

Title: Foundations of Quantum Mechanics - Lecture 8

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URL: <http://pirsa.org/11010047>

Abstract:



perimeter scholars
INTERNATIONAL

Classical complementarity as an epistemic restriction

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A fact about operational quantum theory:

Jointly-measurable observables = a commuting set of observables
(relative to matrix commutator)

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Jointly-measurable observables = a commuting set of observables
(relative to matrix commutator)

This suggests a restriction on a classical statistical theory:

Jointly-knowable variables = a commuting set of variables
(relative to Poisson bracket)

Continuous degrees of freedom

Configuration space: $\mathbb{R}^n \ni (x_1, x_2, \dots, x_n)$

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$$[F, G](m) \equiv \sum_{i=1}^n (a_i d_i - b_i c_i) \quad \text{Independent of } m$$

Discrete degrees of freedom $\mathbb{Z}_d = \{0, 1, \dots, d-1\}$

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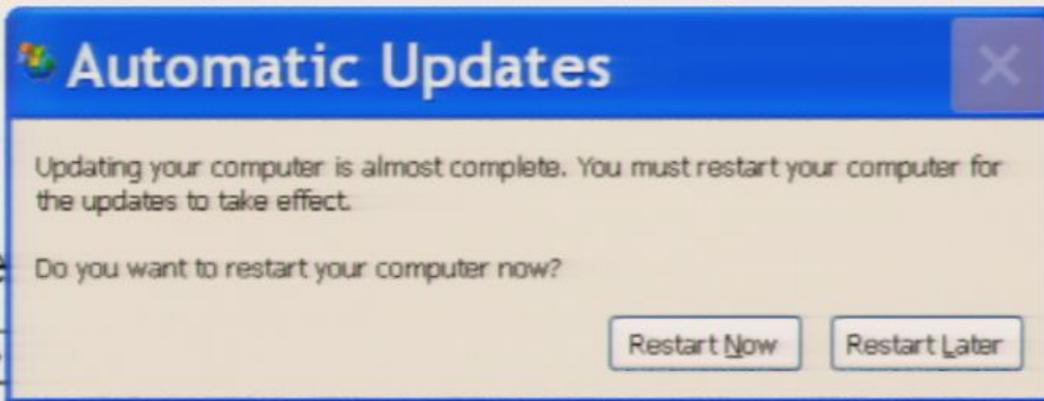
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Poisson brackets

$$[F, G](m) \equiv \sum_i \left(-\left(F[m + e_{p_i}] - F[m] \right) \left(G[m + e_{q_i}] - G[m] \right) \right)$$

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$$[F, G](m) = \sum_{i=1}^n [F(e_{x_i})G(e_{p_i}) - F(e_{p_i})G(e_{x_i})]$$

$$= \sum_{i=1}^n (a_i d_i - b_i c_i)$$

The principle of classical complementarity:

The only statistical distributions that can be prepared correspond to knowing the values of a commuting set of variables (relative to Poisson bracket) and that have maximal entropy otherwise.

Restricted statistical theory of bits (a.k.a. “the toy theory”)

A single bit

X	1		
	0		
		0	1
		P	

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod 2}$$

$$X, P, X + P$$

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Canonical variables

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Statistical distributions

X known

X	1		
	0		
		0	1
		<i>P</i>	

P known

X	1		
	0		
		0	1
		<i>P</i>	

X + *P* known

X	1		
	0		
		0	1
		<i>P</i>	

Nothing known

X	1		
	0		
		0	1
		<i>P</i>	

Convex combination

$$\begin{aligned}
 X \begin{array}{c} 1 \\ 0 \end{array} \begin{array}{|c|c|} \hline \blacksquare & \blacksquare \\ \hline \blacksquare & \blacksquare \\ \hline \end{array} &= X \begin{array}{c} 1 \\ 0 \end{array} \begin{array}{|c|c|} \hline \square & \square \\ \hline \blacksquare & \blacksquare \\ \hline \end{array} +_{\text{cx}} X \begin{array}{c} 1 \\ 0 \end{array} \begin{array}{|c|c|} \hline \blacksquare & \blacksquare \\ \hline \square & \square \\ \hline \end{array} & \frac{1}{2} I = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1| \\
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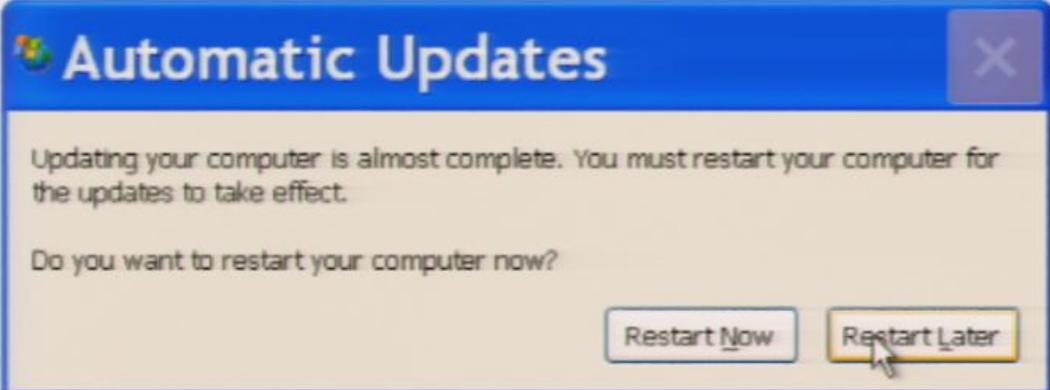
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States of non-maximal knowledge are **mixed**
 States of maximal knowledge are **pure**

There is a multiplicity of decompositions
 of mixed states into pure states

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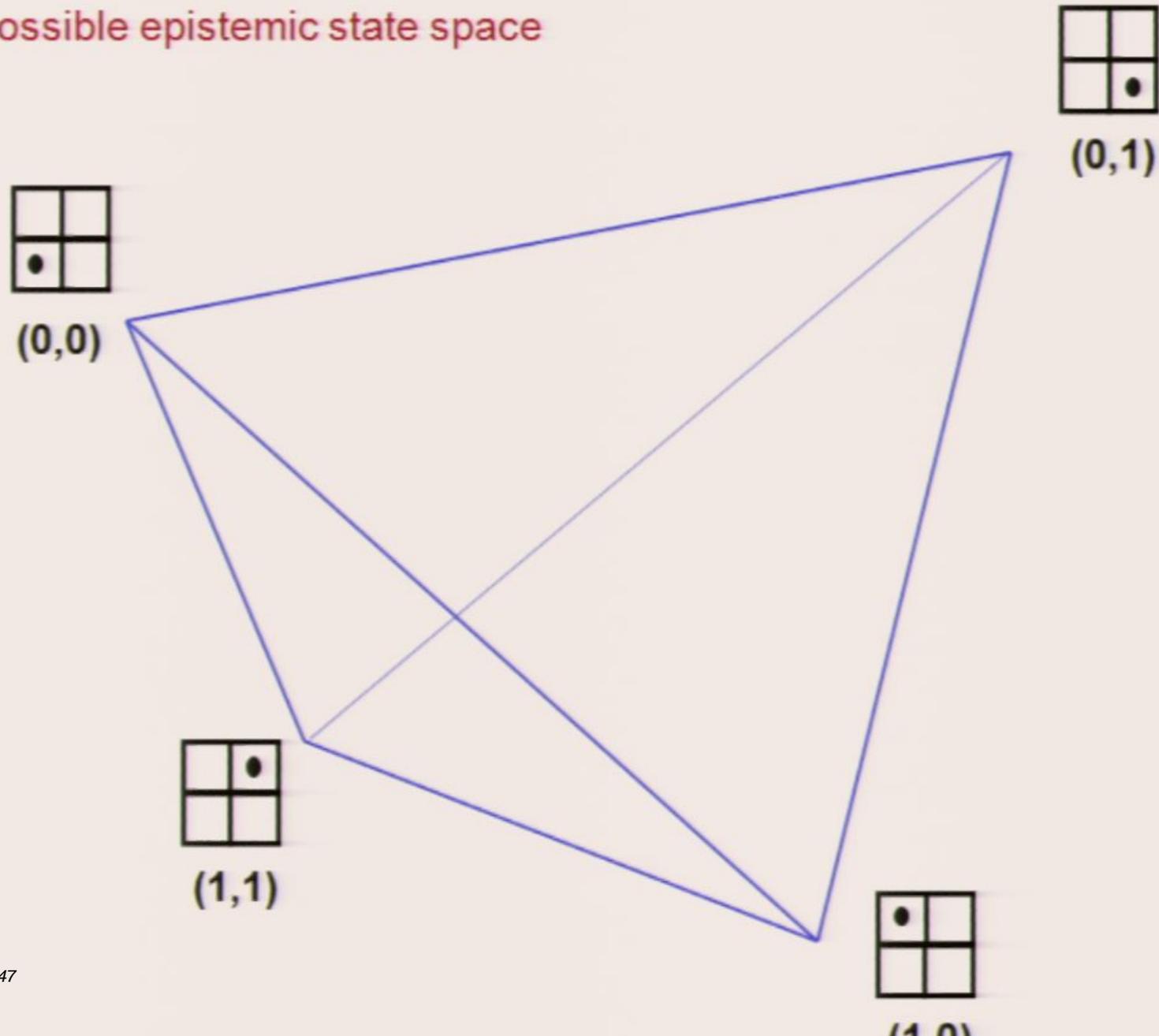
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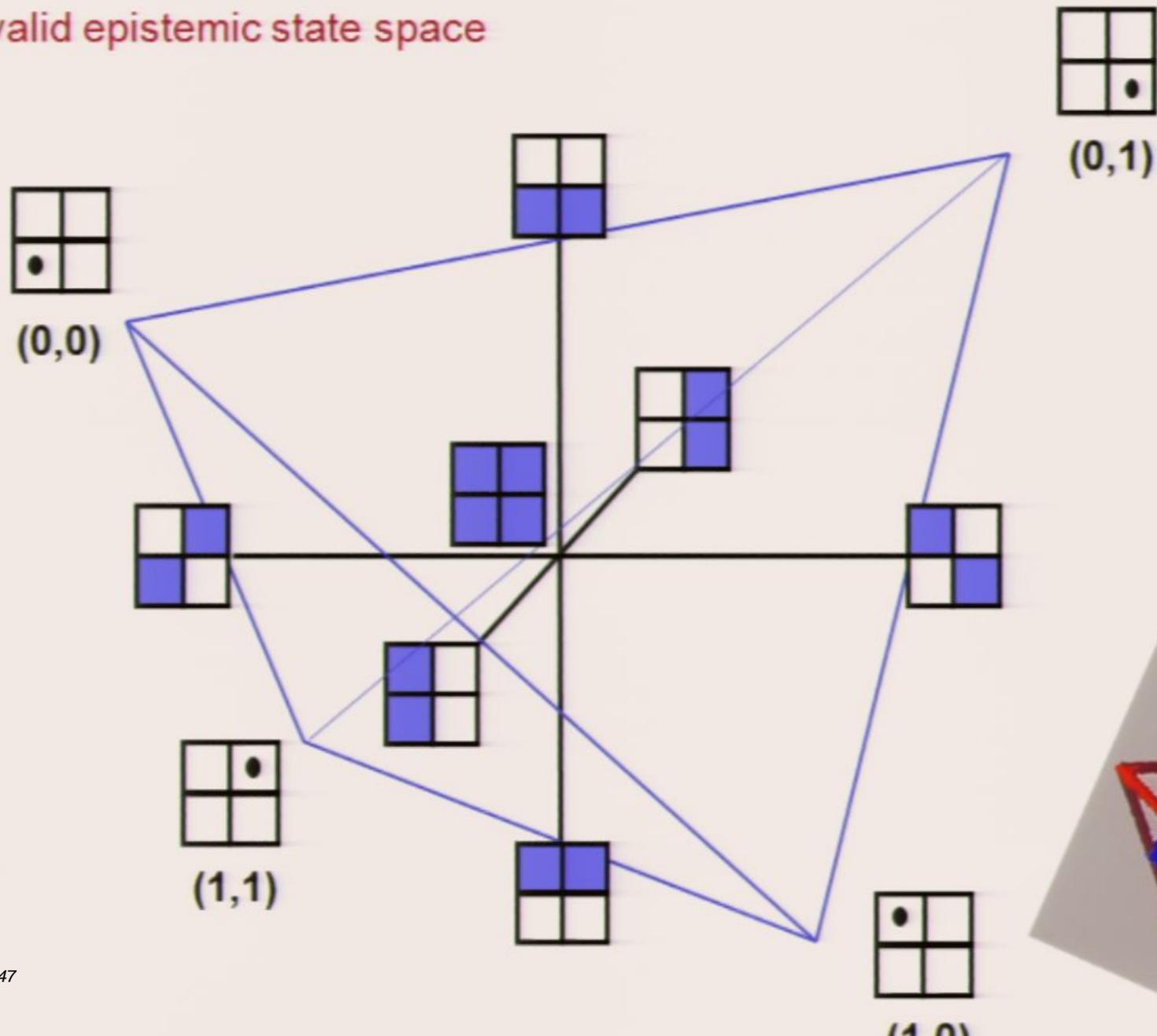
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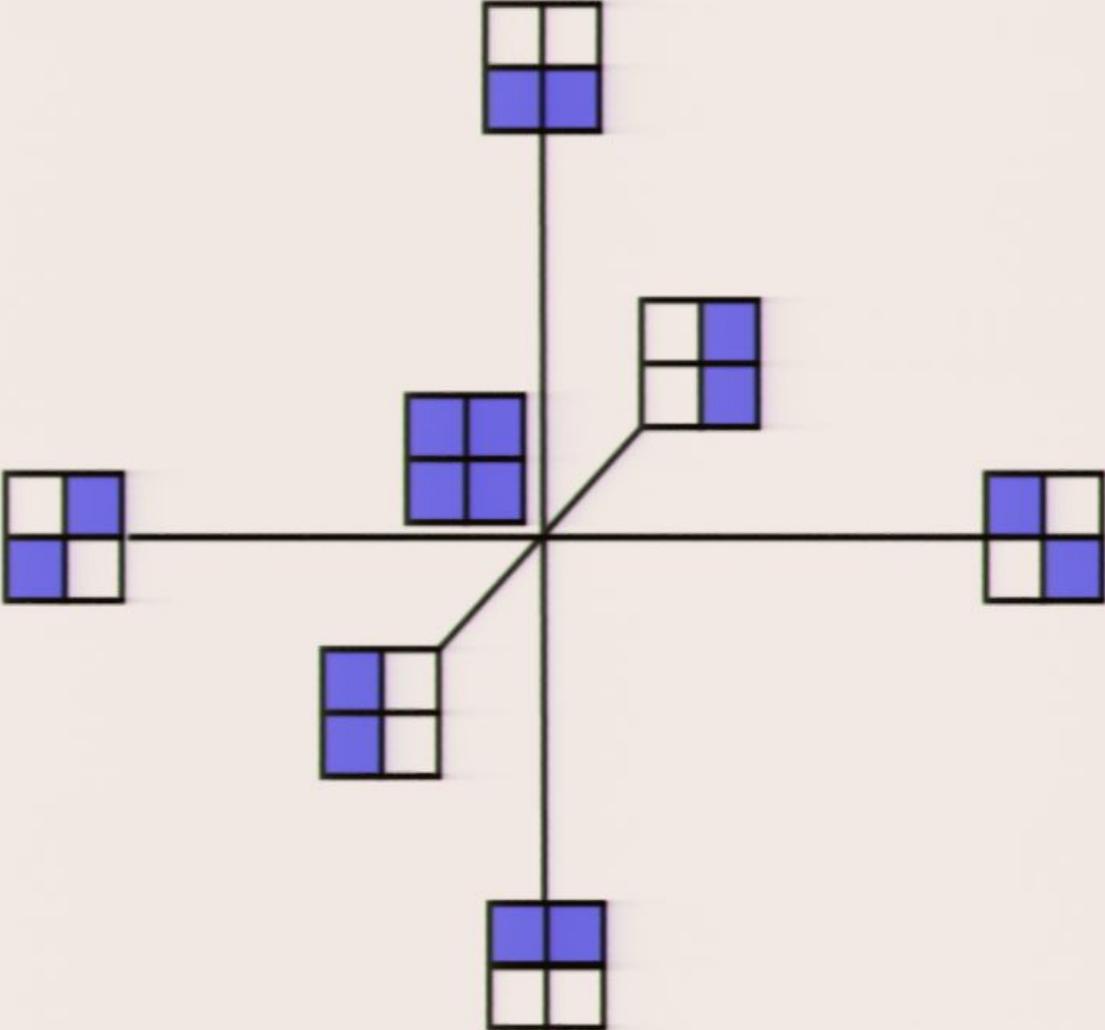
The possible epistemic state space



