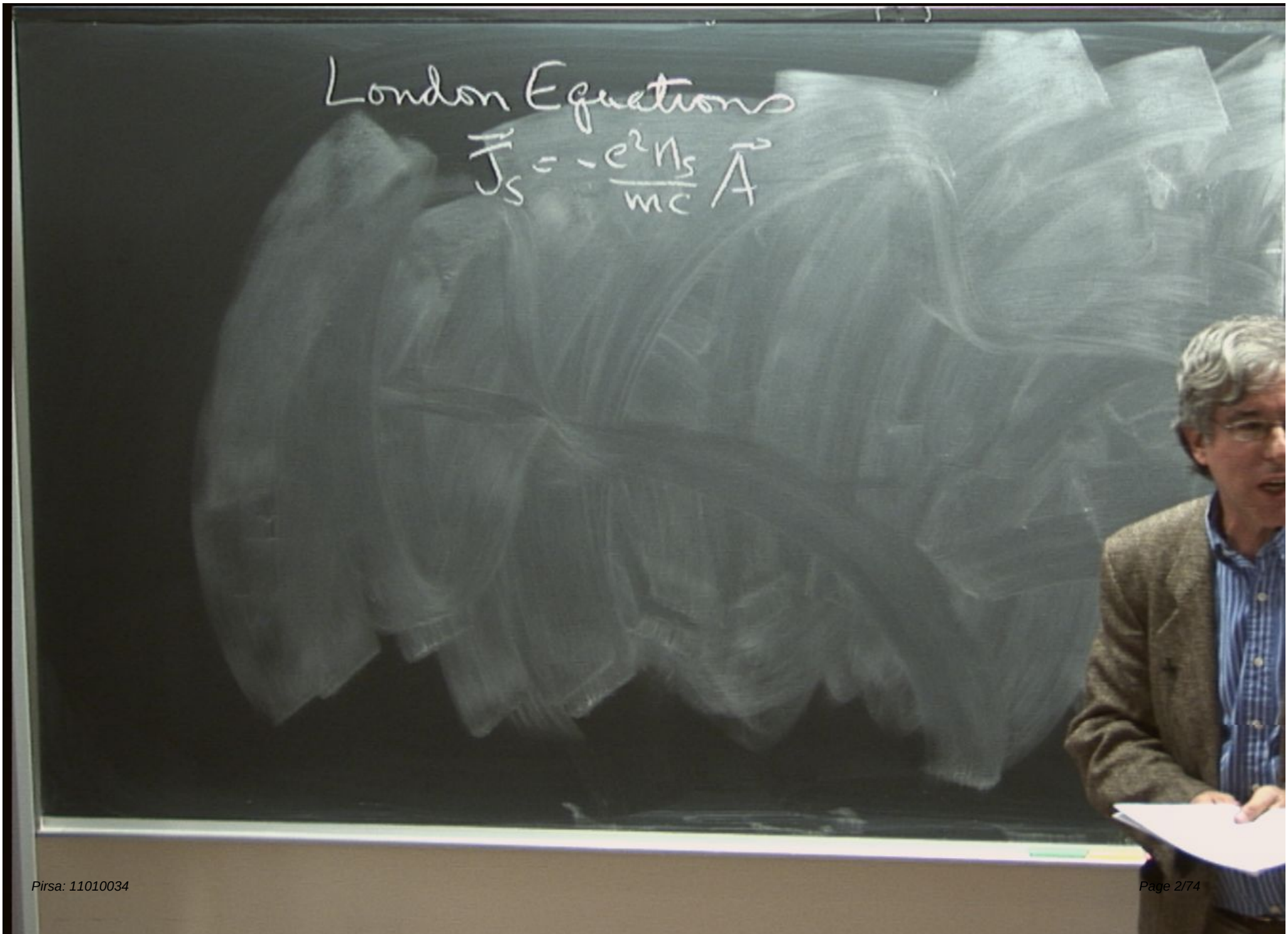
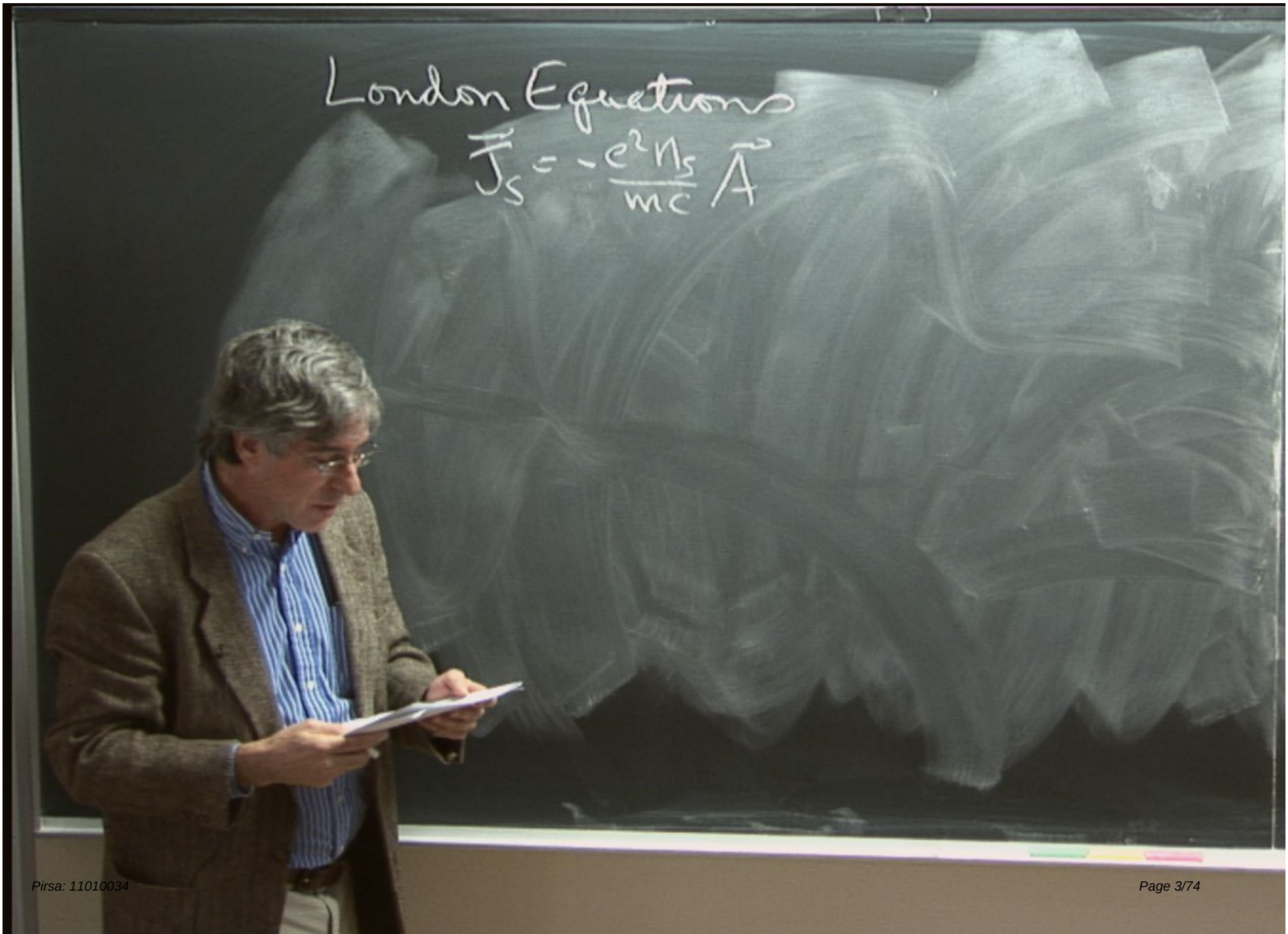
Title: Condensed Matter Review - Lecture 10

Date: Jan 14, 2011 10:15 AM

URL: http://pirsa.org/11010034

Abstract:


London Equations
$\vec{J}_s = -\frac{e^2 n_s}{mc}\vec{A}$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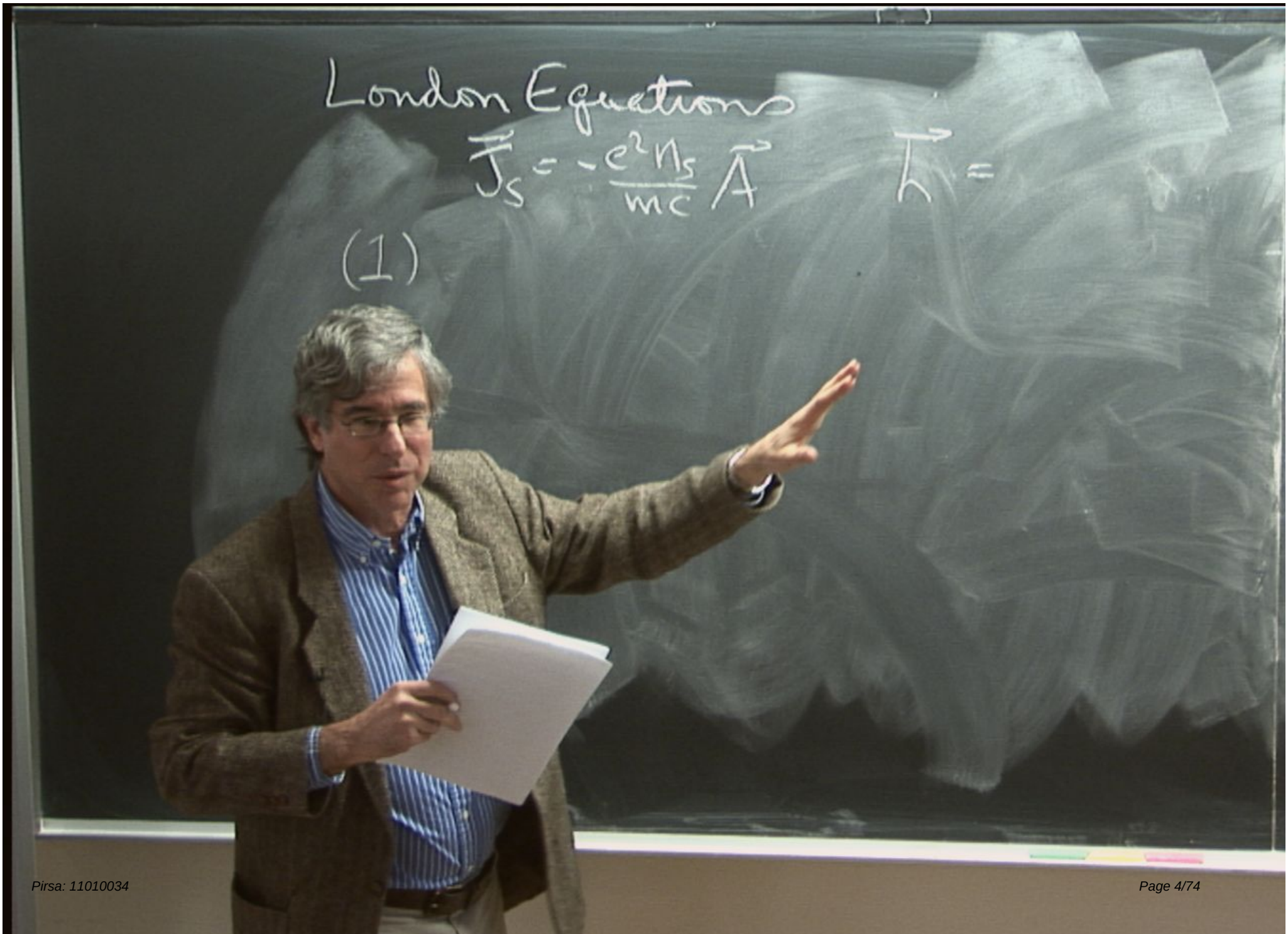
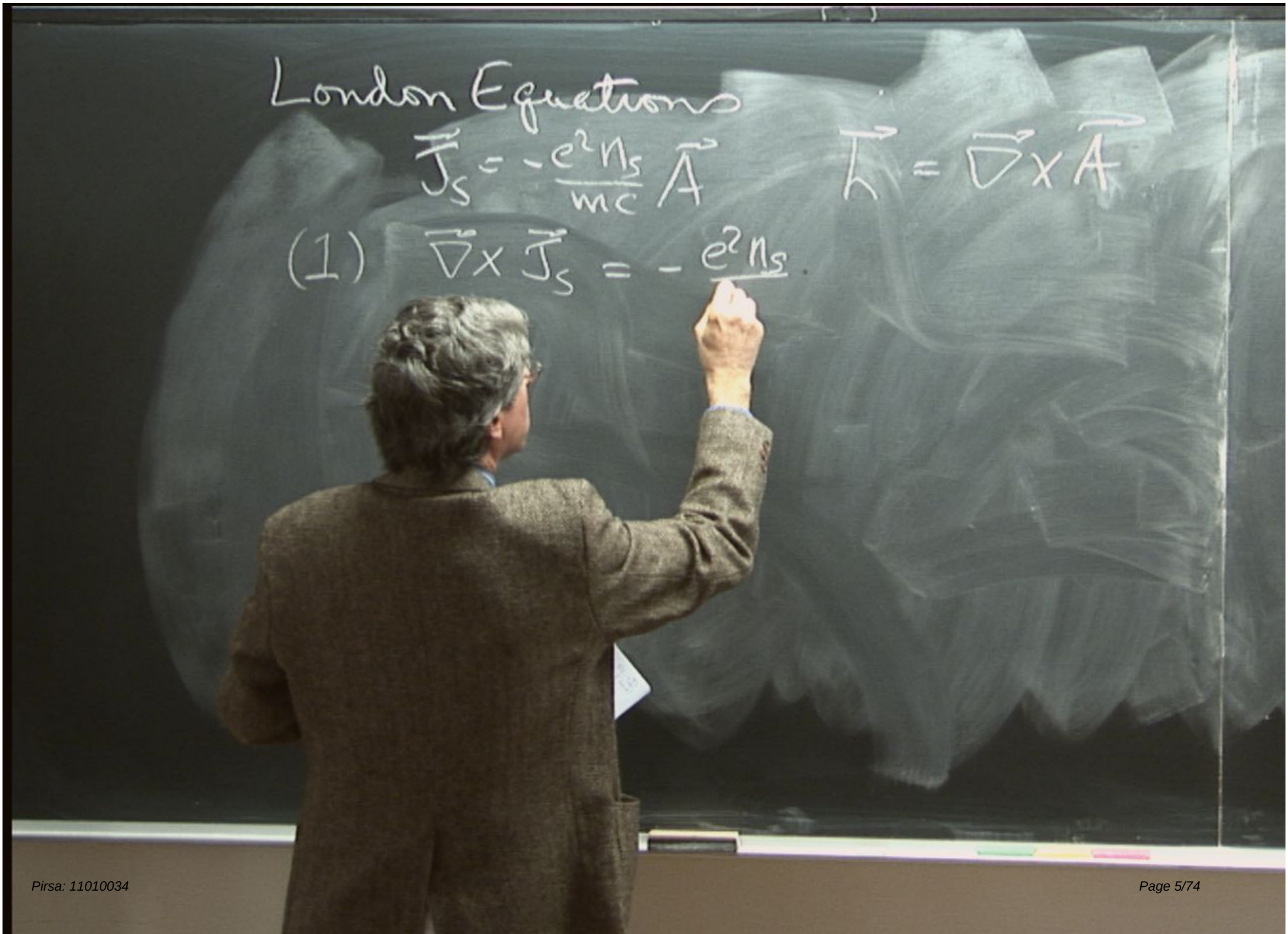
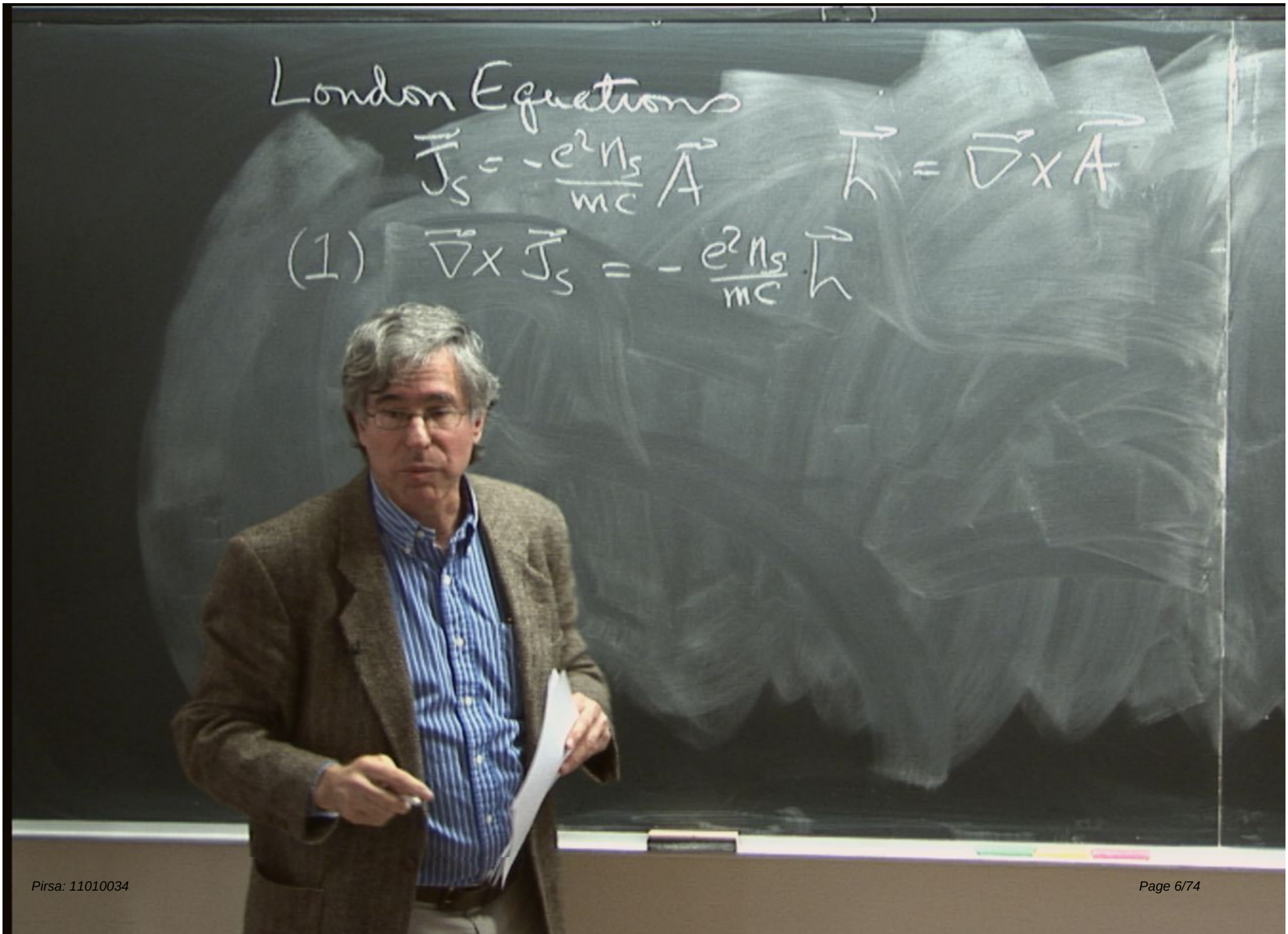

London Equations
$\vec{J}_s = -\frac{e^2 n_s}{mc}\vec{A}$
$\vec{h} =$
(1)


London Equations
$\vec{J}_s = -\frac{e^2 n_s}{mc}\vec{A}$
$\vec{h} = \vec{\nabla}\times\vec{A}$
(1) $\vec{\nabla}\times\vec{J}_s = -\frac{e^2 n_s}{}$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London Equations

$$\vec{J}_s = -\frac{e^2 n_s}{mc}\vec{A} \qquad \vec{h} = \vec{\nabla}\times\vec{A}$$

(1) $$\vec{\nabla}\times\vec{J}_s = -\frac{e^2 n_s}{mc}\vec{h}$$

(2) $$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} = -\frac{m}{e^2 n_s}\frac{\partial\vec{J}_s}{\partial t}$$

