Title: Stampede of the Wild Gluons

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Abstract: To a first approximation, everything that happens at the Large Hadron Collider at CERN is a strong interaction process. If signals of supersymmetric particles or other new states are found at the LHC, the events that produce those signals will represent parts per trillion of the total sample of proton-proton scattering events and parts per billion of the sample of events with hard scattering of quarks and gluons. Can we predict the rates of QCD processes well enough to control their contribution to a tantalizing signal? What physics insights can assist this process? Can string theory help? In this lecture, I will describe the current status of Quantum Chromodynamics and its application to the predictions for hadron colliders. Note: Those who would like more background on the LHC should attend my talk at the Physics Department Colloquium of the University of Waterloo, Thursday, Jan. 13, at 4:00 pm: The Search for New Elementary Particles at the CERN Large Hadron Collider

Stampede of the Wild Gluons

M. E. Peskin
January 2011

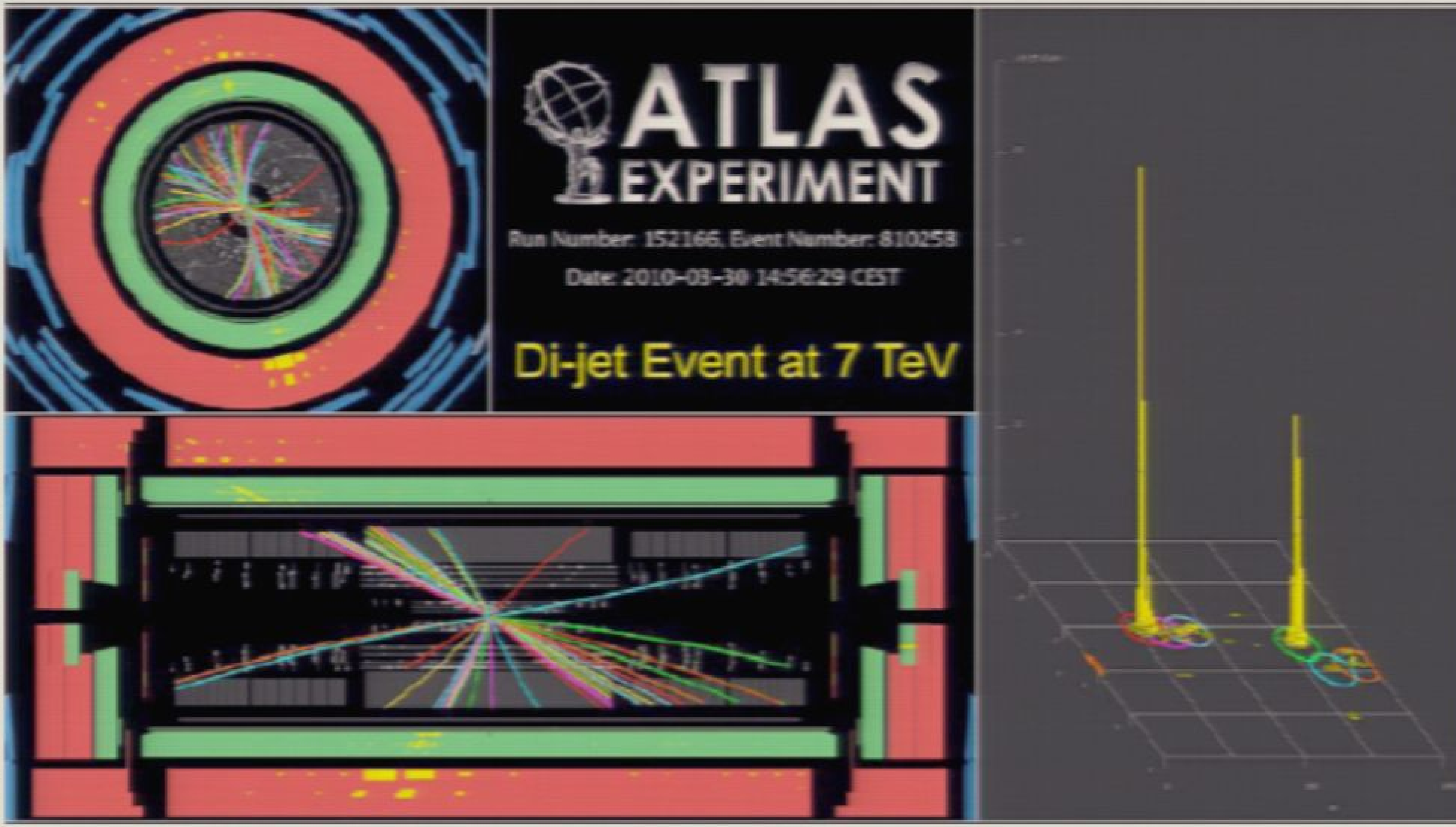
At the Large Hadron Collider, to a first approximation, everything that happens is QCD.

We built the LHC to look for new physics -- supersymmetry, dark matter, the Higgs boson -- but these processes are hidden under a stampede of QCD events. In order to find new physics, we must understand the predictions of QCD precisely.

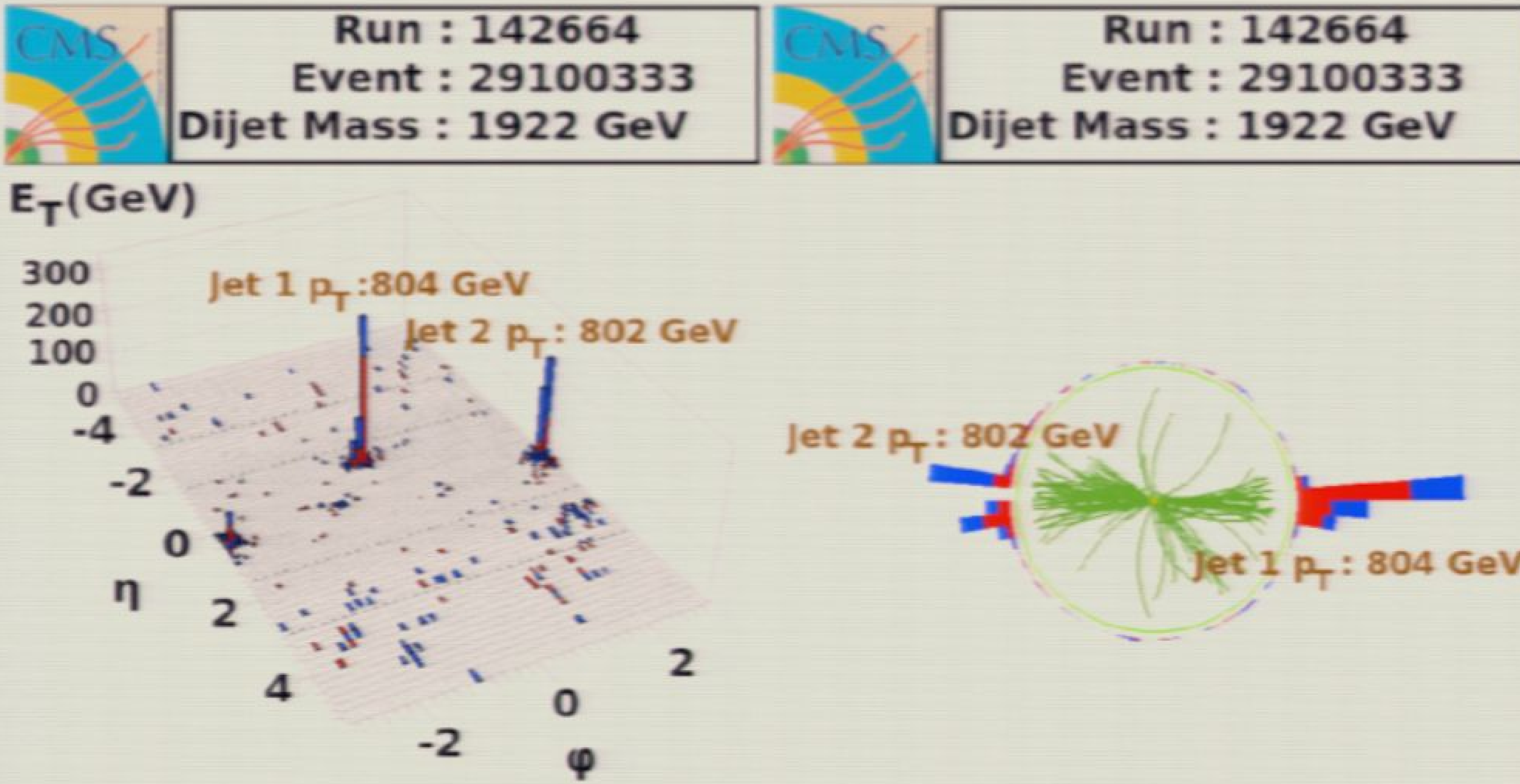
At the level of understanding required, QCD itself becomes a fascinating subject. Certainly, it is worth a colloquium.

Here is an estimate of the sizes of important cross sections at the LHC:

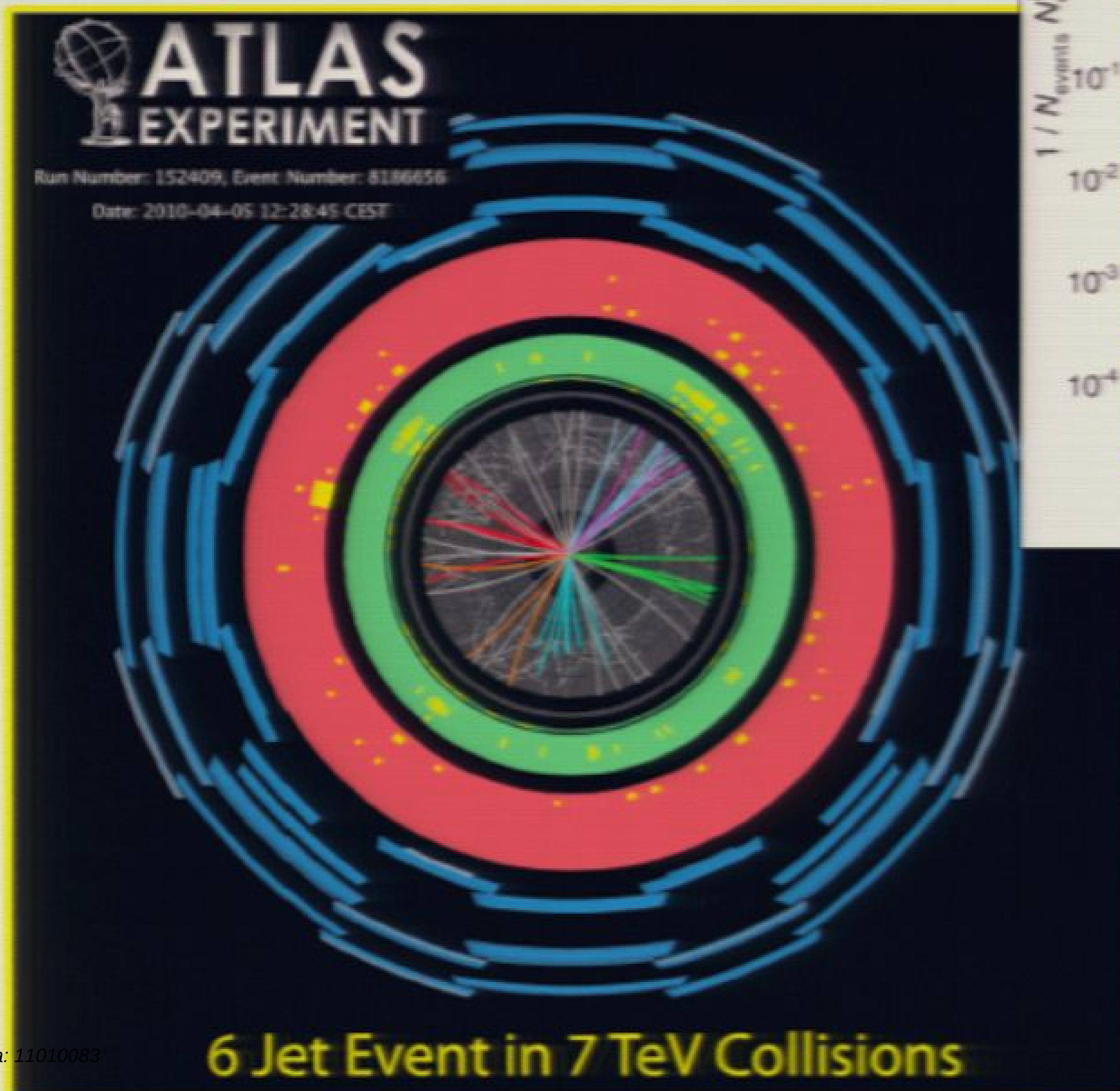
total cross section	100 mb
jets with $E_T > 100$ GeV	$1 \mu b$
$W(\rightarrow \ell\nu)$	10 nb
$Z(\rightarrow \ell^+\ell^-)$	1 nb
$t\bar{t}(\rightarrow \ell\nu jjj)$	0.3 nb
new particles	$1 - 10$ pb

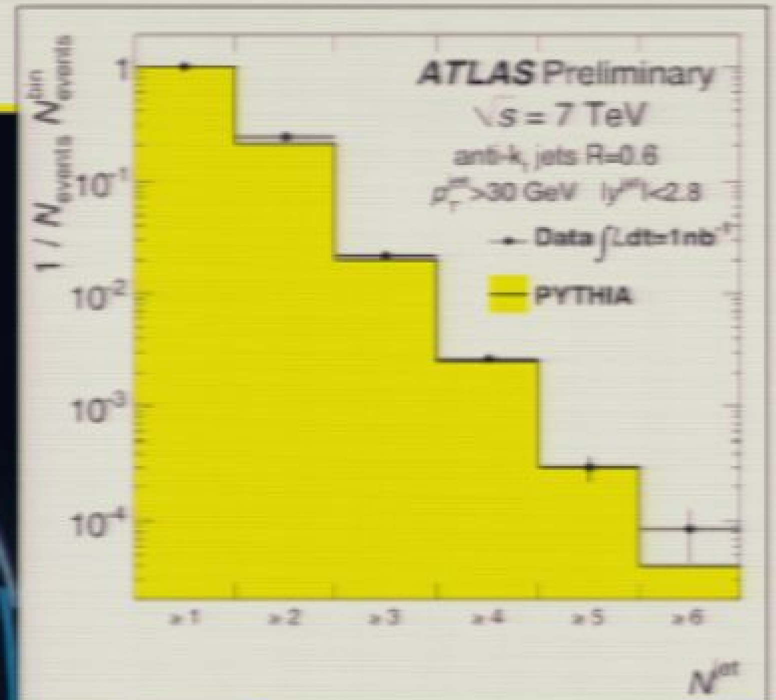


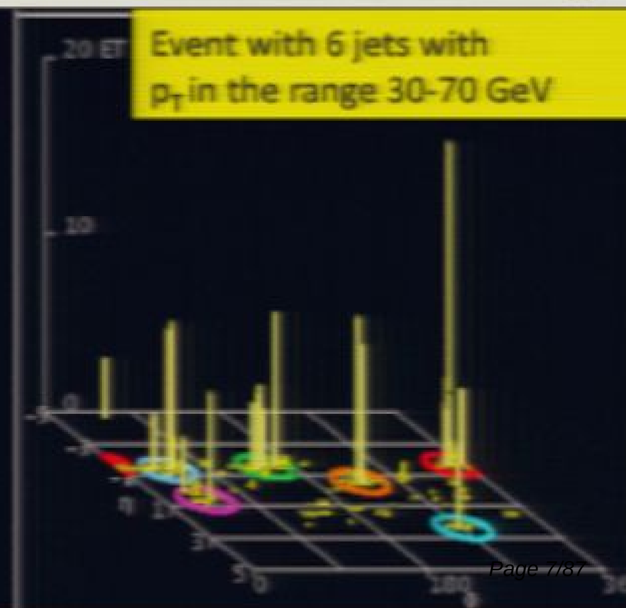
dijet event w. jets of ET = 310 and 350 GeV


Run : 142664
Event : 29100333
Dijet Mass : 1922 GeV
Run : 142664
Event : 29100333
Dijet Mass : 1922 GeV
E_T (GeV)
300
200
100
0
Jet 1 p_T :804 GeV
Jet 2 p_T : 802 GeV
-4
-2
0
η
2
4
-2
0
2
φ
Jet 2 p_T : 802 GeV
Jet 1 p_T : 804 GeV

6 jets event in ATLAS

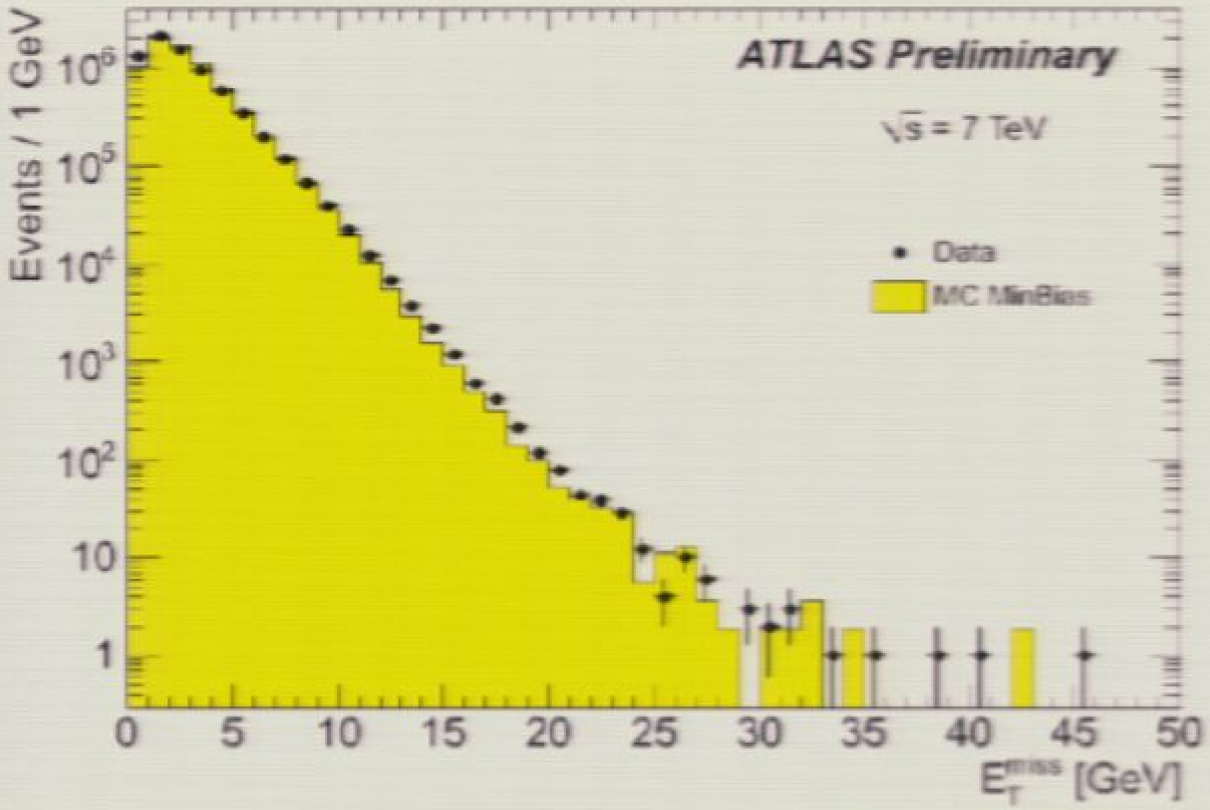




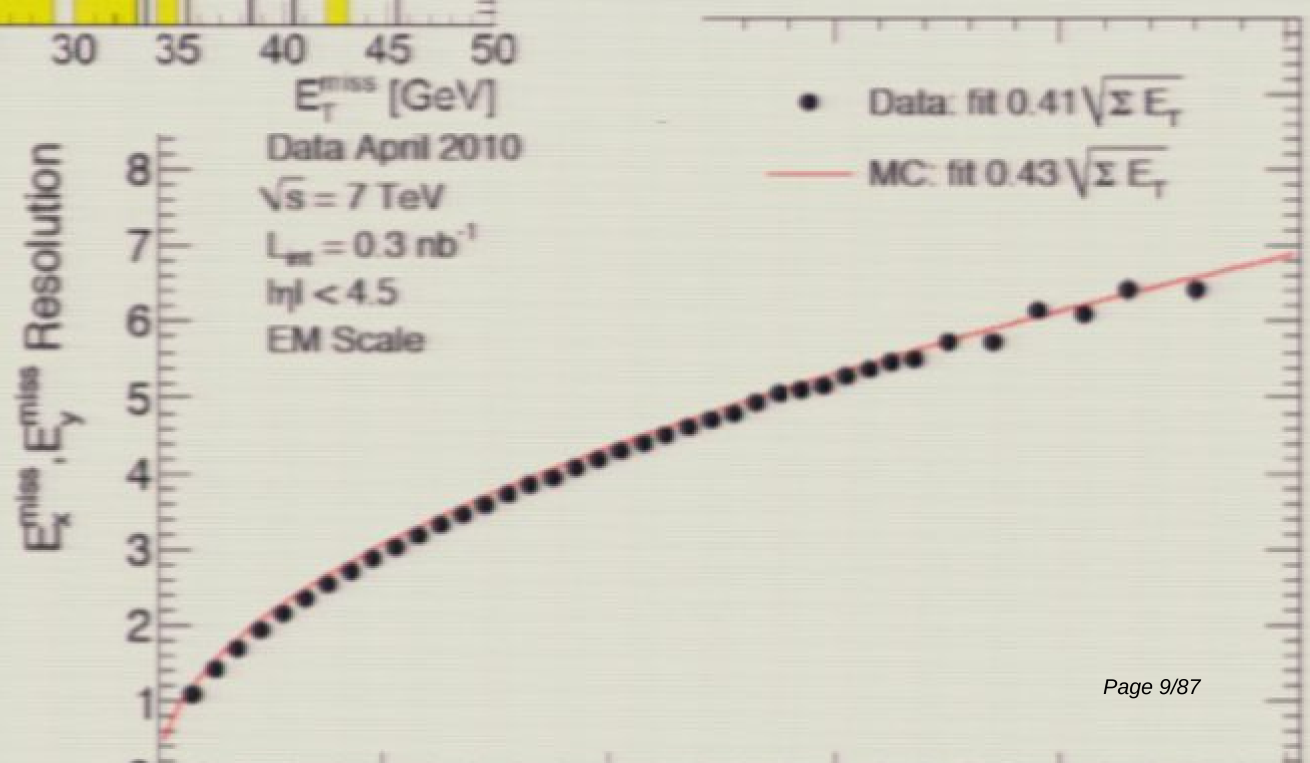


Pure QCD processes are dominantly 2-jet-like and have little unbalanced transverse momentum.

