

Title: Foundations of Quantum Mechanics - Lecture 7

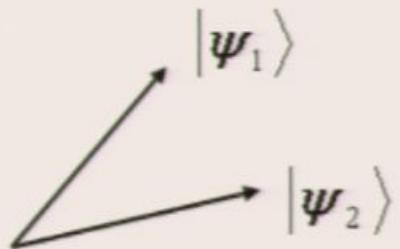
Date: Jan 11, 2011 11:30 AM

URL: <http://pirsa.org/11010046>

Abstract:

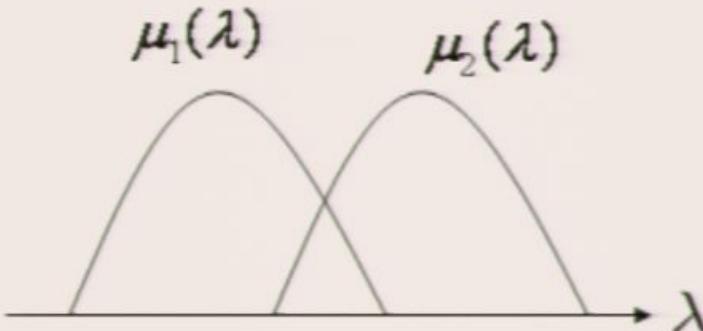
Example 1: The impossibility of discriminating non-orthogonal states with certainty

Consider



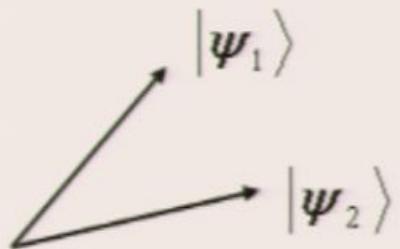
ψ -ontic view: Mysterious. No analogue of non-orthogonality.

ψ -epistemic view: Natural



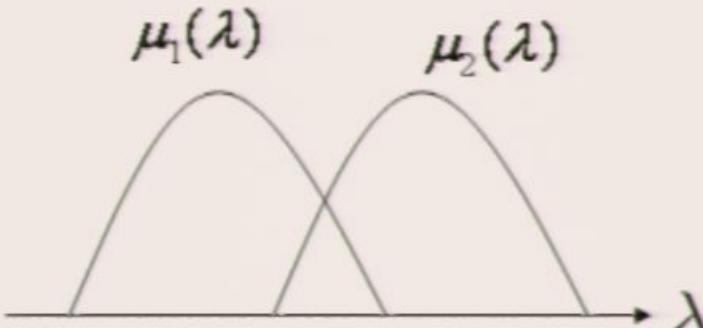
Example 1: The impossibility of discriminating non-orthogonal states with certainty

Consider



ψ -ontic view: Mysterious. No analogue of non-orthogonality.

ψ -epistemic view: Natural



Example 2: Lack of exponential divergence of states under chaotic evolution

In Newtonian mechanics, exponential divergence of ontic states is the signature of chaos

Example 2: Lack of exponential divergence of states under chaotic evolution

In Newtonian mechanics, exponential divergence of ontic states is the signature of chaos

In quantum theory,

$$\begin{aligned}\langle \psi_1(t) | \psi_2(t) \rangle &= \langle \psi_1(0) | U^t U | \psi_2(0) \rangle \\ &= \langle \psi_1(0) | \psi_2(0) \rangle\end{aligned}$$

No divergence!

Example 2: Lack of exponential divergence of states under chaotic evolution

In Newtonian mechanics, exponential divergence of ontic states is the signature of chaos

In quantum theory,

$$\begin{aligned}\langle \psi_1(t) | \psi_2(t) \rangle &= \langle \psi_1(0) | U^t U | \psi_2(0) \rangle \\ &= \langle \psi_1(0) | \psi_2(0) \rangle\end{aligned}$$

No divergence!

ψ -ontic view : This is puzzling

ψ -epistemic view : This is natural, due to Liouville's theorem

$$\int d\lambda \sqrt{\mu_1(\lambda, t)} \sqrt{\mu_2(\lambda, t)} = \int d\lambda \sqrt{\mu_1(\lambda, 0)} \sqrt{\mu_2(\lambda, 0)}$$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|x\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|x\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$
By unitarity, the inner product must be constant

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0 \text{ or } 1$ i.e. orthogonal or identical

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

But $|\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$

while $|\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0$ or 1 i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)\nu(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s = 1, 2$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0 \text{ or } 1$ i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)\nu(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s = 1, 2$

By Liouville's theorem, the classical fidelity must be constant

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0$ or 1 i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)v(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s = 1, 2$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0 \text{ or } 1$ i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)\nu(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s = 1, 2$

By Liouville's theorem, the classical fidelity must be constant

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s = 1, 2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0$ or 1 i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)v(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s = 1, 2$

By Liouville's theorem, the classical fidelity must be constant

$$\text{But } \int dz dy \sqrt{\mu_1(z)v(y)} \sqrt{\mu_2(z)v(y)} = \int dz \sqrt{\mu_1(z)} \sqrt{\mu_2(z)}$$

$$\text{while } \int dz dy \sqrt{\mu_1(z)\mu_1(y)} \sqrt{\mu_2(z)\mu_2(y)} = \left(\int dz \sqrt{\mu_1(z)} \sqrt{\mu_2(z)} \right)^2$$

Example 3: The impossibility of cloning non-orthogonal states

(C. Fuchs, 1996)

Cloning the set $\{|\psi_1\rangle, |\psi_2\rangle\}$ implies $|\psi_s\rangle|\chi\rangle \rightarrow |\psi_s\rangle|\psi_s\rangle$ for $s=1,2$

By unitarity, the inner product must be constant

$$\text{But } |\langle\psi_1|\langle\chi|(\psi_2\rangle|\chi\rangle)| = |\langle\psi_1|\psi_2\rangle|$$

$$\text{while } |\langle\psi_1|\langle\psi_1|(\psi_2\rangle|\psi_2\rangle)| = |\langle\psi_1|\psi_2\rangle|^2$$

These are equal iff $|\langle\psi_1|\psi_2\rangle| = 0 \text{ or } 1$ i.e. orthogonal or identical

ψ -epistemic view:

Cloning the set $\{\mu_1(z), \mu_2(z)\}$ implies $\mu_s(z)v(y) \rightarrow \mu_s(z)\mu_s(y)$ for $s=1,2$

By Liouville's theorem, the classical fidelity must be constant

$$\text{But } \int dz dy \sqrt{\mu_1(z)v(y)} \sqrt{\mu_2(z)v(y)} = \int dz \sqrt{\mu_1(z)} \sqrt{\mu_2(z)}$$

$$\text{while } \int dz dy \sqrt{\mu_1(z)\mu_1(y)} \sqrt{\mu_2(z)\mu_2(y)} = \left(\int dz \sqrt{\mu_1(z)} \sqrt{\mu_2(z)} \right)^2$$

These are equal iff $\int dz \sqrt{\mu_1(z)} \sqrt{\mu_2(z)} = 0 \text{ or } 1$ i.e. disjoint or identical

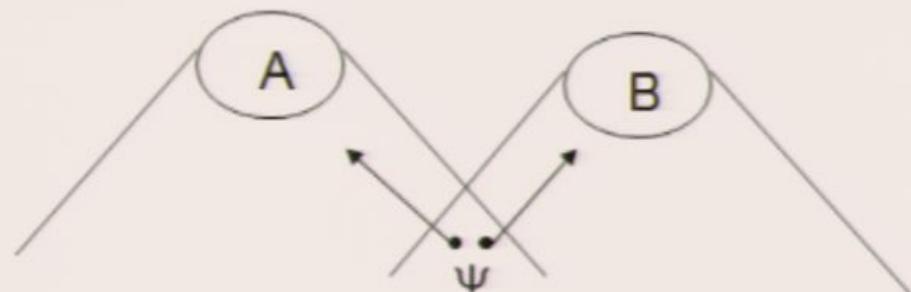
$$F(p_1(z), p_2(z)) \equiv \int dz \sqrt{P_1(z)} \sqrt{P_2(z)}$$



Example 4: The Einstein-Podolsky-Rosen effect

Suppose A and B share

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle)$$



If A measures $\{|+z\rangle, |-z\rangle\}$

B's state becomes $\begin{cases} |+z\rangle & \text{with probability } 1/2 \\ |-z\rangle & \text{with probability } 1/2 \end{cases}$

If A measures $\{|+x\rangle, |-x\rangle\}$

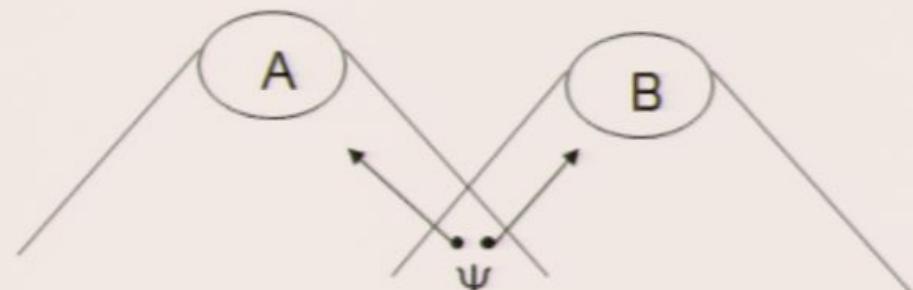
B's state becomes $\begin{cases} |+x\rangle & \text{with probability } 1/2 \\ |-x\rangle & \text{with probability } 1/2 \end{cases}$

“Steering”

Example 4: The Einstein-Podolsky-Rosen effect

Suppose A and B share

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle)$$



If A measures $\{|+z\rangle, |-z\rangle\}$

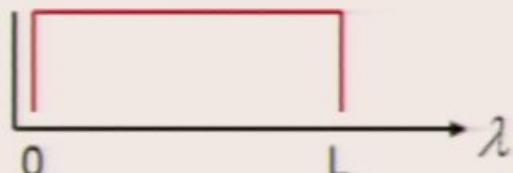
B's state becomes $\begin{cases} |+z\rangle & \text{with probability } 1/2 \\ |-z\rangle & \text{with probability } 1/2 \end{cases}$

If A measures $\{|+x\rangle, |-x\rangle\}$

B's state becomes $\begin{cases} |+x\rangle & \text{with probability } 1/2 \\ |-x\rangle & \text{with probability } 1/2 \end{cases}$

“Steering”

$$p(\lambda', \lambda) = \frac{1}{L} \delta(\lambda - \lambda') \quad \lambda, \lambda' \in [0, L]$$

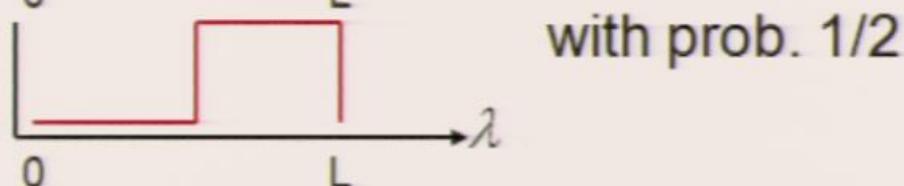


If A measures whether $\lambda' \in [0, \frac{L}{2}]$ or not

Her knowledge of B is updated to



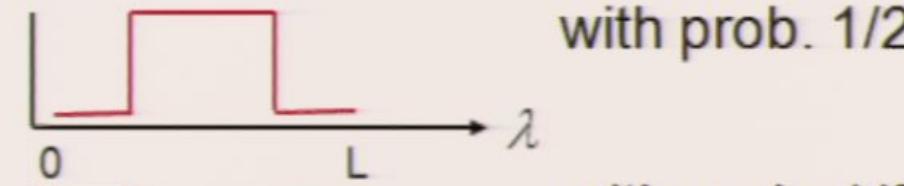
with prob. 1/2



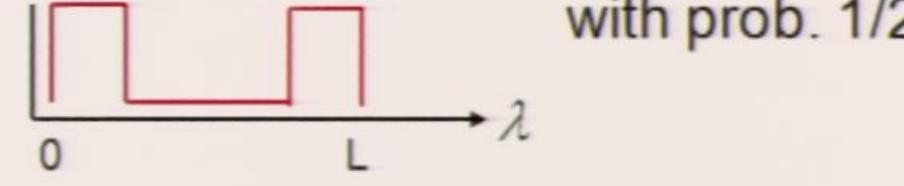
with prob. 1/2

If A measures whether $\lambda' \in [\frac{L}{4}, \frac{3L}{4}]$ or not

Her knowledge of B is updated to



with prob. 1/2

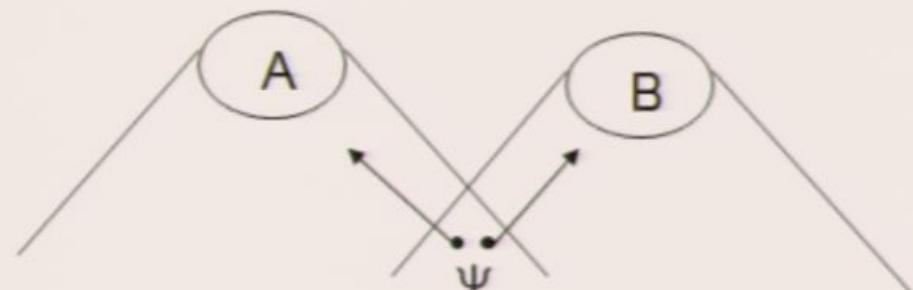


with prob. 1/2

Example 4: The Einstein-Podolsky-Rosen effect

Suppose A and B share

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle)$$



If A measures $\{|+z\rangle, |-z\rangle\}$

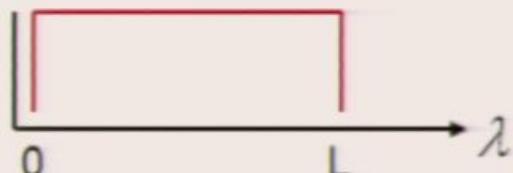
B's state becomes $\begin{cases} |+z\rangle & \text{with probability } 1/2 \\ |-z\rangle & \text{with probability } 1/2 \end{cases}$

