

Title: Foundations of Quantum Mechanics - Lecture 10

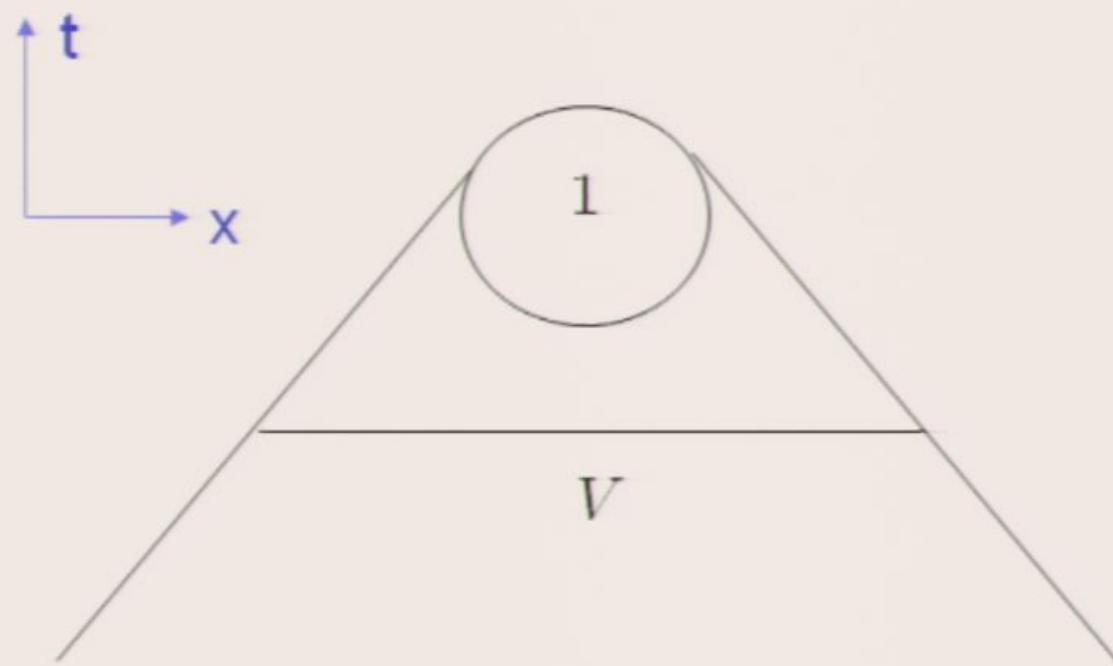
Date: Jan 14, 2011 11:30 AM

URL: <http://pirsa.org/11010049>

Abstract:

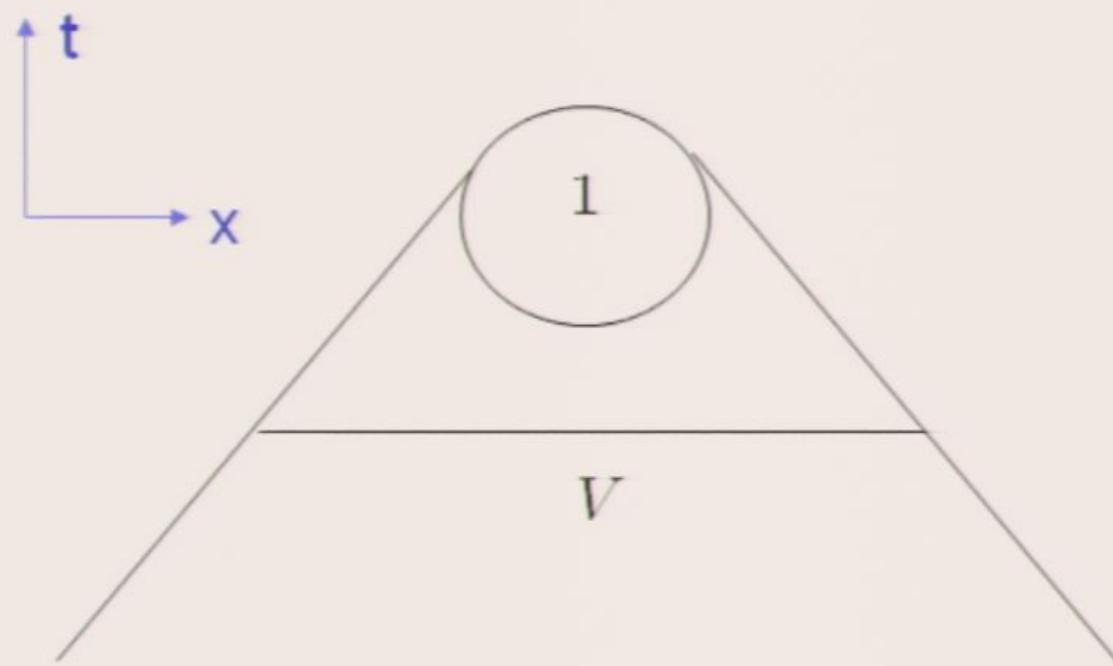
“The [beables] in any space-time region  $1$  are determined by those in any space region  $V$ , at some time  $t$ , which fully closes the backward light cone of  $1$ . Because the region  $V$  is limited, localized, we will say the theory exhibits *local determinism*.

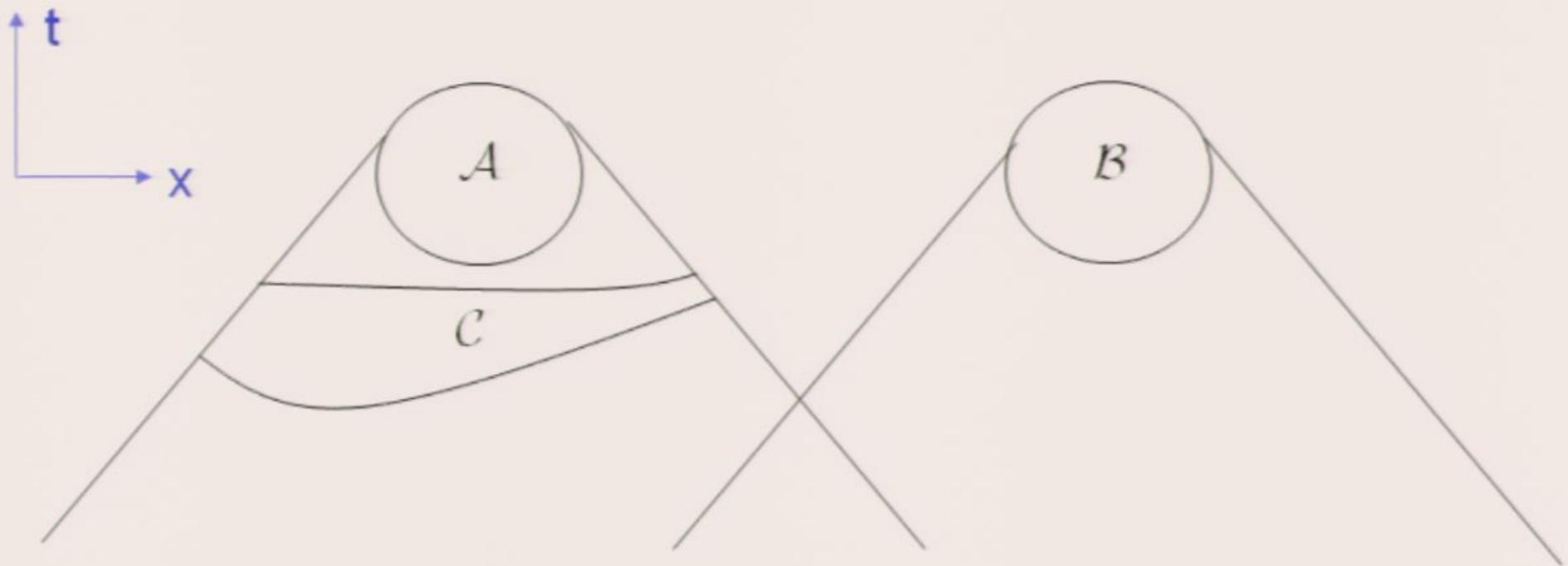
-- J.S. Bell



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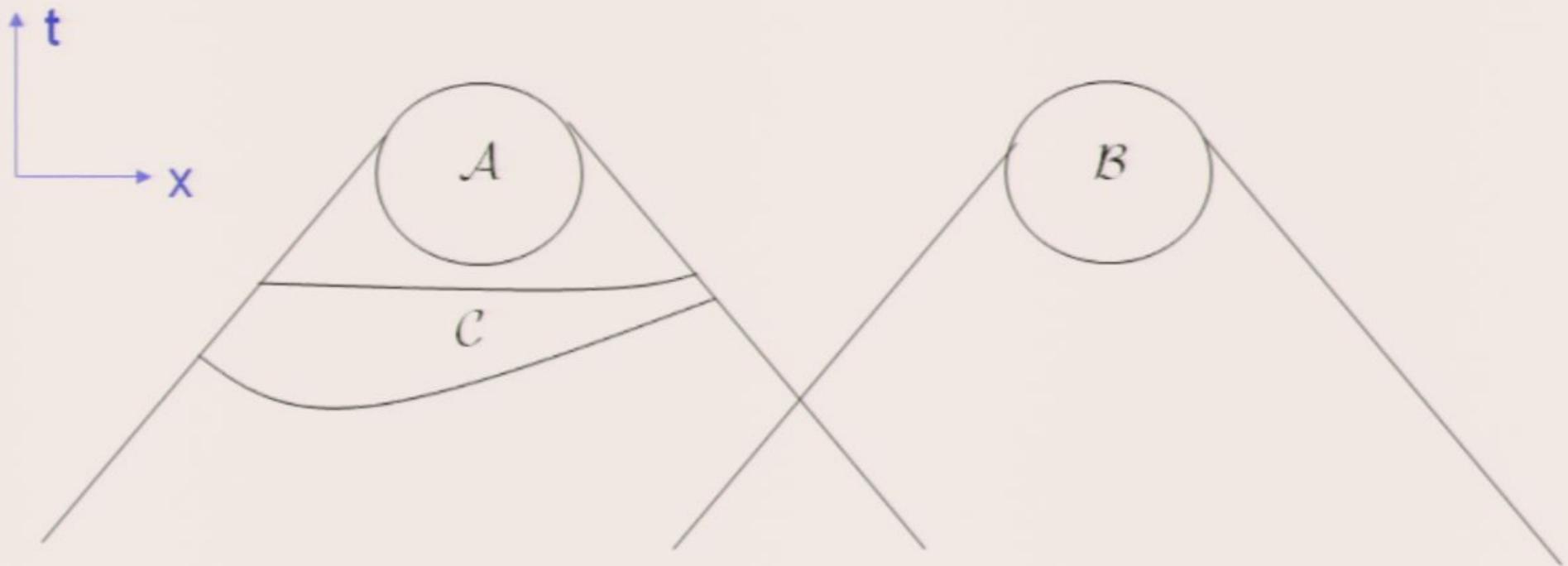
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“A theory will be said to be **locally causal** if the probabilities for the values of local beables in a space-time region A are unaltered by specification of values of local beables in a space-time region B, when what happens in the backward light cone of A is already sufficiently specified, for example by a full specification of local beables in a space-time region C.”

