

Title: Foundations of Quantum Mechanics - Lecture 12

Date: Jan 18, 2011 11:30 AM

URL: <http://pirsa.org/11010053>

Abstract:

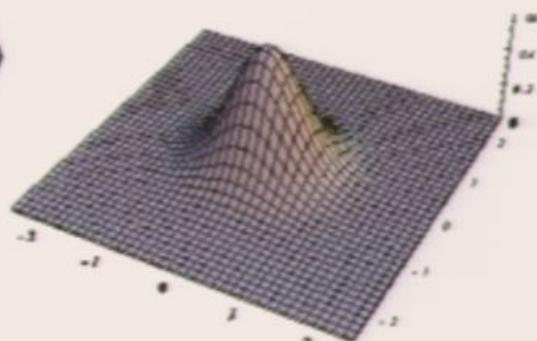
Noncontextuality and the characterization of classicality

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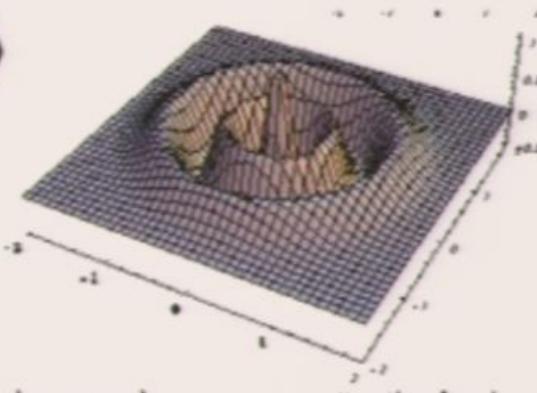
Classicality as non-negativity

Continuous Wigner function
for a harmonic oscillator

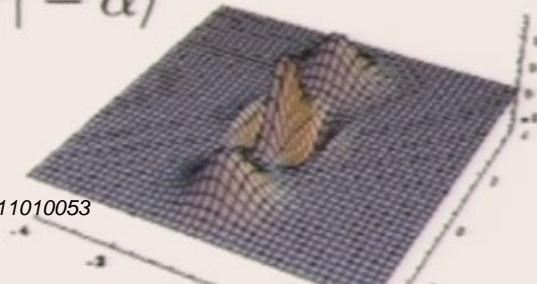
$|0\rangle$



$|4\rangle$



$|\alpha\rangle + |-\alpha\rangle$



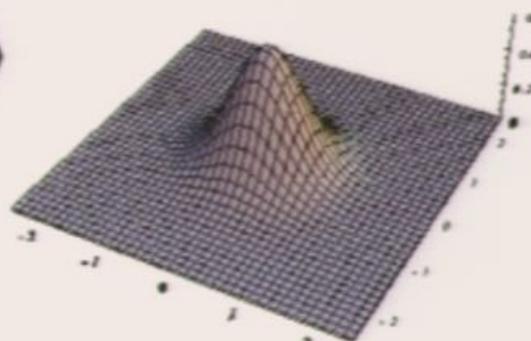
Common slogan:

A quantum state is nonclassical if it has
a negative Wigner representation

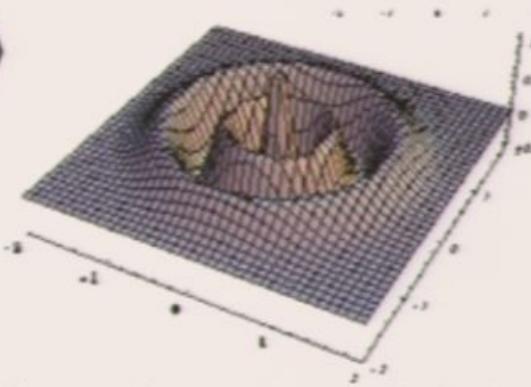
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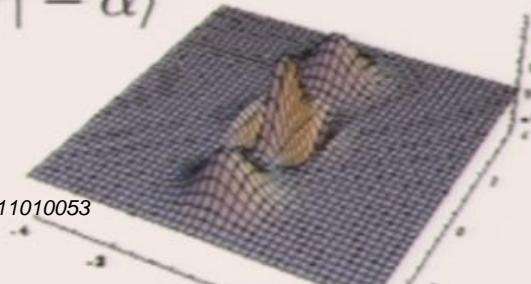
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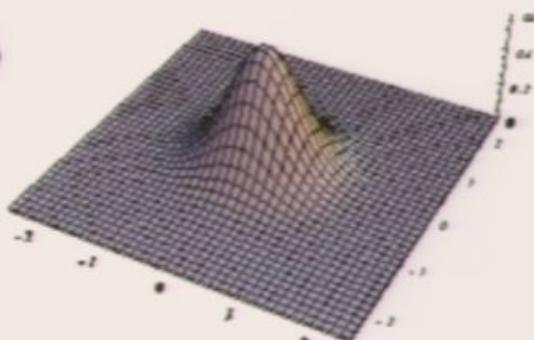
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Better to ask whether a quantum
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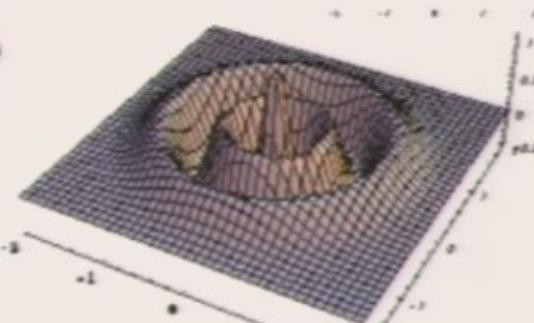
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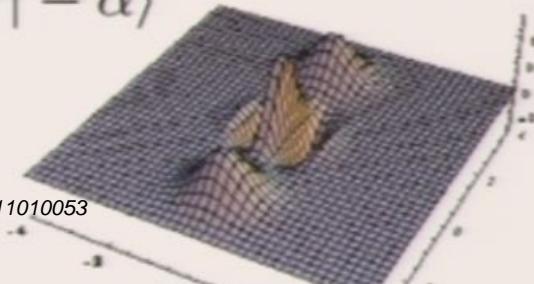
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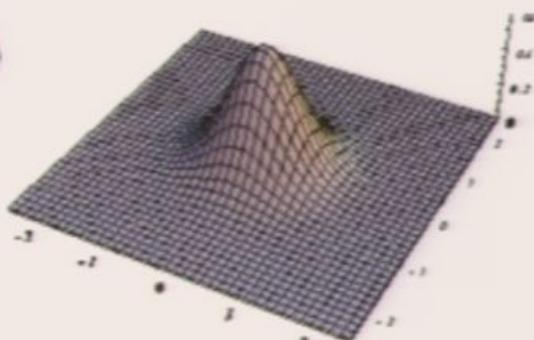
Better to ask whether **a quantum experiment** admits of a classical explanation

Negativity is not necessary for nonclassicality: the nonclassicality could reveal itself in the negativity of the representation of the measurement rather than the state

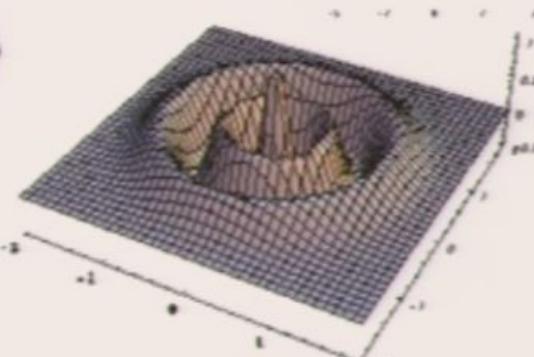
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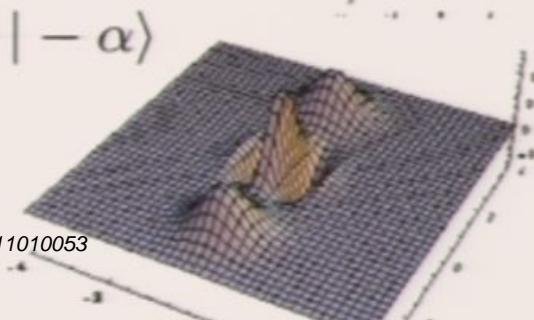
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Negativity is not sufficient for nonclassicality: When considering possibilities for a classical explanation, we need to look at representations other than that of Wigner

Quasi-probability representations of QM:

States

$$\rho \leftrightarrow \mu_\rho(\lambda)$$

$$\mu_\rho : \Lambda \rightarrow \mathbb{R}$$

$$\int \mu_\rho(\lambda) d\lambda = 1$$

Measurements

$$\{E_k\} \leftrightarrow \{\xi_{E_k}(\lambda)\}$$

$$\xi_{E_k} : \Lambda \rightarrow \mathbb{R}$$

$$\sum_k \xi_{E_k}(\lambda) = 1$$

$$\text{Tr}[\rho E_k] = \int d\lambda \mu_\rho(\lambda) \xi_{E_k}(\lambda)$$

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Examples:

- Wigner representation
- discrete Wigner representation (e.g. Wootters, quant-ph/0306135)
- Q representation of quantum optics
- P representation of quantum optics
- etcetera
- ...

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for all ρ

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Classicality from nonnegativity:

A quantum experiment is nonclassical if it fails to admit a quasi-probability representation that is nonnegative for all

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= Noncontextual hidden variable
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Responses to the measurement problem

1. Deny universality of quantum dynamics
 - Quantum-classical hybrid models
 - Collapse models
2. Deny representational completeness of ψ
 - ψ -ontic hidden variable models (e.g. deBroglie-Bohm)
 - ψ -epistemic hidden variable models
3. Deny that there is a unique outcome
 - Everett's relative state interpretation (many worlds)
4. Deny some aspect of classical logic or classical probability theory
 - Quantum logic and quantum Bayesianism
5. Deny some other feature of the realist framework?

The deBroglie-Bohm interpretation



Louis deBroglie
(1892-1987)



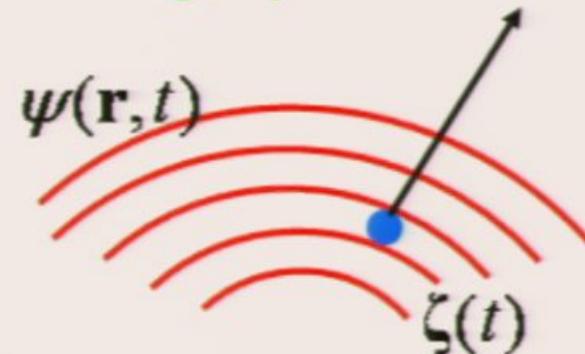
David Bohm
(1917-1992)

"I saw the impossible done..."

The deBroglie-Bohm interpretation for a single particle

The ontic state: $(\psi(\mathbf{r}), \zeta)$

Wavefunction Particle position

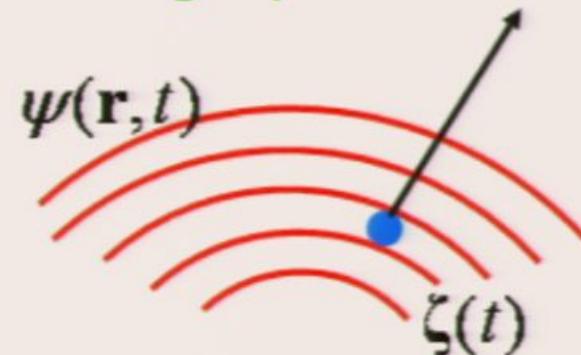


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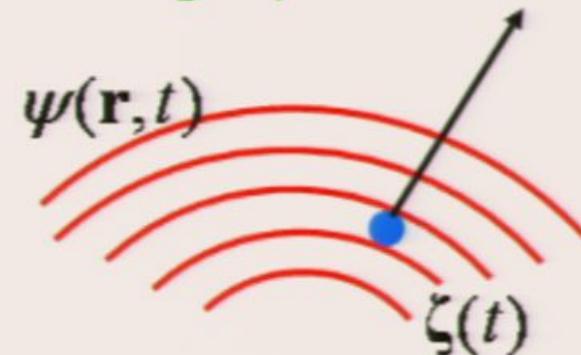
$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) \quad \text{Schrödinger's eq'n}$$

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The guidance eq'n

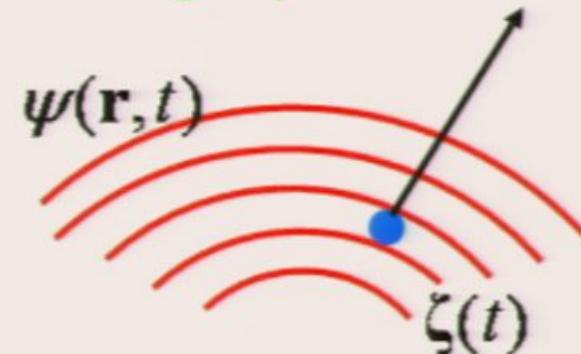
$$\text{where } \psi(\mathbf{r},t) = R(\mathbf{r},t)e^{iS(\mathbf{r},t)/\hbar}$$

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Given $\psi(\mathbf{r}, t) = R(\mathbf{r}, t)e^{iS(\mathbf{r}, t)/\hbar}$

The real part of the Schrodinger eq'n is:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$$

where $Q(\mathbf{r}, t) \equiv -\frac{\hbar^2}{2m} \frac{\nabla^2 R(\mathbf{r}, t)}{R(\mathbf{r}, t)}$

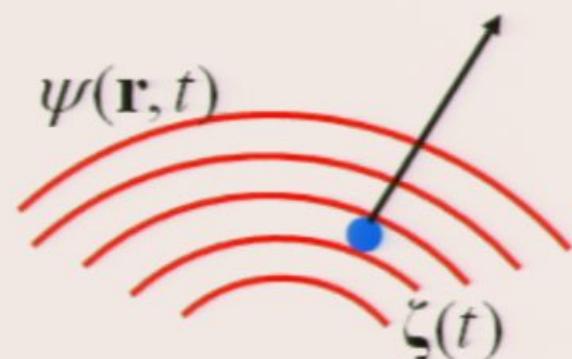
The "quantum potential"

The imaginary part of the Schrodinger eq'n is:

$$\frac{\partial}{\partial t}(R^2) + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0$$

Newtonian form of the particle dynamics:

$$m \frac{d^2\zeta(t)}{dt^2} = -[\nabla V(\mathbf{r}) + \nabla Q(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$



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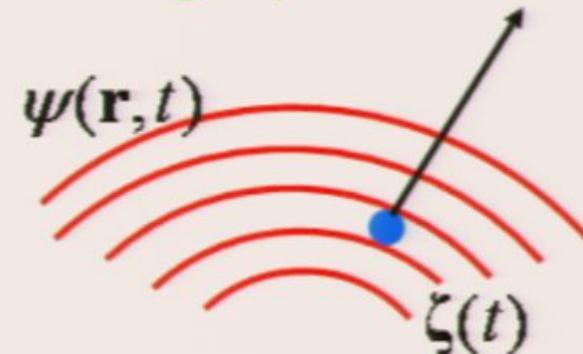
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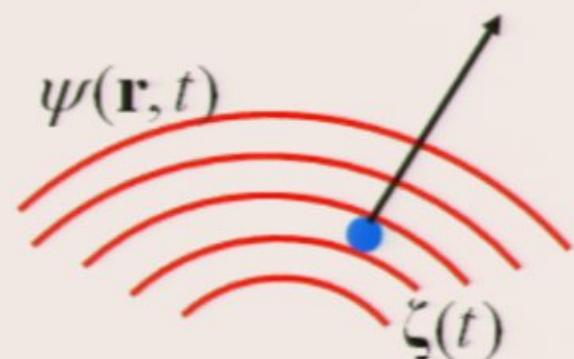
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Acting the ∇ operator on the real part of the Schrodinger eq'n gives:

$$\nabla \left[\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V \right] = 0$$

$$\left(\frac{\partial}{\partial t} + \frac{\nabla S \cdot \nabla}{m} \right) \nabla S = -\nabla(Q + V)$$

Taking the time derivative of the guidance equation gives:

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

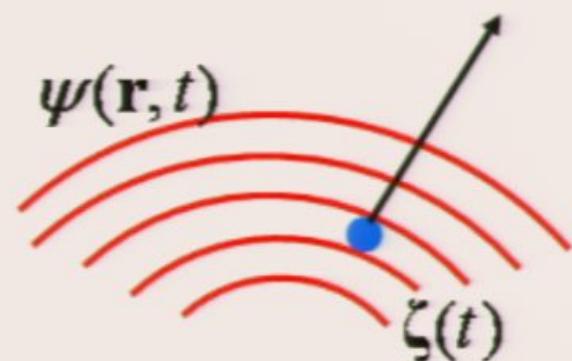
$$\frac{d^2\zeta(t)}{dt^2} = \frac{1}{m} \left(\frac{\partial}{\partial t} + \frac{d\zeta}{dt} \cdot \nabla \right) \nabla S$$

Thus

$$m \frac{d^2\zeta(t)}{dt^2} = -[\nabla V(\mathbf{r}) + \nabla Q(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

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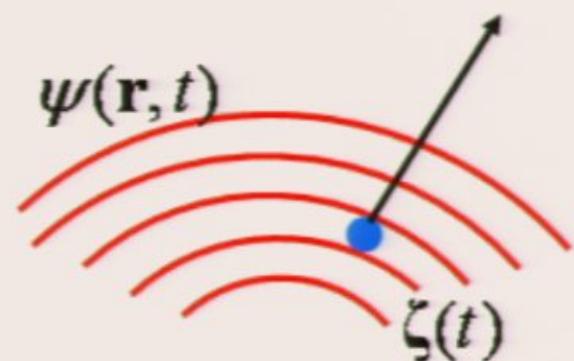


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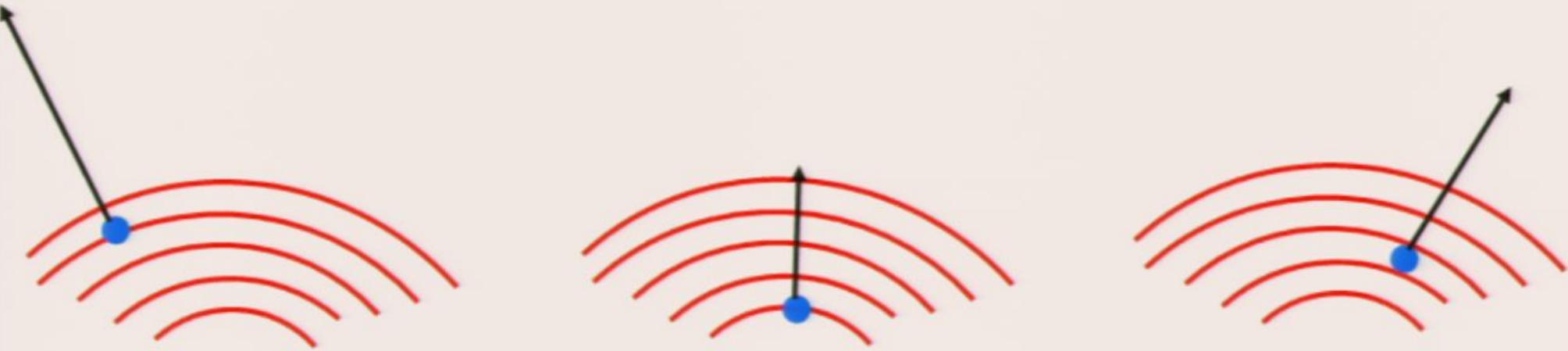


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Nonetheless the dynamics are *fundamentally first order*

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$



Epistemic state (assuming perfect knowledge of $\psi(\mathbf{r}, t)$)

$\rho(\zeta) d\zeta$ = the probability the particle is within $d\zeta$ of ζ .

The "standard distribution"

$$\rho(\zeta, t) = |\psi(\zeta, t)|^2$$

Note: it is preserved by the dynamics:

Pirsa: 11010053 if $\rho(\zeta, 0) = |\psi(\zeta, 0)|^2$ then $\rho(\zeta, t) = |\psi(\zeta, t)|^2$

Proof of the preservation of the standard distribution:

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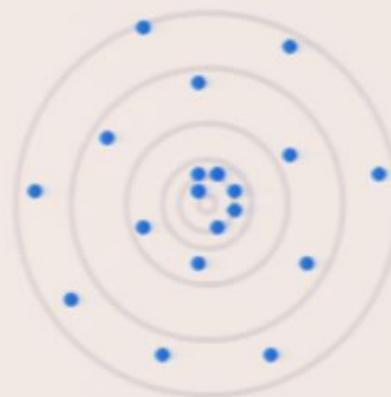
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1s orbital of Hydrogen atom



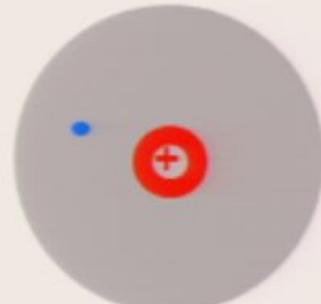
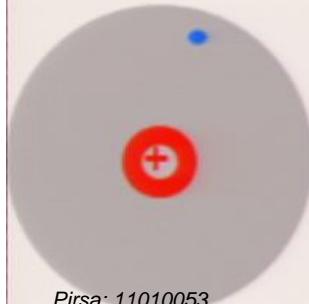
$$|\psi(\mathbf{r})|^2$$



$$\rho(\zeta)$$

$$\psi(\mathbf{r}, t) = R(\mathbf{r})e^{-iEt/\hbar}$$

so $\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} = 0$



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"waves" of the decomposition

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j th wave is empty

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$\zeta \in$ Spatial support of ψ_j

j th wave is occupied

$\zeta \notin$ Spatial support of ψ_j

j th wave is empty

If only the k th wave is occupied

Then the guidance equation depends only on the k th wave

Proof of ineffectiveness of empty waves

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If $R_a R_b \approx 0$

then $R^2 = R_a^2 + R_b^2$

Proof of ineffectiveness of empty waves

$$\psi = \psi_a + \psi_b$$

$$R e^{iS/\hbar} = R_a e^{iS_a/\hbar} + R_b e^{iS_b/\hbar}$$

$$R^2 = R_a^2 + R_b^2 + 2R_a R_b \cos[(S_a - S_b)/\hbar]$$

$$\nabla S = R^{-2} \begin{cases} R_a^2 \nabla S_a + R_b^2 \nabla S_b + R_a R_b \cos[(S_a - S_b)/\hbar] \nabla (S_a + S_b) \\ - \hbar [R_a \nabla R_b - R_b \nabla R_a] \sin[(S_a - S_b)/\hbar] \end{cases}$$

If $R_a R_b \approx 0$

then $R^2 = R_a^2 + R_b^2$ and $\nabla S = \frac{R_a^2 \nabla S_a + R_b^2 \nabla S_b}{R_a^2 + R_b^2}$

Proof of ineffectiveness of empty waves

$$\psi = \psi_a + \psi_b$$

$$R e^{iS/\hbar} = R_a e^{iS_a/\hbar} + R_b e^{iS_b/\hbar}$$

$$R^2 = R_a^2 + R_b^2 + 2R_a R_b \cos[(S_a - S_b)/\hbar]$$

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If $R_a R_b \approx 0$

$$\text{then } R^2 = R_a^2 + R_b^2 \quad \text{and} \quad \nabla S = \frac{R_a^2 \nabla S_a + R_b^2 \nabla S_b}{R_a^2 + R_b^2}$$

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

Proof of ineffectiveness of empty waves

$$\psi = \psi_a + \psi_b$$

$$R e^{iS/\hbar} = R_a e^{iS_a/\hbar} + R_b e^{iS_b/\hbar}$$

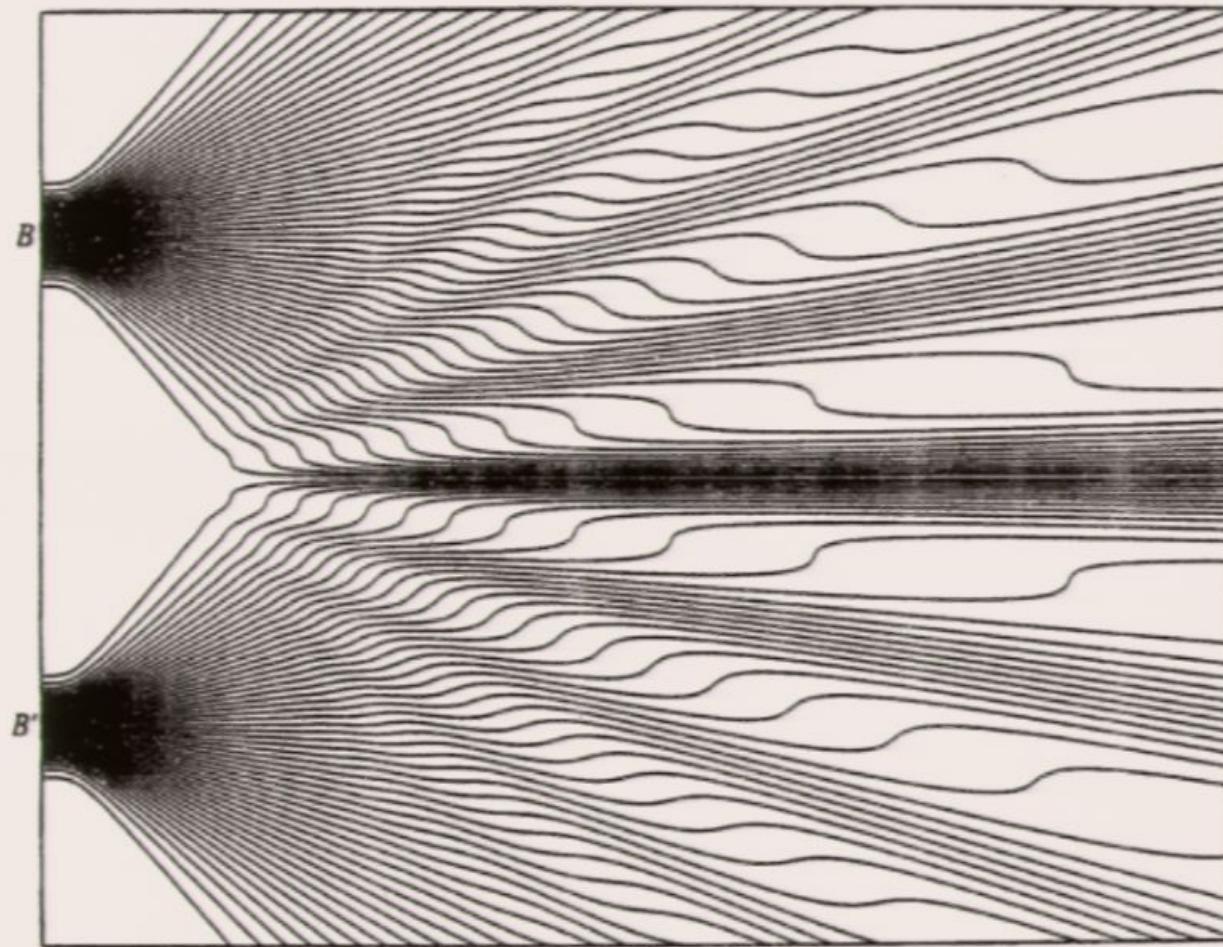
$$R^2 = R_a^2 + R_b^2 + 2R_a R_b \cos[(S_a - S_b)/\hbar]$$

$$\nabla S = R^{-2} \begin{cases} R_a^2 \nabla S_a + R_b^2 \nabla S_b + R_a R_b \cos[(S_a - S_b)/\hbar] \nabla(S_a + S_b) \\ - \hbar [R_a \nabla R_b - R_b \nabla R_a] \sin[(S_a - S_b)/\hbar] \end{cases}$$

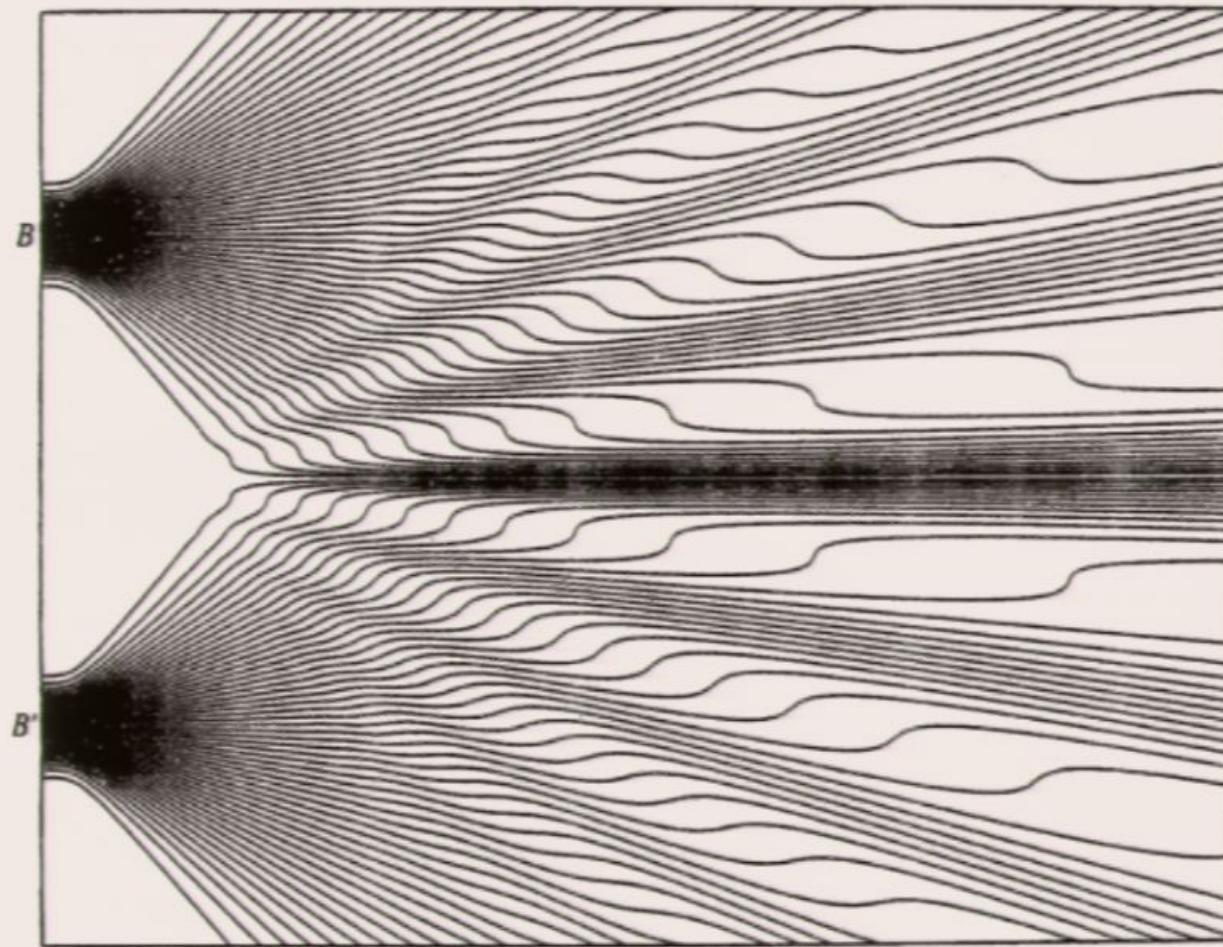
If $R_a R_b \approx 0$

$$\text{then } R^2 = R_a^2 + R_b^2 \quad \text{and} \quad \nabla S = \frac{R_a^2 \nabla S_a + R_b^2 \nabla S_b}{R_a^2 + R_b^2}$$

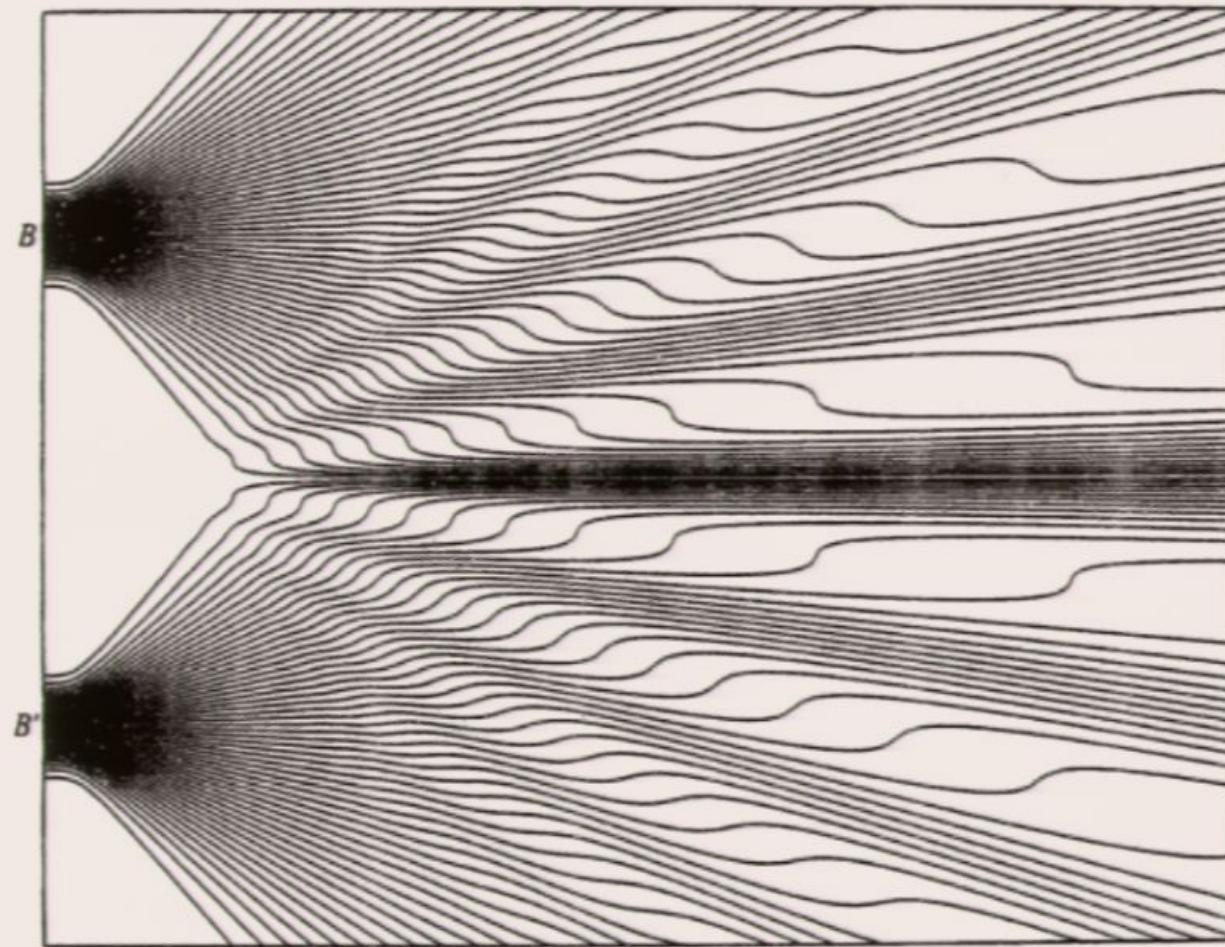
$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} = \frac{\nabla S_a}{m} \quad \text{If } \zeta \in \text{Support of } \psi_a$$



