Title: Condensed Matter Review - Lecture 7

Date: Jan 11, 2011 10:15 AM

URL: http://pirsa.org/11010029

Abstract:

In the presence of Q.P.'s

$$n_{\vec{p},\sigma} = n^0_{\vec{p},\sigma} + \delta n_{\vec{p},\sigma}$$

$$\mathcal{E} = \mathcal{E}_0(\{n^0_{\vec{p},\sigma}\}) + \sum_{\vec{p},\sigma} (\epsilon^0_{\vec{p}\sigma} - \mu)\,\delta n_{\vec{p},\sigma}$$

$$+ \frac{1}{2} \sum_{\substack{\vec{p},\vec{p}' \\ \sigma\sigma'}} f(\vec{p},\sigma;\vec{p}',\sigma')\,\delta n_{\vec{p}} \ldots + \cdots$$

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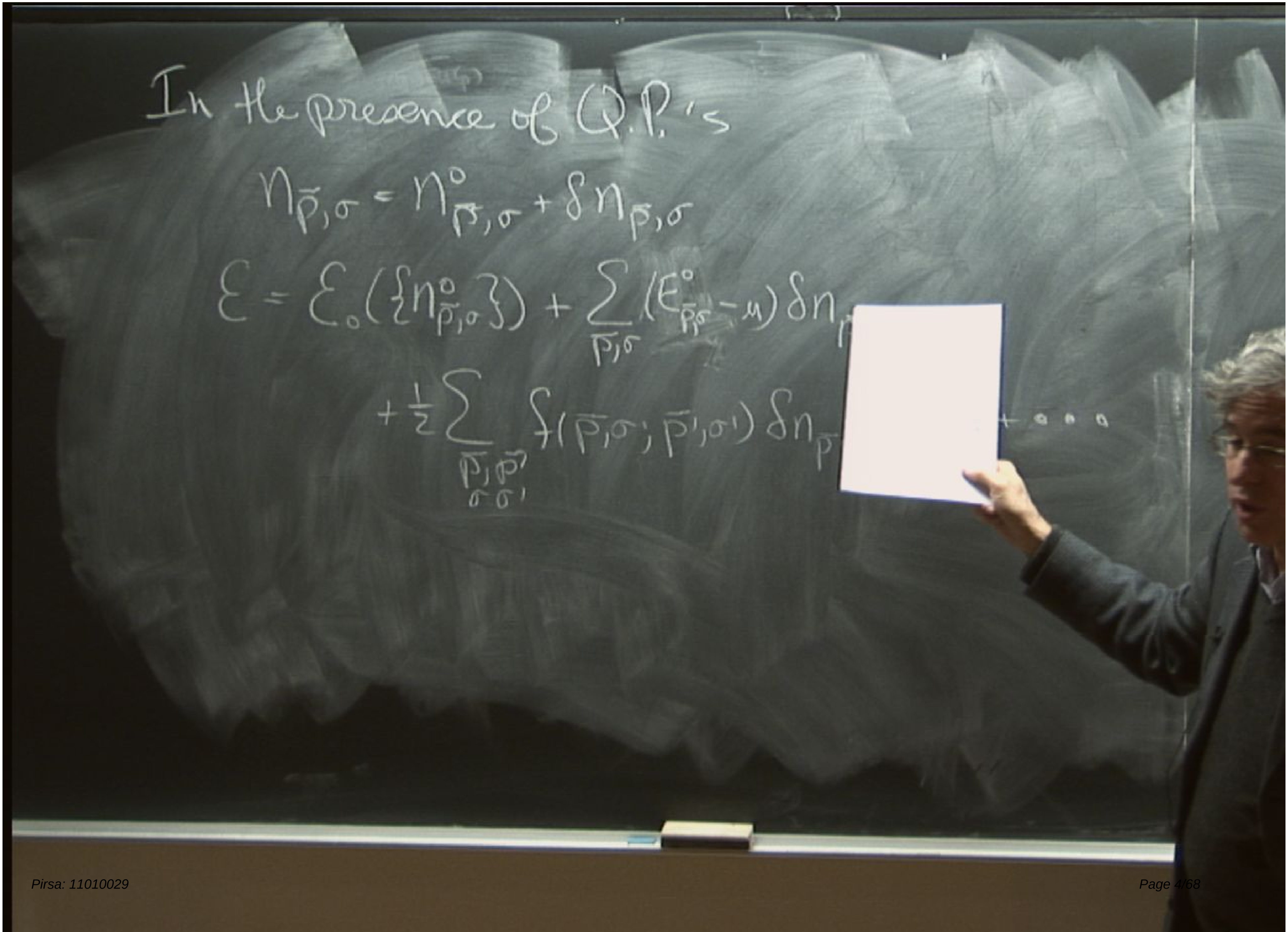
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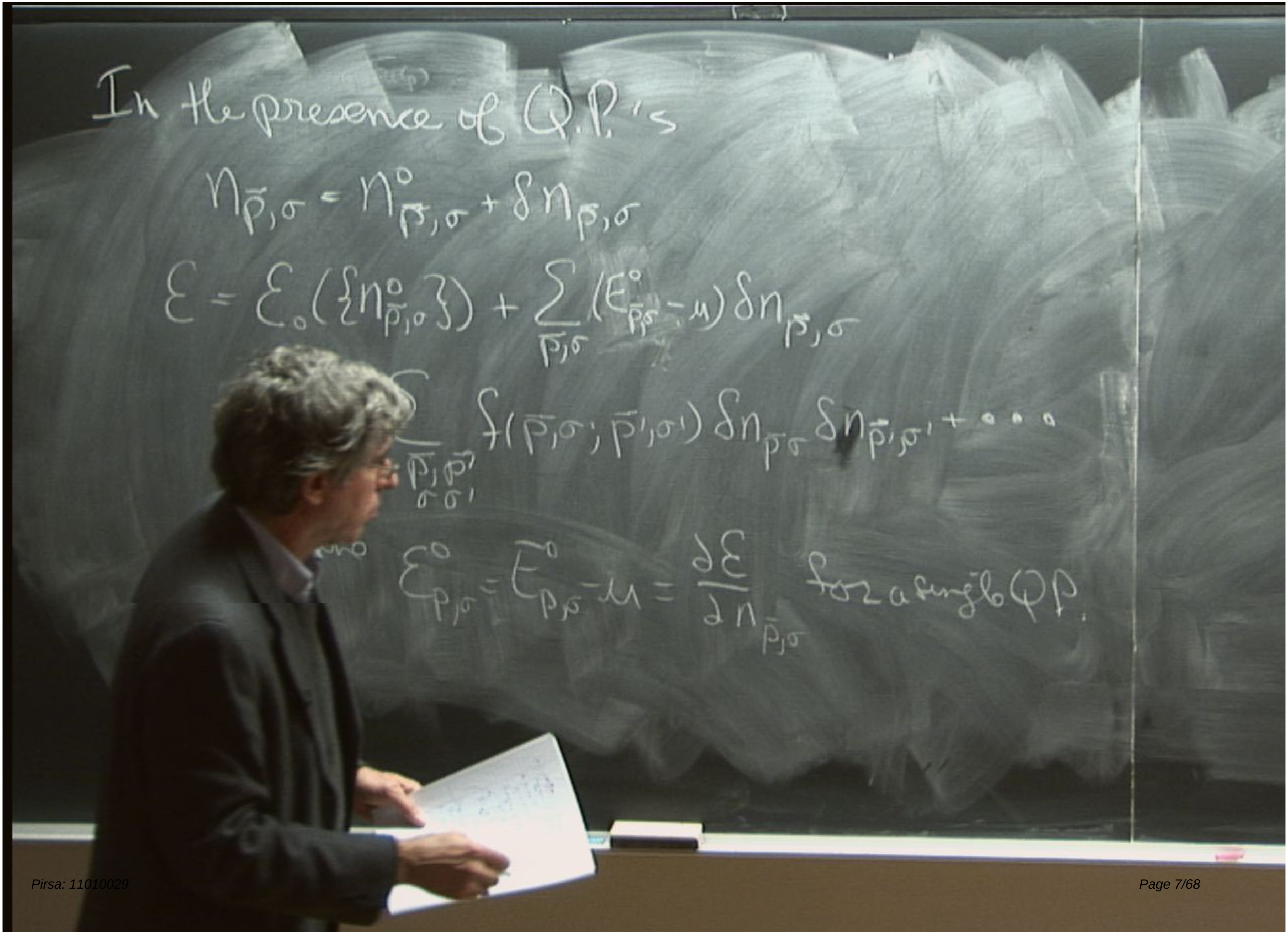
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