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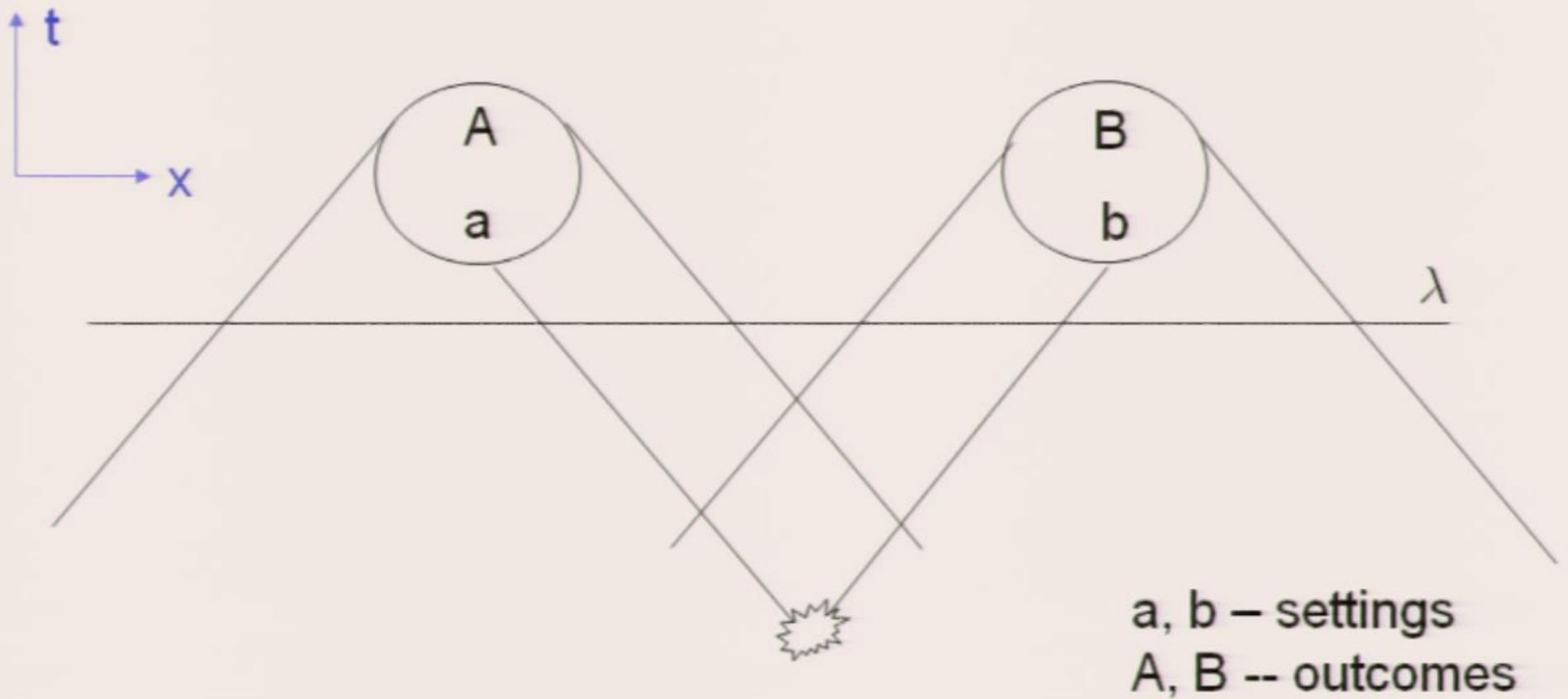


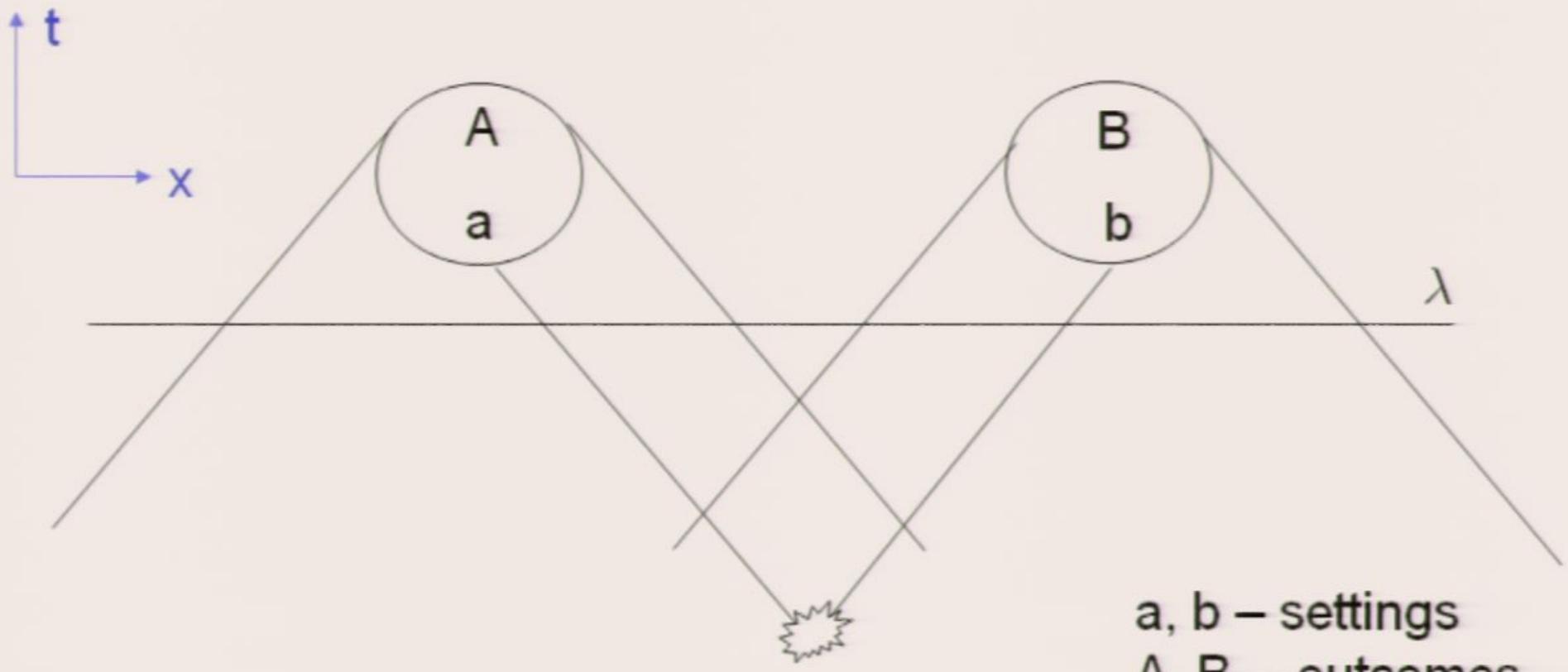
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## Local causality