If A measures $\{|+x\rangle, |-x\rangle\}$

B's state becomes $\begin{cases} |+x\rangle & \text{with probability } 1/2 \\ |-x\rangle & \text{with probability } 1/2 \end{cases}$

“Steering”

$$p(\lambda', \lambda) = \frac{1}{L} \delta(\lambda - \lambda') \quad \lambda, \lambda' \in [0, L]$$

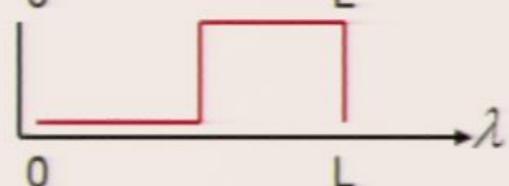


If A measures whether $\lambda' \in [0, \frac{L}{2}]$ or not

Her knowledge of B is updated to



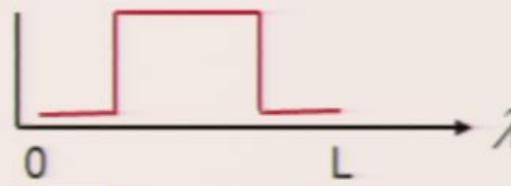
with prob. 1/2



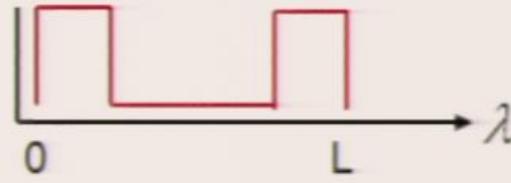
with prob. 1/2

If A measures whether $\lambda' \in [\frac{L}{4}, \frac{3L}{4}]$ or not

Her knowledge of B is updated to



with prob. 1/2



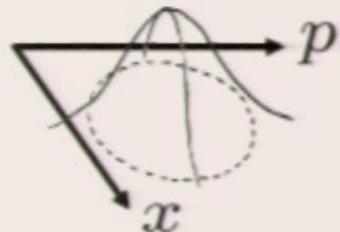
with prob. 1/2

Liouville mechanics with an epistemic restriction is Gaussian Quantum Mechanics

Based primarily on unpublished work
with Stephen Bartlett and Terry Rudolph

Liouville mechanics

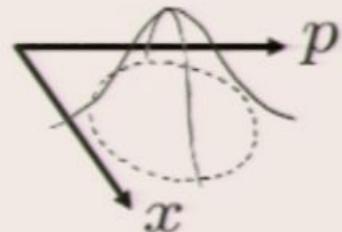
$\mu(x, p)$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Liouville mechanics

$$\mu(x, p)$$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Quantum mechanics

Uncertainty principle:

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

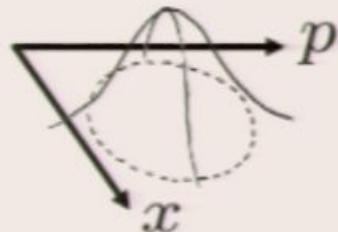
$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$

$$C_{x,p} \equiv \frac{1}{2} \langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$

$$\langle \hat{A} \rangle \equiv \text{Tr}(\hat{A}\hat{\rho})$$

Liouville mechanics

$$\mu(x, p)$$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Quantum mechanics

Uncertainty principle:

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$

$$C_{x,p} \equiv \frac{1}{2} \langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$

$$\langle \hat{A} \rangle \equiv \text{Tr}(\hat{A}\hat{\rho})$$

Liouville mechanics with an epistemic restriction

Uncertainty principle:

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

$$\Delta^2 x \equiv \langle x^2 \rangle - \langle x \rangle^2$$

$$C_{x,p} \equiv \langle xp \rangle - \langle x \rangle \langle p \rangle$$

$$\langle f(x, p) \rangle \equiv \int dx dp f(x, p) \mu(x, p)$$

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

and that have maximal entropy for a given set of second-order moments.

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

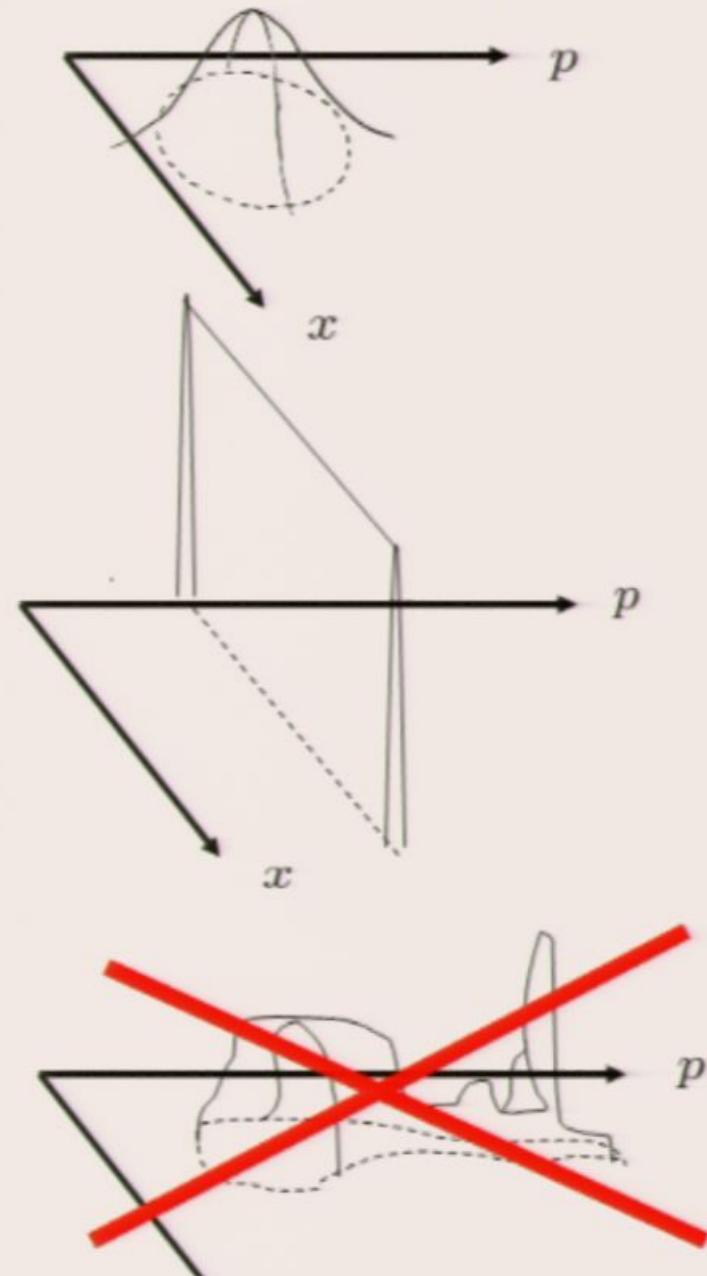
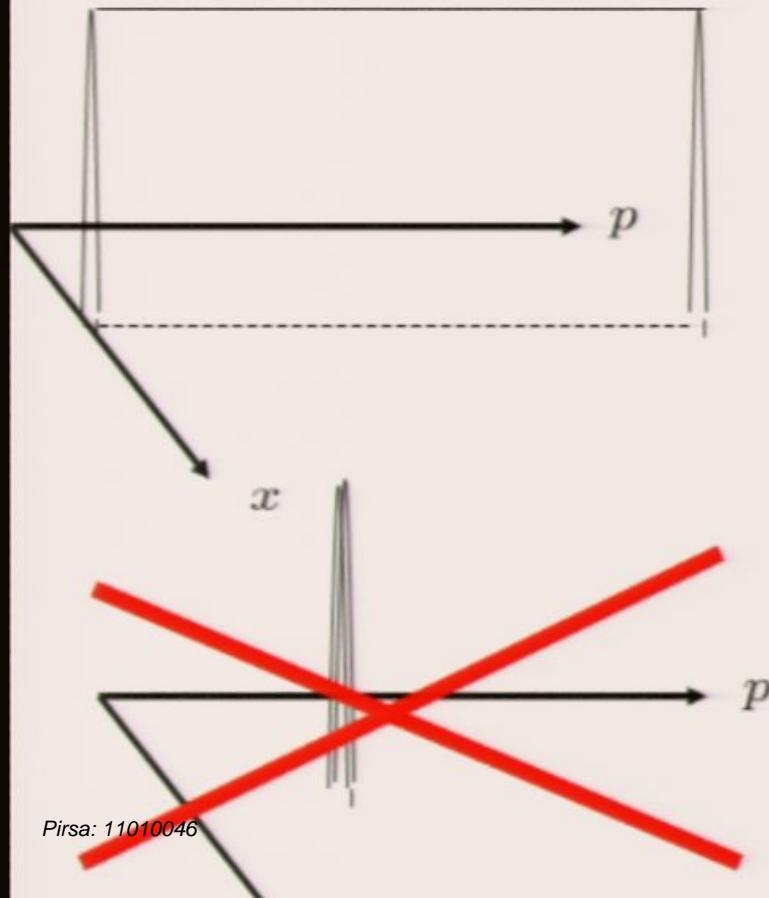
and that have maximal entropy for a given set of second-order moments.

Among $\mu(x,p)$ with a given set of second-order moments, Gaussian distributions maximize the entropy

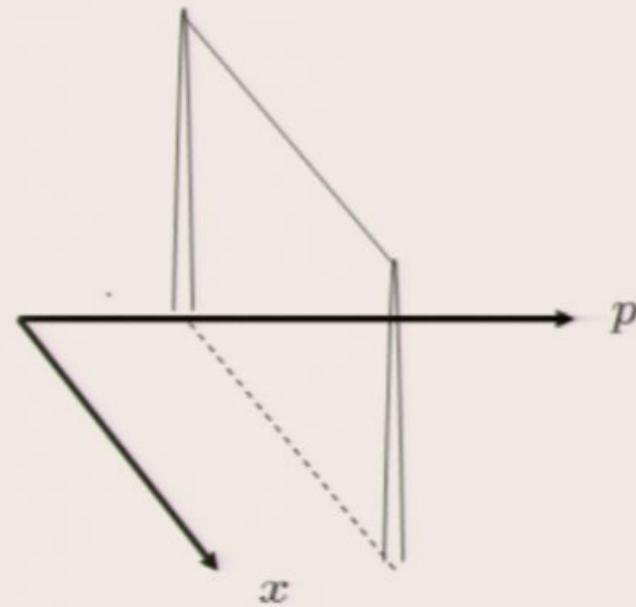
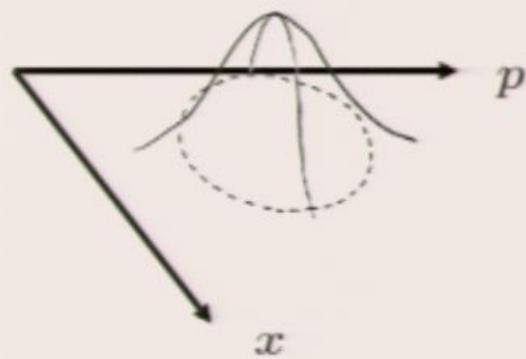
Valid epistemic states for one canonical system

$$\mu(x, p) \geq 0$$

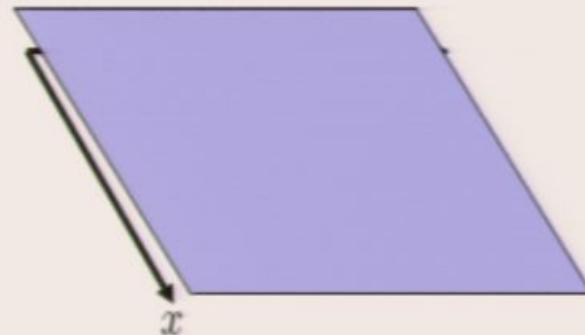
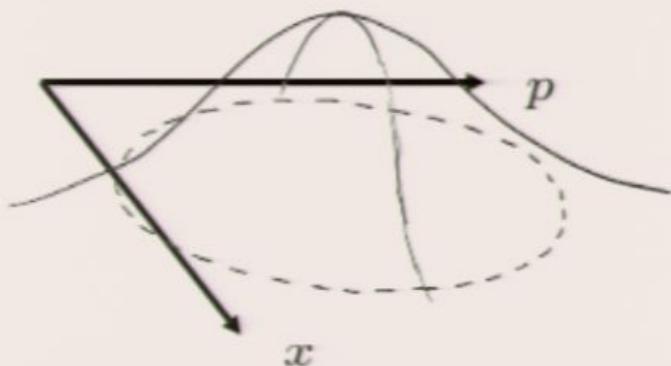
$$\int \mu(x, p) dx dp = 1$$