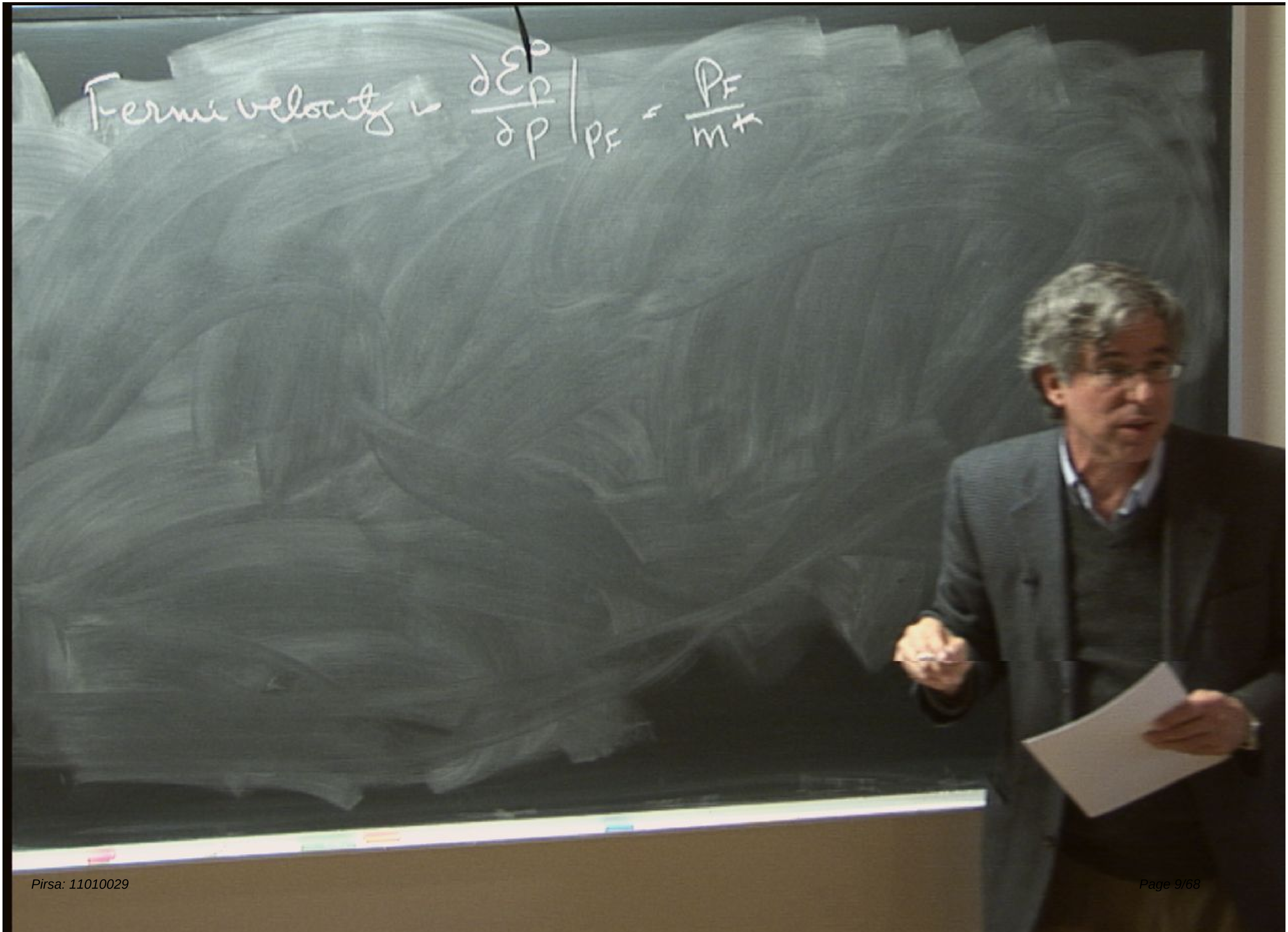
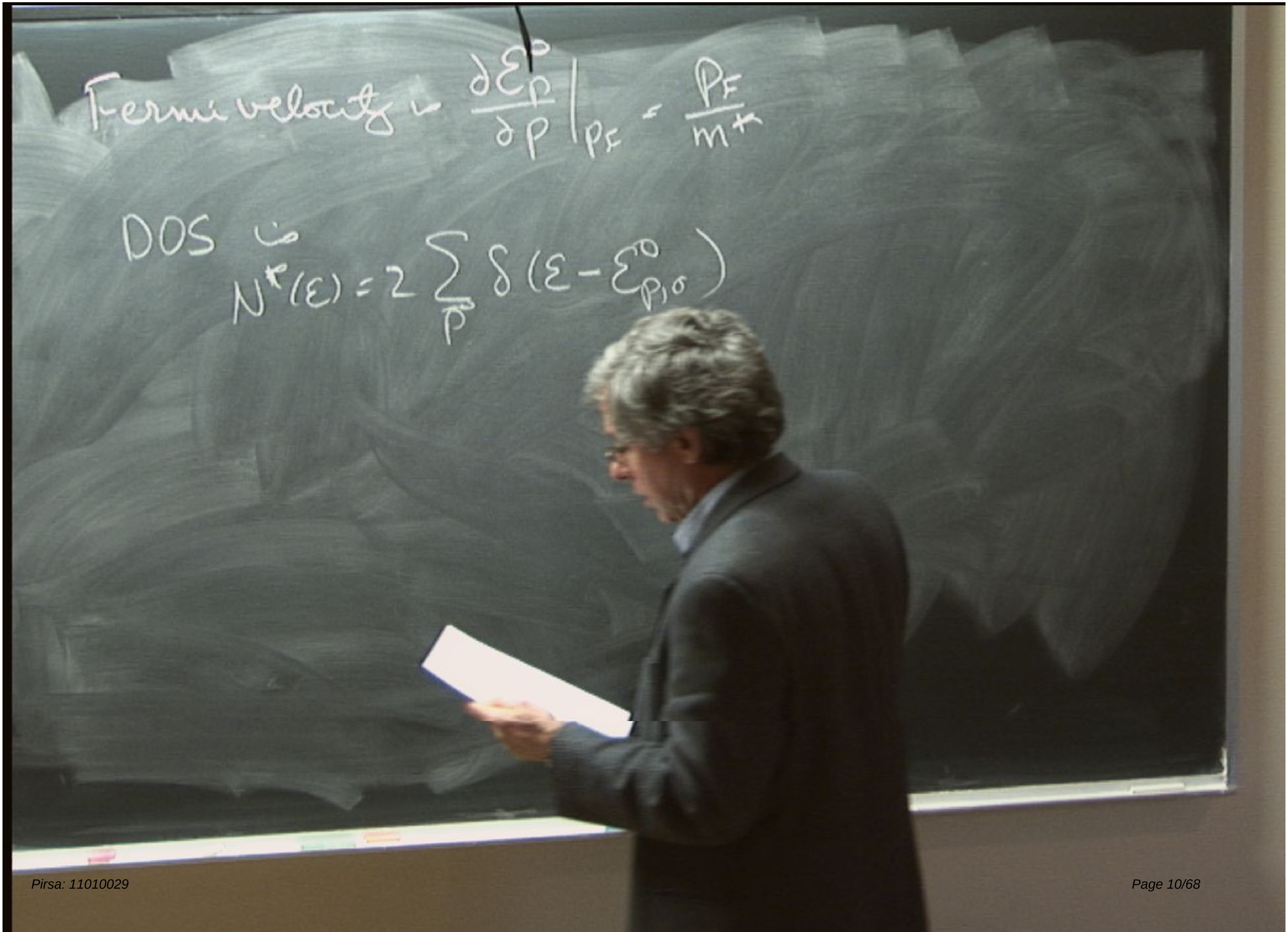

Fermi velocity = $\frac{\partial \mathcal{E}_p^0}{\partial p}\Big|_{p_F} = \frac{p_F}{m^*}$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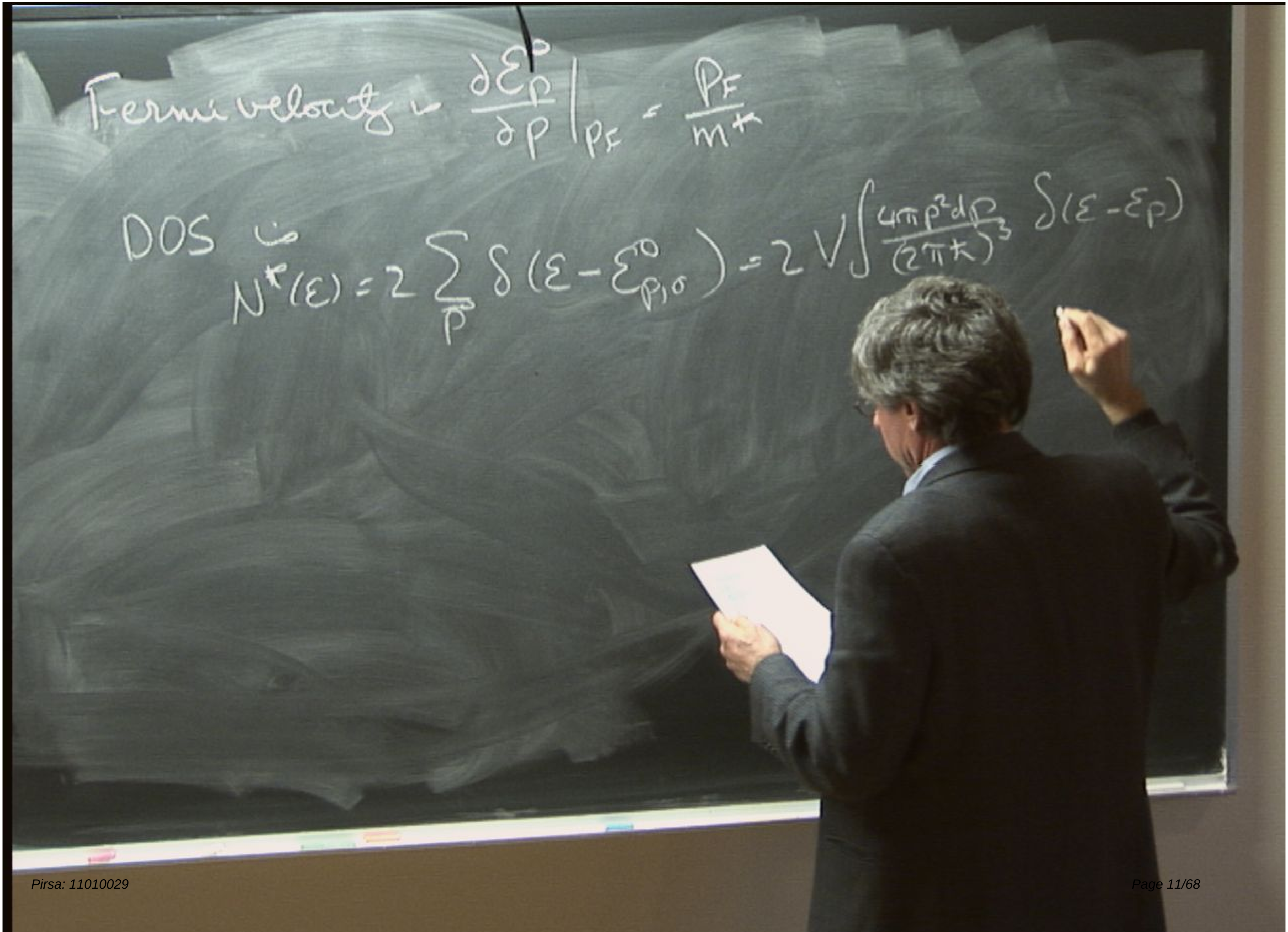
