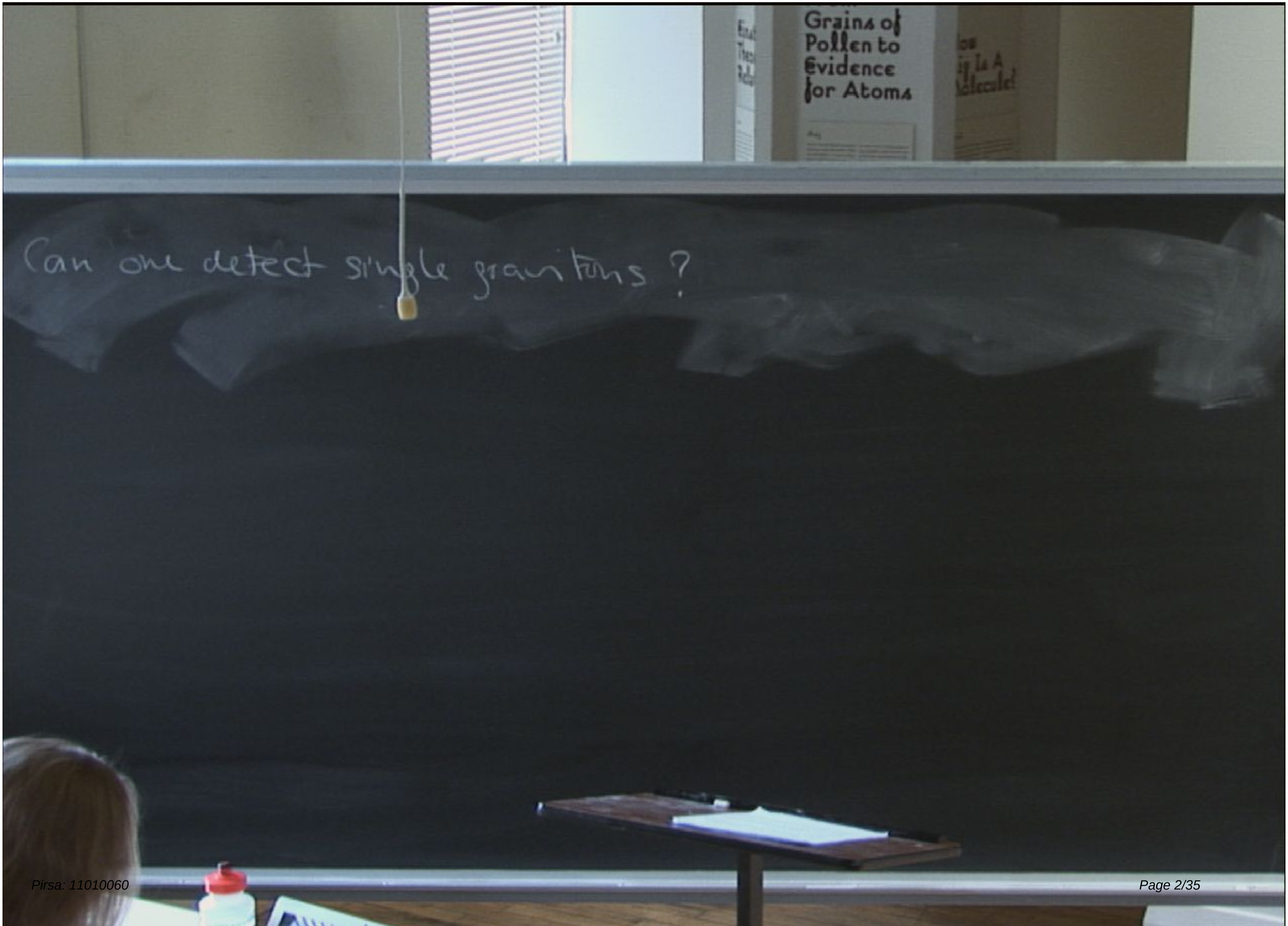
Title: Quantum Gravity Review - Lecture 4

Date: Jan 27, 2011 09:00 AM

URL: http://pirsa.org/11010060

Abstract:


Grains of
Pollen to
Evidence
for Atoms
Can one detect single gravitons ?

Can one detect single gravitons?

typical gravit. wave amplitude $|A| \approx 10^{-20}$

$\omega \approx 1000\ \mathrm{Hz}$, $\lambda \approx 3 \cdot 10^{5}\ m$

graviton number density $\approx \frac{10^{37}}{\lambda^3} \approx 3 \cdot \frac{10^{14}}{cm^3}$

Boughn & Rothman [gr-qc/0601043, 0605052]

Bonghri & Rothman [gr-qc/0601043, 0605052]

transition rate hydrogen atom $3d \to 1s$

$$\Gamma_g \simeq 5.7 \times 10^{-40} / s$$

Gravitational path integral

QM

Gravitational path integral

QM of nonrelat. particle

class. phase space (x_i, p_i), $i=1,2,3$, $\{x_i, p_j\} = \delta_{ij}$

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$$H = \frac{m}{2}\dot{x}^2 + V(\vec{x})$$

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$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$,

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$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}, \quad \hat{x}_i = x_i$$

