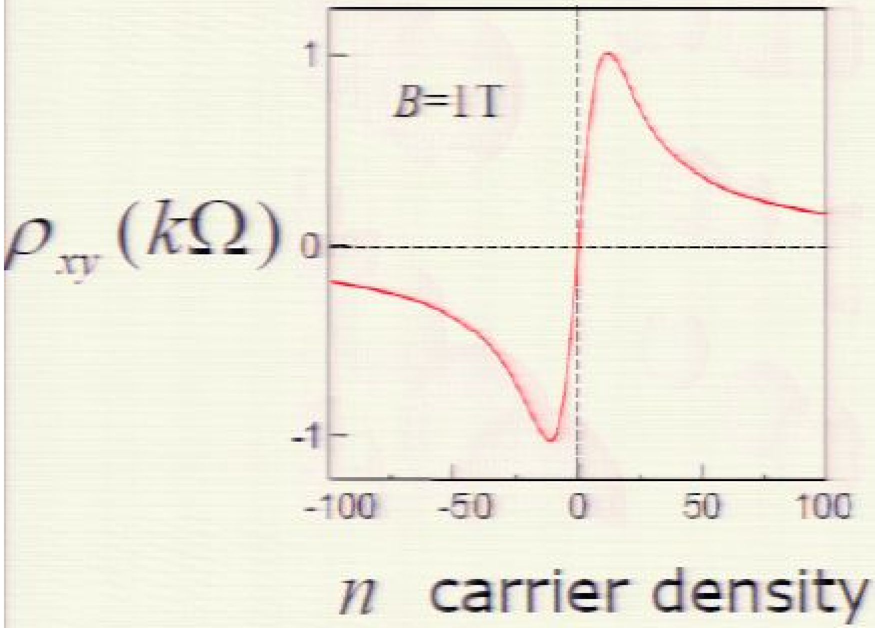
Spin-Hall coefficient

$$\xi = \frac{\rho_{SH}}{\rho_{xx}} \propto \rho_{xy}^{\uparrow} - \rho_{xy}^{\downarrow} \approx \frac{d\rho_{xy}}{d\mu}\Delta$$

Δ -spin splitting

Need to understand Hall resistivity

Steepened ρ_{xy}, giant SHE



Quasiclassical result:

$$\rho_{xy} = -\frac{B}{ne}$$

Diverges at the Dirac point

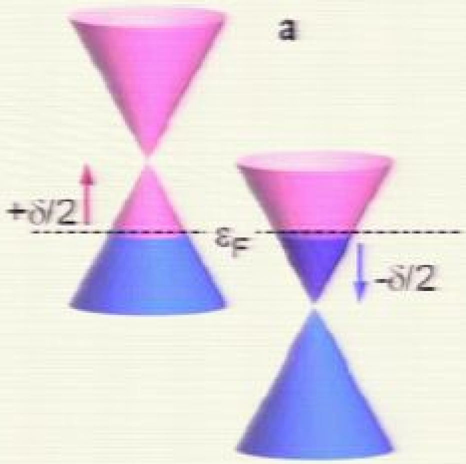
Singularity smeared by disorder and interactions

Steepening→ large $\frac{d\rho_{xy}}{d\mu}$ → giant SHE at the Dirac point

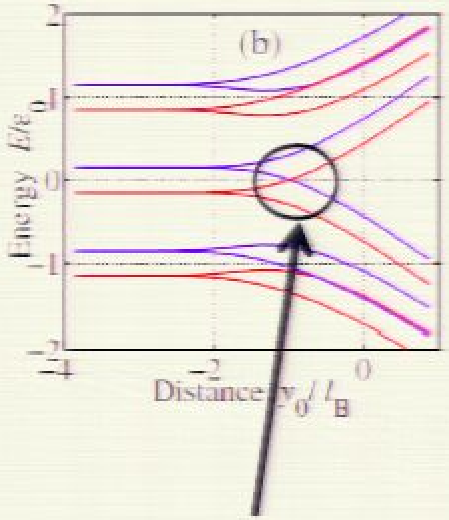
Crossover to QSHE at large B, low T

Quasiclassical

Quantum



B↗ T↘



DA,Lee,Levitov, PRL'06, Brey,Ferti PRB'06

DA, Novoselov,Le Geim,Levitov, PRL'07

-NO Landau levels
-Bulk transport
-giant SHE

$$\xi \sim 0.1$$

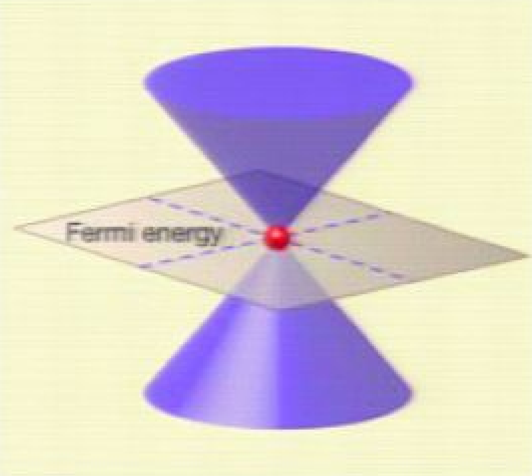
-Landau levels
-Edge states
-quantum (colossal) SHE

$$\xi \sim 1$$

Crossover observed experimentally (Geim et al'10)

Calculating ρ_{xy}

Find $\dfrac{d\rho_{xy}}{d\mu}$ at $\mu = 0$?