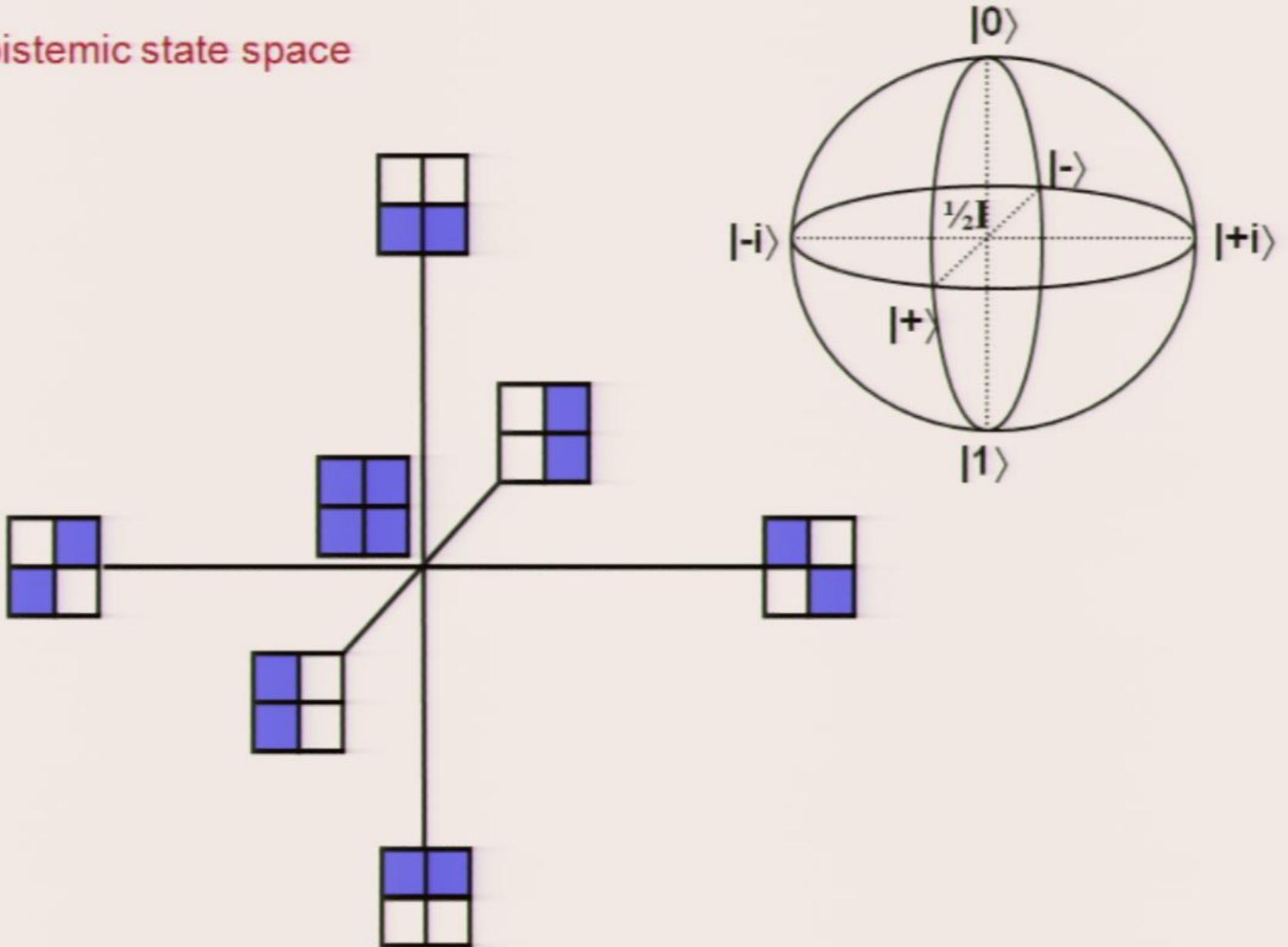
The valid epistemic state space



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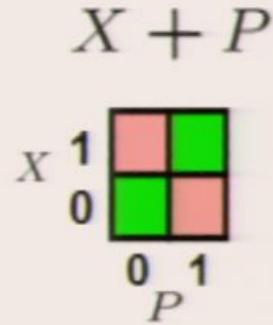
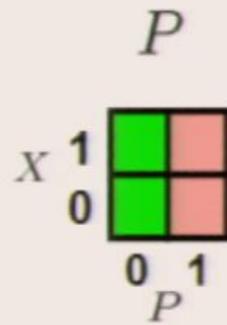
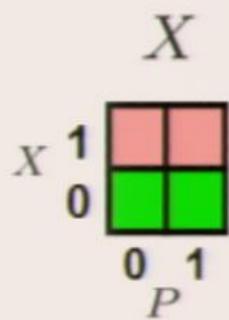


The valid epistemic state space



Valid reproducible measurements:

Any commuting set of canonical variables



Valid reversible transformations:

Those that preserve the Poisson bracket / symplectic inner product:

The **group of symplectic affine transformations (Clifford group)**

for $m \in \Omega$

$$m \mapsto Sm + a$$

where $[Su, Sv] = [u, v]$ **Symplectic**

and $a \in \Omega$ **Affine (Heisenberg-Weyl)**

→ All 24 permutations of the four ontic states

$$[(x_1, p_1), (x_2, p_2)] = x_1 p_2 - p_1 x_2$$

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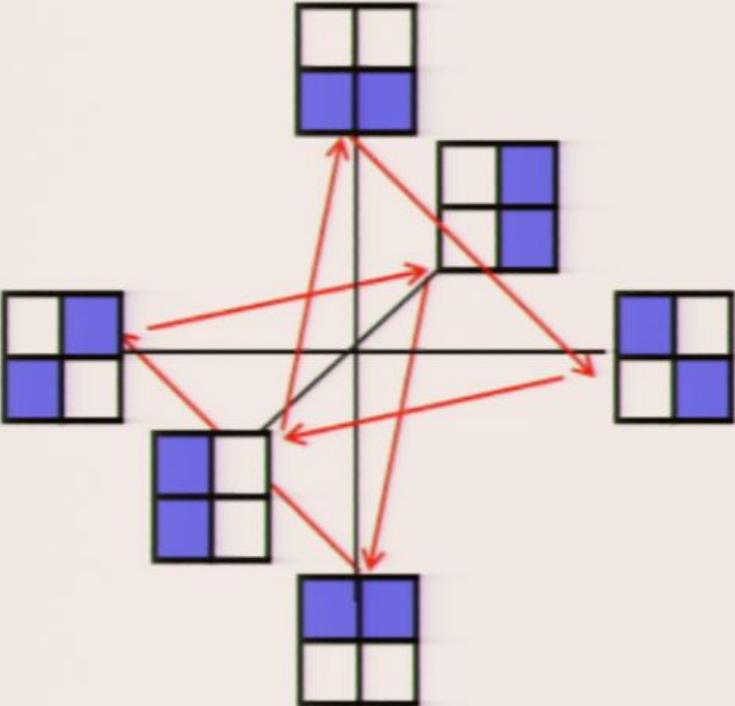
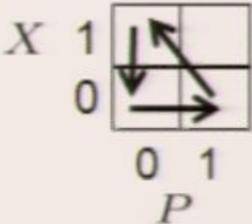
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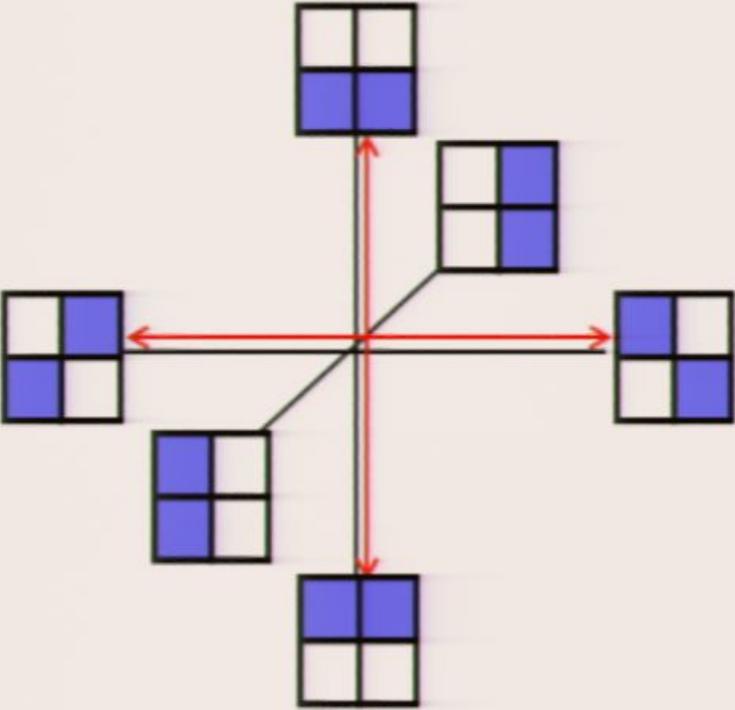
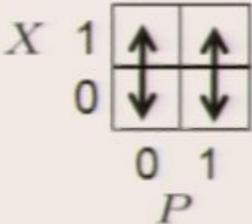
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A 3-cycle



A pair of 2-cycles

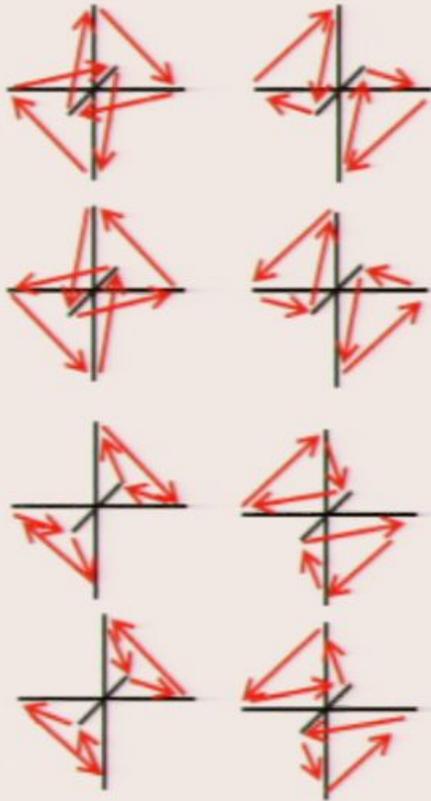


Reversible transformations:

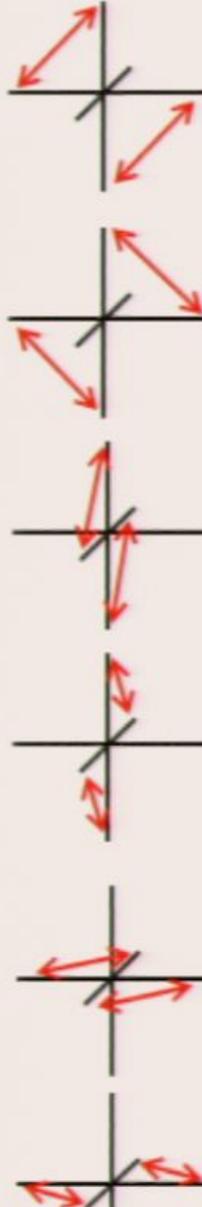
Pairs of 2-cycles



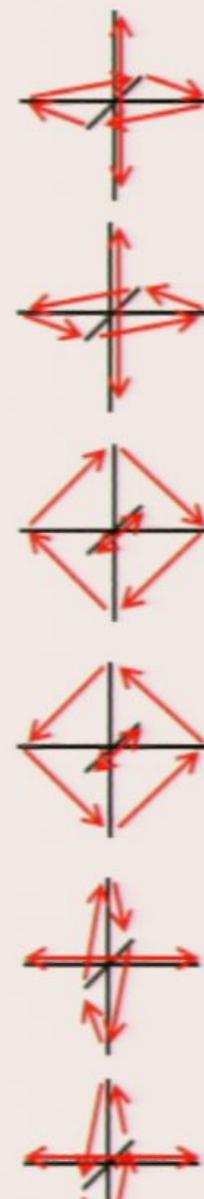
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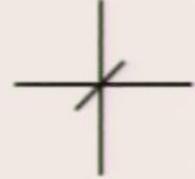
2-cycles



4-cycles

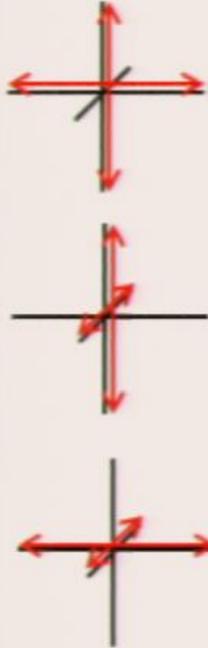


identity

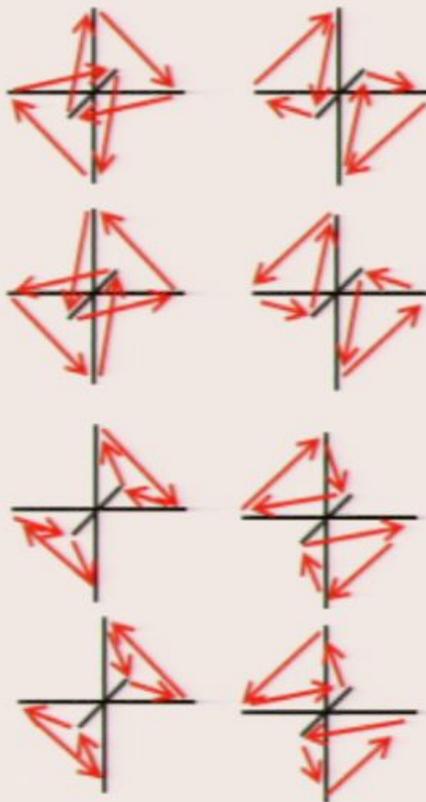


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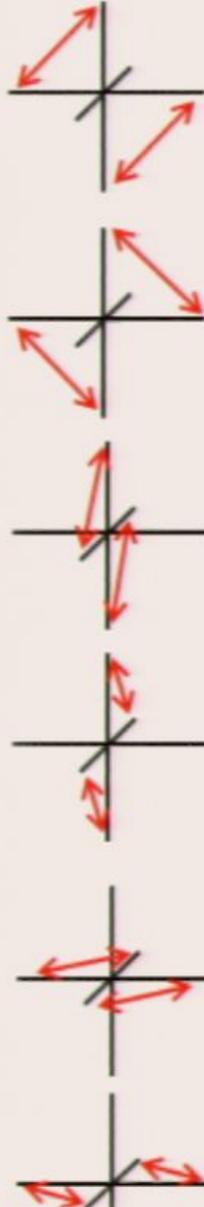
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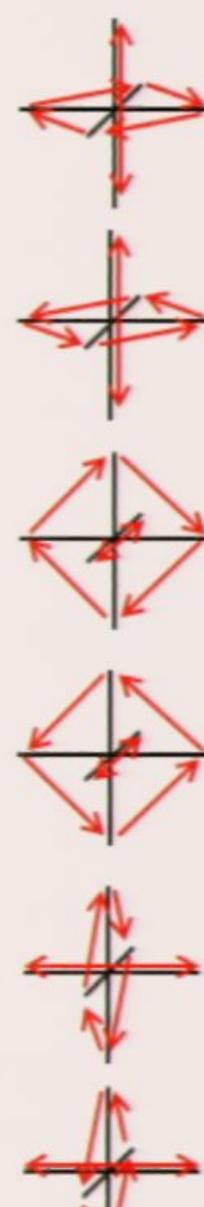
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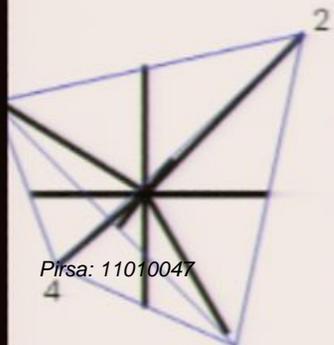
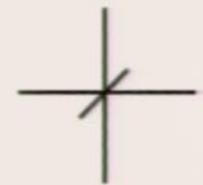
2-cycles



4-cycles



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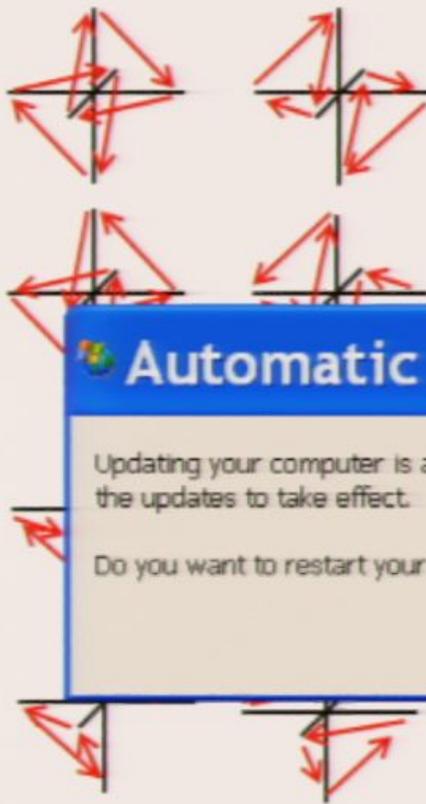
Symmetries of the tetrahedron under rotations and reflections

Reversible transformations:

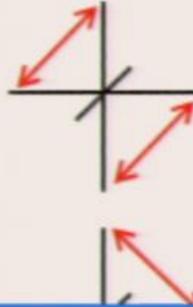
Pairs of 2-cycles



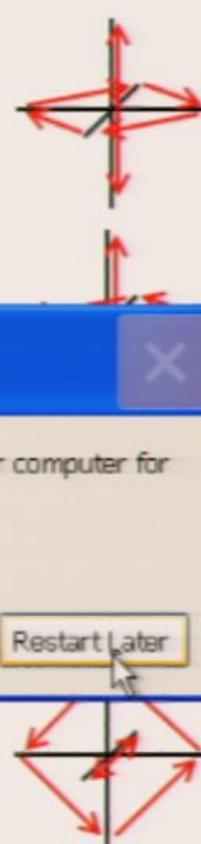
3-cycles



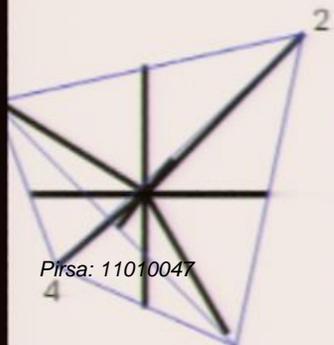
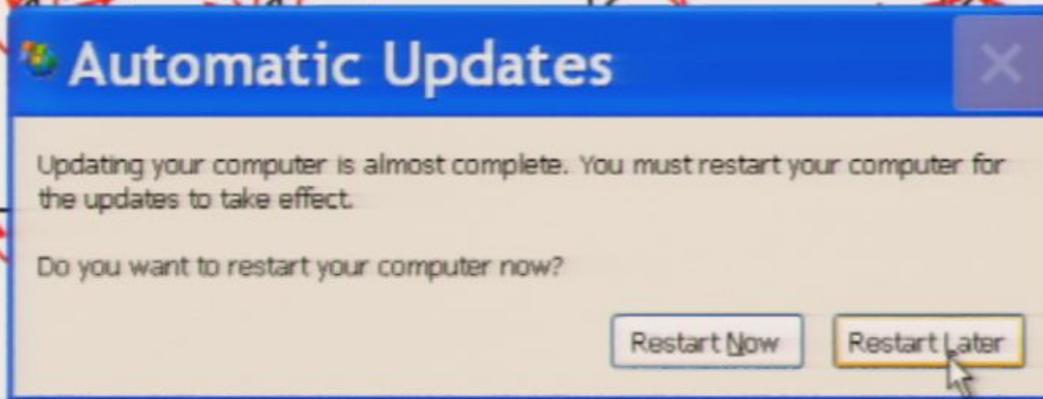
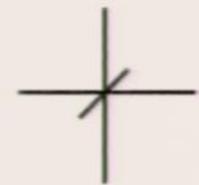
2-cycles