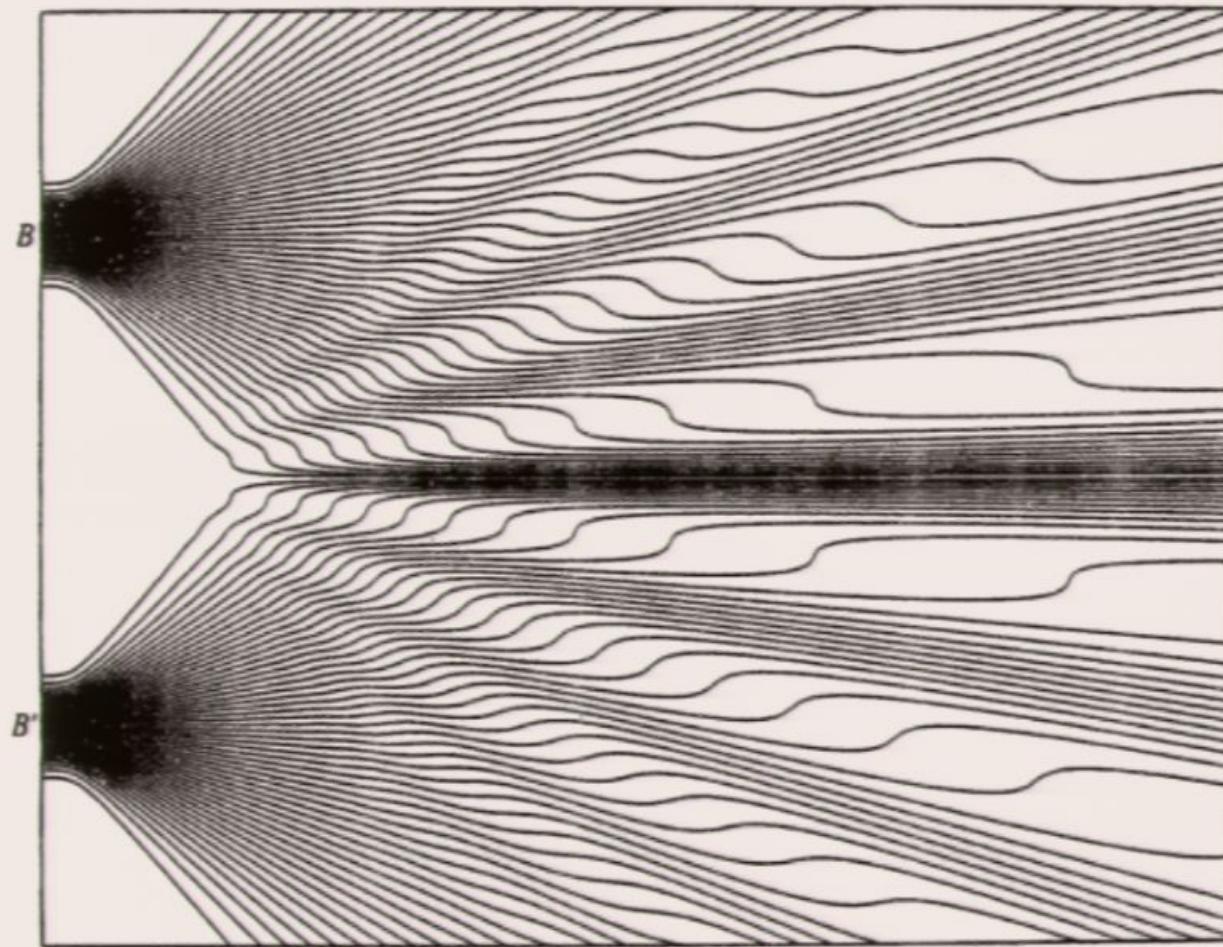
Double slit experiment



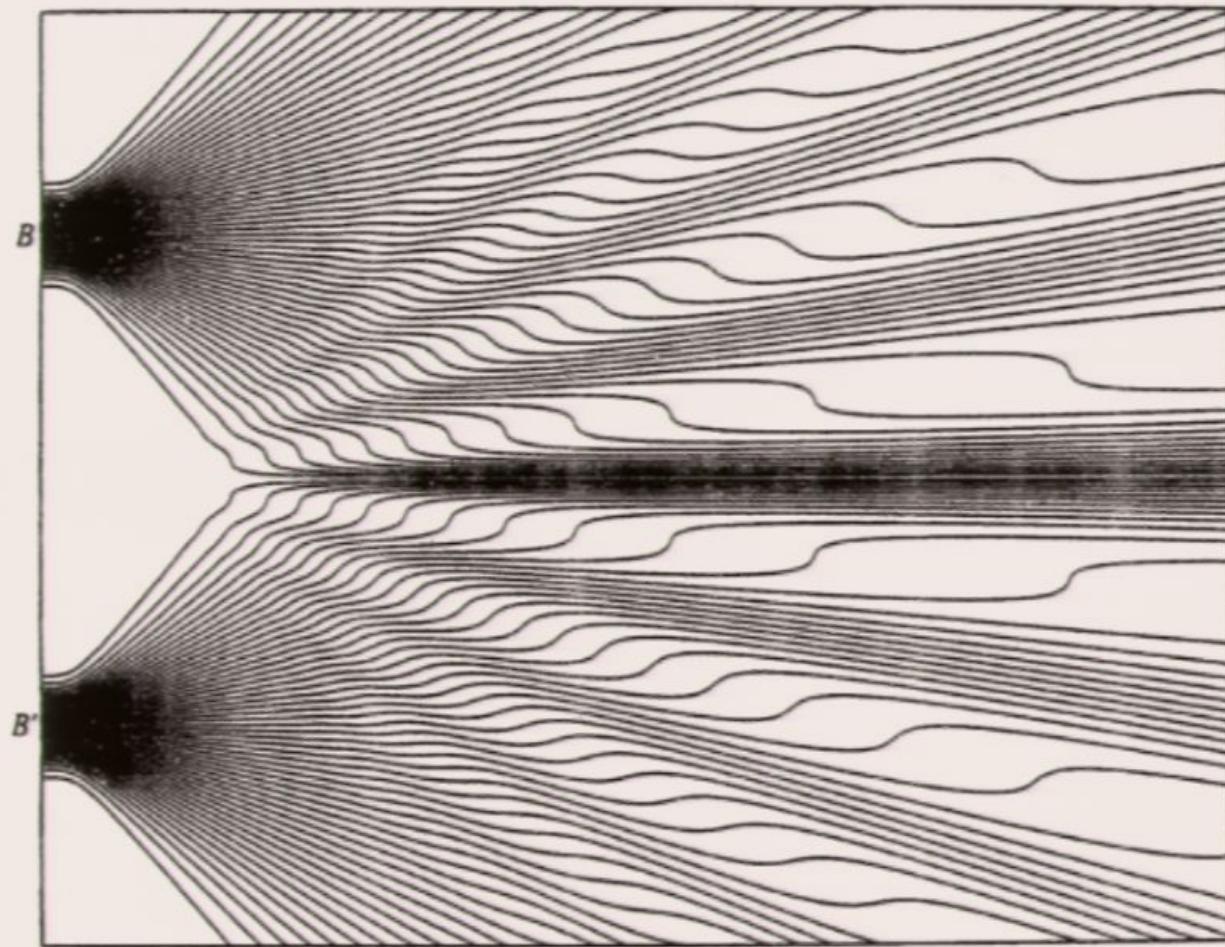
Double slit experiment



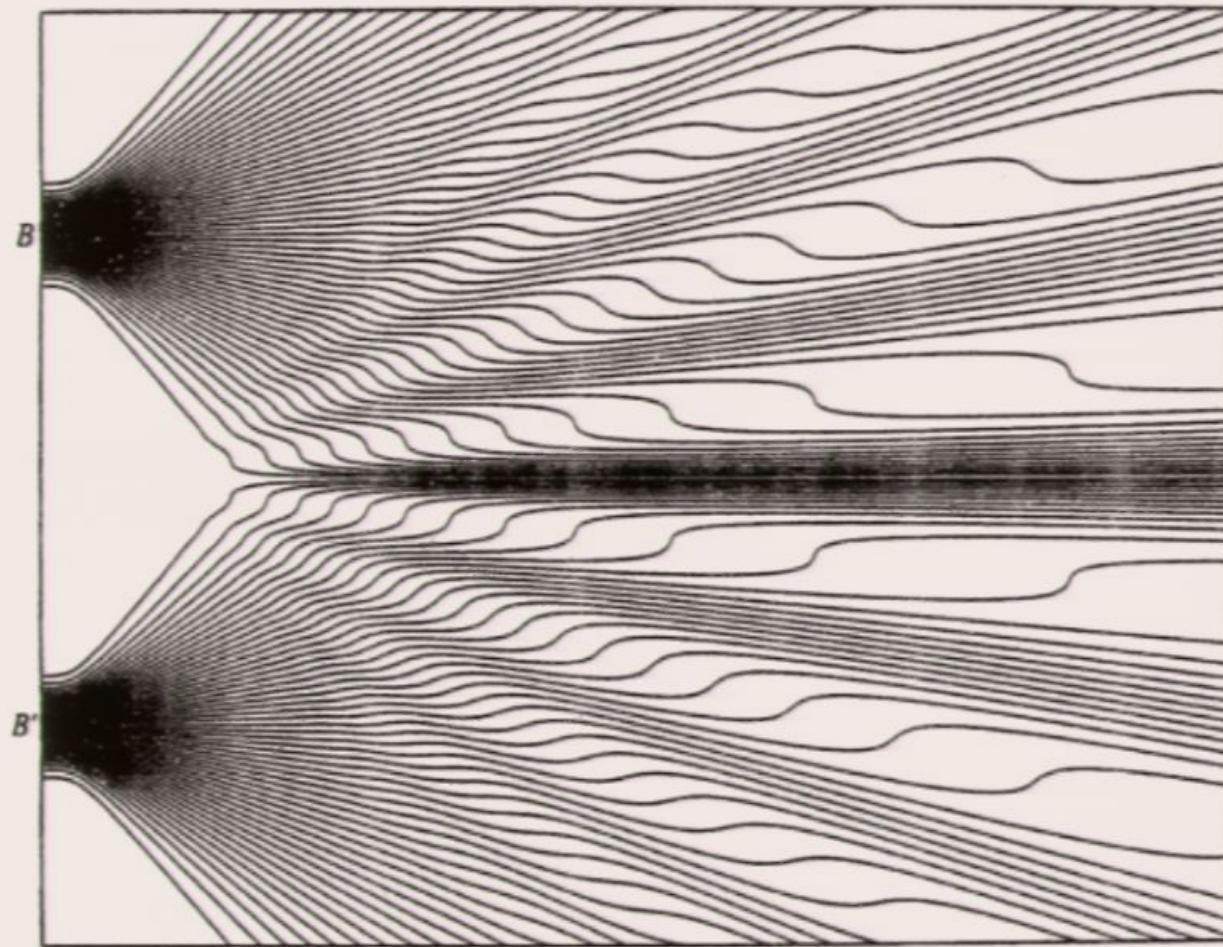
Double slit experiment



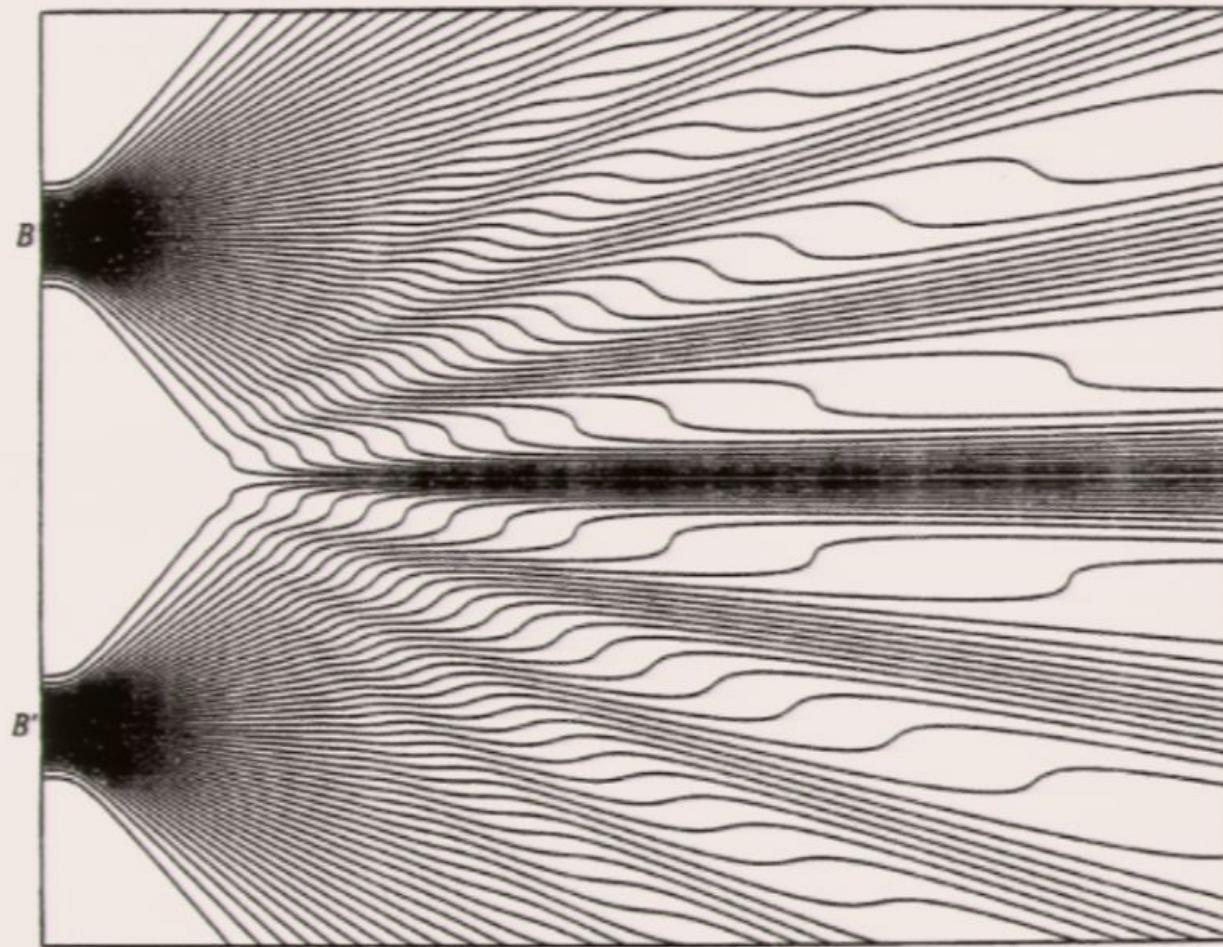
Double slit experiment



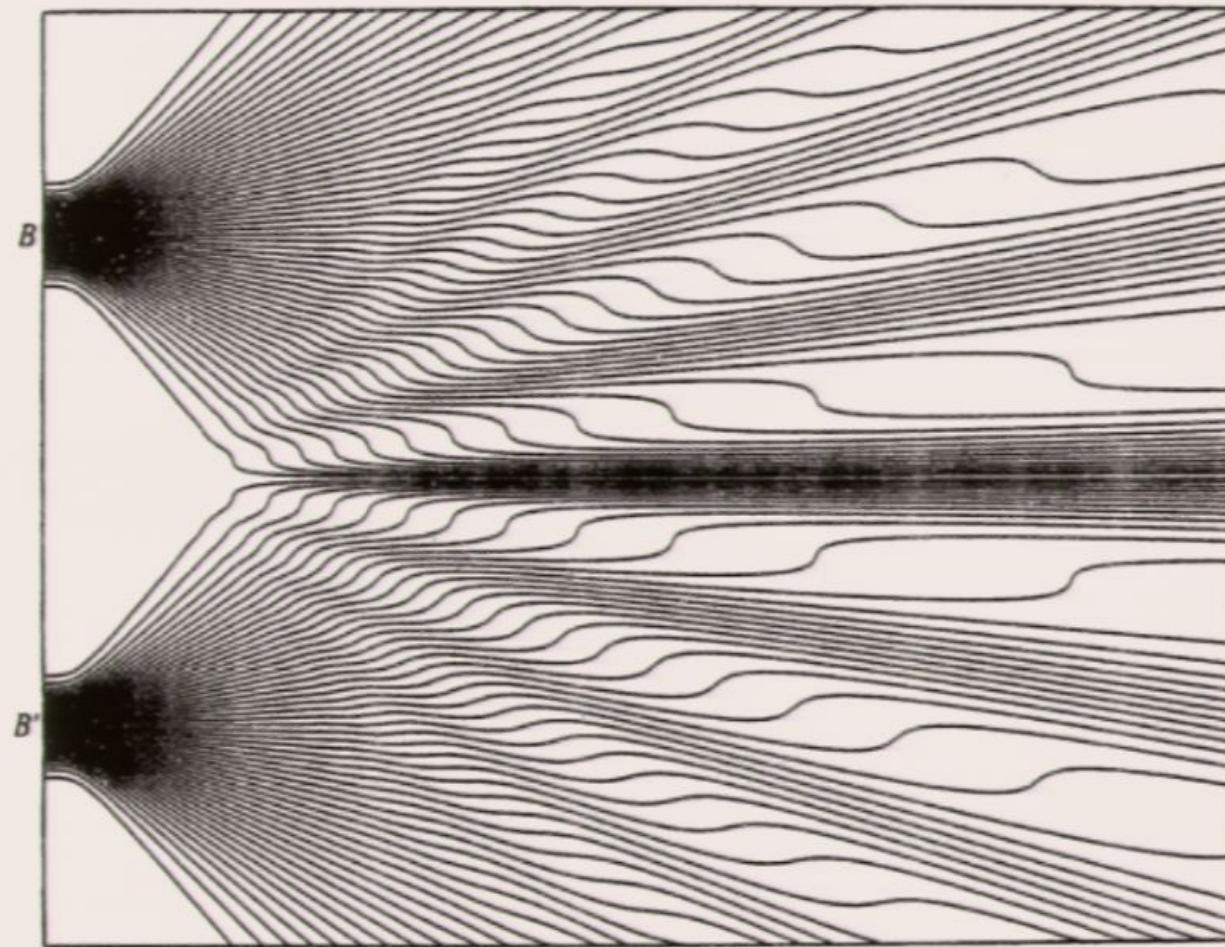
Double slit experiment



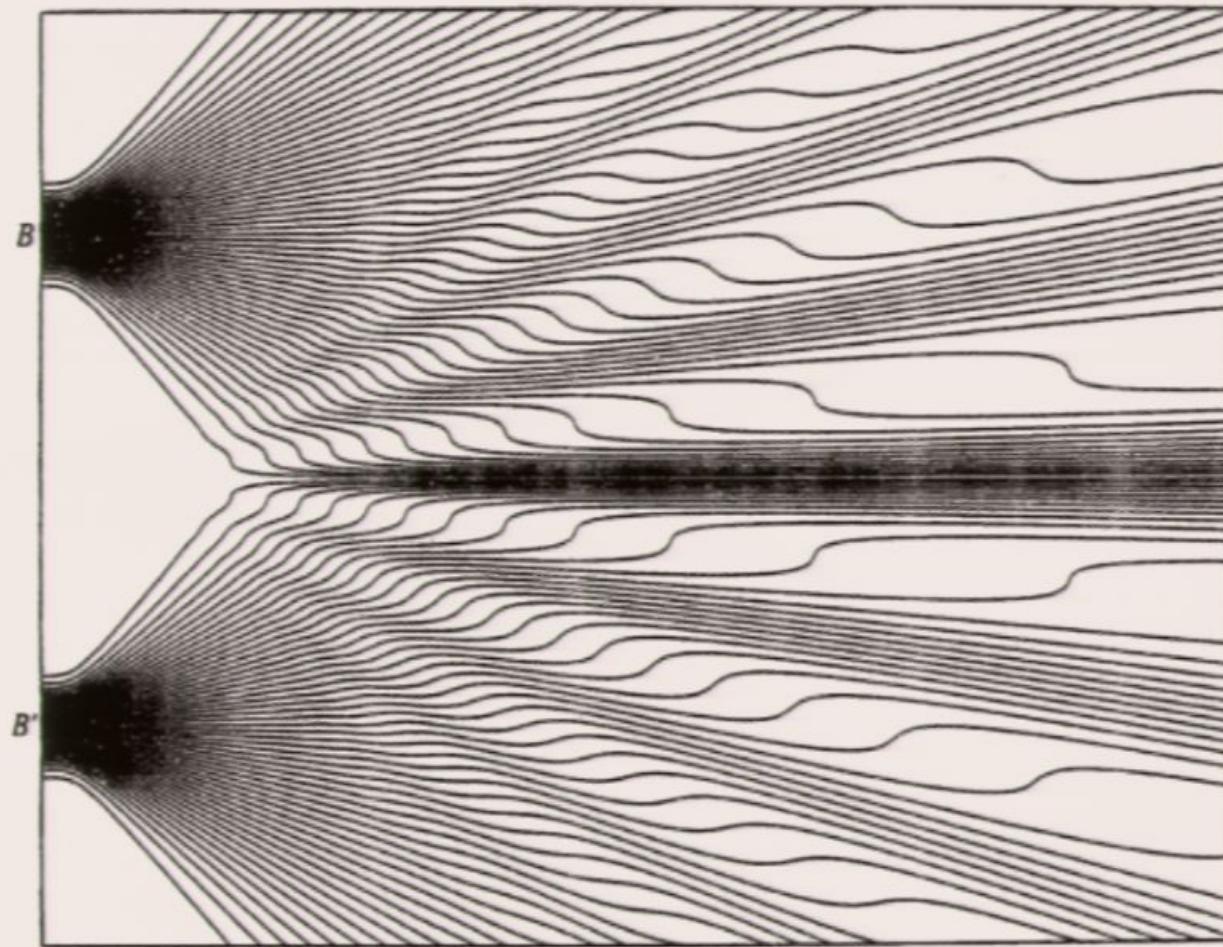
Double slit experiment



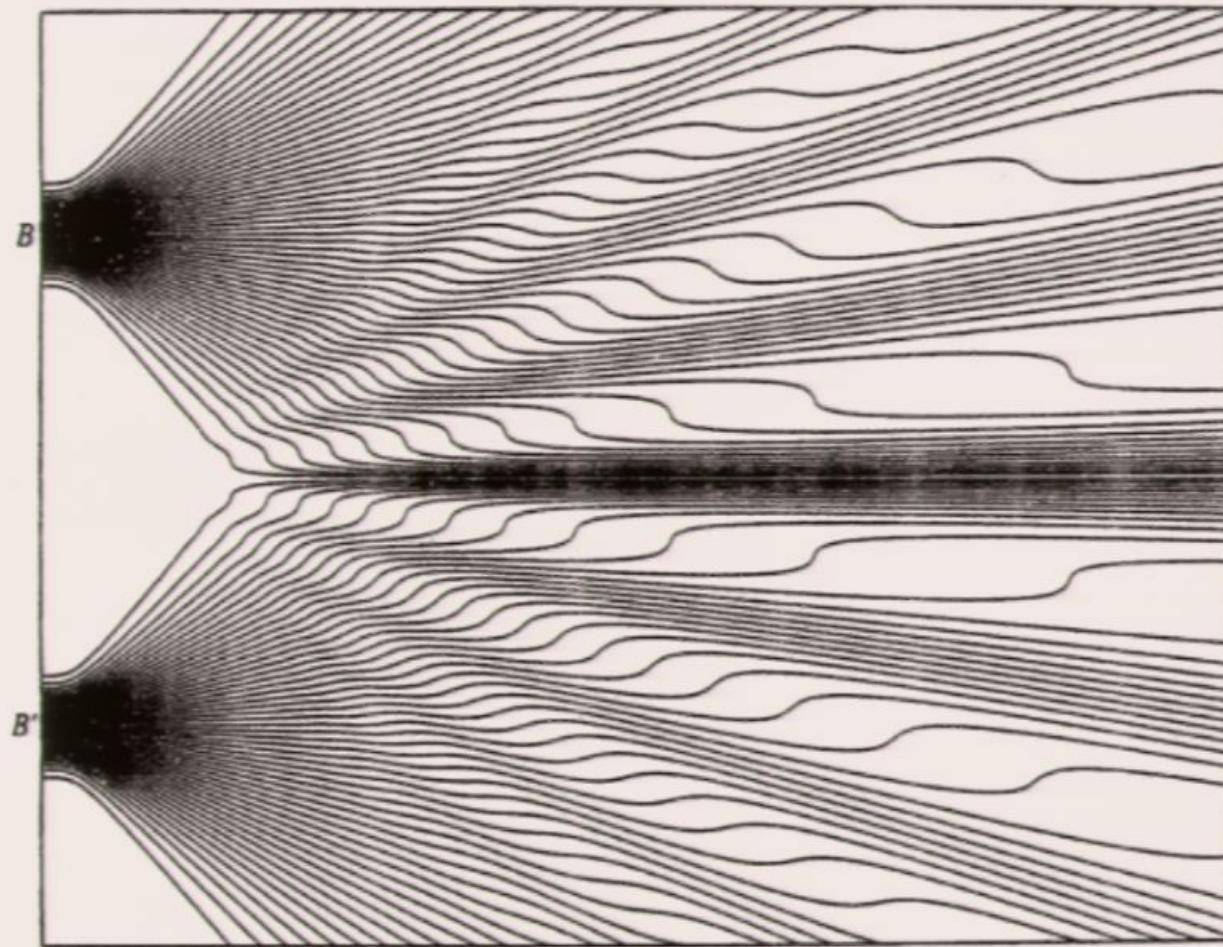
Double slit experiment



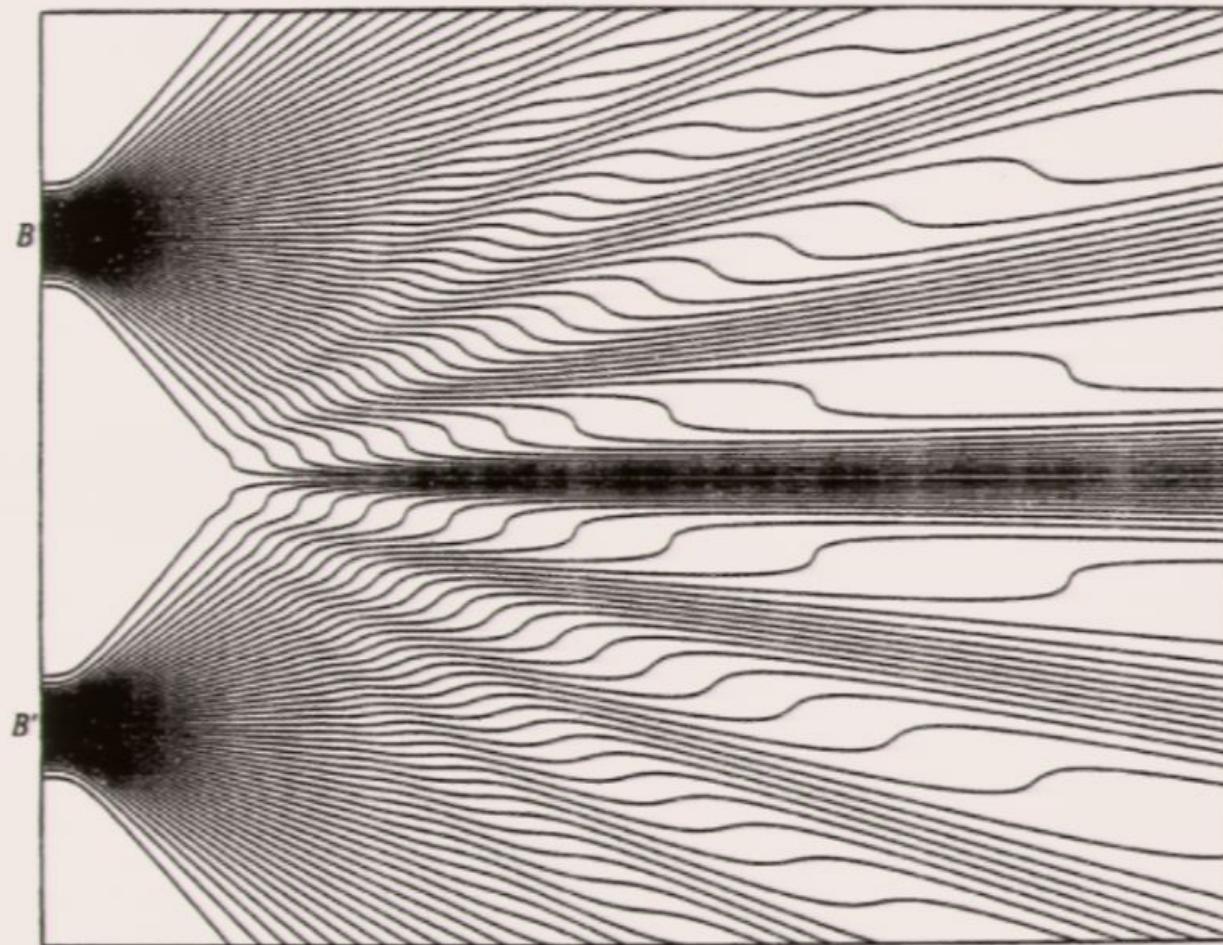
Double slit experiment



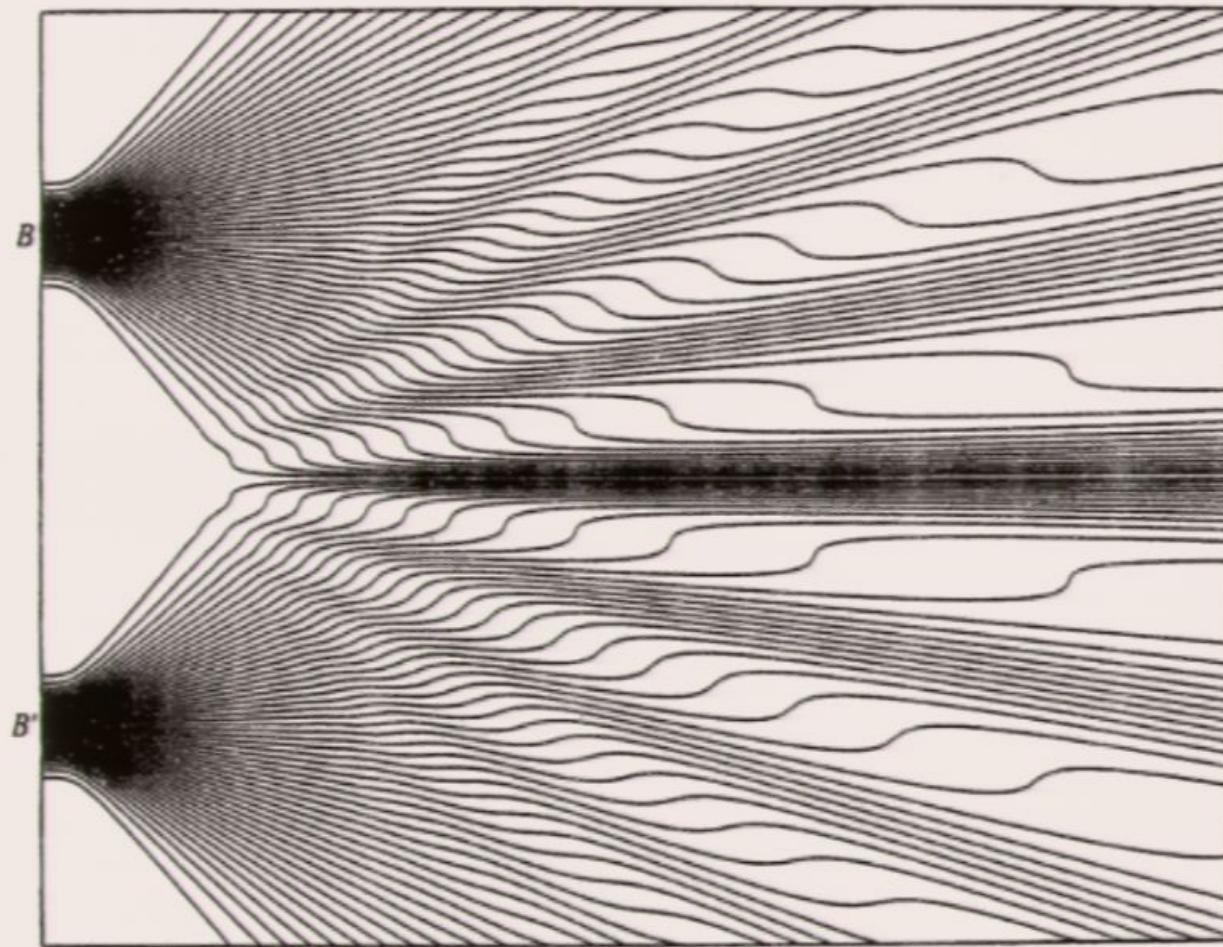
Double slit experiment



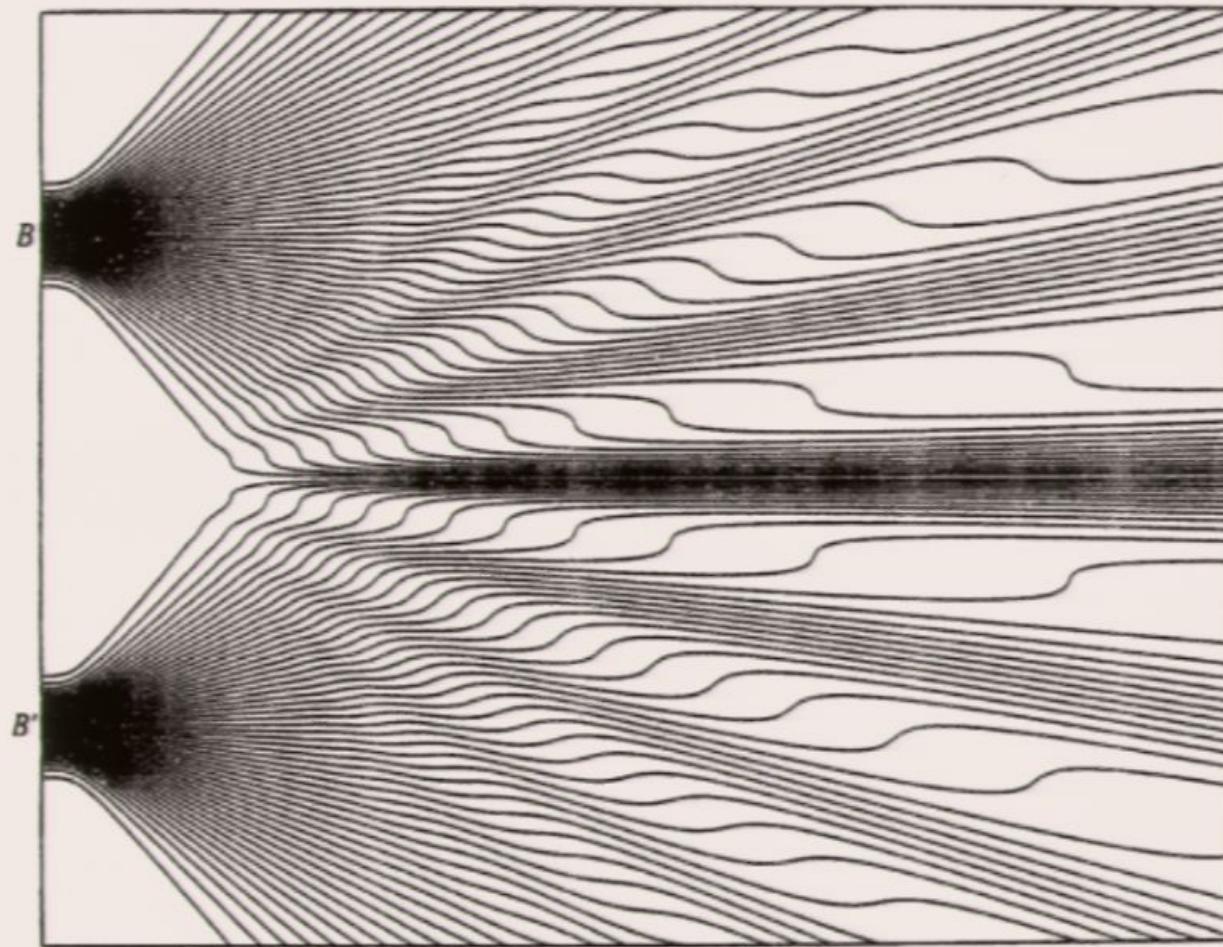
Double slit experiment



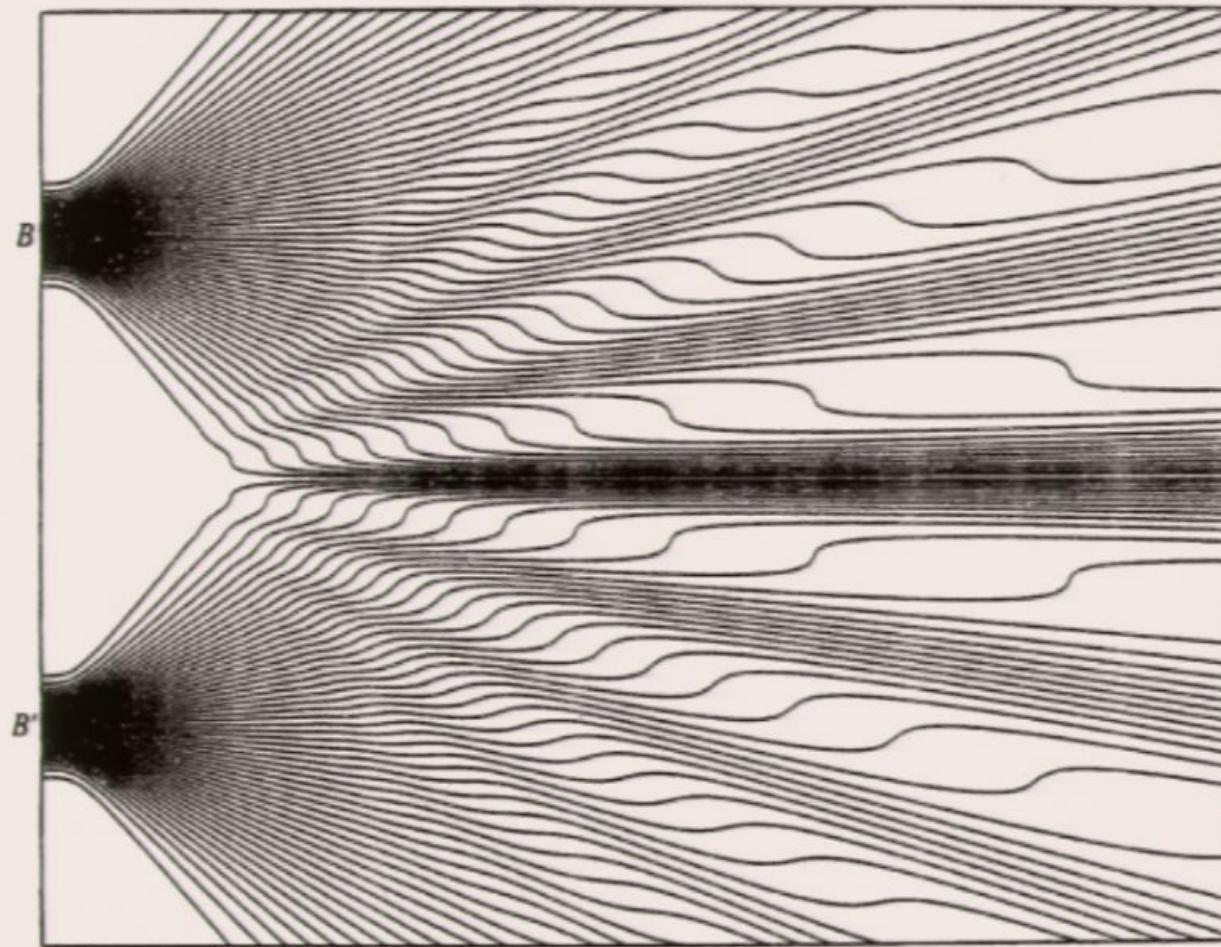
Double slit experiment



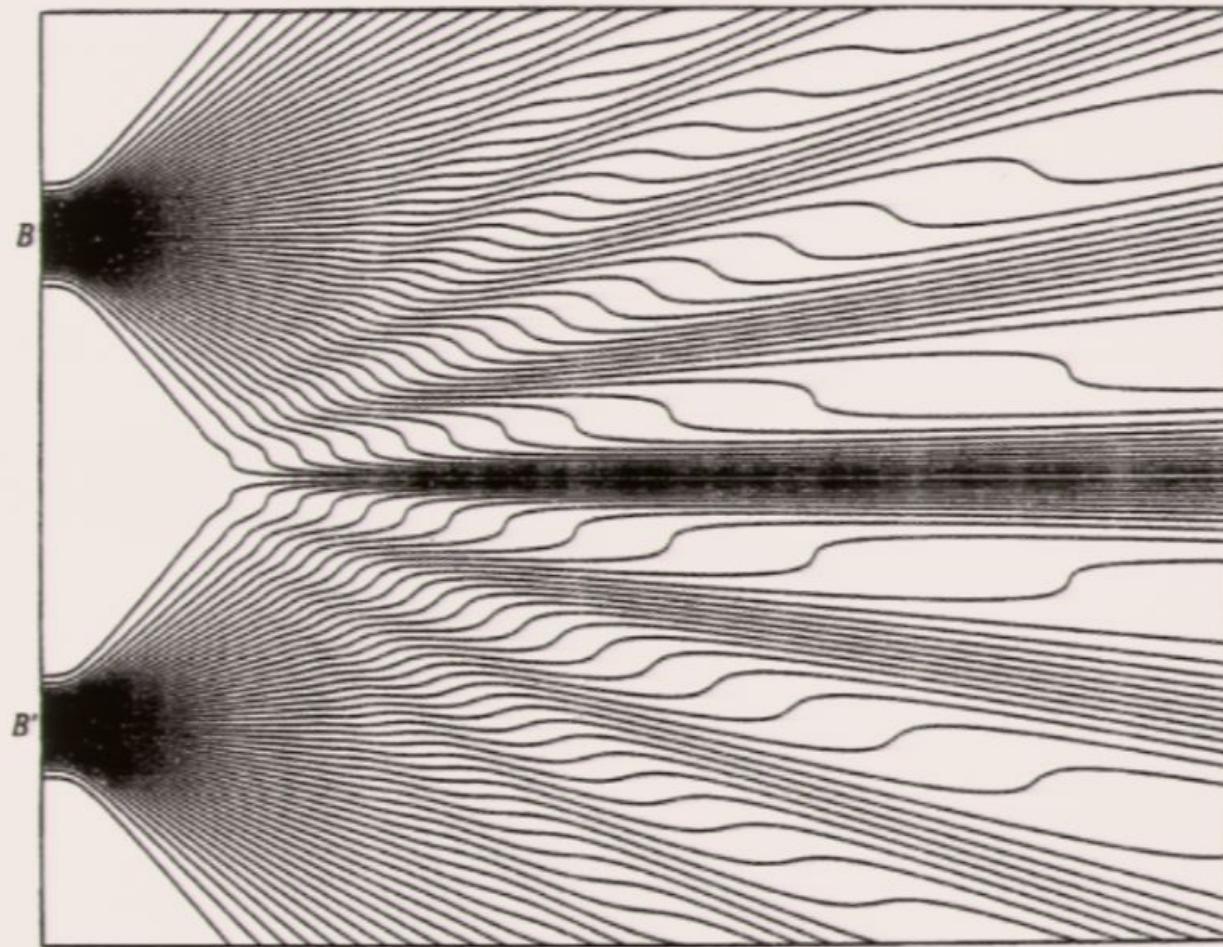
Double slit experiment



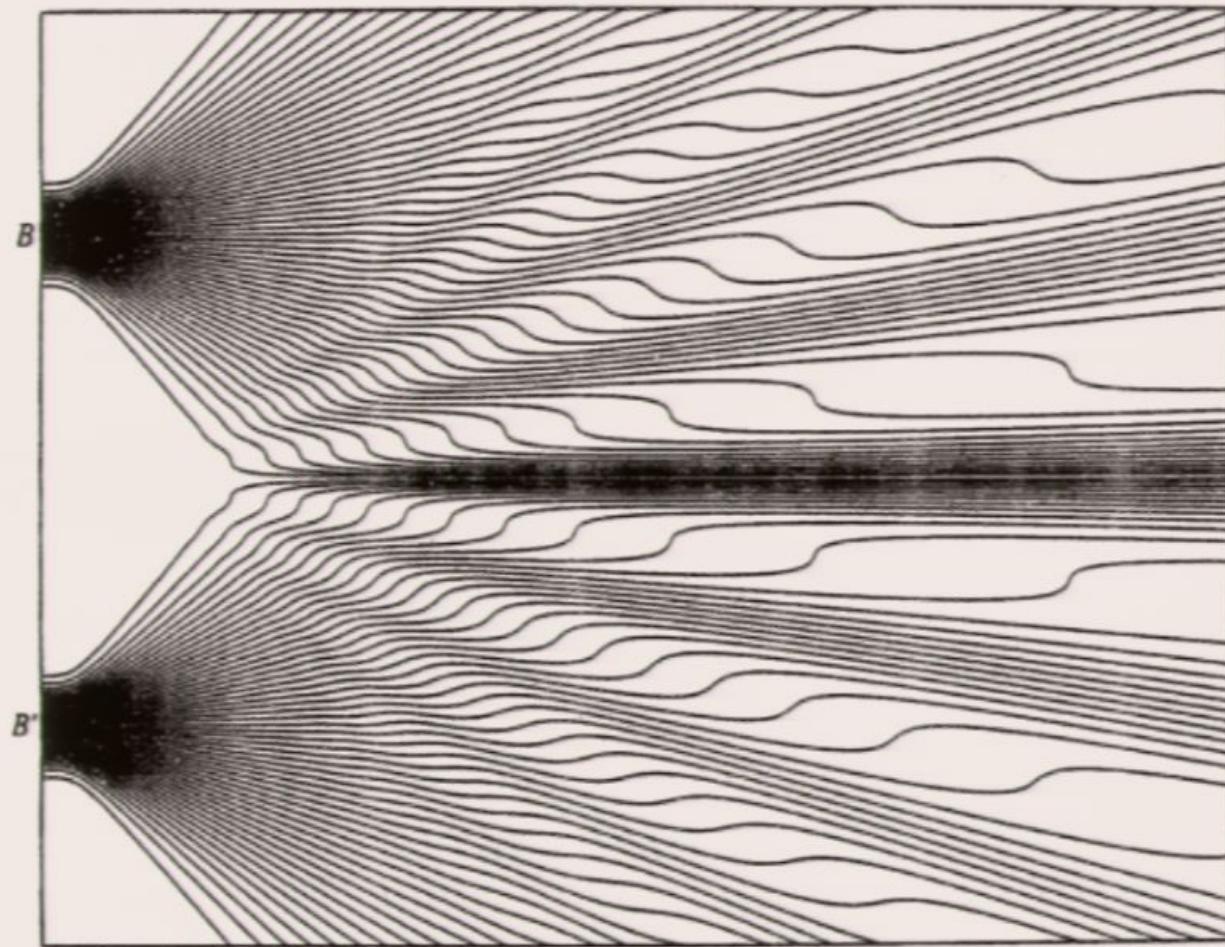
Double slit experiment



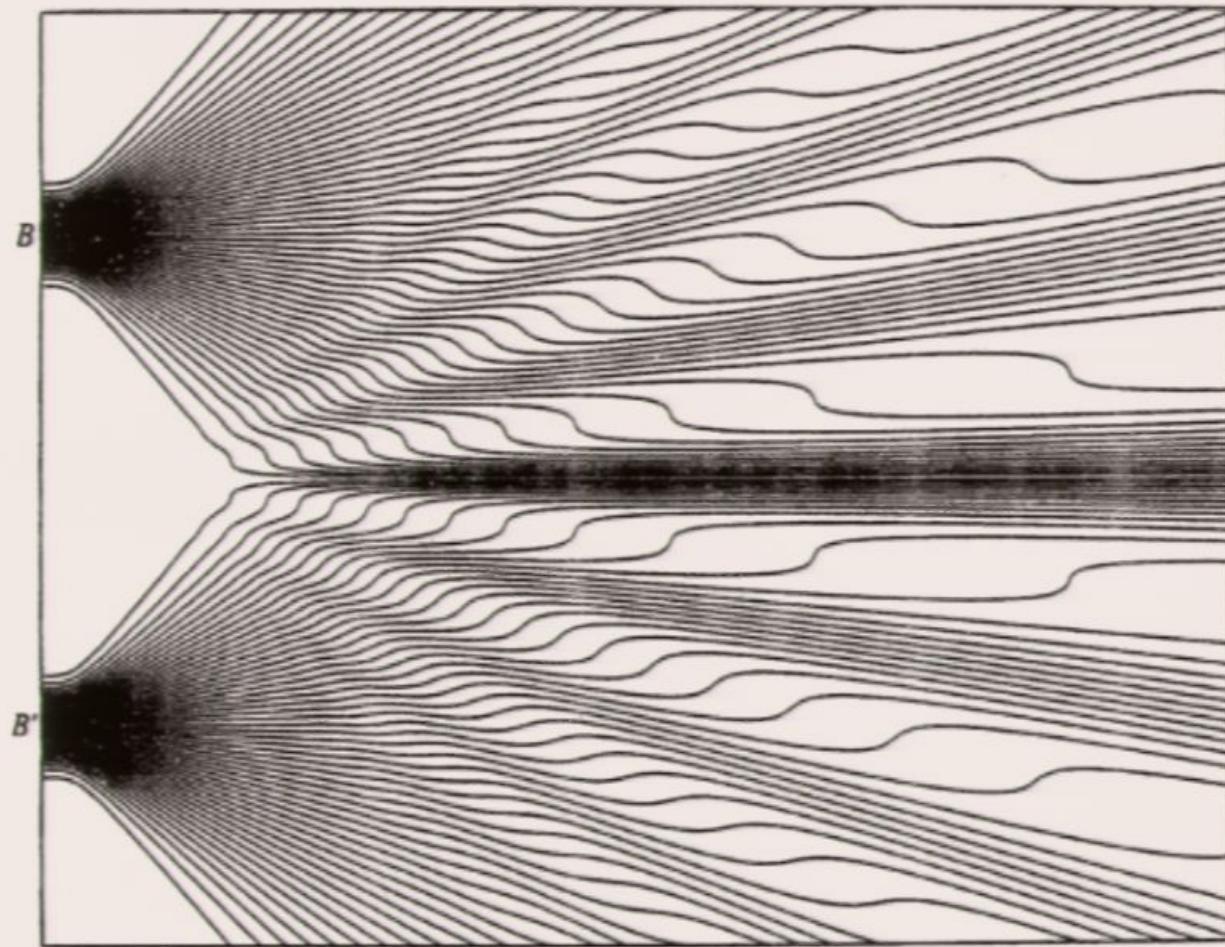
Double slit experiment



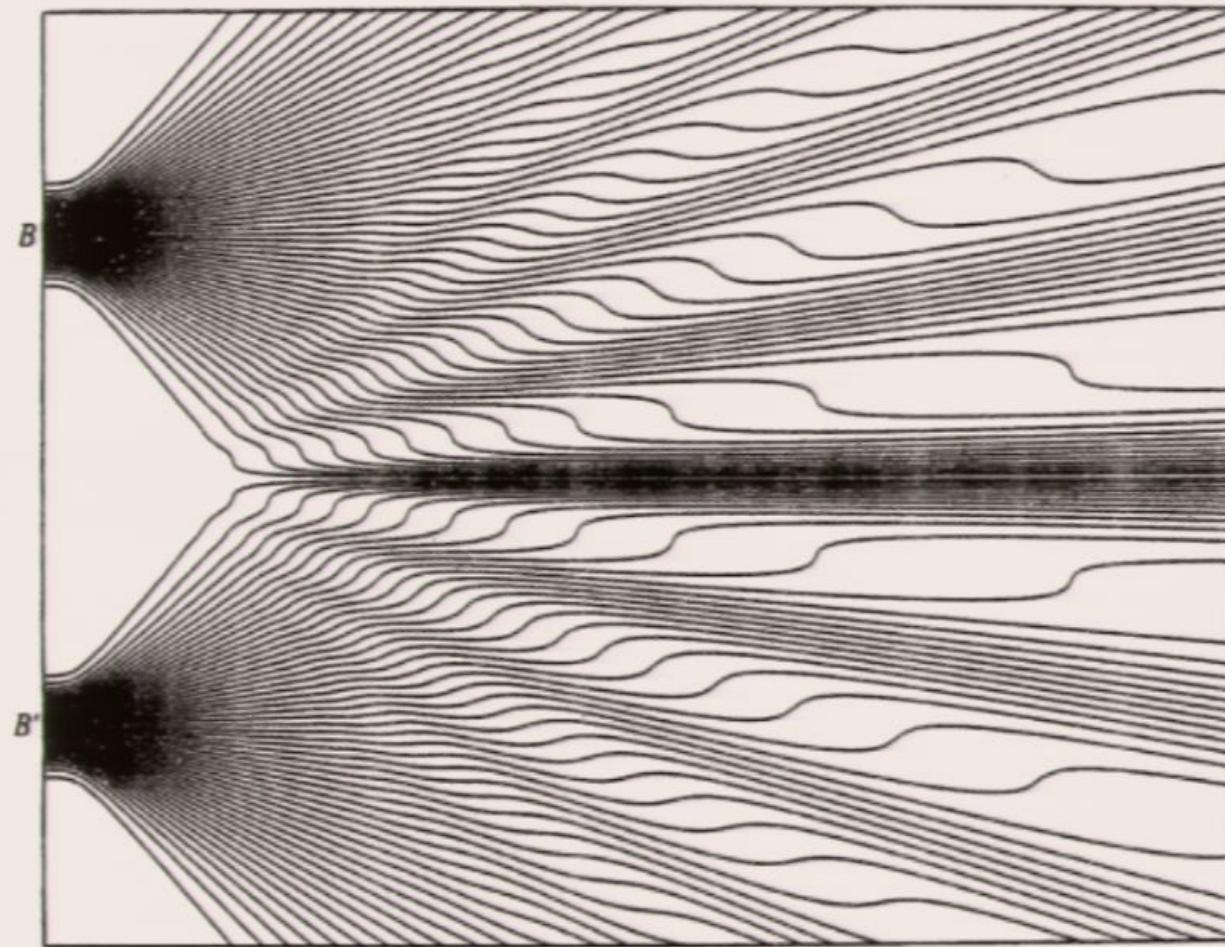
Double slit experiment



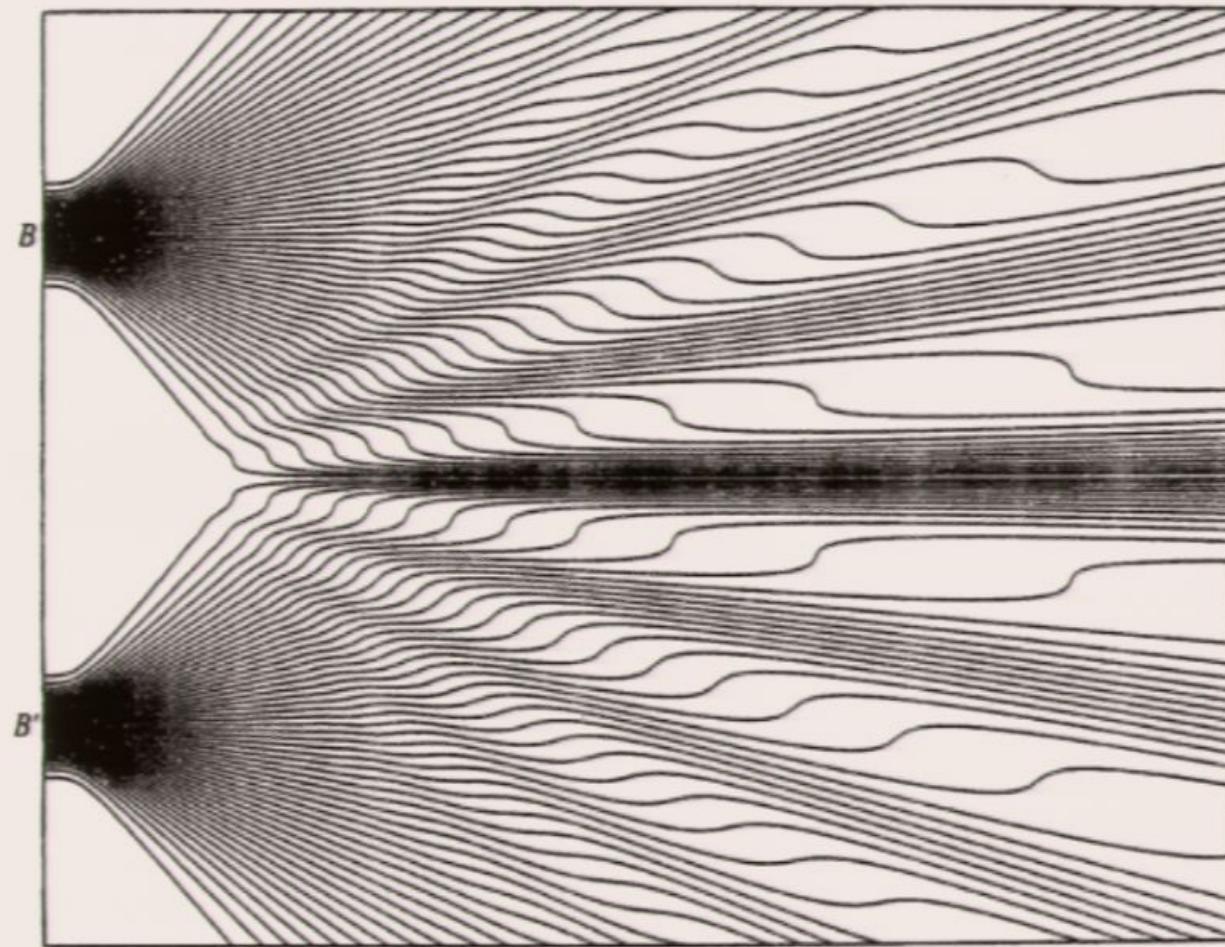
Double slit experiment



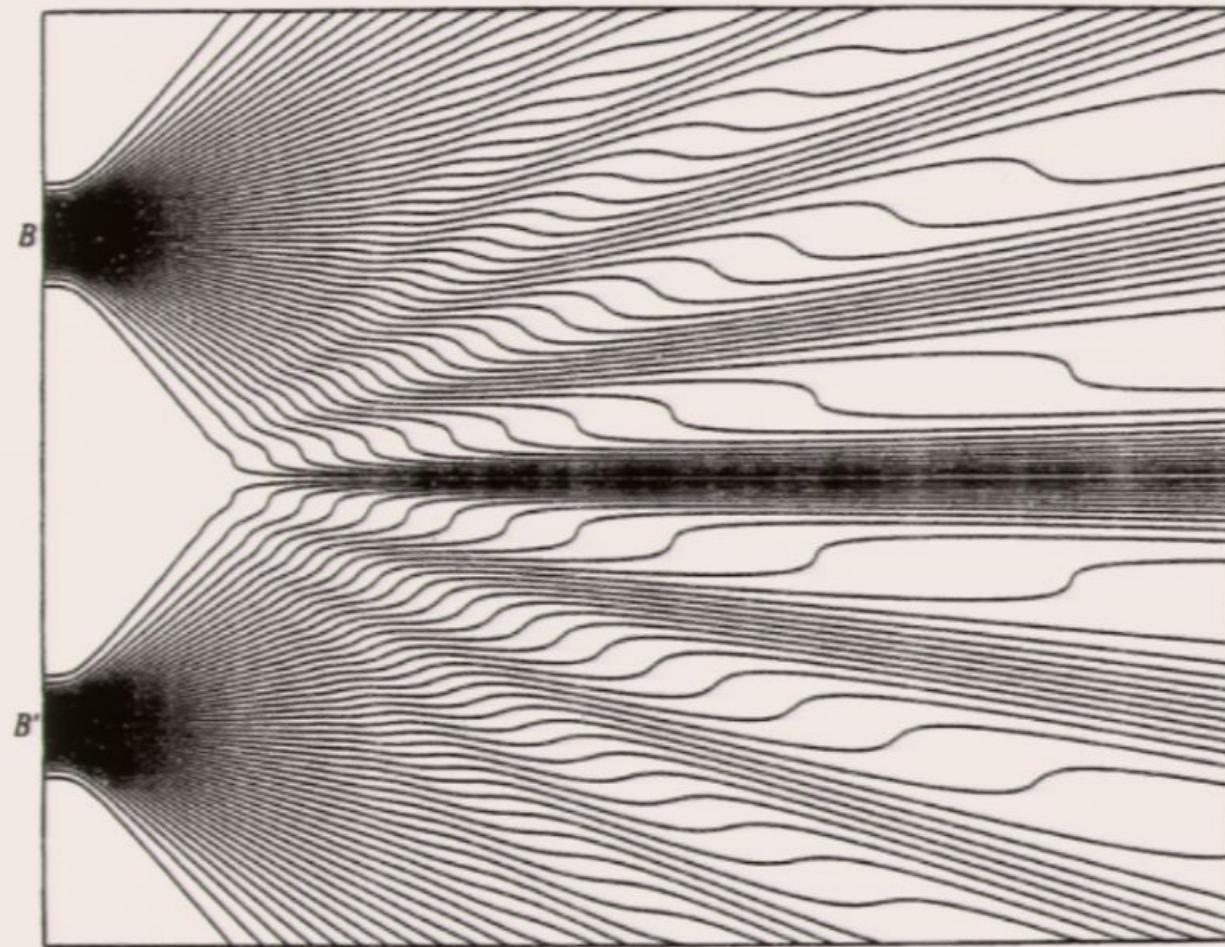
Double slit experiment



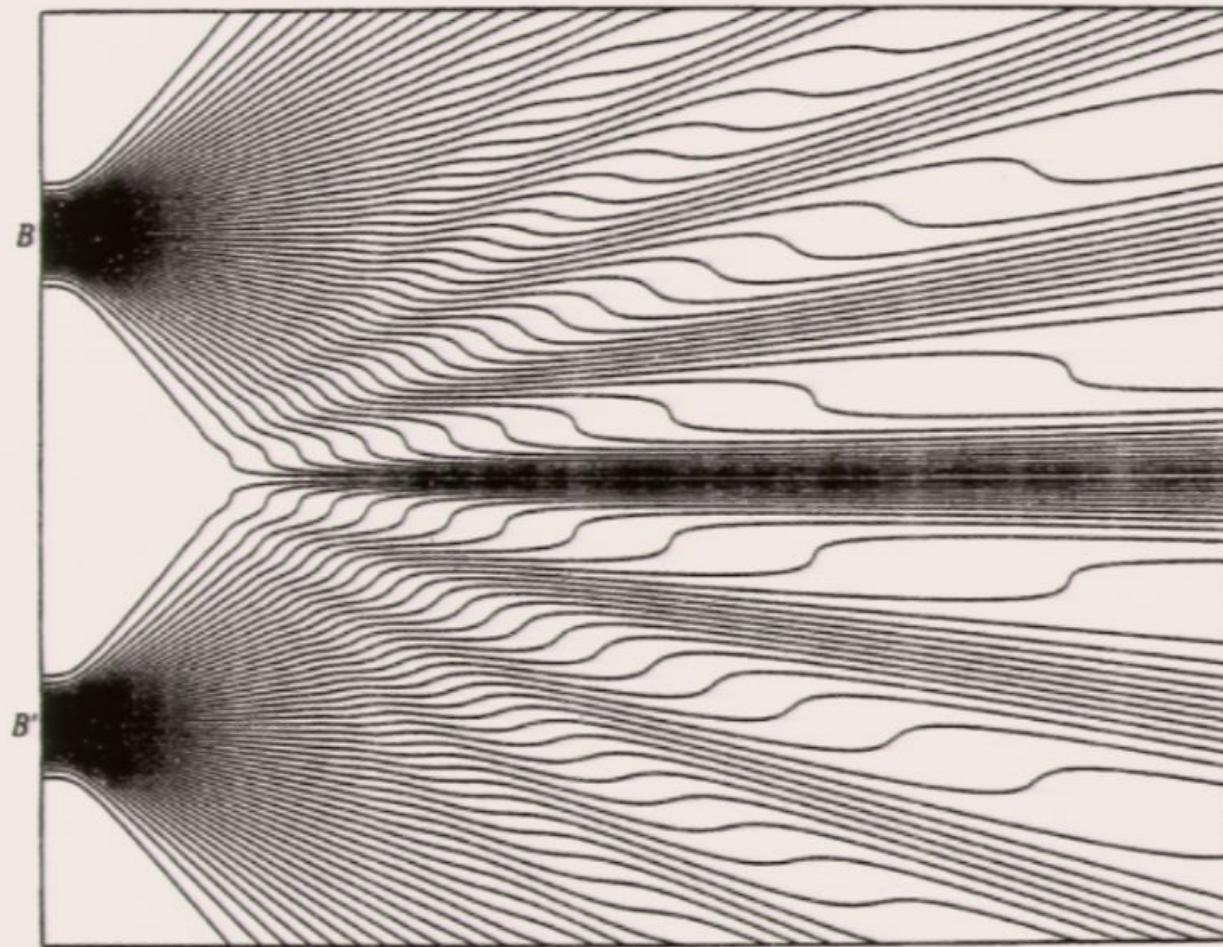
Double slit experiment



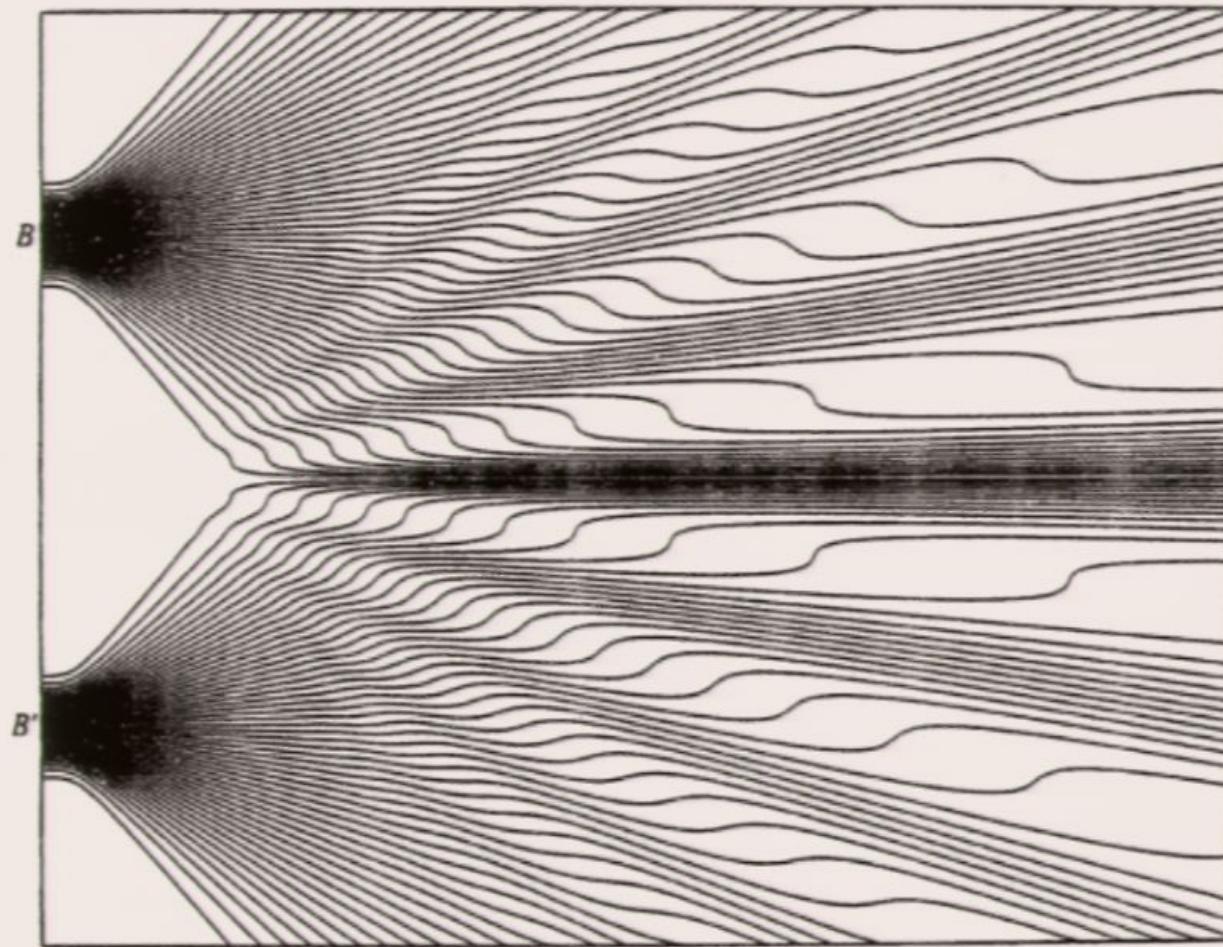
Double slit experiment



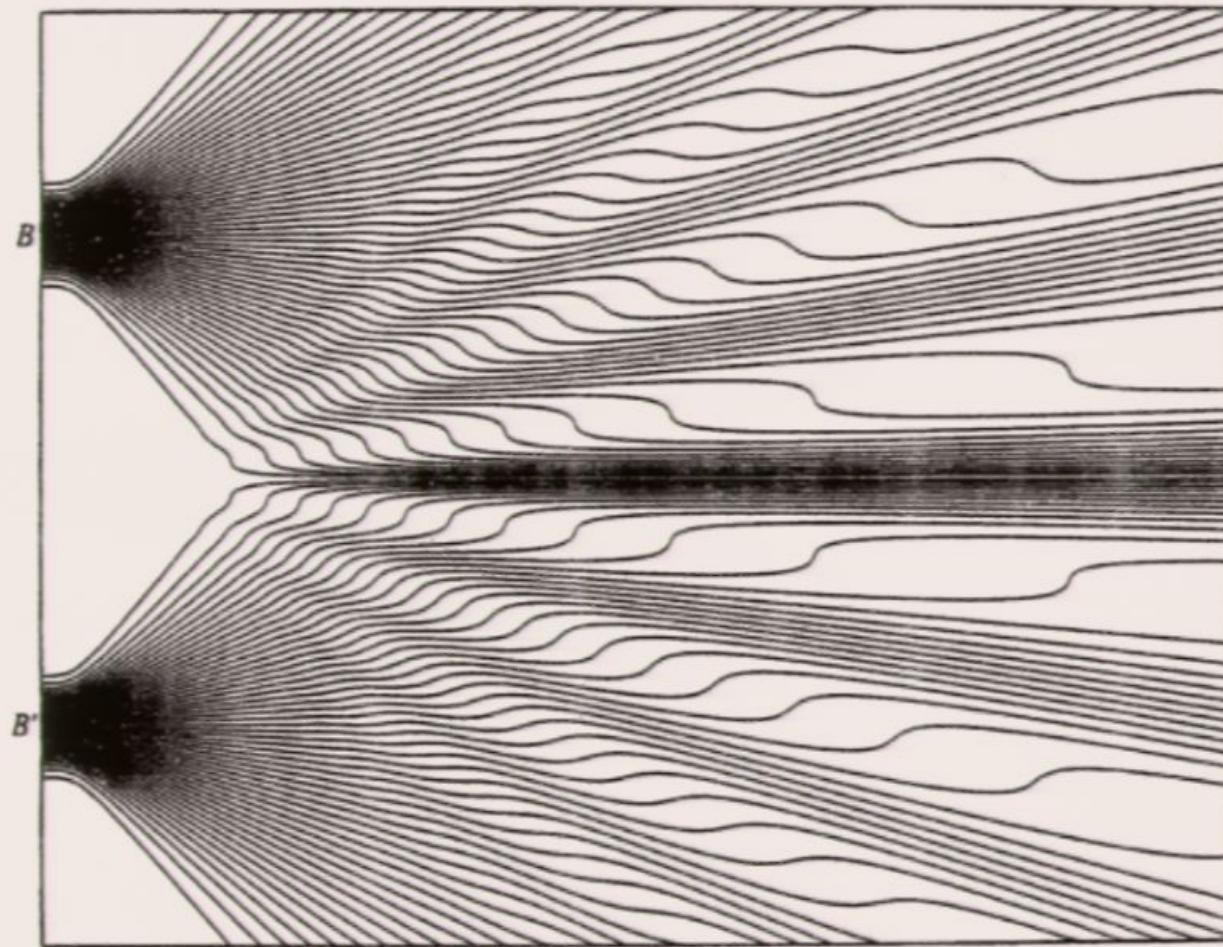
Double slit experiment



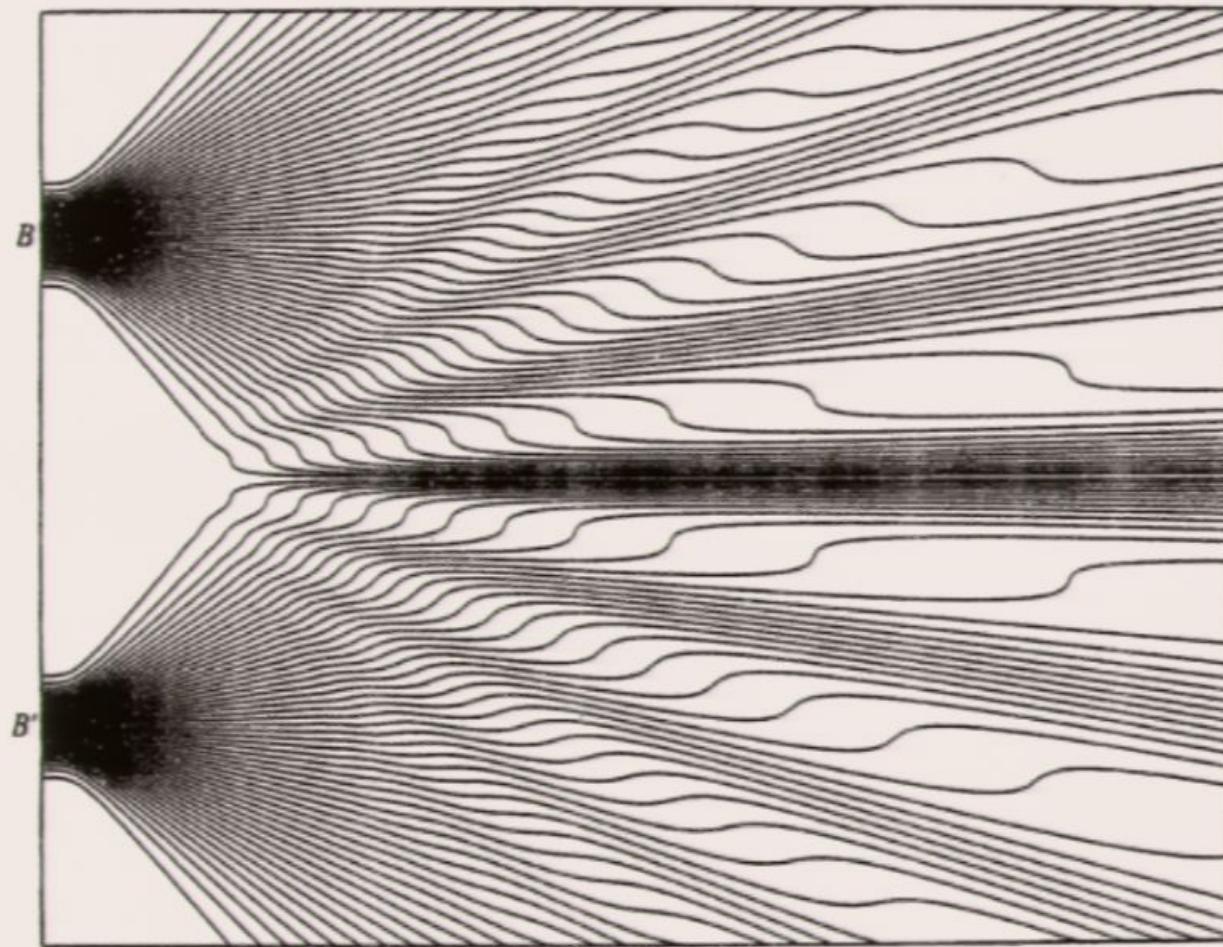
Double slit experiment



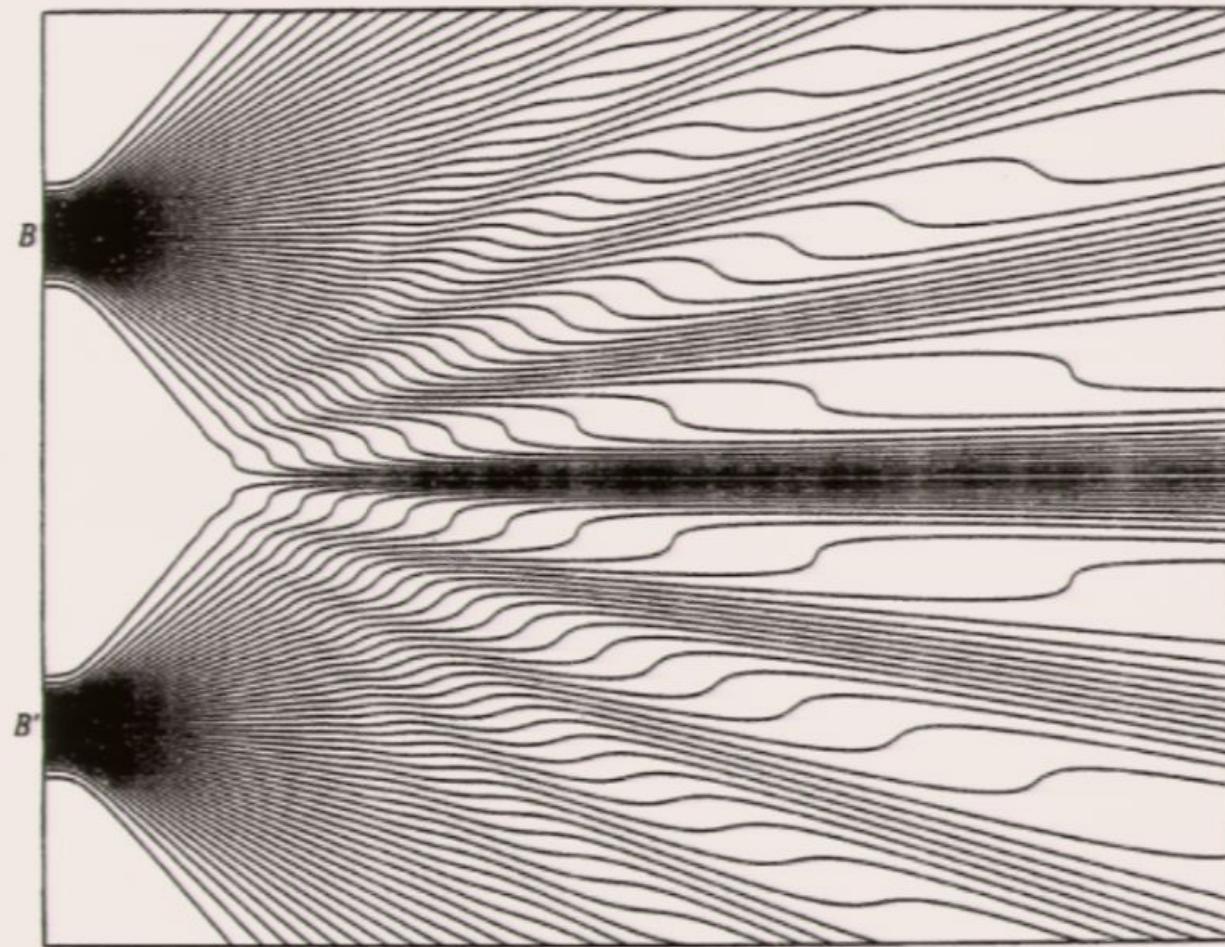
Double slit experiment



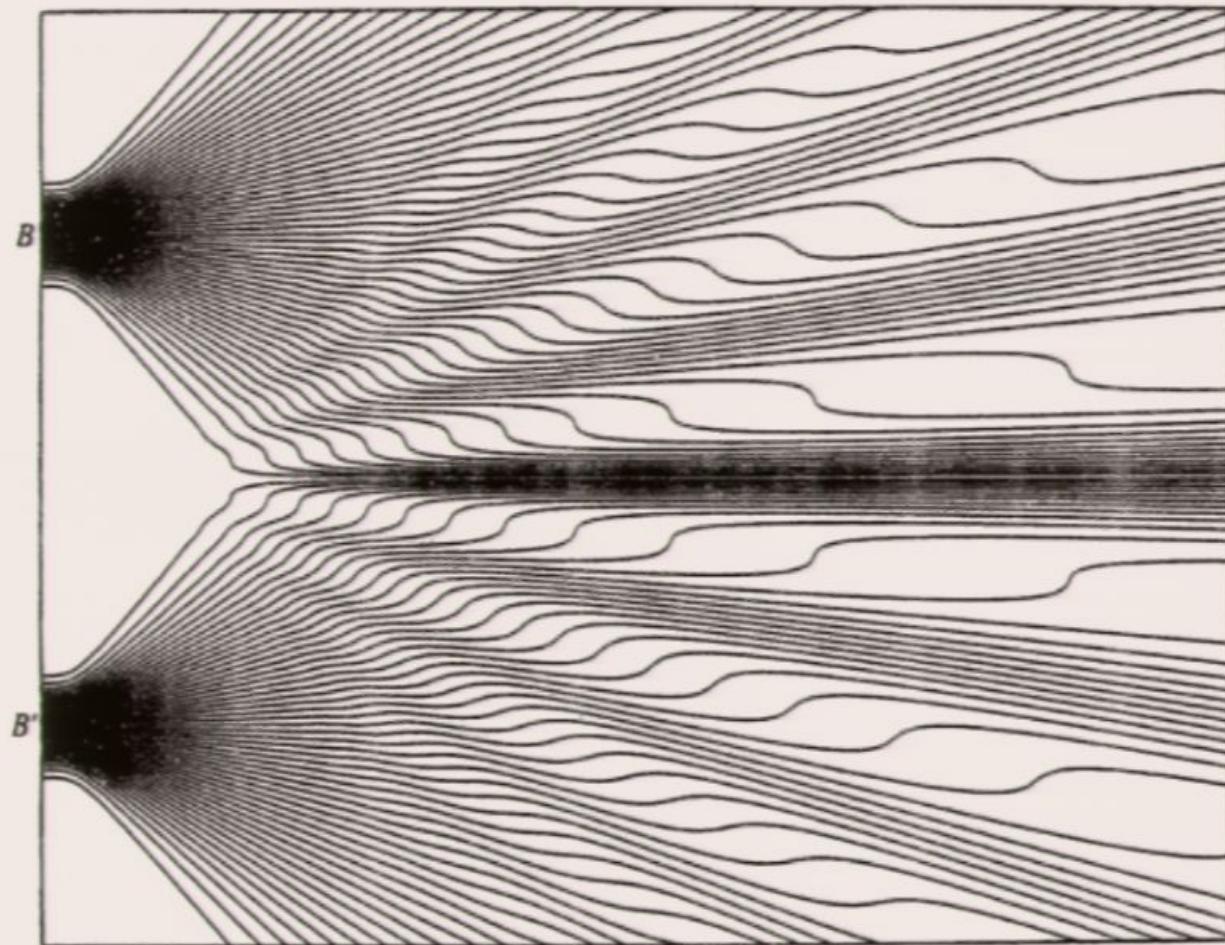
Double slit experiment



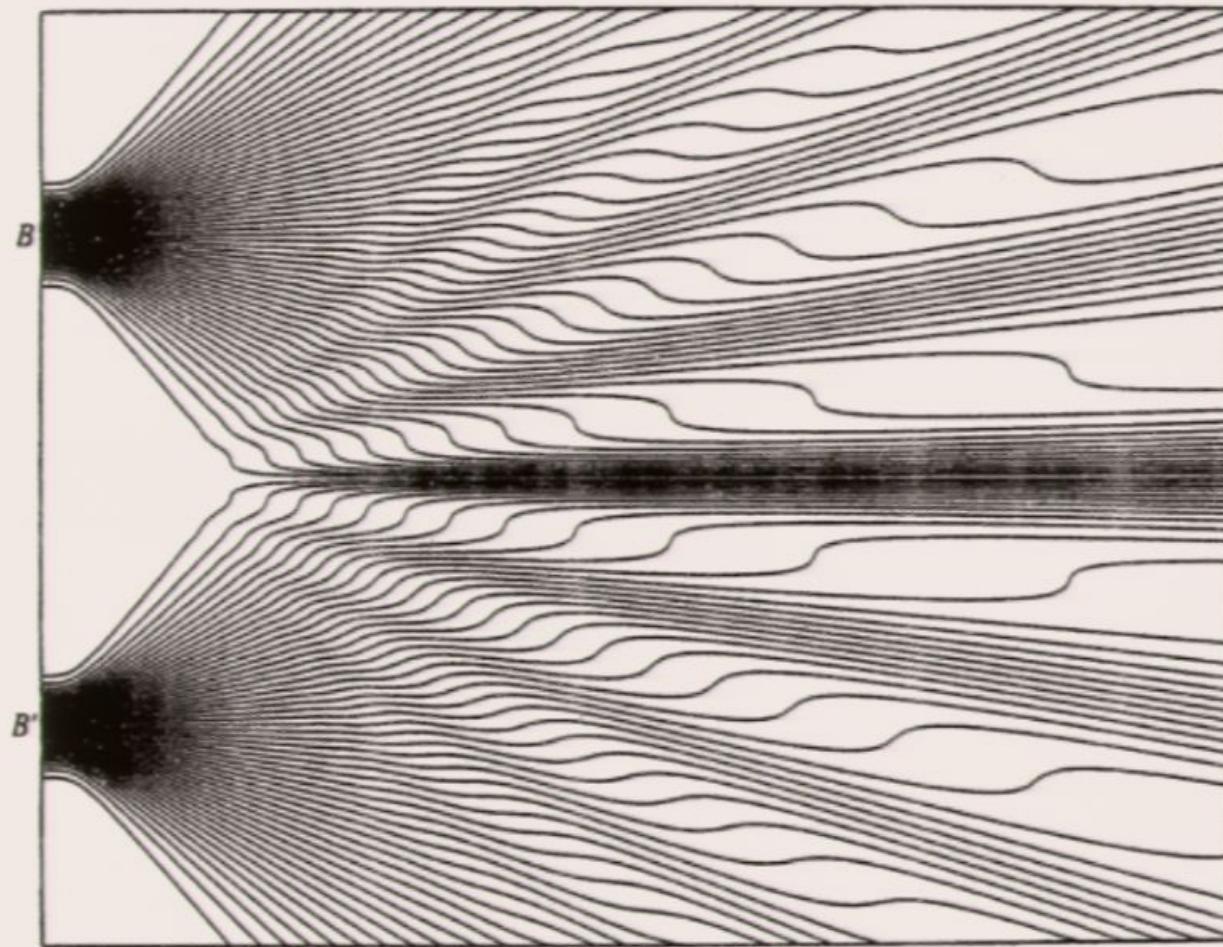
Double slit experiment



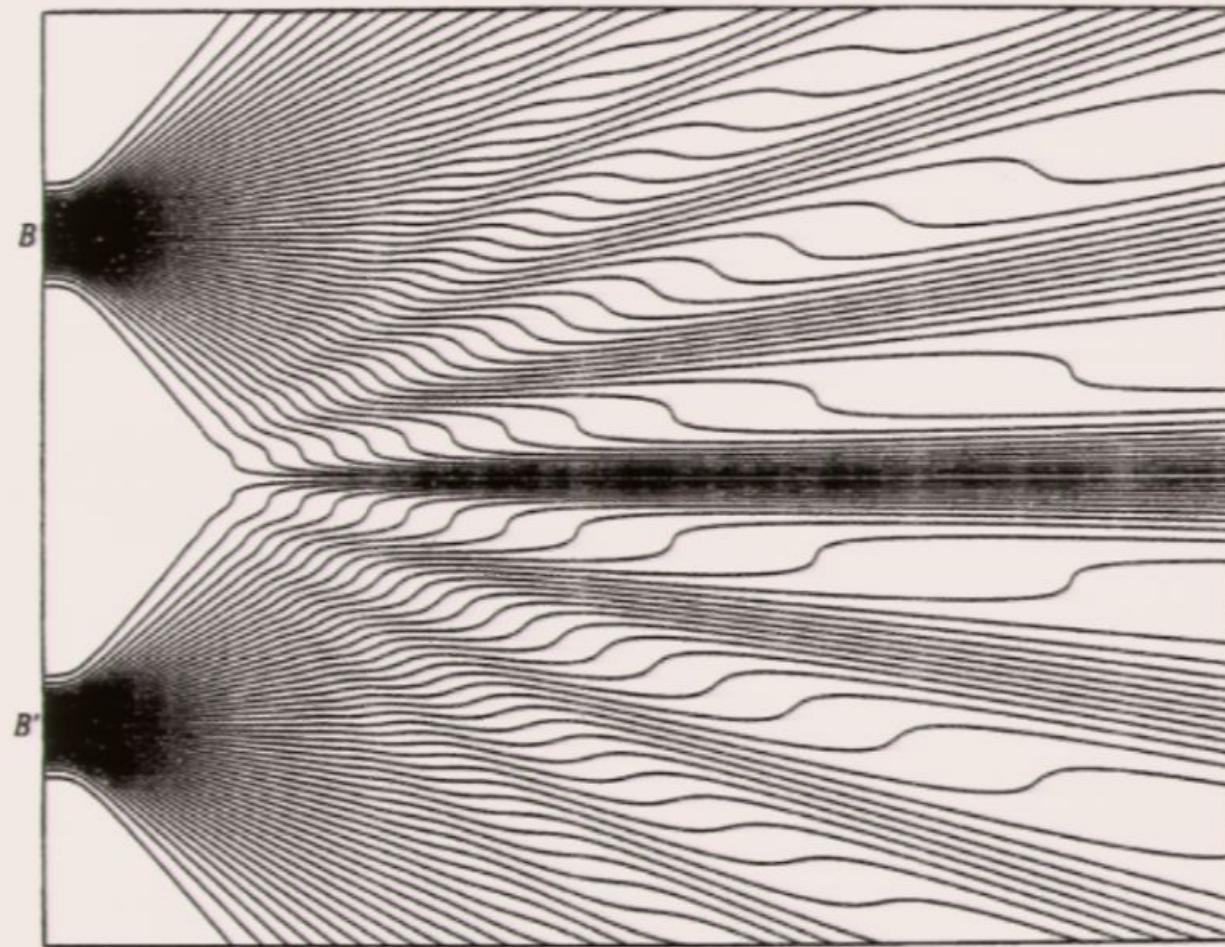
Double slit experiment



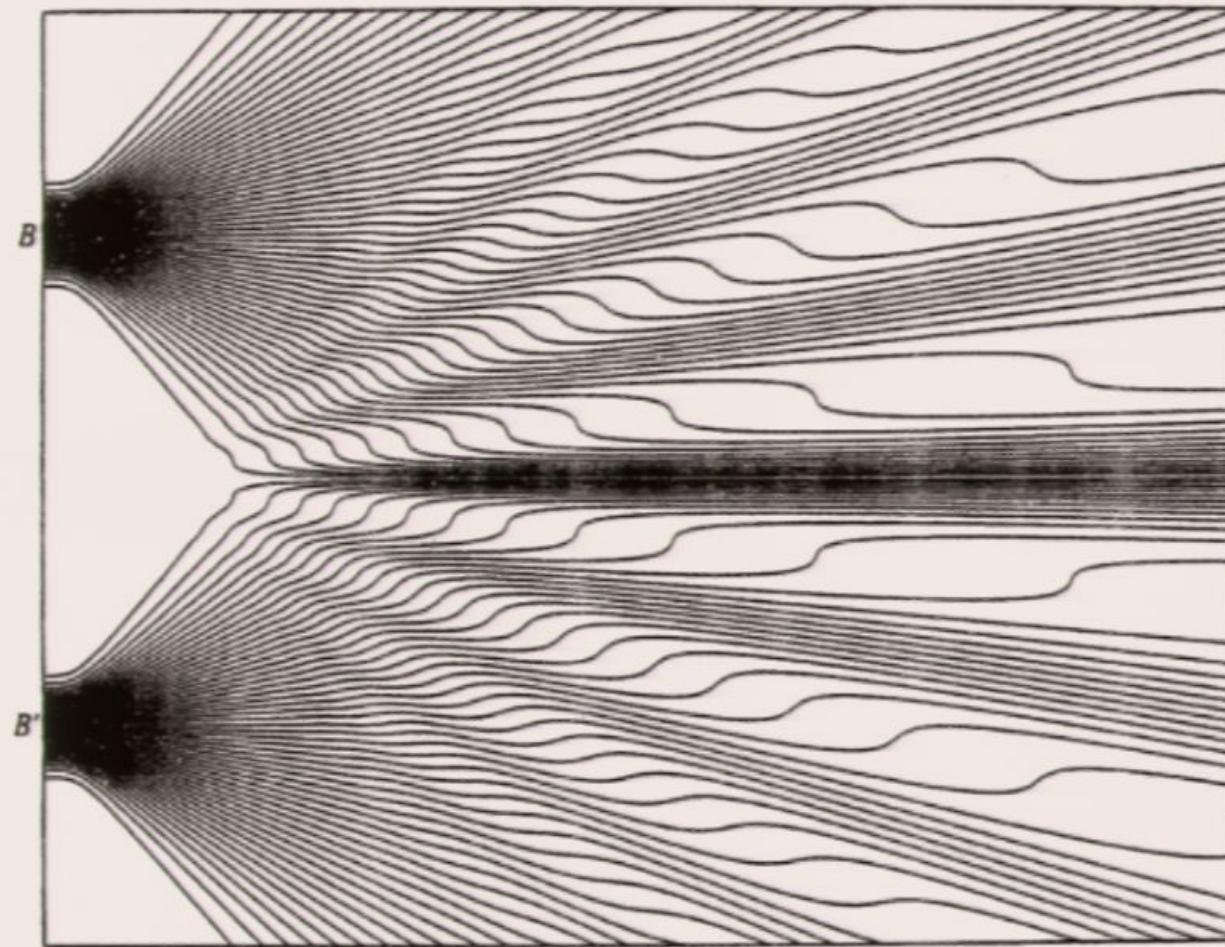
Double slit experiment



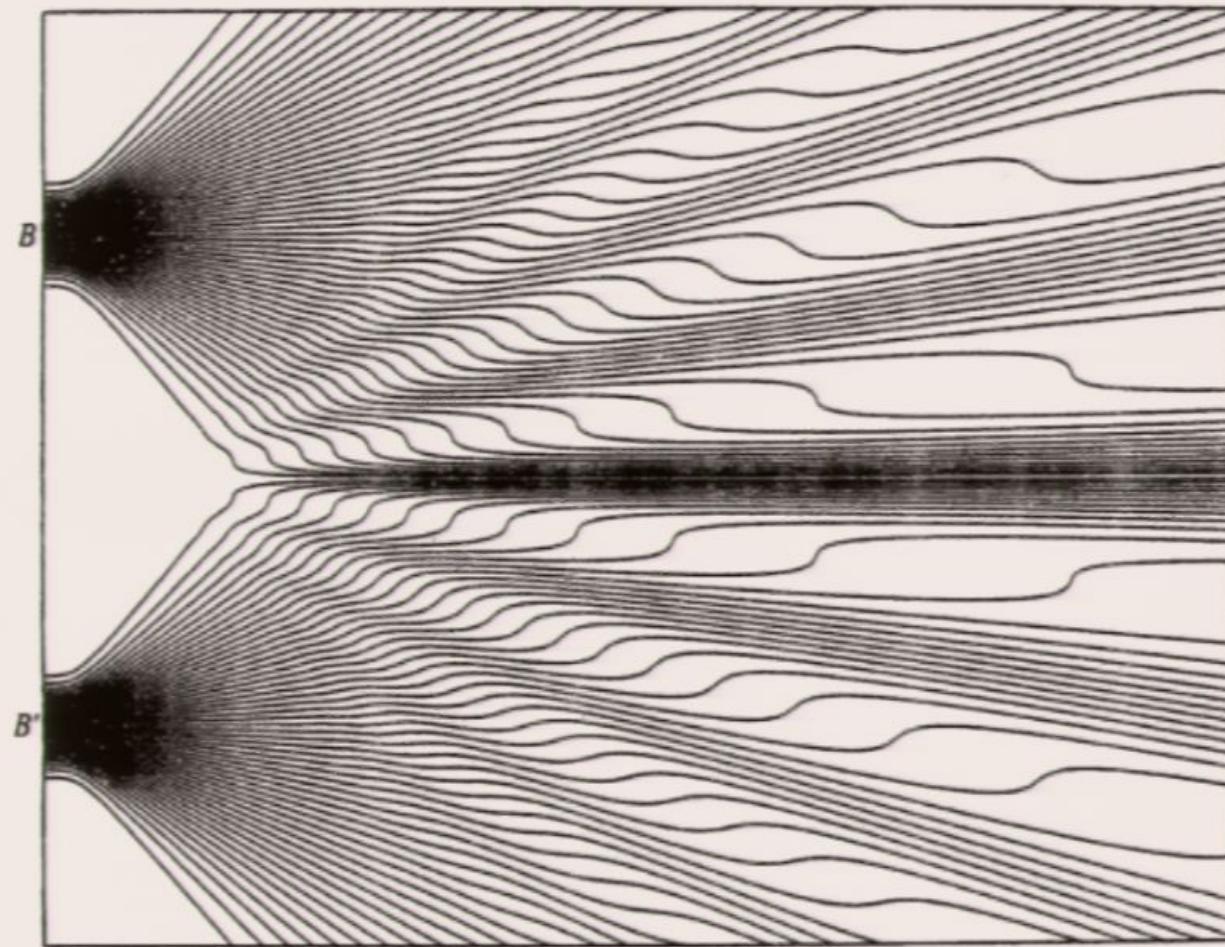
Double slit experiment



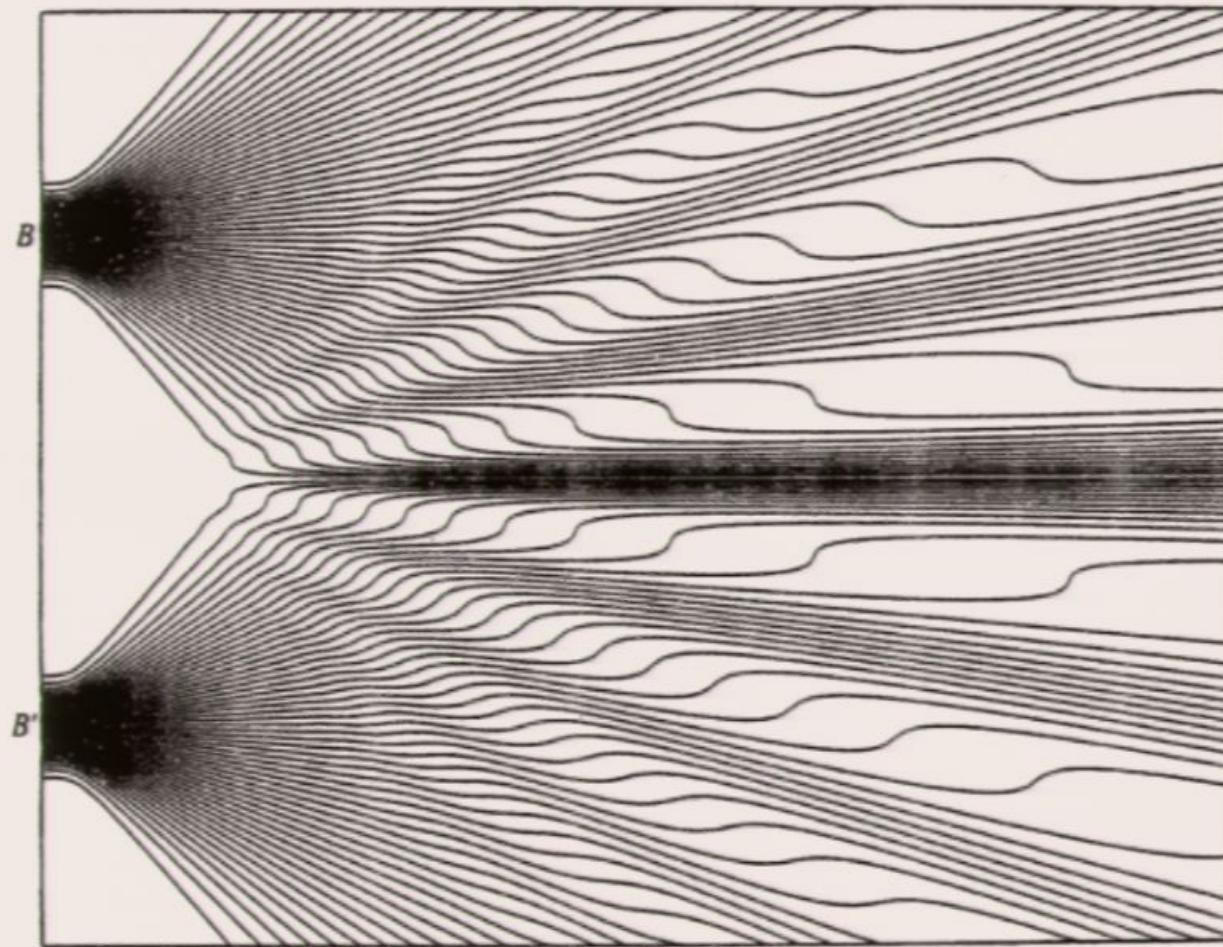
Double slit experiment



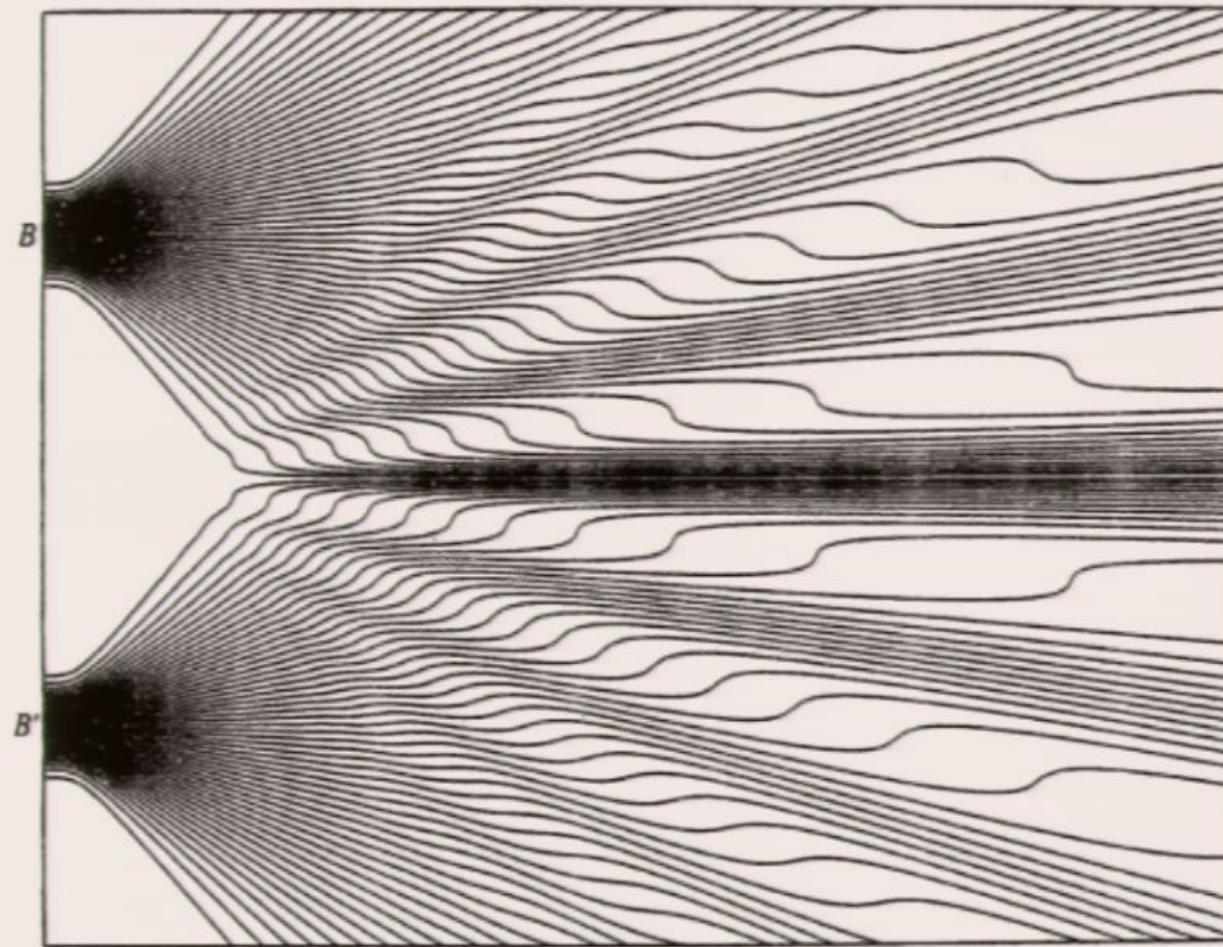
Double slit experiment



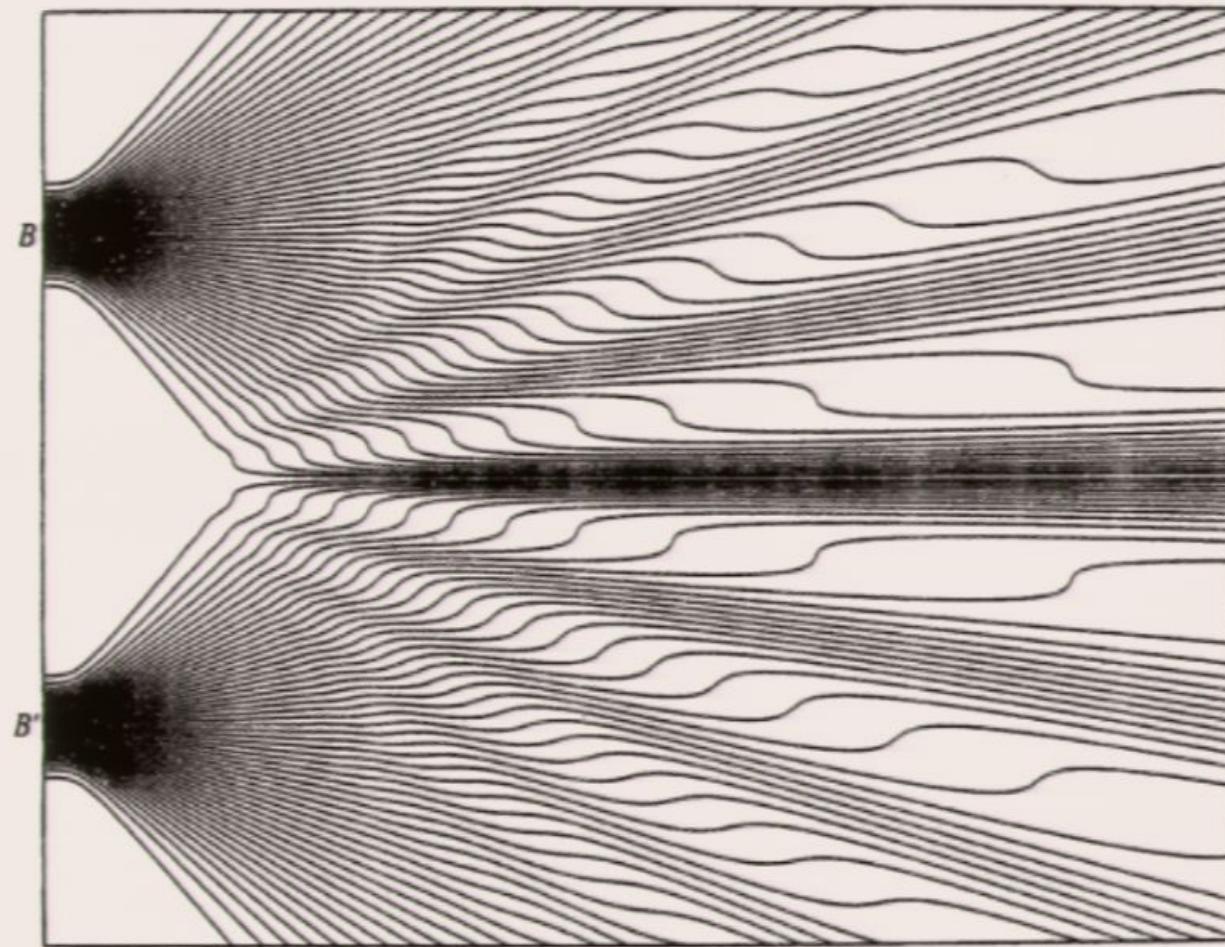
Double slit experiment



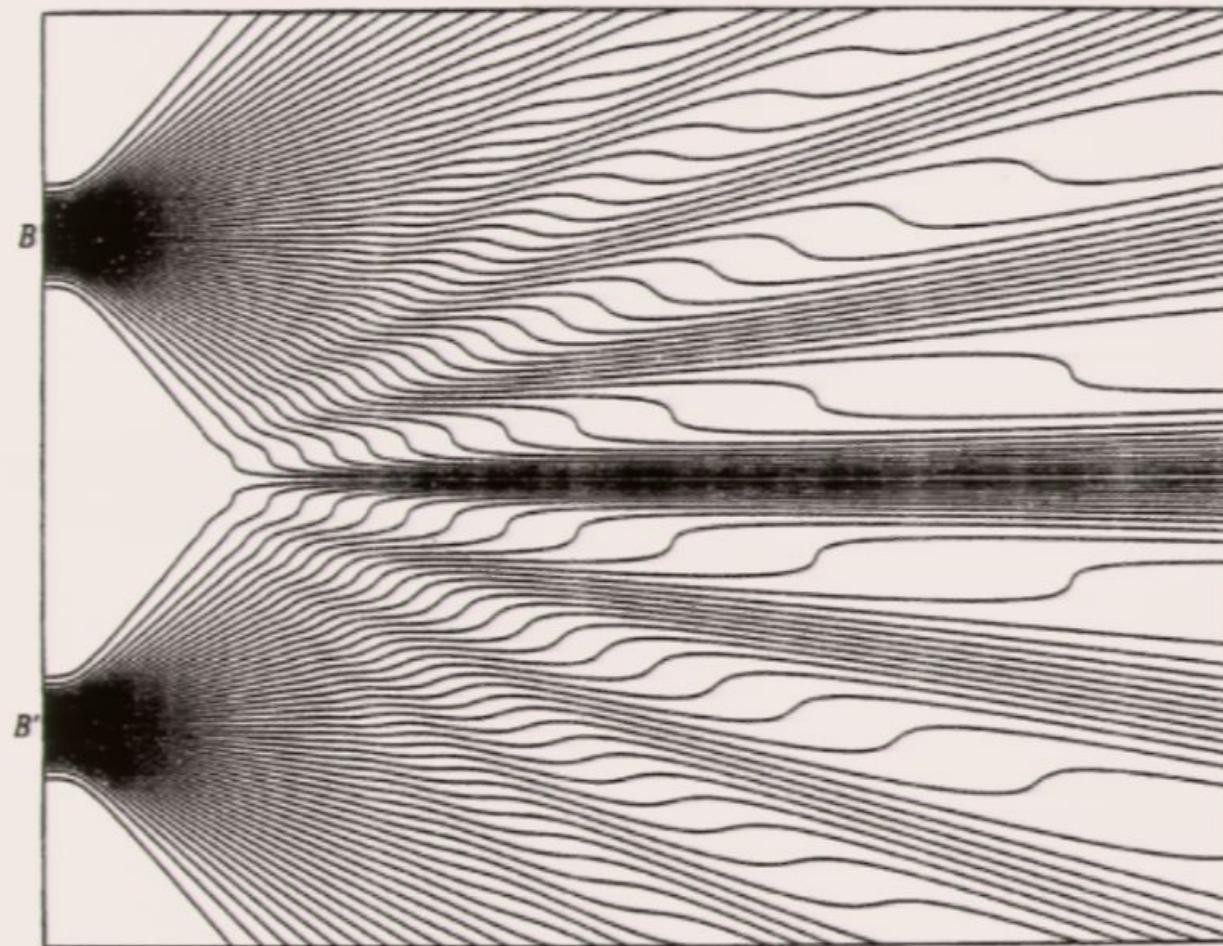
Double slit experiment



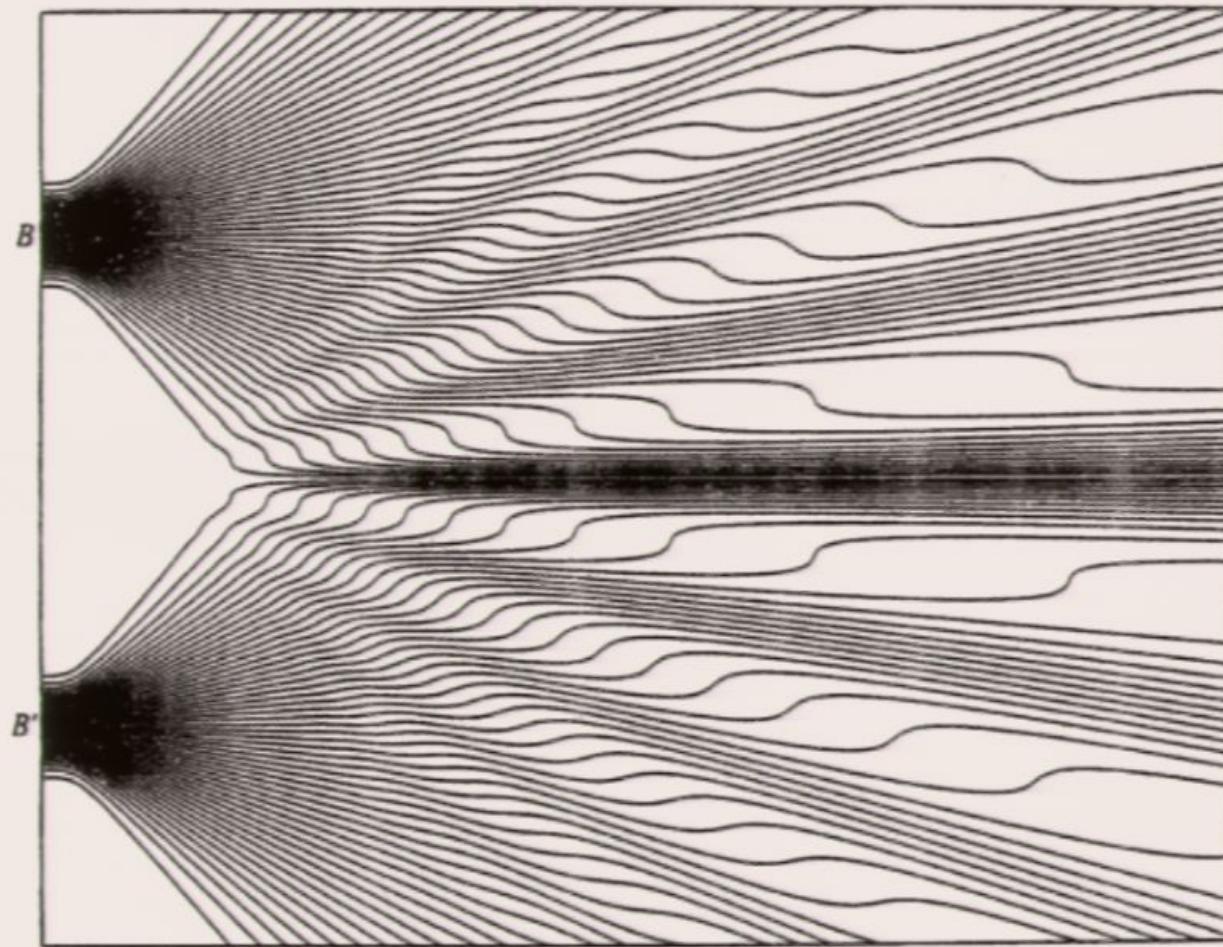
Double slit experiment



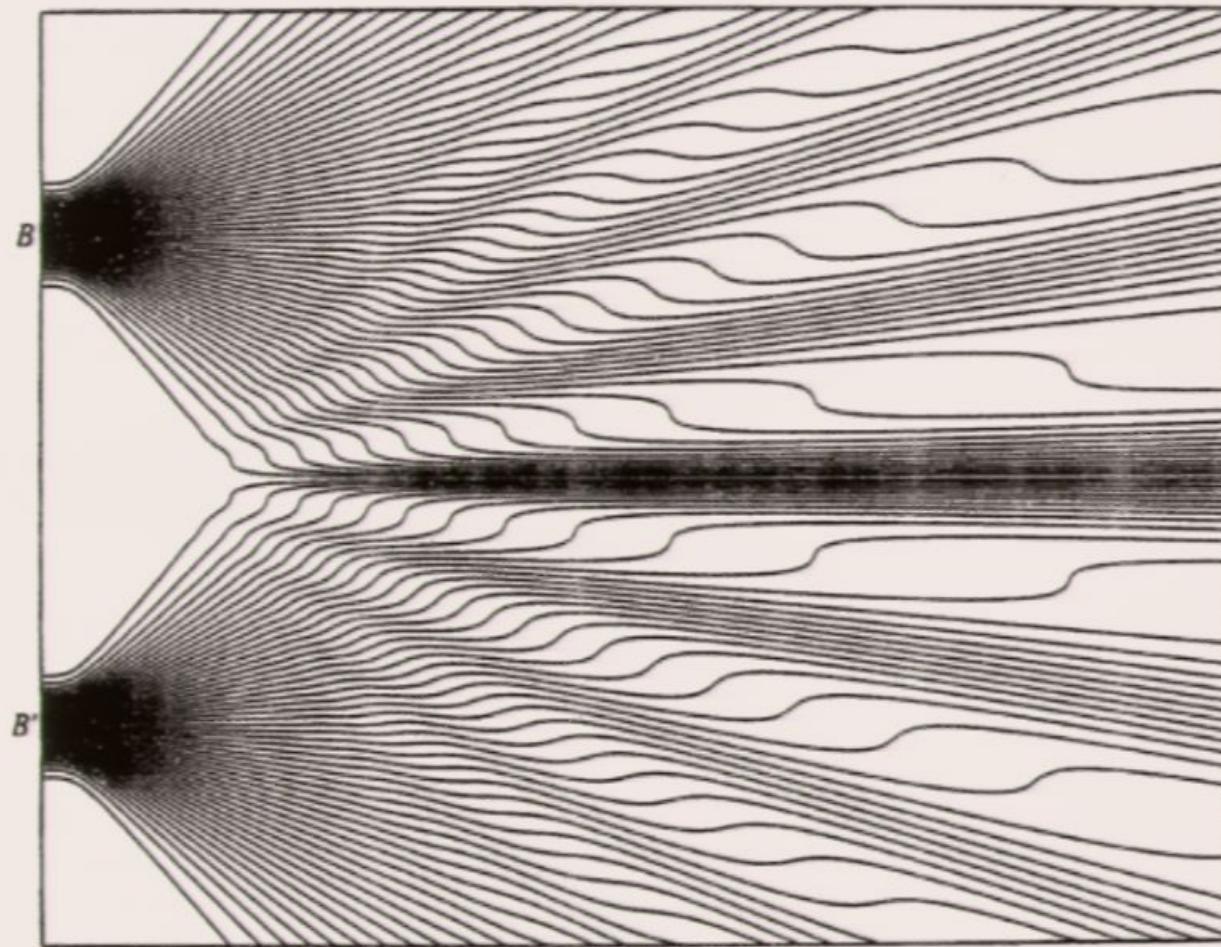
Double slit experiment



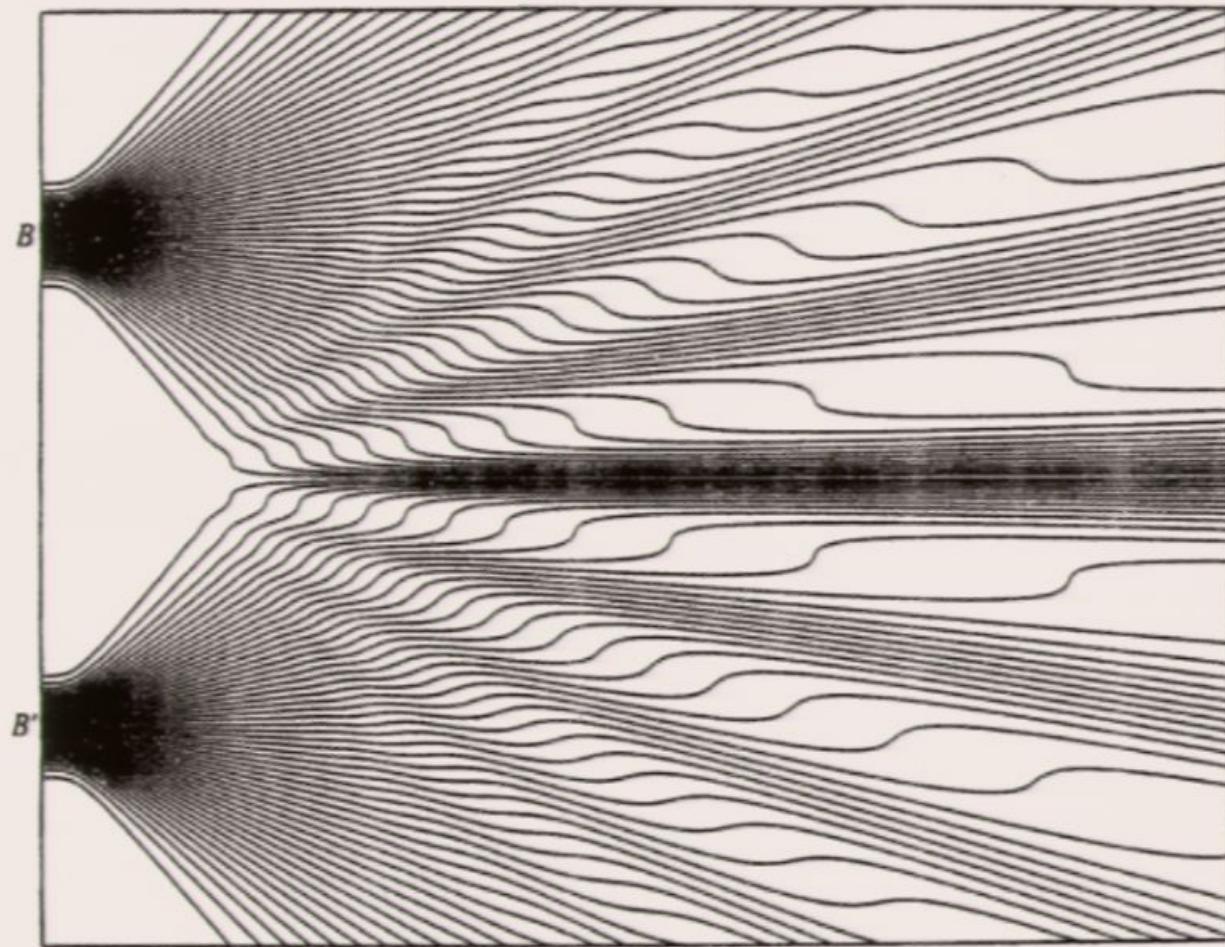
Double slit experiment



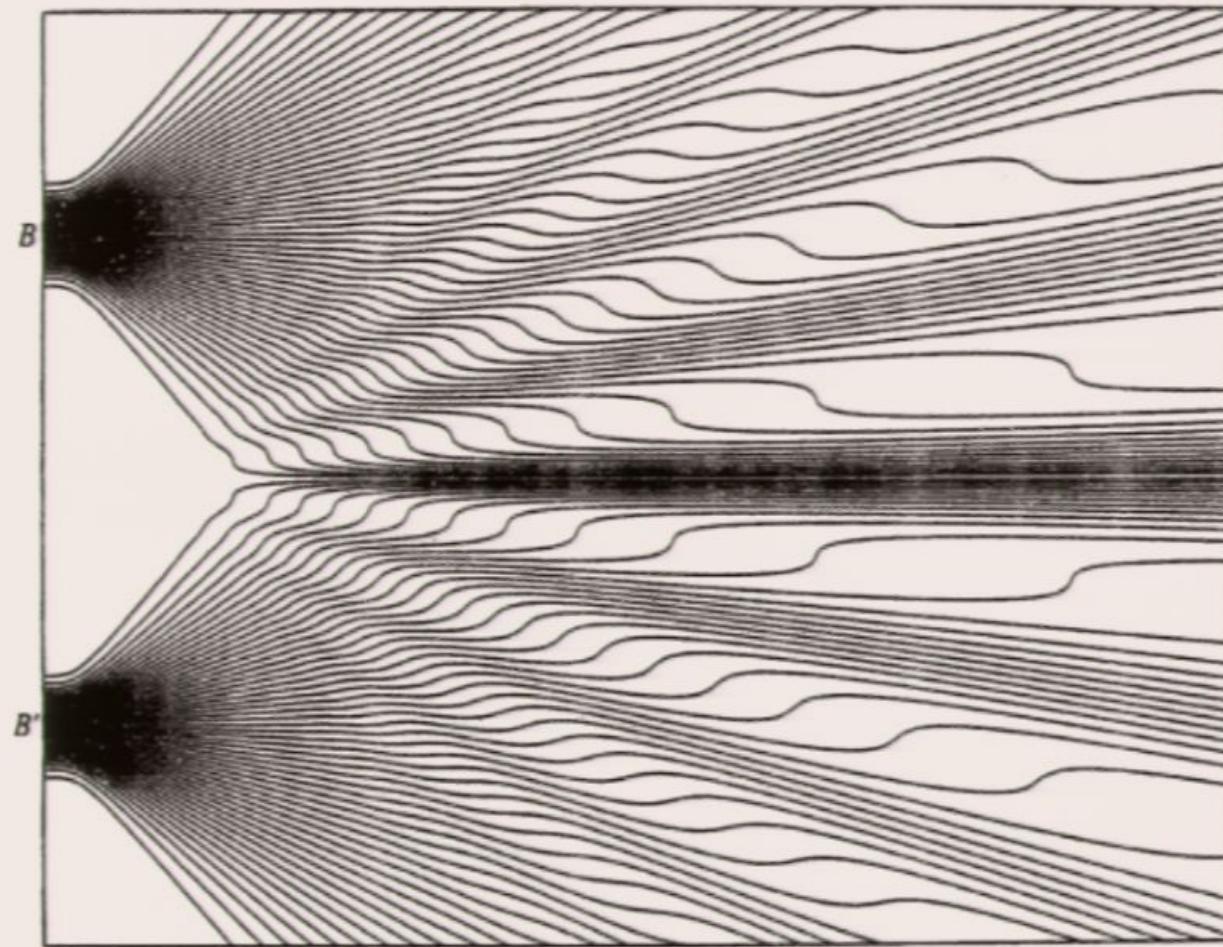
Double slit experiment



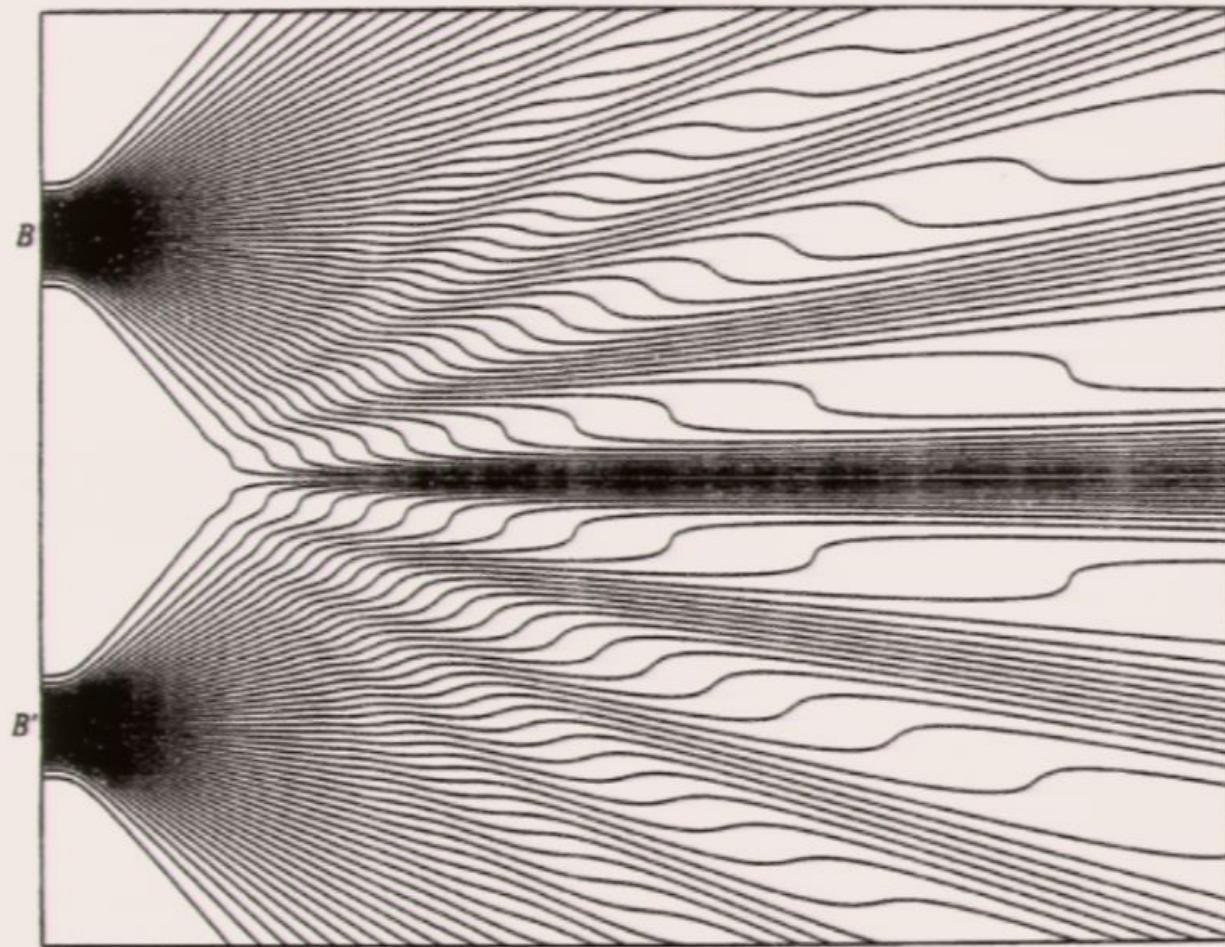