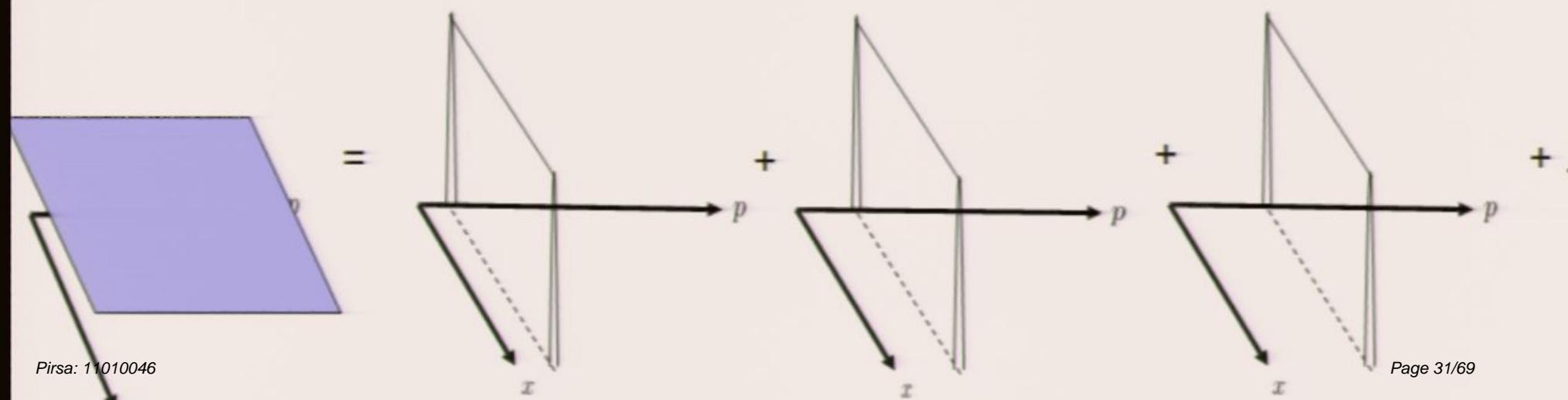
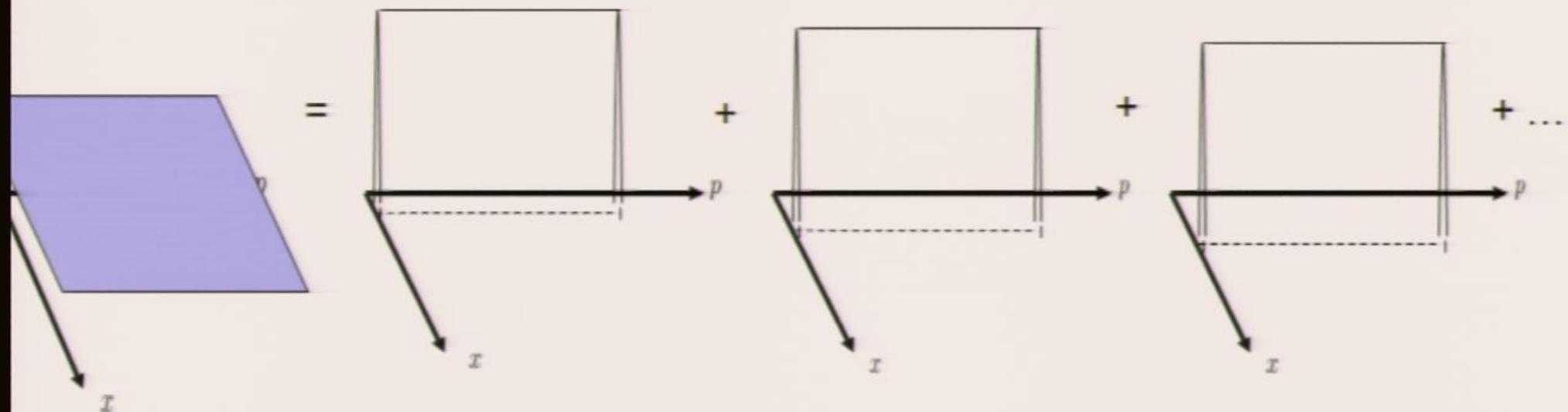
Pure epistemic states



Mixed epistemic states



Multiplicity of convex decompositions of a mixed epistemic state into pure epistemic states



Quantum mechanics

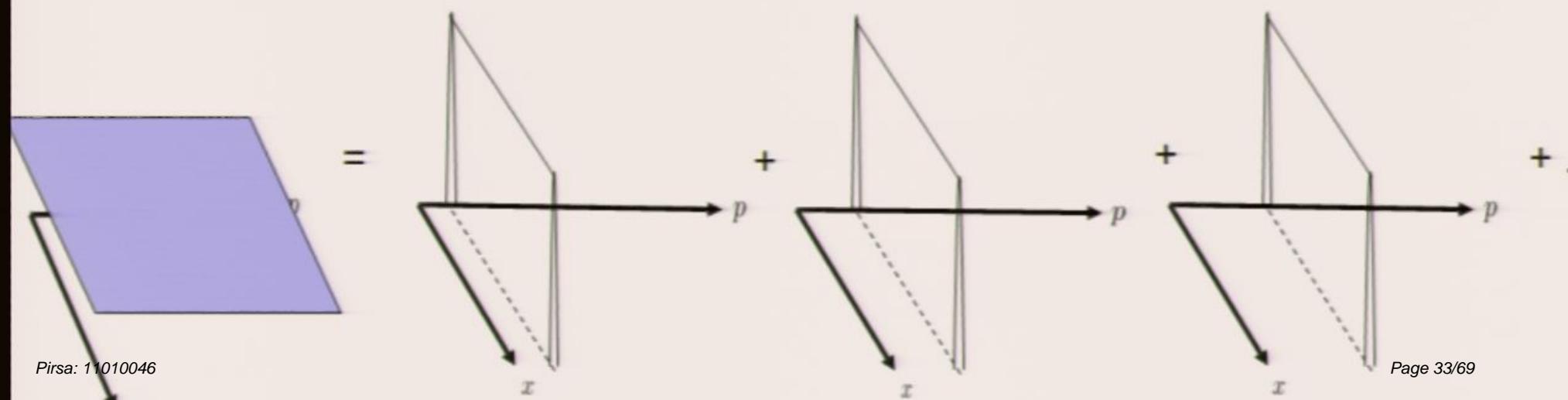
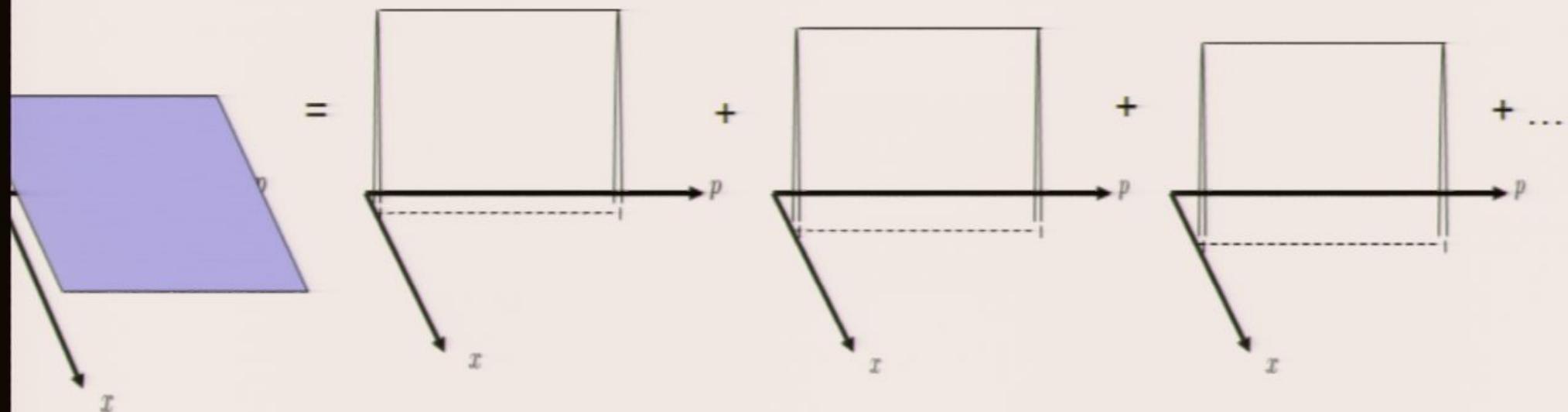
Uncertainty principle:

$$\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0$$

$$(\hat{\rho}) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \dots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0 & -1 & & \dots \\ 1 & 0 & & \\ & 0 & -1 & \\ & 1 & 0 & \\ \vdots & & & \ddots \end{pmatrix}$$

Multiplicity of convex decompositions of a mixed epistemic state into pure epistemic states



Quantum mechanics

Uncertainty principle:

$$\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0$$

$$(\hat{\rho}) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \dots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0 & -1 & & \dots \\ 1 & 0 & & \\ & 0 & -1 & \\ & 1 & 0 & \\ \vdots & & & \ddots \end{pmatrix}$$

Quantum mechanics

Uncertainty principle:

$$\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0$$

$$(\hat{\rho}) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \dots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \dots \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0 & -1 & & \dots \\ 1 & 0 & & \\ & 0 & -1 & \\ & & 1 & 0 & \\ \vdots & & & & \dots \end{pmatrix}$$

Single particle in 1d:

$$2 \begin{pmatrix} \Delta^2 x & C_{x,p} \\ C_{p,x} & \Delta^2 p \end{pmatrix} + i\hbar \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \geq 0$$

$$2 \begin{pmatrix} \Delta^2 x & C_{x,p} - \frac{1}{2}i\hbar \\ C_{p,x} + \frac{1}{2}i\hbar & \Delta^2 p \end{pmatrix} \geq 0$$

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

Quantum mechanics

Uncertainty principle:

$$\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0$$

$$(\hat{\rho}) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \dots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0 & -1 & & \dots \\ 1 & 0 & & \\ & 0 & -1 & \\ & 1 & 0 & \\ \vdots & & & \ddots \end{pmatrix}$$

Liouville mechanics with an epistemic restriction

Uncertainty principle:

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

$$\gamma(\mu) = 2 \begin{pmatrix} \Delta^2 x_1 & C_{x_1,p_1} & C_{x_1,x_2} & C_{x_1,p_2} & \dots \\ C_{p_1,x_1} & \Delta^2 p_1 & C_{p_1,x_2} & C_{p_1,p_2} & \\ C_{x_2,x_1} & C_{x_2,p_1} & \Delta^2 x_2 & C_{x_2,p_2} & \\ C_{p_2,x_1} & C_{p_2,p_1} & C_{p_2,x_2} & \Delta^2 p_2 & \\ \vdots & & & & \ddots \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0 & -1 & & \dots \\ 1 & 0 & & \\ & 0 & -1 & \\ & 1 & 0 & \\ \vdots & & & \ddots \end{pmatrix}$$

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle:

The only Liouville distributions that can be prepared are those that satisfy

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

and that have maximal entropy for a given set of second-order moments.

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle:

The only Liouville distributions that can be prepared are those that satisfy

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

and that have maximal entropy for a given set of second-order moments.

Among valid μ with a given γ , multi-variate Gaussians maximize the entropy

$$\mu(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle)\right)$$

Valid epistemic states for a pair of canonical systems

Uncorrelated distributions

$$\mu(x_1, p_1, x_2, p_2) = \mu(x_1, p_1) \mu(x_2, p_2)$$

Correlated distributions

e.g. $\mu(x_1, p_1, x_2, p_2) = \frac{1}{N} \delta(x_1 - x_2) \delta(p_1 + p_2)$

This corresponds to the **entangled state** of Einstein, Podolsky and Rosen

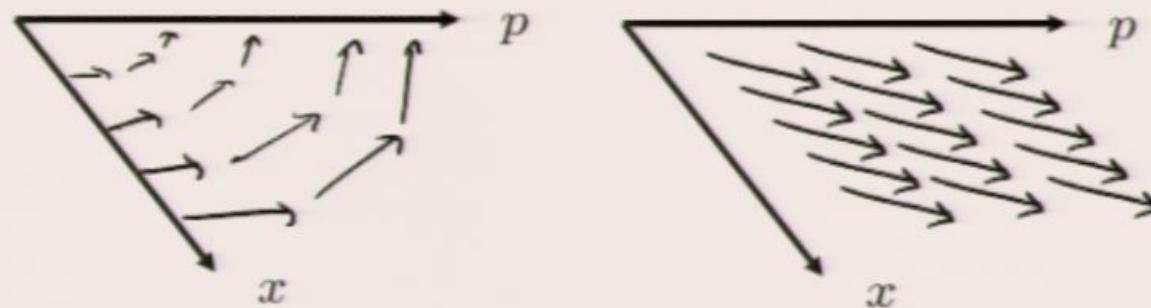
$$\begin{aligned} |\psi\rangle &= \int dx_1 dx_2 \delta(x_1 - x_2) |x_1\rangle |x_2\rangle \\ &= \int dp_1 dp_2 \delta(p_1 + p_2) |p_1\rangle |p_2\rangle \end{aligned}$$