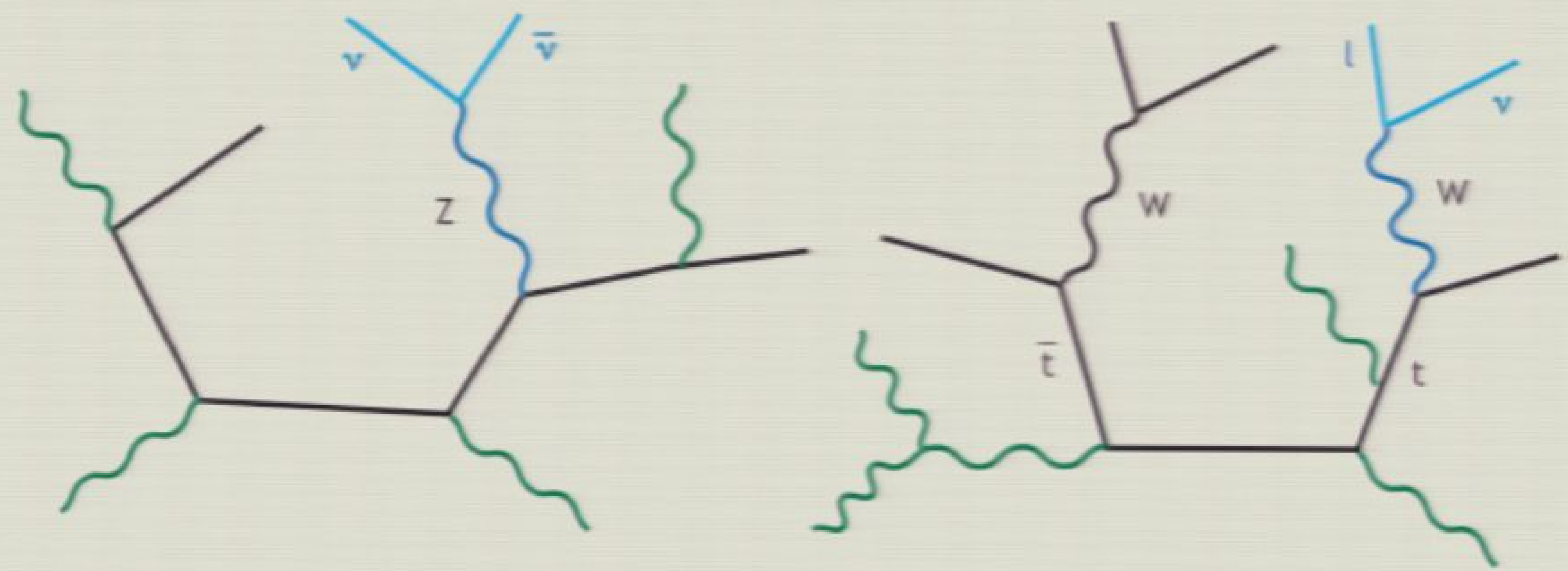
Controlling backgrounds to new physics from these events is mainly a problem of building a detector with no large gaps in solid angle and uniform calibration.



ATLAS missing ET measurement and resolution



Processes with W, Z, t already create leptons and missing pT thorugh Standard Model processes. This is genuinely scary. Processes such as

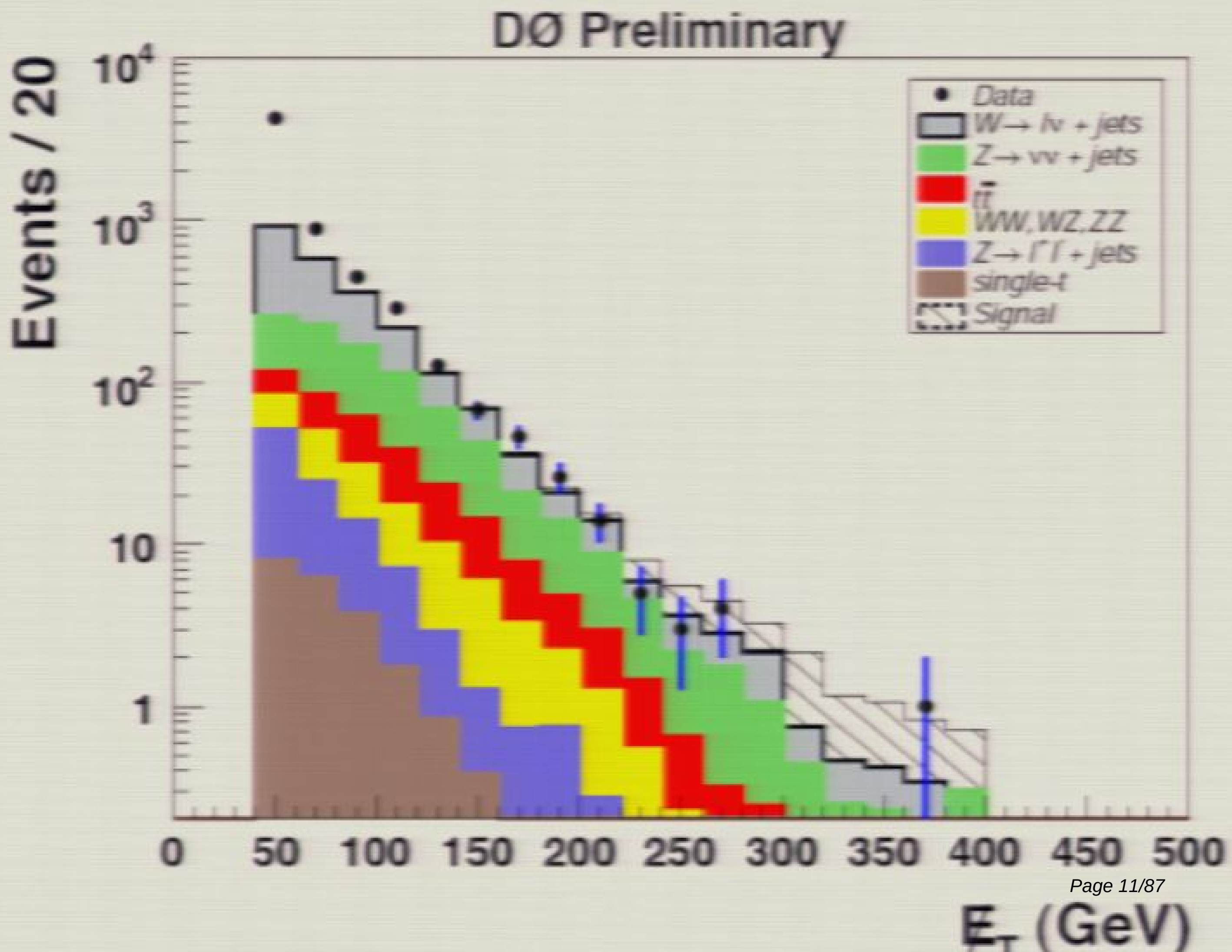


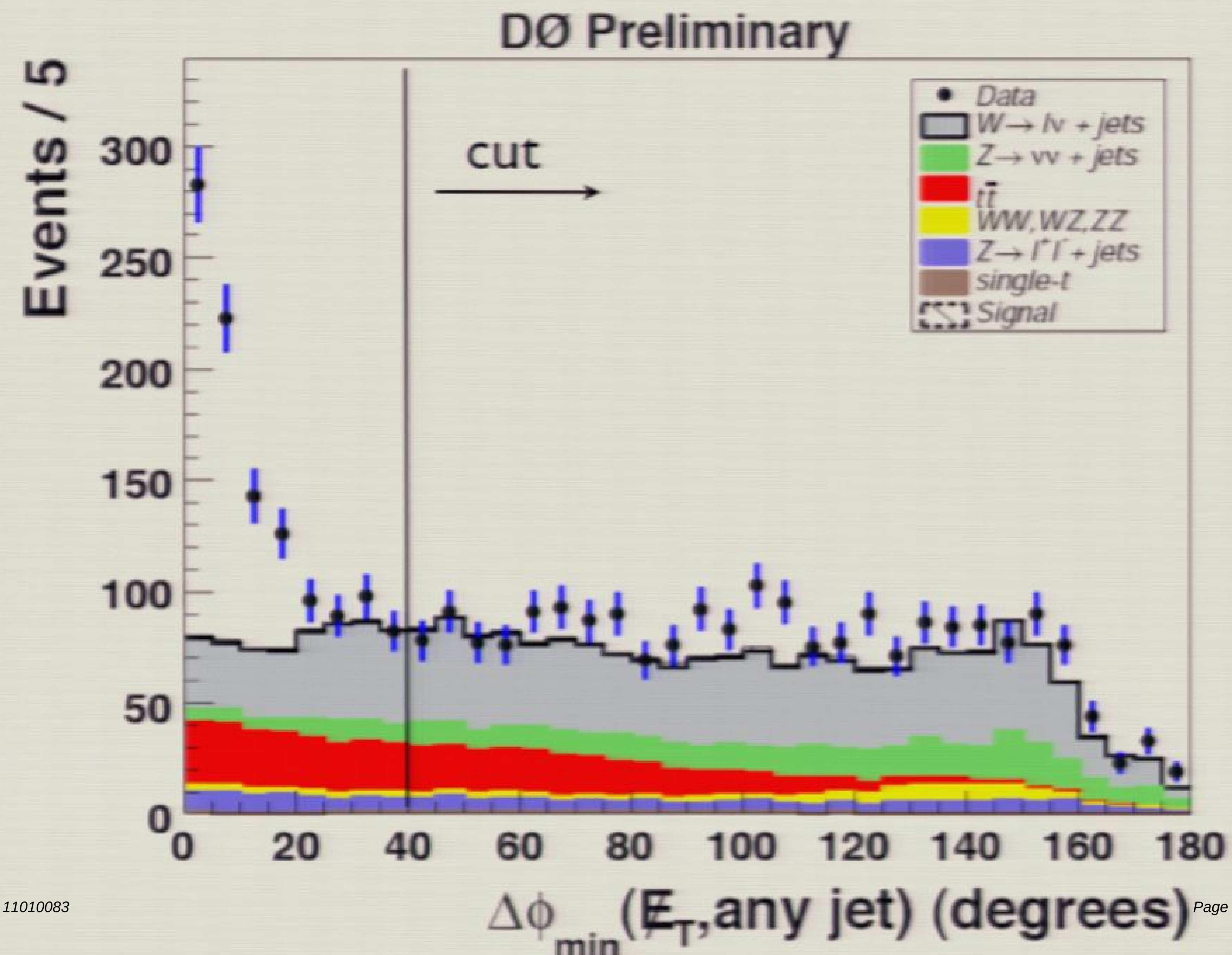
have cross sections comparable to the new physics signal and might compete with it.

At the Tevatron, events with SM heavy particle production (W, Z, t) provide the dominant backgrounds to new physics searches.

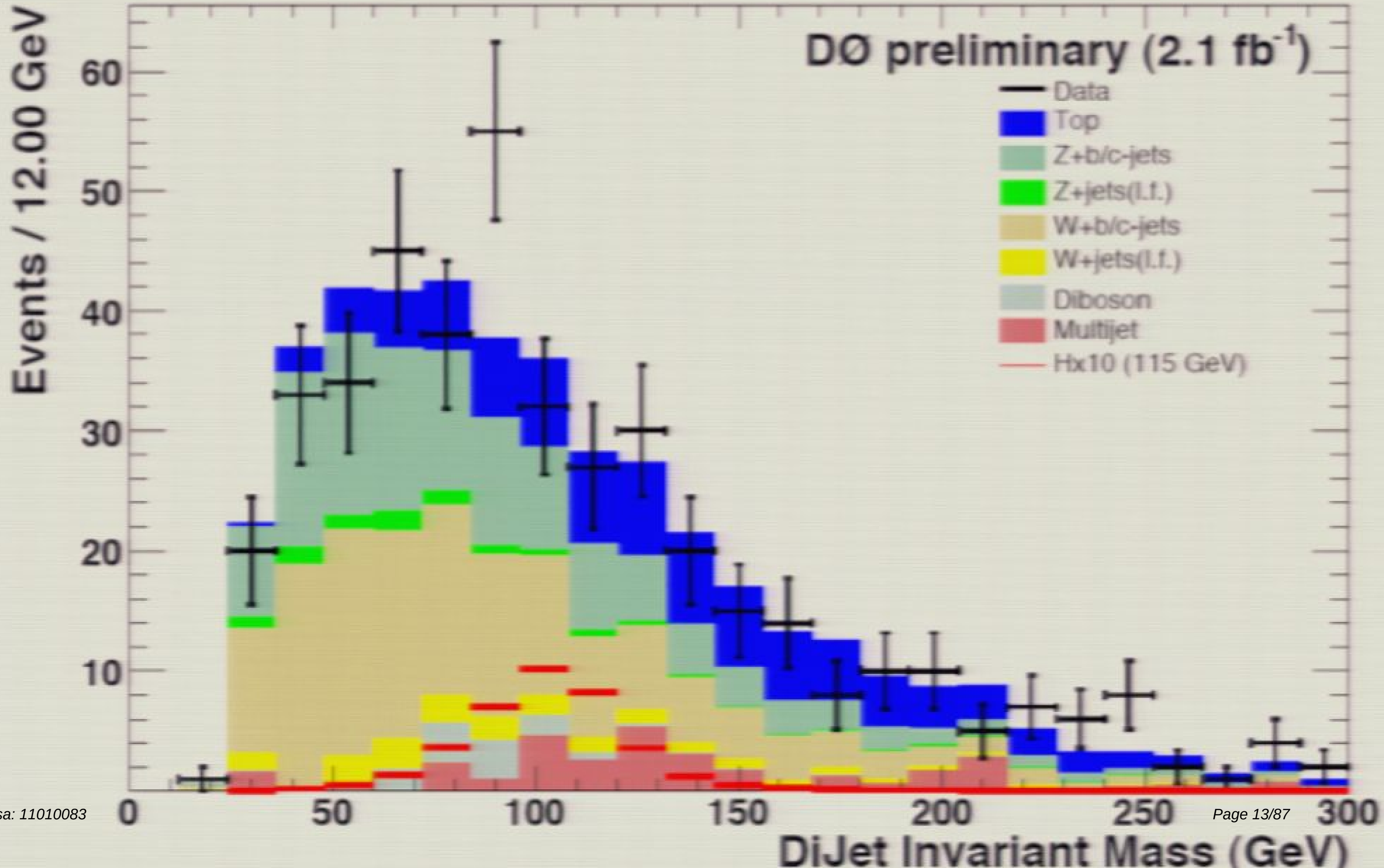
DO SUSY search in acolinear dijets

(2007)

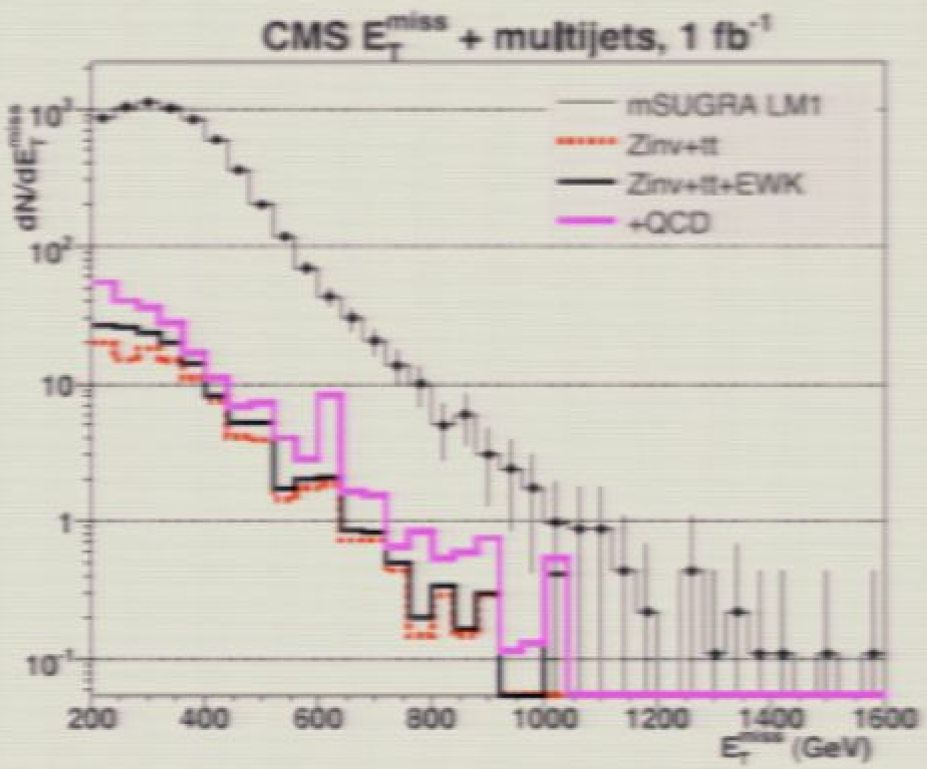


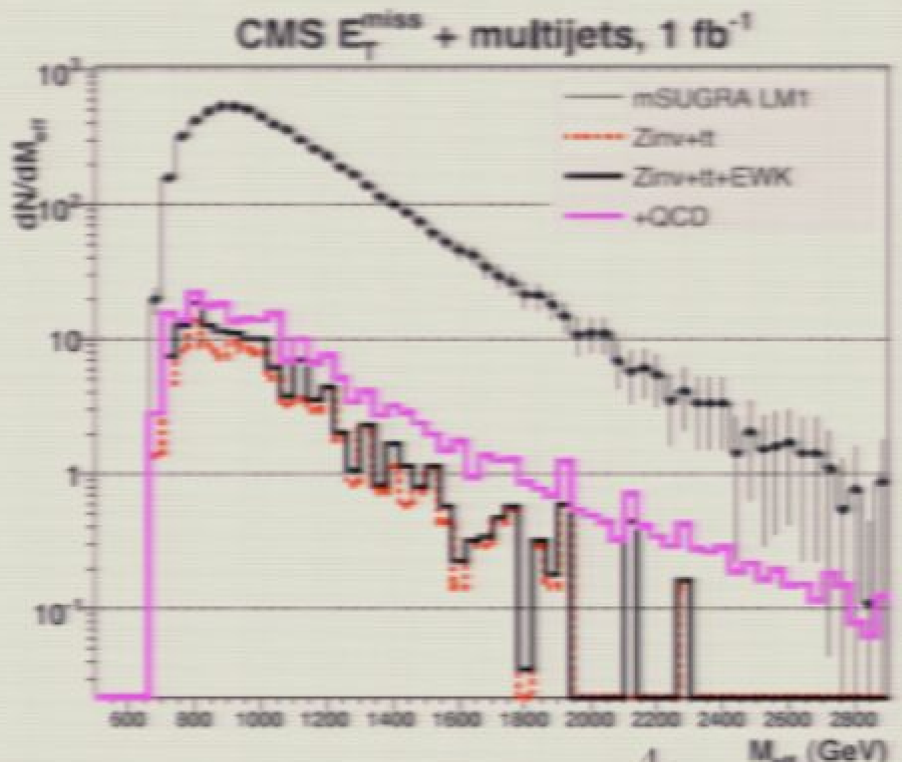

DØ Preliminary
Events / 5
cut
Data
W→ lν + jets
Z→ νν + jets
$t\bar{t}$
WW,WZ,ZZ
Z→ l⁺l⁻ + jets
single-t
Signal
$\Delta\phi_{min}$($\not{E}_T$,any jet) (degrees)

DO Higgs search in $p\bar{p} \rightarrow b\bar{b} + \text{invisible}$ (2008)



$$m(\widetilde{g}) = 600 \text{ GeV}$$



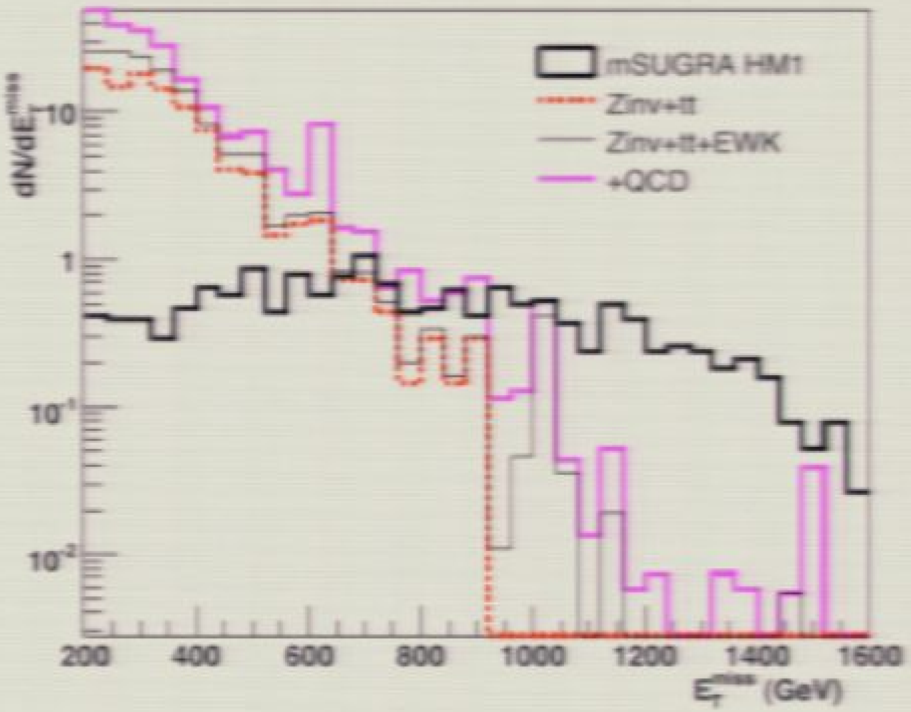


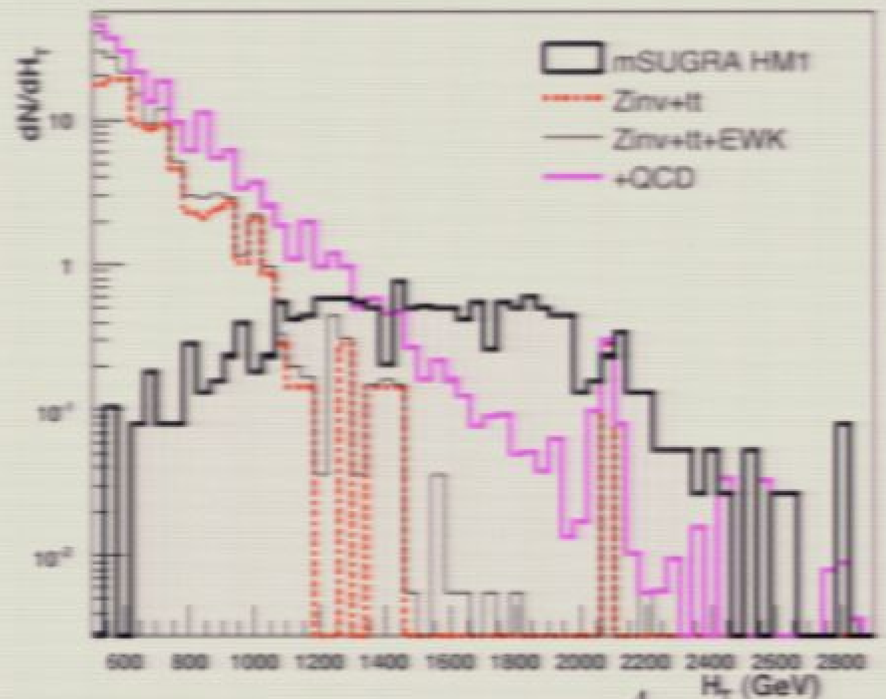
$$\not{E}_T$$

$$M_{\text{eff}} = \not{E} + \sum_i^4 E_{Ti}$$

Yetkin and Spiropulu

$$m(\widetilde{g}) = 1890\ \mathrm{GeV}$$





$$\not{E}_T$$

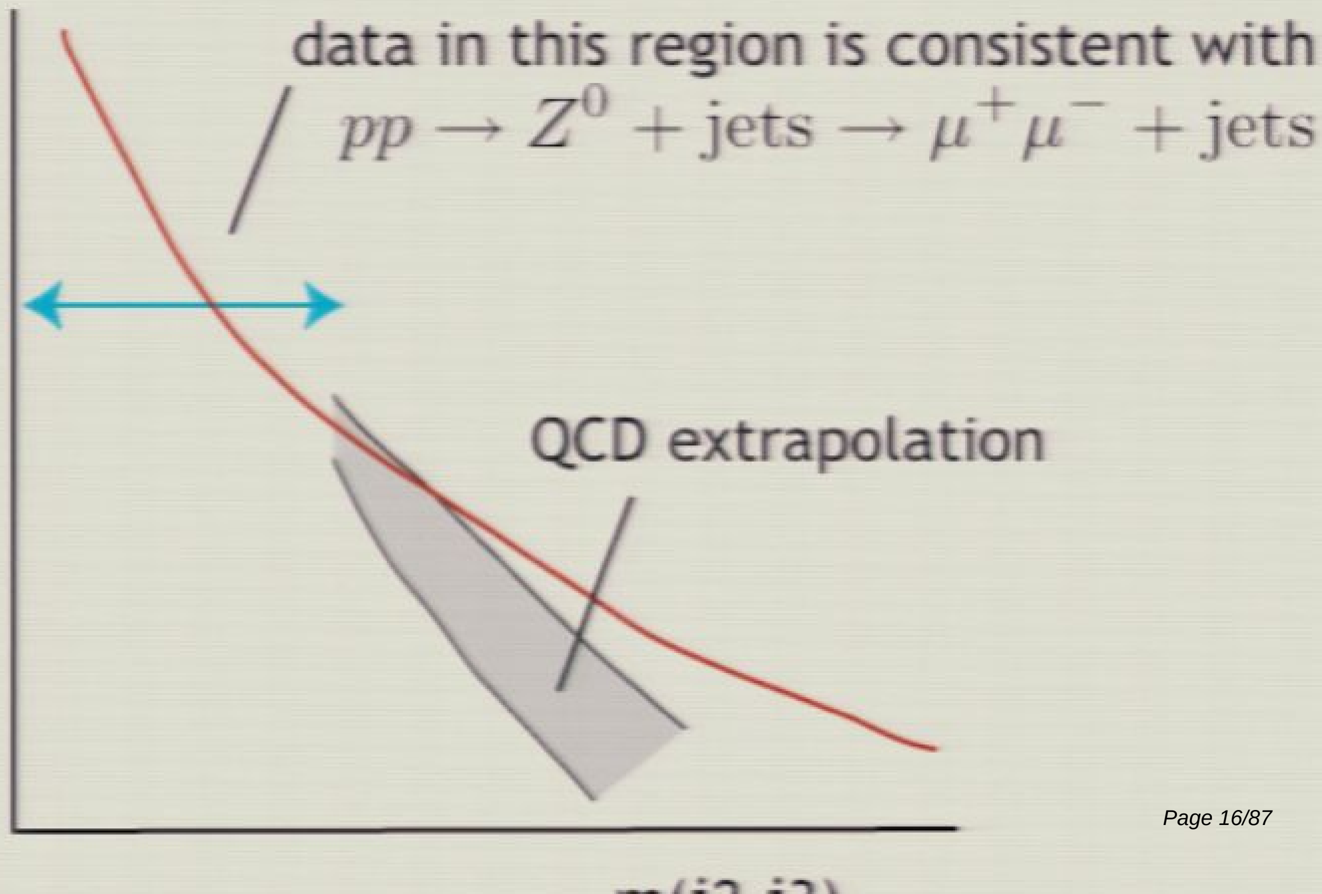
$$M_{\mathrm{eff}} = \not{E} + \sum_{i}^{4} E_{Ti}$$

Yetkin and Spiropulu

It is tempting to say that we can get this control from direct proxies for Standard Model backgrounds that we find in the data. For example: $\sigma(pp \to Z^0 \to \nu\bar{\nu}) \approx 6\,\sigma(pp \to Z^0 \to \mu^+\mu^-)$

However, more typically, we will need to extrapolate from the data using QCD theory. Consider the following distribution in jets + MET events:

Can we claim discovery of new physics ?