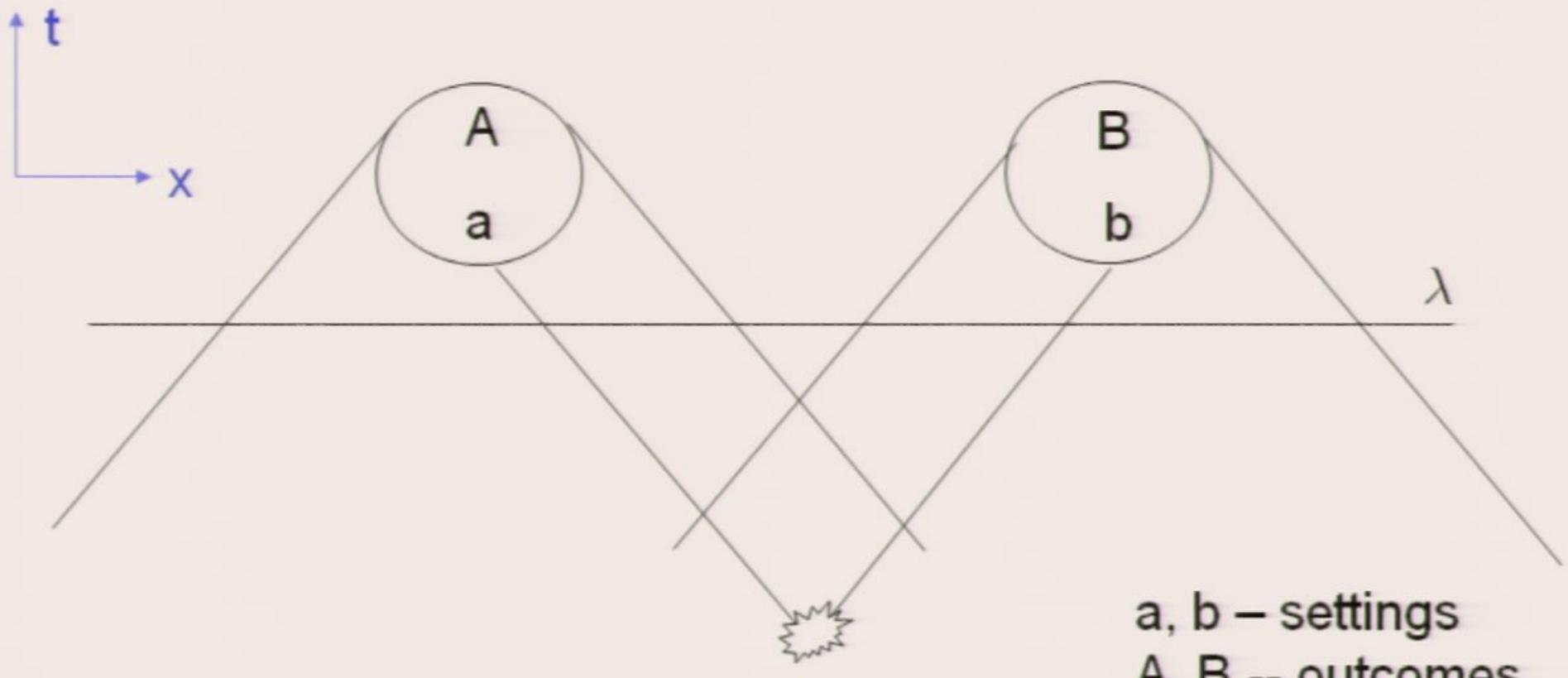
$$p(X_A | X_B, \lambda_C) \equiv p(X_A | \lambda_C)$$





Locality causality implies

$$p(A|a, b, B, \lambda) = p(A|a, \lambda)$$



Locality causality implies

$$p(A|a, b, B, \lambda) = p(A|a, \lambda)$$

$$p(B|a, b, A, \lambda) = p(B|b, \lambda)$$

and implies *factorizability*

$$p(A, B|a, b, \lambda) = p(A|a, \lambda)p(B|b, \lambda)$$

## Factorizability from local causality

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Recall Bayes' rule

$$p(A, B) = p(A|B)p(B)$$

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By local causality

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$$\frac{1}{4} [ p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b') ] \leq 3/4$$



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Define  $C(a, b) = (+1)p(\text{agree}|ab) + (-1)p(\text{disagree}|ab)$



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Define  $C(a, b) = (+1)p(\text{agree}|ab) + (-1)p(\text{disagree}|ab)$

$$|C(a, b) + C(a', b) + C(a, b') - C(a', b')| \leq 2$$

The Clauser-Horn-Shimony-Holt (CHSH) inequality



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# Applications of nonlocality

## Factorizability from local causality

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$$p(A, B) = \boxed{\text{Rescue and Recovery} \quad \text{X}})$$

By local causality



The scheduled backup could not be taken because the mobile system is not connected to AC power.

OK

Thus

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# Applications of nonlocality

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Magic is a natural force that can be used to override the usual laws of nature.

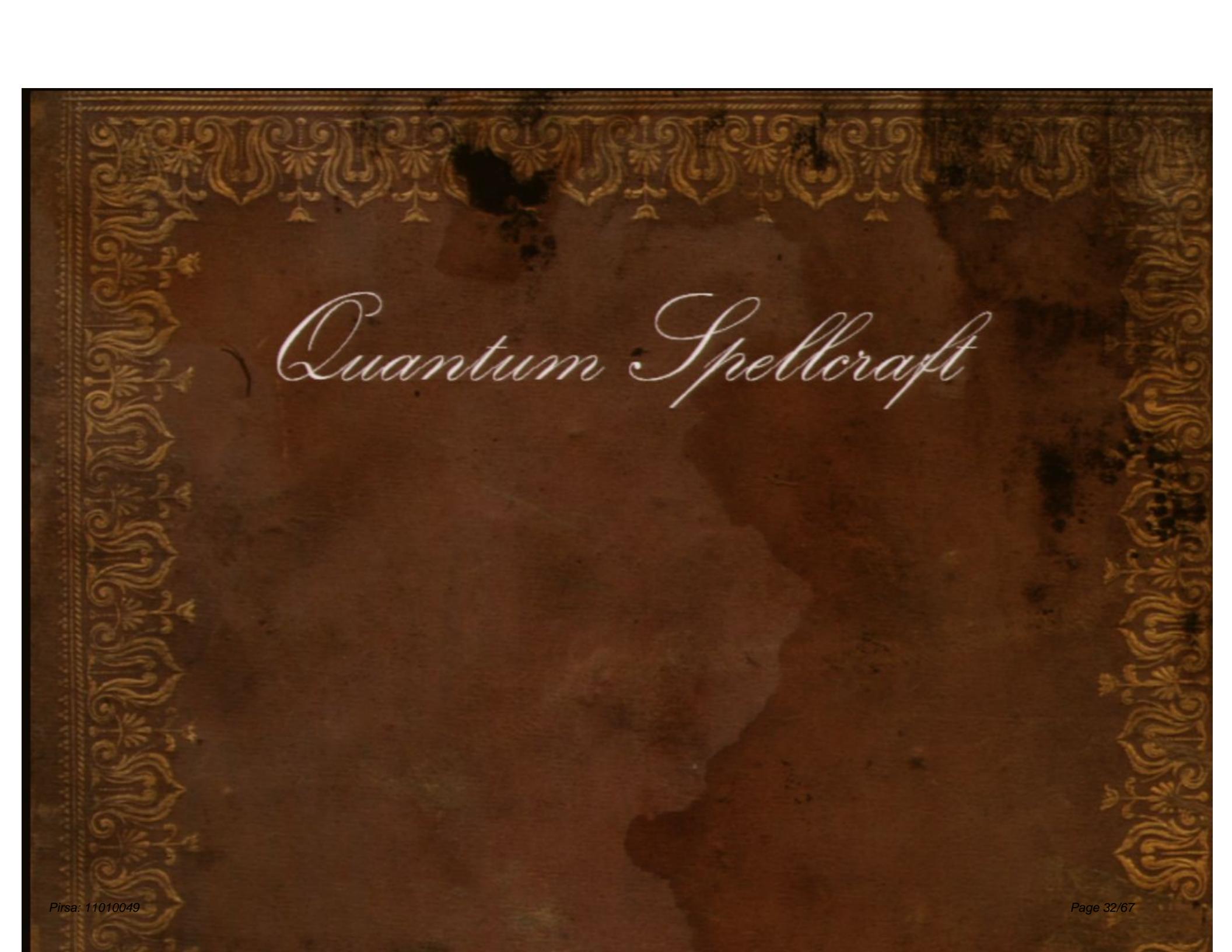
-- Harry Potter entry in wikipedia



**Magic** is a natural force that can be used to override the usual laws of nature.

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**Bell-inequality violations** are natural phenomena that can be used to override the usual (classical-like) laws of nature



*Quantum Spellcraft*

# *Quantum Spellcraft*

*Based on Bell-inequality violation*

**Reduction in communication complexity**

Buhrman, Cleve, van Dam, SIAM J.Comput. 30 1829 (2001)  
Brassard, Found. Phys. 33, 1593 (2003)

**Device-independent secure key distribution**

Barrett, Hardy, Kent, PRL 95, 010503 (2005)

Acin, Gisin, Masanes, PRL. 97, 120405 (2006)

