Title: Foundations of Quantum Mechanics - Lecture 13

Date: Jan 19, 2011 11:30 AM

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Abstract:

The deBroglie-Bohm interpretation

Louis deBroglie
(1892-1987)

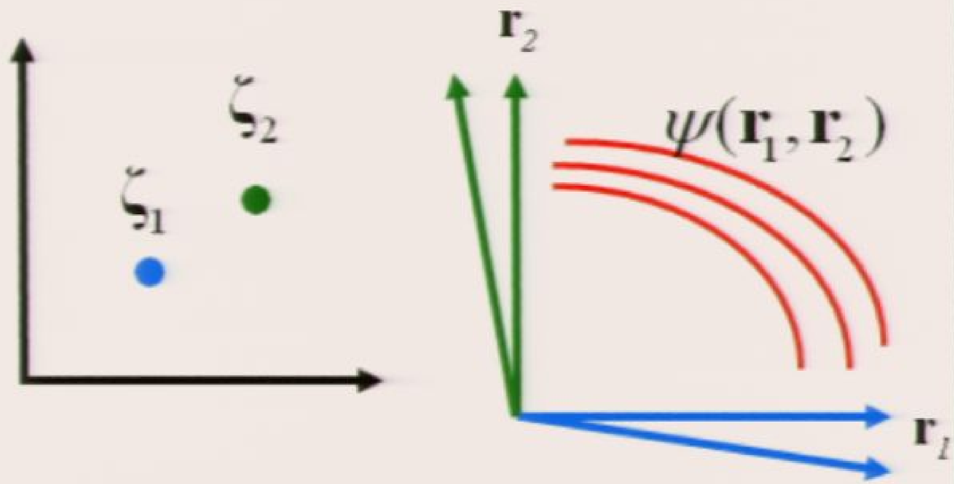
David Bohm
(1917-1992)

"I saw the impossible done..."

The deBroglie-Bohm interpretation for **many particles**

The ontic state: $(\psi(\mathbf{r}_1,\mathbf{r}_2),\zeta_1,\zeta_2)$

Wavefunction on configuration space — Particle positions



The evolution equations:

Schrödinger's equation

$$i\hbar\frac{\partial\psi(\mathbf{r}_1,\mathbf{r}_2,t)}{\partial t}=-\frac{\hbar^2}{2m_1}\nabla_1^2\psi(\mathbf{r}_1,\mathbf{r}_2,t)-\frac{\hbar^2}{2m_2}\nabla_2^2\psi(\mathbf{r}_1,\mathbf{r}_2,t)+V(\mathbf{r}_1,\mathbf{r}_2)\psi(\mathbf{r}_1,\mathbf{r}_2,t)$$

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$$\psi = \sum_j c_j \psi_j$$

"waves" of the decomposition

$\zeta \in$ Spatial support of ψ_j — *j*th wave is **occupied**

$\zeta \notin$ Spatial support of ψ_j — *j*th wave is **empty**

If only the *k*th wave is occupied

Then the guidance equation depends only on the *k*th wave

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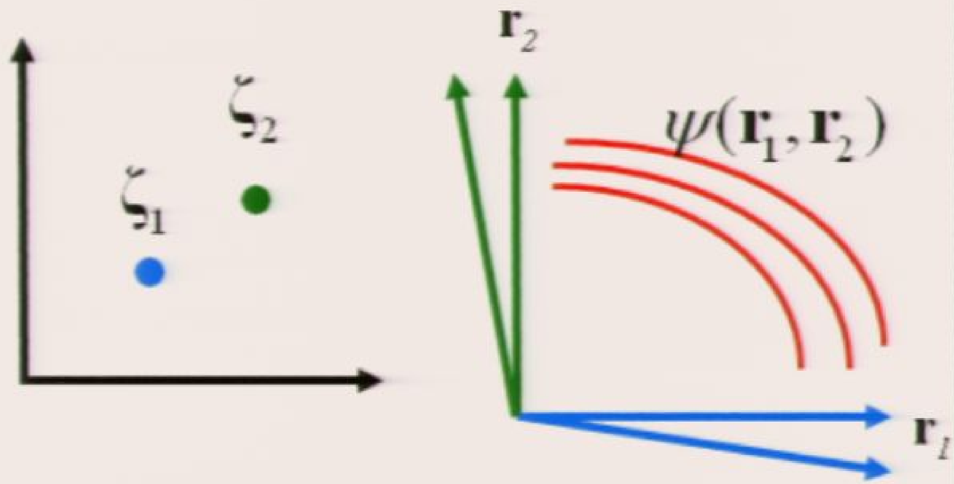