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$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, $\hat{x}_i = x_i$, $\hat{p}_i = i\hbar \partial/\partial x_i$, $\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(\hat{\vec{x}})$

Solve: $\hat{H}\psi(\vec{x}) = E\psi(\vec{x})$

equivalently, unitary time evolution operator

$$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$$

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$$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

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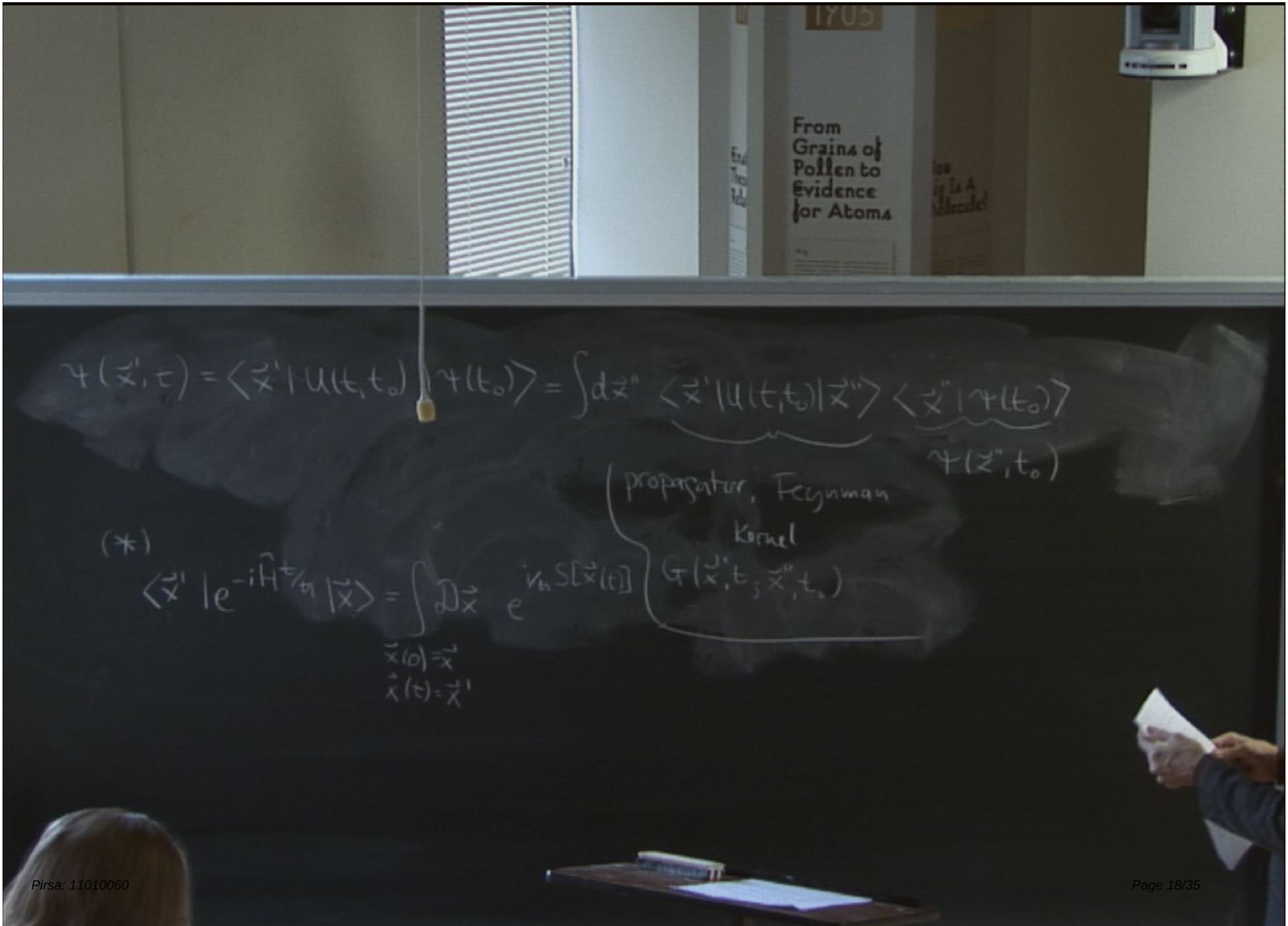
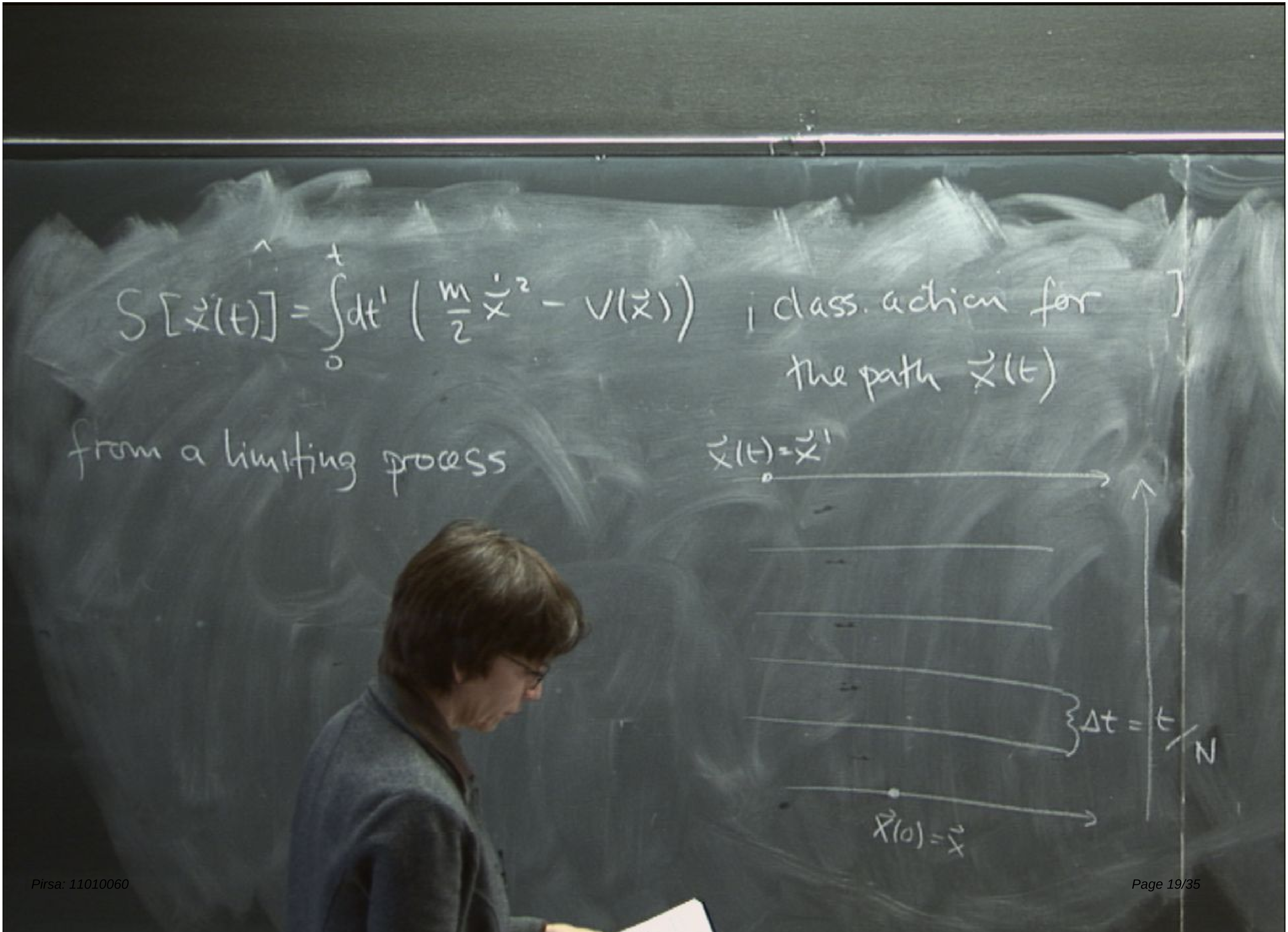
w.r.t. position eigenstates $|\vec{x}\rangle$: $\hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$