**Enhancing zero-error channel capacity**

Cubitt, Loung, Matthews, Winter arXiv:0911.5300

# Monogamy of Bell-inequality violating correlations



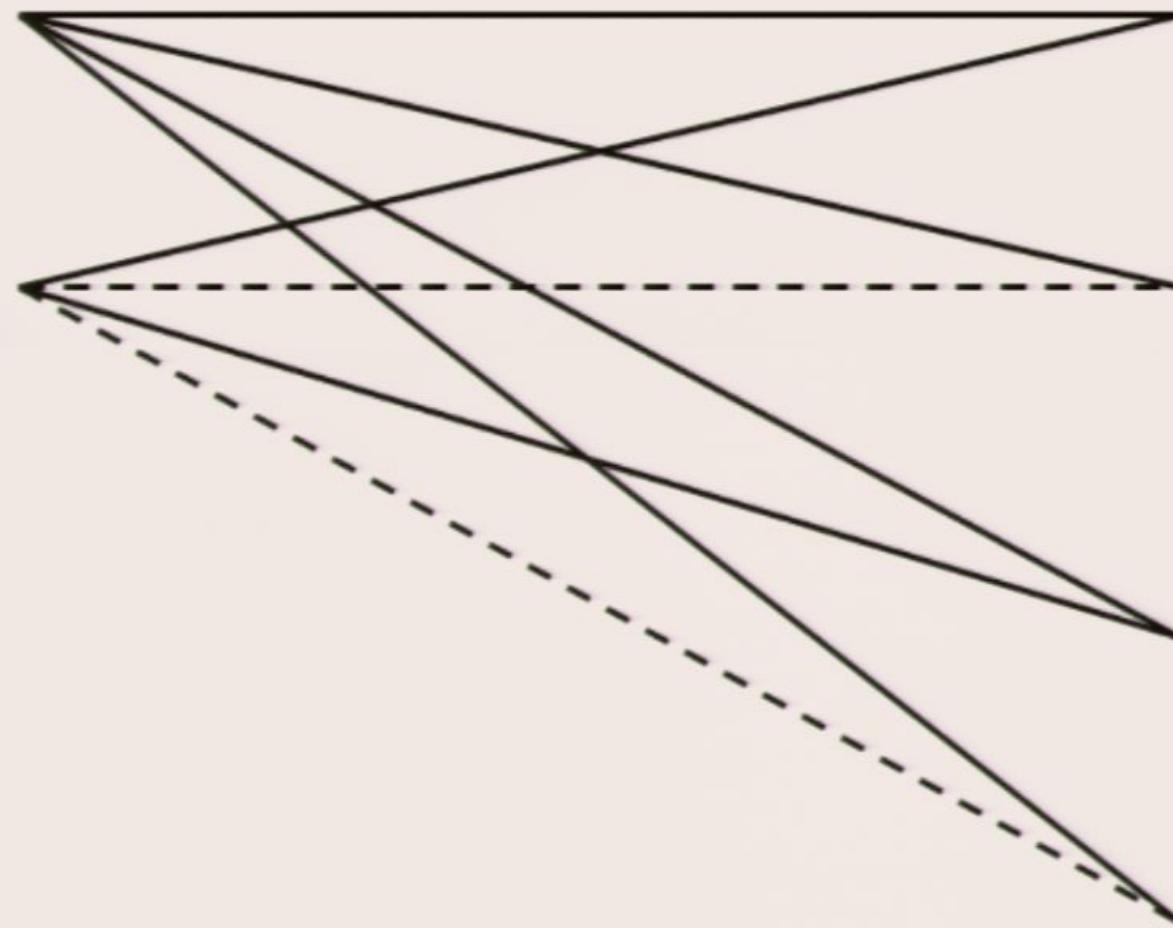
Alice



Bob



Adversary

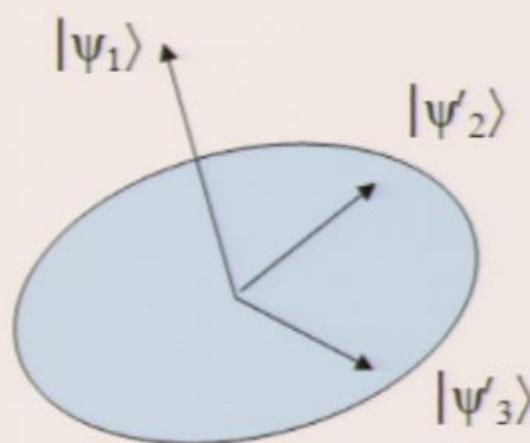
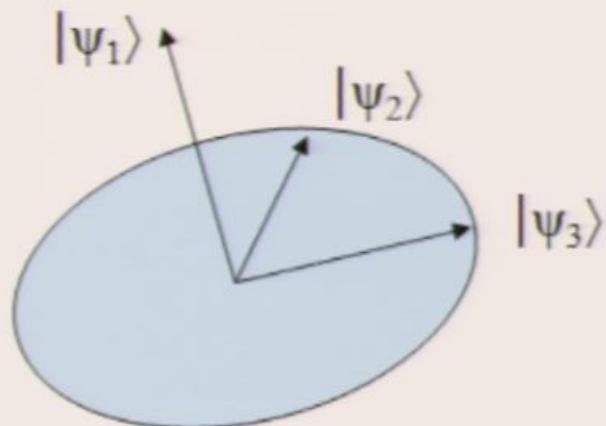


Recent trend in axiomatization:  
Why isn't the world *more* nonlocal?

# The traditional notion of noncontextuality in quantum theory

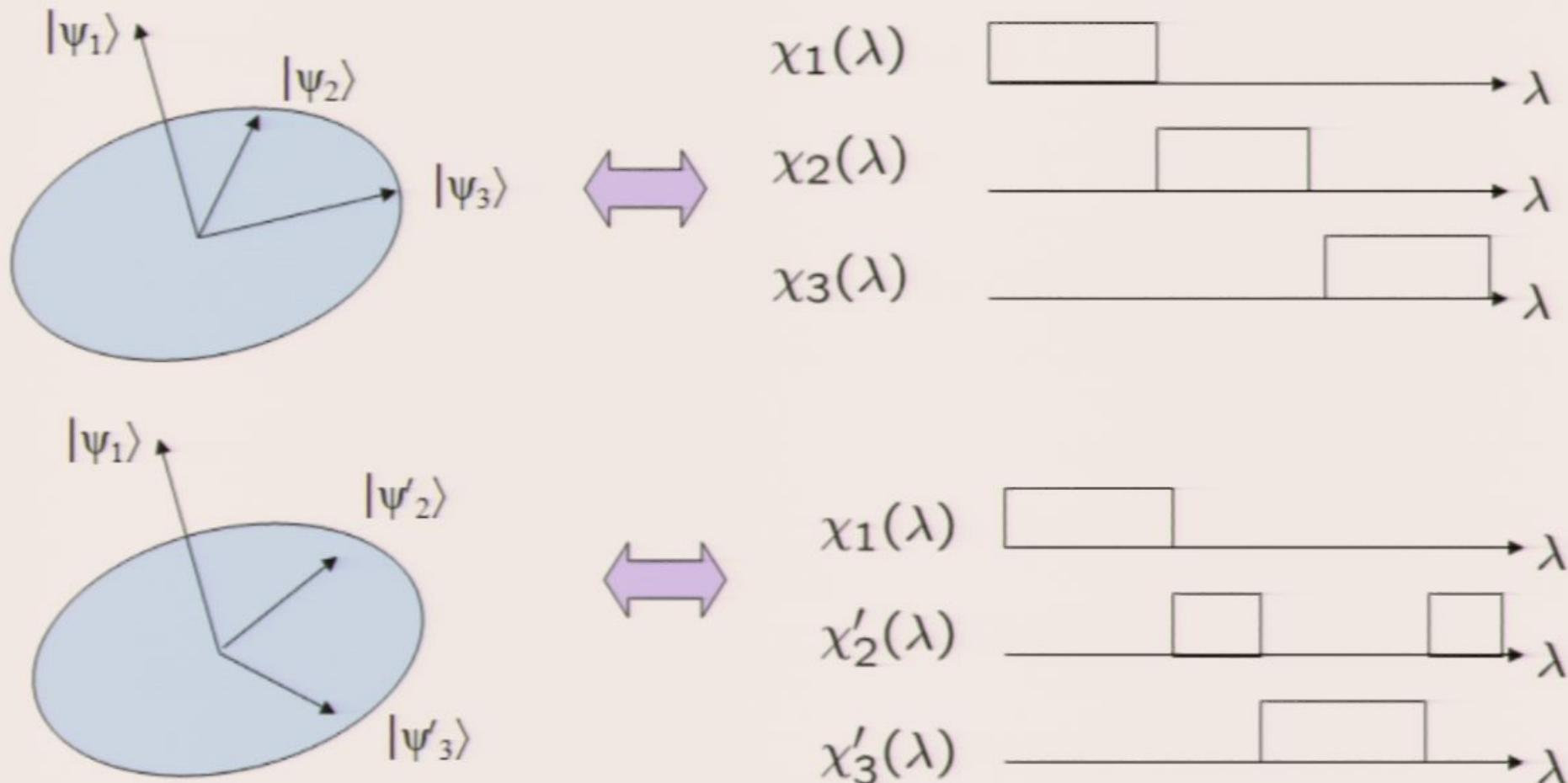
## Traditional notion of noncontextuality