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$\psi(\mathbf{r}_1,\mathbf{r}_2,t)=R(\mathbf{r}_1,\mathbf{r}_2,t)\,e^{iS(\mathbf{r}_1,\mathbf{r}_2,t)/\hbar}$

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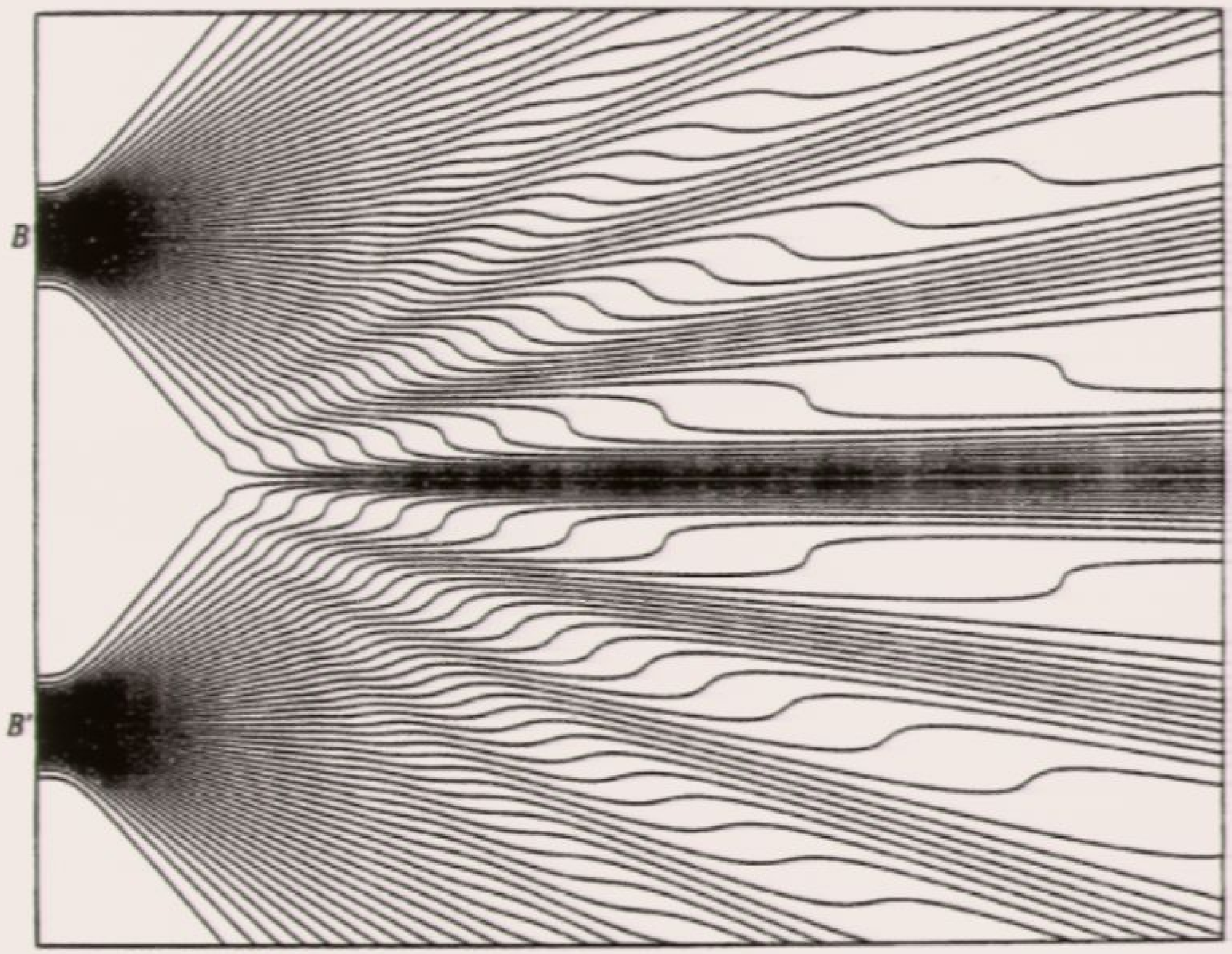