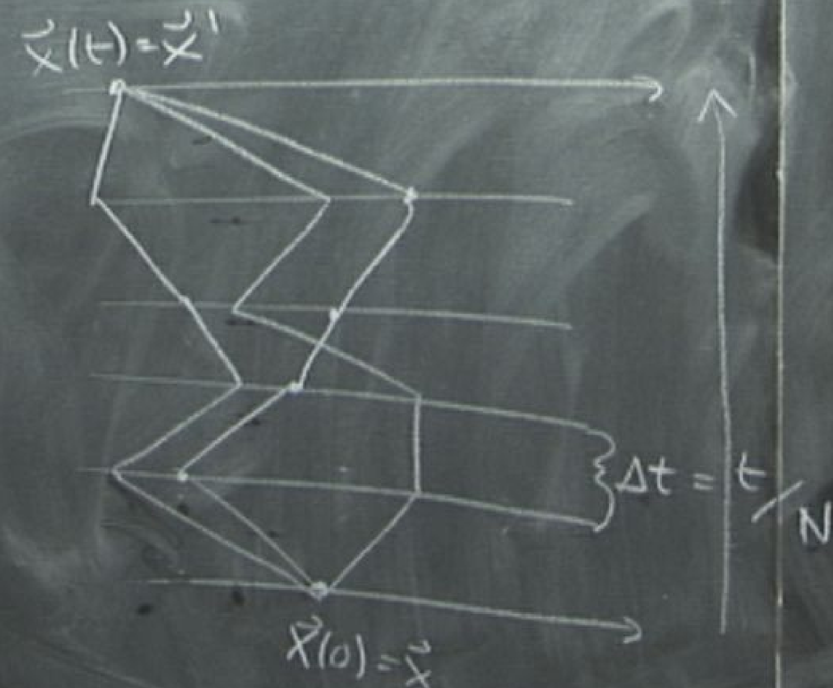
$$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$$


