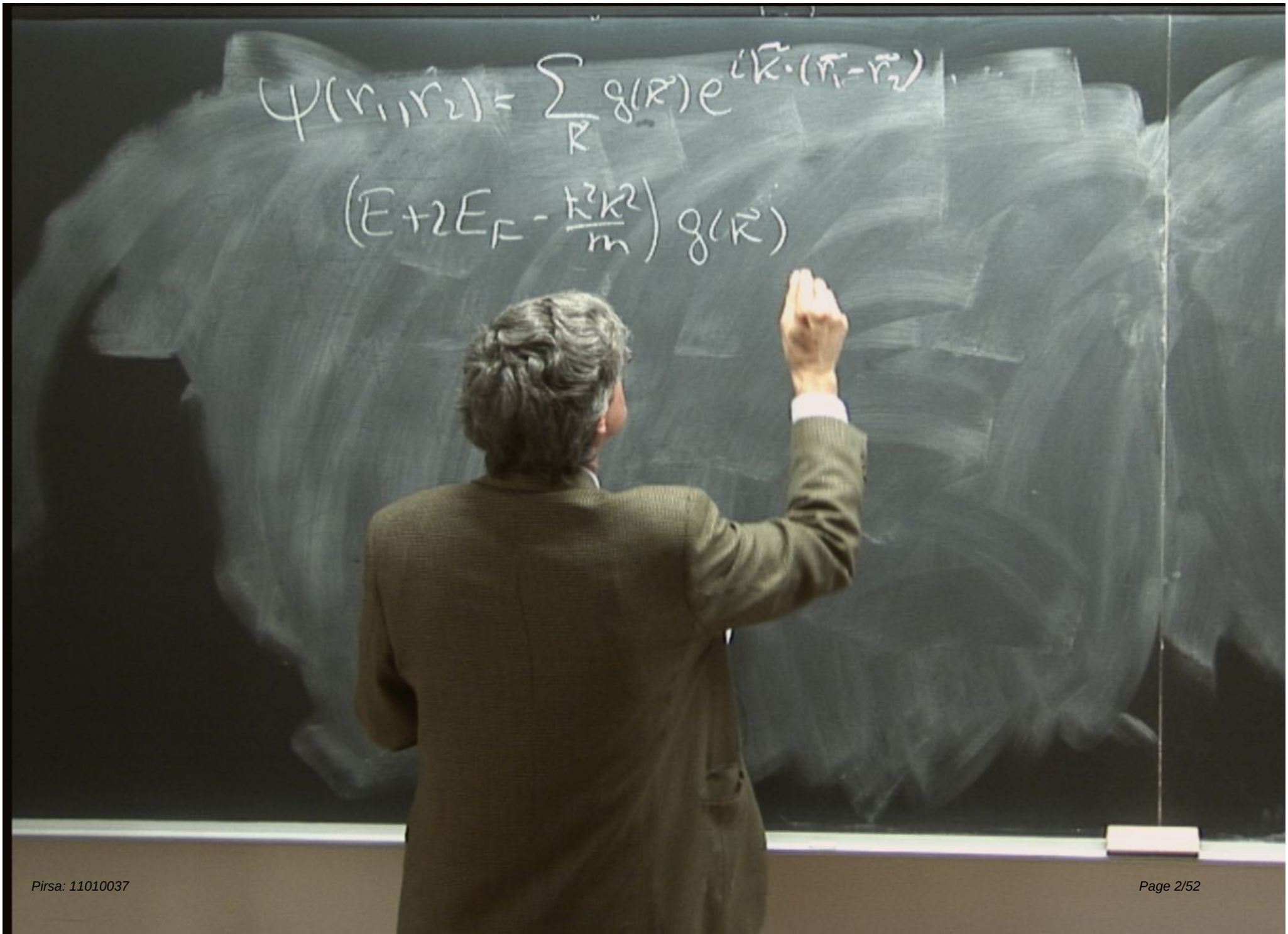
Title: Condensed Matter Review - Lecture 11

Date: Jan 17, 2011 10:15 AM

URL: http://pirsa.org/11010037

Abstract:


$\psi(r_1, r_2) = \sum_{\vec{k}} g(\vec{k}) e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}$
$\left(E + 2E_F - \frac{\hbar^2 k^2}{m}\right) g(\vec{k})$

$$\psi(r_1, r_2) = \sum_{\vec{k}} g(\vec{k})\, e^{i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)}$$

$$\left(E + 2E_F - \frac{\hbar^2 k^2}{m}\right) g(\vec{k}) = -\frac{V}{L^3} \sum_{\vec{k}'}{}' g(\vec{k}')$$

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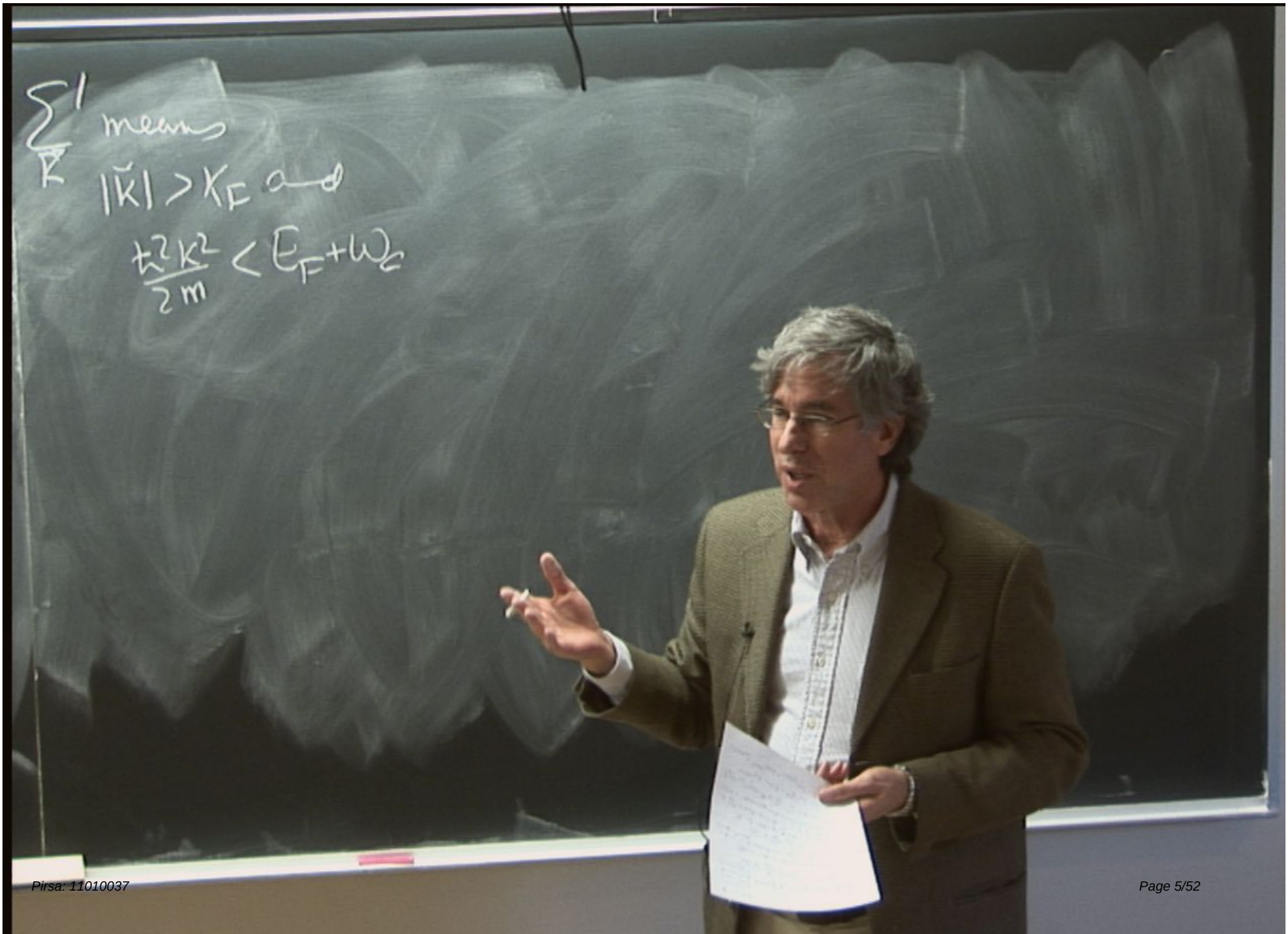
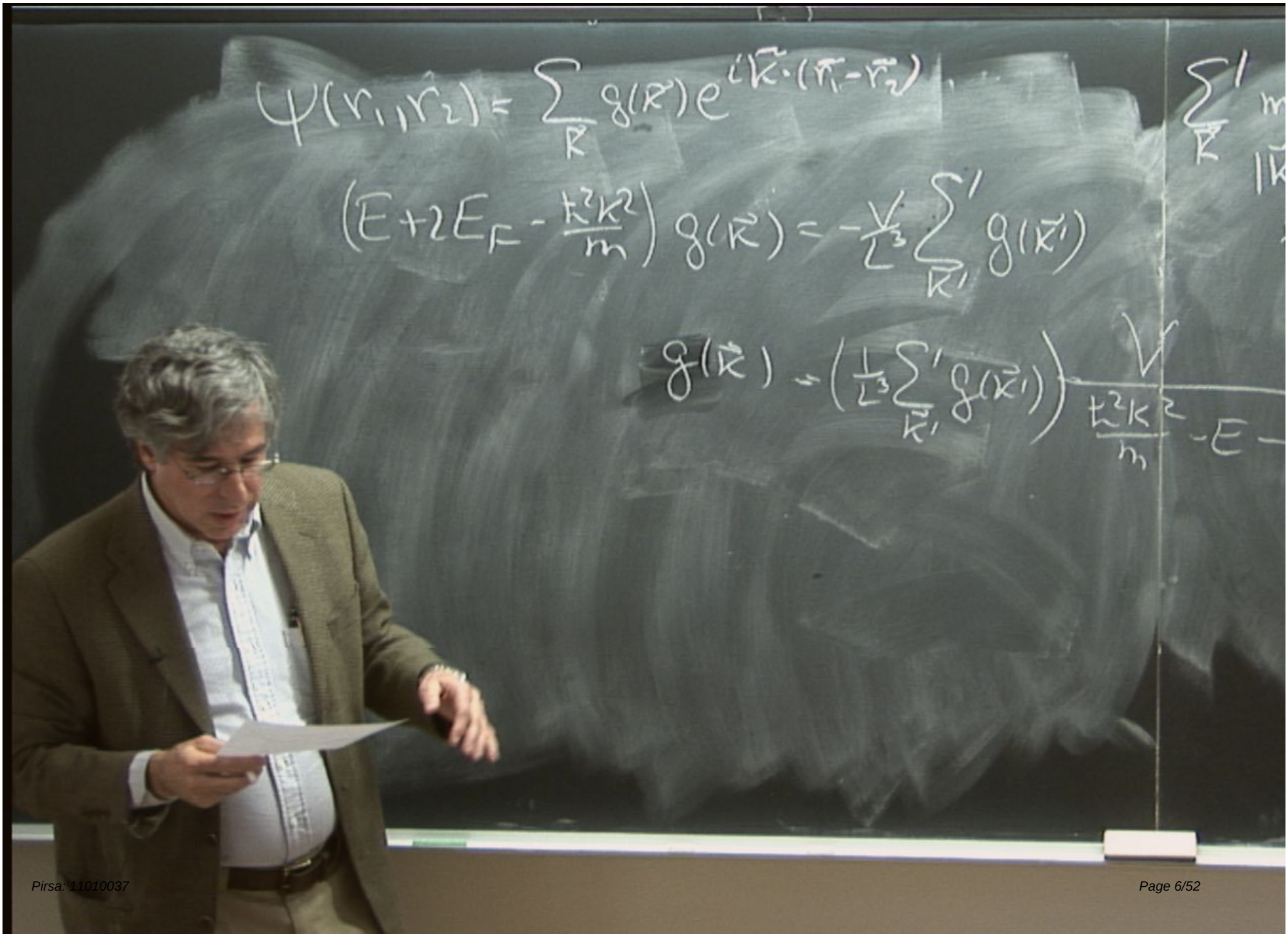
$$\sum{}'_{m}$$