I will discuss this problem in 4 stages:

Computation of tree amplitudes

Computation of loop amplitudes

Modeling of parton showers

Modeling of jet substructure

To begin, note that computations with massless particles can be dramatically simplified by the use of spinors of lightlike momenta

$$1\rangle = u_R(1) \quad 1] = u_L(1) \quad \langle 1 = \overline{u}_L(1) \quad [1 = \overline{u}_R(1)$$

These objects are related to more familiar objects by

$$1\rangle[1 = \frac{1}{2}(1+\gamma^5)\, \not{1}$$

The spinor products are square roots of Lorentz vector products:

$$\langle 12\rangle = \overline{u}_L(1)u_R(2) \qquad [12] = \overline{u}_R(1)u_L(2)$$

$$|\langle 12\rangle|^2 = |[12]|^2 = 2k_1 \cdot k_2$$

The spinor products are antisymmetric. They obey the following useful identities:

$$\langle 1\gamma^\mu 2]\langle 3\gamma_\mu 4] = 2\,\langle 13\rangle[42] \qquad \text{Fierz}$$

$$\langle 12\rangle\langle 34\rangle + \langle 13\rangle\langle 42\rangle + \langle 14\rangle\langle 23\rangle = 0 \quad \text{Schouten}$$

It is simplest to label the helicities as if all particles were outgoing. An incoming L corresponds to an outgoing R. This makes crossing symmetry automatic.

Photon and gluon polarization vectors for momentum k are conveniently written in terms of spinor products involving k and a second lightlike "reference" vector r:

$$\epsilon^{\mu}_{+}(k) = \frac{1}{\sqrt{2}} \frac{\langle r \gamma^{\mu} k]}{\langle r k \rangle} \qquad \epsilon^{\mu}_{-}(k) = -\frac{1}{\sqrt{2}} \frac{[r \gamma^{\mu} k\rangle}{[r k]}$$

The logic of this choice is:

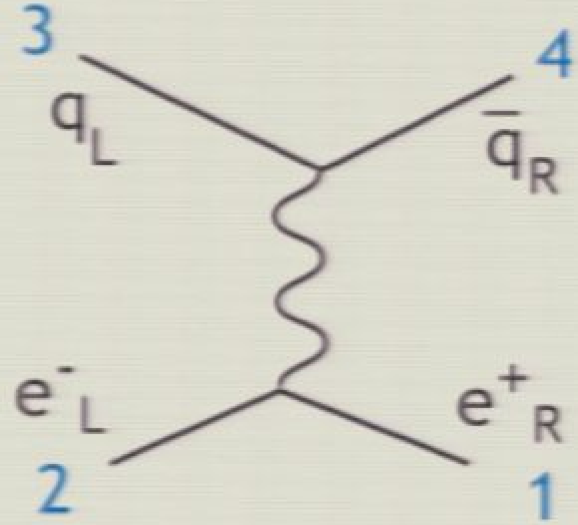
if k = 1 is parallel to z, r = 2 is parallel to -z:

$$\langle 2 \gamma^{\mu} 1] = \overline{u}_L(2) \gamma^{\mu} u_L(1) = \sqrt{s_{12}} \cdot (0, 1, -i, 0)$$

a change in r is like a change of gauge and has no physical effect. We can choose r for maximum convenience, independently in each helicity amplitude.

Here is a very simple example: $e_L^- e_R^+ \to q_L \bar{q}_R$

$$
\begin{aligned}
iM &= (-ie)^2 \langle 1\gamma^\mu 2] \frac{-i}{s_{34}} \langle 3\gamma_\mu 4] \\
&= 2ie^2 \frac{\langle 13\rangle[42]}{\langle 34\rangle[43]} \times \frac{\langle 31\rangle}{\langle 31\rangle} \\
&= -2ie^2 \frac{(\langle 13\rangle)^2}{\langle 34\rangle\langle 12\rangle}
\end{aligned}
$$



Then

$$|M|^2 = 4e^4 \frac{u^2}{s^2} = e^4(1+\cos\theta)^2$$

which is correct !

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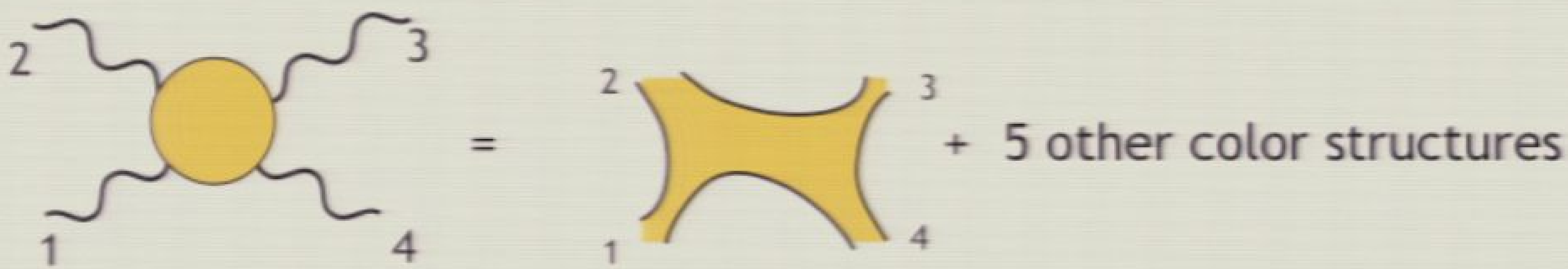
Apply this technology to QCD. It is convenient to decompose QCD amplitudes into color structures.



$$= \mathrm{tr}[T_1 T_2 T_3 T_4]\ i\mathcal{M}_1 + \ 5 \text{ more}$$

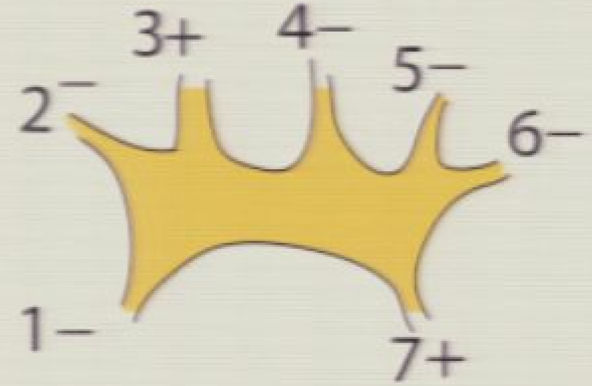
The color-ordered amplitudes can be computed using color-ordered Feynman rules. Using the technology above, we can compute color-ordered amplitudes with definite helicity external particles (all outgoing).

Some remarkable results are obtained.

Parke and Taylor showed that there is a general property here that applies to tree amplitudes with arbitrarily many gluons:

Notate:

$$iM(1^-,2^-,3^+,4^-,5^-,6^-,7^+) =$$



Then:

All amplitudes with all + or only one - vanish. Similarly, all amplitudes with all - or only one + vanish.

Amplitudes with two - and all the rest + have the following simple form:

$$iM(1^+ \dots i^- \dots j^- \dots n^+) = ig^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

These are called Maximum Helicity Violating (MHV) amplitudes.

There are connections here to some deep ideas.

In supersymmetric Yang-Mills theory, amplitudes will all-+ or one - helicity vanish by the supersymmetric Ward Identities.

Ordinary YM is not supersymmetric, but it is an orbifold of a supersymmetric theory. That is good enough for tree amplitudes.

The representation of lightlike momenta by spinors invites the use of twistors as variables.

Witten suggested that MHV amplitudes are holomorphic functions of twistors and that they can be computed by a string theory in twistor space.

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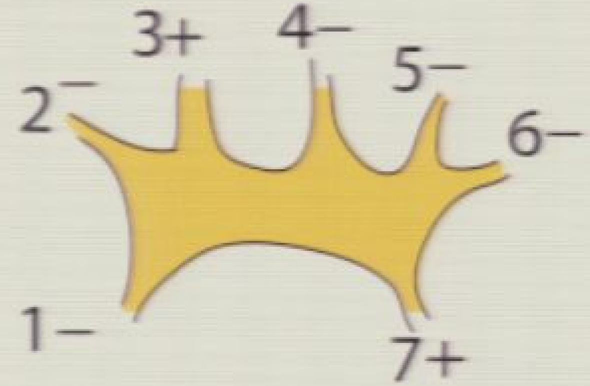

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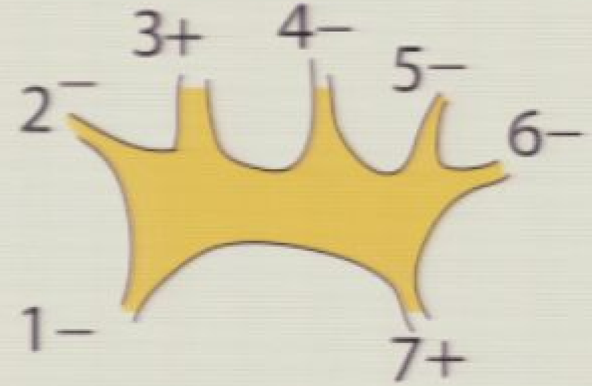

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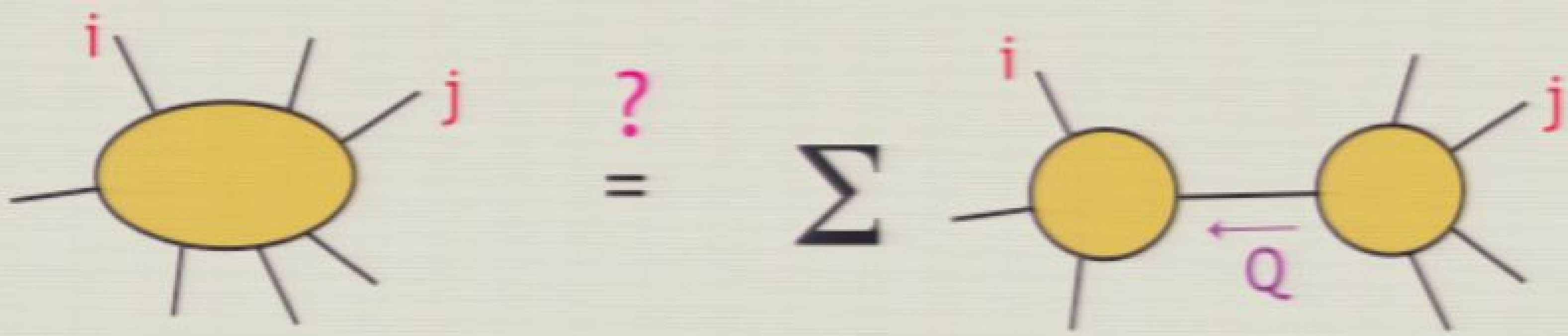
Starting from this insight, Britto, Cachazo, and Feng showed how to use MHV amplitudes as building blocks of more general QCD amplitudes.

It would be wonderful to build up non-MHV amplitudes from the simple MHV expressions. But it is not so obvious how to do this. MHV amplitudes are on-shell expressions, but if we cut the more complex amplitudes, we will have to evaluate them off-shell.



Or do we ? BCF suggested that we pick legs i and j and shift

$$i] \to i] + z\, j] \qquad j\rangle \to j\rangle - z\, i\rangle$$

for z a complex variable. Then the shifted Q can be on-shell.

Now consider

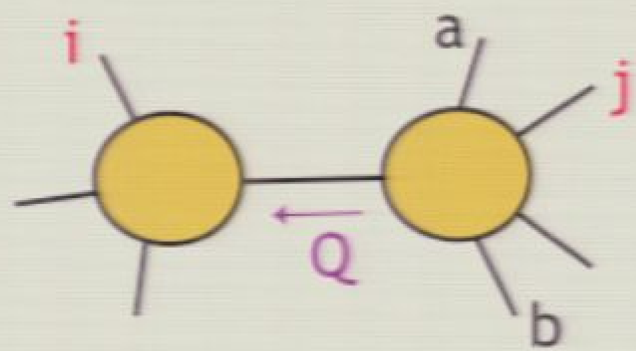
$$\oint \frac{dz}{2\pi i}\ \frac{i\mathcal{M}(z)}{z} = i\mathcal{M}(z=0) + (\text{other poles}) = (\text{contour at } \infty)$$

The first term is the amplitude that we wish to evaluate. The contour at ∞ vanishes if we choose i and j correctly (e.g i a – gluon and j a + gluon). The additional poles result when a momentum on an intermediate line satisfies

$$Q(z)^2 = 0$$

Looking again at the diagram,

$$Q^\mu(z)\gamma_\mu = \sum_{k=a}^{b} k\rangle[k\ -\ z\ i\rangle[j$$



and so

$$z_* = \frac{s_{a\ldots b}}{\langle i(\sum k\rangle[k)j]}$$

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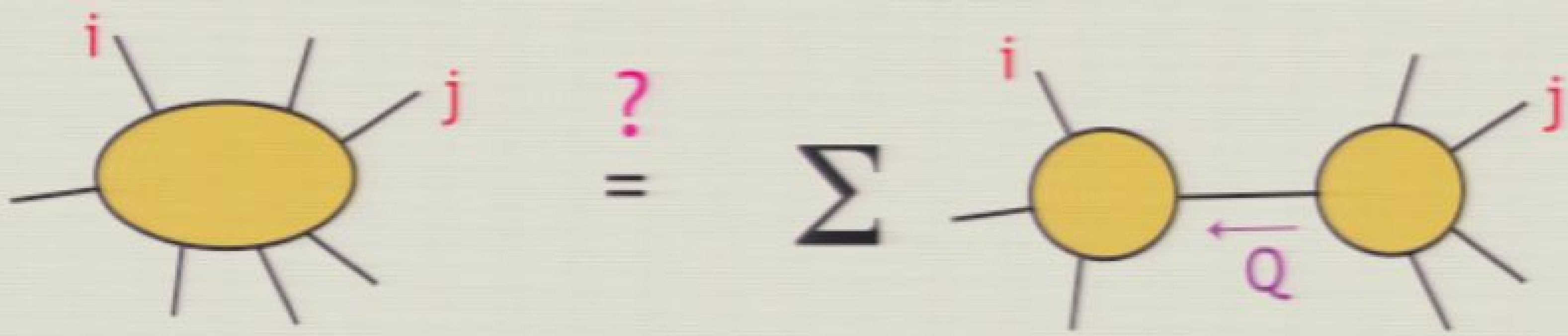

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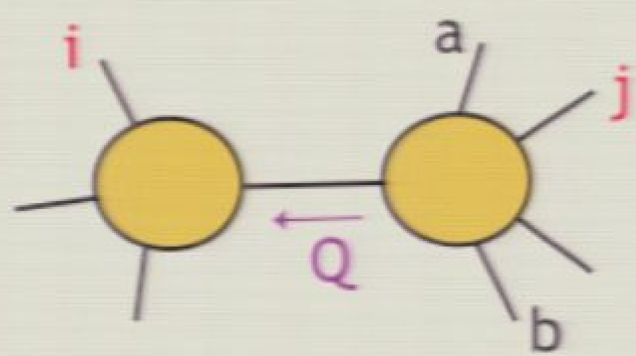

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Tidying up the formula, one finds the following relation:

$$i\mathcal{M}(1\cdots n) = \sum_{splits} i\mathcal{M}(b+1\cdots\hat{i}\cdots a-1\,-\hat{Q}) \cdot \frac{1}{s_{a\cdots b}} \cdot i\mathcal{M}(a\cdots\hat{j}\cdots b\,\hat{Q})$$

called the Britto-Cachazo-Feng (BCF) recursion formula.

Momenta with hats have the shift with z_*. The hatted momenta are complex but satisfy $\hat{Q}^2 = 0$, so the amplitudes on the right-hand side are to be evaluated on shell!

This allows the n-point amplitudes to be recursively evaluated in terms of amplitudes with fewer legs. We can stop when we reach MHV. At 5 points all amplitudes are MHV or anti-MHV.

BCF recursion is a general method for computing QCD tree amplitudes of arbitrarily high order. Even more heavy duty recursion formulae were developed by Berends and Giele, Mangano and Moretti, and others.

BCF recursion can also be used in certain non-renormalizable theories, including gravity and N=8 supergravity.

Unfortunately, the tree level is not good enough.

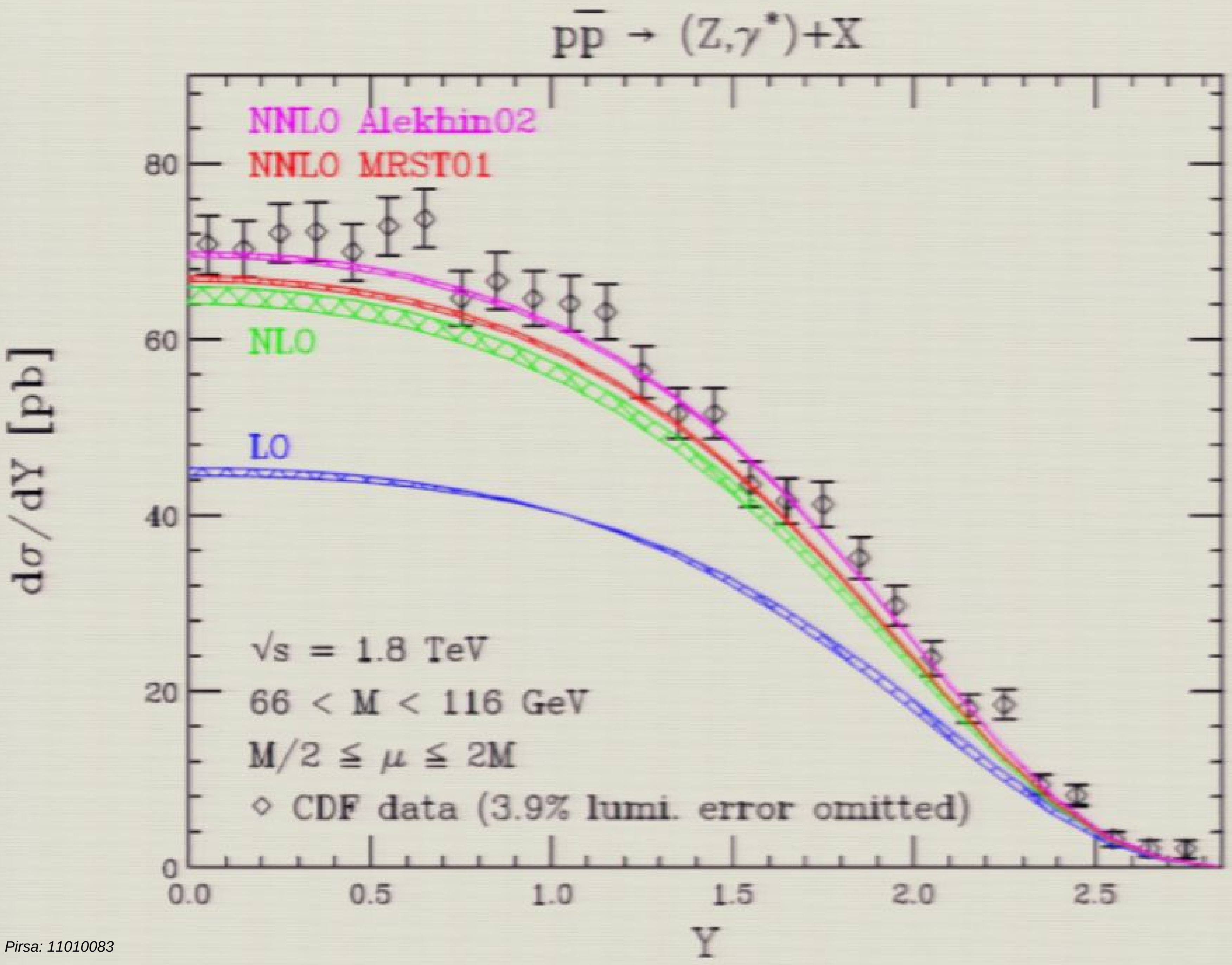
In QCD, the coupling runs according to

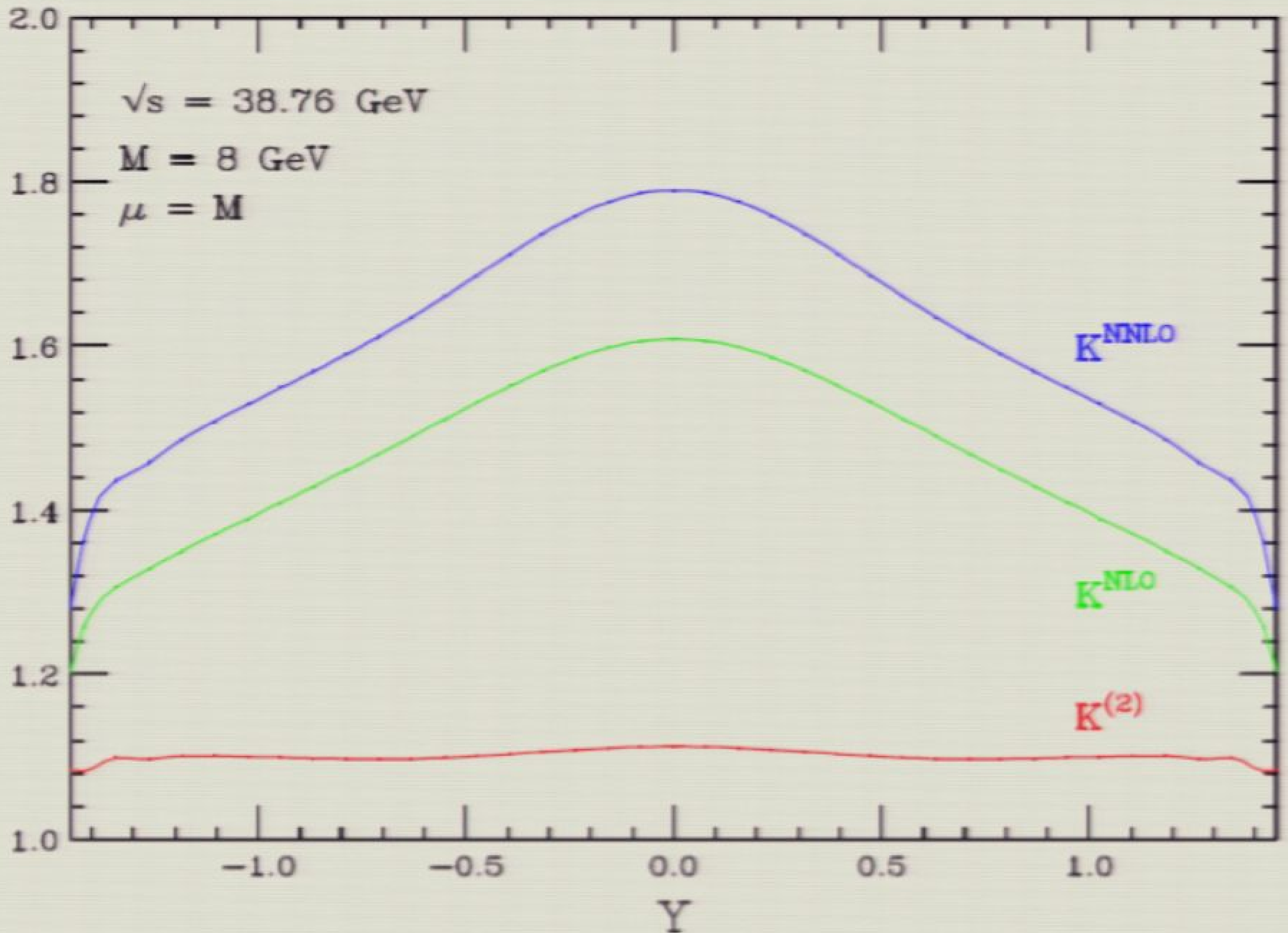
$$\alpha_s(Q) = \frac{\alpha_s(m_Z)}{1 + (b_0 \alpha_s(m_Z)/2\pi)\log(Q/m_Z)}$$

What Q should we use to compute a given cross section. Guesses might differ by more than a factor of 2. In a process with n gluon radiation, $(\alpha_s(Q))^n$ appear. This can lead in practice to 30-50% uncertainty.