Valid deterministic transformations

The group of canonical transformations with quadratic Hamiltonian

Only canonical transformations preserve the uncertainty principle

Only quadratic Hamiltonians preserve the gaussianity

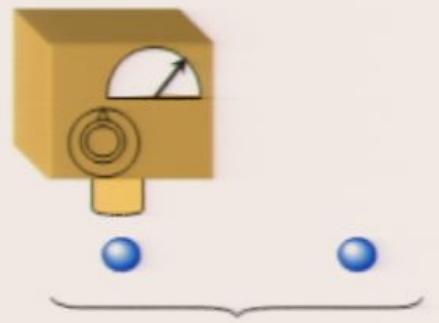


Valid measurements

Sets of indicator functions $\{\xi_k(x, p)\}$

$\xi_k(x, p)$ = probability of k given (x,p)

$$\xi_k(x_1, p_1)$$



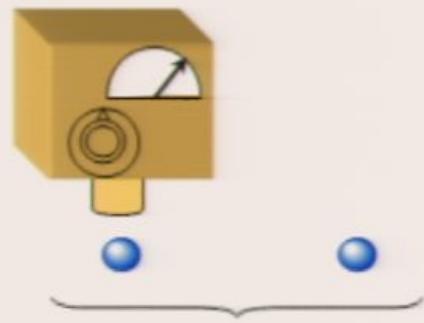
$$\mu(x_1, p_1, x_2, p_2)$$

Valid measurements

Sets of indicator functions $\{\xi_k(x, p)\}$

$\xi_k(x, p)$ = probability of k given (x,p)

$$\xi_k(x_1, p_1)$$



$$\mu(x_1, p_1, x_2, p_2)$$

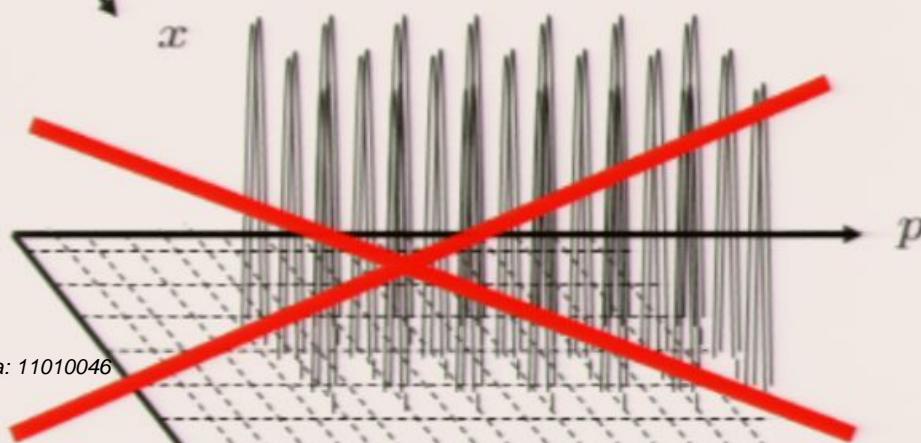
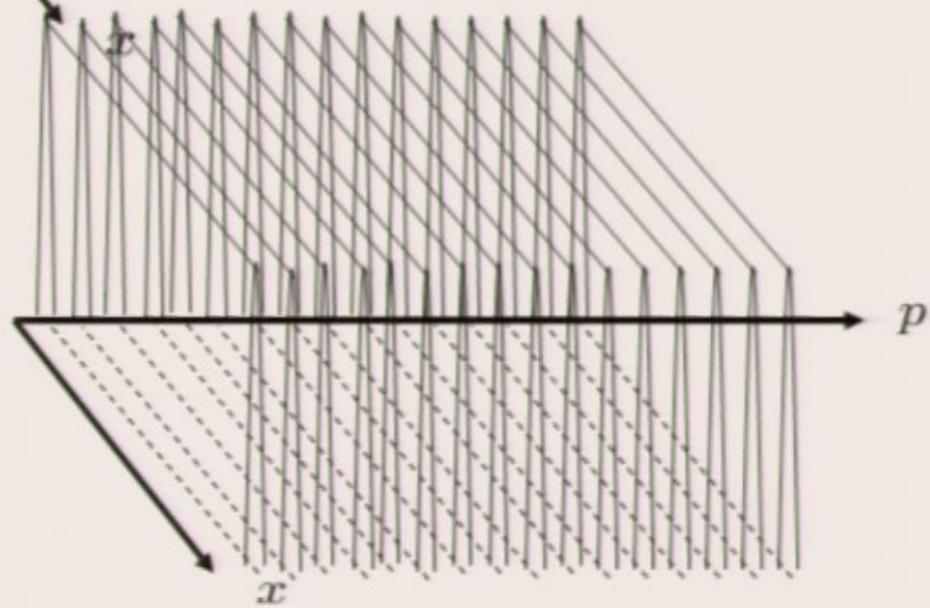
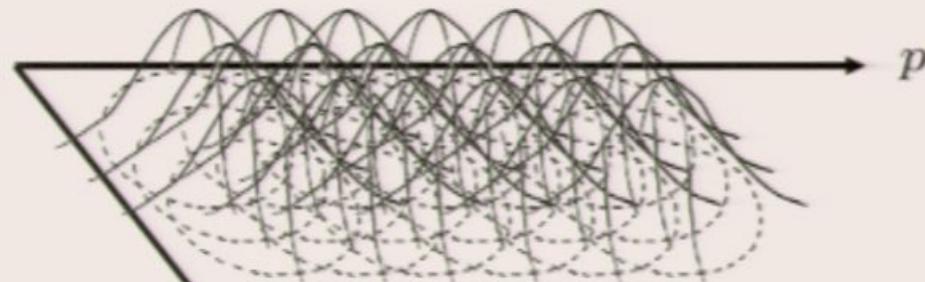
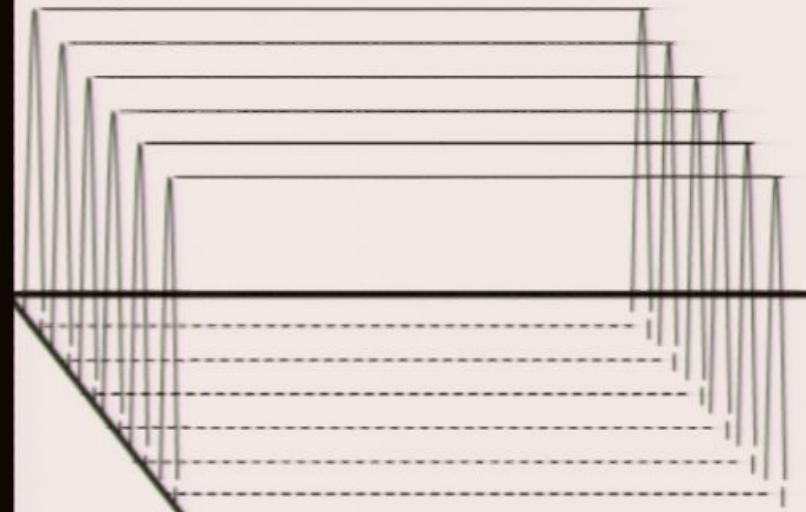
$$\begin{aligned}\mu(x_2, p_2) &\propto \int dx_1 dp_1 \xi(x_1, p_1) \delta(x_1 - x_2) \delta(p_1 + p_2) \\ &= \xi(x_2, -p_2)\end{aligned}$$