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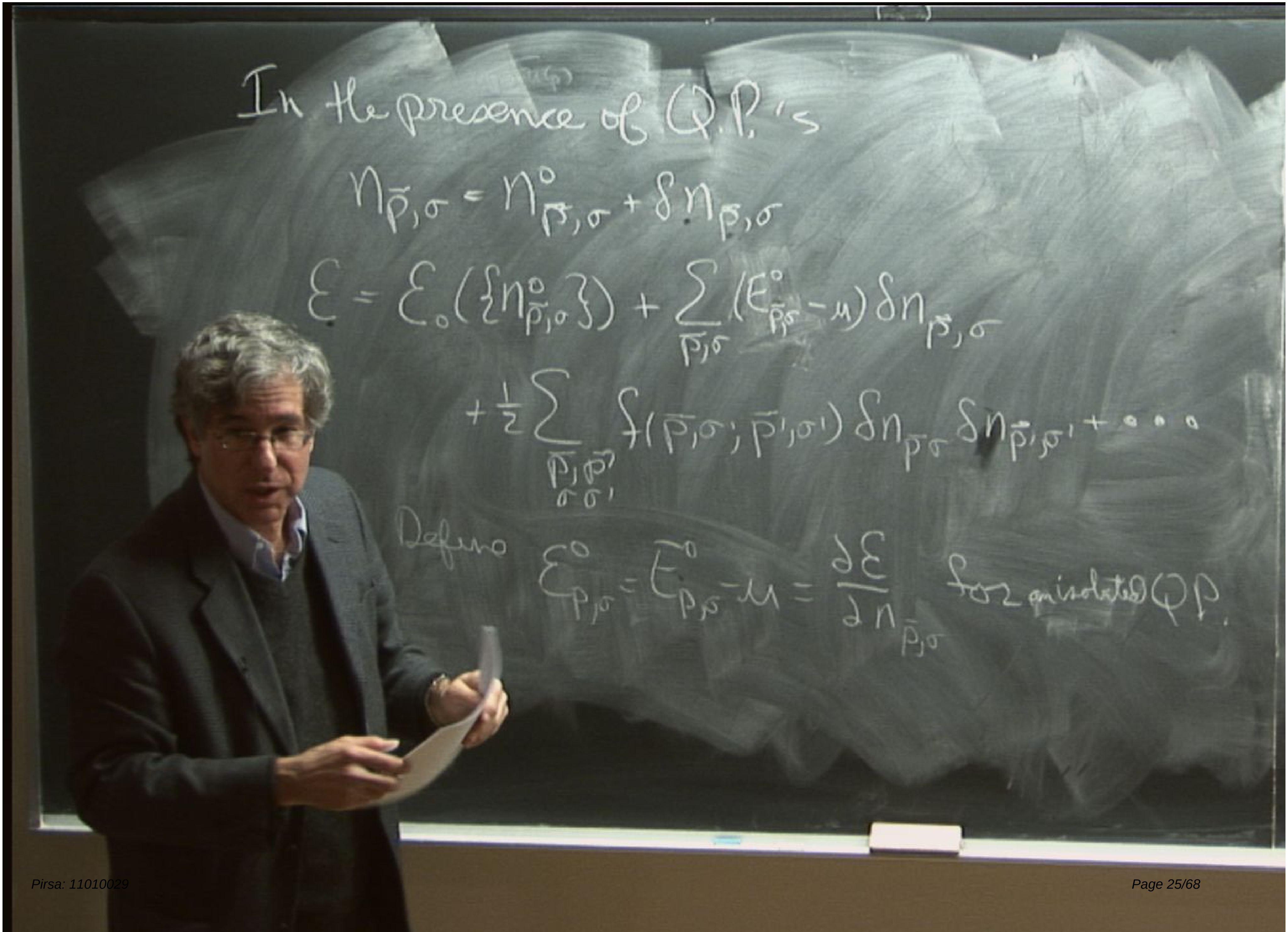
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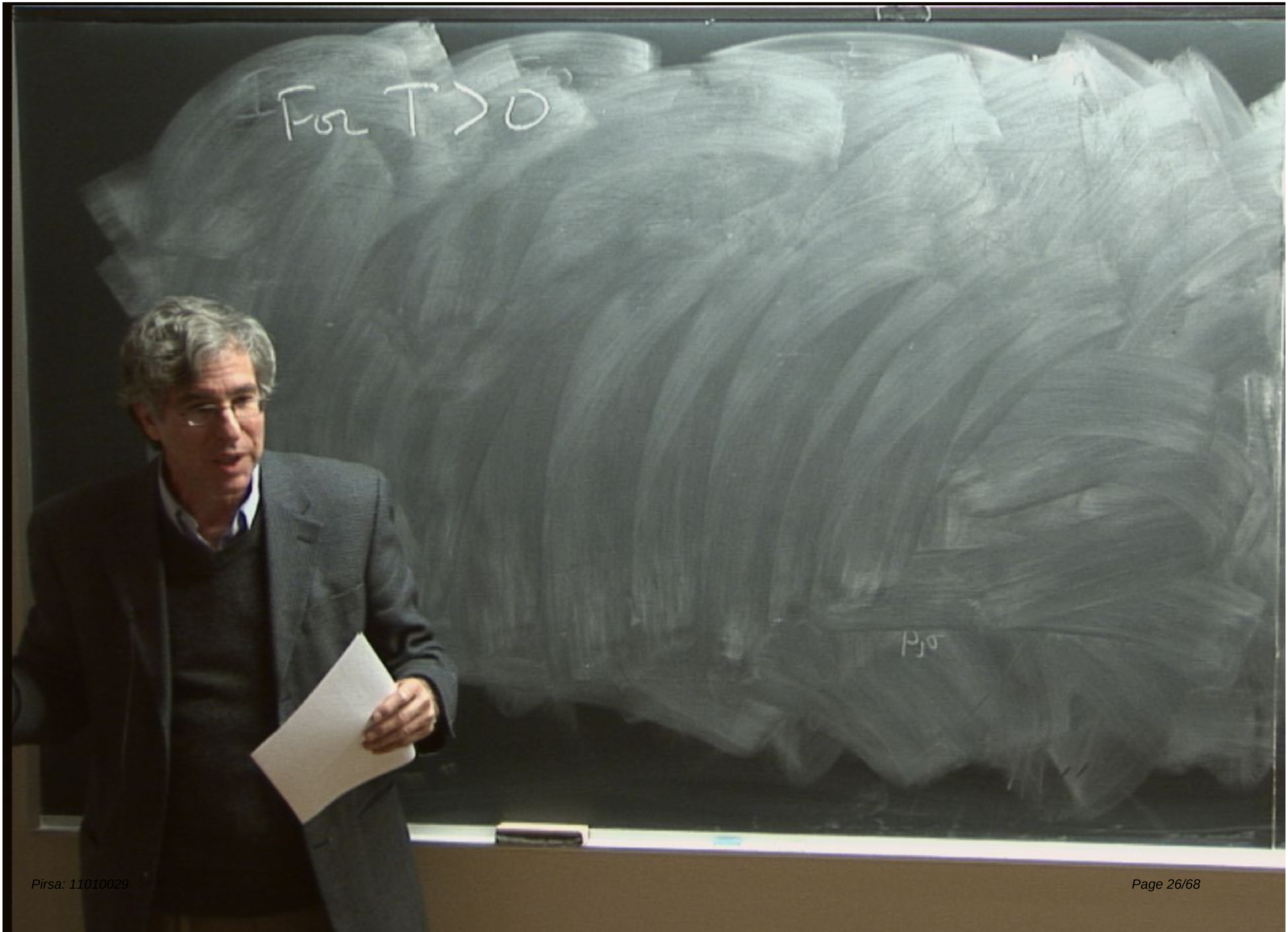
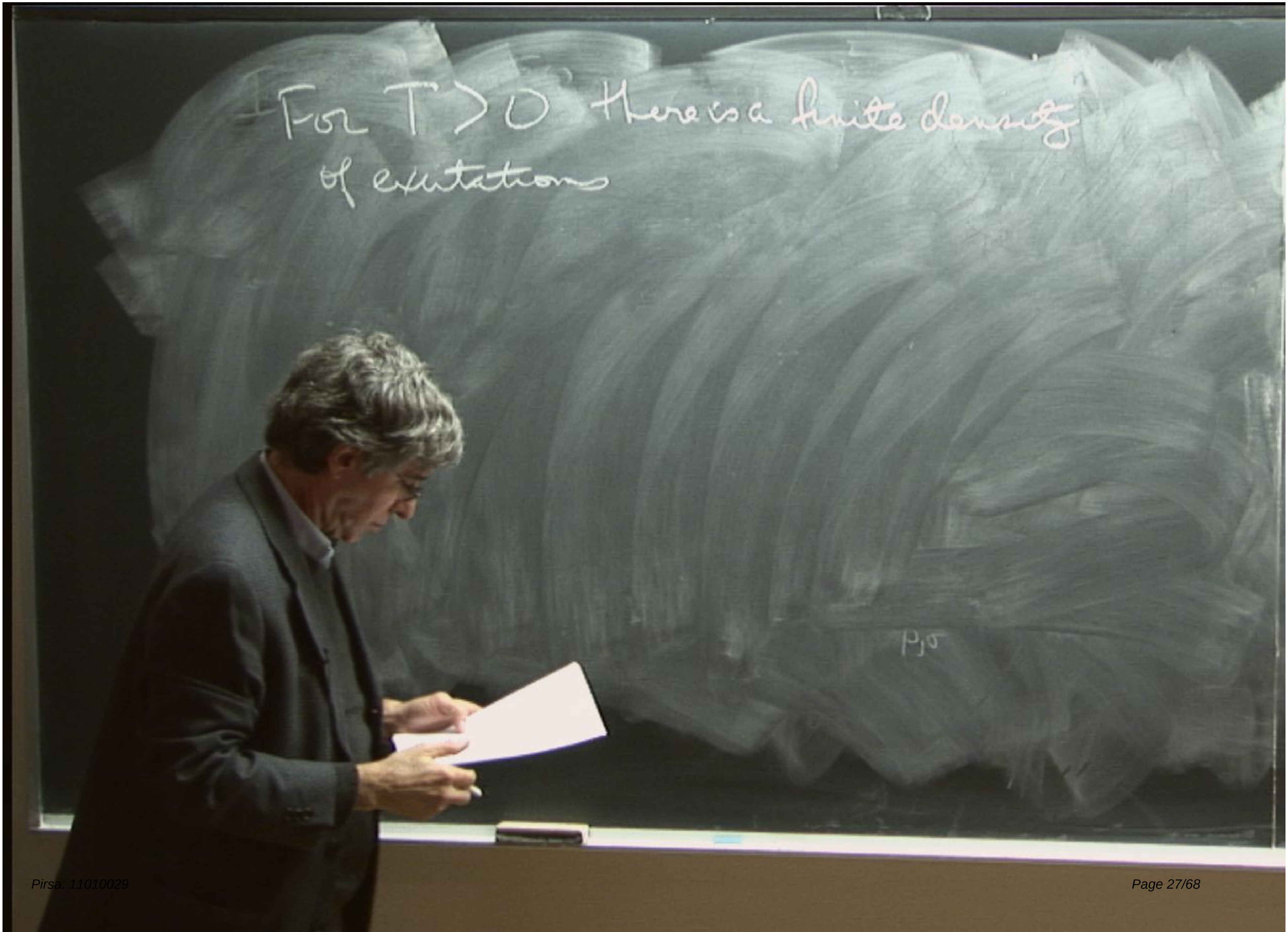