+disorder+interactions

No Fermi surface!

Unconventional transport

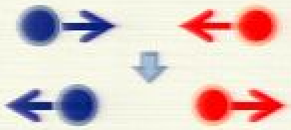
(Drude formula doesn't apply)

Two scattering mechanisms

-Electrons AND holes

-e-e, h-h collisions: ***p, j*** conserved

-e-h collisions: ***p*** conserved,
j NOT conserved



-Disorder scattering

-Both mechanisms contribute to resistivity

Two scattering mechanisms

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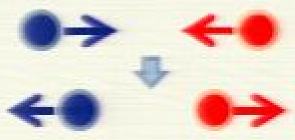
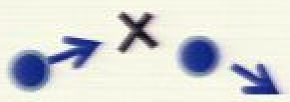


High T

-Disorder scattering



Low T

-Both mechanisms contribute to resistivity

Two scattering mechanisms

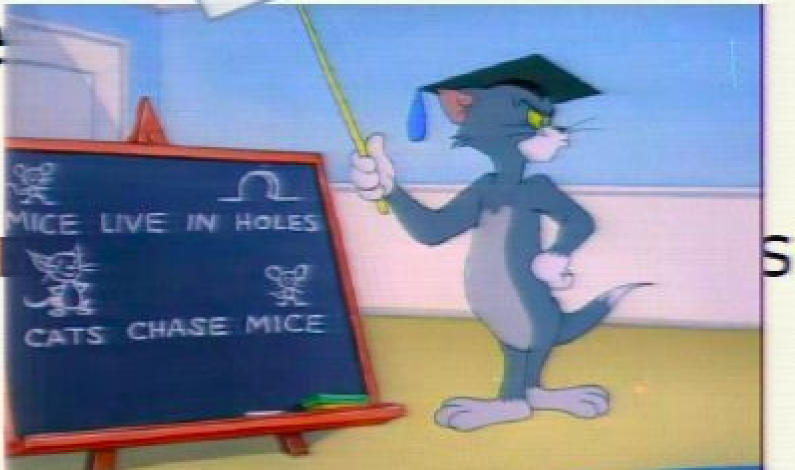
-Electrons AND holes

-e-e, h-h collisi

Analyze high T
Extrapolate to low T

-e-h collisic **High T**
conserved

-Disorder scatte **Low T**



-Both mechanis stivity

High temperature regime

-At the Dirac point

-Finite DC conductivity even without disorder

$$\rho_{xx} = \eta \frac{h}{e^2} \qquad \eta \sim \alpha^2 \qquad \alpha = \frac{e^2}{\hbar v}$$

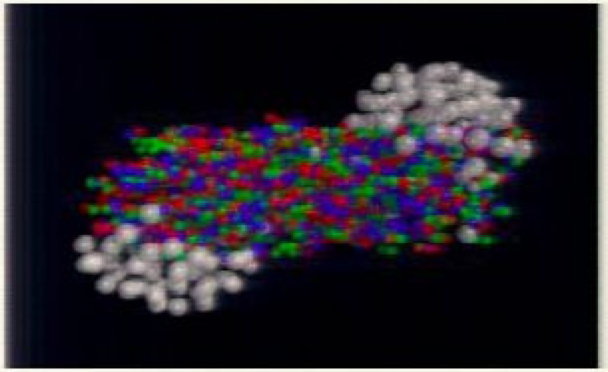
-friction coefficient
between
electrons and holes