$$\mu(x_2, p_2) \propto \int dx_1 dp_1 \xi(x_1, p_1) \mu(x_1, p_1, x_2, p_2)$$

Valid measurements for one canonical system

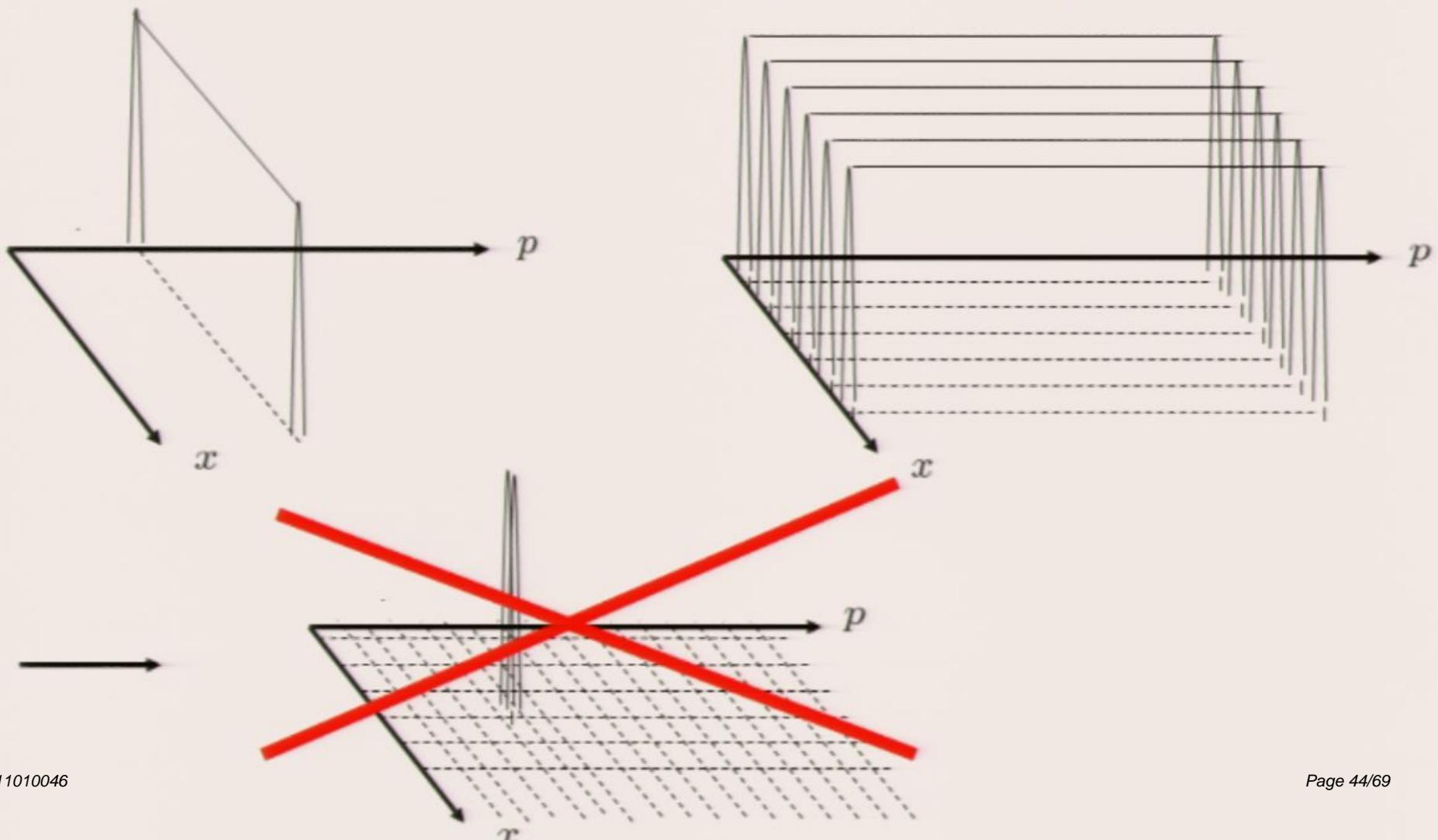
$$\xi_k(x, p) \geq 0$$

$$\sum_k \xi_k(x, p) = 1 \quad \forall x \forall p$$



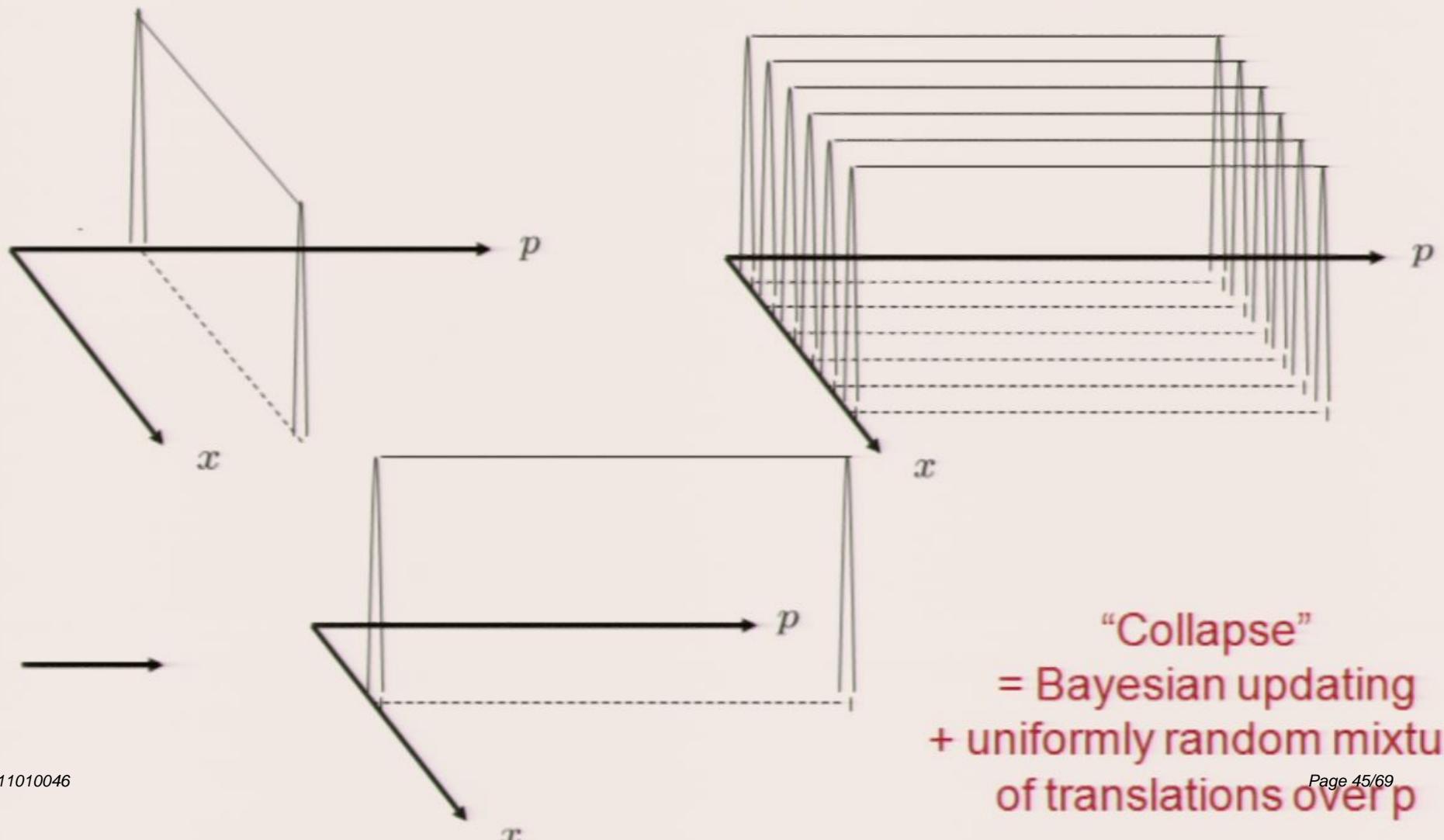
Measurement-induced transformations

Measure x in a
reproducible way



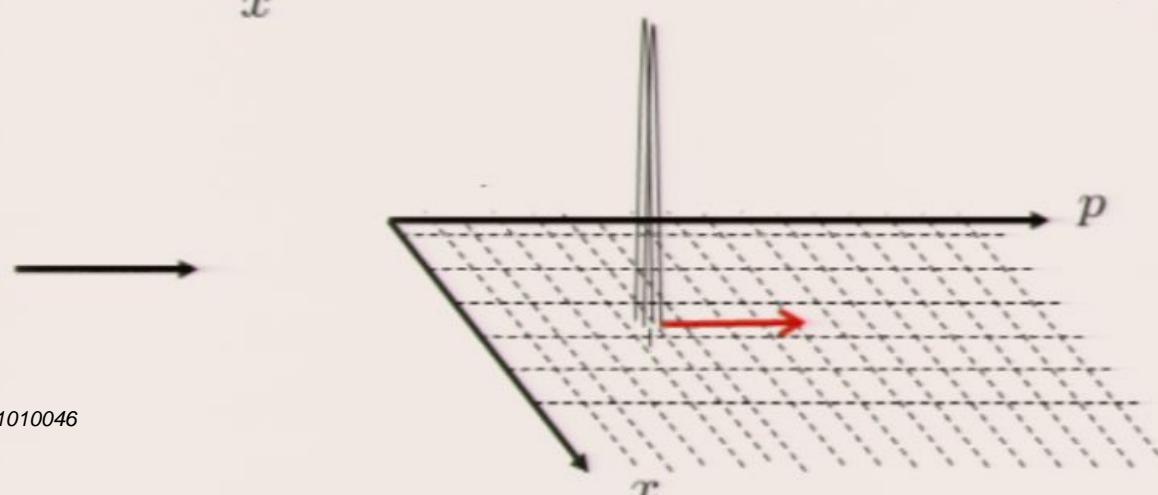
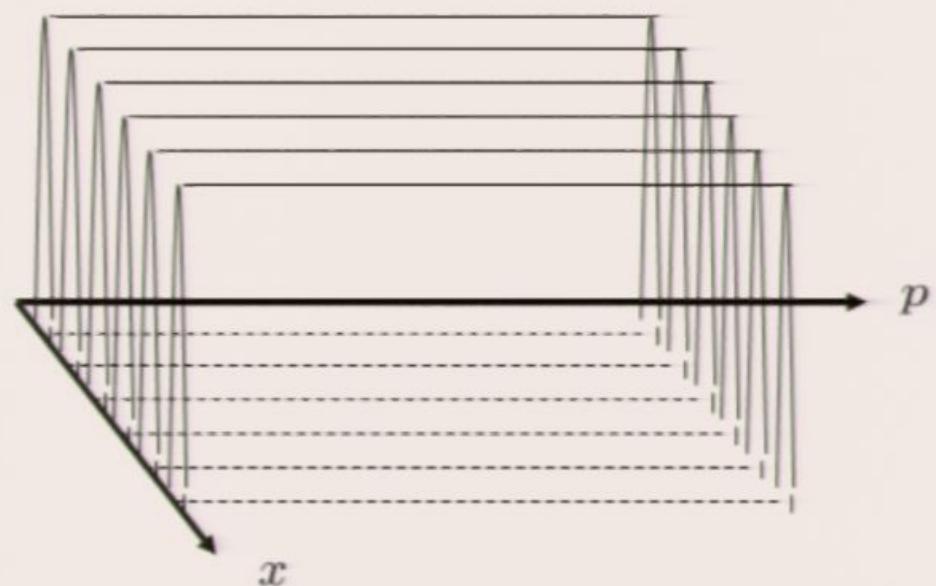
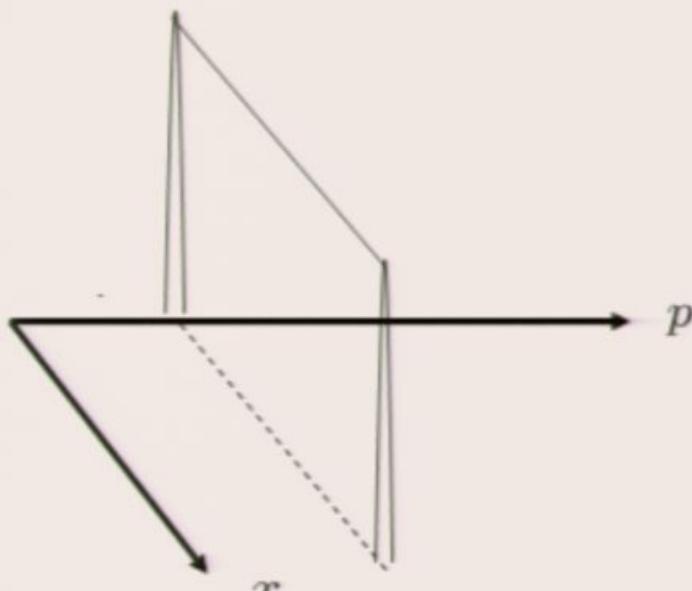
Measurement-induced transformations

Measure x in a
reproducible way



Measurement-induced transformations

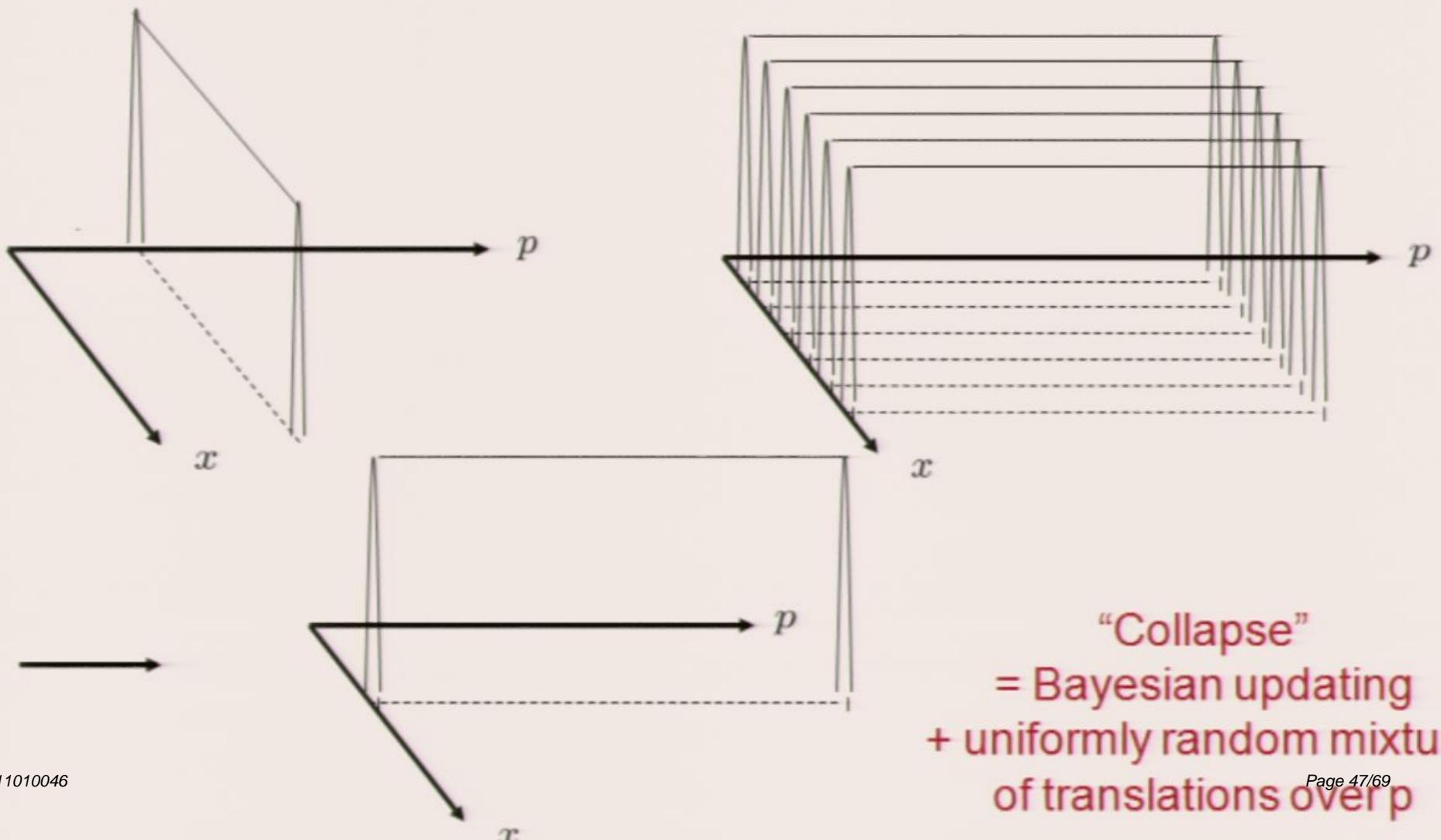
Measure x in a
reproducible way



“Collapse”
= Bayesian updating
+ uniformly random mixture
of translations over p

Measurement-induced transformations

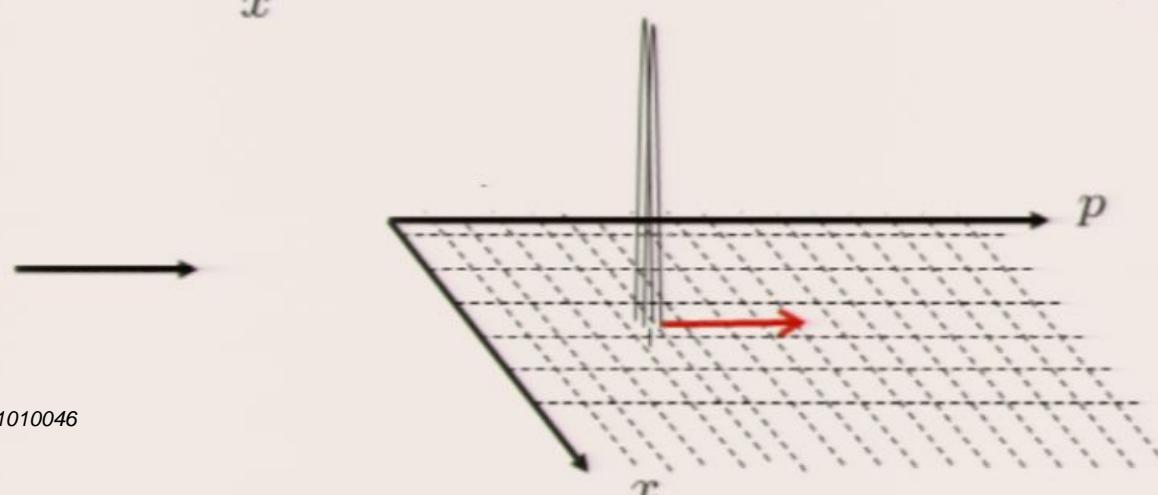
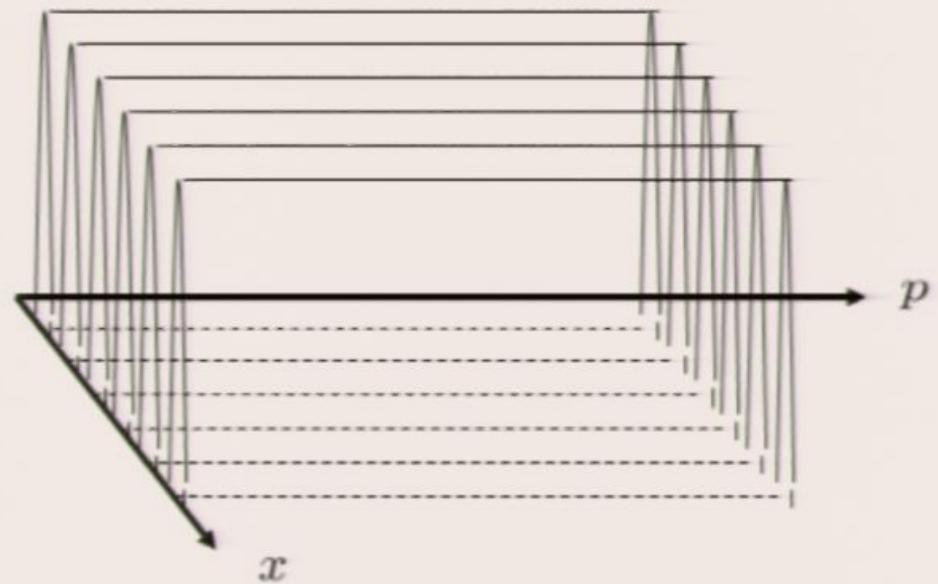
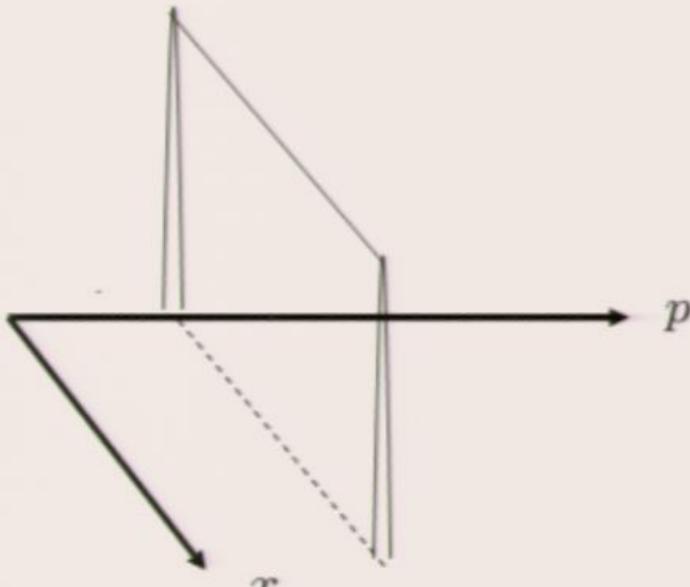
Measure x in a
reproducible way