For T > 0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p,σ

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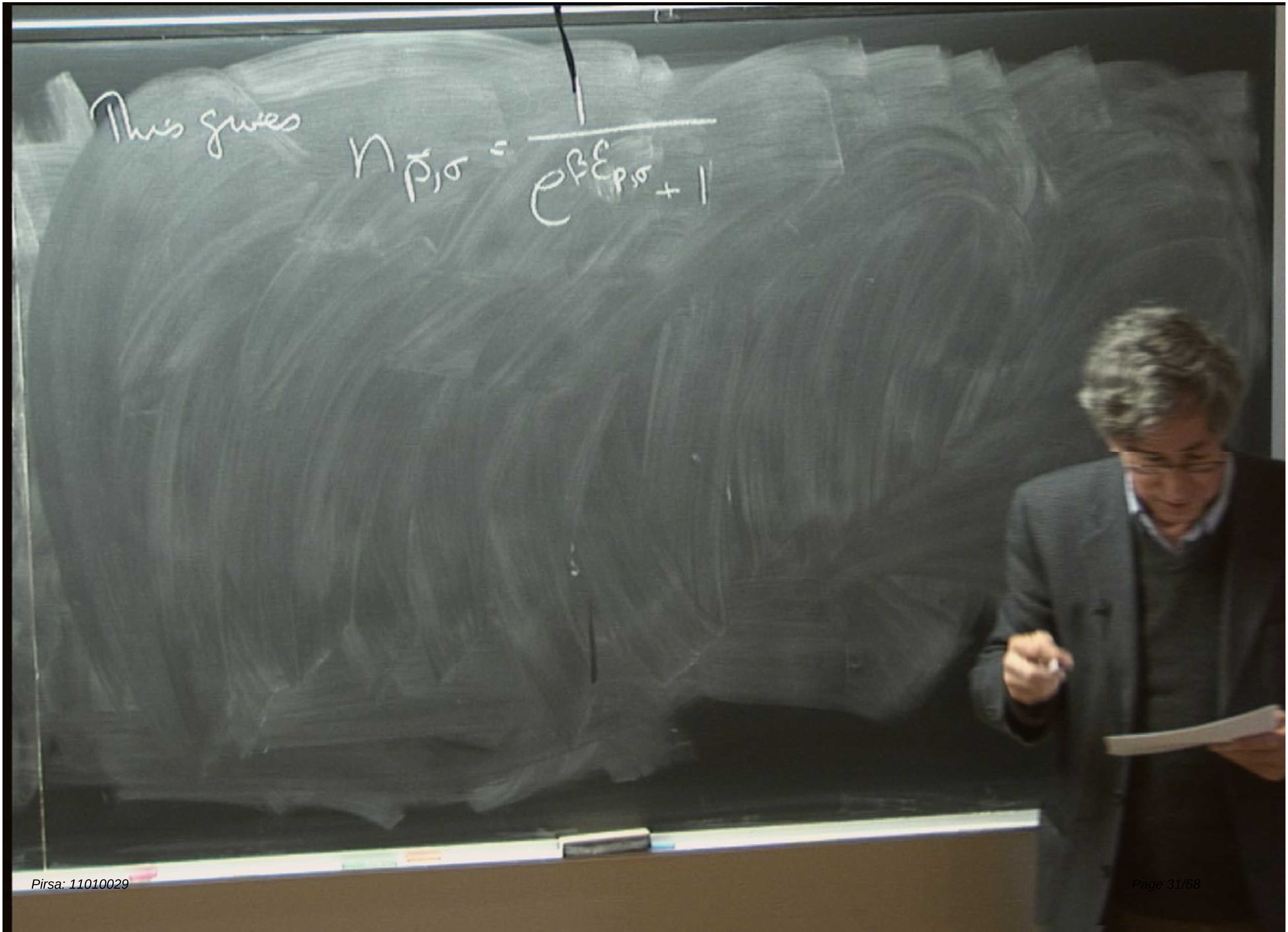
The Free Energy is $F(\{n_{p\sigma}\}) = \mathcal{E} - TS$