This is called scale ambiguity. It is conventional to estimate the accuracy of a perturbation expansion by varying the renormalization scale for $\alpha_s(Q)$ by a factor of 2 in either direction.

In many cases, this underestimates the error!


$p\bar{p} \to (Z,\gamma^*)+X$
NNLO Alekhin02
NNLO MRST01
NLO
LO
$\sqrt{s}$ = 1.8 TeV
66 < M < 116 GeV
M/2 ≤ μ ≤ 2M
◇ CDF data (3.9% lumi. error omitted)
dσ/dY [pb]
Y
0
20
40
60
80
0.0
0.5
1.0
1.5
2.0
2.5


√s = 38.76 GeV
M = 8 GeV
μ = M
K^NNLO
K^NLO
K^(2)
2.0
1.8
1.6
1.4
1.2
1.0
−1.0
−0.5
0.0
0.5
1.0
Y

Explicit perturbative corrections gives a large coefficient of $\alpha_s(Q)$:

$$\frac{8\pi^2}{9} - \frac{7}{3} = 6.44$$

so

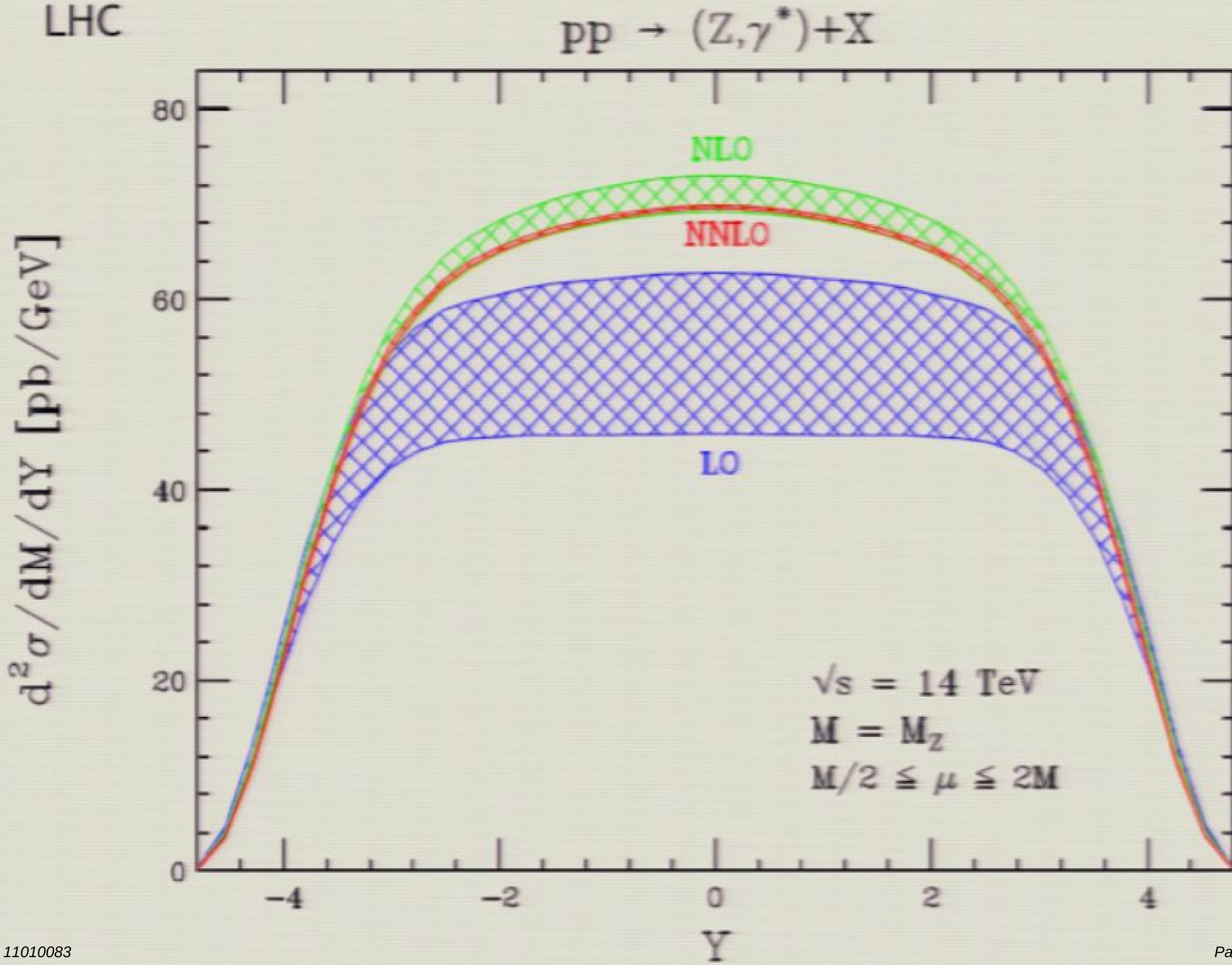
$$1 + (\frac{8\pi^2}{9} - \frac{7}{3})\frac{\alpha_s(Q)}{\pi} = \begin{cases} 1.25 & Q = m_W \\ 1.6 & Q = 2\ \mathrm{GeV} \end{cases}$$

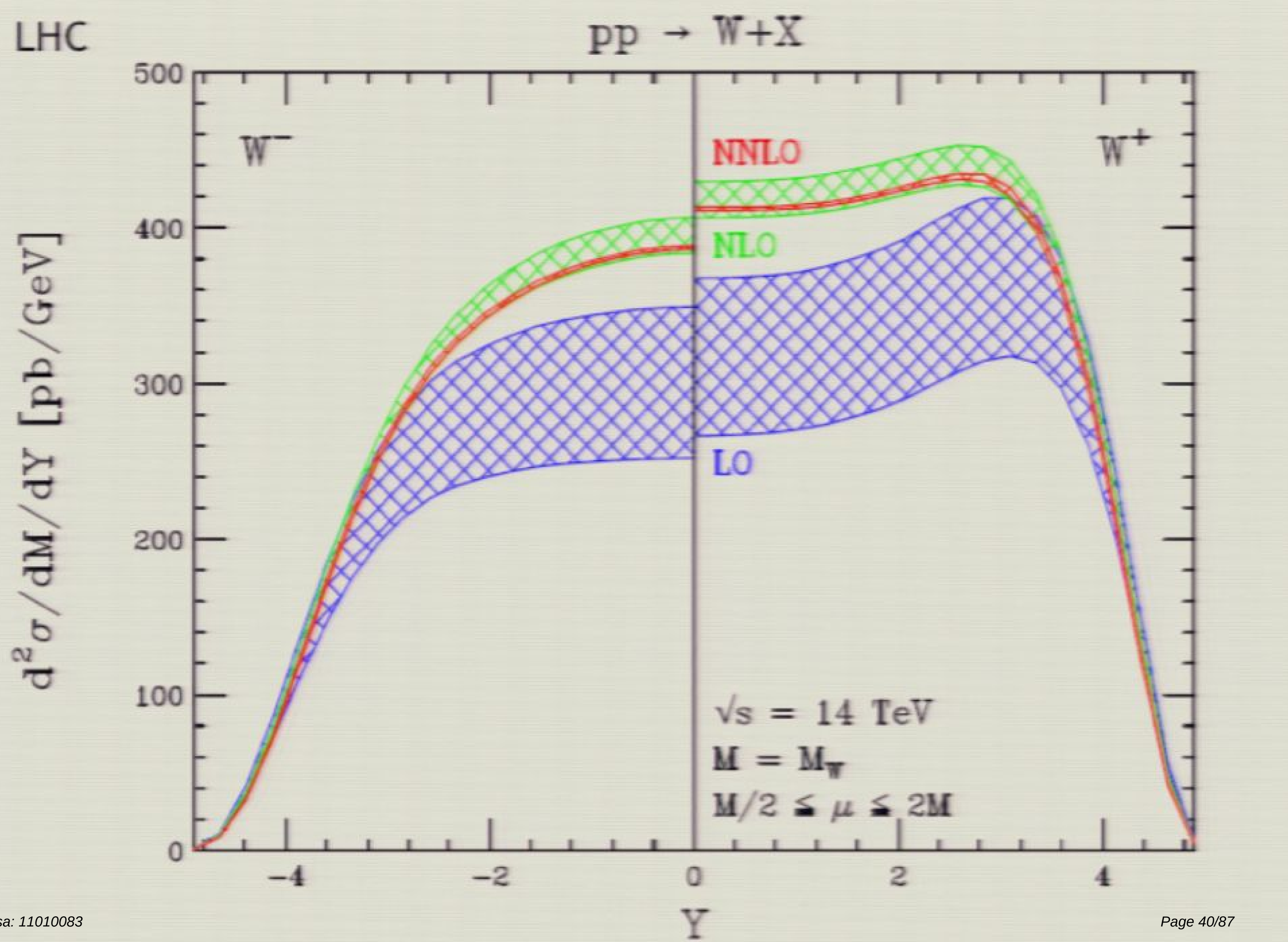
The actual correction to the Drell-Yan rate is even larger, because in higher orders there is a new contribution proportional to the proton's gluon distribution.

It is typical the the NLO corrections to hadron collider cross sections give large postive corrections to the rate.

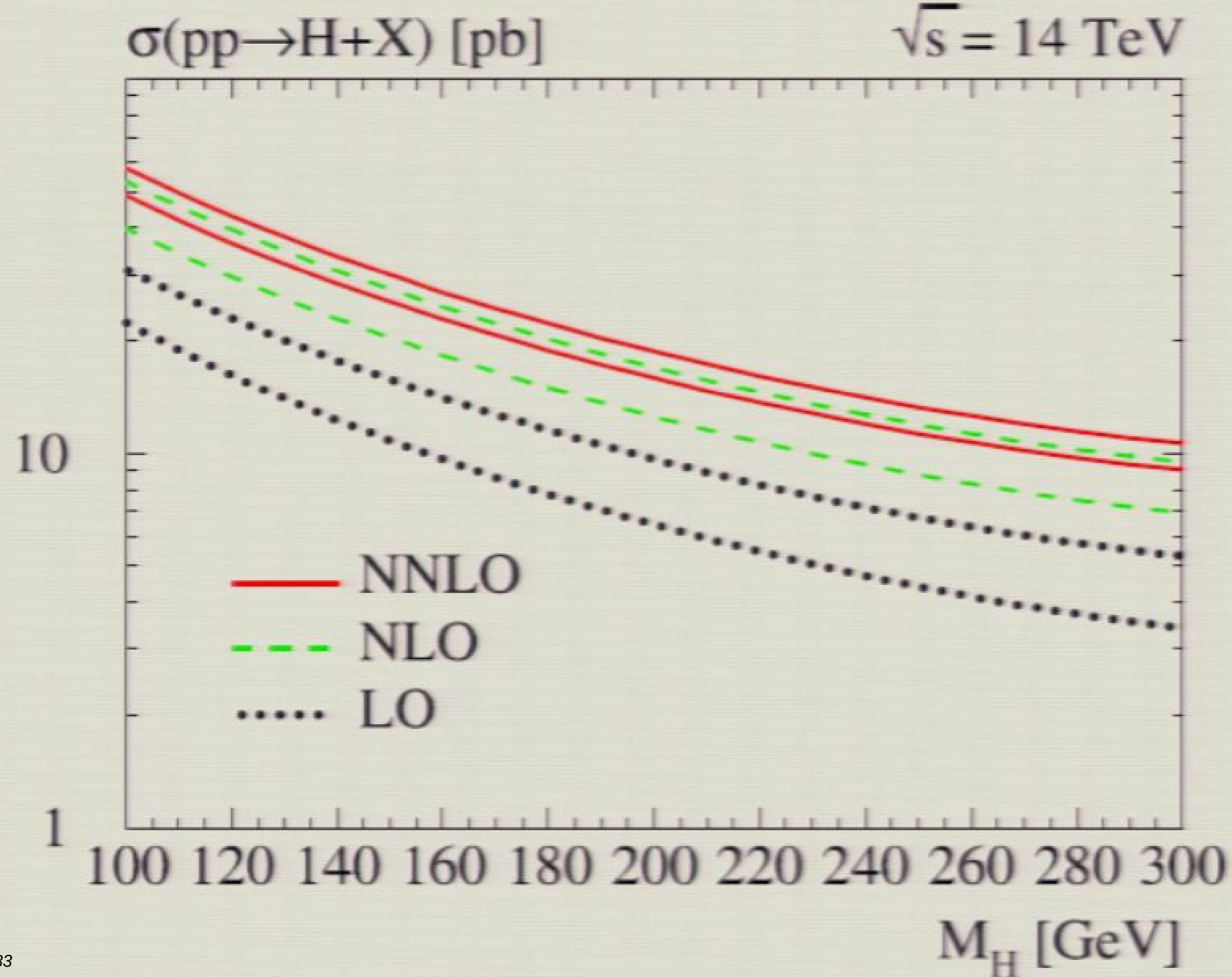
This correction is often expressed as a multiplicative factor on top of the leading order cross section, called a K-factor.

LHC


pp → (Z,γ*)+X
NLO
NNLO
LO
√s = 14 TeV
M = M_Z
M/2 ≤ μ ≤ 2M
d²σ/dM/dY [pb/GeV]
Y
0
20
40
60
80
−4
−2
0
2
4

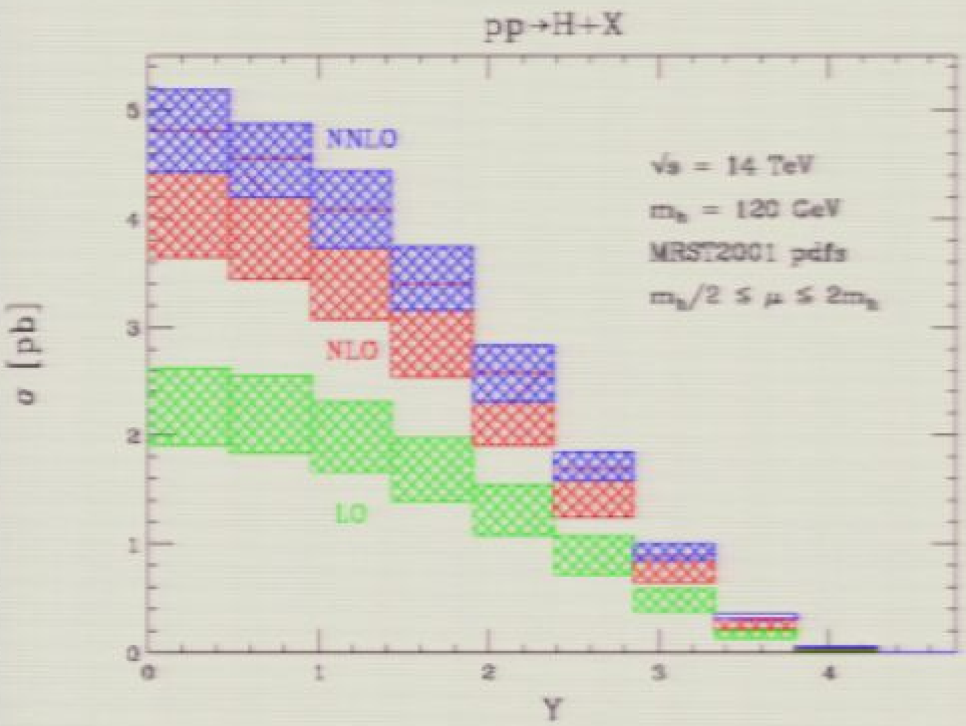

LHC
pp → W+X
d²σ/dM/dY [pb/GeV]
W⁻
W⁺
NNLO
NLO
LO
√s = 14 TeV
M = M_W
M/2 ≤ μ ≤ 2M
Y
0
100
200
300
400
500
−4
−2
0
2
4

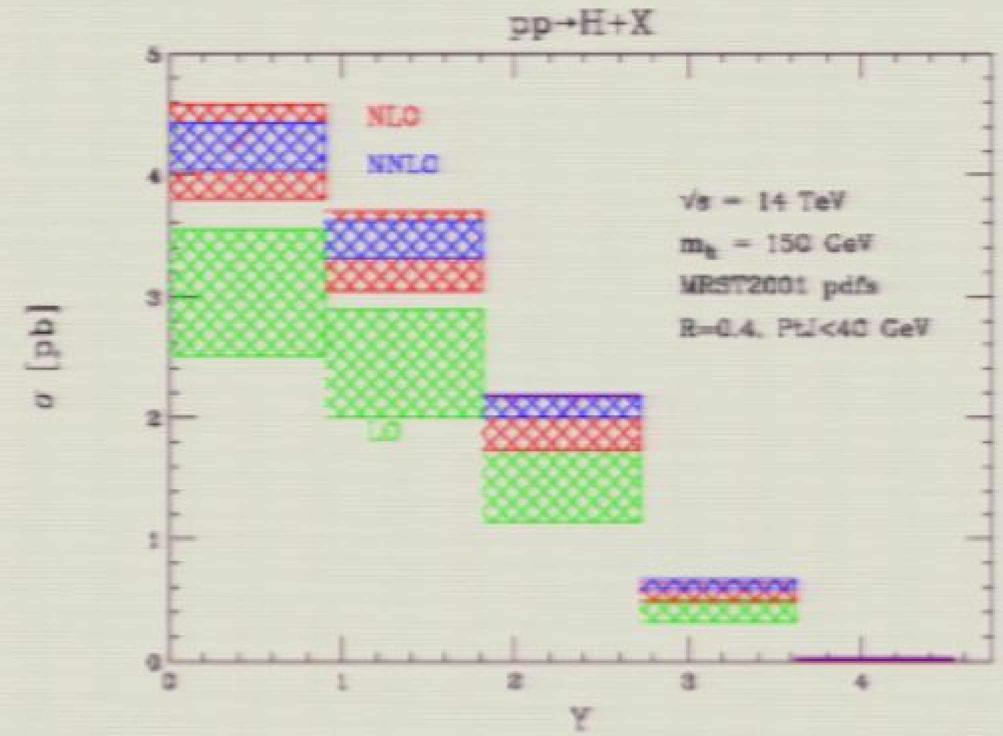
gluon fusion cross section (pb)



Harlander and Kilgore

rapidity distribution of gg->Higgs,
without and with a jet veto





Anastasiou, Melnikov, Petriello

For 2 > 2 processes, these NLO calculations can be done by hand. For 2 > 3 processes, they already involve thousands of diagrams, with thousands of terms each.

At NNLO, or at NLO for 2 > 4 and beyond, brute force Feynman diagram evaluation fails even when done by computers. New methods are needed.

Passarino and Veltman:

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k_1^2 k_2^2 \cdots k_n^2} = \sum A_\pi \int \frac{d^4k}{(2\pi)^4} \frac{1}{k_{\pi(1)}^2 k_{\pi(2)}^2 k_{\pi(3)}^2 k_{\pi(4)}^2}$$

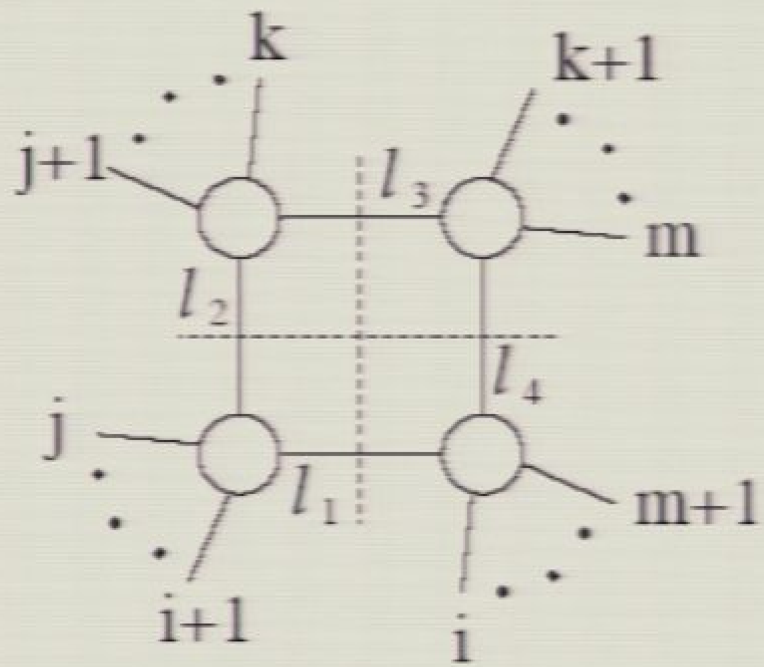
There are a finite number of integrals; do them once and for all.

Bern, Dixon, Dunbar, Kosower:

In N=4 super-Yang-Mills, a much smaller set of integrals appear. These include only boxes, no triangles or bubbles. Identity these by unitary cuts.