Double slit experiment

$$\psi(\mathbf{r}_1,\mathbf{r}_2,t)=\phi^{(1)}(\mathbf{r}_1,t)\ \chi^{(2)}(\mathbf{r}_2,t)$$ Product state

$$=R_1(\mathbf{r}_1,t)e^{iS_1(\mathbf{r}_1,t)/\hbar}\ R_2(\mathbf{r}_2,t)e^{iS_2(\mathbf{r}_2,t)/\hbar}$$

$$S(\mathbf{r}_1,\mathbf{r}_2,t)=S_1(\mathbf{r}_1,t)+S_2(\mathbf{r}_2,t)$$

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The two particles evolve independently

Reproducing the operational predictions

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

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Assumption: different outcomes of a measurement correspond to disjoint regions of the configuration space of the apparatus

$$\chi_j(\mathbf{r}')\chi_k(\mathbf{r}') \simeq 0 \text{ if } \quad j \neq k$$

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To have re-interference with the empty waves, it would be necessary to map all the η_k back to η

Do measurements reveal attributes of the particles?

Position measurements are "statistically faithful":

$$\langle\psi|F(\mathbf{R})|\psi\rangle = \int d\zeta\, F(\zeta)\, \rho(\zeta)$$

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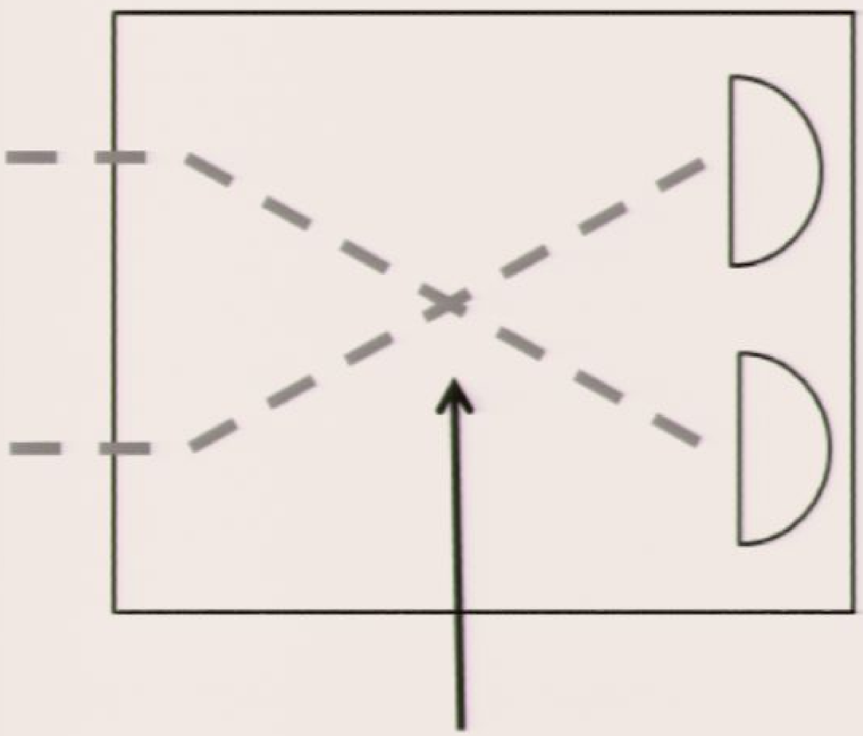
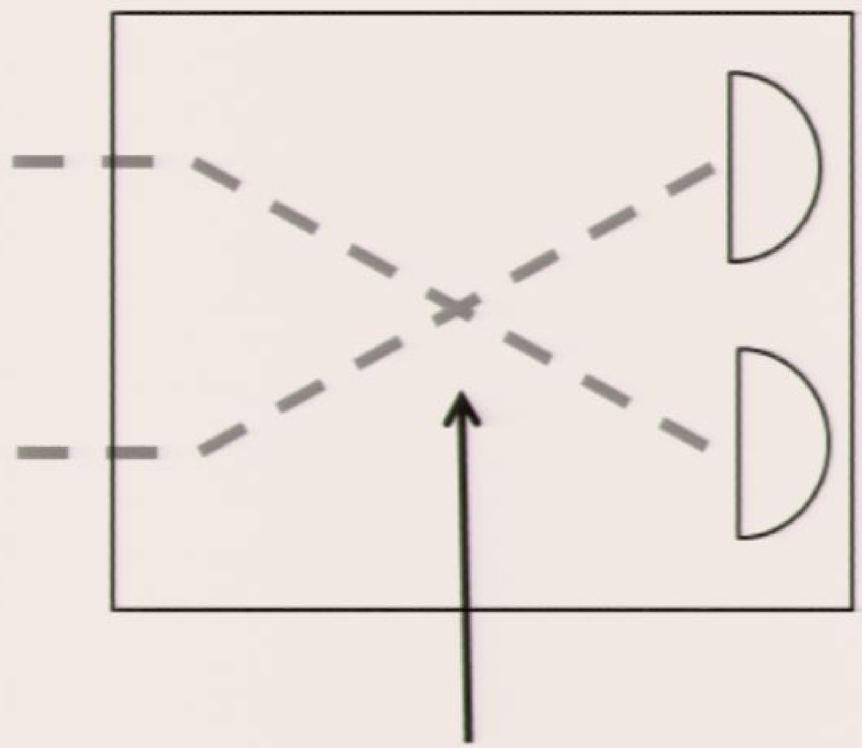
$$= \int d\mathbf{r}\,\psi^*(\mathbf{r},t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r},t)$$