The correct $n_{\bar{p},\sigma} = n^0_{p\sigma} + \delta n_{p\sigma}$ are the ones that minimize F.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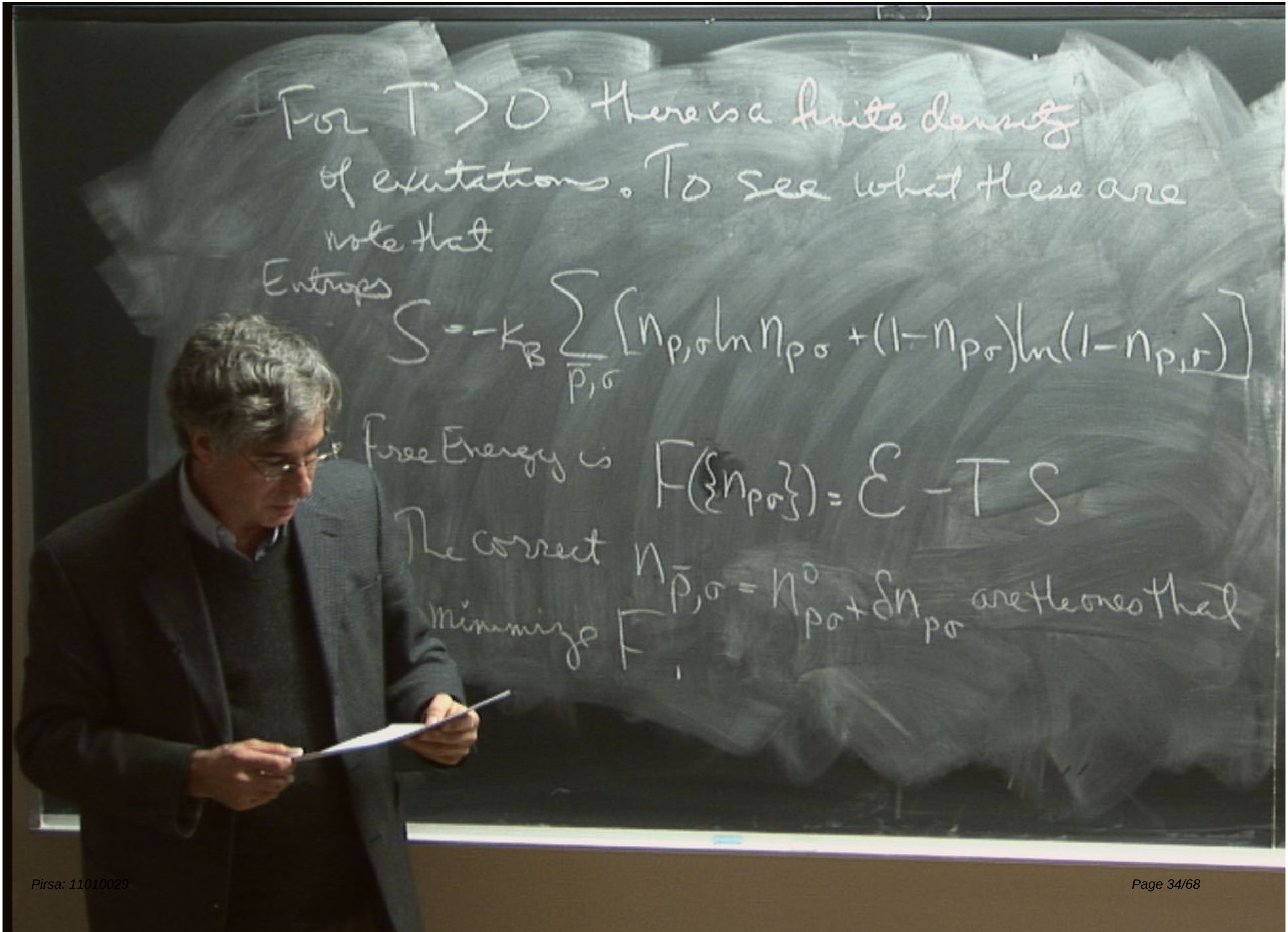
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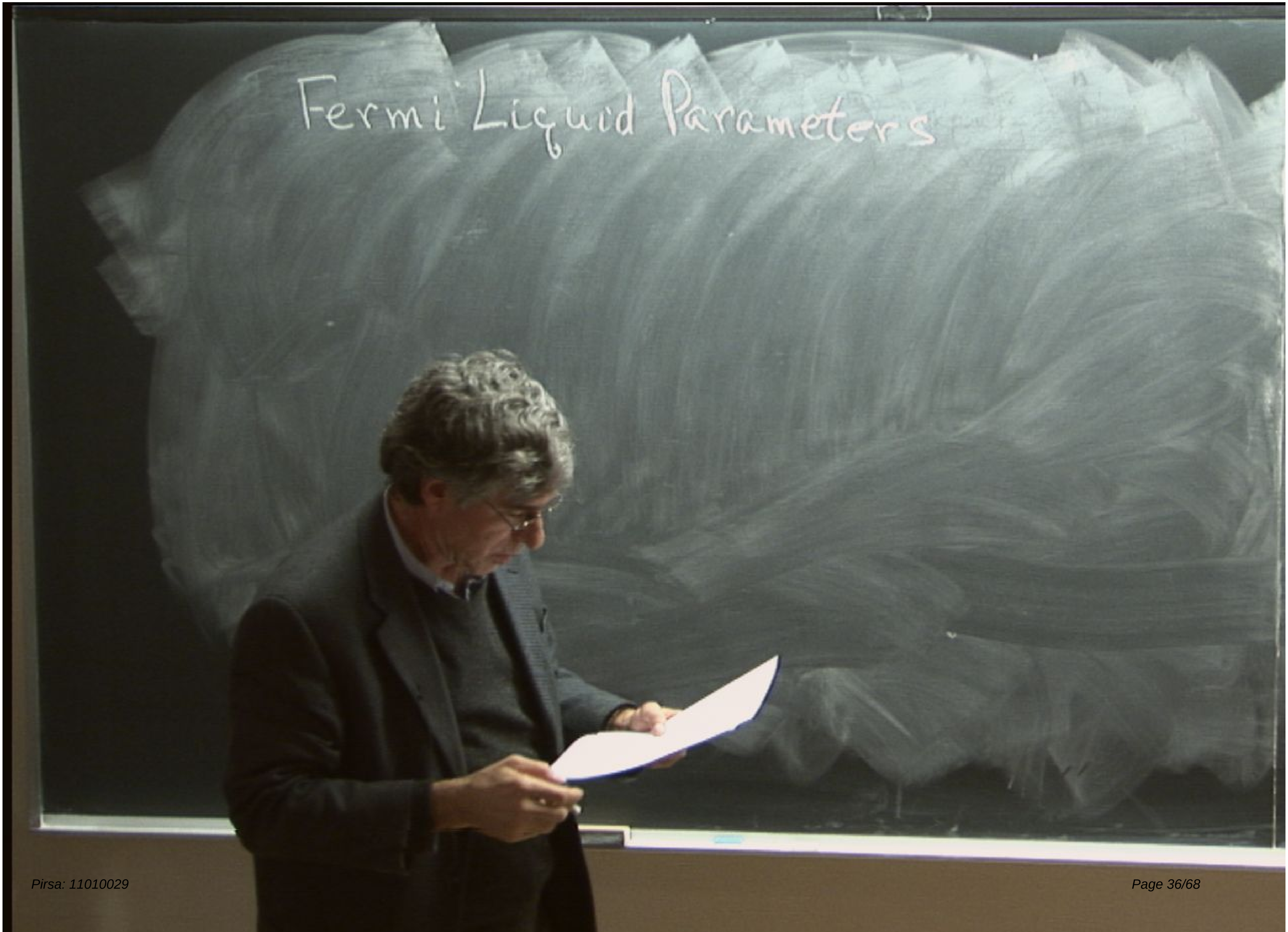