4-cycles



identity



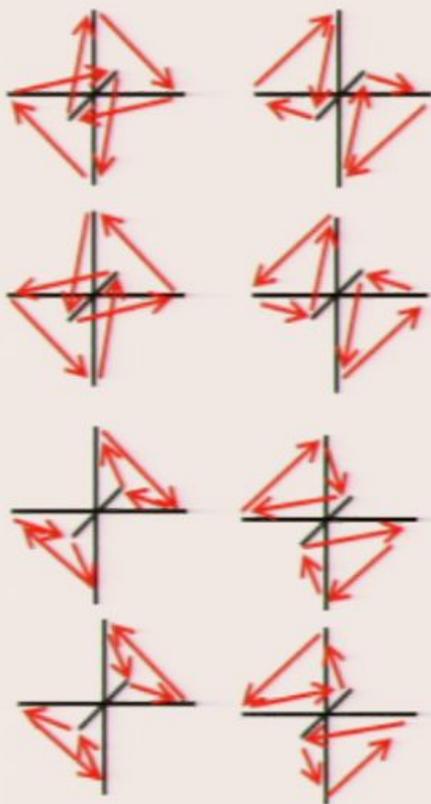
Symmetries of the tetrahedron under rotations and reflections

Reversible transformations:

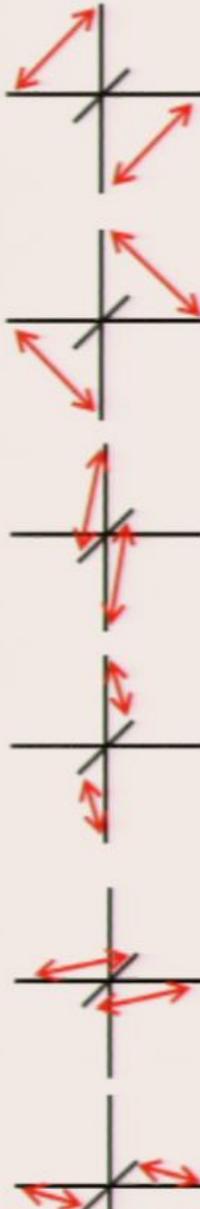
Pairs of 2-cycles



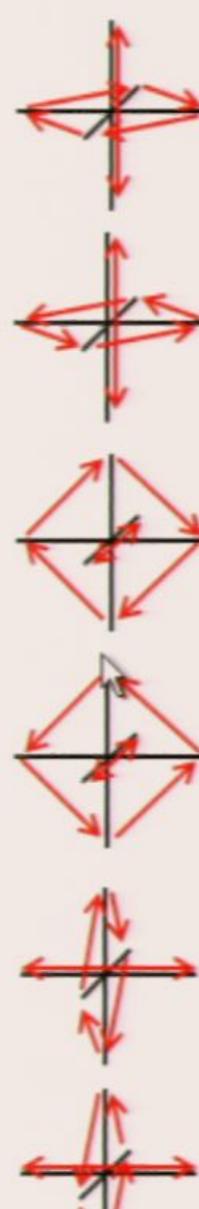
3-cycles



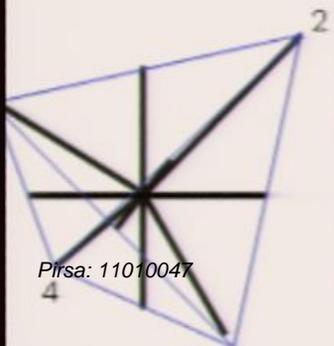
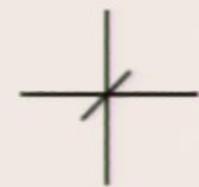
2-cycles



4-cycles



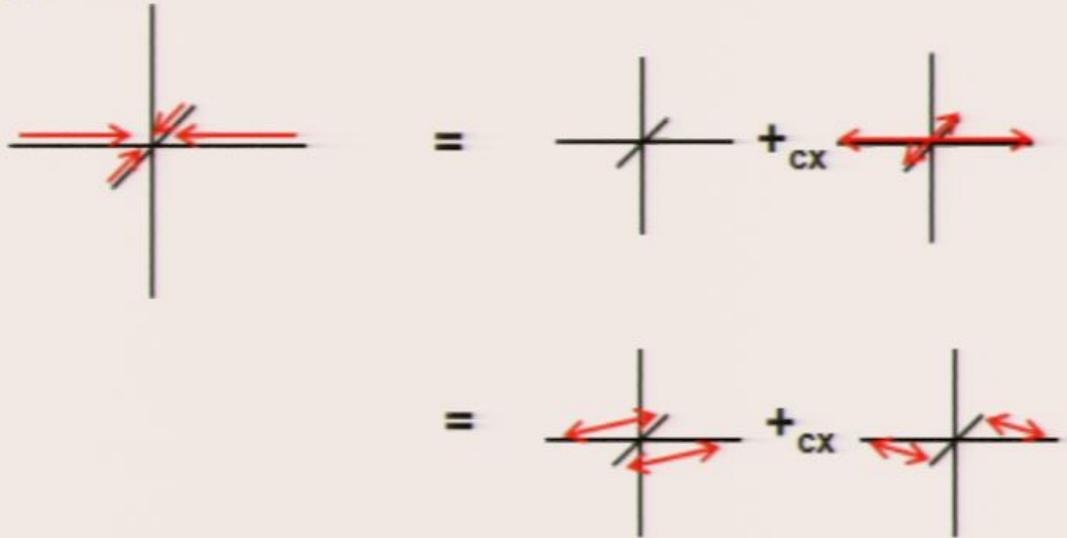
identity



Symmetries of the tetrahedron under rotations and reflections

Irreversible transformations:

Example:



Updating the epistemic state after a reproducible measurement

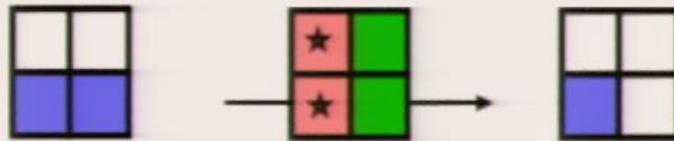
Suppose no disturbance



But this is too much knowledge!

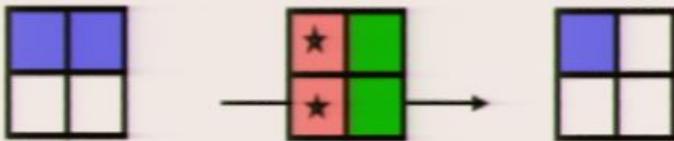
Updating the epistemic state after a reproducible measurement

Suppose no disturbance



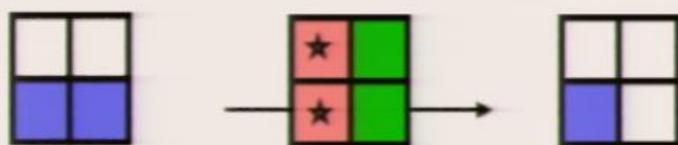
But this is too much knowledge!

Similarly,



Updating the epistemic state after a reproducible measurement

Suppose no disturbance

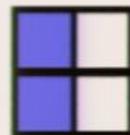


But this is too much knowledge!

Similarly,

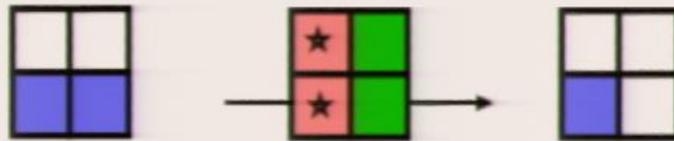


Reproducibility implies a final epistemic state



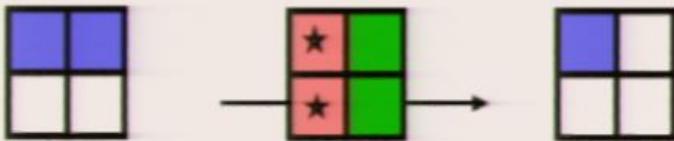
Updating the epistemic state after a reproducible measurement

Suppose no disturbance

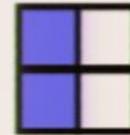


But this is too much knowledge!

Similarly,



Reproducibility implies a final epistemic state



So we must assume that



or



occurs

Updating the epistemic state after a reproducible measurement

Suppose no disturbance

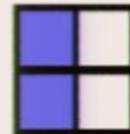


But this is too much knowledge!

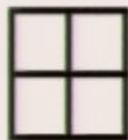
Similarly,



Reproducibility implies a final epistemic state



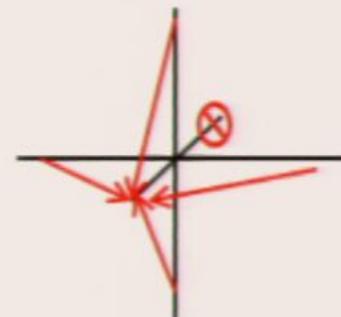
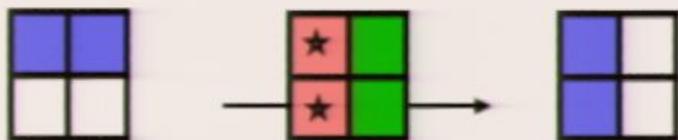
So we must assume that



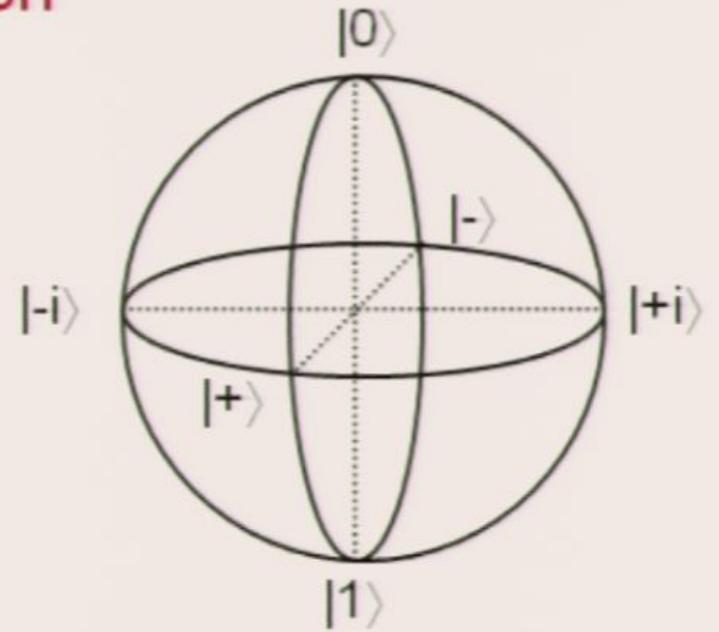
or



occurs



Coherent superposition



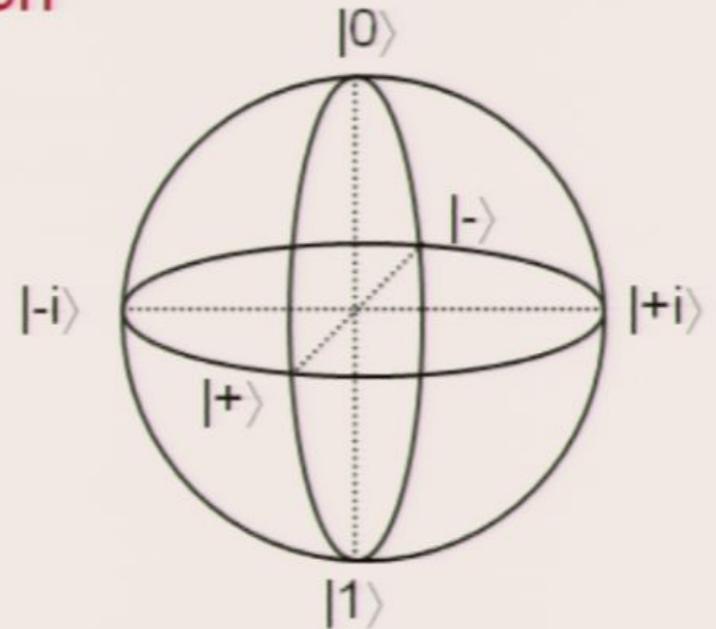
Coherent superposition

An equal superposition of $|0\rangle$ and $|1\rangle$ is obtained by implementing on $|0\rangle$

the square root of a U that maps one to the other

$$R_y(\pi)$$

$$R_x(-\pi)$$



Coherent superposition

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$$R_y(\pi)$$

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$$R_y(-\pi/2)$$

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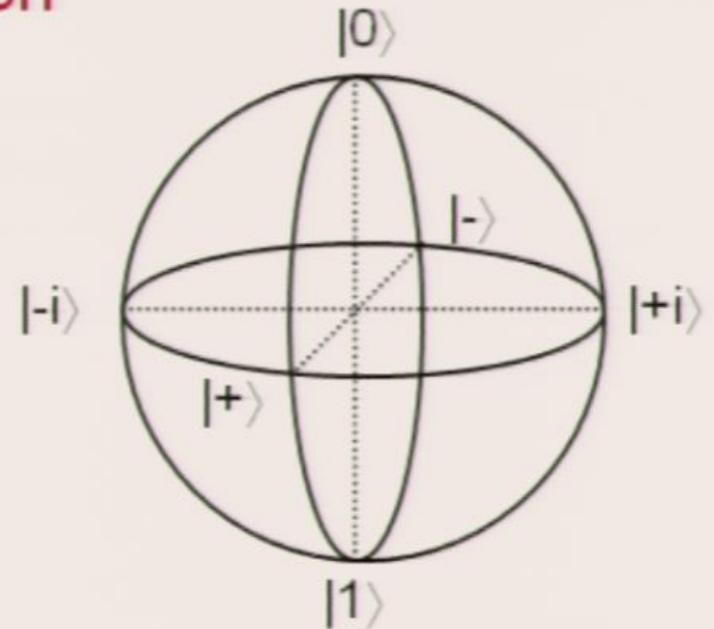
$$R_x(-\pi/2)$$

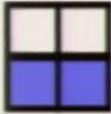
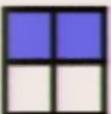
$$|+\rangle$$

$$|-\rangle$$

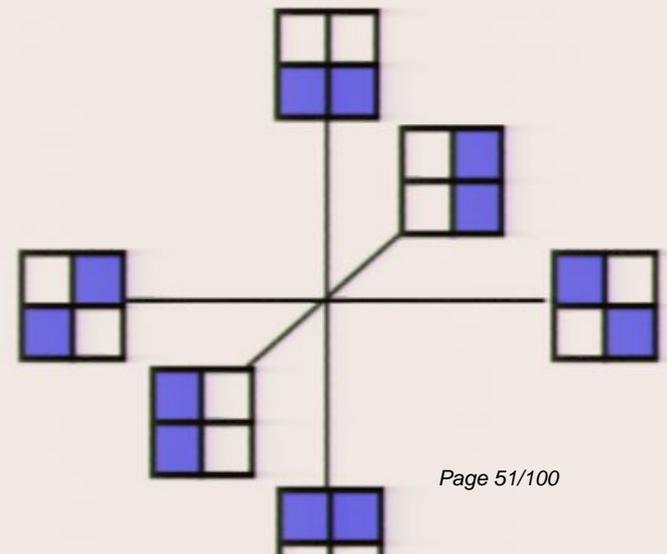
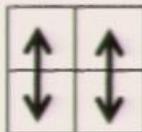
$$|+i\rangle$$

$$|-i\rangle$$



An equal superposition of  and  is obtained by implementing

the square root of a P that maps one to the other



Coherent superposition

An equal superposition of $|0\rangle$ and $|1\rangle$ is obtained by implementing on $|0\rangle$

the square root of a U that maps one to the other

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$$R_y(-\pi/2)$$

$$R_y(\pi/2)$$

$$R_x(\pi/2)$$

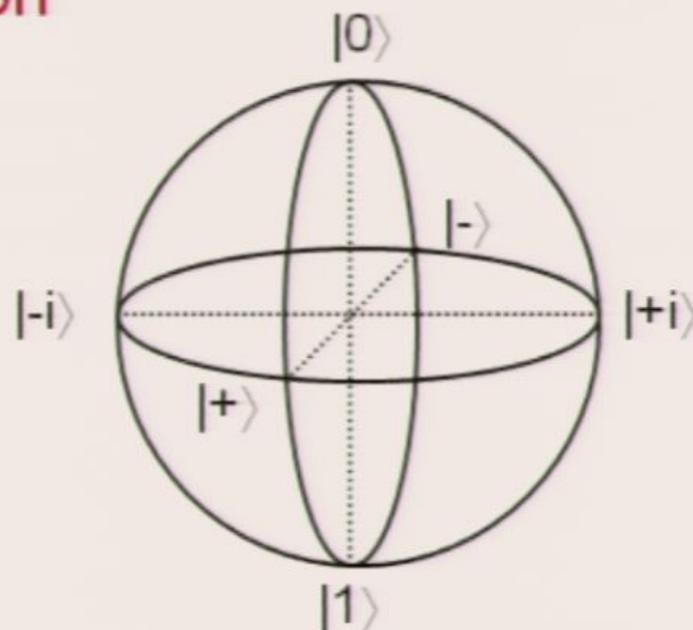
$$R_x(-\pi/2)$$

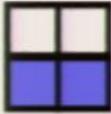
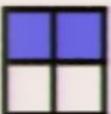
$$|+\rangle$$

$$|-\rangle$$

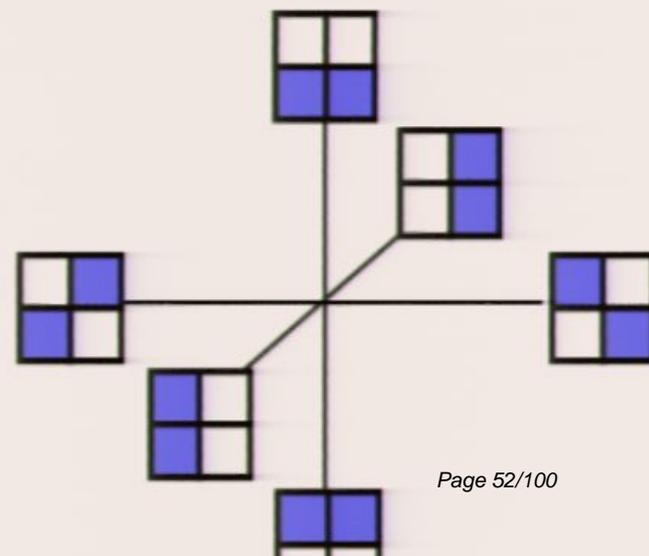
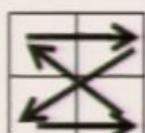
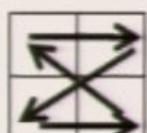
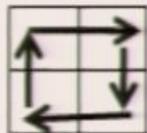
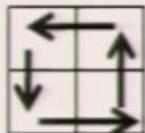
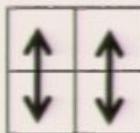
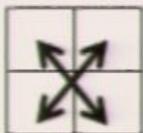
$$|+i\rangle$$