A given vector may appear in many different measurements



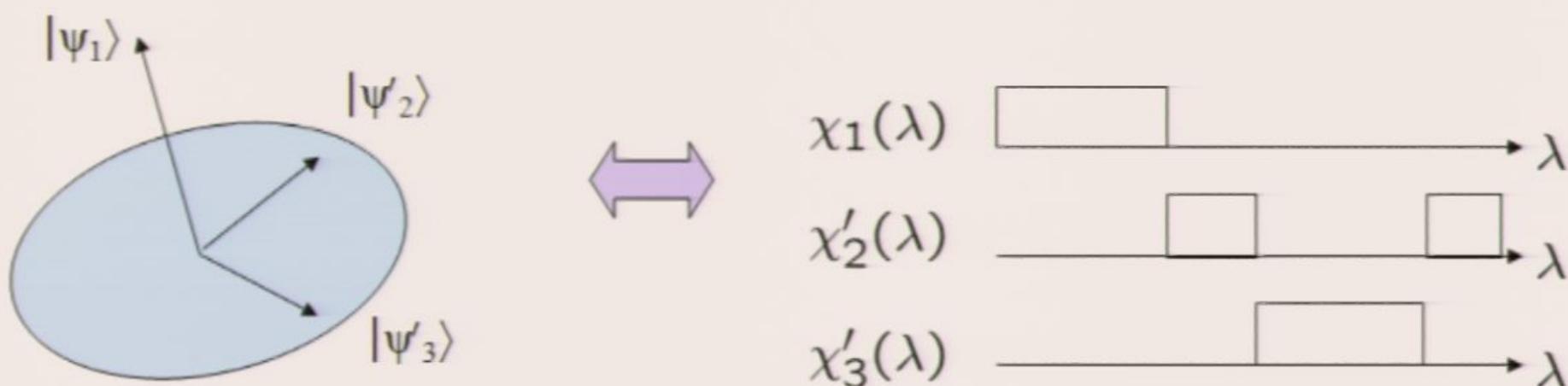
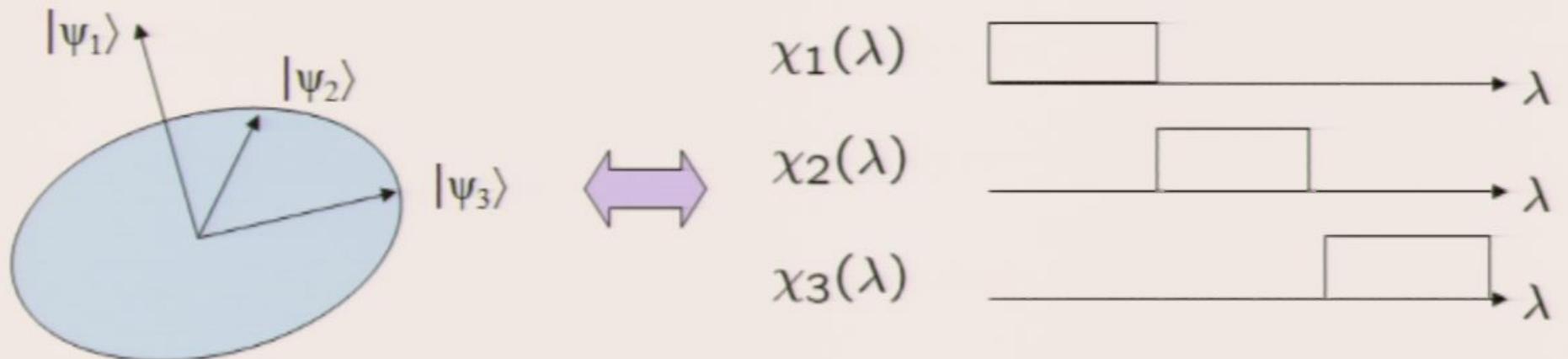
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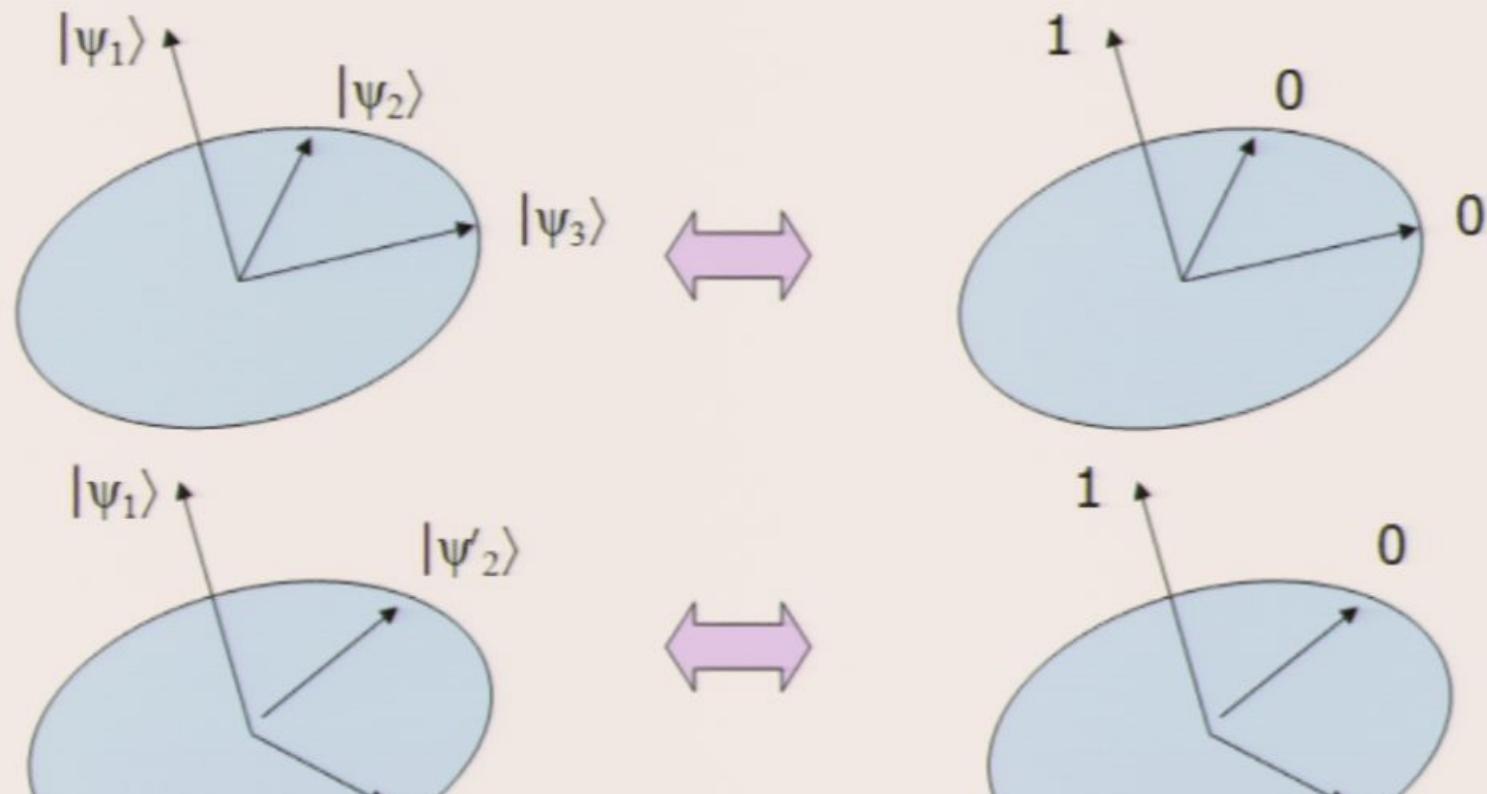
The traditional notion of noncontextuality:

Every vector is associated with the same

$\chi(\lambda)$

### The traditional notion of noncontextuality:

For every  $\lambda$ , every basis of vectors receives a 0-1 valuation, wherein exactly one element is assigned the value 1 (corresponding to the outcome that would occur for  $\lambda$ ), and every vector is assigned the same value regardless of the basis it is considered a part (i.e. **the context**).





John S. Bell

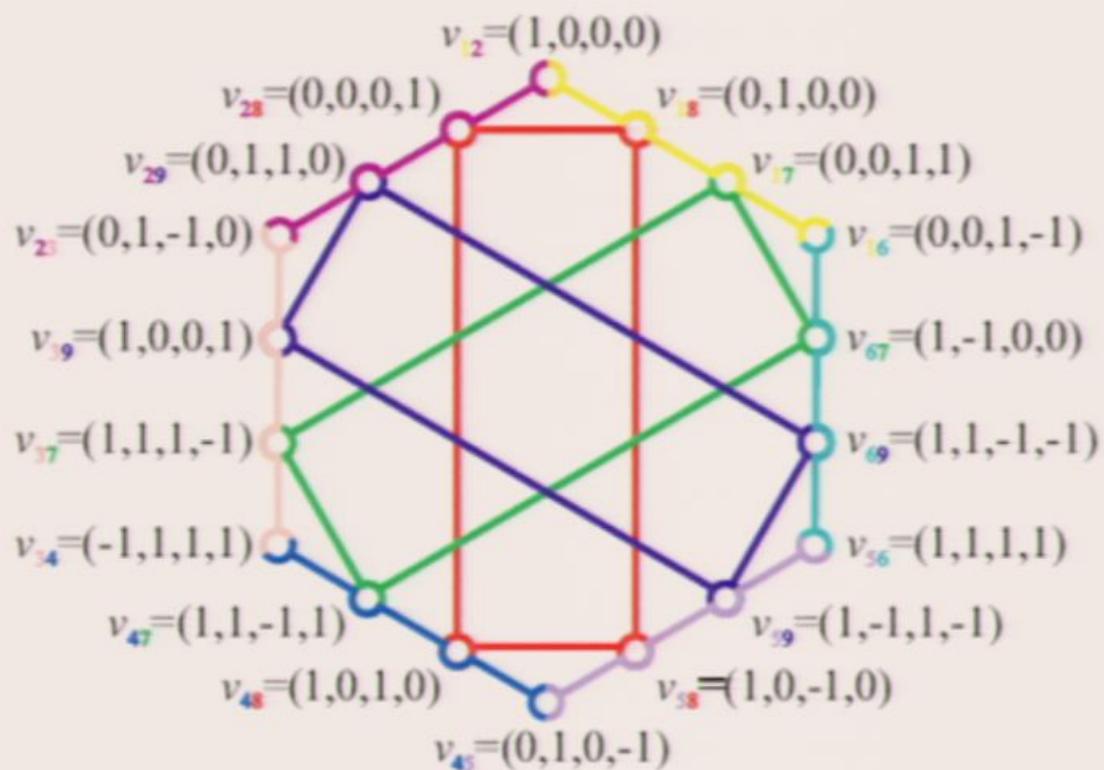


Ernst Specker (with son) and  
Simon Kochen

**Bell-Kochen-Specker theorem:** A noncontextual hidden variable model of quantum theory for Hilbert spaces of dimension 3 or greater is **impossible**.