$$|\vec{k}$$


$\sum_{\vec{k}}'$ means
$|\vec{k}| > k_F$ and
$\frac{\hbar^2 k^2}{2m} < E_F + \omega_c$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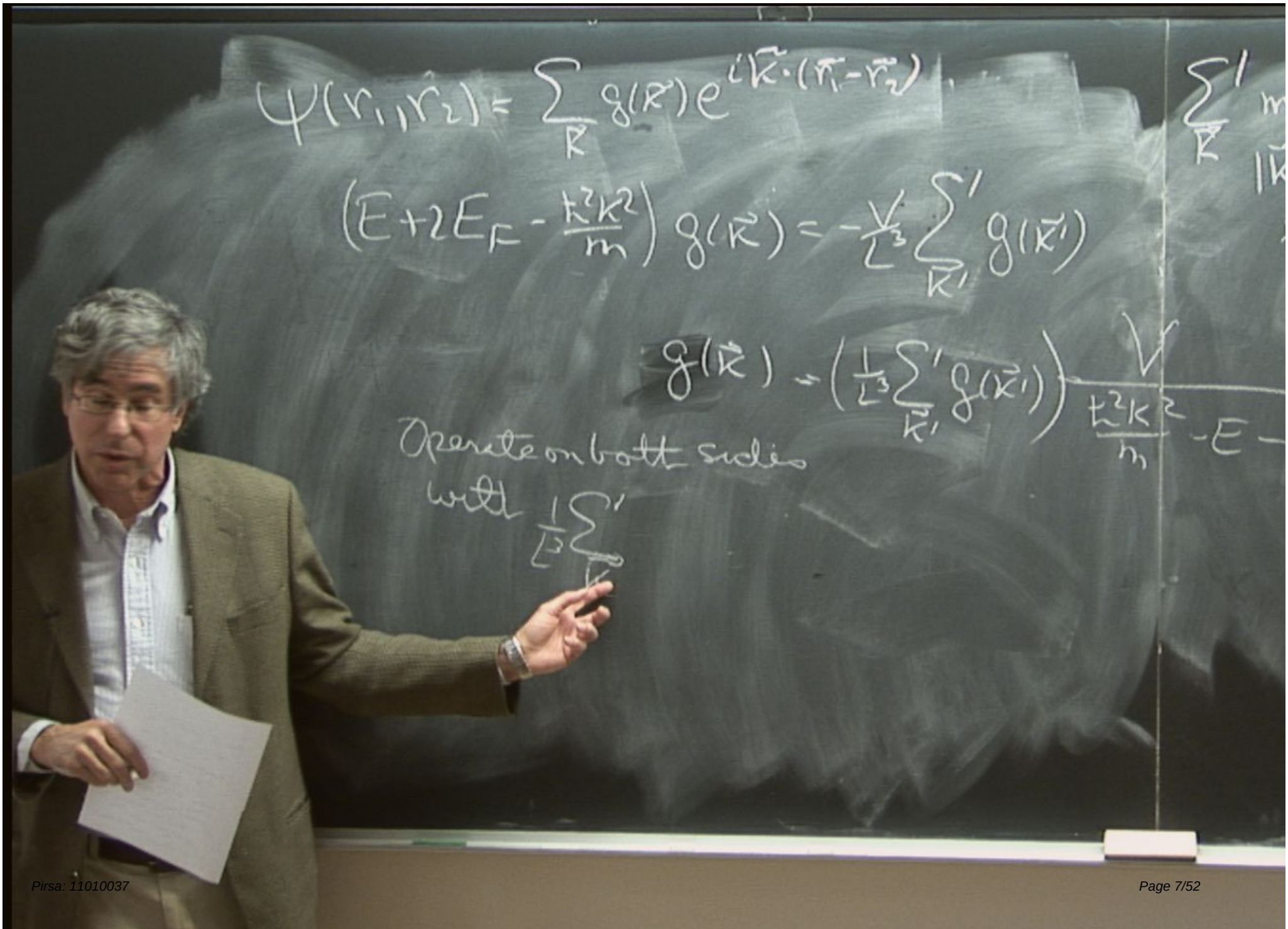
$g(\vec{k}) = \left(\frac{1}{L^3} \sum_{\vec{k}'}' g(\vec{k}')\right) \frac{V}{\frac{\hbar^2 k^2}{m} - E -}$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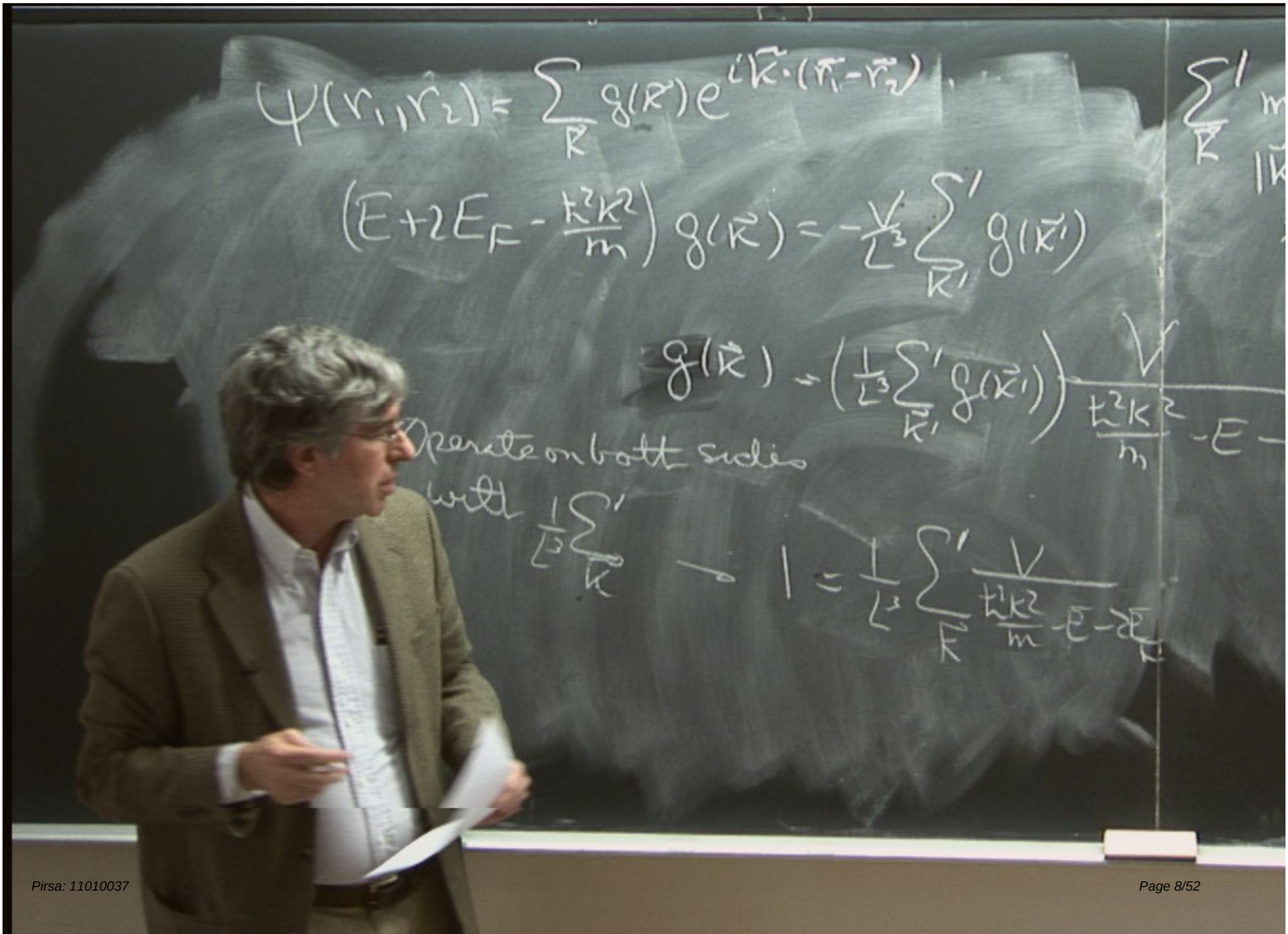
Operate on both sides
with $\frac{1}{L^3} \sum_{\vec{k}}'$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Operate on both sides
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$1 = \frac{1}{L^3} \sum_{\vec{k}}' \frac{V}{\frac{\hbar^2 k^2}{m} - E - 2E_F}$