Double slit experiment



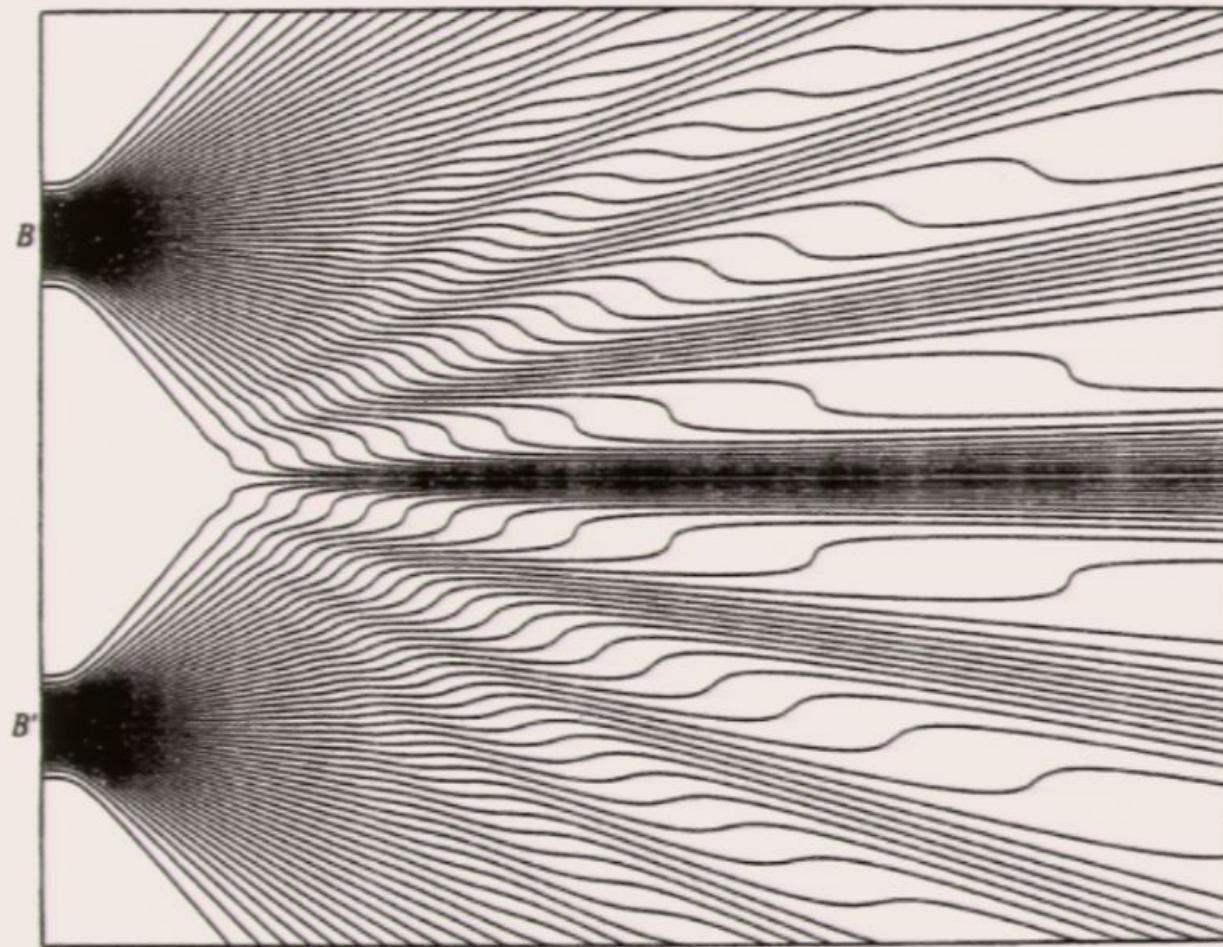
Double slit experiment



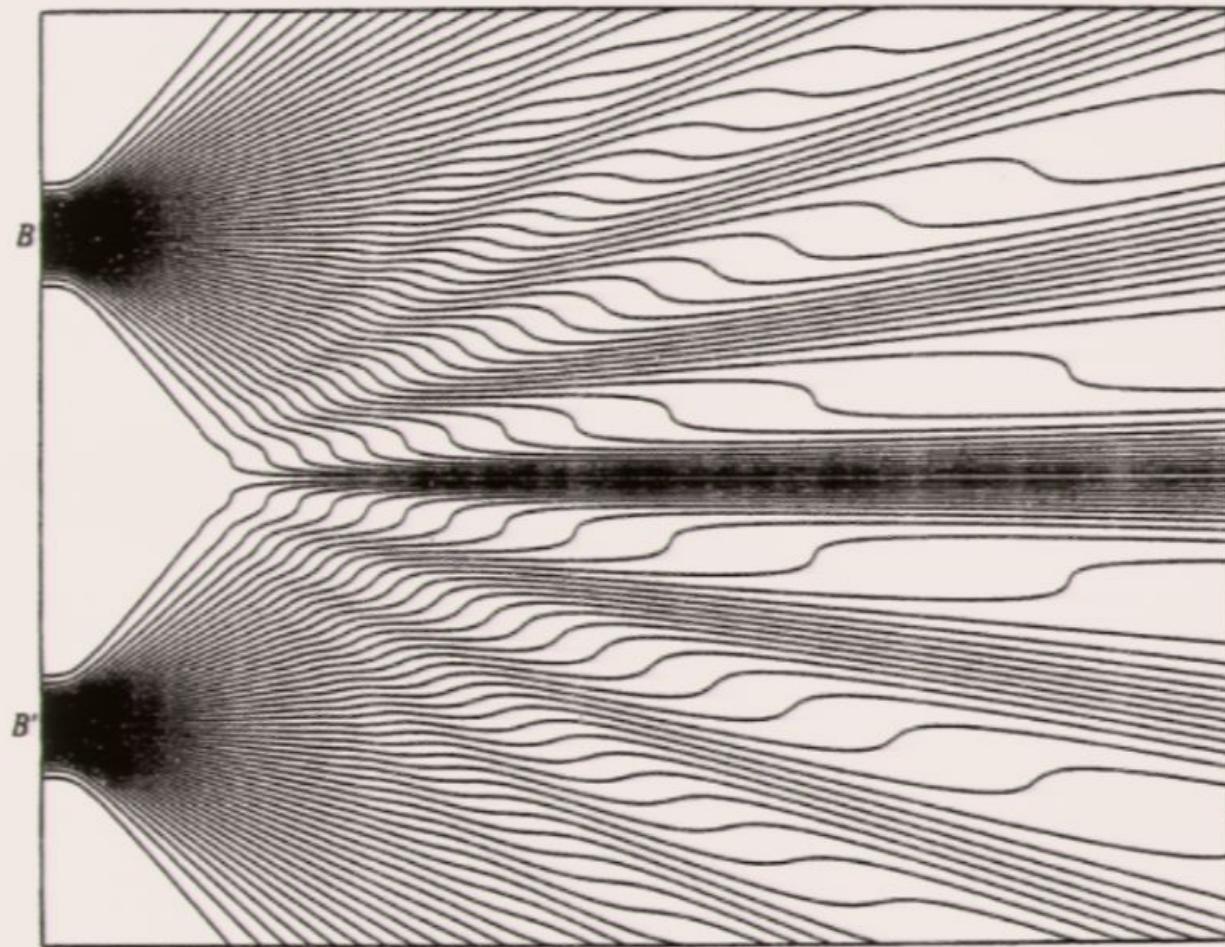
Double slit experiment



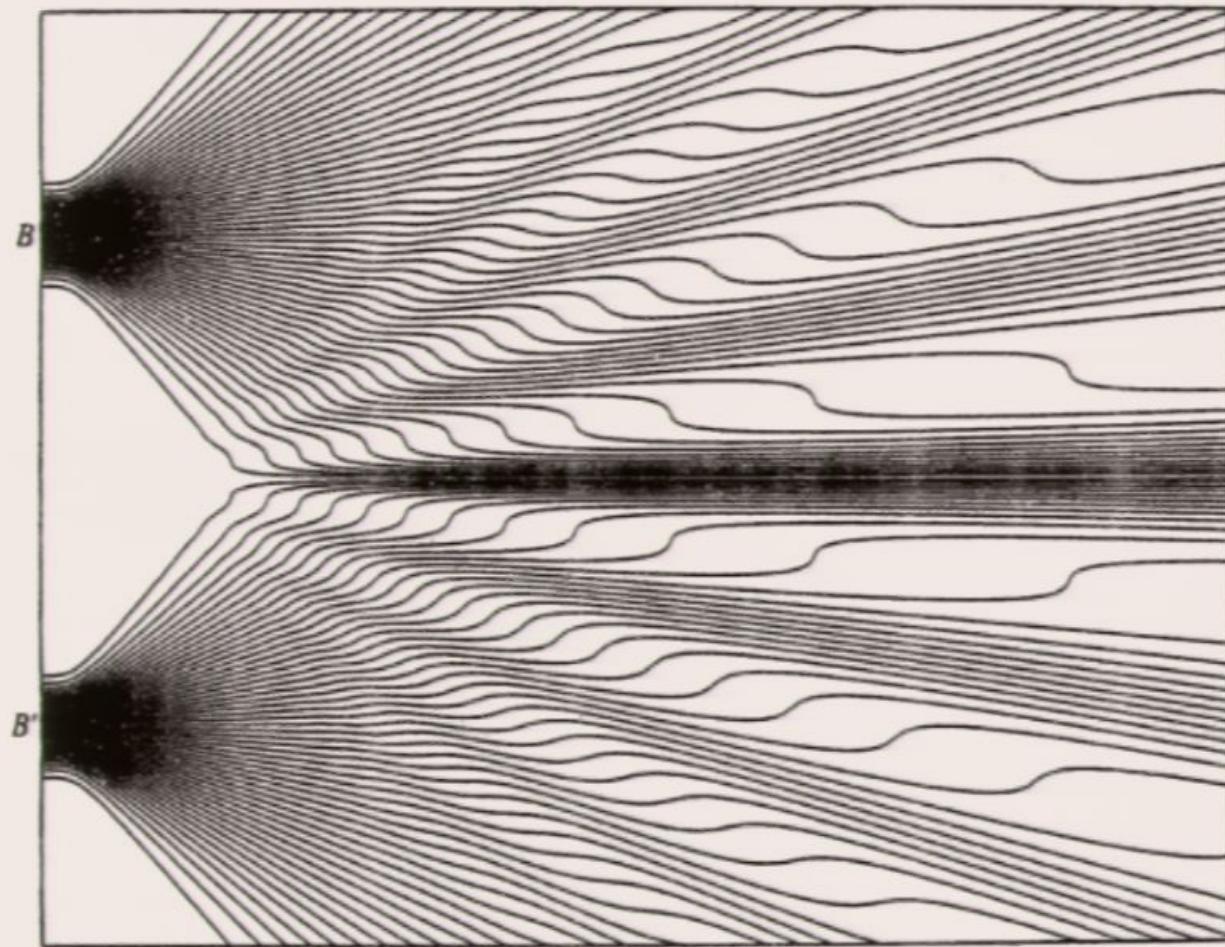
Double slit experiment



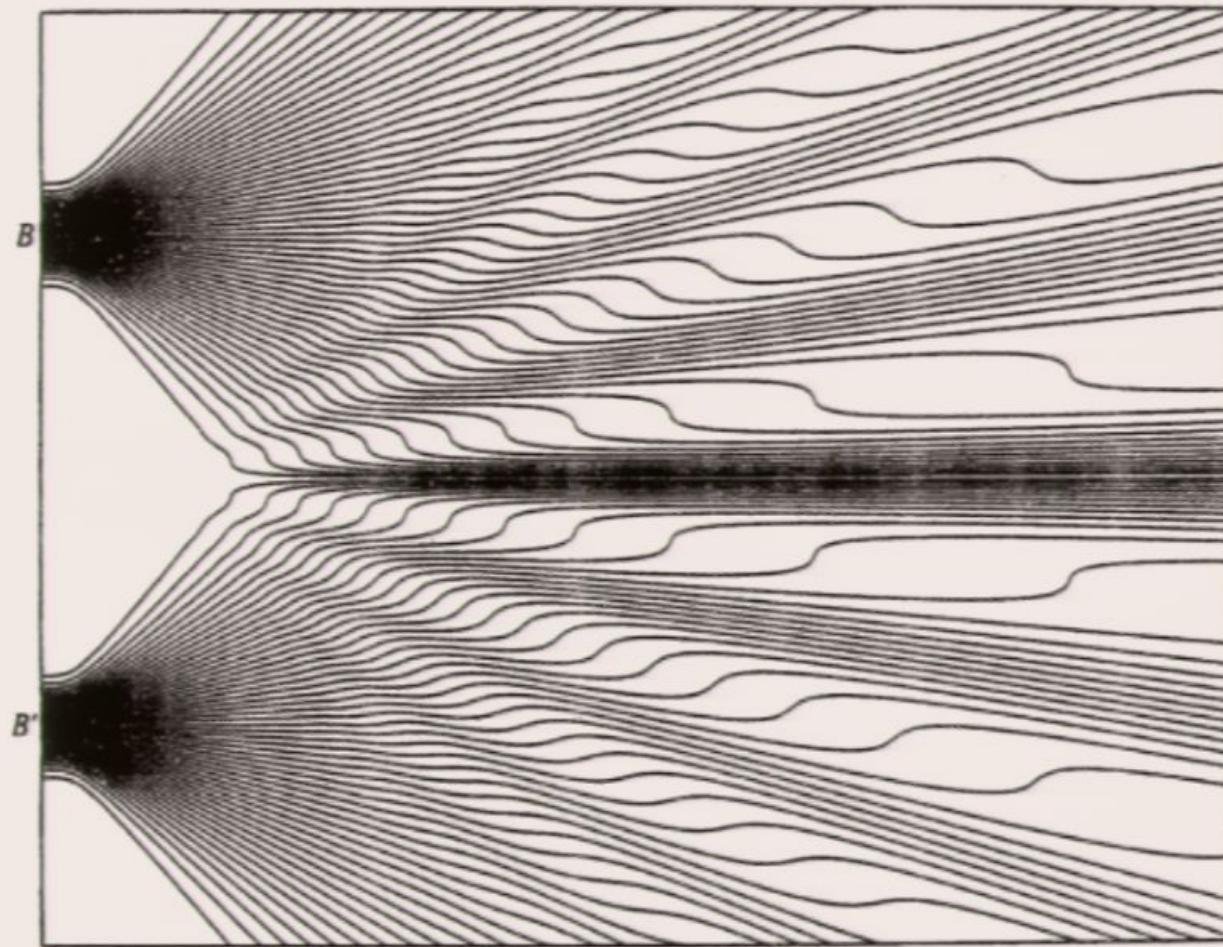
Double slit experiment



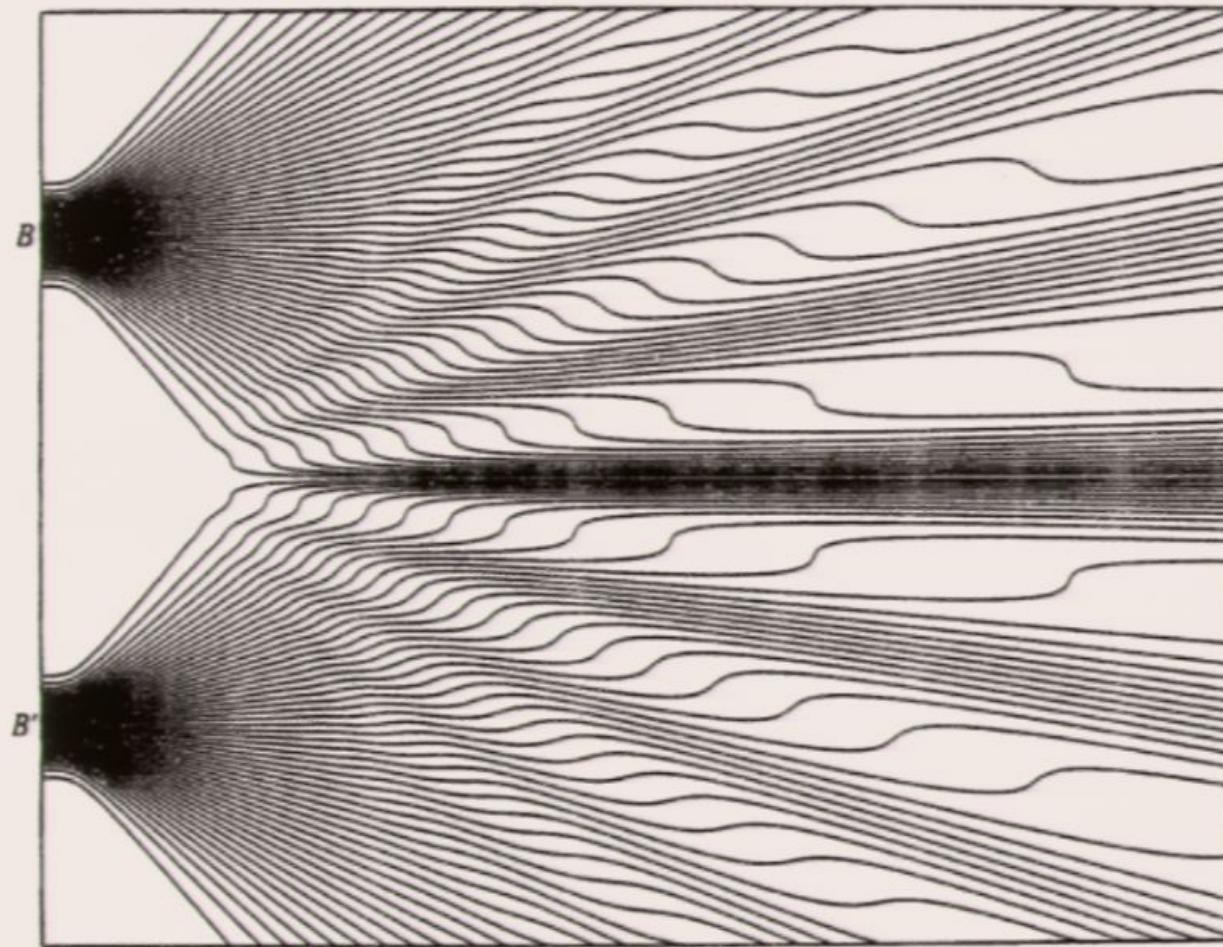
Double slit experiment



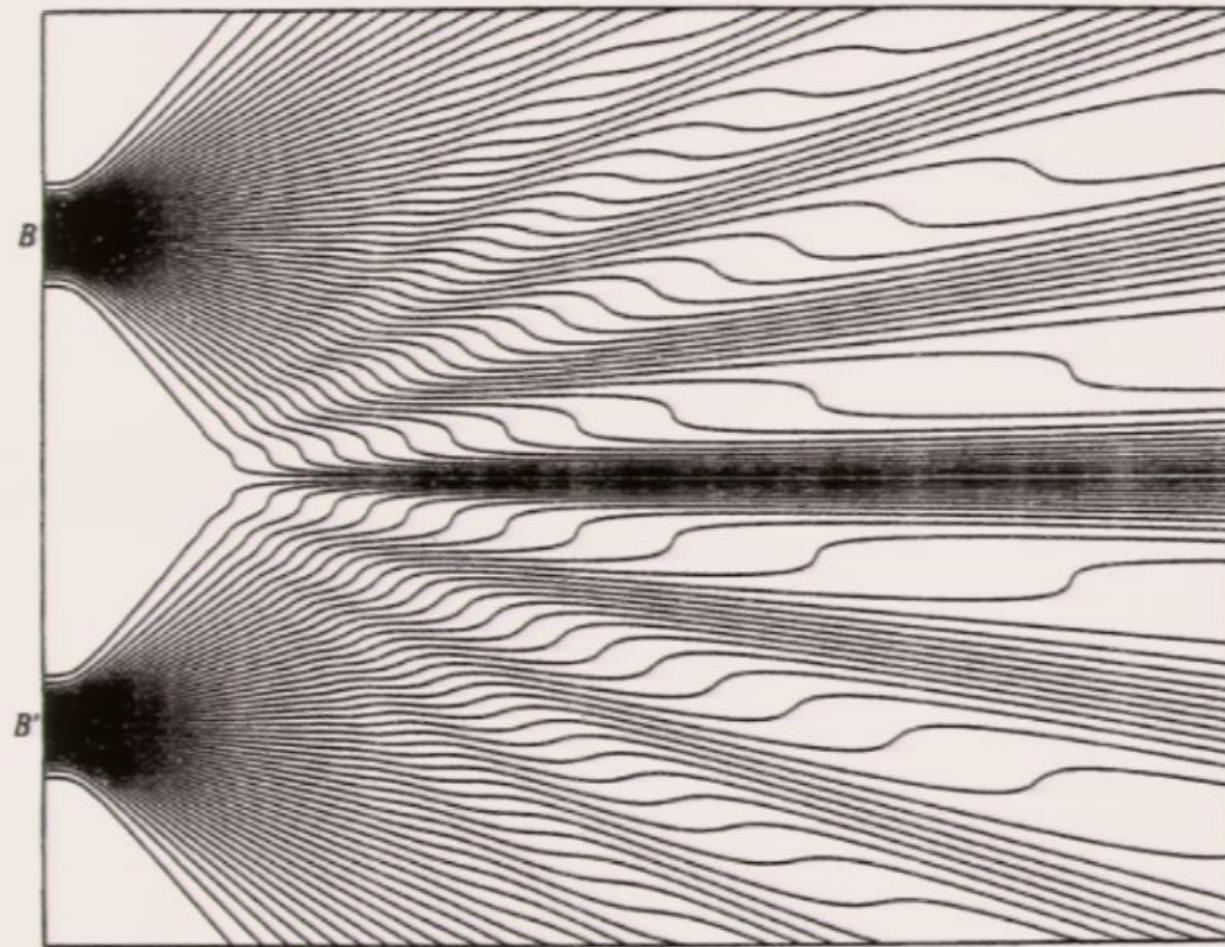
Double slit experiment



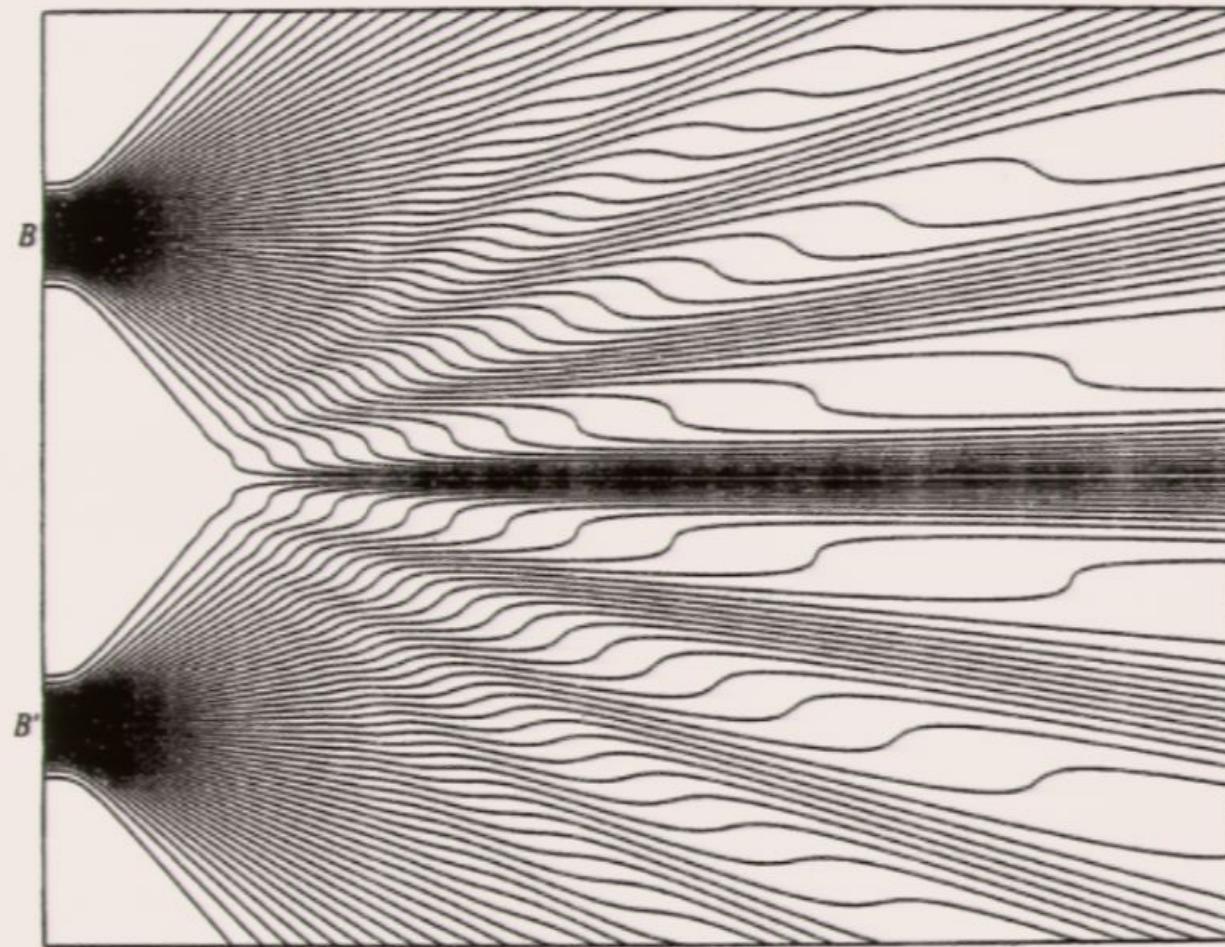
Double slit experiment



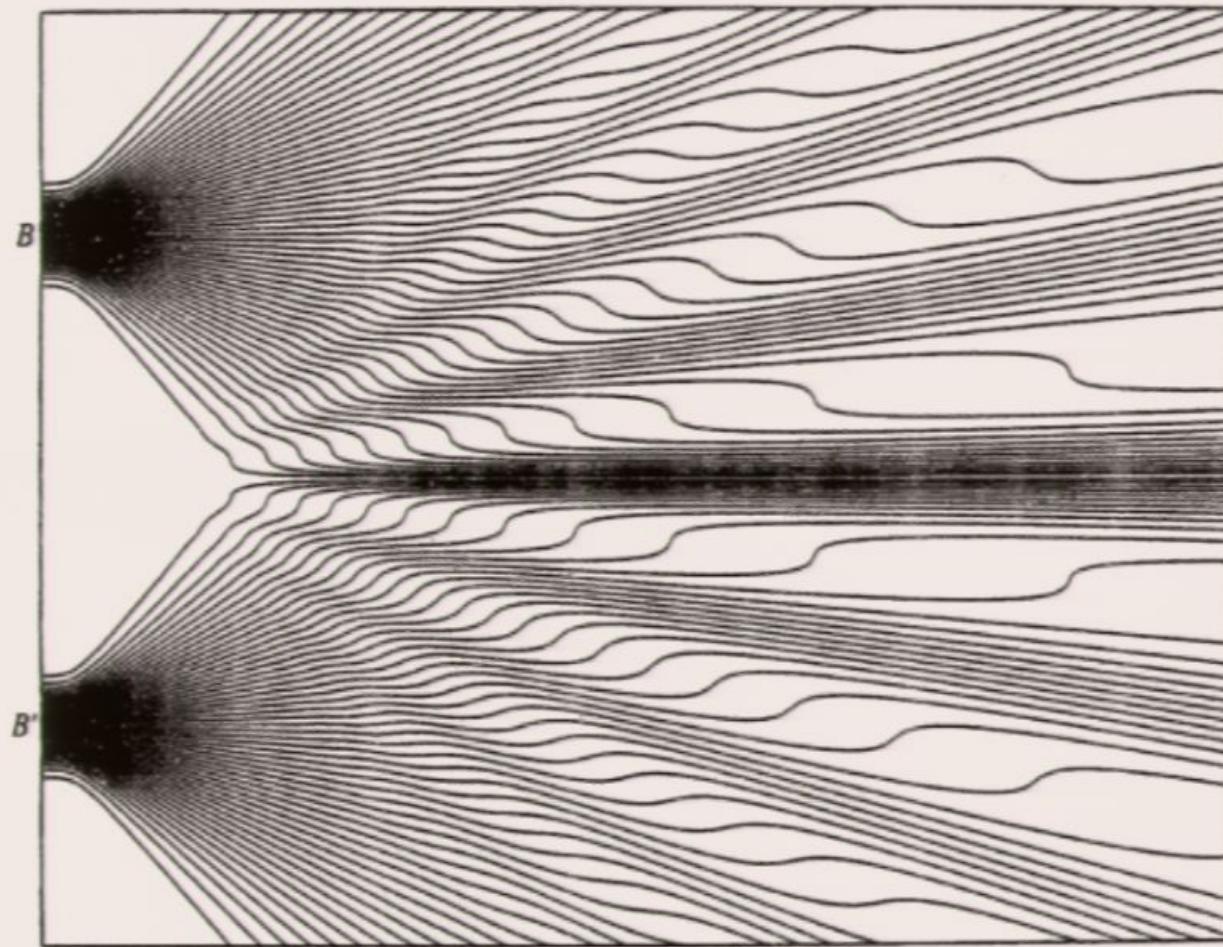
Double slit experiment



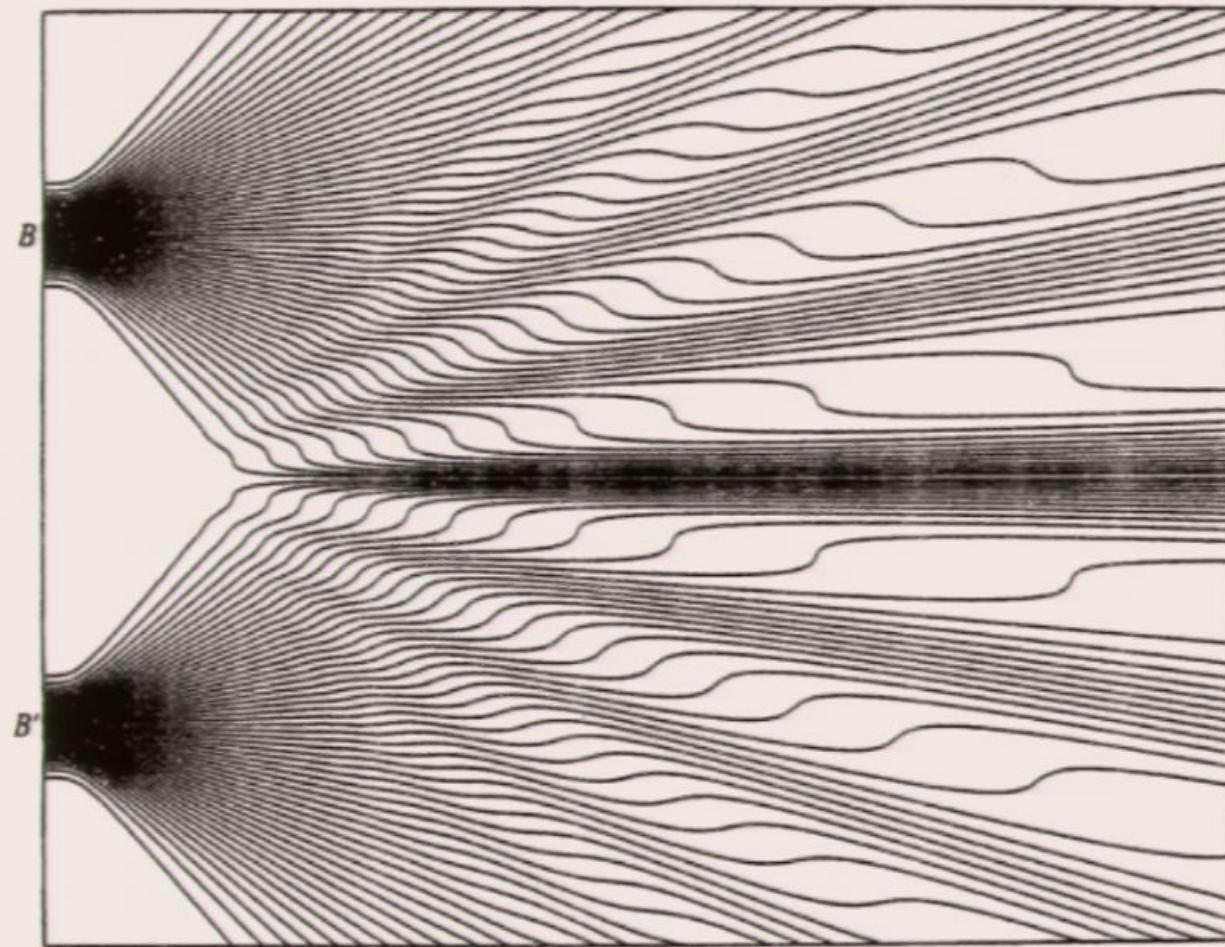
Double slit experiment



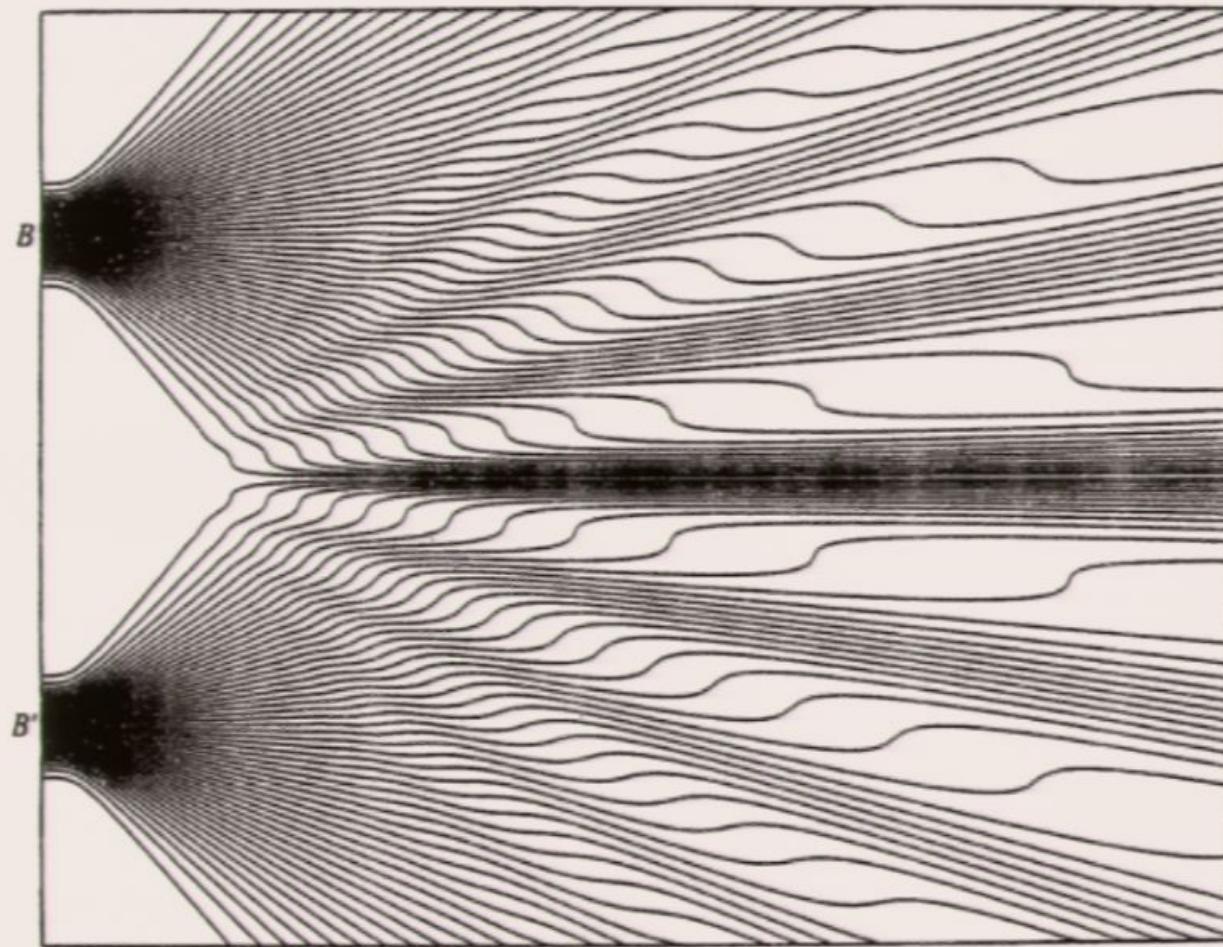
Double slit experiment



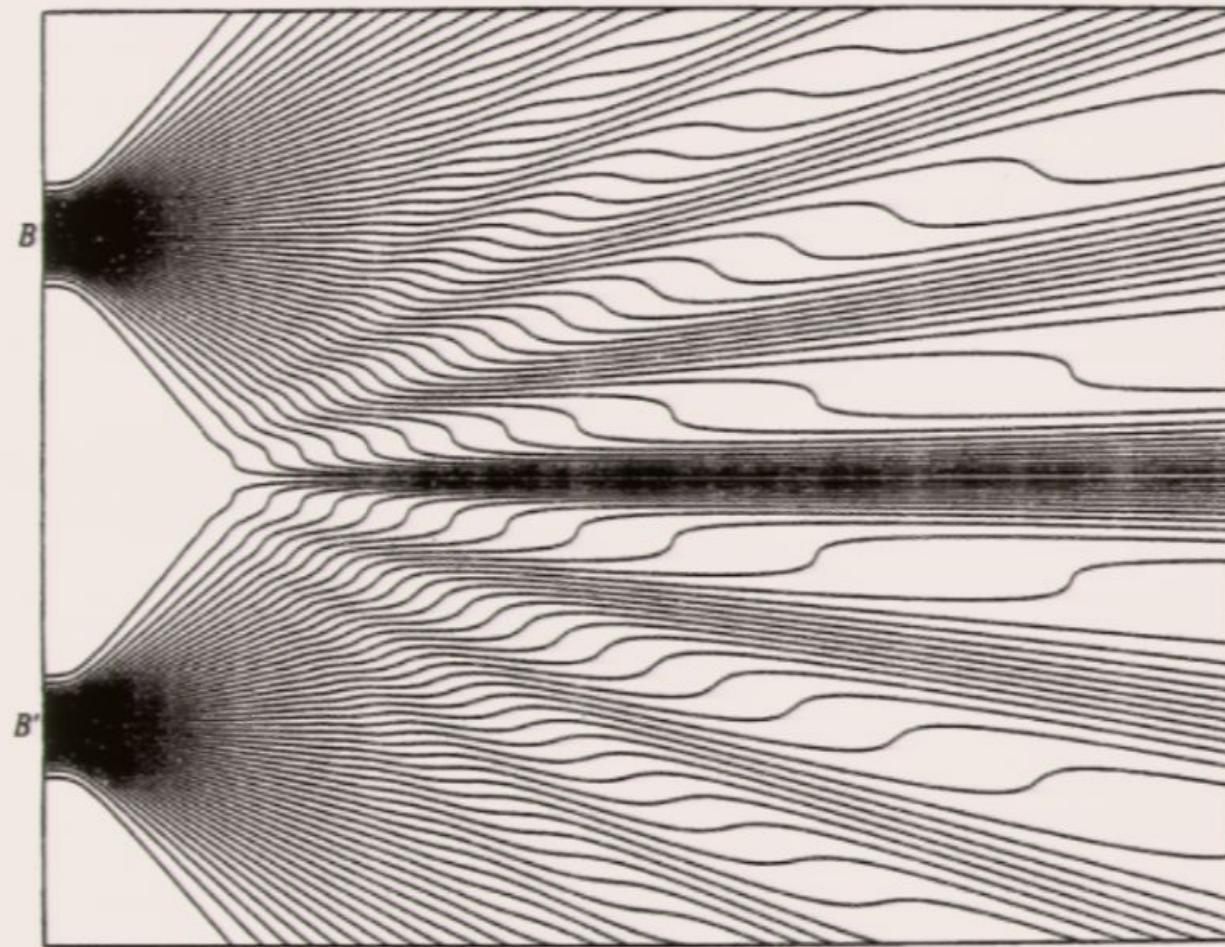
Double slit experiment



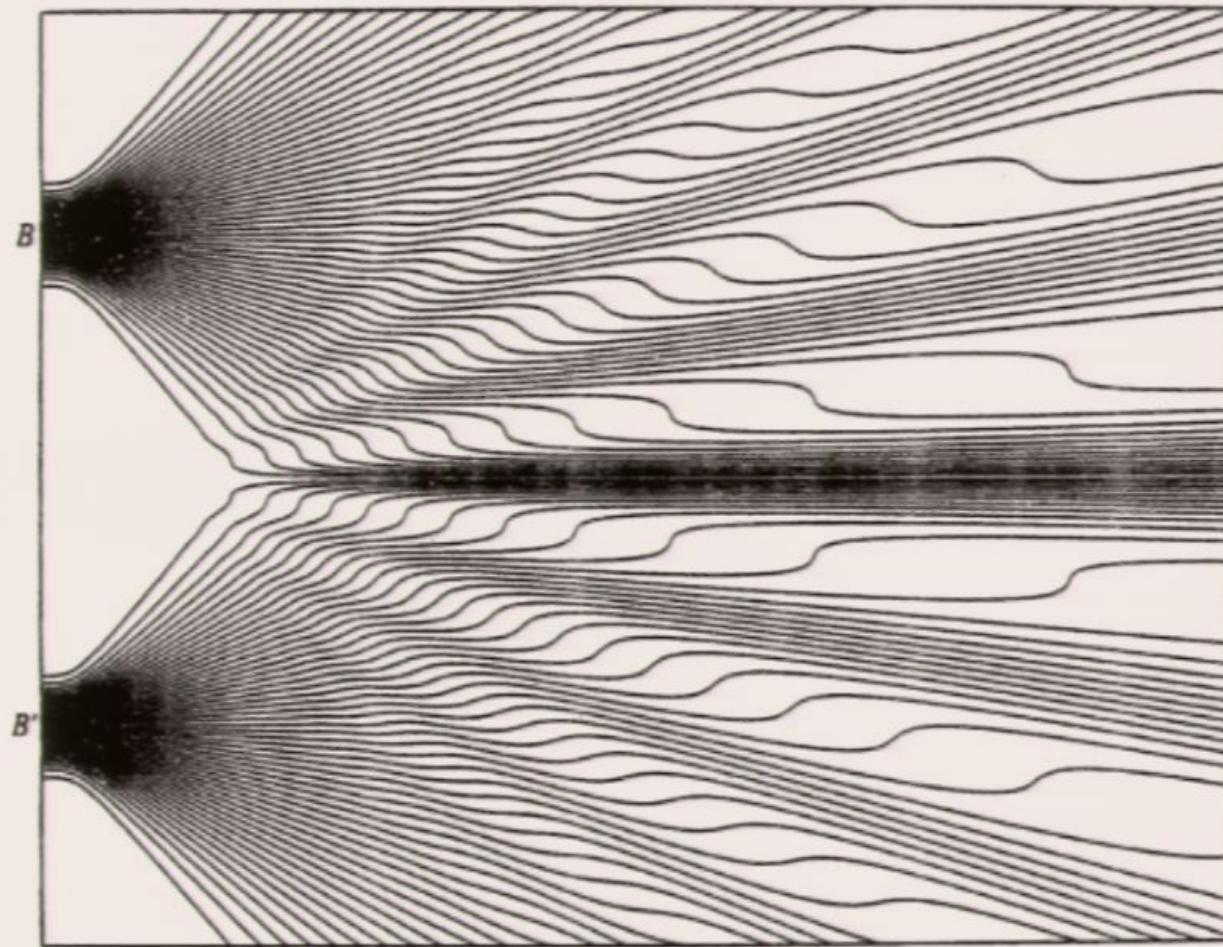
Double slit experiment



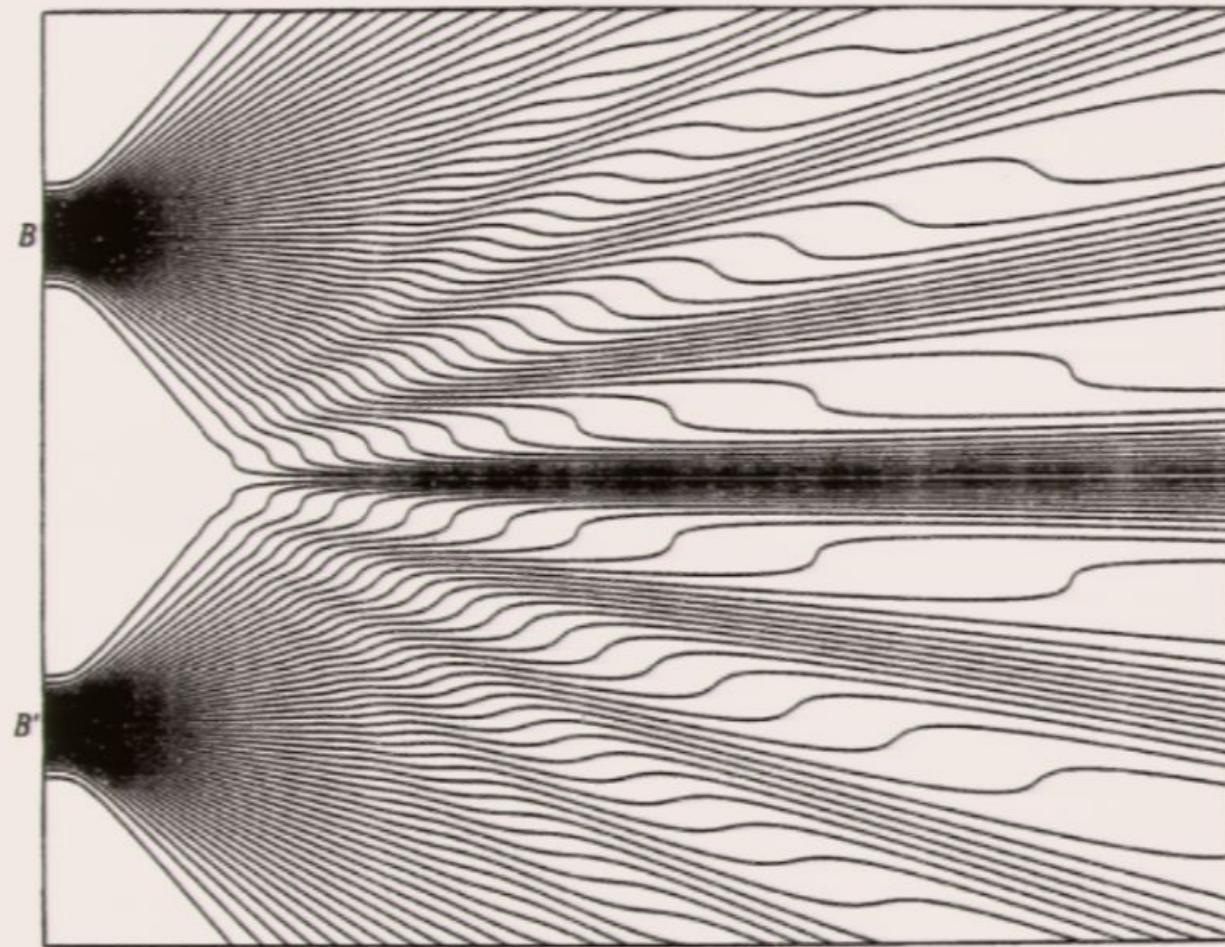
Double slit experiment



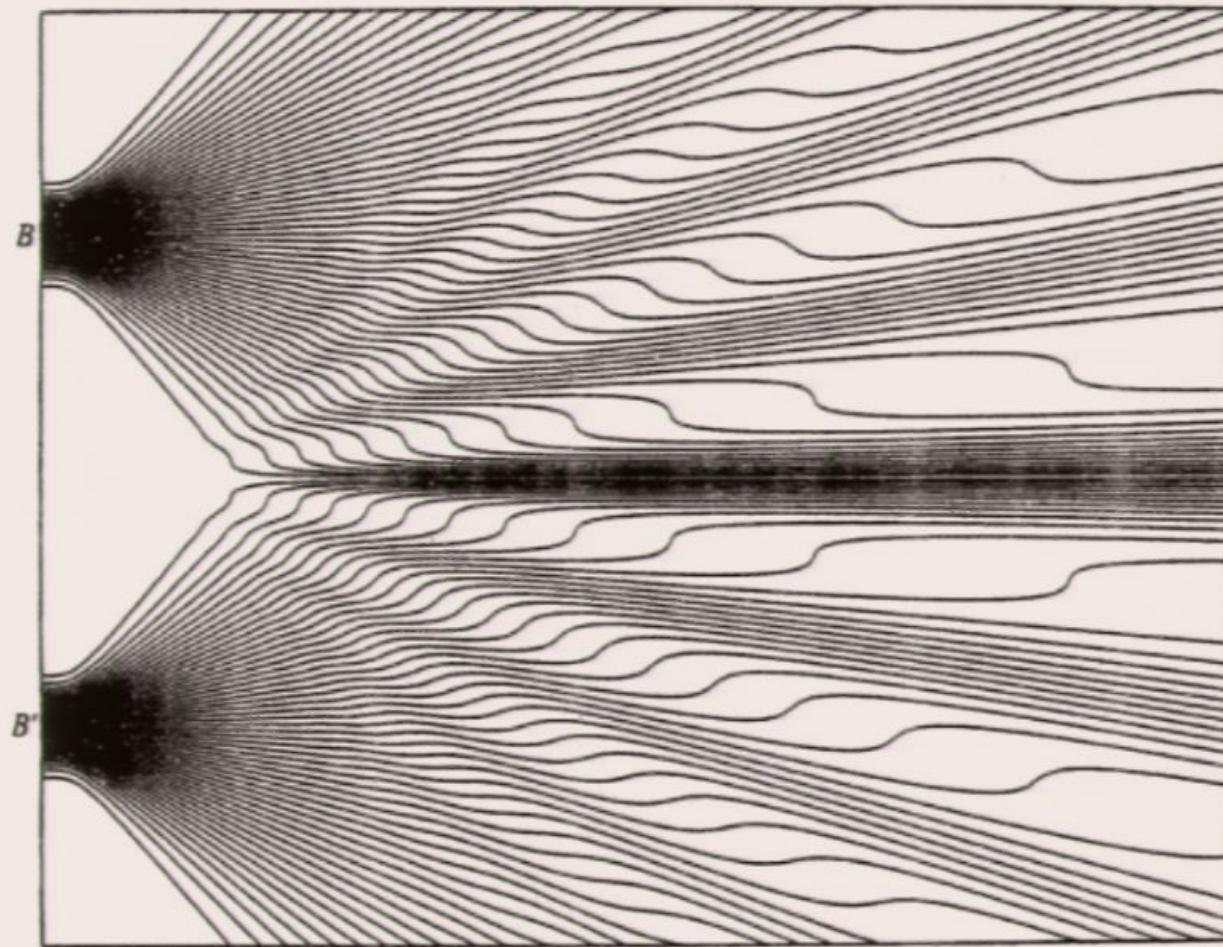
Double slit experiment



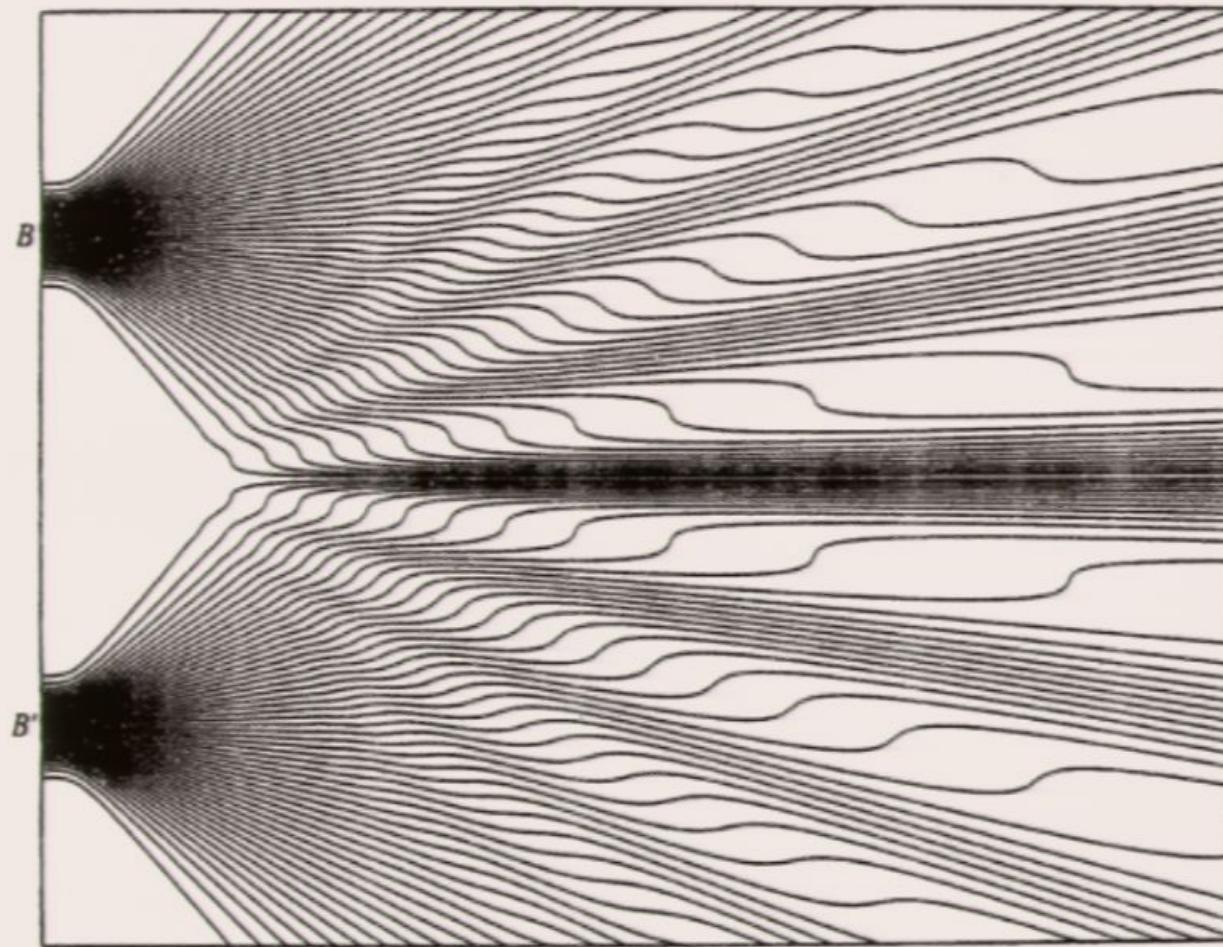
Double slit experiment



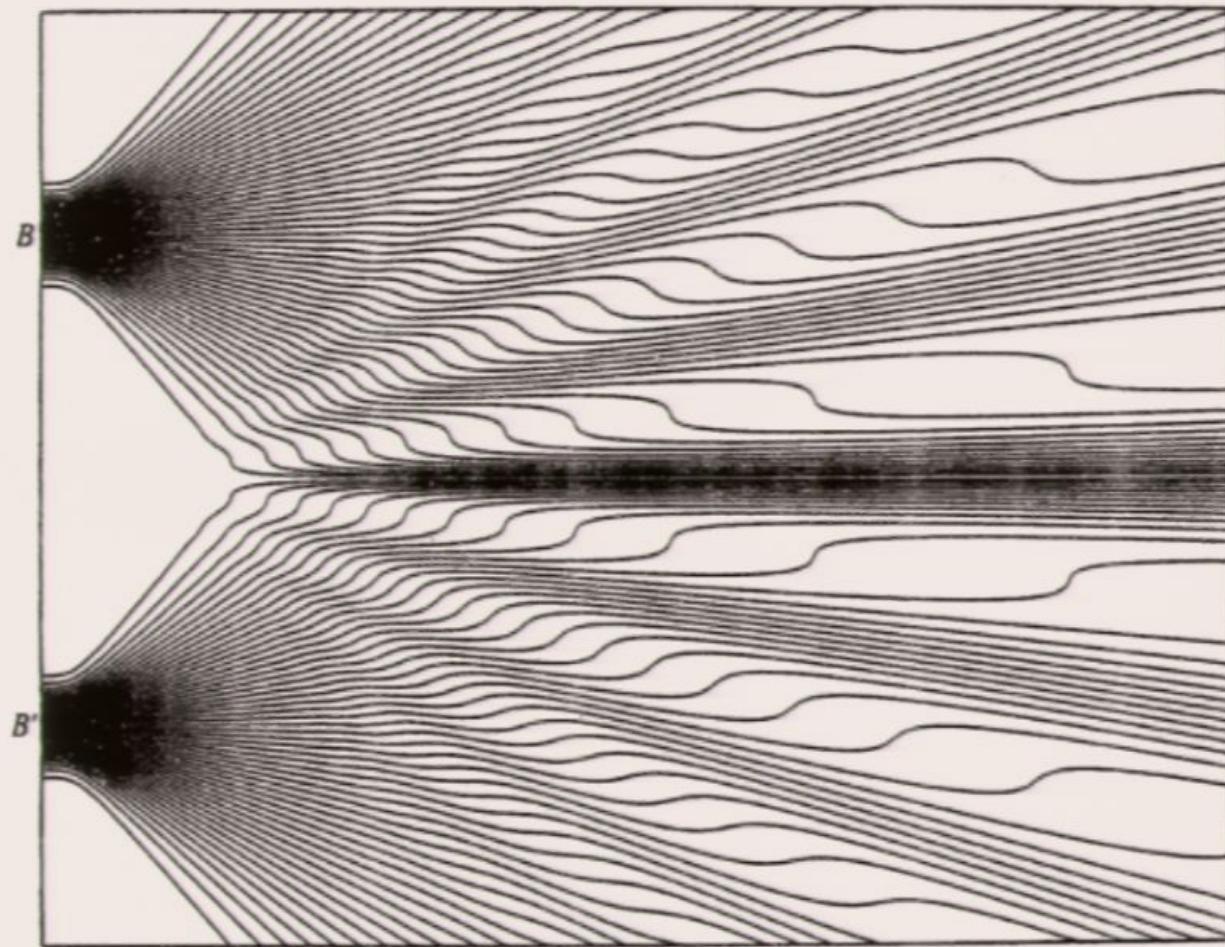
Double slit experiment



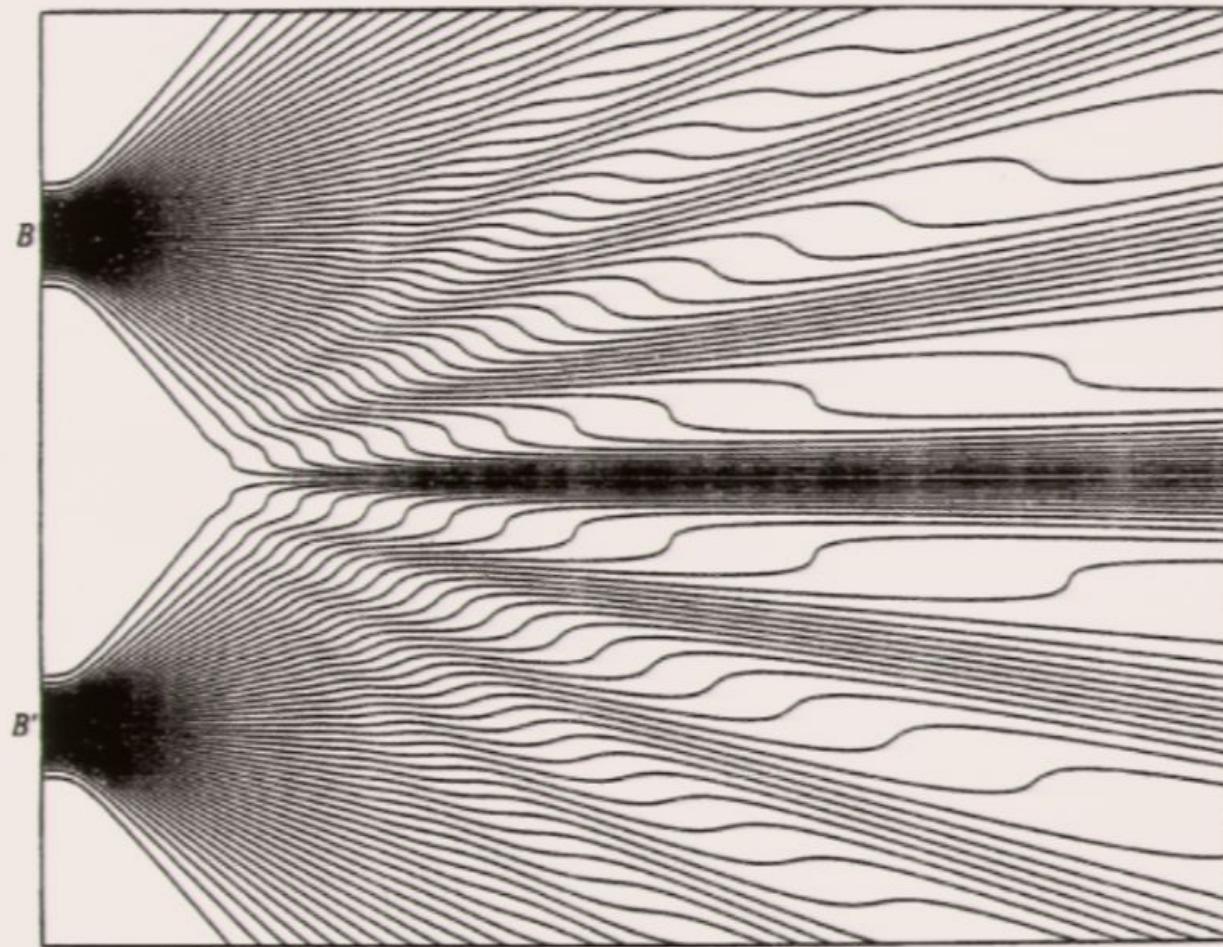
Double slit experiment



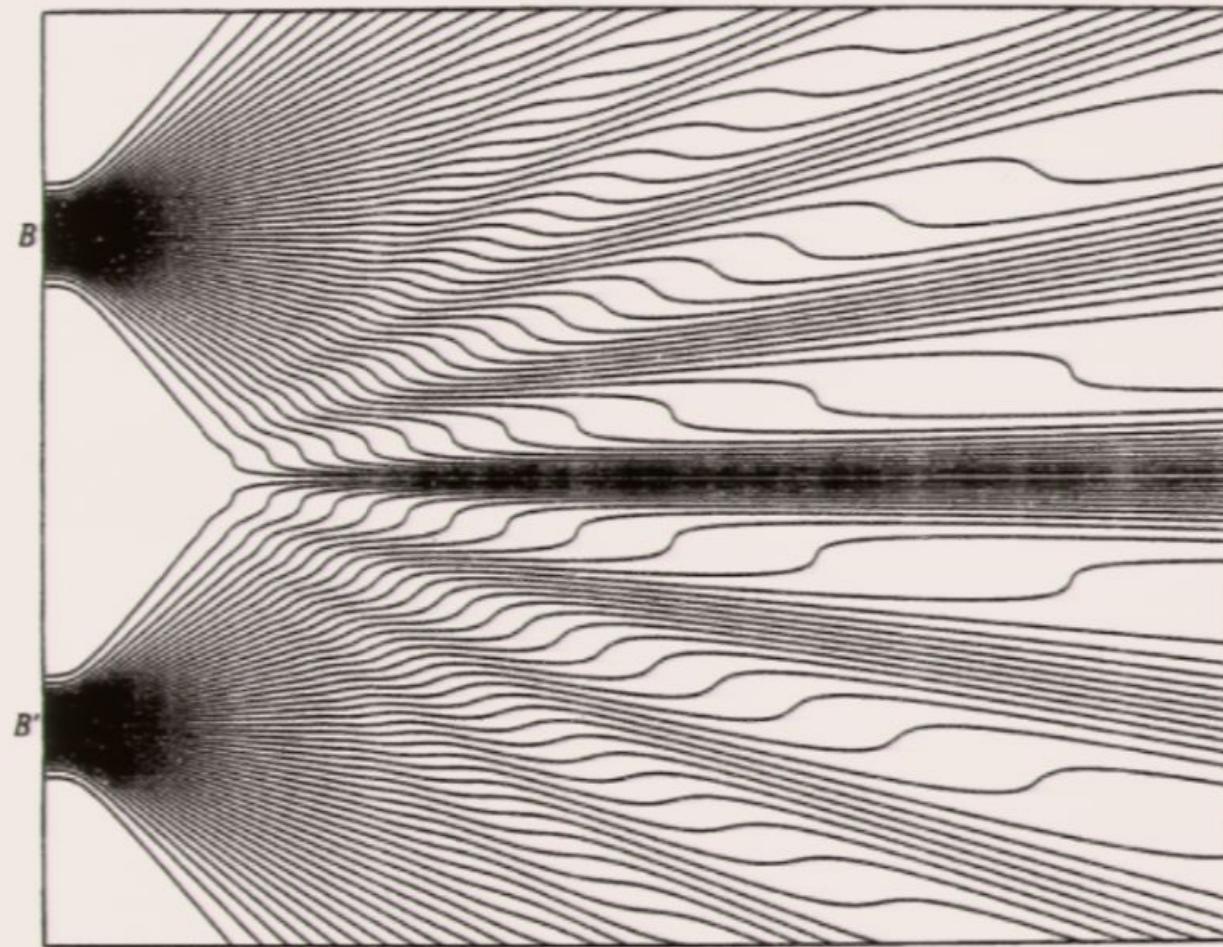
Double slit experiment



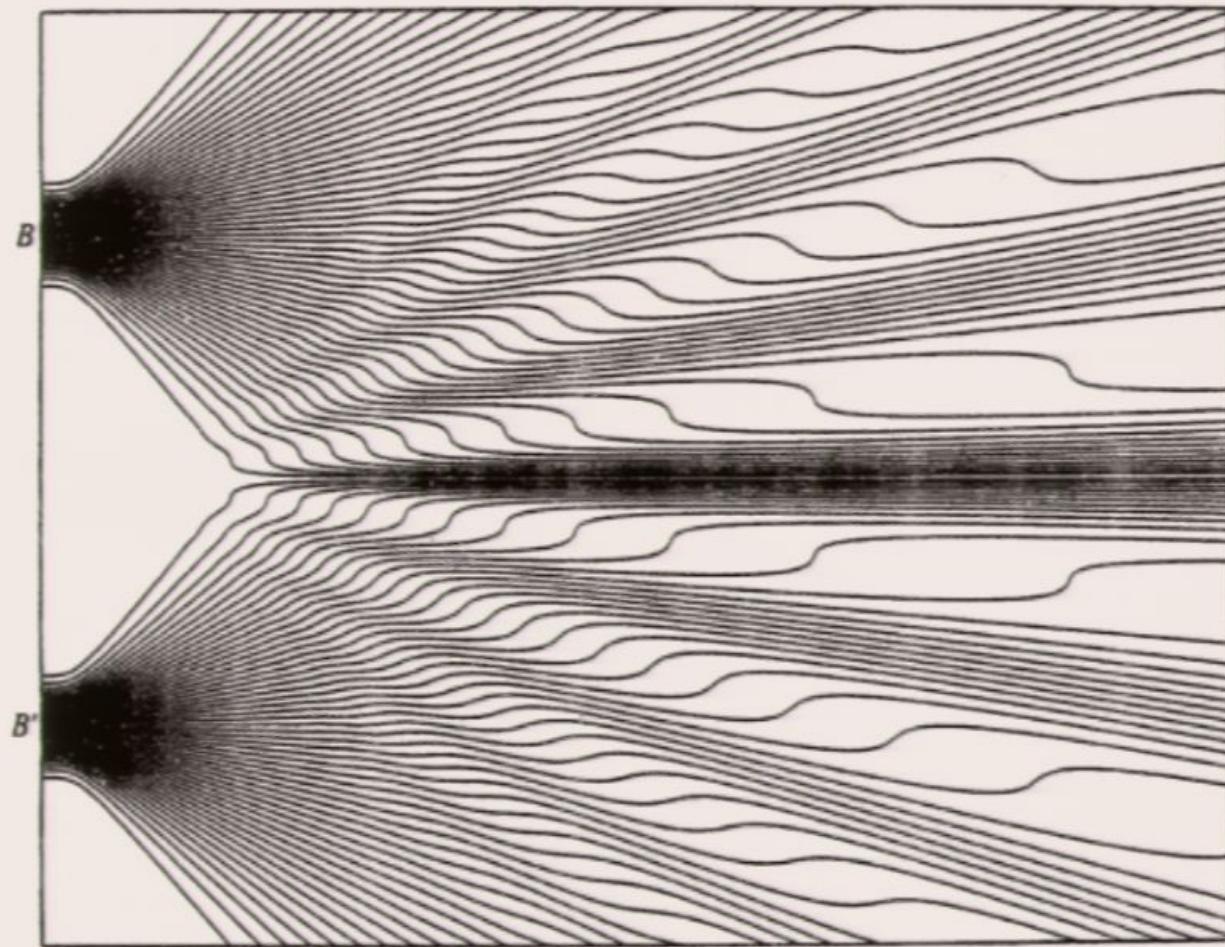
Double slit experiment



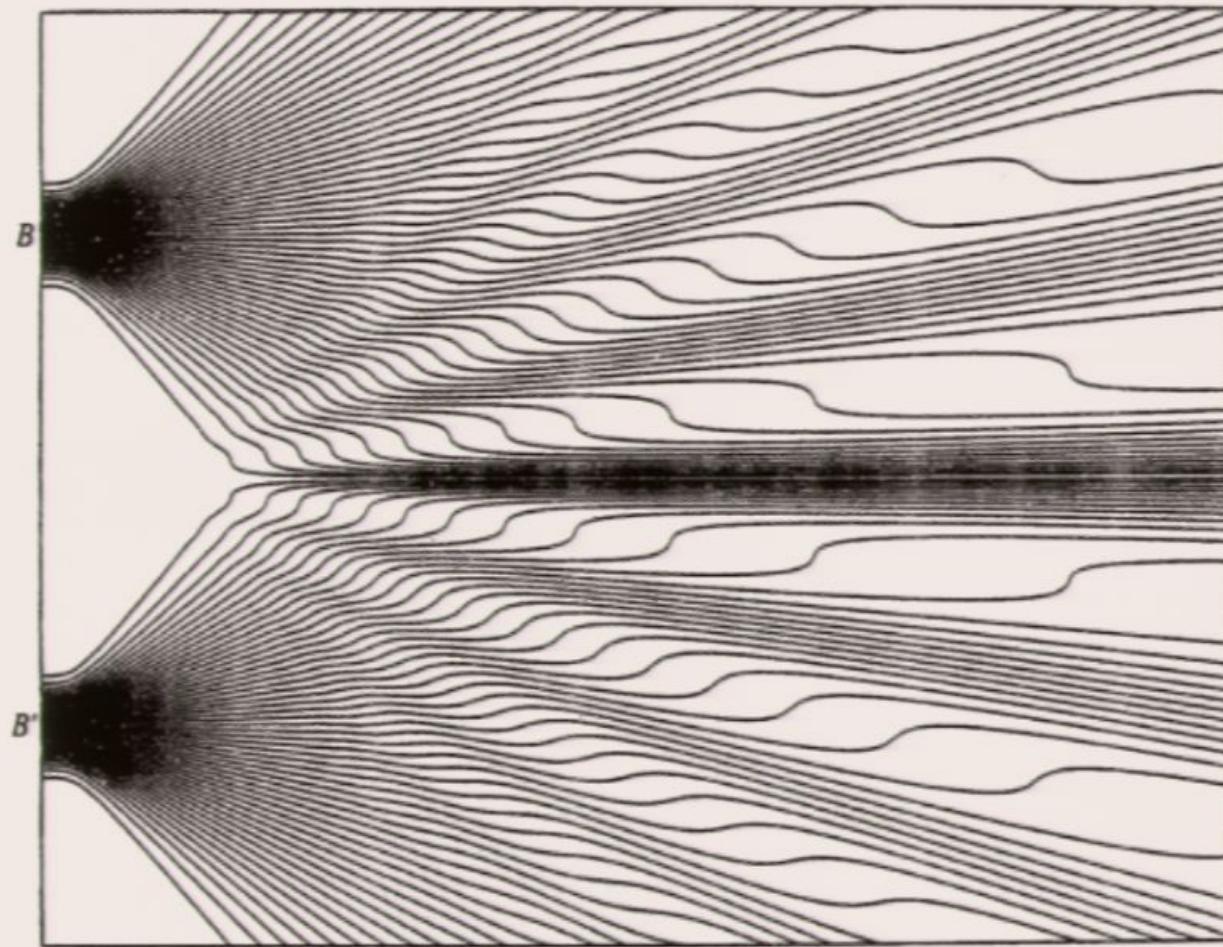
Double slit experiment



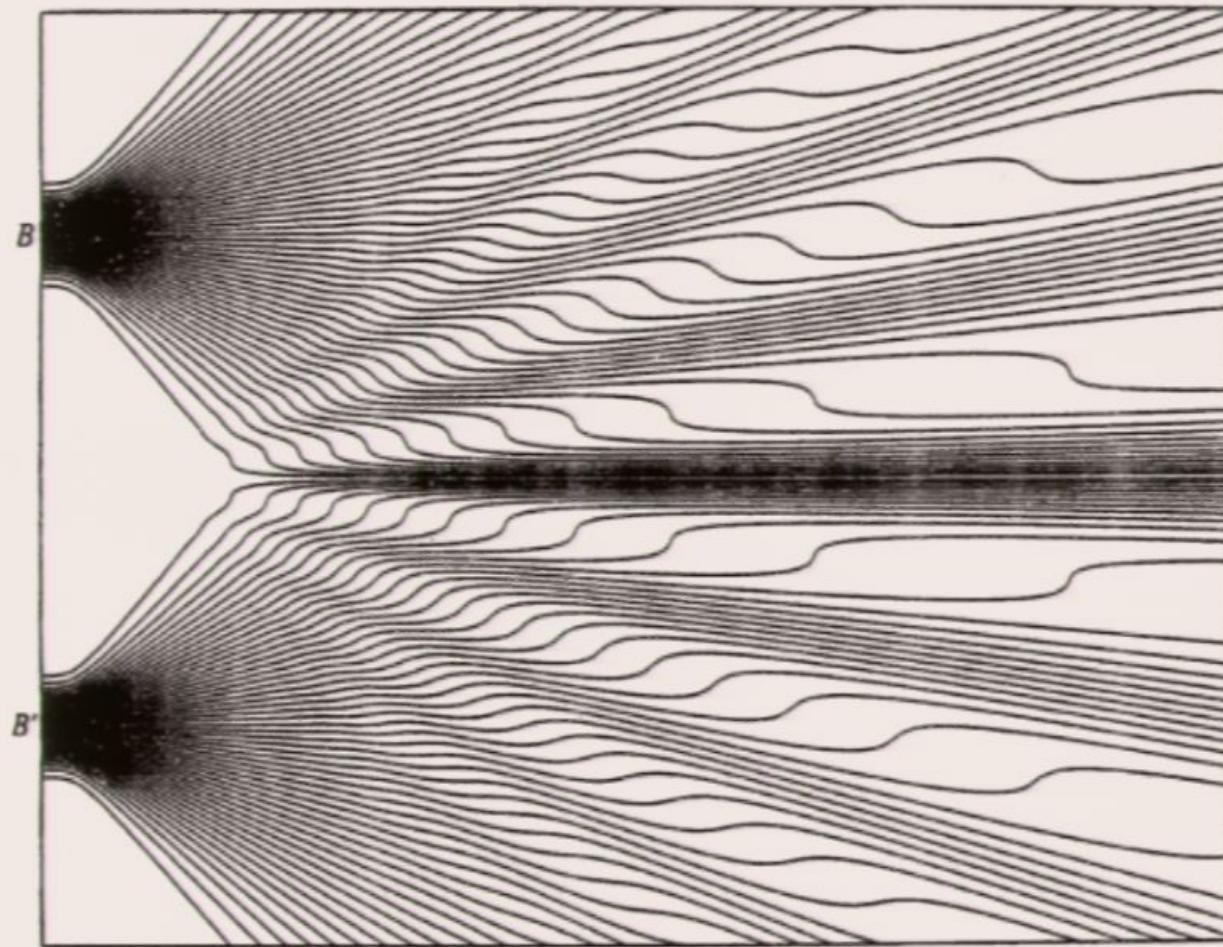
Double slit experiment



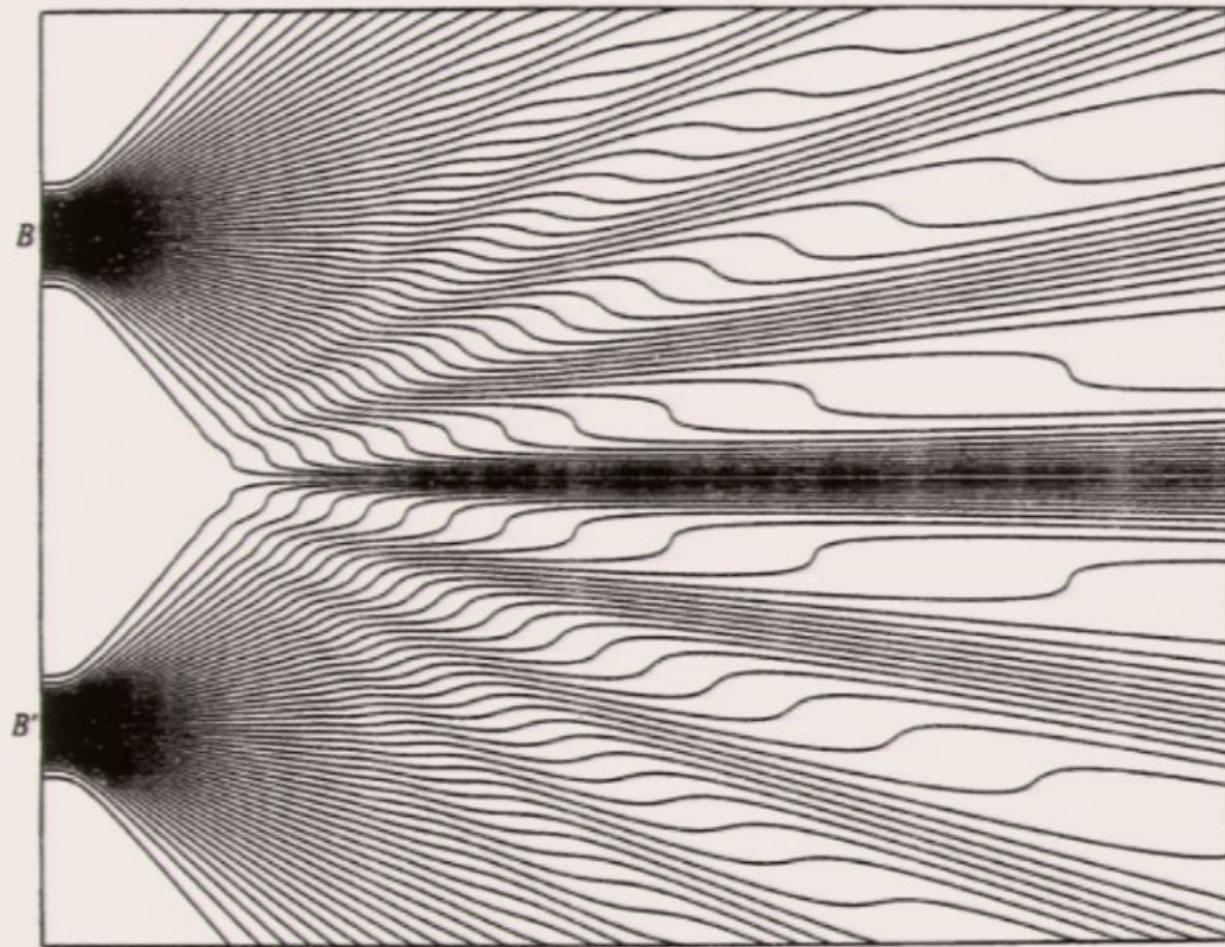
Double slit experiment



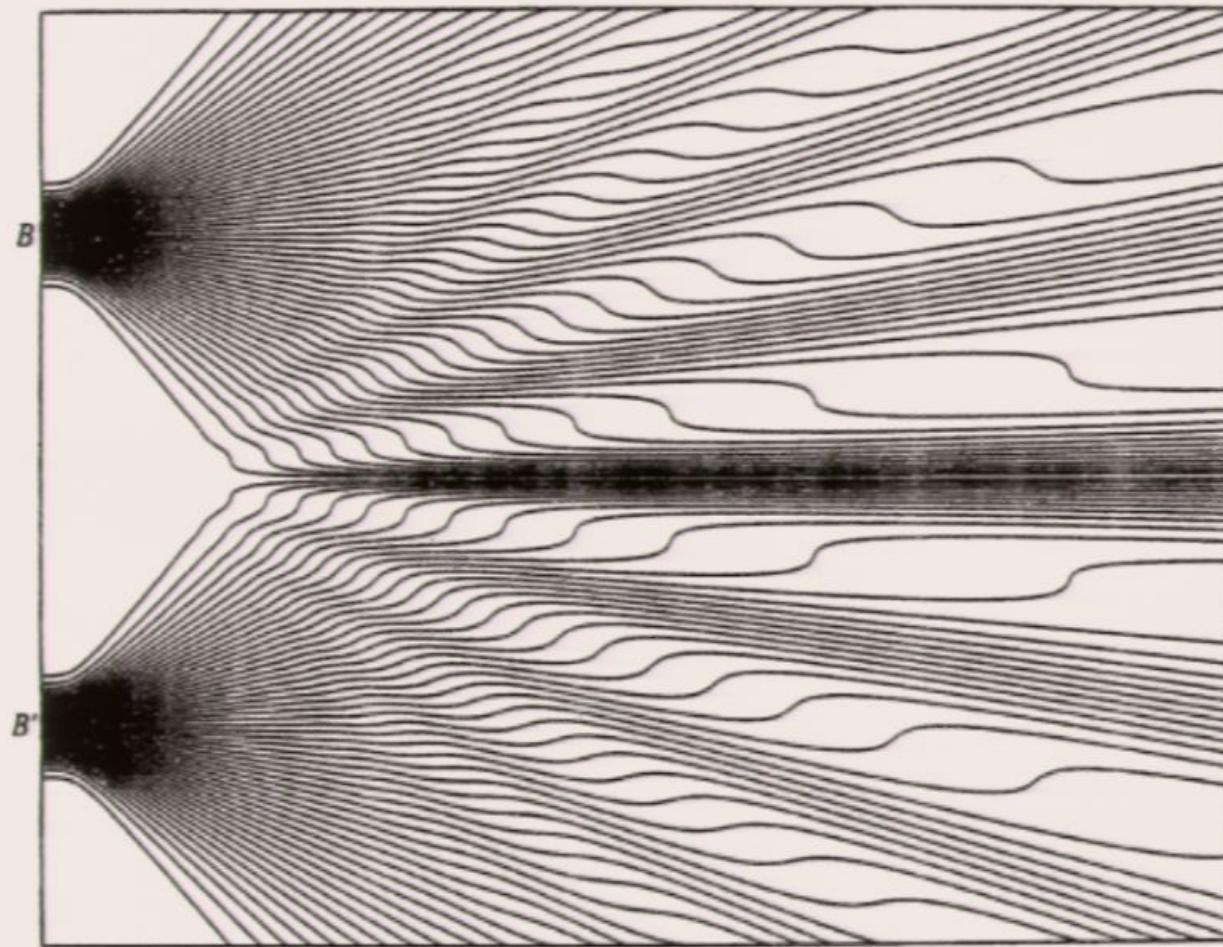
Double slit experiment



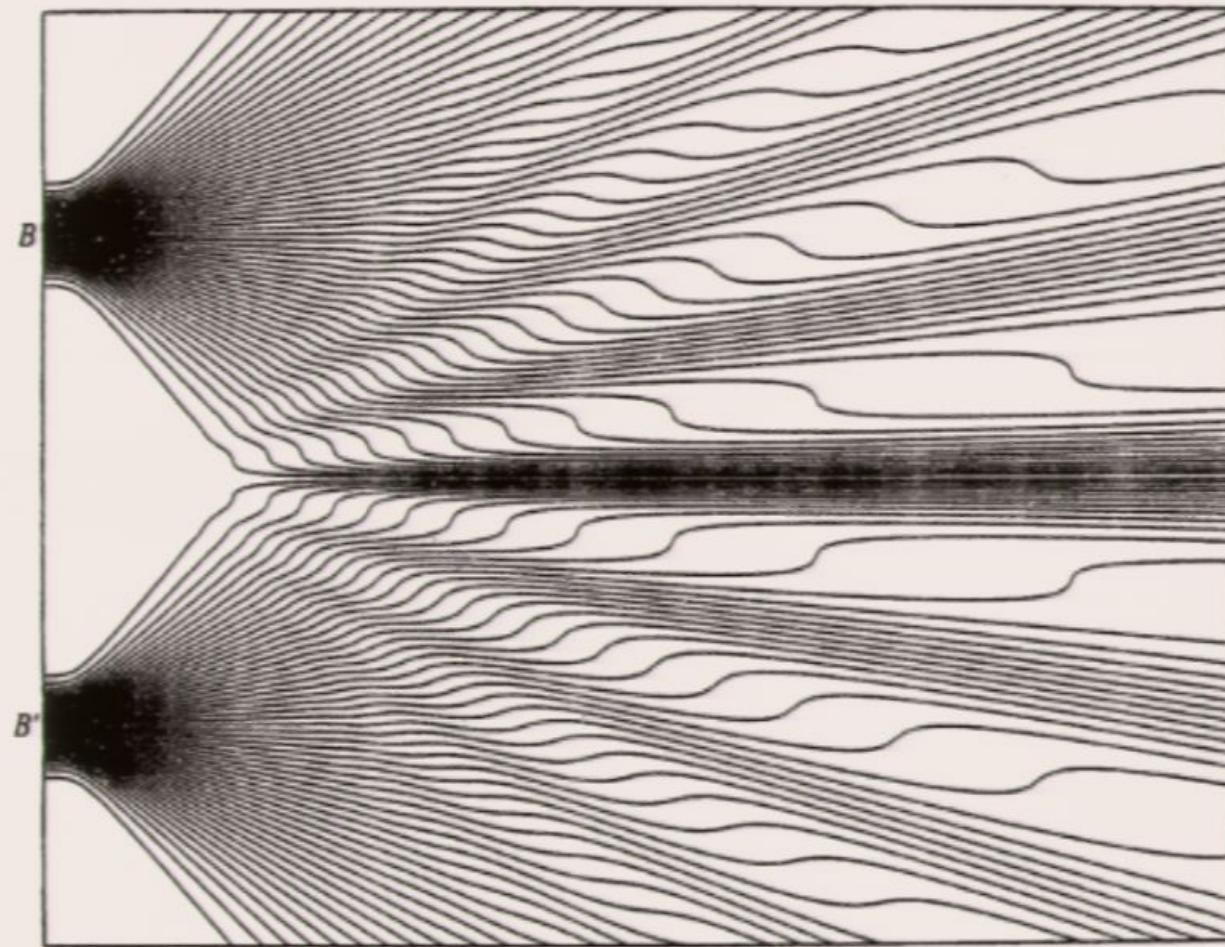
Double slit experiment



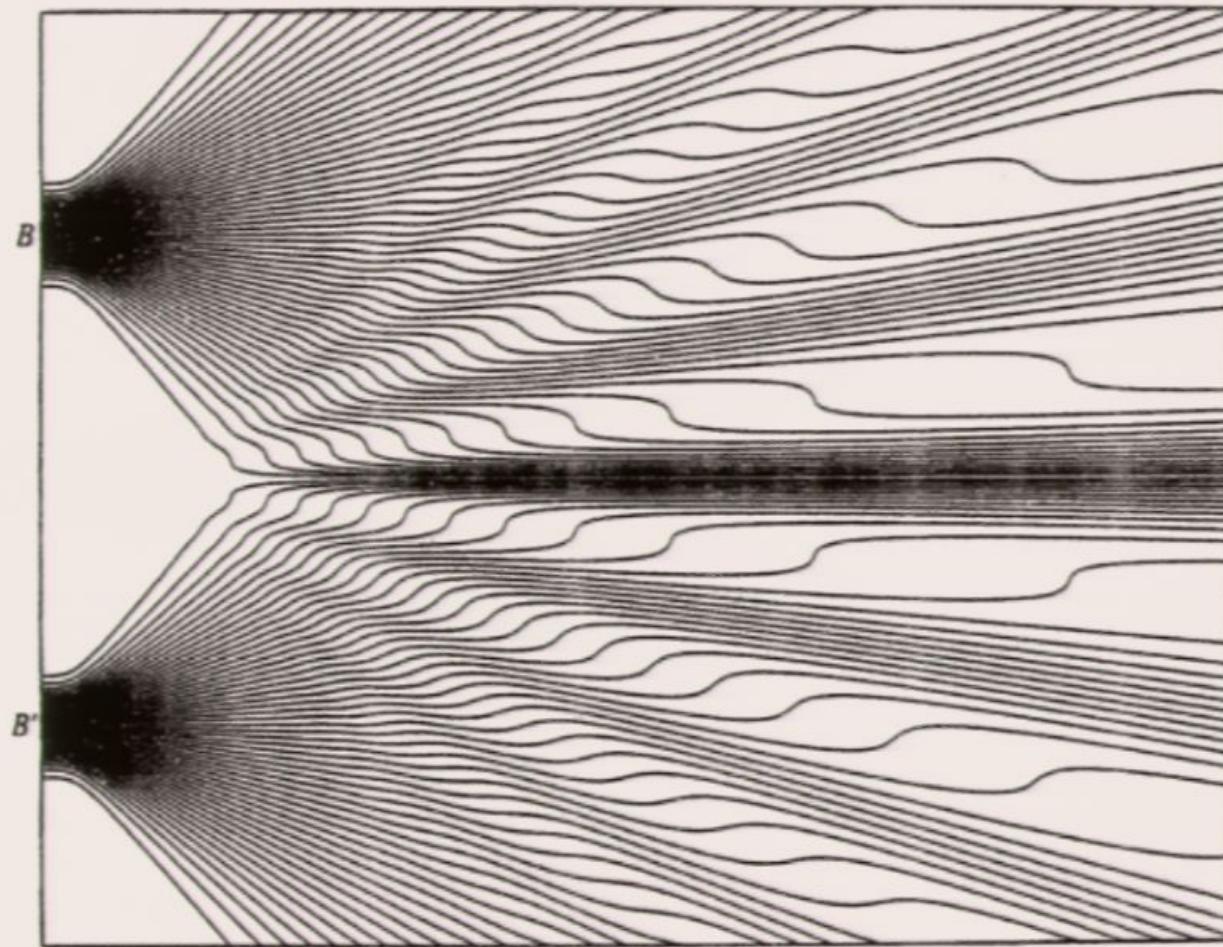
Double slit experiment



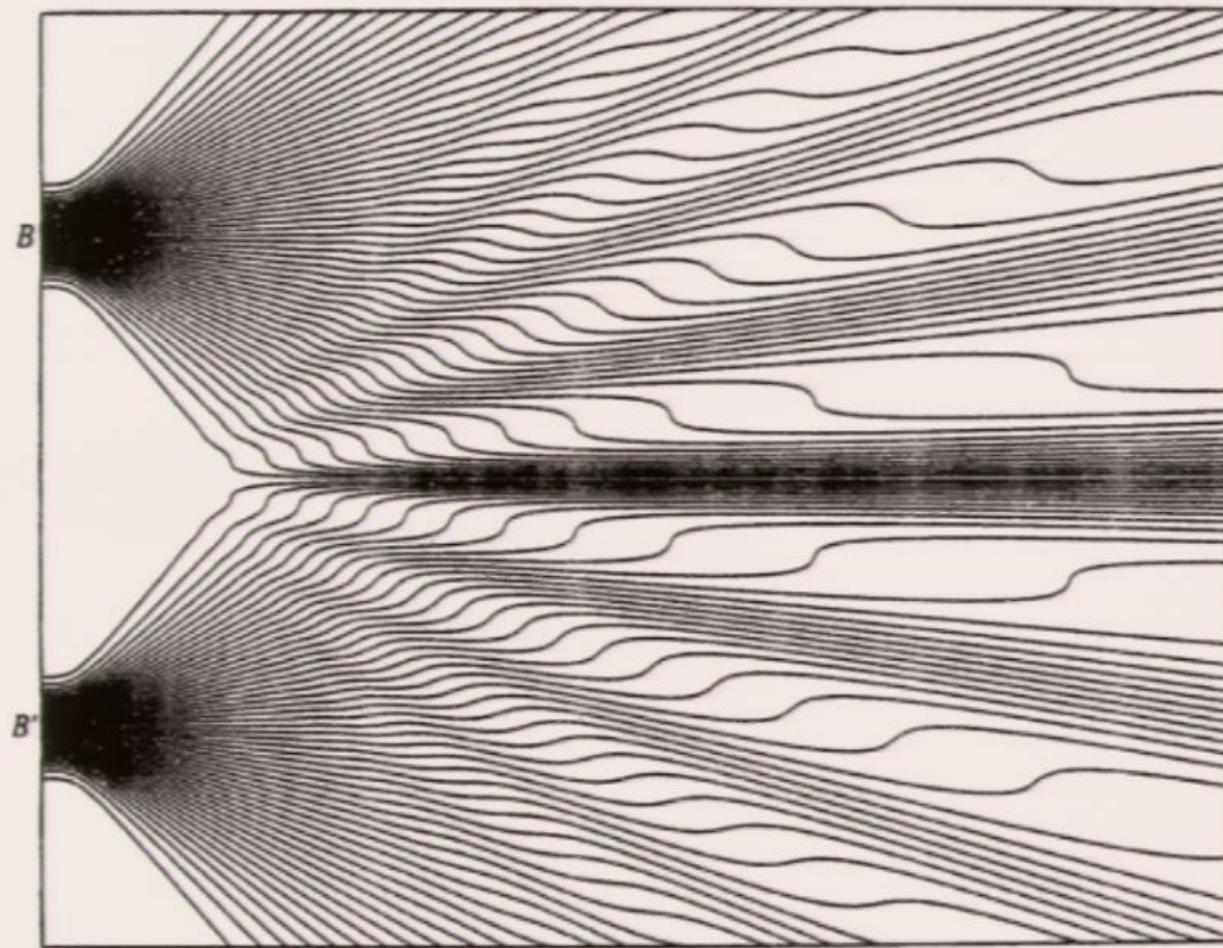
Double slit experiment



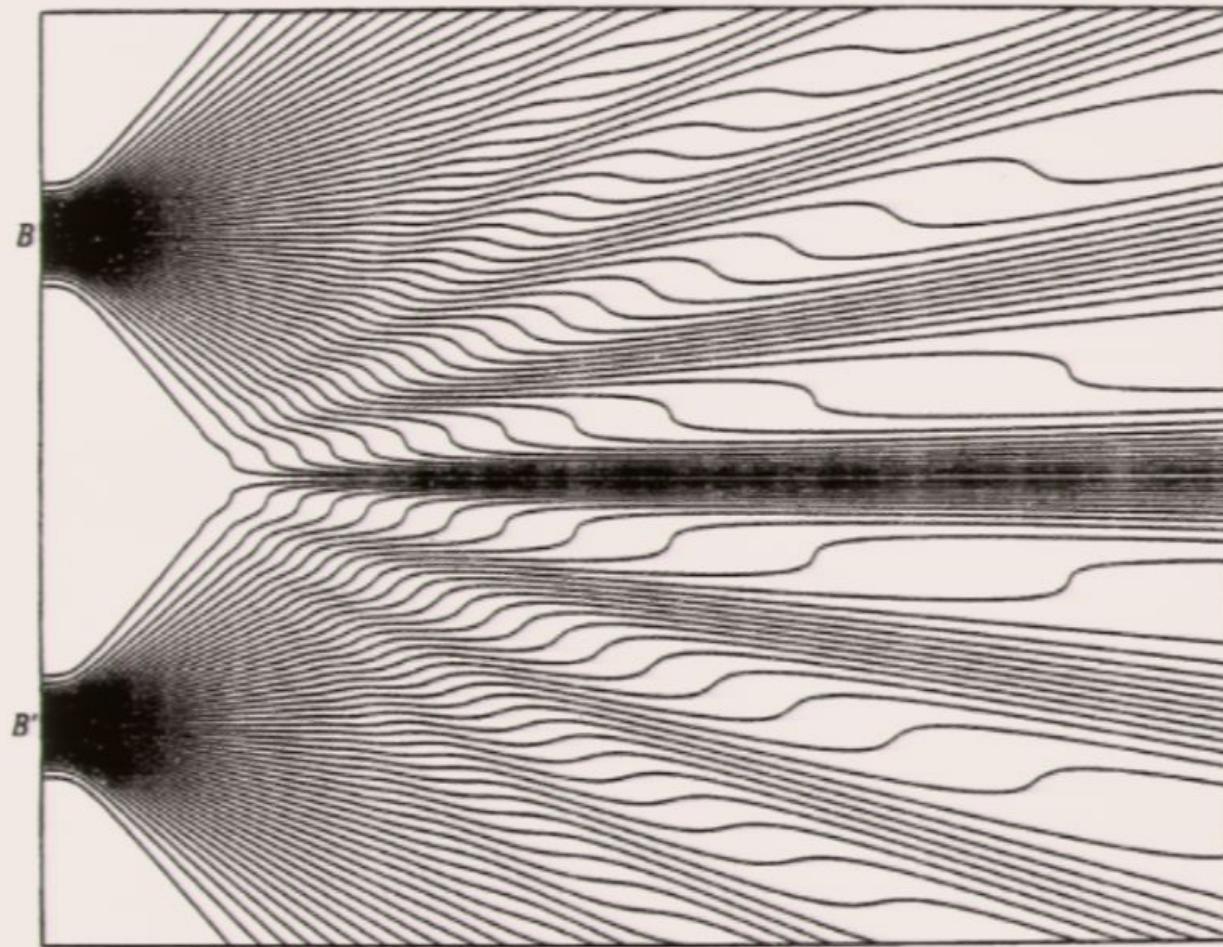
Double slit experiment



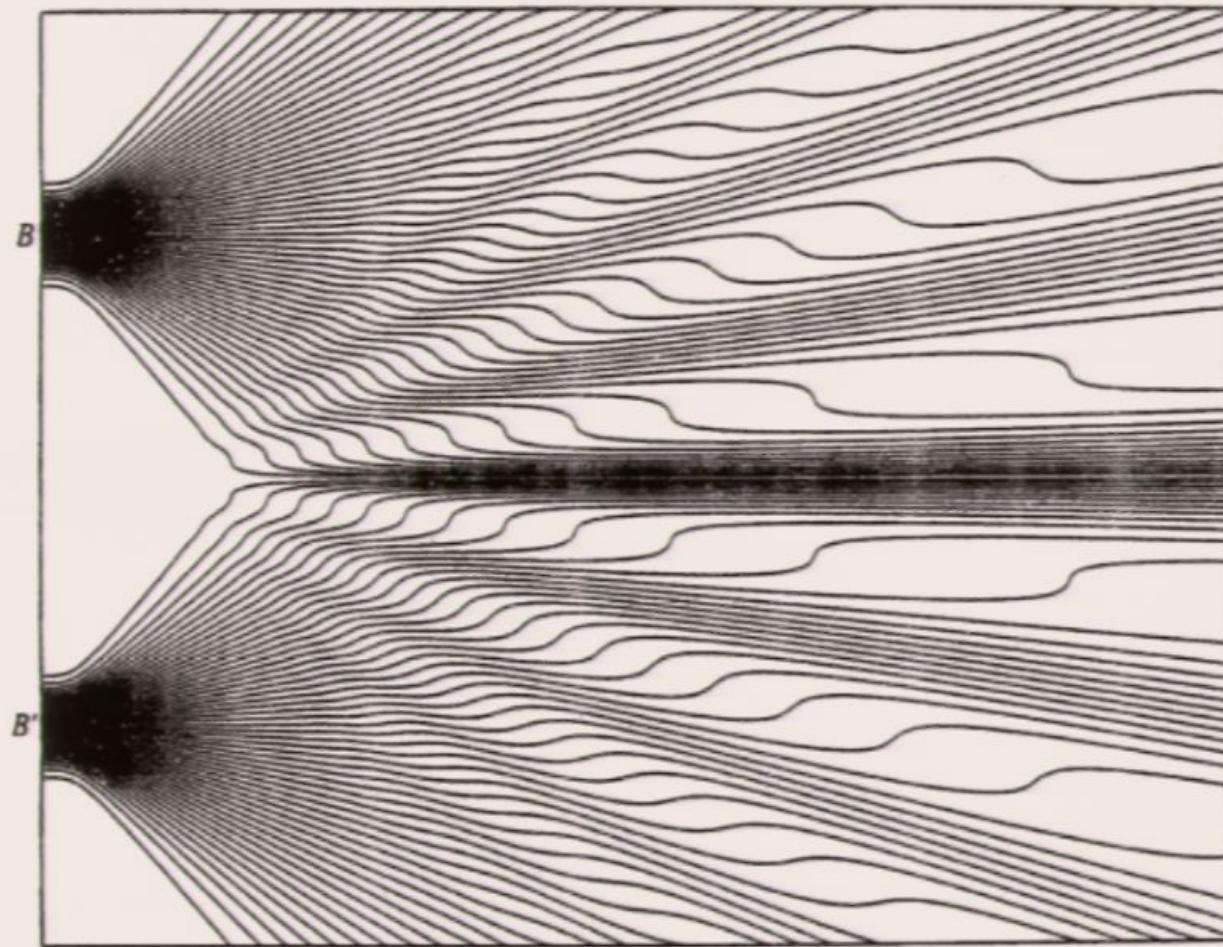
Double slit experiment



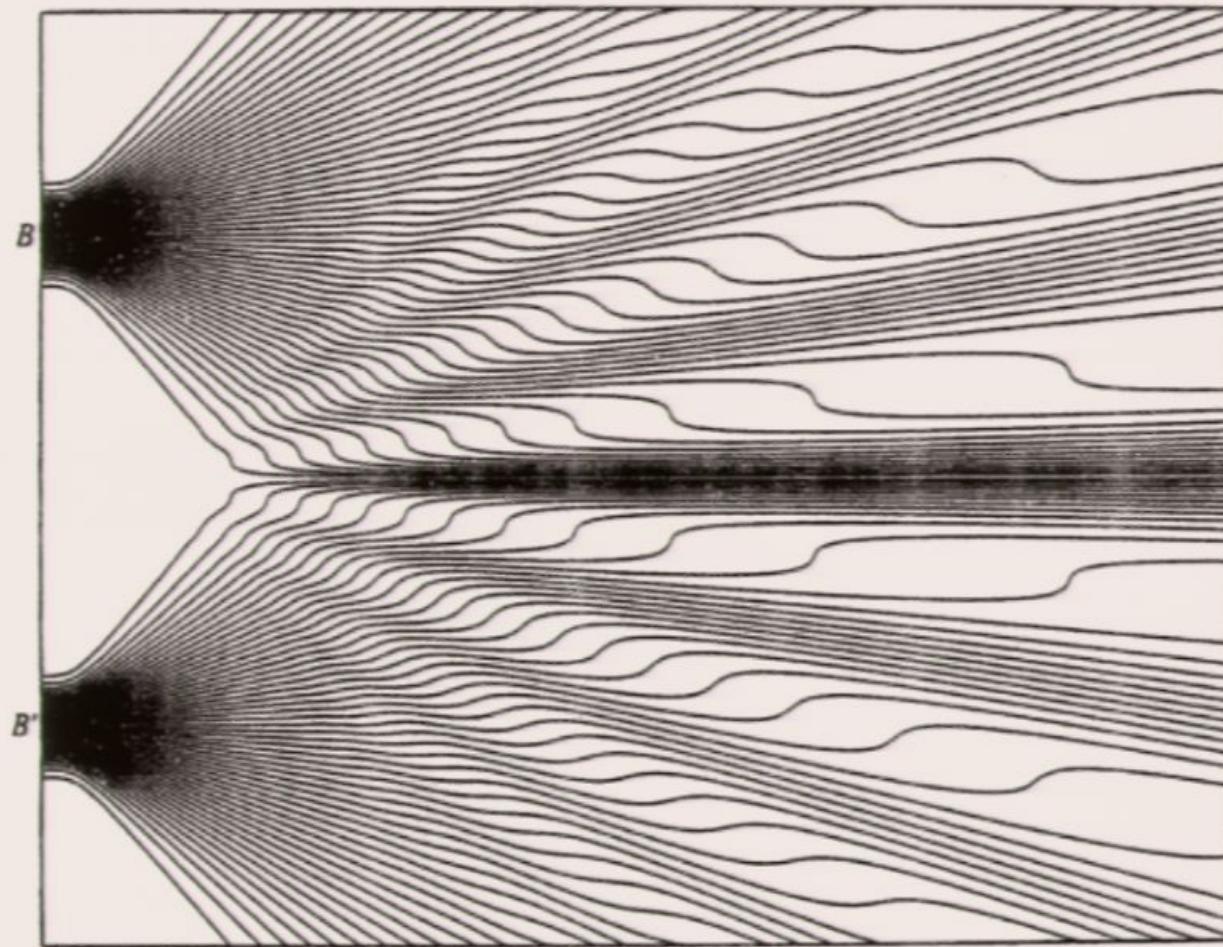
Double slit experiment



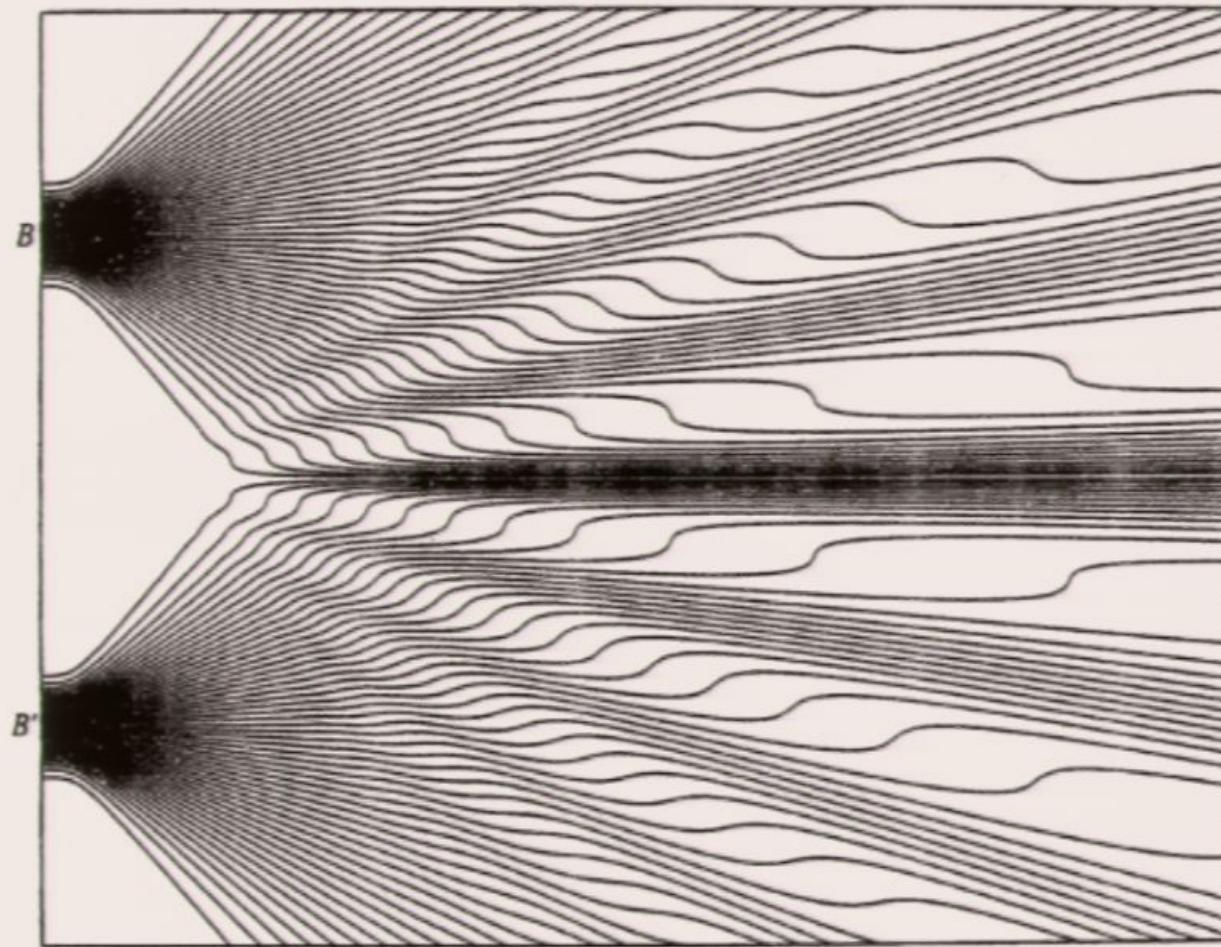
Double slit experiment



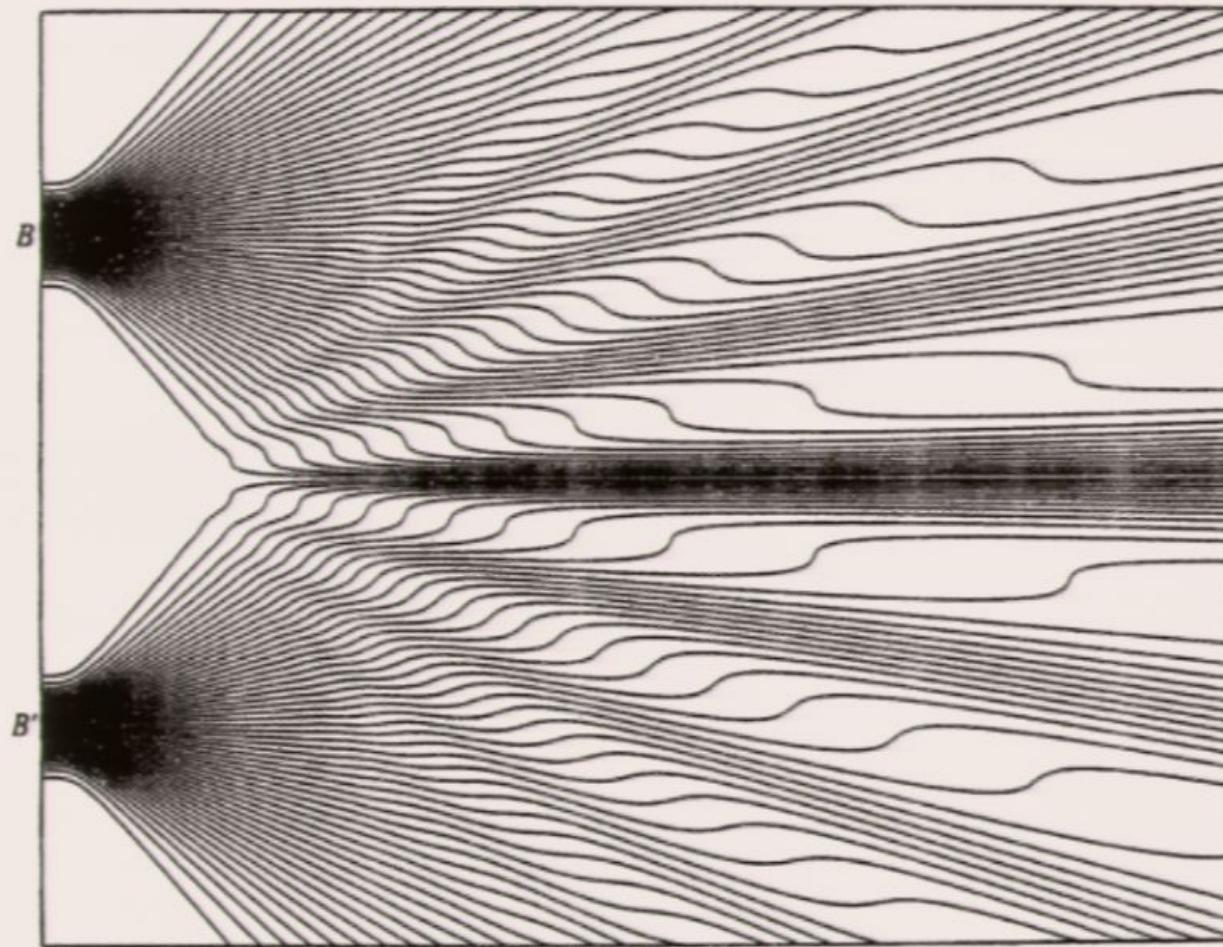
Double slit experiment



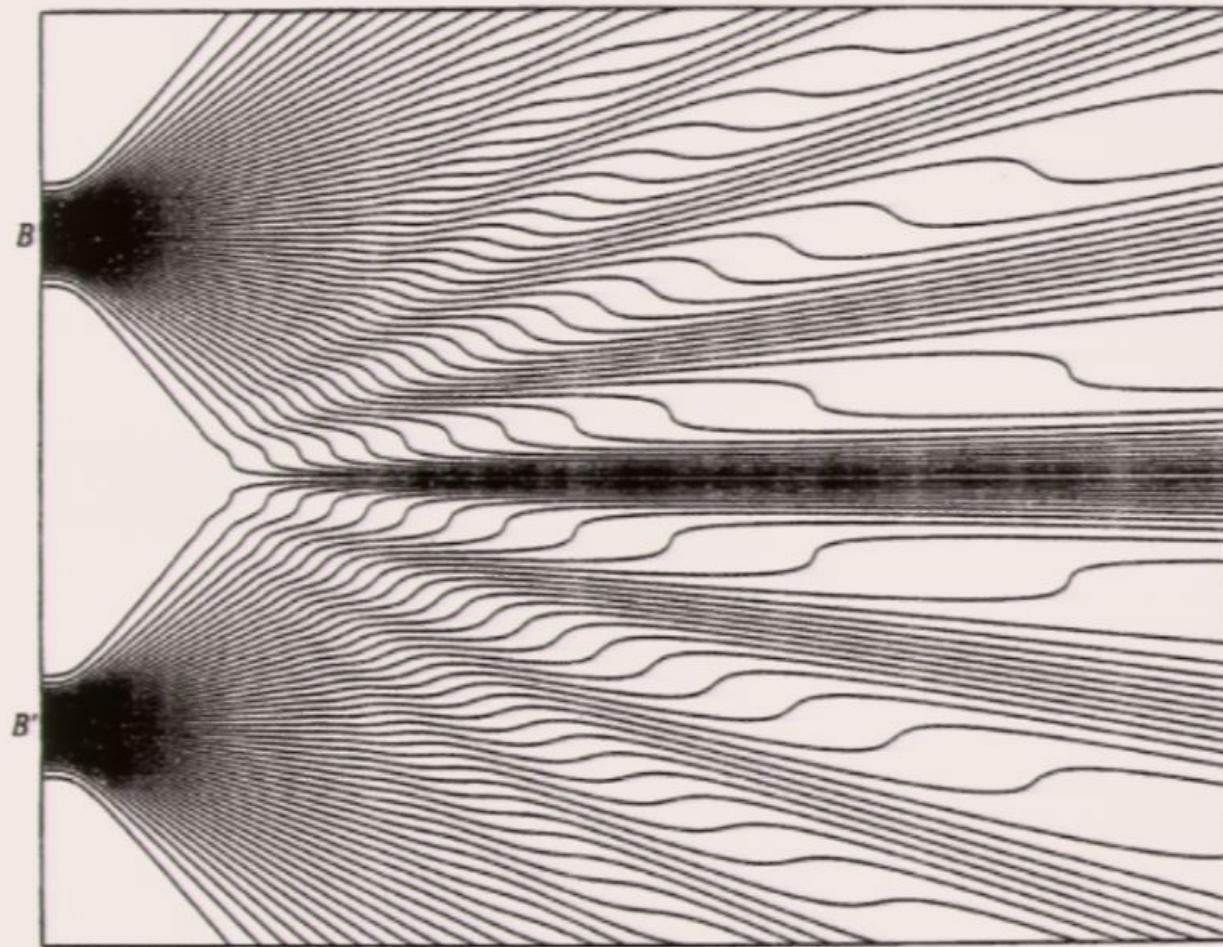
Double slit experiment



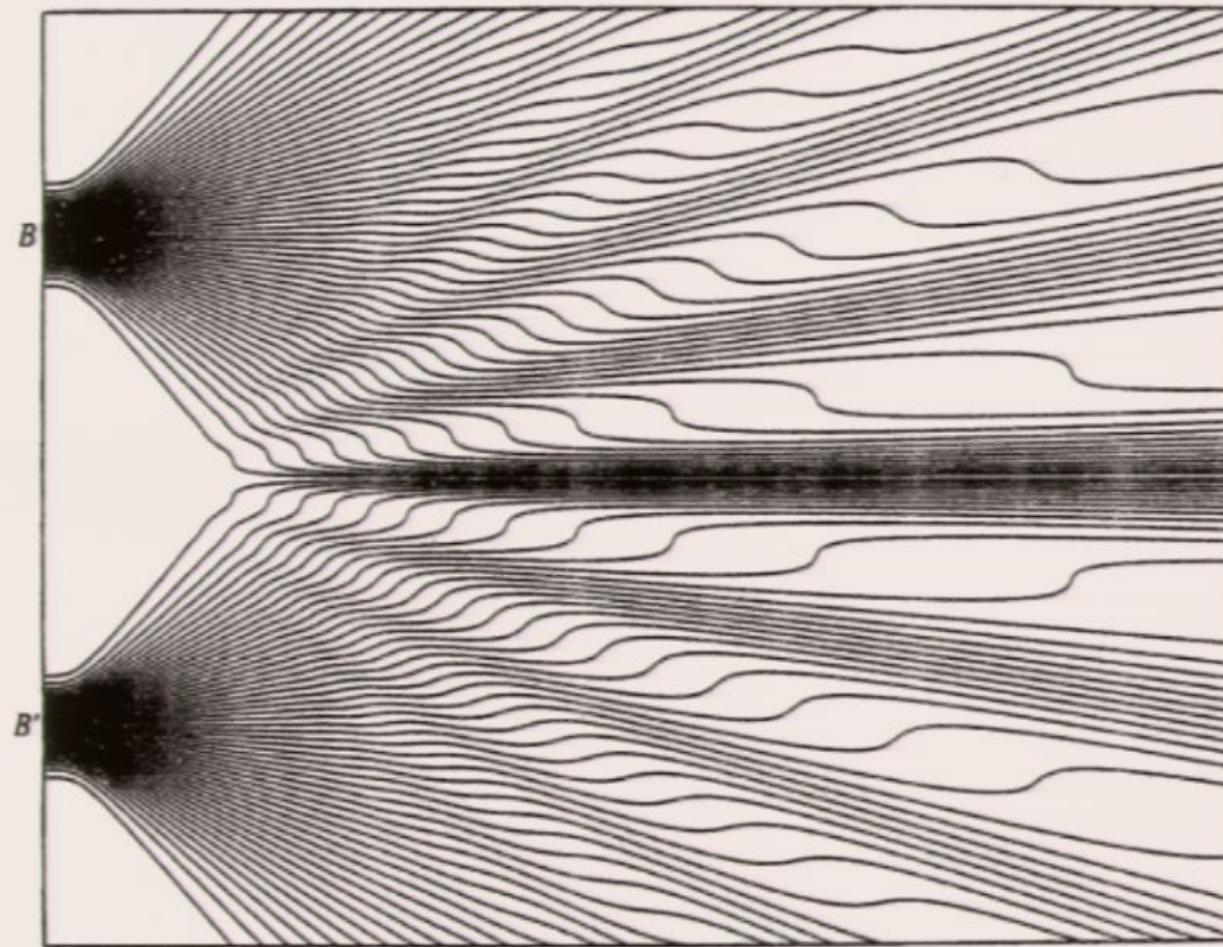
Double slit experiment



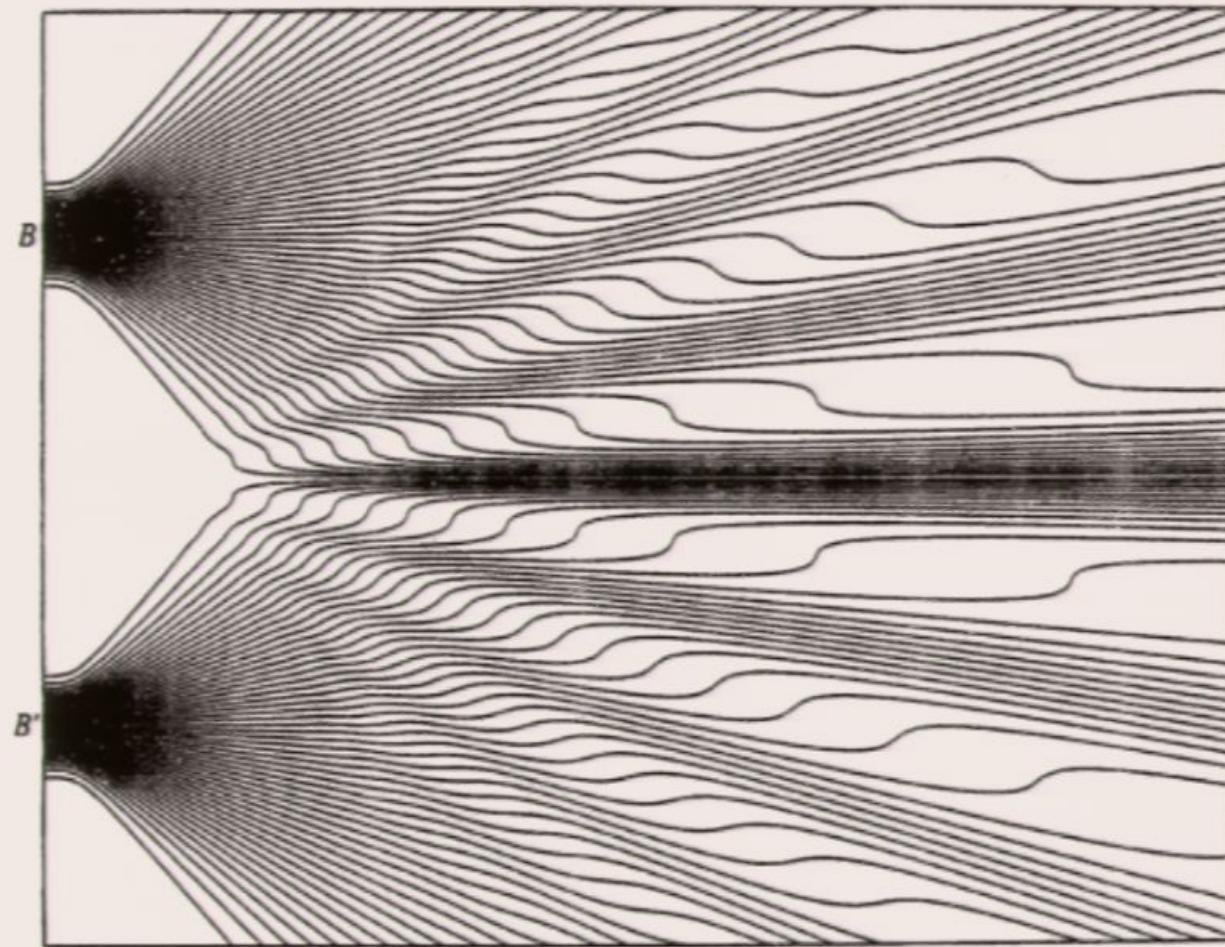
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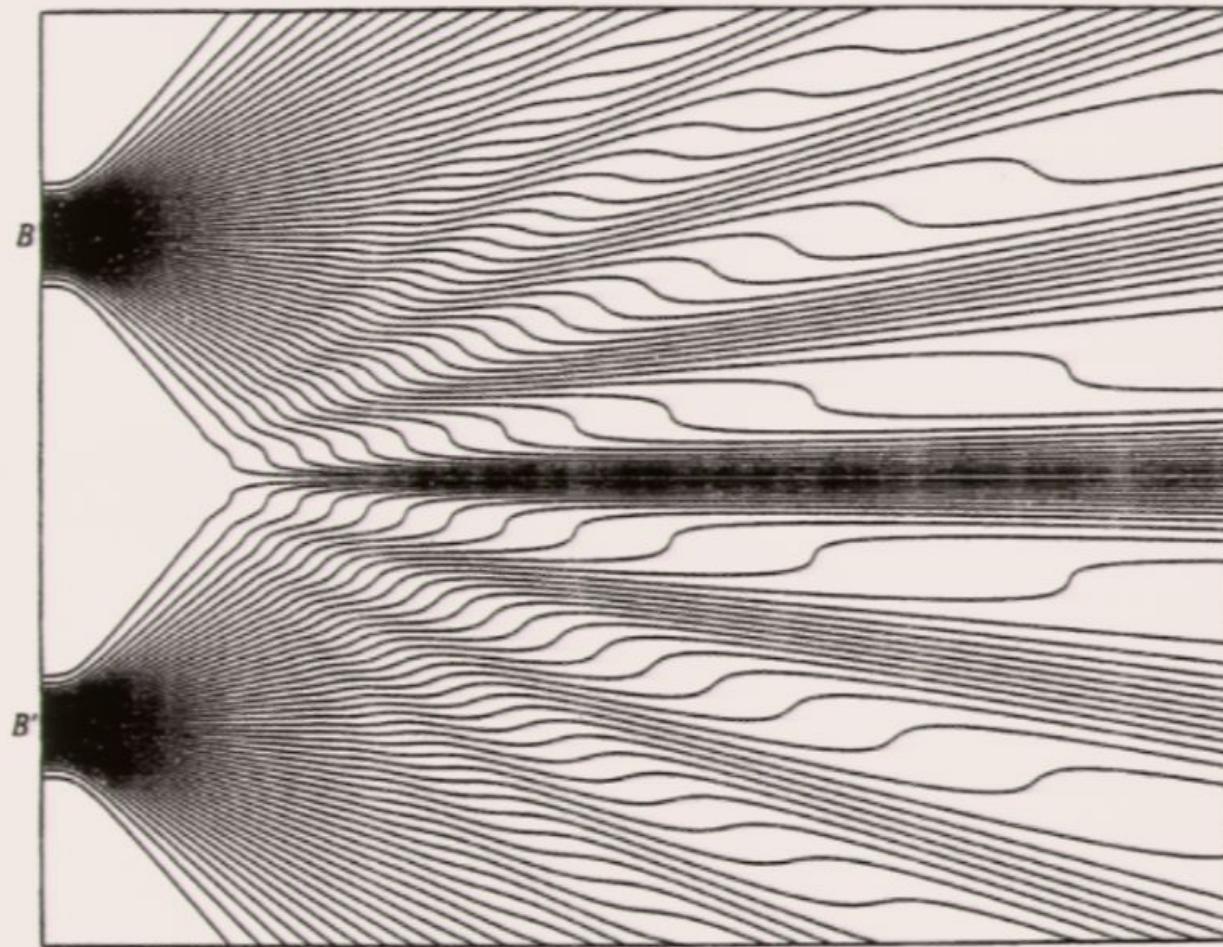
Double slit experiment