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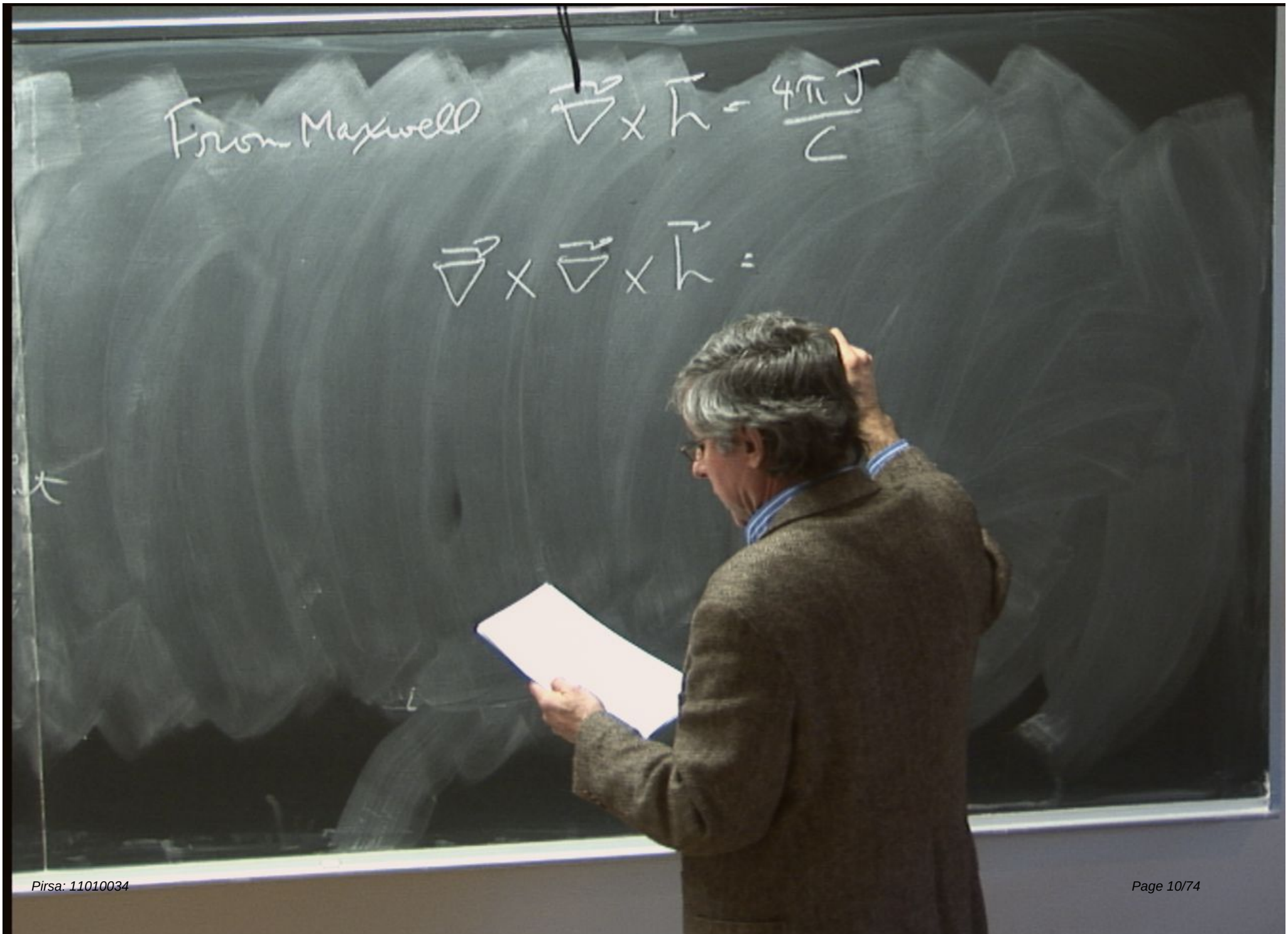
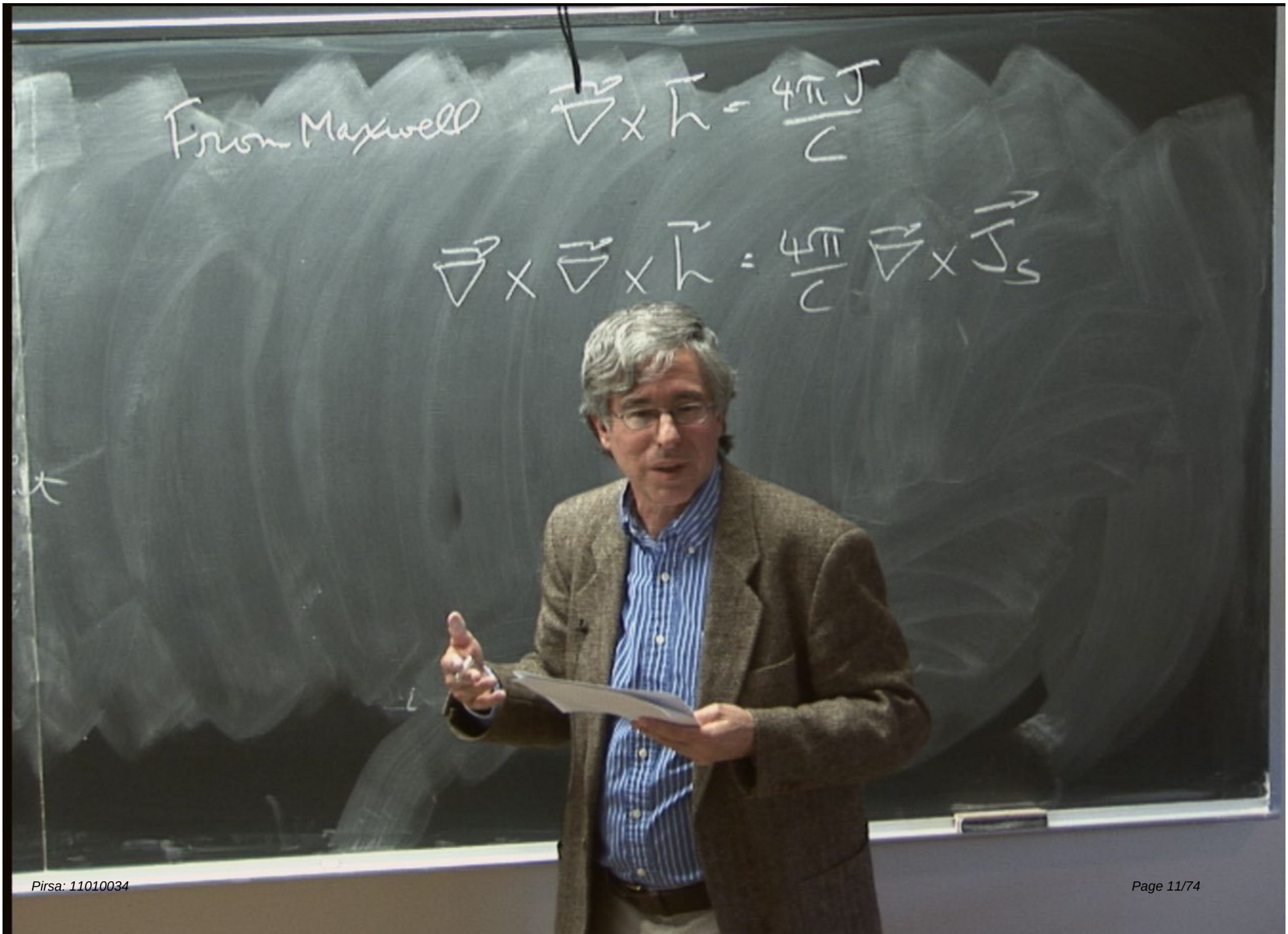

From Maxwell $\vec{\nabla} \times \vec{h} = \frac{4\pi J}{c}$
$\vec{\nabla} \times \vec{\nabla} \times \vec{h} =$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$$\underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{h}} = \frac{4\pi}{c} \vec{\nabla} \times \vec{J}_s$$

Since $\vec{\nabla} \cdot \vec{h} = 0$, $-\nabla^2 \vec{h} = +\frac{4\pi}{c} \frac{e^2 n_s}{mc} \vec{h}$

$$\equiv \frac{1}{\lambda_L^2} \vec{h}$$

λ_L = London Penetration Depth

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$|\vec{h}_{\parallel}|$

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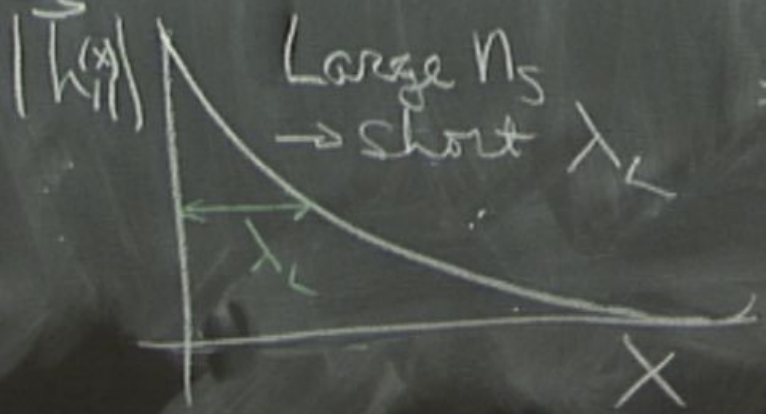

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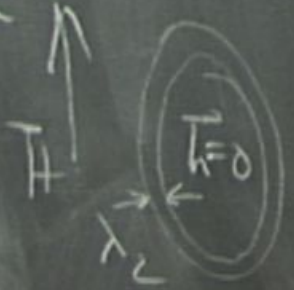

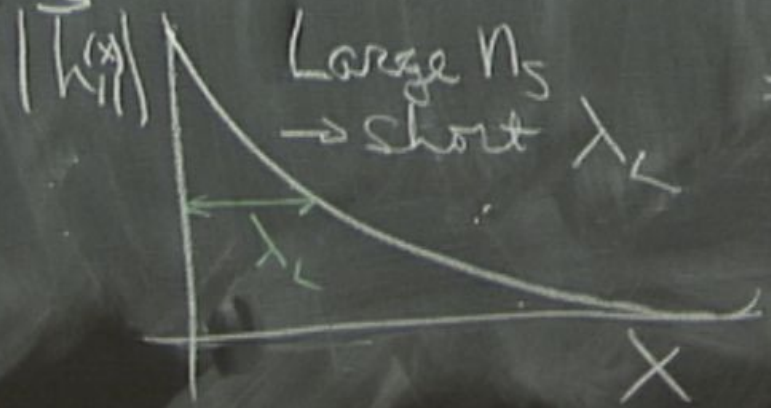

$\times \vec{A}$

E-field accelerates the current

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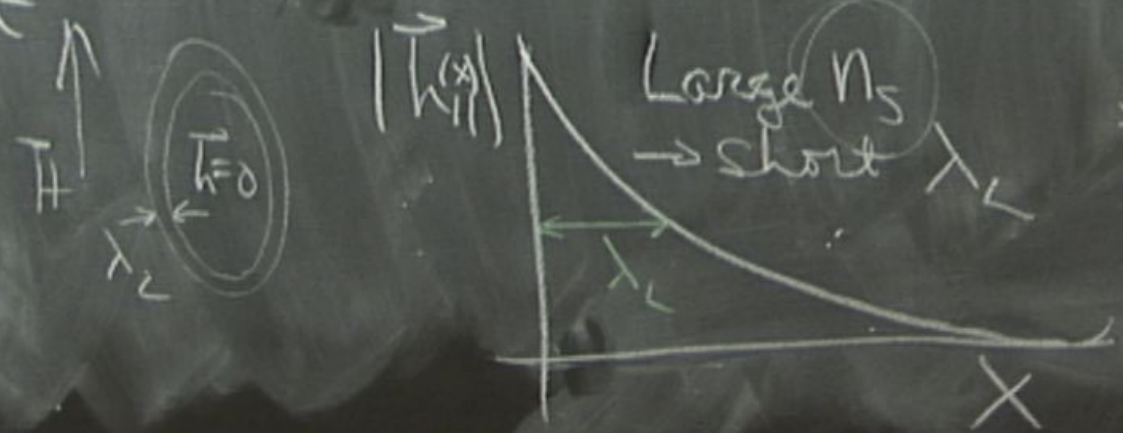

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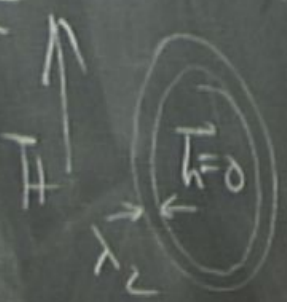

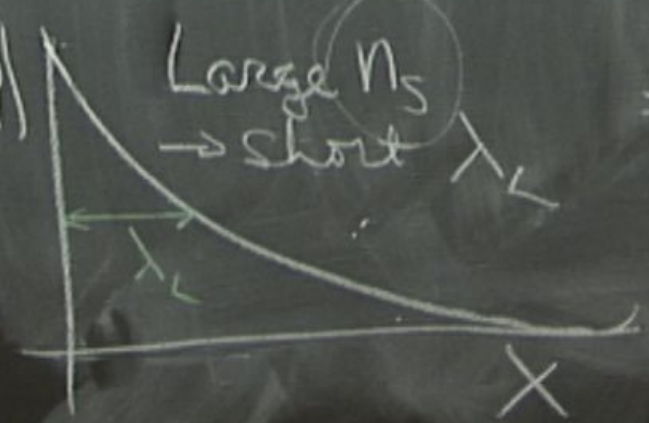

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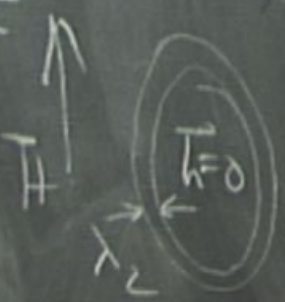

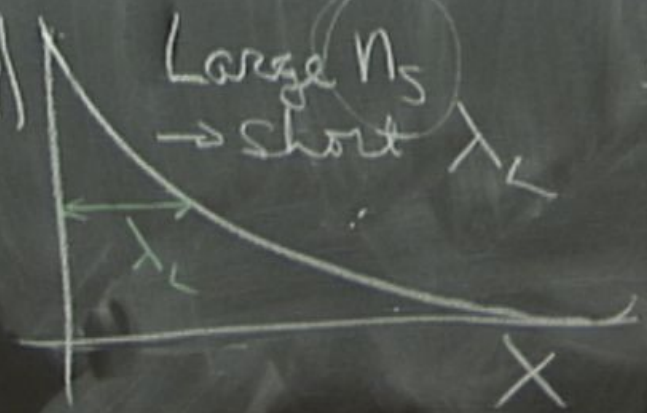

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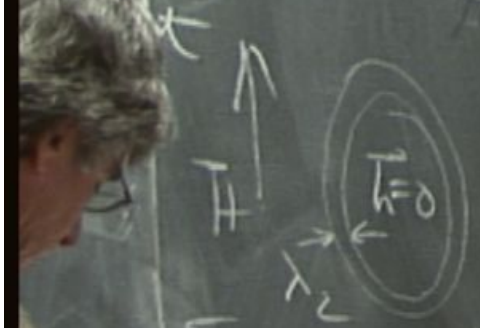
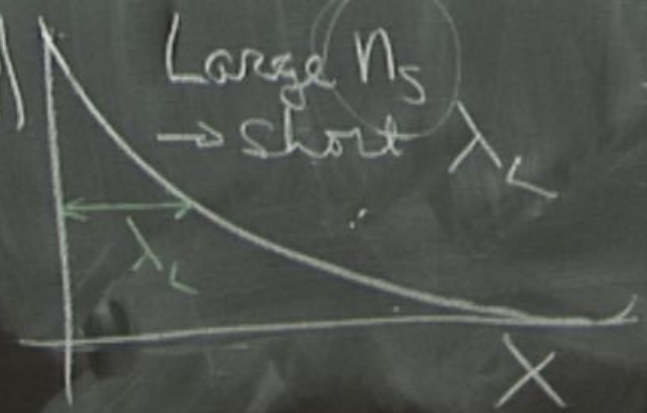


Experimentally

$\lambda_L =$



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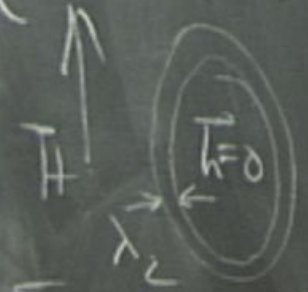
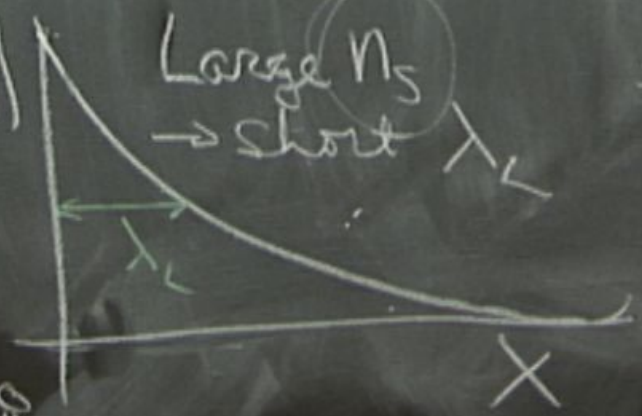
London