Contextuality



No overlap in 3D



Overlap

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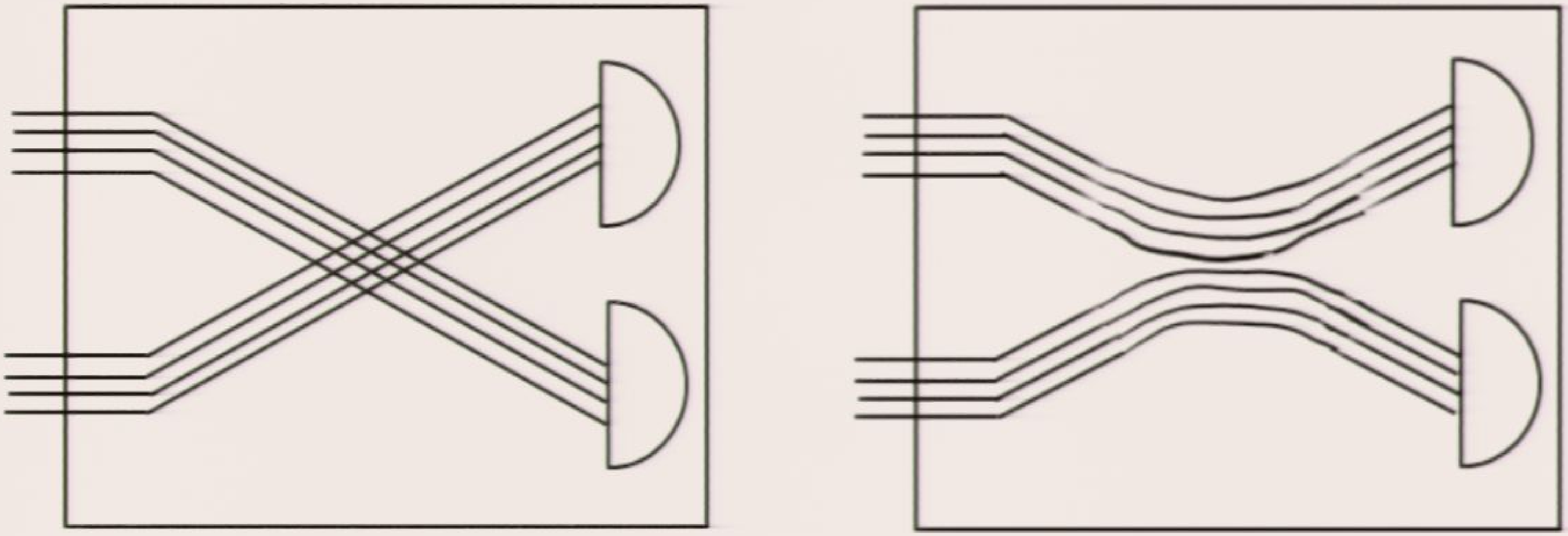