$$|-i\rangle$$



An equal superposition of  and  is obtained by implementing

the square root of a P that maps one to the other



Coherent superposition

An equal superposition of $|0\rangle$ and $|1\rangle$ is obtained by implementing on $|0\rangle$

the square root of a U that maps one to the other

$$R_y(\pi)$$

$$R_x(-\pi)$$

$$R_y(-\pi/2)$$

$$R_y(\pi/2)$$

$$R_x(\pi/2)$$

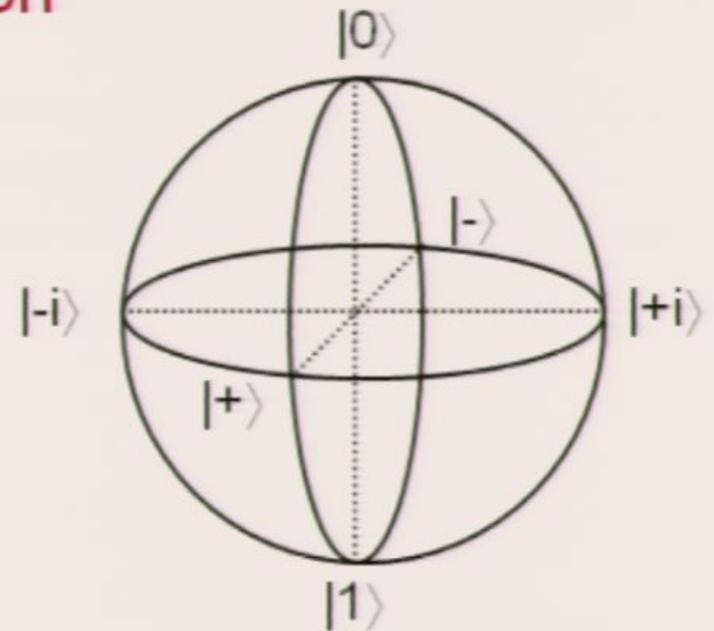
$$R_x(-\pi/2)$$

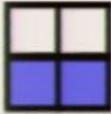
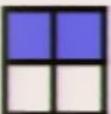
$$|+\rangle$$

$$|-\rangle$$

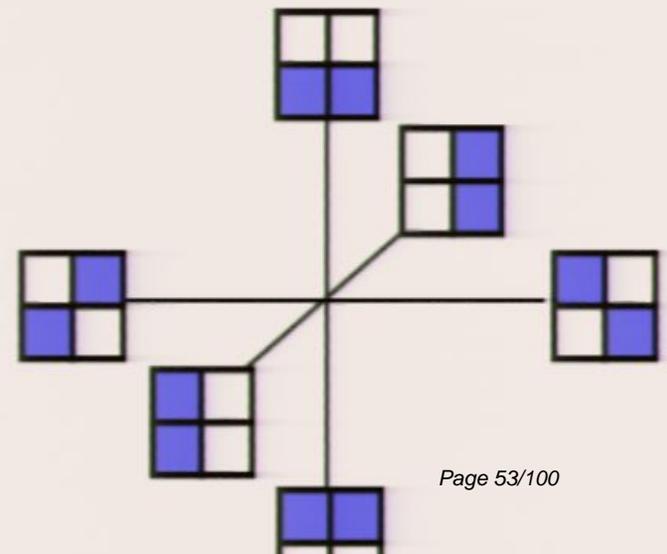
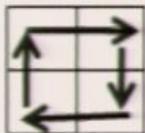
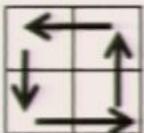
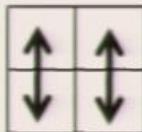
$$|+i\rangle$$

$$|-i\rangle$$



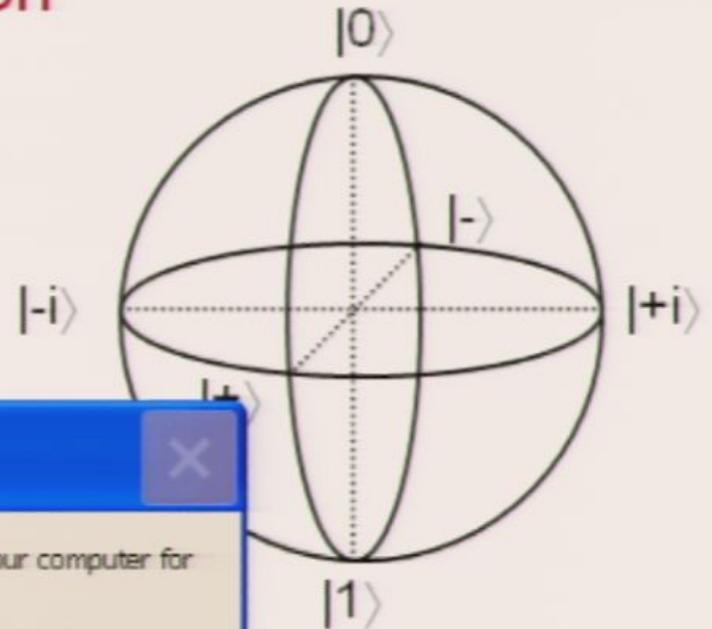
An equal superposition of  and  is obtained by implementing

the square root of a P that maps one to the other



Coherent superposition

An equal superposition of $|0\rangle$ and $|1\rangle$ is obtained by implementing on $|0\rangle$ the square root of a U that maps one to the other



$$R_y(\pi)$$

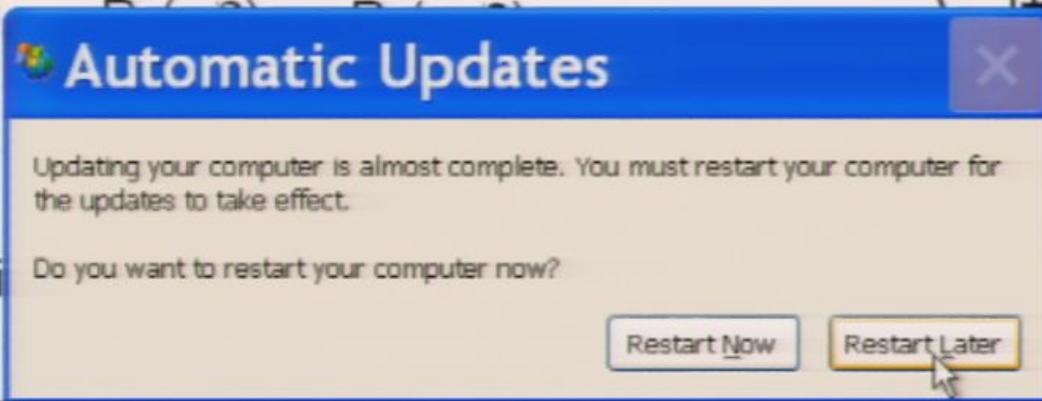
$$R_x(-\pi)$$

$$R_y(-\pi/2)$$

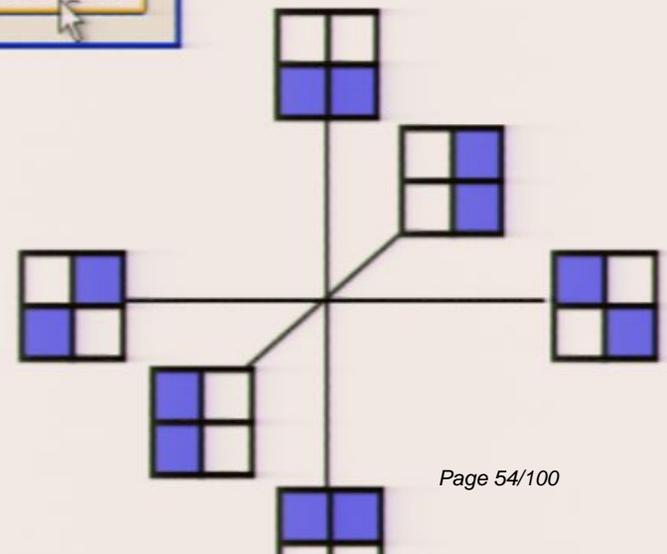
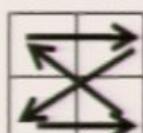
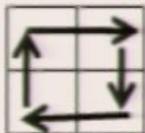
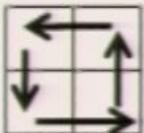
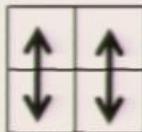
$$R_y(\pi/2)$$

$$|+\rangle$$

$$|-\rangle$$



An equal superposition by implementing the square root of a P that maps one to the other



Coherent superposition

An equal superposition of $|0\rangle$ and $|1\rangle$ is obtained by implementing on $|0\rangle$

the square root of a U that maps one to the other

$$R_y(\pi)$$

$$R_x(-\pi)$$

$$R_y(-\pi/2)$$

$$R_y(\pi/2)$$

$$R_x(\pi/2)$$

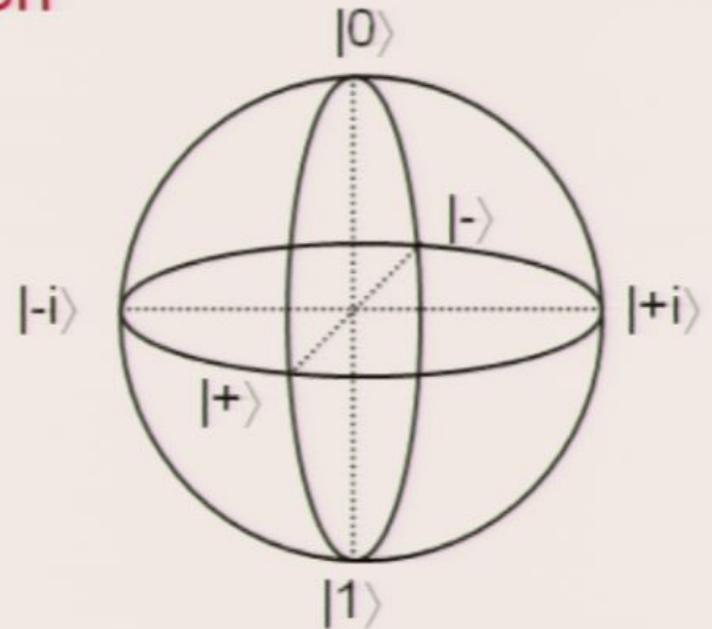
$$R_x(-\pi/2)$$

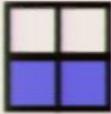
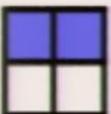
$$|+\rangle$$

$$|-\rangle$$

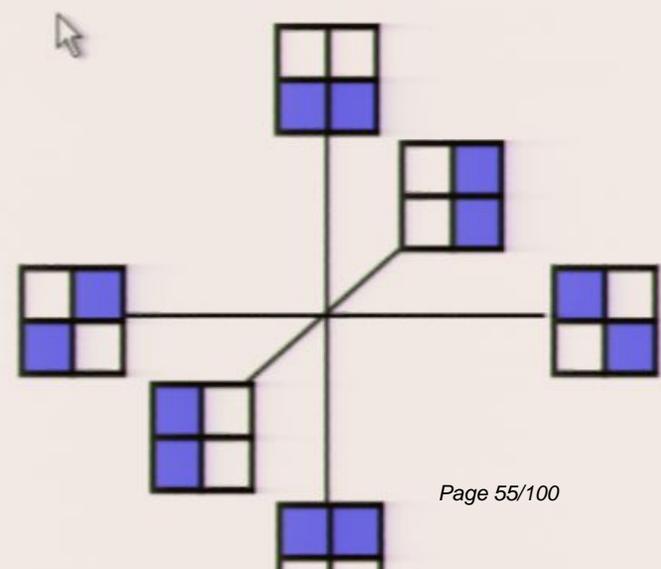
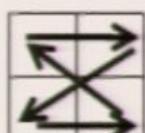
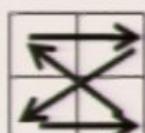
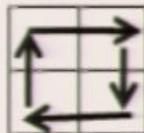
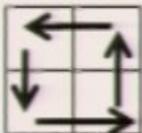
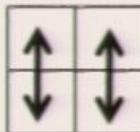
$$|+i\rangle$$

$$|-i\rangle$$



An equal superposition of  and  is obtained by implementing

the square root of a P that maps one to the other



Suppose the basis is $a \vee b$ and $c \vee d$
where $a \neq b \neq c \neq d$,

(ad)(bc)		(ac)(bd)	
(acdb)	(abdc)	(abcd)	(adcb)
$a \vee c$	$b \vee d$	$b \vee c$	$a \vee d$

Interference

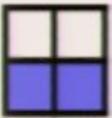
Quantum theory:

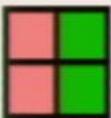
Prepare $|0\rangle$ measure $\{|+\rangle, |-\rangle\}$ get 50/50 probabilities

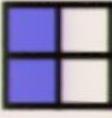
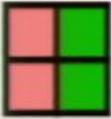
Prepare $|1\rangle$ measure $\{|+\rangle, |-\rangle\}$ get 50/50 probabilities

Prepare $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ measure $\{|+\rangle, |-\rangle\}$ get 100/0 probabilities

Toy theory:

Prepare  measure  get 50/50 probabilities

Prepare  measure  get 50/50 probabilities

Prepare  measure  get 100/0 probabilities

Superposition is neither “or” nor “and”
but “one possibility from each set”.

Suppose the basis is $a \vee b$ and $c \vee d$
 where $a \neq b \neq c \neq d$,

(ad)(bc)		(ac)(bd)	
(acdb)	(abdc)	(abcd)	(adcb)
$a \vee c$	$b \vee d$	$b \vee c$	$a \vee d$

Coherent superposition:

A set of binary operations that take two pure states to a third

$$(a \vee b) \dagger_1 (c \vee d) \equiv (acdb)(a \vee b) = a \vee c$$

$$(a \vee b) \dagger_2 (c \vee d) \equiv (abdc)(a \vee b) = b \vee d$$

$$(a \vee b) \dagger_3 (c \vee d) \equiv (abcd)(a \vee b) = b \vee c$$

$$(a \vee b) \dagger_4 (c \vee d) \equiv (adcb)(a \vee b) = a \vee d$$

Interference

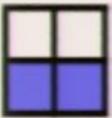
Quantum theory:

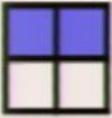
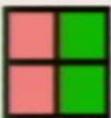
Prepare $|0\rangle$ measure $\{|+\rangle, |-\rangle\}$ get 50/50 probabilities

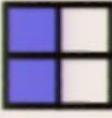
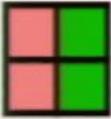
Prepare $|1\rangle$ measure $\{|+\rangle, |-\rangle\}$ get 50/50 probabilities

Prepare $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ measure $\{|+\rangle, |-\rangle\}$ get 100/0 probabilities

Toy theory:

Prepare  measure  get 50/50 probabilities

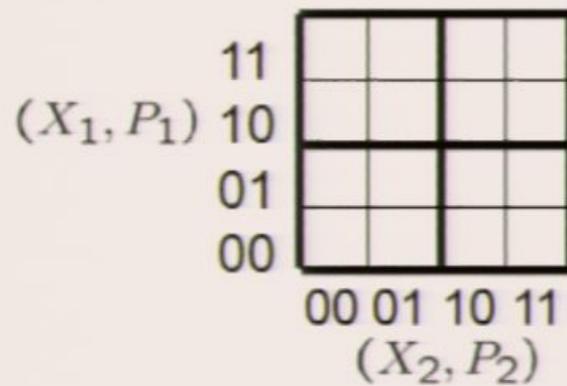
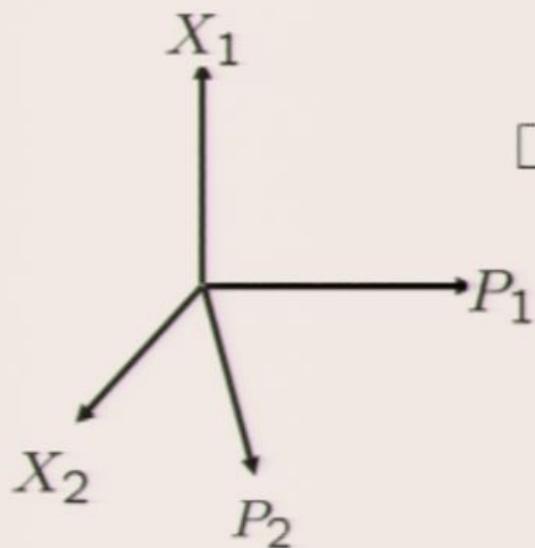
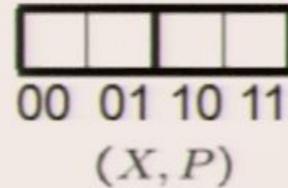
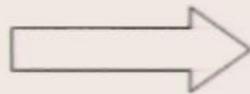
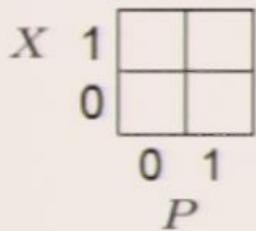
Prepare  measure  get 50/50 probabilities