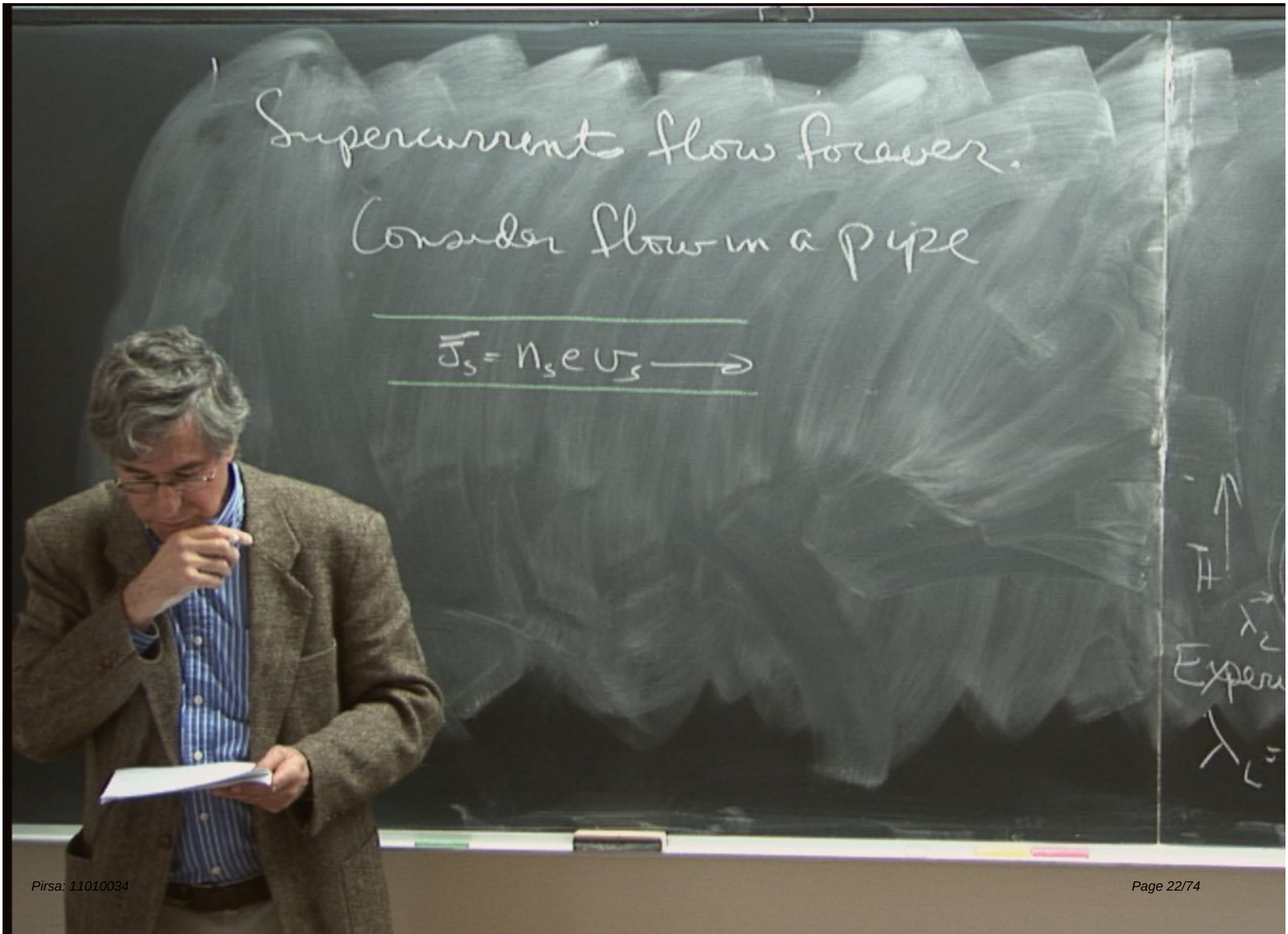
λ_L = Penetration Depth

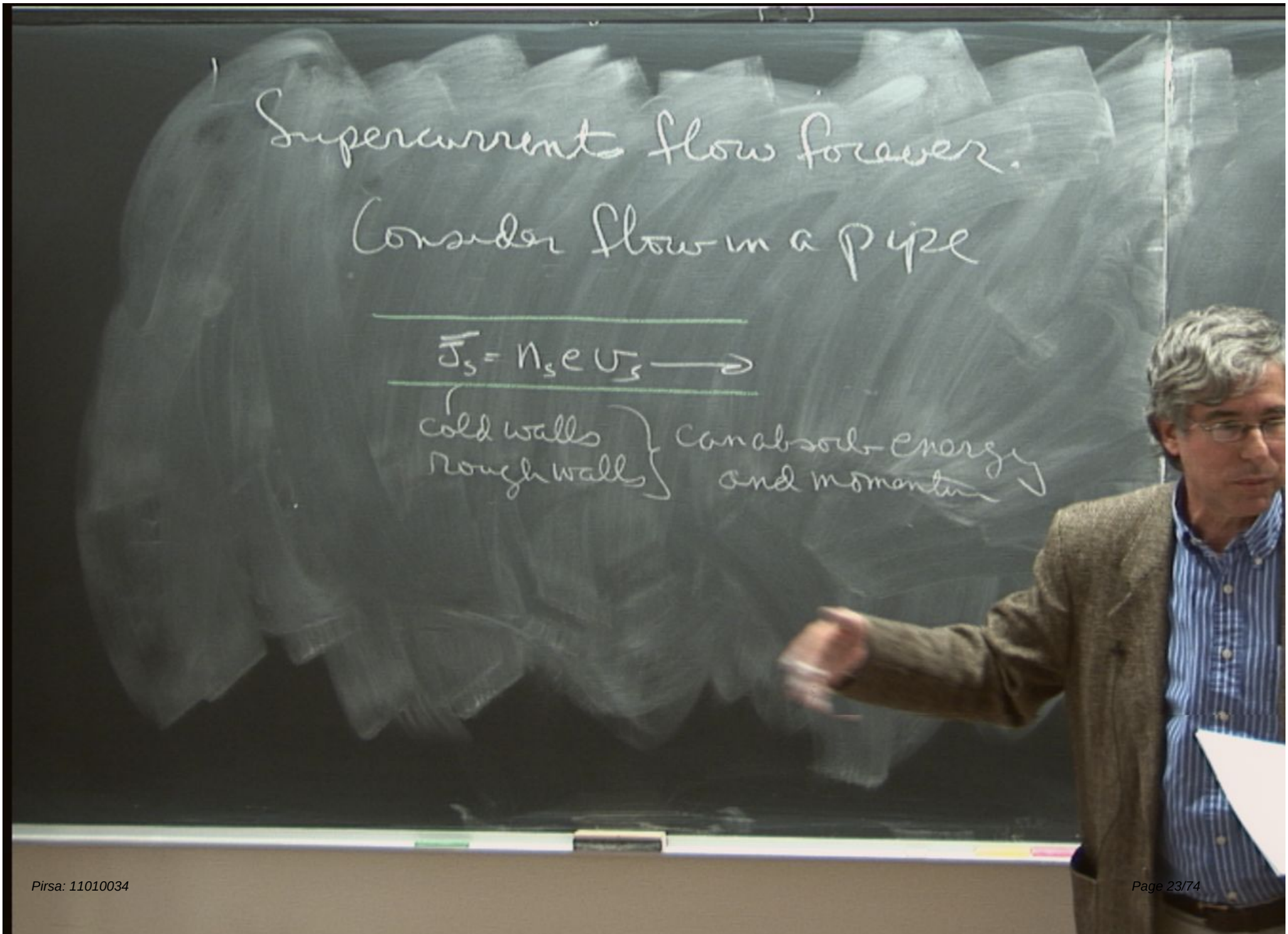


Experimentally

$\lambda_L = 10^2 - 10^4$ Å




Supercurrents flow forever.
Consider flow in a pipe
$\vec{J}_s = n_s e v_s \longrightarrow$
$\vec{H}$
$\vec{\lambda}_L$
Experi
$\lambda_L =$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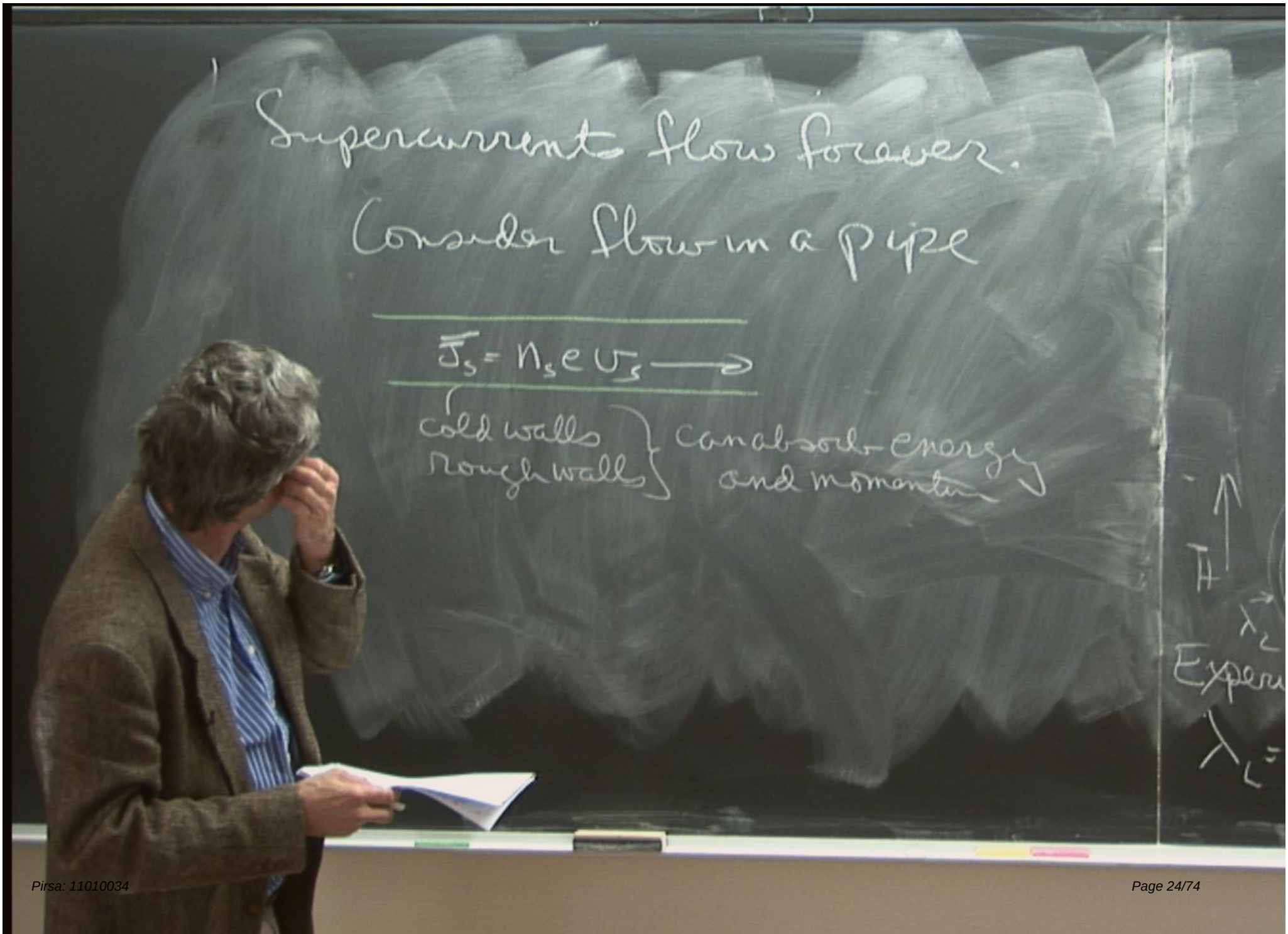
Consider flow in a pipe
$\bar{J}_s = n_s e v_s \longrightarrow$
cold walls
rough walls
can absorb energy and momentum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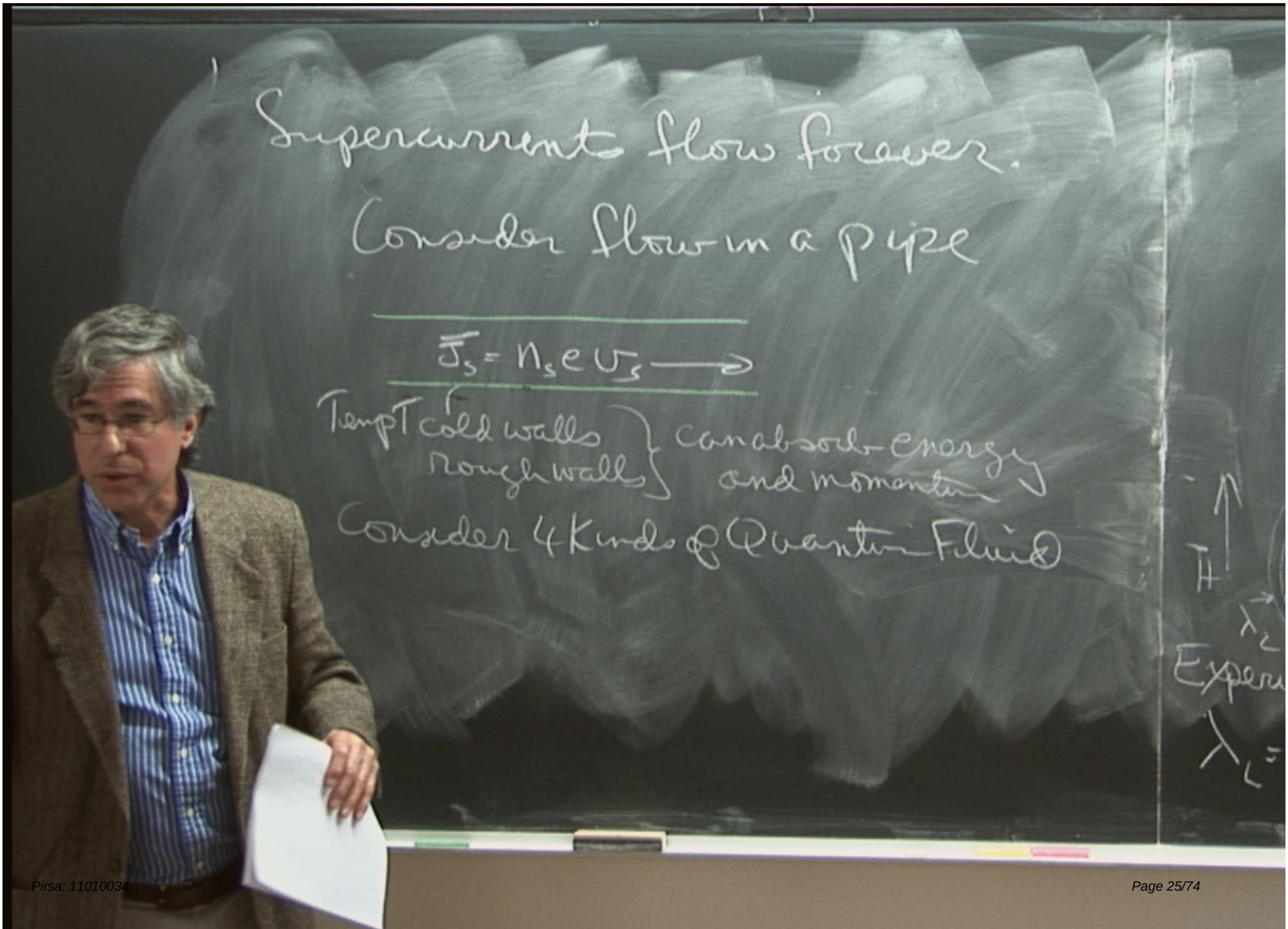
Supercurrents flow forever.
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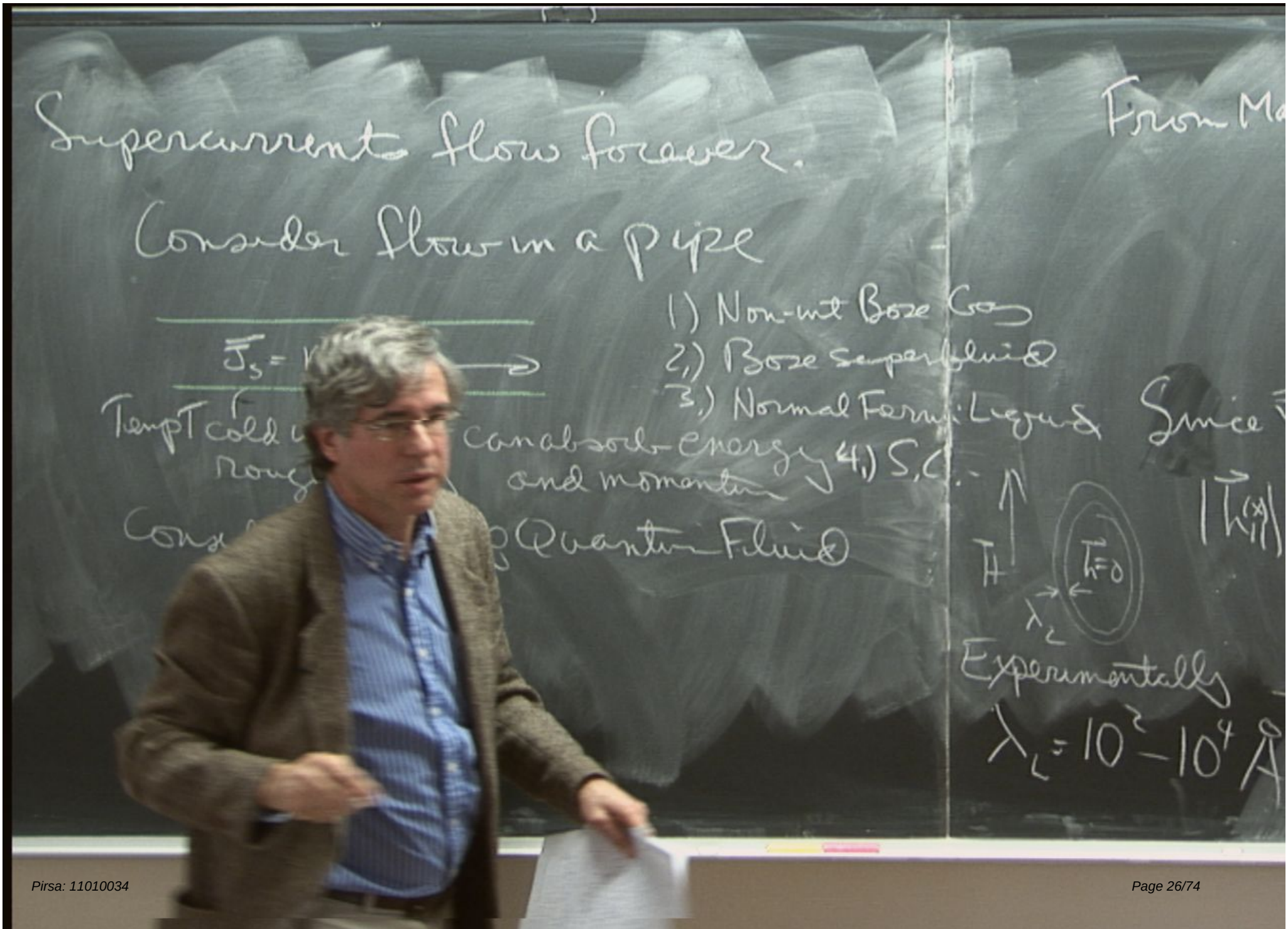
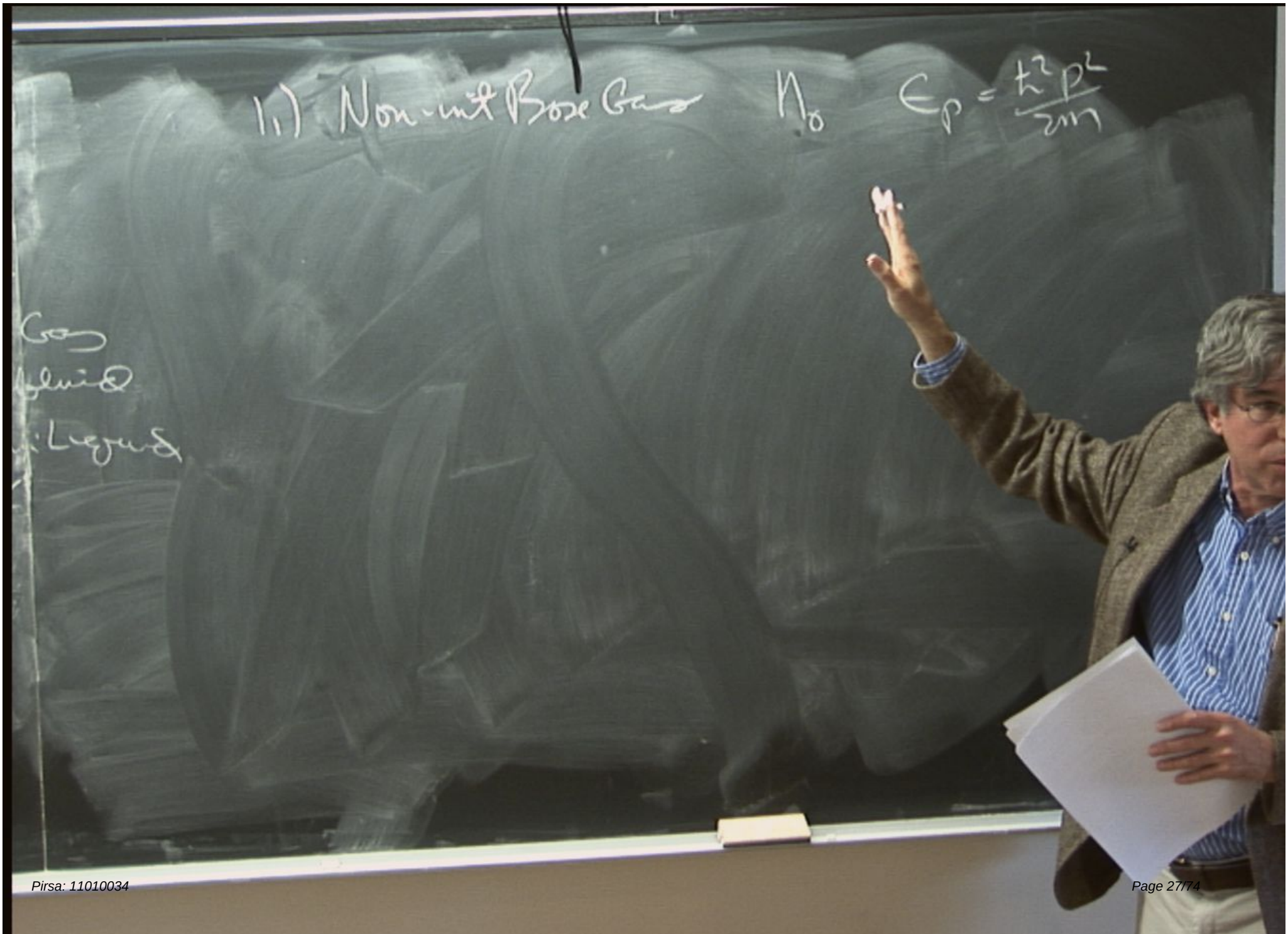
$\vec{J}_s = n_s e v_s \longrightarrow$
Temp T cold walls
rough walls
can absorb energy and momentum
Consider 4 Kinds of Quantum Fluid
H
$\vec{\lambda}_L$
Experi
λ_L