## Example: The CEGA algebraic 18 ray proof in 4d:

Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)



## Example: The CEGA algebraic 18 ray proof in 4d:

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If we list all 9 orthogonal quadruples, each ray appears twice in the list

0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

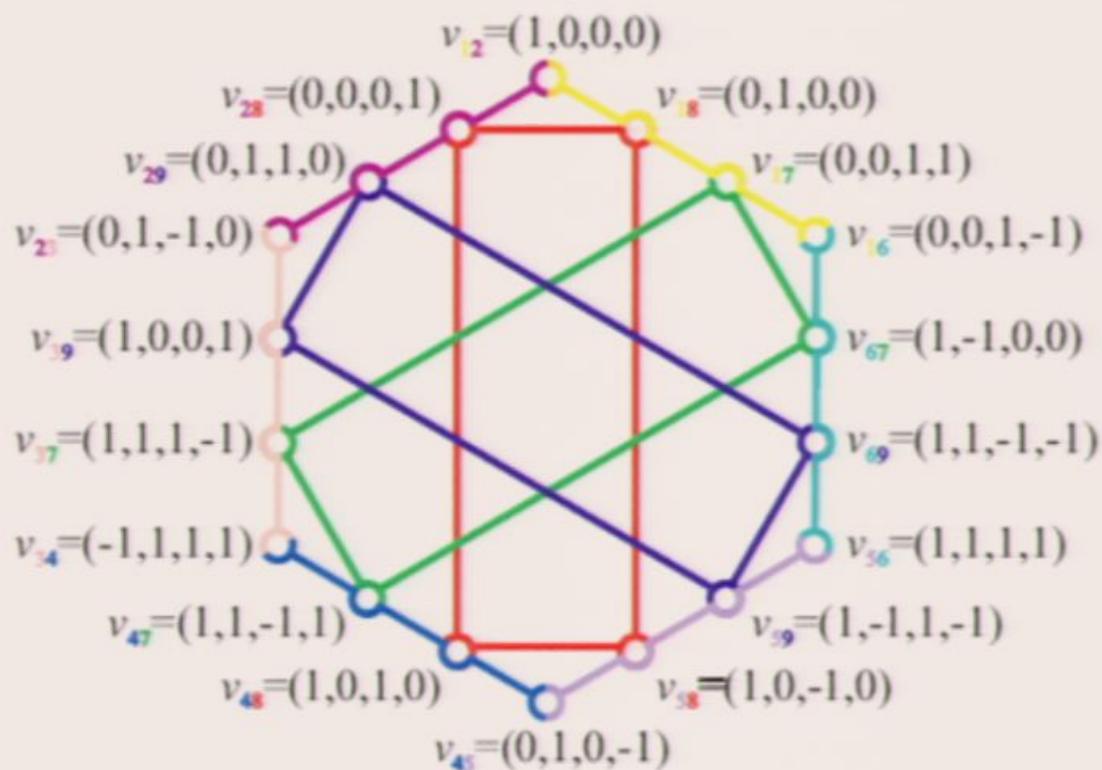
In each of the 9 quadruples, one ray is assigned 1, the other three 0  
Therefore, 9 rays must be assigned 1

But each ray appears twice and so there must be an even number  
of rays assigned 1

**CONTRADICTION!**

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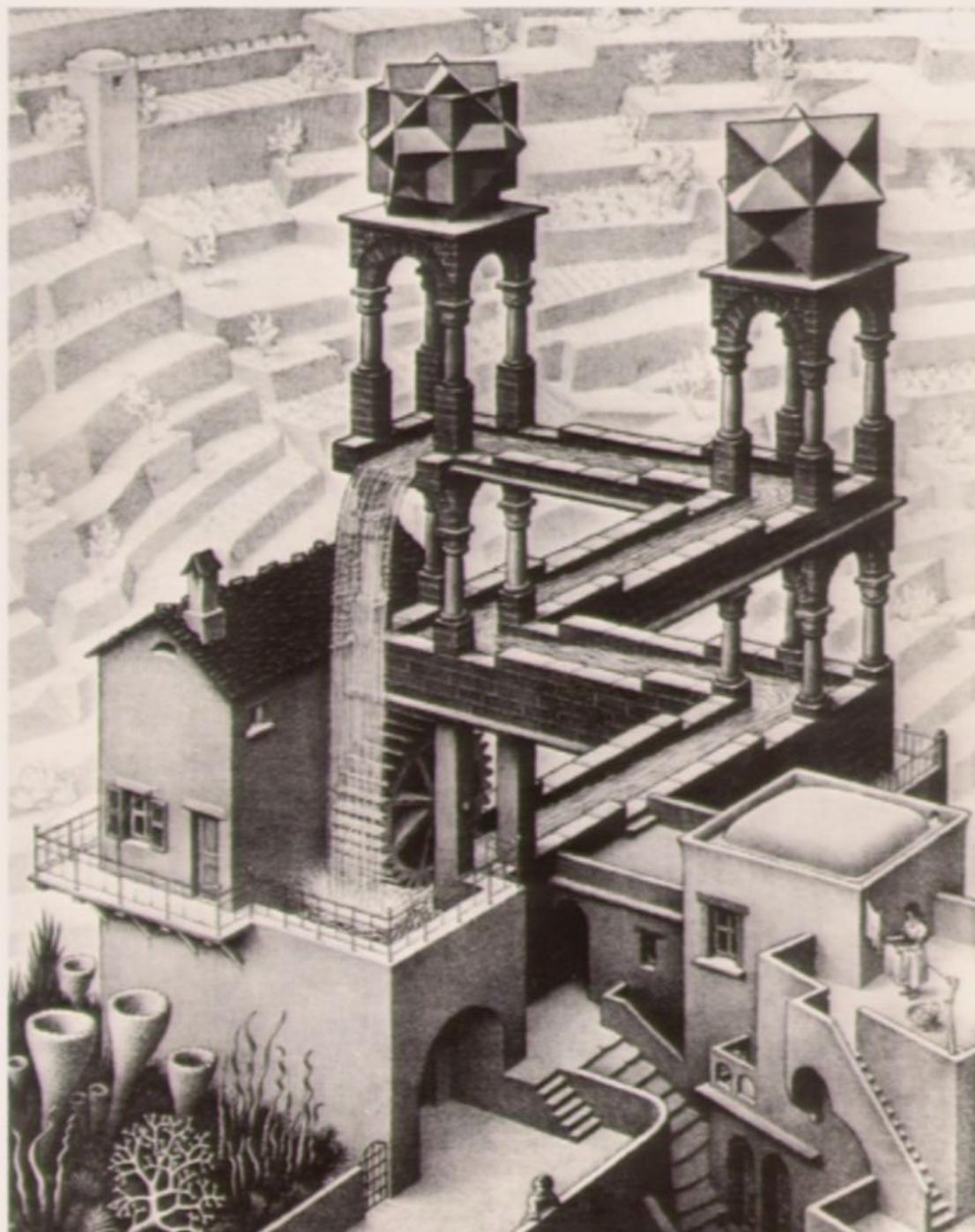
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0,0,0,1	0,0,0,1	1,-1,1,-1	1,-1,1,-1	0,0,1,0	1,-1,-1,1	1,1,-1,1	1,1,-1,1	1,1,1,-1
0,0,1,0	0,1,0,0	1,-1,-1,1	1,1,1,1	0,1,0,0	1,1,1,1	1,1,1,-1	-1,1,1,1	-1,1,1,1
1,1,0,0	1,0,1,0	1,1,0,0	1,0,-1,0	1,0,0,1	1,0,0,-1	1,-1,0,0	1,0,1,0	1,0,0,1
1,-1,0,0	1,0,-1,0	0,0,1,1	0,1,0,-1	1,0,0,-1	0,1,-1,0	0,0,1,1	0,1,0,-1	0,1,-1,0

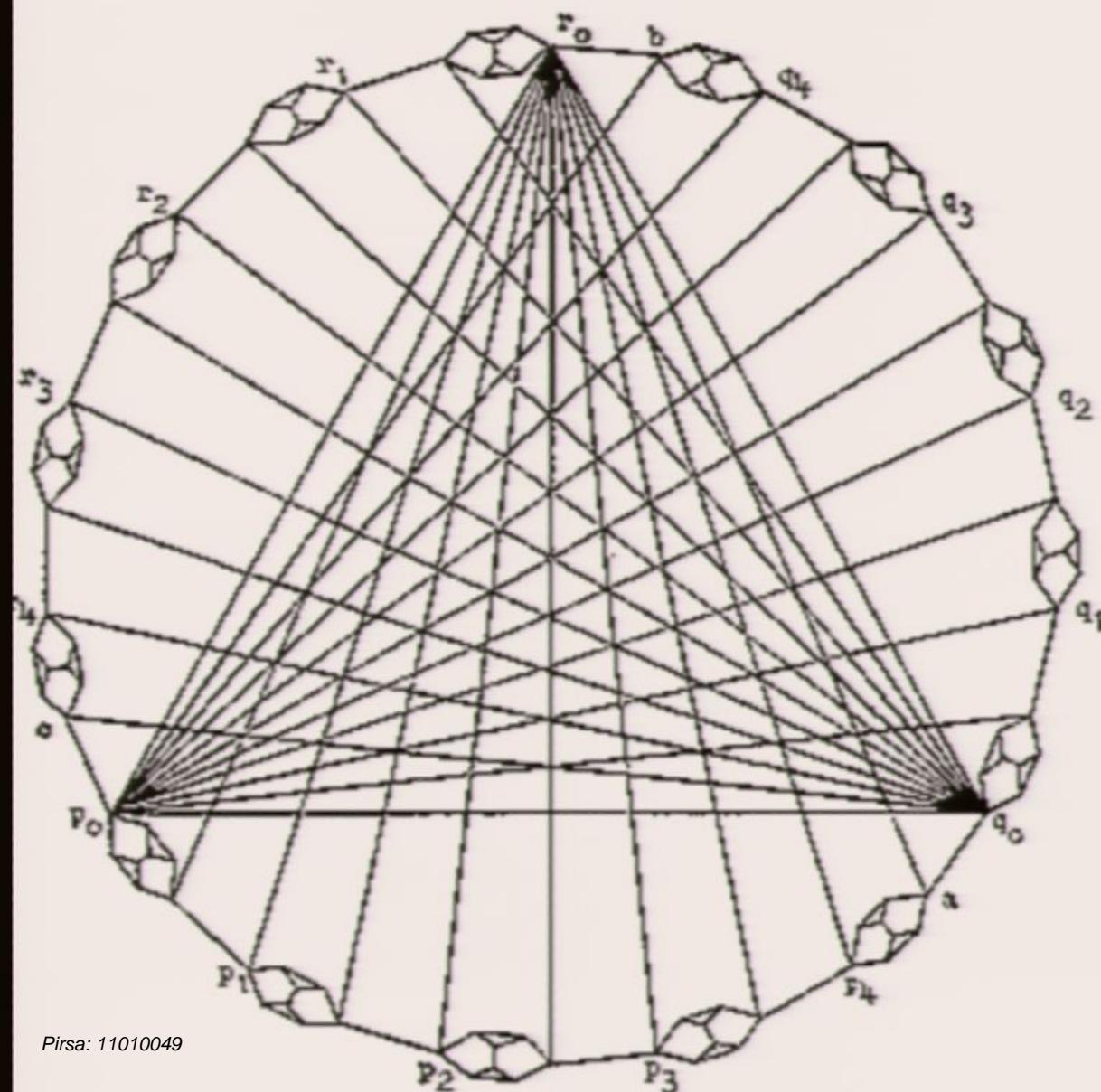
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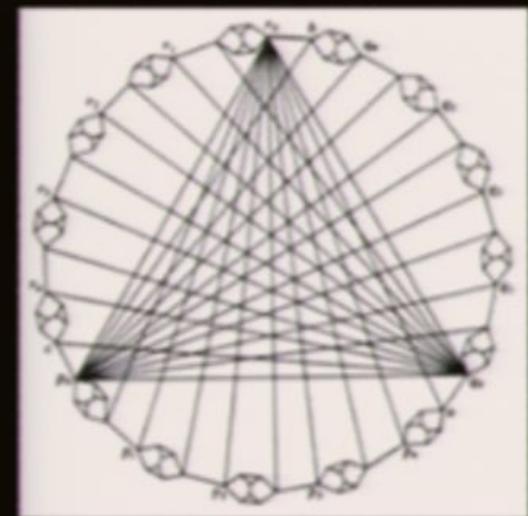