-Hydrodynamic regime

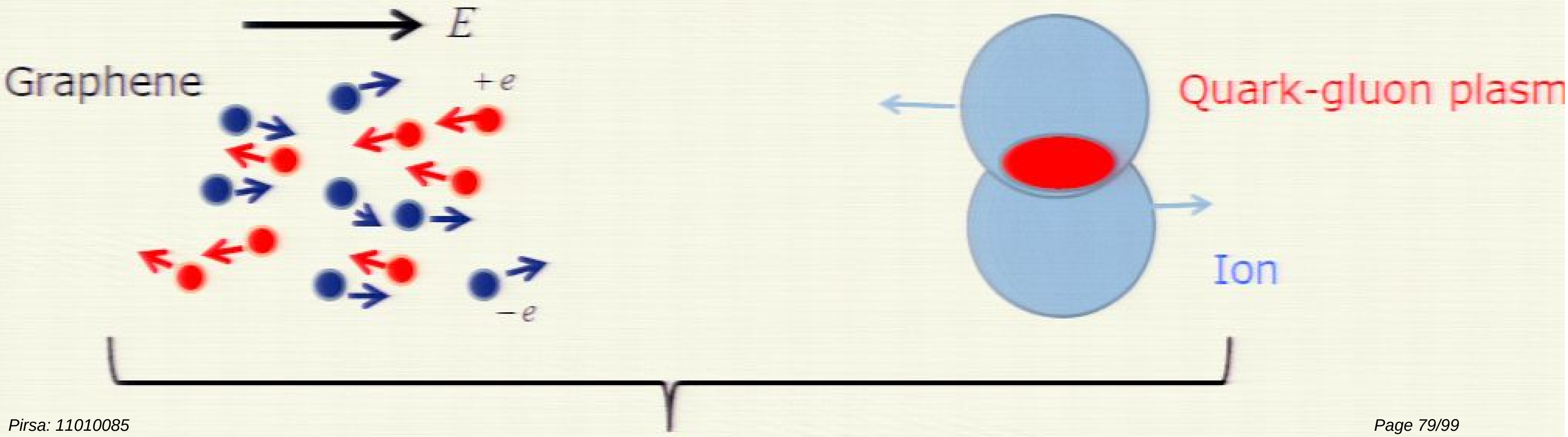
-Scale-invariant

Unexpected analogy

RHIC, Brookhaven



- -Quark-gluon plasma
- -Hot, relativistic liquid of quarks
- -Concept of viscosity



Nearly perfect fluids

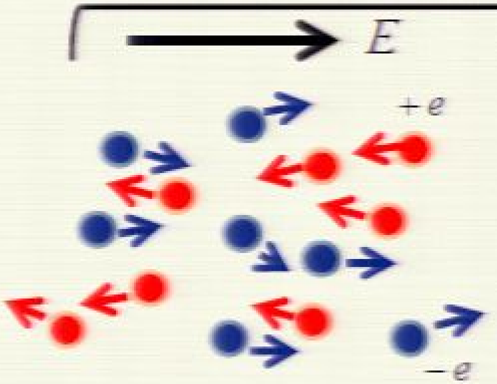
String theory conjecture: minimum shear viscosity
(Kovtun, Son, Starinets '05)

$$\frac{\nu}{s} \geq \frac{1}{4\pi}$$

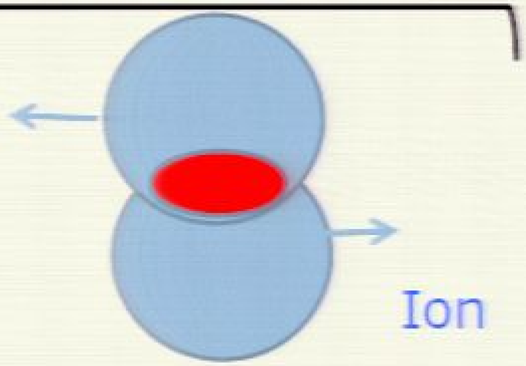
ν	shear viscosity
s	entropy density

Near-minimum viscosity

Mueller, Fritz, Schmalian PRL'09



Graphene



Quark-gluon plasma

Romatschke, Luzum PRL'0

Nearly perfect fluids

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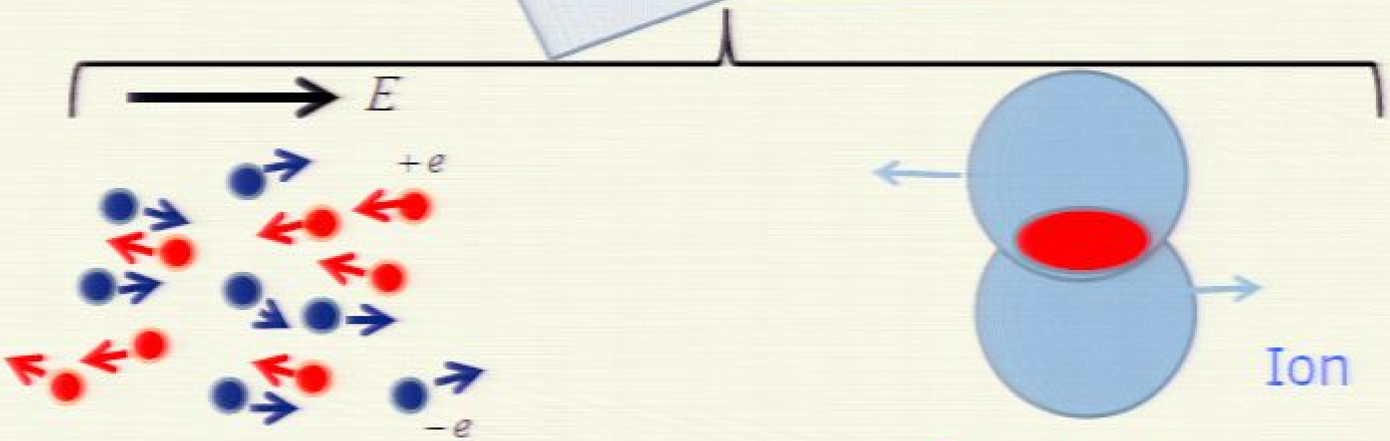
(Kovtun, Son, Starinets '05)

Q: An experiment to observe viscosity in graphene?