“Collapse”
= Bayesian updating
+ uniformly random mixture
of translations over p

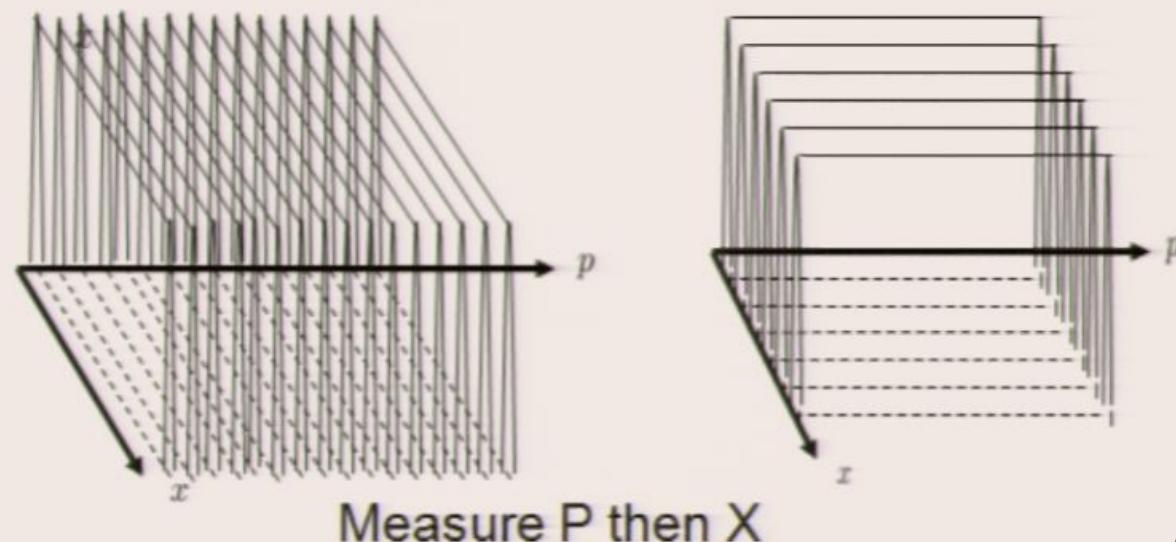
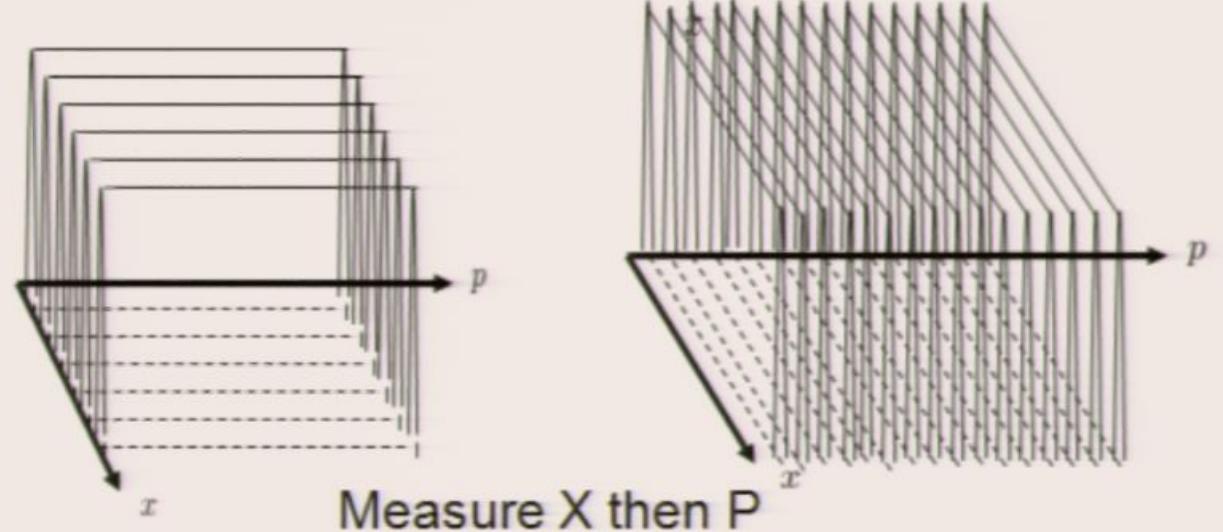
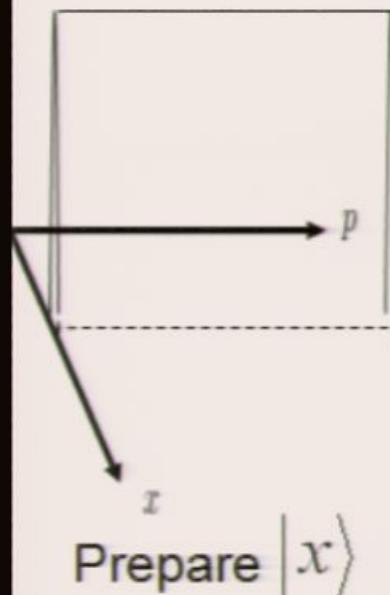
Measurement-induced transformations