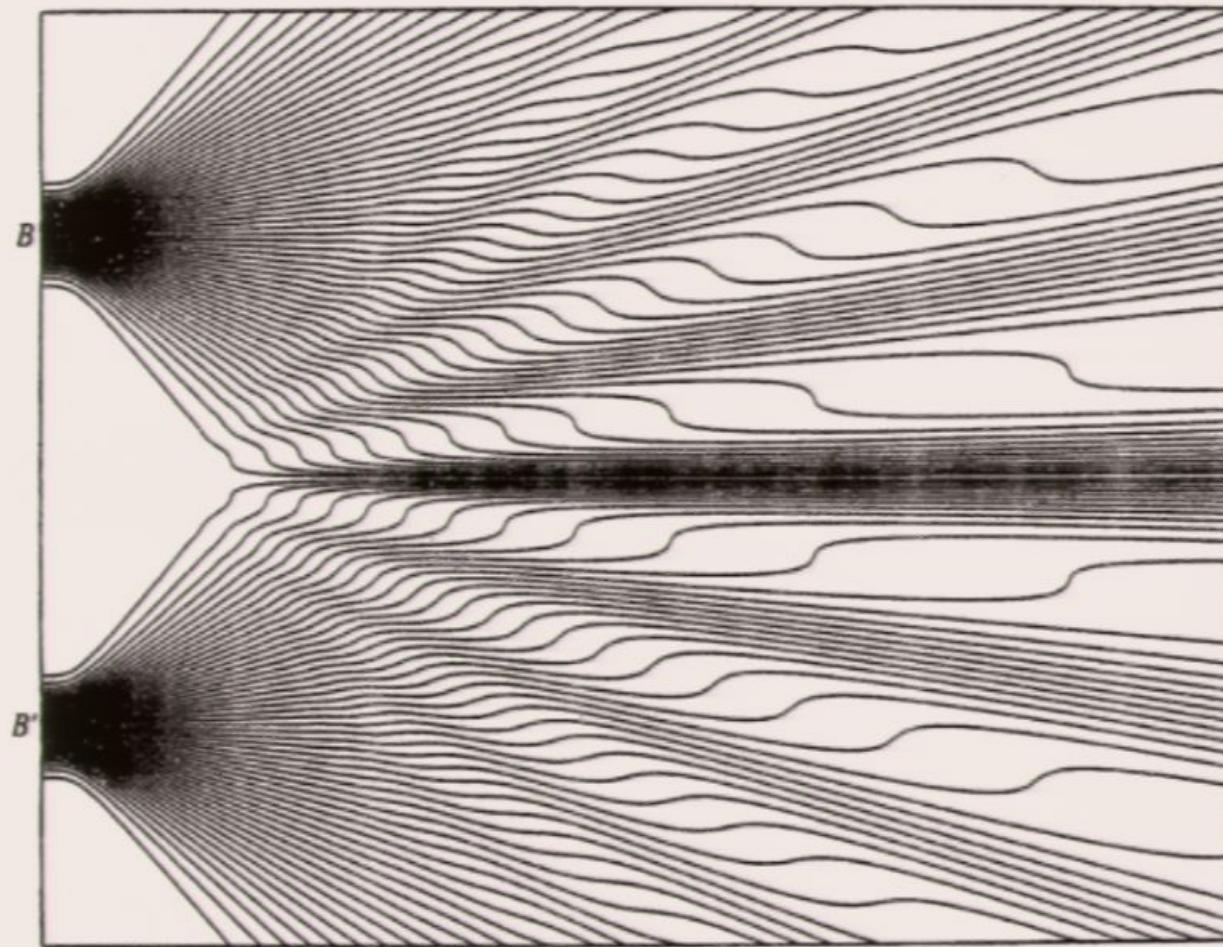
Double slit experiment



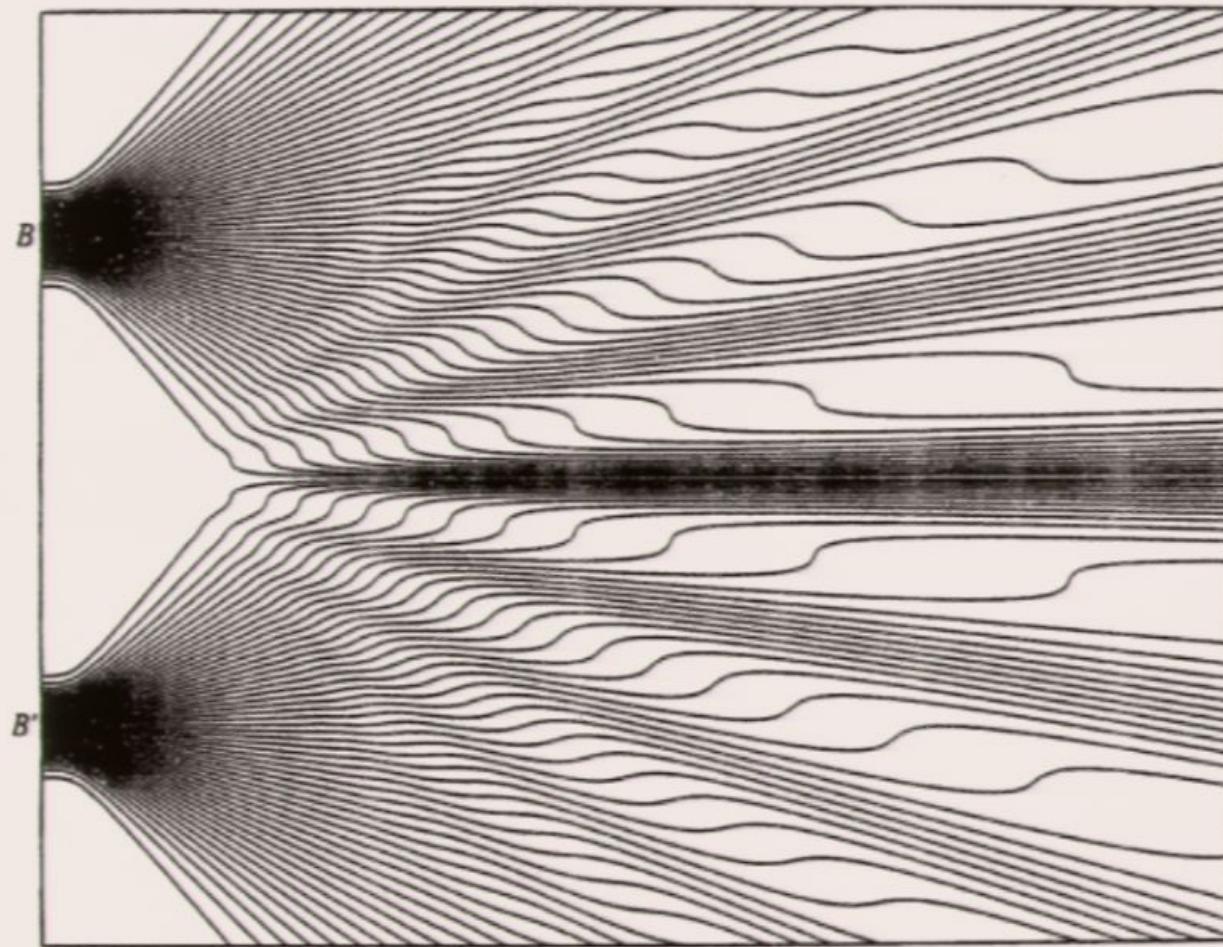
Double slit experiment



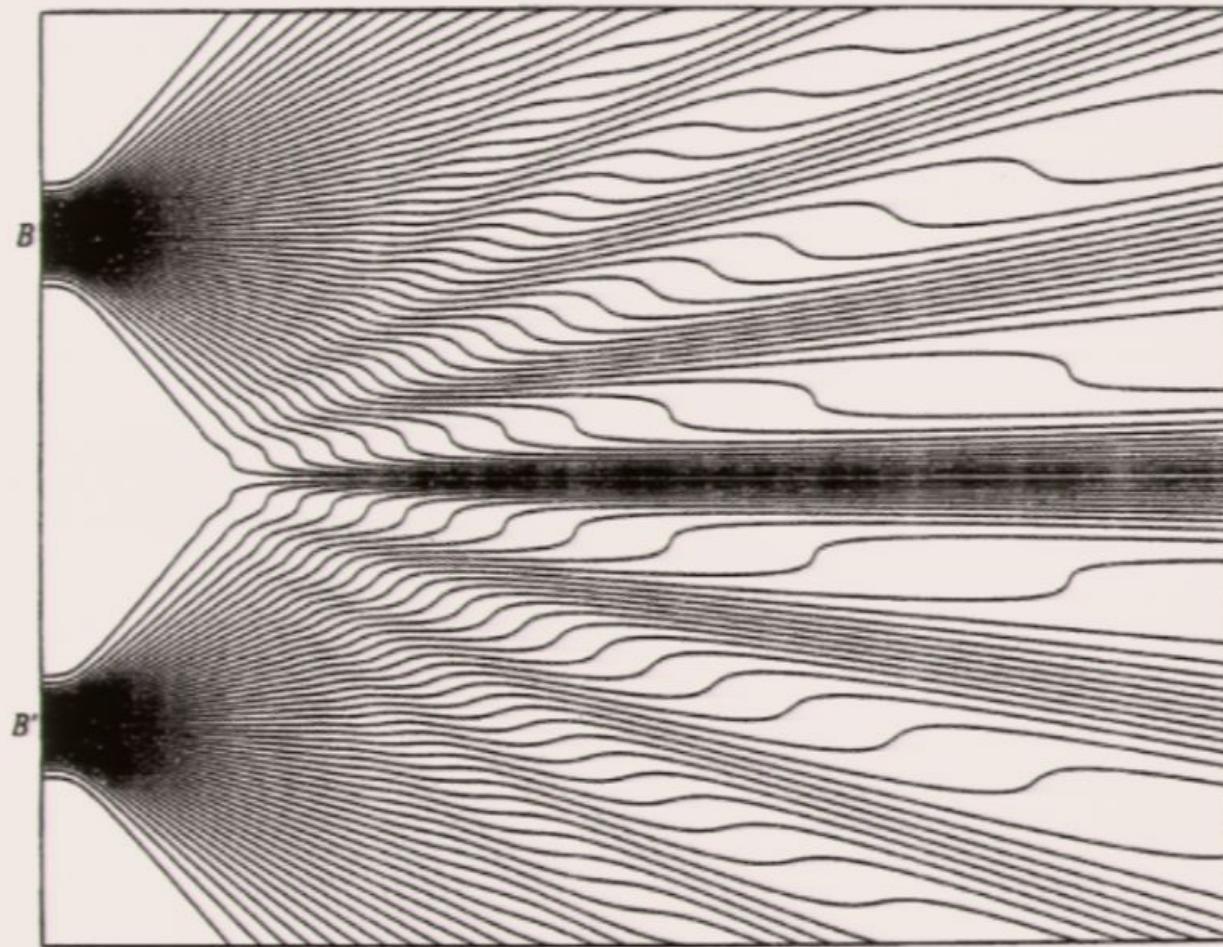
Double slit experiment



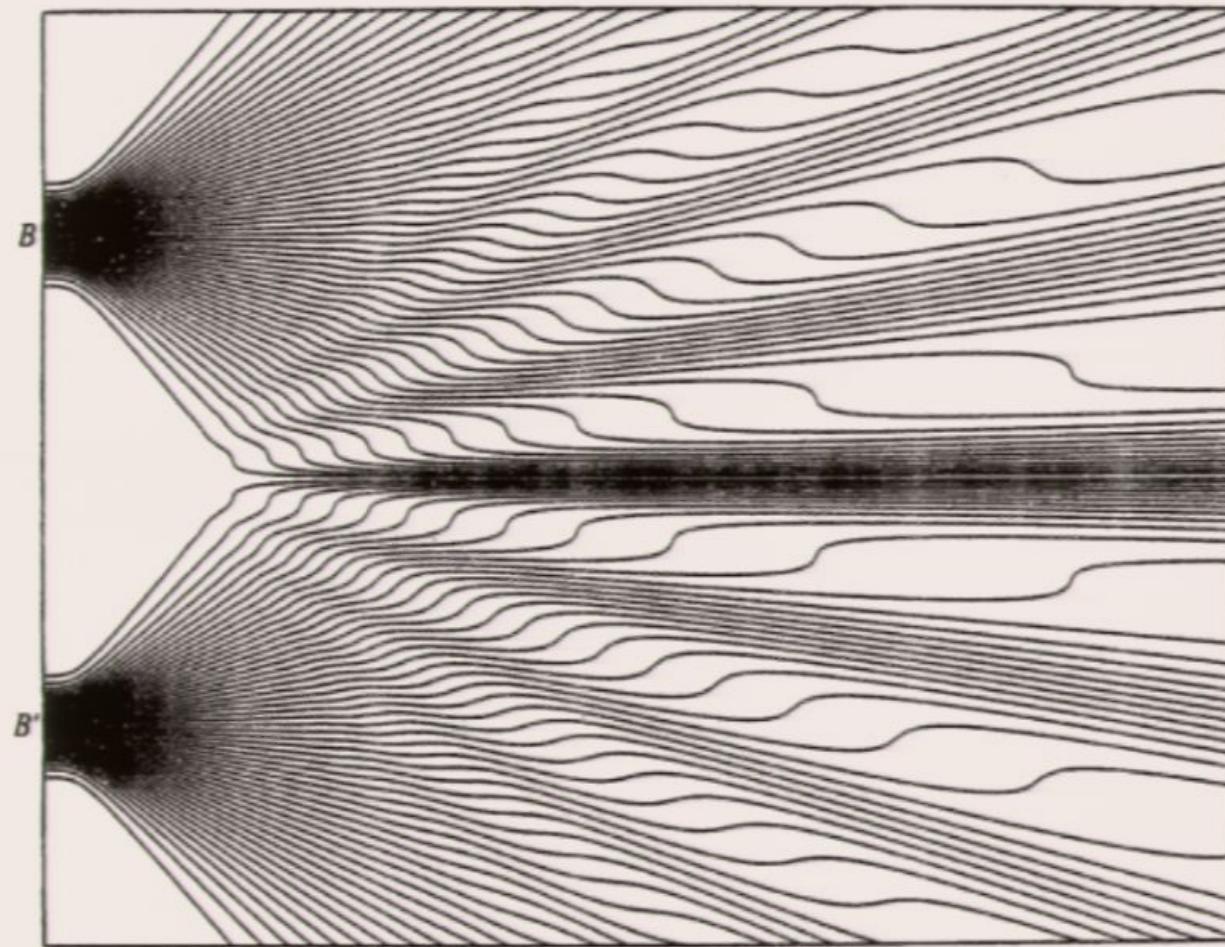
Double slit experiment



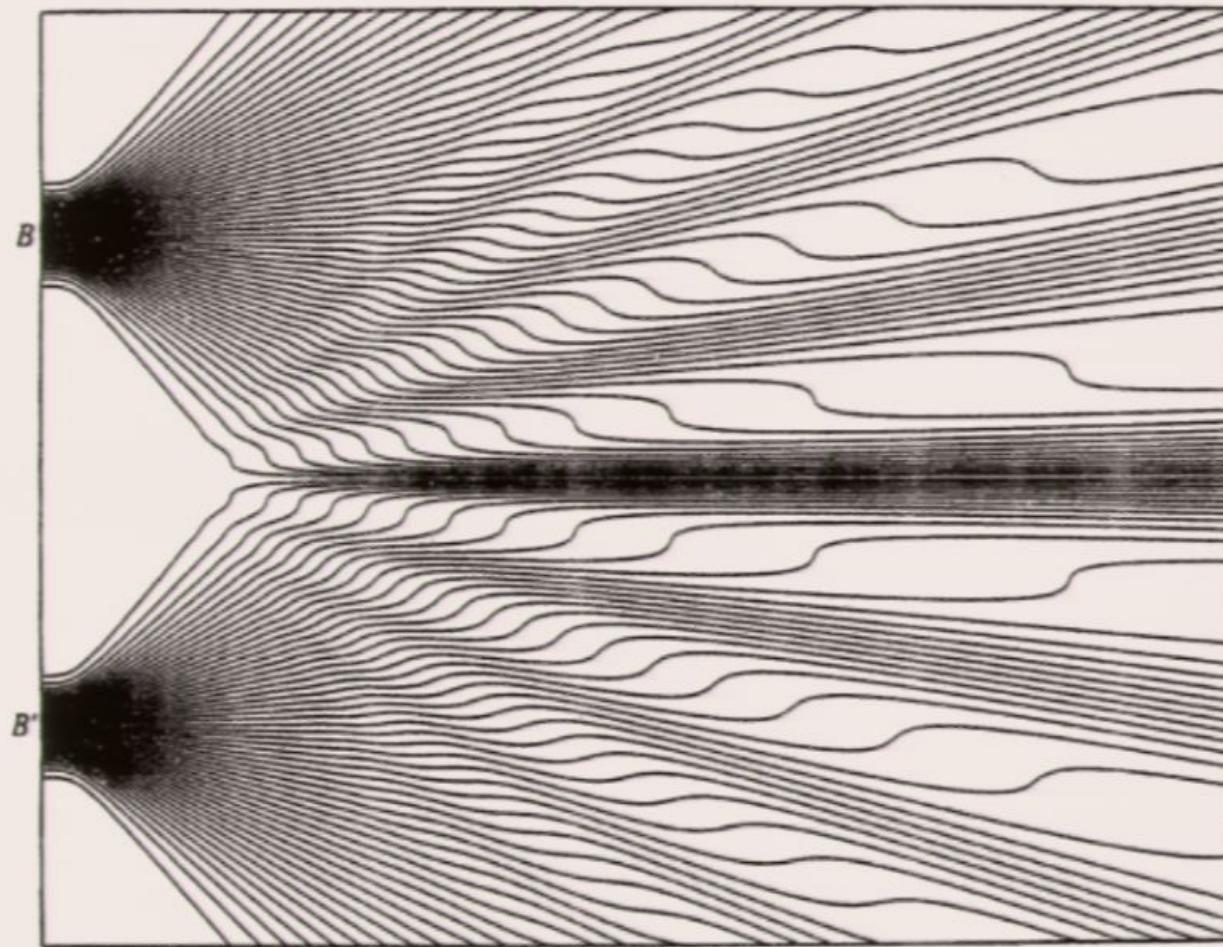
Double slit experiment



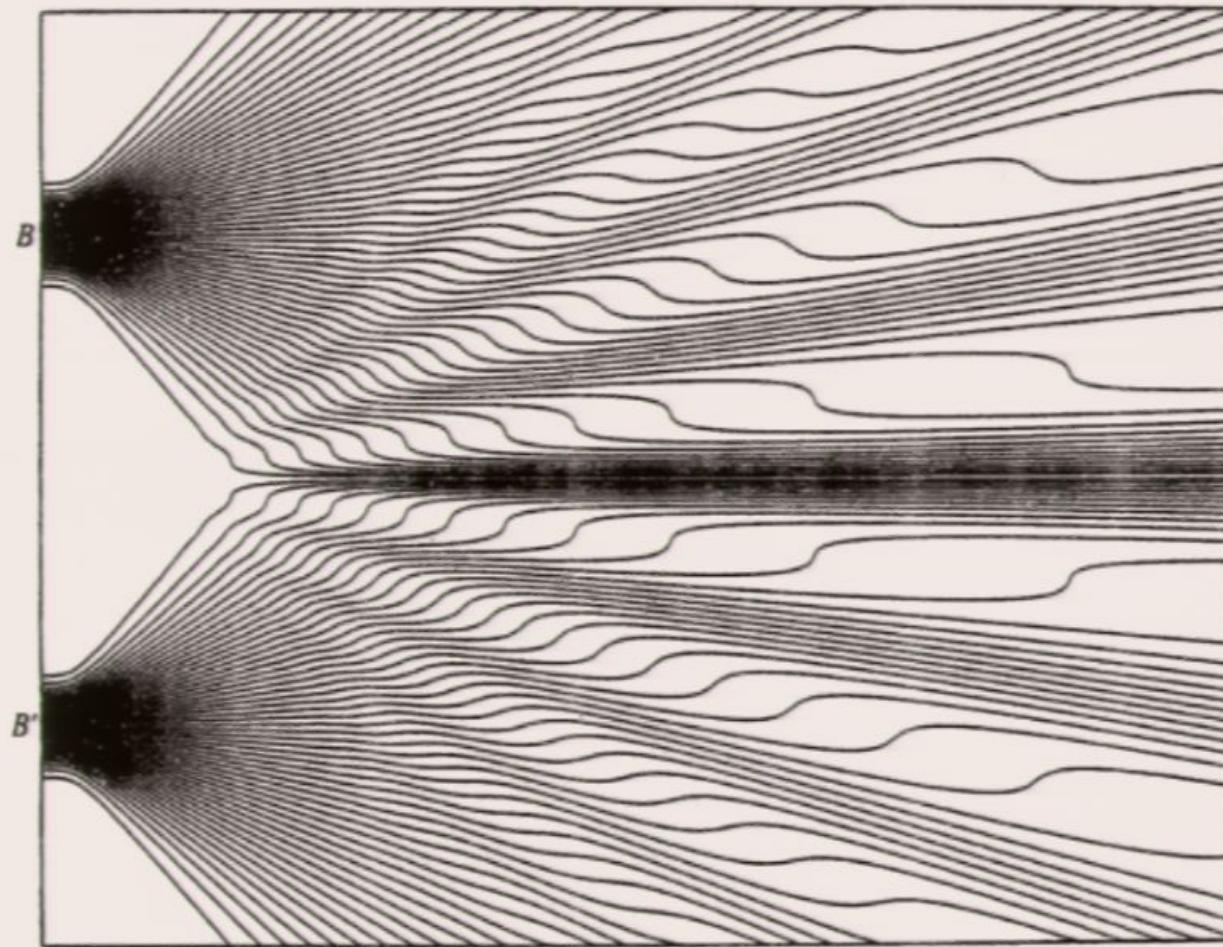
Double slit experiment



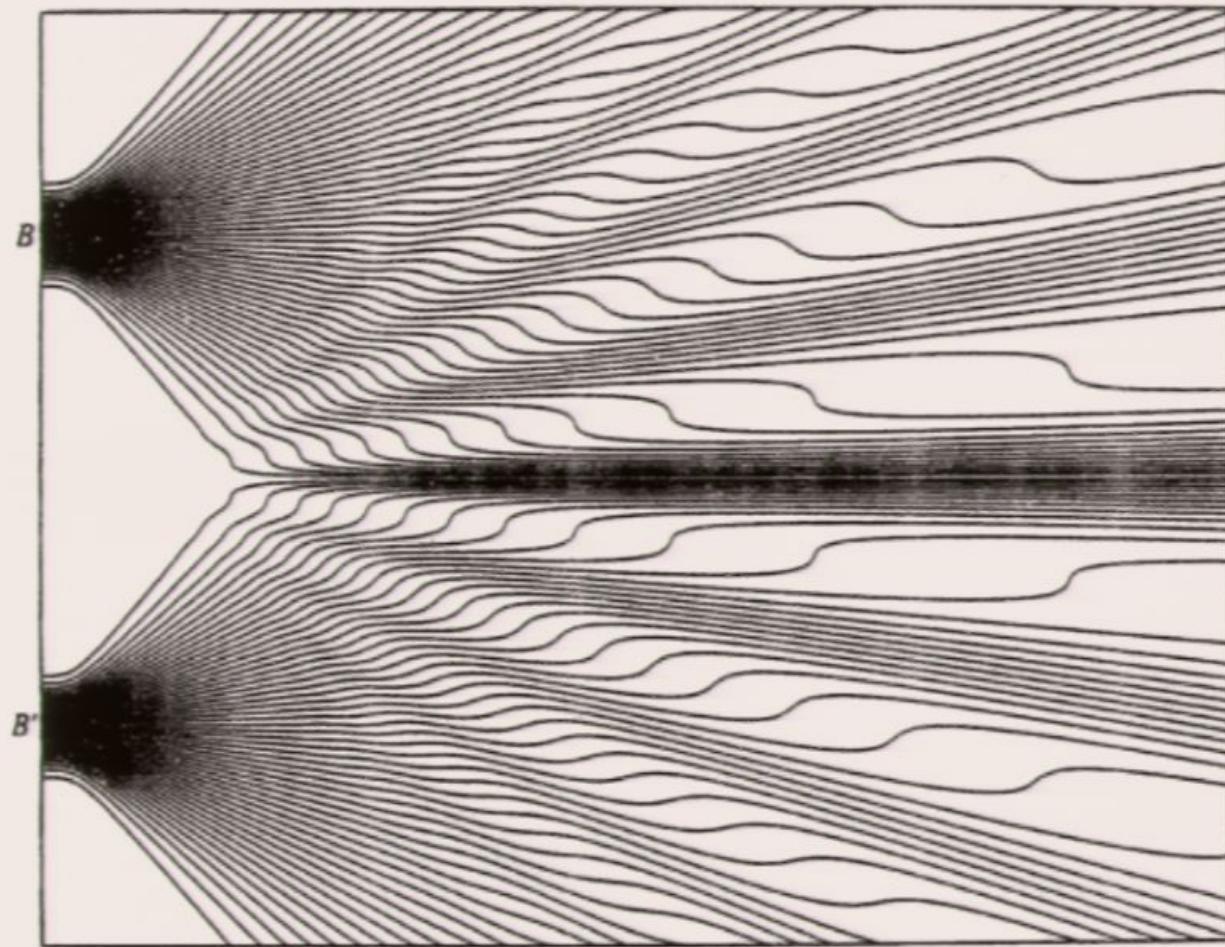
Double slit experiment



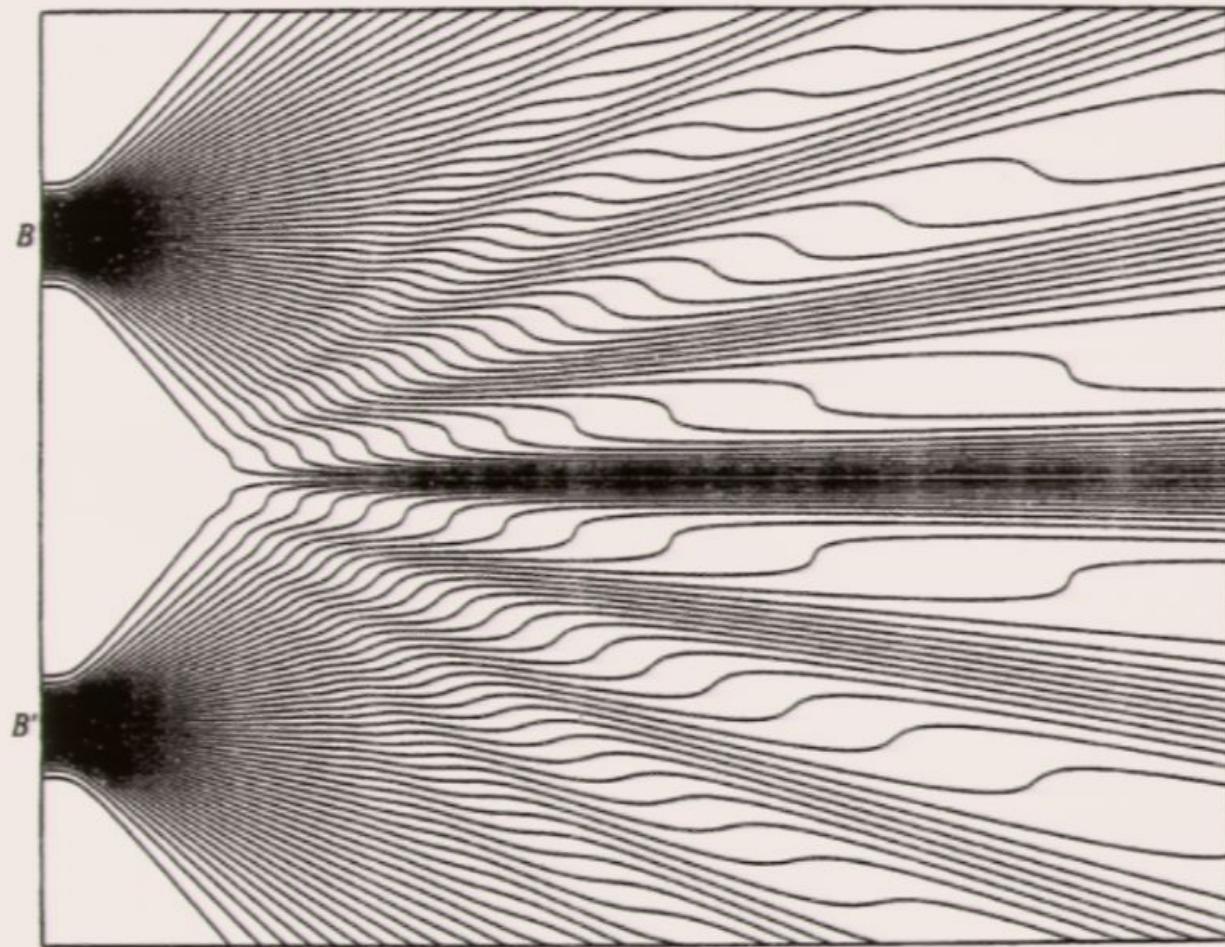
Double slit experiment



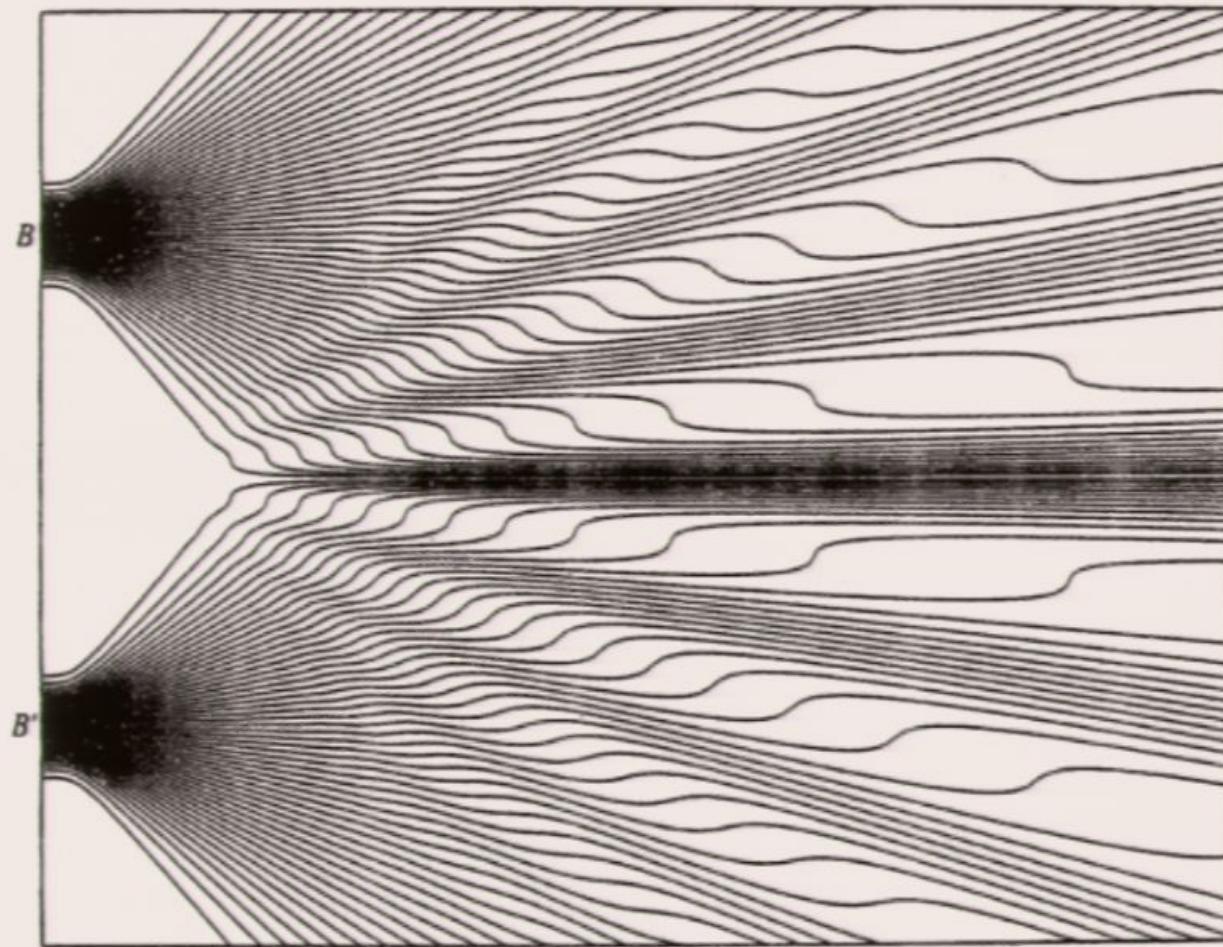
Double slit experiment



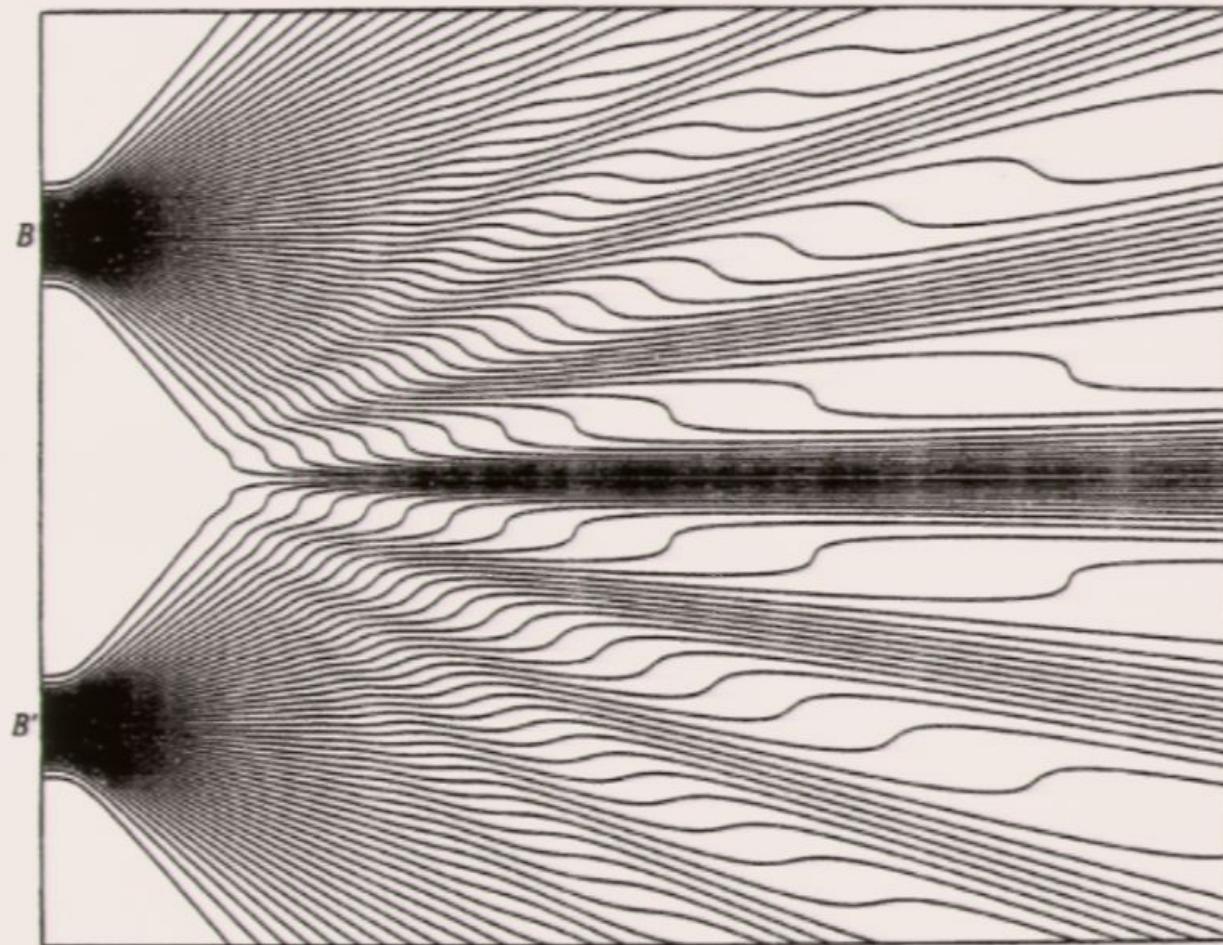
Double slit experiment



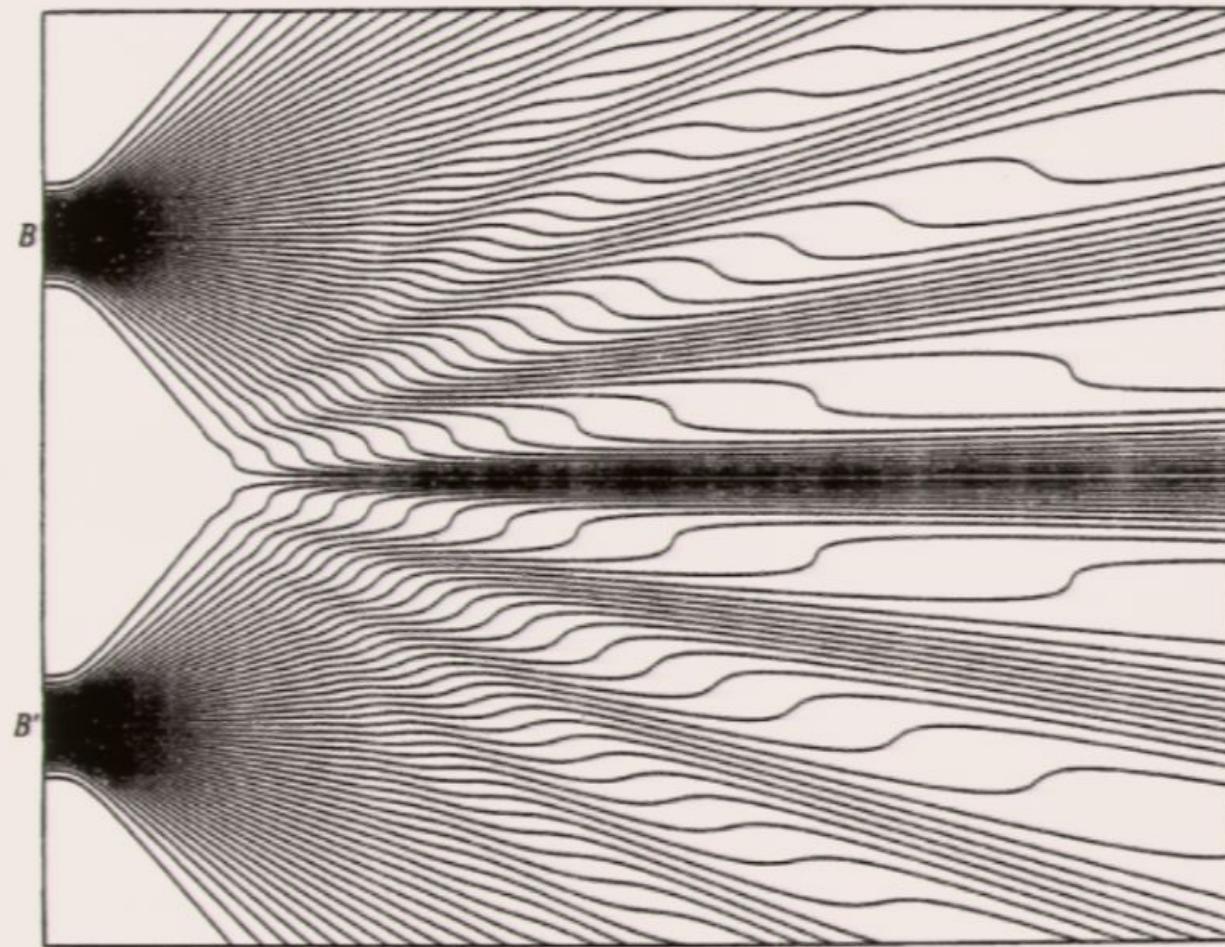
Double slit experiment



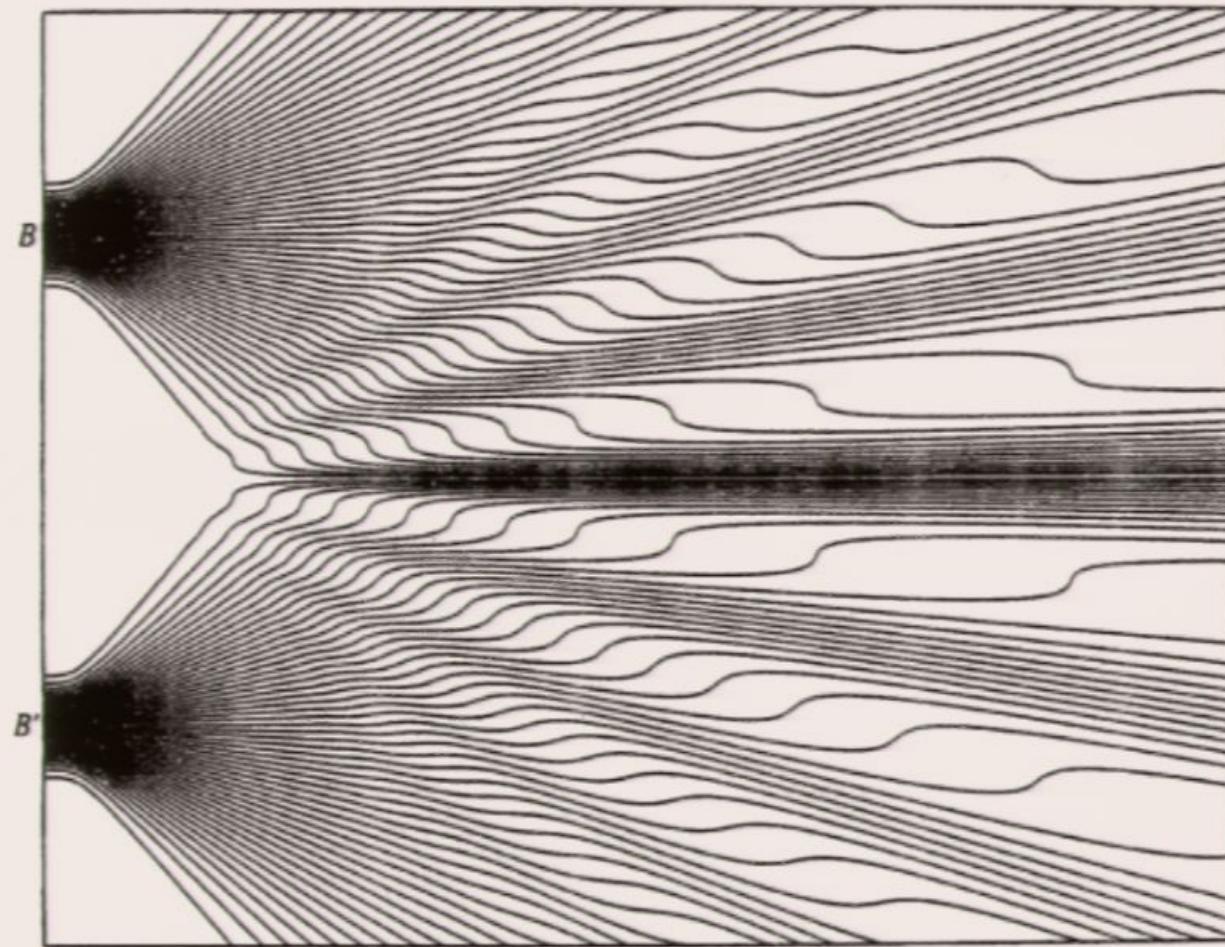
Double slit experiment



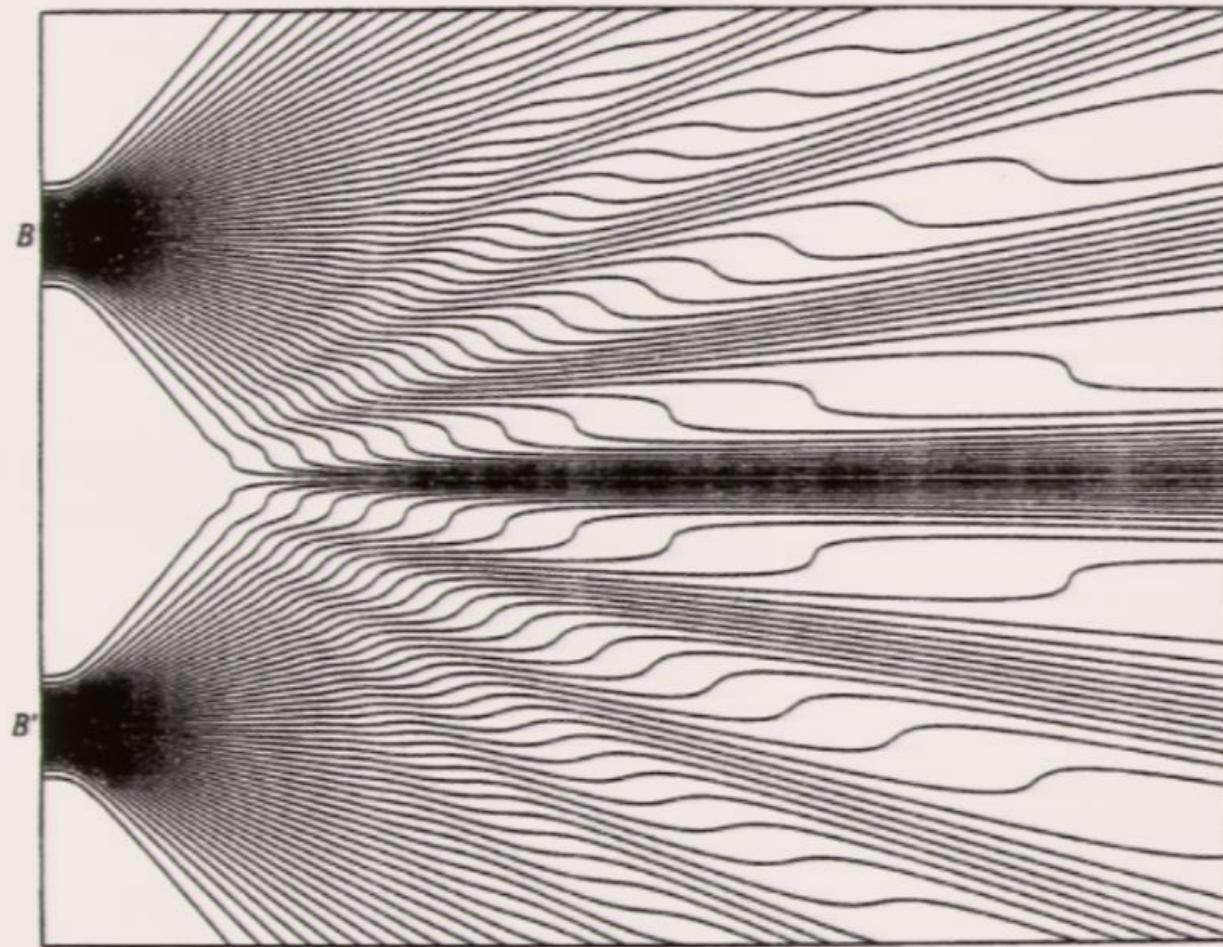
Double slit experiment



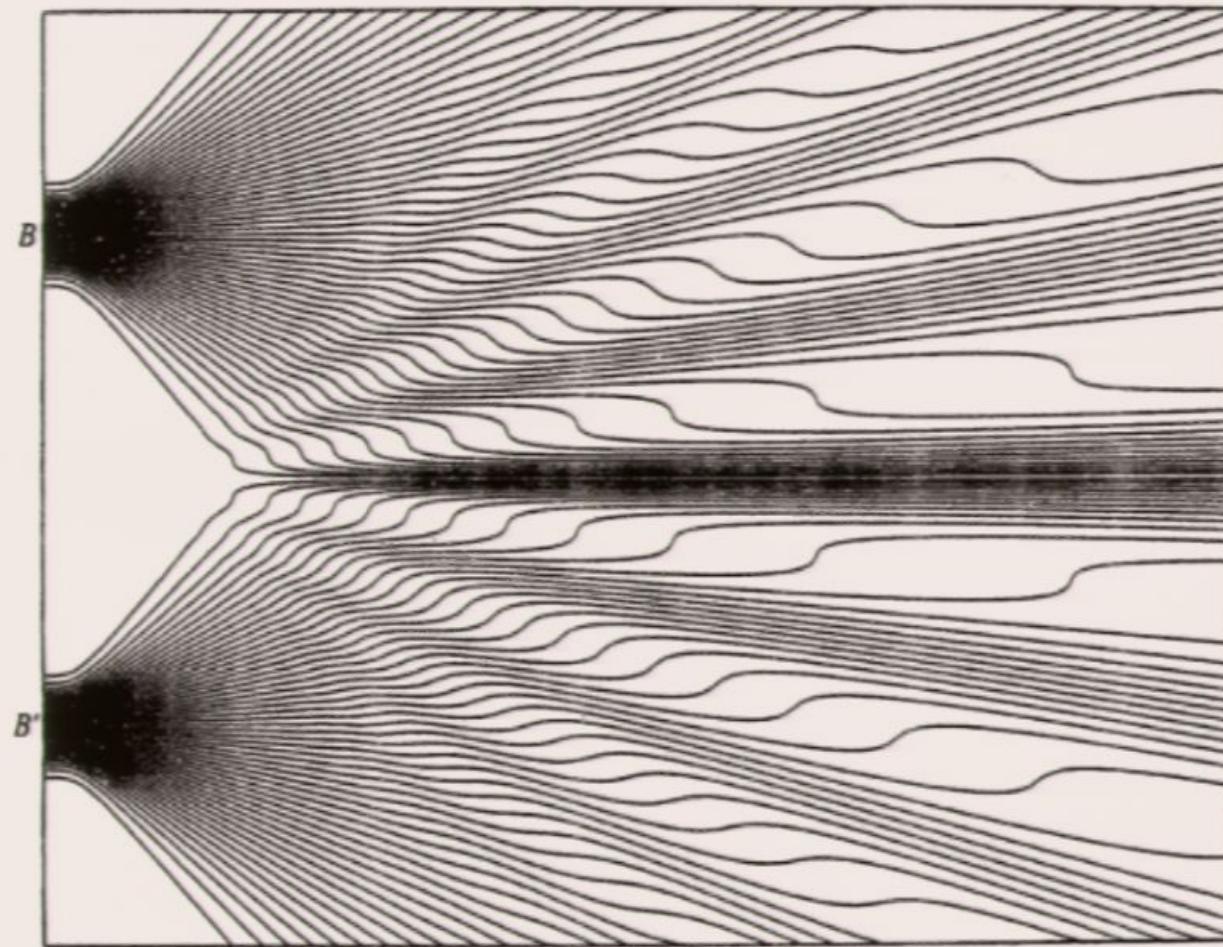
Double slit experiment



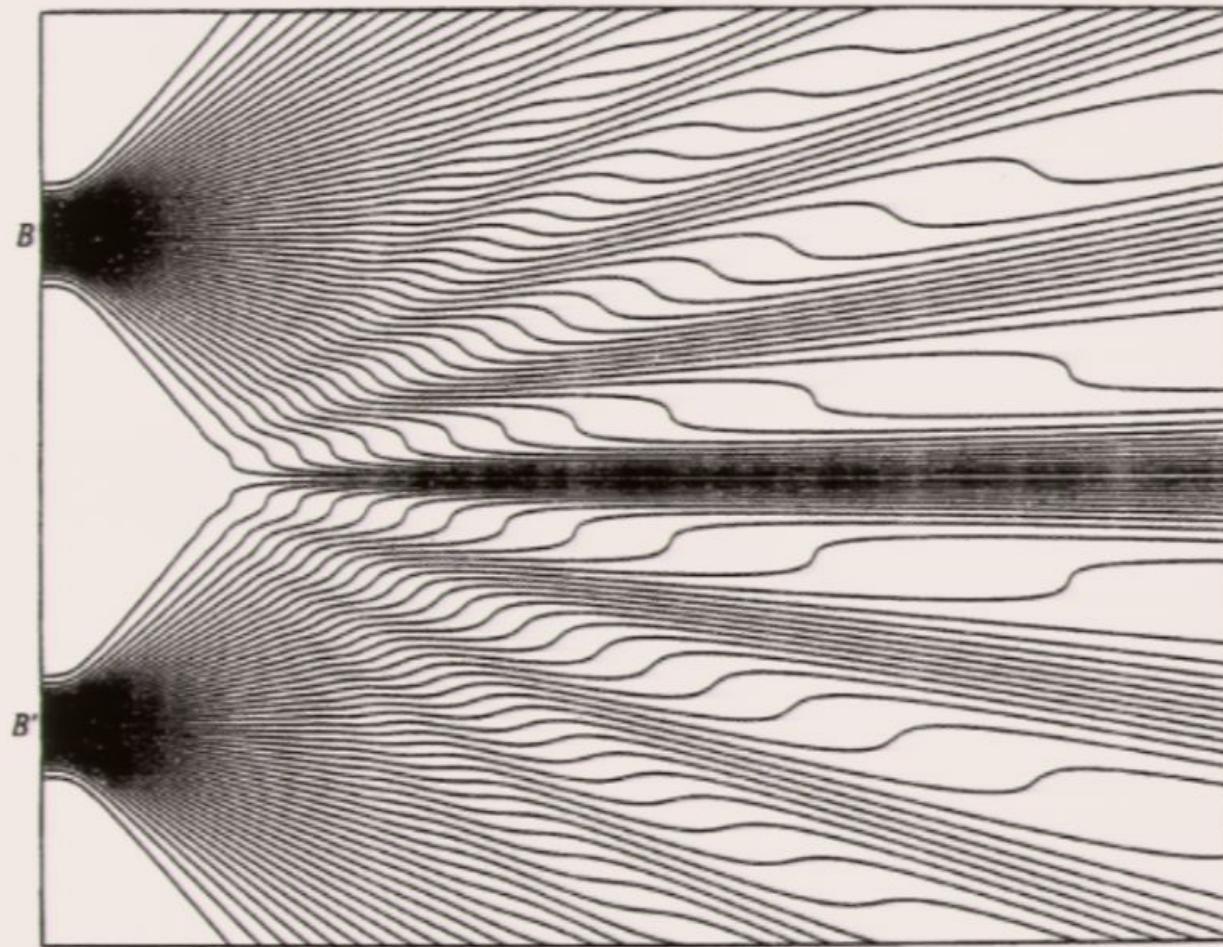
Double slit experiment



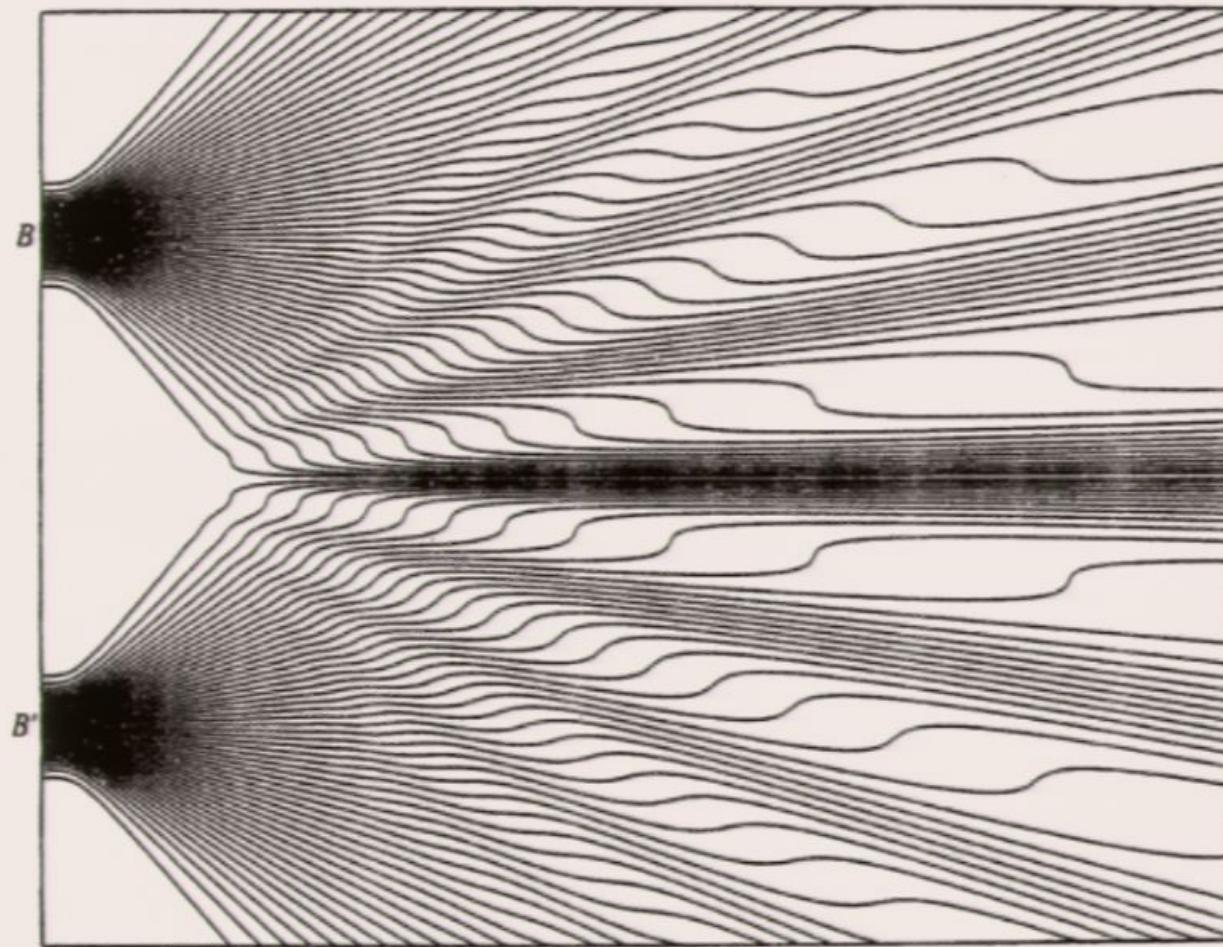
Double slit experiment



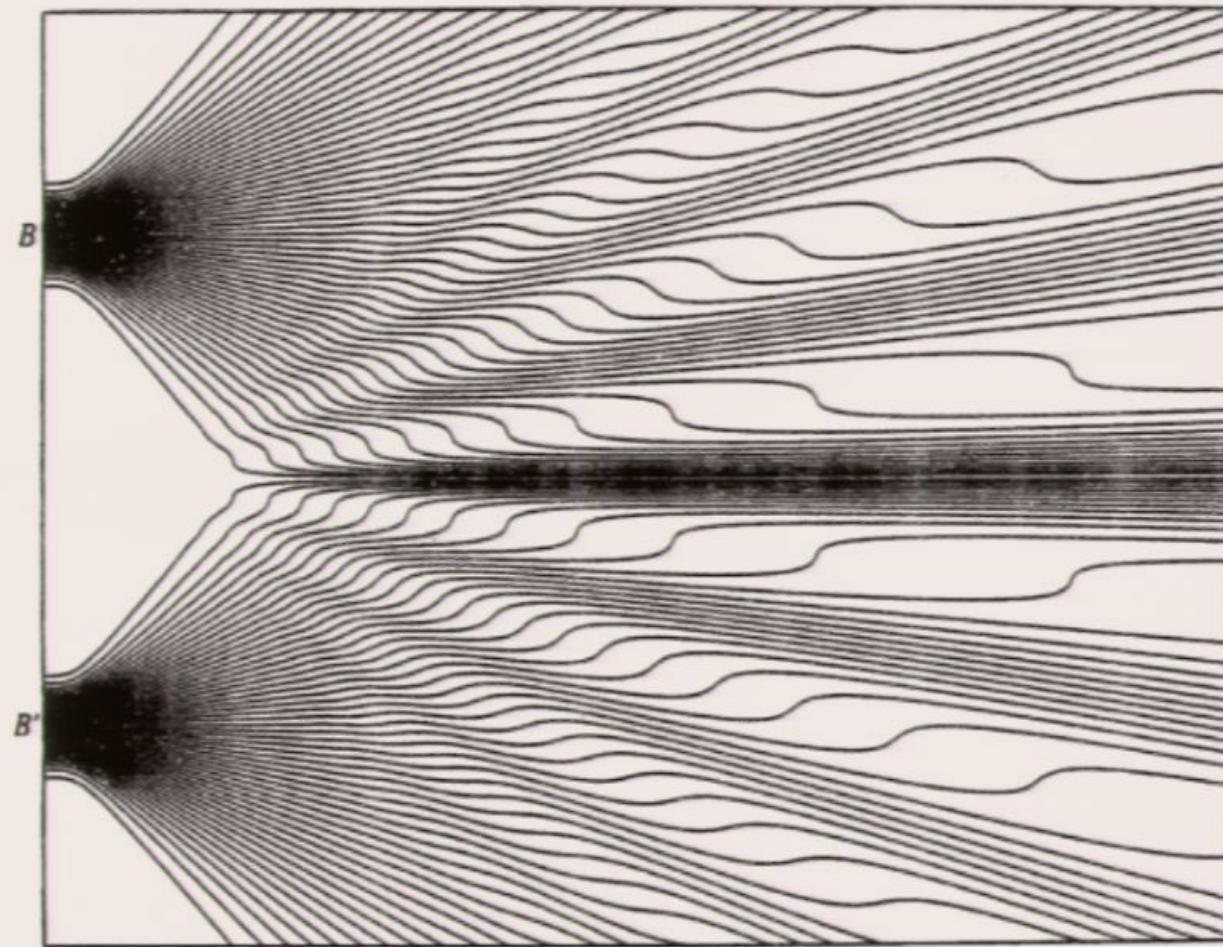
Double slit experiment



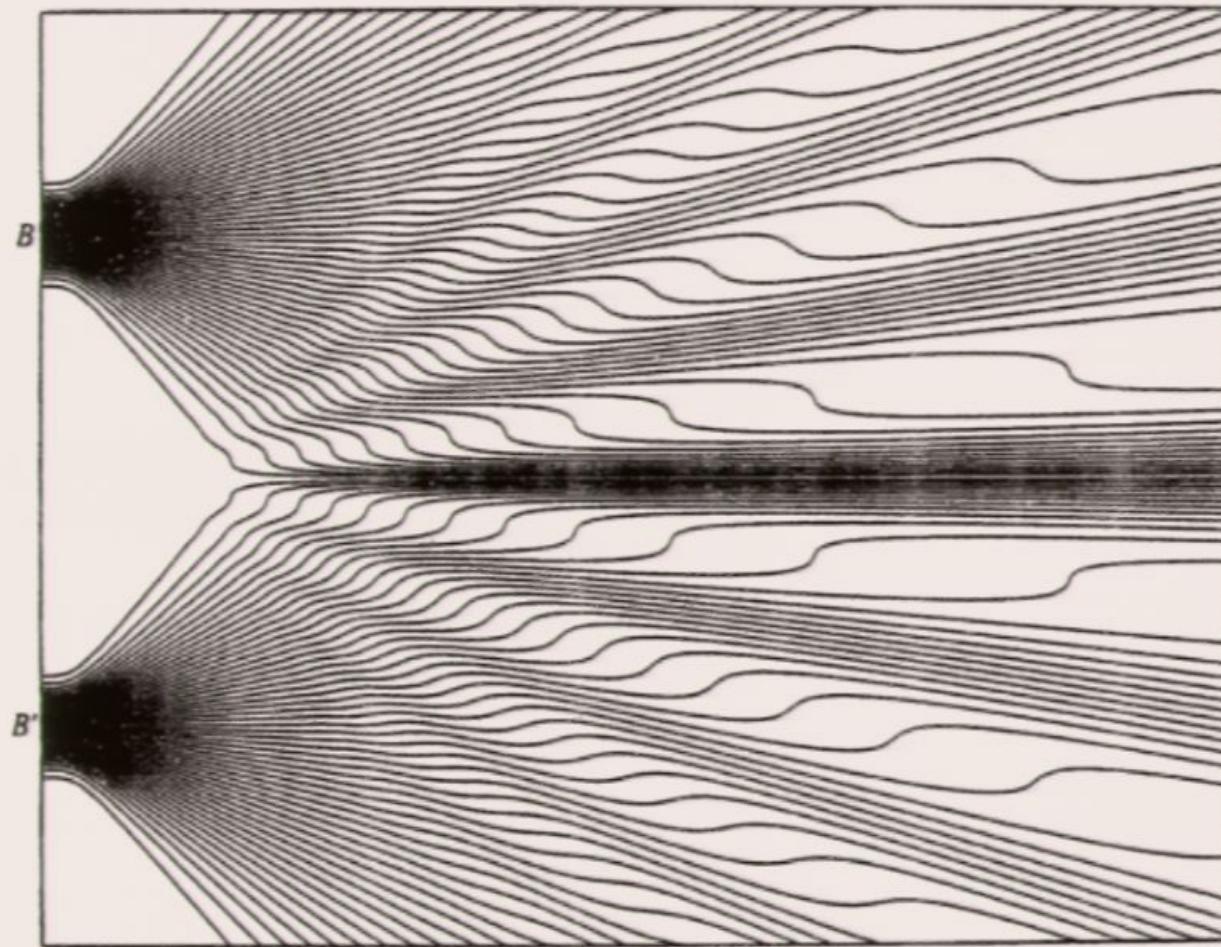
Double slit experiment



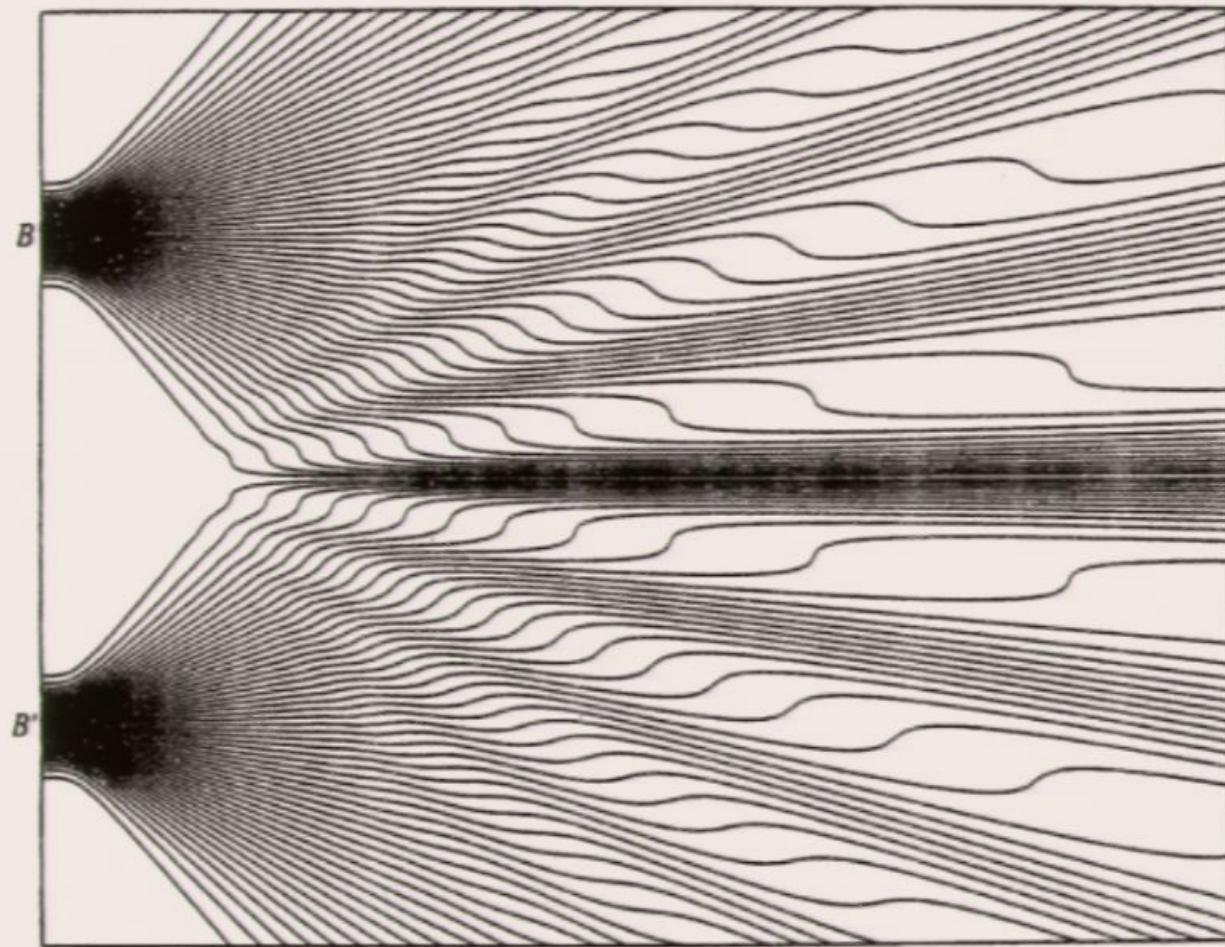
Double slit experiment



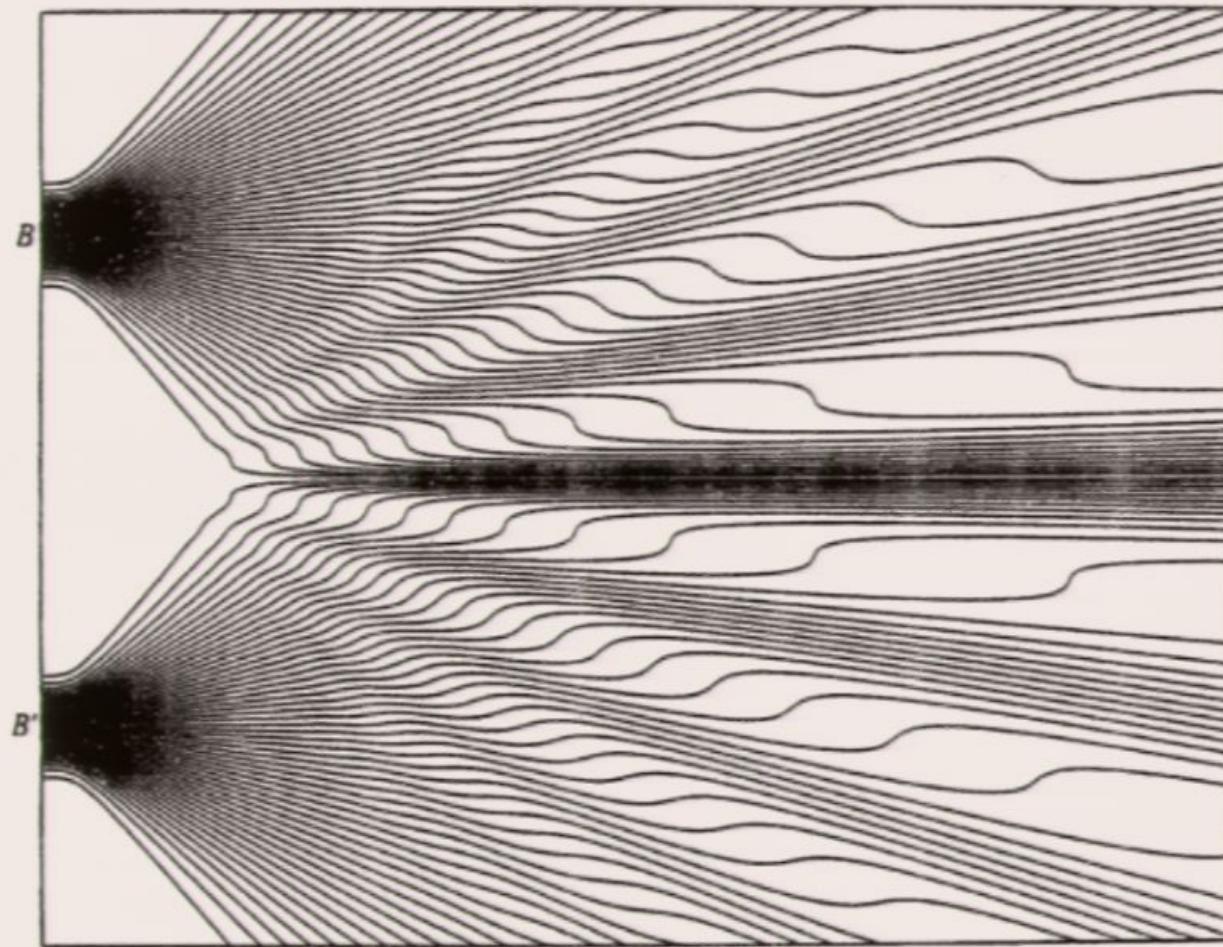
Double slit experiment



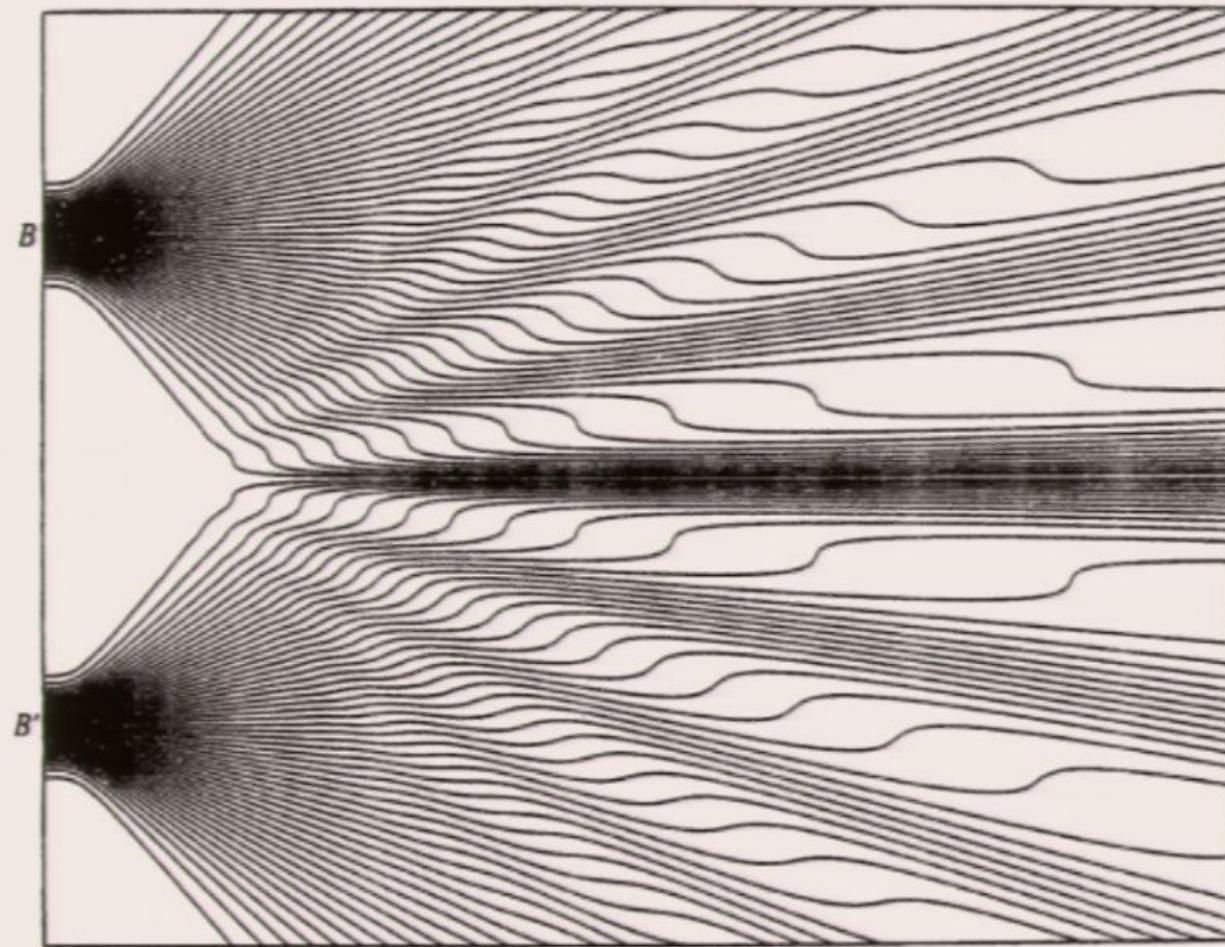
Double slit experiment



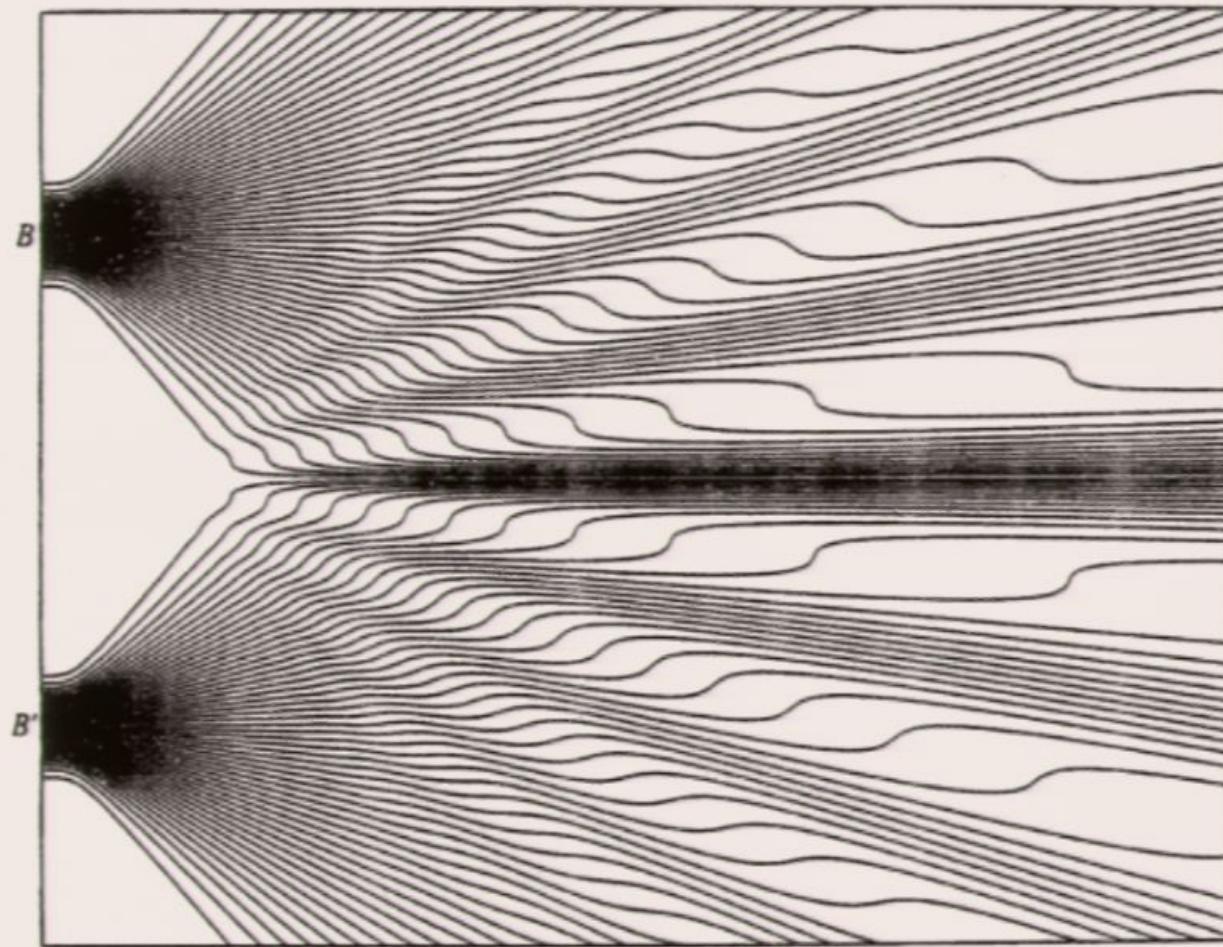
Double slit experiment



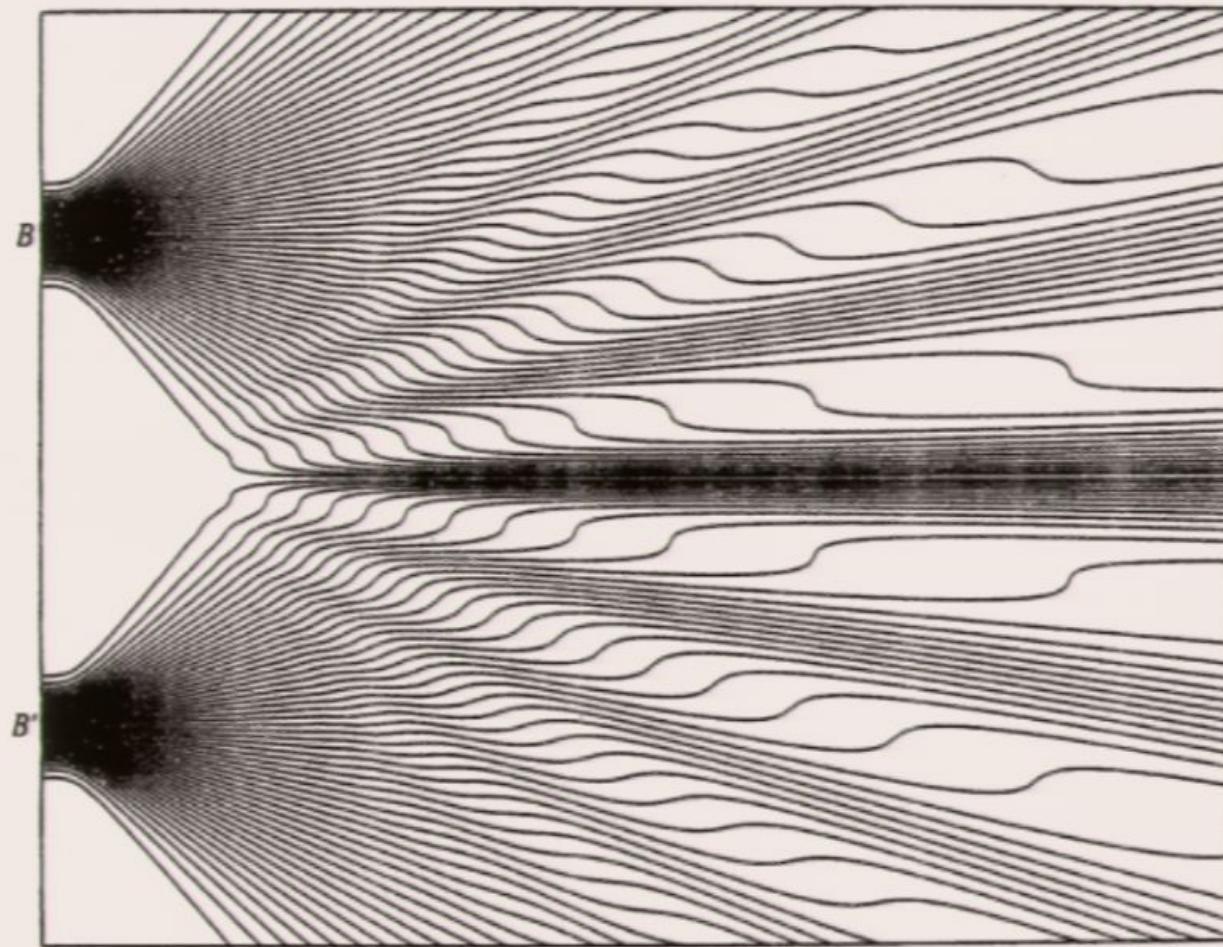
Double slit experiment



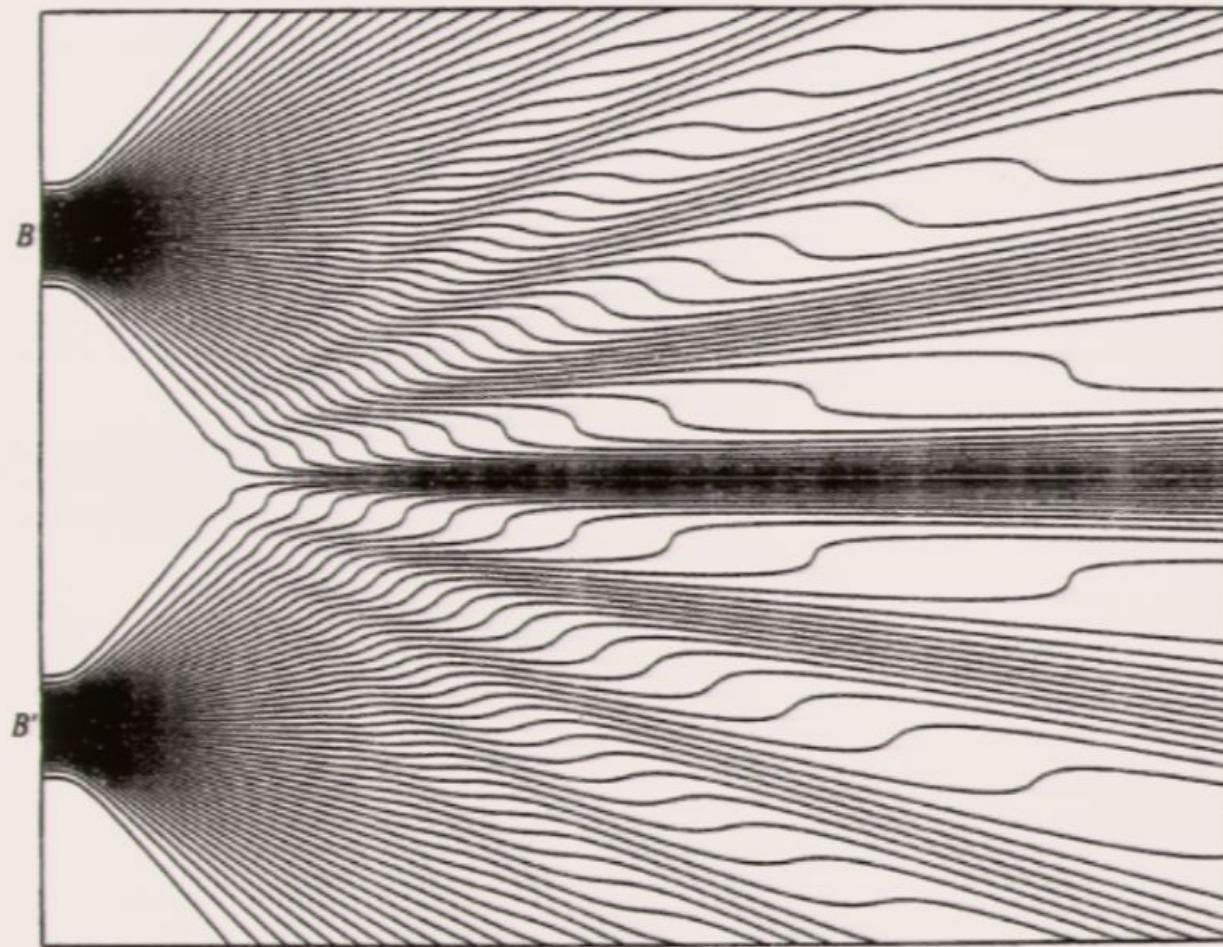
Double slit experiment



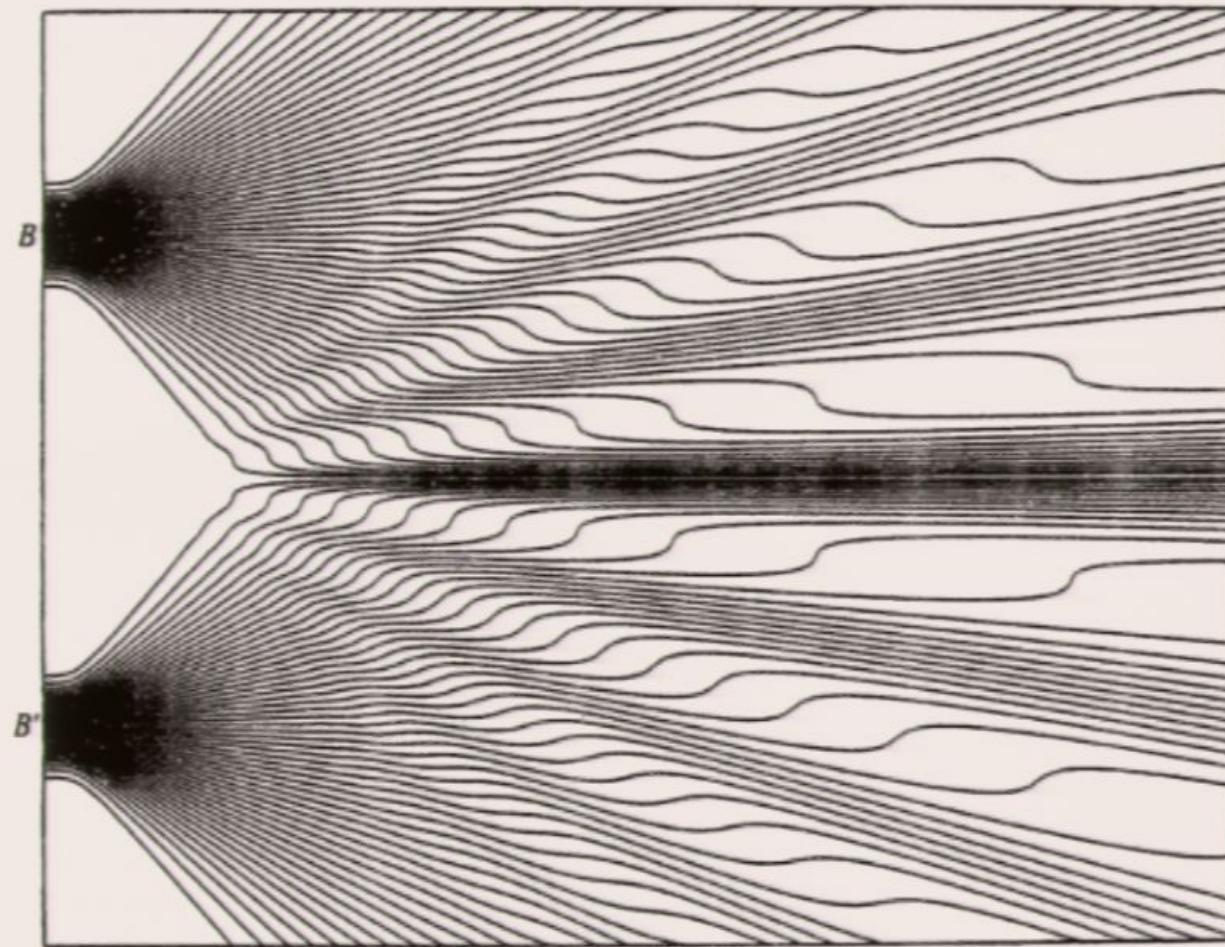
Double slit experiment



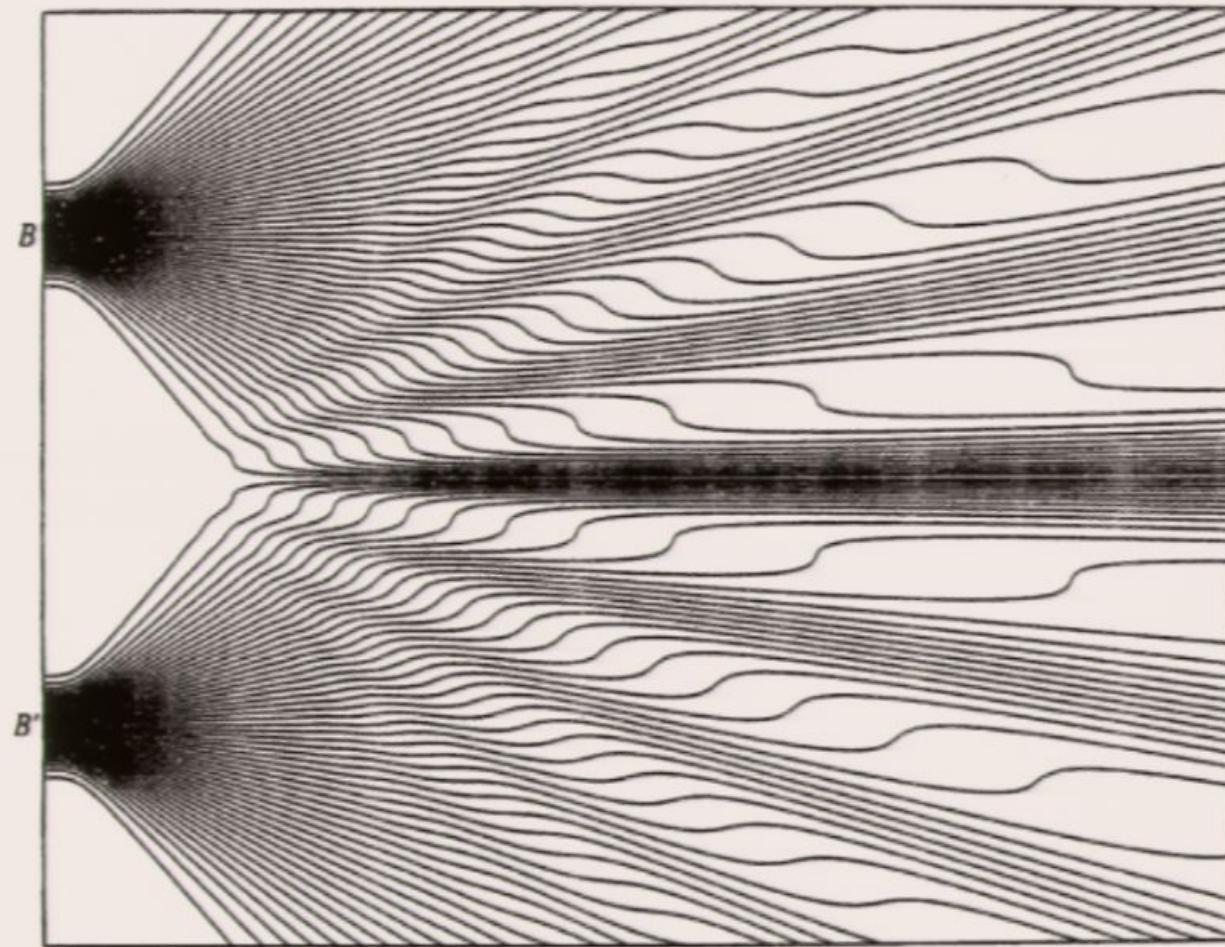
Double slit experiment



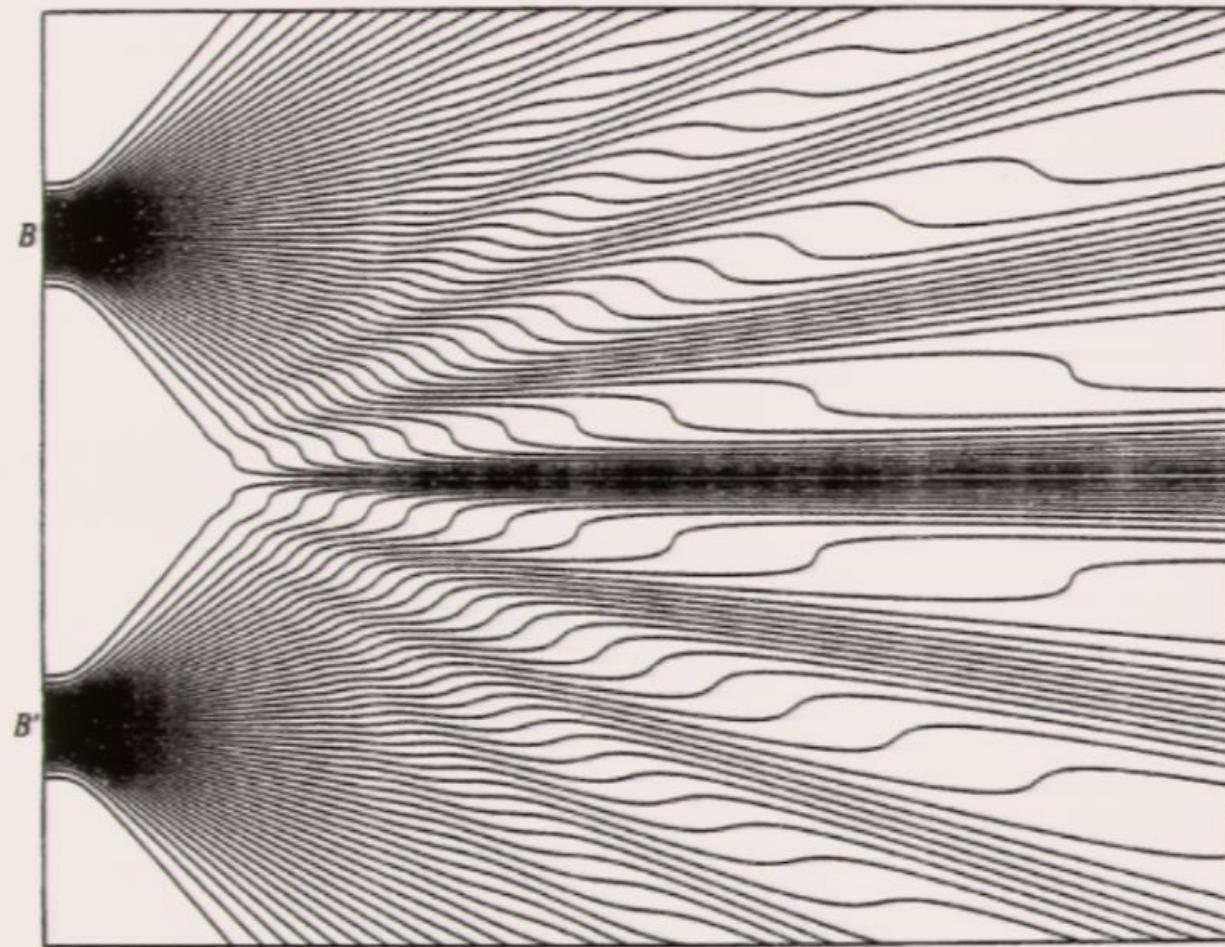
Double slit experiment



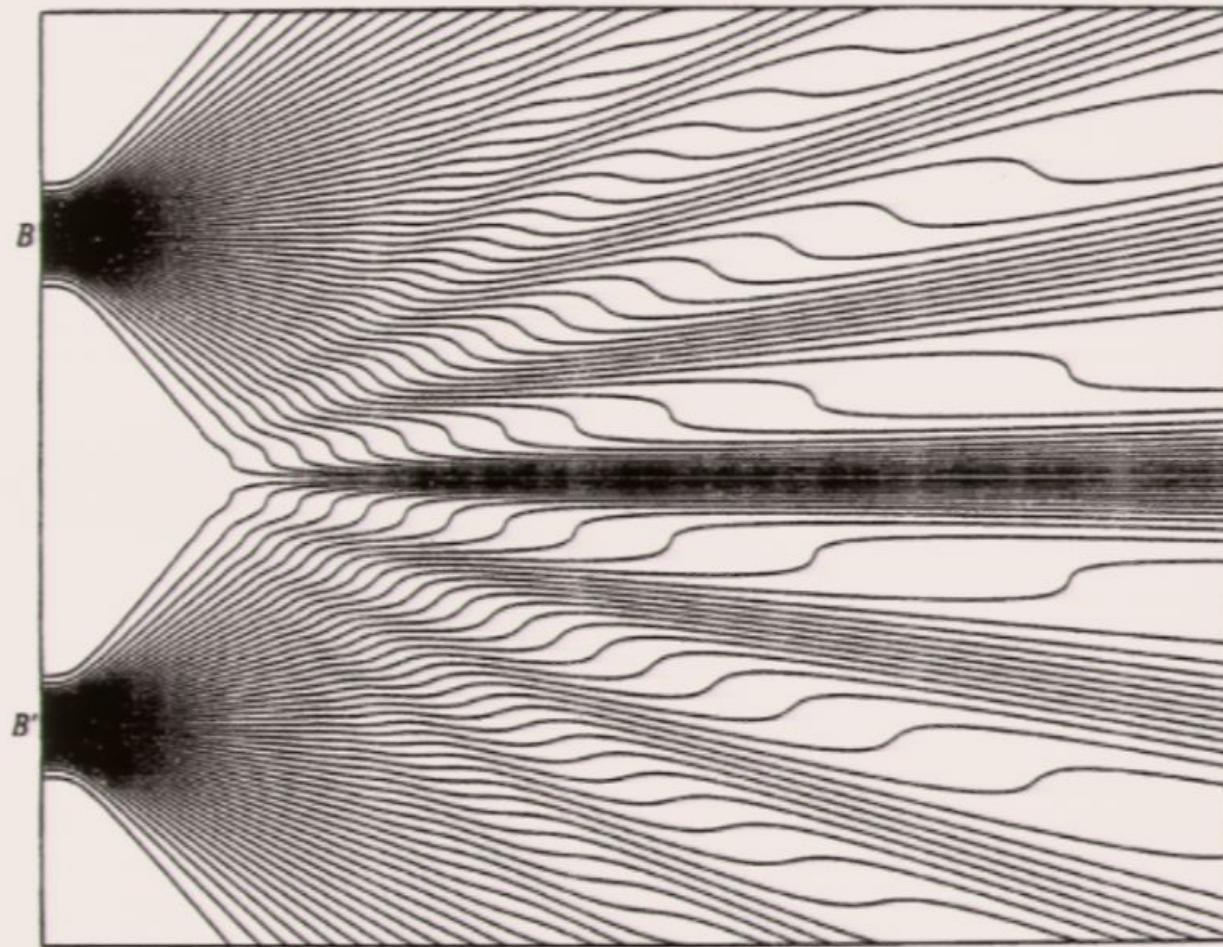
Double slit experiment



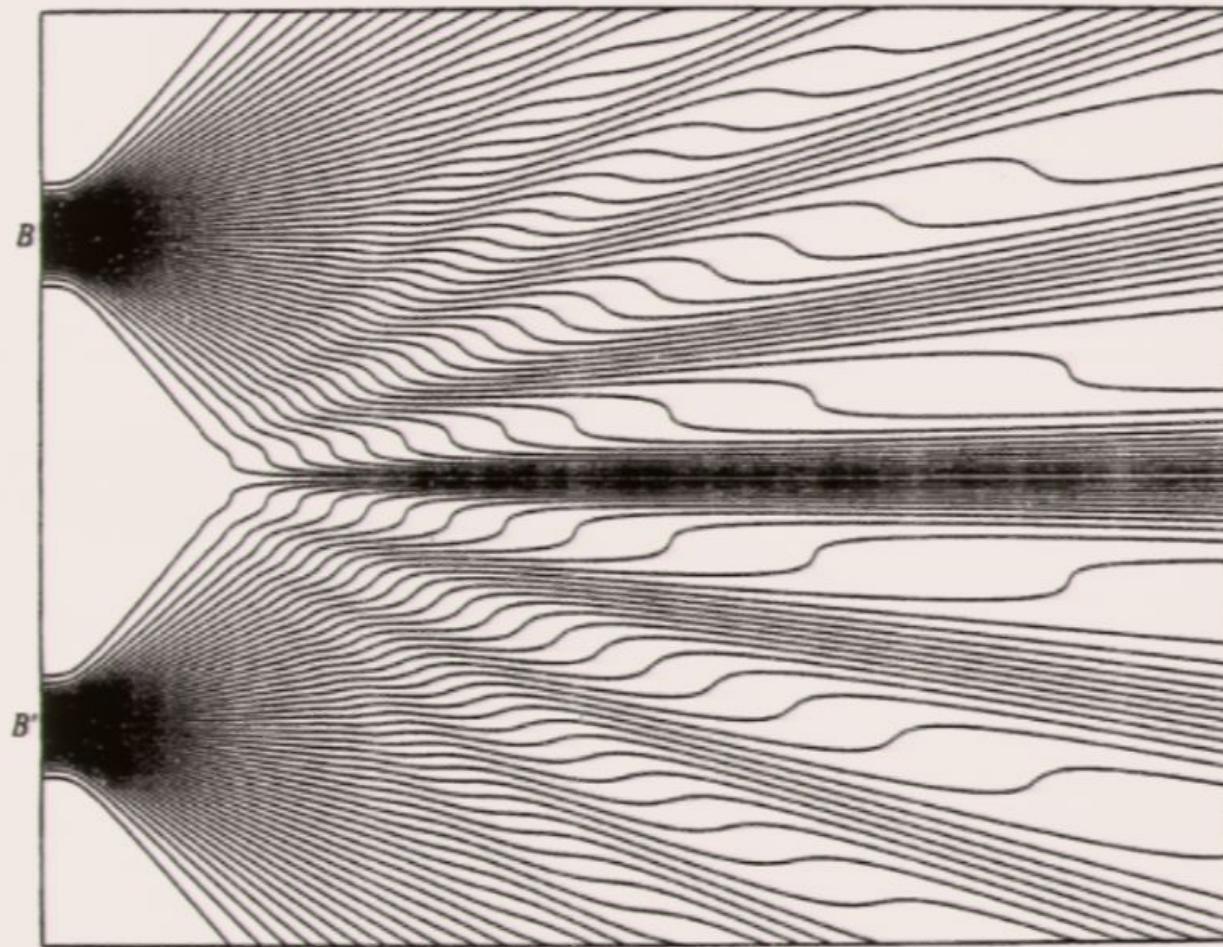
Double slit experiment



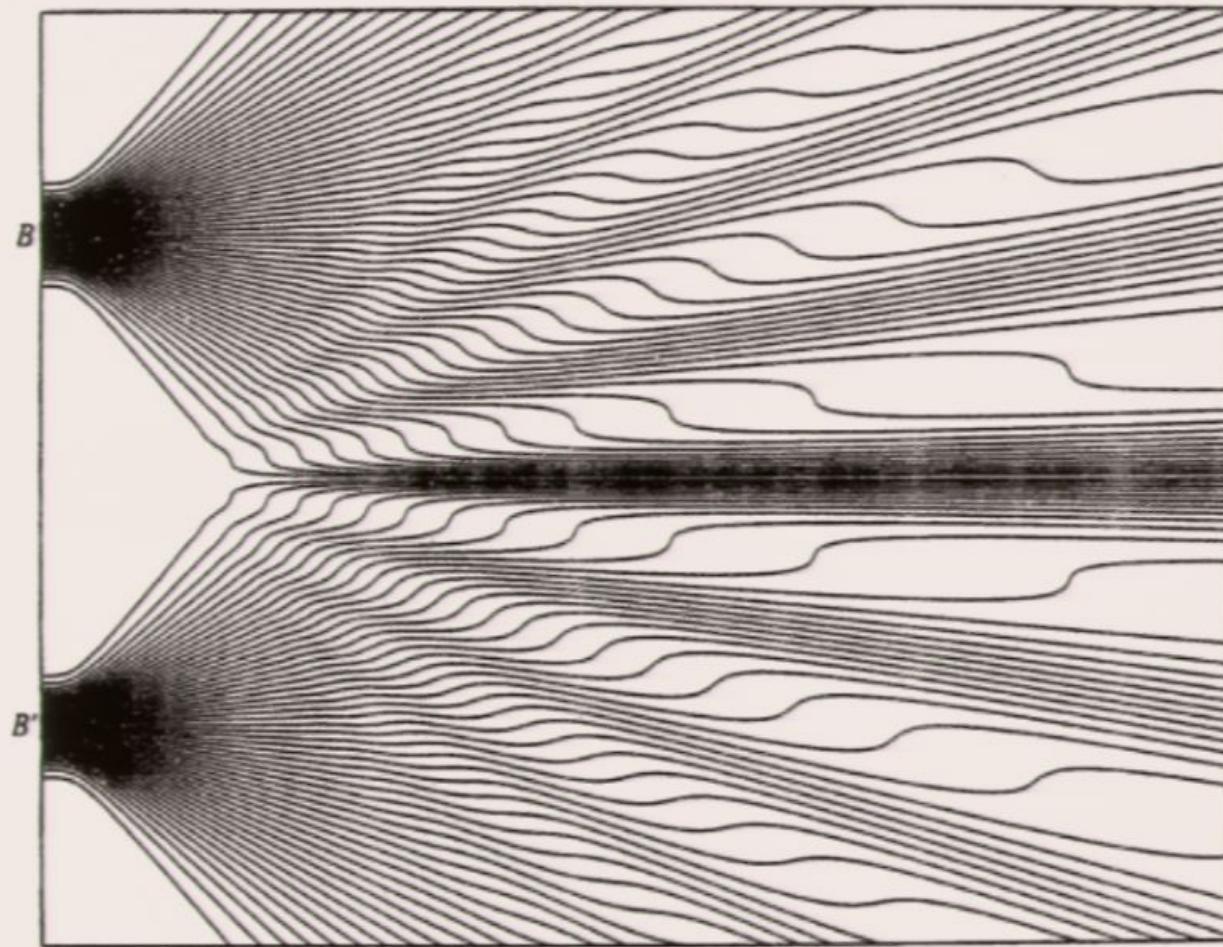
Double slit experiment



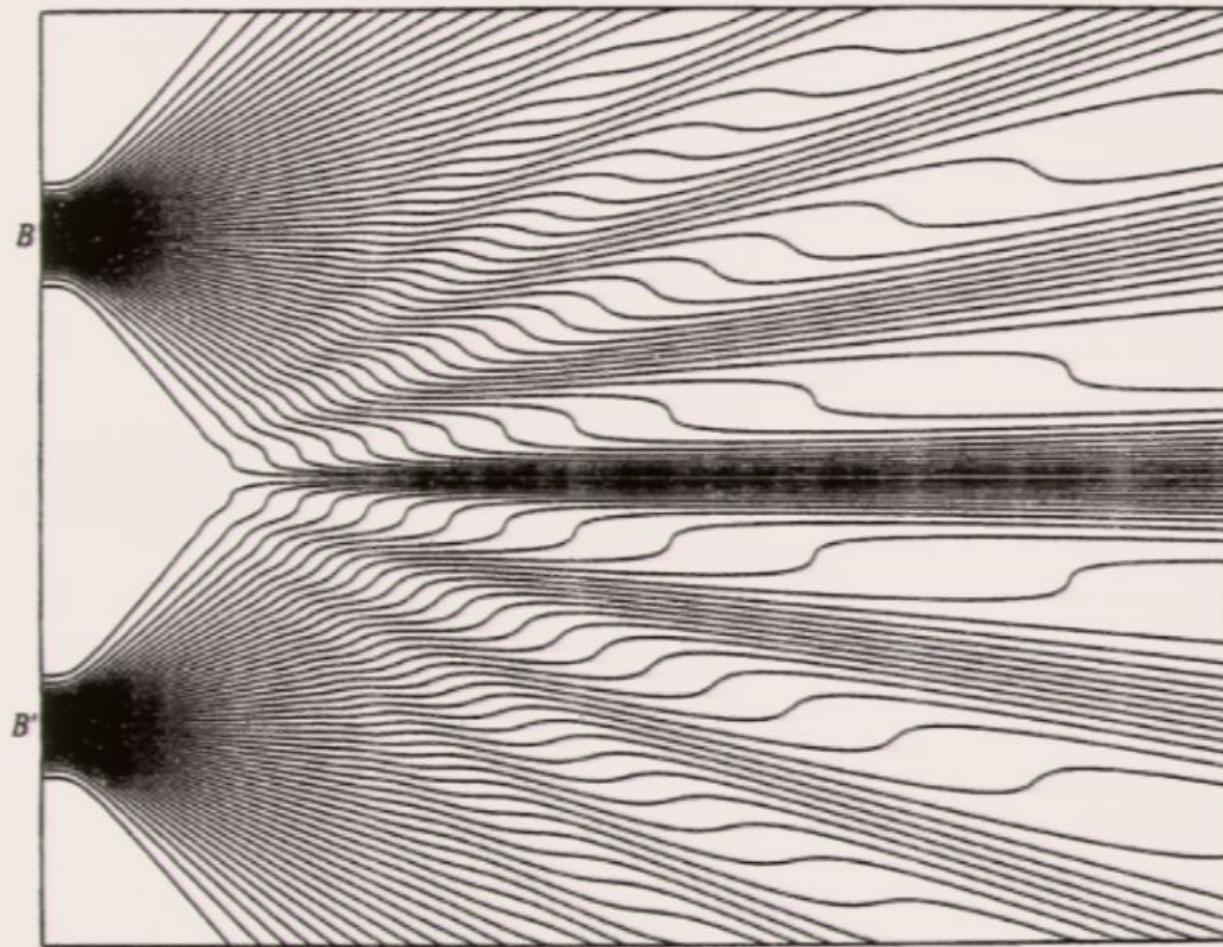
Double slit experiment



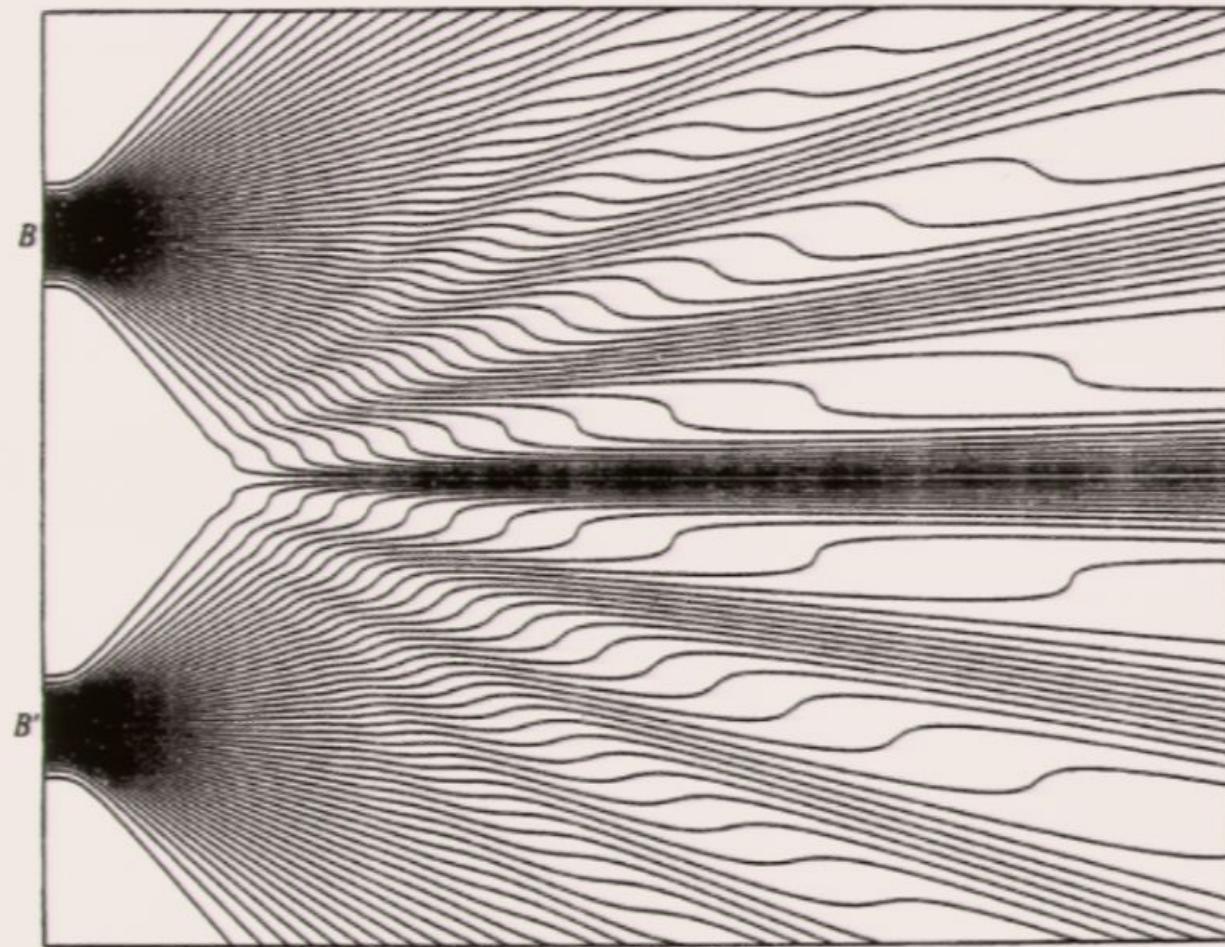
Double slit experiment



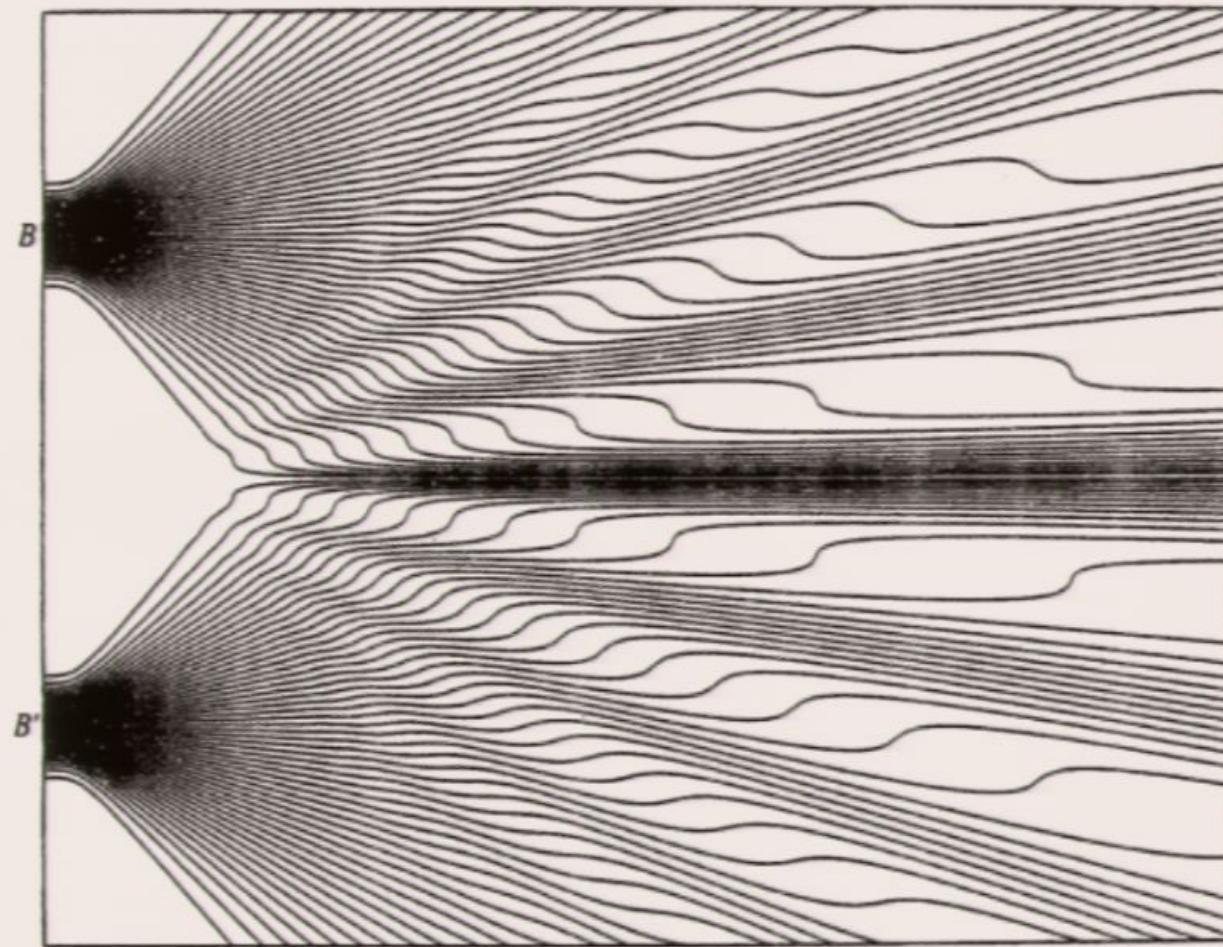
Double slit experiment



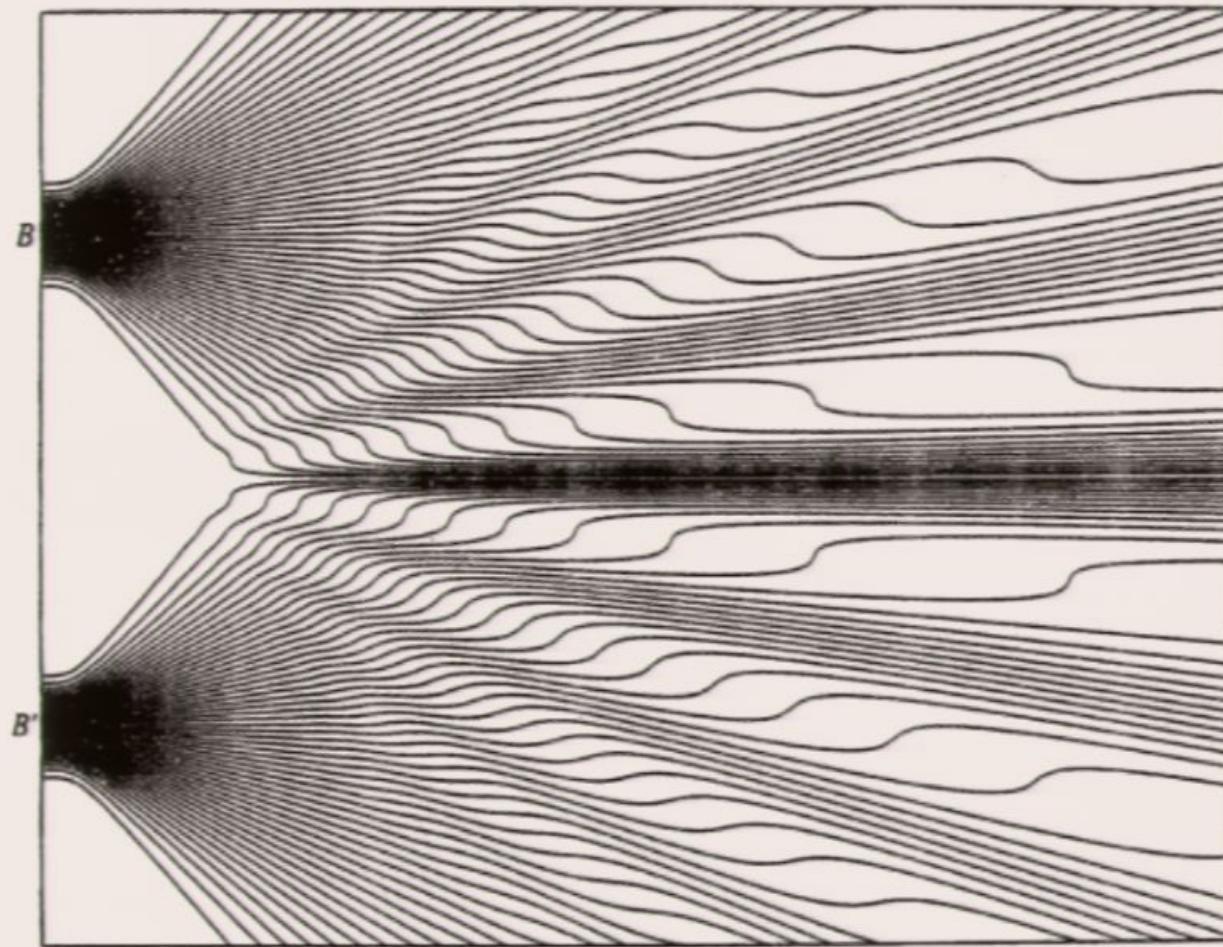
Double slit experiment



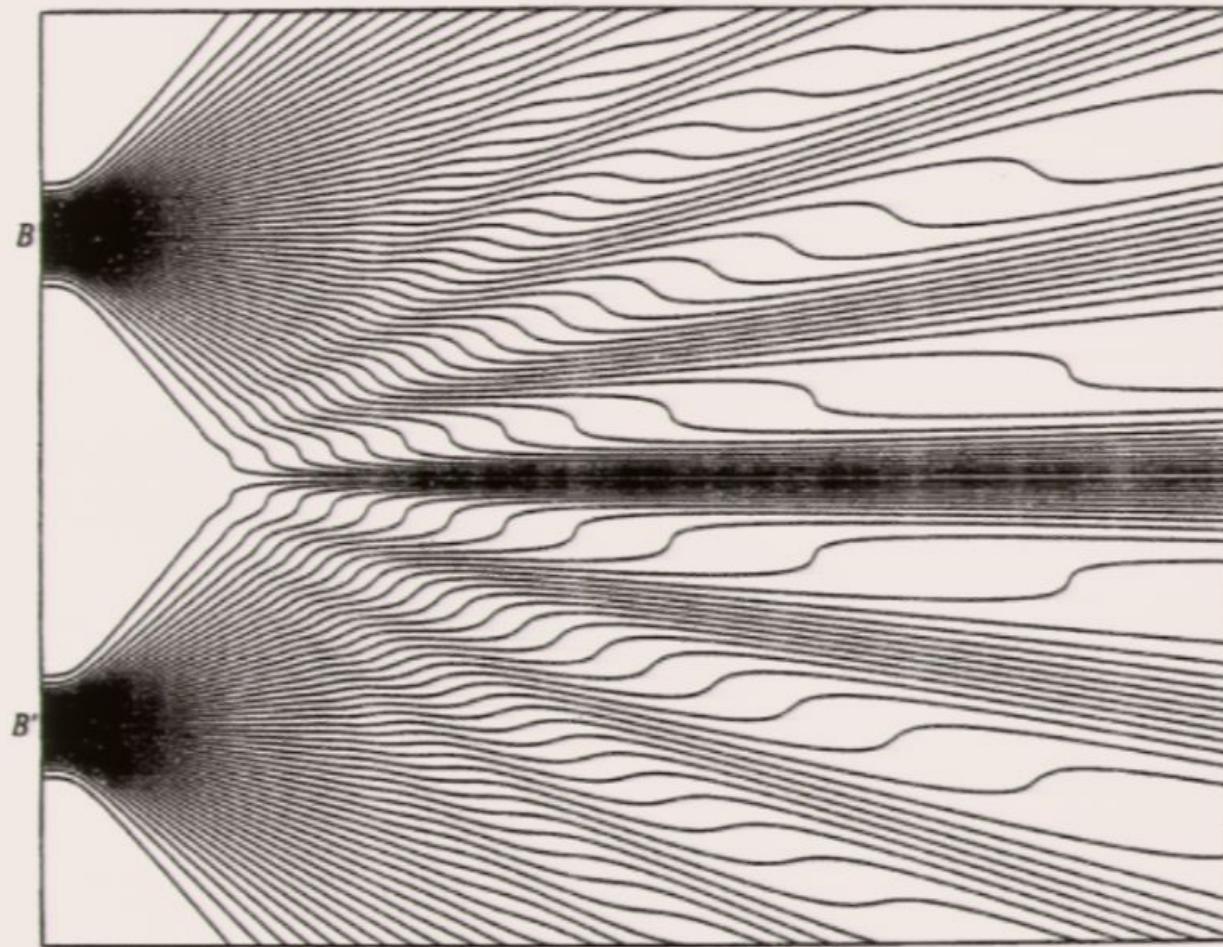
Double slit experiment



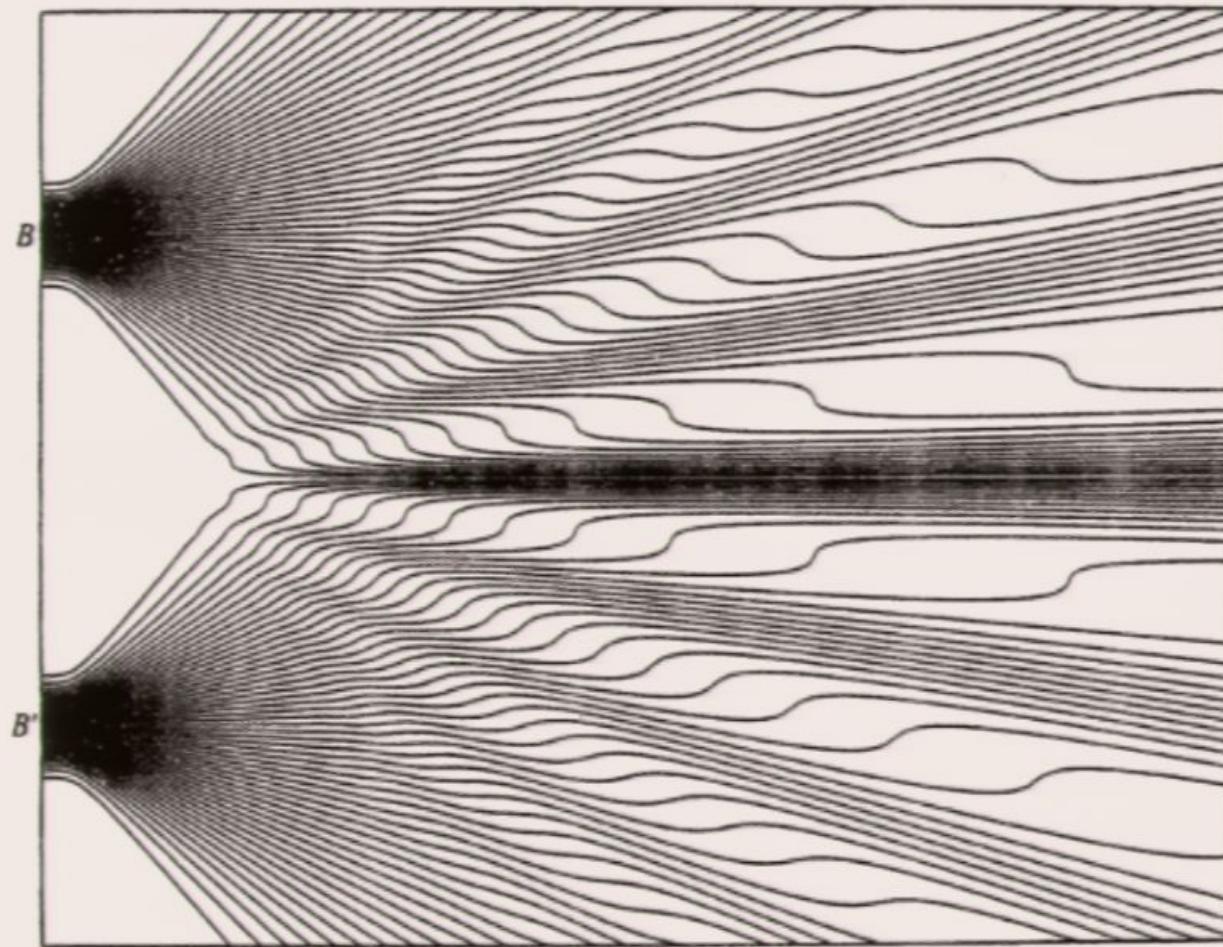