From Grains of Pollen to Evidence for Atoms
$\psi(\vec{x}',t) = \langle \vec{x}'|U(t,t_0)|\psi(t_0)\rangle = \int d\vec{x}'' \underbrace{\langle \vec{x}'|U(t,t_0)|\vec{x}''\rangle}\, \underbrace{\langle \vec{x}''|\psi(t_0)\rangle}_{\psi(\vec{x}'',t_0)}$
propagator, Feynman
Kernel
$G(\vec{x}',t;\vec{x}'',t_0)$
(*)
$\langle \vec{x}'|e^{-i\hat{H}t/\hbar}|\vec{x}\rangle = \int \mathcal{D}\vec{x}\; e^{i/\hbar\, S[\vec{x}(t)]}$
$\vec{x}(0)=\vec{x}$
$\vec{x}(t)=\vec{x}'$


$S[\hat{\vec{x}}(t)] = \int_0^t dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right)$; class. action for the path $\vec{x}(t)$
from a limiting process
$\vec{x}(t) = \vec{x}'$
$\Delta t = t/N$
$\vec{x}(0) = \vec{x}$

$$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right) \quad ; \text{class. action for the path } \vec{x}(t)$$

from a limiting process



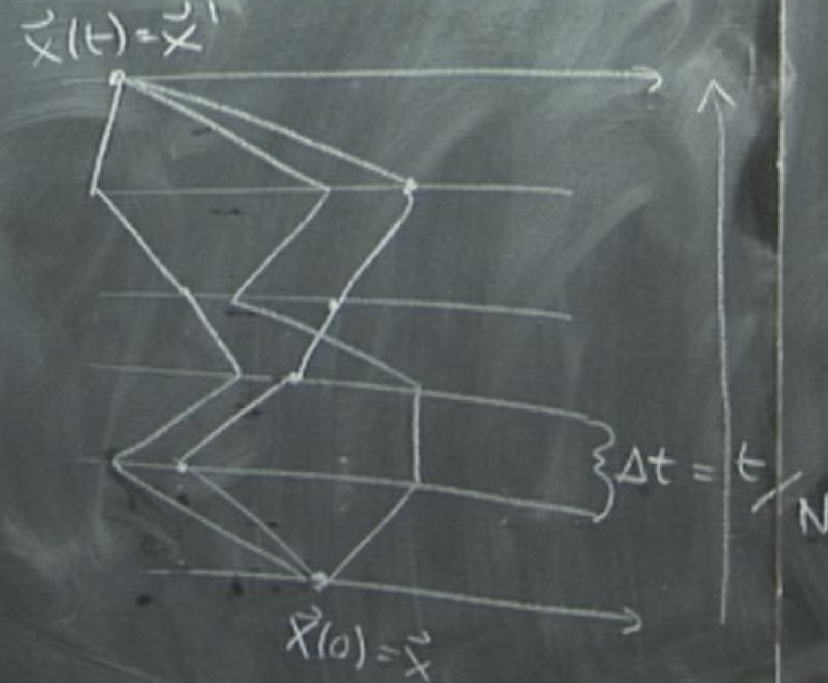
$$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right)$$; class. action for the path $\vec{x}(t)$]