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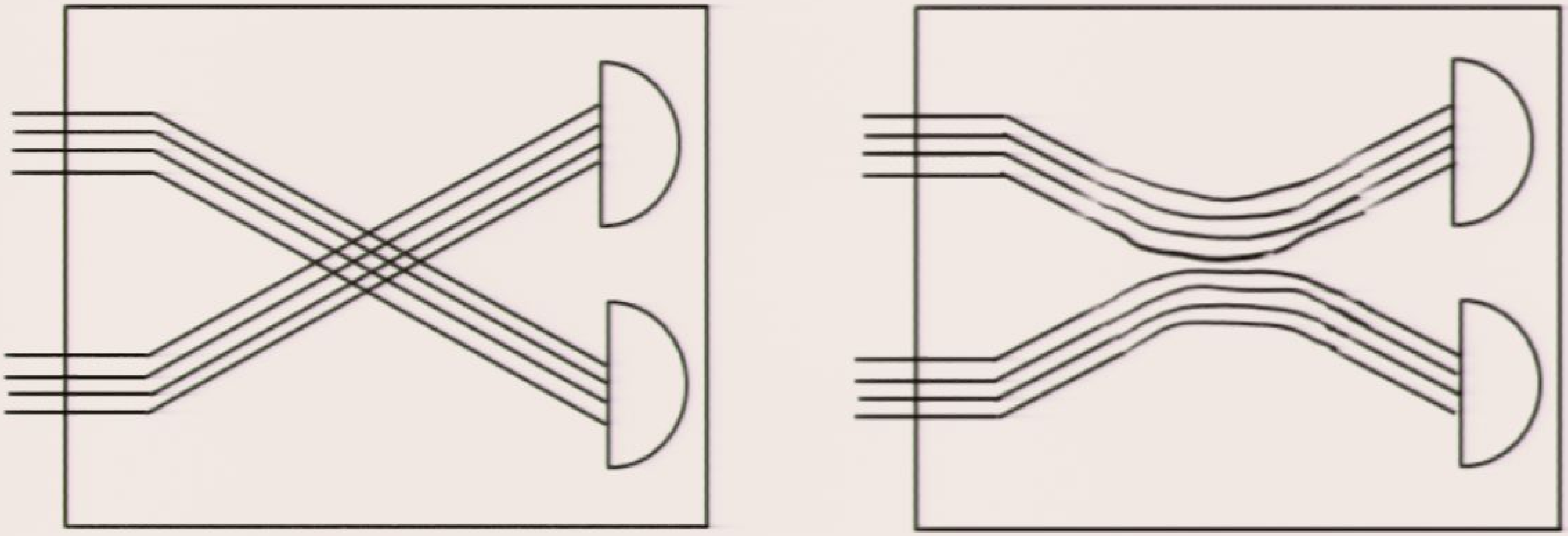

"Surrealistic" trajectories?

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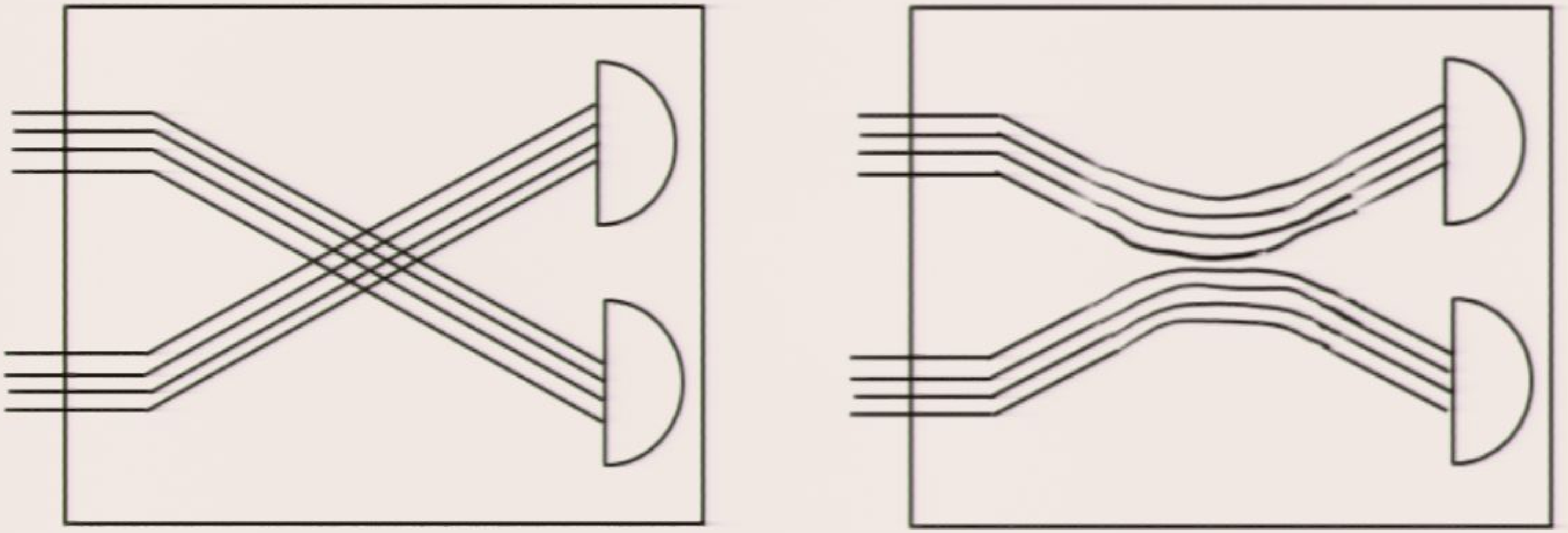