Double slit experiment



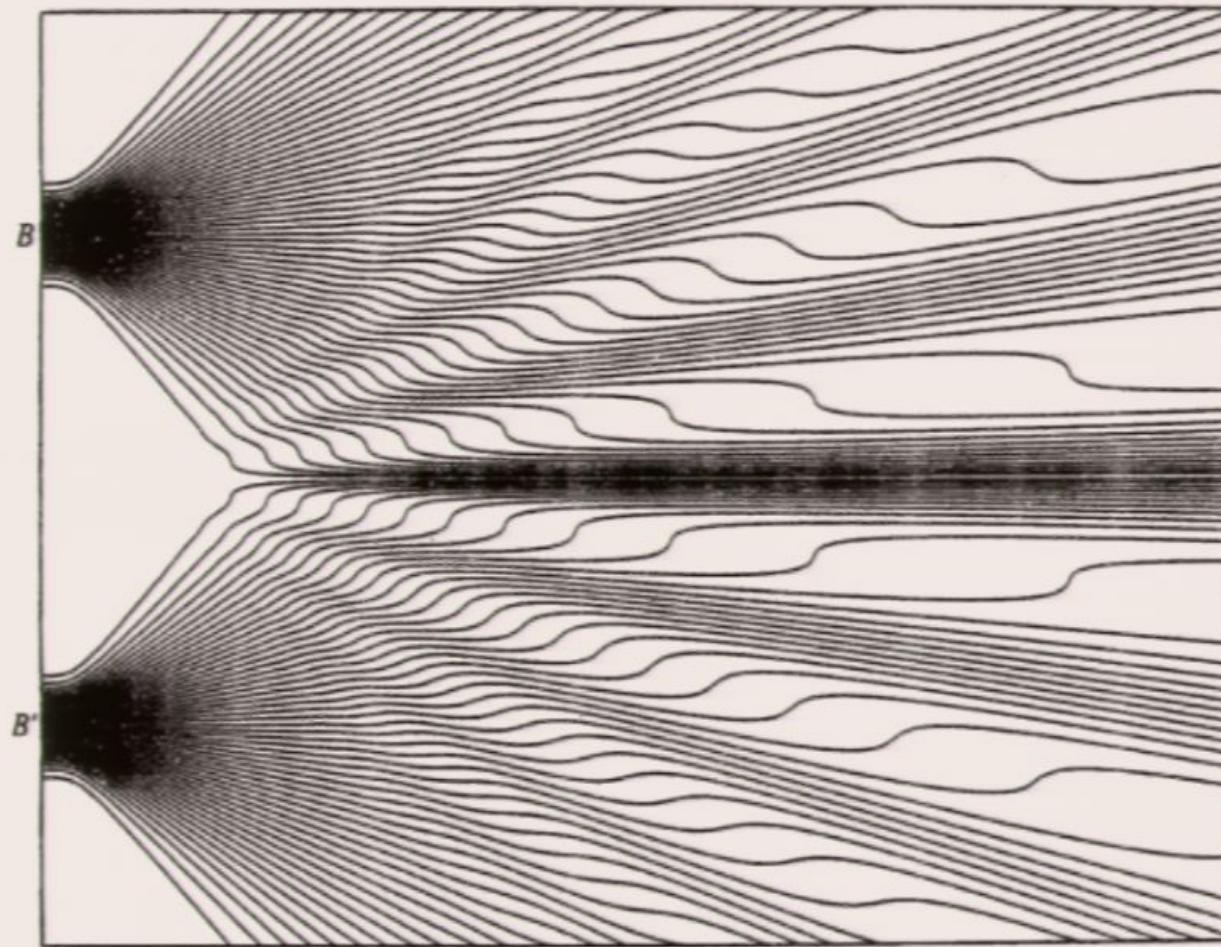
Double slit experiment



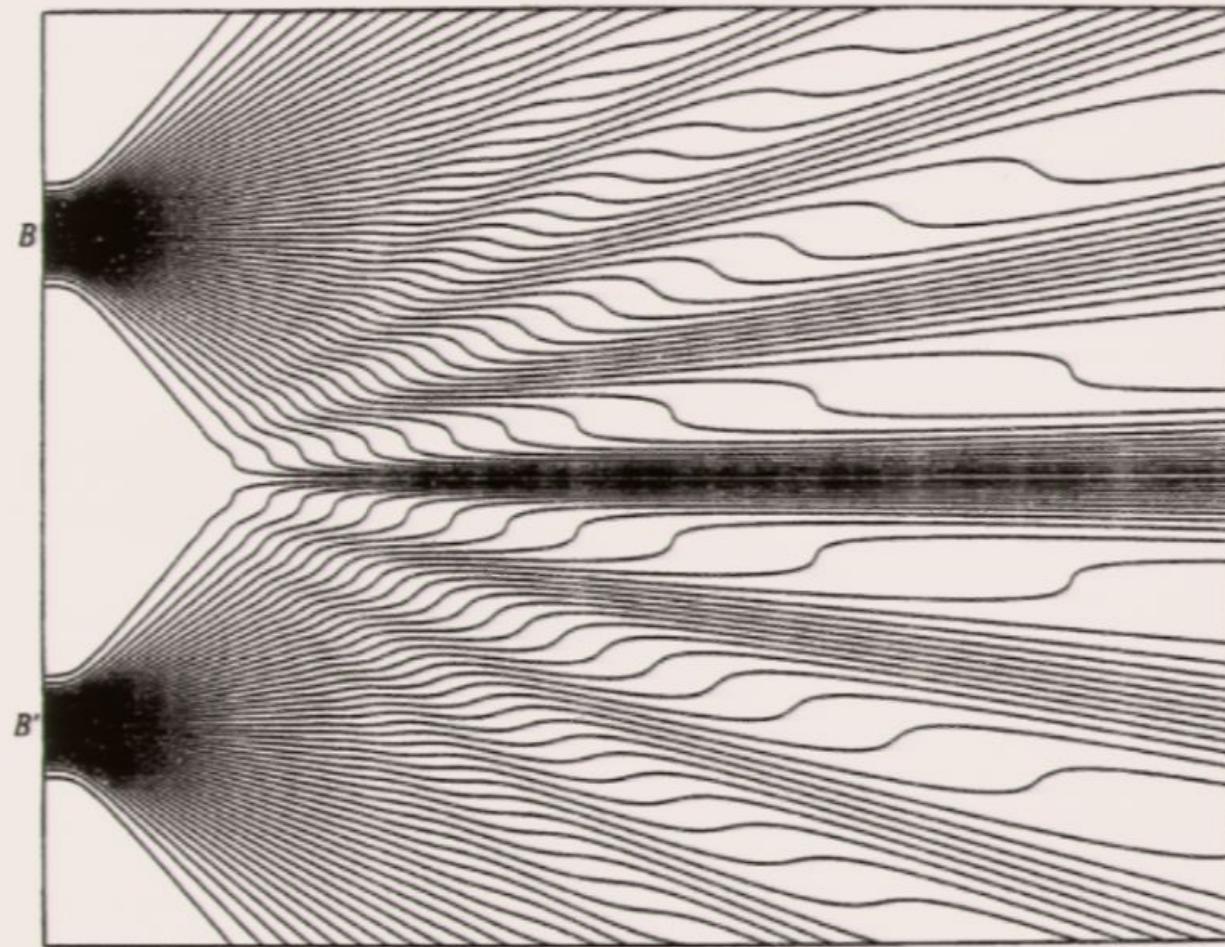
Double slit experiment



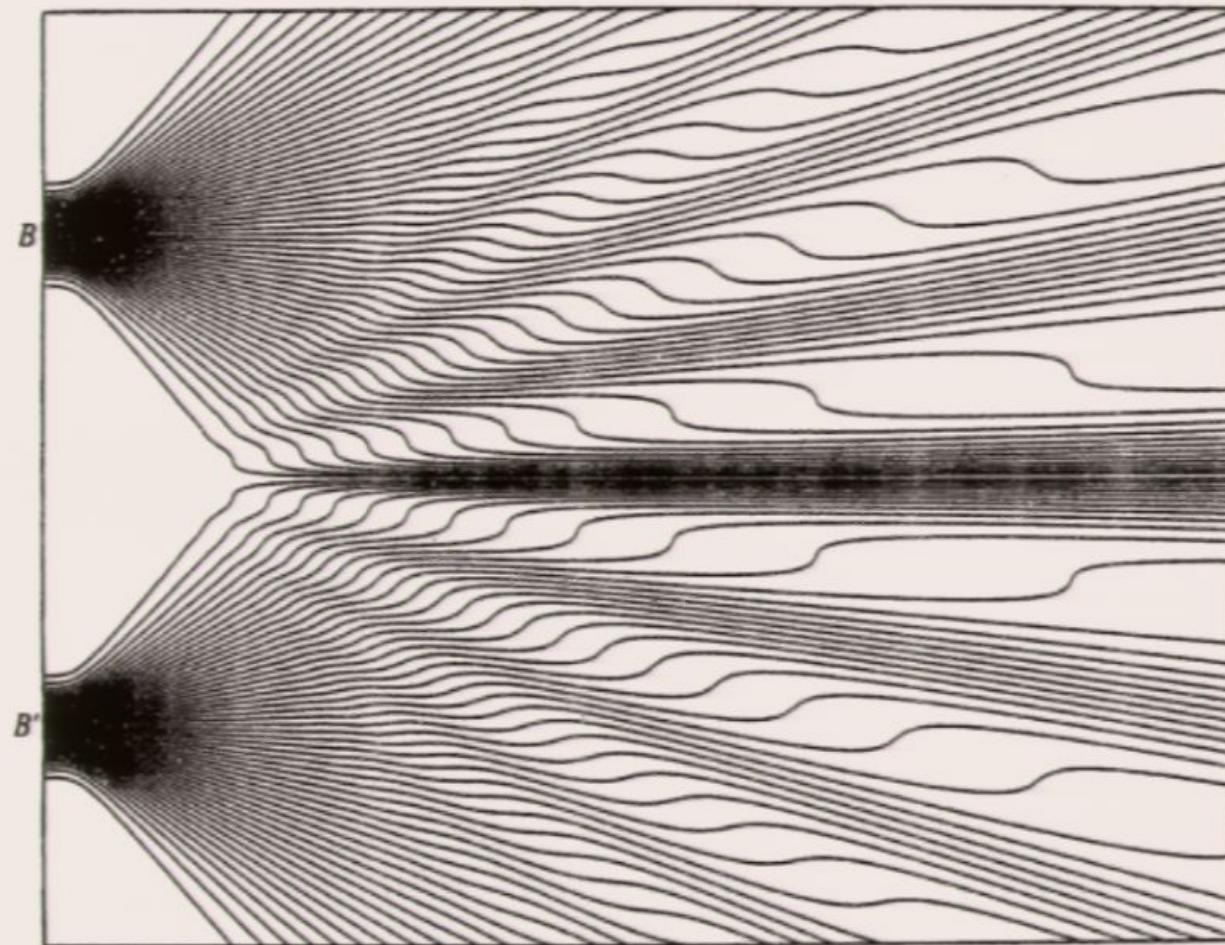
Double slit experiment



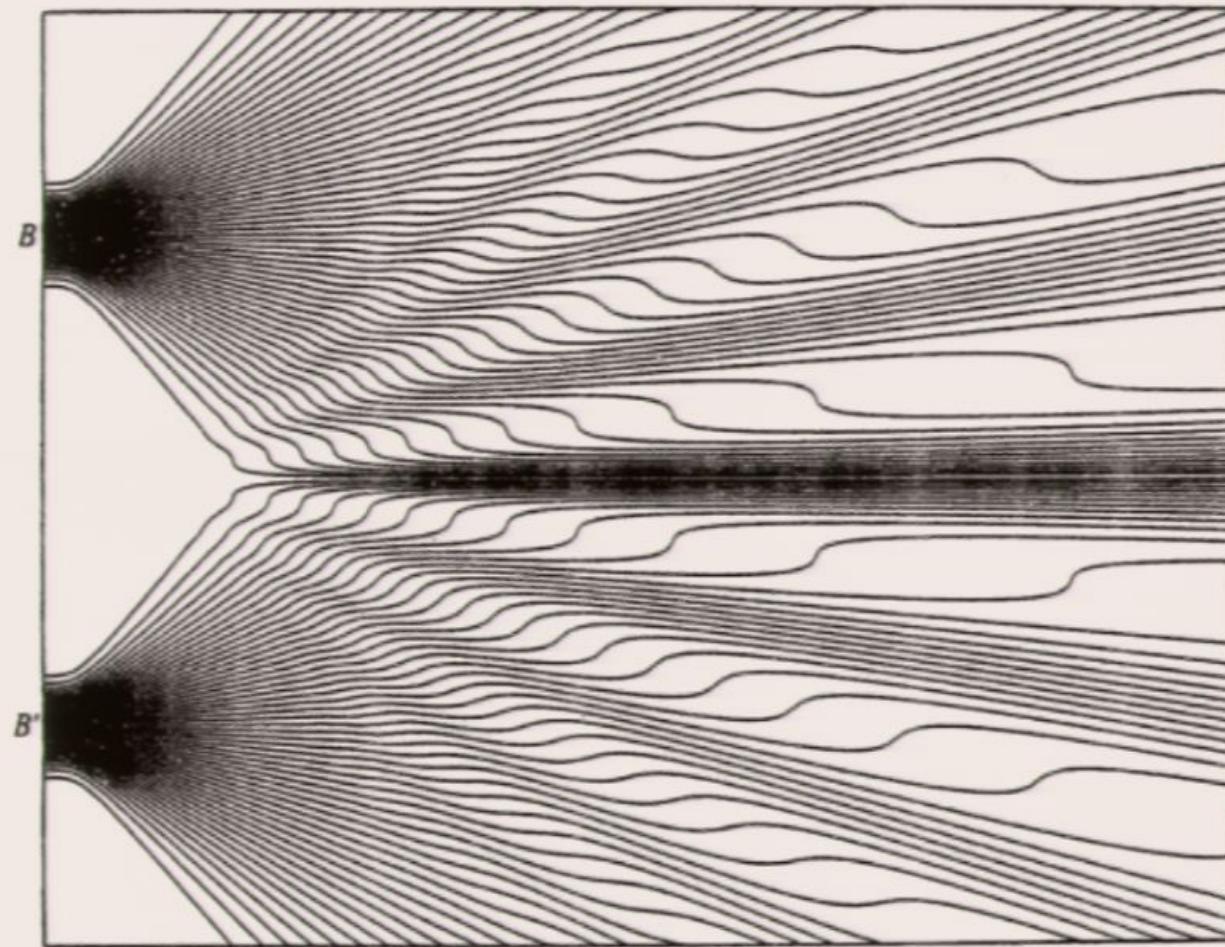
Double slit experiment



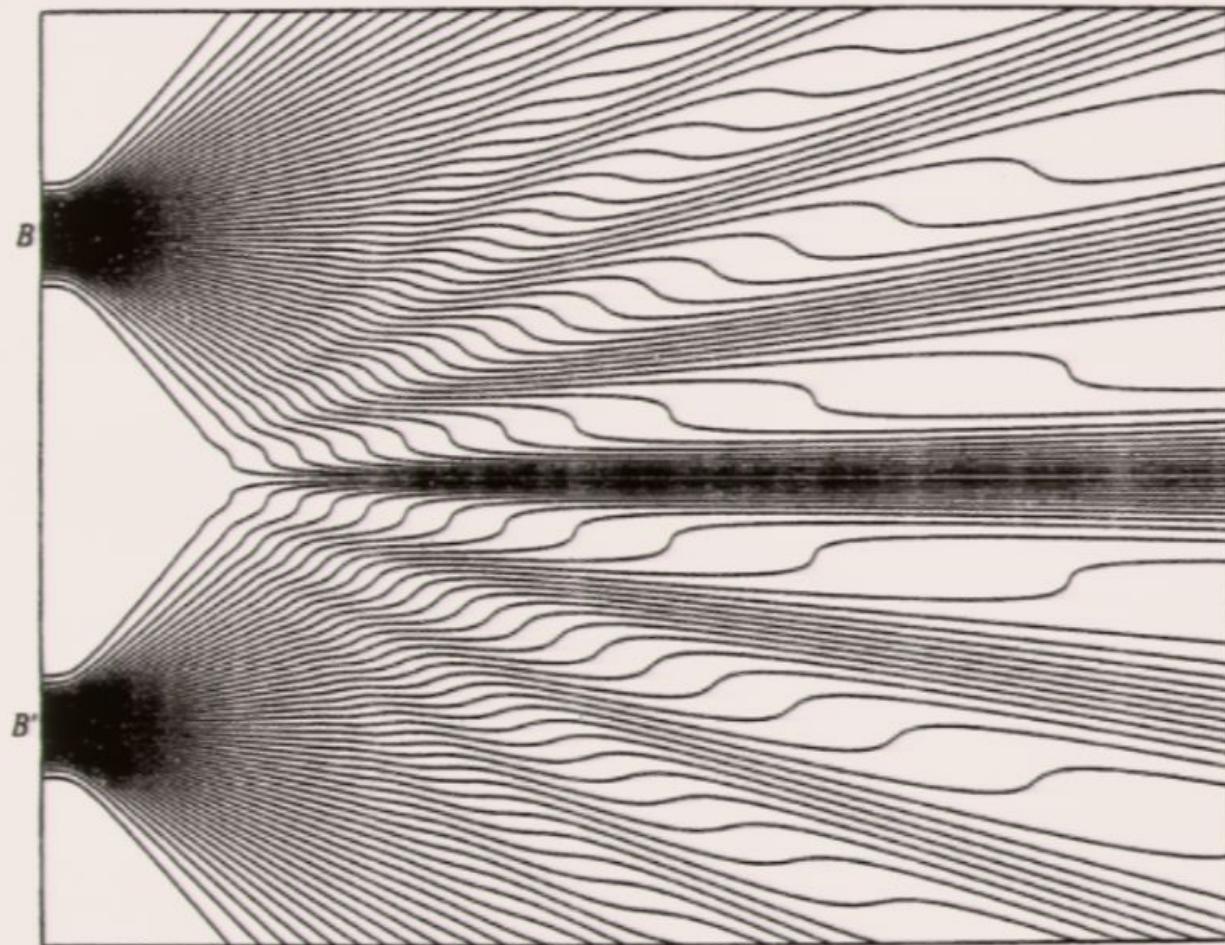
Double slit experiment



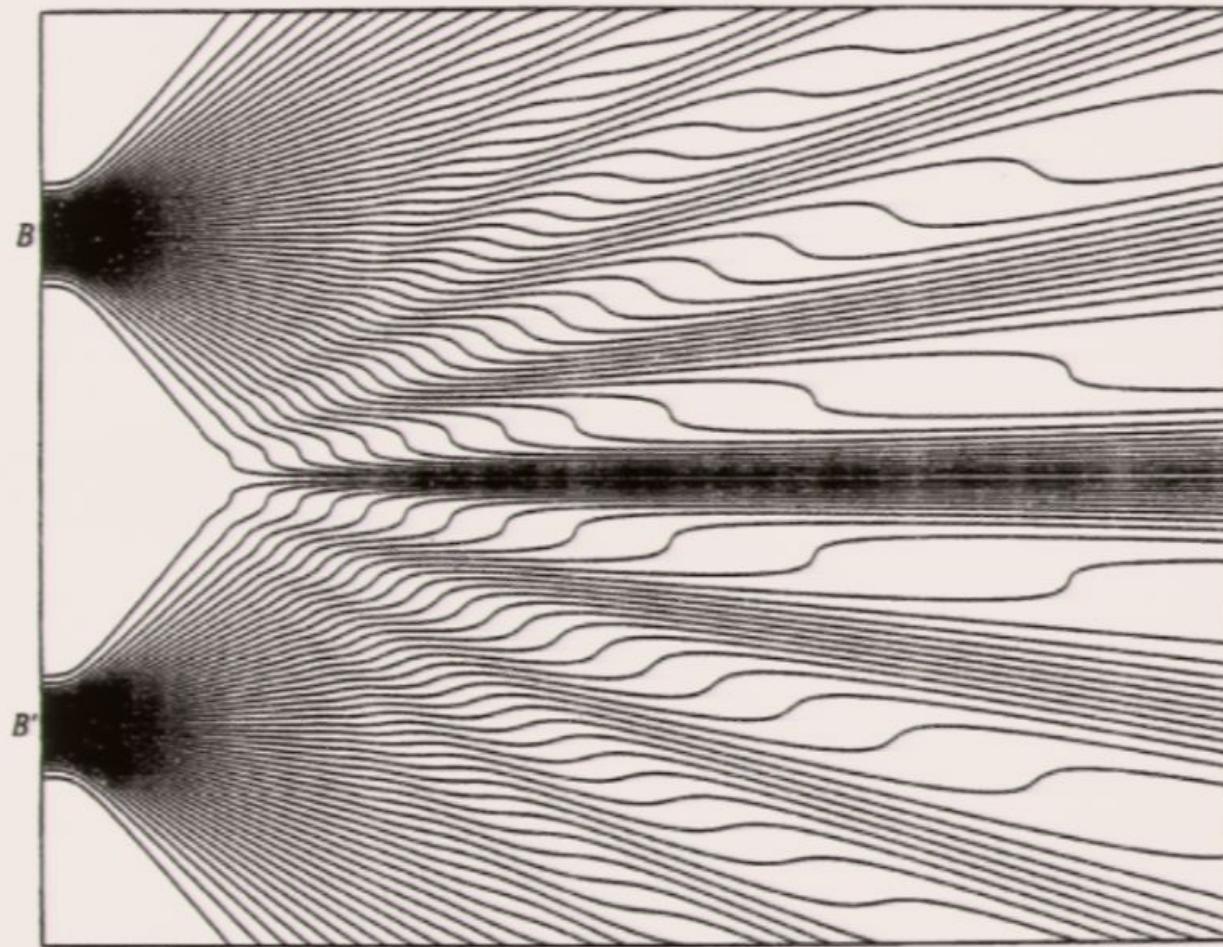
Double slit experiment



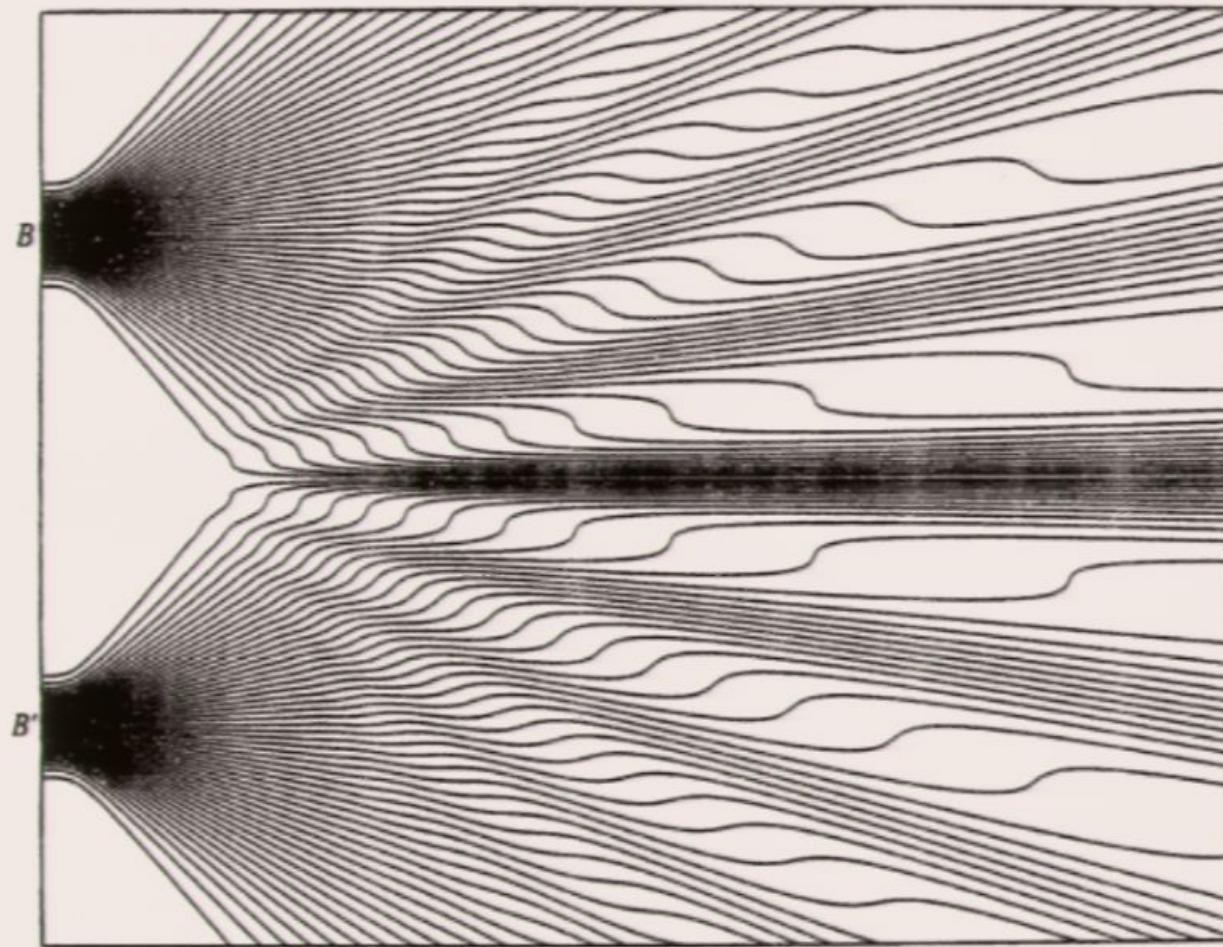
Double slit experiment



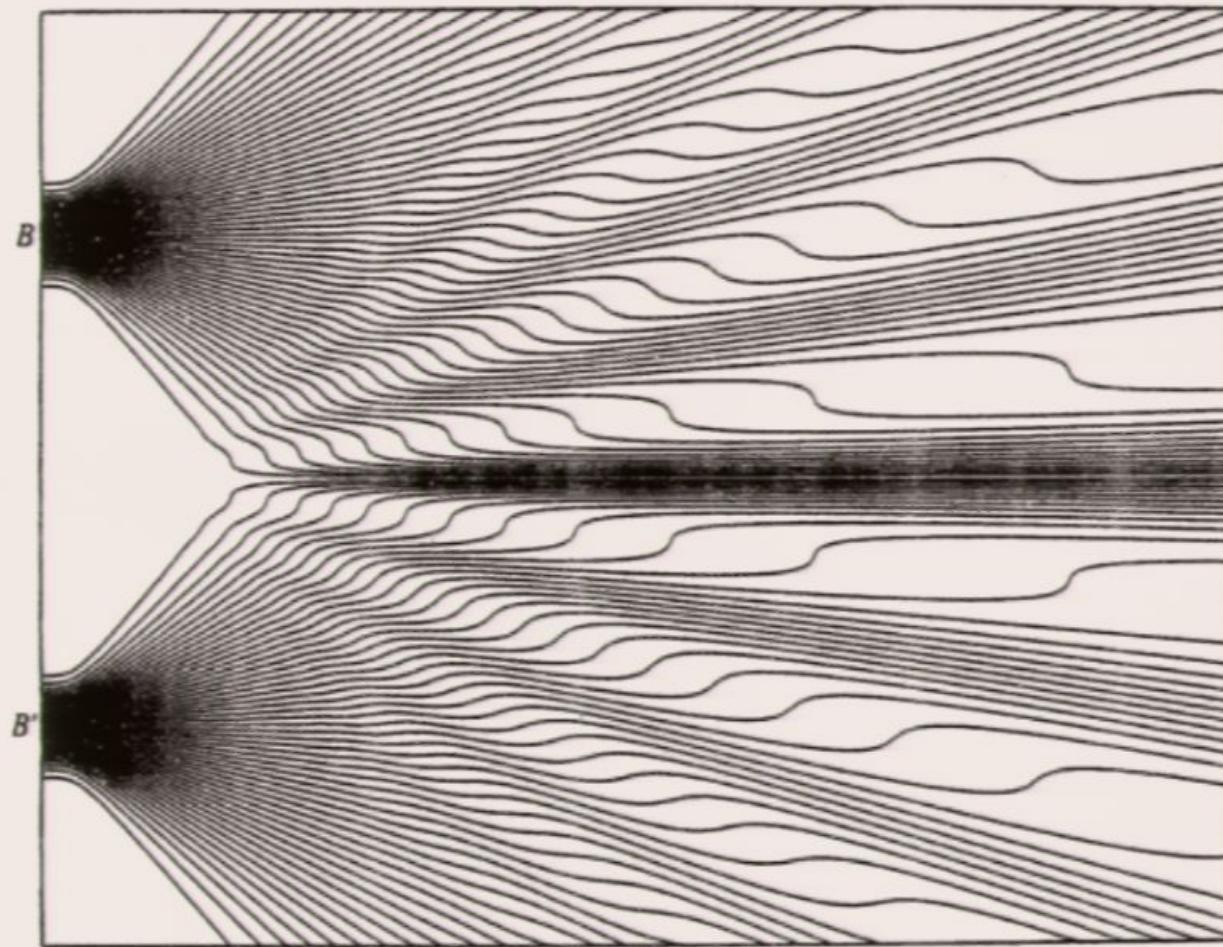
Double slit experiment



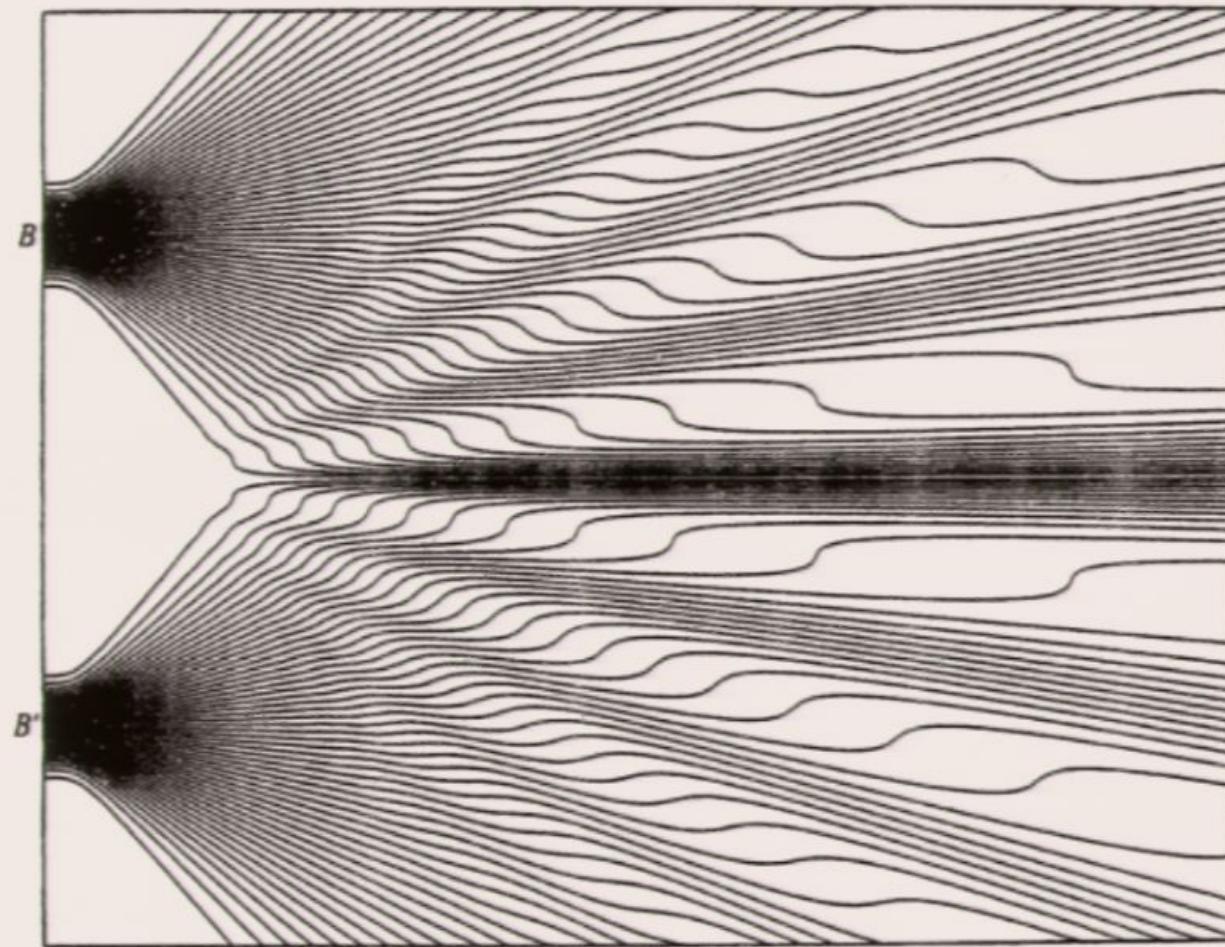
Double slit experiment



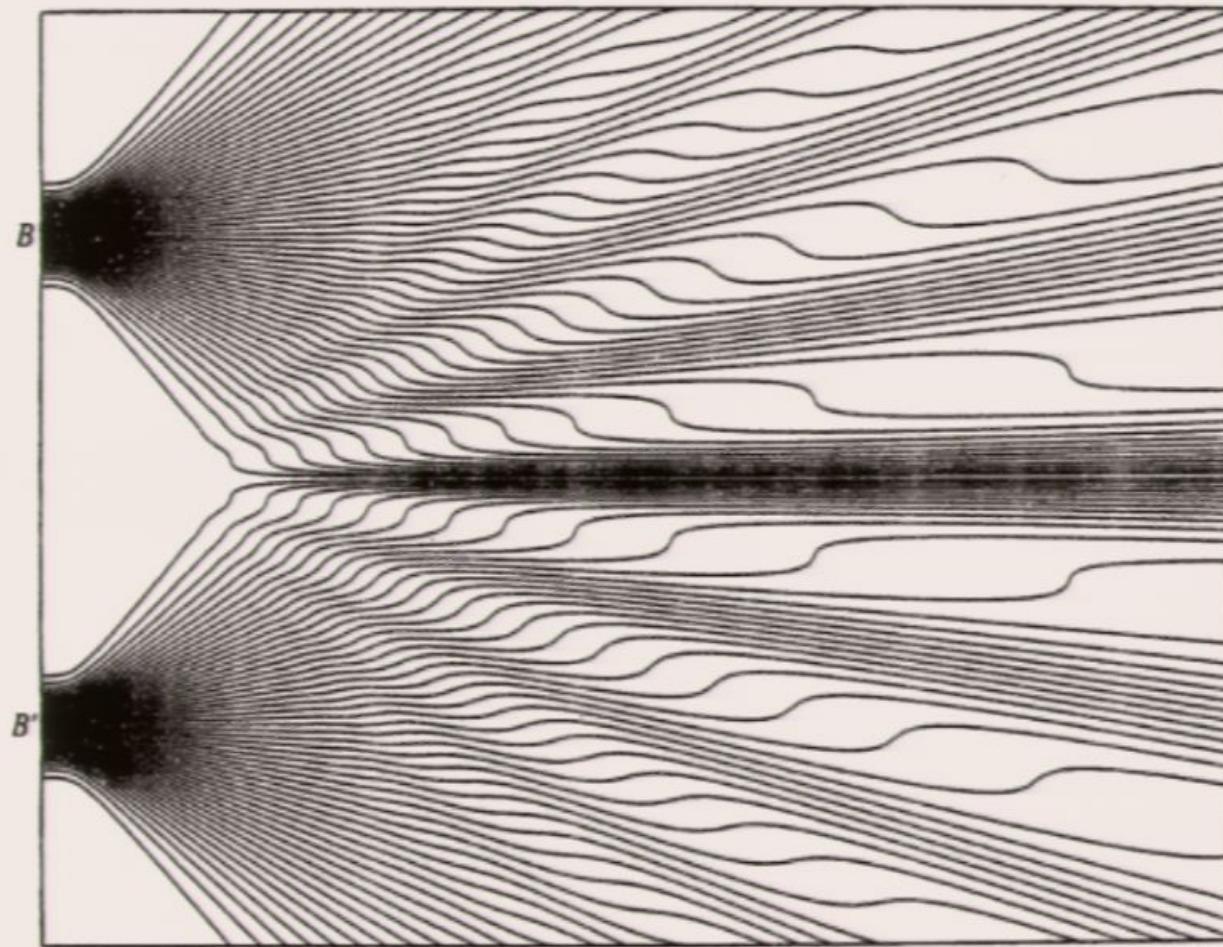
Double slit experiment



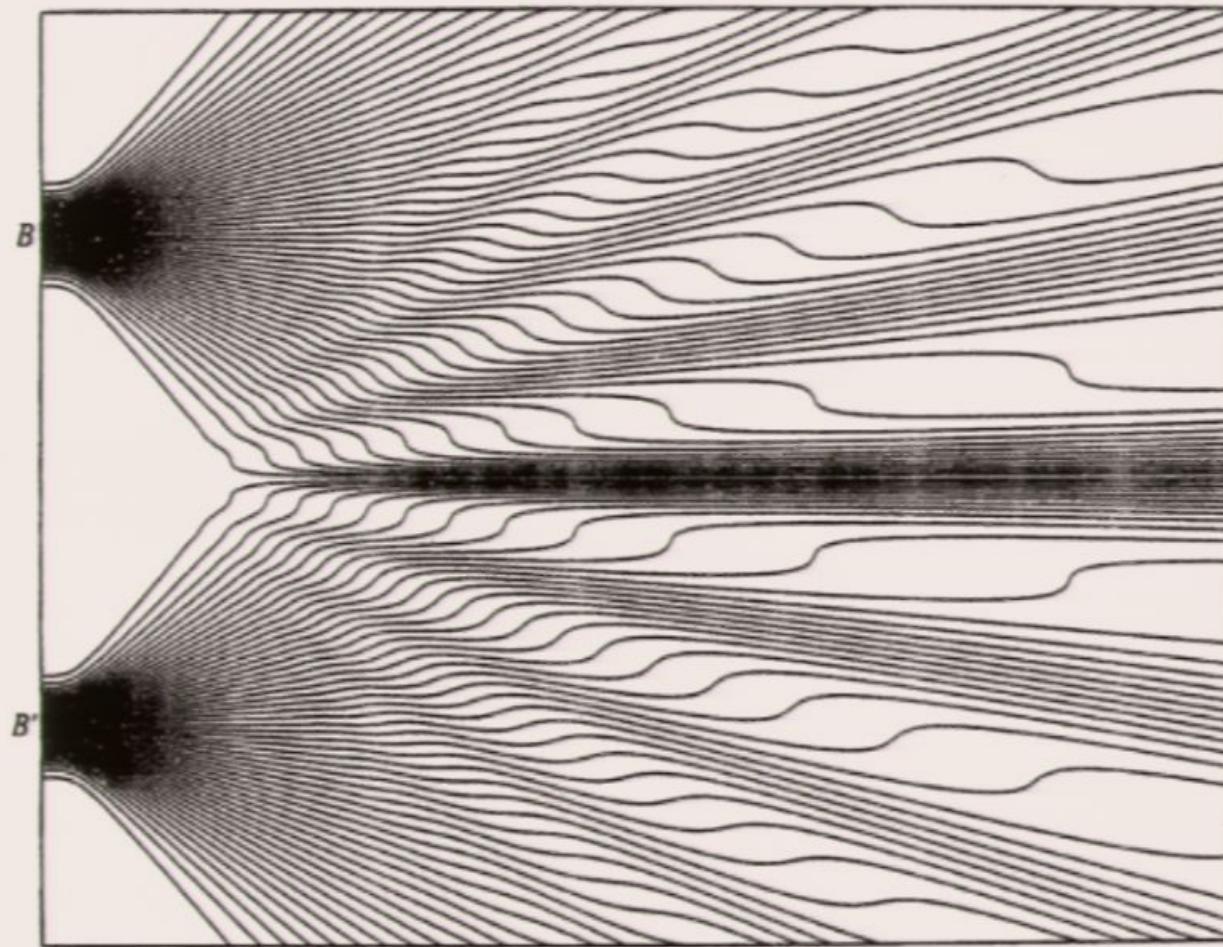
Double slit experiment



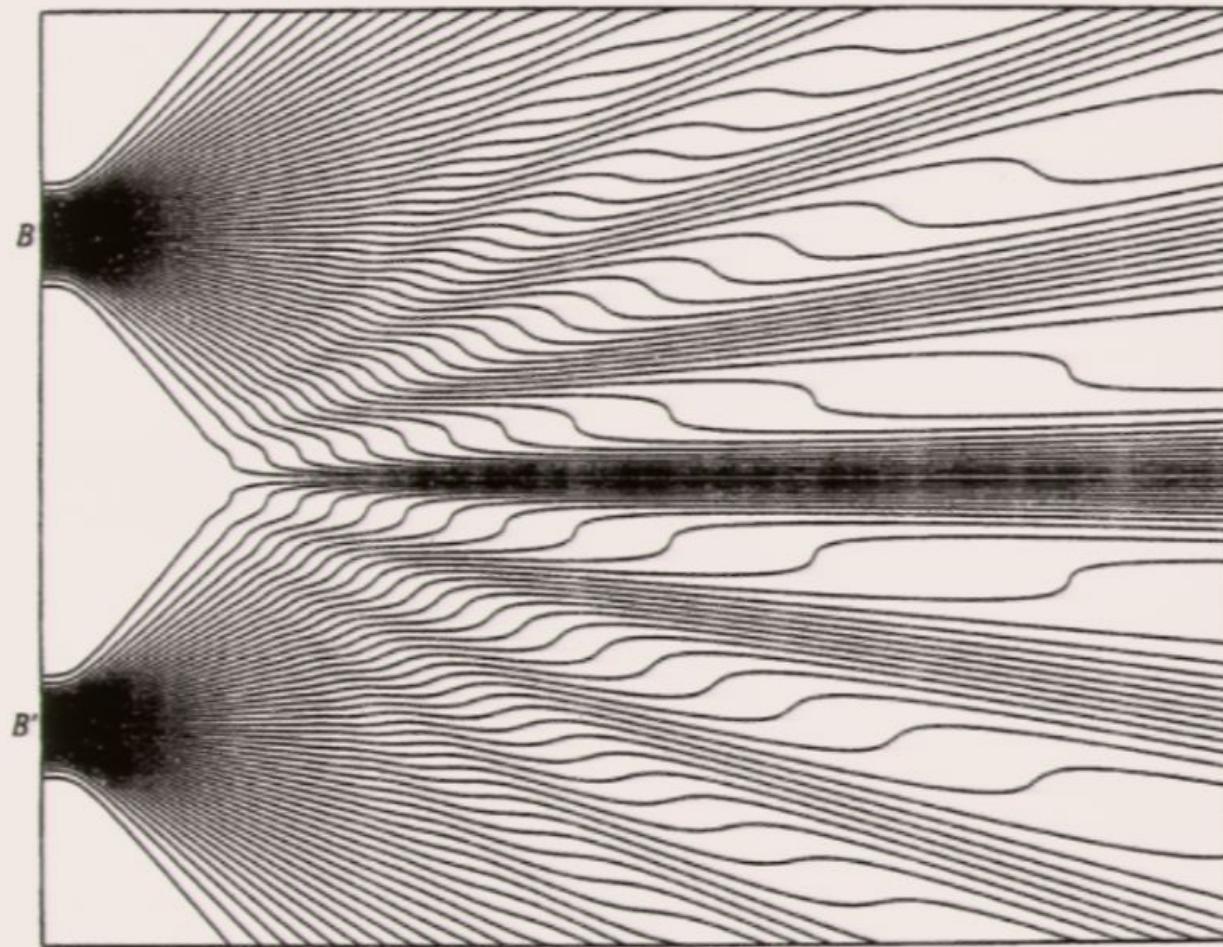
Double slit experiment



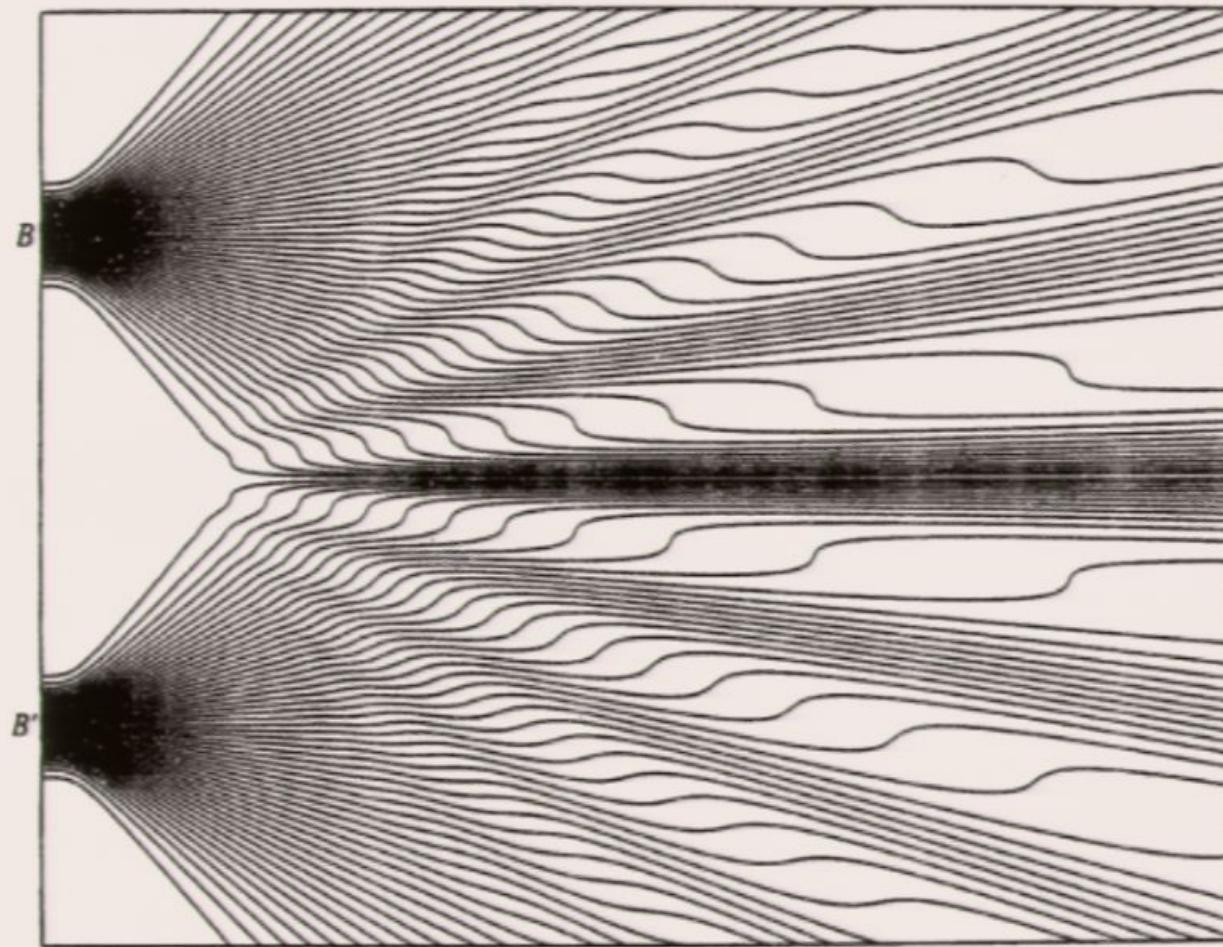
Double slit experiment



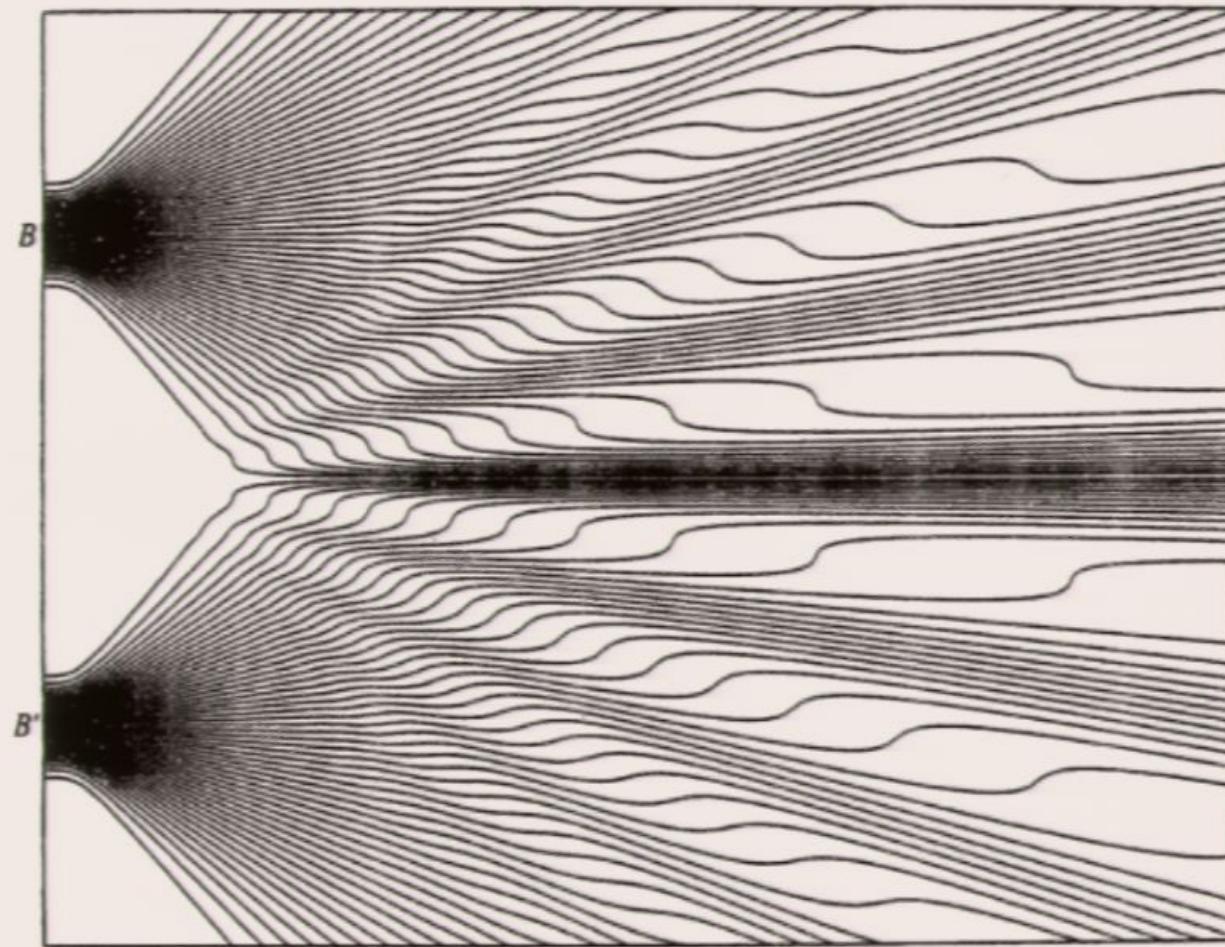
Double slit experiment



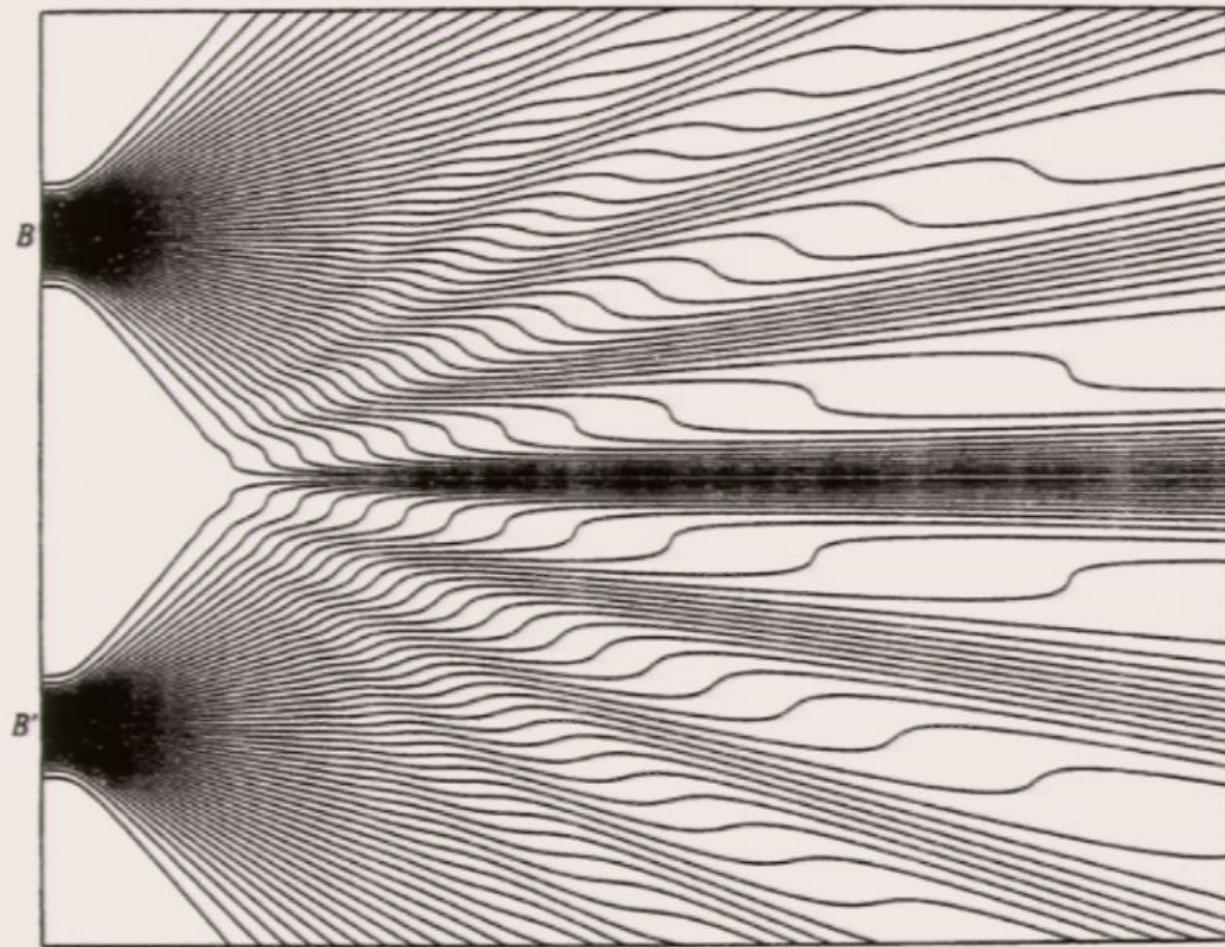
Double slit experiment



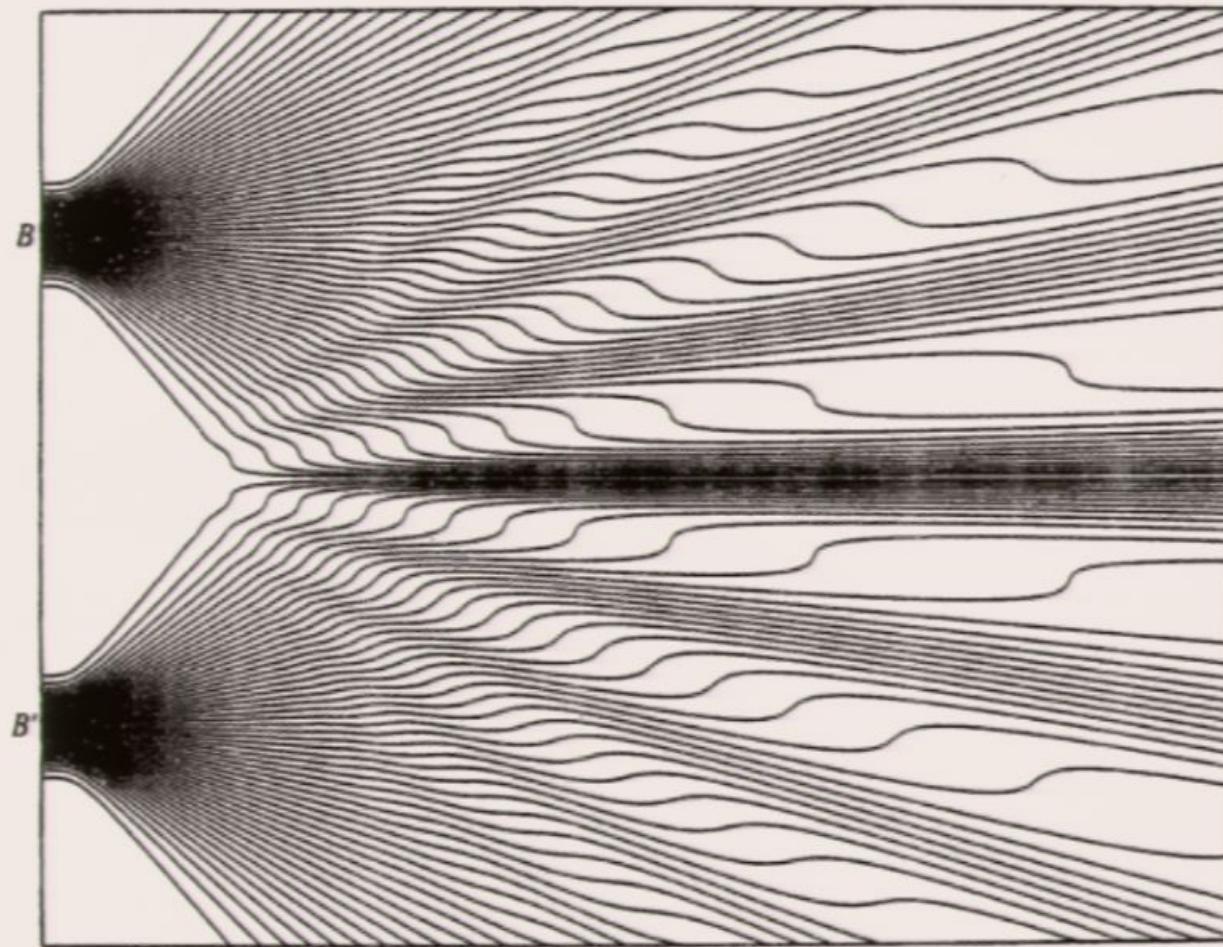
Double slit experiment



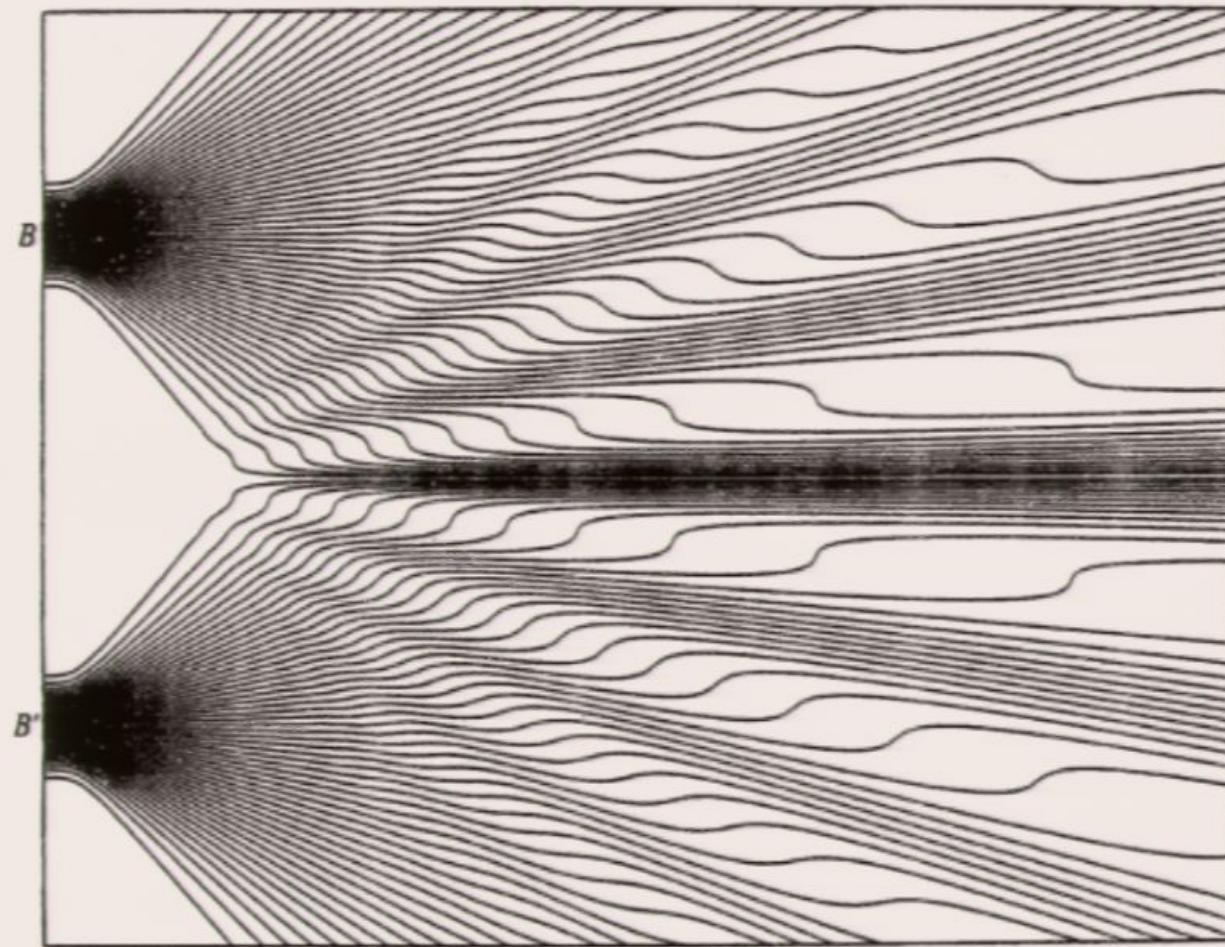
Double slit experiment



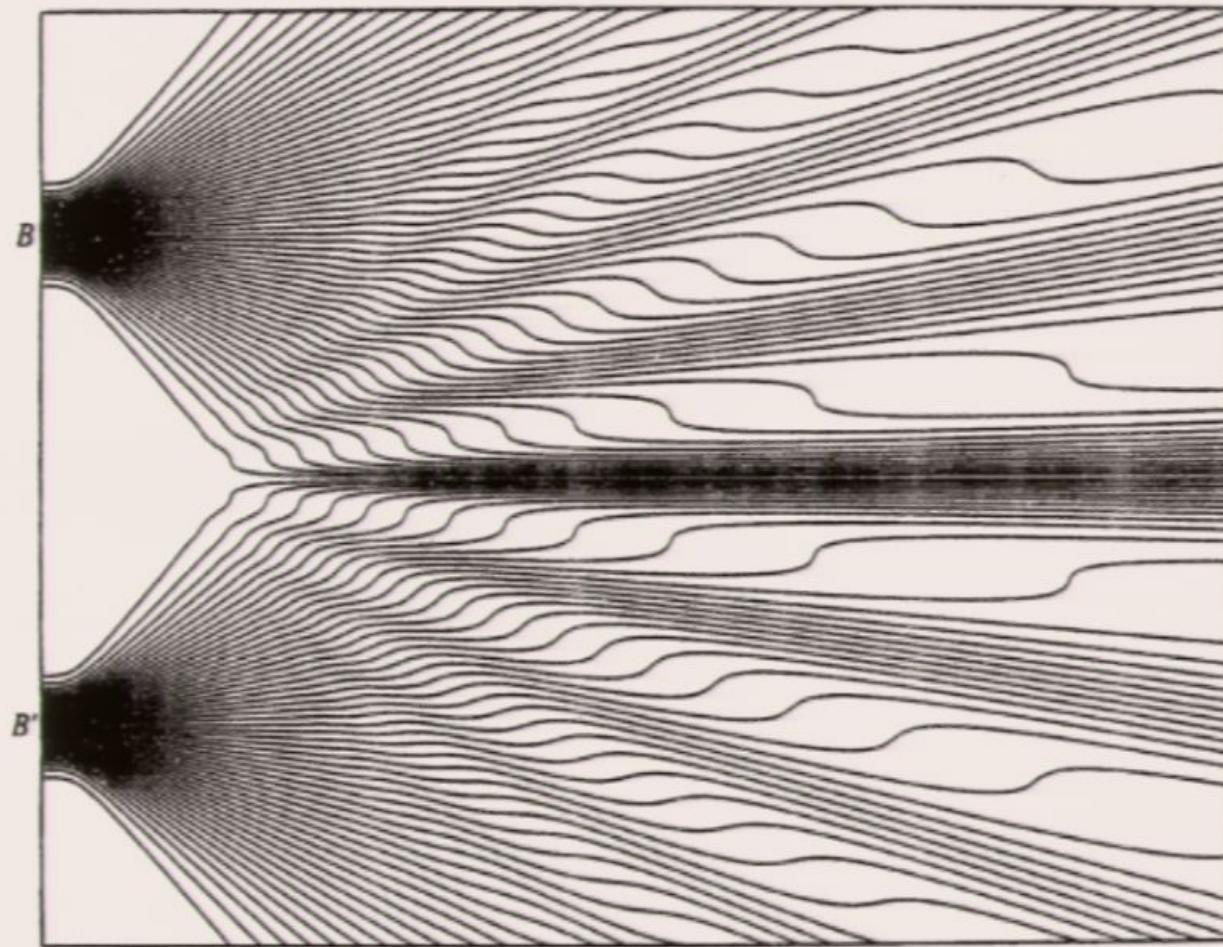
Double slit experiment



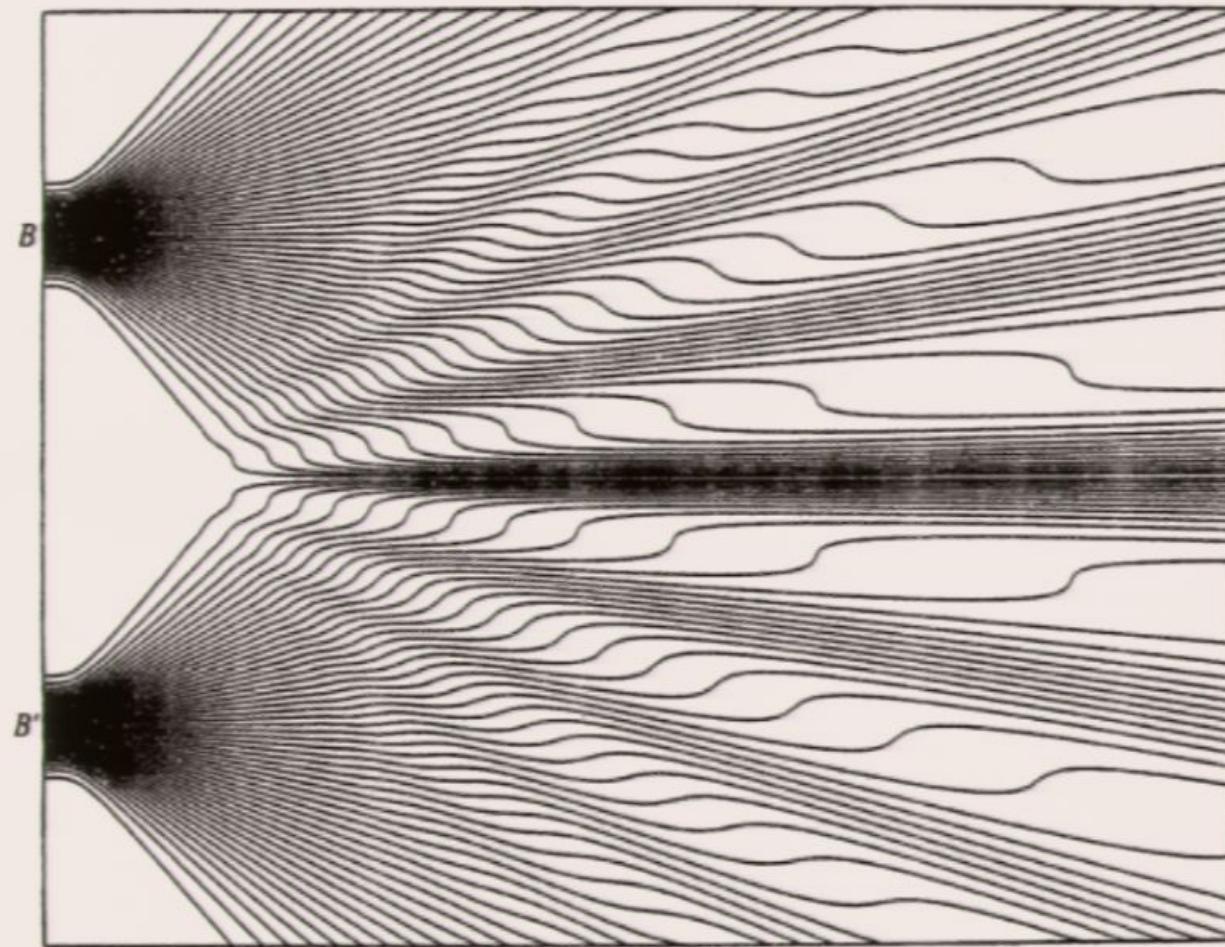
Double slit experiment



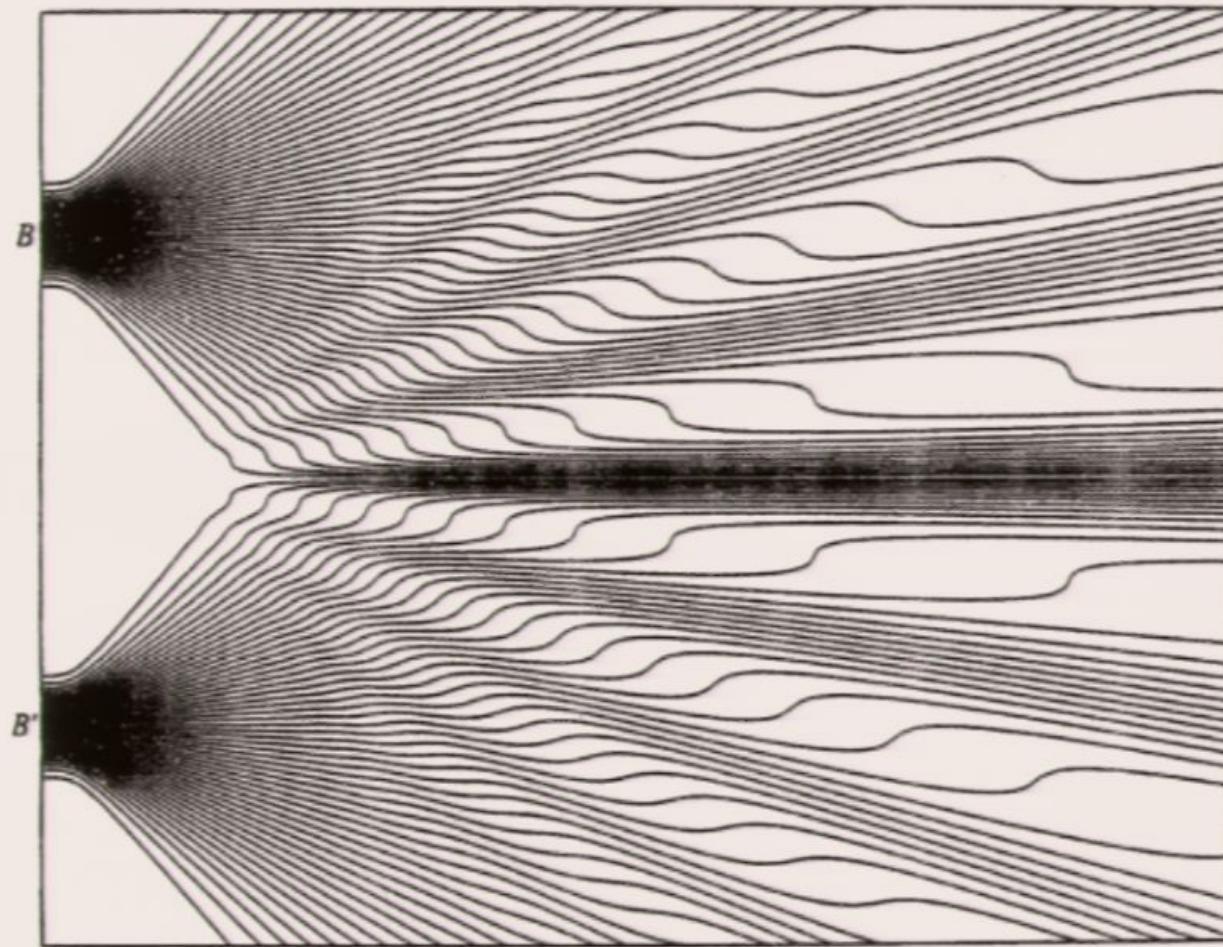
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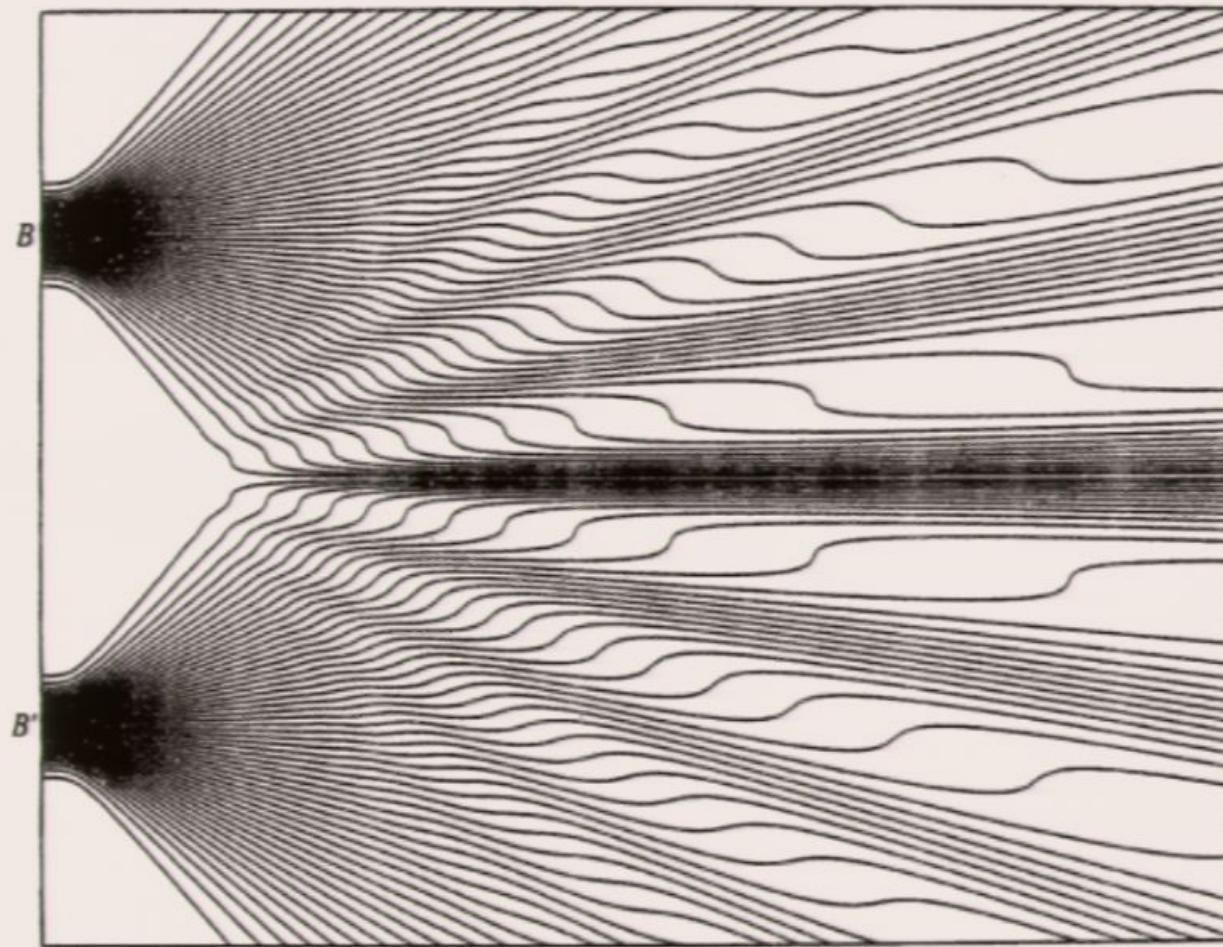
Double slit experiment



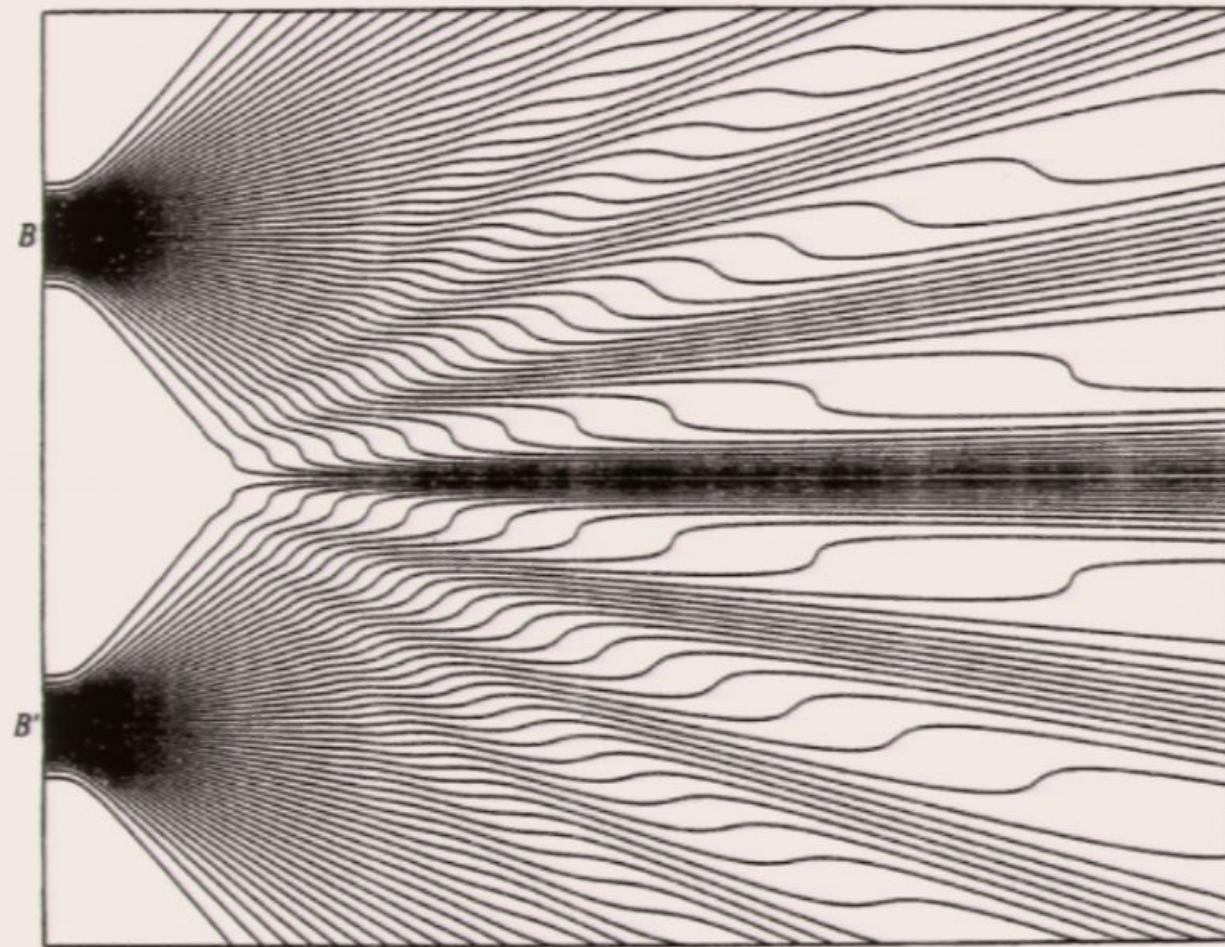
Double slit experiment



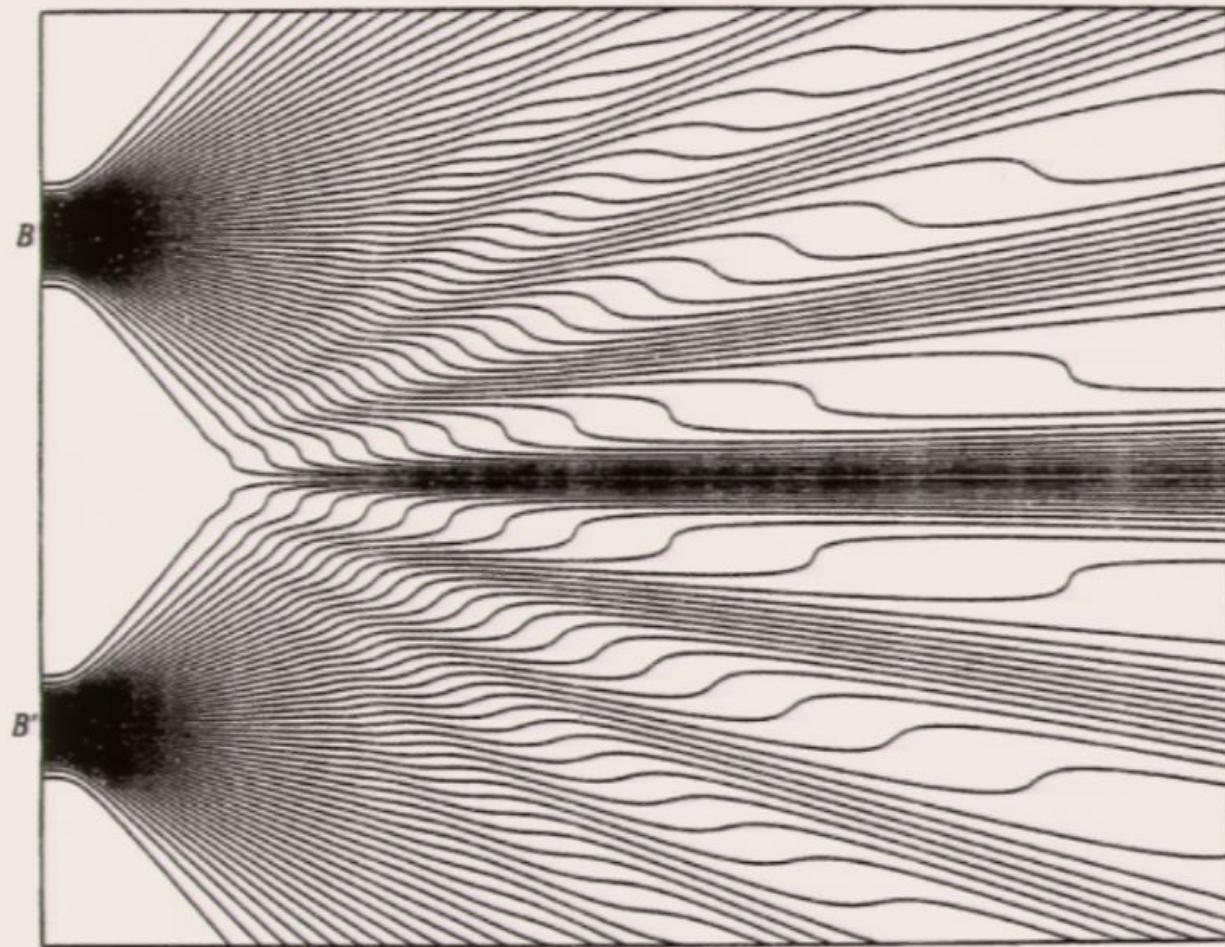
Double slit experiment



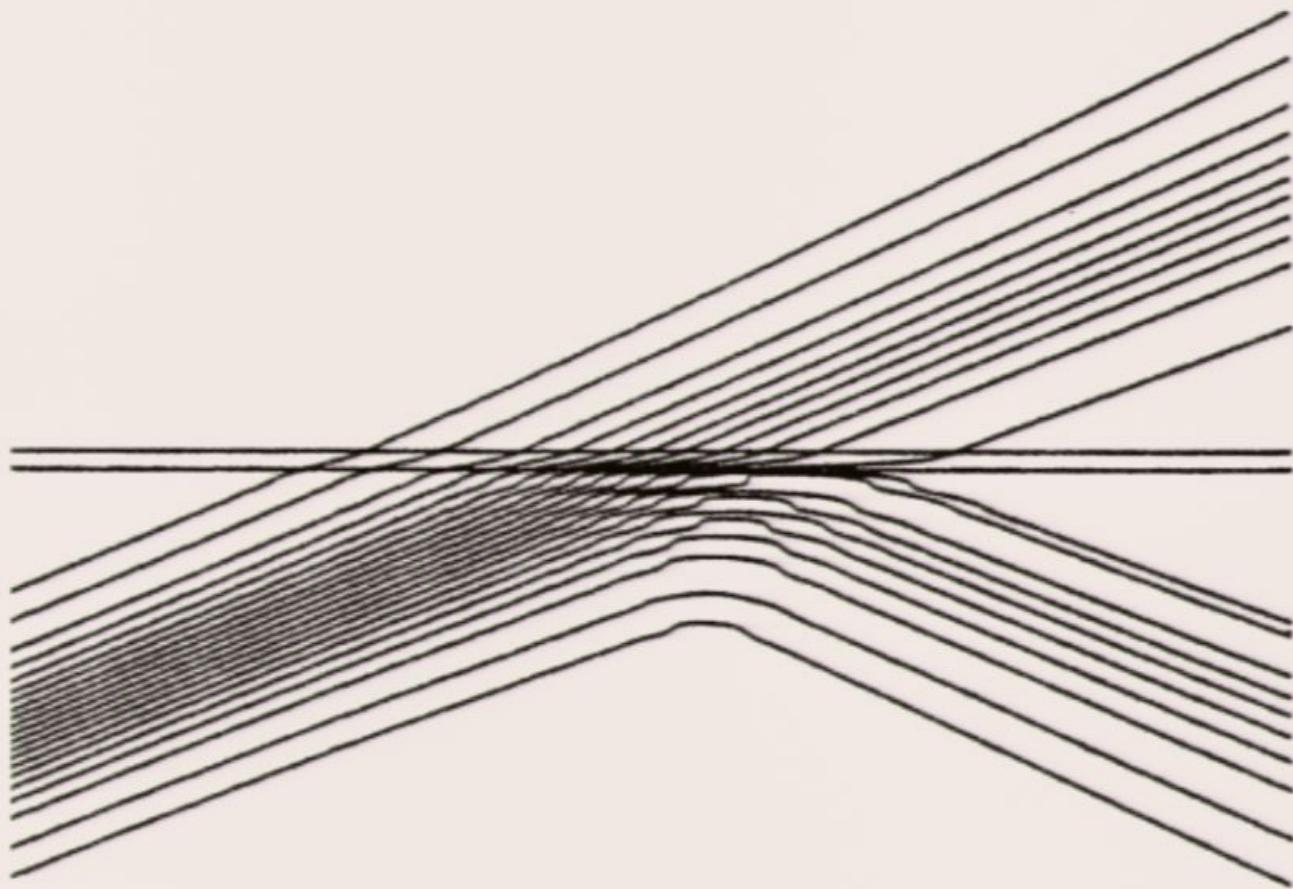
Double slit experiment



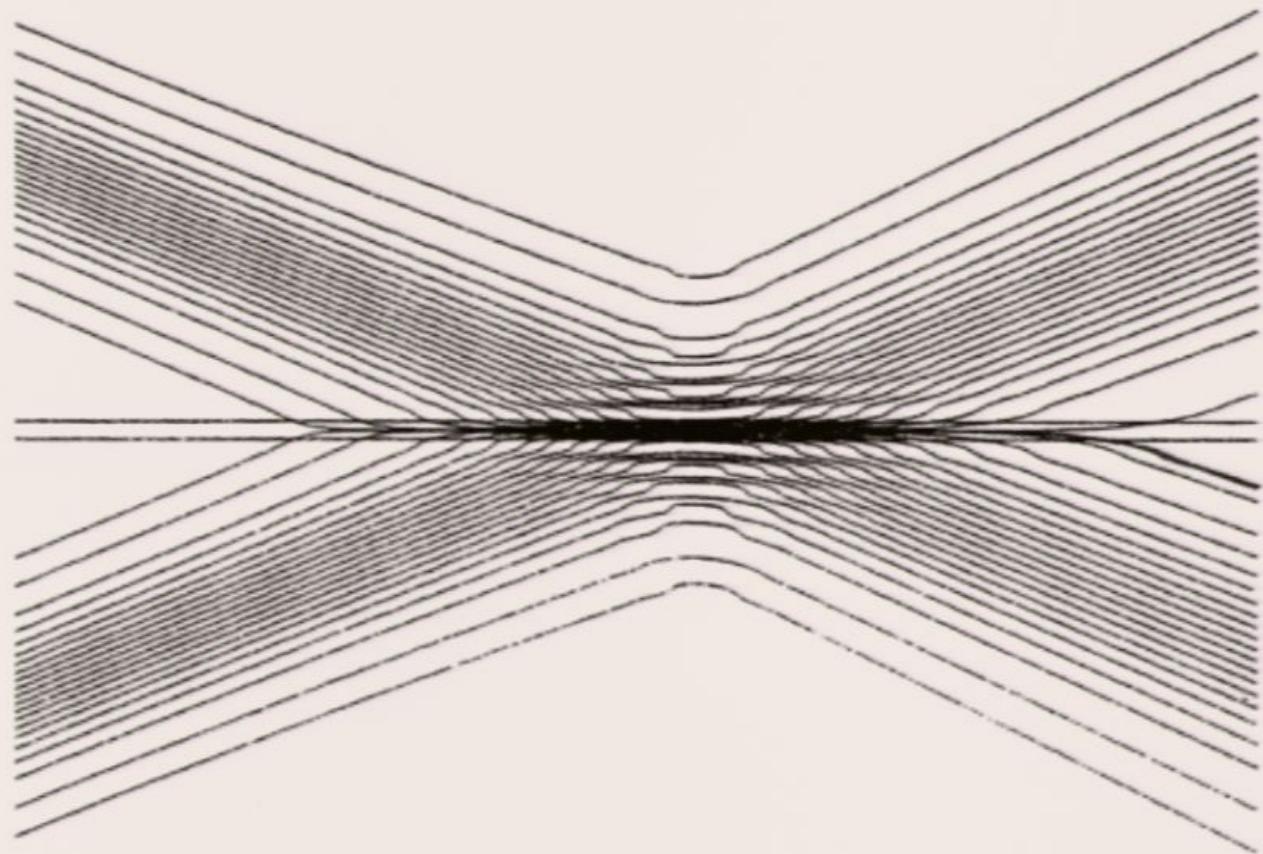
Double slit experiment



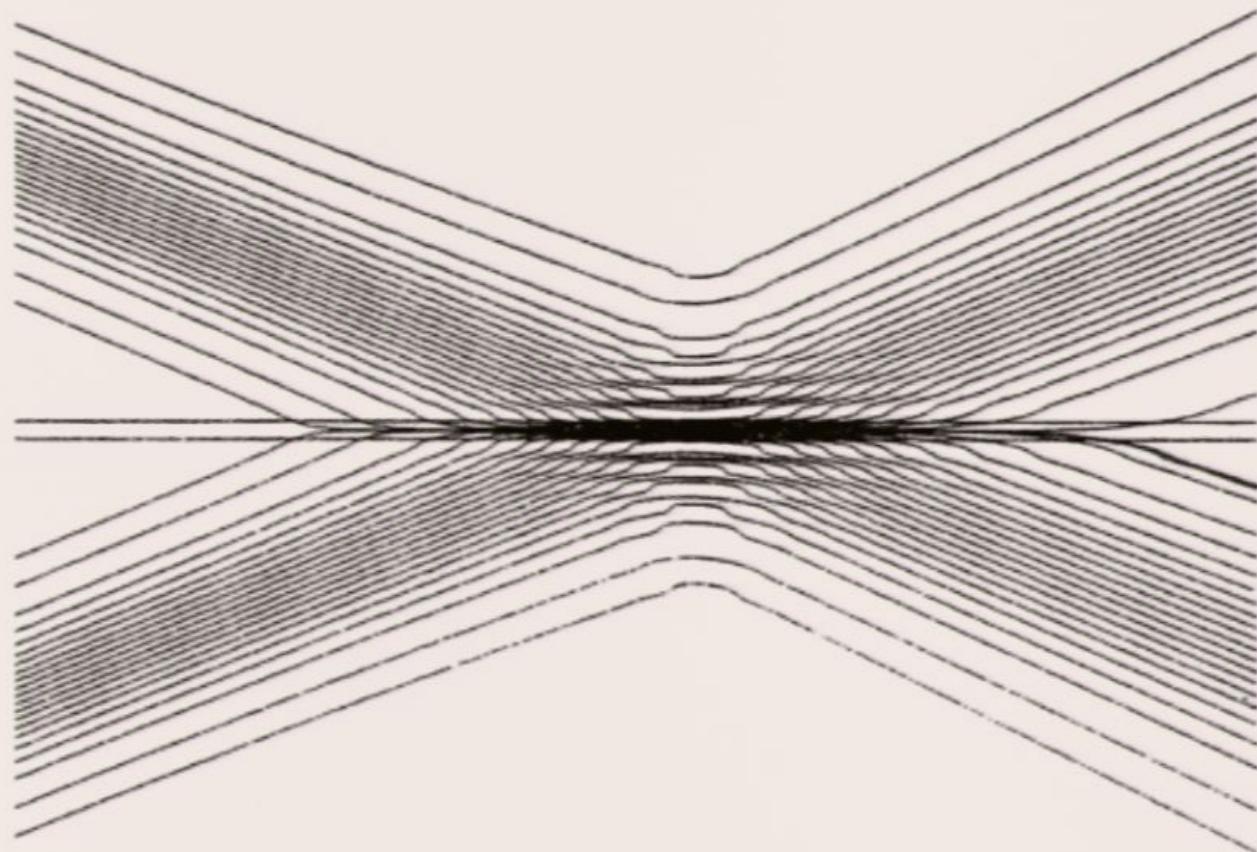
Double slit experiment



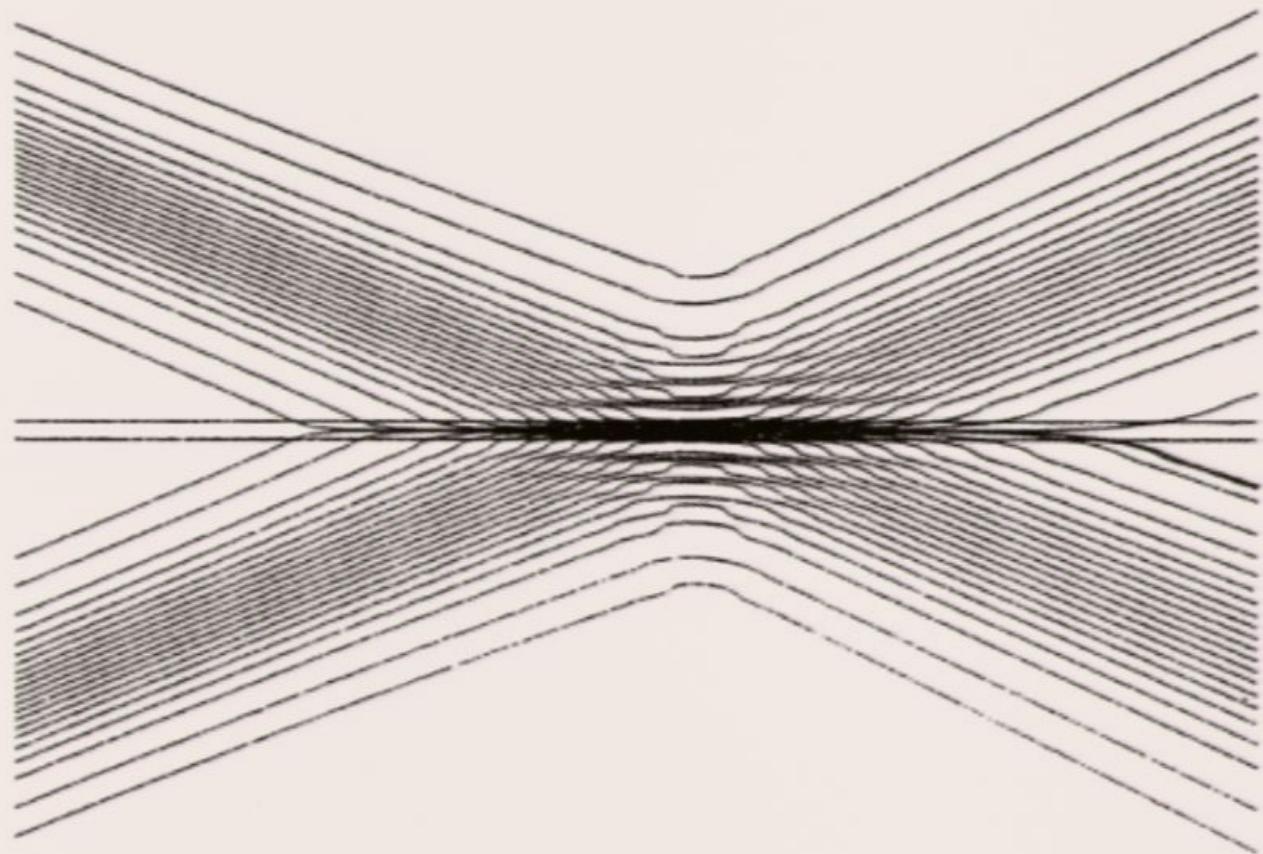
Transmission through a barrier (probability $\frac{1}{2}$)



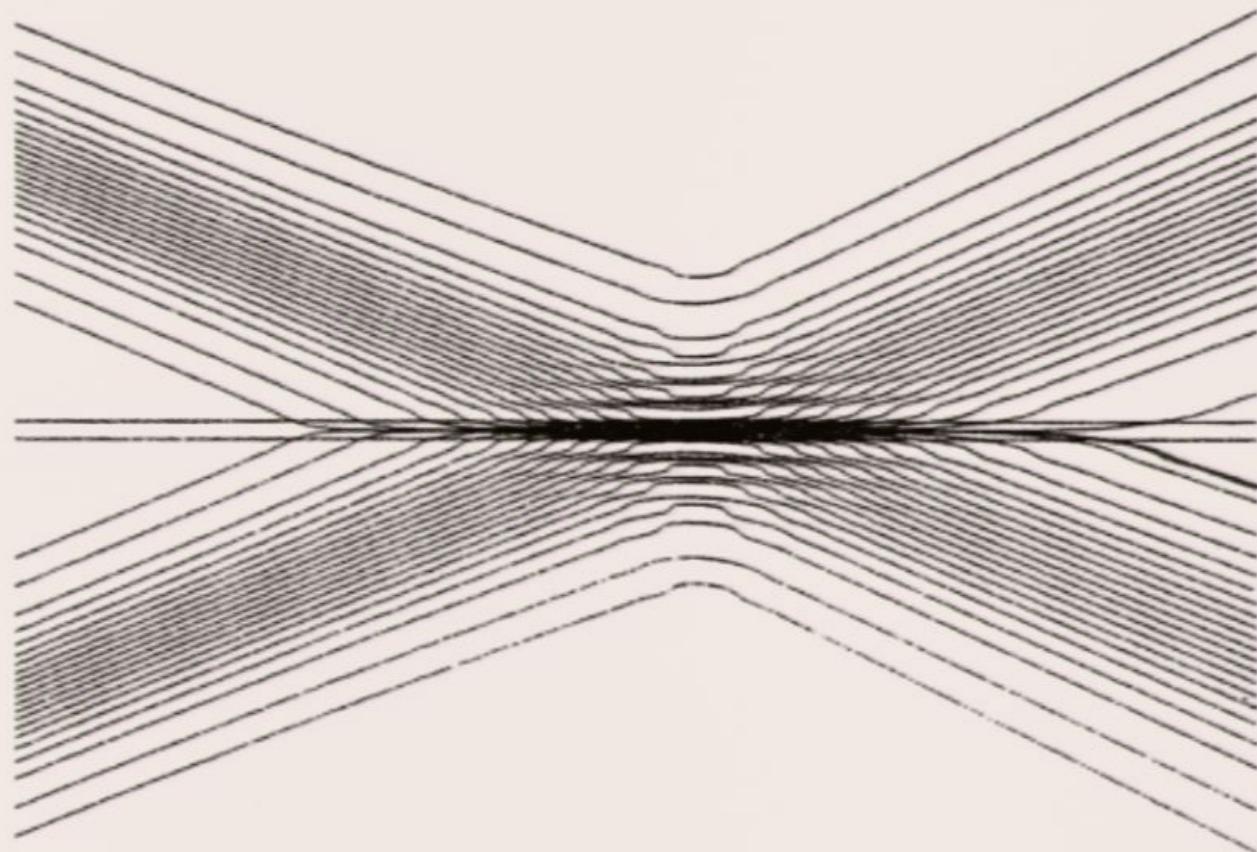
Beam splitter experiment



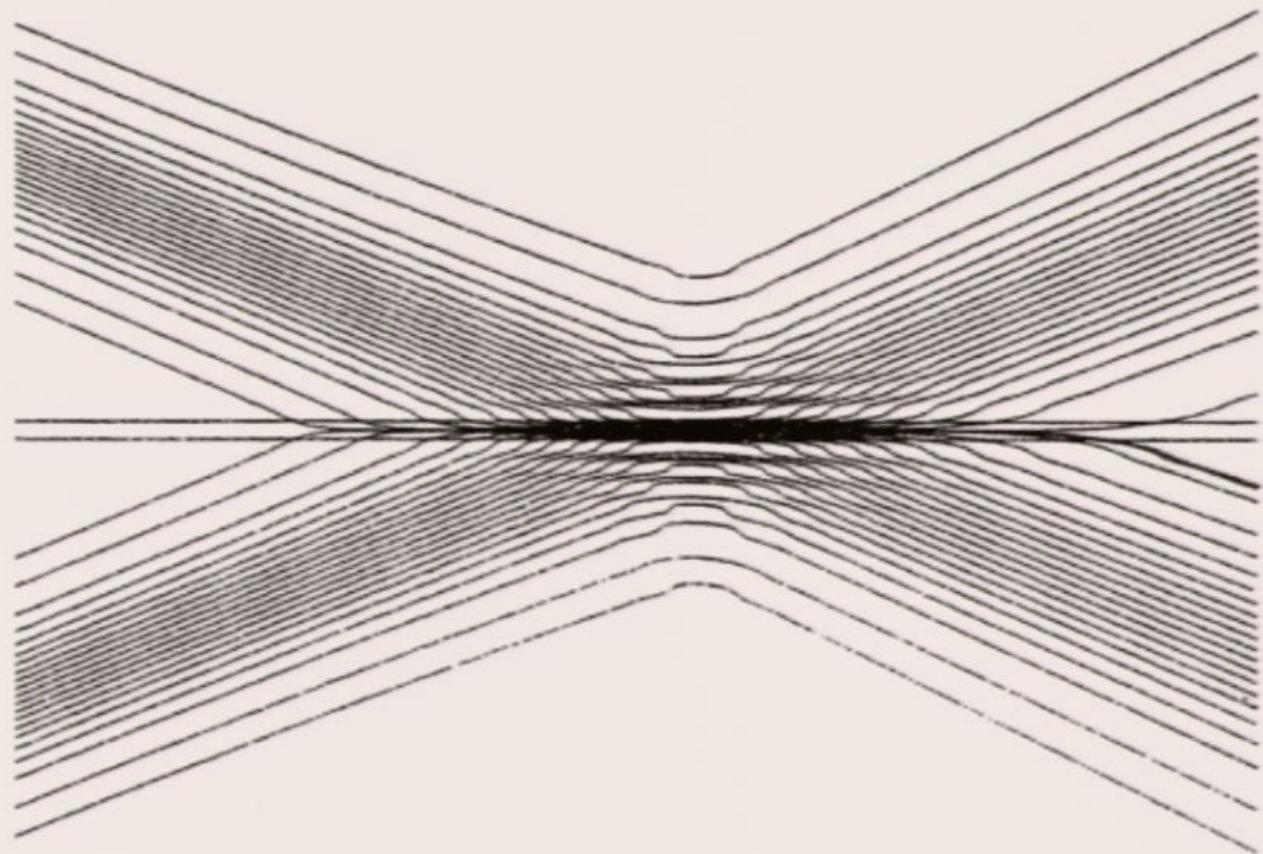
Beam splitter experiment



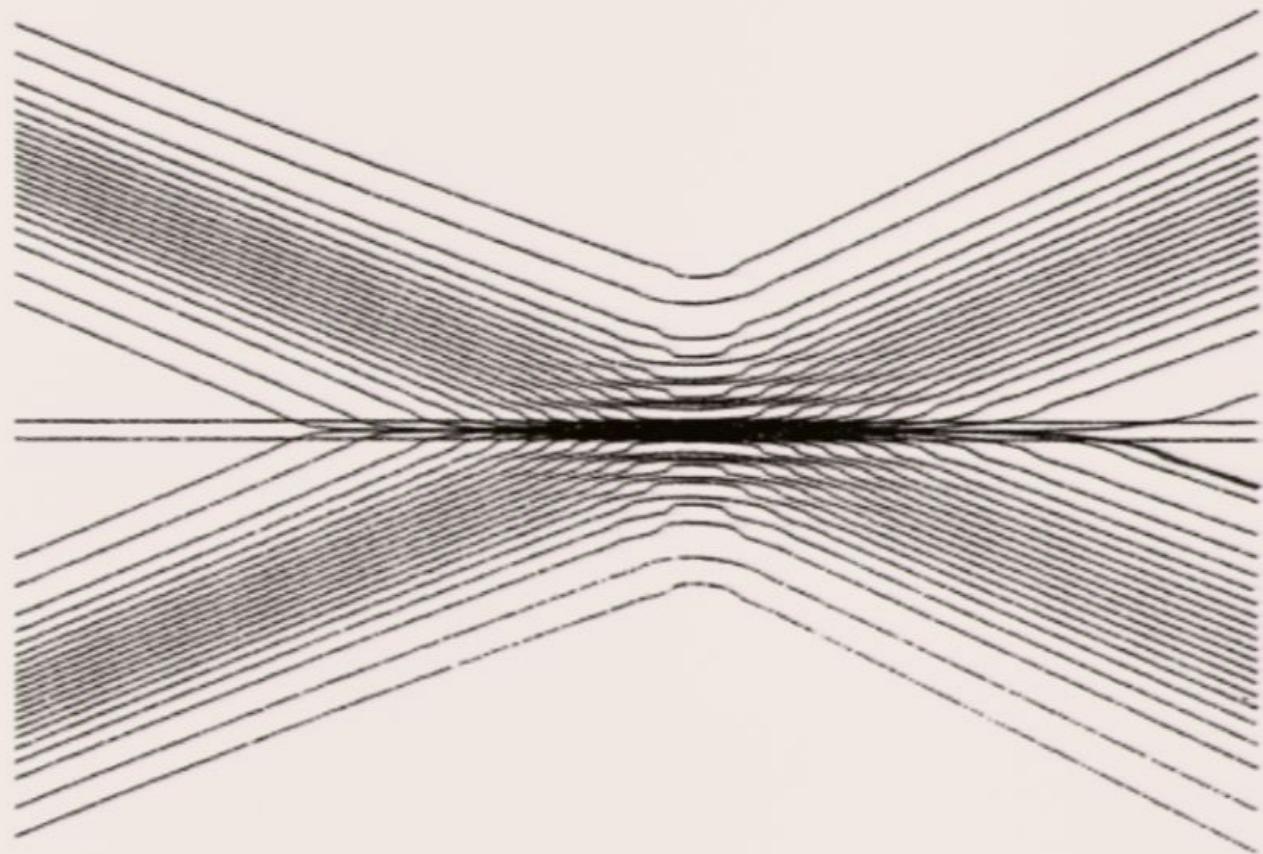
Beam splitter experiment



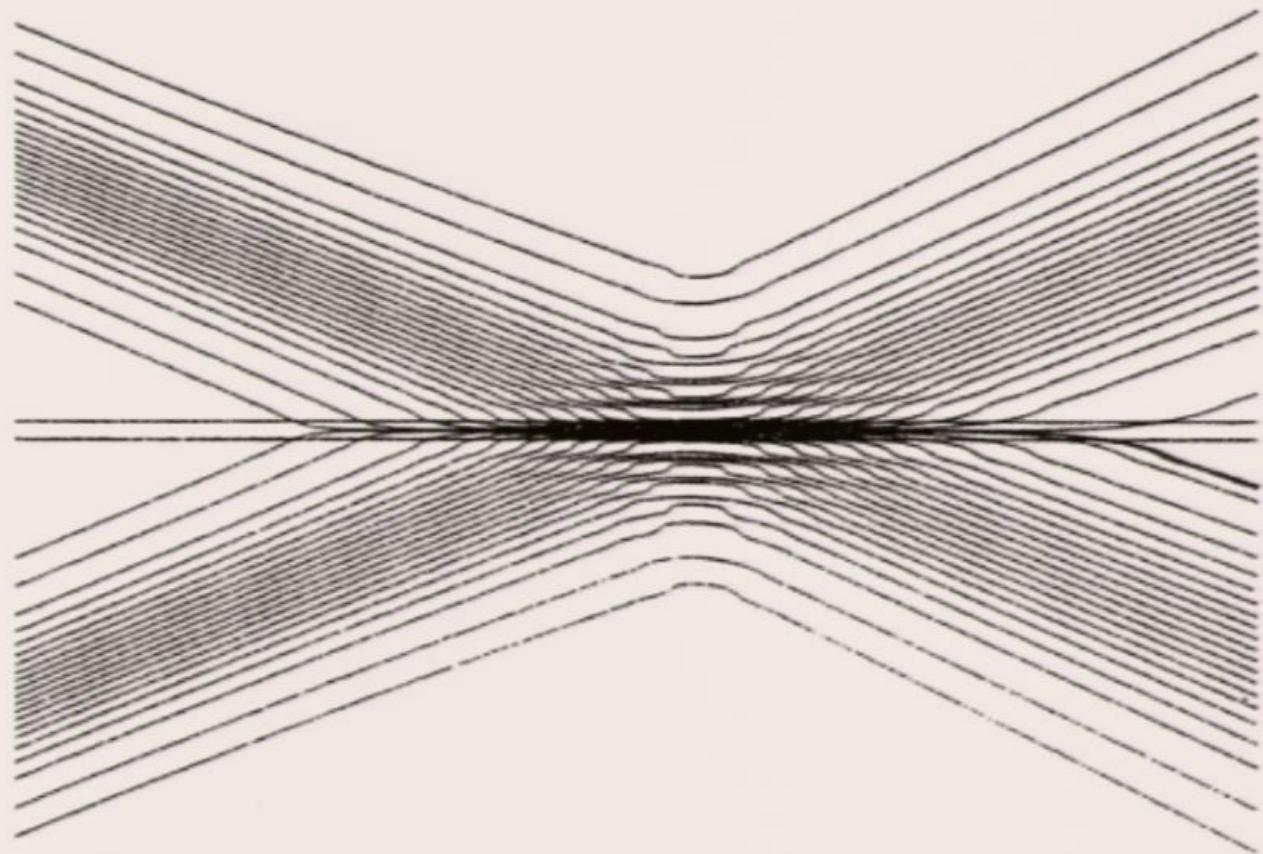
Beam splitter experiment



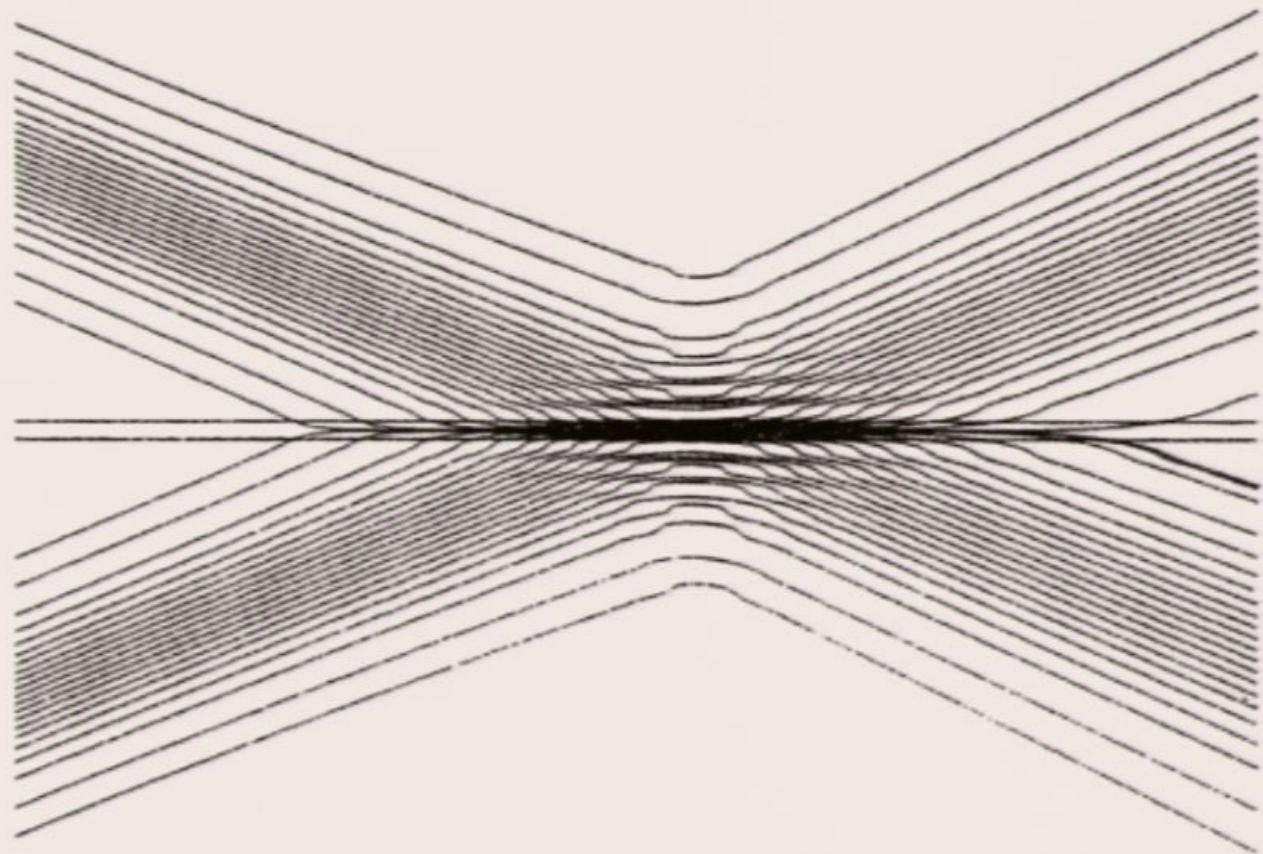
Beam splitter experiment



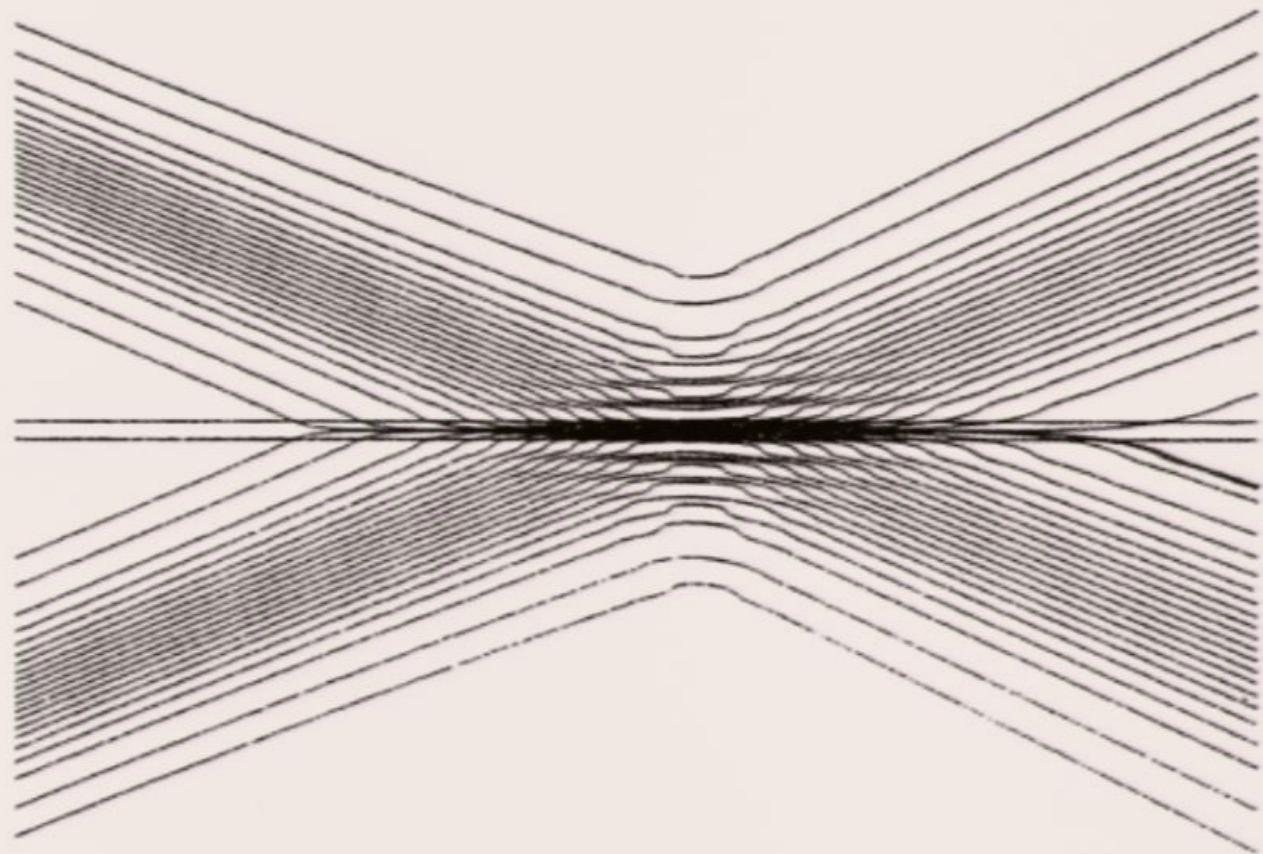
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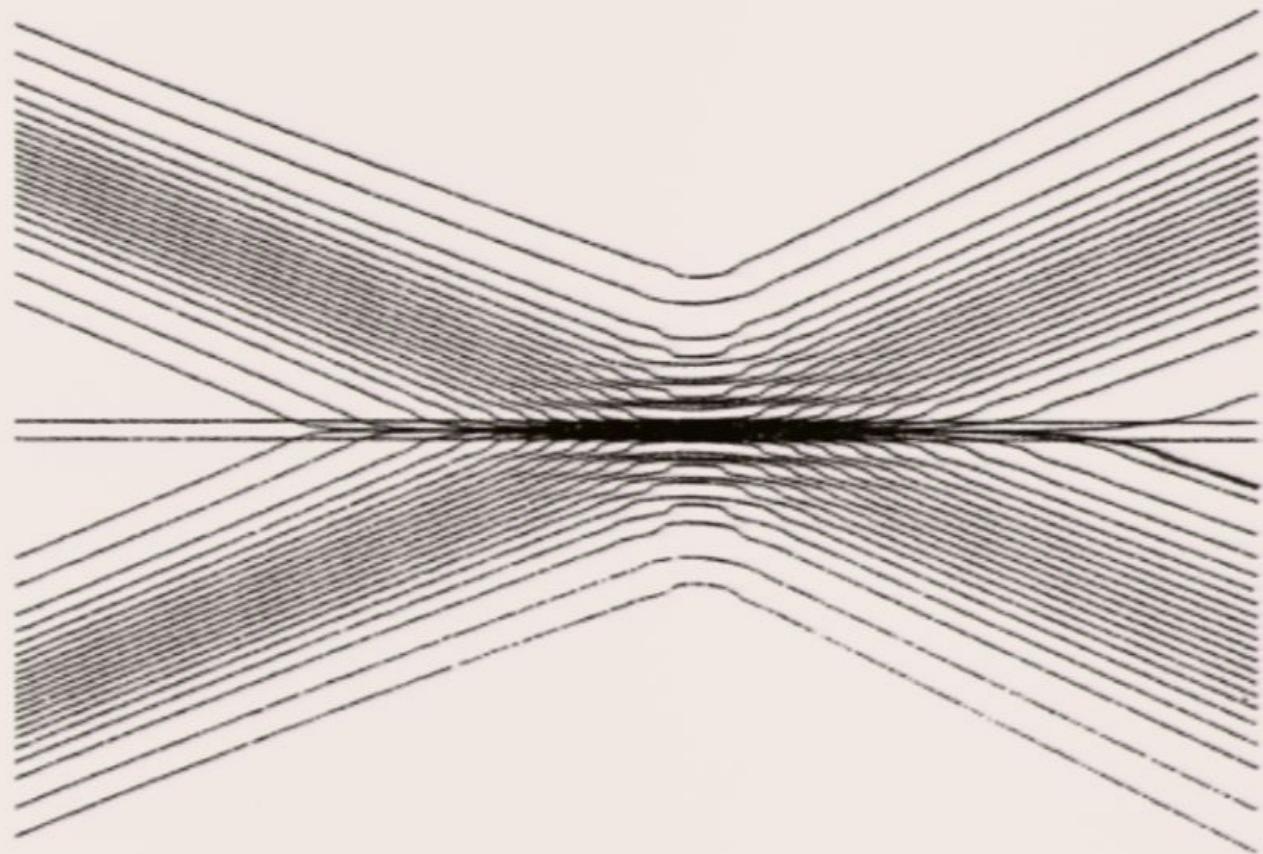
Beam splitter experiment