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Operate on both sides with $\frac{1}{L^3} \sum_{\vec{k}}{}'$

$$\rightarrow 1 = \frac{1}{L^3} \sum_{\vec{k}}{}' \frac{V}{\frac{\hbar^2 k^2}{m} - E - 2E_F}$$

$$\sum_{\vec{k}}{}' \; m \; |\vec{k}$$

$\sum_{\vec{k}}'$ means

$|\vec{k}| > k_F$ and

$\frac{\hbar^2 k^2}{2m} < E_F + \omega_c$

Write $\frac{1}{L^3}\sum_k' = \int_0^{\omega_c} d\xi\, N(\xi)$ where

$\xi = \frac{\hbar^2 k^2}{2m} - E_F$

$-E - 2E_F$

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is $\approx N(0)$

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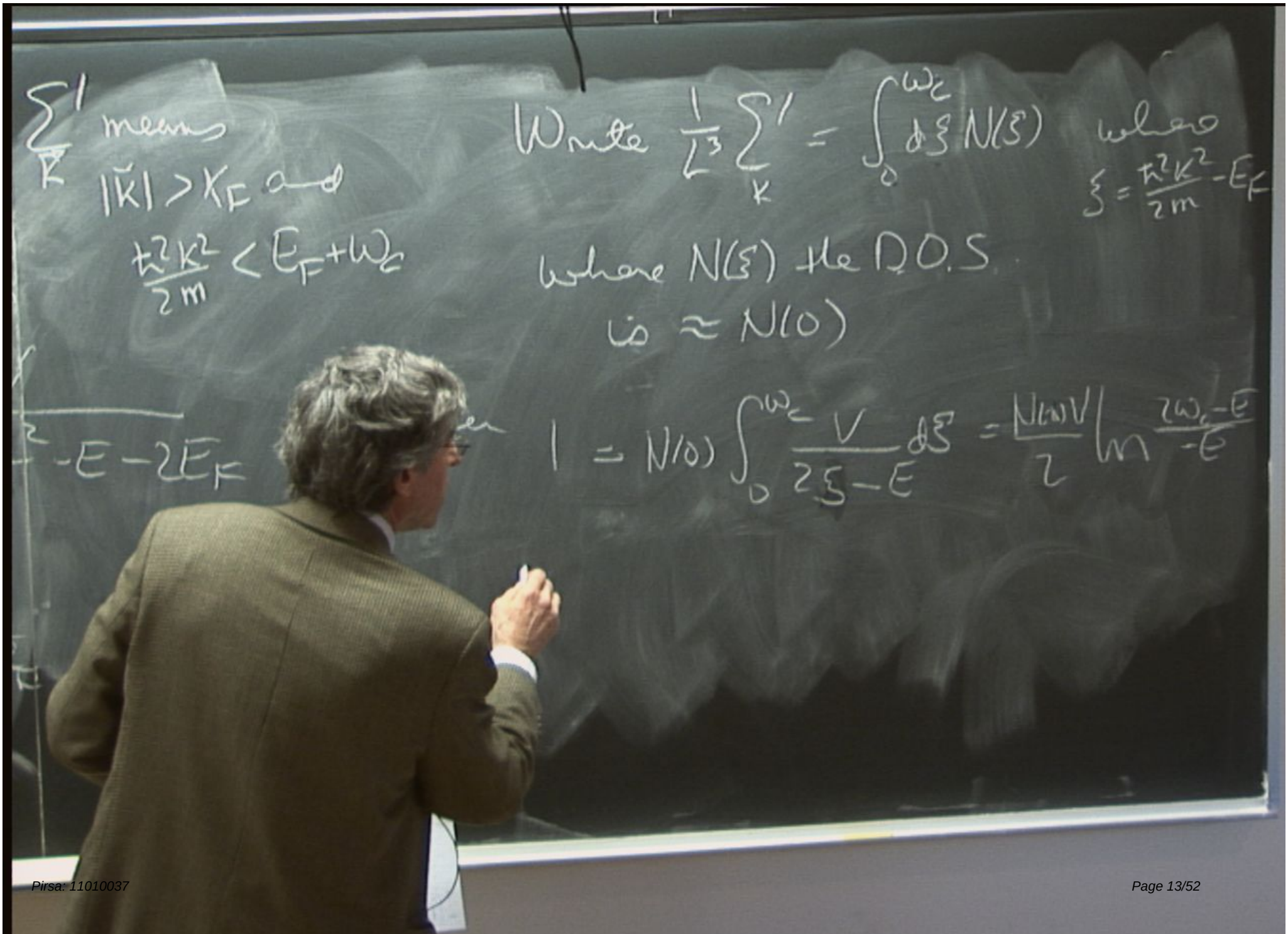
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$1 = N(0)\int_0^{\omega_c} \frac{V}{2\xi - E} d\xi = \frac{N(0)V}{2} \ln \frac{2\omega_c - E}{-E}$

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Then $1 = N(0)\int_0^{\omega_c} \frac{V}{2\xi - E}\, d\xi = \frac{N(0)V}{2}\ln\frac{2\omega_c - E}{-E}$

$\therefore \frac{2}{N(0)V} = \ln\frac{E - 2\omega_c}{E} \longrightarrow E - 2\omega_c = E\, e^{2/N(0)V}$

$E = \frac{2\omega_c}{1 - e^{2/N(0)V}} \approx -2\omega_c e^{-2/N(0)V}$

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$V < \omega_c < E_F$

Then $1 = N(0)\int_0^{\omega_c} \frac{V}{2\xi - E}\, d\xi = \frac{N(0)V}{2}\ln\frac{2\omega_c - E}{-E}$

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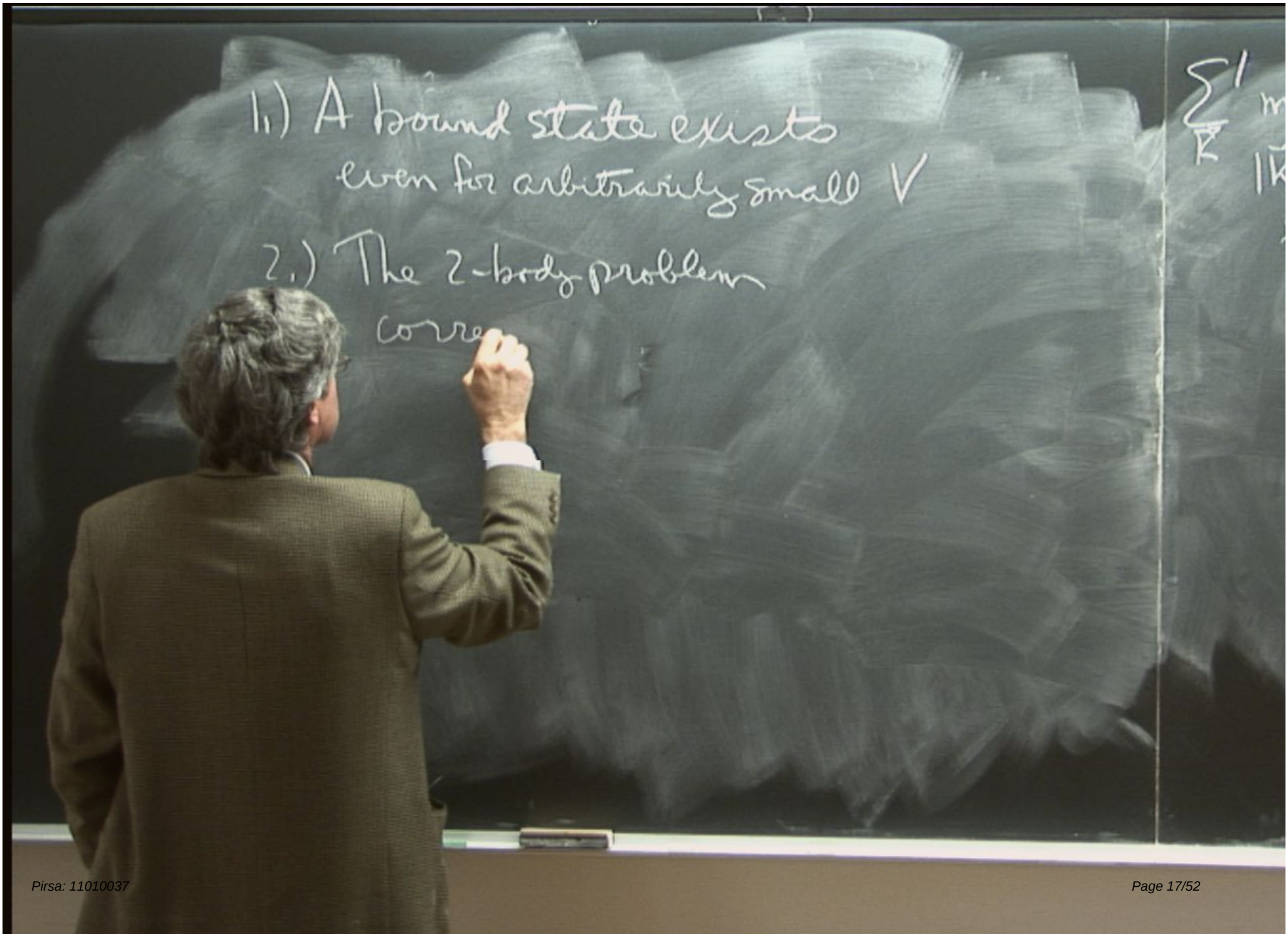
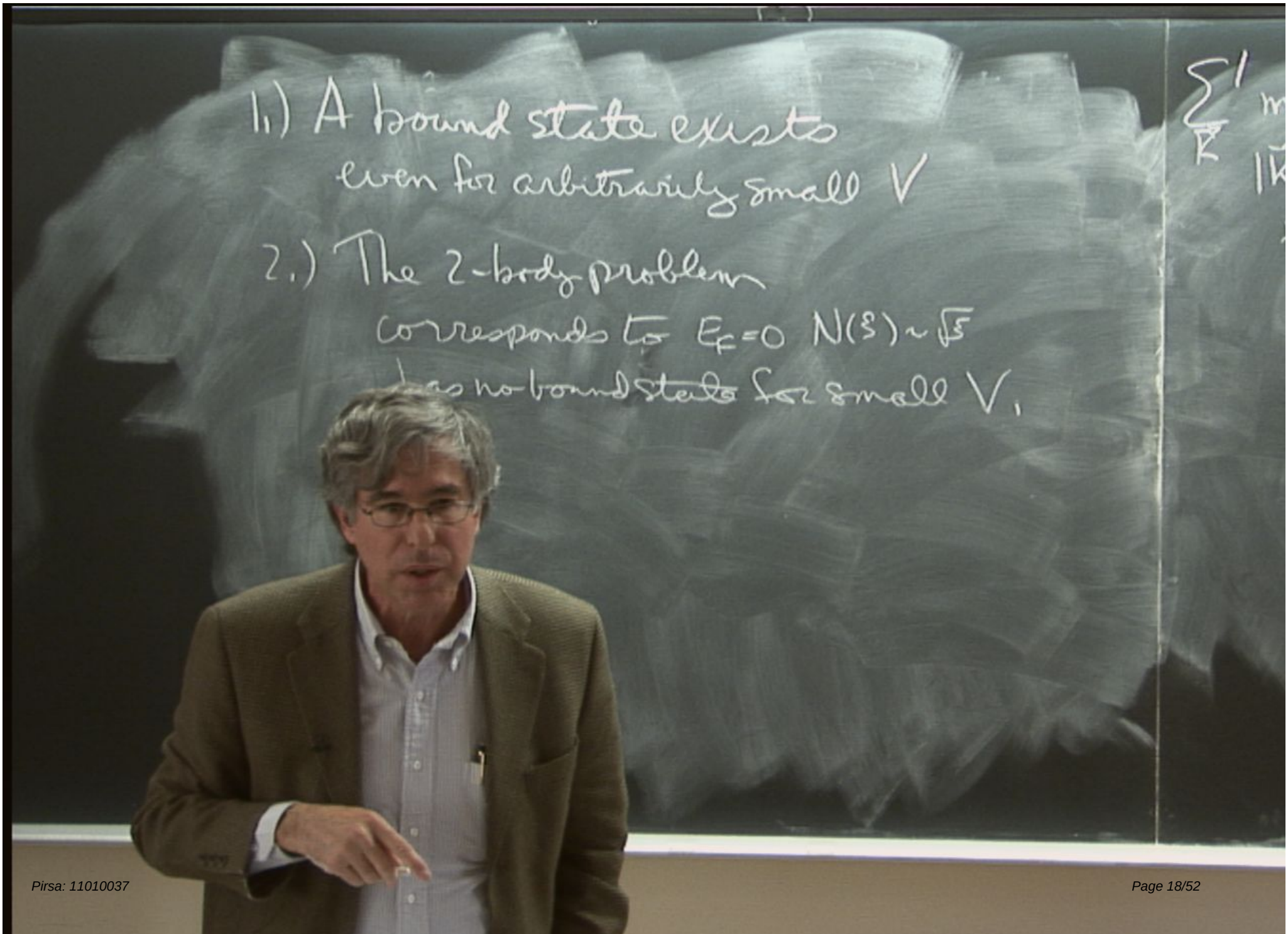