Britto, Cachazo, and Feng:

The coefficients can be identified easily by putting 4 intermediate momenta on shell:



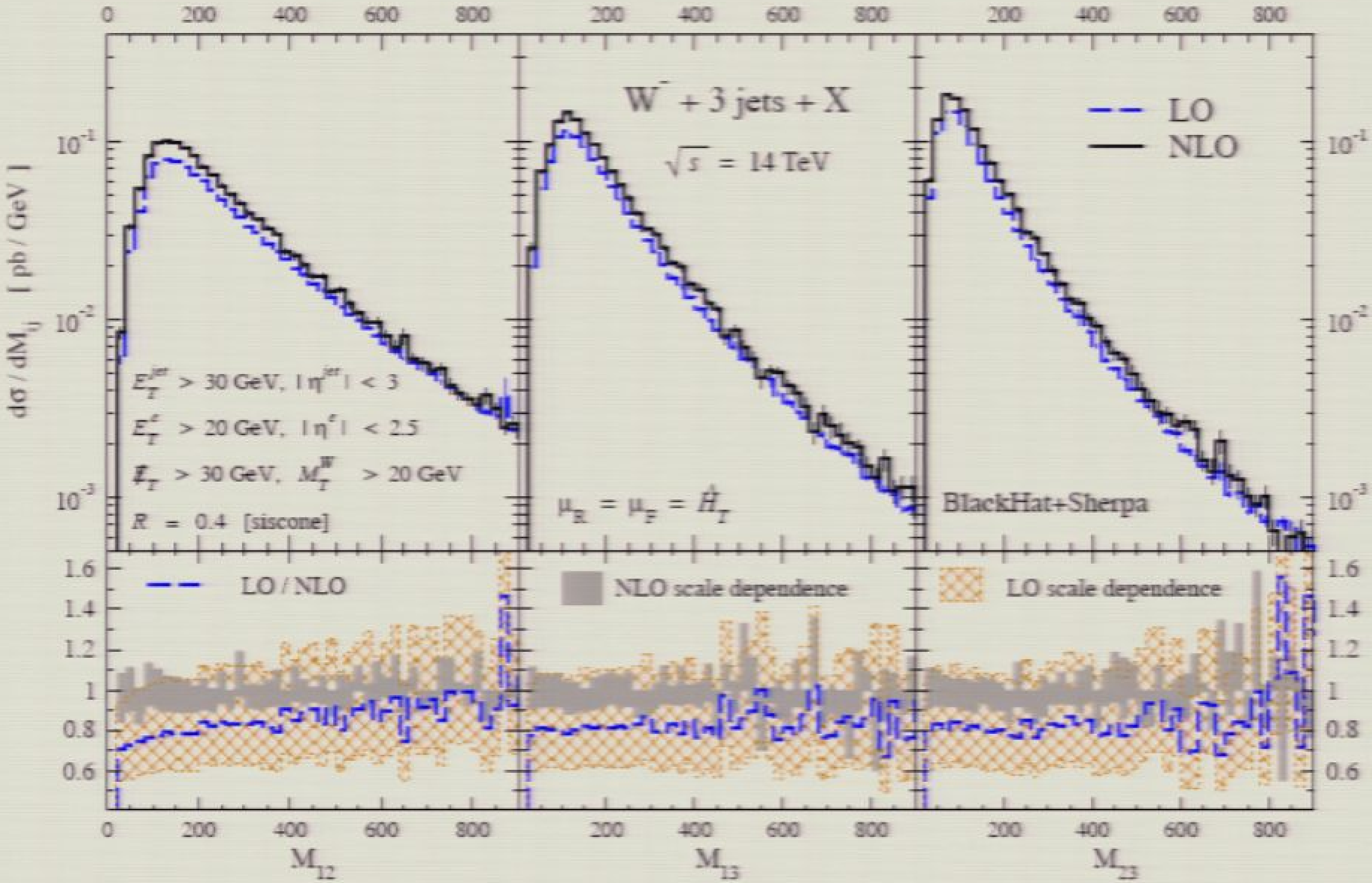
Ossola, Papadopoulos, Pittau,   Forde:

Systematic algorithms for extracting the coefficients of triangle and bubble integrals needed in non-supersymmetric QCD.

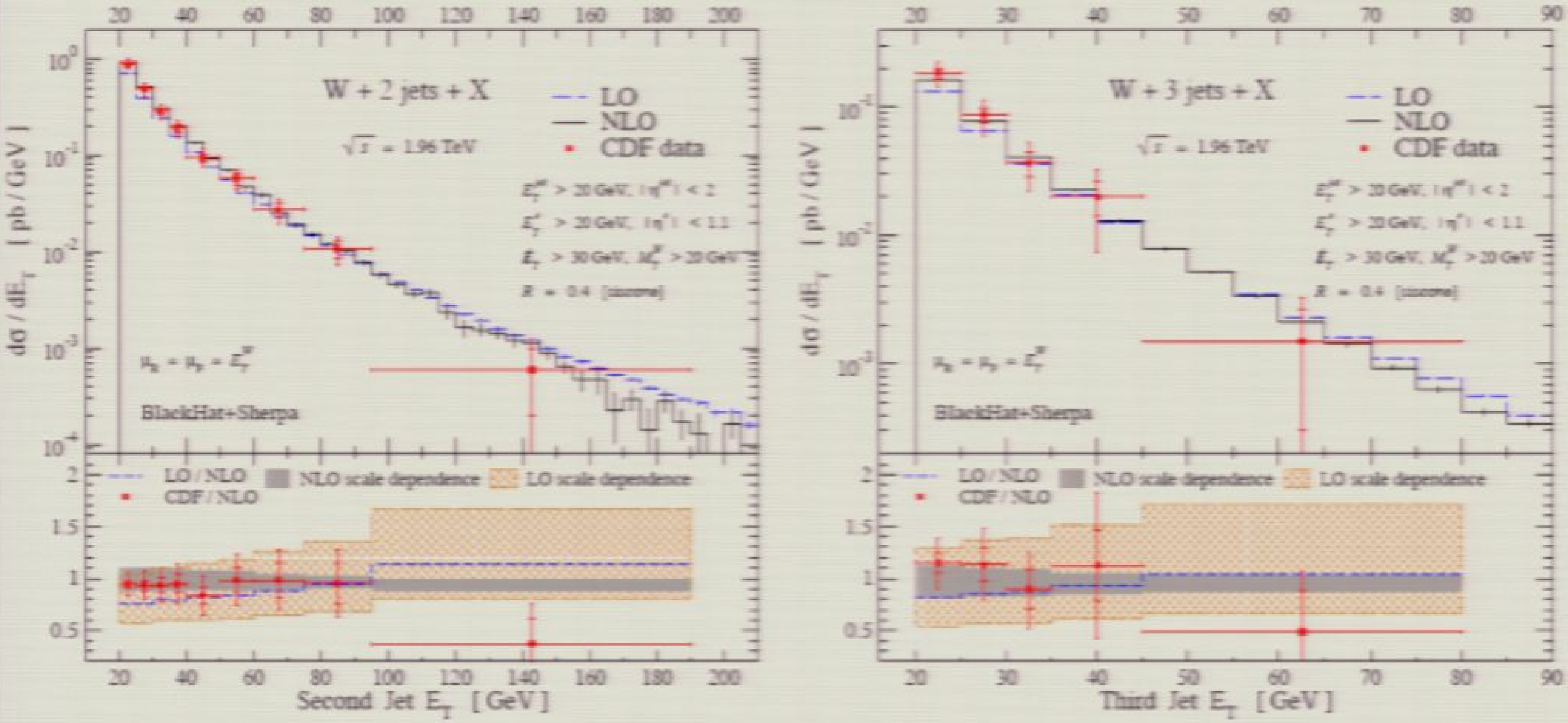
Berger, Forde, ...   Ellis, Kunszt, Zanderighi

Methods for evaluation nonzero rational (cut = 0) pieces.

At this point, it is possible to automate the calcuation of NLO contributions to multiparticle processes.



C.Berger et al (BlackHat)



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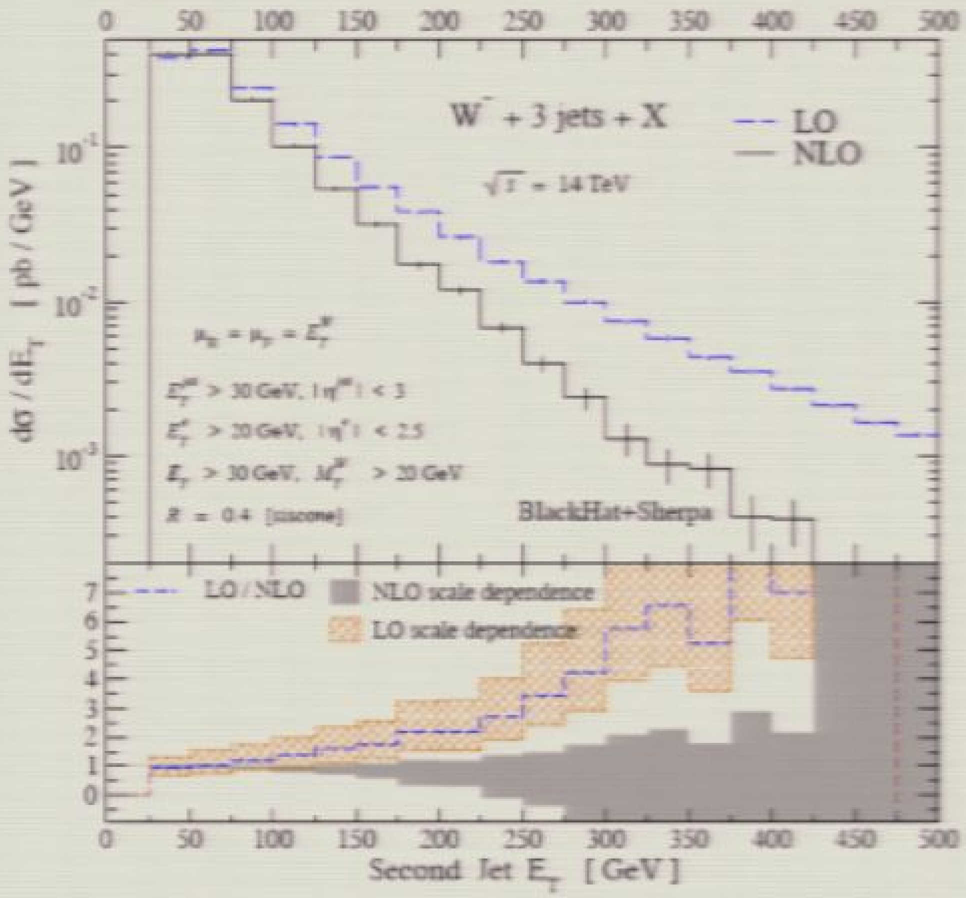
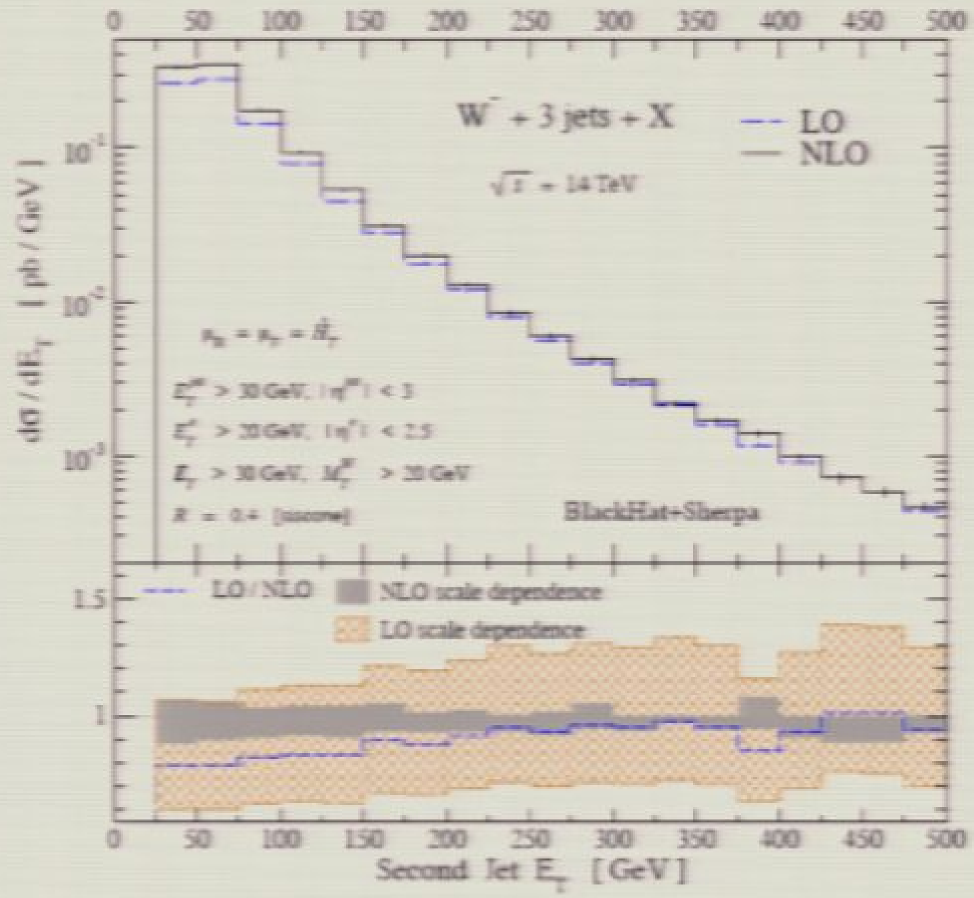




C.Berger et al (BlackHat)

So, approximate our formula for $e^+e^- \to q\bar{q}g$ in the region of collinear qg emission.

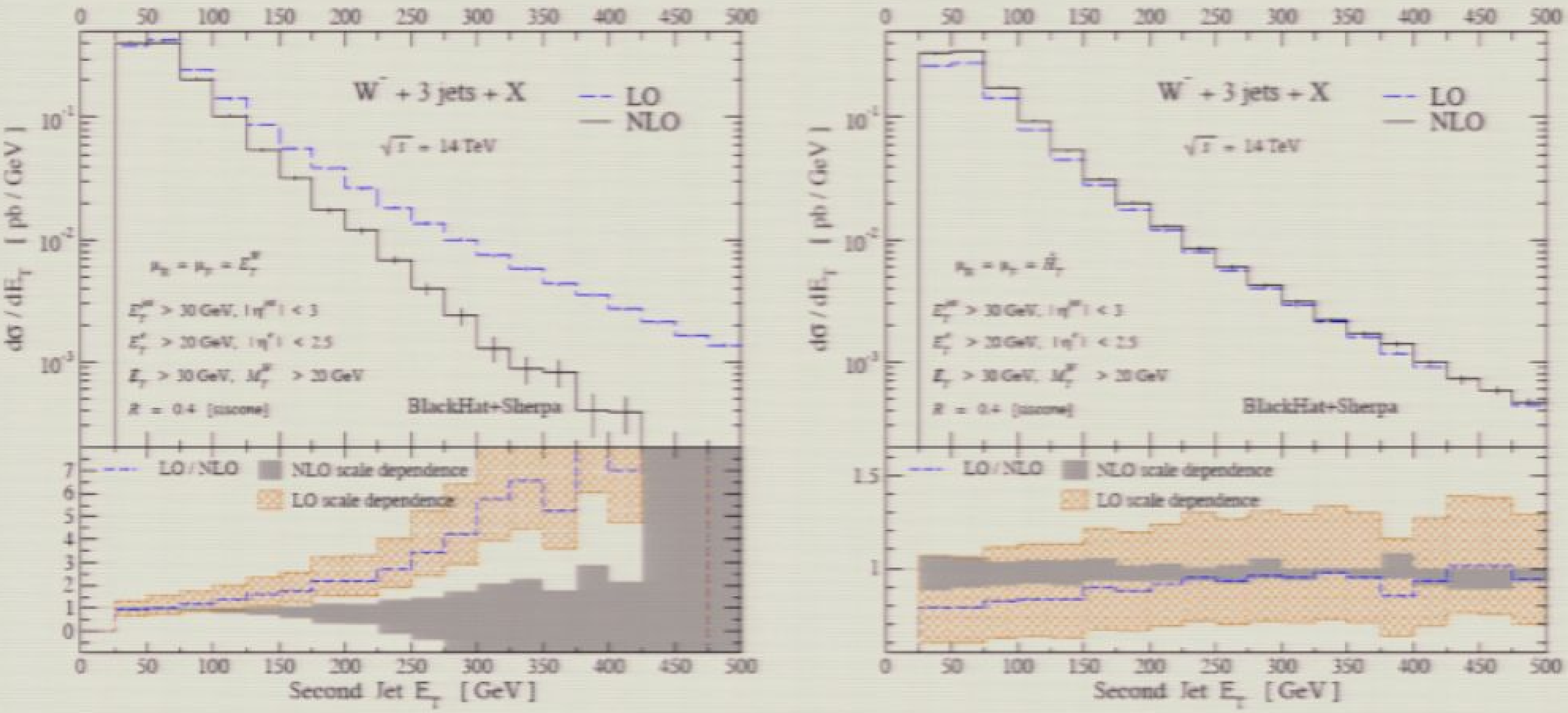
$$x_{\bar{q}} \approx 1 \quad (1 - x_{\bar{q}}) \sim \frac{p_T^2}{s} \quad x_q \approx (1 - x_g)$$

we find

$$\begin{aligned} \frac{\sigma(q\bar{q}g)}{\sigma_0} &= \frac{2\alpha_s}{3\pi} \int dx_q dx_{\bar{q}} \frac{x_q^2 + x_{\bar{q}}^2}{(1 - x_q)(1 - x_{\bar{q}})} \\ &= \frac{2\alpha_s}{3\pi} \int \frac{dp_T^2}{p_T^2} \int dx_g \frac{1 + (1 - x_g)^2}{x_g} \end{aligned}$$

From this, we can compute the probability of finding a gluon at large transverse momentum:

$$\frac{\sigma(p_T > P_T)}{\sigma_0} = \frac{2\alpha_s}{3\pi} log \frac{s}{P_T^2} \cdot \int dx_g \frac{1 + (1 - x_g)^2}{x_g}$$



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$$= \frac{2\alpha_s}{3\pi} \int \frac{dp_T^2}{p_T^2} \int dx_g \frac{1 + (1 - x_g)^2}{x_g}$$

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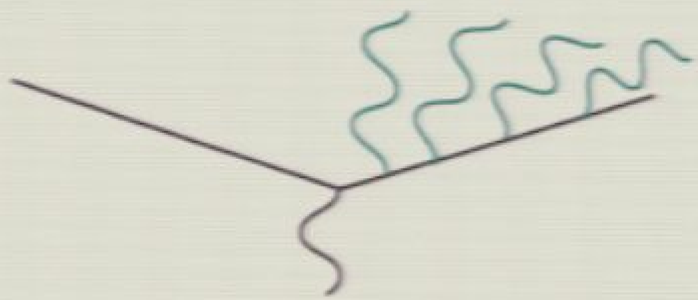
This formula raises an issue noted earlier:

$$\frac{\alpha_s}{\pi} \log \frac{s}{P_T^2}$$

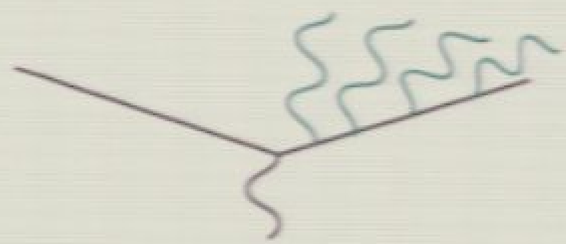
can be of order 1 for sufficiently small P_T . This factor needs to be summed to all orders.

In other words, there is high probability of emitting many almost collinear gluons.



Can we do the accounting for these emissions ?

Using the formula for one collinear gluon emission, we can organize the series of gluon emissions. In analyzing the diagram



assume that each successive gluon is emitted at a lower value of p_T. Only then will each successive propagator be singular and give a logarithmic enhancement.

This condition, $p_{T1} \gg p_{T2} \gg p_{T3} \gg \cdots$ is called strong ordering.

Consider adding the last gluon in the p_T region $(P_T, P_T + dP_T)$ Then if $\mathcal{P}_g(z, P_T)$ is the probability to find a gluon with the fraction z of the original quark momentum and $p_T > P_T$,

$$P_T^2 \frac{d}{dP_T^2} \mathcal{P}_g(z, P_T) = \int dx \frac{\alpha_s}{2\pi} \int dw P_{q\to gq}(w) \mathcal{P}_q(x, P_T) \delta(z - xw)$$

Integrating over the delta function, we find

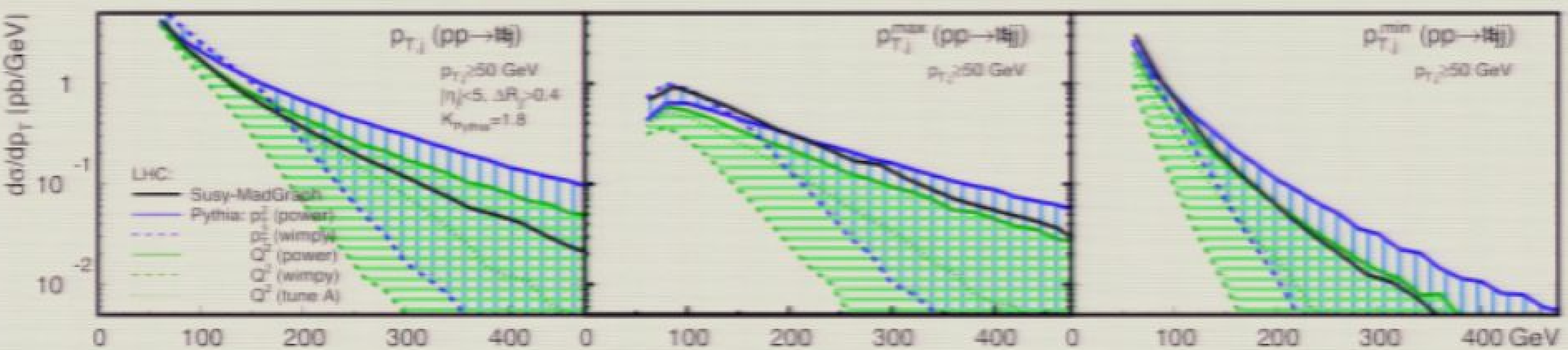
$$\frac{d}{d\log P_T}\mathcal{P}_g(z,P_T) = \frac{\alpha_s}{\pi}\int \frac{dw}{w}\mathcal{P}_q(\frac{z}{w},P_T)P_{q\to gq}(w)$$

This is called the Altarelli-Parisi equation. For the complete equation, we should sum over all species that could radiate a collinear gluon:

$$\frac{d}{d\log P_T}\mathcal{P}_g(z,P_T) = \frac{\alpha_s}{\pi}\int \frac{dw}{w}\sum_f \mathcal{P}_f(\frac{z}{w},P_T)P_{f\to g}(w)$$

By integrating this equation, we can find the full gluon content of the final state of e+e- annihilation to quarks.

One can use these various methods to estimate the rates for producing 1, 2 extra jets in association with $t\bar{t}$ at the LHC. Finding agreement between theory and experiment for these rates will give us confidence that we can predict backgrounds to new physics due to the tails of SM top production.