from a limiting process

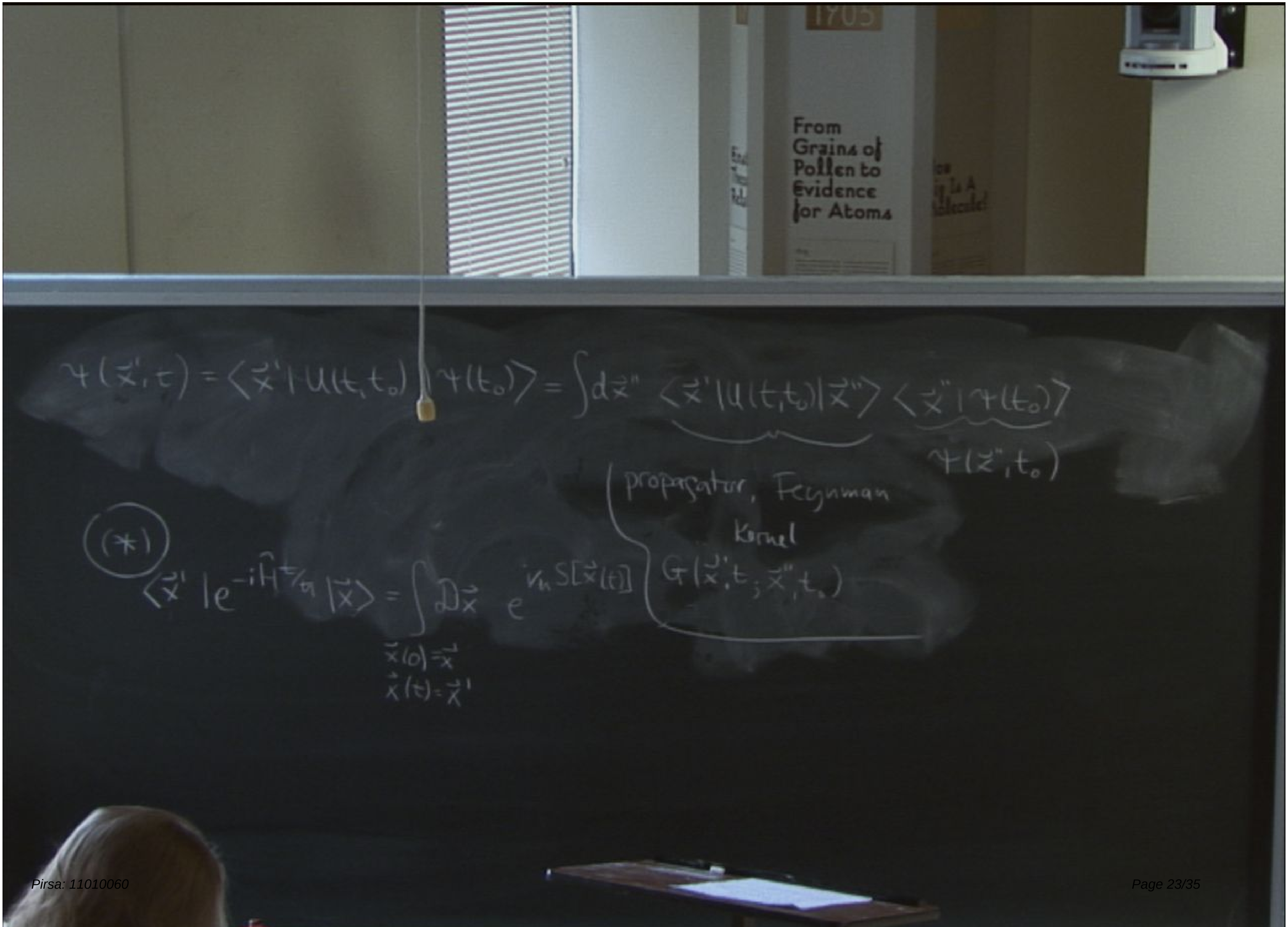
$\Delta t \to 0$, $N \to \infty$ from

integrating over all piecewise

straight paths of N segments



- analytically continue to imaginary time $\tau = it$


1905
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$\psi(\vec{x}',t) = \langle \vec{x}' | U(t,t_0) | \psi(t_0) \rangle = \int d\vec{x}'' \, \underbrace{\langle \vec{x}' | U(t,t_0) | \vec{x}'' \rangle} \, \underbrace{\langle \vec{x}'' | \psi(t_0) \rangle}_{\psi(\vec{x}'',t_0)}$
propagator, Feynman
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$\langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int_{\substack{\vec{x}(0)=\vec{x} \\ \vec{x}(t)=\vec{x}'}} \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$

QFT

PI / covariant : Lorentz invariance is "manifest"