Supercurrents flow forever.
Consider flow in a pipe
1) Non-int Bose Gas
2,) Bose Superfluid
3,) Normal Fermi Liquid
4,) S.C.
Quantum Fluid
Experimentally
$\lambda_L = 10^2 - 10^4$ Å

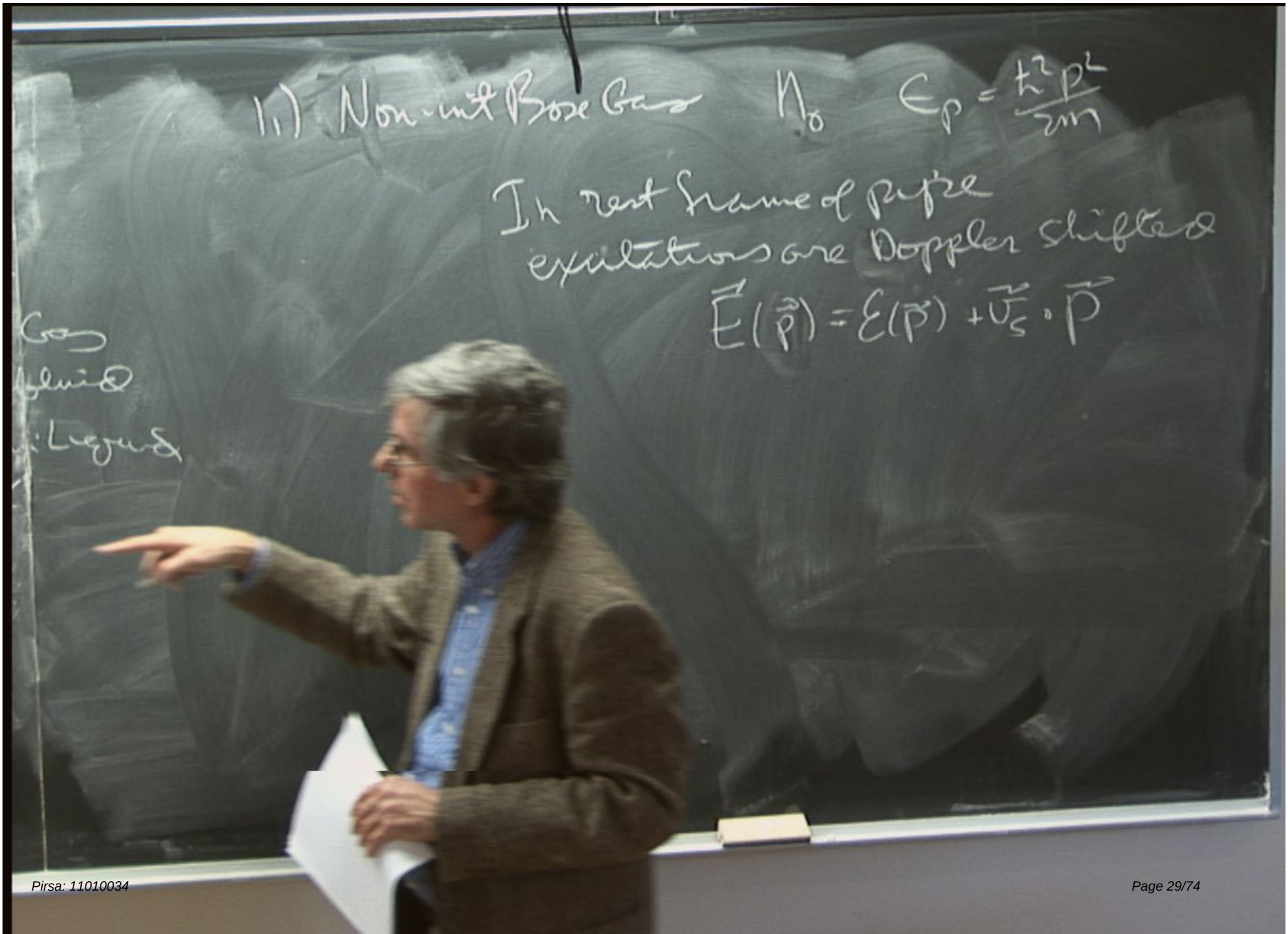

1.1) Non-int Bose Gas
n_0
$\epsilon_p = \frac{\hbar^2 p^2}{2m}$
Gas
Liquid

1.) Non-int Bose Gas n_0 $\epsilon_p = \frac{\hbar^2 p^2}{2m}$

In rest frame of pipe
excitations are Doppler shifted

$$E(\vec{p}) = \mathcal{E}(\vec{p}) + \vec{v}_s \cdot \vec{p}$$

Gas
fluid
Liquid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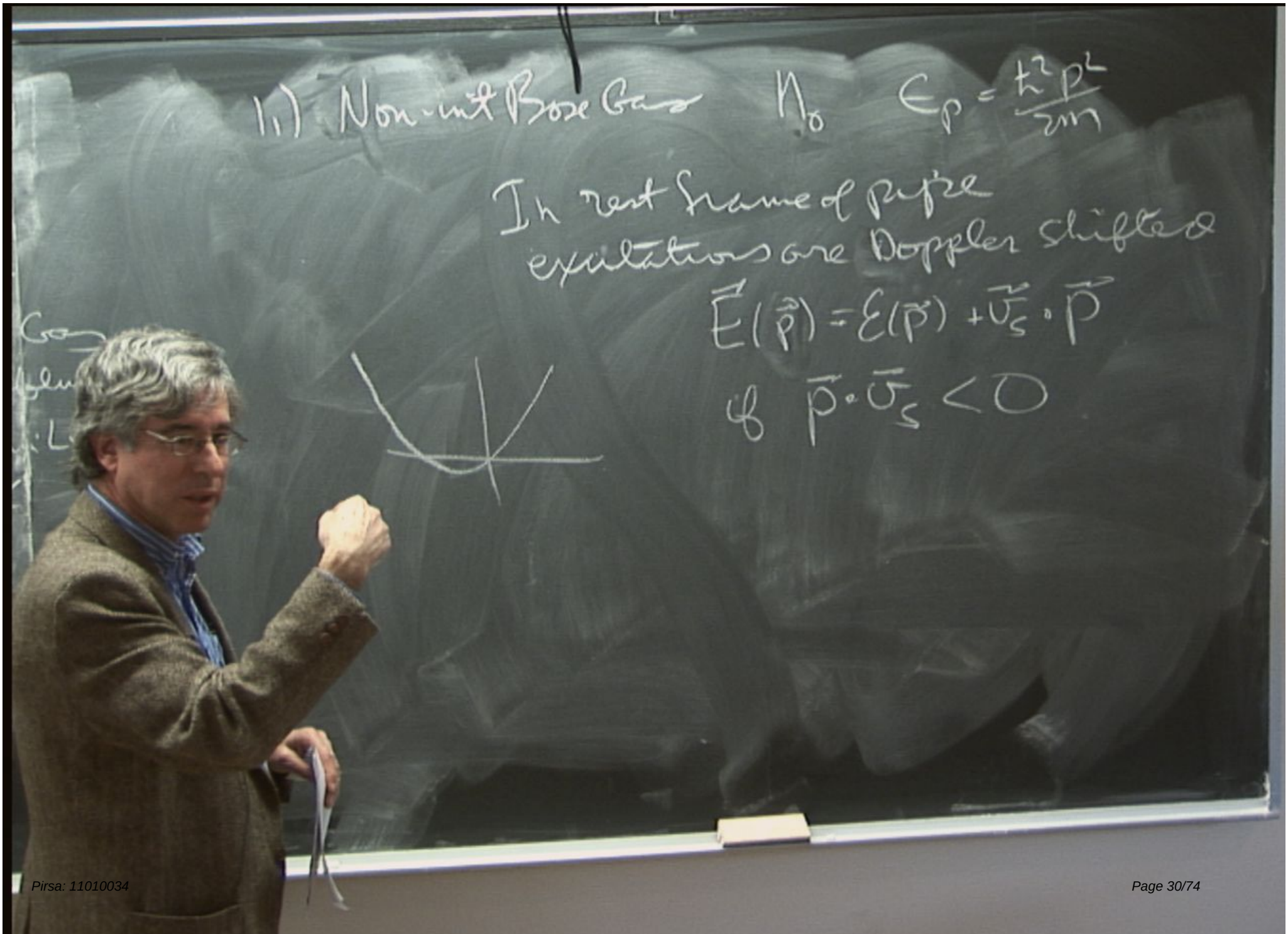
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In rest frame of pipe
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$\vec{E}(\vec{p}) = \mathcal{E}(\vec{p}) + \vec{v}_s \cdot \vec{p}$
if $\vec{p} \cdot \vec{v}_s < 0$

1.) Non-int Bose Gas $\quad n_0 \quad \epsilon_p = \frac{\hbar^2 p^2}{2m}$

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Gas
fluid
Liquid

2. Weakly repulsive Bose gas.

1.) Non-int Bose Gas $\quad n_0 \quad \epsilon_p = \frac{\hbar^2 p^2}{2m}$

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Gas
fluid
Liquid

2. Weakly repulsive Bose gas.

$$\mathcal{E}(\vec{p}) = c_s |\vec{p}|$$

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Gas
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2. Weakly repulsive Bose gas

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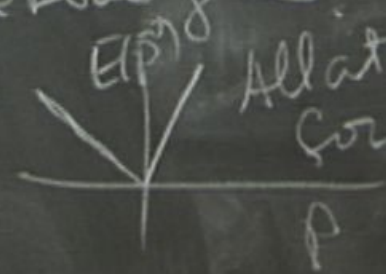


All at $E(\vec{p}) > 0$
for $|v_s| < c_s$

1.) Non-int Bose Gas $\quad n_0 \quad \epsilon_p = \frac{\hbar^2 p^2}{2m}$

In rest frame of pipe excitations are Doppler shifted

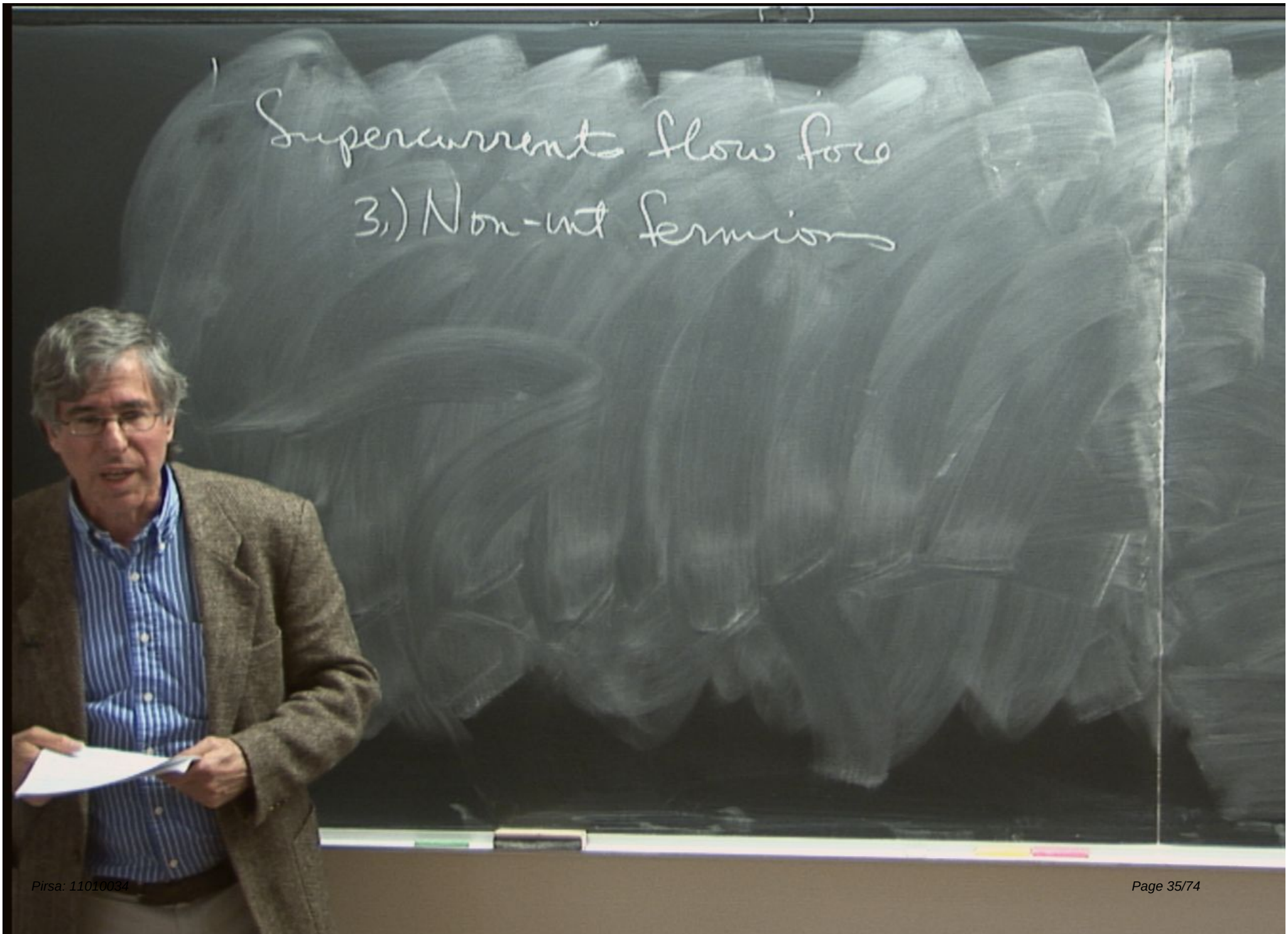
$$\vec{E}(\vec{p}) = \mathcal{E}(\vec{p}) + \vec{v}_s \cdot \vec{p}$$

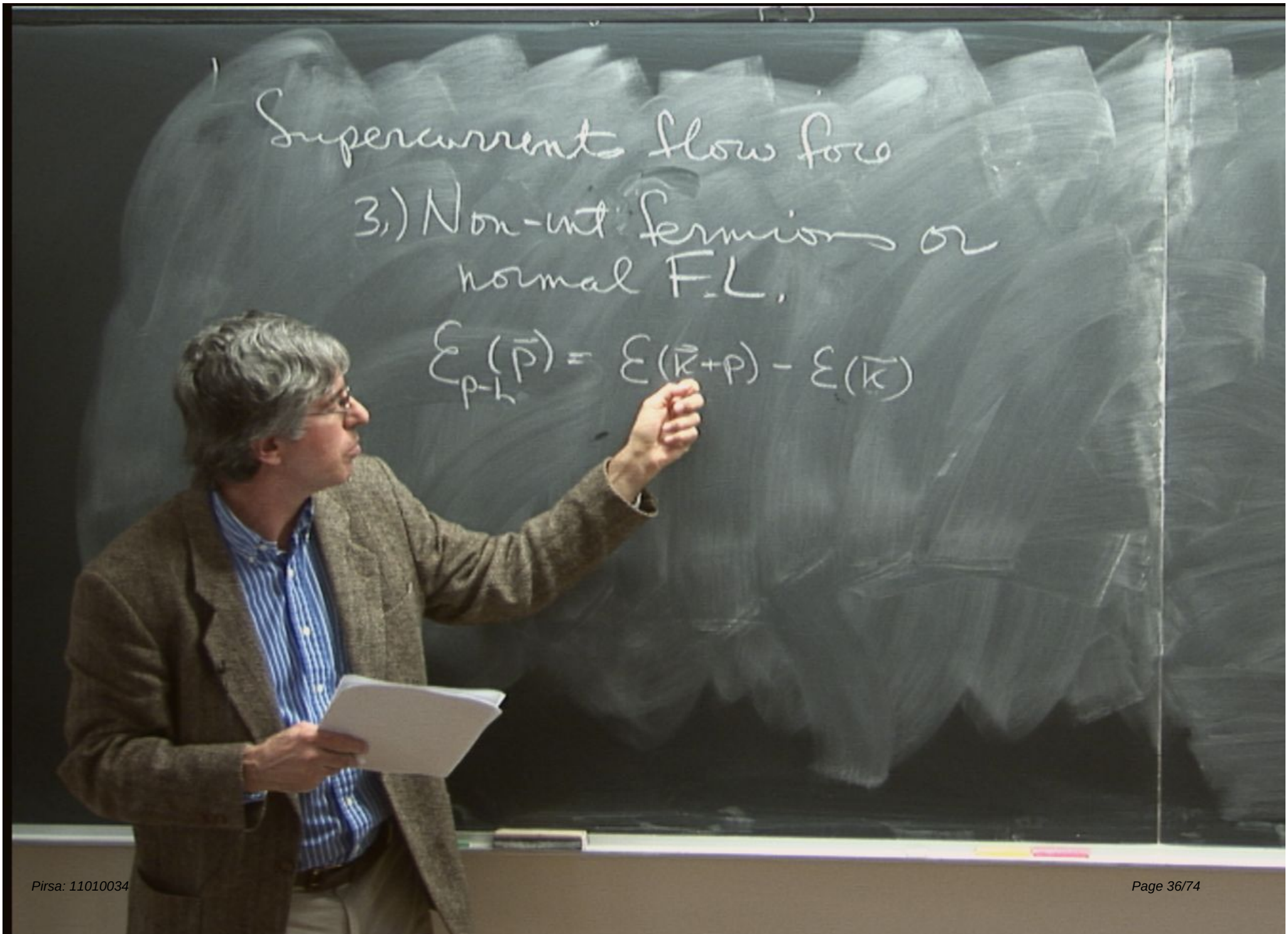
if $\vec{p} \cdot \vec{v}_s < 0$

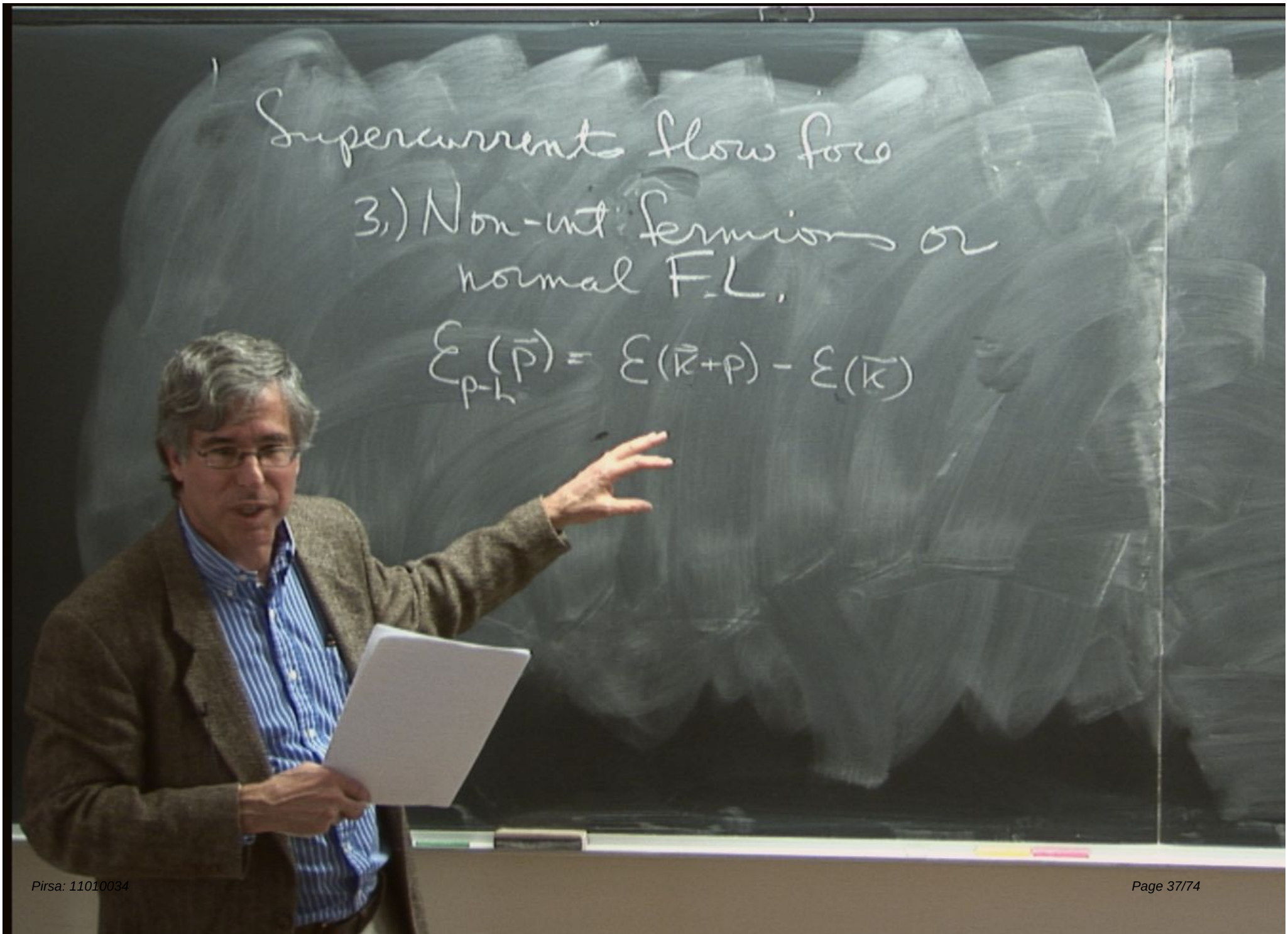
2.) Weakly repulsive Bose gas:

$$\mathcal{E}(\vec{p}) = c_s |\vec{p}|$$

All at $E(\vec{p}) > 0$ for $|v_s| < c_s$


Supercurrents flow free
3.) Non-int fermions


Supercurrents flow for
3.) Non-int fermions or
normal F.L.
$\mathcal{E}_{p-h}(\vec{p}) = \mathcal{E}(\vec{k}+p) - \mathcal{E}(\vec{k})$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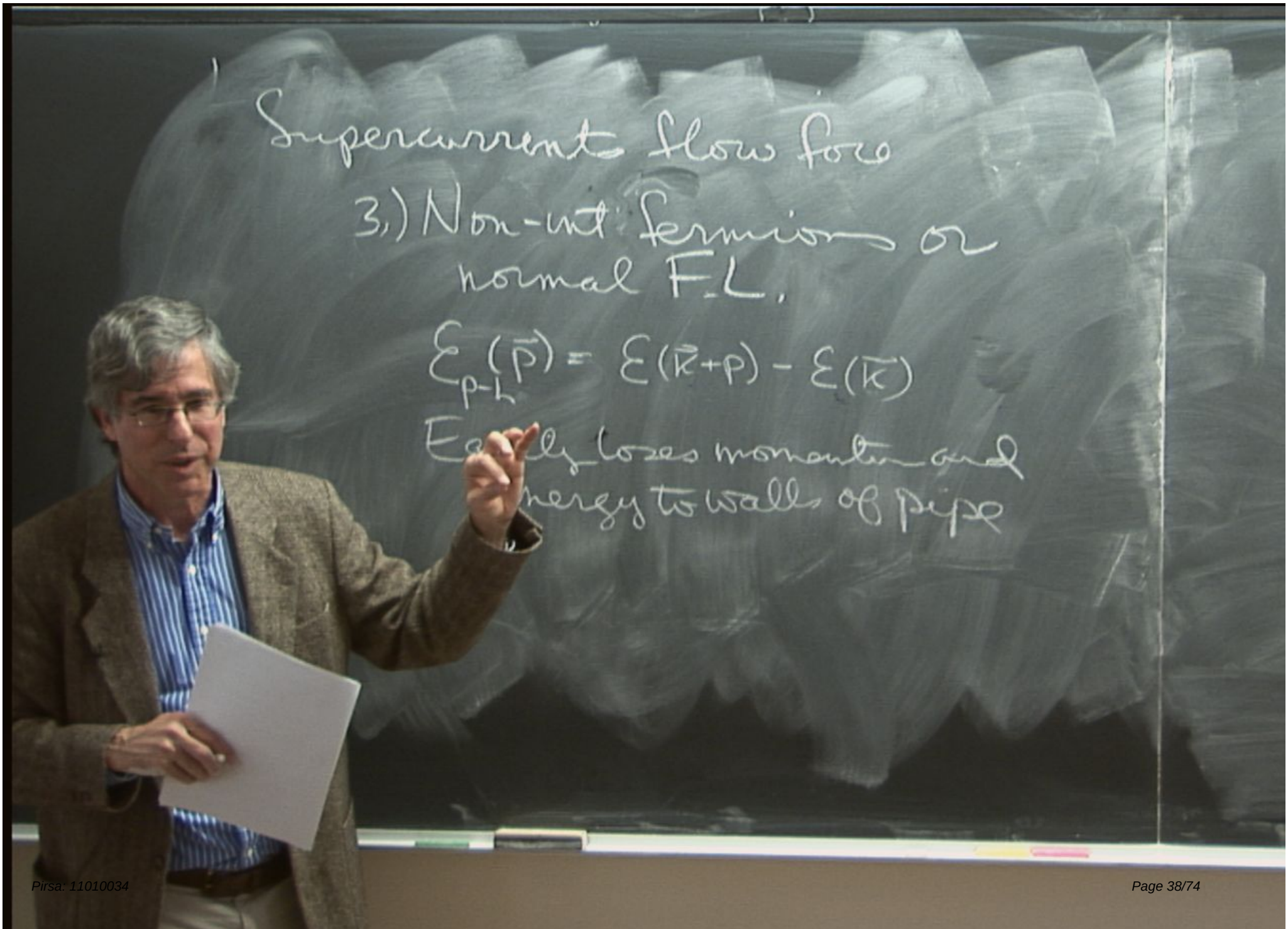
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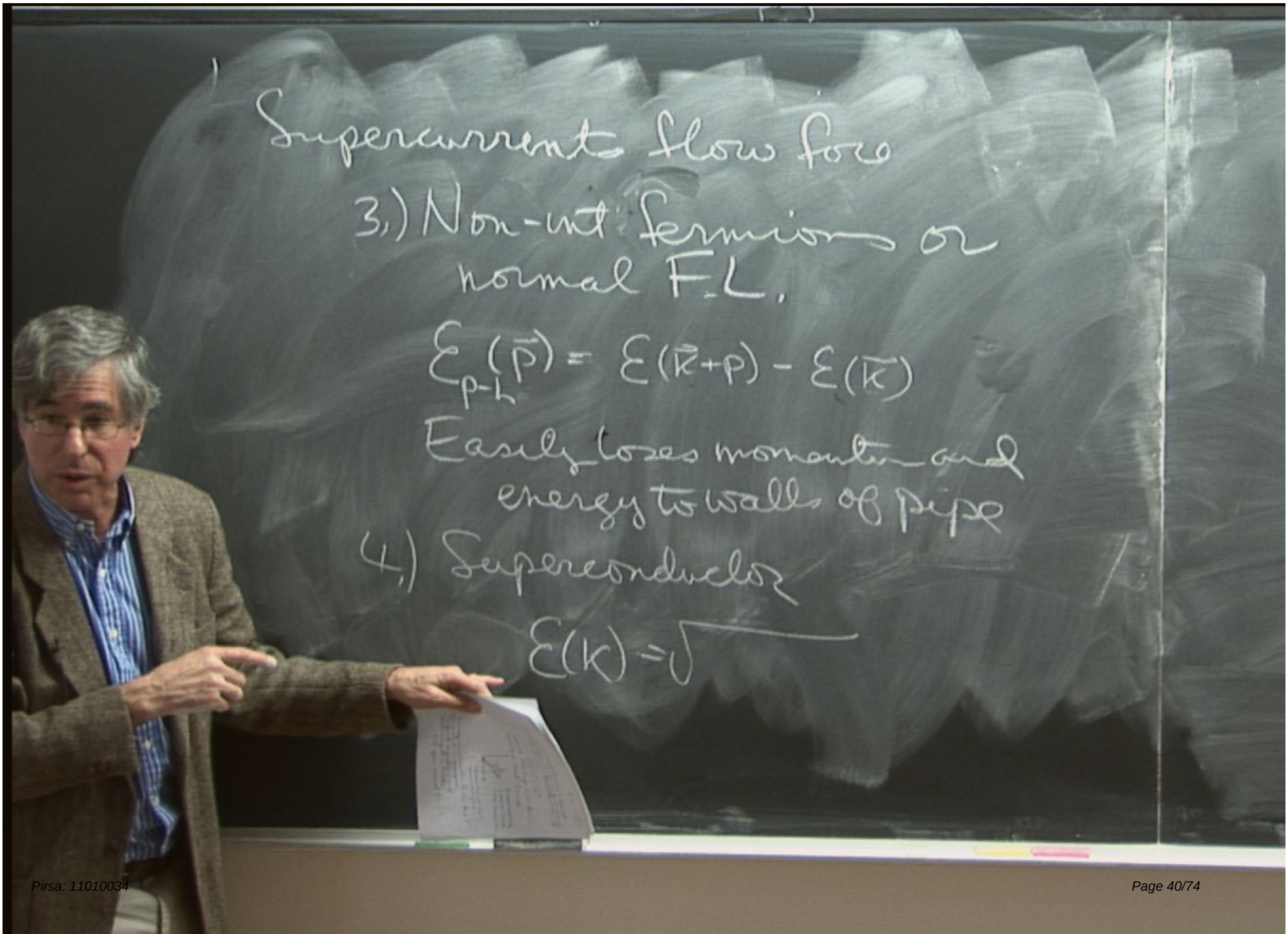
Easily loses momentum and
energy to walls of pipe

Supercurrents flow forever

3.) Non-int fermions or normal F.L.

$$\mathcal{E}_{p\text{-}h}(\vec{p}) = \mathcal{E}(\vec{k}+p) - \mathcal{E}(\vec{k})$$

Easily loses momentum and energy to walls of pipe


Supercurrents flow free
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1.) Non-int Bose Gas $\quad n_0 \quad \epsilon_p = \frac{\hbar^2 p^2}{2m}$

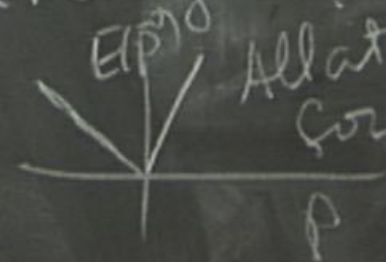
In rest frame of pipe excitations are Doppler shifted

$$\vec{E}(\vec{p}) = \mathcal{E}(\vec{p}) + \vec{v}_s \cdot \vec{p}$$

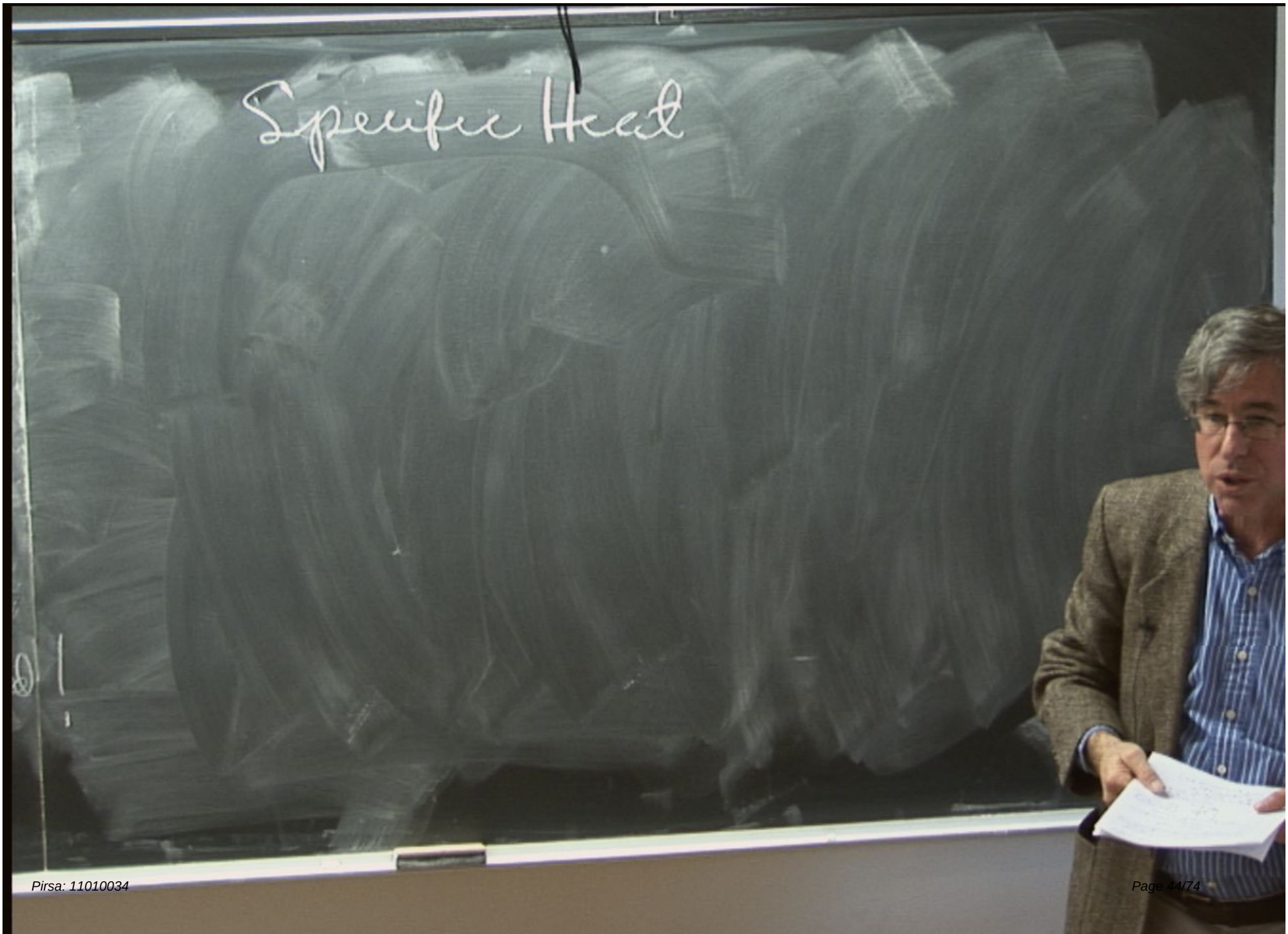
if $\vec{p} \cdot \vec{v}_s < 0$

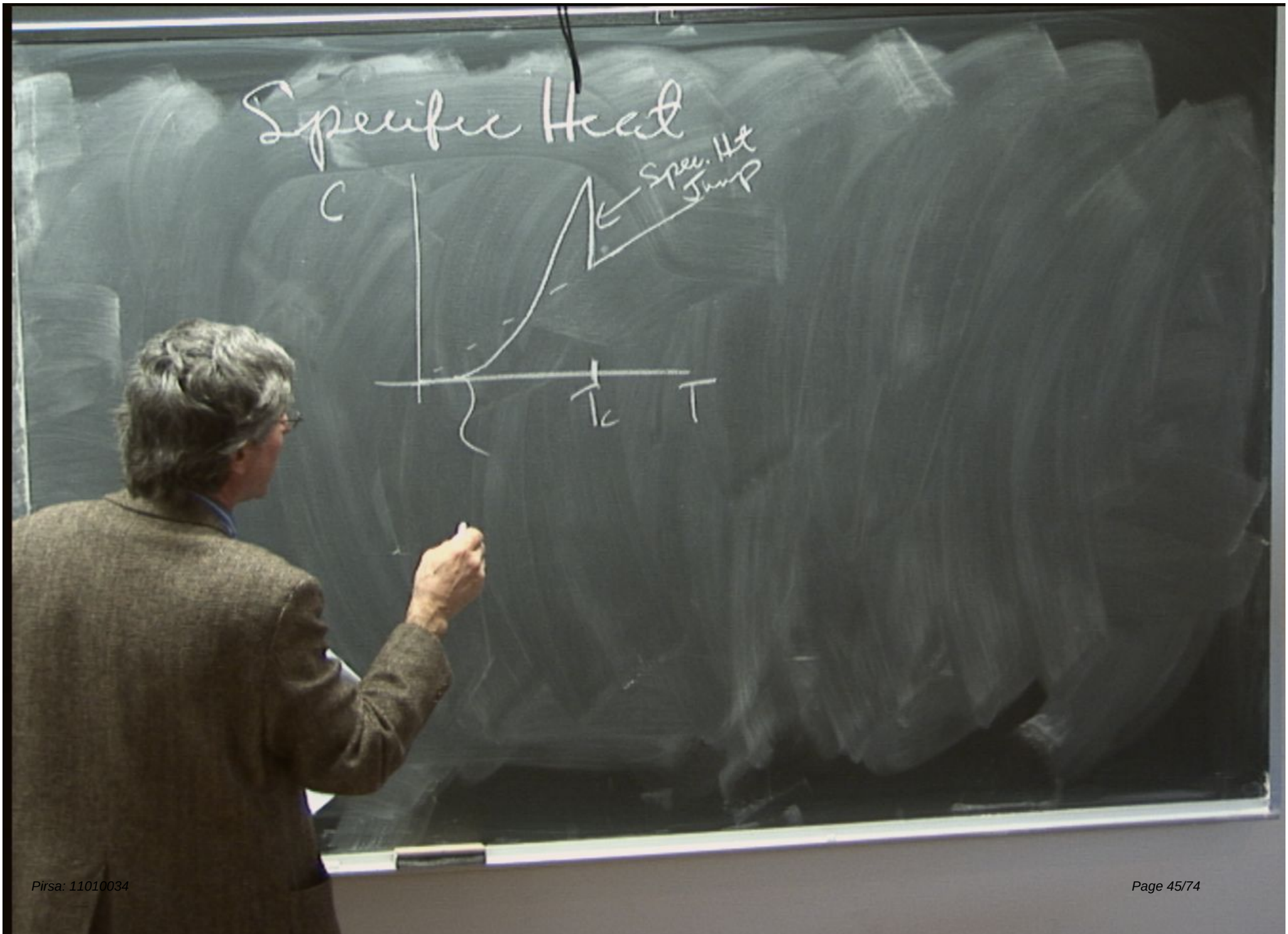
2. Weakly repulsive Bose gas

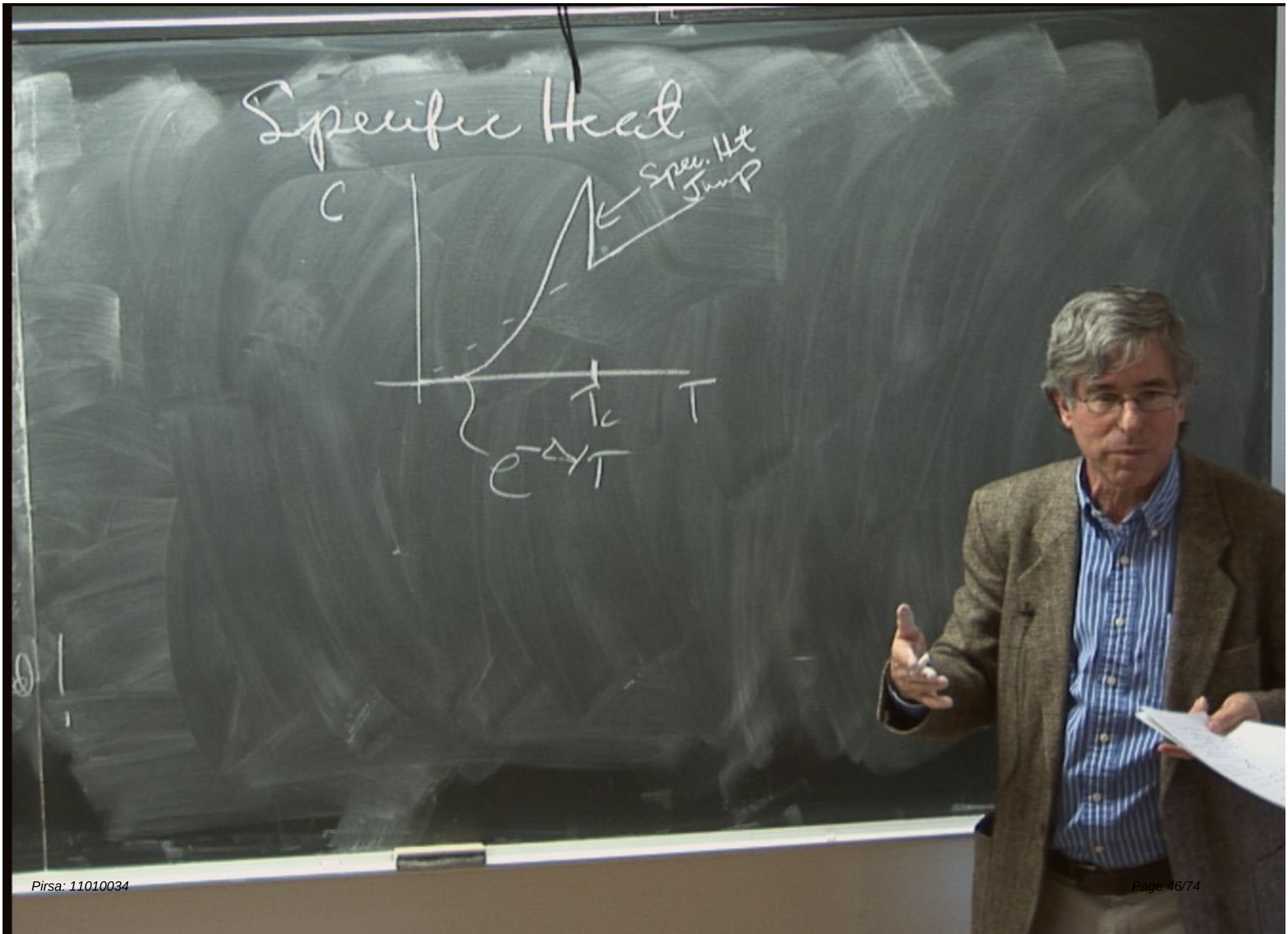
$$\mathcal{E}(\vec{p}) = c_s |\vec{p}|$$