Measure x in a
reproducible way

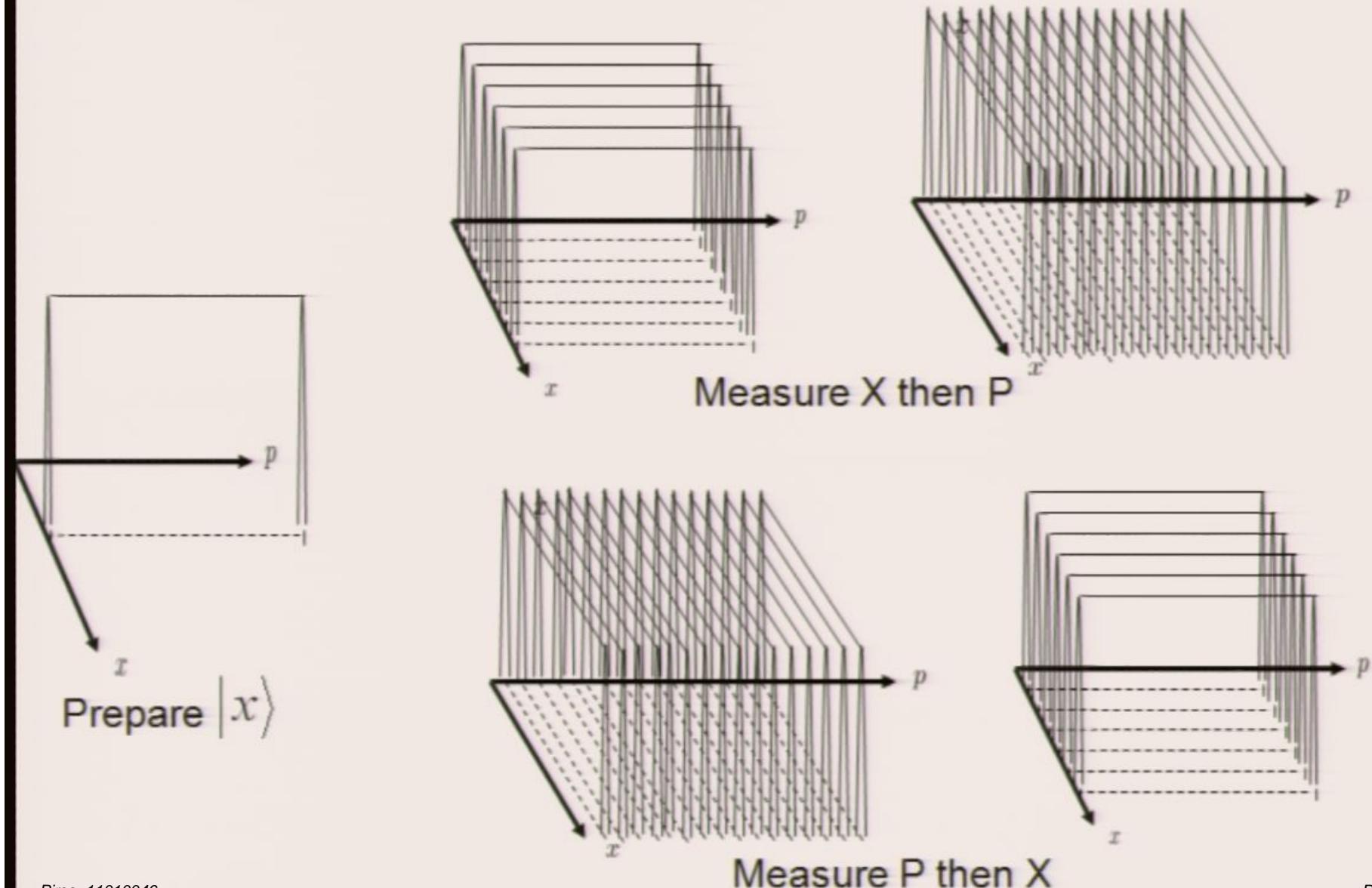


“Collapse”
= Bayesian updating
+ uniformly random mixture
of translations over p

Non-commutativity of measurements



Non-commutativity of measurements



Note: the evolution is *deterministic* if the apparatus is treated internally

External apparatus



Unknown disturbance to p

Internal apparatus

Measure x

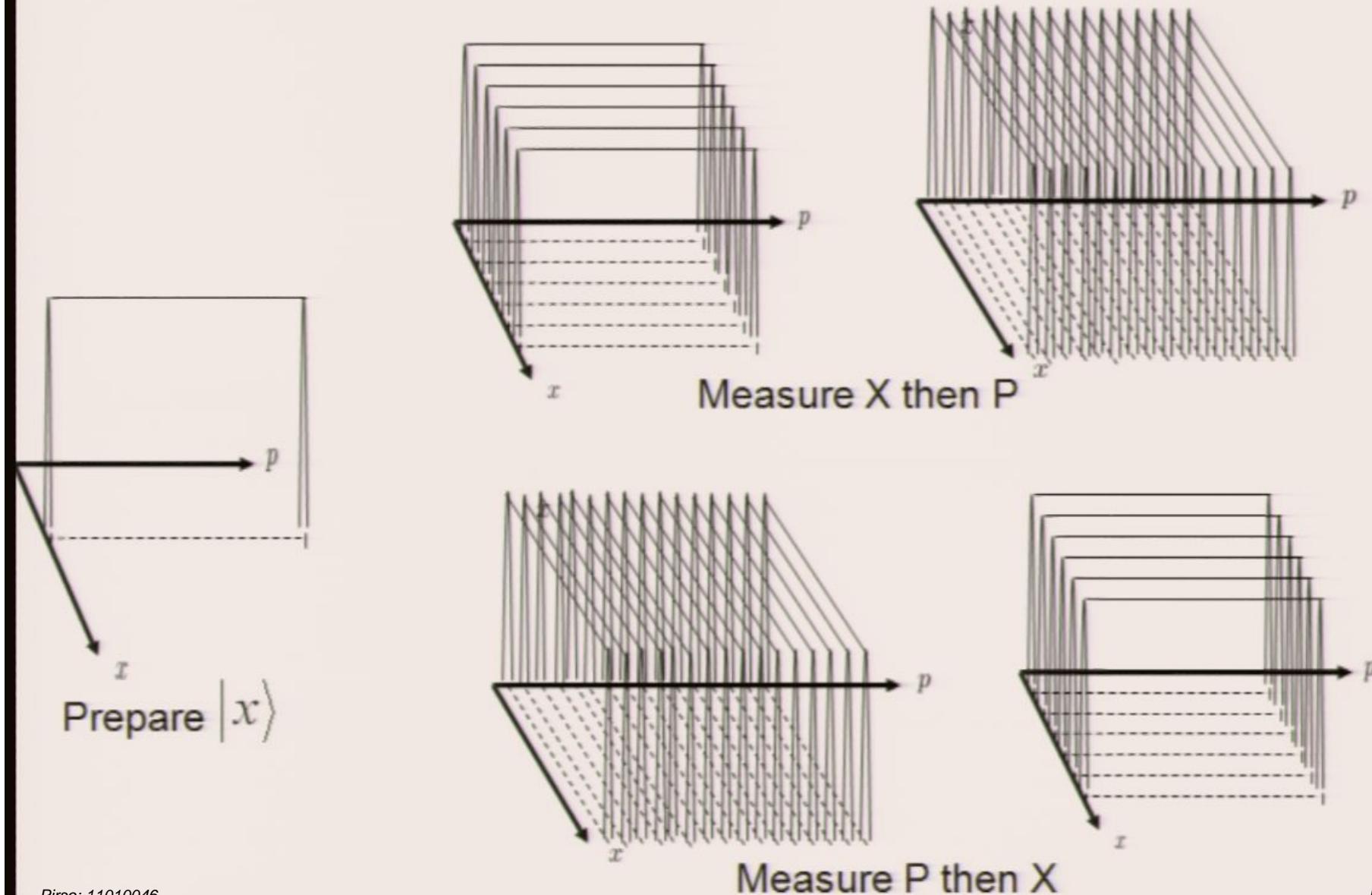


Interact by $H_{int} = x_{sys} p_{app}$



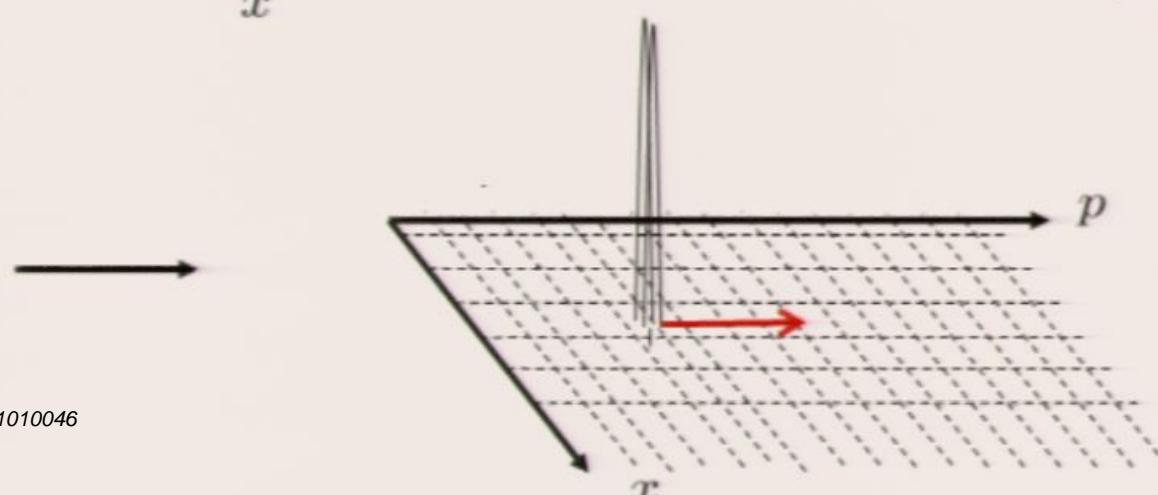
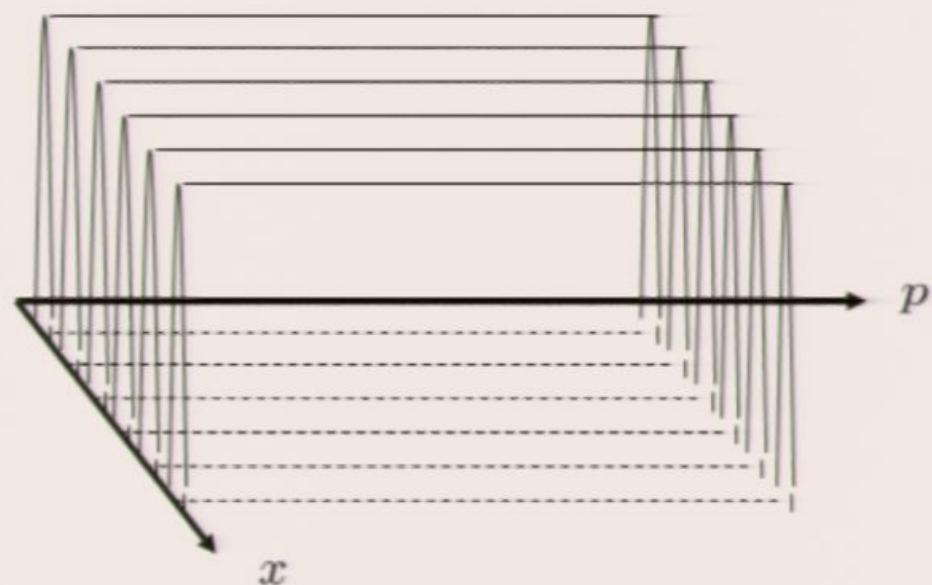
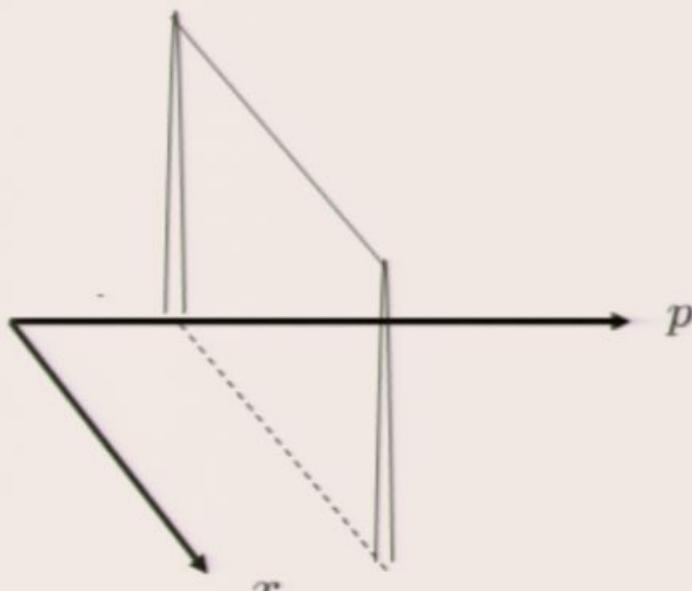
Final x of apparatus reflects initial x of system
Final p of system reflects initial p of apparatus

Non-commutativity of measurements



Measurement-induced transformations

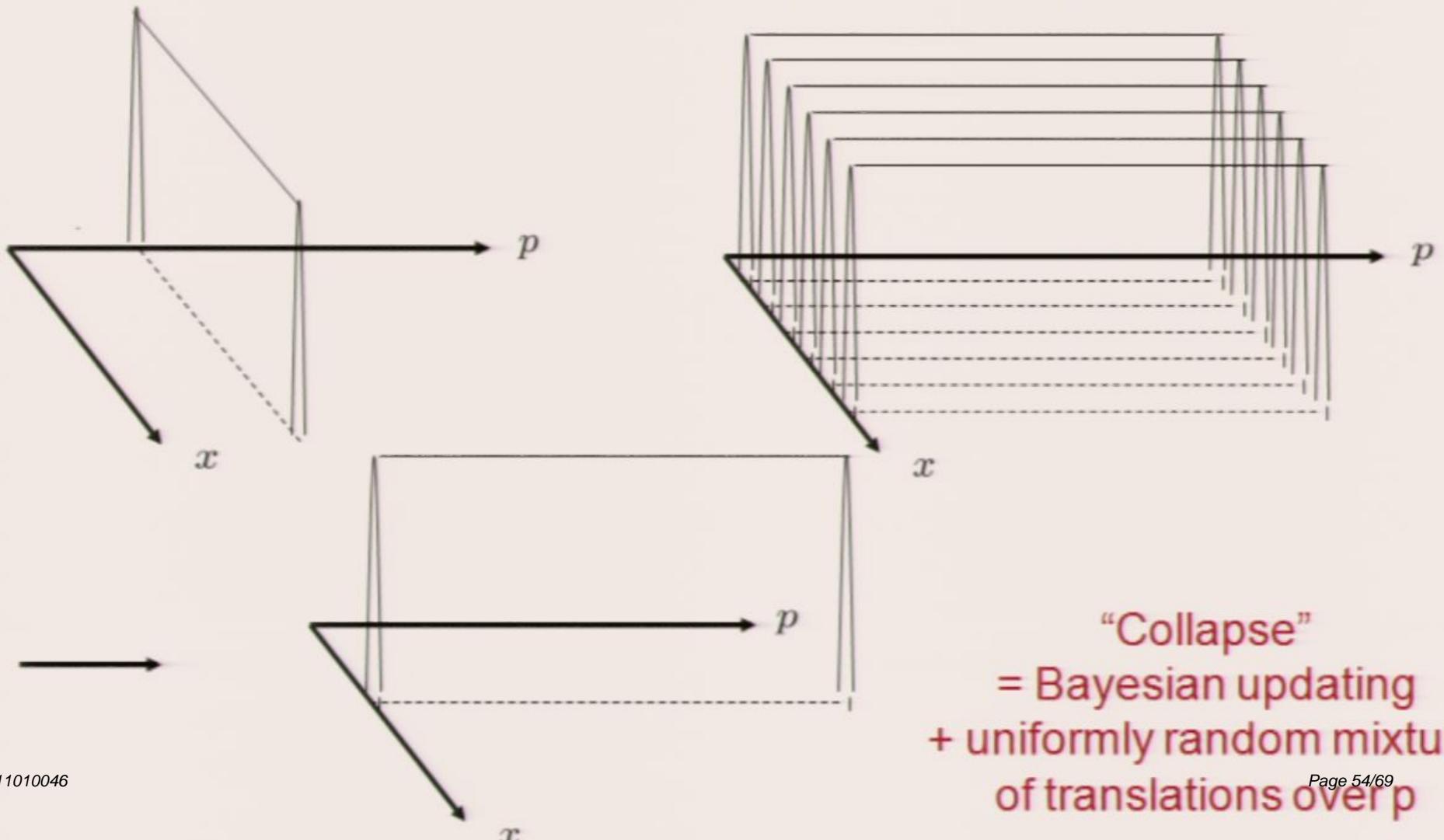
Measure x in a
reproducible way



“Collapse”
= Bayesian updating
+ uniformly random mixture
of translations over p

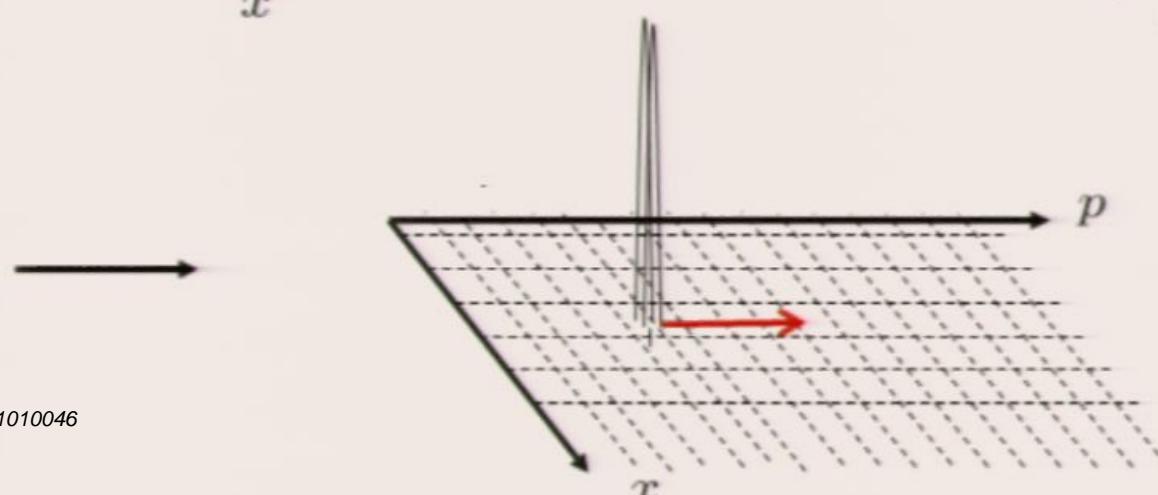
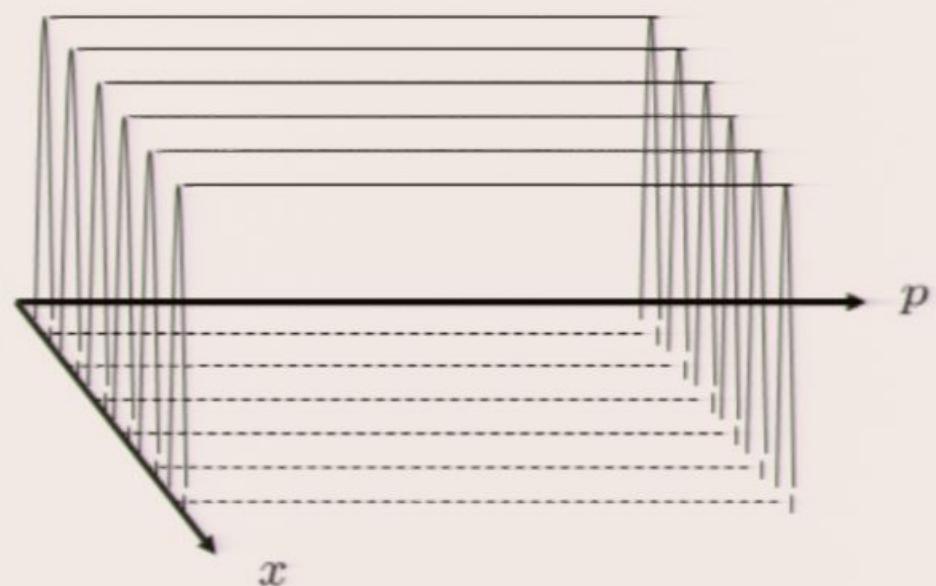
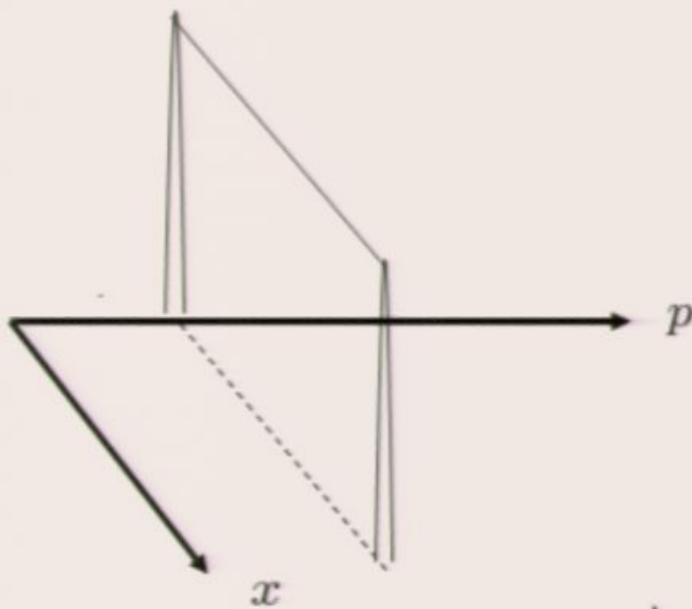
Measurement-induced transformations