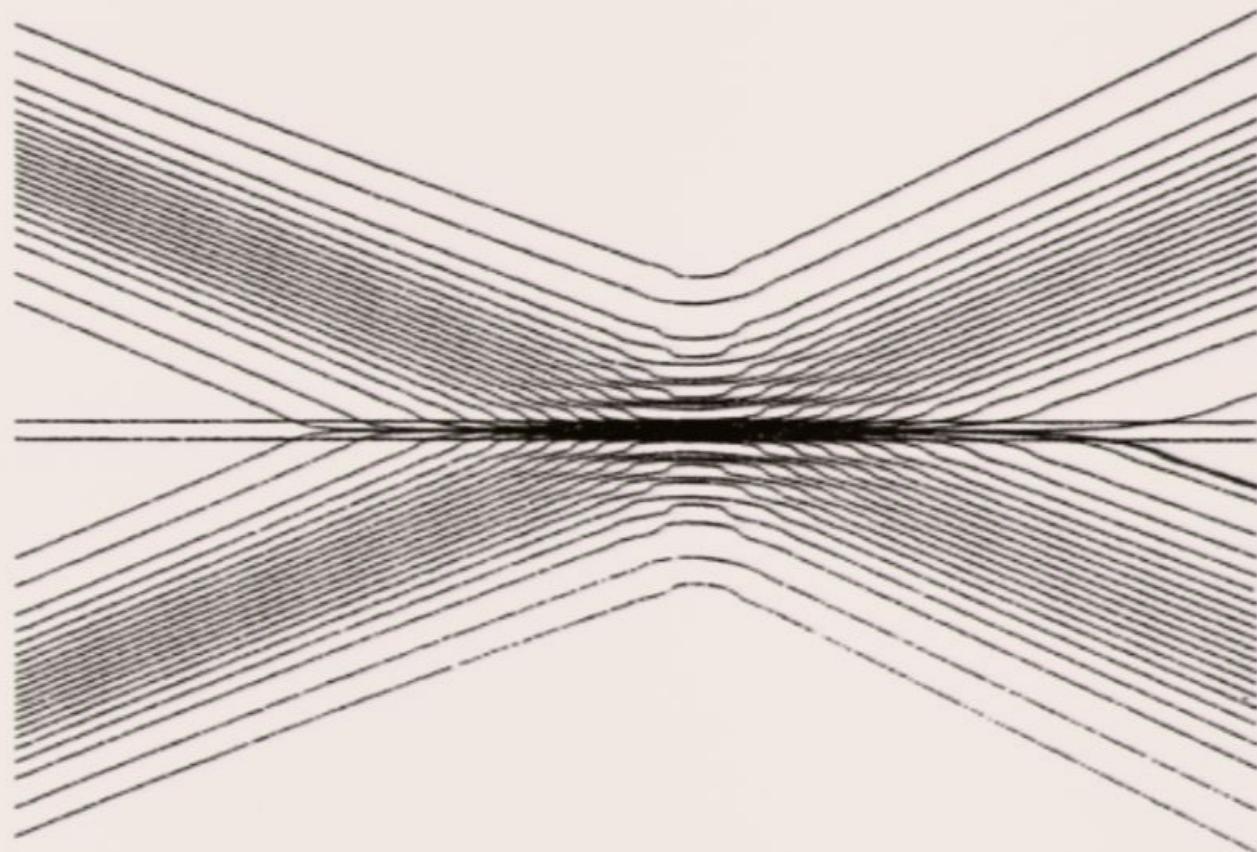
Beam splitter experiment



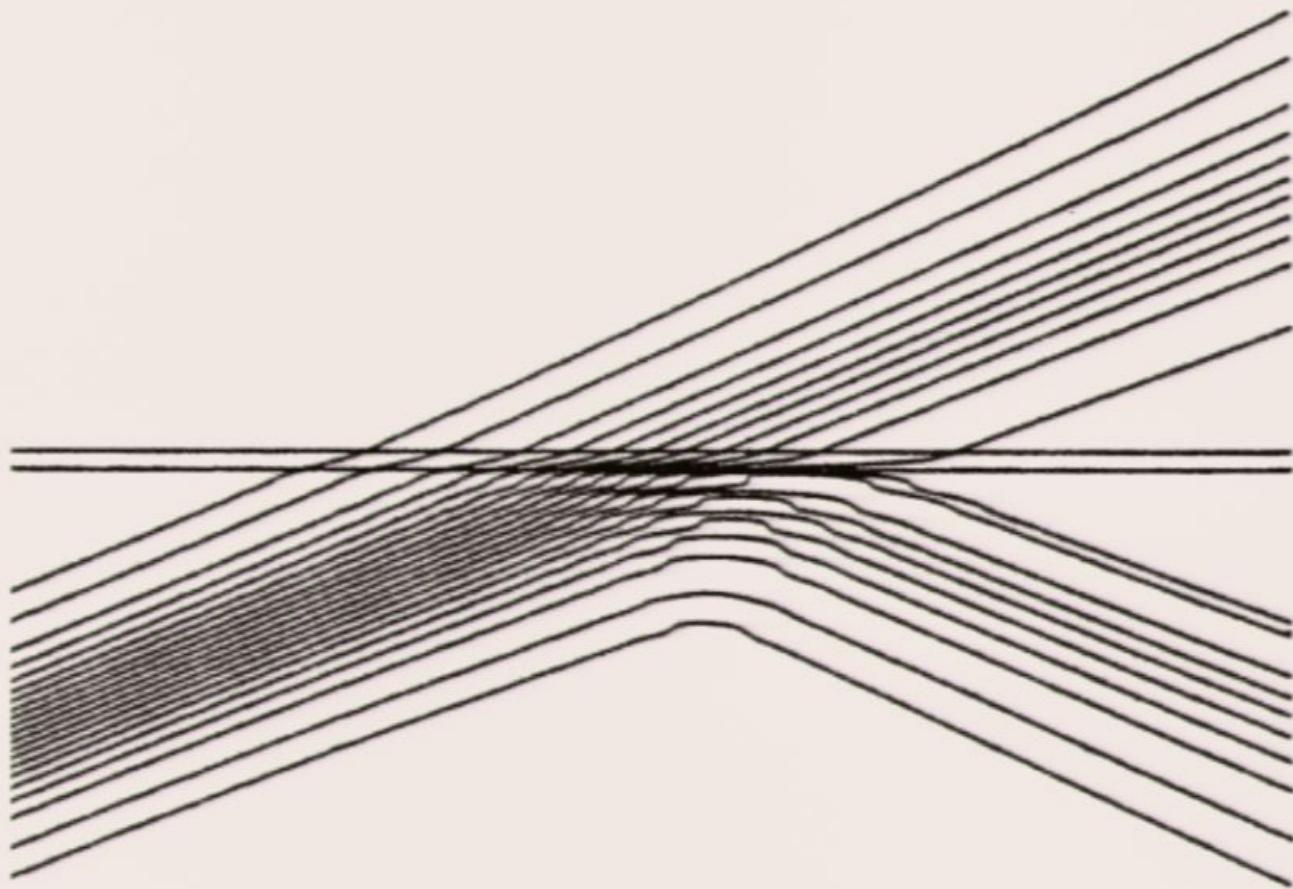
Beam splitter experiment



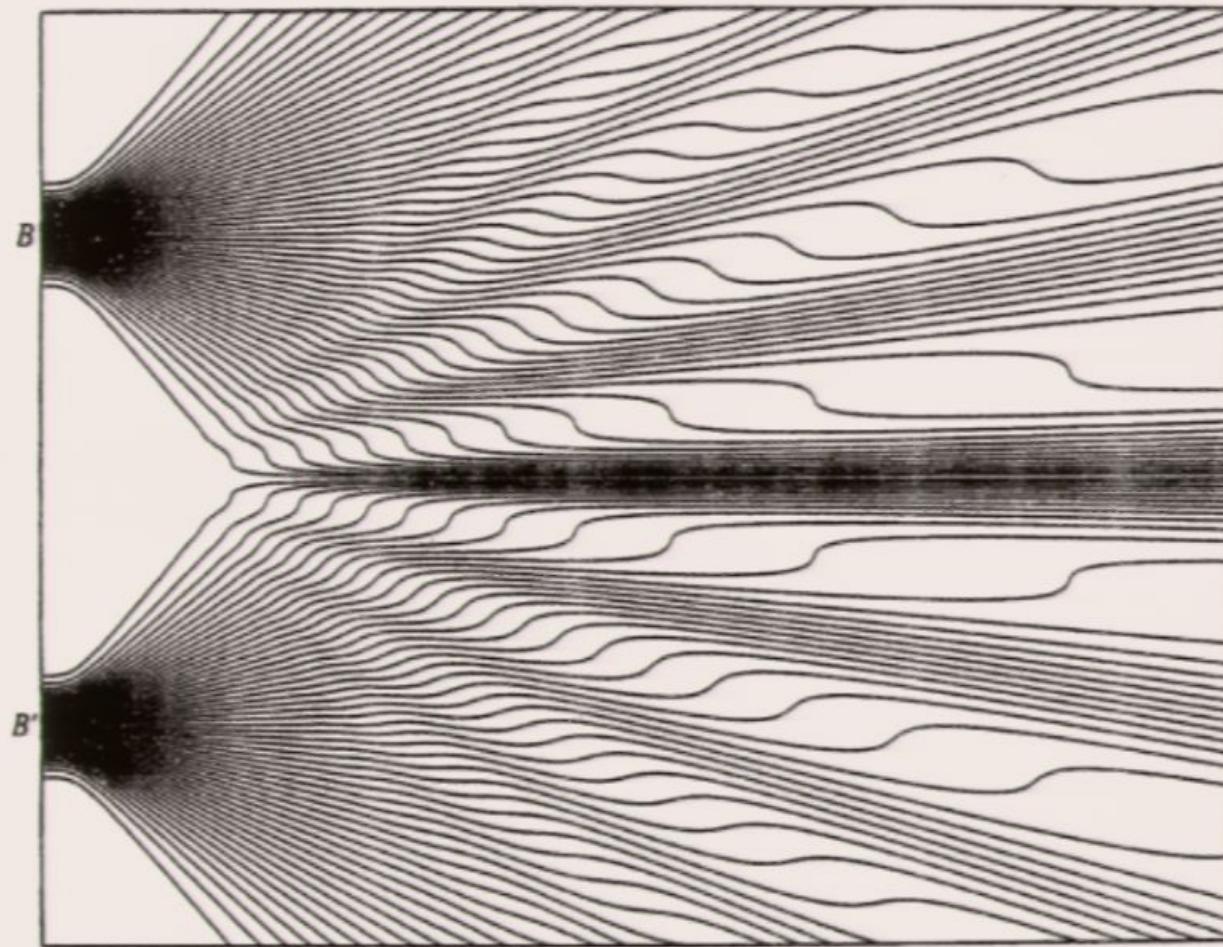
Beam splitter experiment



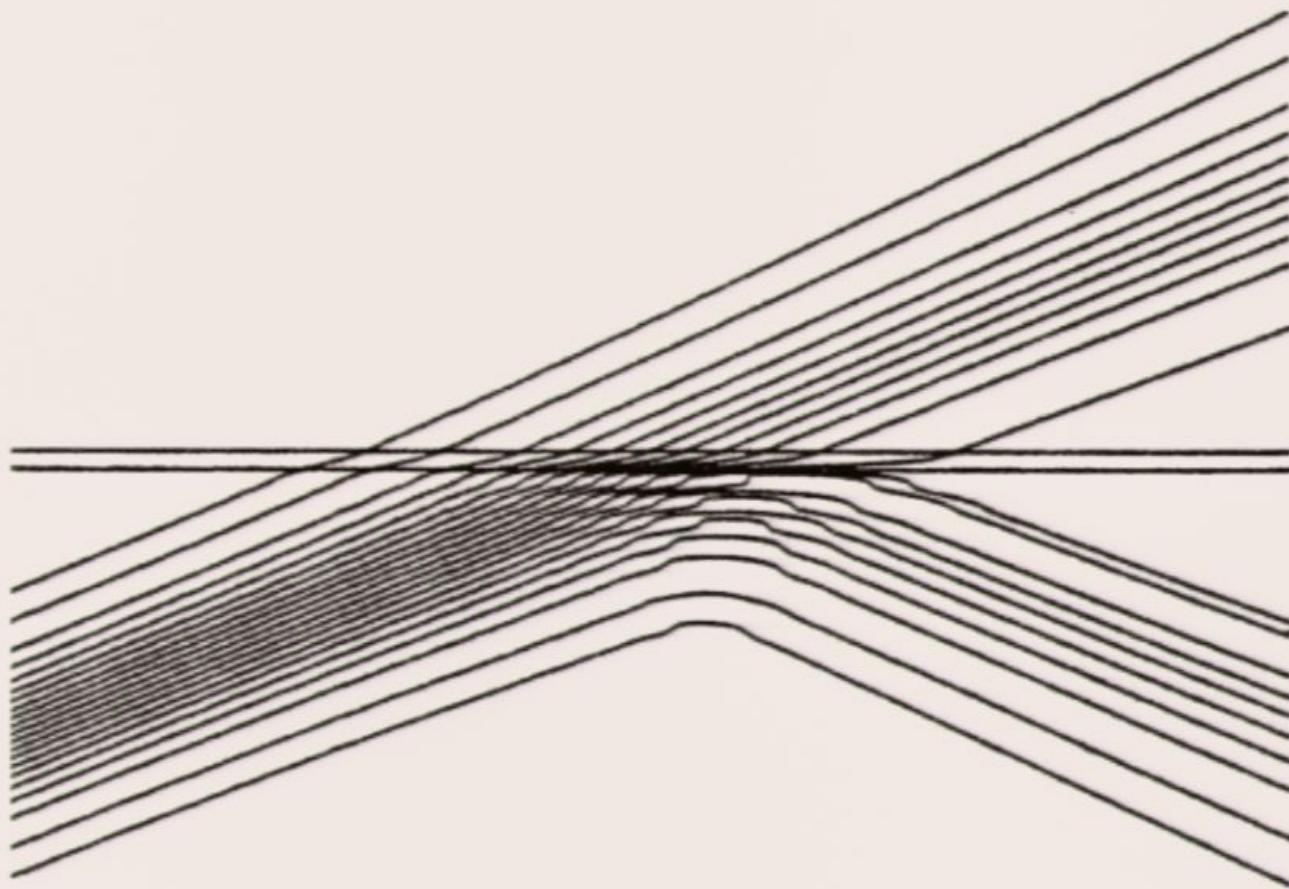
Beam splitter experiment



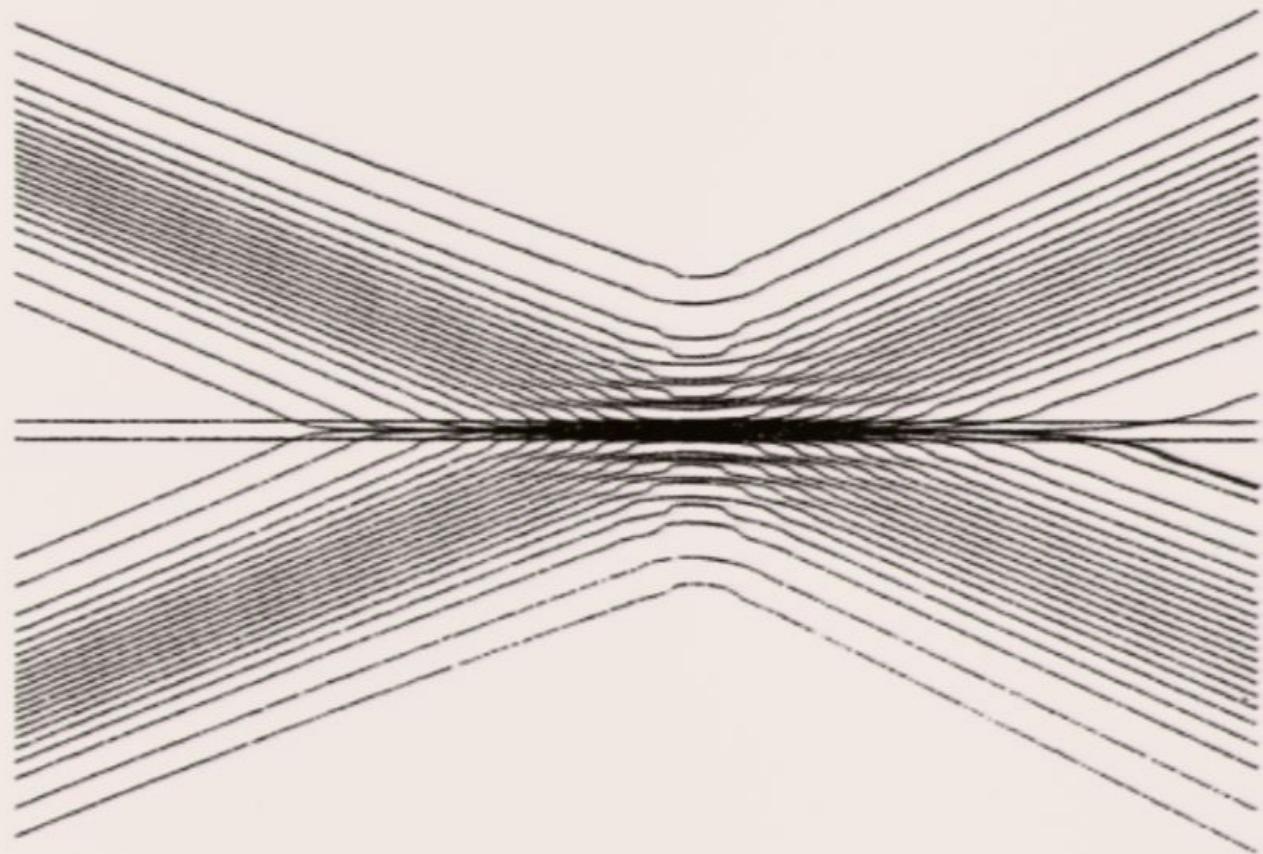
Transmission through a barrier (probability $\frac{1}{2}$)



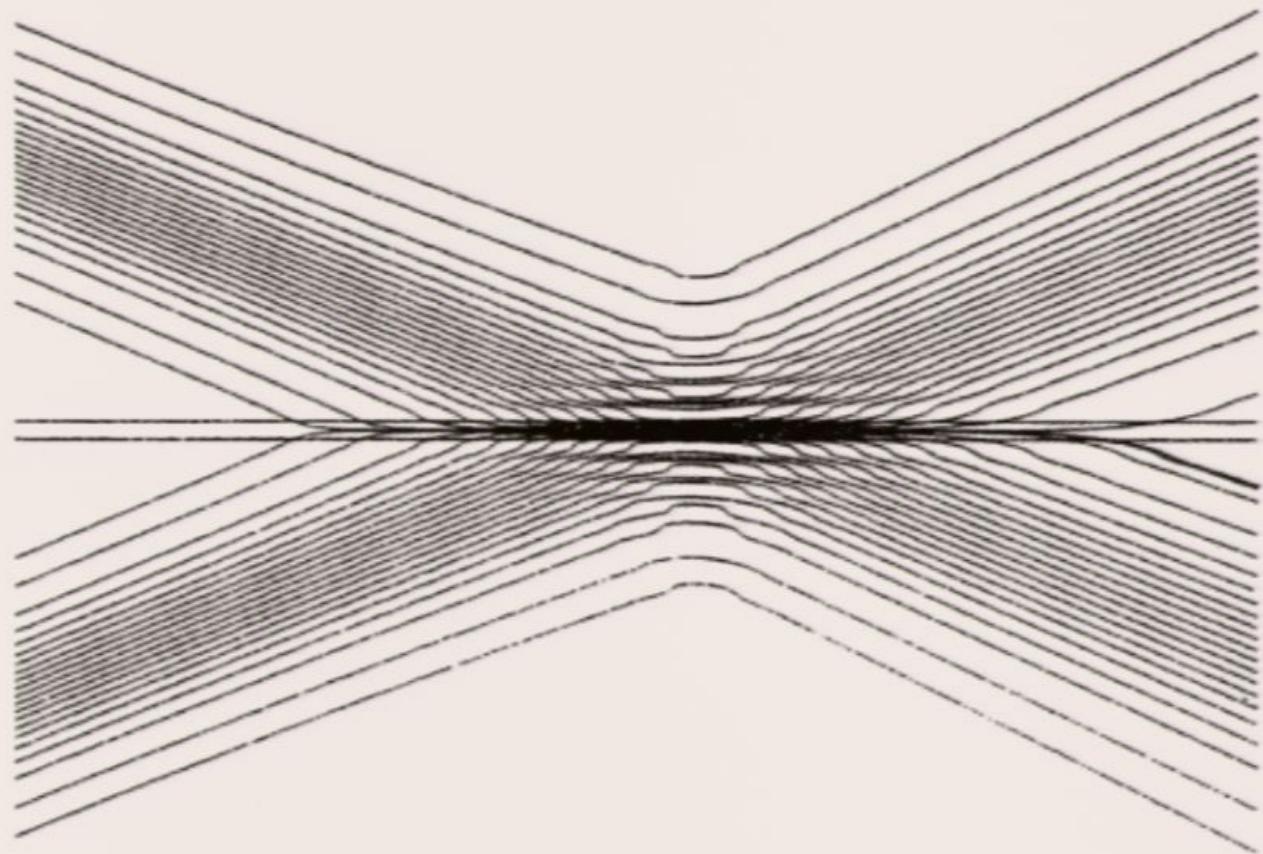
Double slit experiment



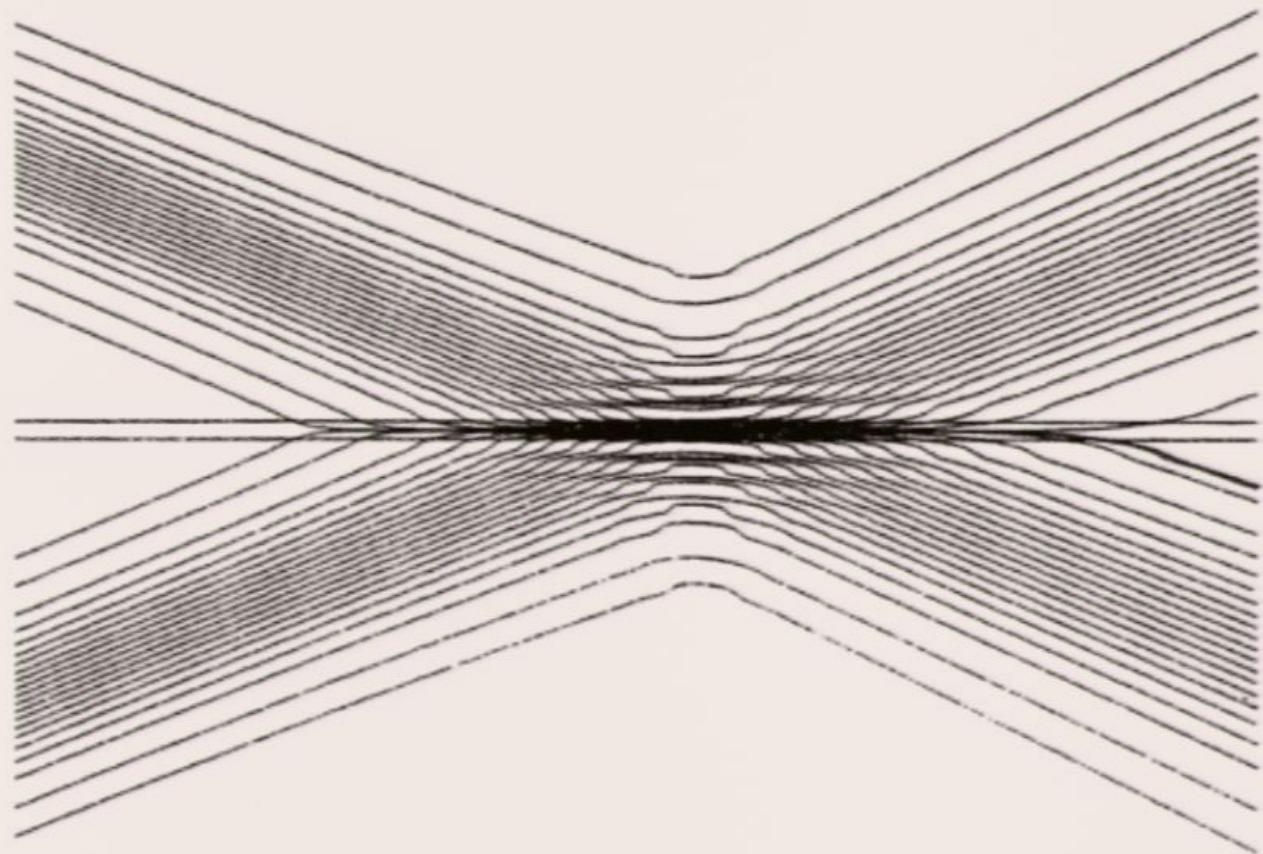
Transmission through a barrier (probability $\frac{1}{2}$)



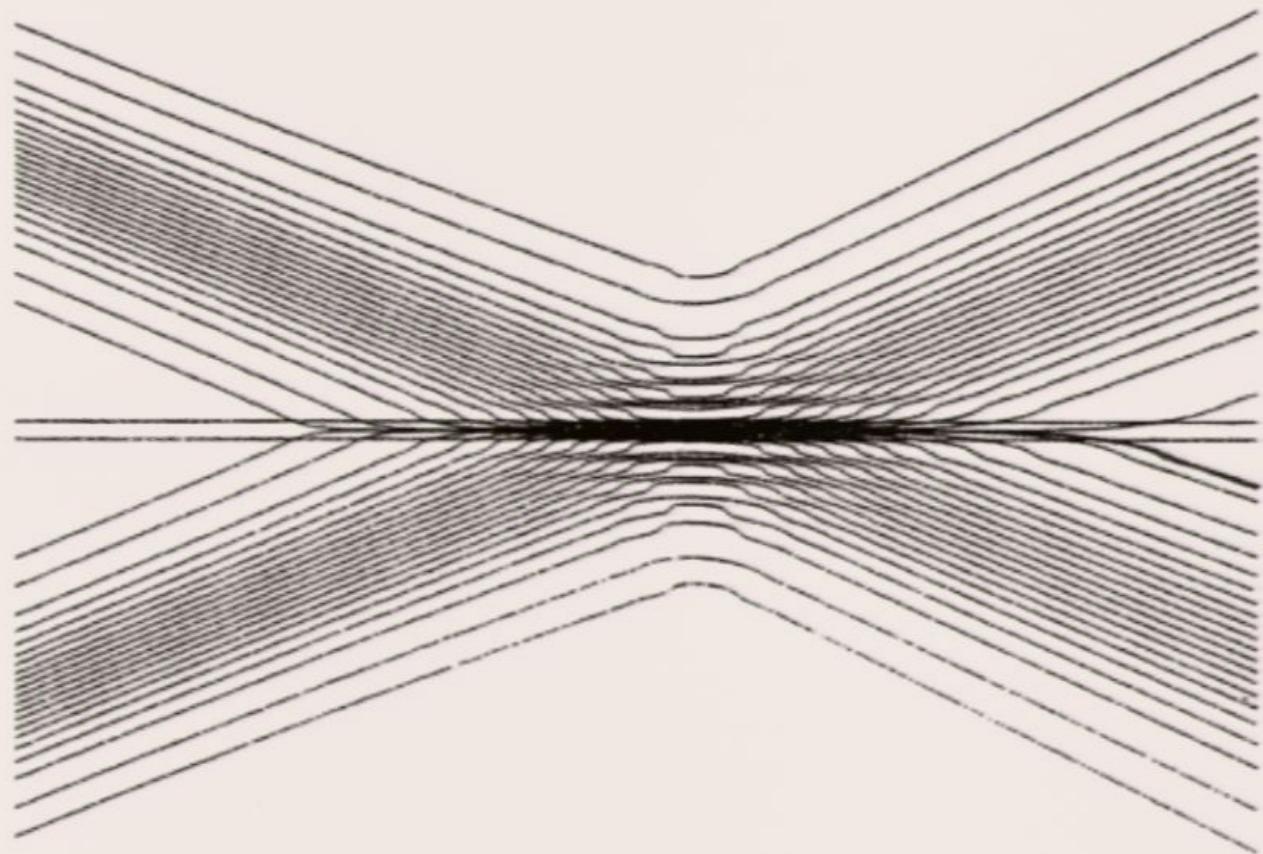
Beam splitter experiment



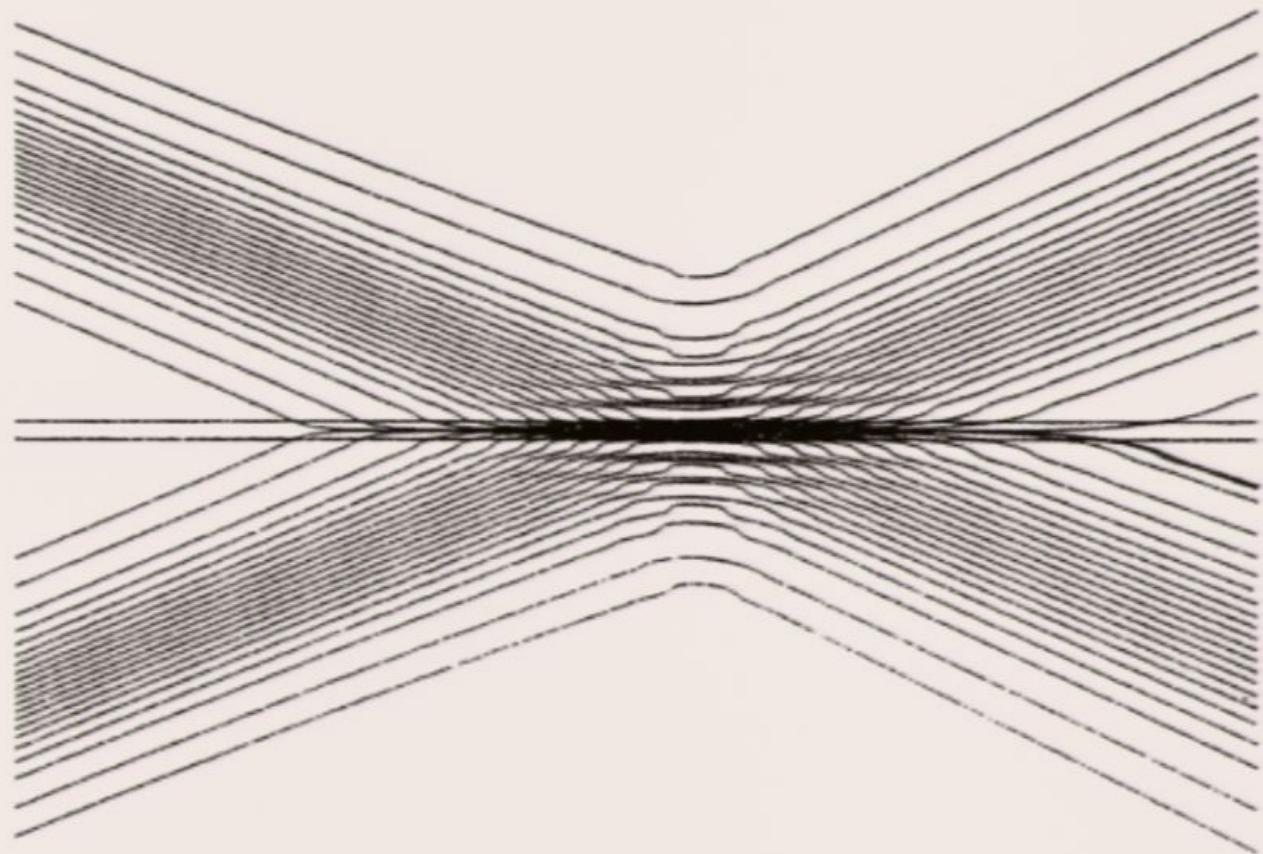
Beam splitter experiment



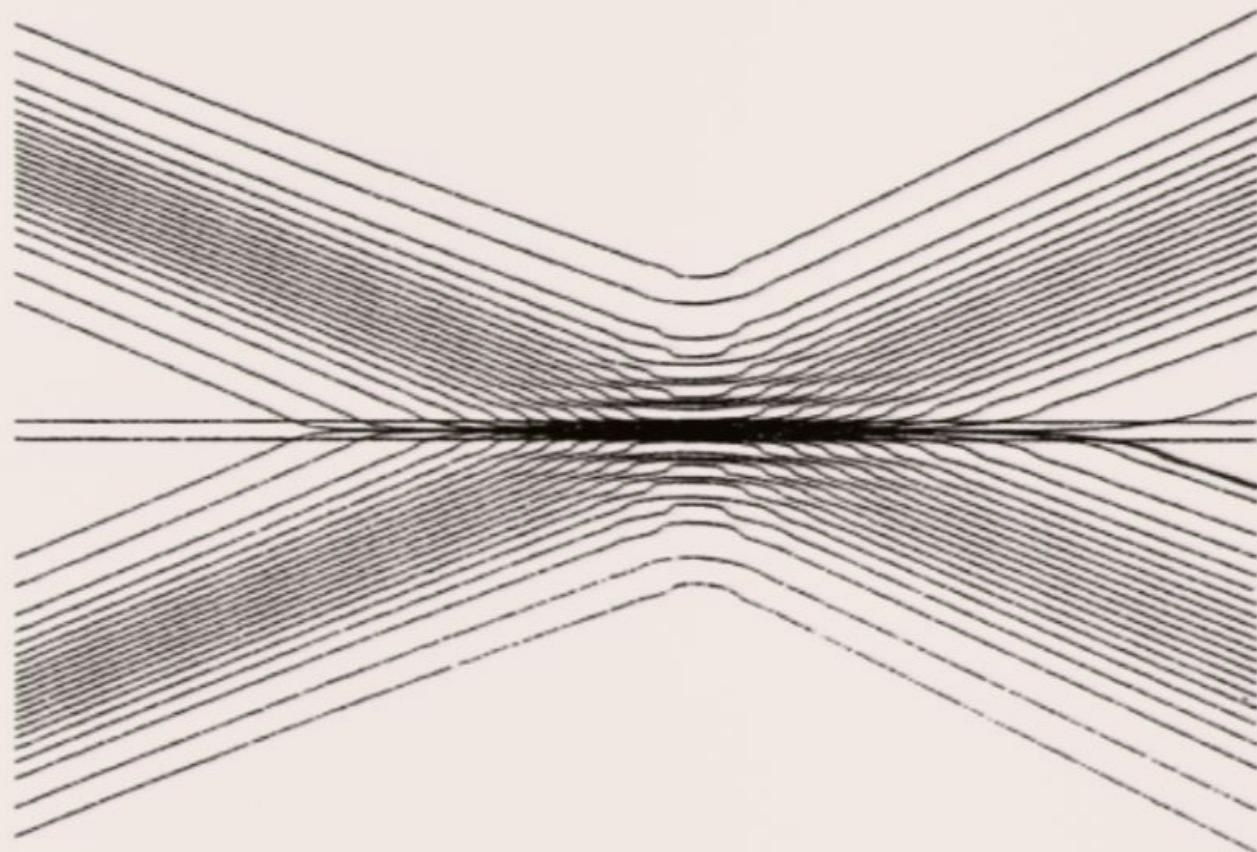
Beam splitter experiment



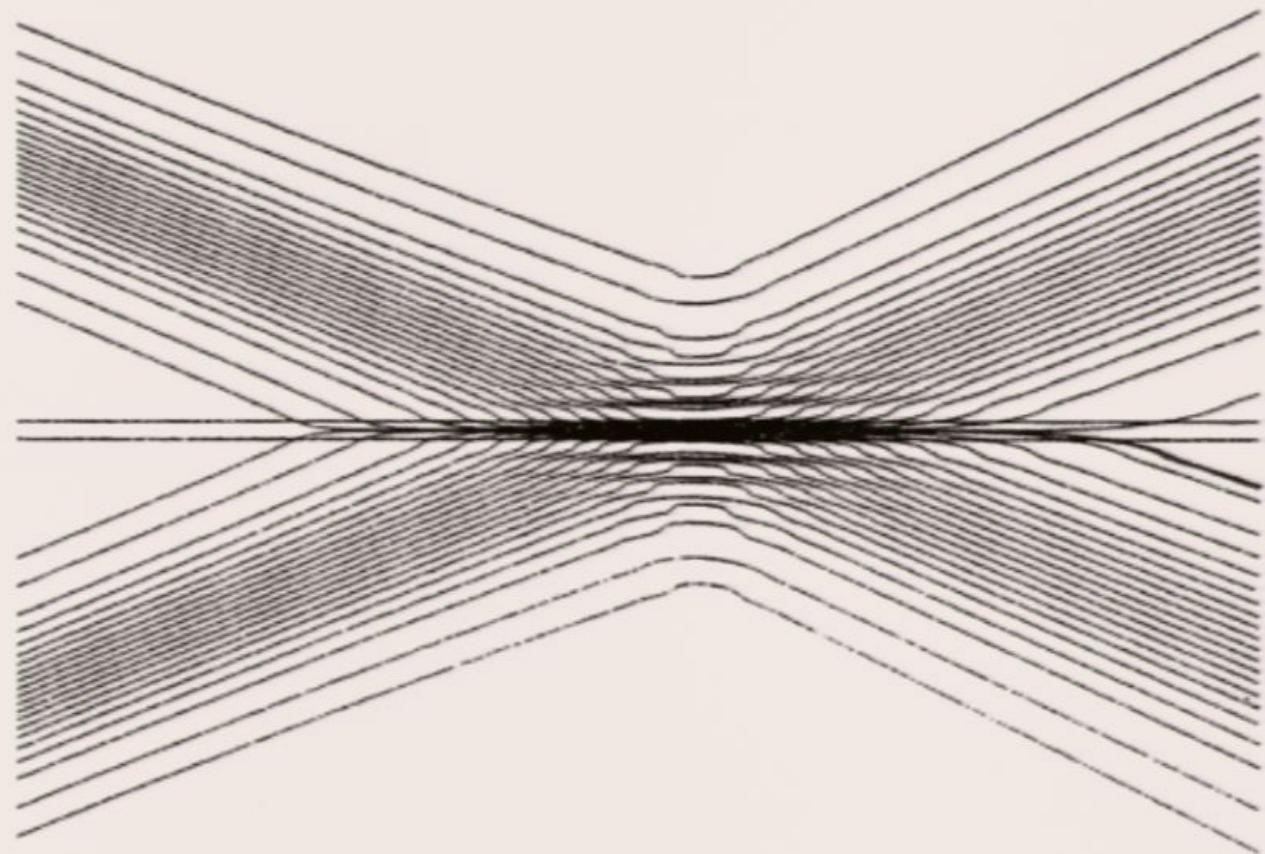
Beam splitter experiment



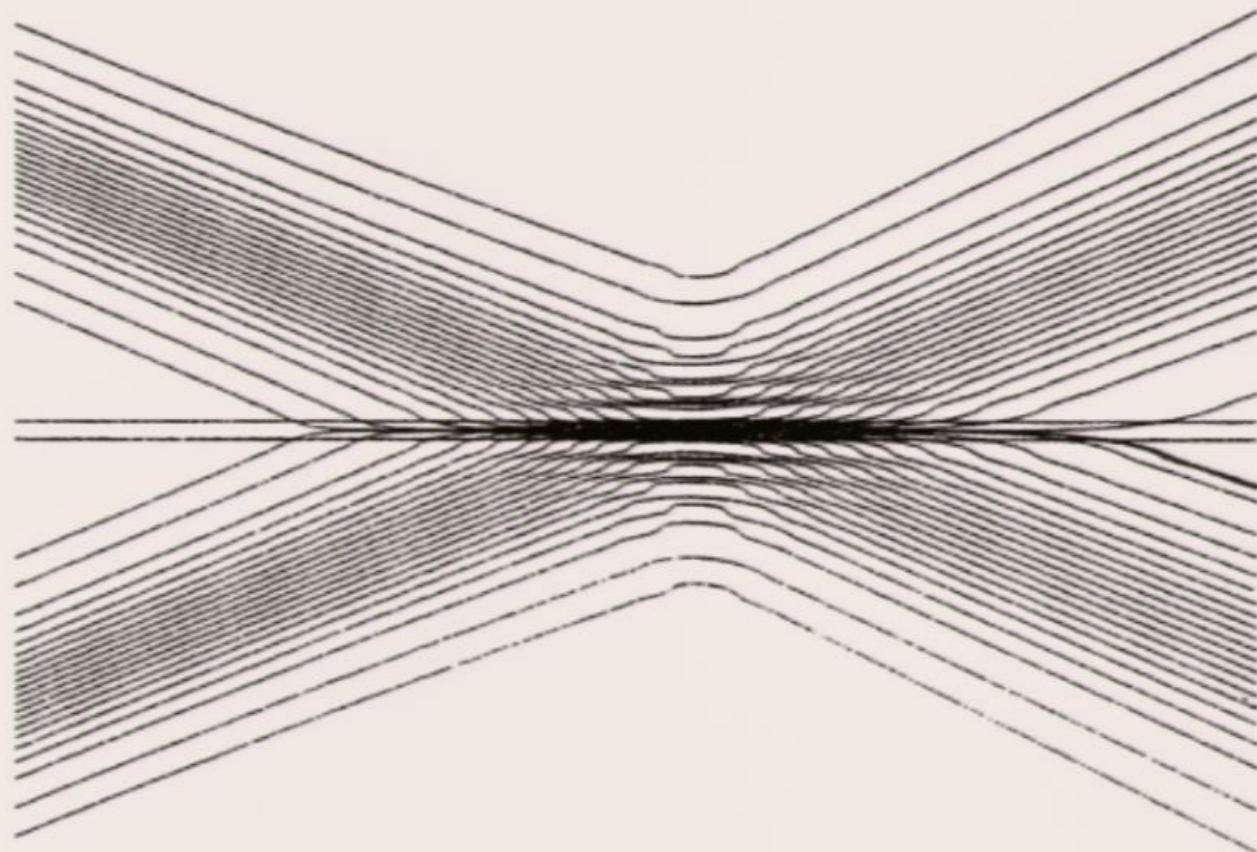
Beam splitter experiment



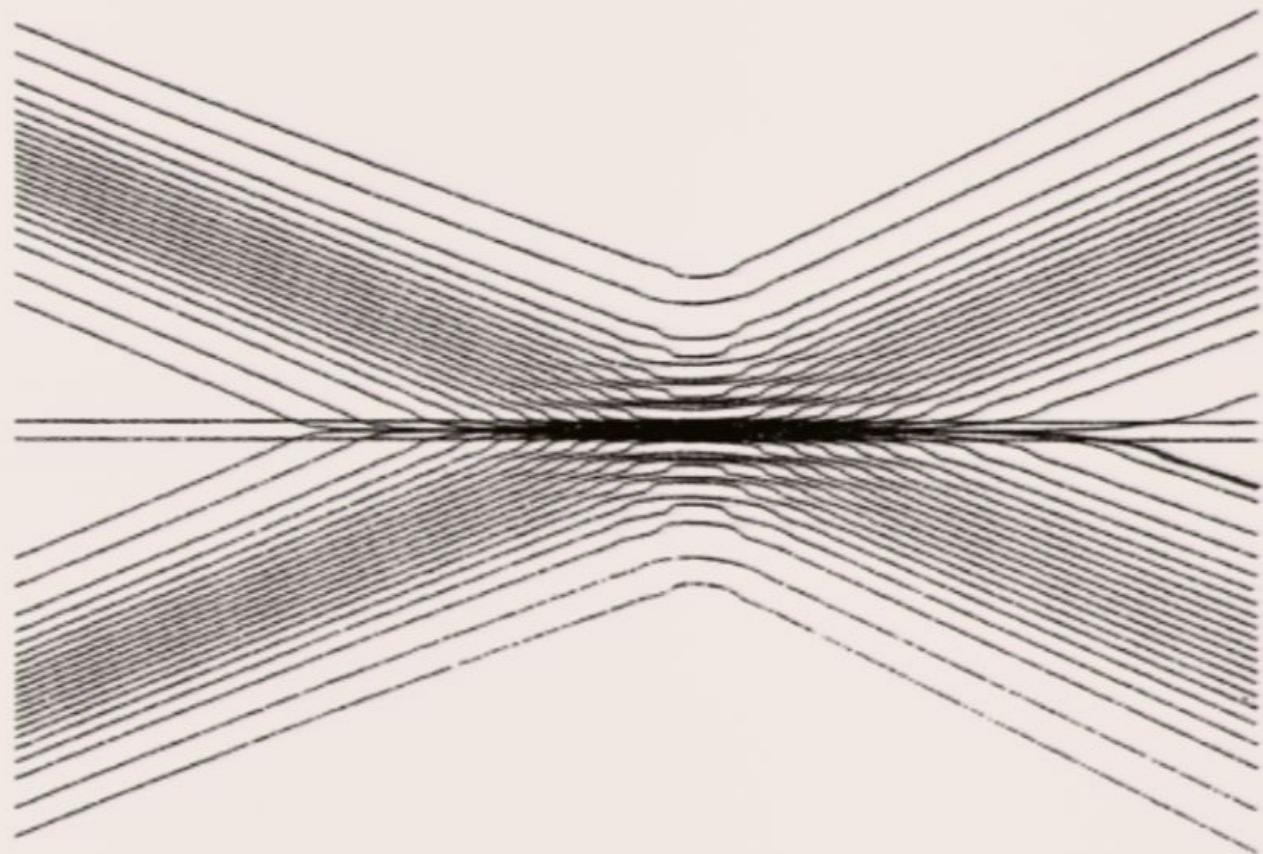
Beam splitter experiment



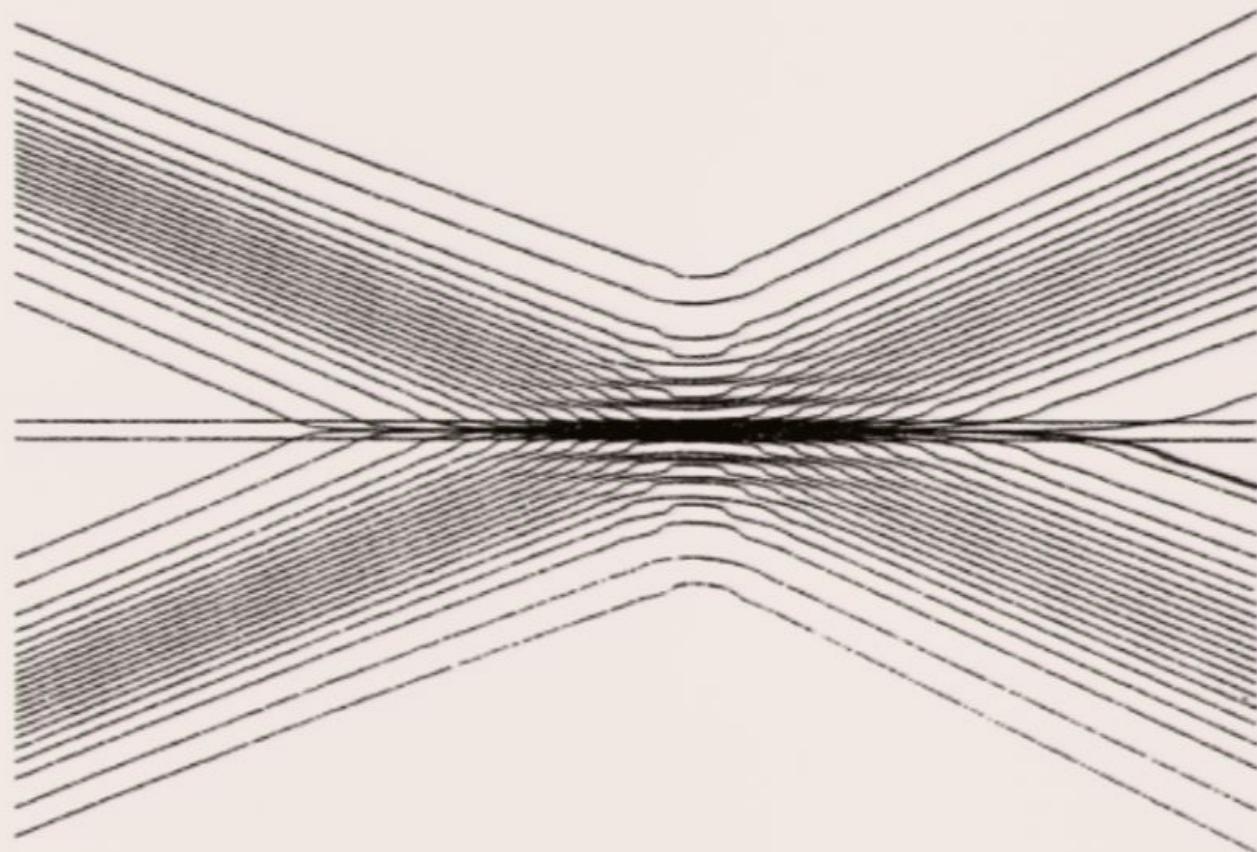
Beam splitter experiment



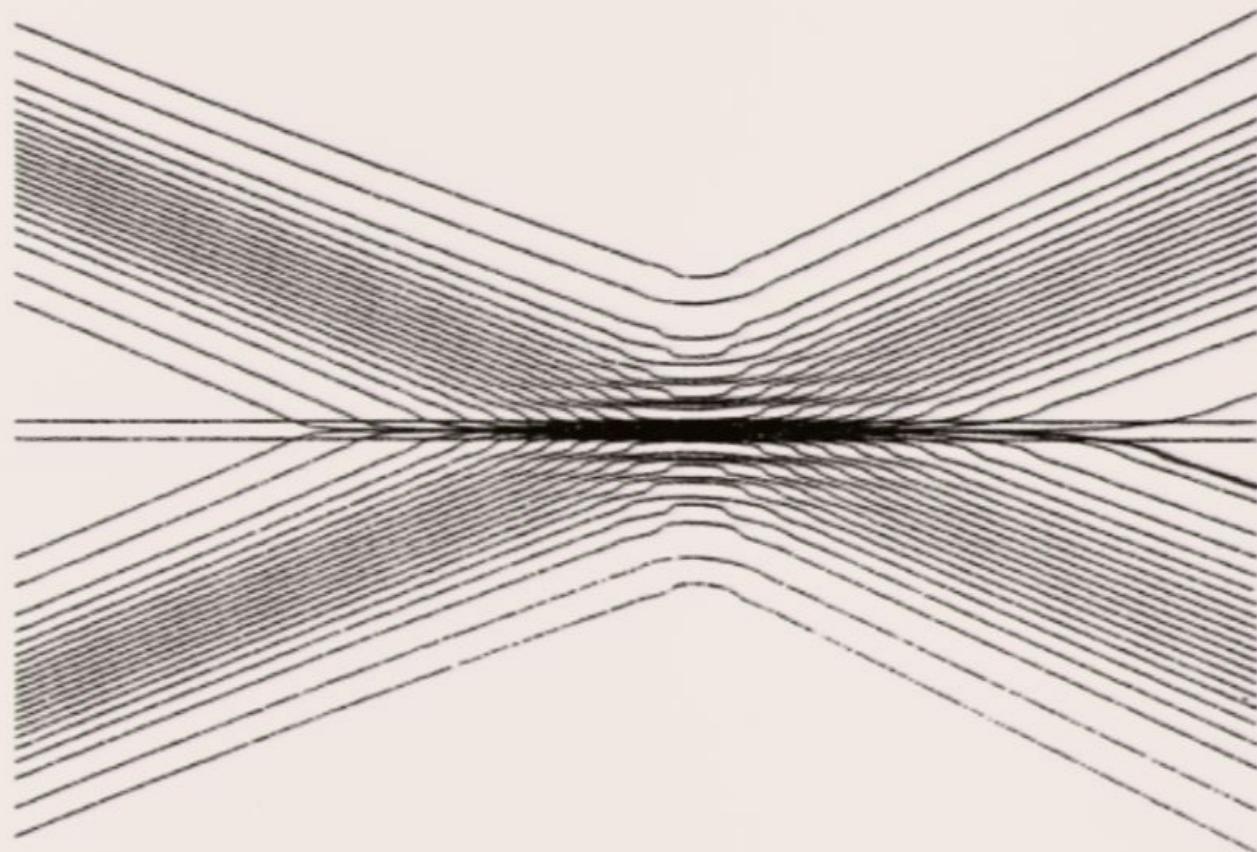
Beam splitter experiment



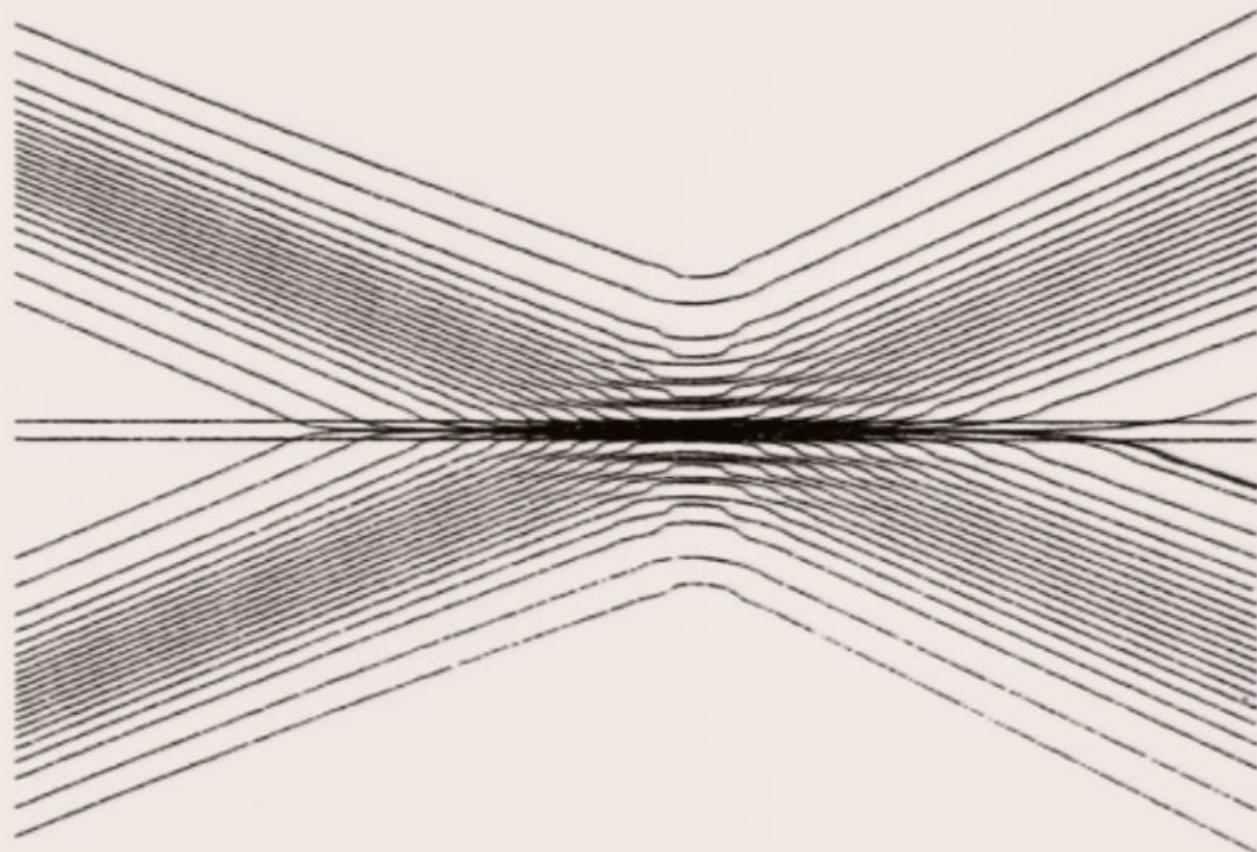
Beam splitter experiment



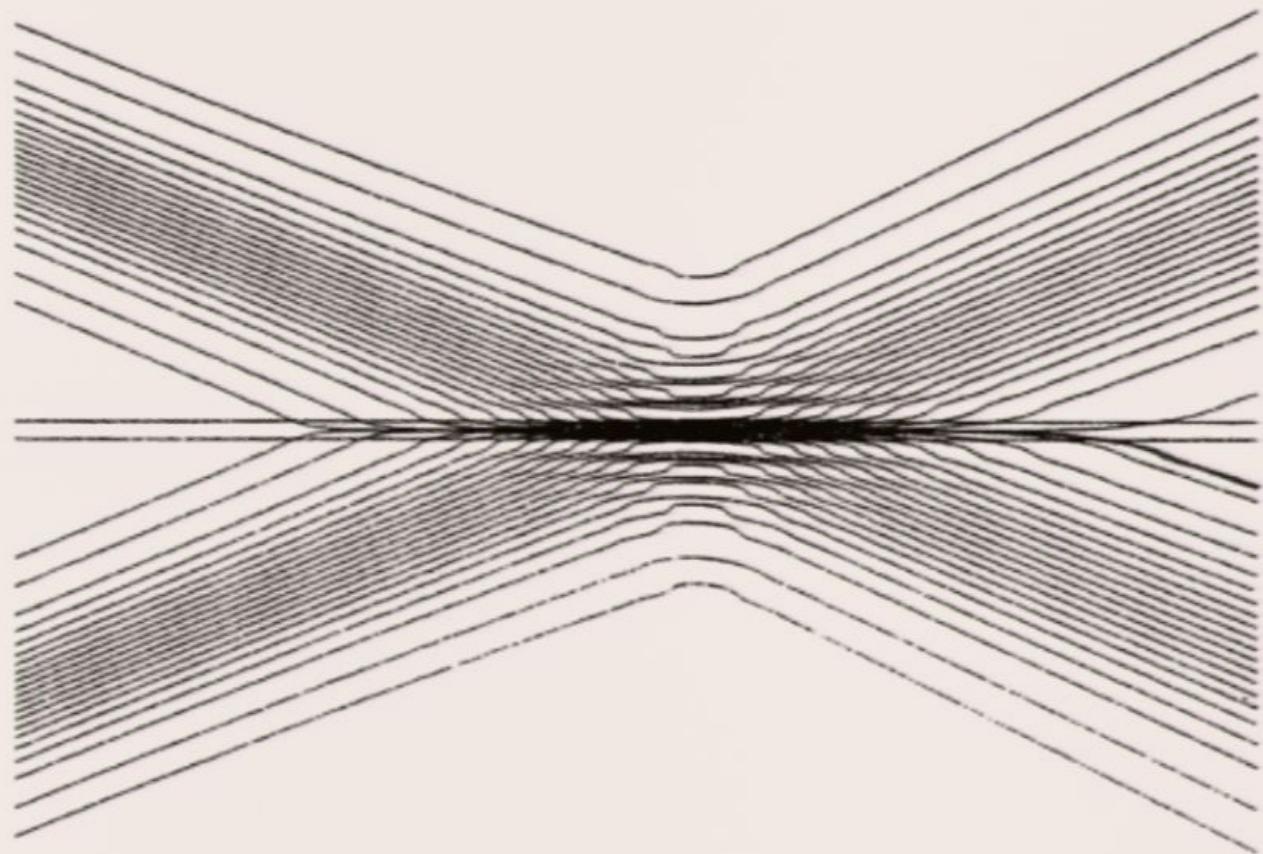
Beam splitter experiment



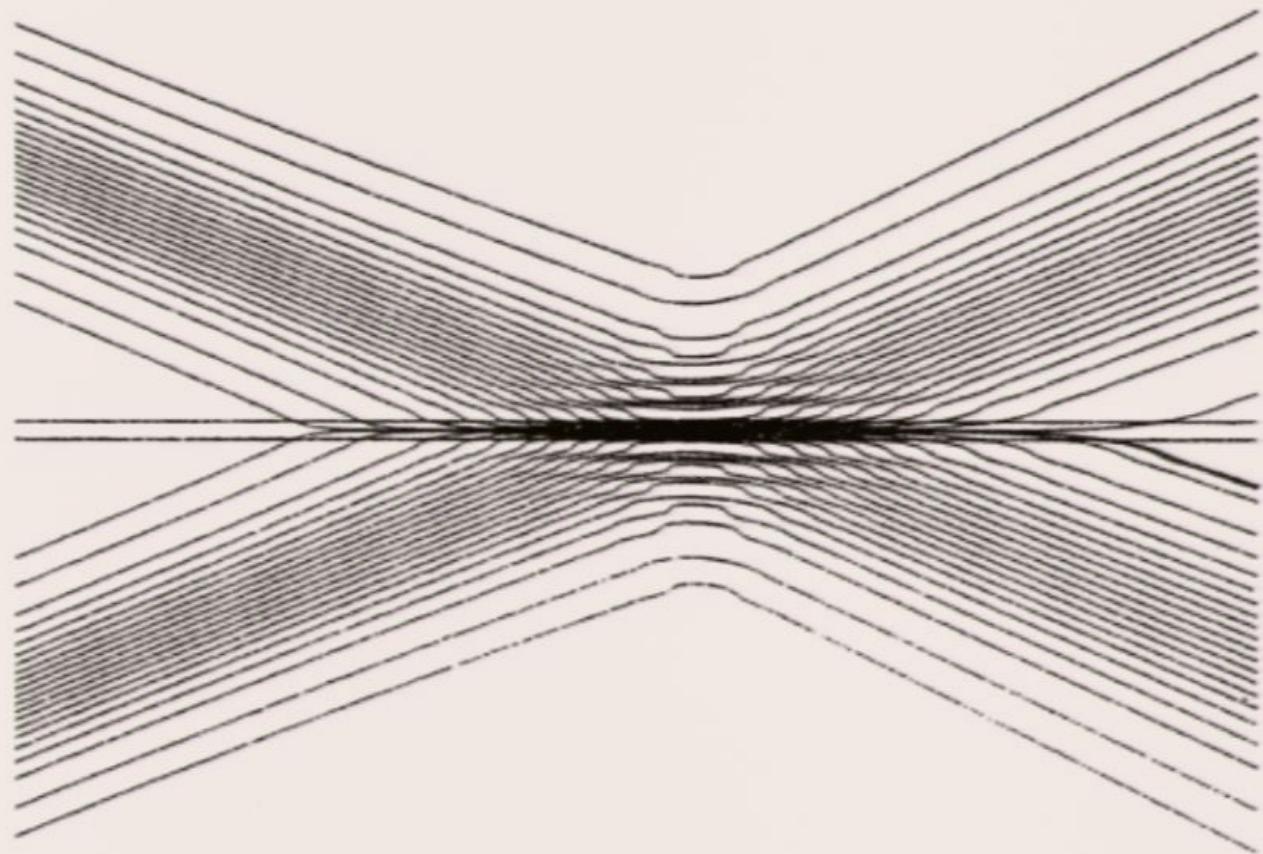
Beam splitter experiment



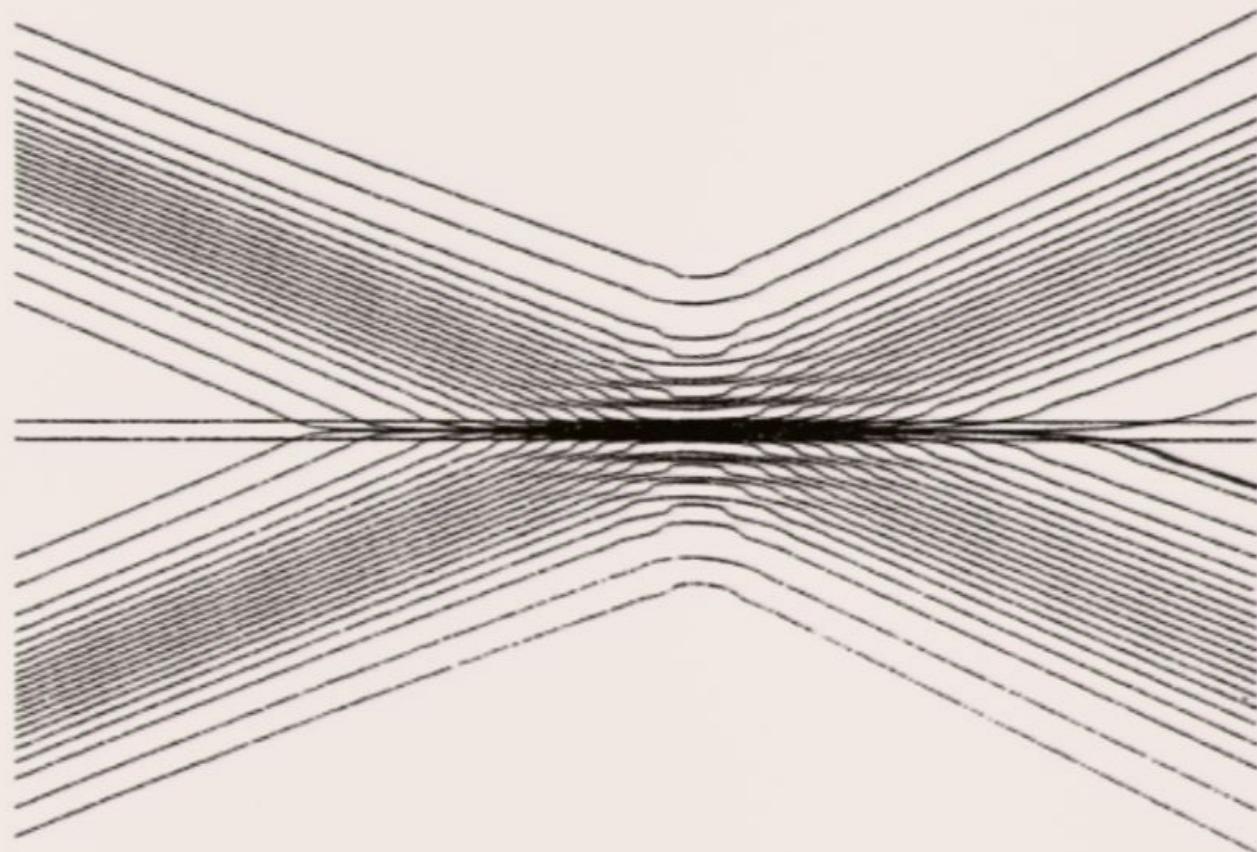
Beam splitter experiment



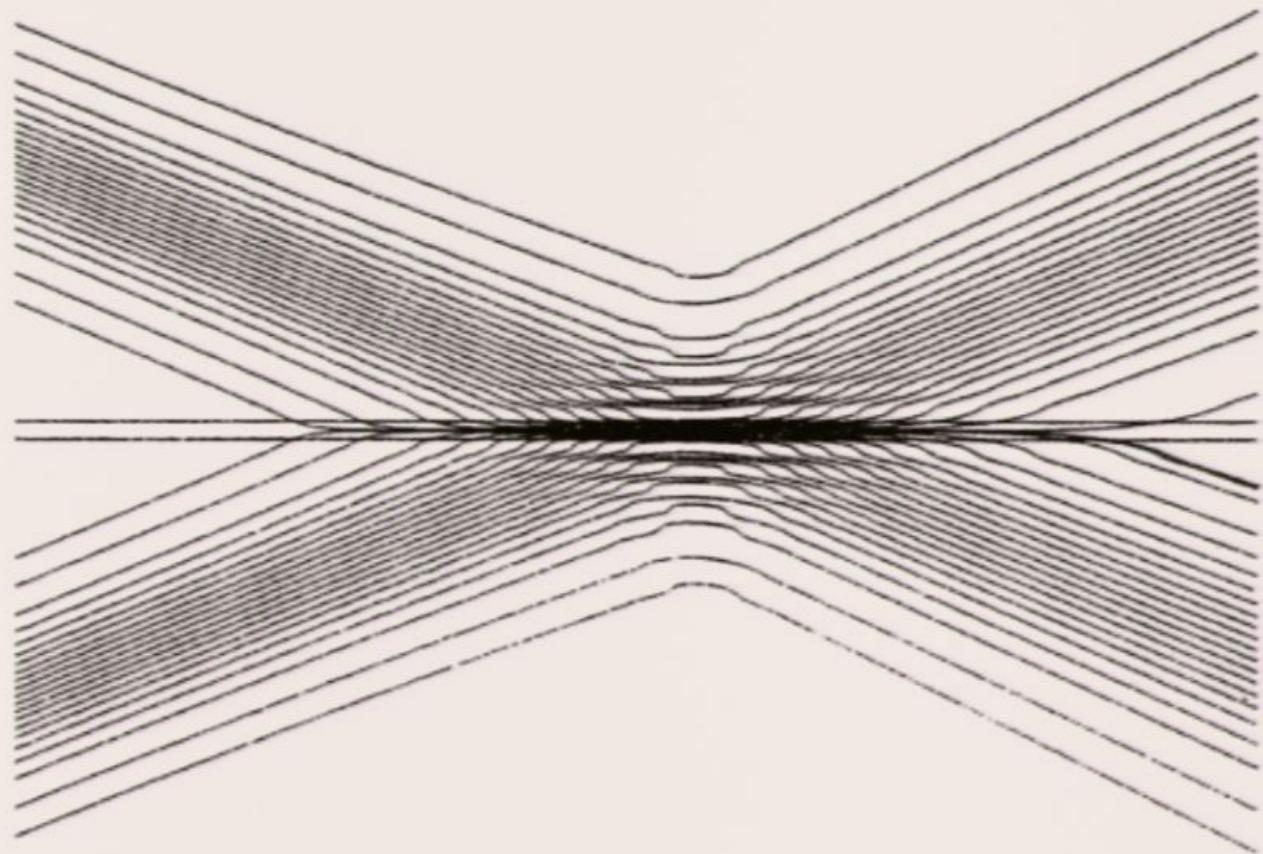
Beam splitter experiment



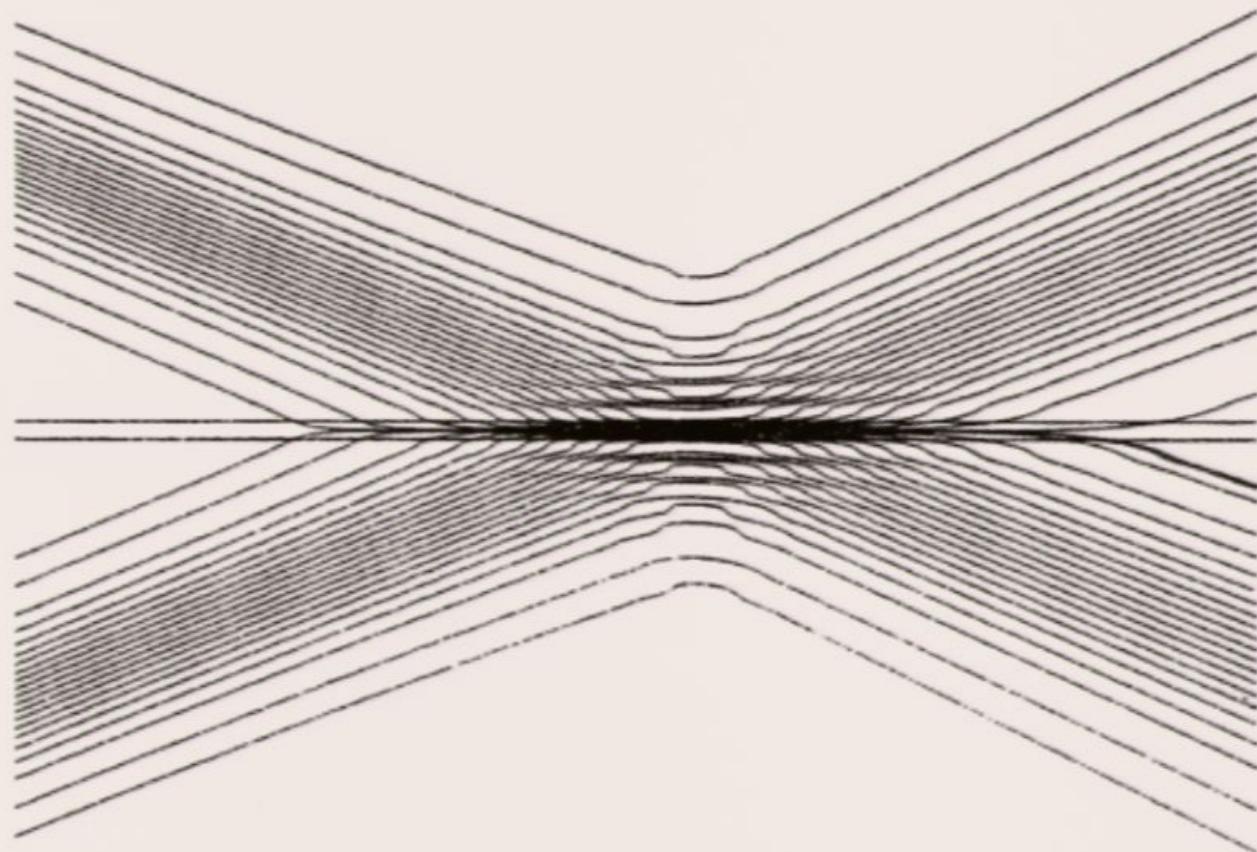
Beam splitter experiment



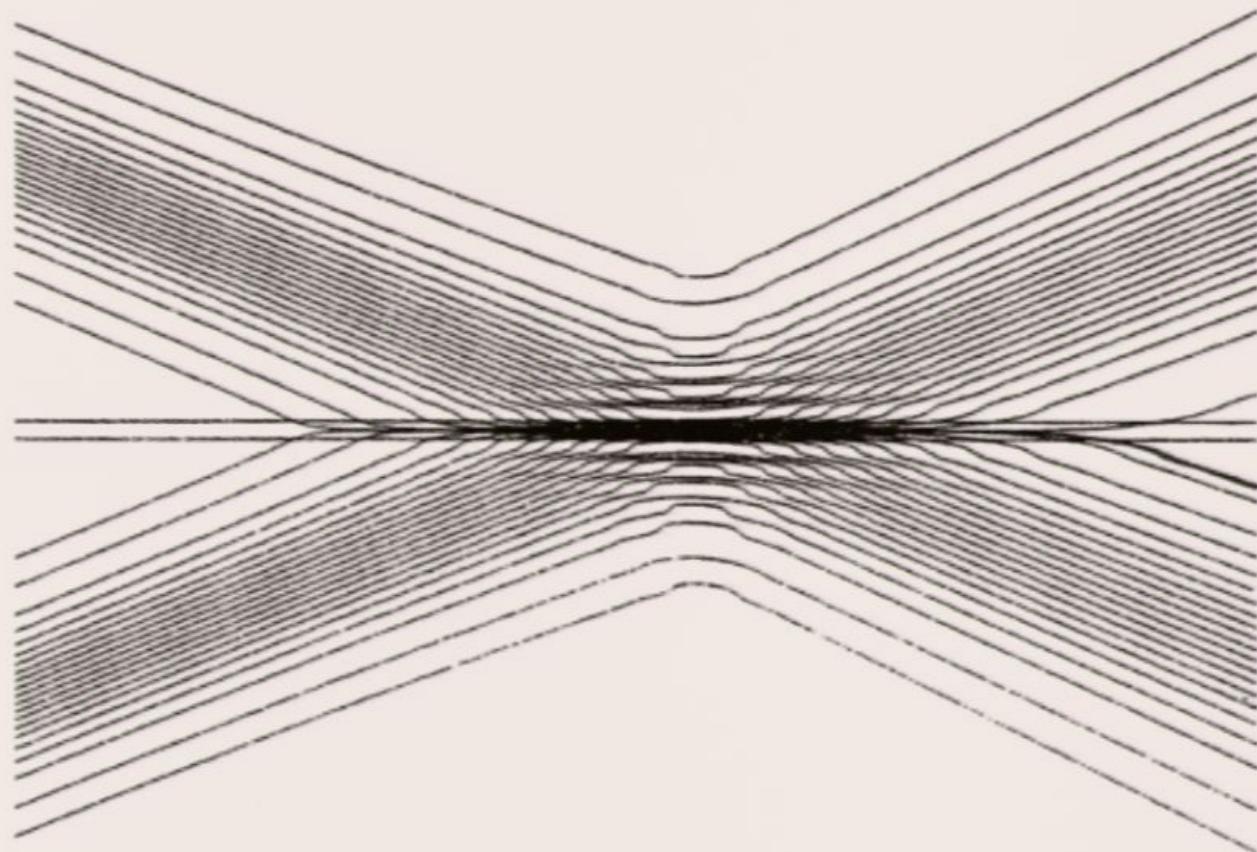
Beam splitter experiment



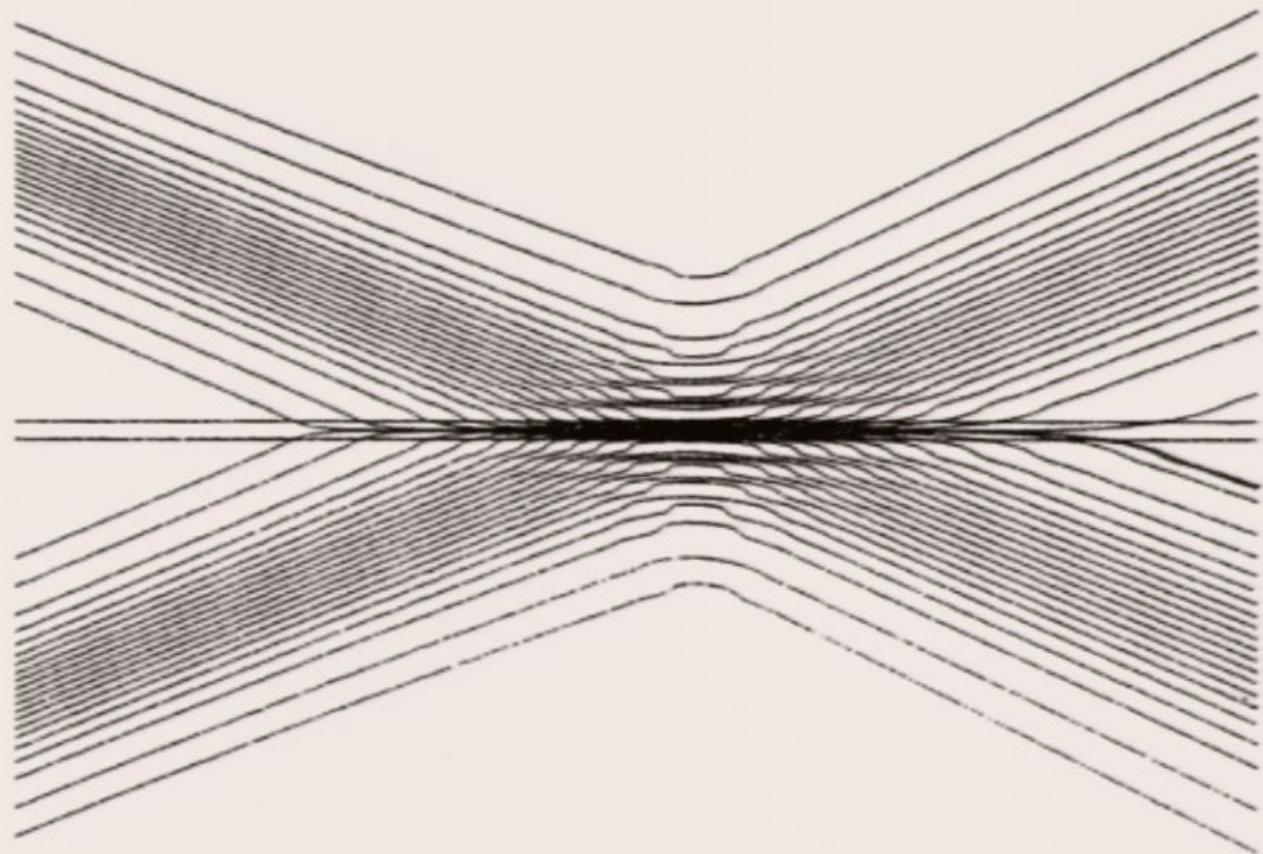
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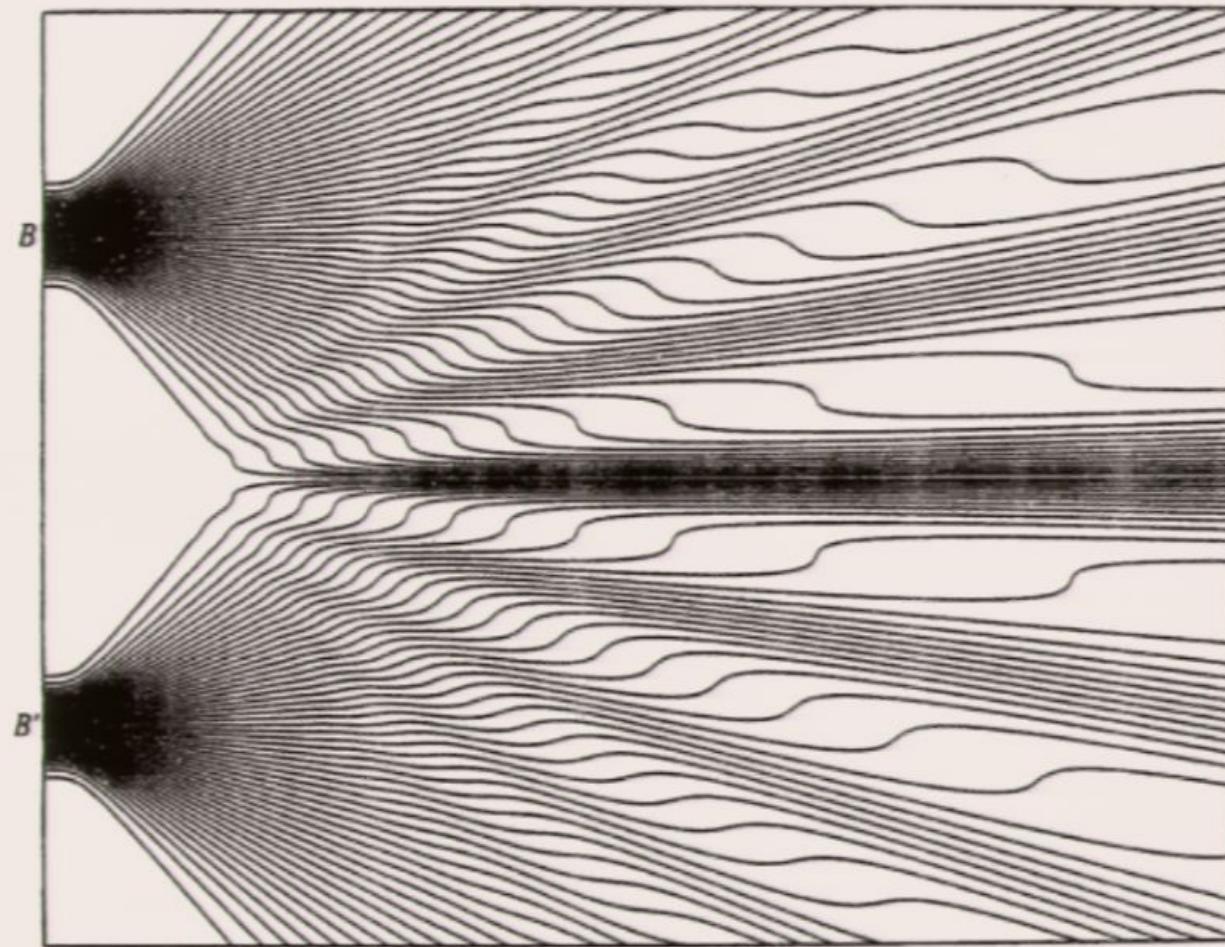
Beam splitter experiment



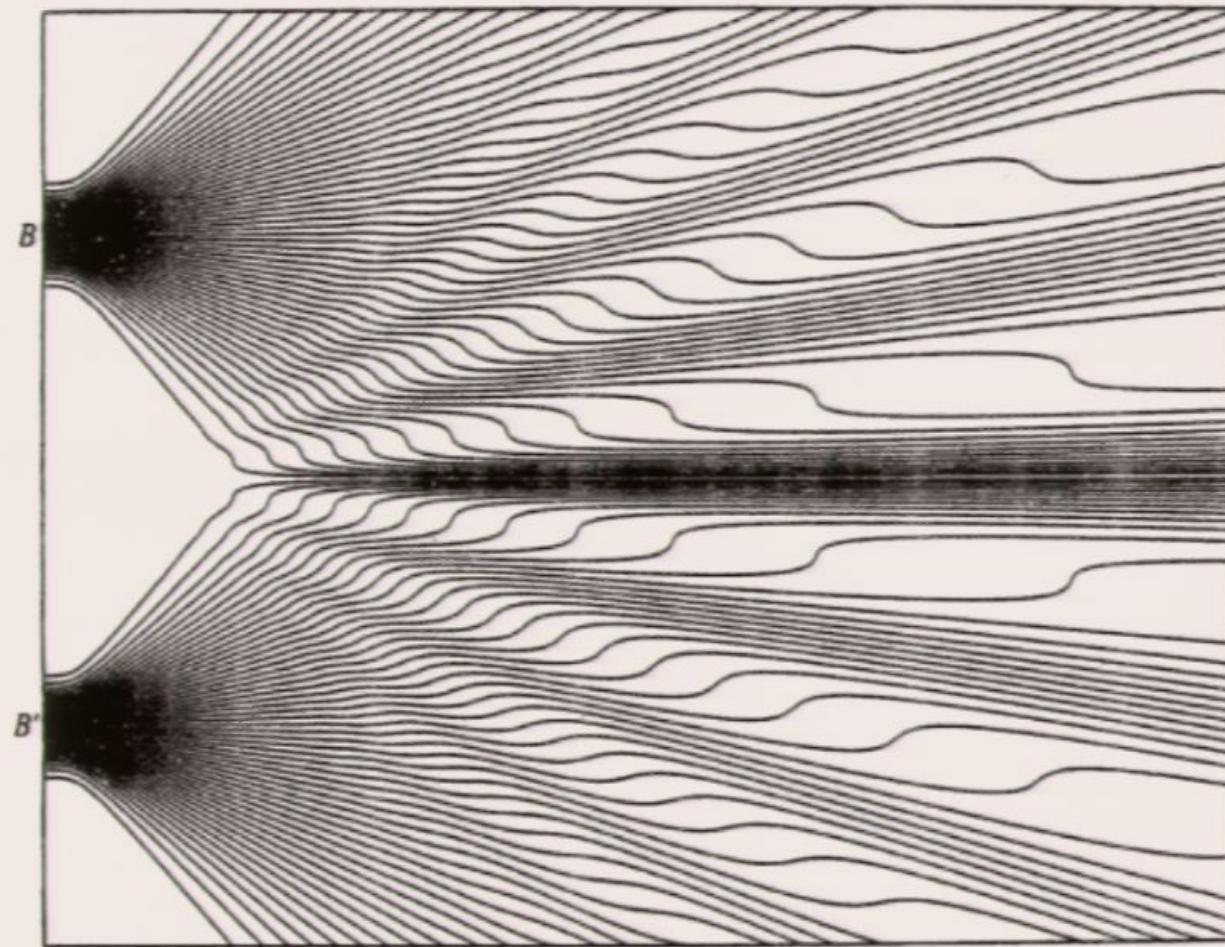
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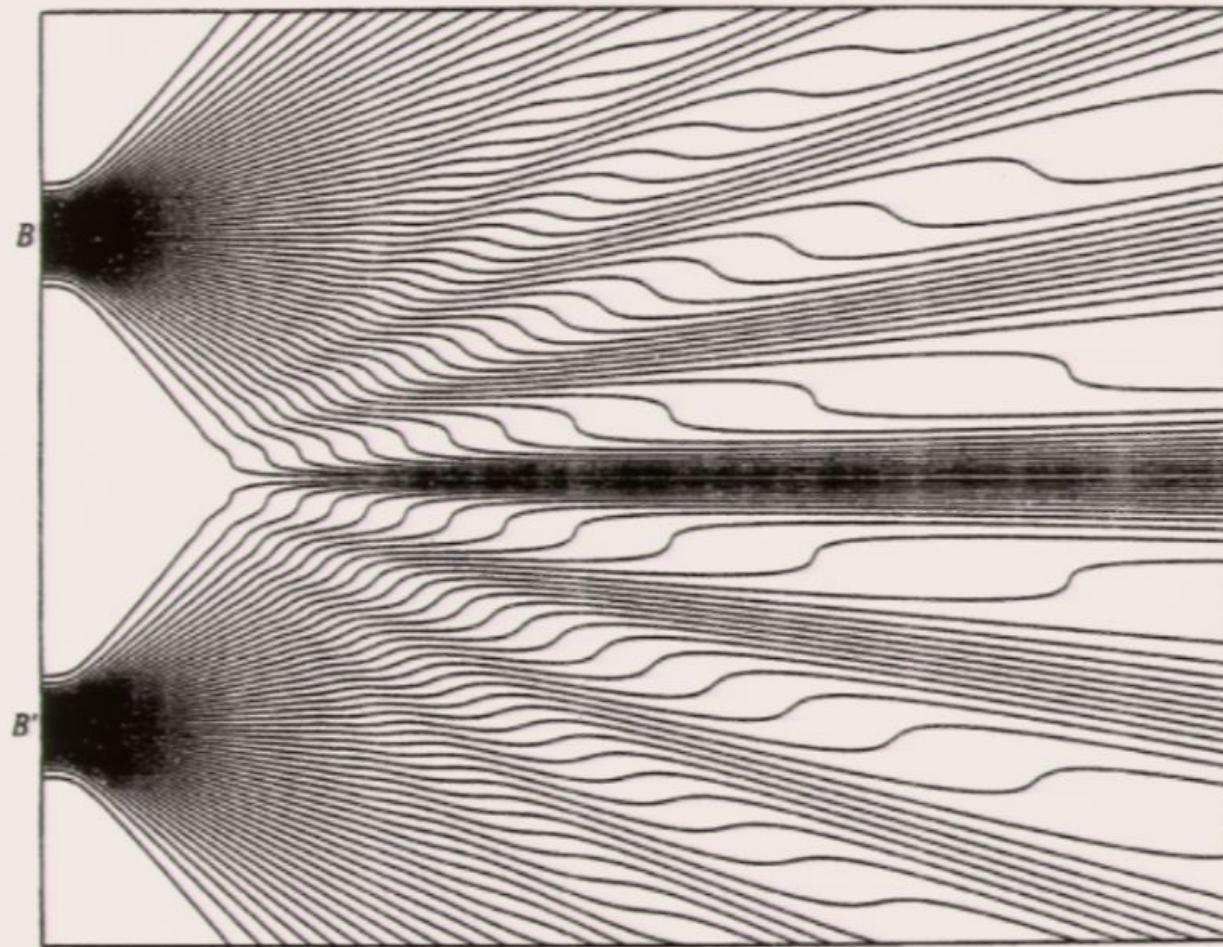
Beam splitter experiment