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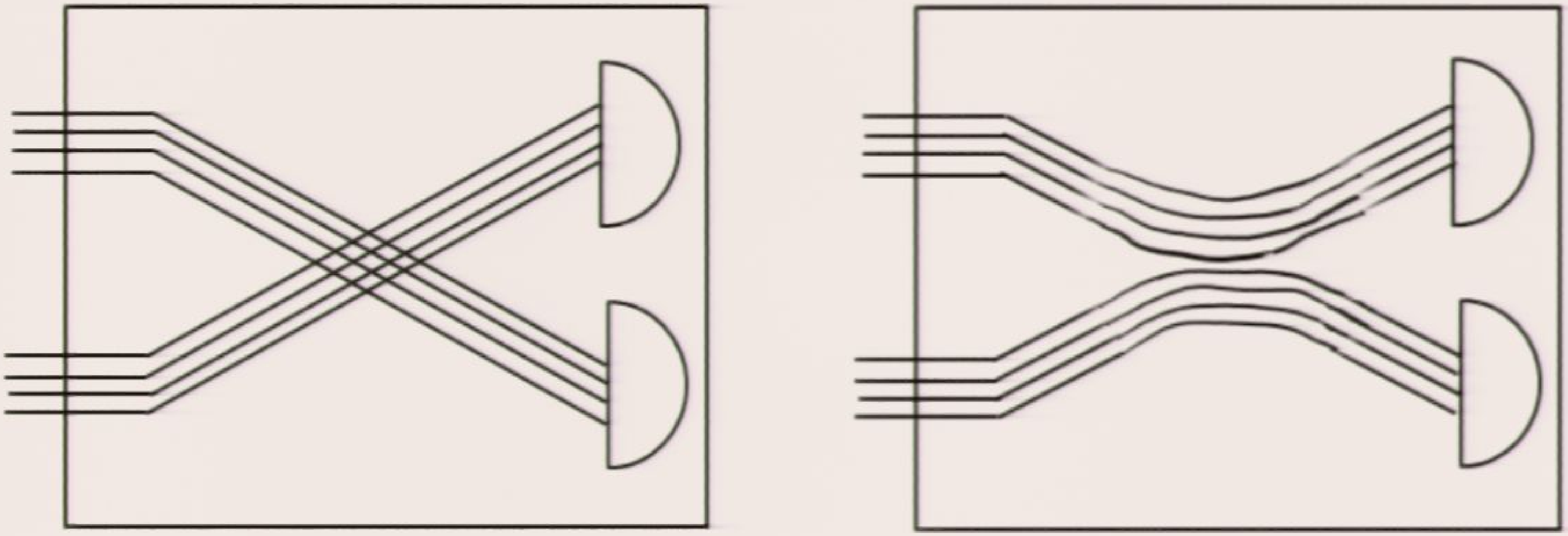

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Note however that a criticism remains: deBroglie-Bohm has

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Operational correspondence vs. Ontological correspondence