1.) A bound state exists
even for arbitrarily small V
2.) The 2-body problem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1.) A bound state exists
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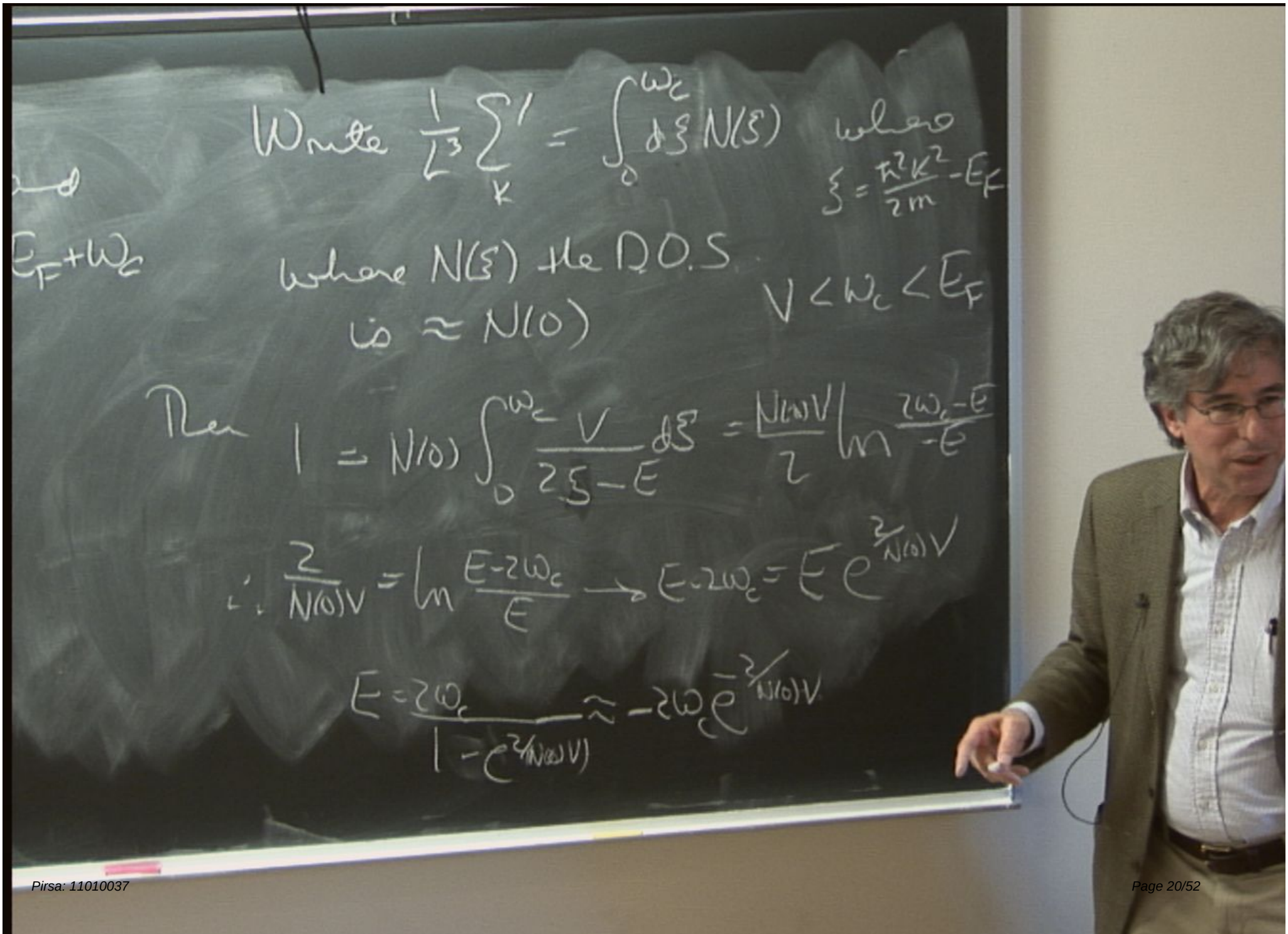
2.) The 2-body problem
corresponds to $E_F = 0$ $N(\xi) \sim \sqrt{\xi}$
has no bound states for small V.

3.) The binding energy is non-analytic in V.

4.) Chose spatially form for pot. and for $\Psi(r_1, r_2)$

$\sum_{\vec{k}}'$


Write $\frac{1}{L^3}\sum_k' = \int_0^{\omega_c} d\xi\, N(\xi)$ where
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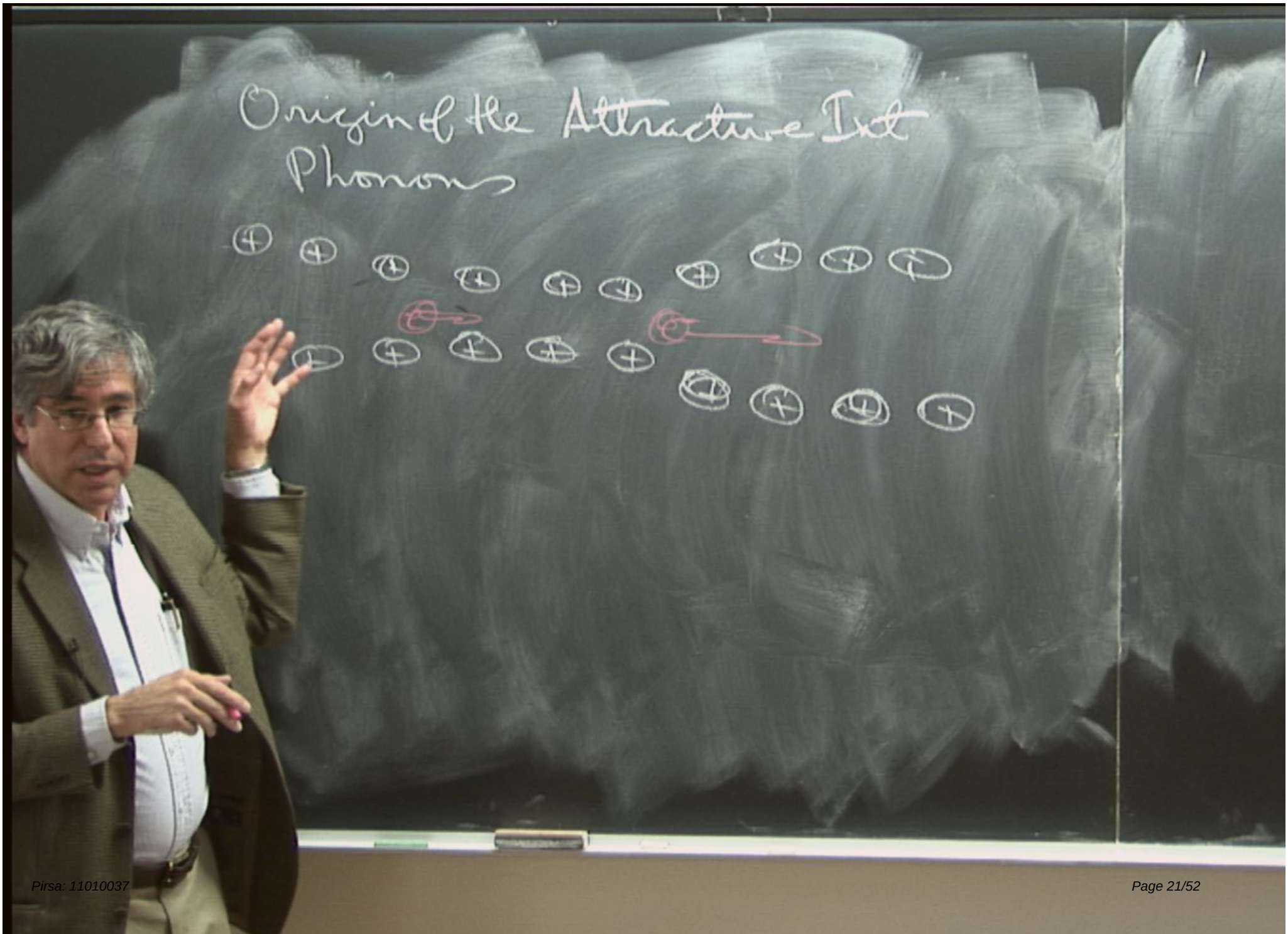
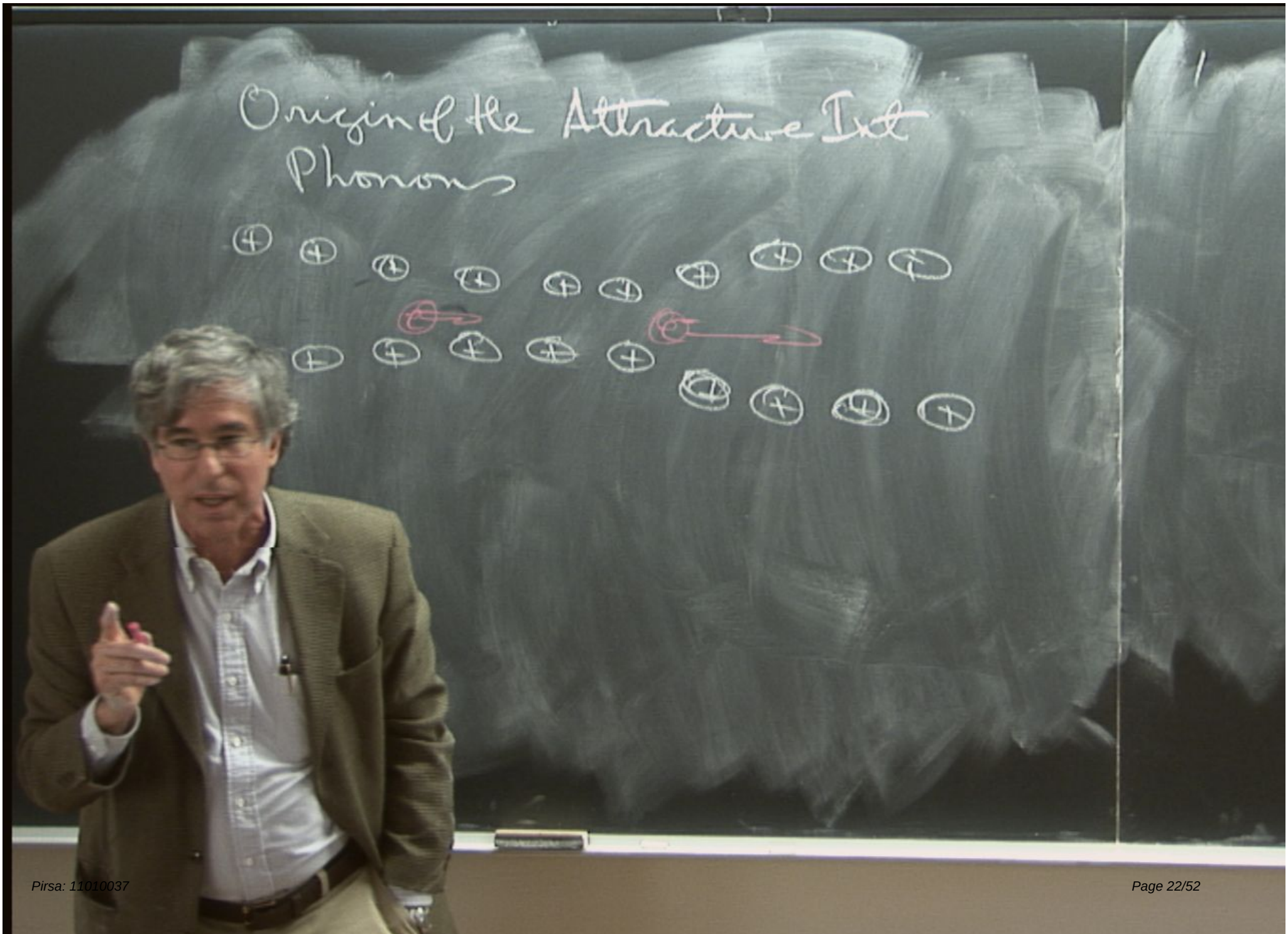