Mueller, Fritz, Schmalian PRL'09



Romatschke, Luzum PRL'0

Graphene

Quark-gluon plasma

Solving quantum kinetic equation

-Two groups of carriers, electrons and holes $n_e \neq n_h$

-Two-component ansatz *(Gantmakher,Levinson '78)*

$$f_{e(h)} = \frac{1}{e^{(\varepsilon(p) - V_{e(h)} \cdot p - \mu)/T} + 1}$$

$V_{e(h)}$

electron (hole)
velocity

-Coupled equations for $V_{e(h)}$

DA, Novoselov,
Geim, Levitov '11

-Mutual friction

-*Assumption: weak Coulomb interaction,* $\alpha = \frac{e^2}{\hbar v} << 1$

Resistivity tensor

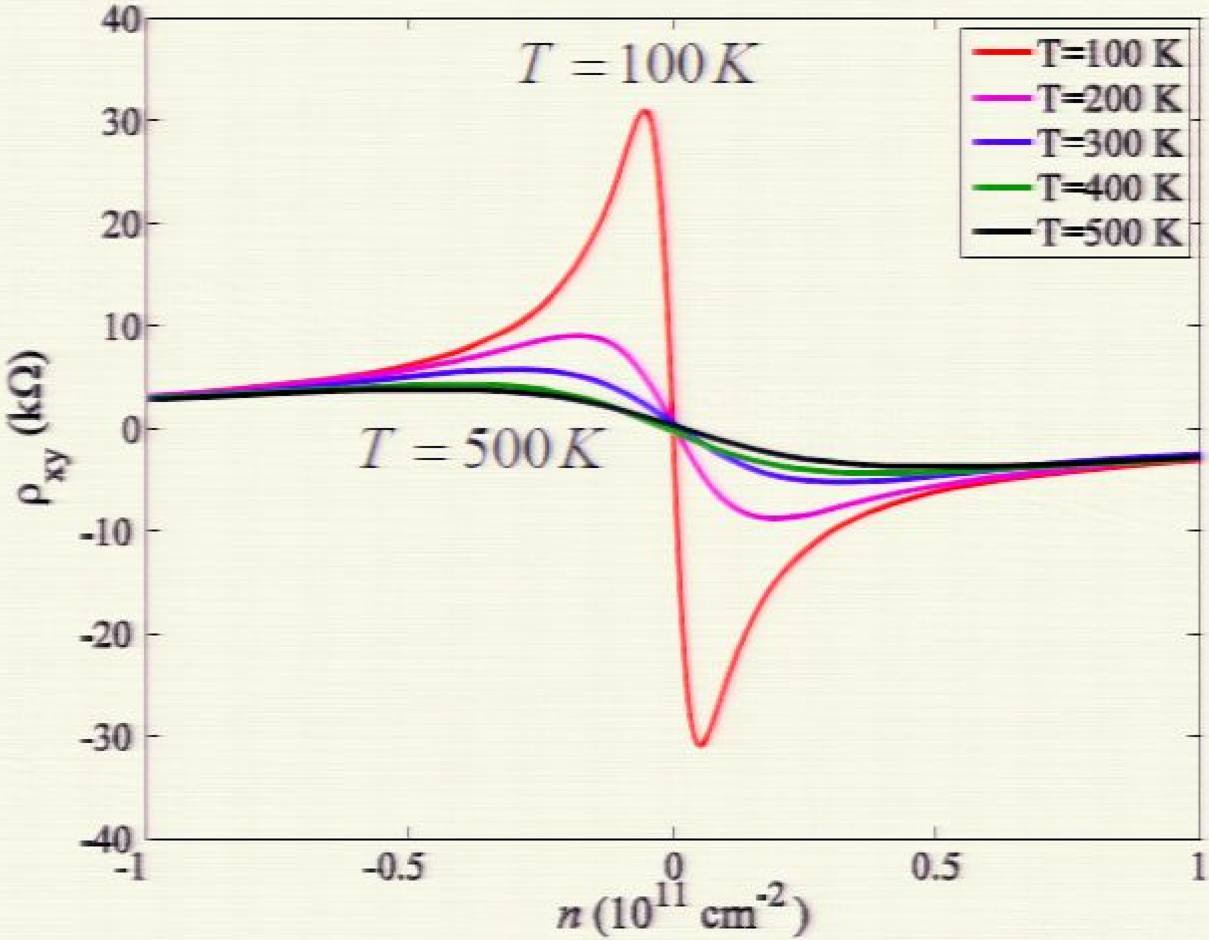
$$\rho_{xx} + i\rho_{xy} = \frac{D}{C}\frac{\hbar}{e^2}$$

$$D = \frac{1}{\tilde{\tau}_e}\frac{1}{\tilde{\tau}_h} + \eta\frac{n_e}{m_h}\frac{1}{\tilde{\tau}_e} + \eta\frac{n_h}{m_e}\frac{1}{\tilde{\tau}_h}, \quad \frac{1}{\tilde{\tau}_i} = \frac{1}{\tau_i} - \alpha_i i\Omega_i \qquad \Omega_i = eB/m_i$$

$$C = \frac{n_e}{m_e}\frac{1}{\tilde{\tau}_h} + \frac{n_h}{m_h}\frac{1}{\tilde{\tau}_e} + \frac{\eta}{m_e m_h}(n_e - n_h)^2.$$

$$\rho_{xy} = \mathrm{Im}\hat{\rho} \approx \eta\frac{\Omega_T T}{\gamma^2}\frac{\delta n}{n_T}\frac{\hbar}{e^2}, \quad \Omega_T = \frac{eBv_0^2}{T} \qquad n_T = n_i(0,T) \sim \frac{T^2}{v_0^2}$$