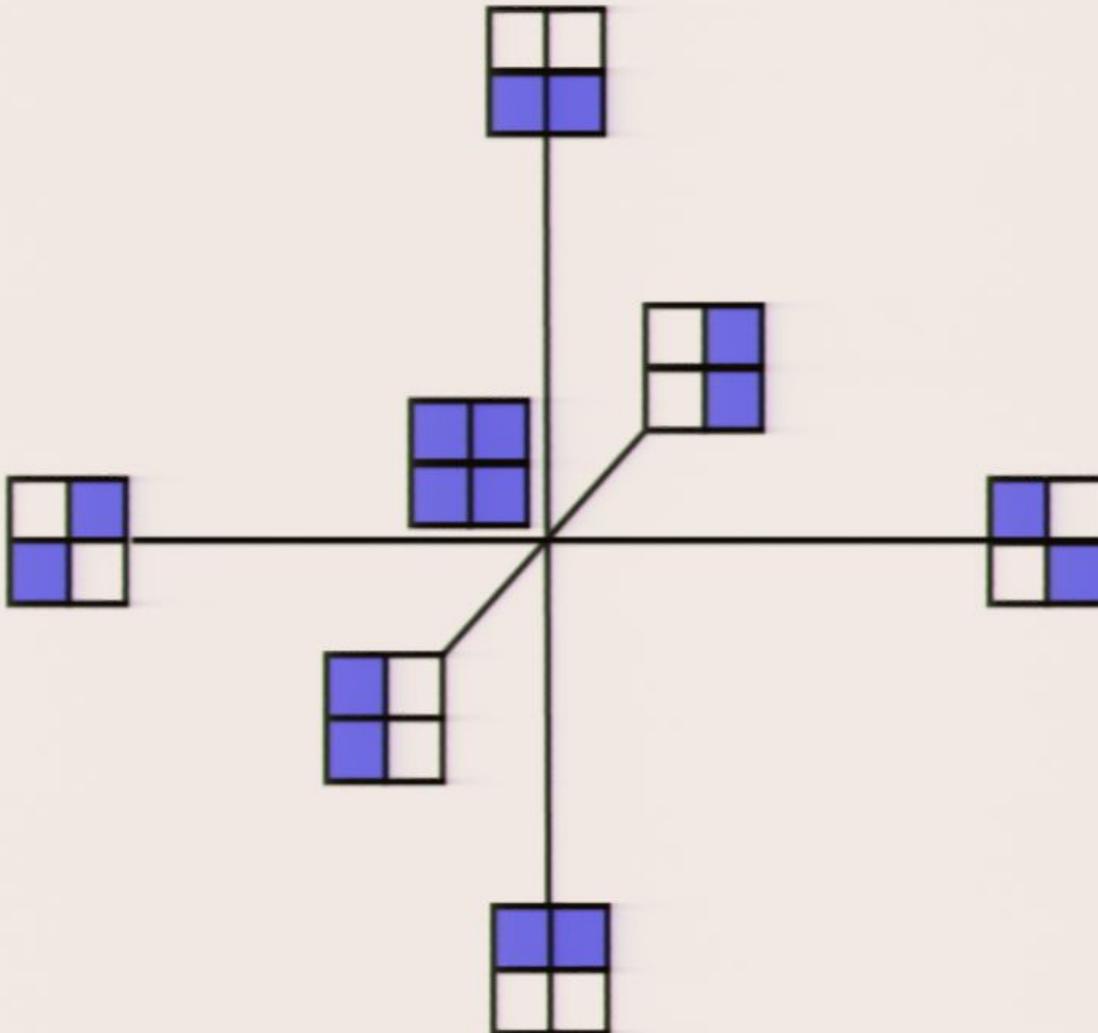
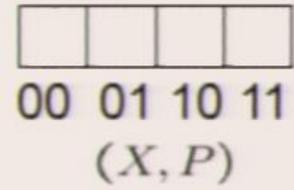
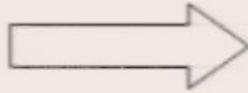
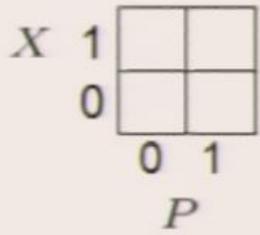
Prepare  measure  get 100/0 probabilities

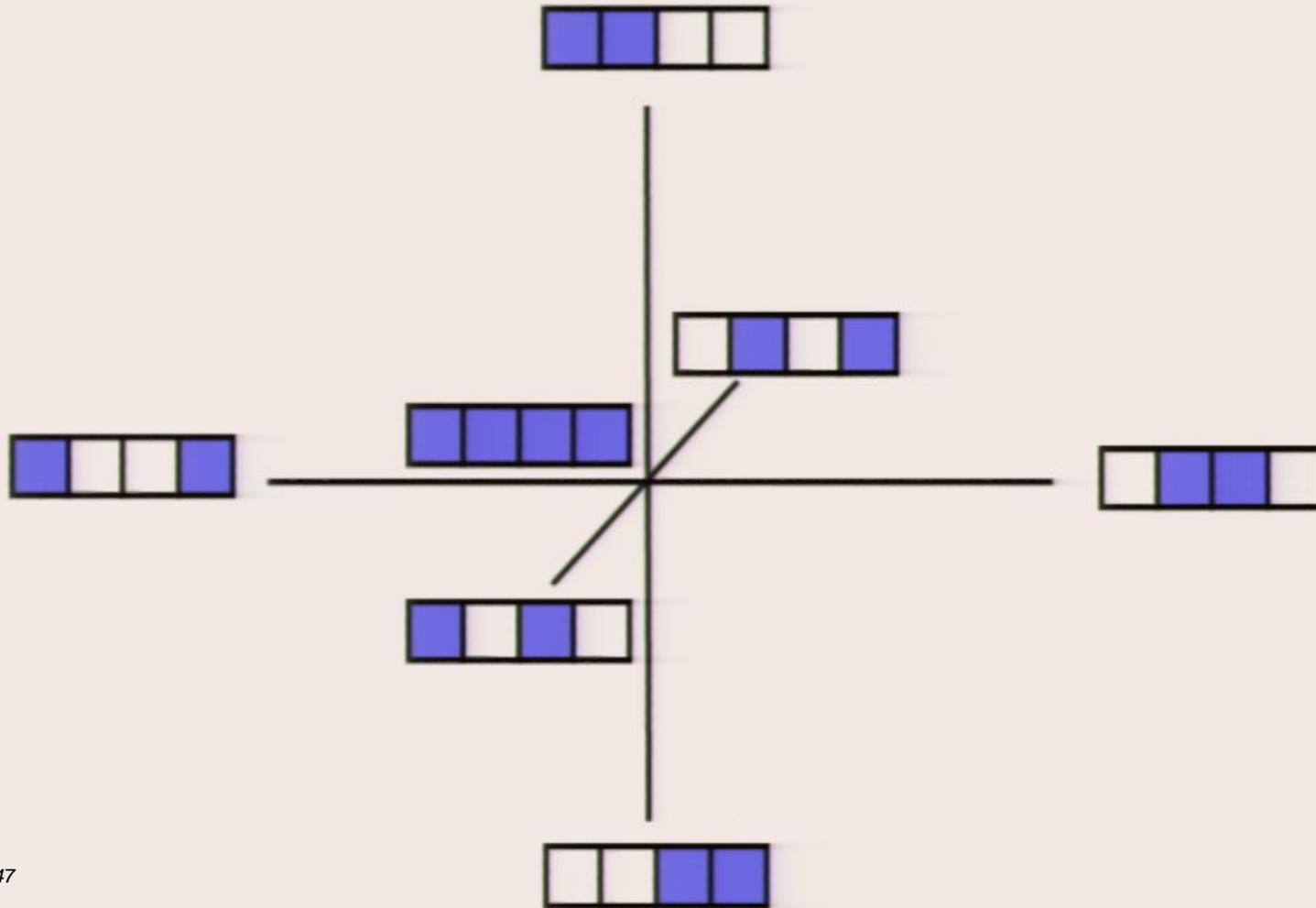
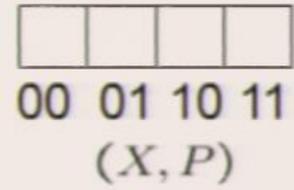
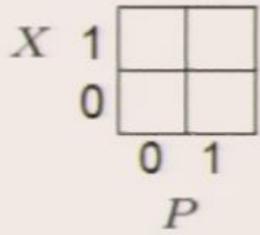
Superposition is neither “or” nor “and”
but “one possibility from each set”.

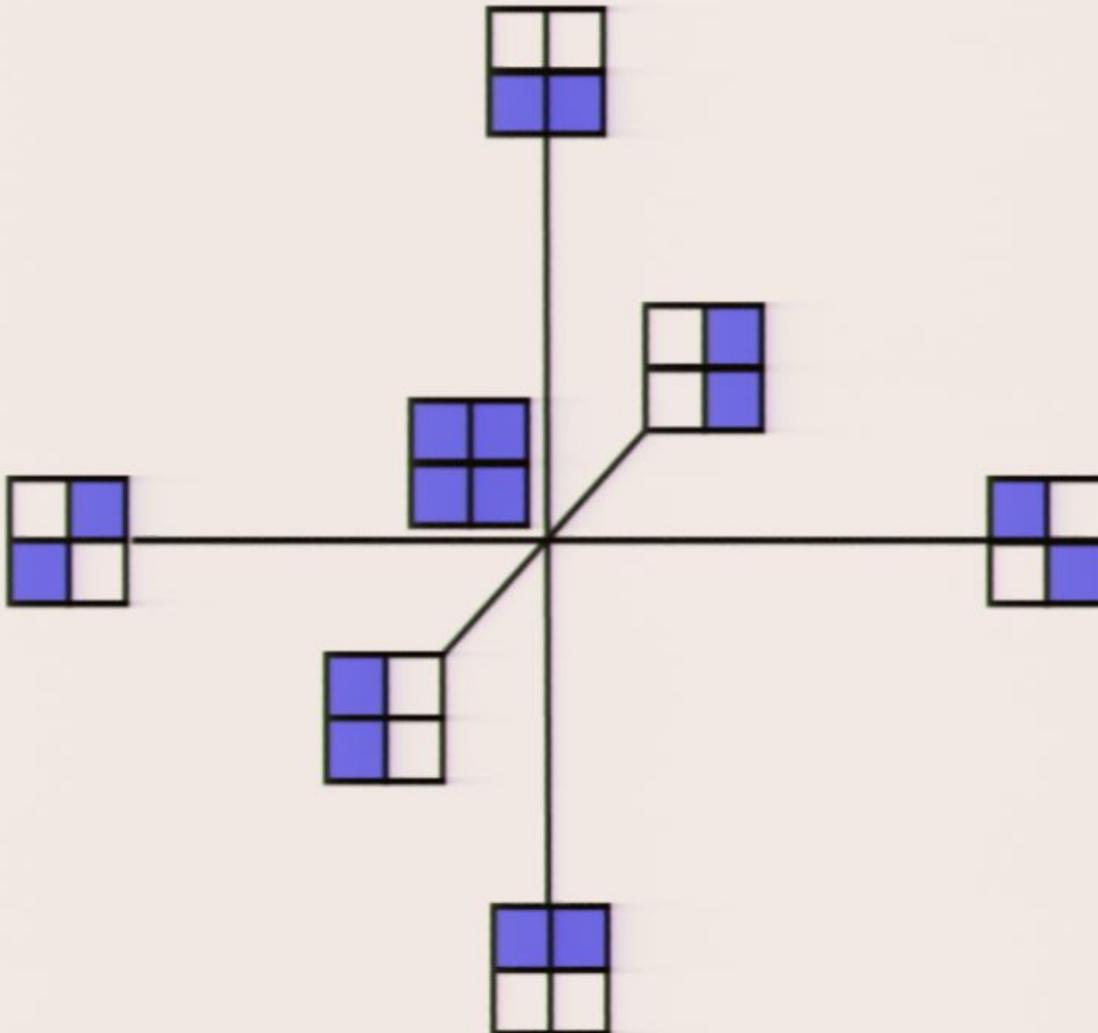
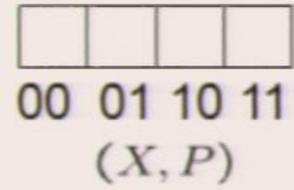
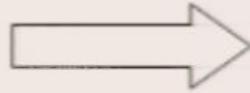
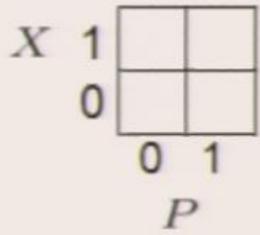
A pair of bits

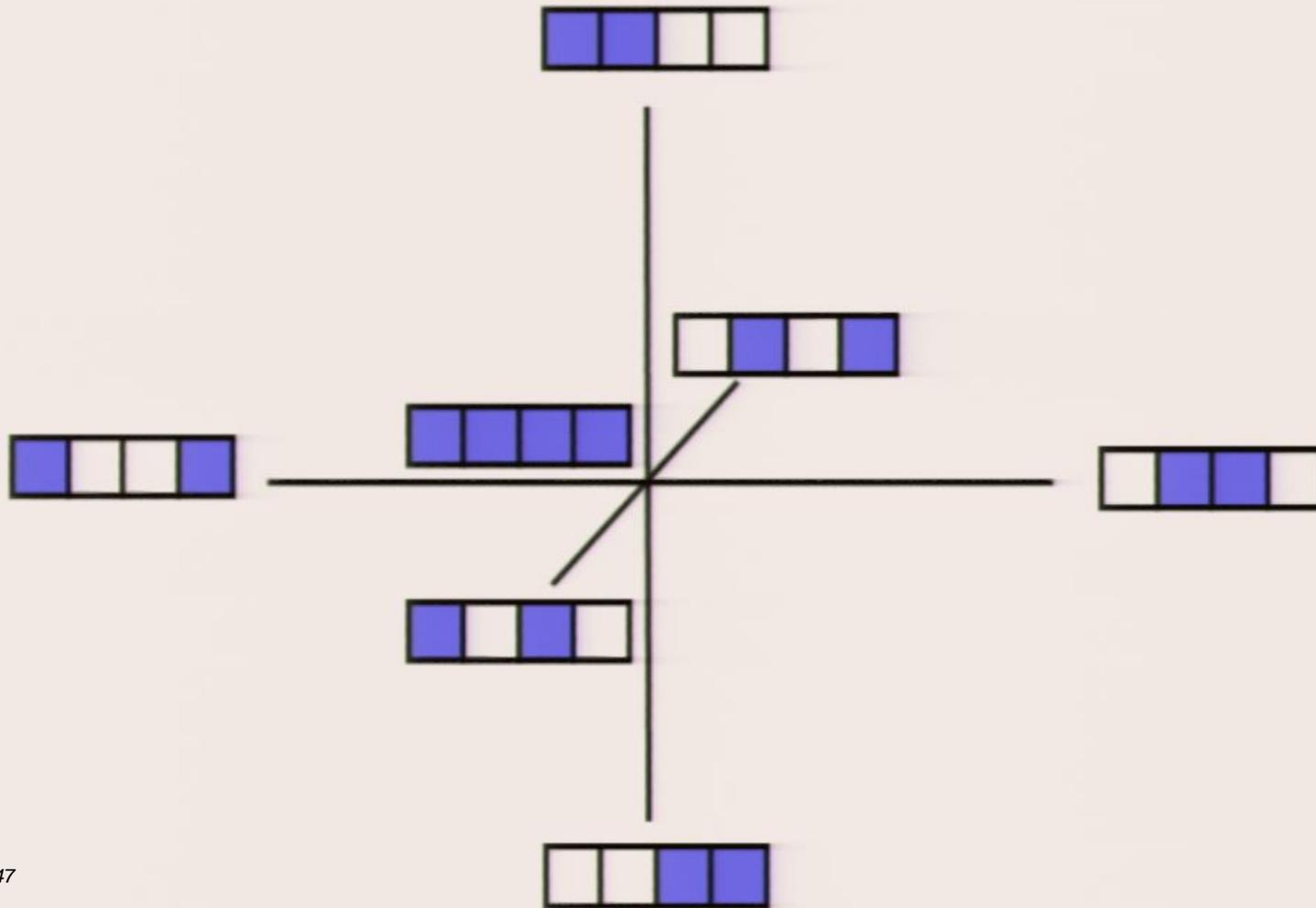
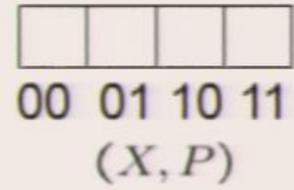
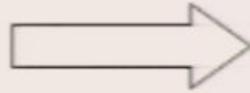
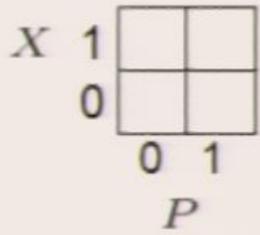
Canonical variables $a_1X_1 + b_1P_1 + a_2X_2 + b_2P_2$ $a_1, b_1, a_2, b_2 \in \mathbb{Z}_2$





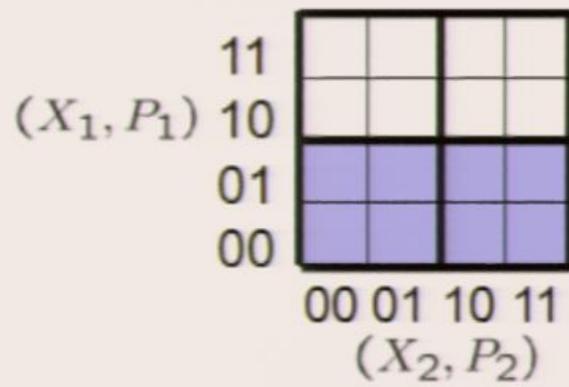




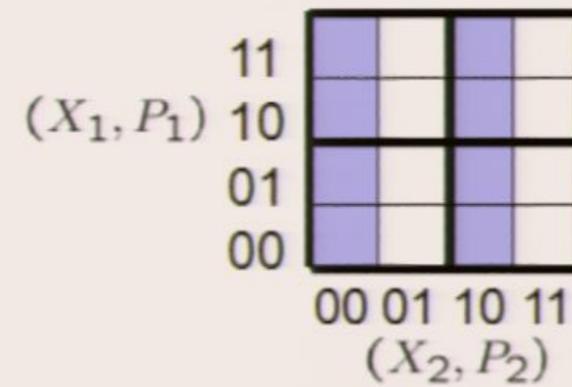


1 variable known

X_1 known

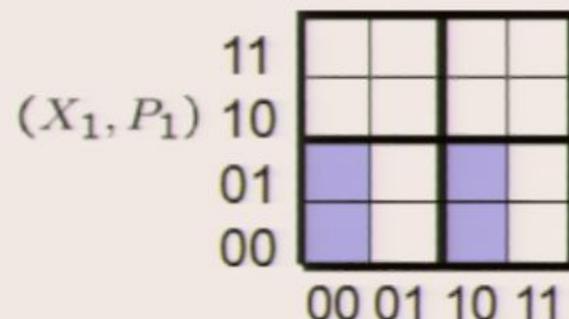


P_2 known



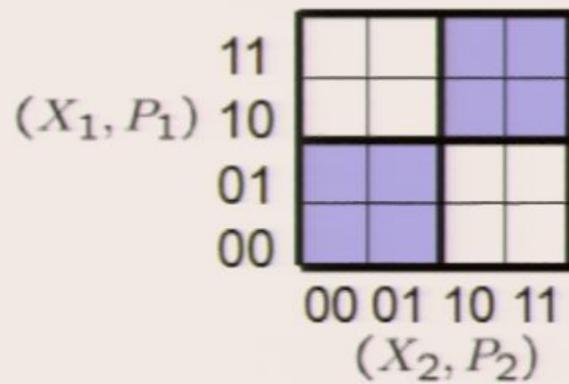
2 variables known

X_1 and P_2 known

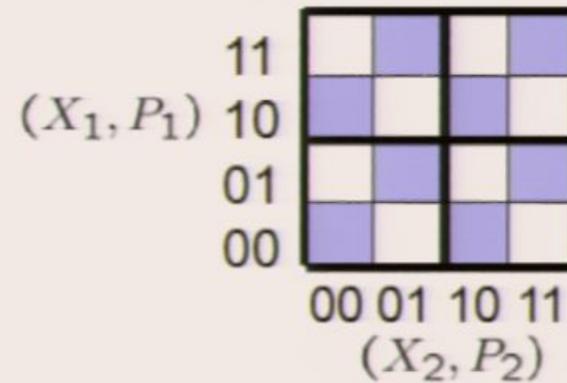


1 variable known

$X_1 + X_2$ known

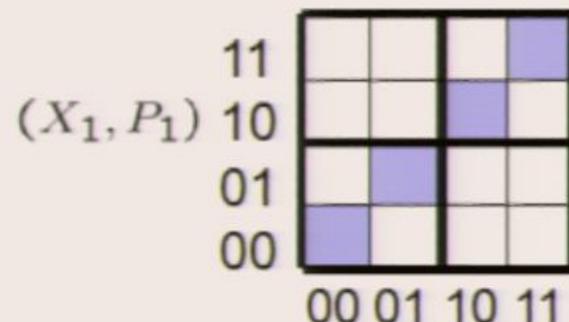


$P_1 + P_2$ known



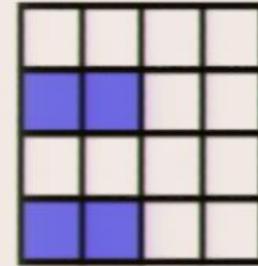
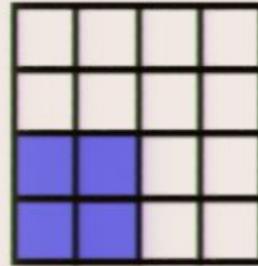
2 variables known