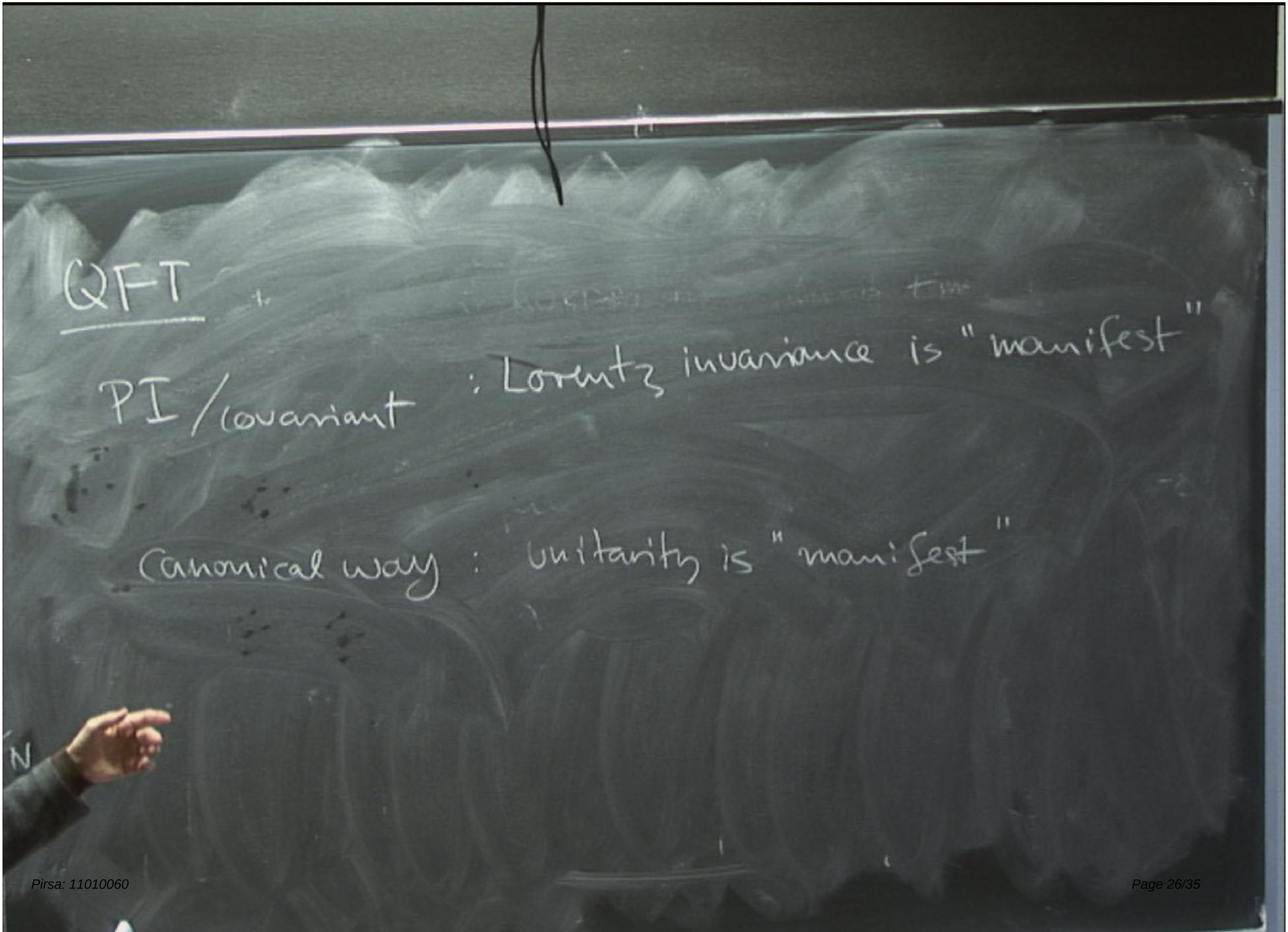
N

QFT

PI / covariant : Lorentz invariance is "manifest"

Canonical way

N


QFT
PI / covariant : Lorentz invariance is "manifest"
Canonical way : unitarity is "manifest"

QFT

PI / covariant : Lorentz invariance is "manifest"

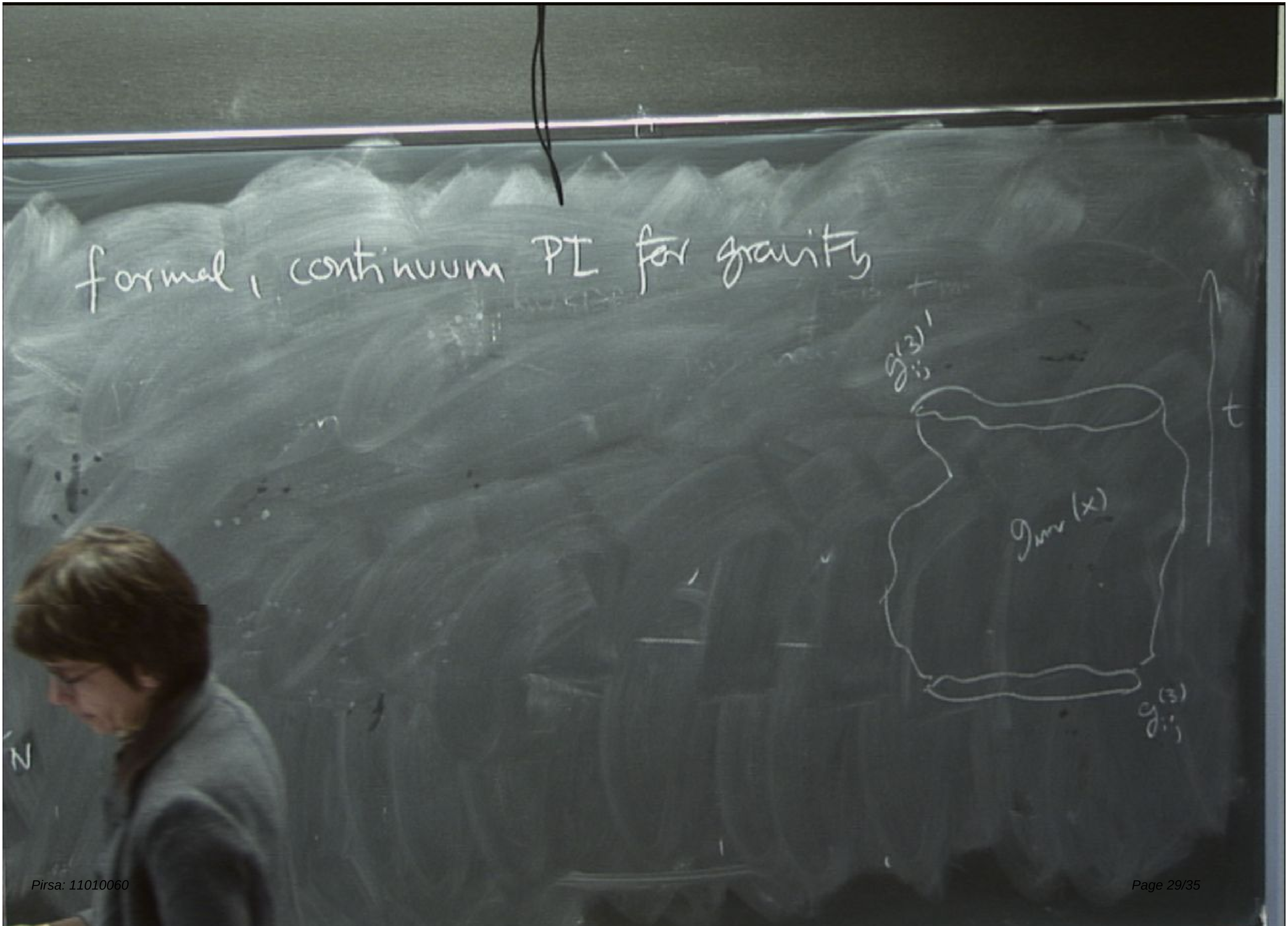
Canonical way : unitarity is "manifest"