Results: ρ_{xy}



- Describes T-dependence
- Successfully models experiment

DA, Novoselov, Geim, Levitov, '11

Results: spin-Hall effect

$\Delta = g\mu_B B$ Zeeman splitting γ -disorder scale $T_* = \gamma / \alpha$

$$\xi = \begin{cases} \dfrac{eBv^2}{TT_*^2}\Delta, T >> T_* \\ \dfrac{eBv^2}{T^3}\Delta, \gamma << T << T_* \\ \dfrac{eBv^2}{\gamma^3}\Delta, T \le \gamma \end{cases}$$

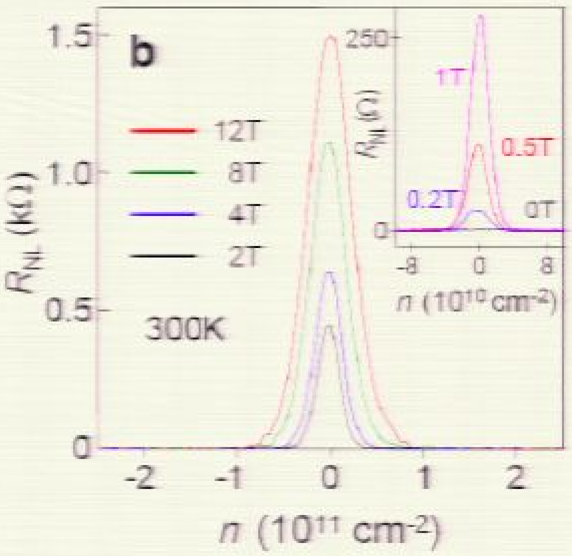
Collisions dominate, $\xi \propto 1/T$

Disorder dominates, $\xi \propto 1/T^3$

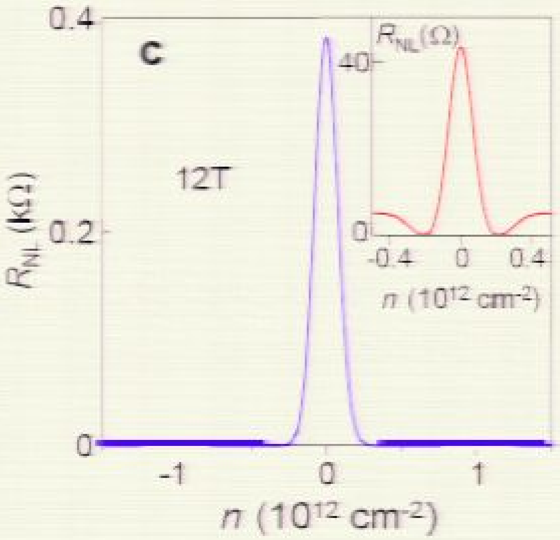
Saturates at $T \sim \gamma$ $\xi \propto 1/\gamma^3$

Experiment confirms theory

Experiment



Theory



Theory captures all the main features:

-Peak at the Dirac point

-Growth as a function of 1/T, B

-Magnitude

Summary

- Nonlocal transport: a new tool to create and measure neutral currents
- Giant flavor-Hall effect predicted and observed in Dirac materials

DA, Morozov,Novoselov,Levitov,Geim,
submitted;+in preparation (2010)

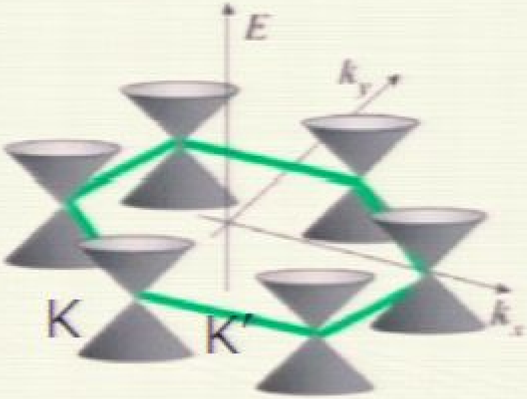
What's next?

-Ideas apply to other systems

(top. insulators, gapless spin liquids,..)

-Valley-Hall effect

NEEDED: valley splitting



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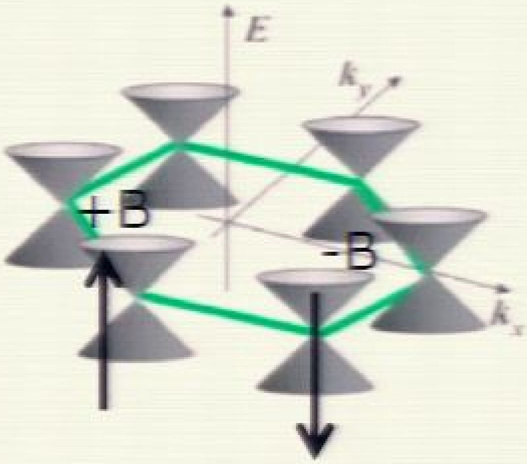