Origin of the Attractive Int
Phonons


Origin of the Attractive Int
Phonons

Origin of the Attractive Int

Phonons

retardation

Origin of the Attractive Int

Phonons

retardation

Bose ops

$$\mathcal{H}_{phonon} = \sum_{q} \omega_q \left(b_q^\dagger b_q + \frac{1}{2} \right)$$

$$\mathcal{H}_{el\text{-}ph} = \frac{i}{N} \sum_{k,\sigma} \sum_{q} D_q \, c^\dagger_{k+q,\sigma} \, c_{k\sigma} \left(b_q - b_{-q}^\dagger \right)$$

Bose ops

$$\mathcal{H}_{phonon} = \sum_{q} \omega_q \left(b_q^{+} b_q + \frac{1}{2} \right)$$

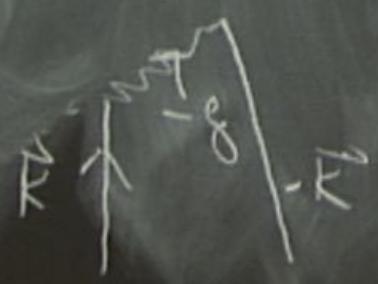
$$\mathcal{H}_{el\text{-}ph} = \frac{i}{N} \sum_{k,\sigma} \sum_{q} D_q \, c^{+}_{k+q,\sigma} \, c_{k\sigma} \left(b_q - b_q^{+} \right)$$

Bose ops

$$\mathcal{H}_{phonon} = \sum_{q} \omega_q \left(b_q^\dagger b_q + \frac{1}{2} \right)$$

$$\mathcal{H}_{el\text{-}ph} = \frac{i}{N} \sum_{k,\sigma} \sum_{q} D_q \, c^\dagger_{k+q,\sigma} \, c_{k\sigma} \left(b_q - b_q^\dagger \right)$$

Effective e-e int



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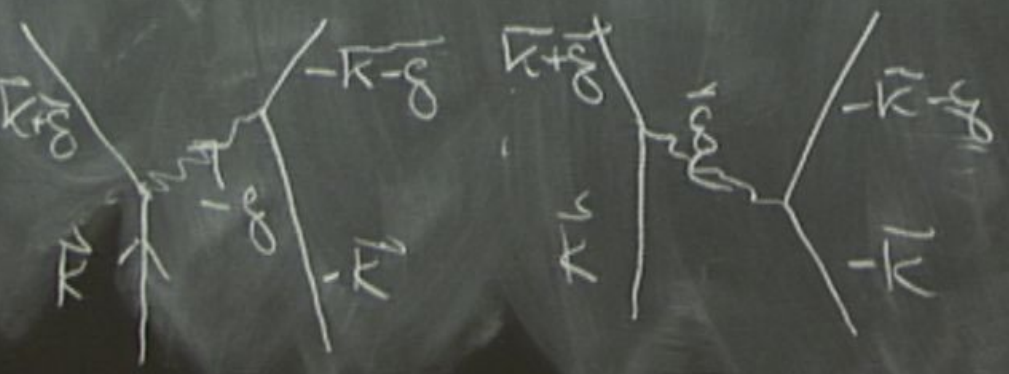

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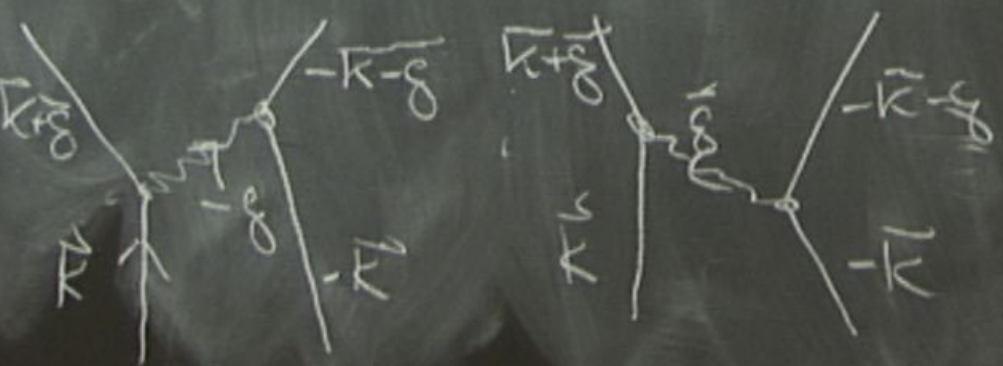


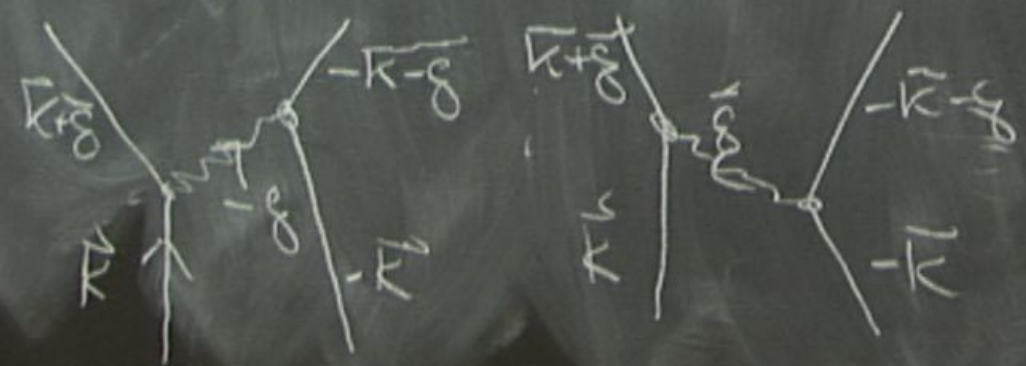
Bose ops

$$\mathcal{H}_{phonon} = \sum_{\vec{q}} \omega_q \left(b^\dagger_q b_q + \tfrac{1}{2} \right)$$

$$\mathcal{H}_{el\text{-}ph} = \frac{i}{N} \sum_{k,\sigma} \sum_{q} D_q \, c^\dagger_{k+q,\sigma} \, c_{k\sigma} \left(b_q - b^\dagger_q \right)$$

Effective e-e int

$$V_{eff}(\vec{q}) = \frac{|D_q|^2 \, \omega_q}{(\varepsilon_{k+q} - \varepsilon_k)^2 - \omega_q^2}$$



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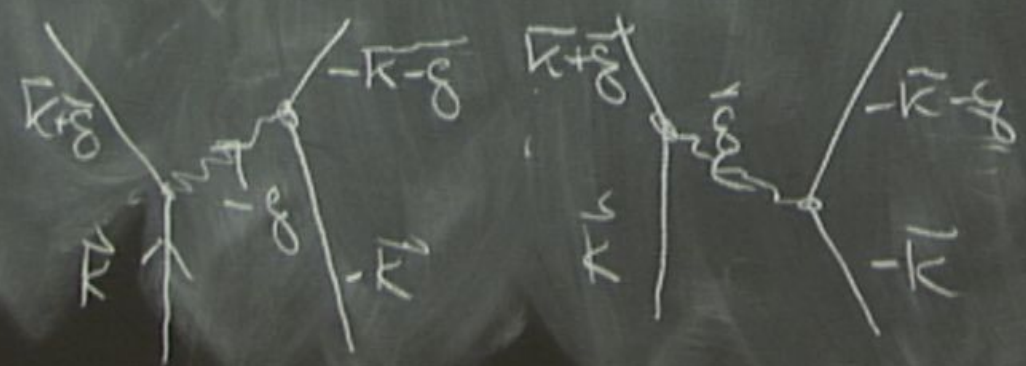