Plehn, Rainwater, Skands

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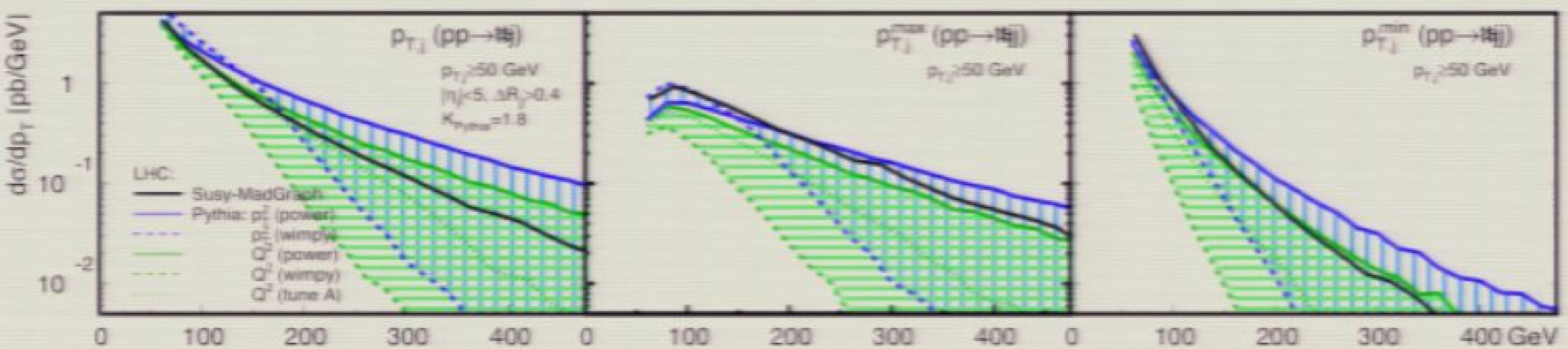
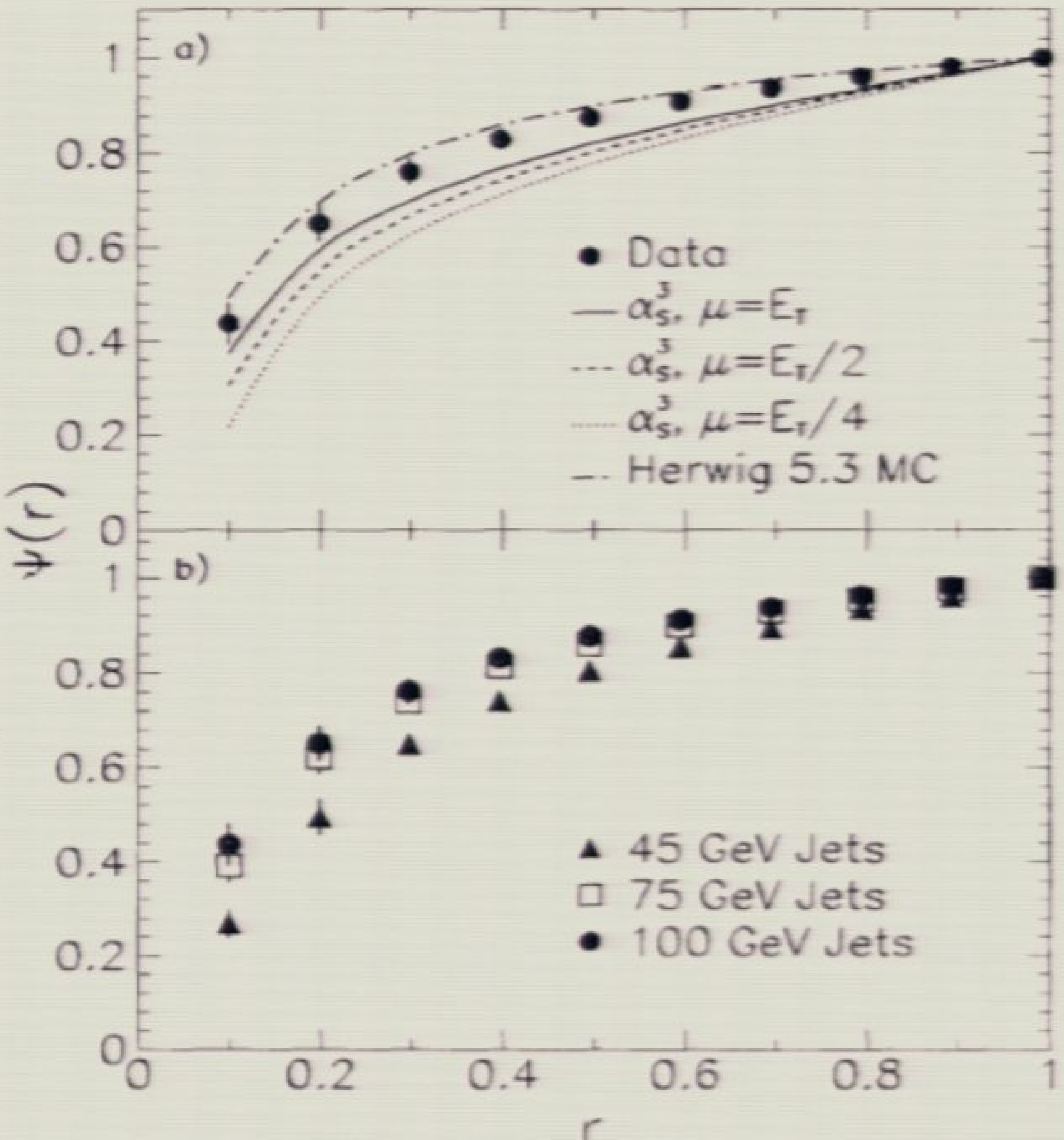
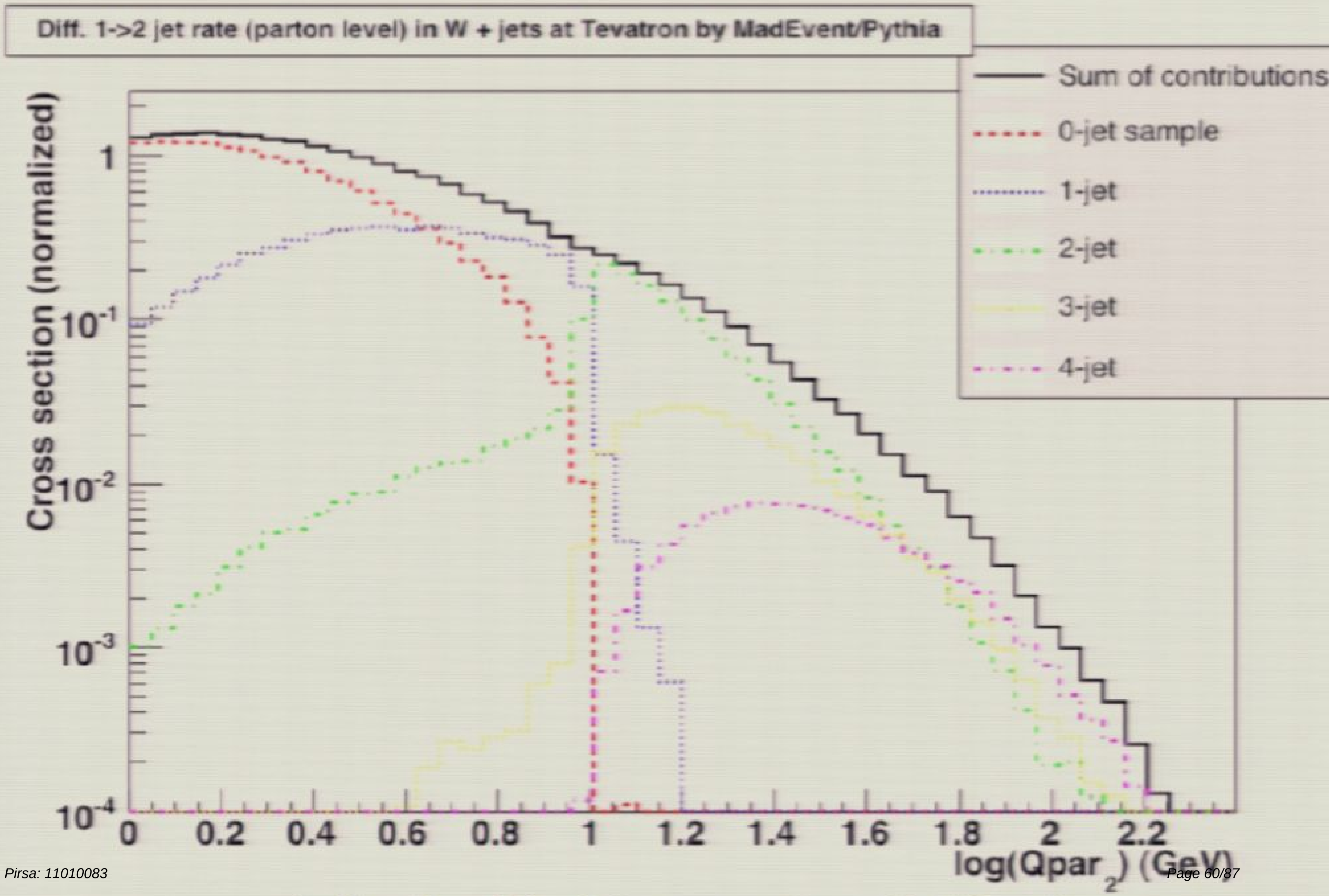


Plehn, Rainwater, Skands

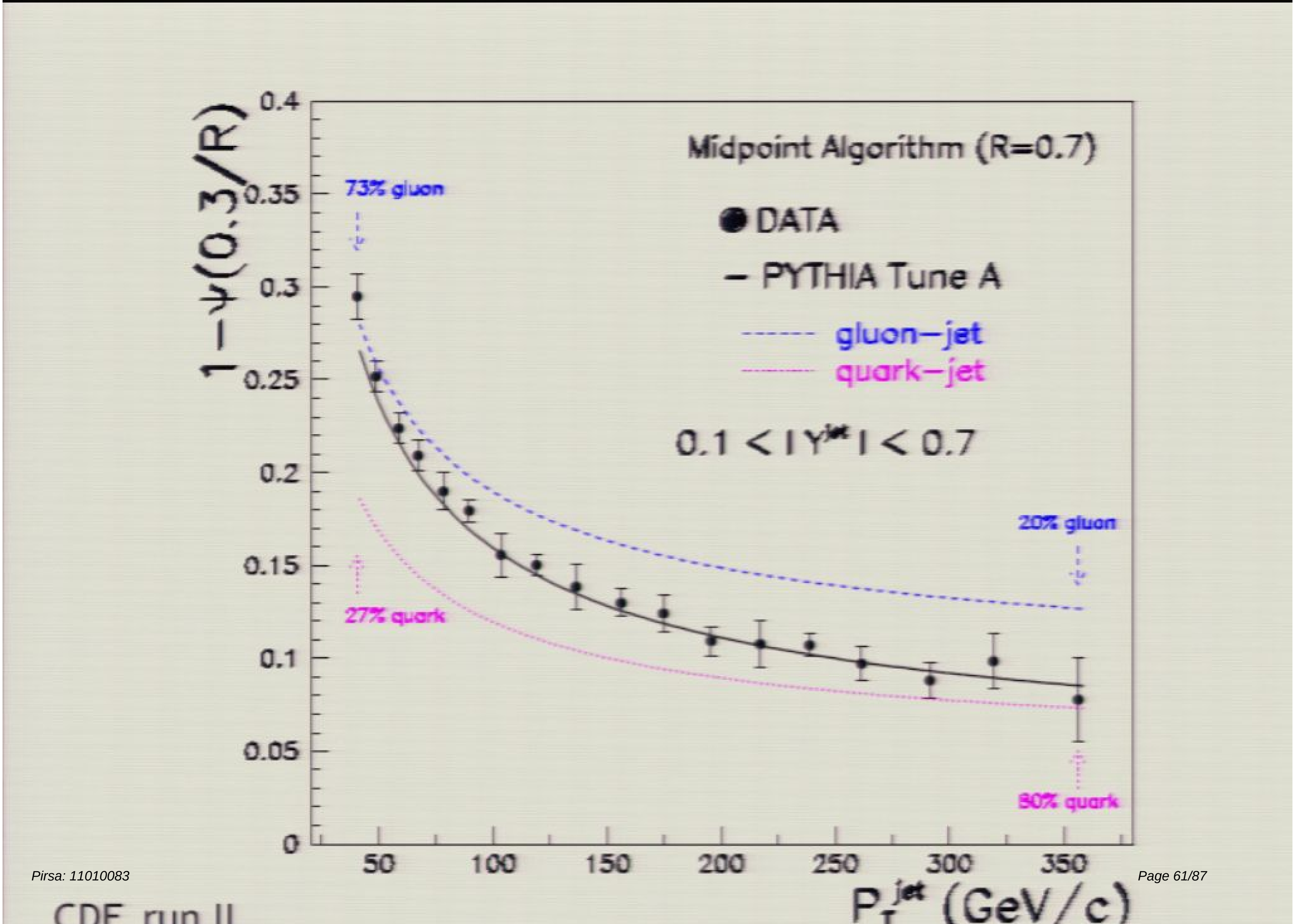


data: CDF

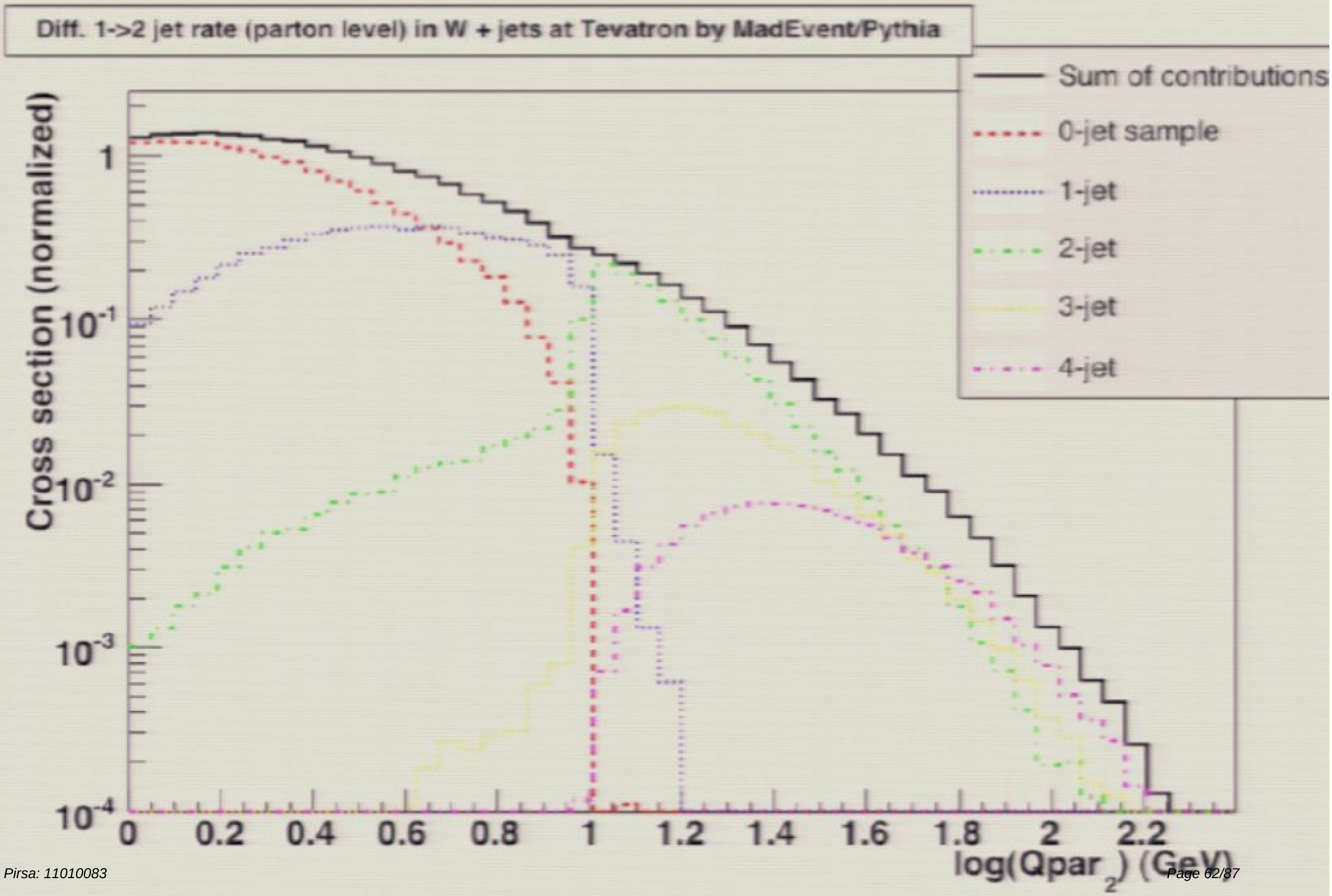
theory:
S. Ellis, Kunszt, and Soper



J. Alwall: W + 2 jet rates from MadGraph



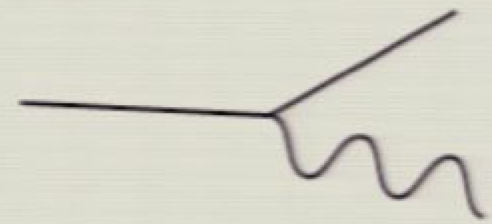
Midpoint Algorithm (R=0.7)
73% gluon
DATA
PYTHIA Tune A
gluon−jet
quark−jet
0.1 < I Y^jet I < 0.7
20% gluon
27% quark
80% quark
1−Ψ(0.3/R)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
50
100
150
200
250
300
350
P_T^jet (GeV/c)
CDF run II



J. Alwall: W + 2 jet rates from MadGraph

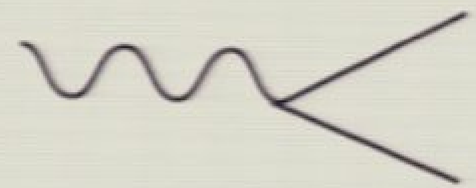
It is a straightforward exercise to work out the other Altarelli-Parisi splitting functions by using definite polarization vectors with the QCD vertices:

$$P_{q_L\to g_L}(z) = \frac{4}{3}\frac{1}{z}$$



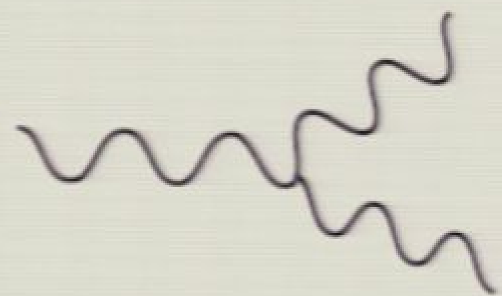
$$P_{q_L\to g_R}(z) = \frac{4}{3}\frac{(1-z)^2}{z}$$

$$P_{q_L\to q_L}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(z-1)\right]$$



$$P_{g_L\to q_L}(z) = \frac{1}{2}z^2$$

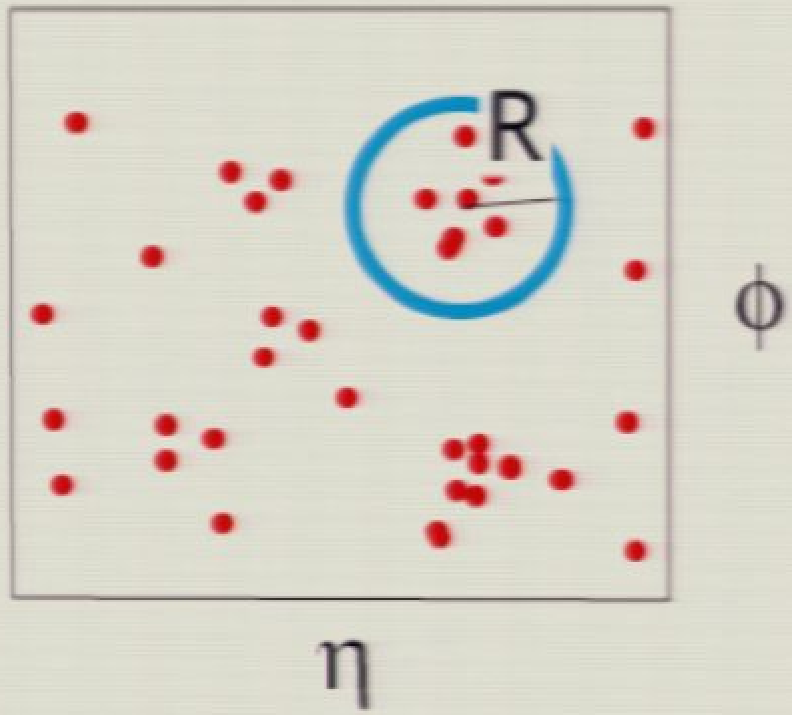
$$P_{g_L\to q_R}(z) = \frac{1}{2}(1-z)^2$$



$$P_{g_L\to g_R}(z) = 3\frac{(1-z)^3}{z}$$

$$P_{g_L\to g_L}(z) = 3\left[\frac{1}{z(1-z)_+} + \frac{z^4}{(1-z)_+} + \left(\frac{11}{6} - \frac{n_f}{9}\right)\delta(z-1)\right]$$

A cone algorithm places circles of fixed radius R on the (η, ϕ) plane so that each circle contains as much p_T as possible



A seeded cone algorthm that starts the cones at the positions of hadrons or quarks and gluons is infrared unsafe.

Global search for cone positions without specific seeds gives the SIScone algorithm, which is infrared safe.

2nd method: iterative algorithms

Define a distance measure, including distance from the beam direction. Combine objects i,j at the smallest distance.

If the iB distance is the smallest, consider i as a jet, and drop it from the list of particles.

Continue until no particles remain.

kt algorithm:

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2) \cdot \Delta R_{ij}^2 / R^2 \ , \qquad d_{iB} = p_{Ti}^2$$

Cambridge-Aachen:

$$d_{ij} = \Delta R_{ij}^2 / R^2 \ , \qquad d_{iB} = 1$$

anti-kt:

$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2}) \cdot \Delta R_{ij}^2 / R^2 \ , \qquad d_{iB} = p_{Ti}^{-2}$$

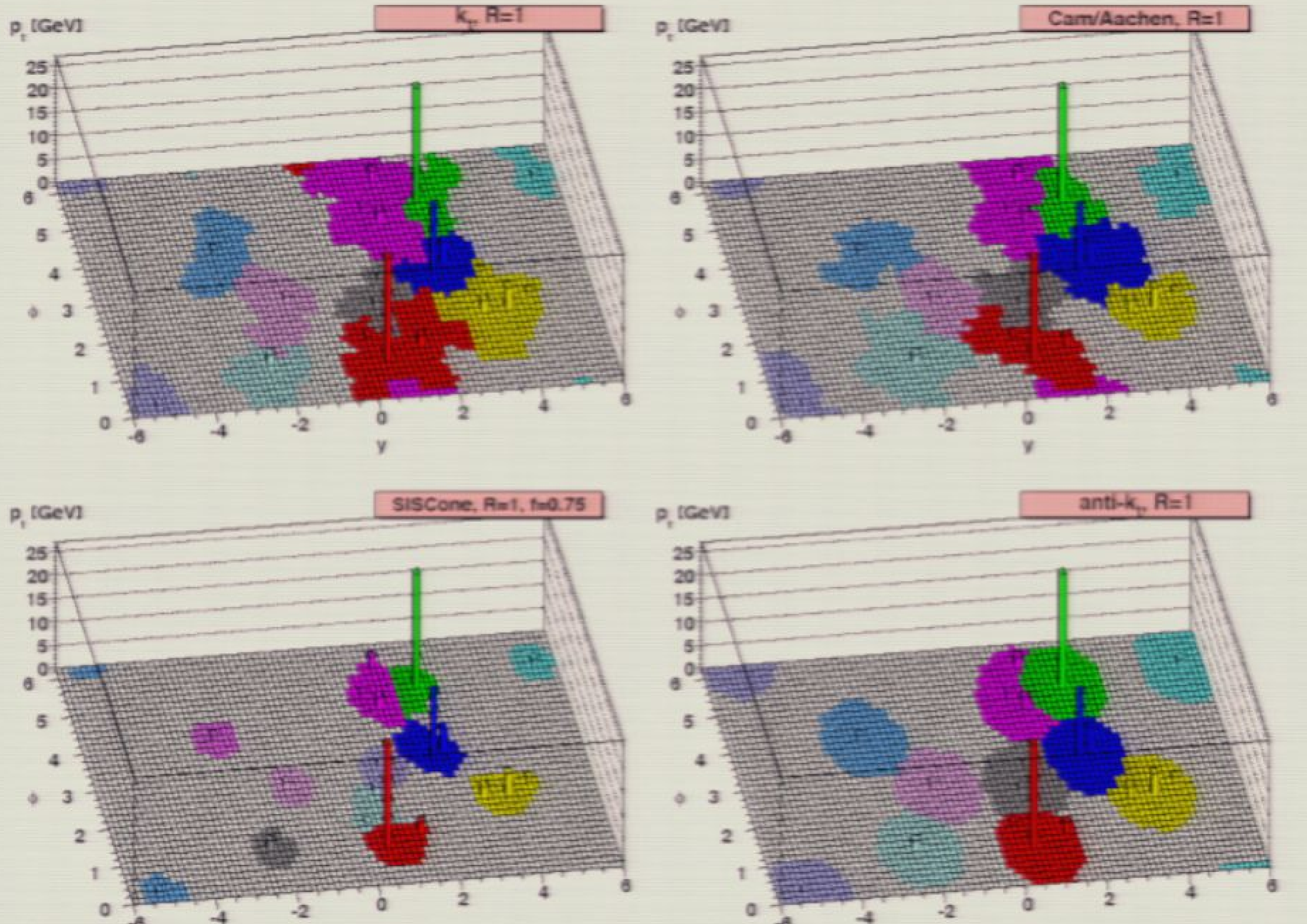
All of these algorithms are infrared-safe.