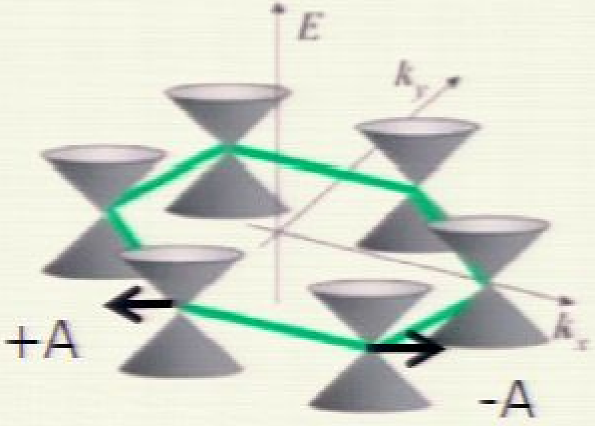
NEEDED: valley splitting

1)STRAIN → pseudomagnetic field

2) VALLEY-SELECTIVE EDGE SCATTERING

Giant pseudomagnetic fields

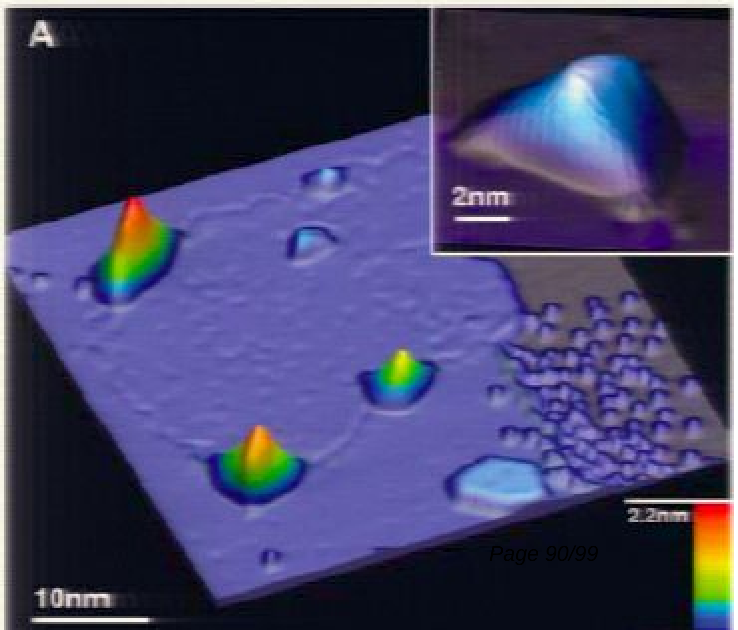
- -Strain
- -Dirac points shift in momentum space
- -Vector-potential → Pseudo-magnetic field



Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2*†] S. A. Burke,[1*‡] K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2§]

30 JULY 2010 VOL 329 SCIENCE www.sciencemag.org



Fractional quantum Hall effect

Graphene

-strong interactions

(DA, Skachko, Levitov, Andrei PRB'10)

-Intera

-Robus
state

DA,Papi
submitte

GaAs

-weak interaction

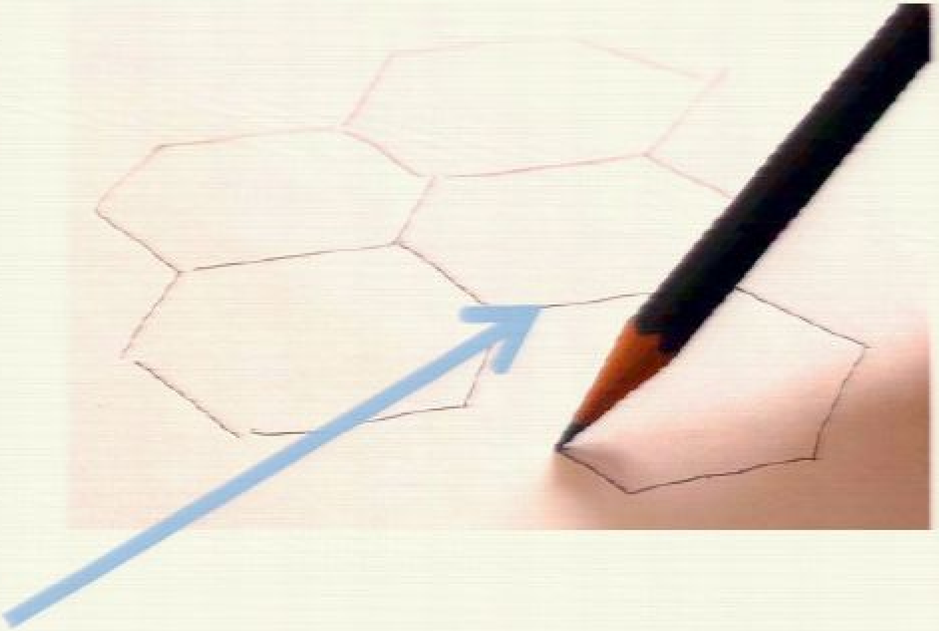
And more:

-Other exotic states

-SU(4)-symmetric FQH states

-Strong Landau level mixing → new interactions, states

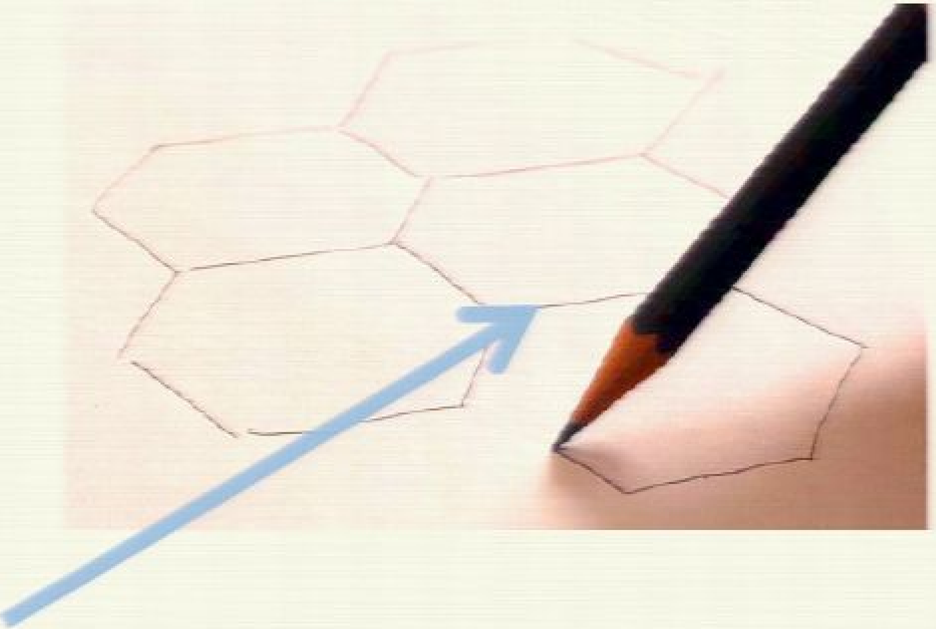
Summary



- Dirac fermions
- Klein tunneling
- Half-integer QHE
- Giant spin-Hall effect
- Non-abelian fractional QHE
- Skyrmions

*……… **and much more***

Summary



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……… *and much more*

Acknowledgements

Giant flavor-Hall effect (this talk)

MIT: Leonid Levitov, Patrick Lee

Manchester: Kostya Novoselov, Andre Geim

Exeter: Andrey Shytov

Harvard: Bert Halperin

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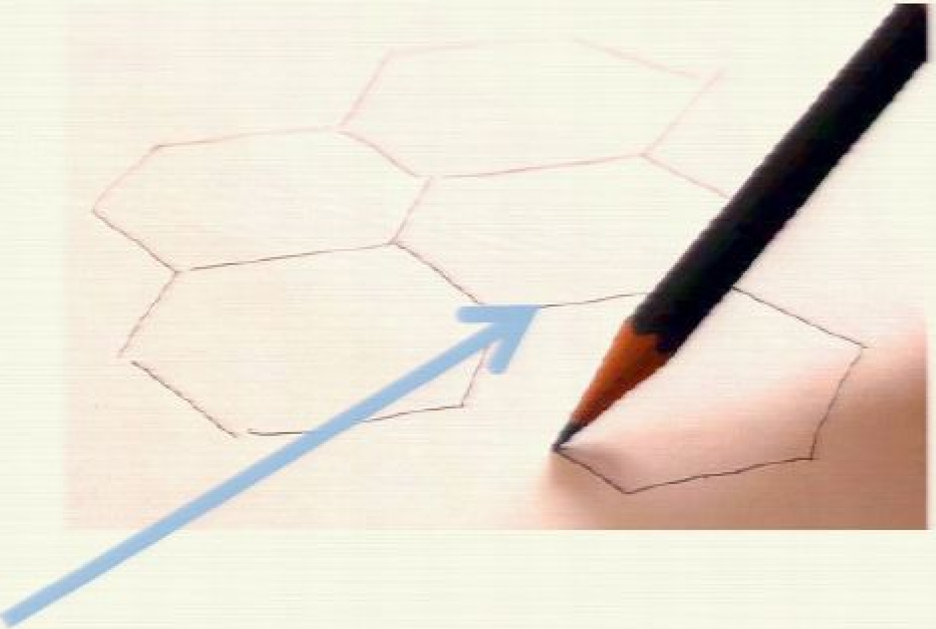
Rutgers: Eva Andrei, Ivan Skachko

Stony Brook: Xu Du

Columbia: Philip Kim, Pablo Jarillo-Herrero, Barbaros Ozyilmaz

Stanford: Steven Kivelson

Summary



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……… *and much more*

Disorder put to good use

-Functionalization with adatoms, e.g., H, F

(Elias et al Science'09) – band gap?

-Stron ms

U~1,

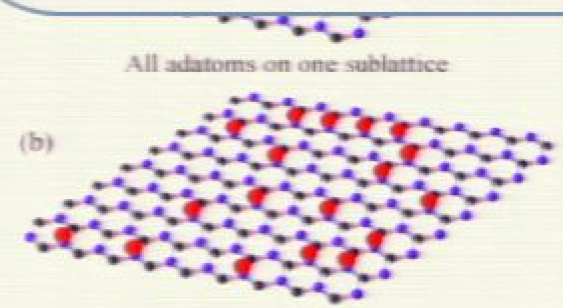
-Order *'10)*

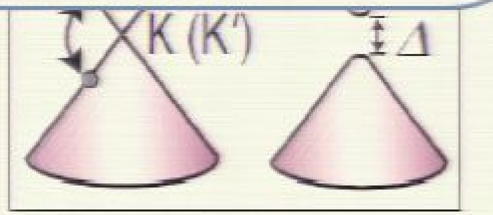
Future:

Other types or ordering

Magnetic adatoms

Heavy fermion physics?