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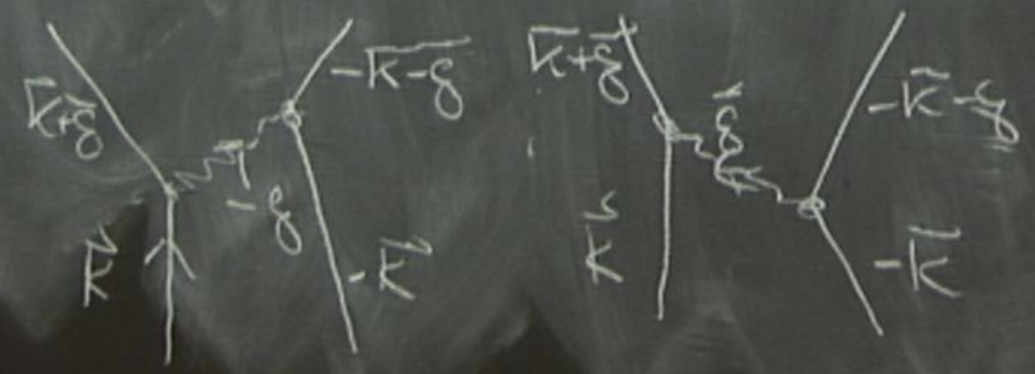

Attractive for

$$|\varepsilon_{k+q} - \varepsilon_k| < \omega_q$$

Bose ops

$$\mathcal{H}_{phonon} = \sum_{\vec{q}} \omega_q \left(b_q^\dagger b_q + \tfrac{1}{2} \right)$$

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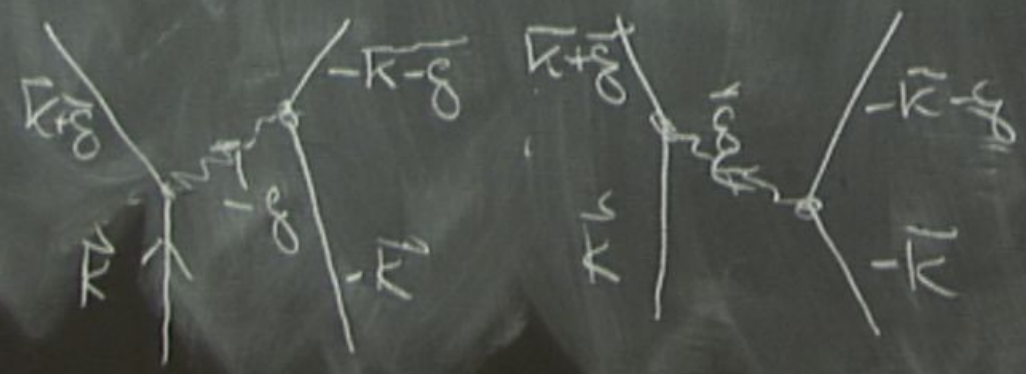
Effective e-e int

$$V_{eff}(\vec{q}) = \frac{|D_q|^2 \, \omega_q}{(\varepsilon_{k+q} - \varepsilon_k)^2 - \omega_q^2}$$



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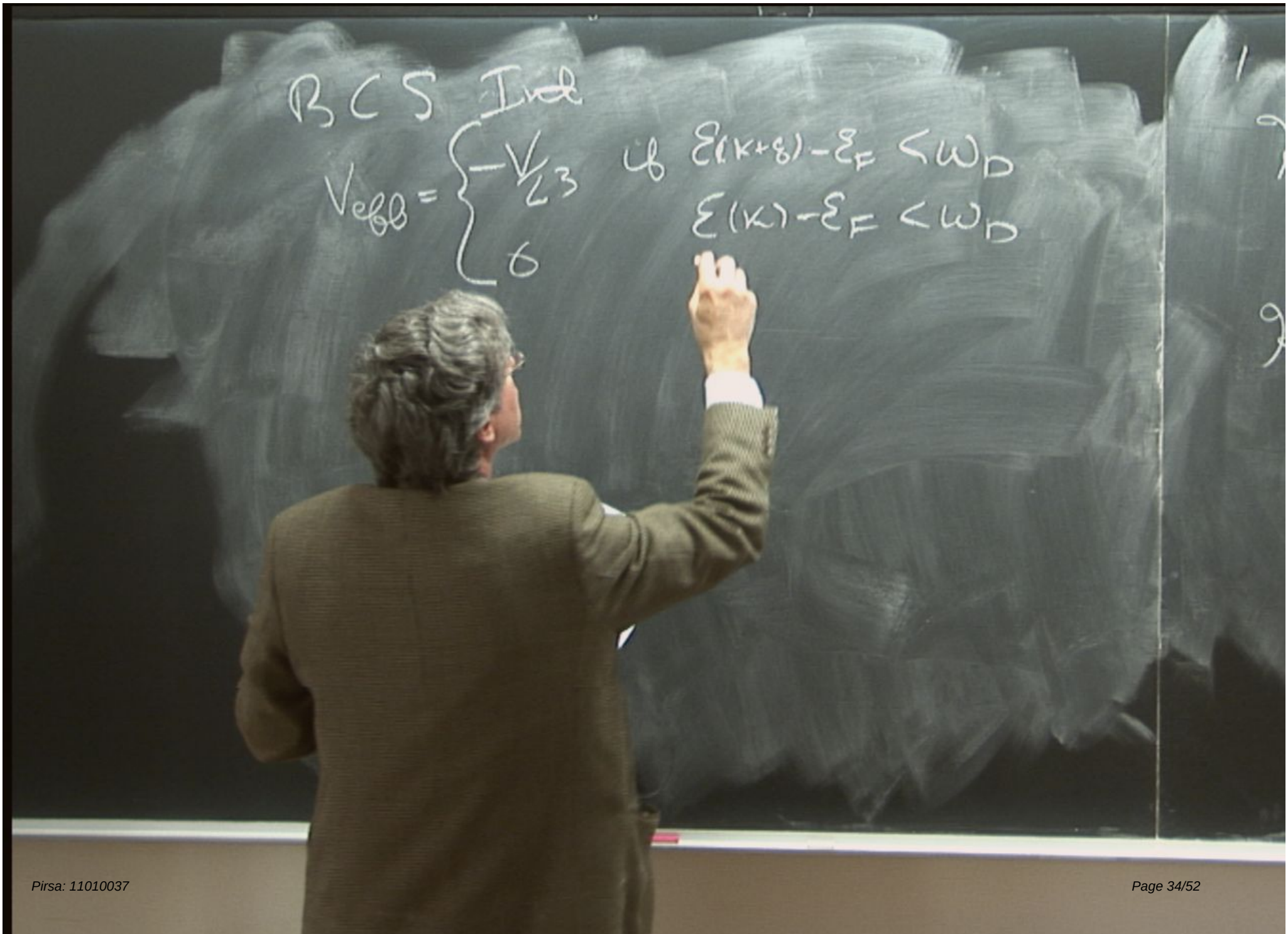
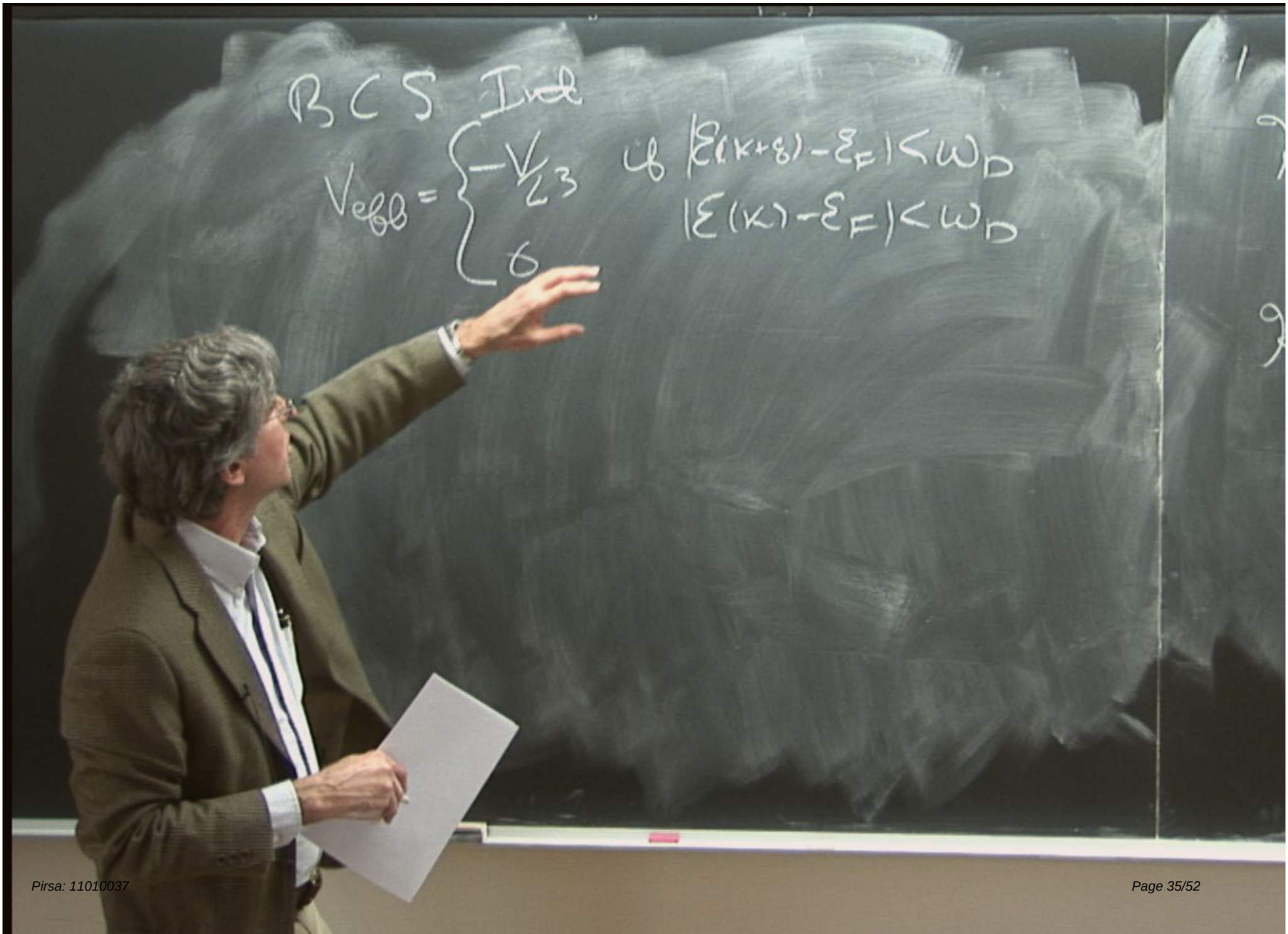