The three algorithms collect particles into jets in different ways. The variation might suit different purposes in analysis.

Anti-kt has the nice property of producing round, cone-like jets.

ATLAS and CMS now use anti-kt jets with R = 0.4 - 0.7 for most jet-analysis purposes.

With a definite jet definition, we can compute n-jet diffential cross sections and compare to experiment.



recommended reference: G. Salam, arXiv 0906.1833

Finally, it would be good to be able to tag exotic Standard Model objects such as W, top, and Higgs efficiently in their hadronic decay modes. The hope has been raised recently that this can be possible when the heavy particle is highly boosted in the lab, as the result of being emitted in a high momentum transfer reaction.

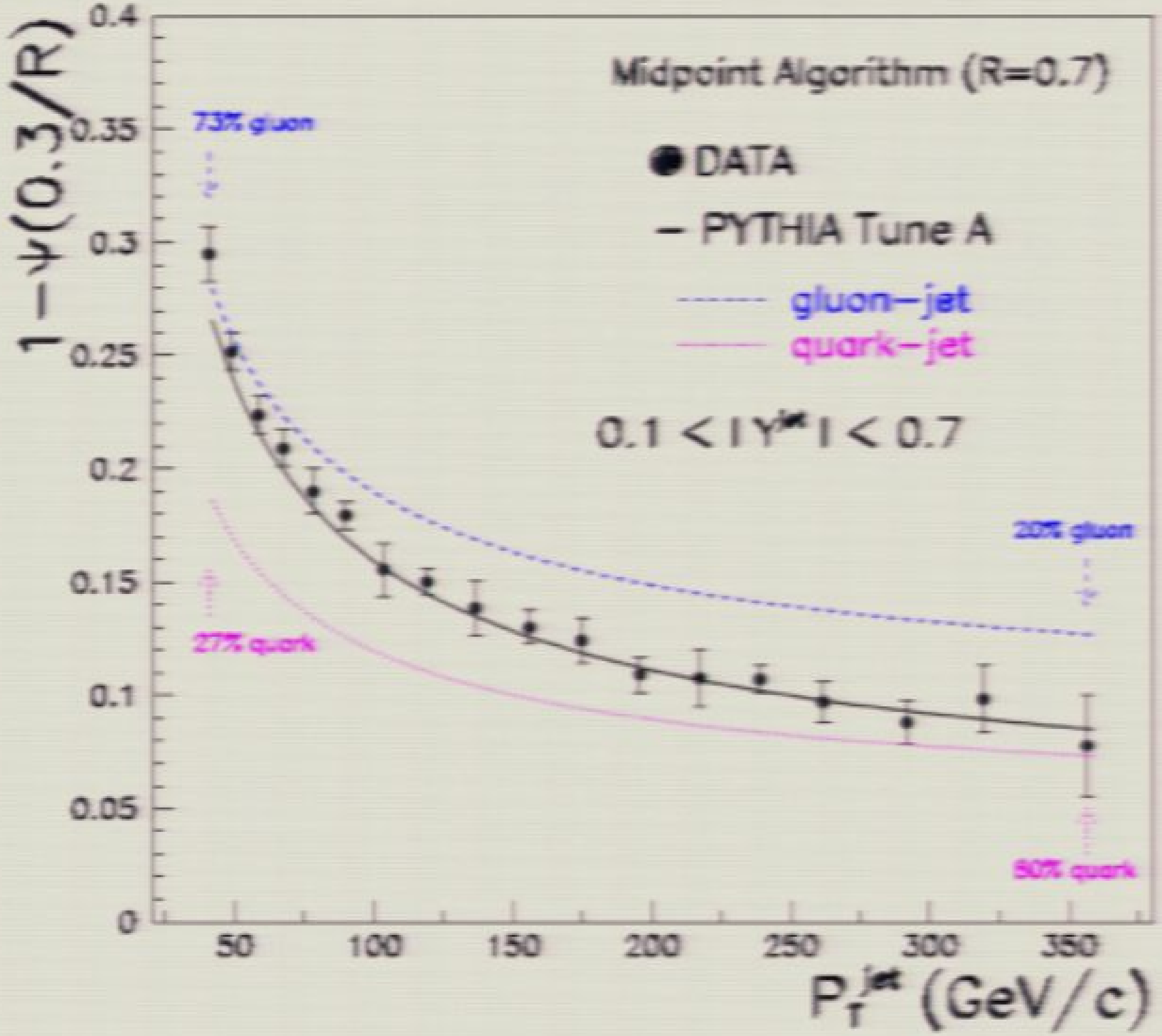
This is a special situation, but one of intrinsic interest.

Some of the players are:

ATLAS - Brooijmans, Butterworth
CMS - Hadley, Swartz
theory - Ellis, Perez, Kaplan, Salam, Schwartz, Wang

They consider the situation in which a heavy particle is boosted so that is fits in an R= 0.4 cone and so looks like a single jet.

It is well known that gluon jets are fatter than quark jets. This effect has been used at the Tevatron to verify the predicted crossover from gg to $q\bar{q}$ scattering as ET is increased.



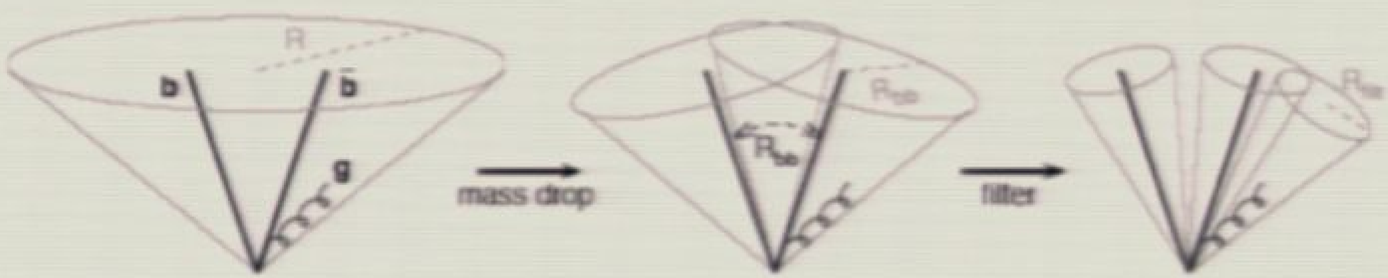
CDF, run II

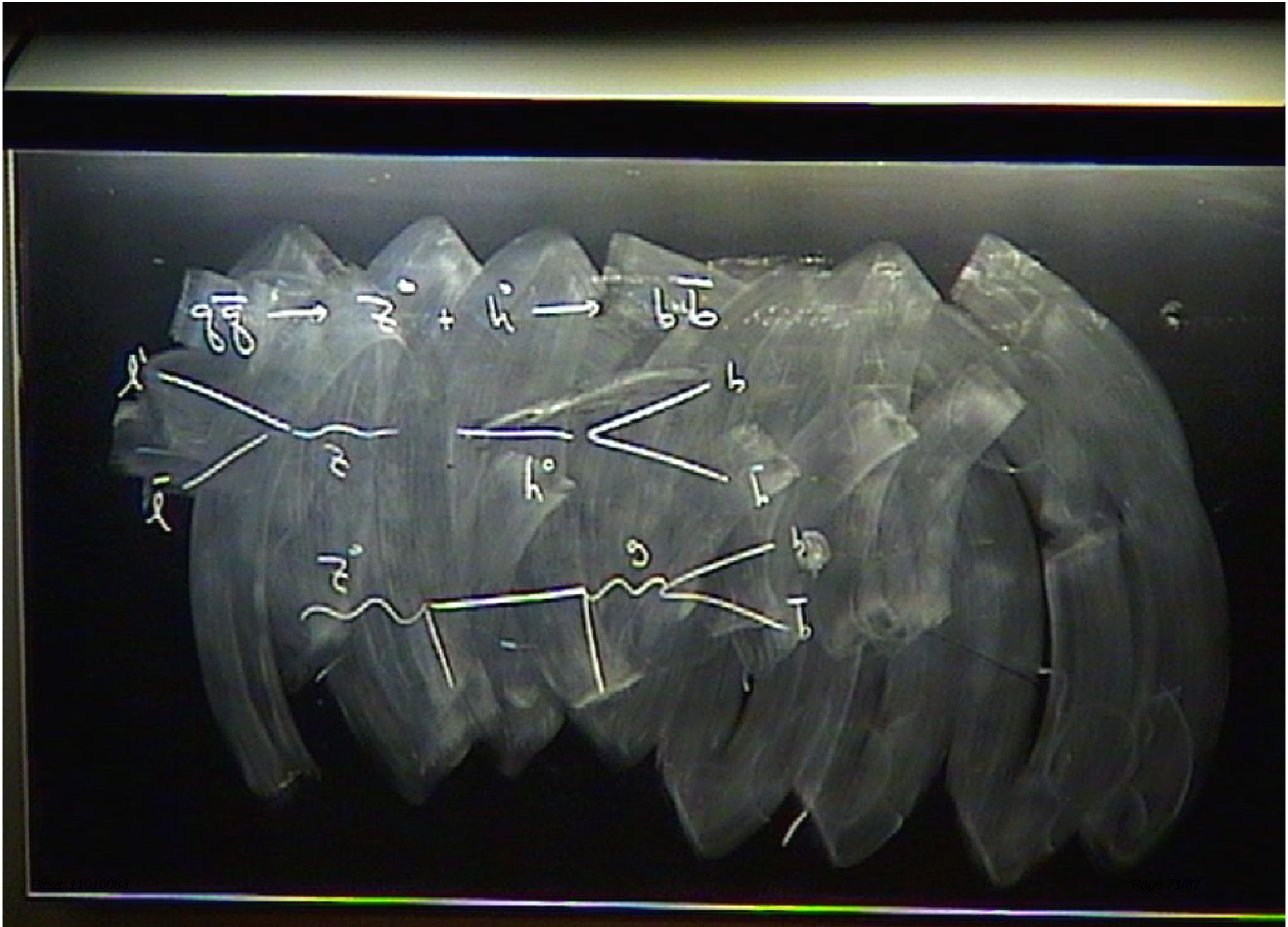
Try to be more sophisticated about this.

Jets are assembled from particles by jet algorithms. Some jet algorithms (kT, Cambridge-Aachen, but NOT anti-kT) systematically create clusters of increasing mass or internal kT. These clusters can be considered as subjets.

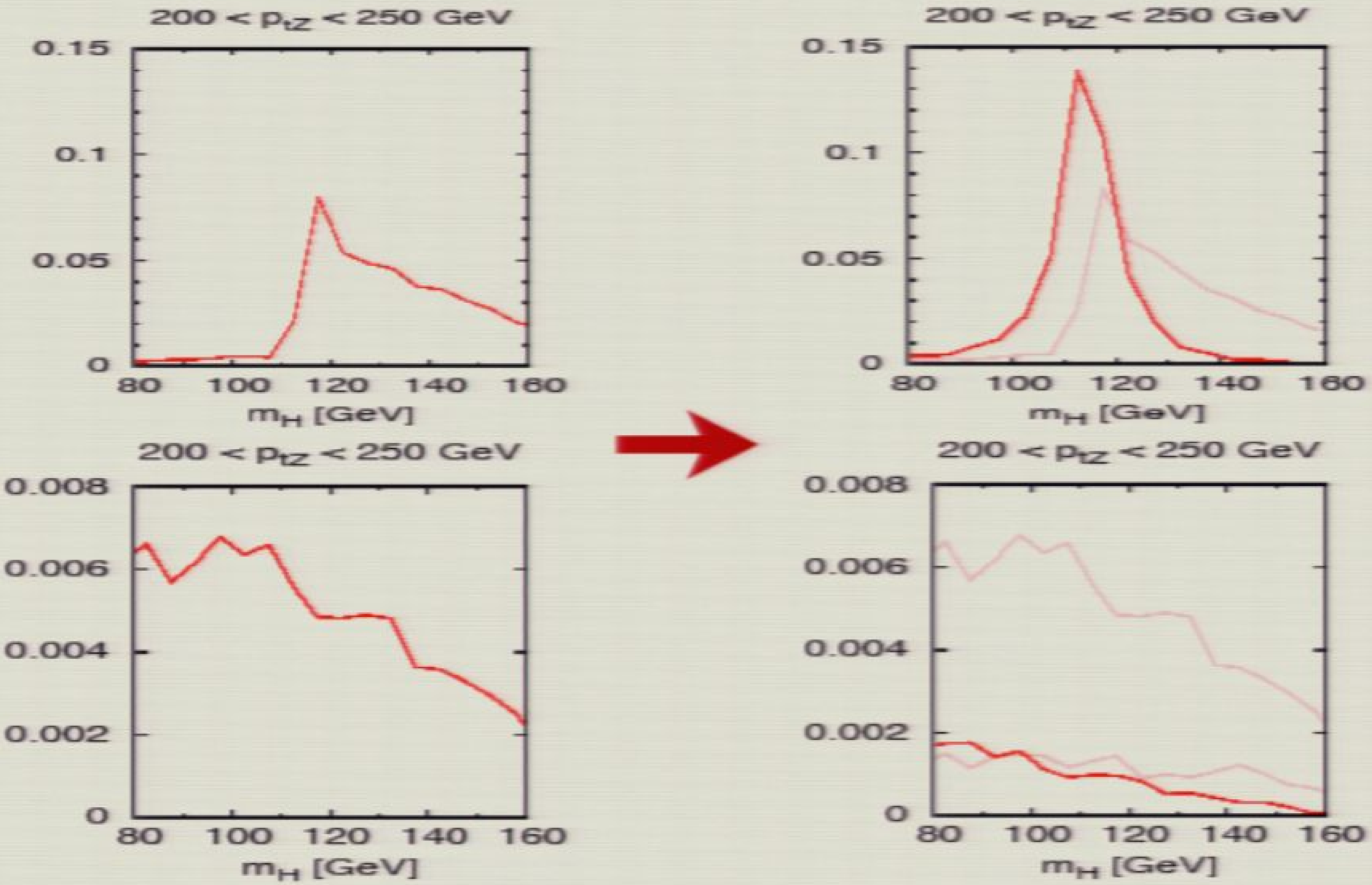
Heavy-particle decay leads to two subjects of relatively equal z. Gluon radiation leads to a subject that typically has a small z. So, to isolate heavy particles, look at the tree of subjets and remove ('prune') subjects of small z.

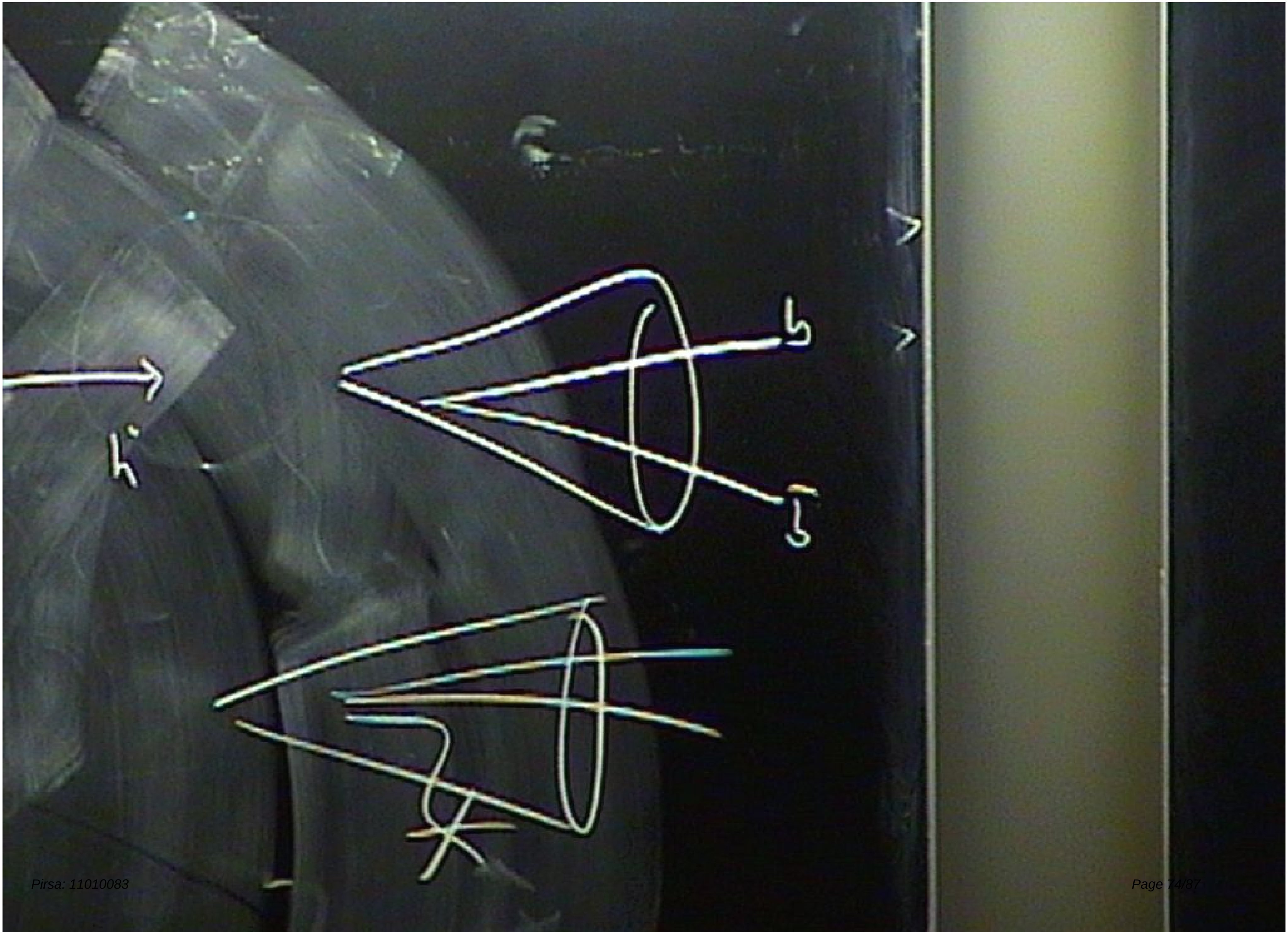
This destroys jets built from successive gluon radiations, but it has little effect on 'exotic jets' formed from a boosted heavy particle.