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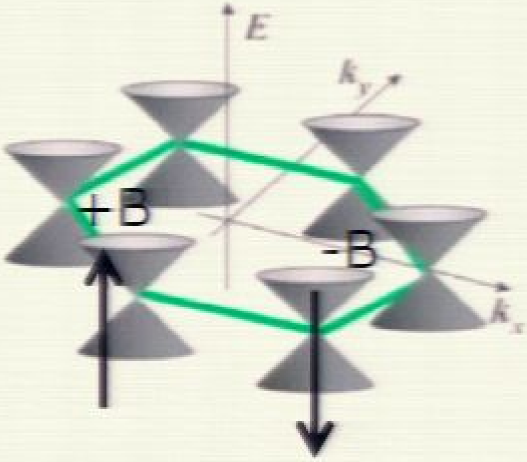

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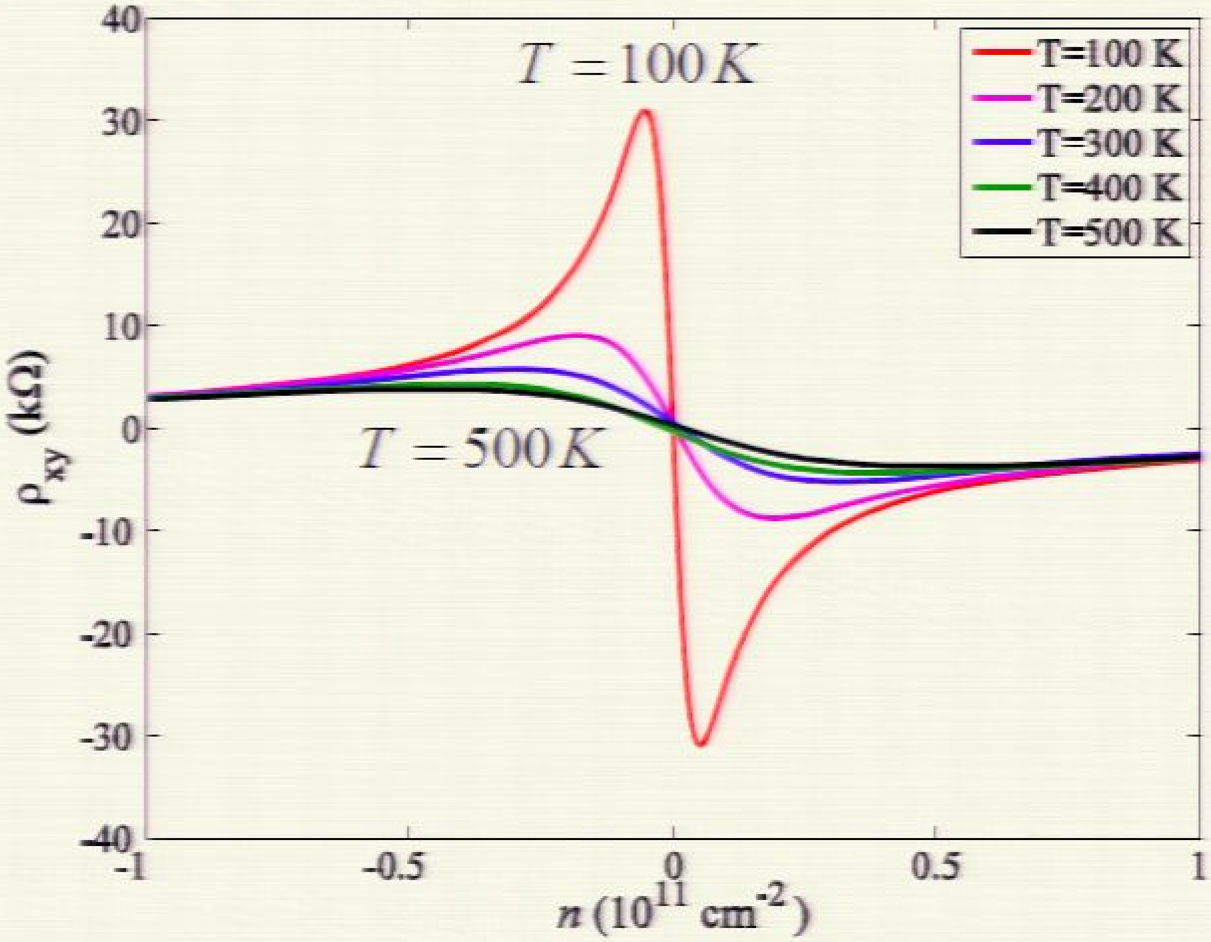

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High temperature regime

-At the Dirac point

-Finite DC conductivity even without disorder

$$\rho_{xx} = \eta \frac{h}{e^2} \qquad \eta \sim \alpha^2 \qquad \alpha = \frac{e^2}{\hbar v}$$

-friction coefficient
between
electrons and holes

-Hydrodynamic regime

-Scale-invariant