All at $E(\vec{p}) > 0$ for $|v_s| < c_s$


Specific Heat


Specific Heat
C
Spec. Ht Jump
T_c
T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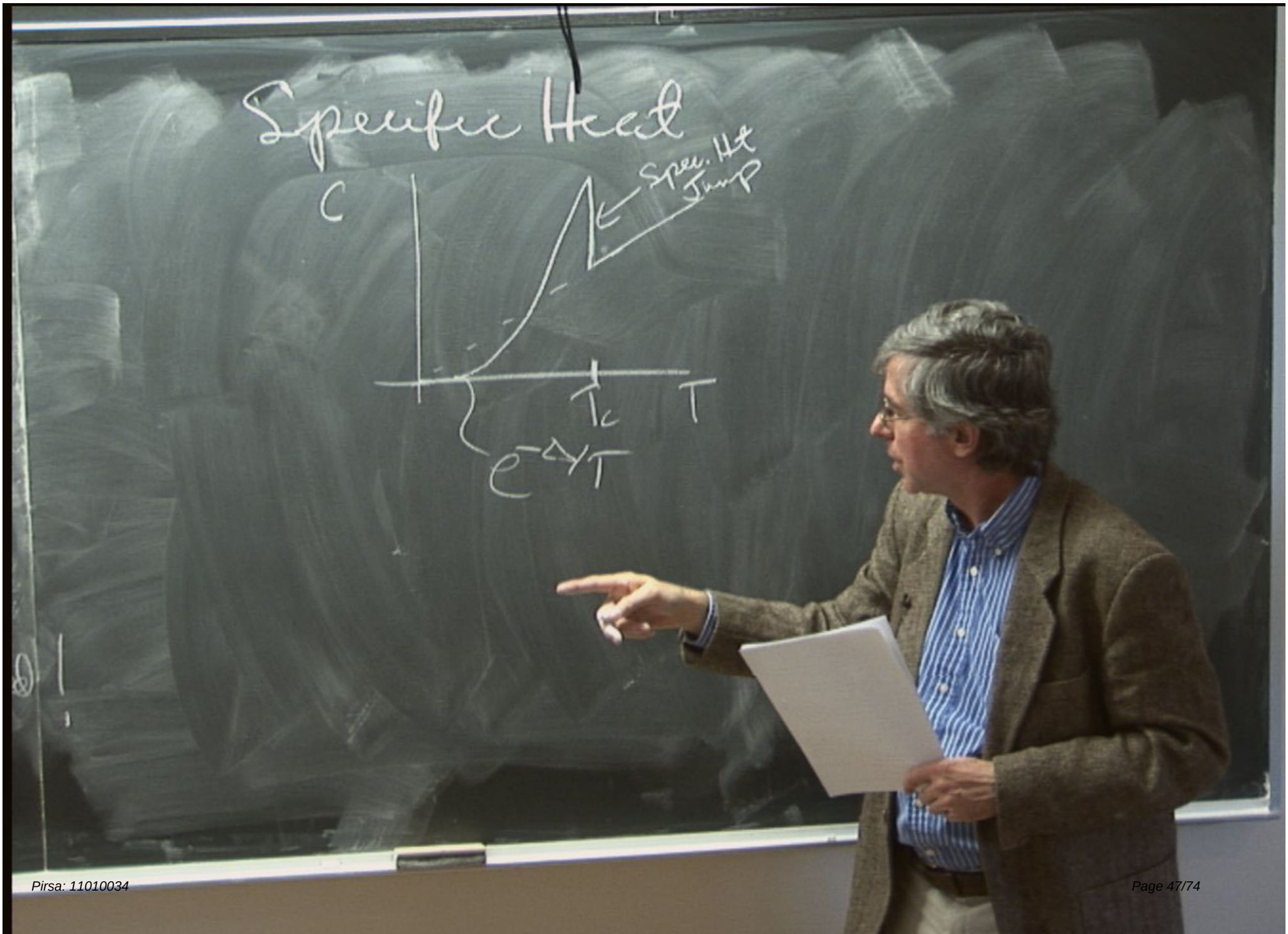
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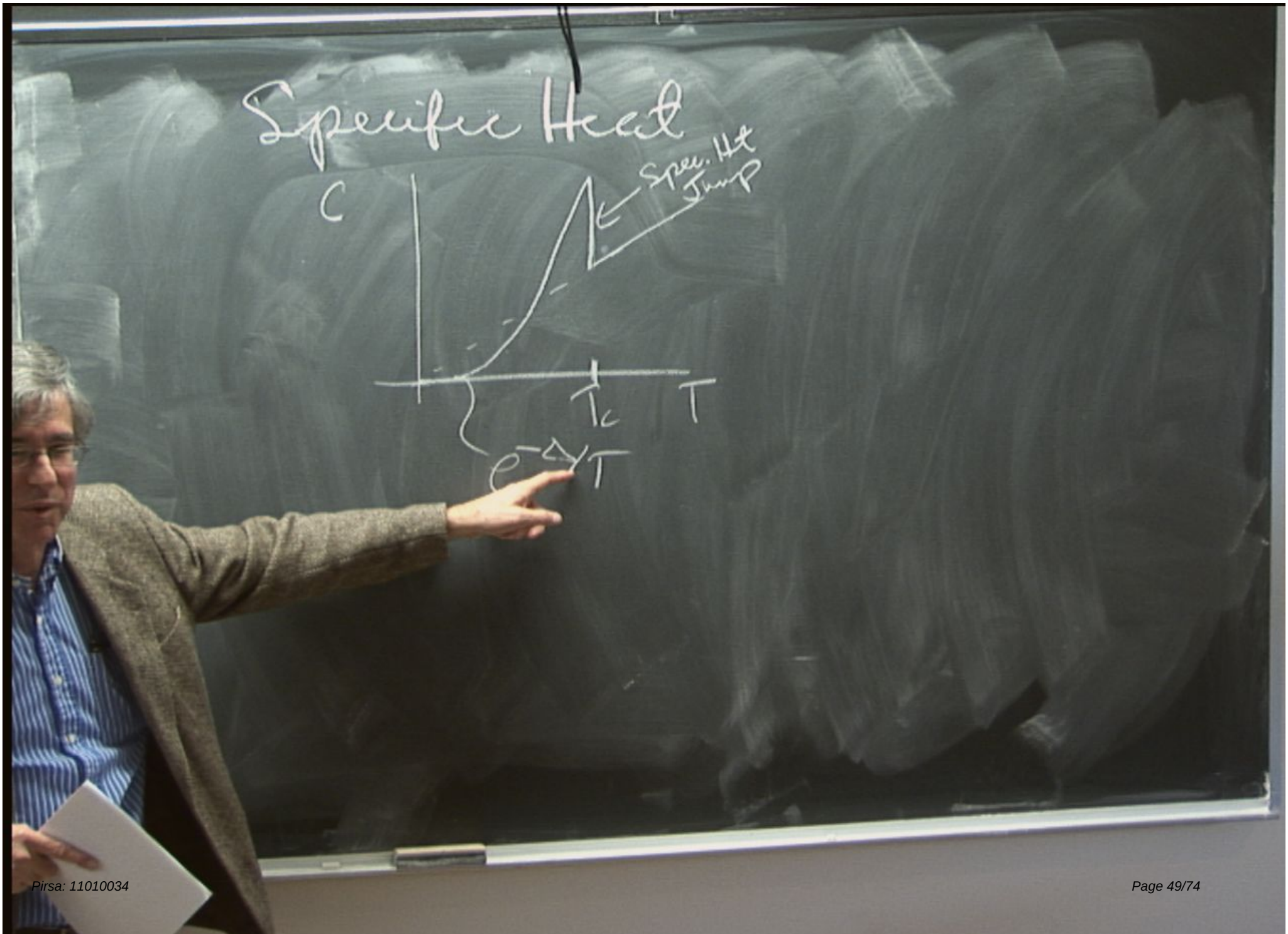
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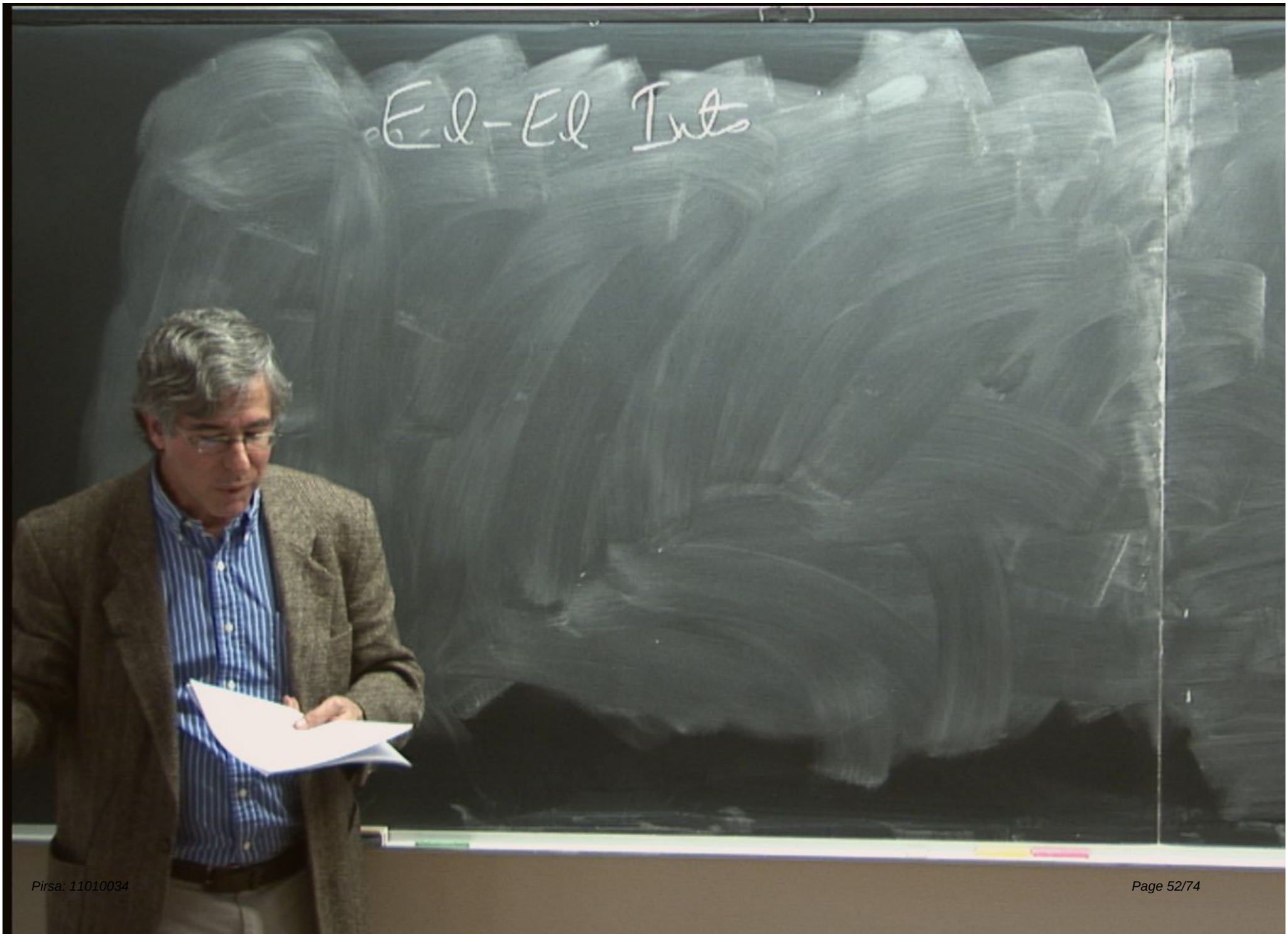
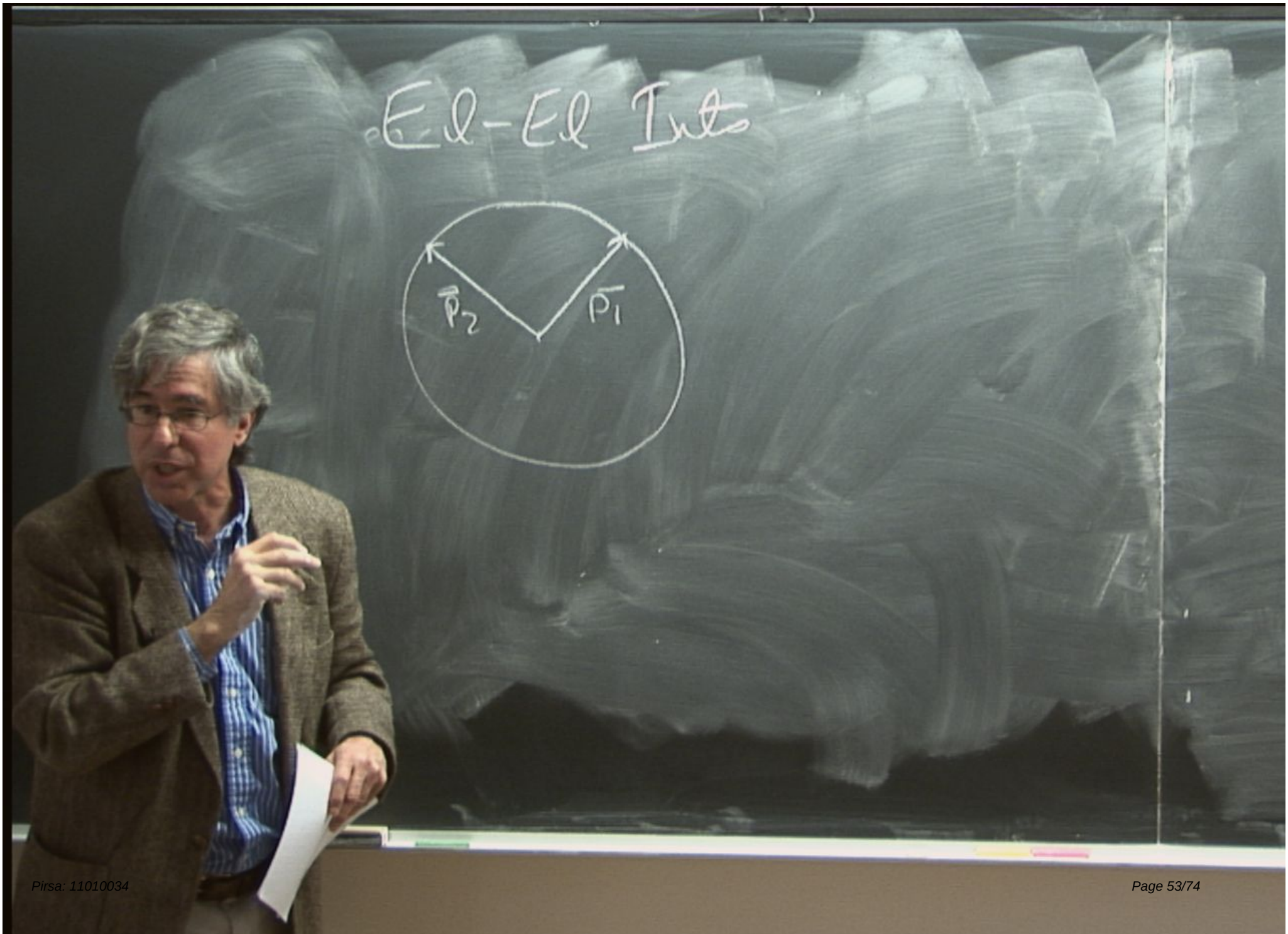
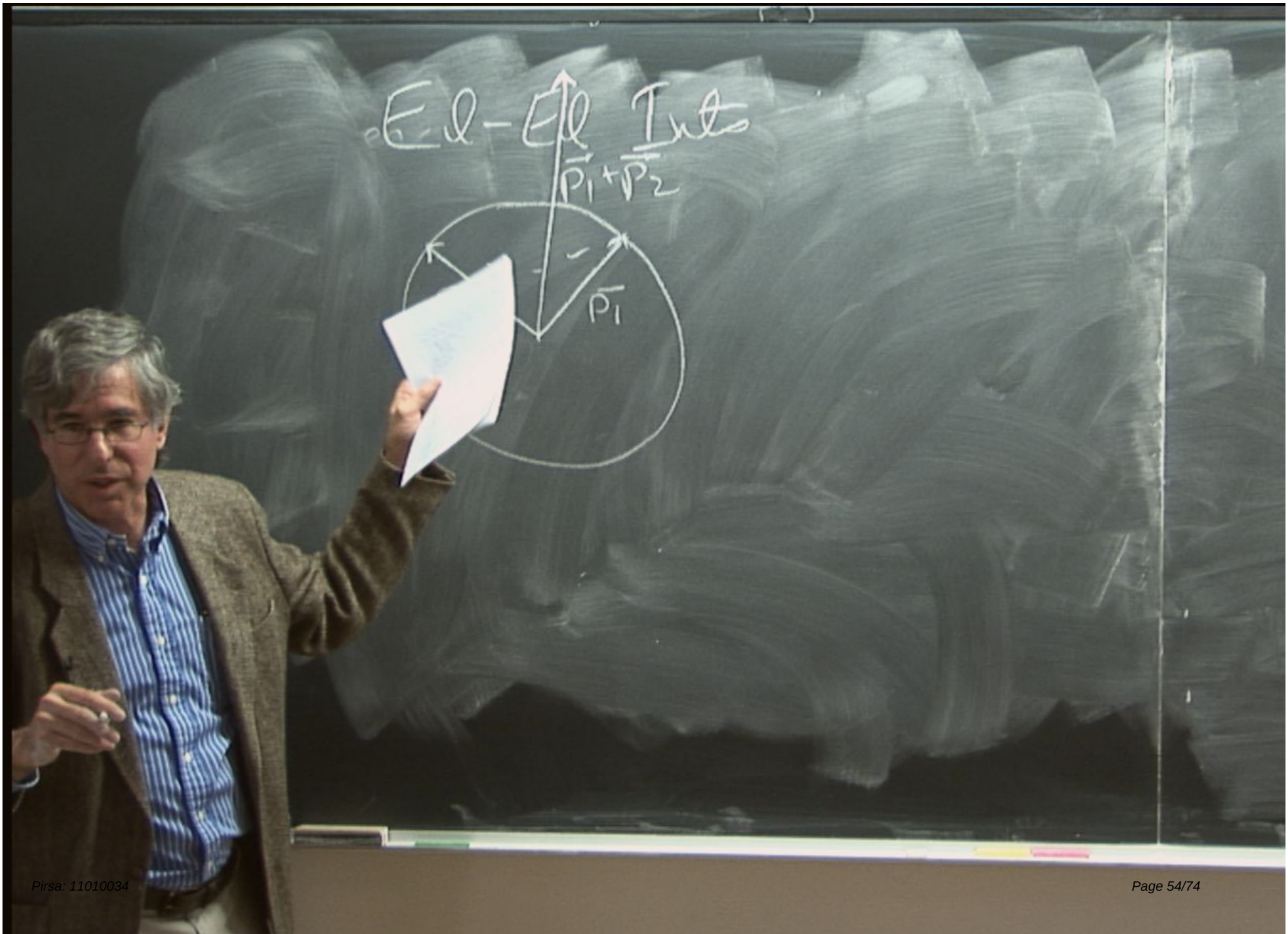