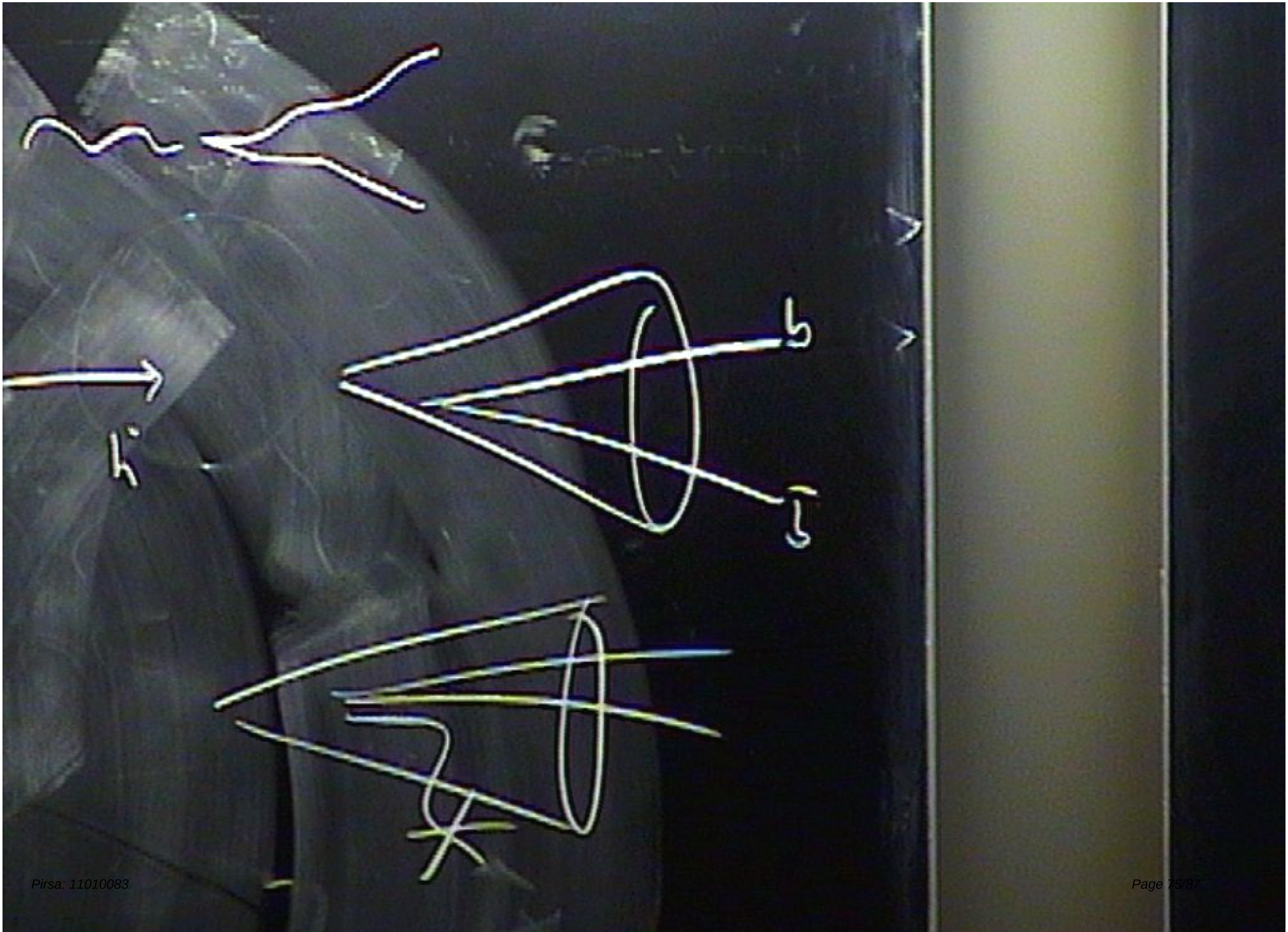


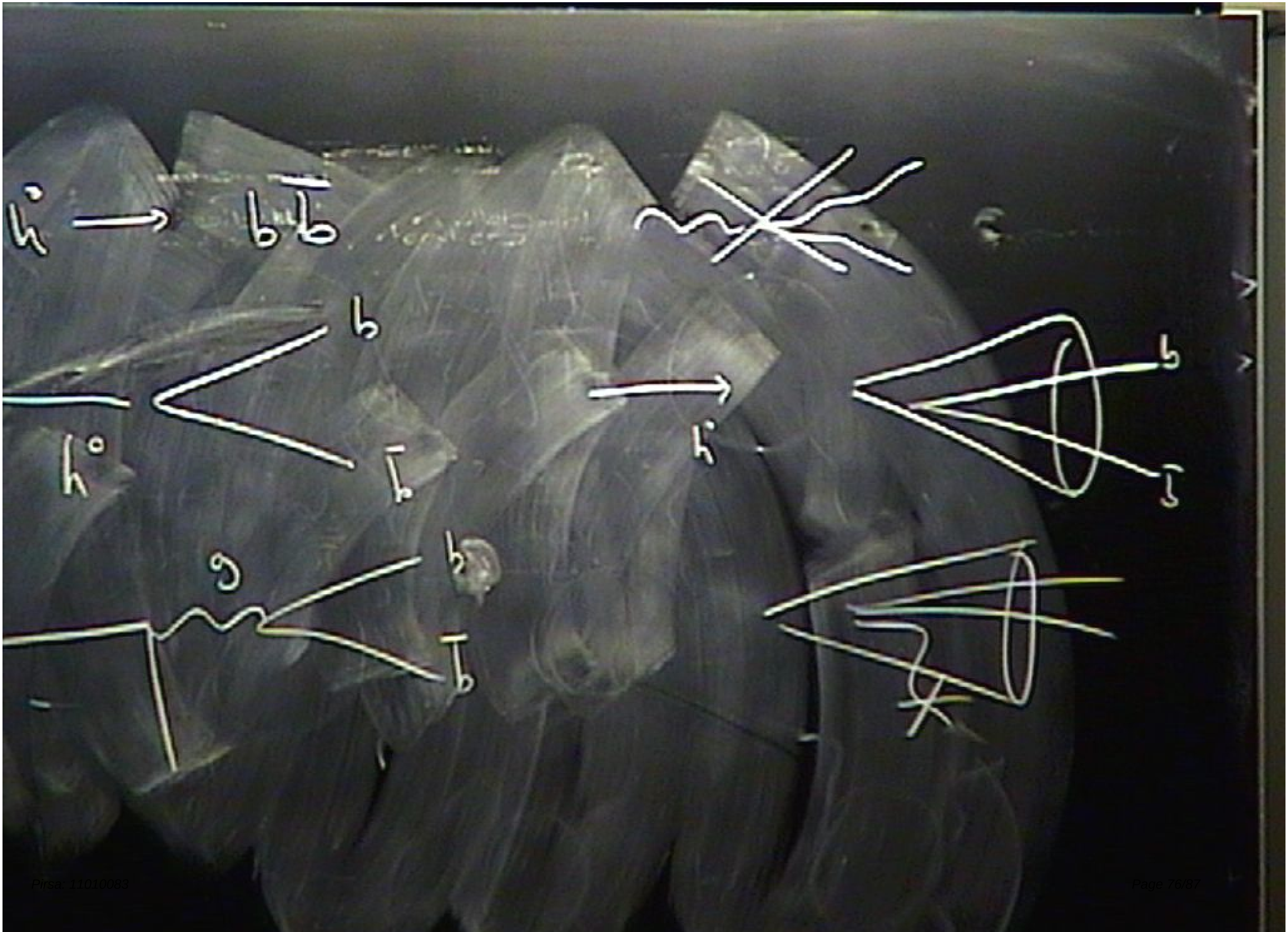

q q̄ → Z° + h° → b b̄
l
l̄
Z
h°
b
b̄
Z°
g
b
b̄

Here is an example from Butterworth, Davison, Rubin, Salam, for the boosted Higgs in $q\bar{q} \to Wh$






b
b


h → b b̄
b
b̄
h°
h
b
b̄
g
b
b̄

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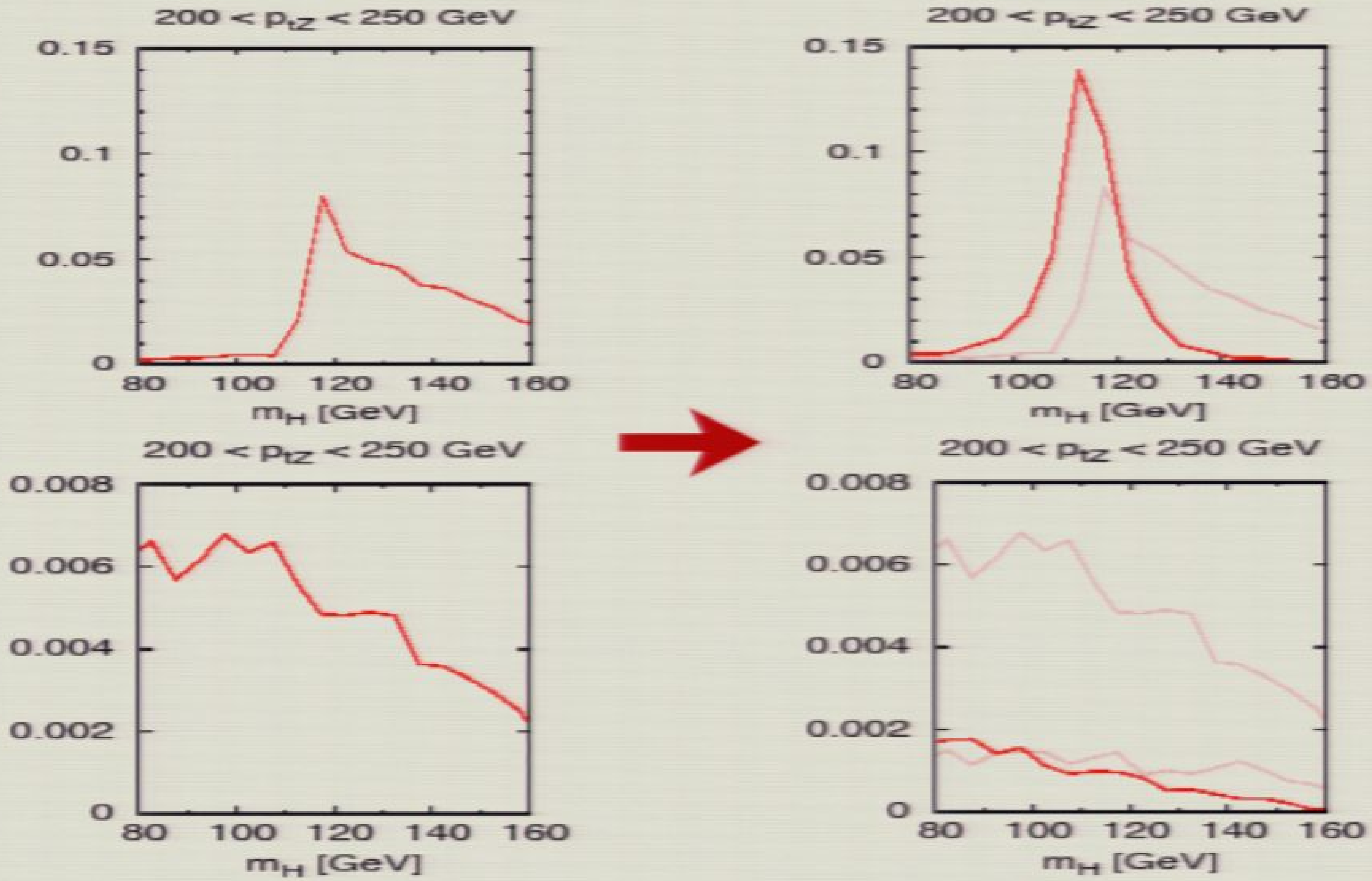

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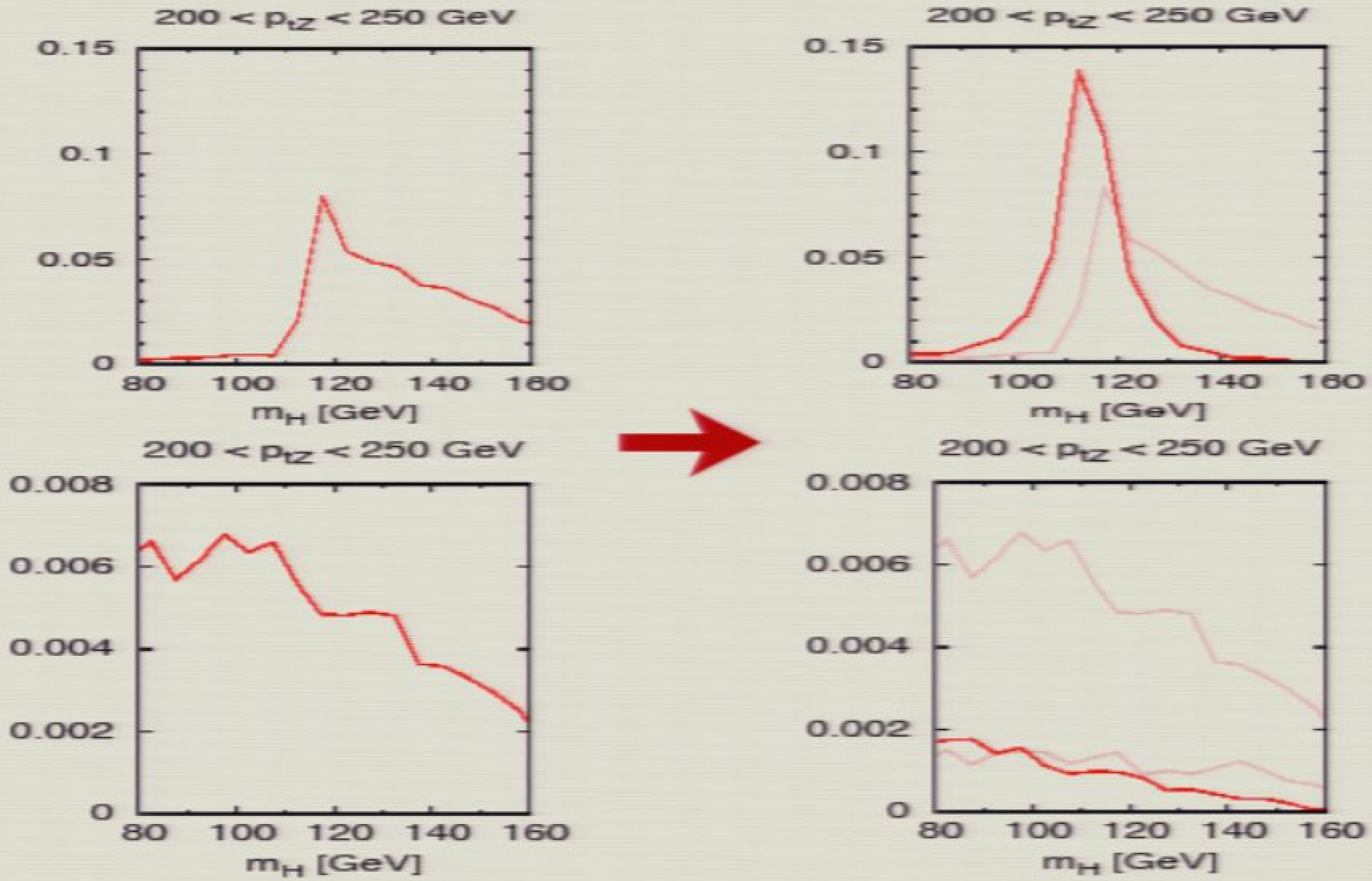


The main effect on the Higgs mass search turns out to come from the sharpening of the Higgs mass distribution. Here pruning has the effect of measuring the color of the boosted system.

These methods work in Monte Carlo, but do they work on real jets ? These methods can be tested at the LHC (and, still, at the Tevatron) on jet samples with different color content. For example, at LHC,

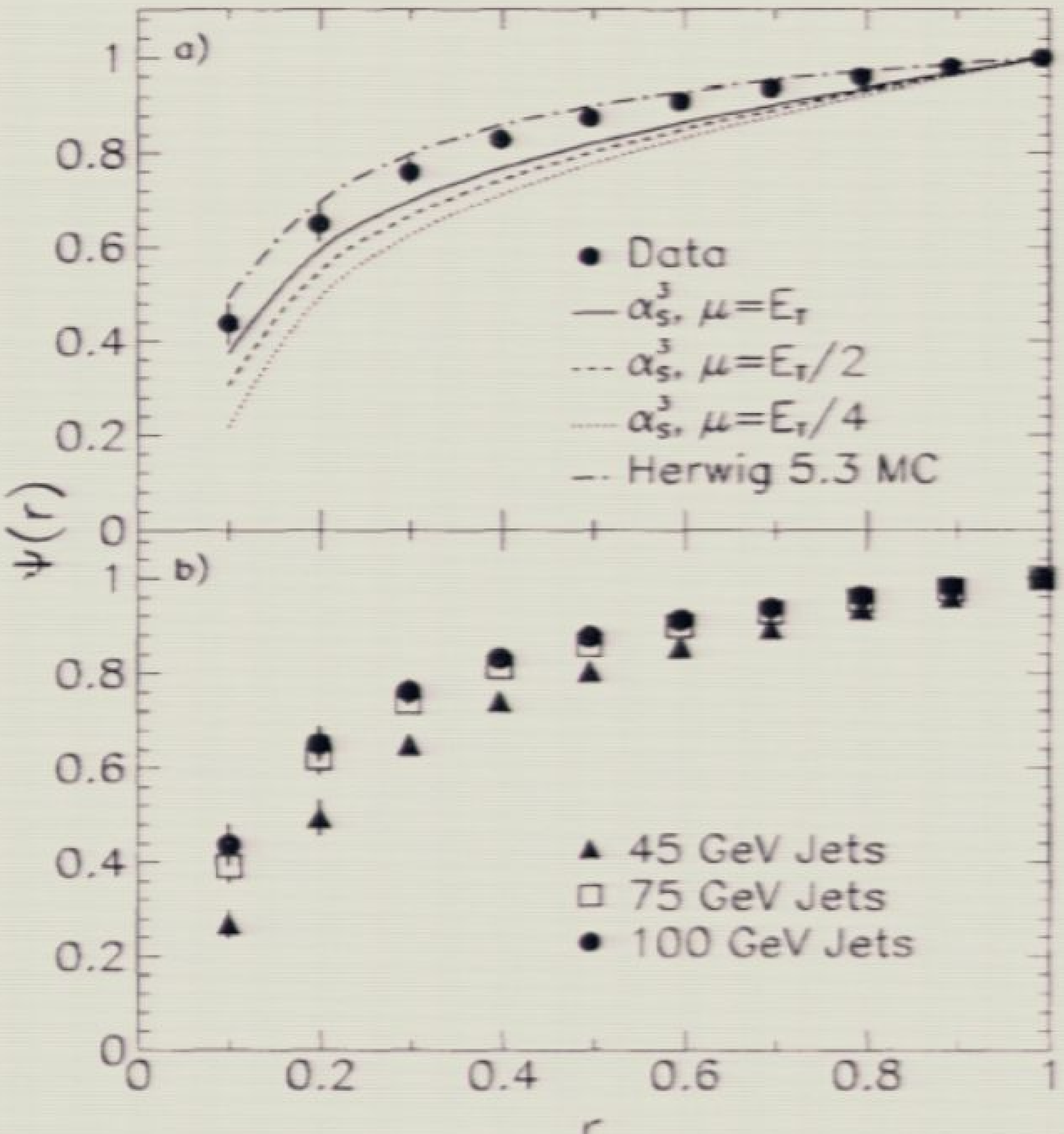
generic 2-jets with ET < 200 are typically gg
generic 2-jets with ET > 300 are typically qg
jets in $pp \to \gamma + \text{jet}$ are typically q
jets in top quark decay are typically q

Does pruning have the same effect on real jets as on Monte Carlo jets ?

Can we measure the color of the parton responsible for a jet with a high probability per event ?

Pursuit of these ideas in early LHC data could have major implications in later discovery stages.

I hope this talk gives you a feeling for recent progress in QCD and its application to LHC physics. There is much more work to be done. We hope that this work will be useful for the discoveries of the LHC era.



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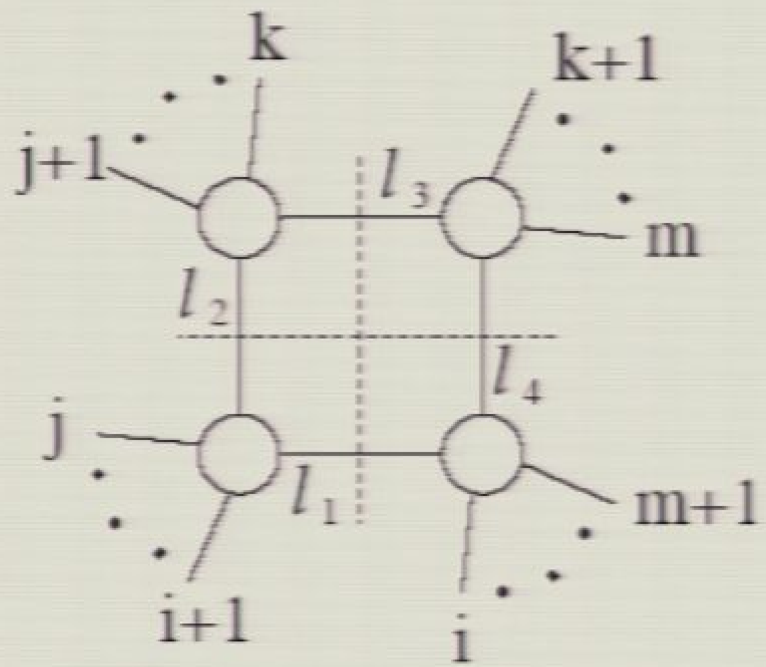
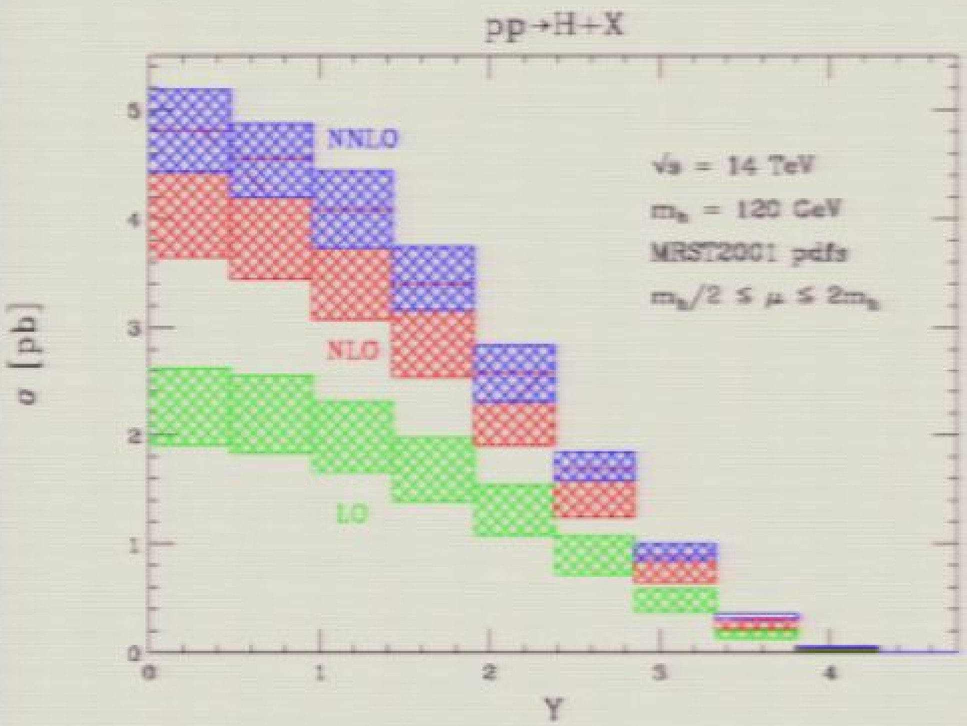
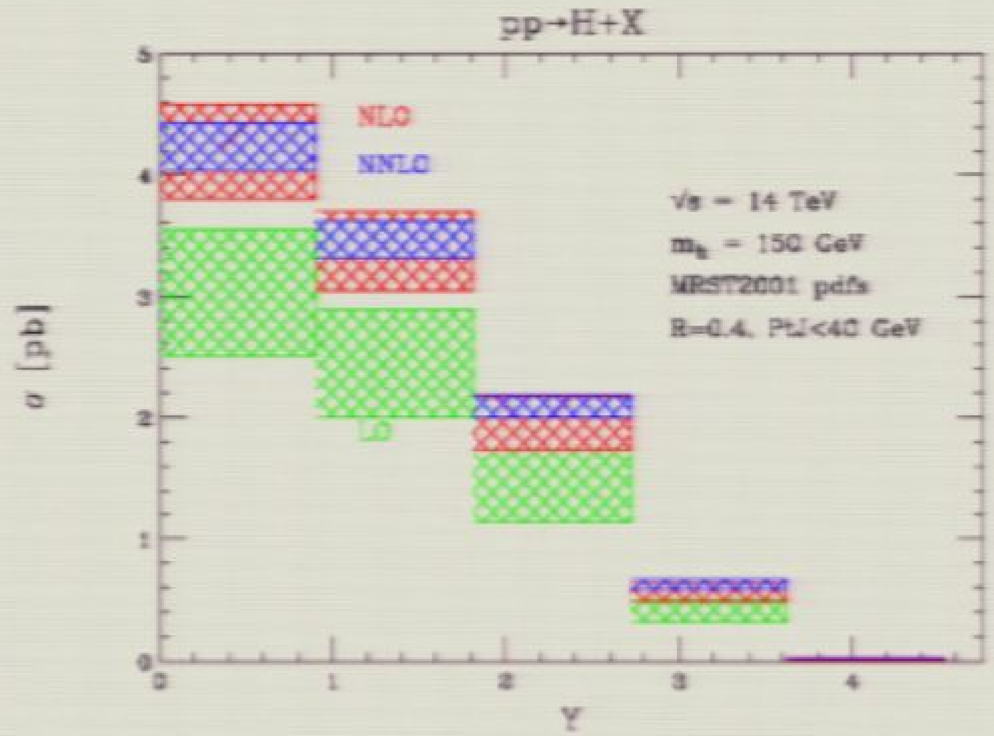


rapidity distribution of gg->Higgs, without and with a jet veto





Anastasiou, Melnikov, Petriello

Tidying up the formula, one finds the following relation:

$$i\mathcal{M}(1\cdots n) = \sum_{splits} i\mathcal{M}(b+1\cdots\hat{i}\cdots a-1\; -\hat{Q}) \cdot \frac{1}{s_{a\cdots b}} \cdot i\mathcal{M}(a\cdots\hat{j}\cdots b\,\hat{Q})$$

called the Britto-Cachazo-Feng (BCF) recursion formula.

Momenta with hats have the shift with z_* . The hatted momenta are complex but satisfy $\hat{Q}^2 = 0$, so the amplitudes on the right-hand side are to be evaluated on shell!

This allows the n-point amplitudes to be recursively evaluated in terms of amplitudes with fewer legs. We can stop when we reach MHV. At 5 points all amplitudes are MHV or anti-MHV.

I will discuss this problem in 4 stages:

Computation of tree amplitudes

Computation of loop amplitudes

Modeling of parton showers

Modeling of jet substructure

I will discuss this problem in 4 stages:

Computation of tree amplitudes

Computation of loop amplitudes

Modeling of parton showers

Modeling of jet substructure

It is tempting to say that we can get this control from direct proxies for Standard Model backgrounds that we find in the data. For example: $\sigma(pp \to Z^0 \to \nu\bar{\nu}) \approx 6\,\sigma(pp \to Z^0 \to \mu^+\mu^-)$

However, more typically, we will need to extrapolate from the data using QCD theory. Consider the following distribution in jets + MET events:

Can we claim discovery of new physics ?