Measure x in a
reproducible way



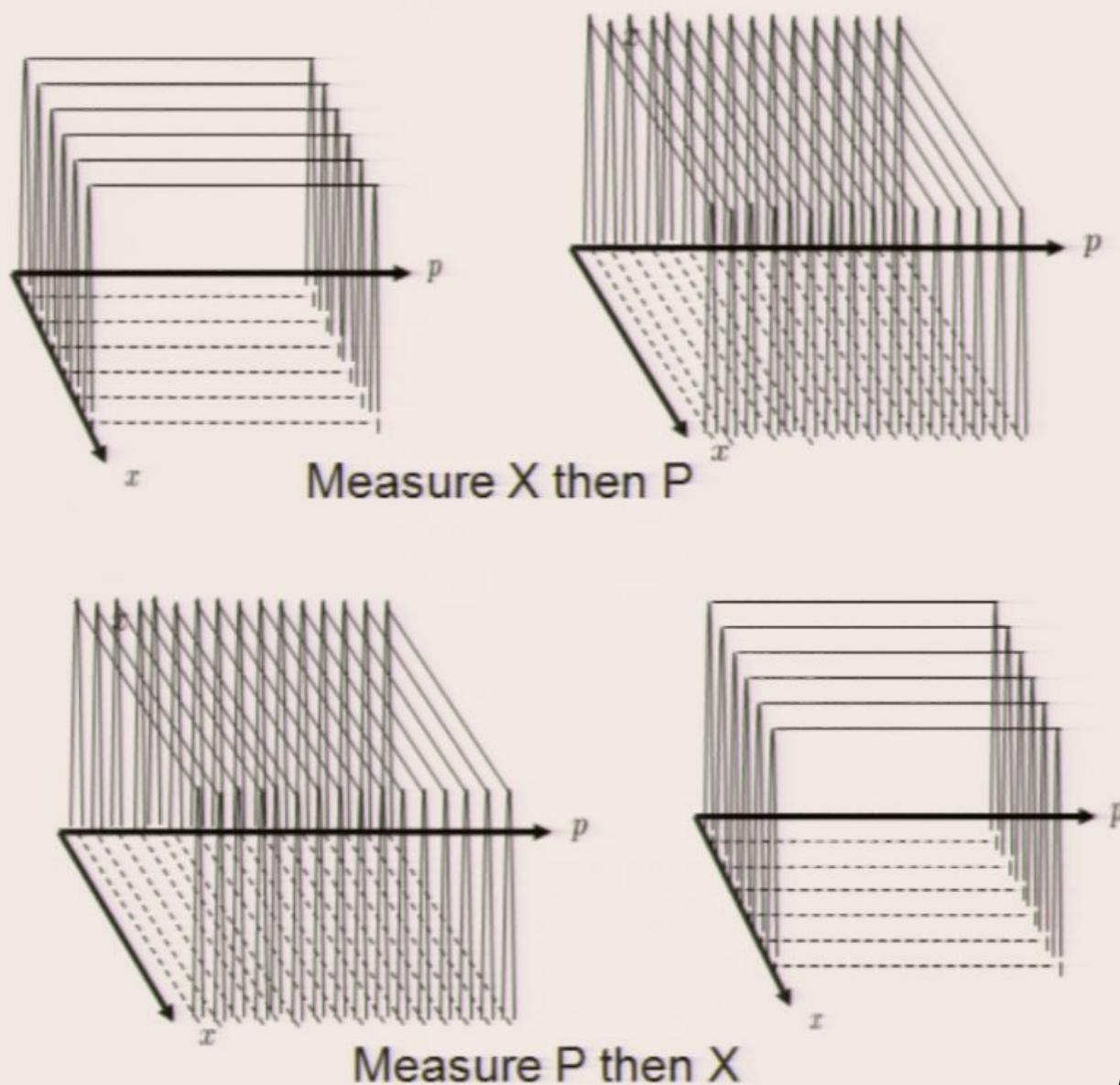
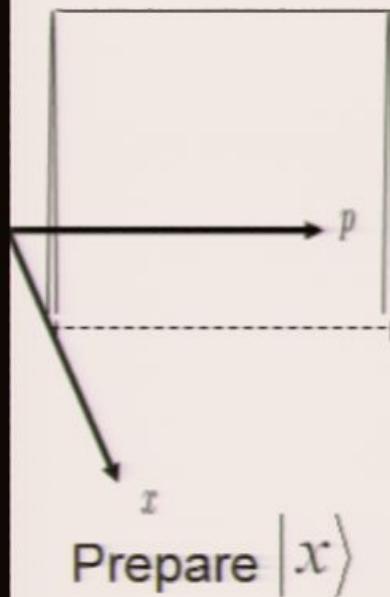
Measurement-induced transformations

Measure x in a
reproducible way



“Collapse”
= Bayesian updating
+ uniformly random mixture
of translations over p

Non-commutativity of measurements



Note: the evolution is *deterministic* if the apparatus is treated internally

External apparatus



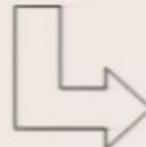
Unknown disturbance to p

Internal apparatus

Measure x

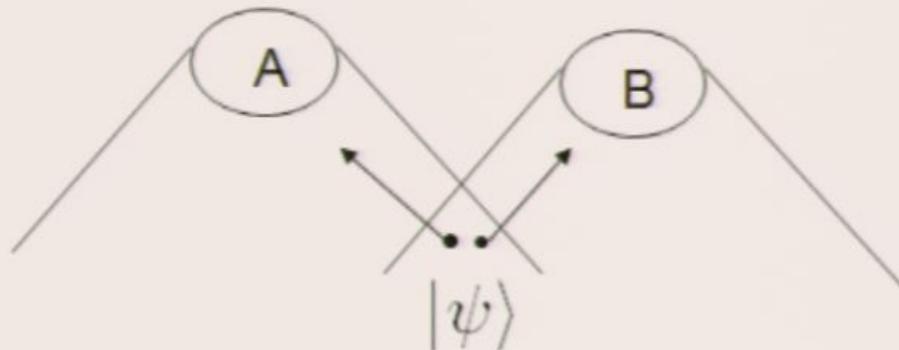


Interact by $H_{int} = x_{sys} p_{app}$



Final x of apparatus reflects initial x of system
Final p of system reflects initial p of apparatus

The Einstein-Podolsky-Rosen argument



$$\begin{aligned} |\psi\rangle &= \int dx_1 dx_2 \delta(x_1 - x_2) |x_1\rangle |x_2\rangle \\ &= \int dp_1 dp_2 \delta(p_1 + p_2) |p_1\rangle |p_2\rangle \end{aligned}$$

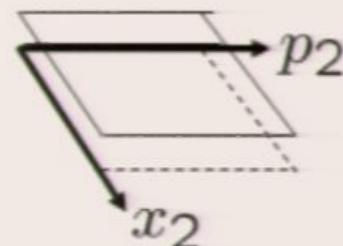
On particle 1, measure either X or P

Outcomes for measurements of X or P on particle 2 become certain

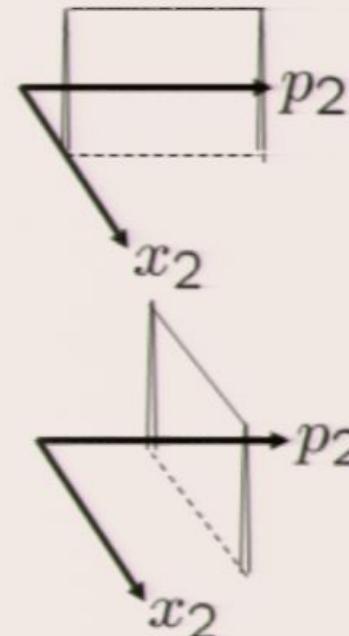
$$\mu(x_1, p_1, x_2, p_2) = \frac{1}{N} \delta(x_1 - x_2) \delta(p_1 + p_2)$$

$$\mu(x_2, p_2) = \frac{1}{N}$$

Initially A is completely ignorant of 2



If A measures x on 1, she infers x of 2



If A measures p on 1, she infers p of 2

A's decision does not affect the reality at 2,
the x and p were already elements of reality

The Wigner representation

Weyl operators $\hat{w}(u, v) = \exp(-iv\hat{x} - iu\hat{p})$

The Wigner representation

Weyl operators $\hat{w}(u, v) = \exp(-iv\hat{x} - iu\hat{p})$

Point operators $\hat{A}(x, p) = \frac{1}{(2\pi)^2} \int du dv \exp(ivx + iup) \hat{w}(u, v)$

The Wigner representation

Weyl operators $\hat{w}(u, v) = \exp(-iv\hat{x} - iu\hat{p})$

Point operators $\hat{A}(x, p) = \frac{1}{(2\pi)^2} \int du dv \exp(ivx + iup) \hat{w}(u, v)$

Wigner representation $W_{\hat{\rho}}(x, p) = \text{Tr}[\hat{\rho}\hat{A}(x, p)]$

$W_{\hat{E}}(x, p) = \text{Tr}[\hat{E}\hat{A}(x, p)]$

The Wigner representation

Weyl operators $\hat{w}(u, v) = \exp(-iv\hat{x} - iu\hat{p})$

Point operators $\hat{A}(x, p) = \frac{1}{(2\pi)^2} \int du dv \exp(ivx + iup) \hat{w}(u, v)$

Wigner representation $W_{\hat{\rho}}(x, p) = \text{Tr}[\hat{\rho}\hat{A}(x, p)]$

$W_{\hat{E}}(x, p) = \text{Tr}[\hat{E}\hat{A}(x, p)]$

$$\int dx dp W_{\hat{\rho}}(x, p) W_{\hat{E}}(x, p) = \text{Tr}[\hat{\rho}\hat{E}]$$

This can be generalized

$$W_{\hat{\rho}}(x_1, p_1, x_2, p_2) = \text{Tr}[\hat{\rho}\hat{A}(x_1, p_1) \otimes \hat{A}(x_2, p_2)]$$

Gaussian quantum mechanics

Gaussian state ρ : one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp \left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle) \right)$$

The Wigner representation

Weyl operators $\hat{w}(u, v) = \exp(-iv\hat{x} - iu\hat{p})$

Point operators $\hat{A}(x, p) = \frac{1}{(2\pi)^2} \int du dv \exp(ivx + iup) \hat{w}(u, v)$

Wigner representation $W_{\hat{\rho}}(x, p) = \text{Tr}[\hat{\rho}\hat{A}(x, p)]$

$W_{\hat{E}}(x, p) = \text{Tr}[\hat{E}\hat{A}(x, p)]$

$$\int dx dp W_{\hat{\rho}}(x, p) W_{\hat{E}}(x, p) = \text{Tr}[\hat{\rho}\hat{E}]$$

This can be generalized

$$W_{\hat{\rho}}(x_1, p_1, x_2, p_2) = \text{Tr}[\hat{\rho}\hat{A}(x_1, p_1) \otimes \hat{A}(x_2, p_2)]$$

Gaussian quantum mechanics

Gaussian state ρ : one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp \left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle) \right)$$

Gaussian quantum mechanics

Gaussian state ρ : one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle)\right)$$

Note: $\langle \hat{x}^k \hat{p}^l \rangle_{\hat{\rho}} = \langle x^k p^l \rangle_{W_{\hat{\rho}}}$ therefore $\gamma(\hat{\rho}) = \gamma(W_{\hat{\rho}})$

Therefore, the Wigner rep'n satisfies the classical uncertainty principle

Gaussian quantum mechanics

Gaussian state ρ : one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle)\right)$$

Note: $\langle \hat{x}^k \hat{p}^l \rangle_{\hat{\rho}} = \langle x^k p^l \rangle_{W_{\hat{\rho}}}$ therefore $\gamma(\hat{\rho}) = \gamma(W_{\hat{\rho}})$

Therefore, the Wigner rep'n satisfies the classical uncertainty principle

Gaussian measurements and transformations: preserve Gaussianity

Gaussian quantum mechanics

Gaussian state ρ : one that has a Gaussian Wigner rep'n

$$W_\rho(\mathbf{z}) = \frac{1}{(2\pi)^{n/2}|\gamma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \langle \mathbf{z} \rangle)^T \gamma^{-1} (\mathbf{z} - \langle \mathbf{z} \rangle)\right)$$

Note: $\langle \hat{x}^k \hat{p}^l \rangle_{\hat{\rho}} = \langle x^k p^l \rangle_{W_{\hat{\rho}}}$ therefore $\gamma(\hat{\rho}) = \gamma(W_{\hat{\rho}})$

Therefore, the Wigner rep'n satisfies the classical uncertainty principle

Gaussian measurements and transformations: preserve Gaussianity

One can prove

Theorem: Liouville mechanics with an epistemic restriction is empirically equivalent to Gaussian quantum mechanics