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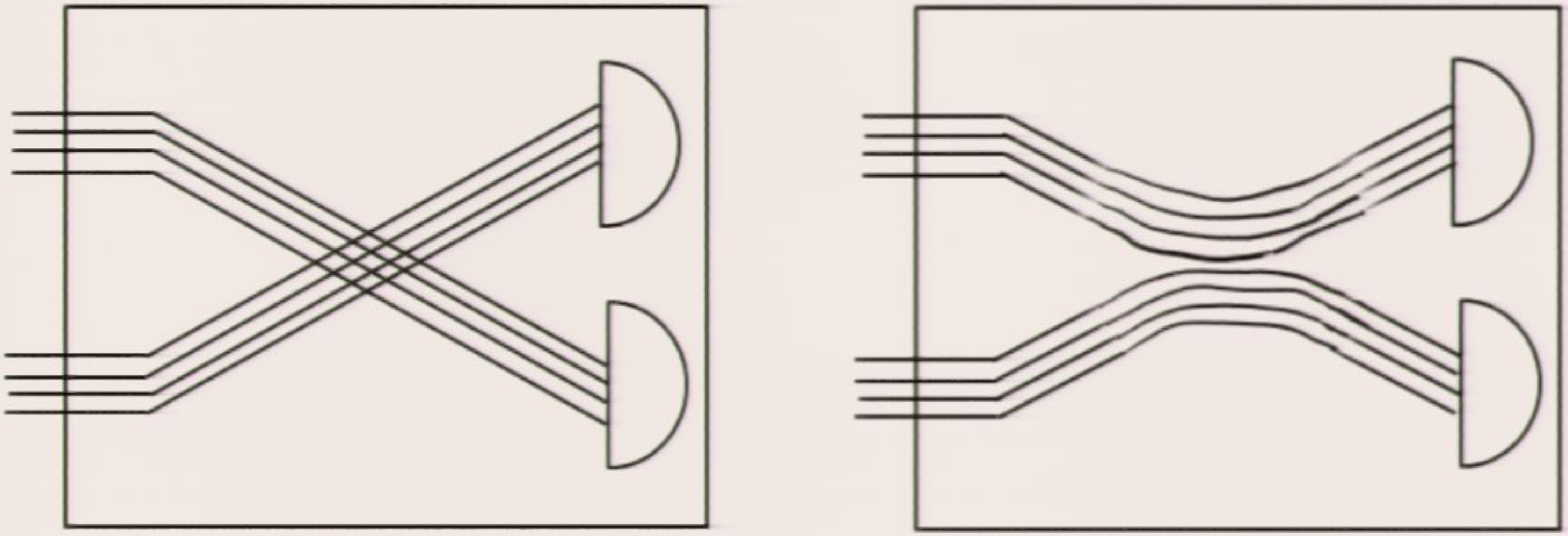

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Response:
Lorentz invariance is an emergent symmetry - a statistical consequence of quantum equilibrium

The underdetermination criticism

Underdetermination: when there are many possible choices of ontological structure that are consistent with observations

Underdetermination of the supplementary variables

Standard approach – Position preferred

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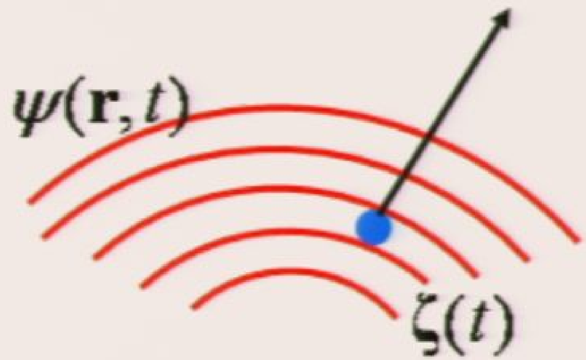
Wavefunction in position rep'n — Particle position



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Alternative approach - Momentum preferred (Epstein, 1952)

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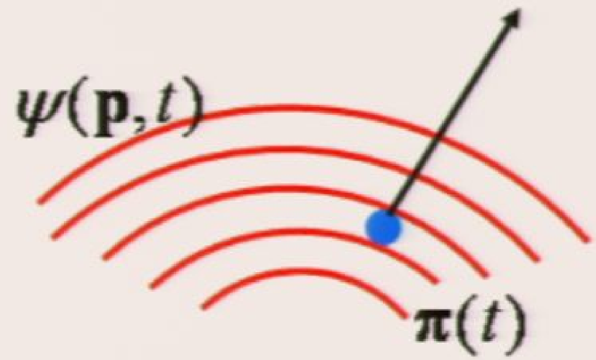
Wavefunction in momentum rep'n

Particle momentum



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Schrödinger's eq'n

$$\frac{d\boldsymbol{\pi}(t)}{dt} = \frac{1}{m}\left[\nabla_{\mathbf{p}} S(\mathbf{p},t)\right]_{\mathbf{p}=\boldsymbol{\pi}(t)}$$

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The problem returns for the case of a deBroglie-Bohm theory of the electromagnetic field:

Supplementary variable: **Electric field** or **Magnetic field**?