El-El Ints


El-El Ints
$\vec{p}_2$
$\vec{p}_1$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$\vec{p}_1+\vec{p}_2$
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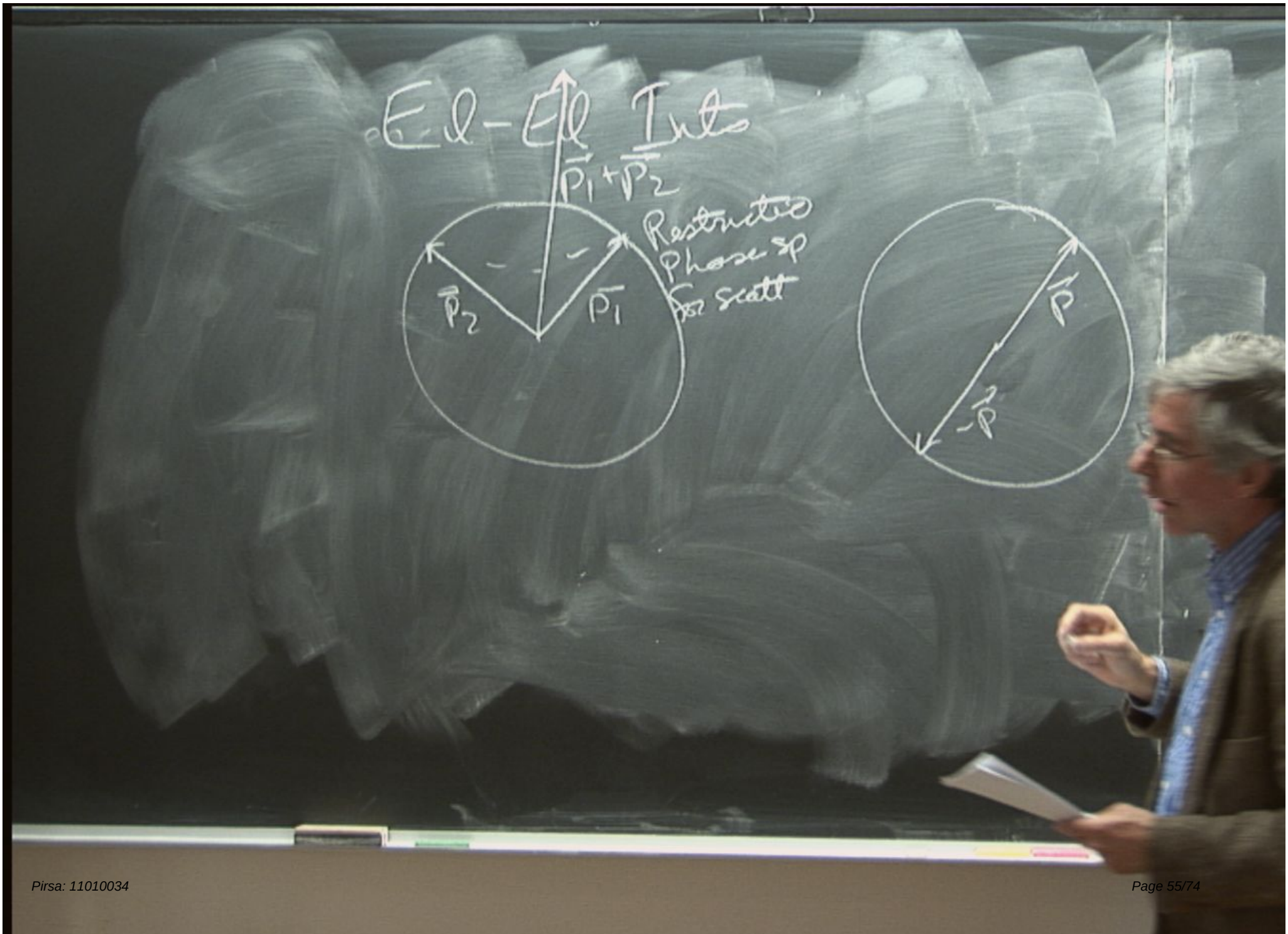
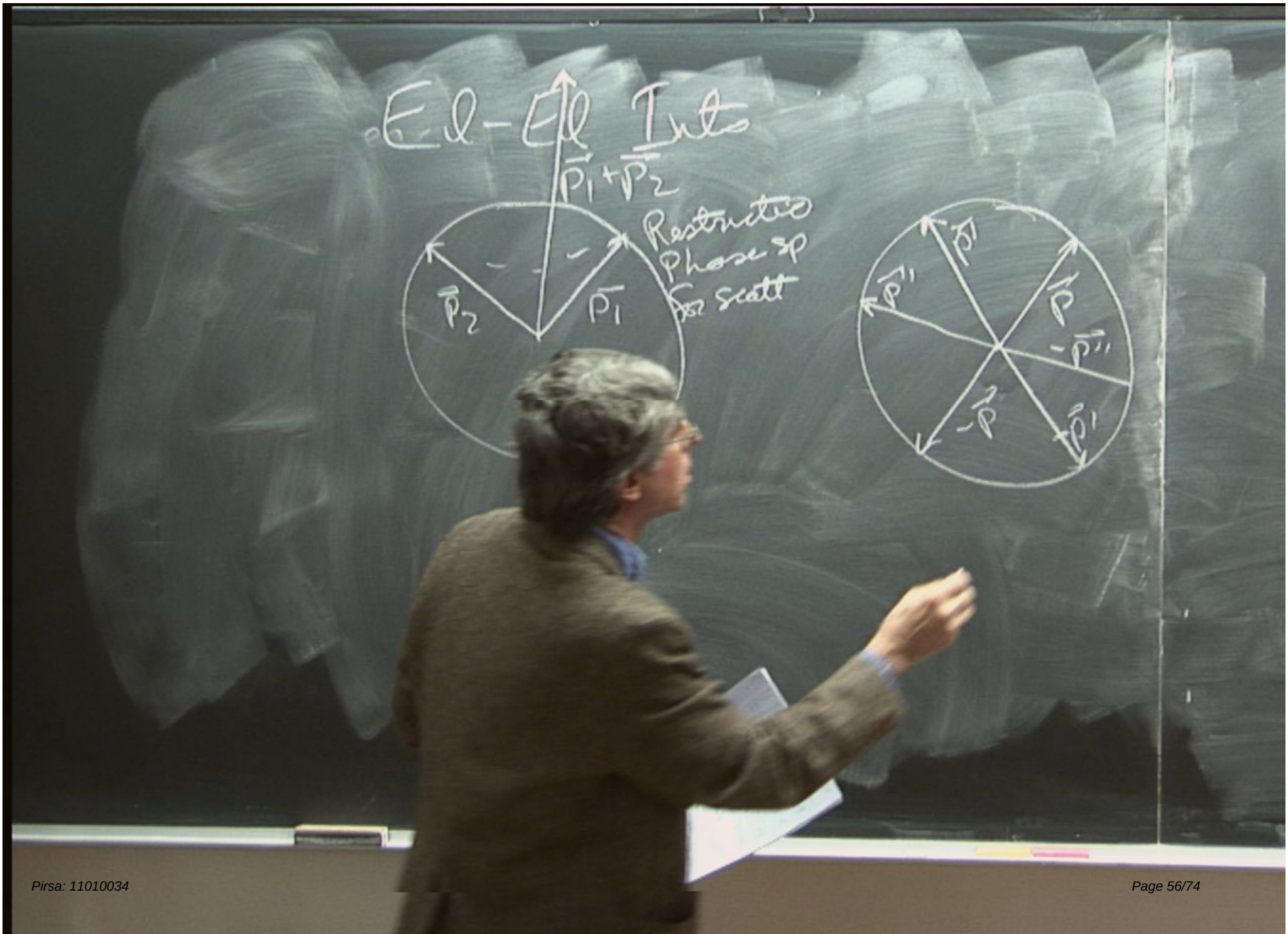

E.l-El Ints
$\vec{p}_1+\vec{p}_2$
$\vec{p}_2$
$\vec{p}_1$
Restricts
Phase sp
for scatt
$\vec{p}$
$-\vec{p}$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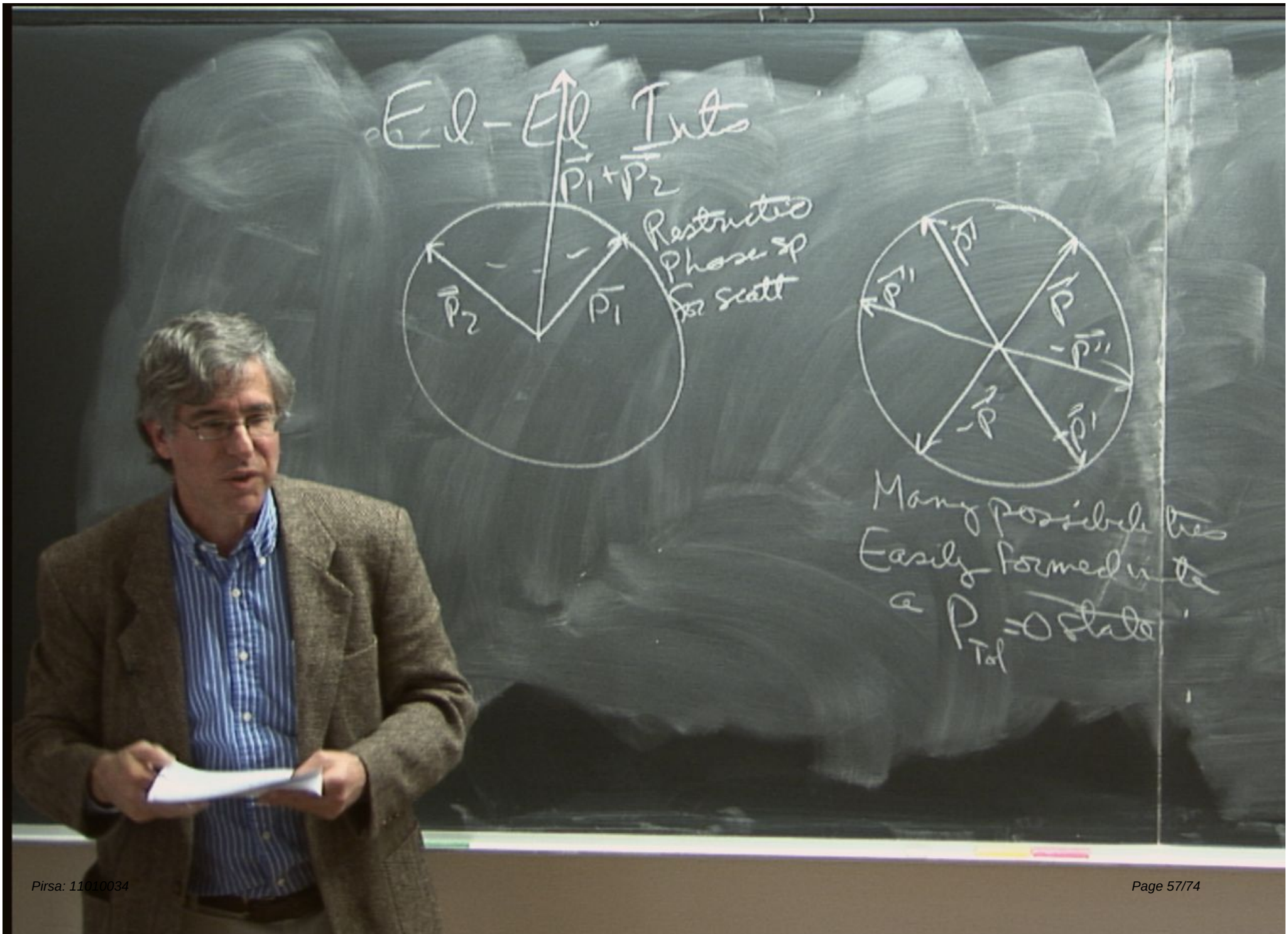
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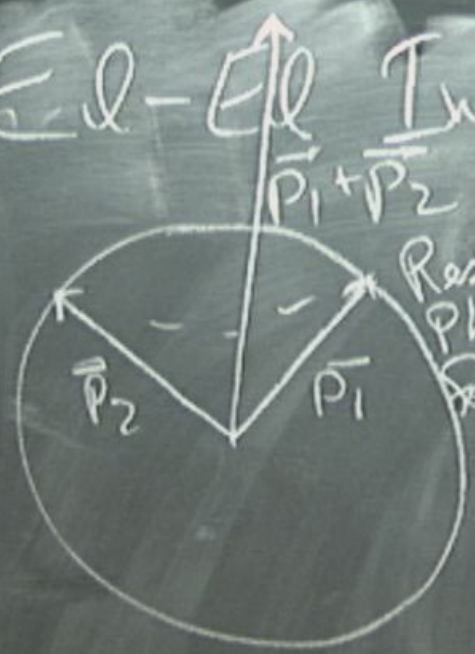

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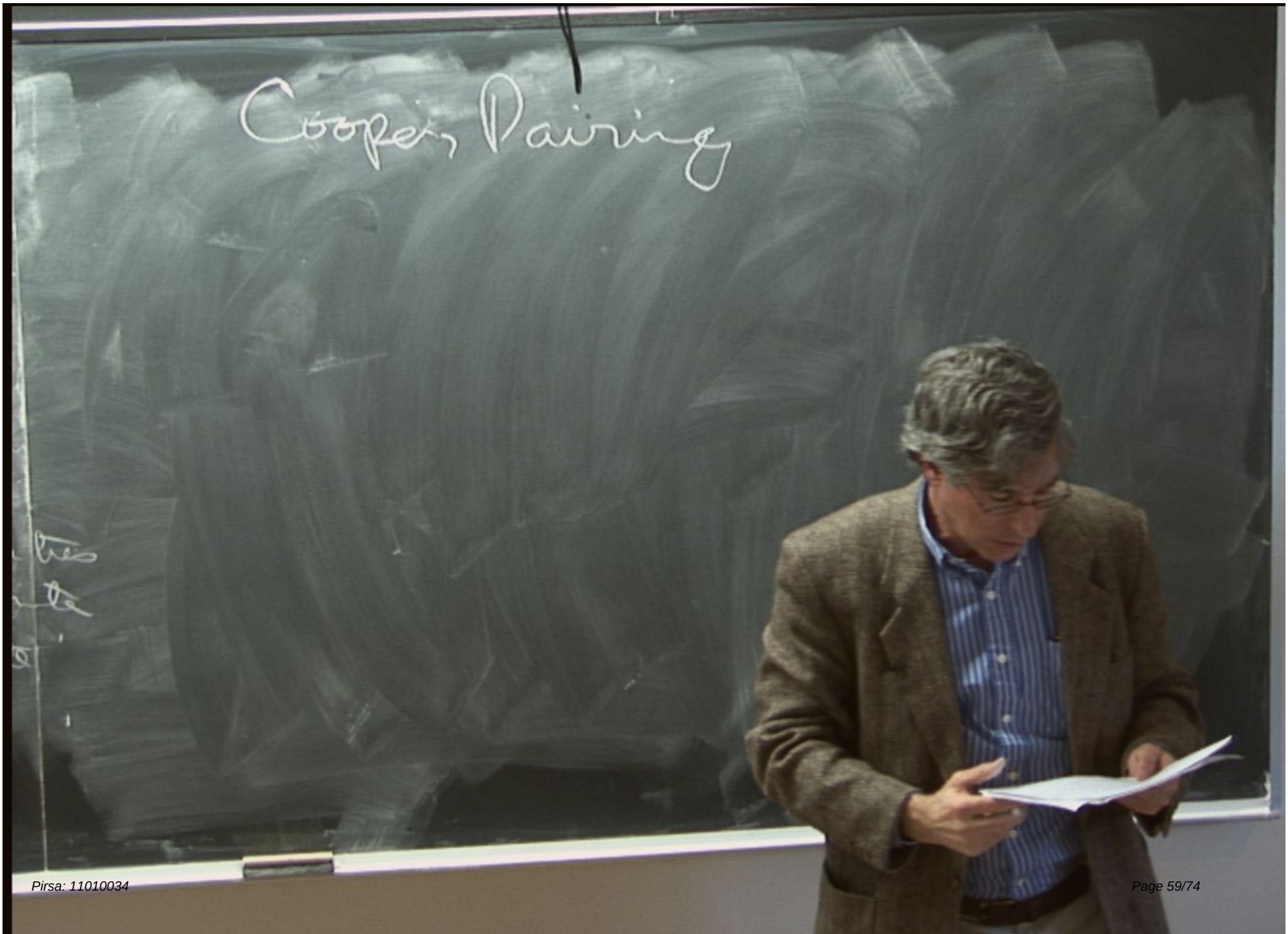
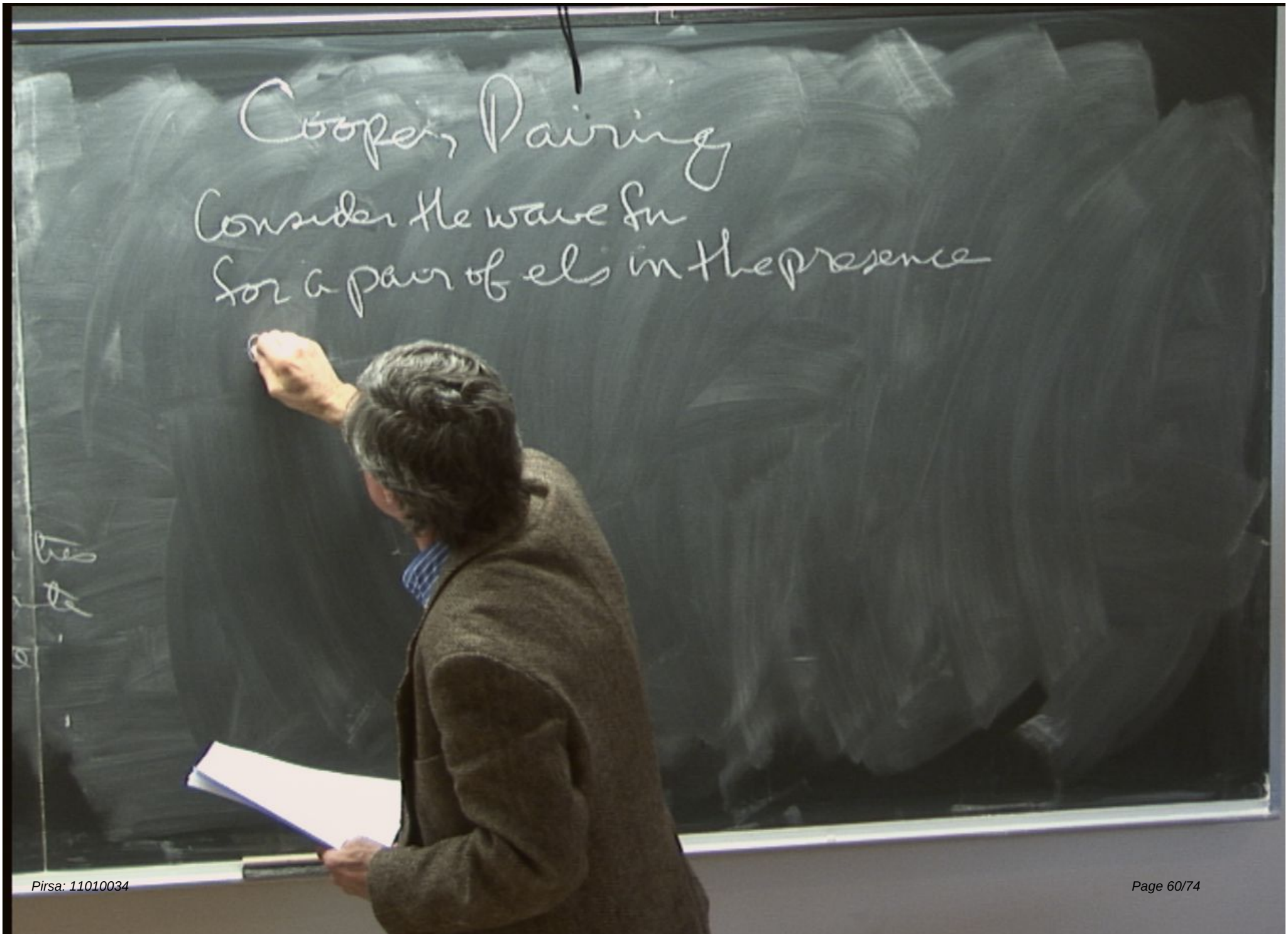