$X_1 + X_2$ and $P_1 + P_2$ known



uncorrelated

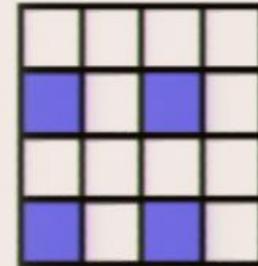
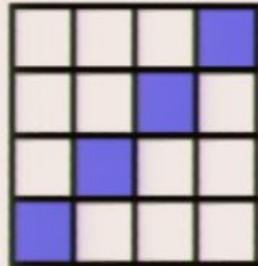
If A measures



$$|0\rangle|0\rangle \rightarrow |+\rangle|0\rangle$$

correlated

If A measures



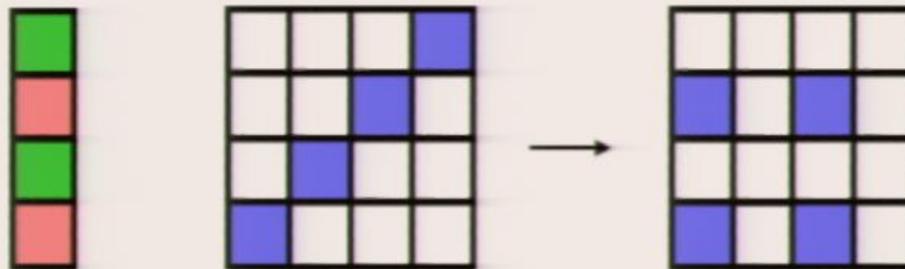
$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \rightarrow |+\rangle|+\rangle$$

uncorrelated

If A measures

The diagram illustrates a quantum state evolution. On the left, a vertical stack of four colored boxes (green, red, green, red) represents the state of system A. To its right is a 4x4 grid representing system B, with the bottom two rows (rows 3 and 4) shaded blue. An arrow points to a second 4x4 grid where the top two rows (rows 1 and 2) are shaded blue. Overlaid on this diagram is a blue dialog box titled "Automatic Updates". The dialog box contains the text: "Updating your computer is almost complete. You must restart your computer for the updates to take effect. Do you want to restart your computer now?" At the bottom right of the dialog box are two buttons: "Restart Now" and "Restart Later". A mouse cursor is pointing at the "Restart Later" button.

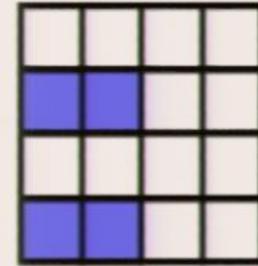
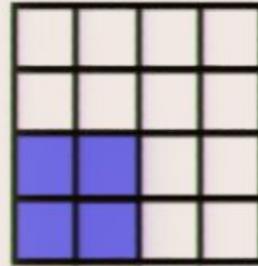
If A measures



$$\frac{1}{\sqrt{2}} (|0\rangle|0\rangle + |1\rangle|1\rangle) \rightarrow |+\rangle|+\rangle$$

uncorrelated

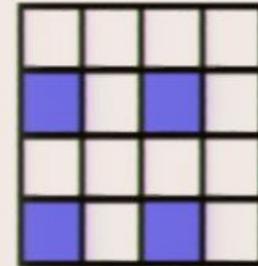
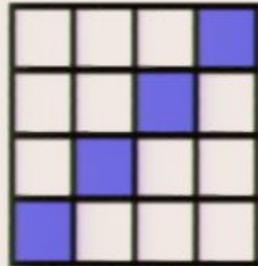
If A measures



$$|0\rangle|0\rangle \rightarrow |+\rangle|0\rangle$$

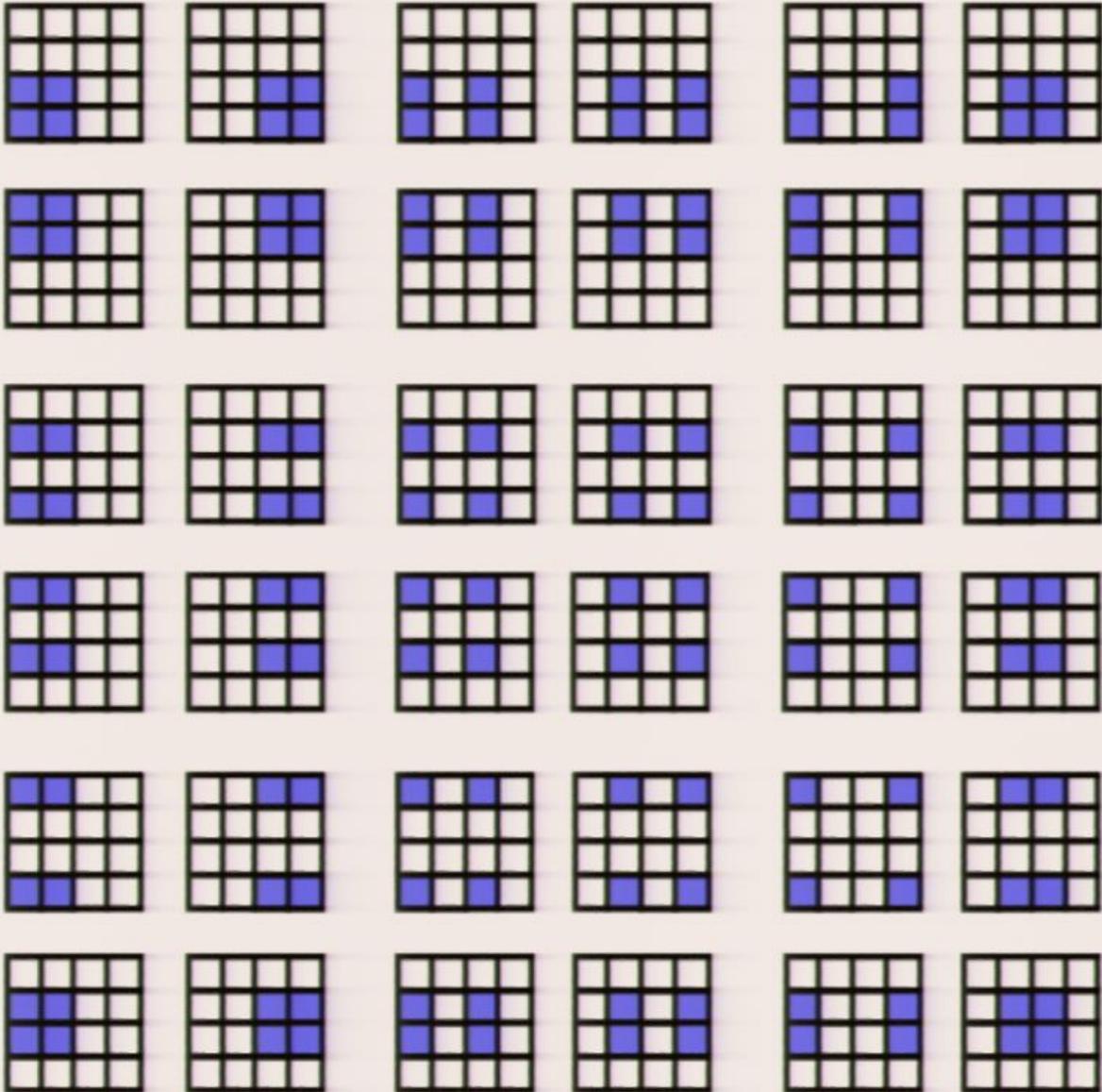
correlated

If A measures

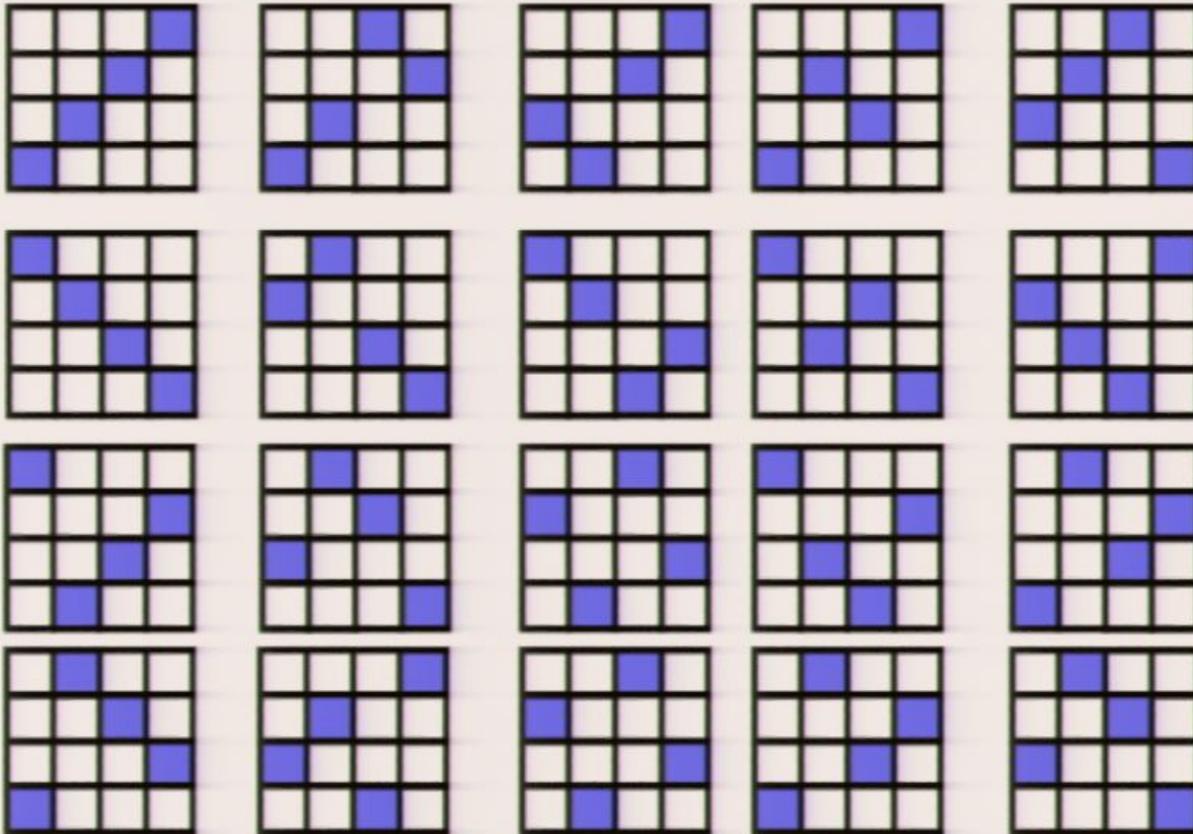


$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) \rightarrow |+\rangle|+\rangle$$

Uncorrelated pure epistemic states

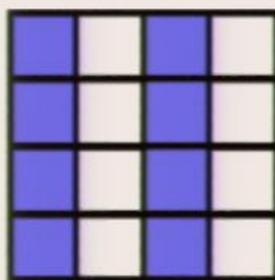
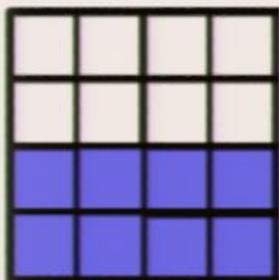


Correlated pure epistemic states

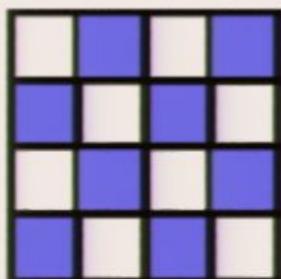
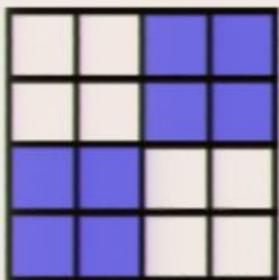


Epistemic states of non-maximal knowledge

Uncorrelated

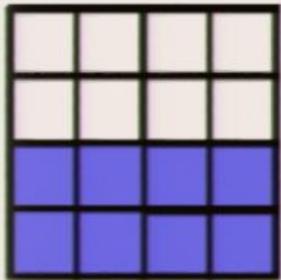


Correlated

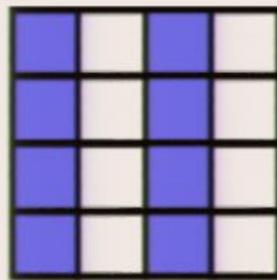


Epistemic states of non-maximal knowledge

Uncorrelated

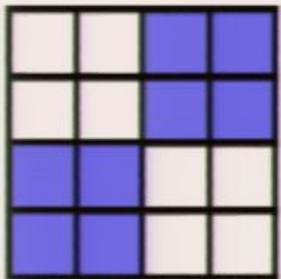


$$|0\rangle\langle 0| \otimes \frac{1}{2}I$$

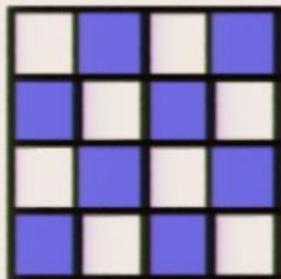


$$\frac{1}{2}I \otimes |+\rangle\langle +|$$

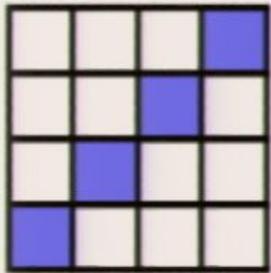
Correlated



$$\frac{1}{2}|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|$$

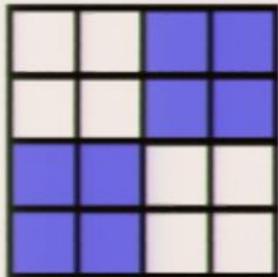


$$\frac{1}{2}|+\rangle\langle +| \otimes |+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -| \otimes |-\rangle\langle -|$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |1\rangle$$

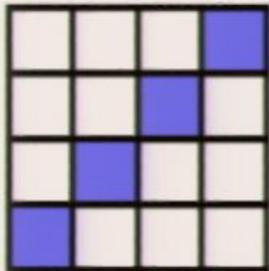
“quantum correlation”



$$\rho = \frac{1}{2}|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \otimes |1\rangle\langle 1|$$

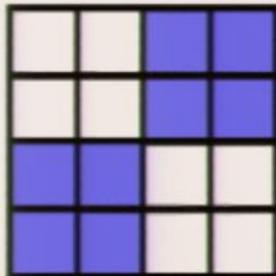
“classical correlation”

A distinction between
maximal and nonmaximal correlation



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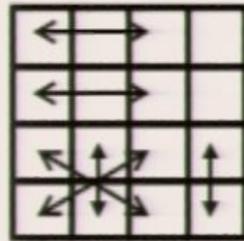
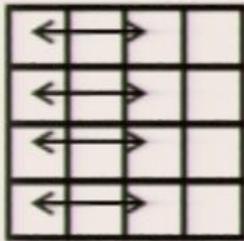
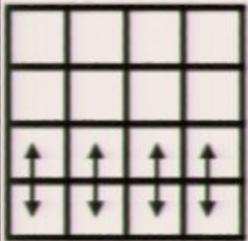


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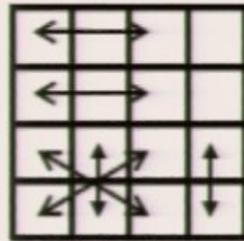
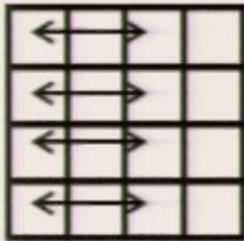
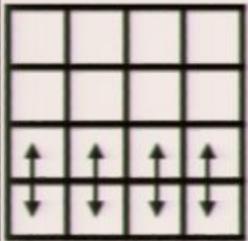
Local transformations: conjunction of permutations on each system