Example: Kochen and Specker's original algebraic 117 ray proof in 3d



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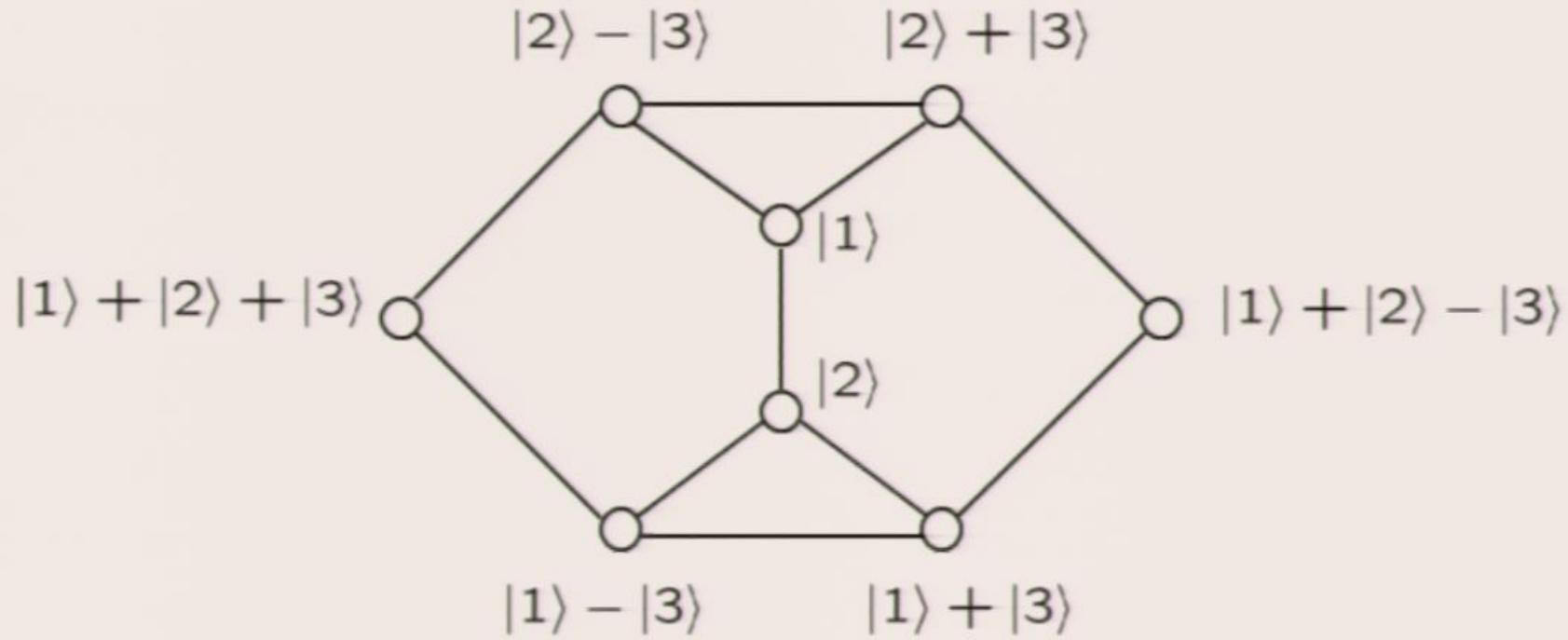
MICHAEL REDHEAD  
**INCOMPLETENESS  
NONLOCALITY  
AND REALISM**  
A Prolegomenon to the Philosophy of  
Quantum Mechanics



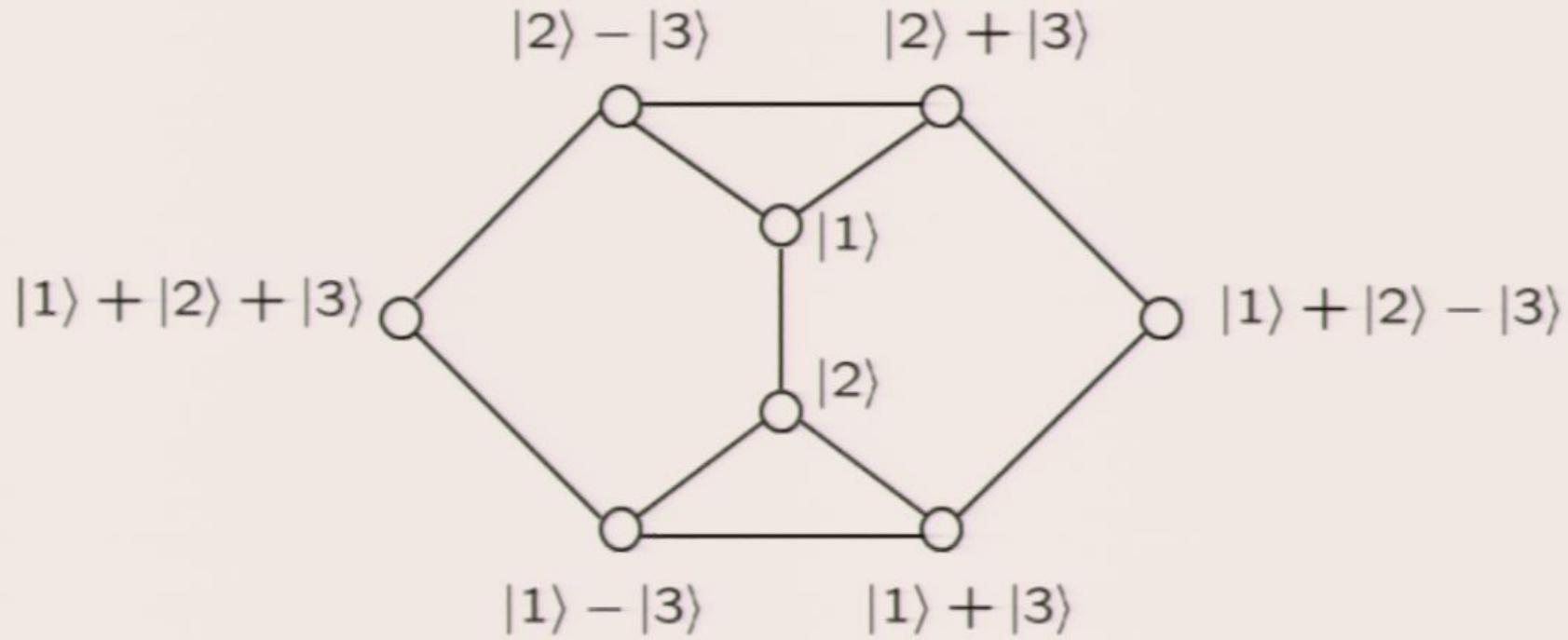
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## Example: Clifton's state-specific 8 ray proof in 3d

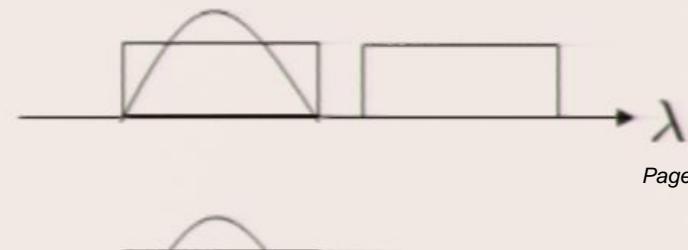


## Example: Clifton's state-specific 8 ray proof in 3d



$|1\rangle + |2\rangle + |3\rangle$

$|1\rangle + |2\rangle - |3\rangle$



The traditional notion of noncontextuality:

For every  $\lambda$ , every projector  $\Pi$  is assigned a value 0 or 1 regardless of how it is measured (i.e. **the context**)

$$v(\Pi) = 0 \text{ or } 1 \quad \text{for all } \Pi$$

Coarse-graining of a measurement implies a coarse-graining of the value (because it is just post-processing)

$$v(\sum_k \Pi_k) = \sum_k v(\Pi_k)$$

Every measurement has *some* outcome

$$v(I) = 1$$

The traditional notion of noncontextuality:

For Hermitian operators A, B, C satisfying

$$[A, B] = 0 \quad [A, C] = 0 \quad [B, C] \neq 0$$

the value assigned to A should be independent of whether it is measured together with B or together with C (i.e. **the context**)

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Measure A = measure projectors onto eigenspaces of A,  $\{\Pi_a\}$

$$A = \sum_a a \Pi_a \quad \rightarrow \quad v(A) = \sum_a a v(\Pi_a)$$

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$$A = \sum_a a \Pi_a \quad \rightarrow \quad v(A) = \sum_a a v(\Pi_a)$$

Measure A with B

= measure projectors onto joint eigenspaces of A and B,  $\{\Pi_{ab}\}$

then coarse-grain over B outcome  $\Pi_a = \sum_b \Pi_{ab}$

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Then coarse-grain over C outcome  $\Pi_a = \sum_b \Pi_{ac}$

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Functional relationships among commuting Hermitian operators must be respected by their values

If  $f(L, M, N, \dots) = 0$

then  $f(v(L), v(M), v(N), \dots) = 0$

Functional relationships among commuting Hermitian operators must be respected by their values

$$\text{If } f(L, M, N, \dots) = 0$$

$$\text{then } f(v(L), v(M), v(N), \dots) = 0$$

Proof: the possible sets of eigenvalues one can simultaneously assign to L, M, N,... are specified by their joint eigenstates. By acting the first equation on each of the joint eigenstates, we get the second.