Cooper Pairing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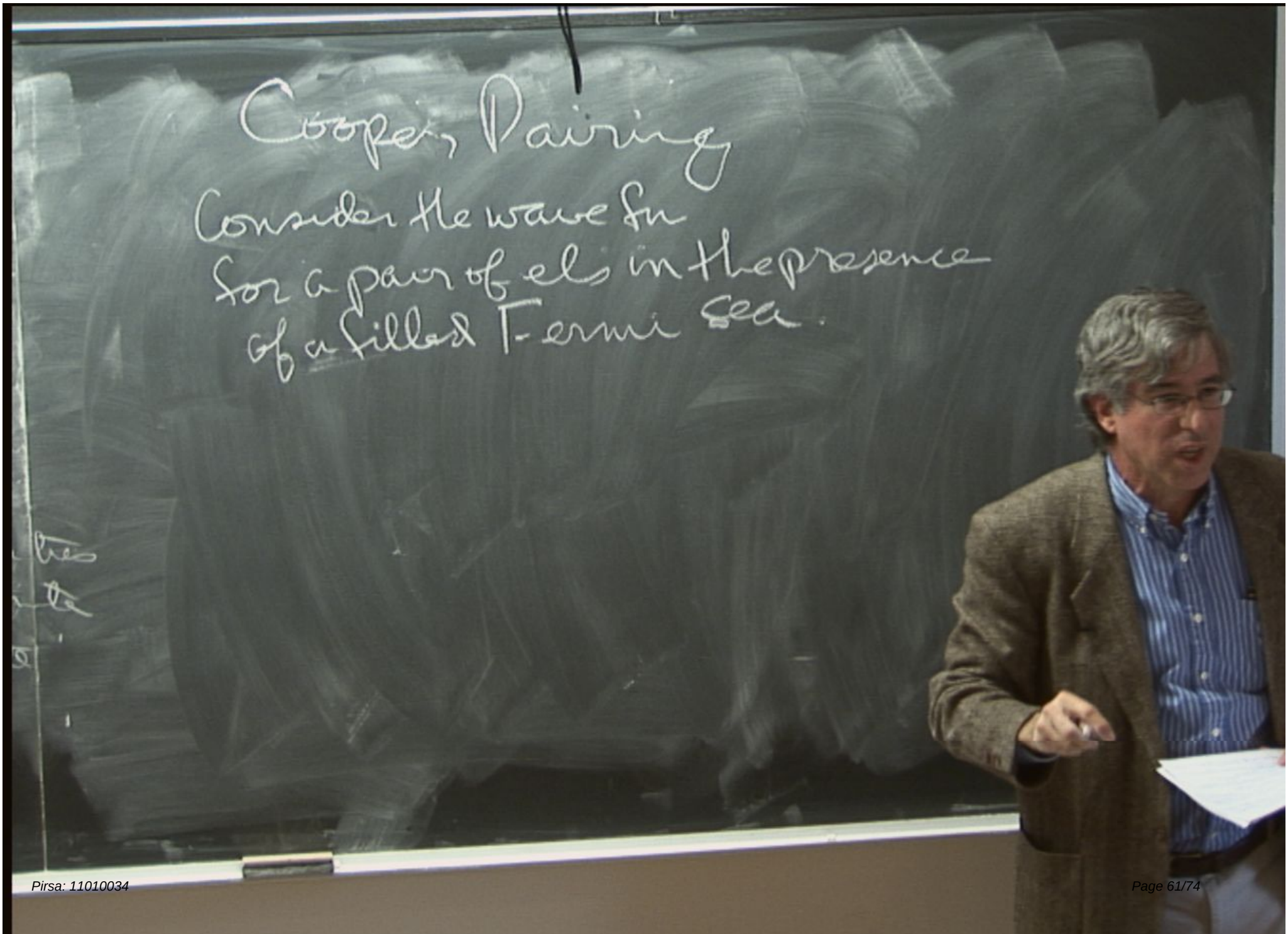
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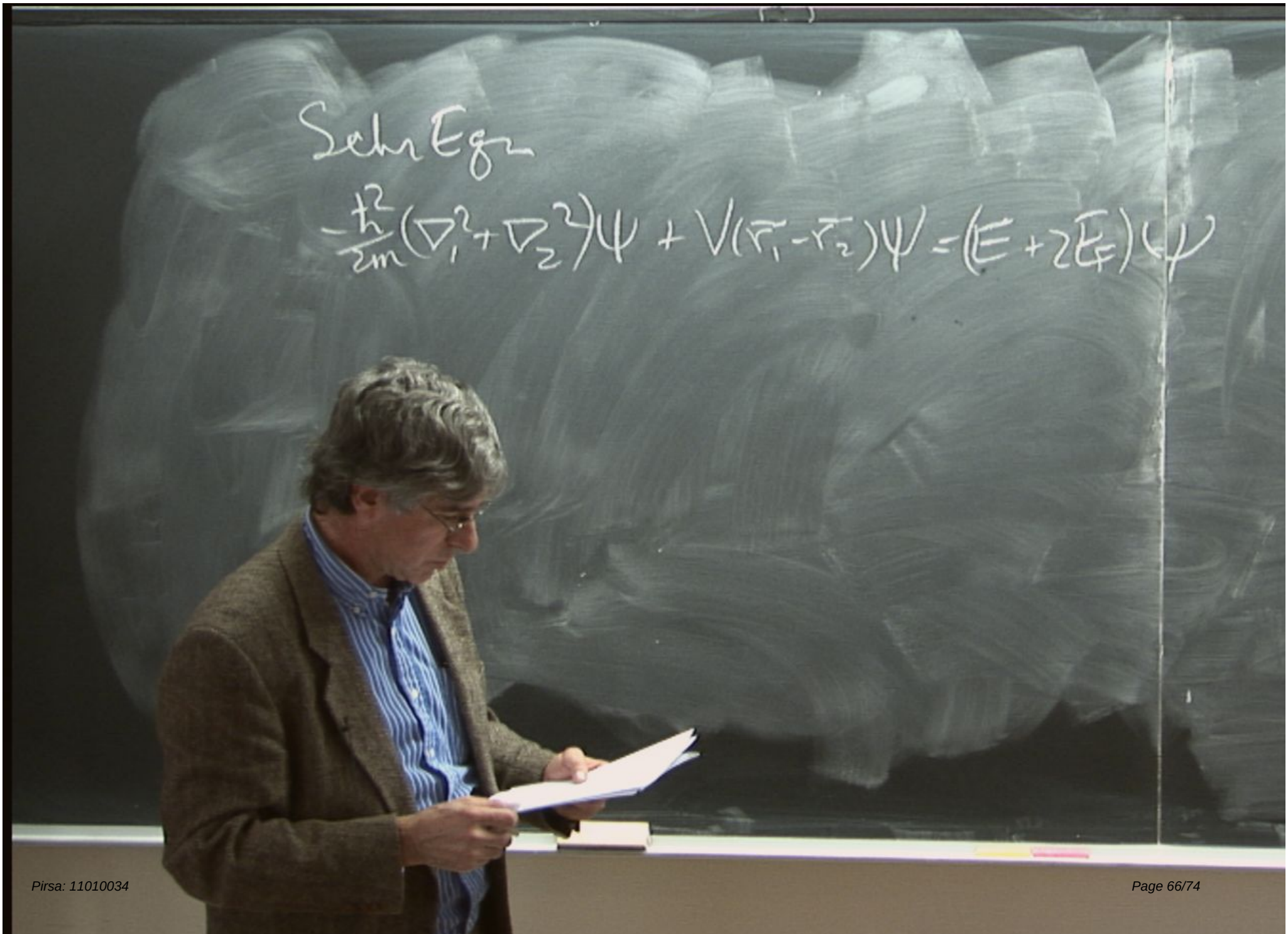
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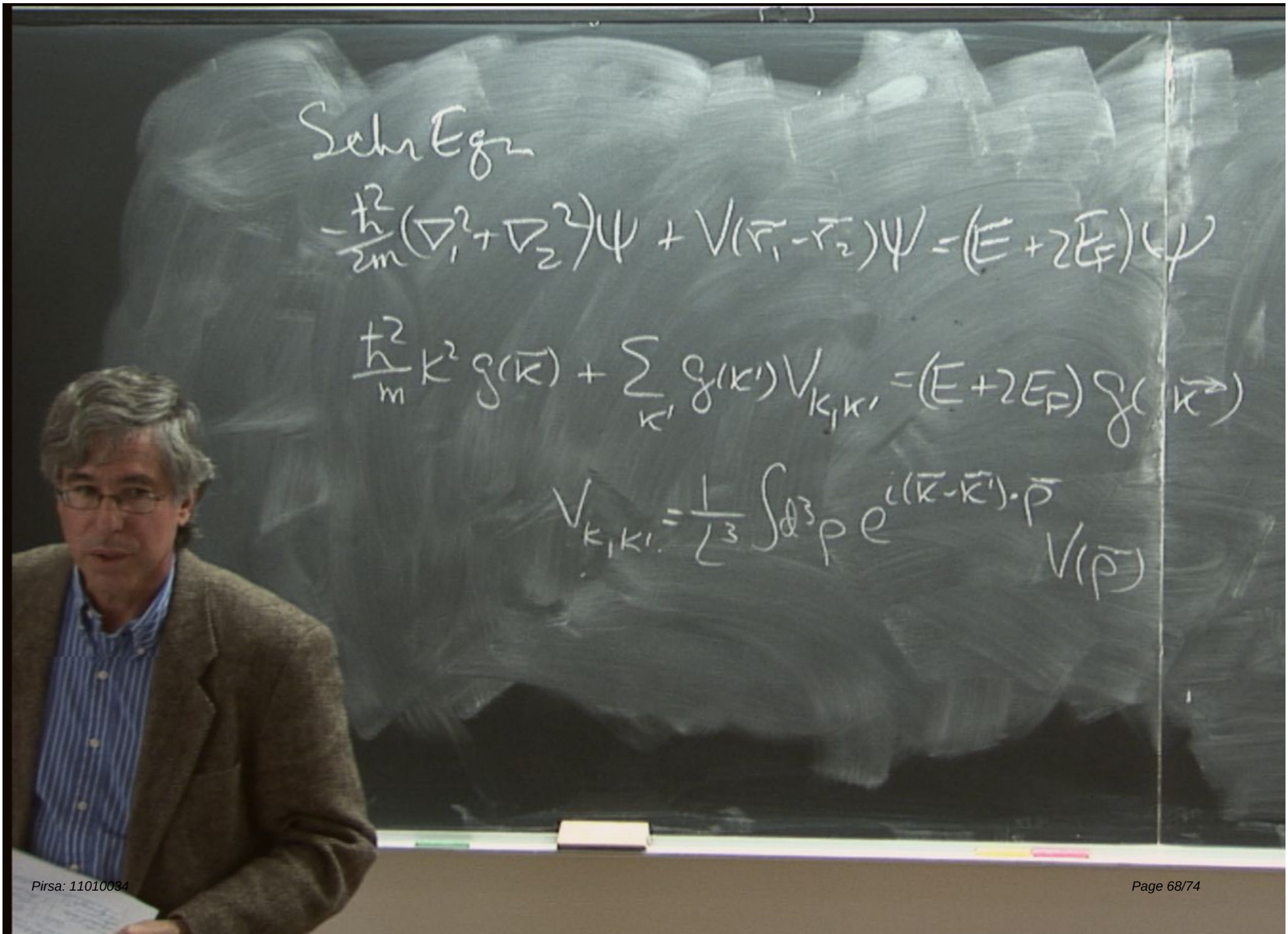
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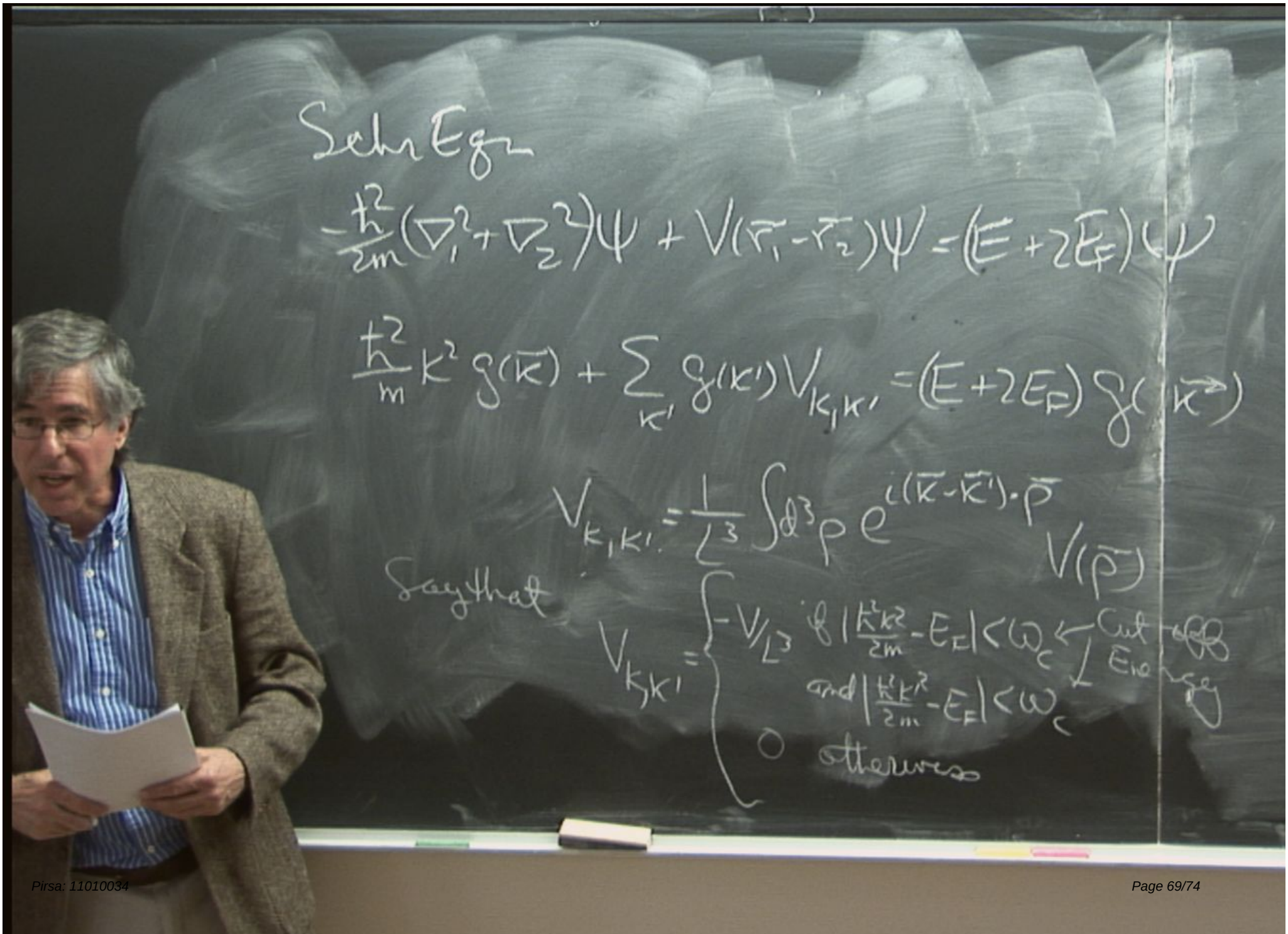
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Say that
$V_{k,k'} = \begin{cases} -V/L^3 & \text{if } \left|\frac{\hbar^2k^2}{2m}-E_F\right|<\omega_c \text{ and } \left|\frac{\hbar^2k'^2}{2m}-E_F\right|<\omega_c \\ 0 & \text{otherwise} \end{cases}$
Cut off energy

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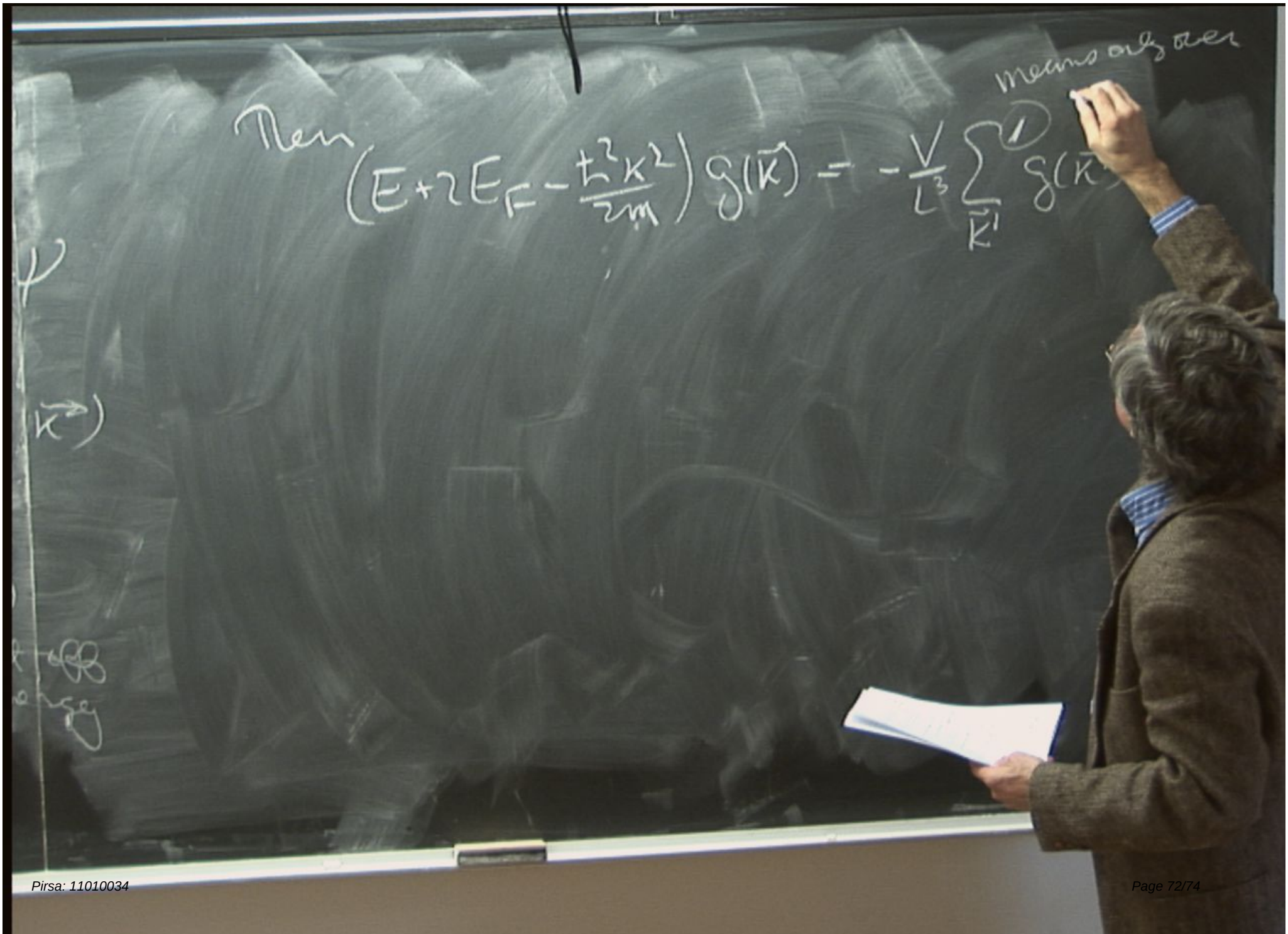
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means only over $\vec{k}'$ for which $V_{\vec{k}\vec{k}'} \neq 0$

Then

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