Takes uncorrelated to
uncorrelated
correlated to correlated

$$U_{AB} = U_A \otimes U_B$$

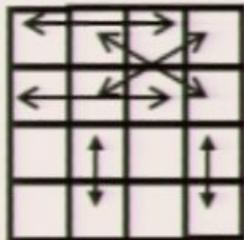
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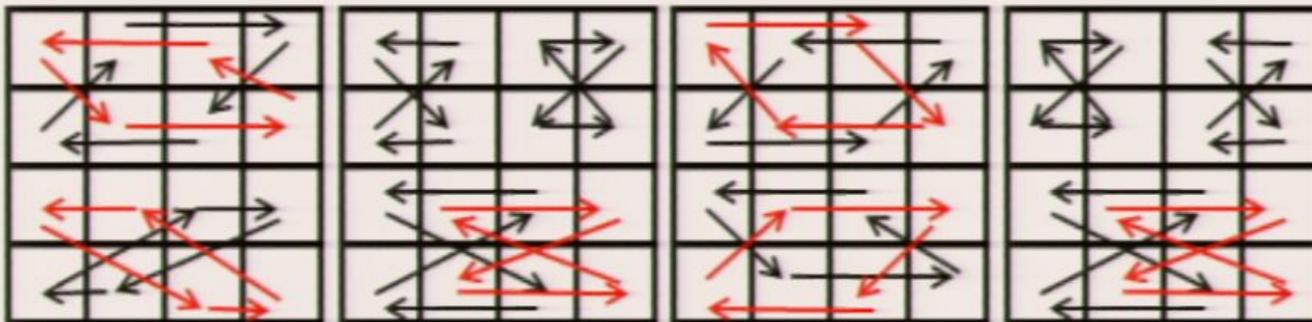
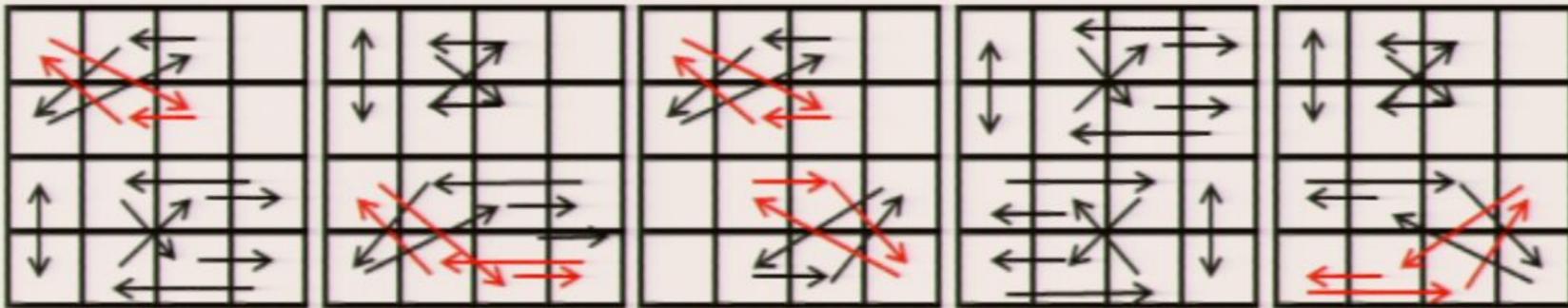
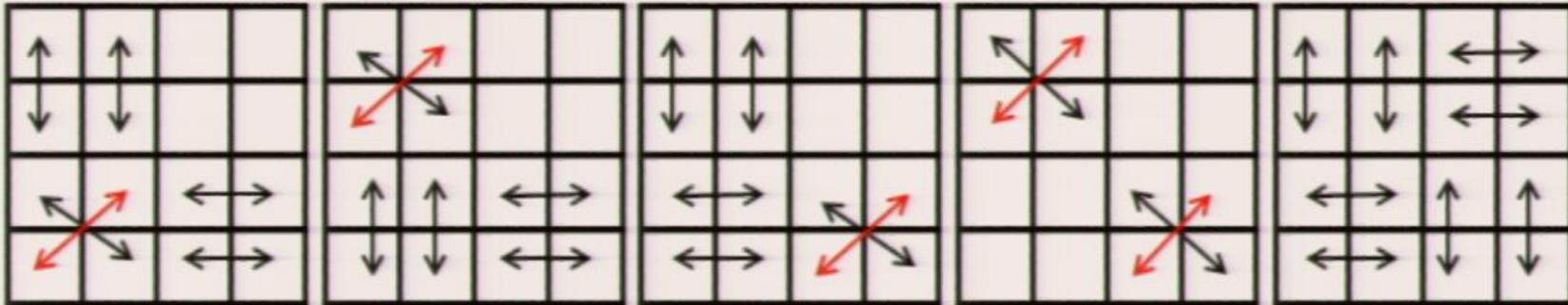
Joint transformations:

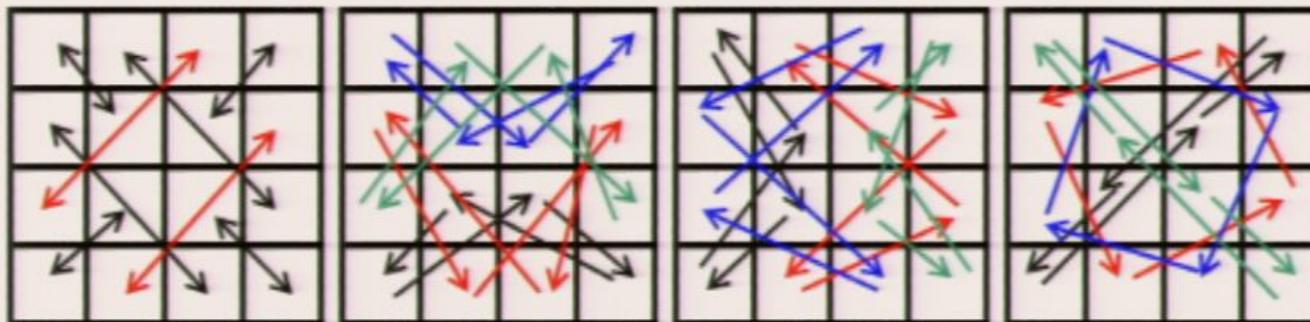
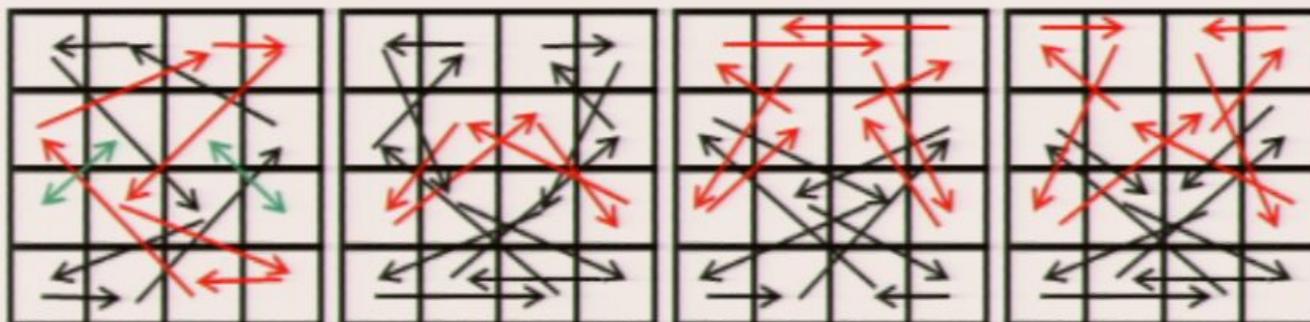
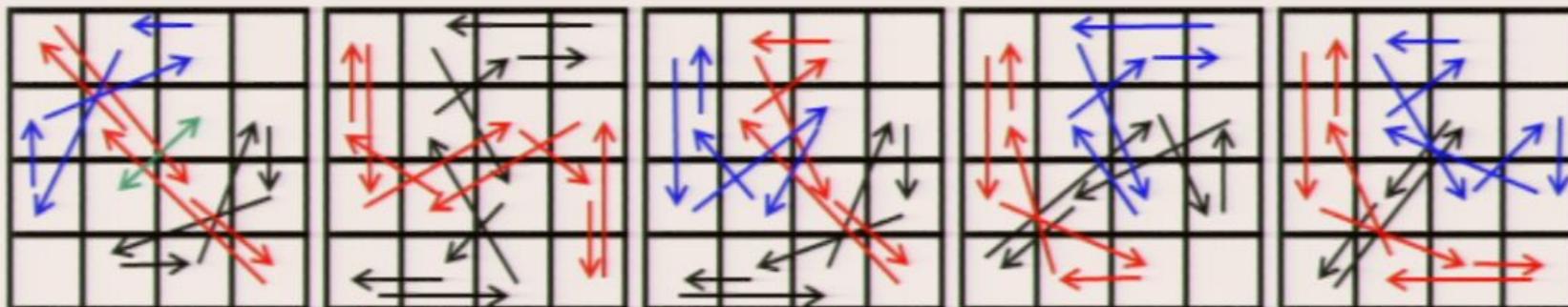


Can change the degree of correlation

$$U_{AB} \neq U_A \otimes U_B$$

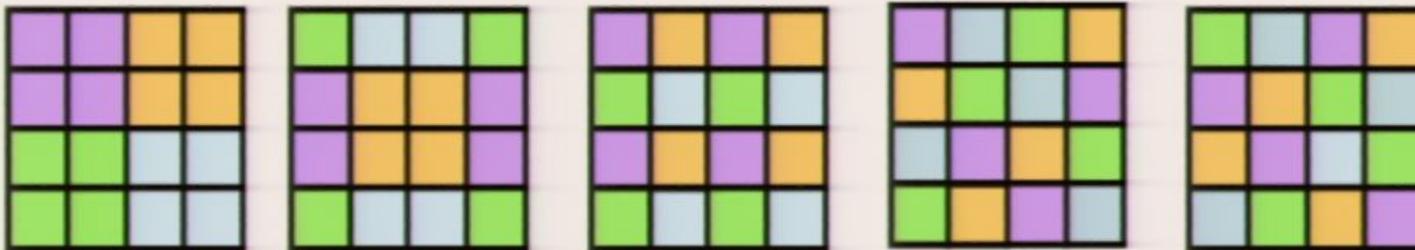
Joint transformations





Valid reproducible measurements:

Any commuting set of canonical variables



No Cloning

Quantum: A cloning process
for a set $\{|\psi_i\rangle\}$ satisfies

$$|\psi_i\rangle |\chi\rangle \rightarrow |\psi_i\rangle |\psi_i\rangle$$

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Example: $\{|1\rangle, |+\rangle\}$

$$|1\rangle |0\rangle \rightarrow |1\rangle |1\rangle$$

$$|+\rangle |0\rangle \rightarrow |+\rangle |+\rangle$$

Overlaps are:

$$\langle 1|+\rangle \langle 0|0\rangle^2 \neq |\langle 1|+\rangle \langle 1|+\rangle|^2$$

$$1/2 \neq 1/4$$

But unitary maps preserve overlaps, therefore this map is not unitary

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$$(a_i \vee b_i) \cdot (c \vee d) \rightarrow (a_i \vee b_i) \cdot (a_i \vee b_i)$$

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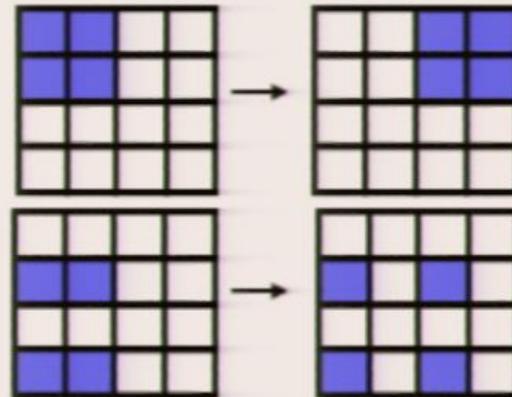
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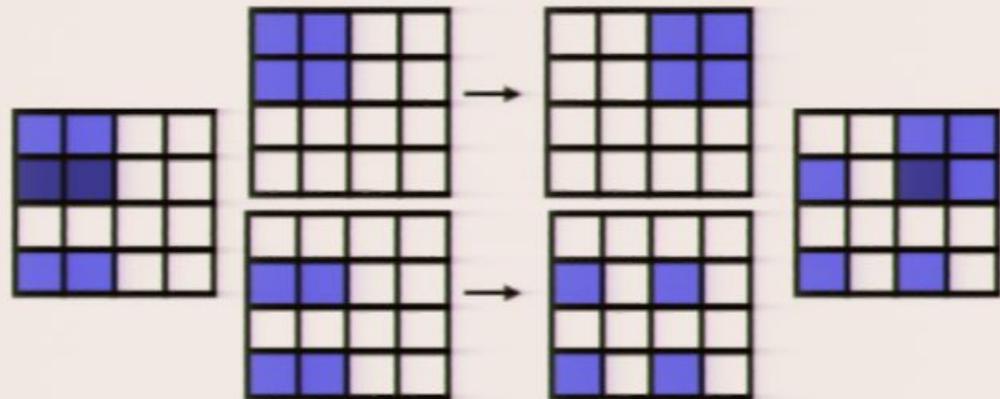
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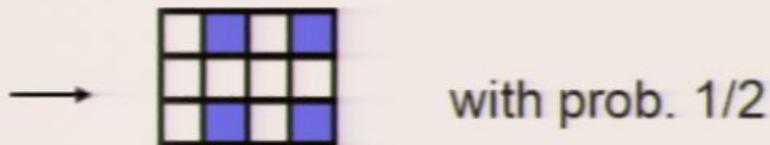
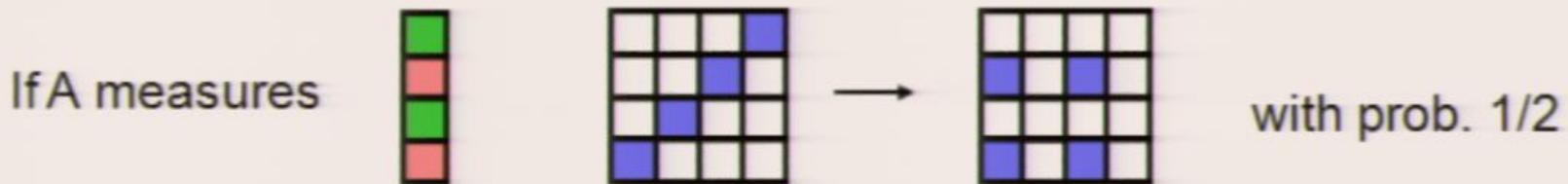
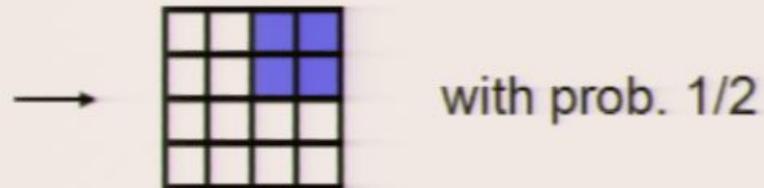
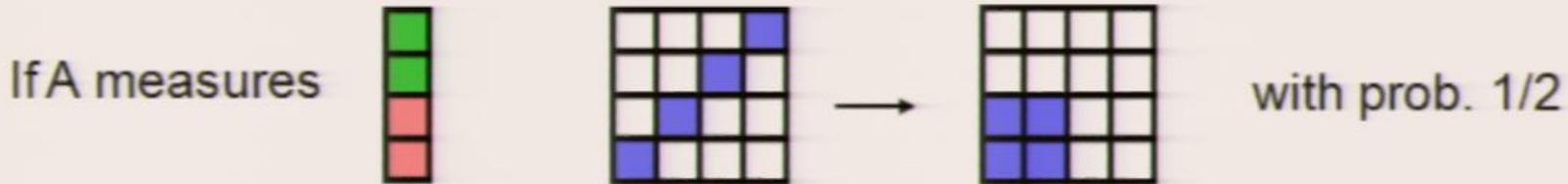
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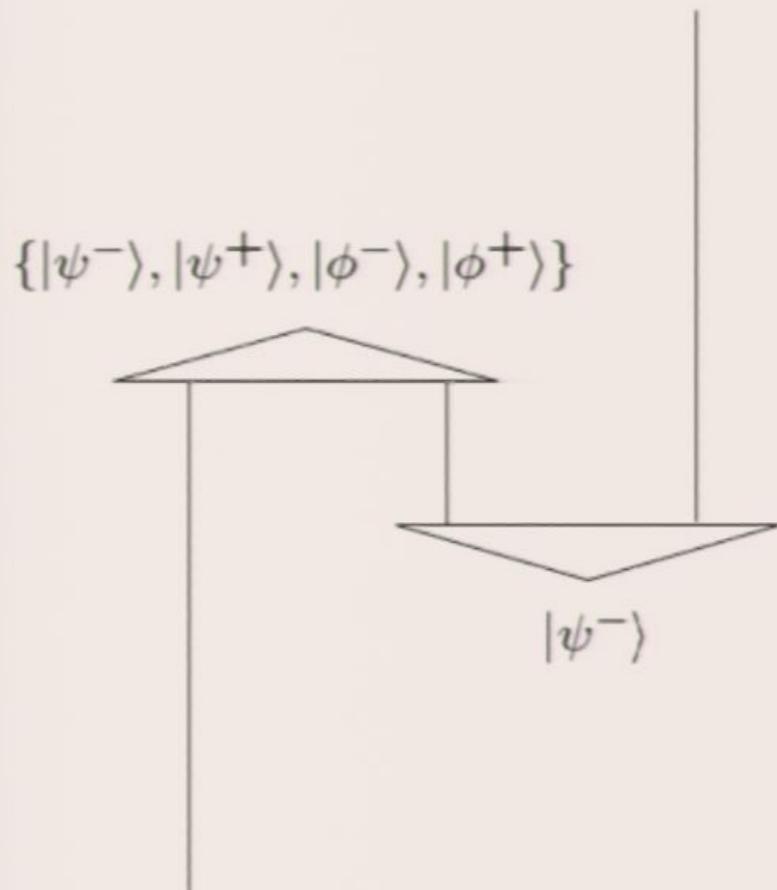
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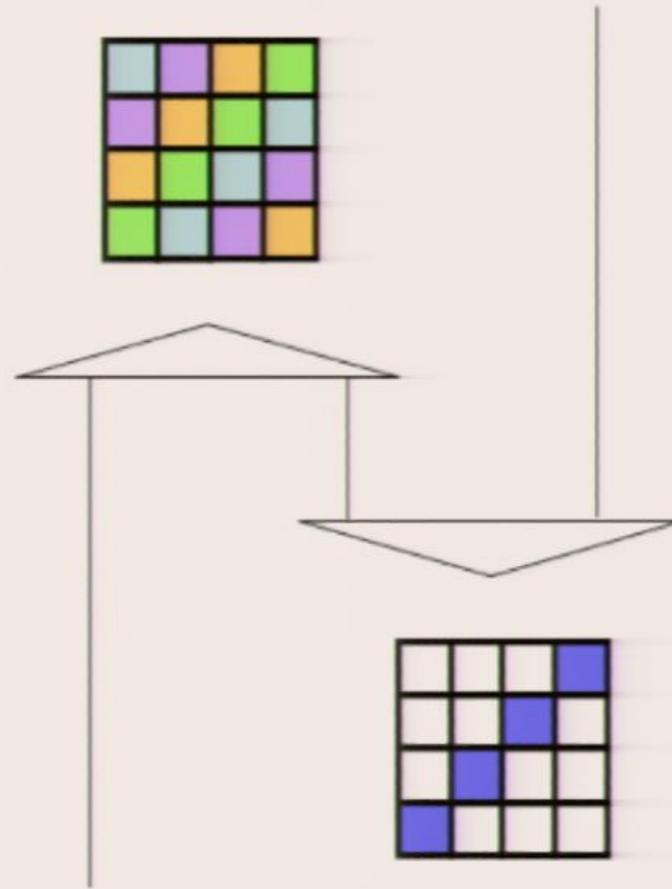
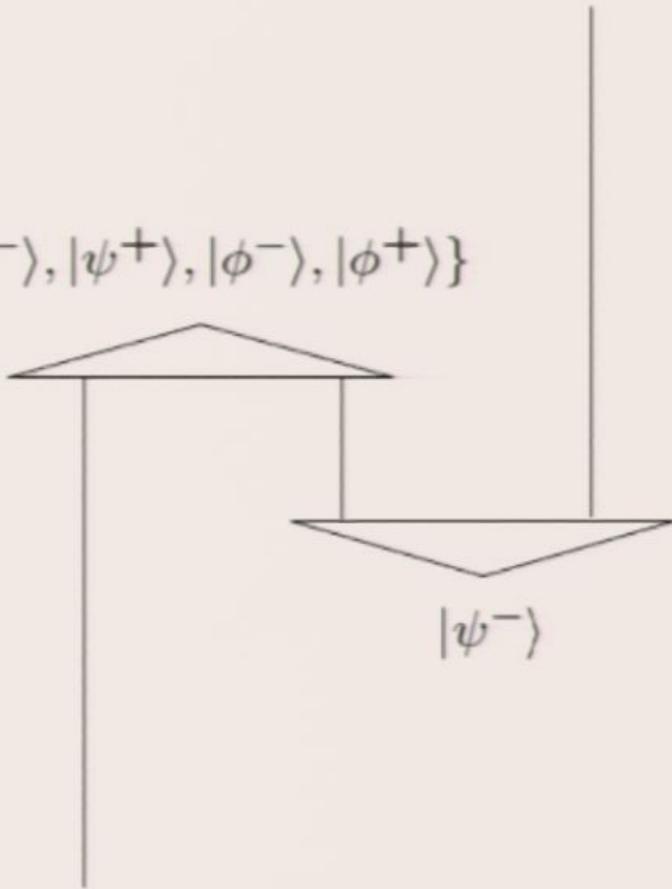