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Multiple treatments of spin:

Underdetermination of the supplementary variables

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Bohm, Schiller and Tiomno approach
Supplementary variables: particle position and orientation

The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

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The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

or

Bell's minimalist approach
Supplementary variables: particle position

The effect of spin is seen only in the dynamics of the particle positions
The operational predictions are reproduced by virtue of localization of pointers

Bell's minimalist approach to spin

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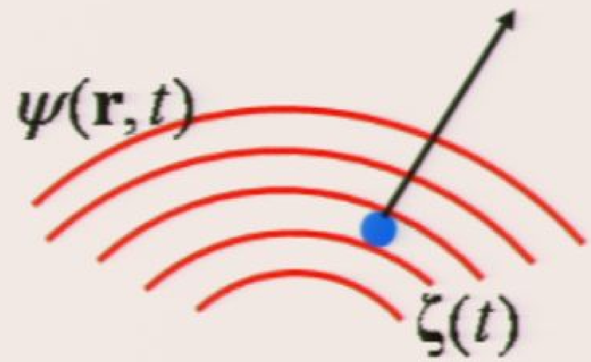
The ontic state: $(\psi(\mathbf{r}), \zeta)$

Two-component wavefunction

Particle position



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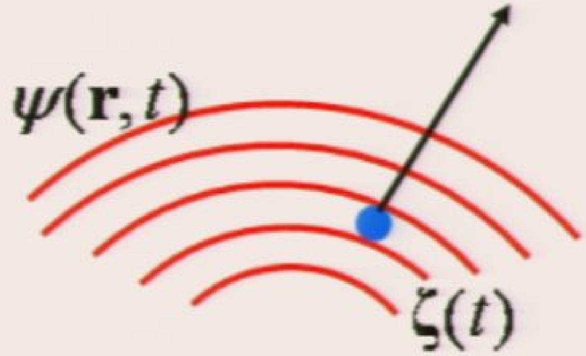

$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\left(\nabla - \frac{ie}{\hbar c}\mathbf{A}\right)^2 + \boldsymbol{\sigma}\cdot\mathbf{B} + V(\mathbf{r}) \right]\psi(\mathbf{r},t)$$

Pauli eq'n

Bell's minimalist approach to spin

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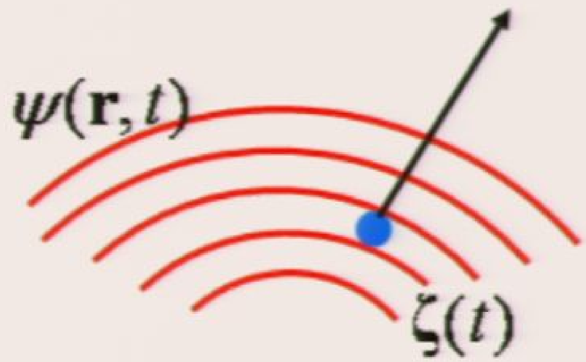
Two-component wavefunction — Particle position



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Pauli eq'n

$$\frac{d\zeta(t)}{dt}=\left.\frac{\mathbf{j}(\mathbf{r},t)}{R(\mathbf{r},t)}\right|_{\mathbf{r}=\zeta(t)}$$

The guidance eq'n

where $R(\mathbf{r},t)=\sum_s\left|\psi_s(\mathbf{r},t)\right|^2$

$$\mathbf{j}(\mathbf{r},t)=\sum_s\left(\frac{\hbar}{2mi}\left(\psi_s^*\nabla\psi_s-\psi_s\nabla\psi_s^*\right)-\frac{e}{mc}\mathbf{A}\,\psi_s^*\psi_s\right)(\mathbf{r},t)$$

Underdetermination of the supplementary variables

Multiple treatments of quantum electrodynamics:

Underdetermination of the supplementary variables

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Bohm's model of the free electromagnetic field
Supplementary variables: electric field (or magnetic field)

combined with

Bell's model of fermions (indeterministic, discrete) or Colin's continuum version of it
Supplementary variables: fermion number at each lattice point
(Note: field variables for fermions have been problematic)

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Bohm's model of the free electromagnetic field
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Bell's model of fermions (indeterministic, discrete) or Colin's continuum version of it
Supplementary variables: fermion number at each lattice point
(Note: field variables for fermions have been problematic)

or

Struyve and Westman's minimalist model of QED:
Supplementary variables : magnetic field (or electric field)
(the classical EM field carries an image of pointer positions)