BCS Ind
$V_{eff} = \begin{cases} -V/L^3 & \text{if } \mathcal{E}(k+q) - \mathcal{E}_F < \omega_D \\ & \mathcal{E}(k) - \mathcal{E}_F < \omega_D \\ 0 & \end{cases}$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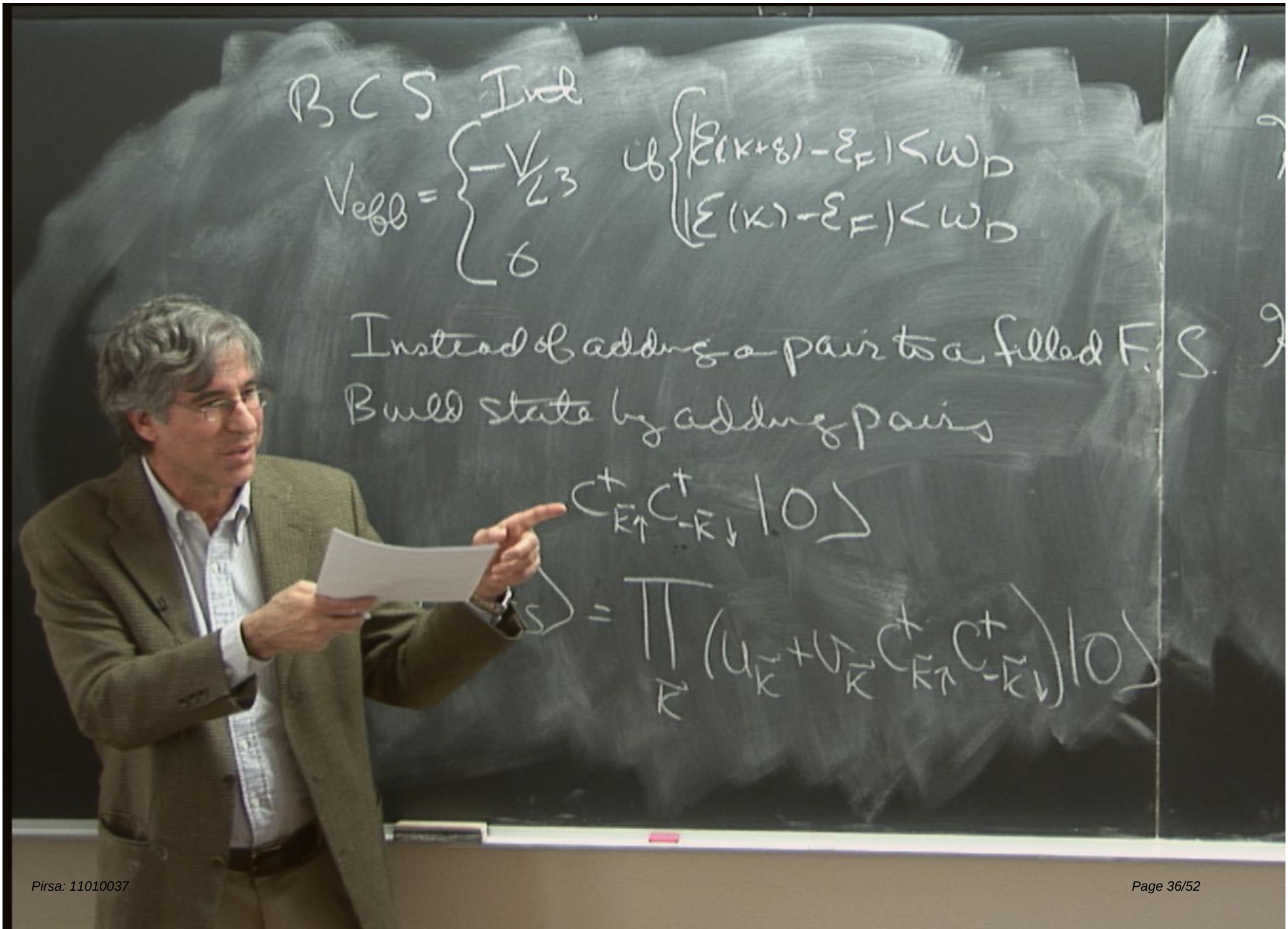
BCS Ind
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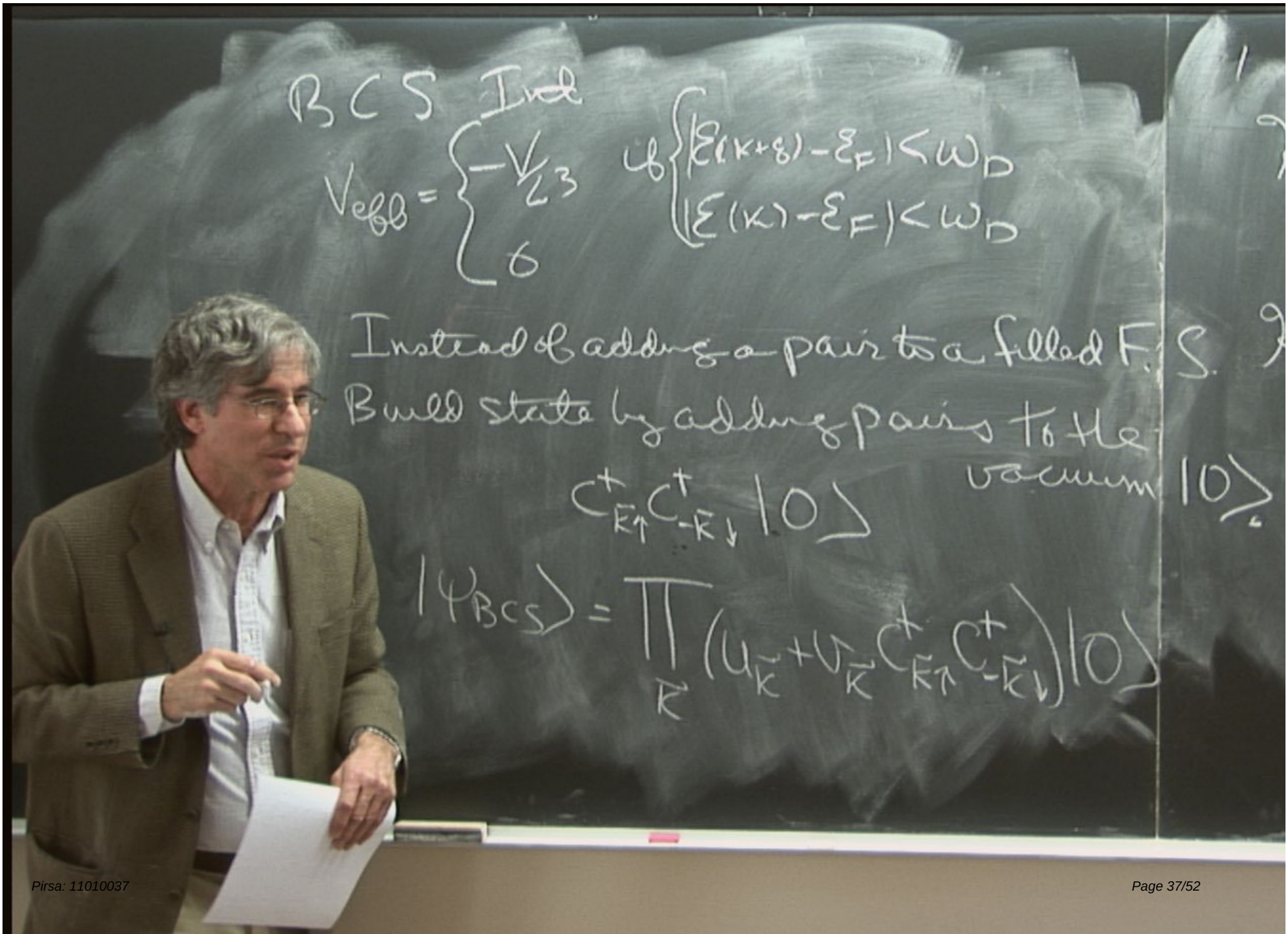
Instead of adding a pair to a filled F.S.
Build state by adding pairs
$c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow} |0\rangle$
$|S\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}) |0\rangle$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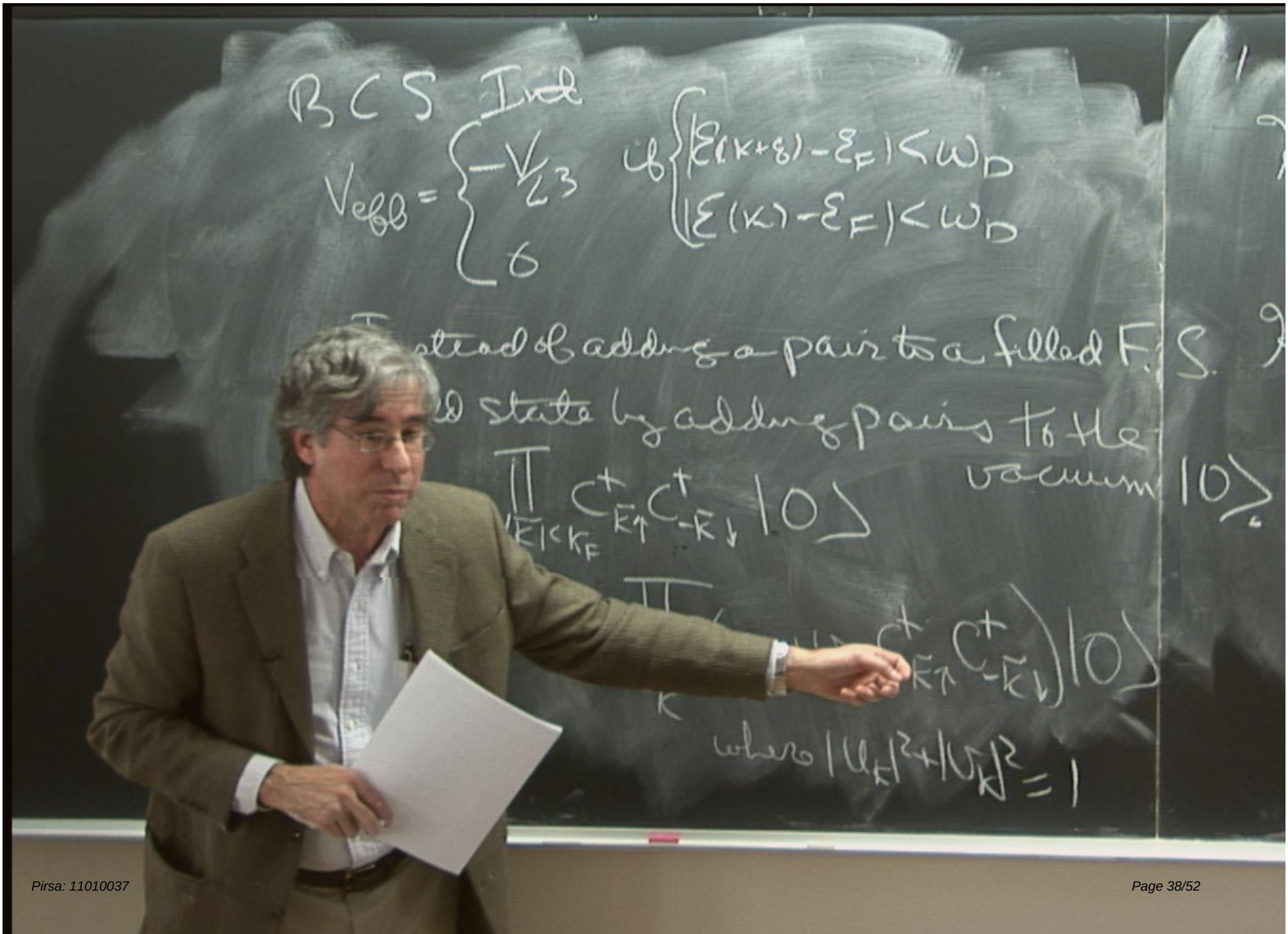
Instead of adding a pair to a filled F.S.
Build state by adding pairs to the vacuum $|0\rangle$
$c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow} |0\rangle$
$|\psi_{BCS}\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}) |0\rangle$