formal, continuum PI for gravity

t

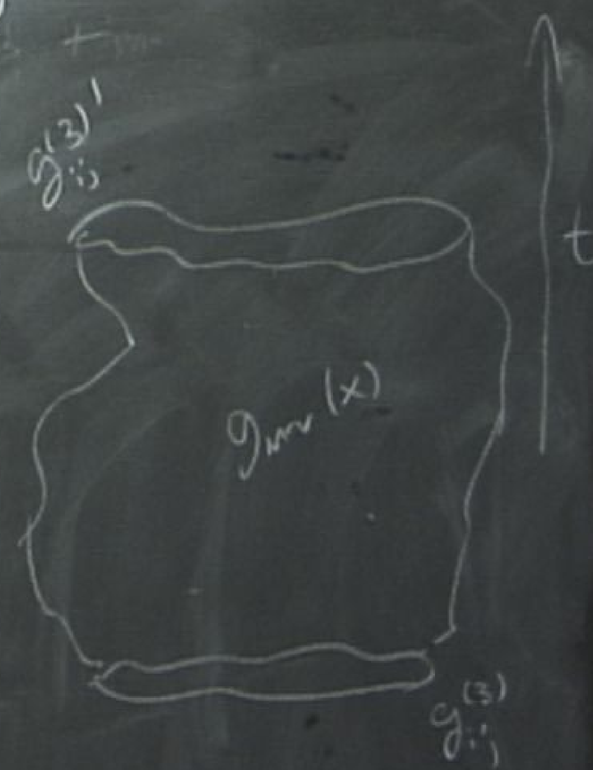
$g^{(3)}$

N


formal, continuum PI for gravity
$g^{(3)'}_{ij}$
$g_{\mu\nu}(x)$
t
$g^{(3)}_{ij}$

formal, continuum PI for gravity

$$Z\left([g^{(3)}_{ij}], [g'^{(3)}_{ij}]\right)$$

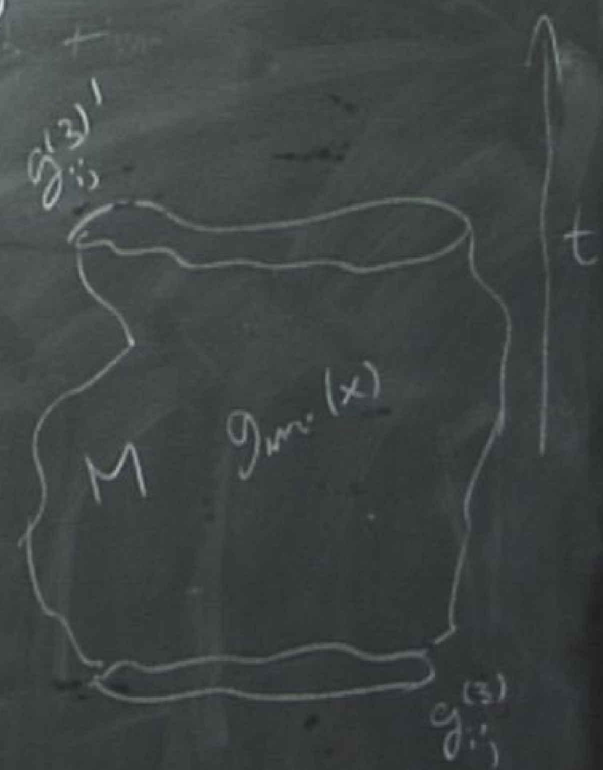


N

formal, continuum PI for gravity

$$Z\left([g^{(3)}_{ij}],\,[g'^{(3)}_{ij}]\right) = \int \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E-H}[g_{\mu\nu}]}$$

$$S^{E-H} = \frac{\Lambda}{16\pi G_N}\int_M d^4x \sqrt{|g|}$$



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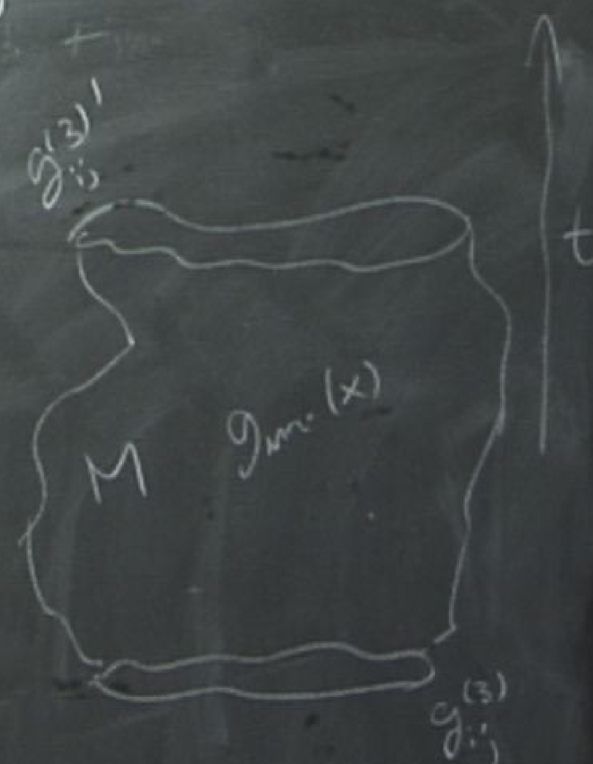