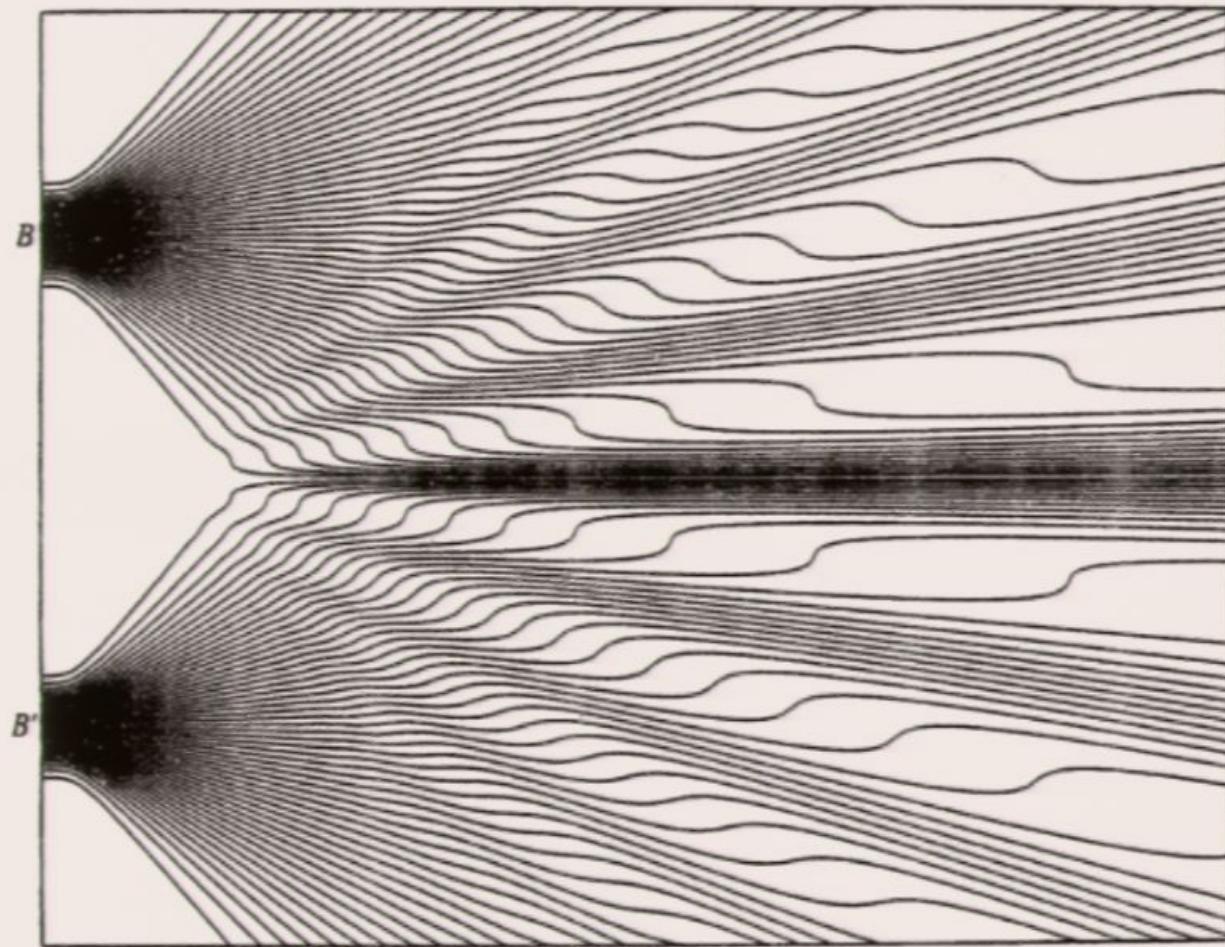
Double slit experiment



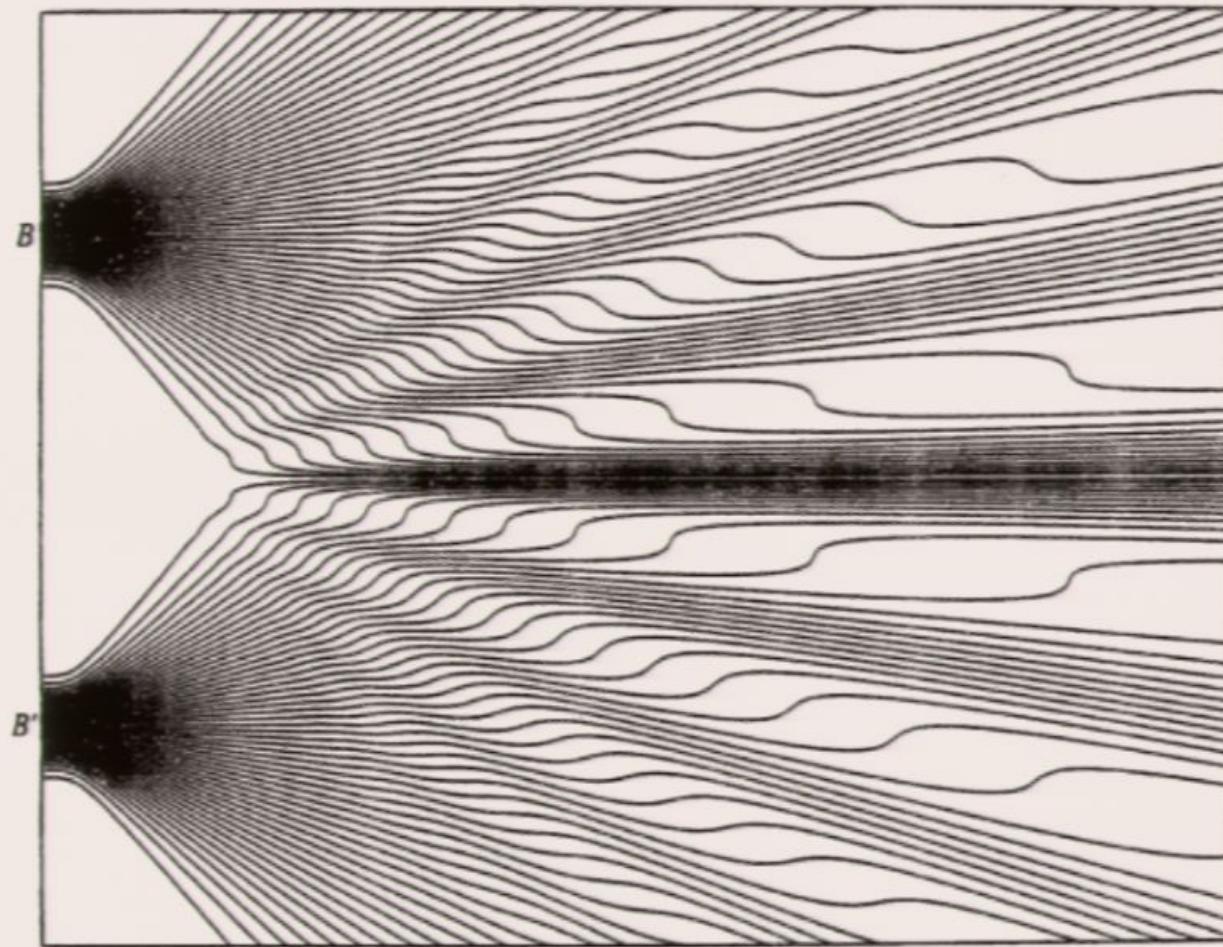
Double slit experiment



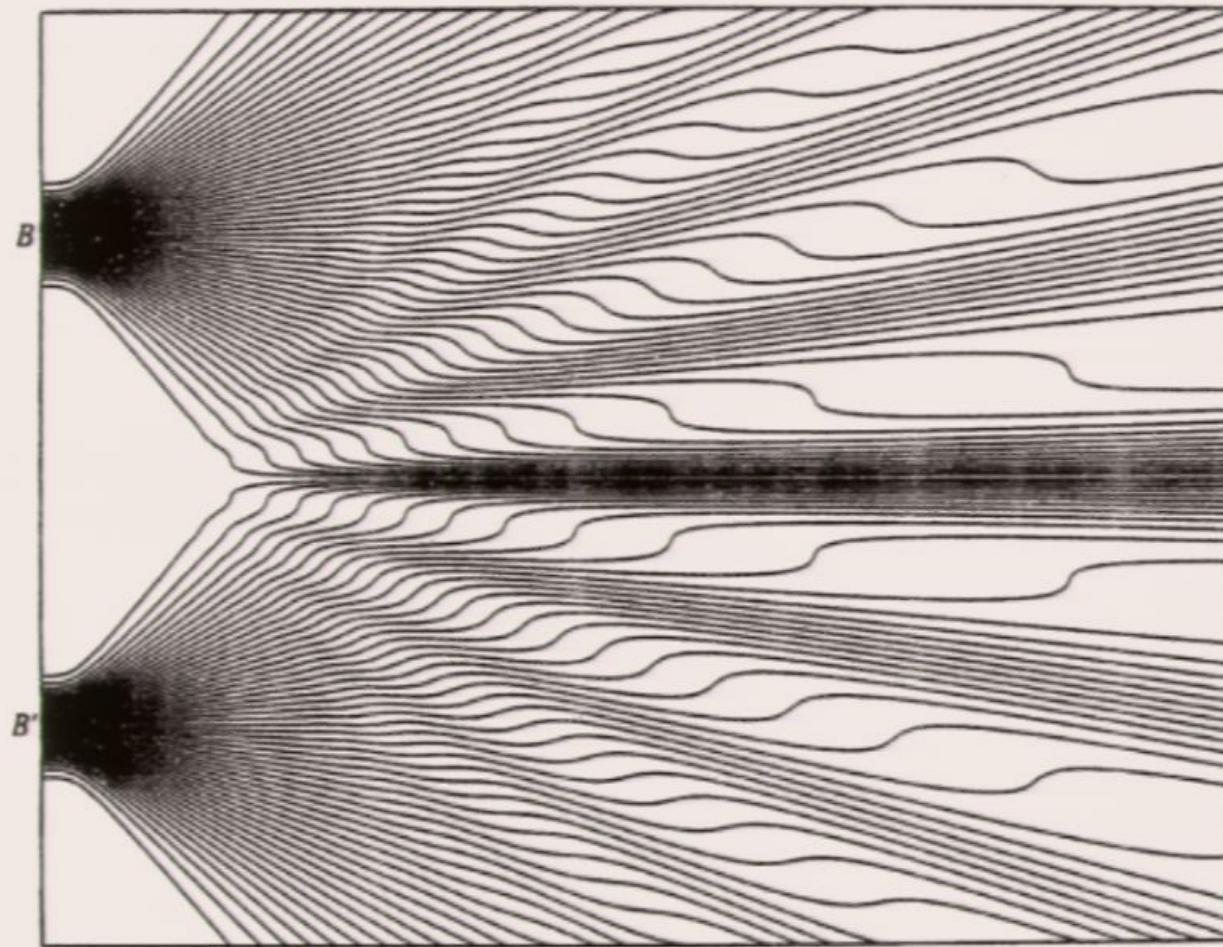
Double slit experiment



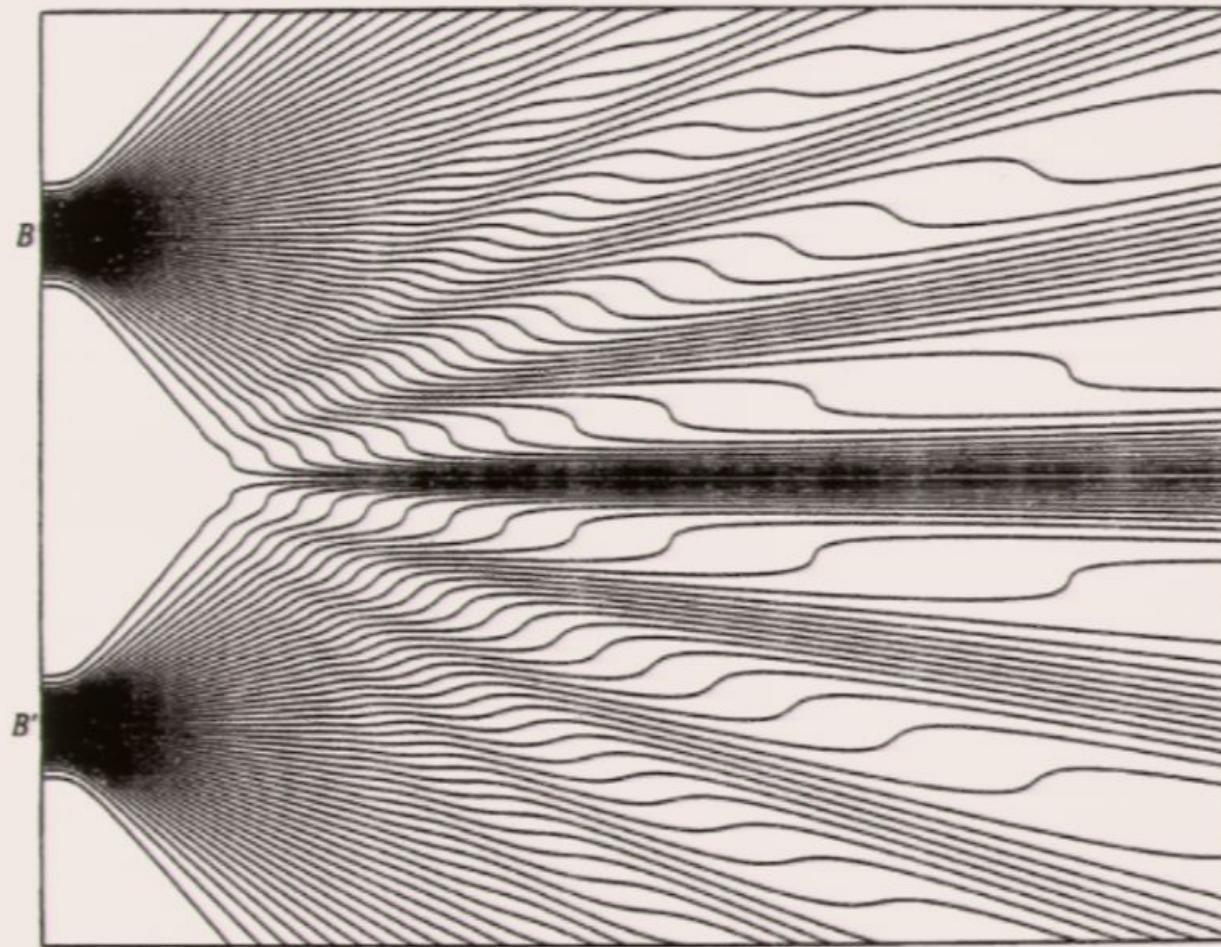
Double slit experiment



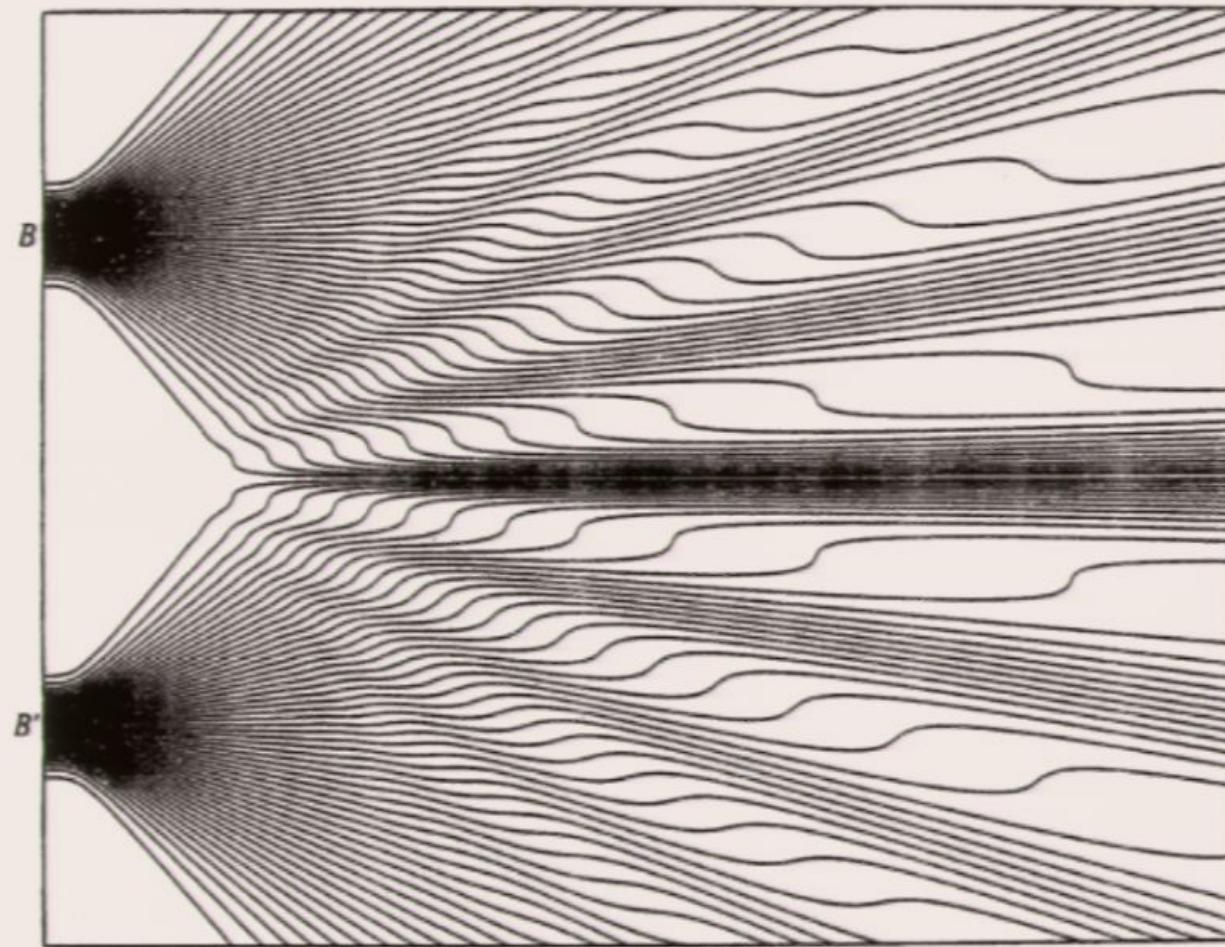
Double slit experiment



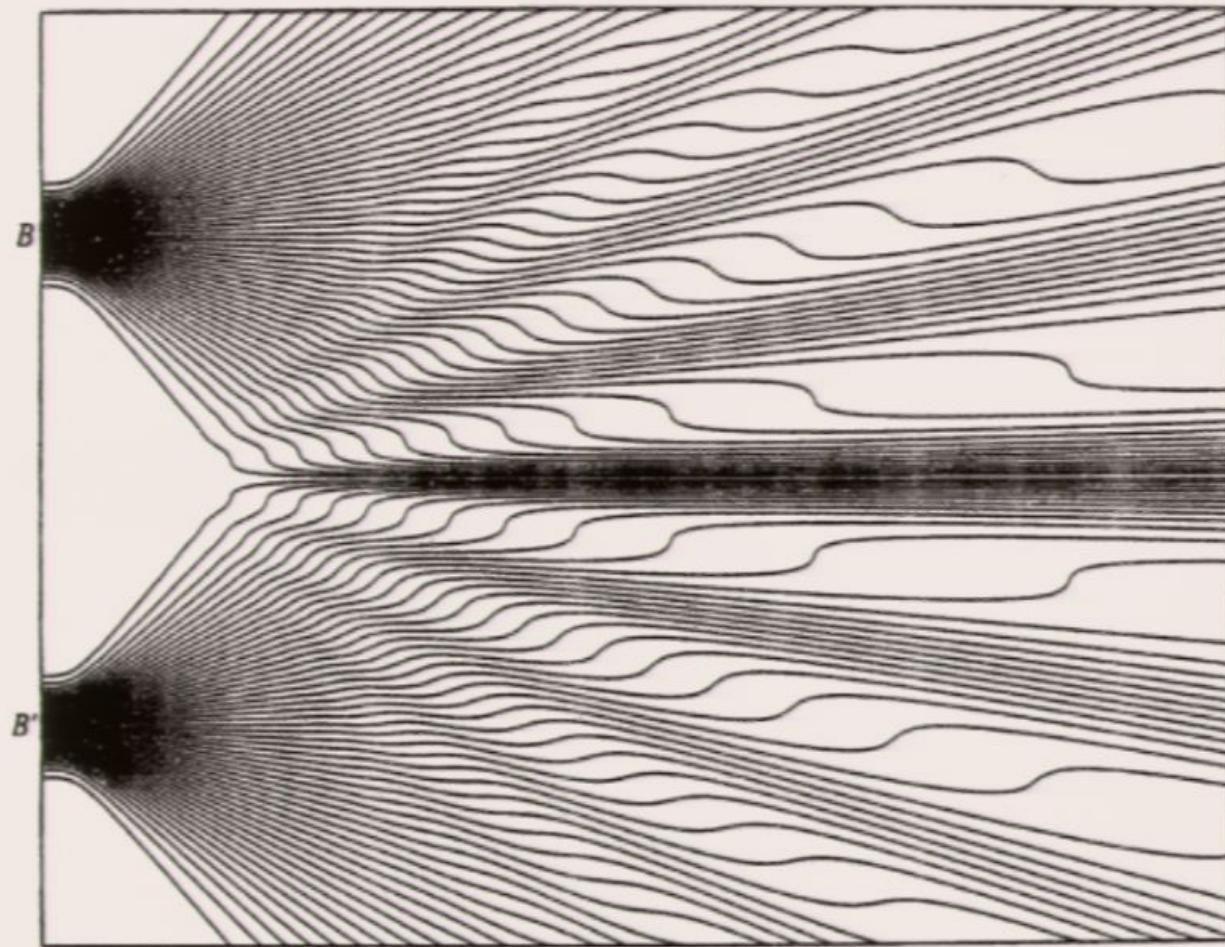
Double slit experiment



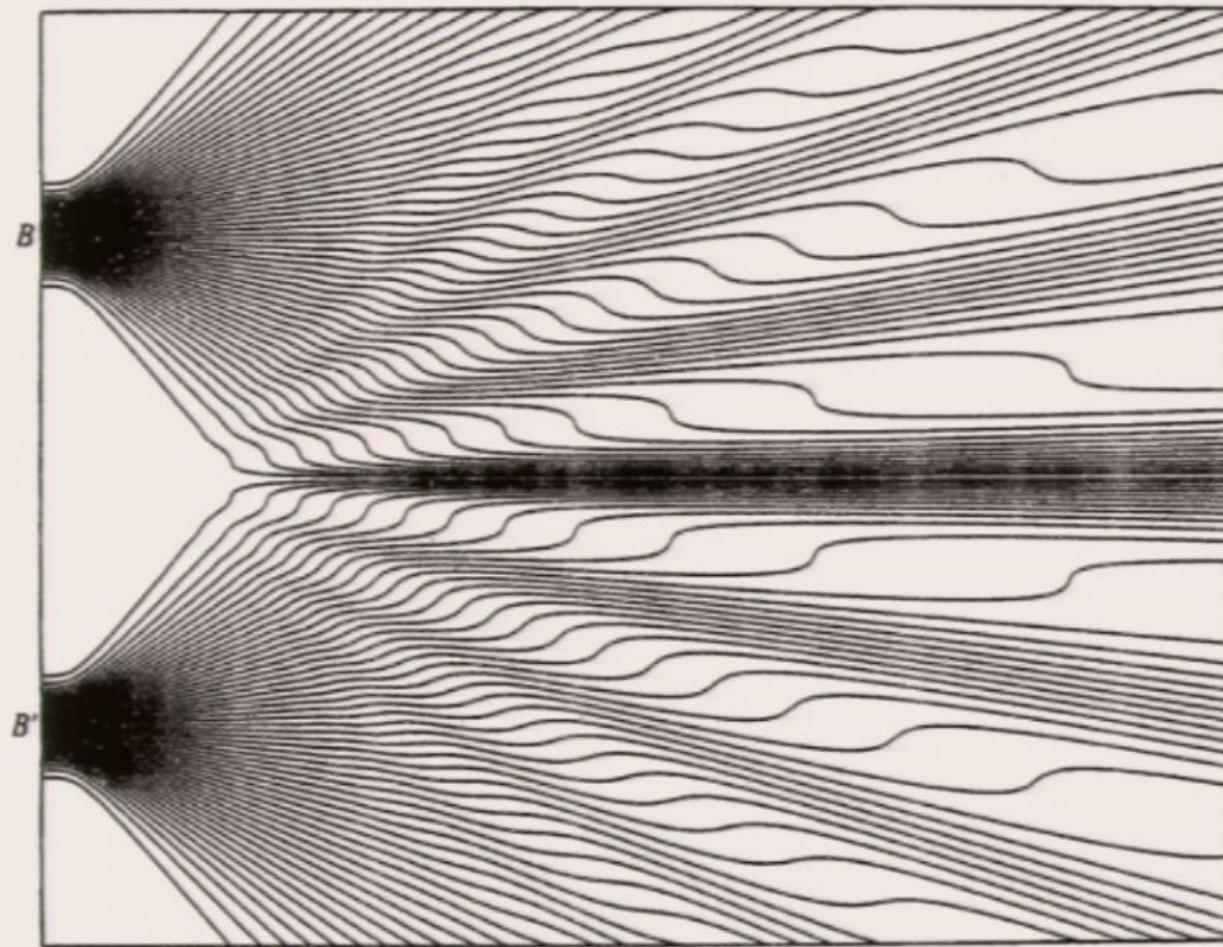
Double slit experiment



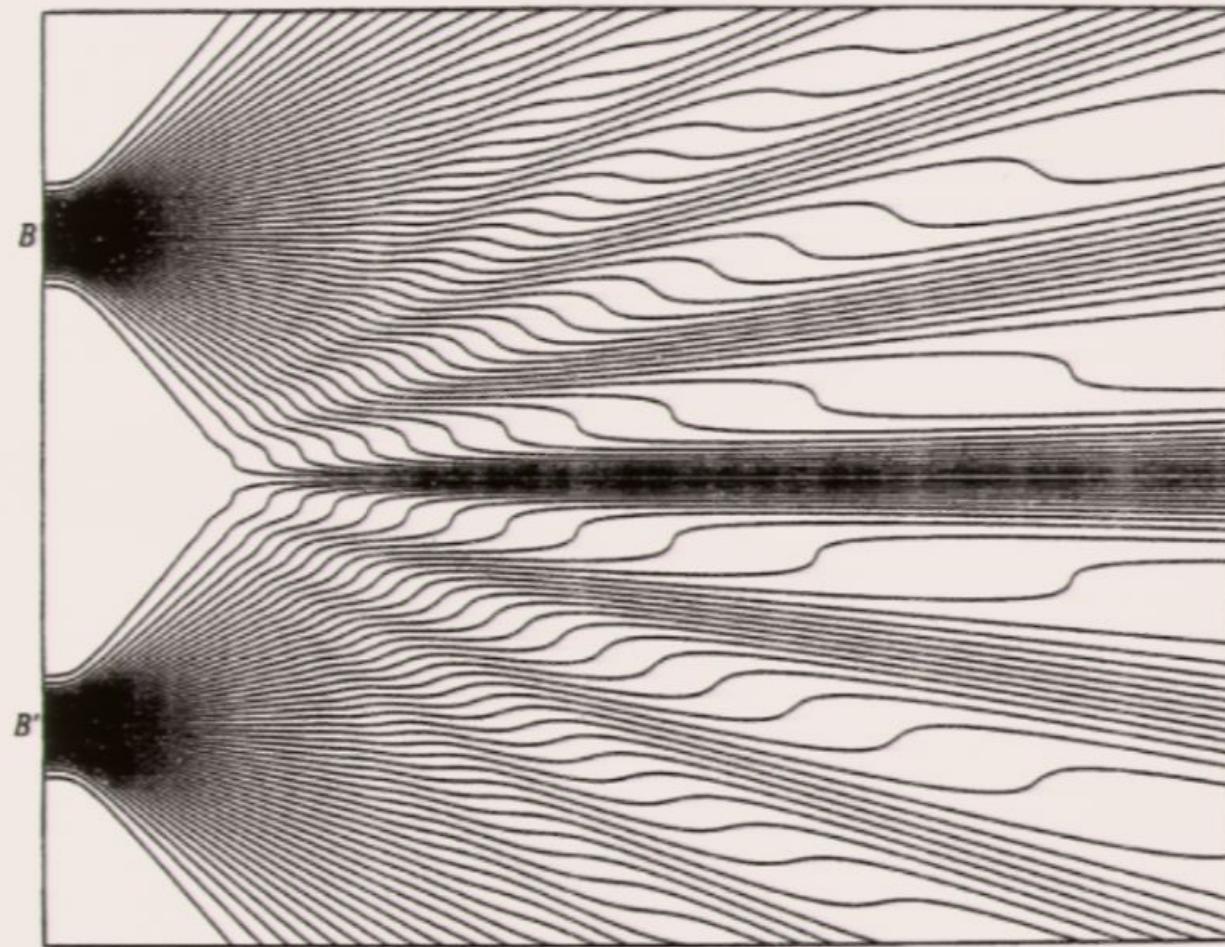
Double slit experiment



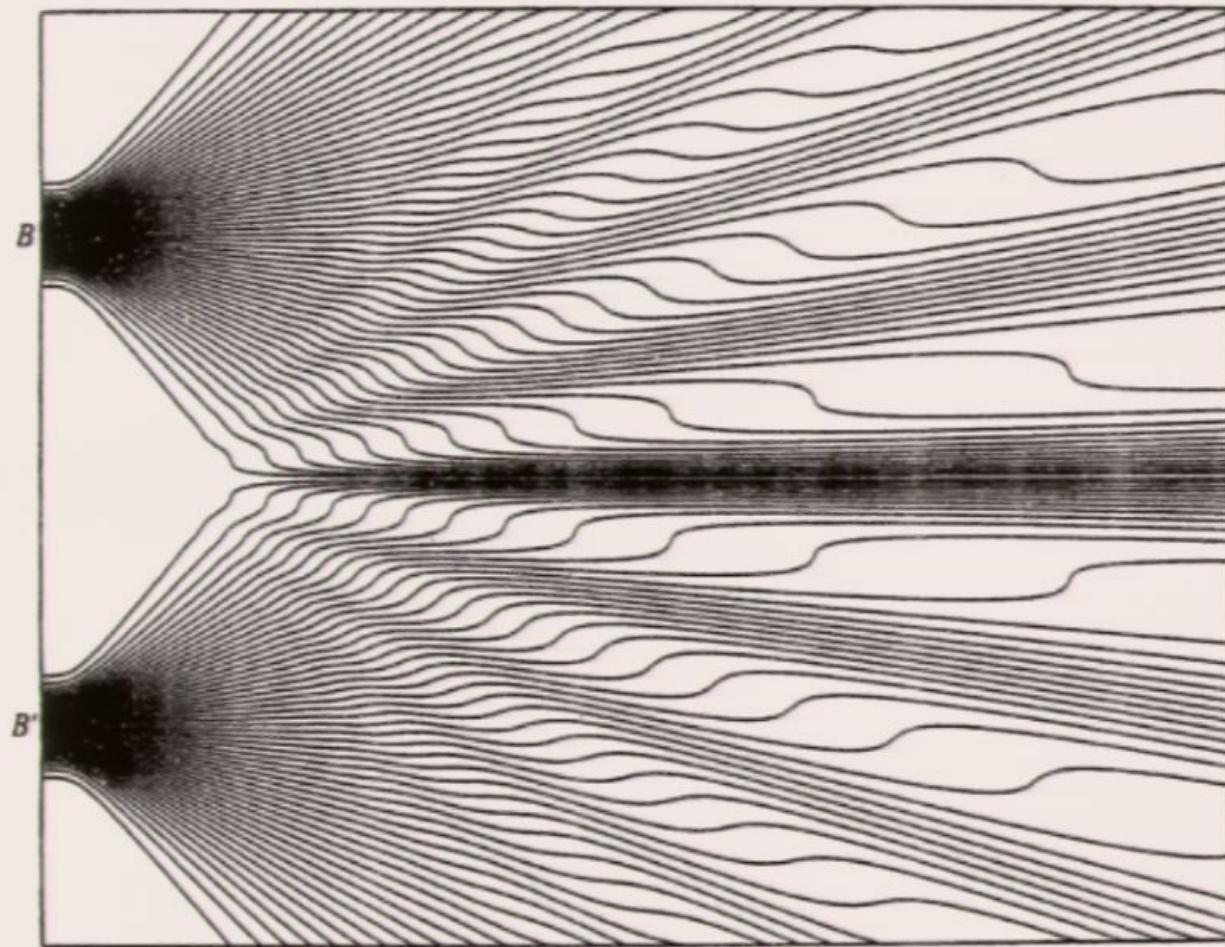
Double slit experiment



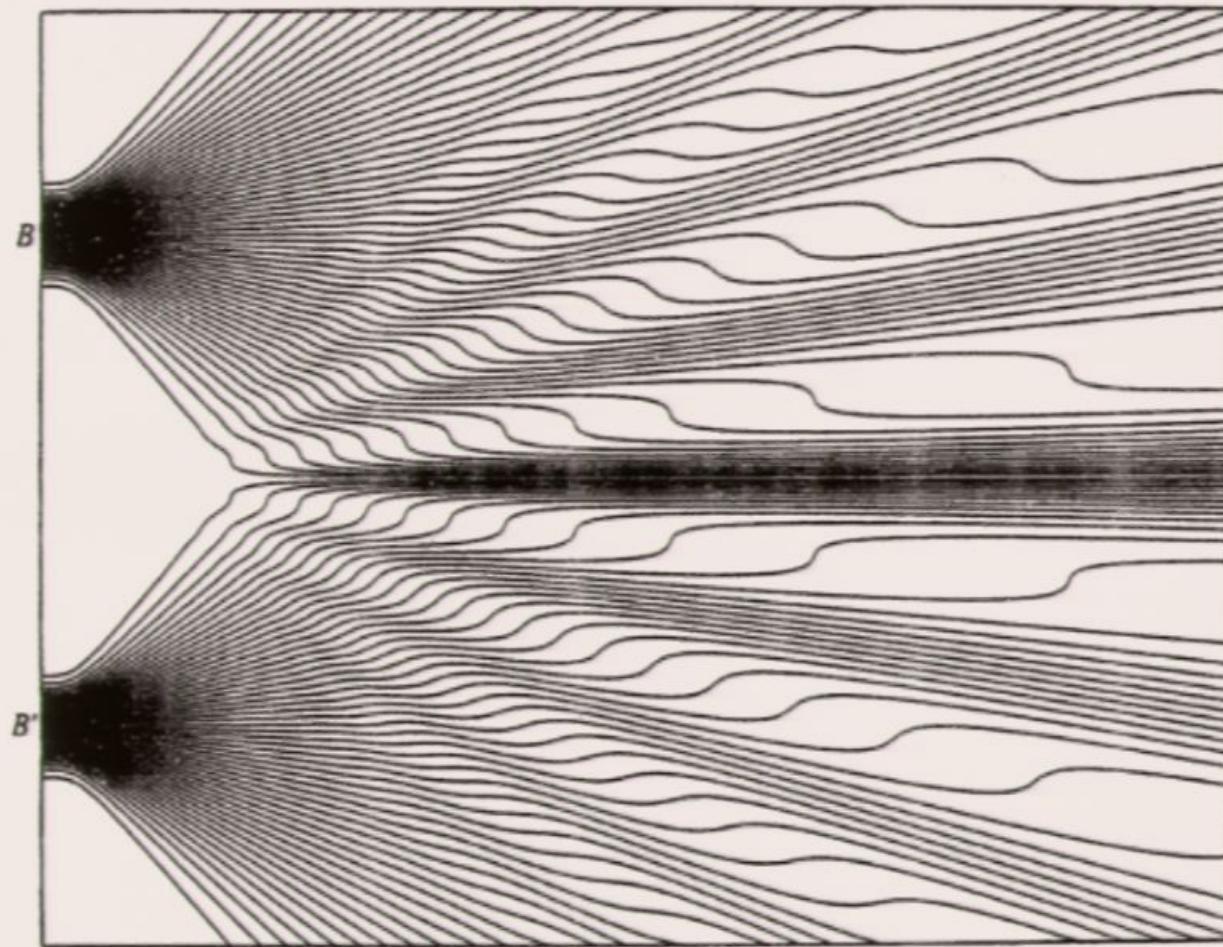
Double slit experiment



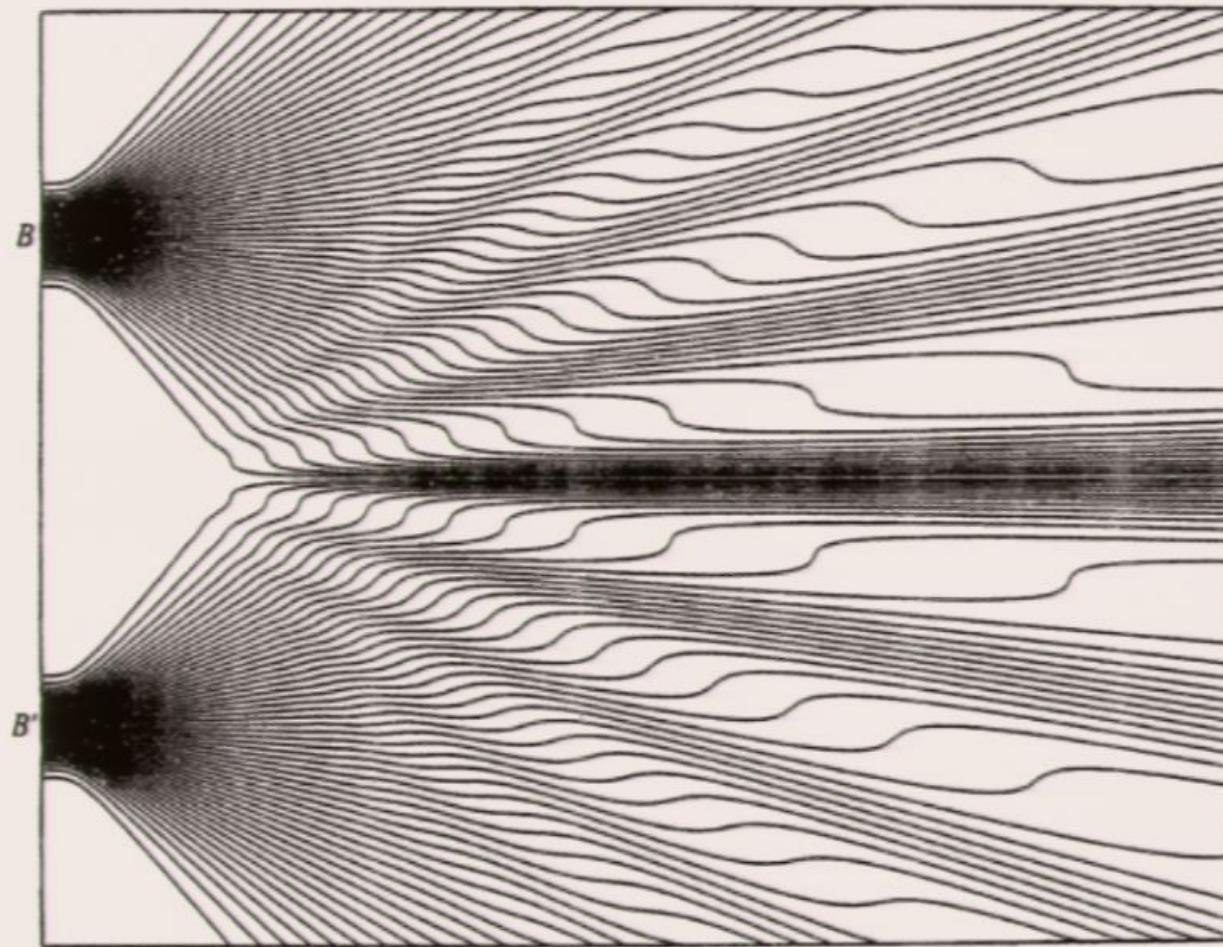
Double slit experiment



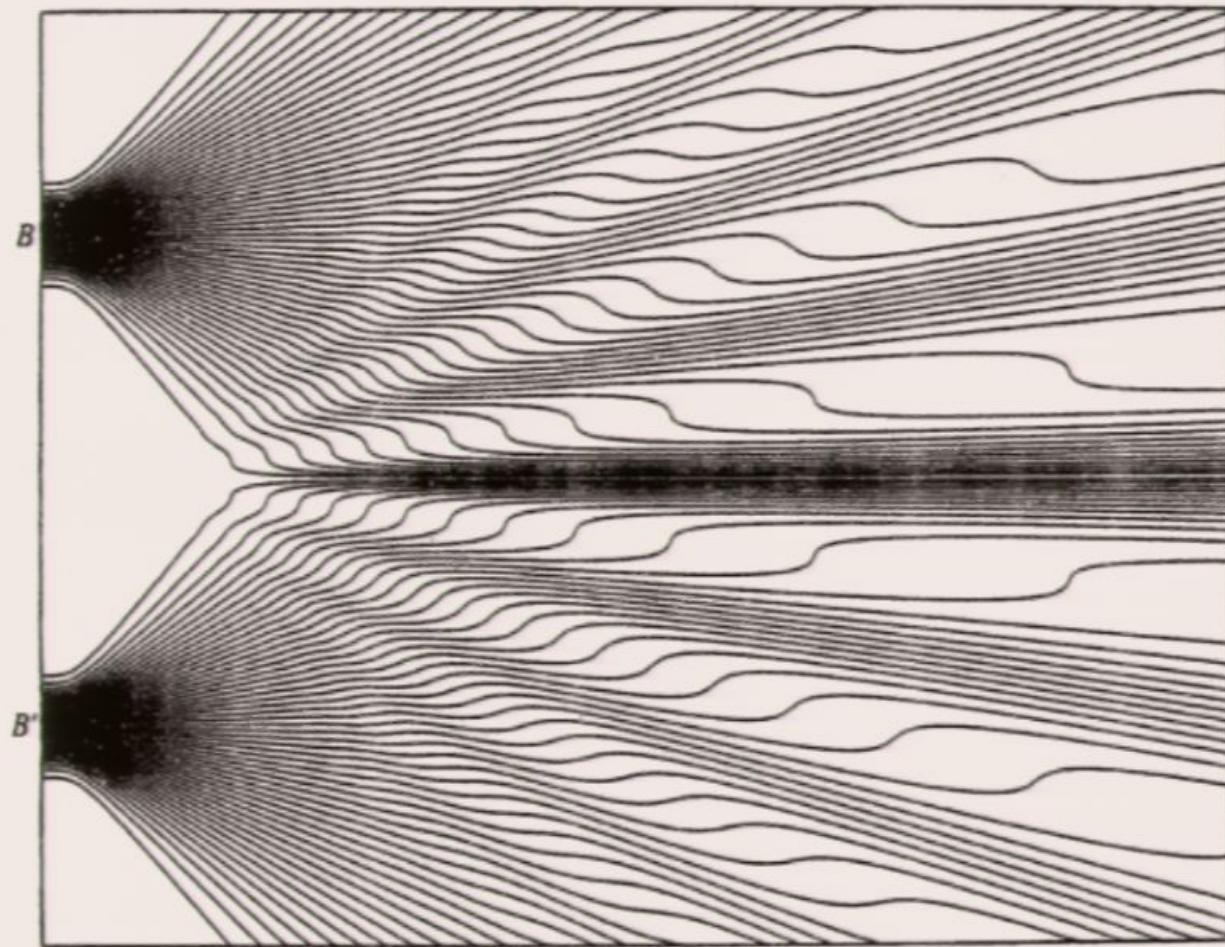
Double slit experiment



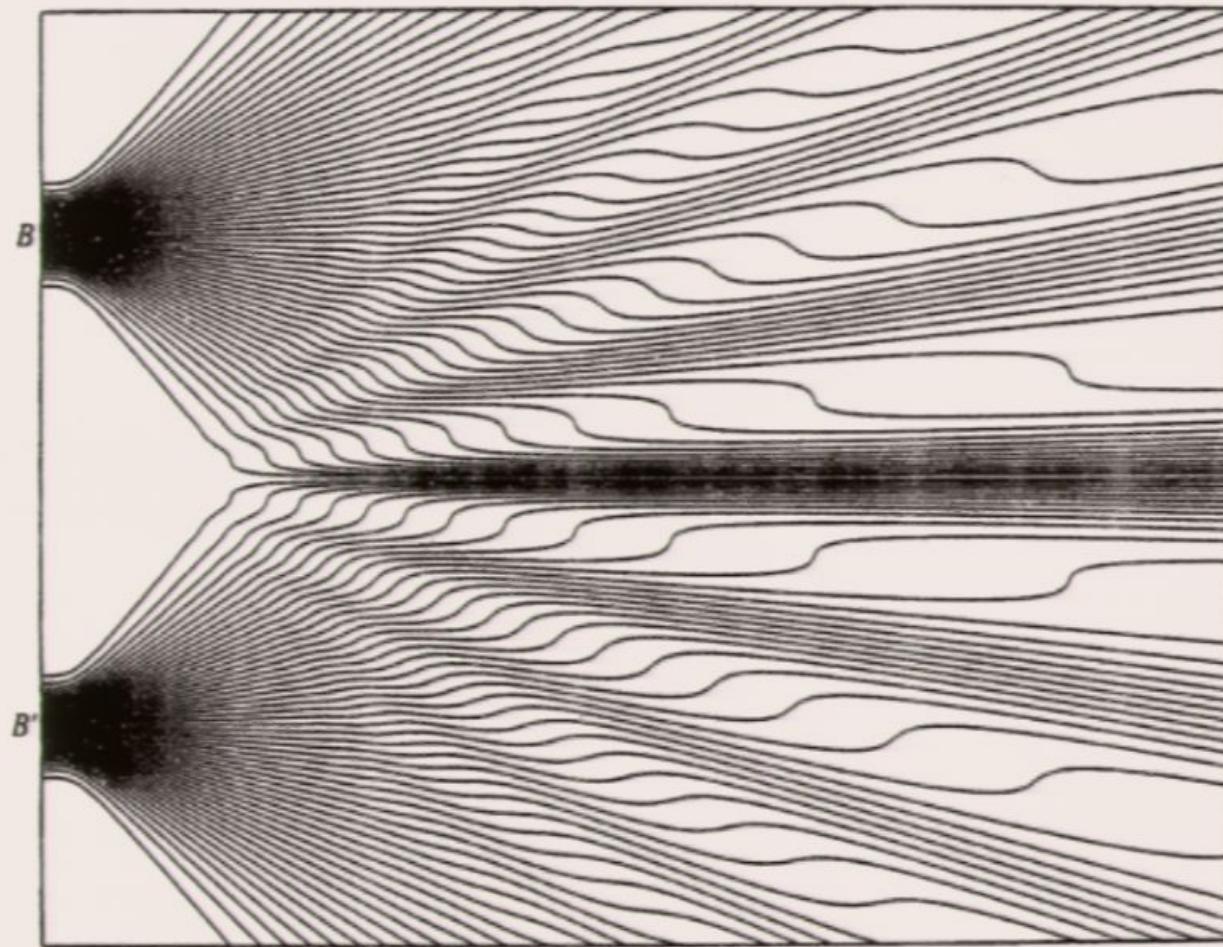
Double slit experiment



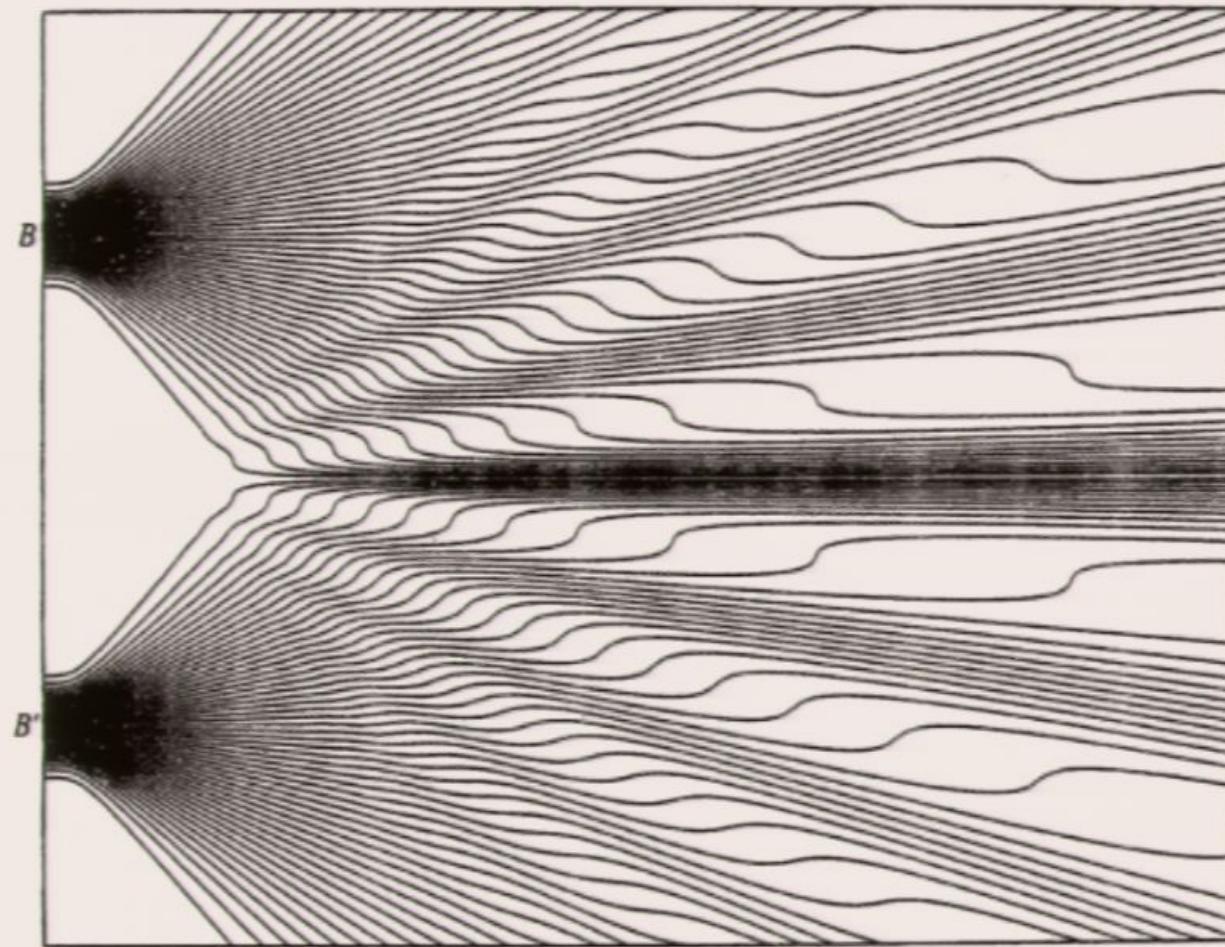
Double slit experiment



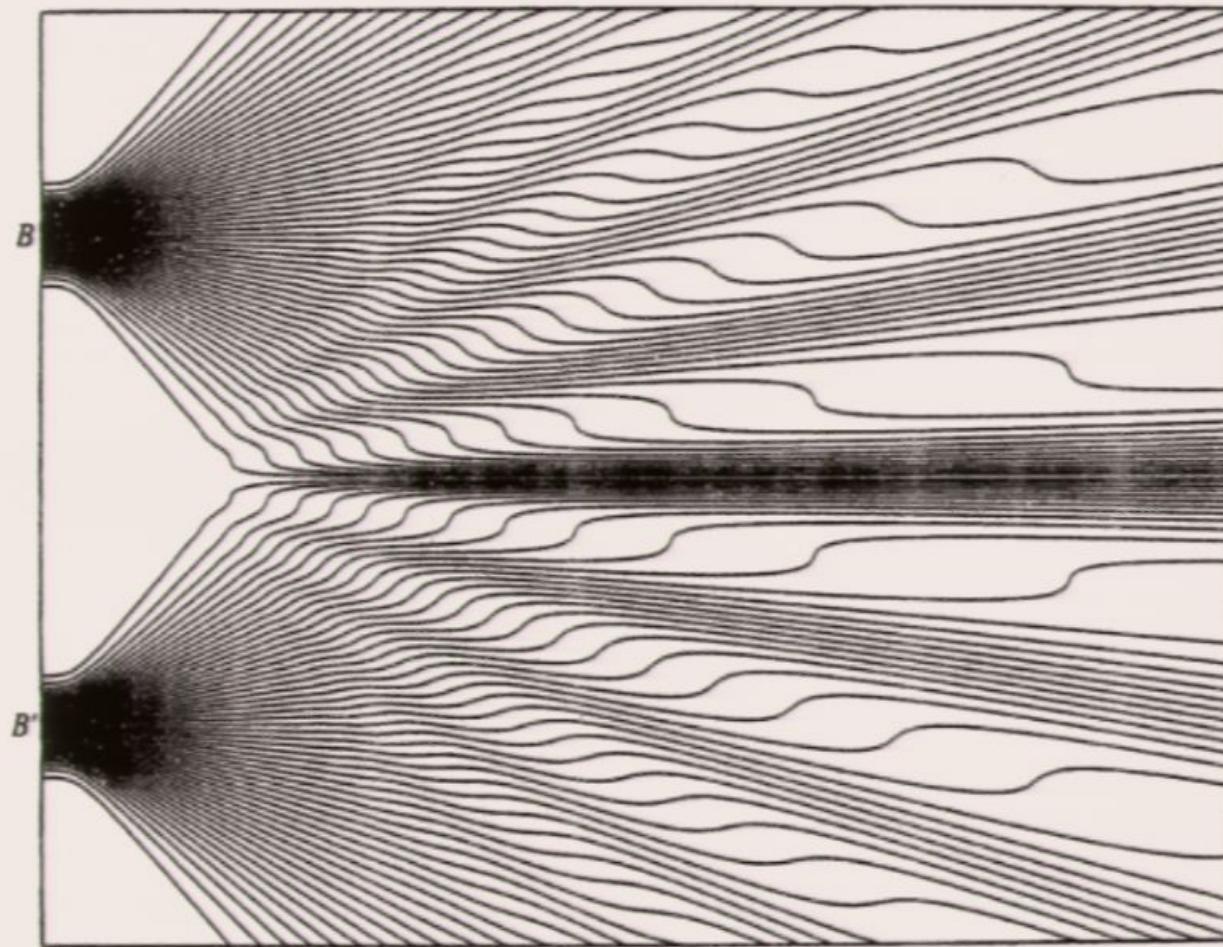
Double slit experiment



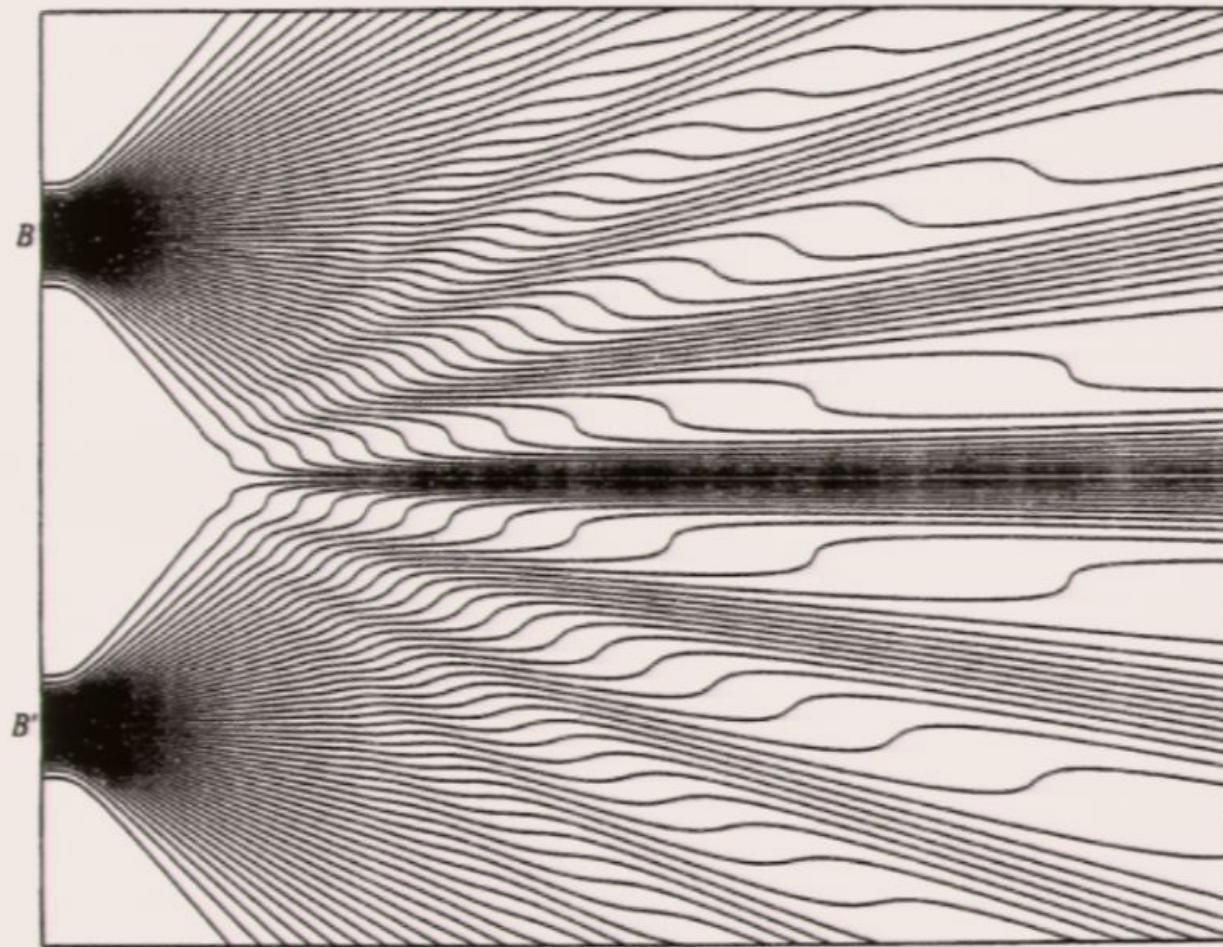
Double slit experiment



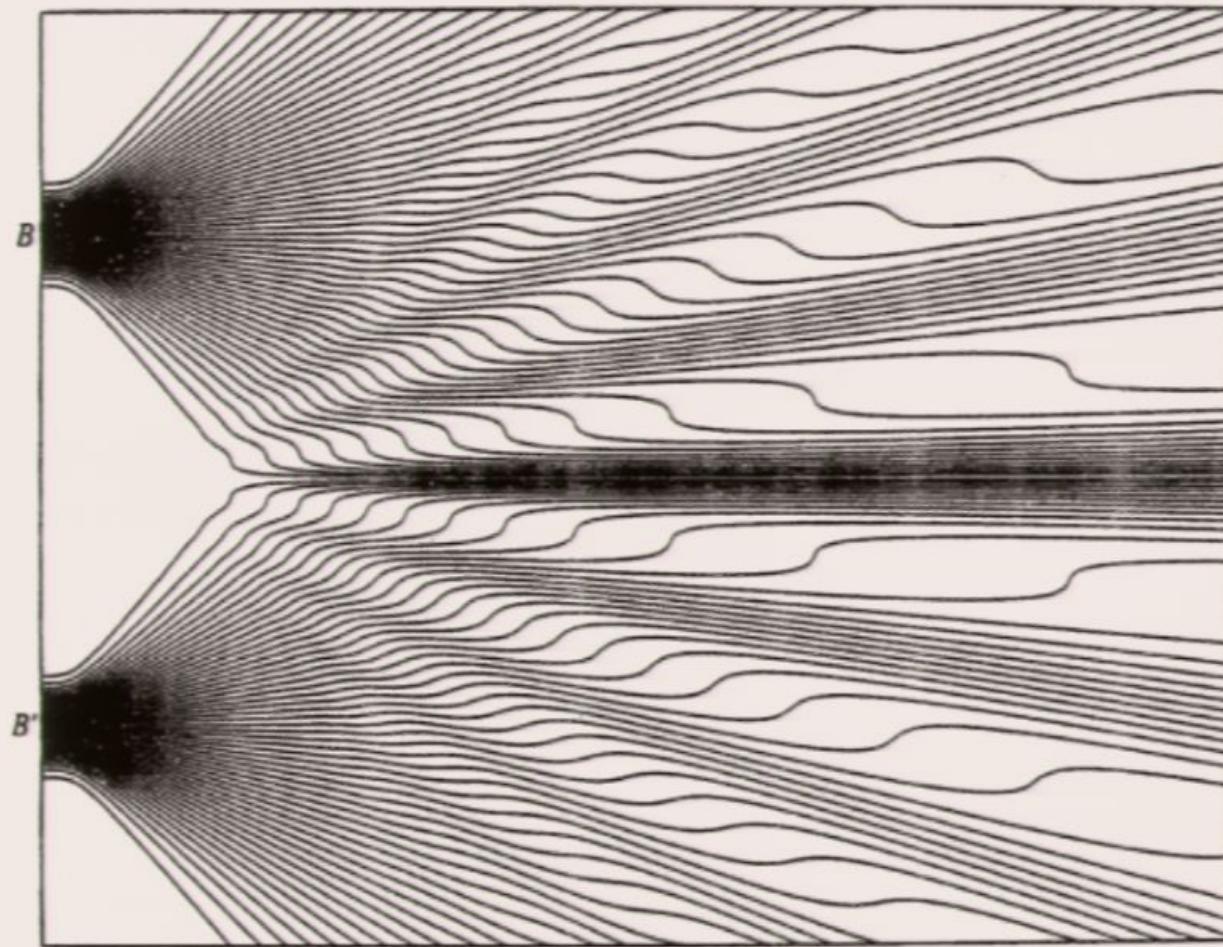
Double slit experiment



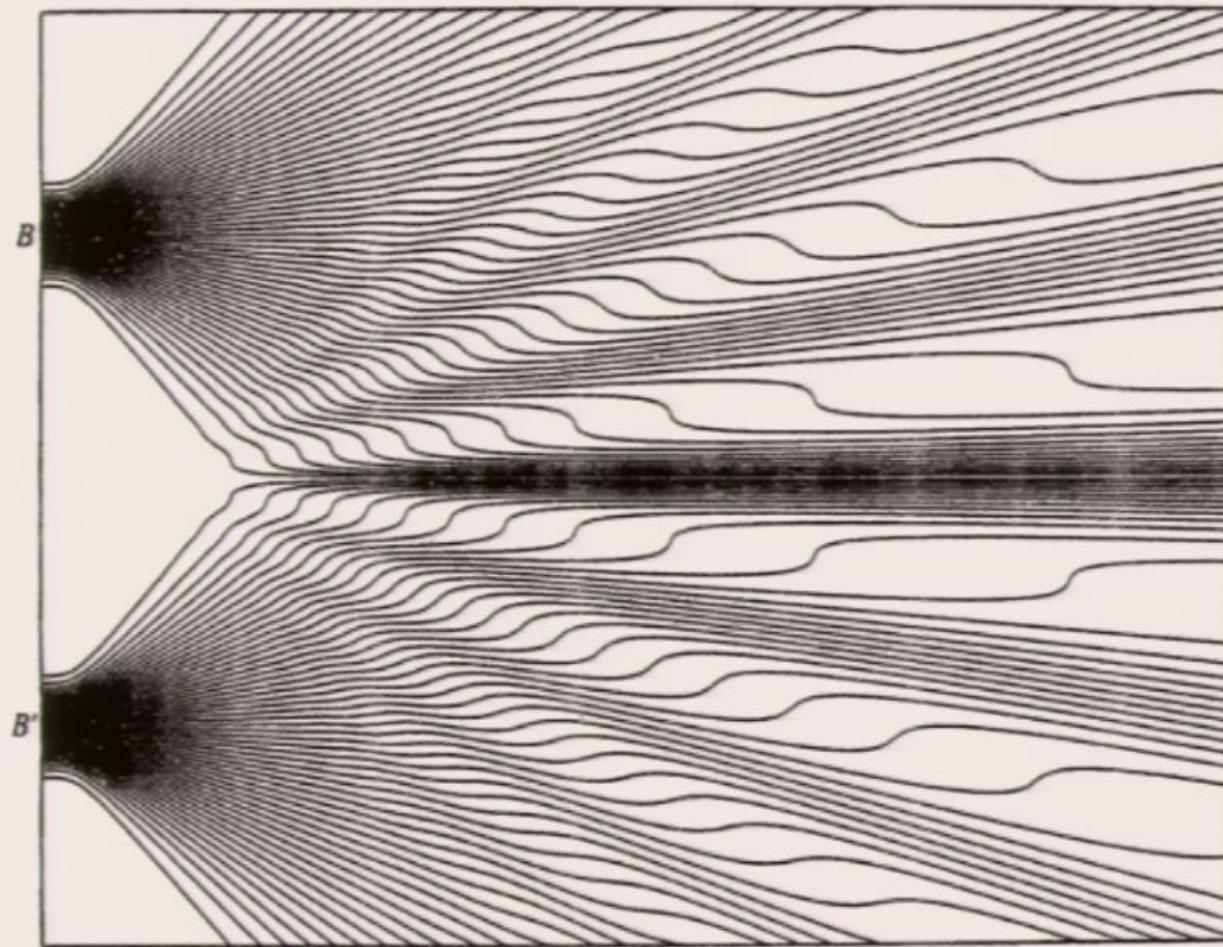
Double slit experiment



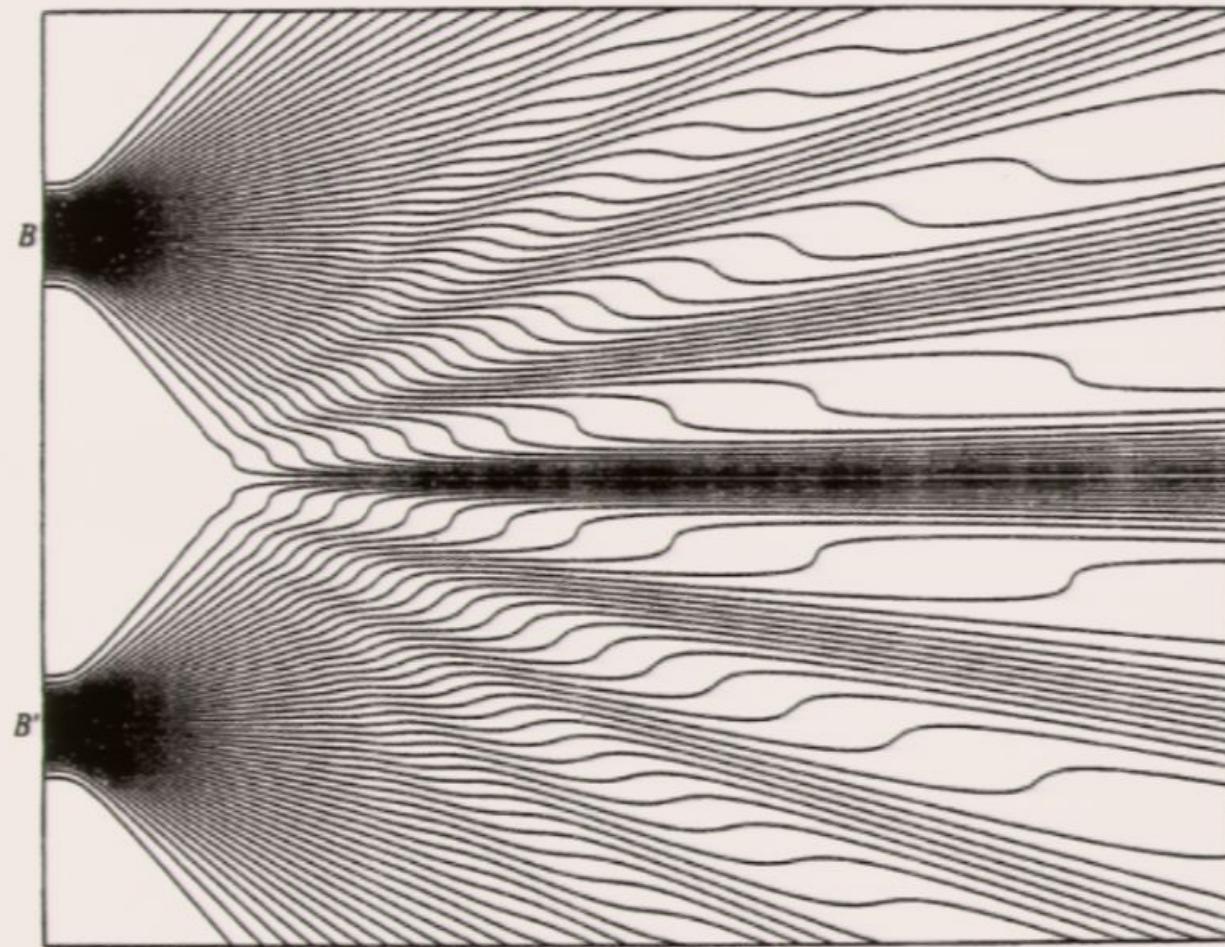
Double slit experiment



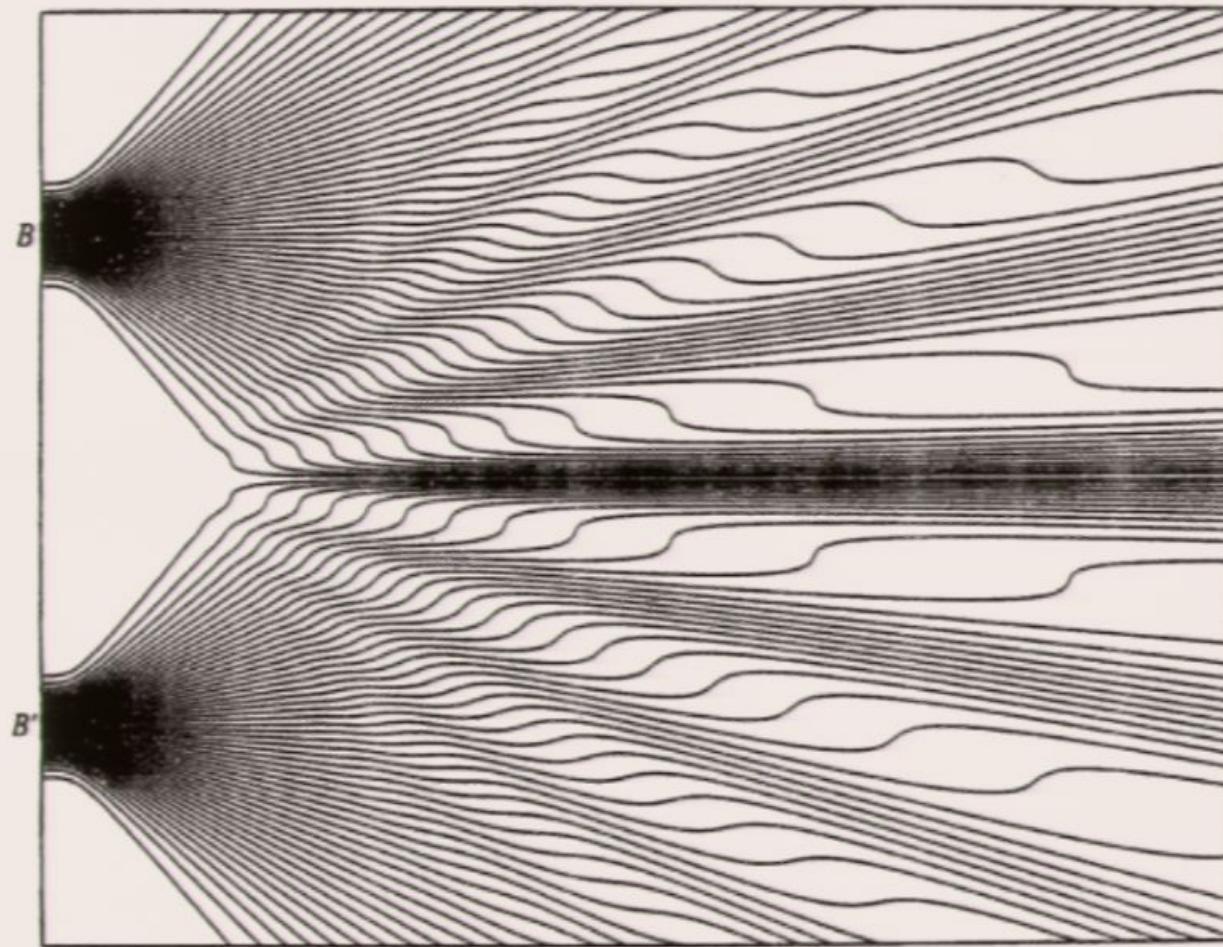
Double slit experiment



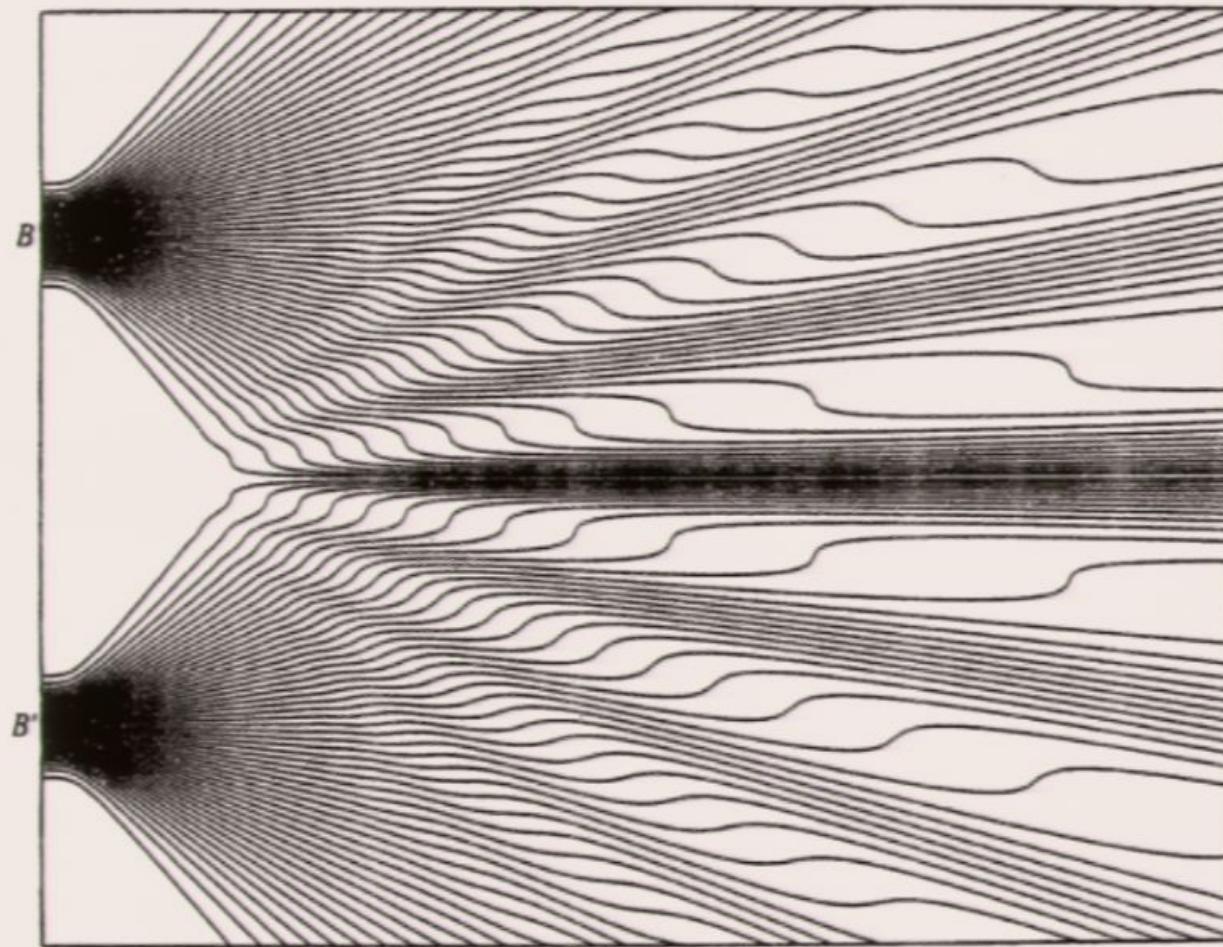
Double slit experiment



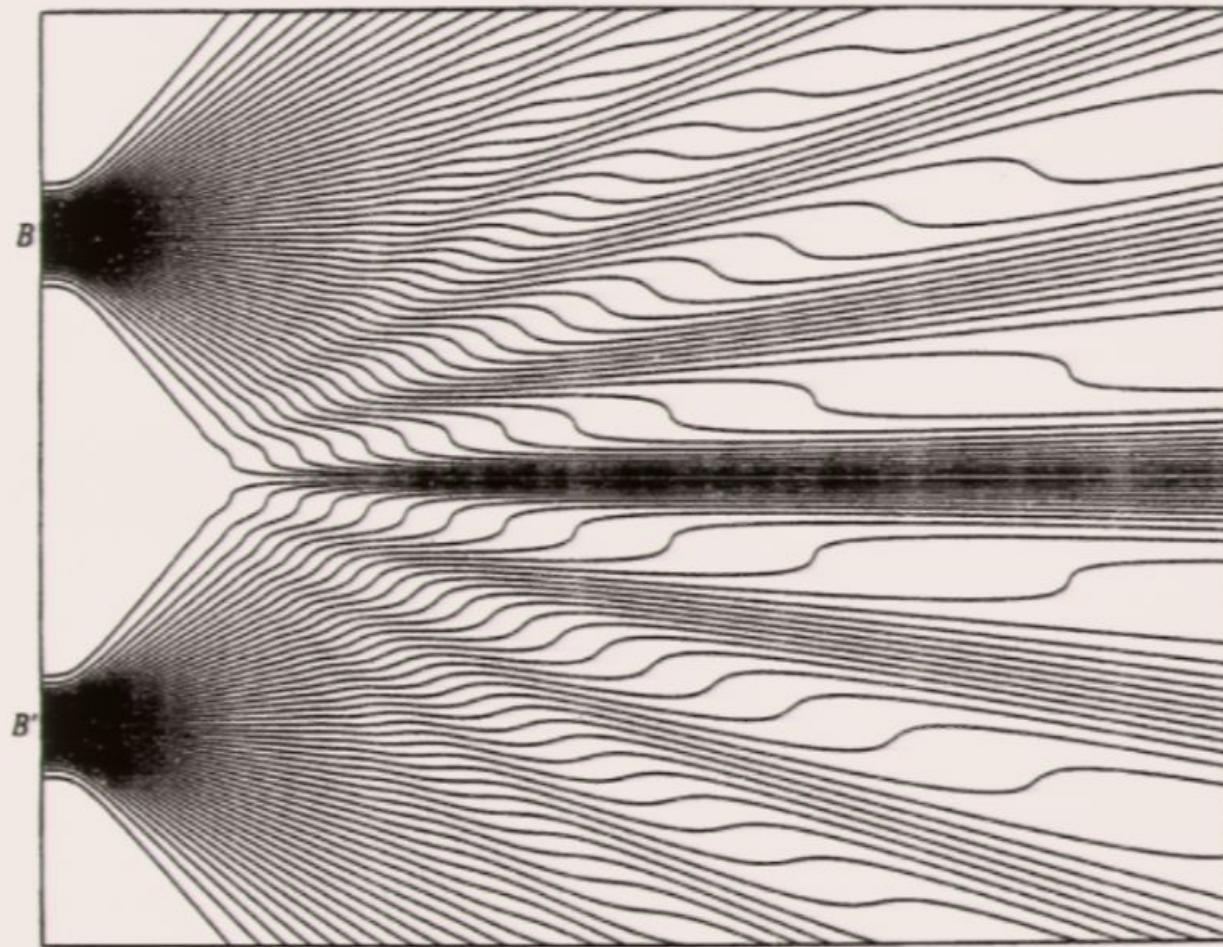
Double slit experiment



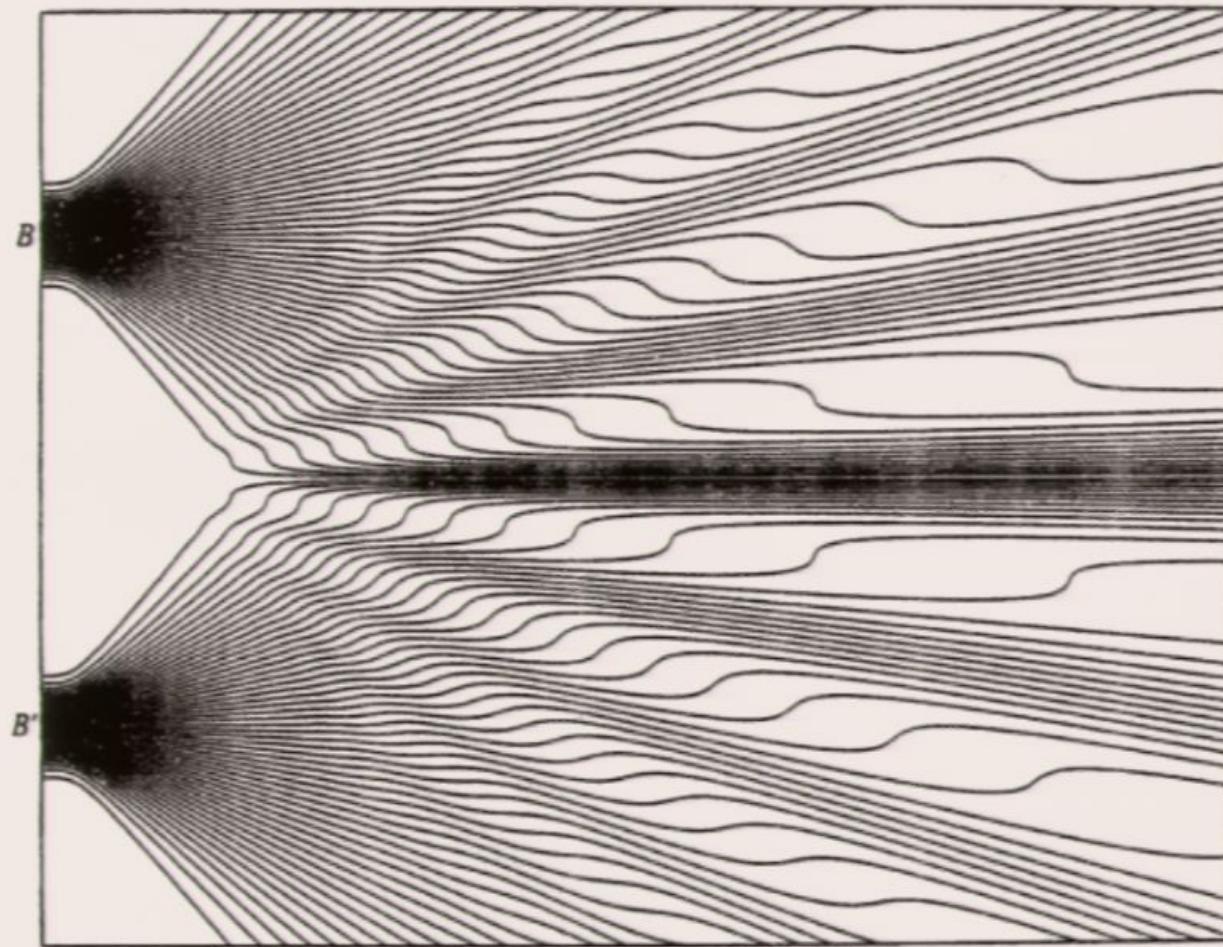
Double slit experiment



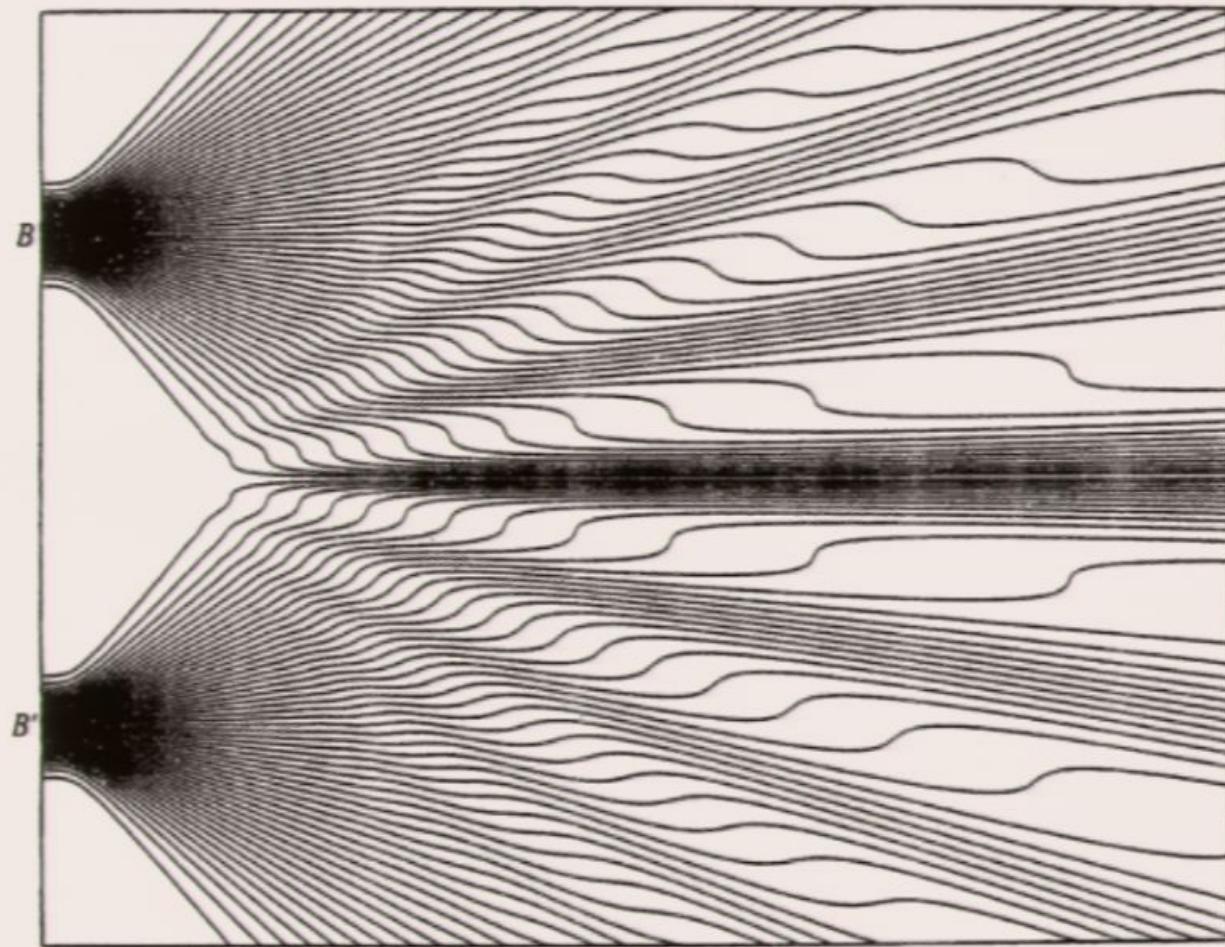
Double slit experiment



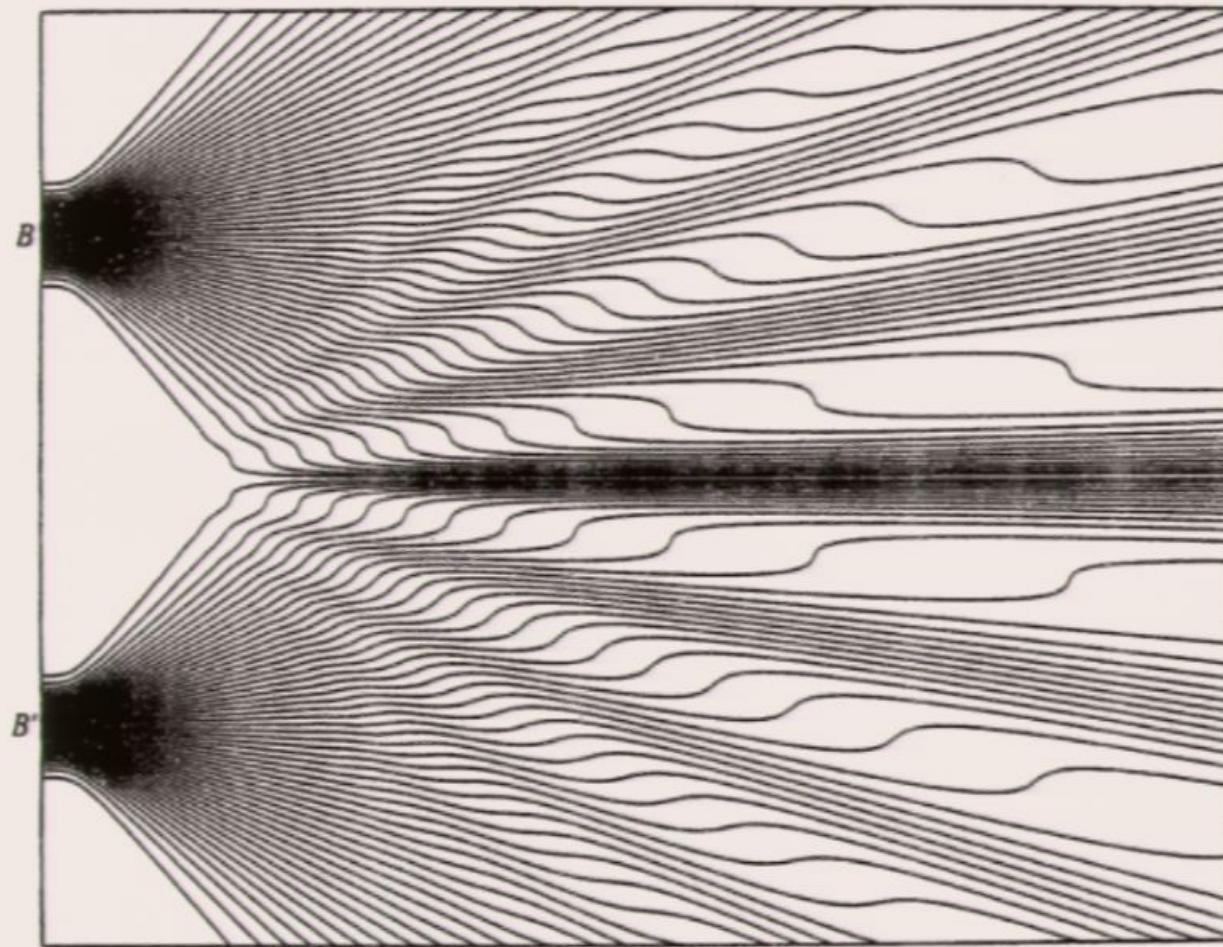
Double slit experiment



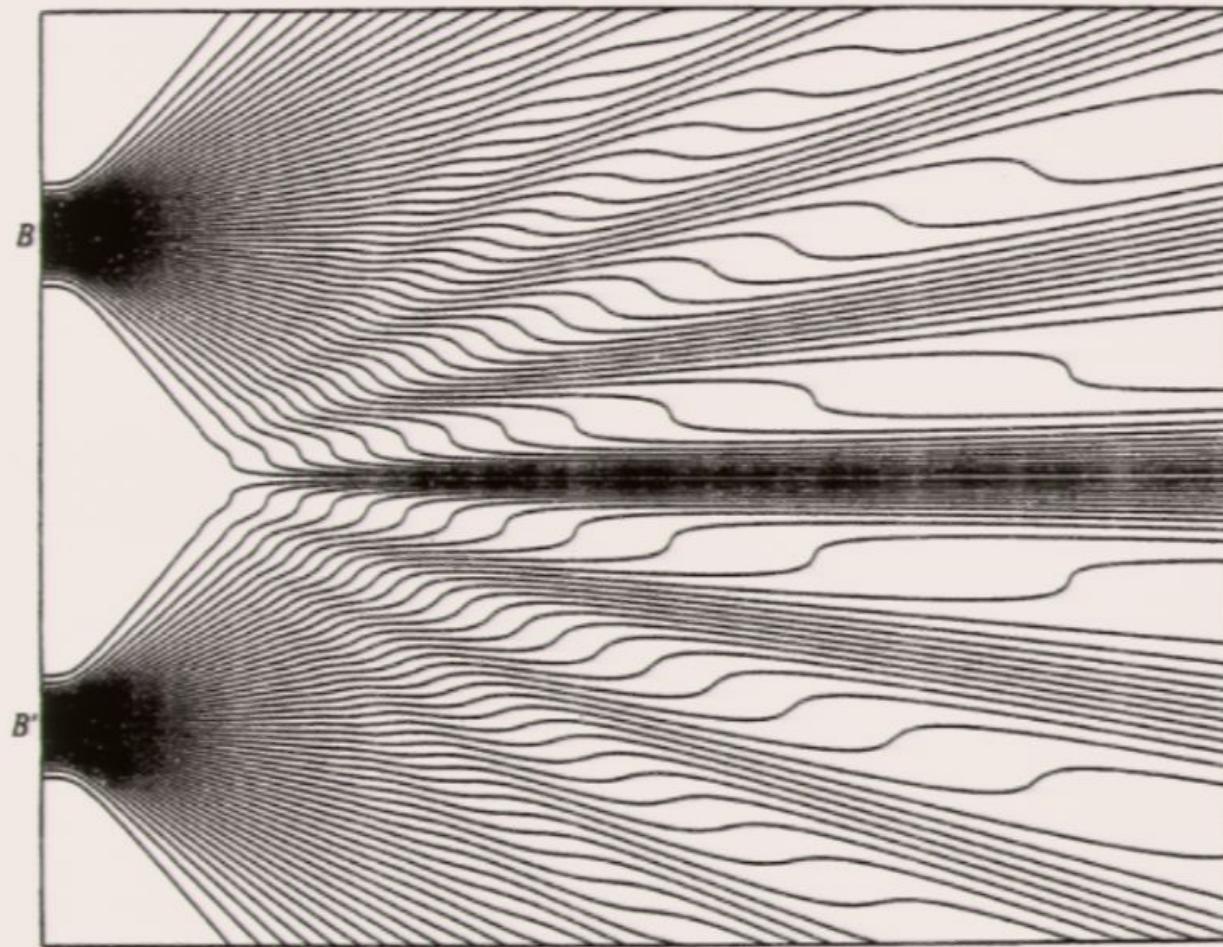
Double slit experiment



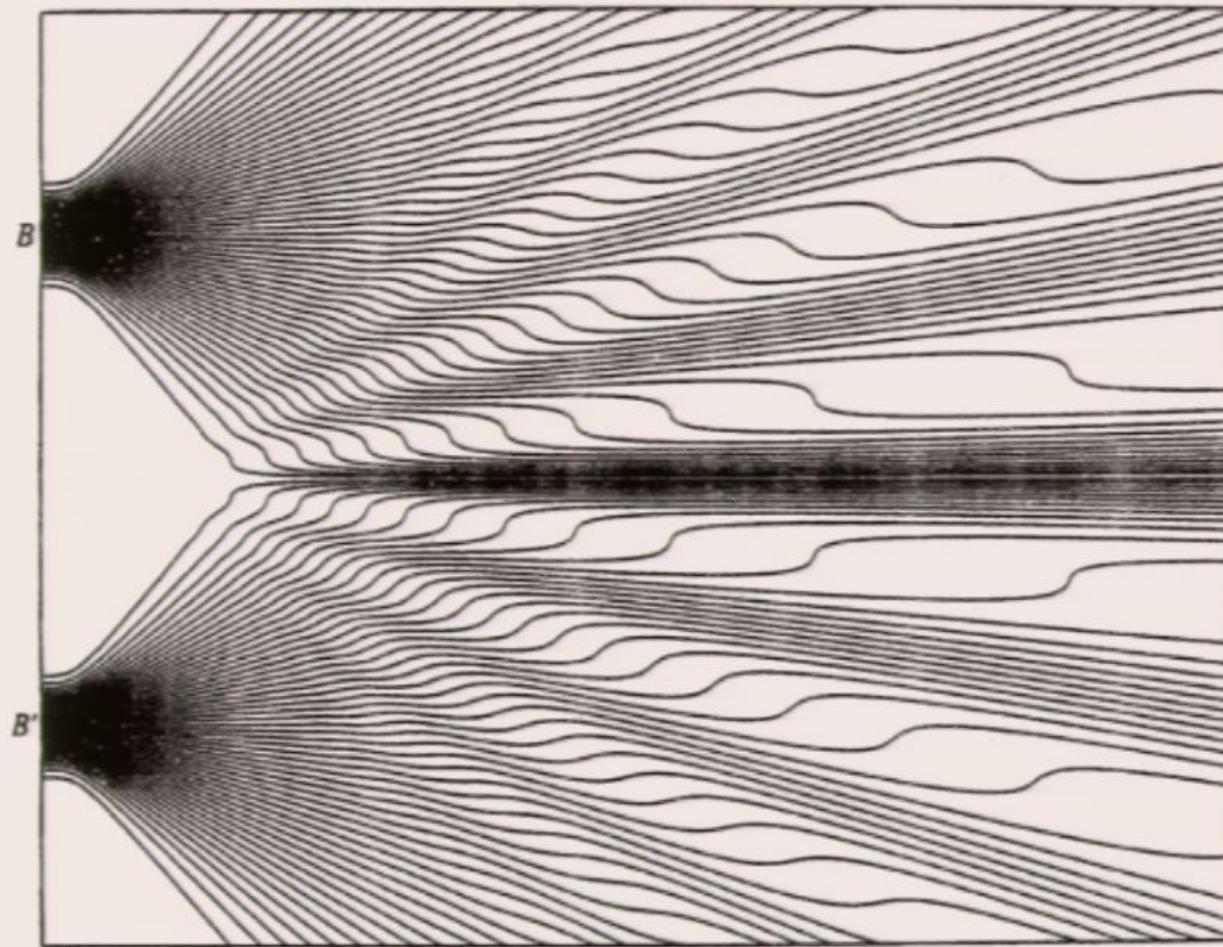
Double slit experiment



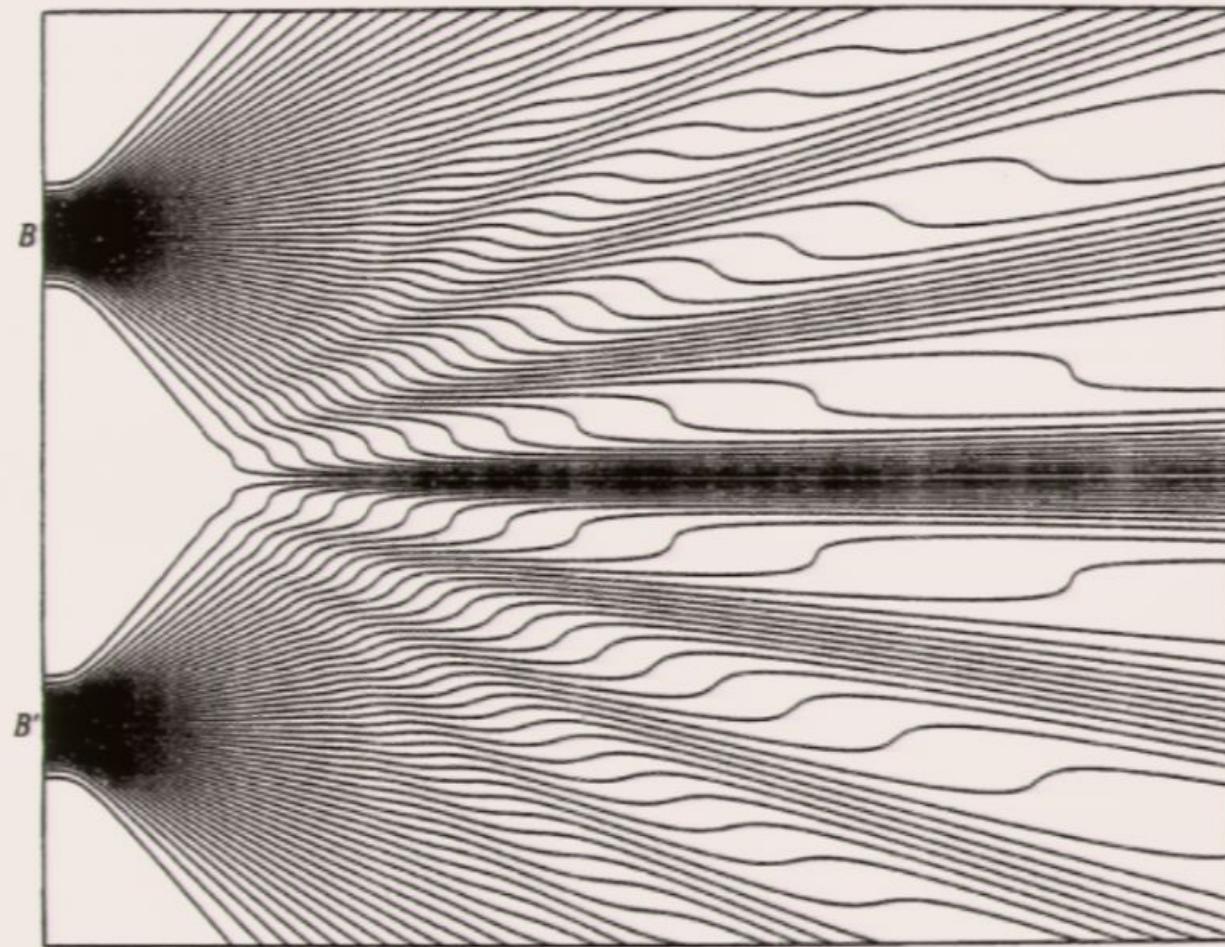
Double slit experiment



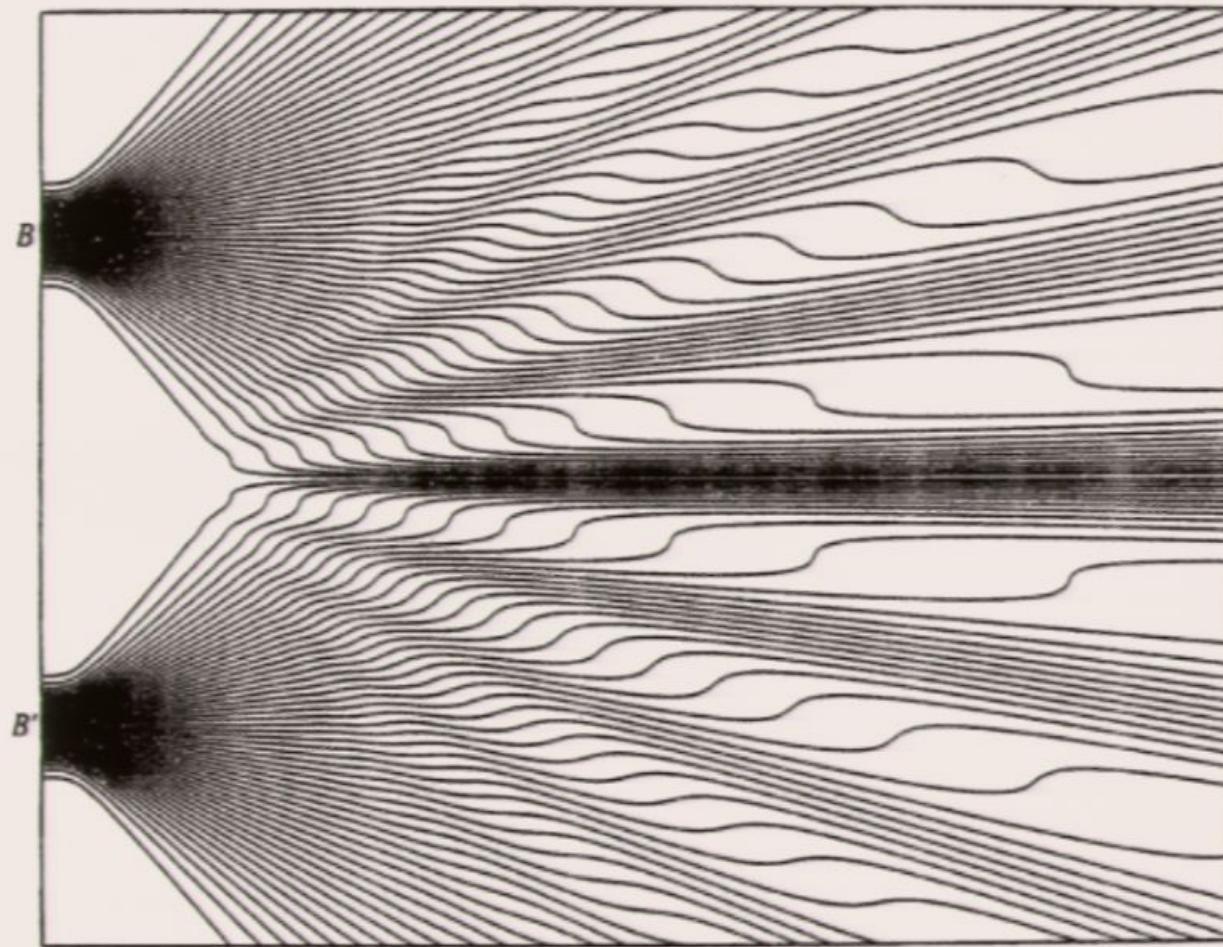
Double slit experiment



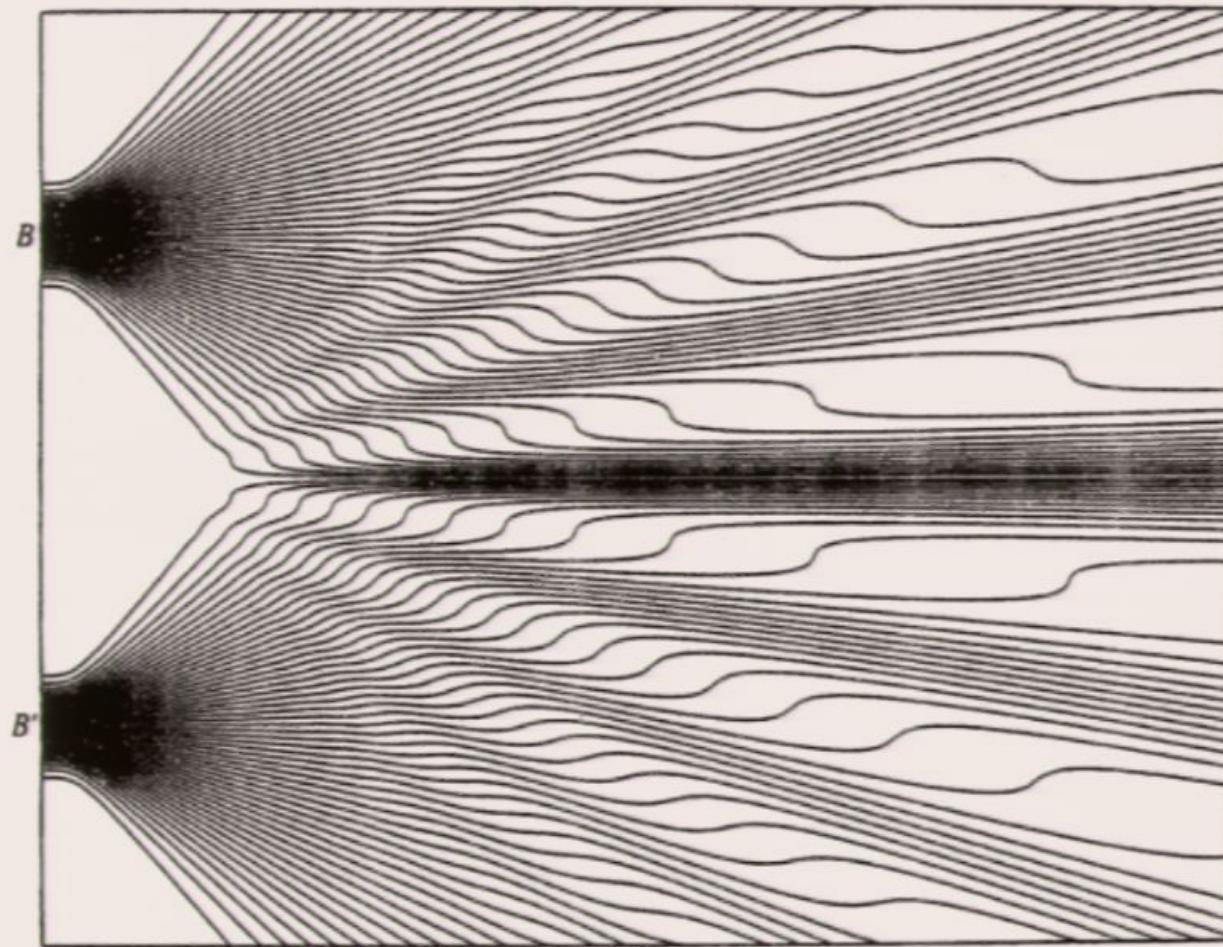
Double slit experiment



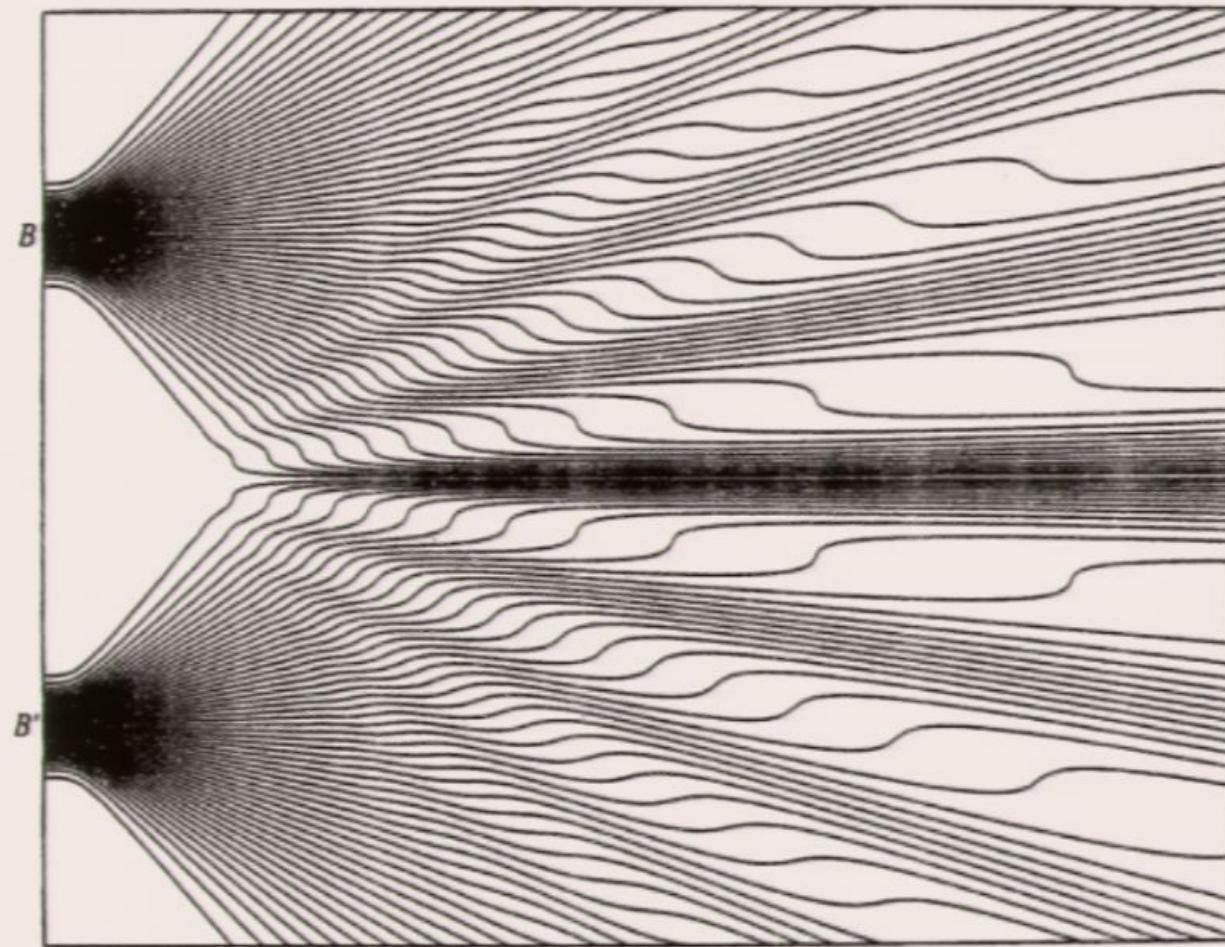
Double slit experiment



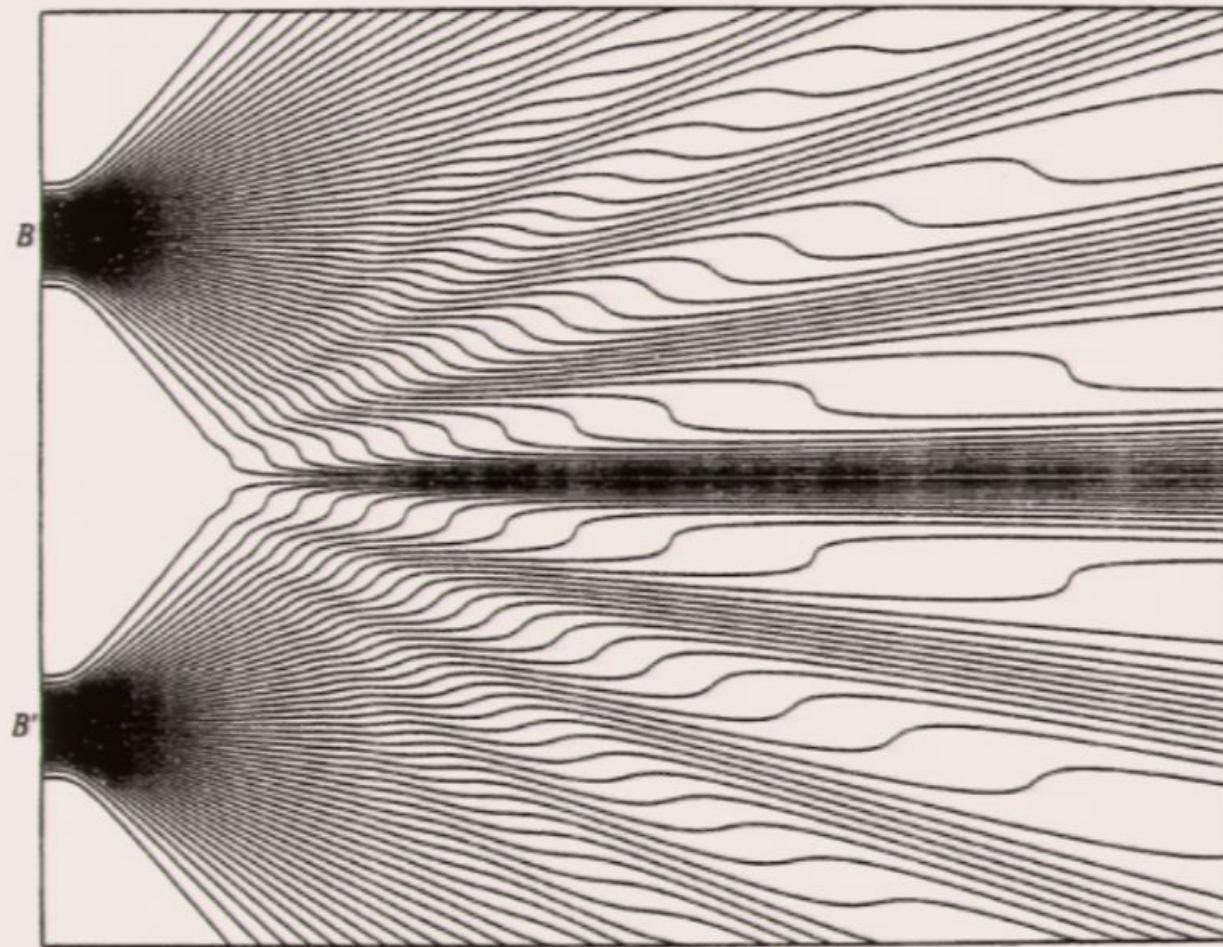
Double slit experiment



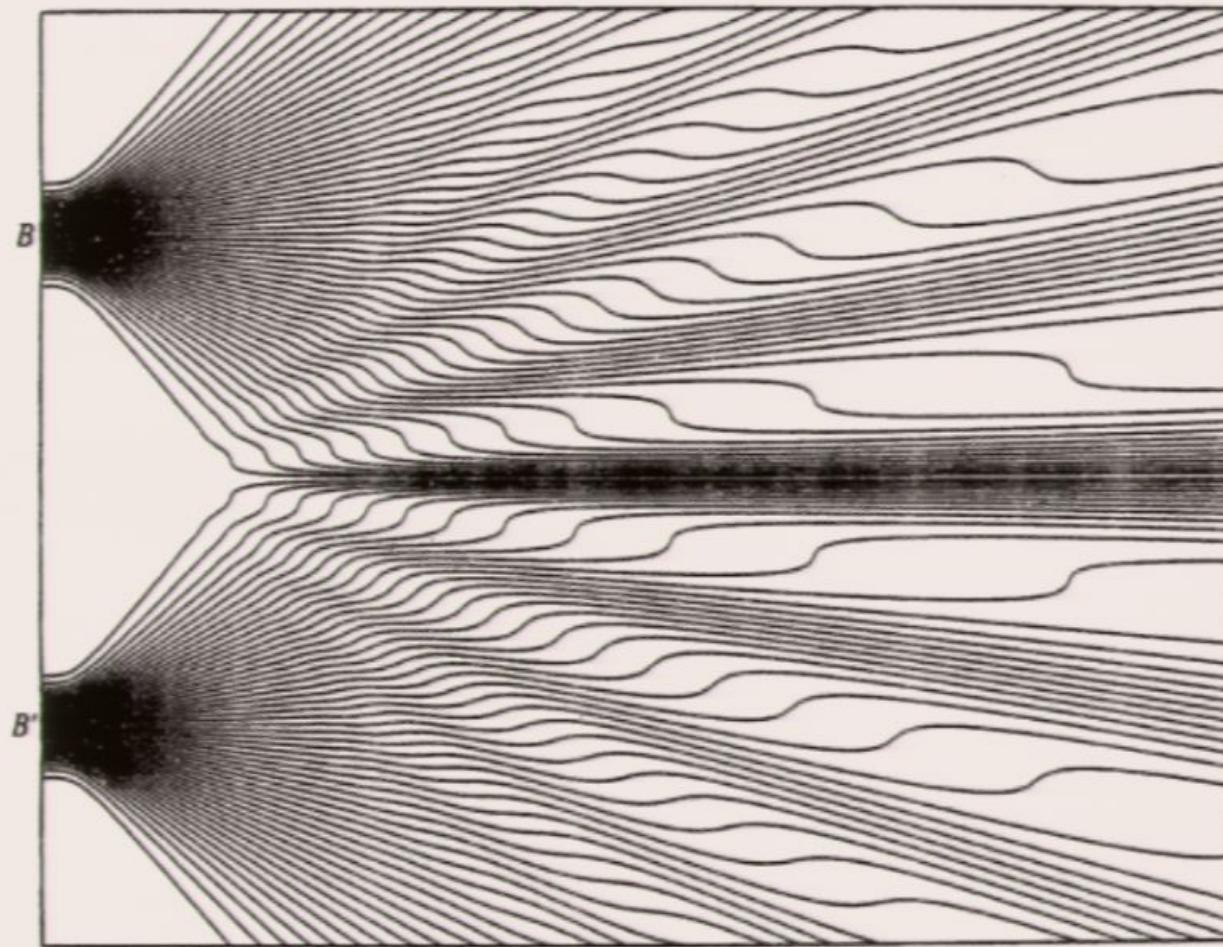
Double slit experiment



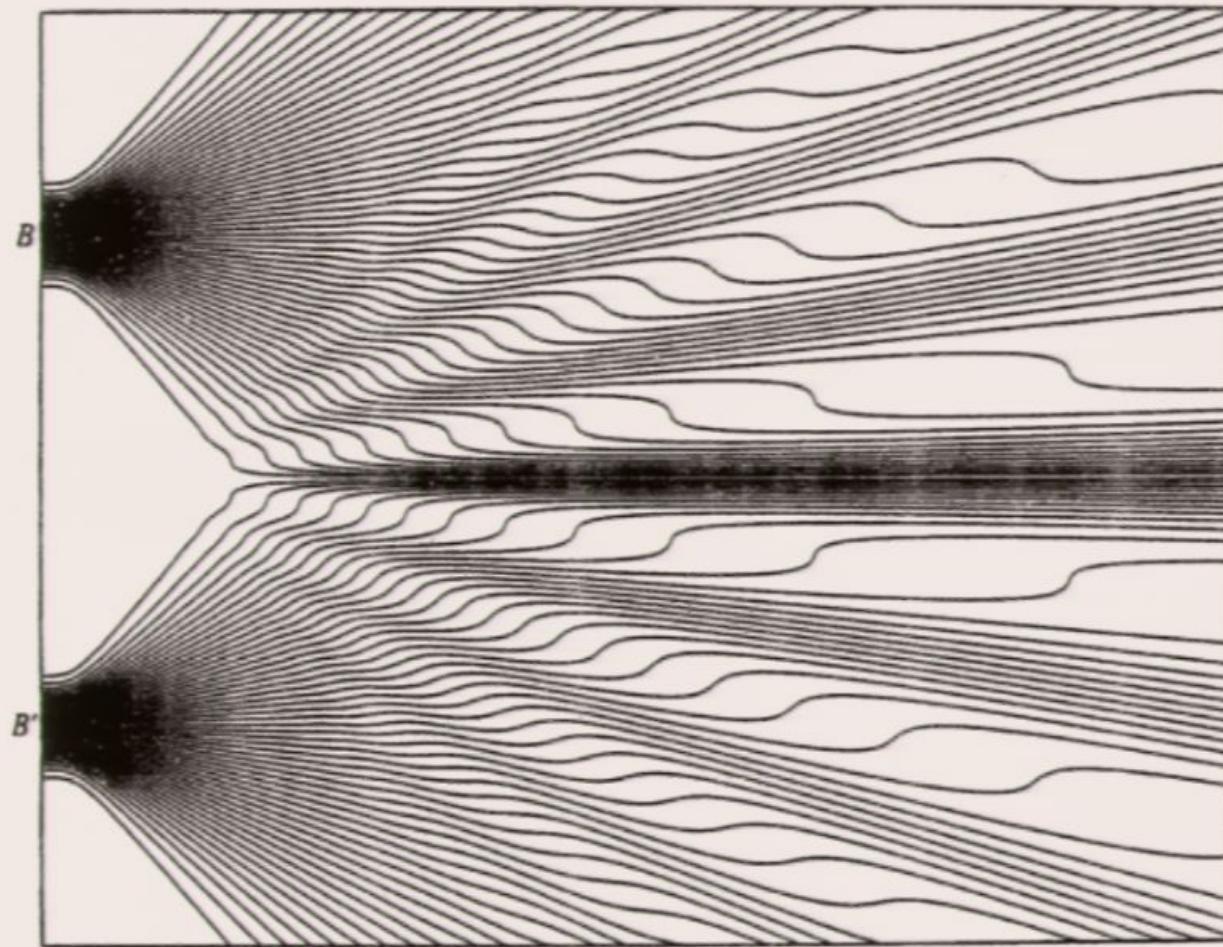
Double slit experiment



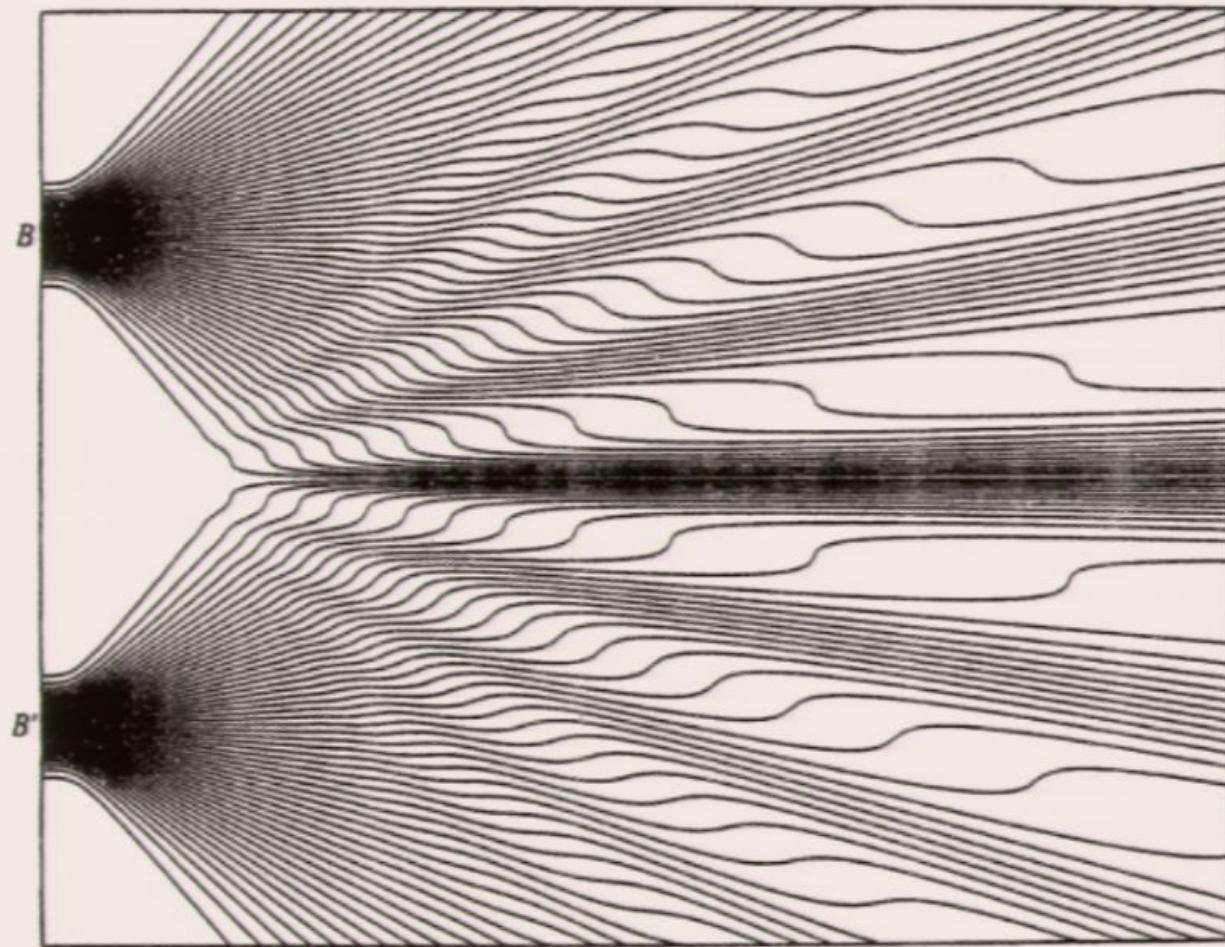
Double slit experiment



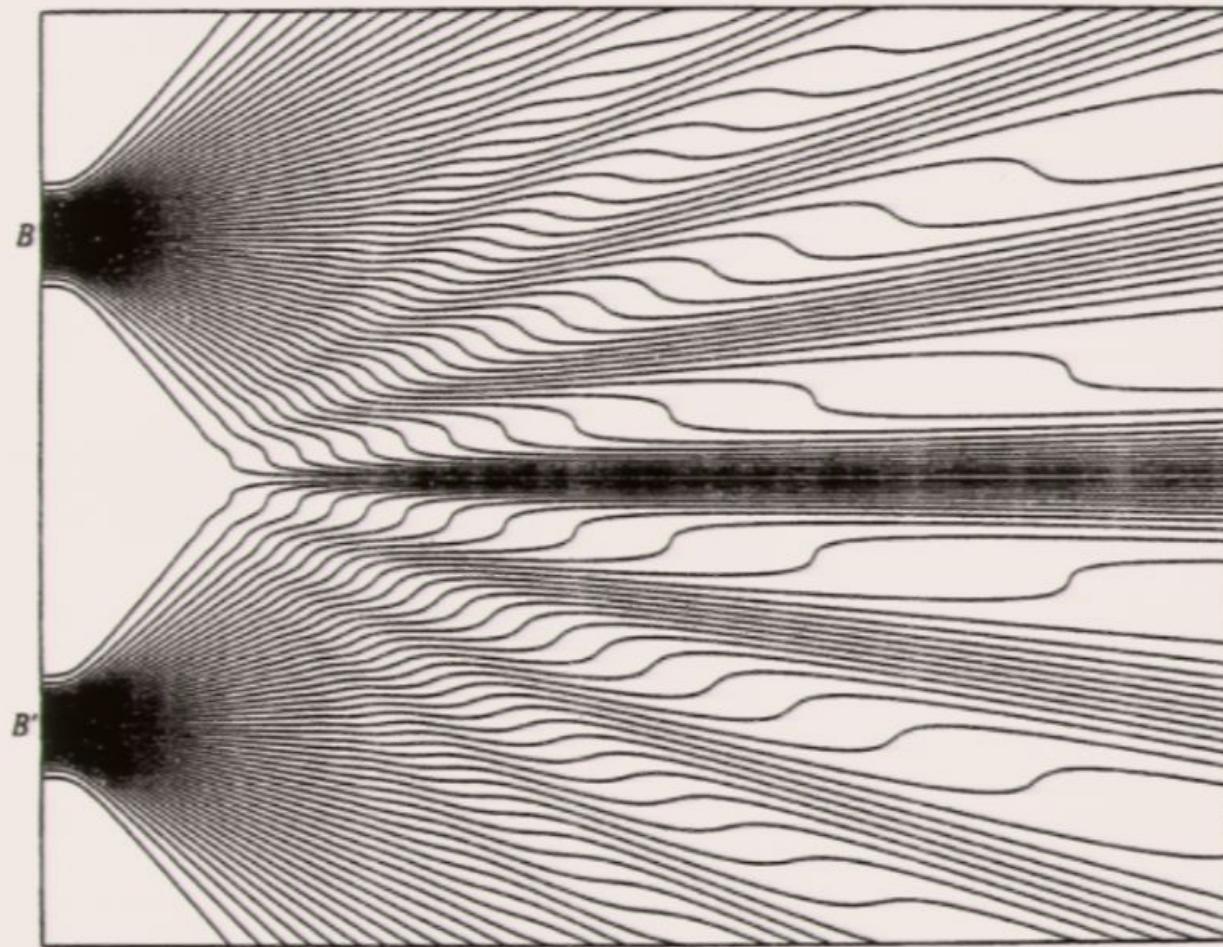
Double slit experiment



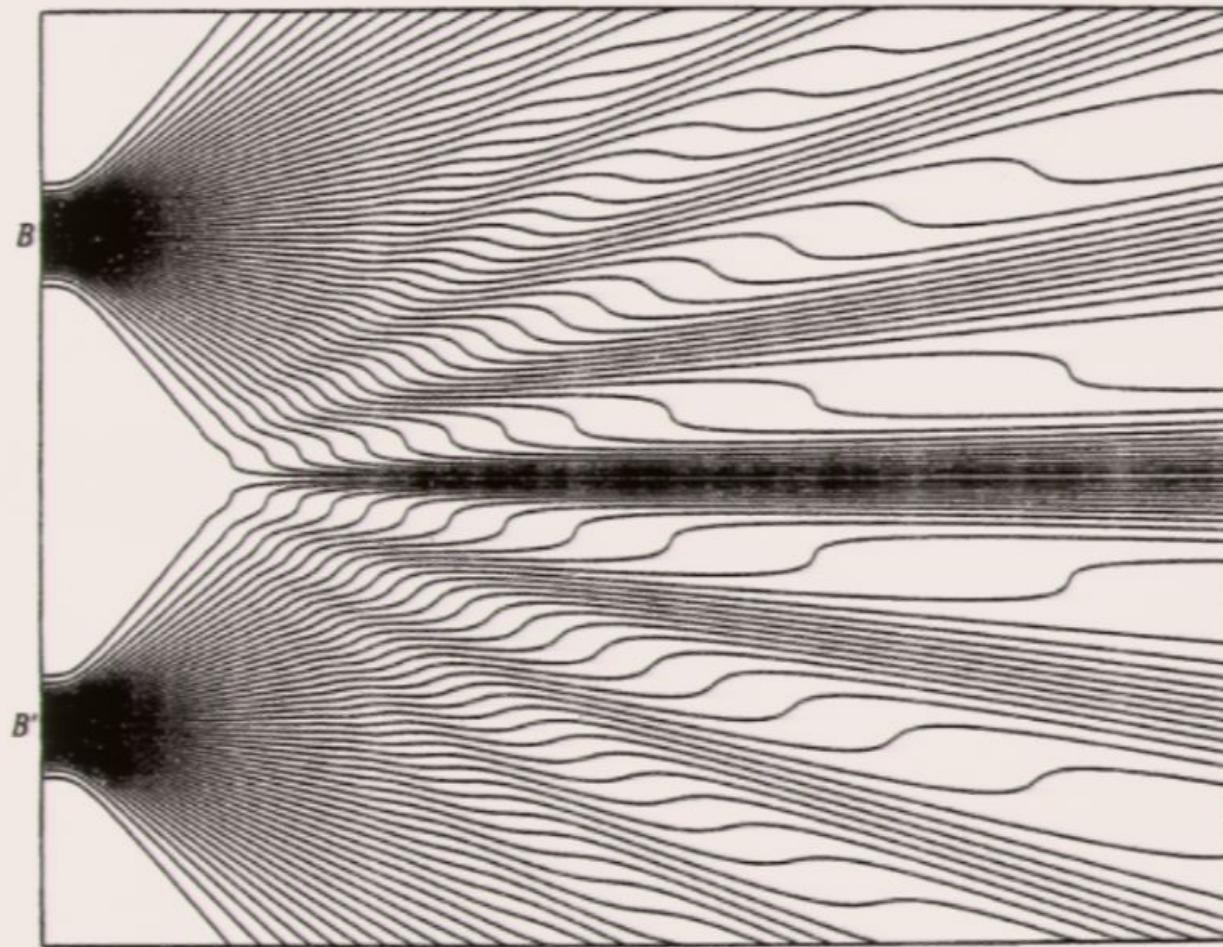
Double slit experiment



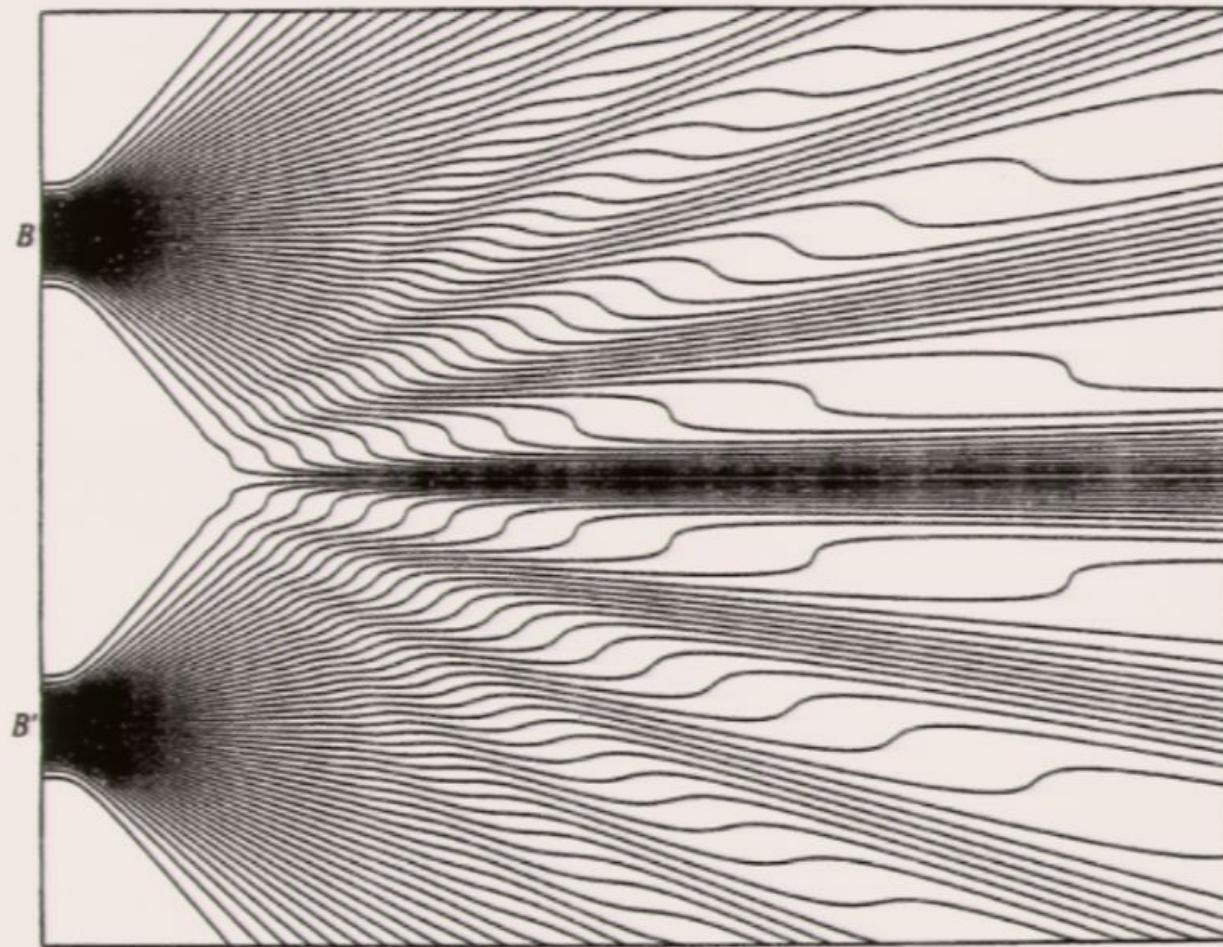
Double slit experiment



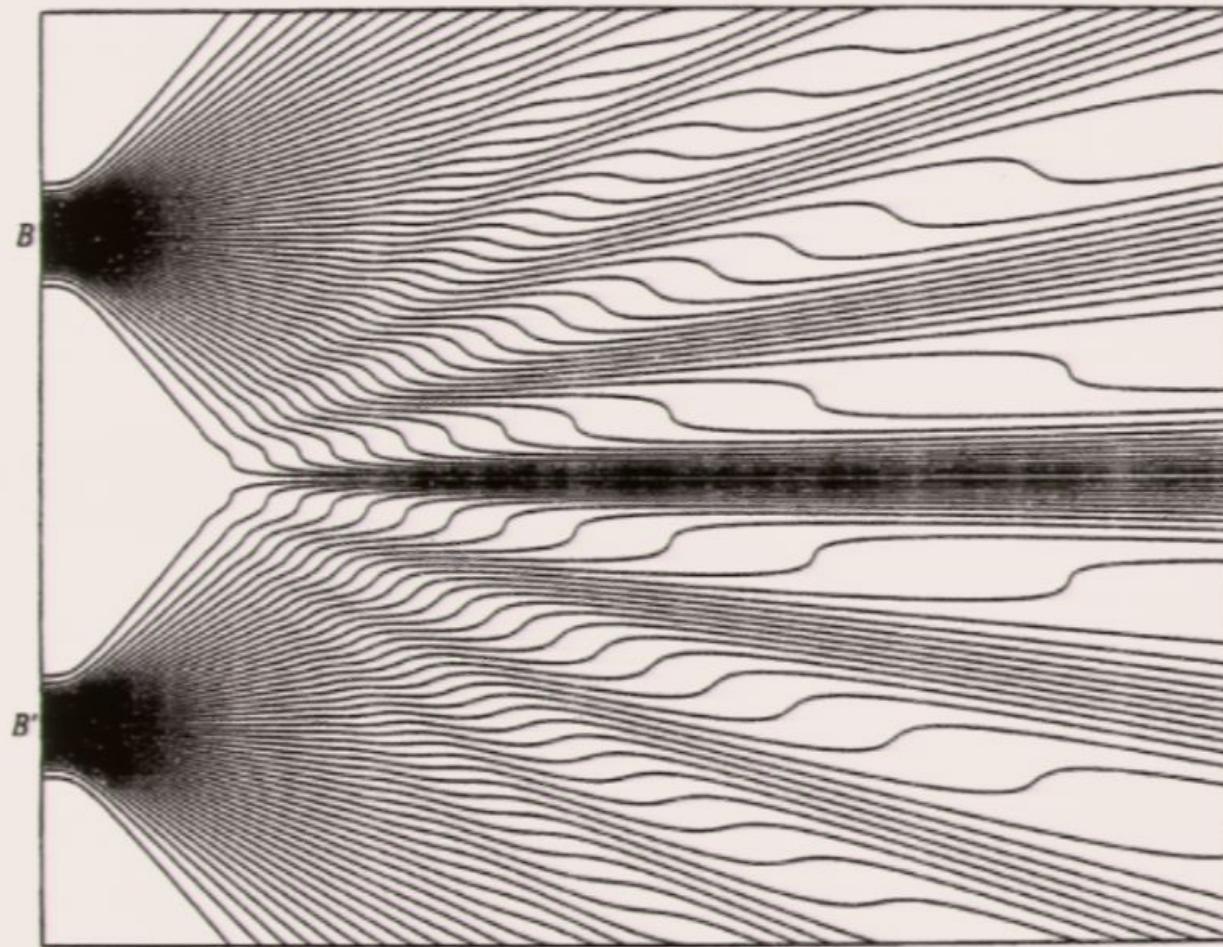
Double slit experiment



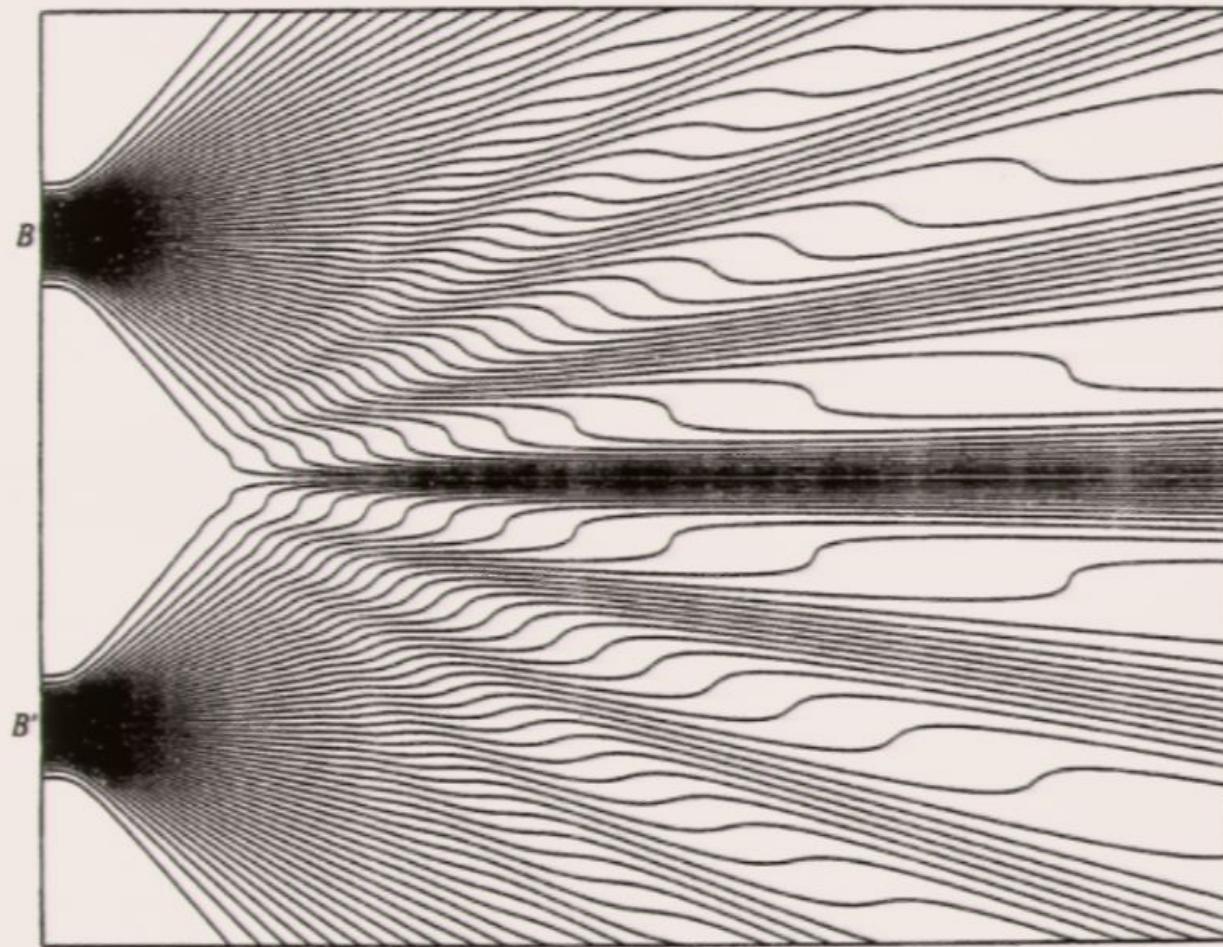
Double slit experiment



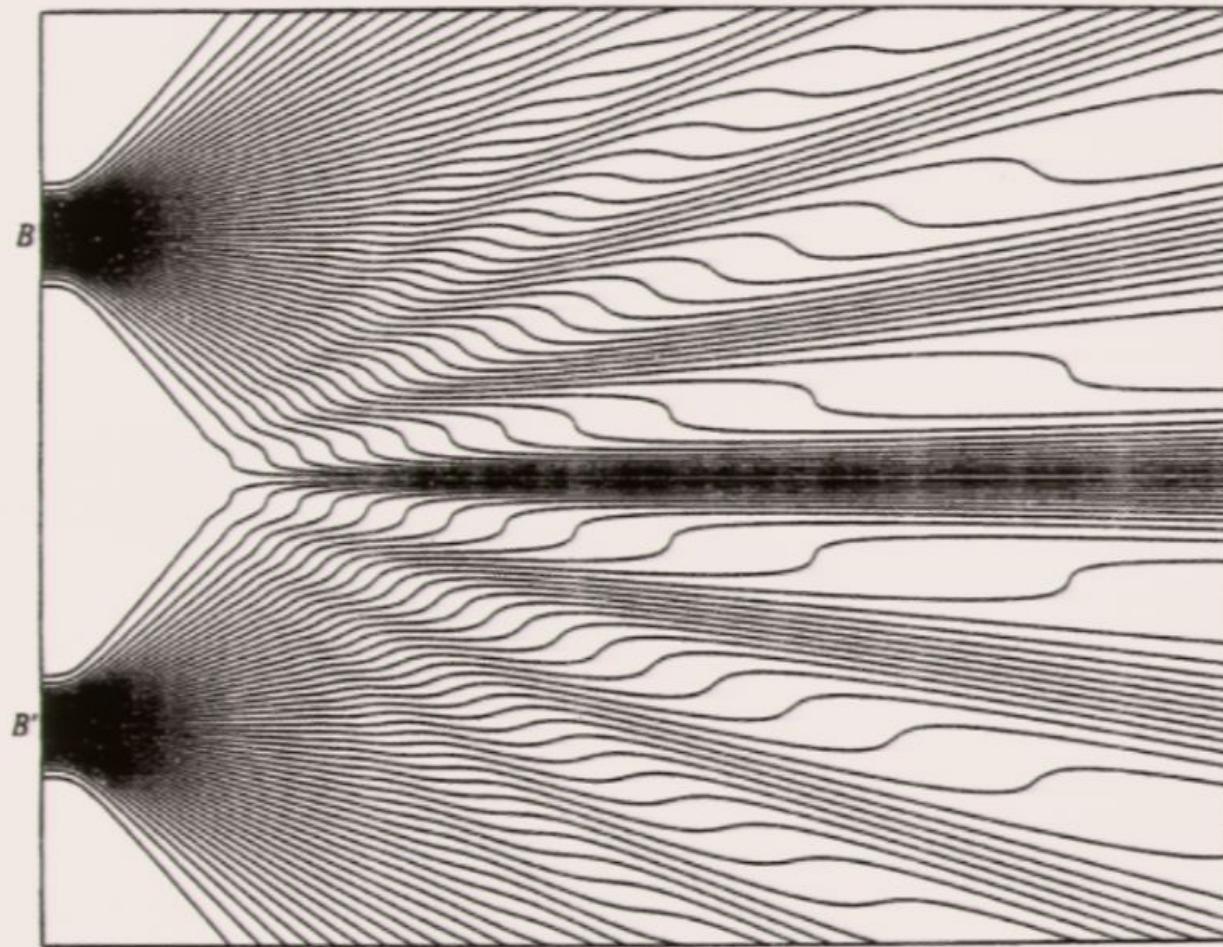
Double slit experiment



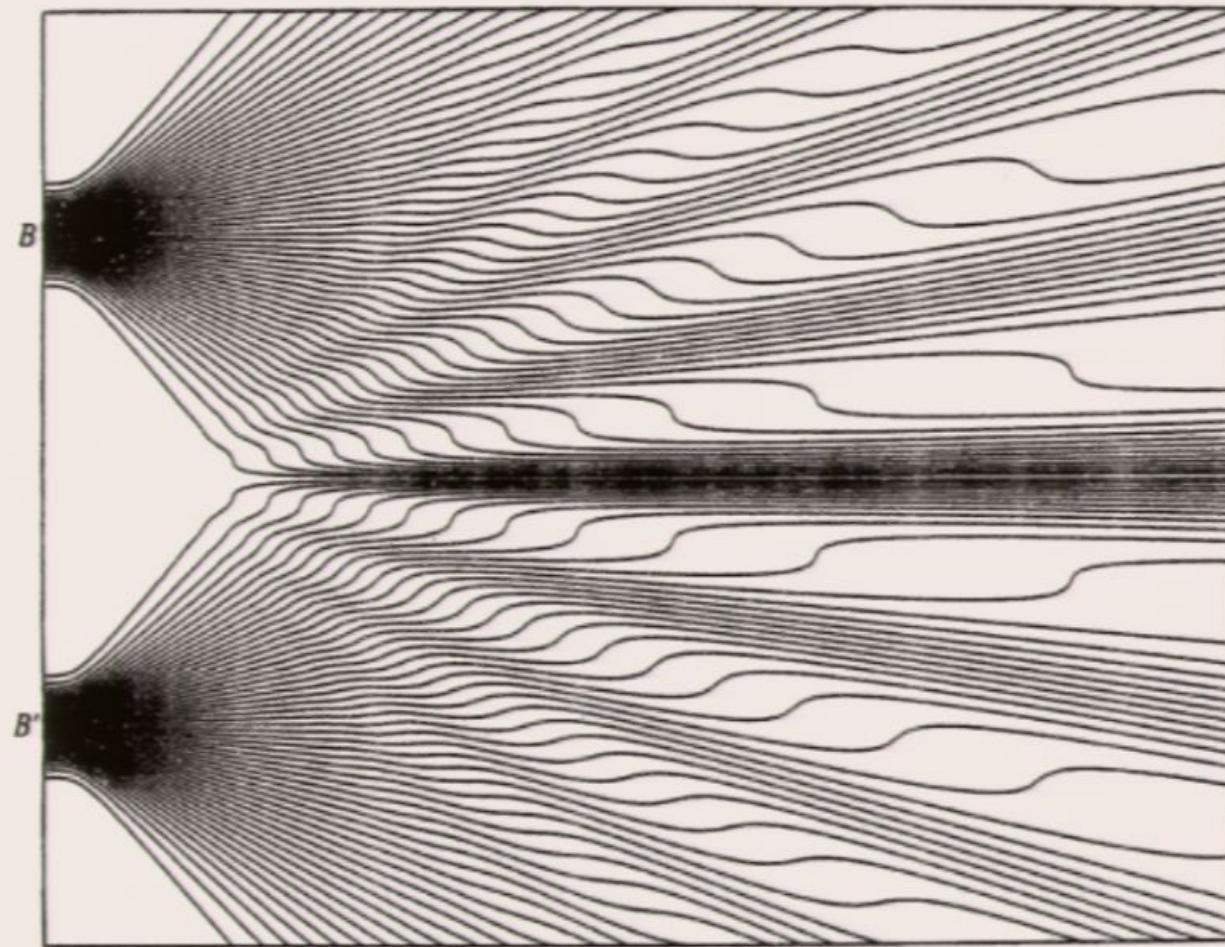
Double slit experiment



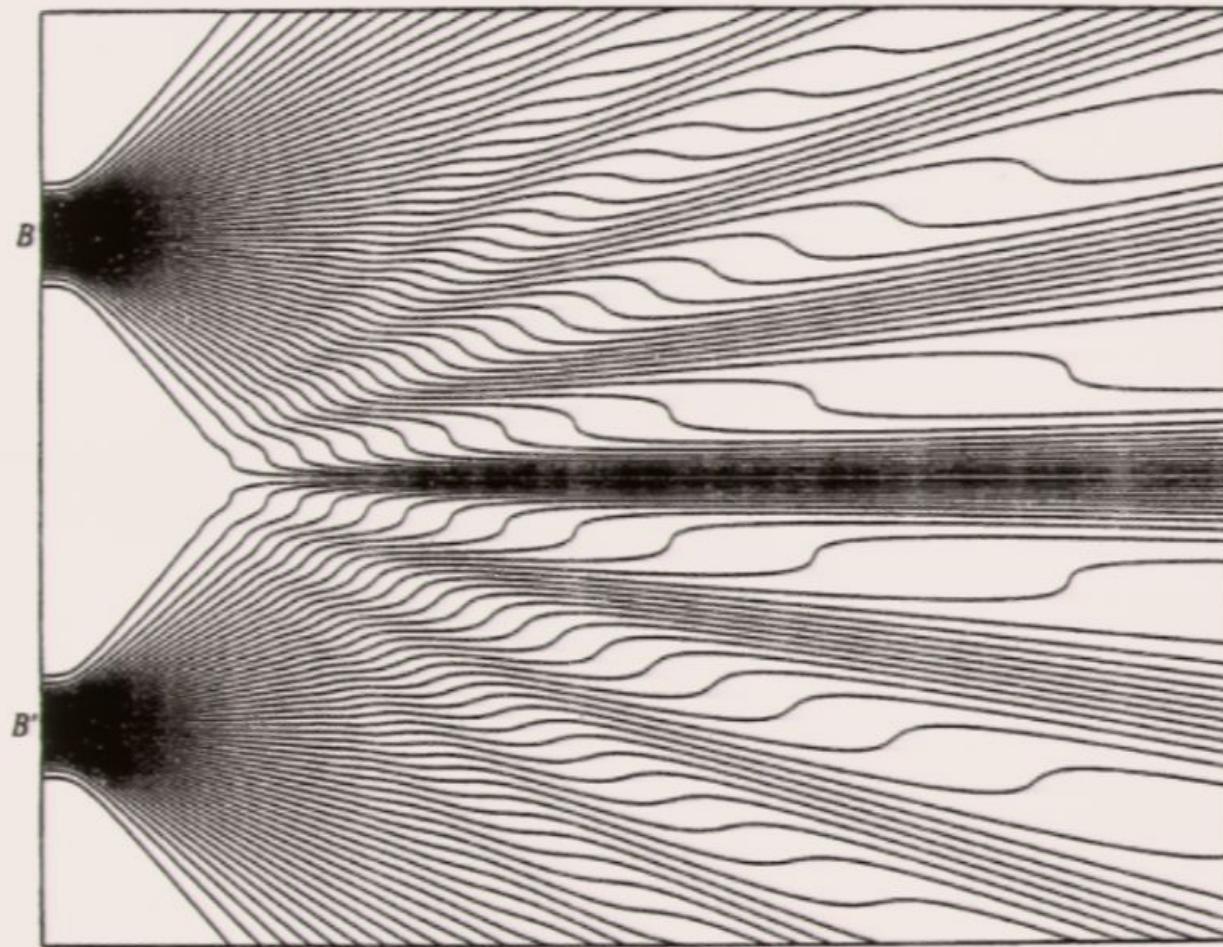
Double slit experiment



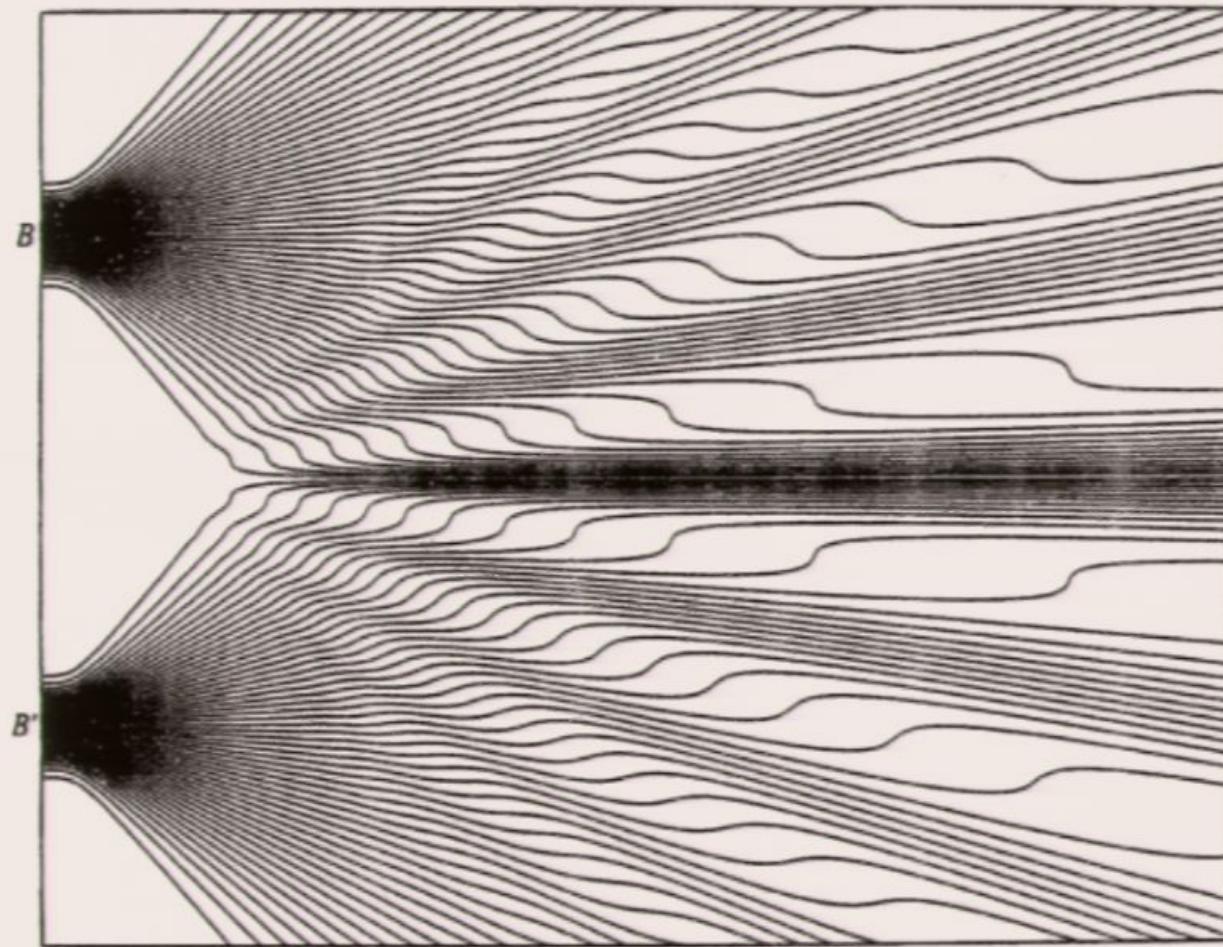
Double slit experiment



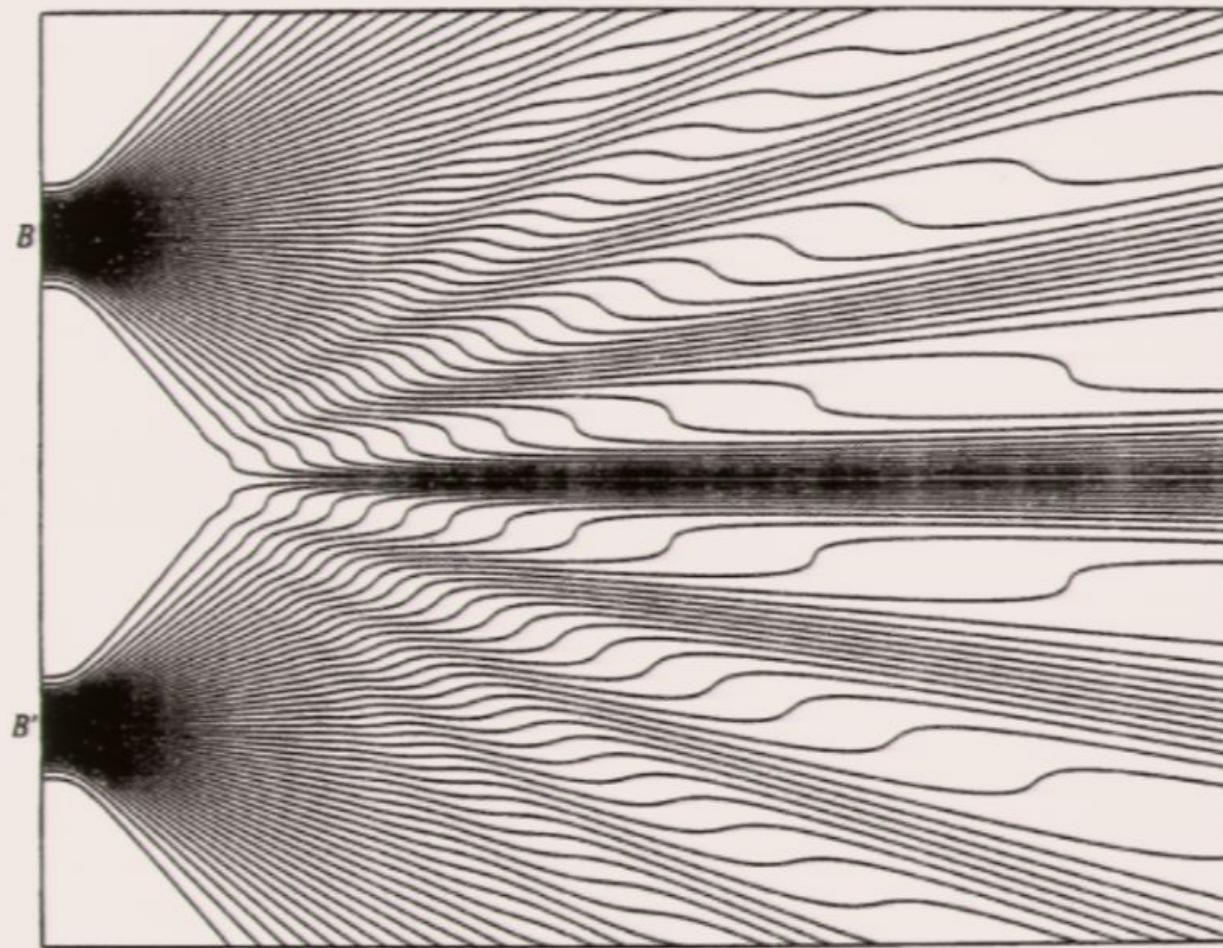
Double slit experiment



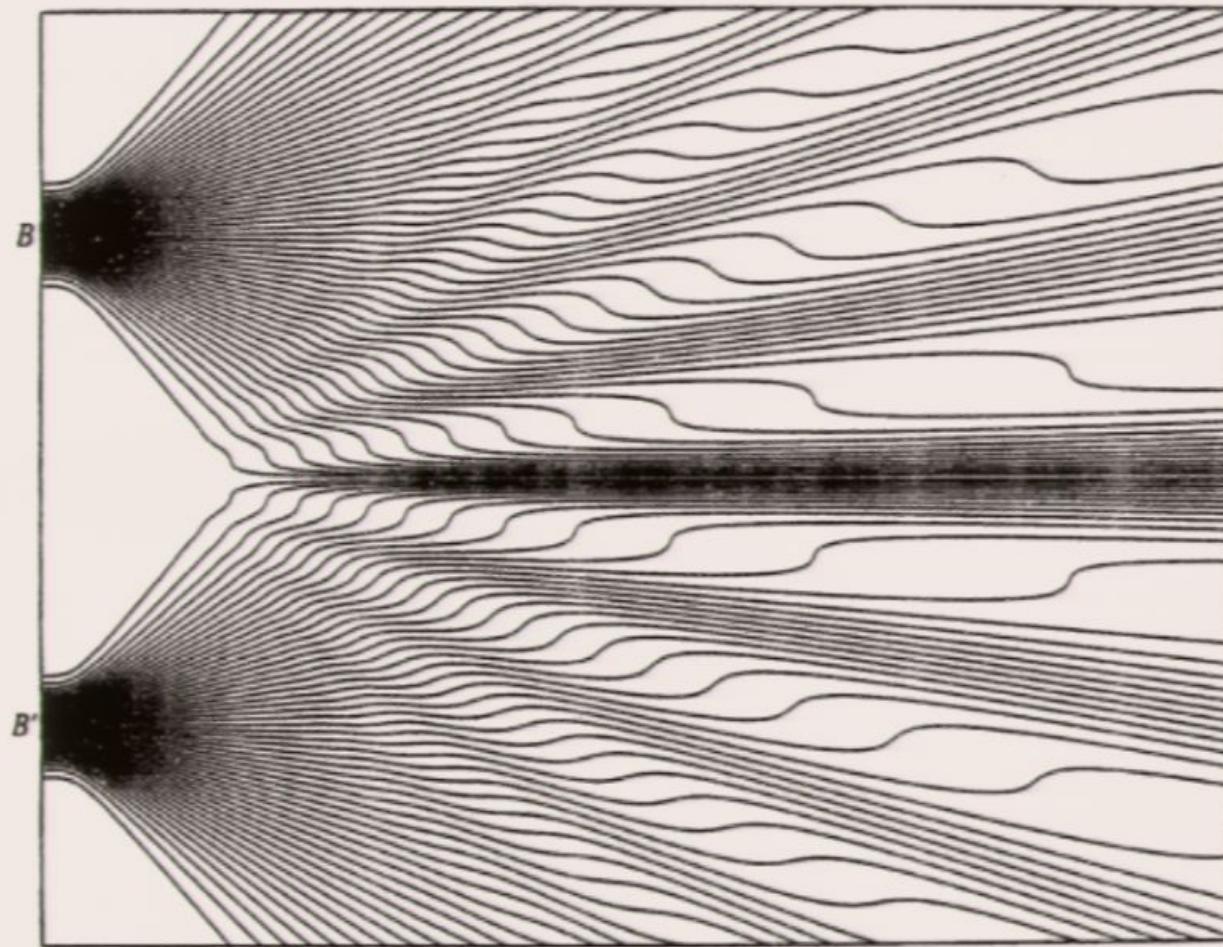
Double slit experiment



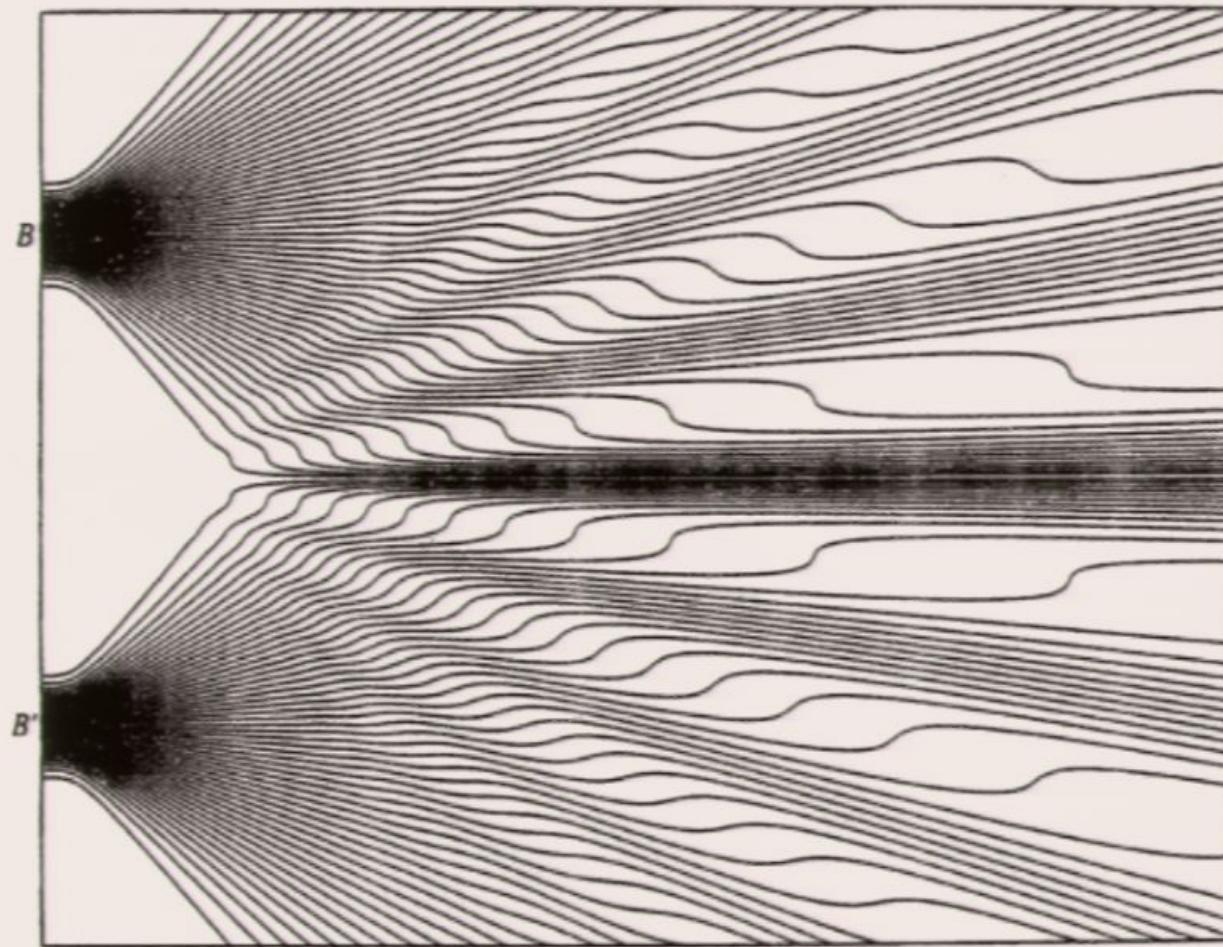
Double slit experiment



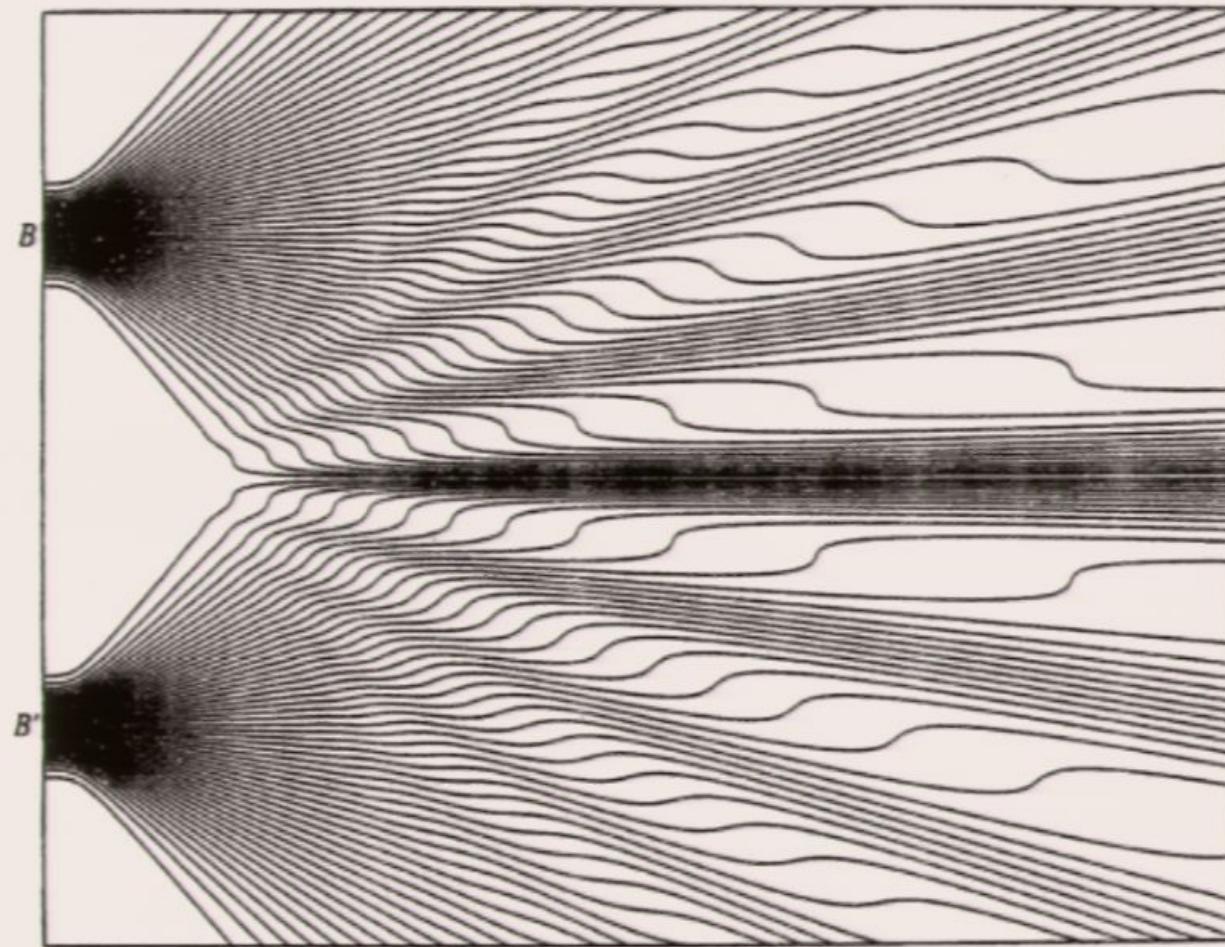
Double slit experiment



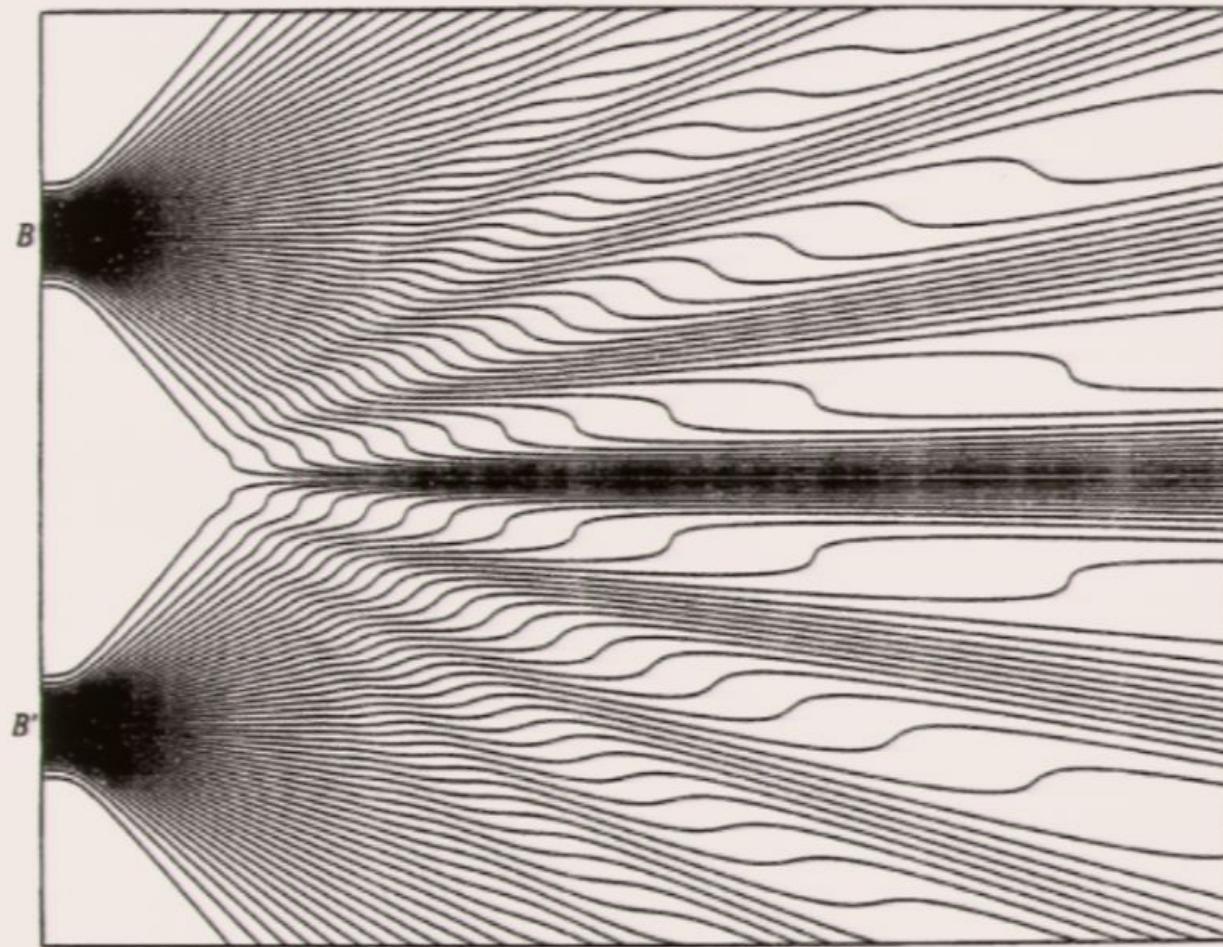
Double slit experiment



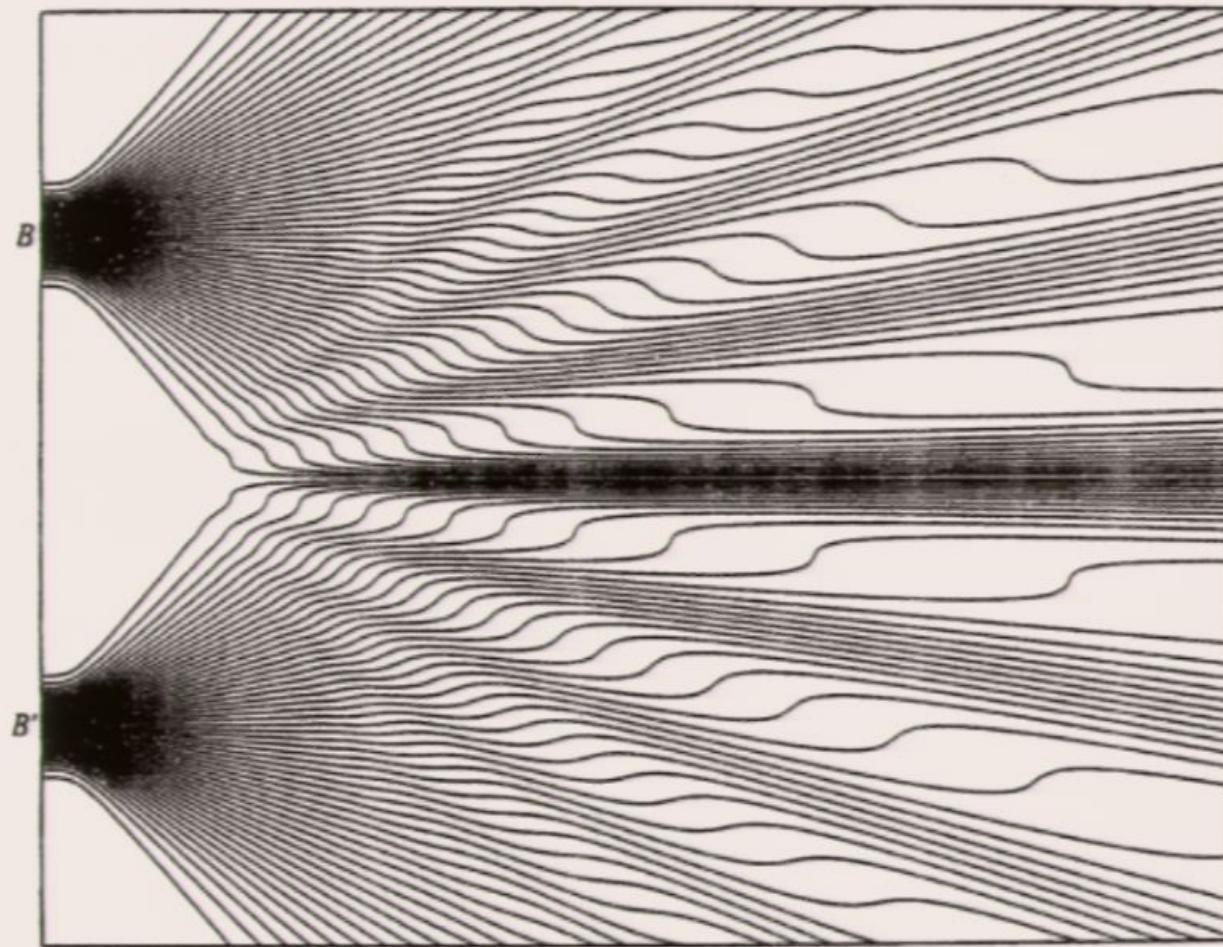
Double slit experiment



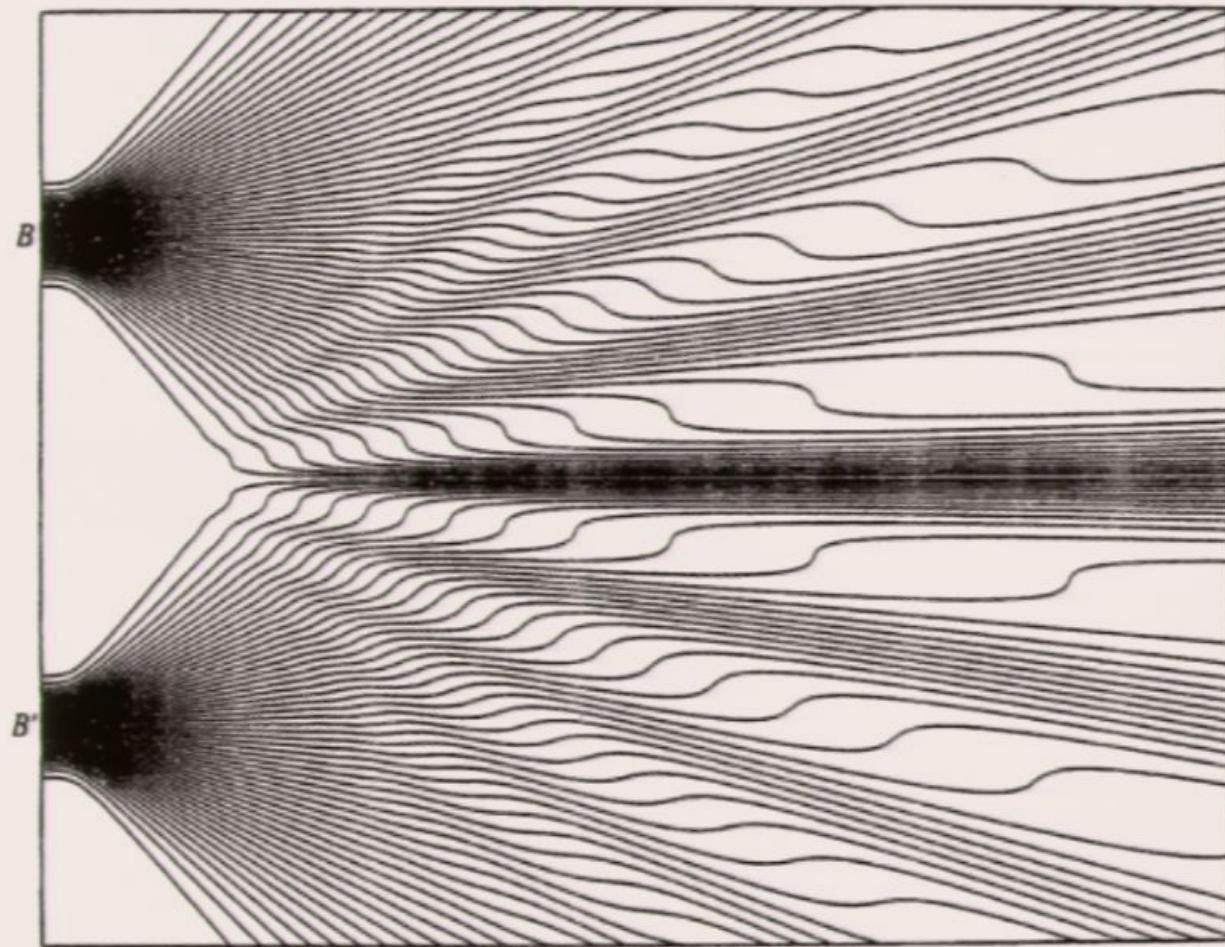
Double slit experiment



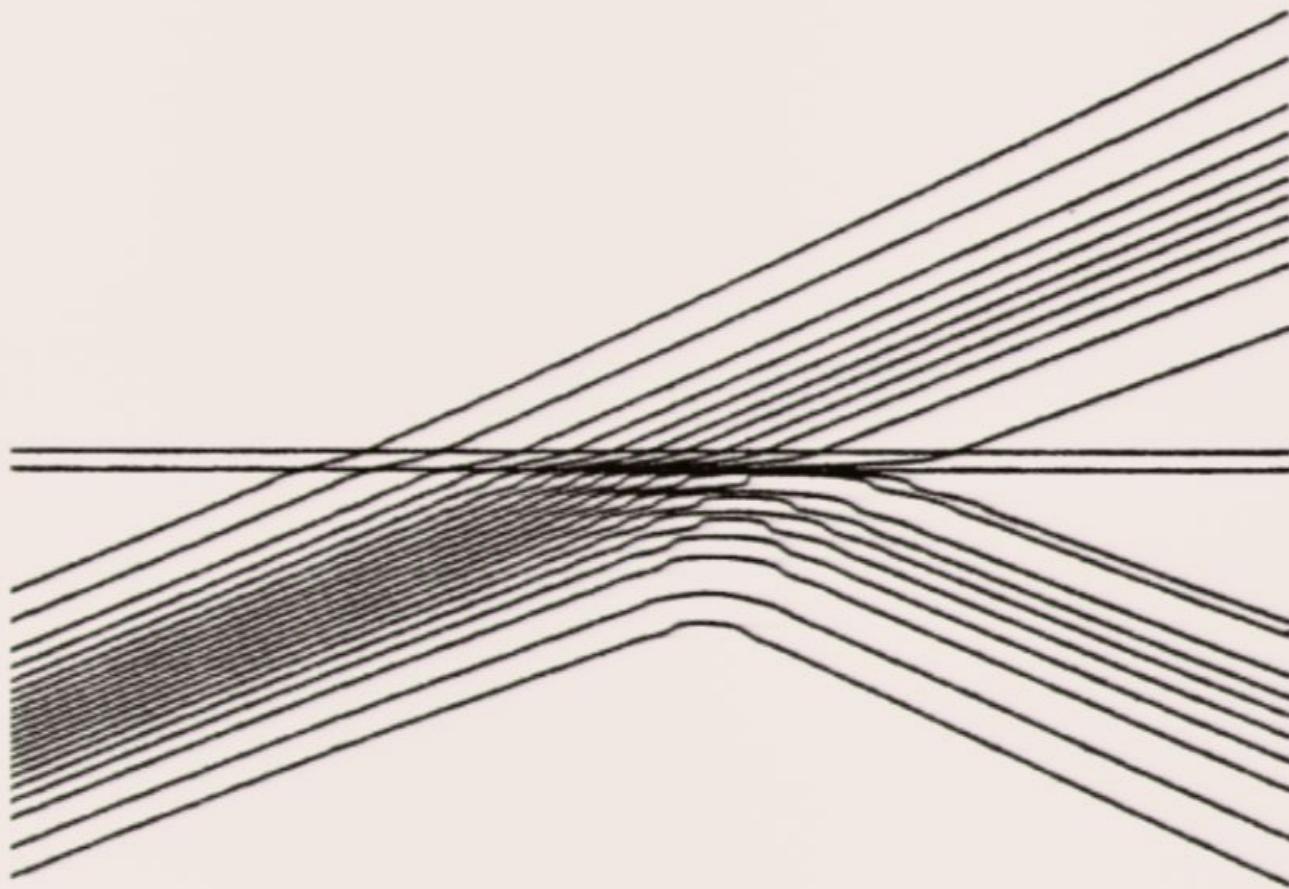
Double slit experiment



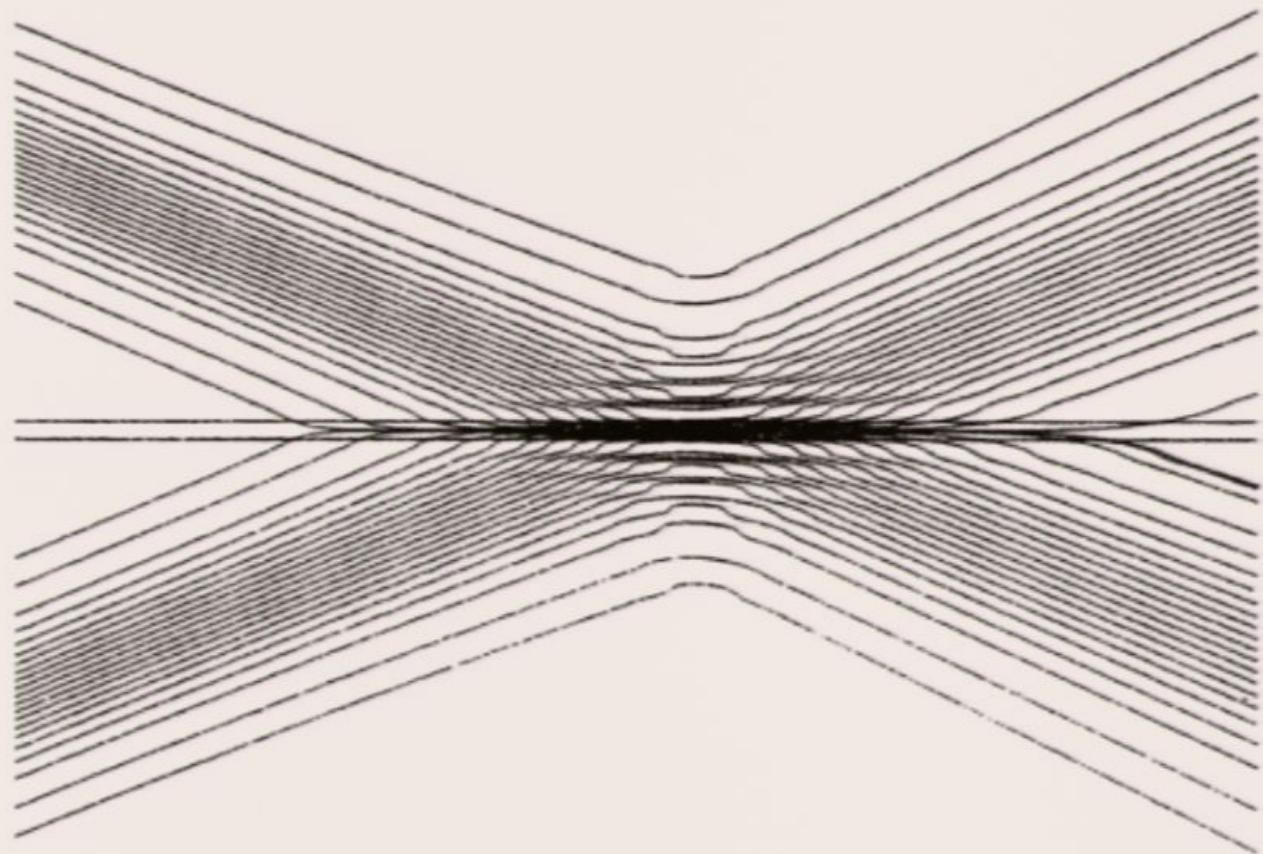
Double slit experiment



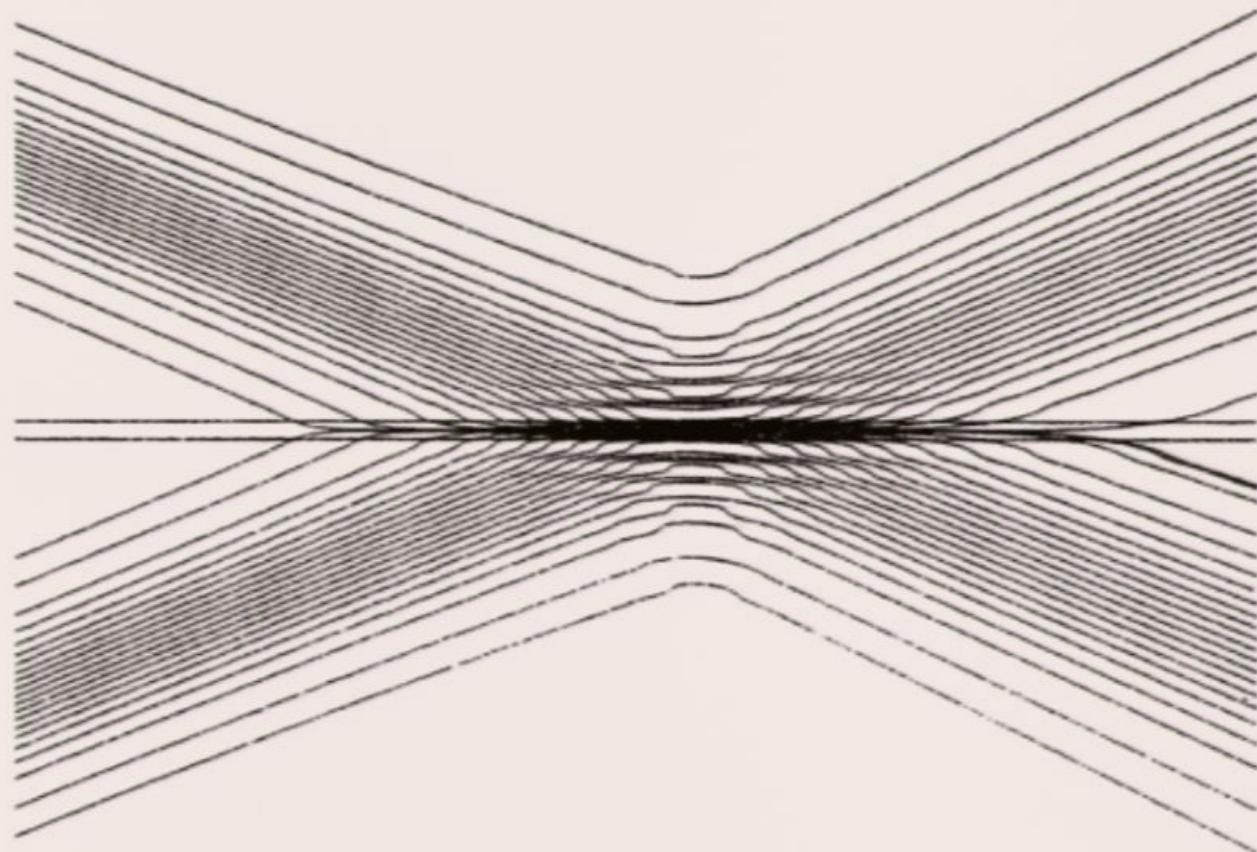
Double slit experiment



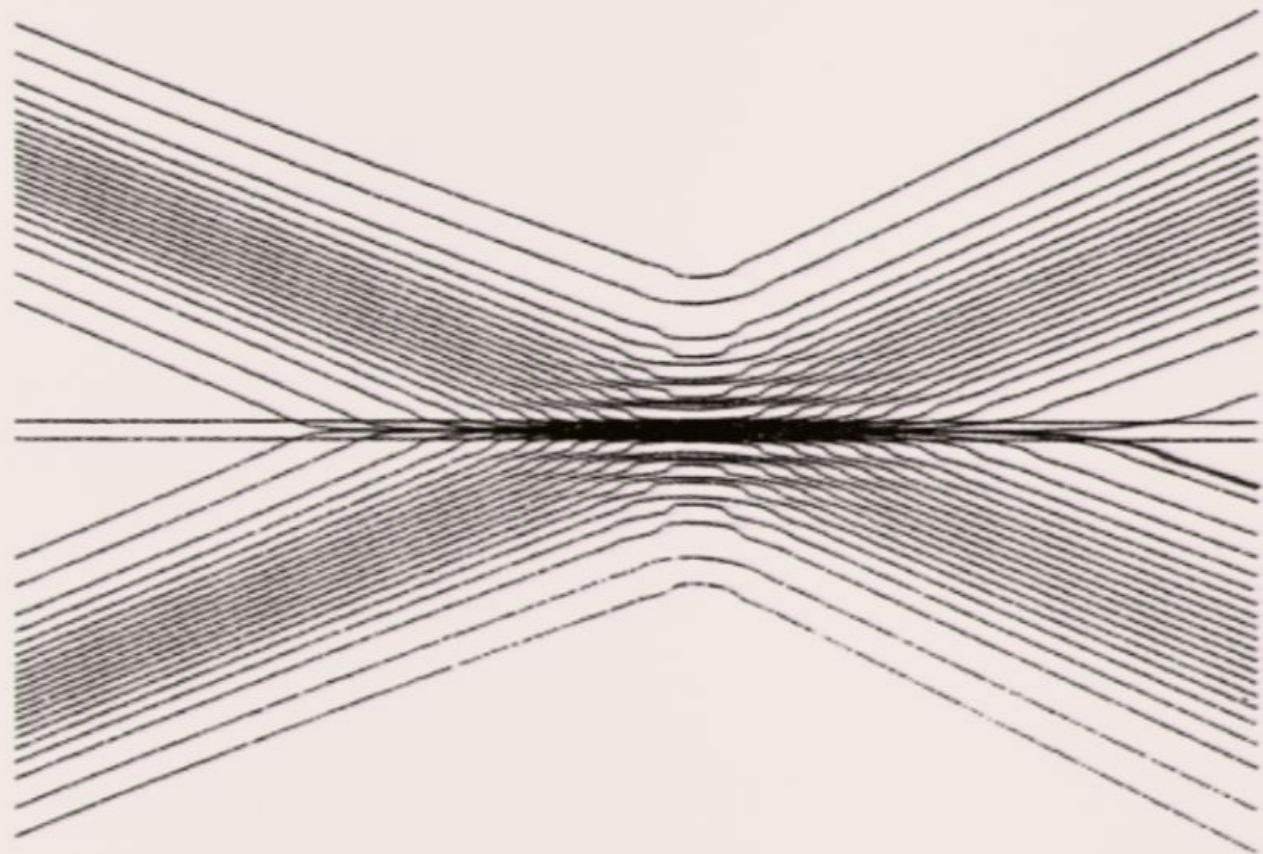
Transmission through a barrier (probability $\frac{1}{2}$)



Beam splitter experiment



Beam splitter experiment



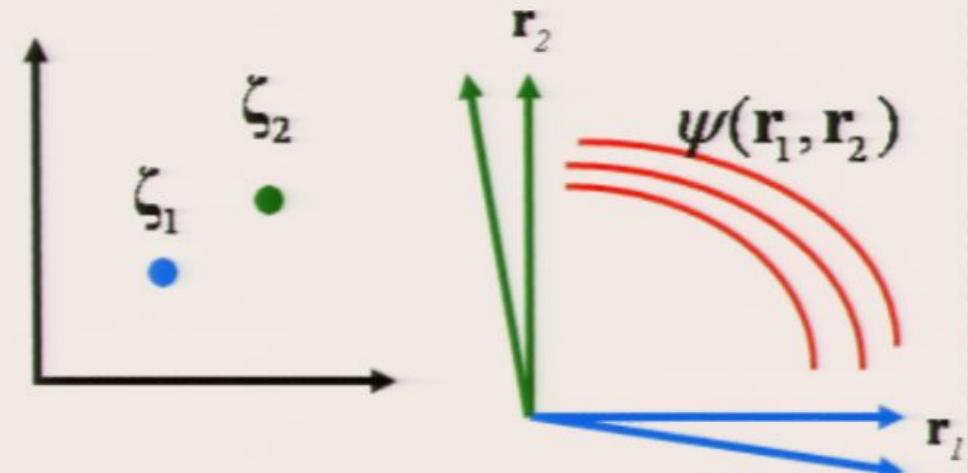
Beam splitter experiment

The deBroglie-Bohm interpretation for many particles

The ontic state: $(\psi(\mathbf{r}_1, \mathbf{r}_2), \zeta_1, \zeta_2)$

↑
Wavefunction on
configuration space

↑
Particle
positions

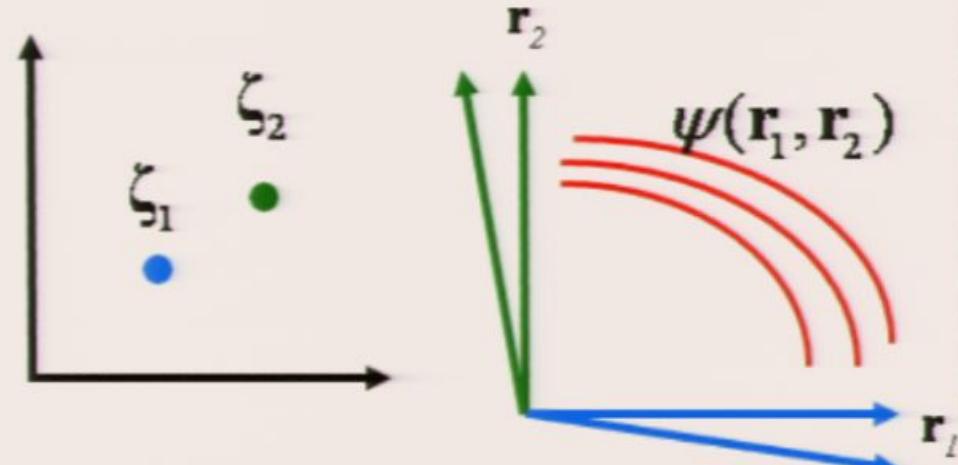


The deBroglie-Bohm interpretation for many particles

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The evolution equations:

Schrödinger's equation

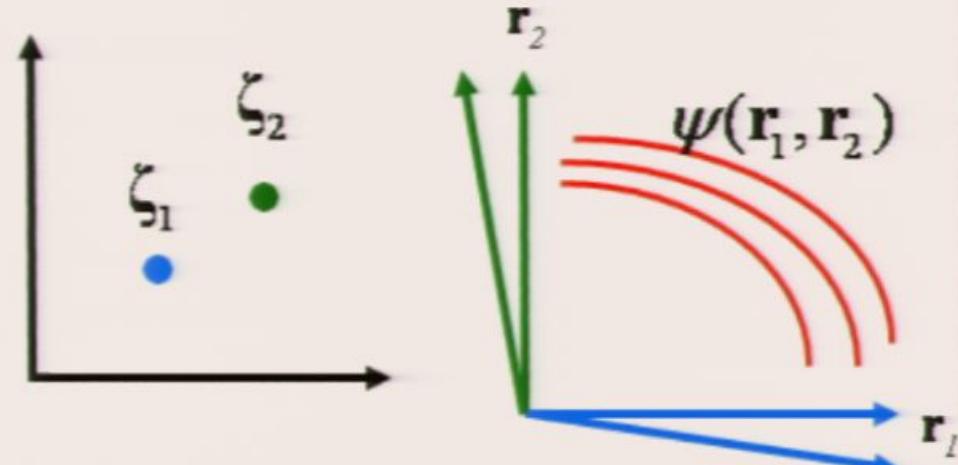
$$i\hbar \frac{\partial \psi(\mathbf{r}_1, \mathbf{r}_2, t)}{\partial t} = -\frac{\hbar^2}{2m_1} \nabla_1^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) - \frac{\hbar^2}{2m_2} \nabla_2^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) + V(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_1, \mathbf{r}_2, t)$$