BCS Ind
$V_{eff} = \begin{cases} -V/L^3 & \text{if } \begin{cases} |\varepsilon(k+q) - \varepsilon_F| < \omega_D \\ |\varepsilon(k) - \varepsilon_F| < \omega_D \end{cases} \\ 0 \end{cases}$
stead of adding a pair to a filled F.S.
d state by adding pairs to the vacuum $|0\rangle$
$\prod_{|k|<k_F} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} |0\rangle$
$c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$
where $|u_k|^2 + |v_k|^2 = 1$

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Instead of adding a pair to a filled F.S.

Build state by adding pairs to the vacuum $|0\rangle$.

$$\prod_{|\vec{k}|<k_F} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow} |0\rangle$$

$$|\psi_{BCS}\rangle = \prod_{\vec{k}} \left(u_{\vec{k}} + v_{\vec{k}} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}\right)|0\rangle$$

Variational wave fn for u_k, v_k

where $|u_k|^2 + |v_k|^2 = 1$

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Variational wave fn for u_k, v_k

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Can work with definite N

$$|\psi(N)\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-\frac{iN\varphi}{2}} \prod_k \left(|u_k| + |v_k| e^{i\varphi} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

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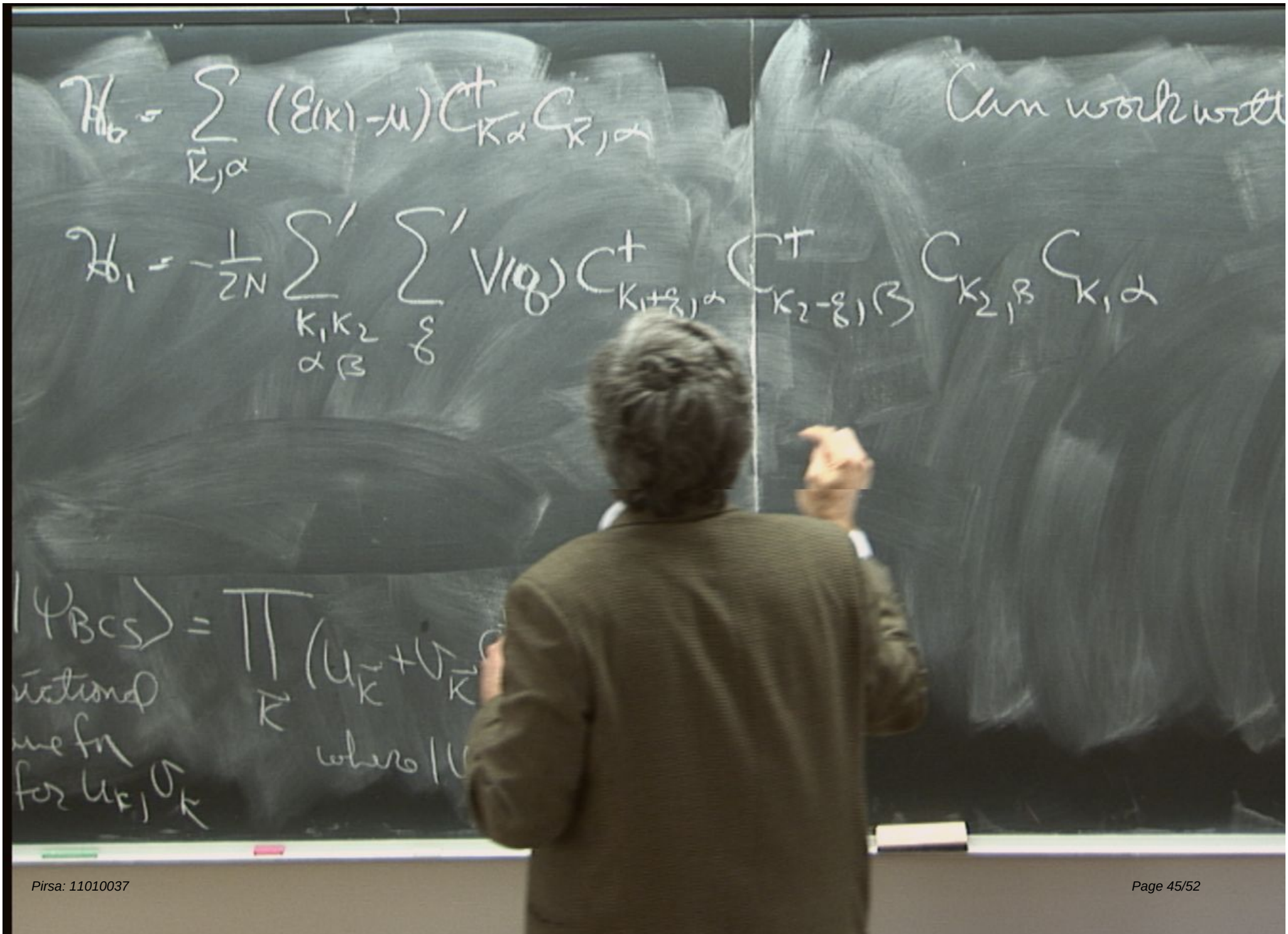
$\mathcal{H}_0 = \sum_{\vec{k},\alpha} (\varepsilon(k) - \mu) c^\dagger_{k\alpha} c_{k,\alpha}$
$\mathcal{H}_1 = -\frac{1}{2N} \sum'_{\substack{k_1 k_2 \\ \alpha\beta}} \sum'_{q} V(q) c^\dagger_{k_1+q,\alpha} c^\dagger_{k_2-q,\beta} c_{k_2,\beta} c_{k_1,\alpha}$
Can work with
$|\psi_{BCS}\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}}$
where $|u$
for u_k, v_k

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 \qquad \mathcal{H}_0 = \sum_{\vec{k},\alpha} (\varepsilon(k) - \mu)\, c^\dagger_{\vec{k}\alpha} c_{\vec{k},\alpha}$$

$$\mathcal{H}_1 = -\frac{1}{2N} \sum_{\substack{k_1 k_2 \\ \alpha\beta}}{}' \sum_{q}{}' V(q)\, c^\dagger_{k_1+q,\alpha}\, c^\dagger_{k_2-q,\beta}\, c_{k_2,\beta}$$

Variational problem is equivalent to

$$\mathcal{H}_v = \sum_{\vec{k},\alpha} (\varepsilon_v(\vec{k}) - \alpha)\, c^\dagger_{\vec{k}\alpha} c_{\vec{k}\alpha} +$$

$$|\psi_{BCS}\rangle = \prod_{\vec{k}} \left(u_{\vec{k}} + v_{\vec{k}}\, c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}\right)|0\rangle$$

Variational wave fn for u_k, v_k

where $|u_k|^2 + |v_k|^2 = 1$

$-\mu) C^{\dagger}_{k\alpha} C_{k,\alpha}$

$$\sum_{q}{}' V(q)\, C^{\dagger}_{k_1+q,\alpha}\, C^{\dagger}_{k_2-q,\beta}\, C_{k_2,\beta}\, C_{k_1,\alpha}$$

$-\alpha) C^{\dagger}_{k\alpha} C_{k\alpha} + \sum_{k} \left(\Delta_k C^{\dagger}_{k\uparrow} C^{\dagger}_{-k\downarrow} + \Delta^{*}_{k} C_{-k\downarrow} C_{k\uparrow} \right)$

$+ v_k C^{\dagger}_{k\uparrow} C^{\dagger}_{-k\downarrow}) |0\rangle$

$\text{ero } |u_k|^2 + |v_k|^2 = 1$

Can work with definite N

$$e^{-\frac{iN\varphi}{2}} \prod_{k}$$

Define new fermion ops.

$$\gamma^{\dagger}_{k\uparrow} = u_k c^{\dagger}_{k\uparrow} + v_k c_{-k\downarrow}$$

$$c^{\dagger}_{k_2-q,\beta}\, c_{k_2,\beta}\, c_{k,\alpha}$$

$$\left(\Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \Delta^{*}_k c_{-k\downarrow} c_{k\uparrow}\right)$$

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$$c^{\dagger}_{k_2-q,\beta} c_{k_2,\beta} c_{k_1,\alpha}$$

$$\left(\Delta_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + \Delta^{*}_k c_{-k\downarrow} c_{k\uparrow}\right)$$

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The cond

$$\left\{\gamma^{\dagger}_{k\uparrow}, \gamma_{k'\uparrow}\right\} = \delta_{kk'}$$

requires that $|u_k|^2 + |v_k|^2 = 1$

$$c^{\dagger}_{k_2-q,\beta}\, c_{k_2,\beta}\, c_{k,\alpha}$$

$$\Delta_k \to c_{k\uparrow\, -k}$$

Define new fermion ops.

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$$\{\gamma^{\dagger}_{k\uparrow}, \gamma_{k'\uparrow}\} = \delta_{kk'}$$

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Same for u'_k, v'_k

Define new fermion ops.

$$\gamma^{+}_{k\uparrow} = u_k c^{+}_{k\uparrow} + v_k c_{-k\downarrow}$$

$$\gamma_{-k\downarrow} = u'_k c_{-k\downarrow} + v'_k c^{+}_{k\uparrow}$$

The cond

$$\{\gamma^{+}_{k\uparrow}, \gamma_{k'\uparrow}\} = \delta_{kk'}$$

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Same for u'_k, v'_k