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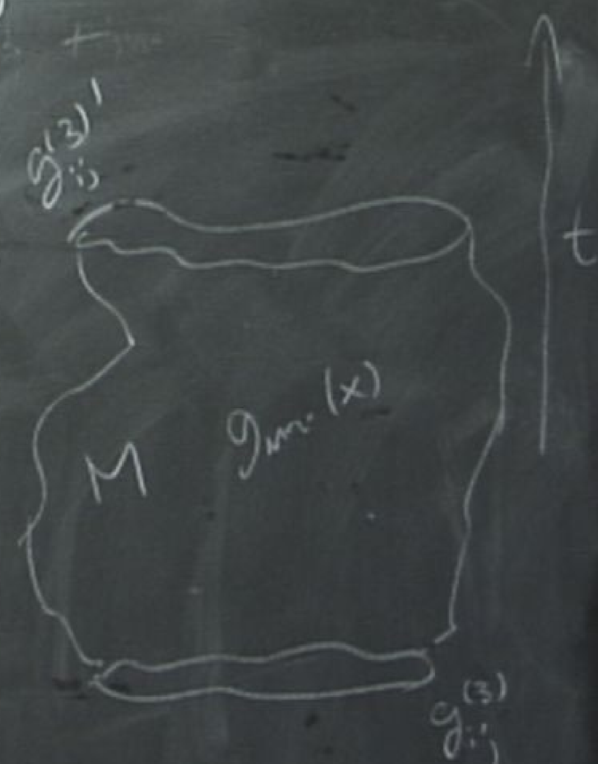

$$S^{E-H} = \frac{1}{16\pi G_N}\int_M d^4x \sqrt{|g|}\,(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{\partial M} d^3x \sqrt{\det h}\; K$$

$[g_{\mu\nu}]$ ~ diffeo-equivalence class

formal, continuum PI for gravity

$$Z\left([g^{(3)}_{ij}],[g'^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E\text{-}H}[g_{\mu\nu}]}$$

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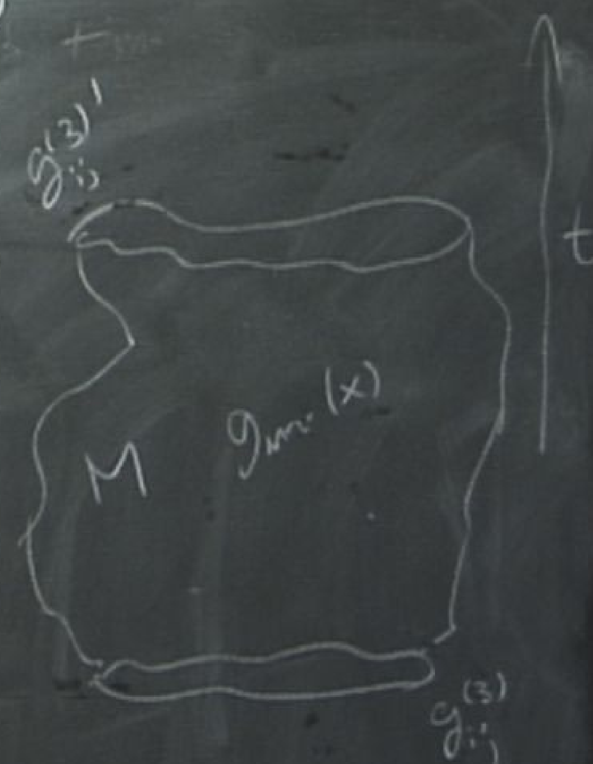

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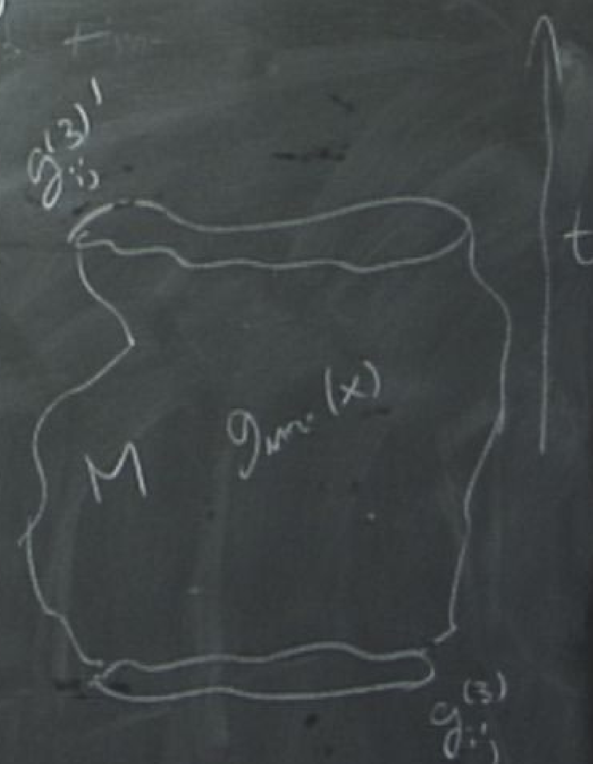

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