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$$\left. \begin{aligned} \frac{d\zeta_1(t)}{dt} &= \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \\ \frac{d\zeta_2(t)}{dt} &= \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \end{aligned} \right\}$$

The guidance equation

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t)$$

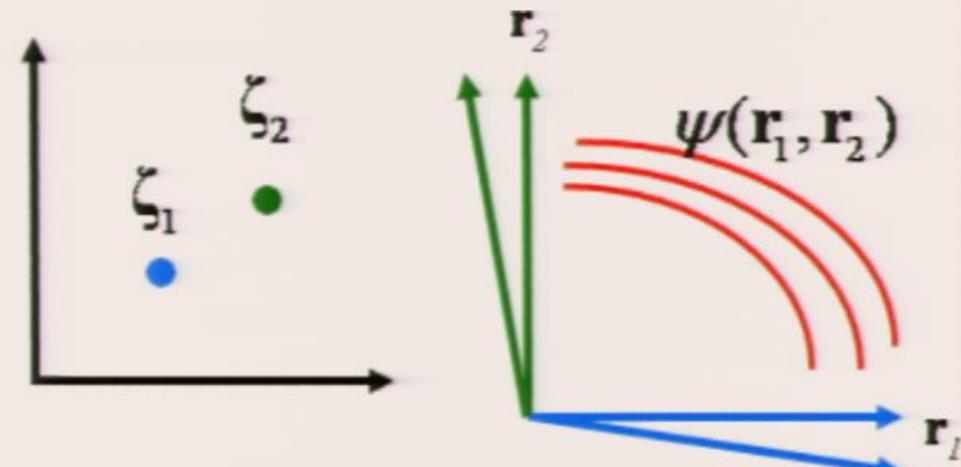
Product state

The deBroglie-Bohm interpretation for many particles

The ontic state: $(\psi(\mathbf{r}_1, \mathbf{r}_2), \zeta_1, \zeta_2)$

↑
Wavefunction on
configuration space

↑
Particle
positions



The evolution equations:

Schrödinger's equation

$$i\hbar \frac{\partial \psi(\mathbf{r}_1, \mathbf{r}_2, t)}{\partial t} = -\frac{\hbar^2}{2m_1} \nabla_1^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) - \frac{\hbar^2}{2m_2} \nabla_2^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) + V(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_1, \mathbf{r}_2, t)$$

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$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t)$$

Product state

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

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$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_2} [\nabla_2 S_2(\mathbf{r}_2, t)]_{\mathbf{r}_2=\zeta_2(t)}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_2} [\nabla_2 S_2(\mathbf{r}_2, t)]_{\mathbf{r}_2=\zeta_2(t)}$$

The two particles evolve independently

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$$

Entangled state

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is occupied

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is occupied

$(\zeta_1, \zeta_2) \notin$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is empty

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is occupied

$(\zeta_1, \zeta_2) \notin$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is empty

If only the k th wave is occupied

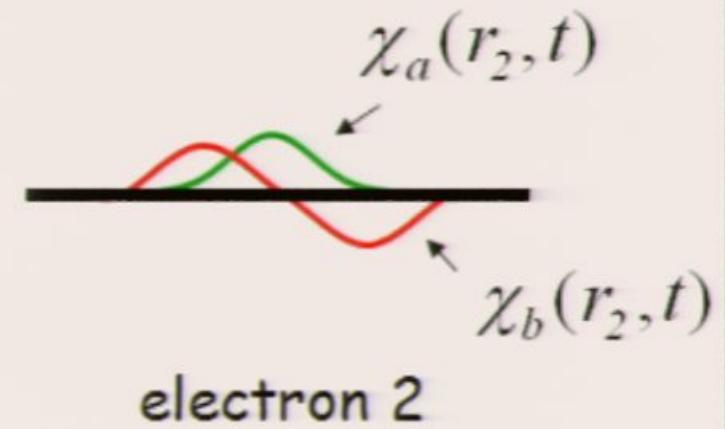
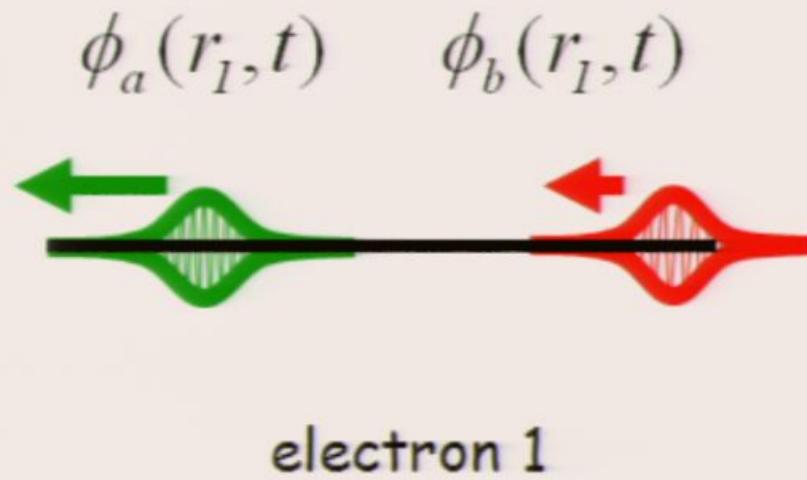
$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is occupied

$(\zeta_1, \zeta_2) \notin$ support of $\phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is empty

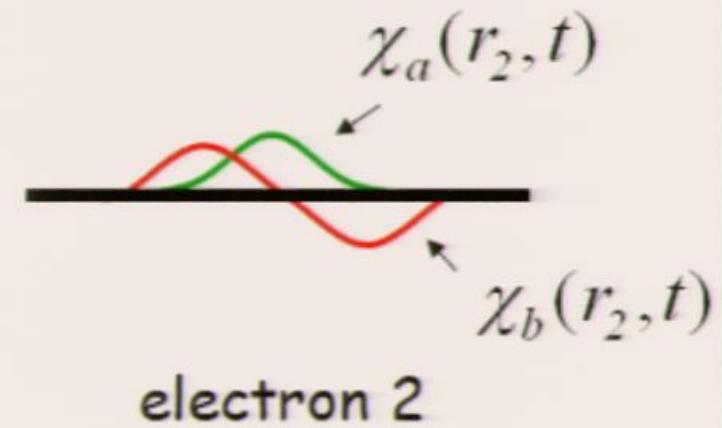
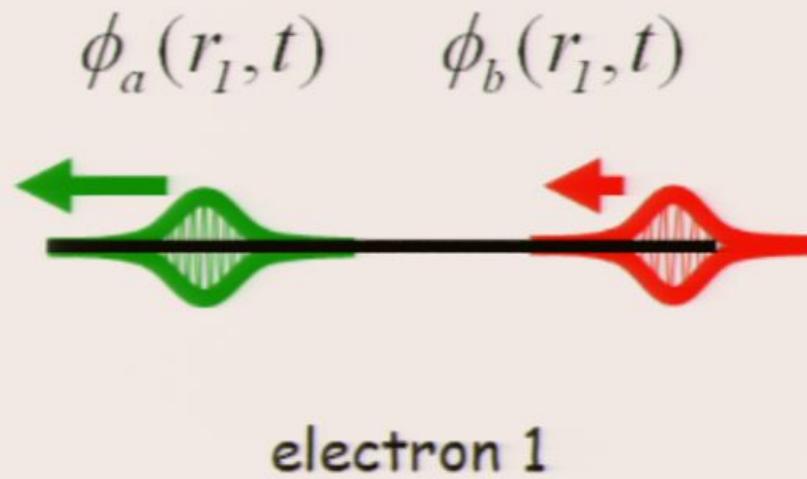
If only the k th wave is occupied

Then the particles evolve independently



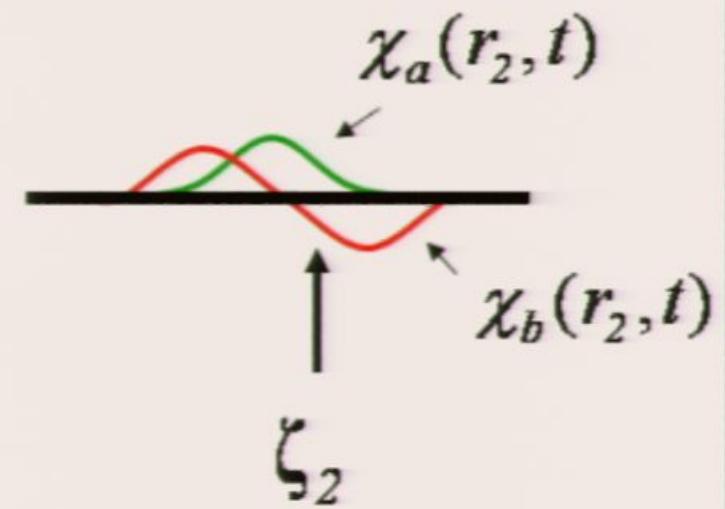
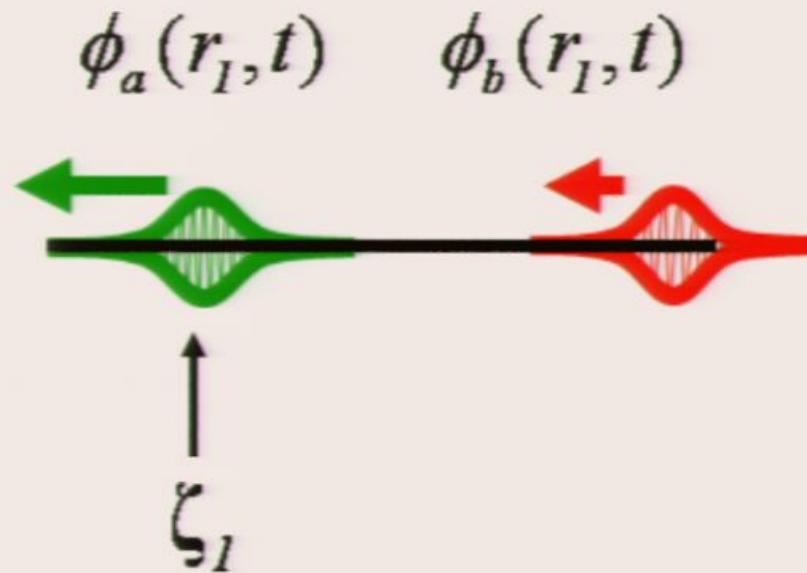
$$\psi(r_1, r_2; t) =$$

$$c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$



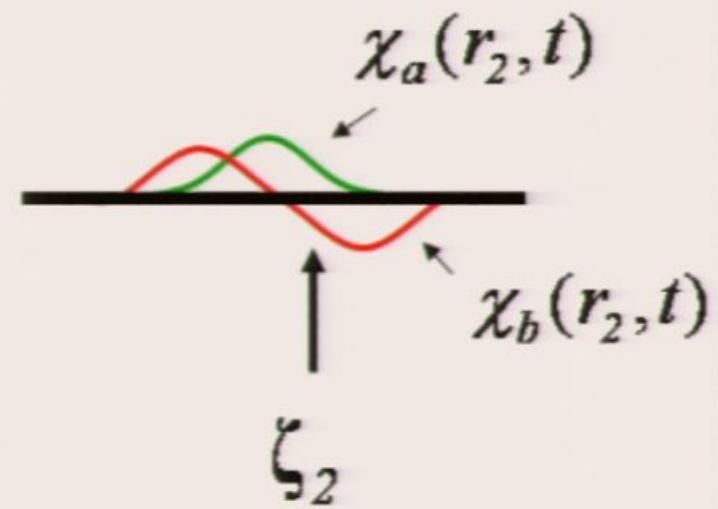
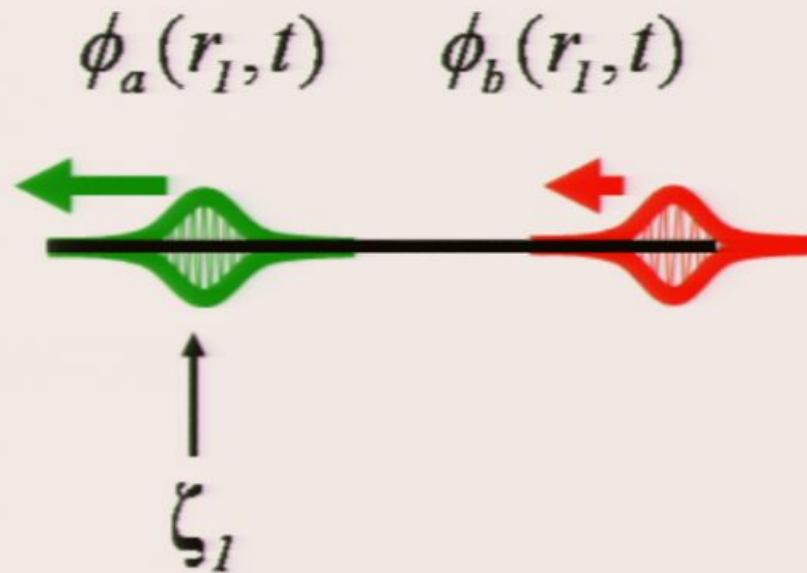
$$\psi(r_1, r_2; t) =$$

$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$



$$\psi(r_1, r_2; t) =$$

$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$

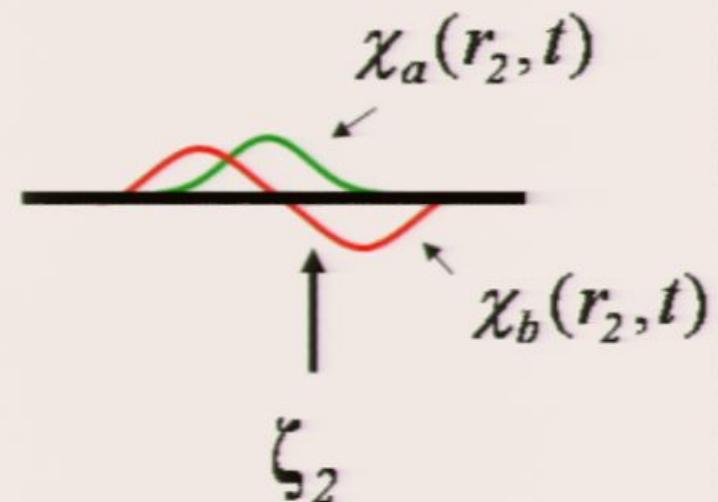
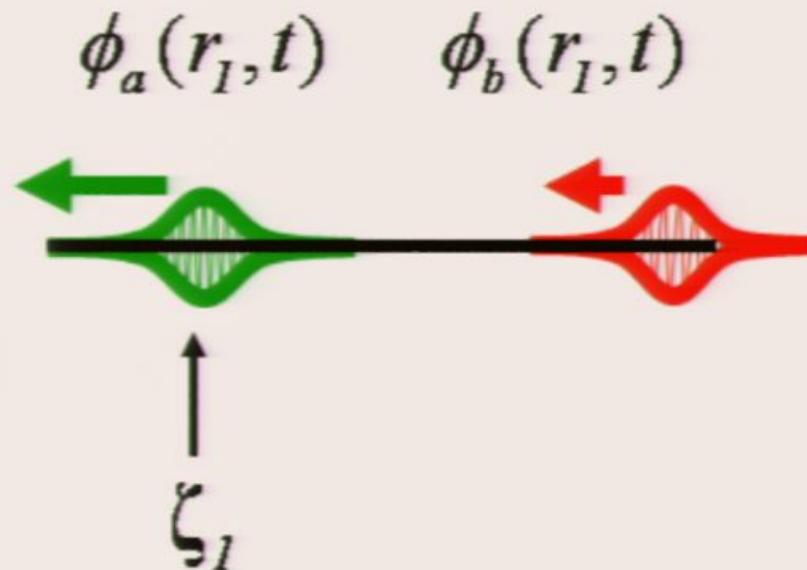


$$\psi(r_1, r_2; t) =$$

$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$

}

occupied wave

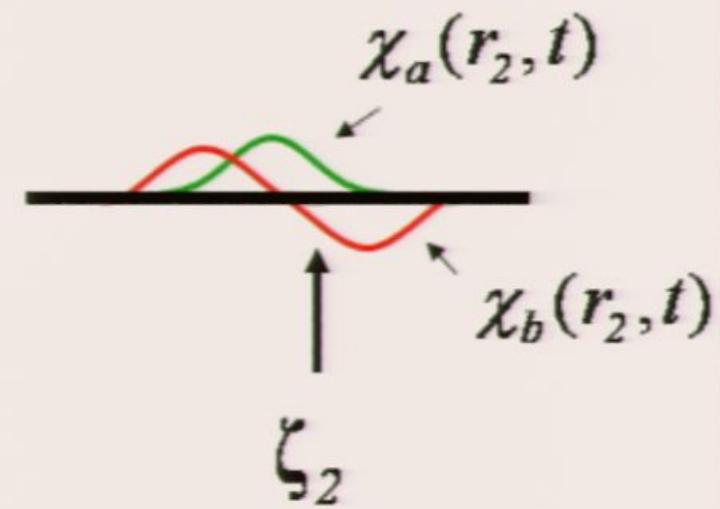
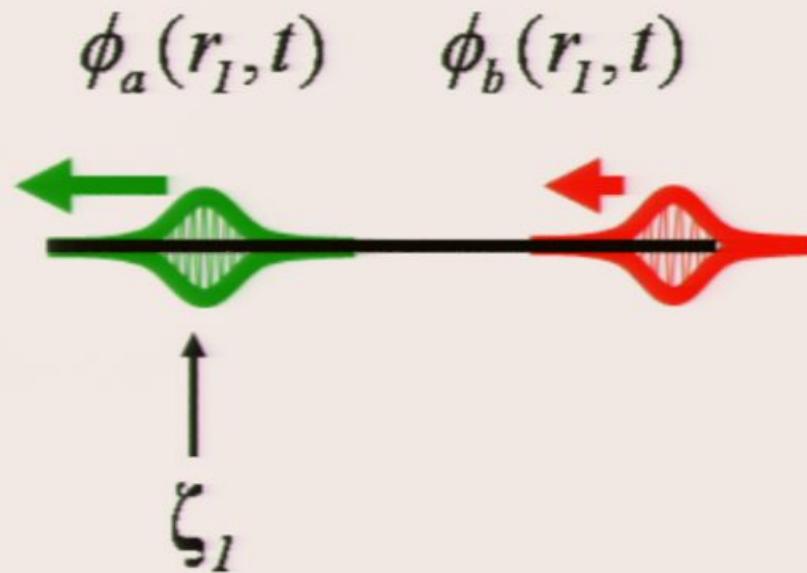

 $\psi(r_1, r_2; t) =$

$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$

}

occupied wave

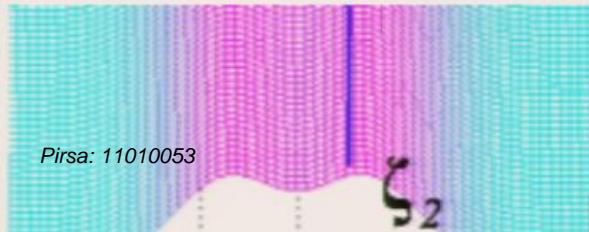
$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)}$$



$$\psi(r_1, r_2; t) =$$

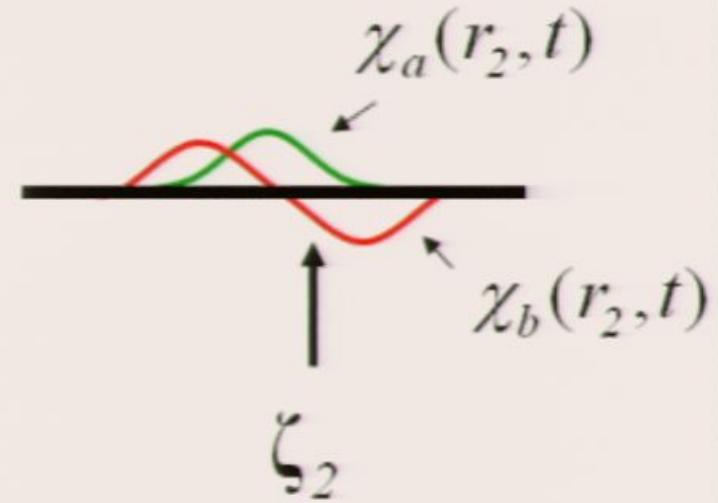
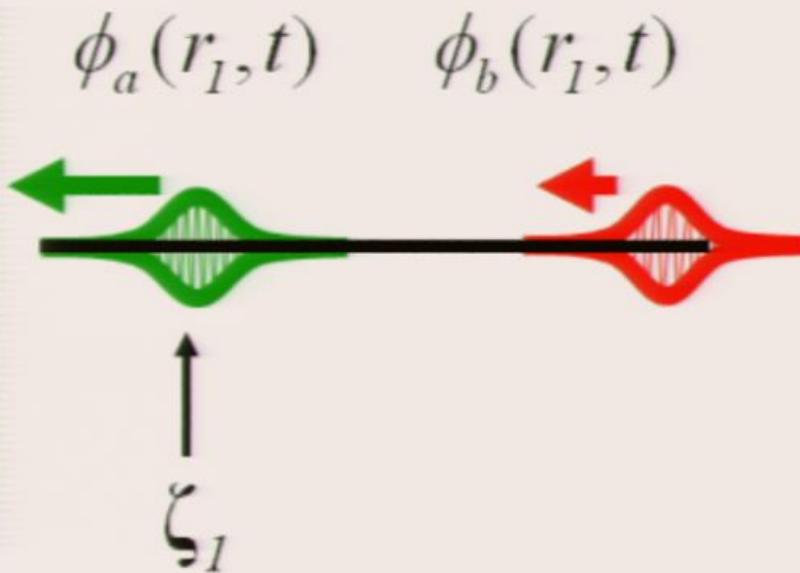
$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$

occupied wave



time

$$\frac{d\xi_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\xi_1(t), \mathbf{r}_2=\xi_2(t)}$$

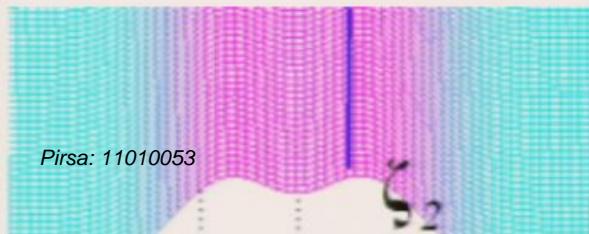


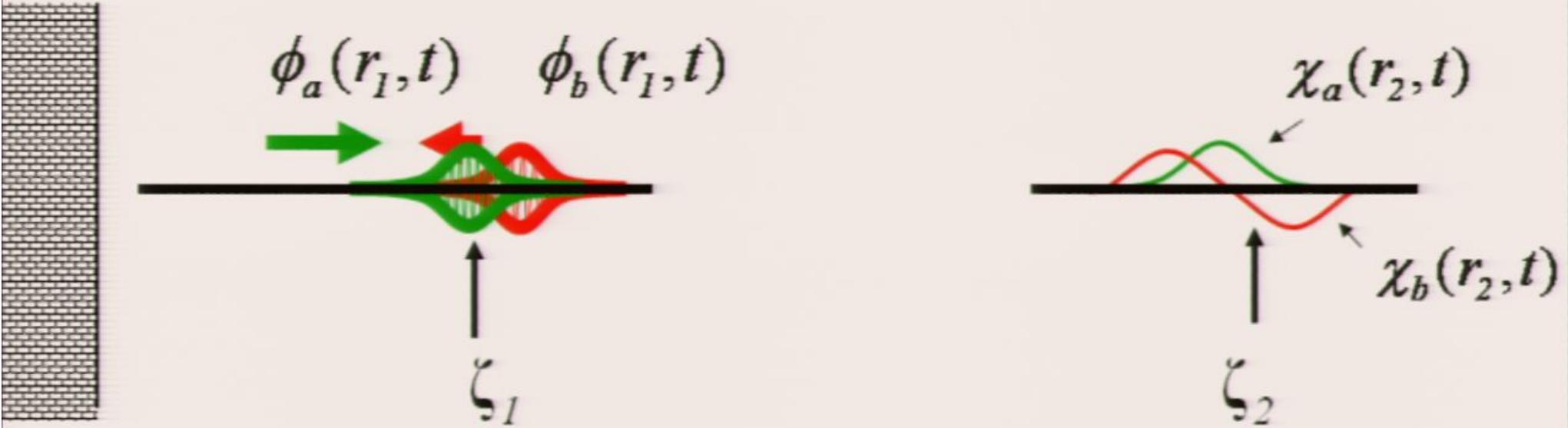
$$\psi(r_1, r_2; t) =$$

$$c_a \boxed{\phi_a(r_1, t) \chi_a(r_2, t)} + c_b \boxed{\phi_b(r_1, t) \chi_b(r_2, t)}$$



occupied wave

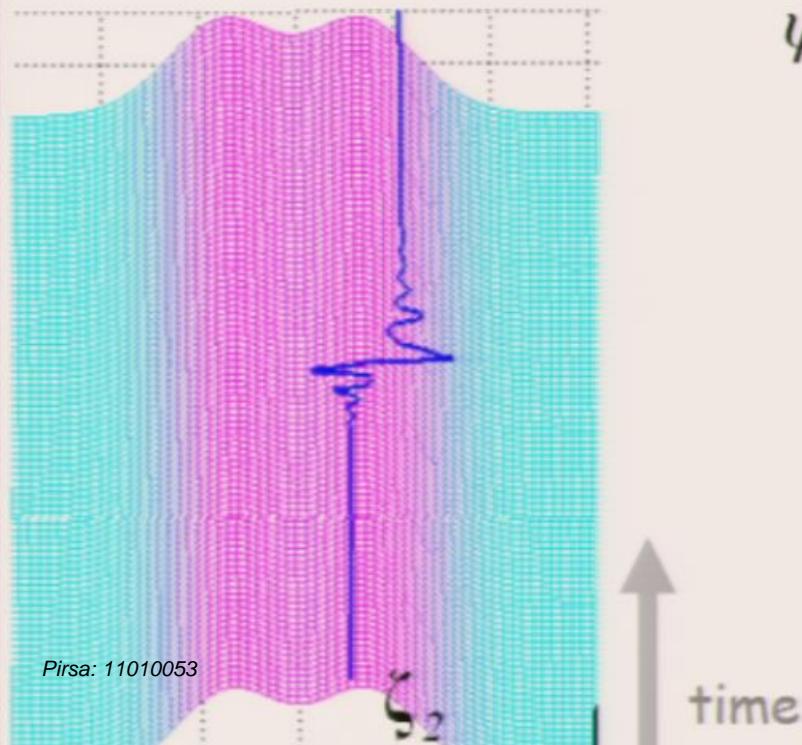


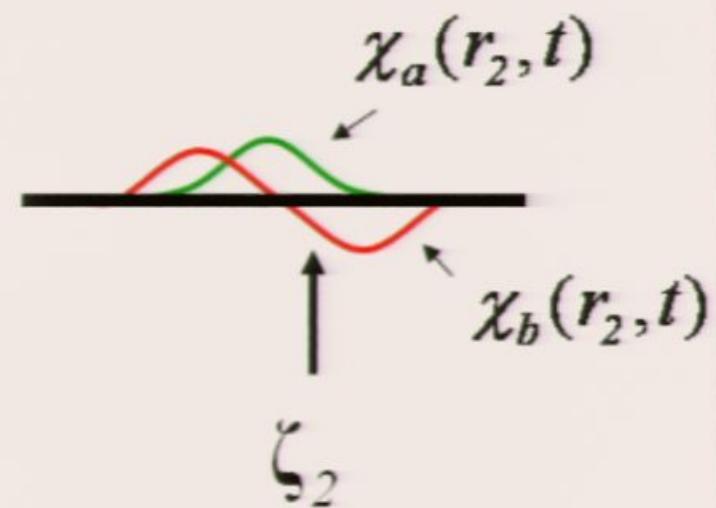
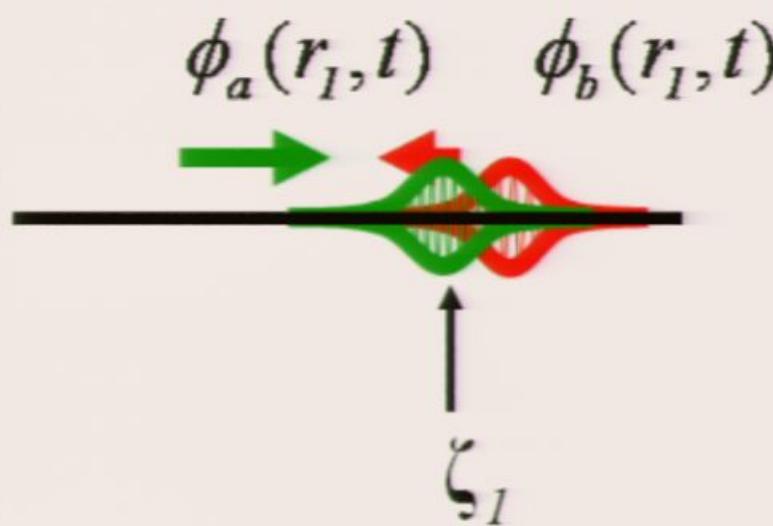


$$\psi(r_1, r_2; t) =$$

$$c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$

both waves occupied

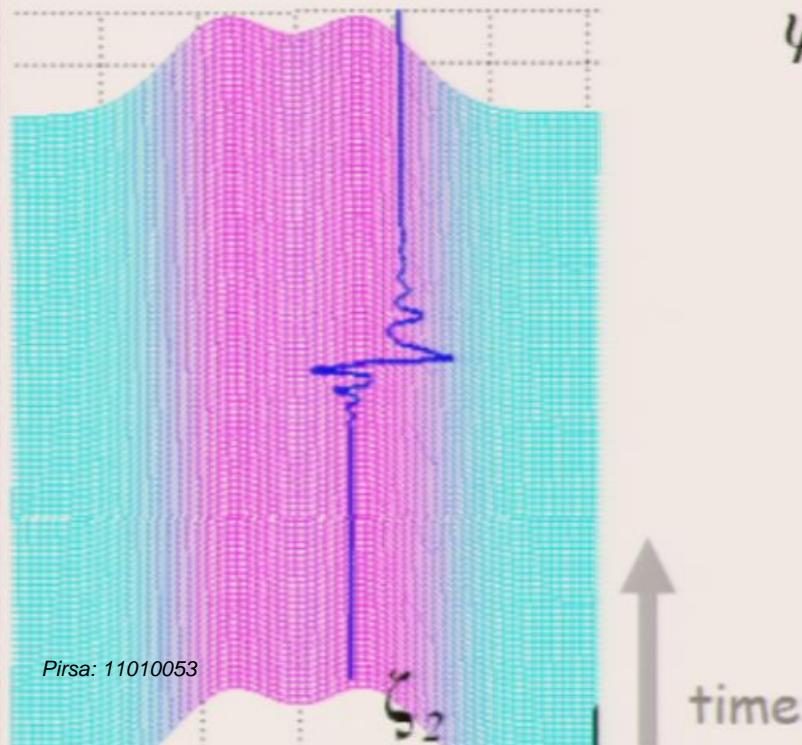




$$\psi(r_1, r_2; t) =$$

$$c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$

both waves occupied



The "standard distribution" as quantum equilibrium

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A. Valentini and H. Westman

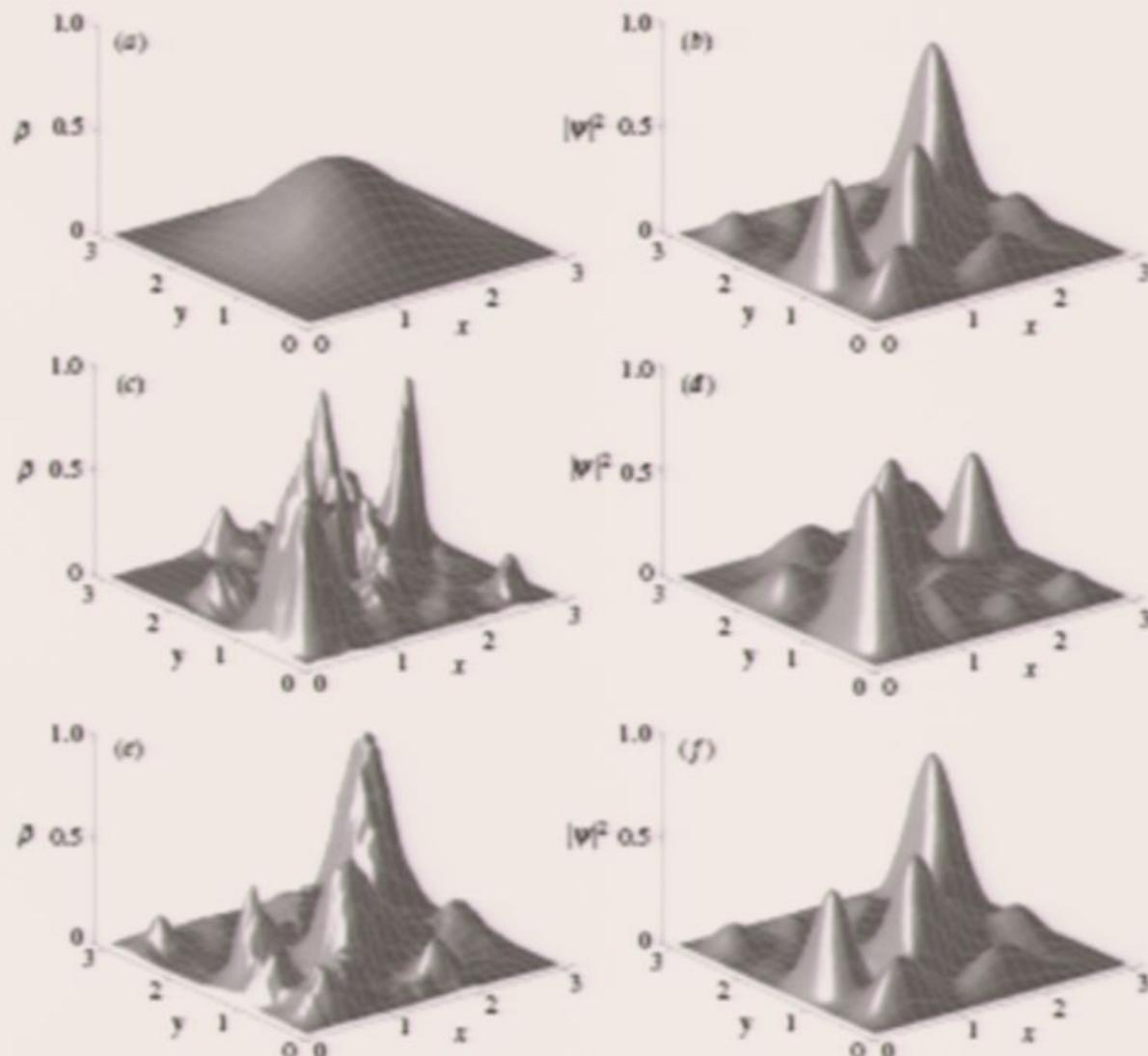


Figure 7. Smoothed ρ ((a), (c) and (e)), compared with $|\psi|^2$ ((b), (d) and (f)), at times $t = 0$ ((a), (b)), 2π ((c), (d)) and 4π ((e), (f)). While $|\psi|^2$ returns to its initial value, the smoothed ρ