## Example: Mermin's magic square proof in 4d

$X_1$	$X_2$	$X_1X_2$
$Y_2$	$Y_1$	$Y_1Y_2$
$X_1Y_2$	$Y_1X_2$	$Z_1Z_2$

$I$

$I$

$I$

$I$

$I$

$-I$

$$X_1 X_2 (X_1 X_2) = I$$

$$Y_1 Y_2 (Y_1 Y_2) = I$$

$$(X_1 Y_2) (Y_1 X_2) (Z_1 Z_2) = I$$

$$X_1 Y_2 (X_1 Y_2) = I$$

$$Y_1 X_2 (Y_1 X_2) = I$$

$$(X_1 X_2) (Y_1 Y_2) (Z_1 Z_2) = -I$$

## Example: Mermin's magic square proof in 4d

$X_1$	$X_2$	$X_1X_2$
$Y_2$	$Y_1$	$Y_1Y_2$
$X_1Y_2$	$Y_1X_2$	$Z_1Z_2$

 $I$  $I$  $I$  $I$  $I$  $-I$ 

$$X_1 X_2 (X_1 X_2) = I$$

$$Y_1 Y_2 (Y_1 Y_2) = I$$

$$(X_1 Y_2) (Y_1 X_2) (Z_1 Z_2) = I$$

$$X_1 Y_2 (X_1 Y_2) = I$$

$$Y_1 X_2 (Y_1 X_2) = I$$

$$(X_1 X_2) (Y_1 Y_2) (Z_1 Z_2) = -I$$

$$v(X_1) v(X_2) v(X_1 X_2) = 1$$

$$v(Y_1) v(Y_2) v(Y_1 Y_2) = 1$$

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## Example: Mermin's magic square proof in 4d

$X_1$	$X_2$	$X_1X_2$
$Y_2$	$Y_1$	$Y_1Y_2$
$X_1Y_2$	$Y_1X_2$	$Z_1Z_2$

 $I$  $I$  $I$  $I \quad I \quad -I$ 

$$X_1 X_2 (X_1 X_2) = I$$

$$Y_1 Y_2 (Y_1 Y_2) = I$$

$$(X_1 Y_2) (Y_1 X_2) (Z_1 Z_2) = I$$

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$$(X_1 X_2) (Y_1 Y_2) (Z_1 Z_2) = -I$$

$$v(X_1) v(X_2) v(X_1 X_2) = 1$$

$$v(Y_1) v(Y_2) v(Y_1 Y_2) = 1$$

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$$v(X_1) v(Y_2) v(X_1 Y_2) = 1$$

$$v(Y_1) v(X_2) v(Y_1 X_2) = 1$$

Product of LHSs = +1

Product of RHSs = -1

**CONTRADICTION**

Aside: Local determinism is an instance of traditional noncontextuality where the context is remote

$S_a^A \otimes I^B$  is either measured with  $I^A \otimes S_b^B$   
or with  $I^A \otimes S_{b'}^B$

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Recall traditional noncontextuality:

For Hermitian operators A, B, C satisfying

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the value assigned to A should be independent of whether it is measured together with B or together with C (i.e. **the context**)

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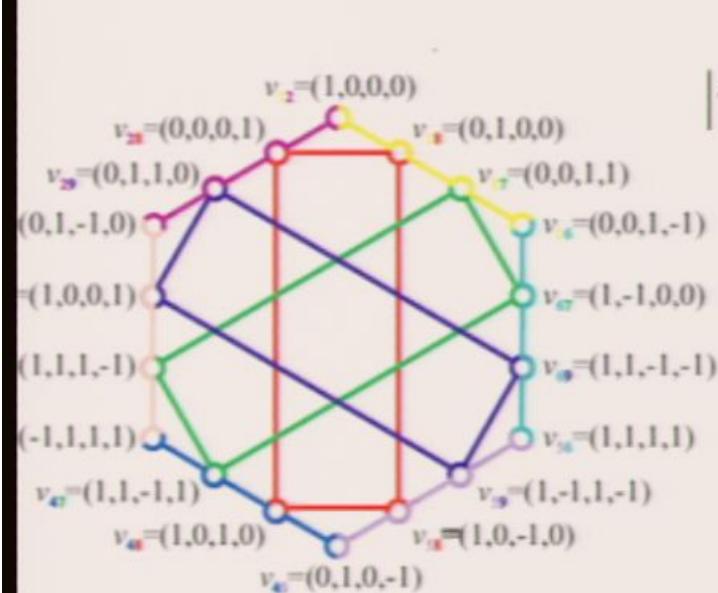
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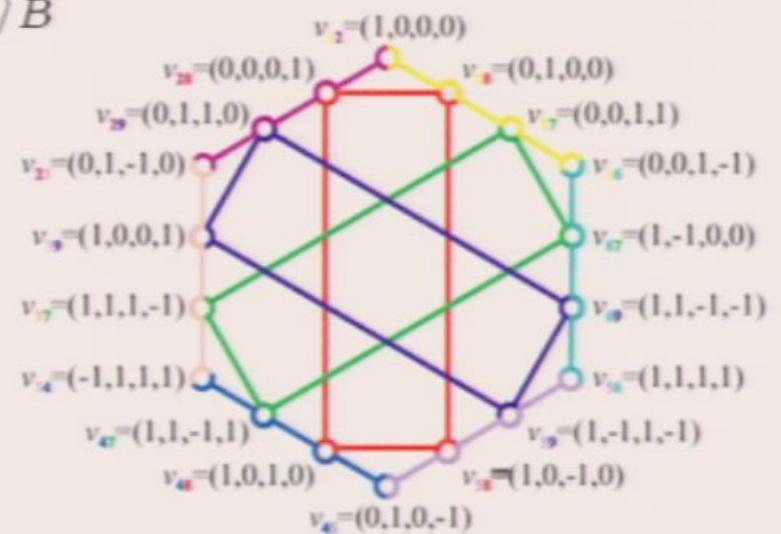
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## Aside: Traditional noncontextuality can sometimes be justified by local causality

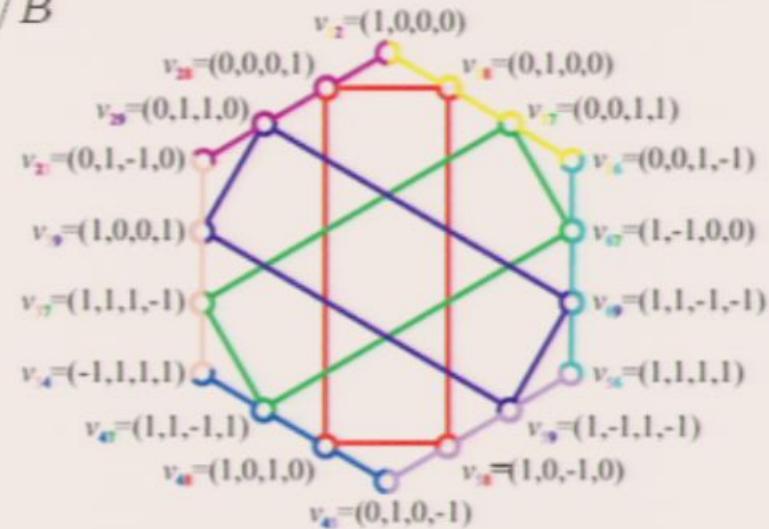
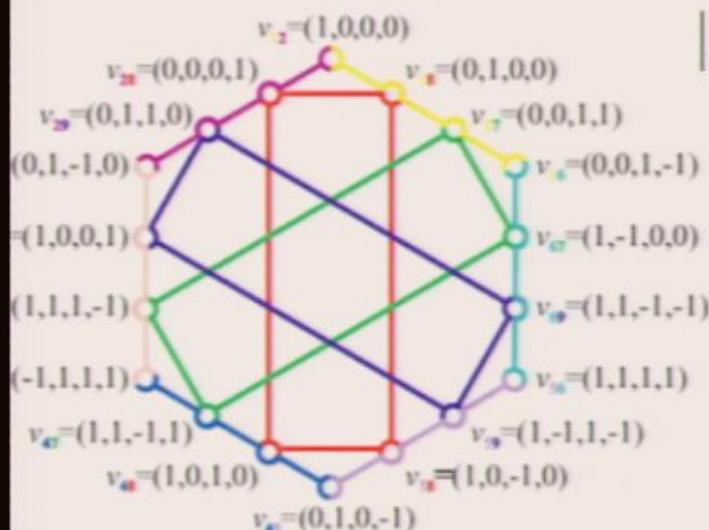


$$|\psi\rangle_{AB} = \frac{1}{2} \sum_{i=1}^4 |i\rangle_A |i\rangle_B$$



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Perfect correlation when same mmt is made on both wings  
+ local causality

→ Traditional noncontextual hidden variable model for mmnts on one wing

# The generalized notion of noncontextuality