Teleportation



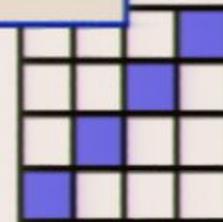
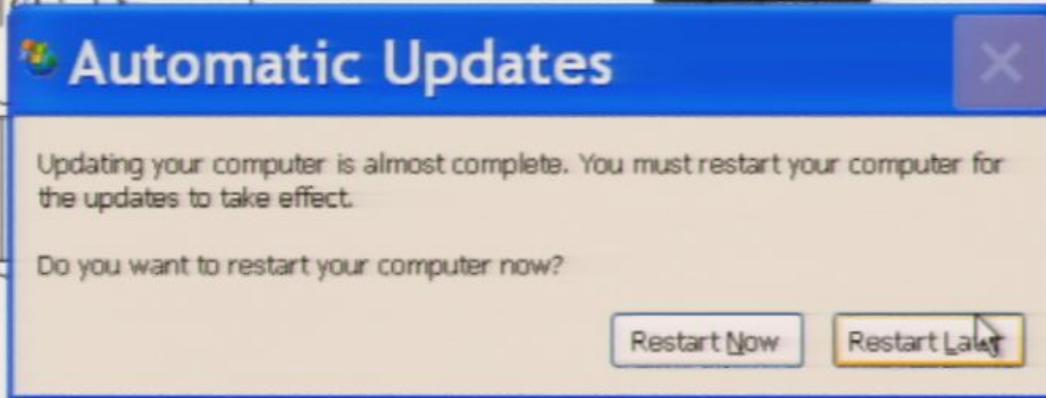
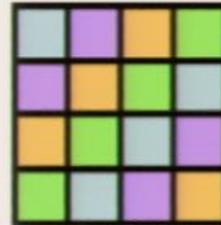
Teleportation

$\{|\psi^-\rangle, |\psi^+\rangle, |\phi^-\rangle, |\phi^+\rangle\}$



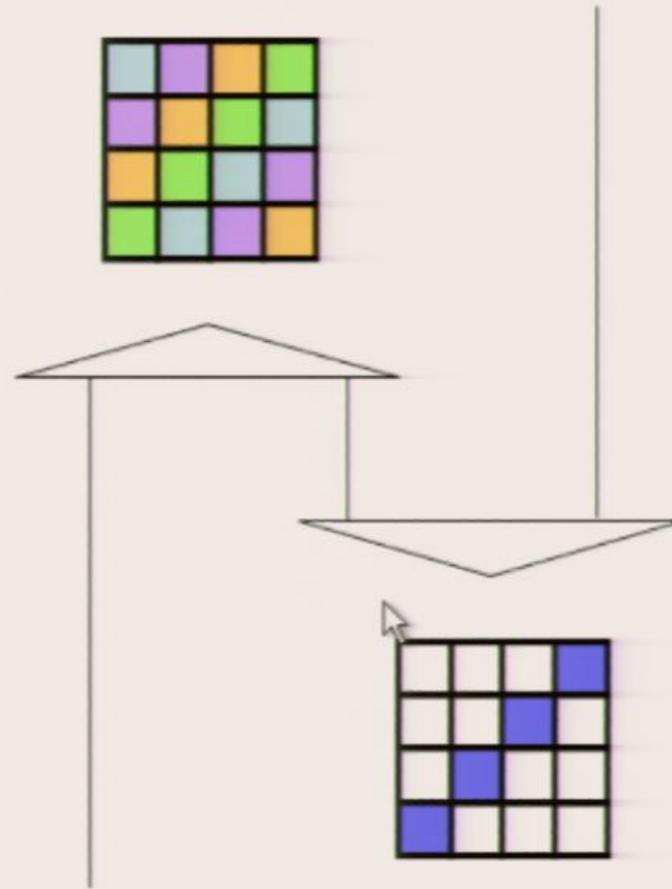
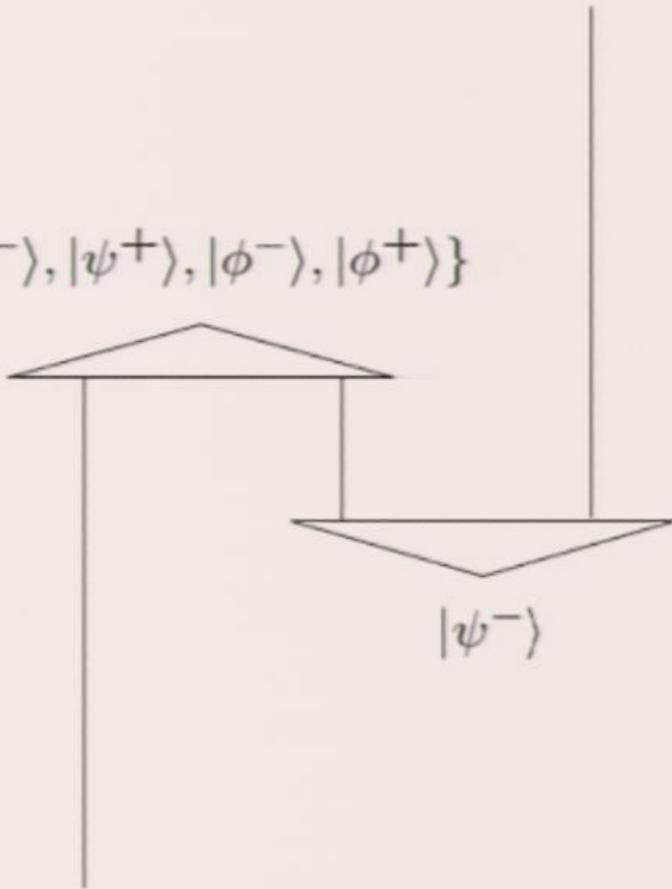
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Consider the stabilizer theory for qubits

It is possible to define a discrete Wigner function for qubits

But: This Wigner function can be negative for qubit stabilizer states

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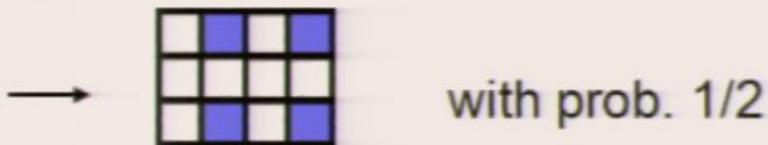
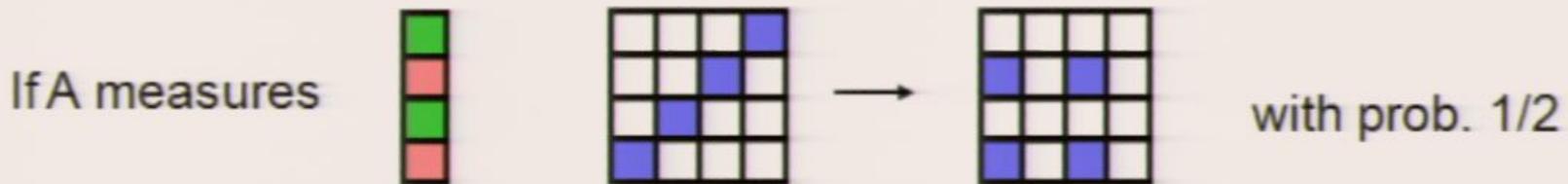
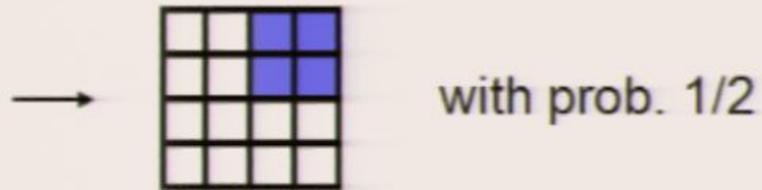
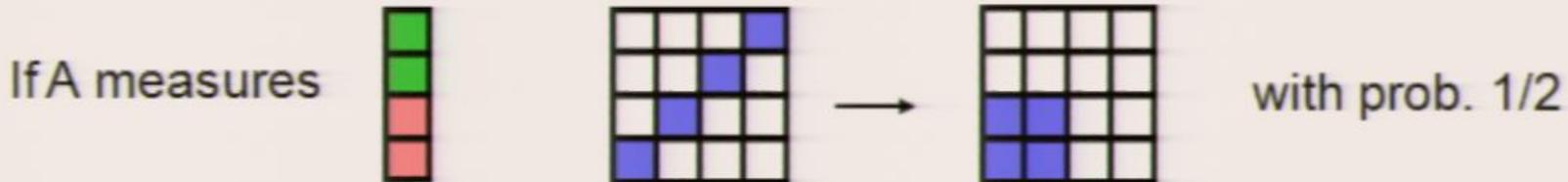
But: This Wigner function can be negative for qubit stabilizer states

The restricted statistical theory of bits is similar but not equivalent to the Stabilizer theory for qubits

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No Cloning

Quantum: A cloning process for a set $\{|\psi_i\rangle\}$ satisfies

$$|\psi_i\rangle |\chi\rangle \rightarrow |\psi_i\rangle |\psi_i\rangle$$

Example: $\{|1\rangle, |+\rangle\}$

$$|1\rangle |0\rangle \rightarrow |1\rangle |1\rangle$$

$$|+\rangle |0\rangle \rightarrow |+\rangle |+\rangle$$

Overlaps are:

$$\langle 1|+\rangle \langle 0|0\rangle|^2 \neq |\langle 1|+\rangle \langle 1|+\rangle|^2$$

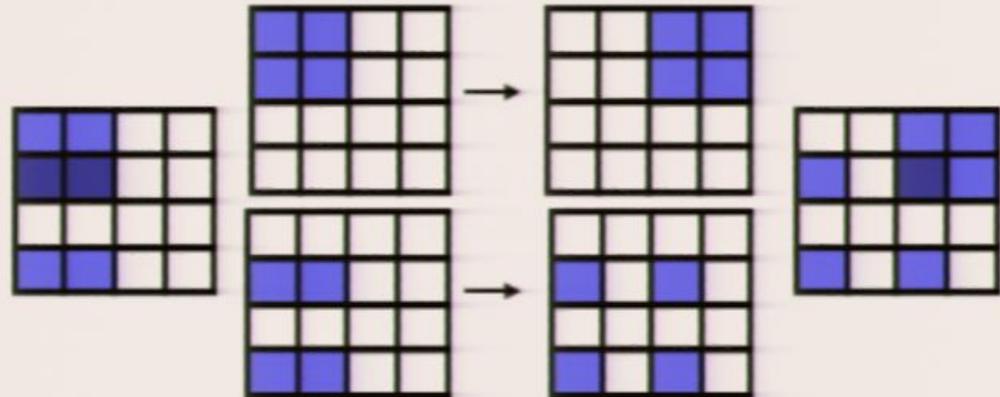
$$1/2 \neq 1/4$$

But unitary maps preserve overlaps, therefore this map is not unitary

Toy theory: A cloning process for a set $\{(a_i \vee b_i)\}$ satisfies

$$(a_i \vee b_i) \cdot (c \vee d) \rightarrow (a_i \vee b_i) \cdot (a_i \vee b_i)$$

Example:



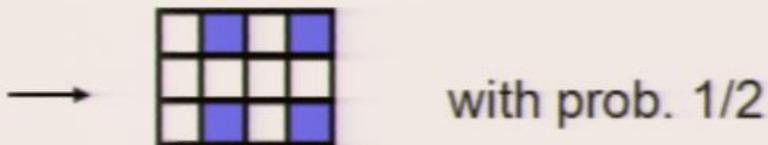
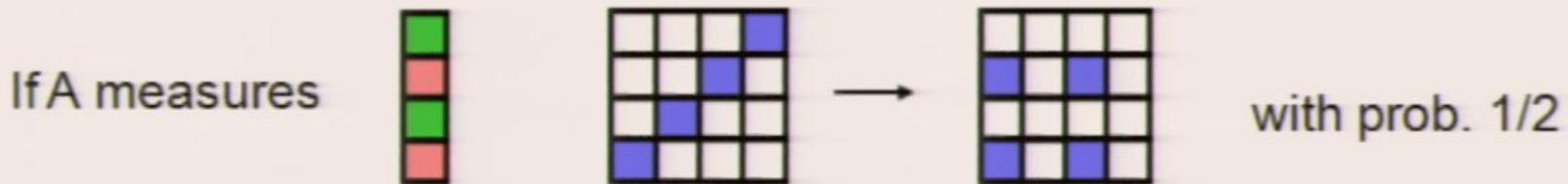
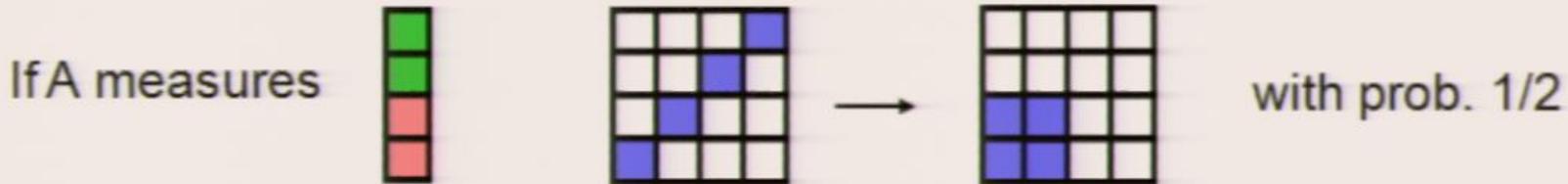
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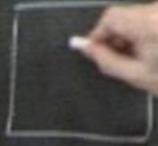
$$(1+x)$$

$$[(x_1, \beta_1), (x_2, \beta_2)]$$

$$x_2 - \beta_1 x_1$$

$$\beta_1 \beta_2$$

$$S_3$$



$$T_3$$

$$(1+x)$$

$(+x)$

$$[(x_1, p_1), (x_2, p_2)] = x_1 p_2 - x_2 p_1$$

S_z

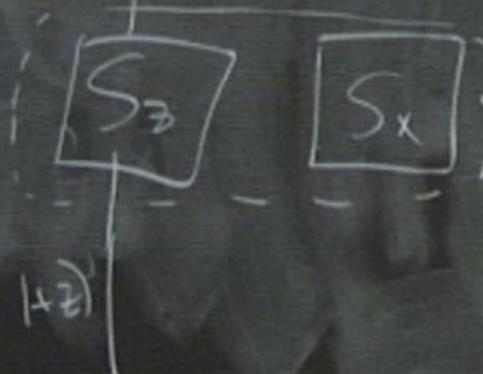
S_x

$(+y)$

$(+z)$

$(+x)$

$$[(x_1, p_1), (x_2, p_2)] = x_1 p_2 - p_1 x_2 \\ = x_1 x_2 + p_1 p_2$$



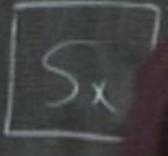
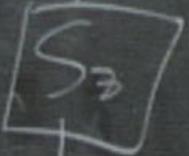
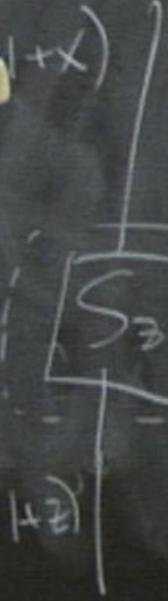
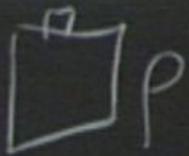
α β

ρ

$(1+x)$

S

$$\begin{aligned} (x_1, \beta_1), (x_2, \beta_2) &= x_1 \beta_2 - \beta_1 x_2 \\ &= x_1 x_2 + \beta_1 \beta_2 \end{aligned}$$



$$[(x, p_1), (x, p_2 - p_1 x_2 + p_1 p_2)]$$