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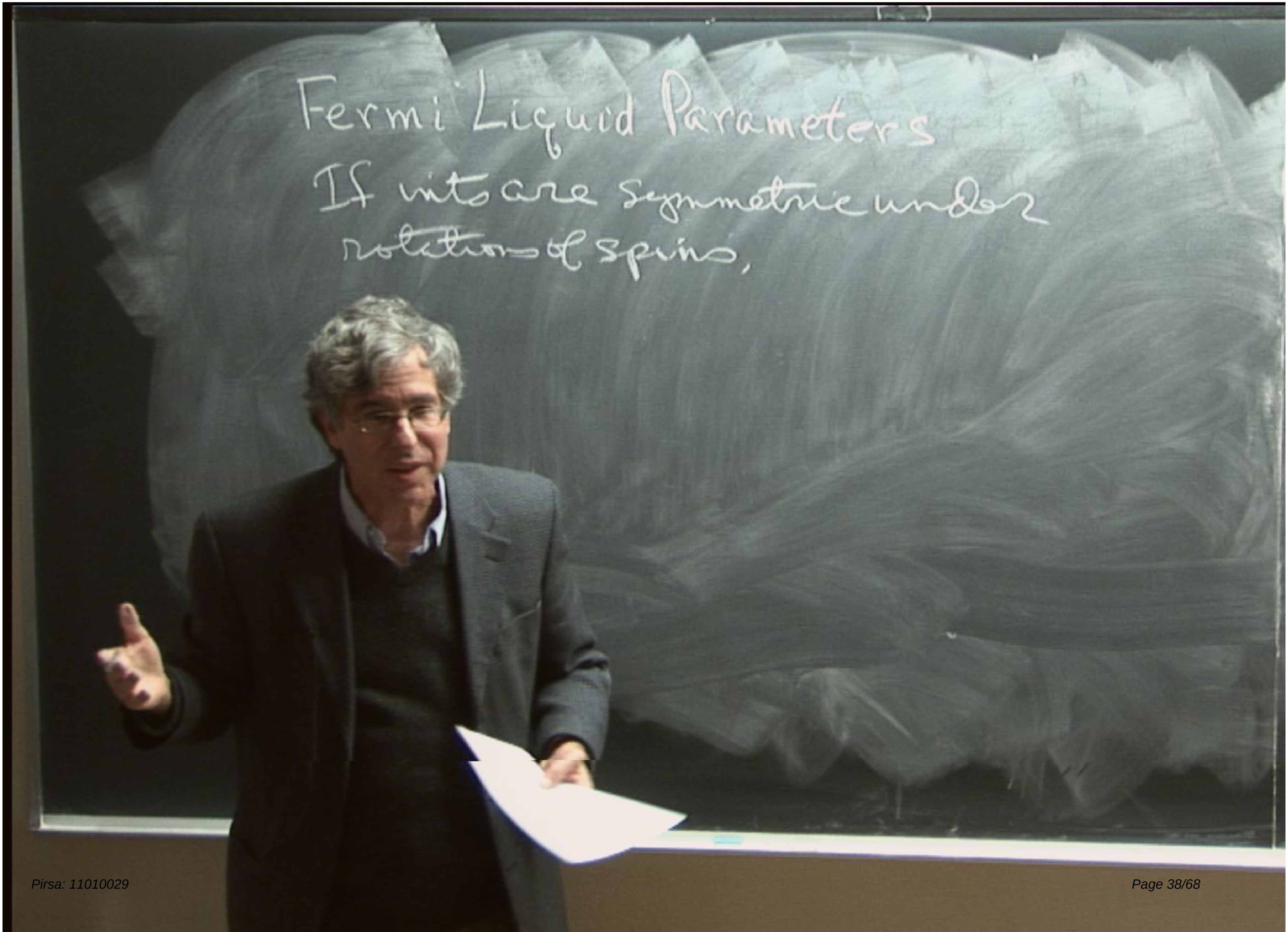
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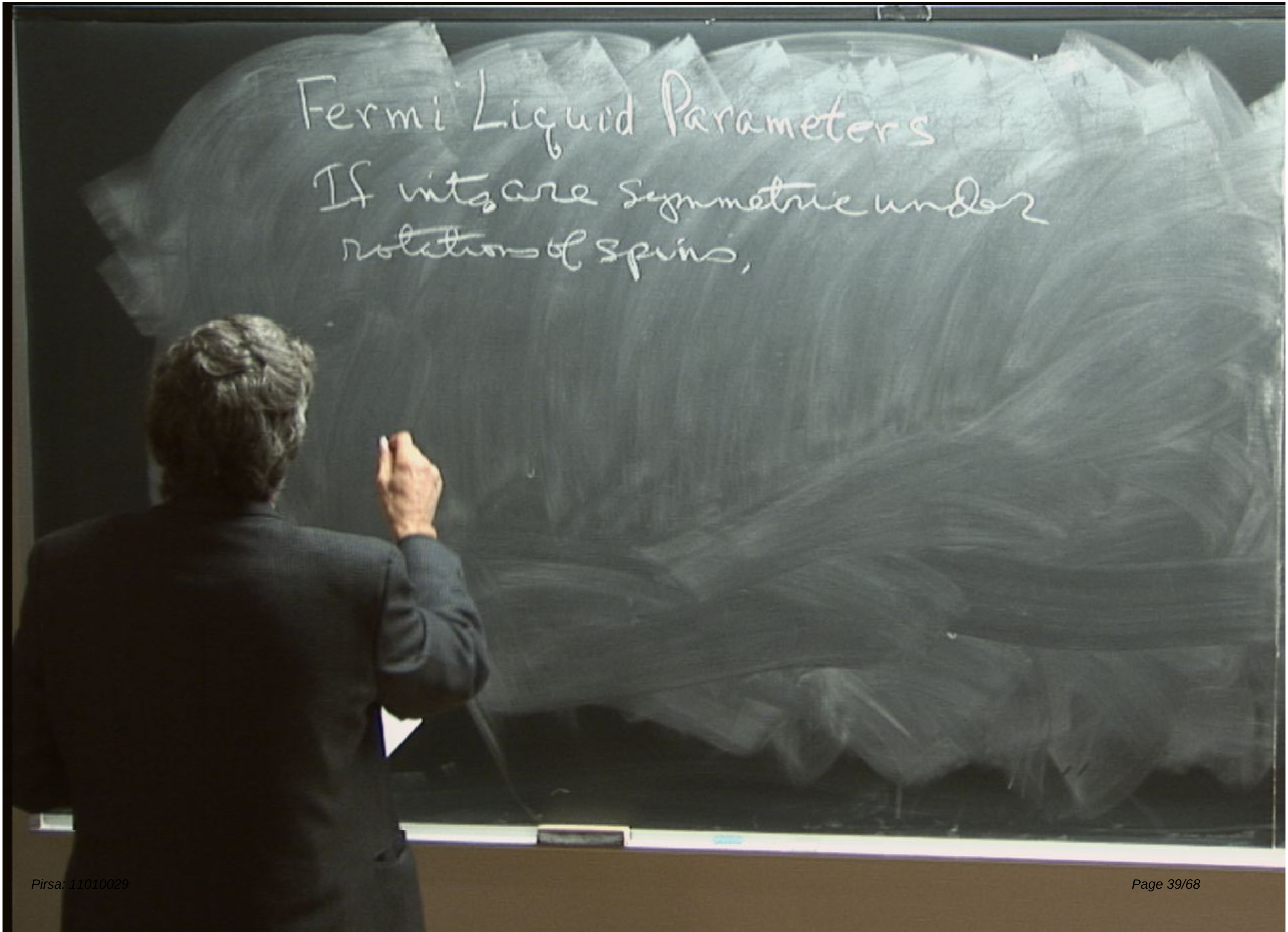
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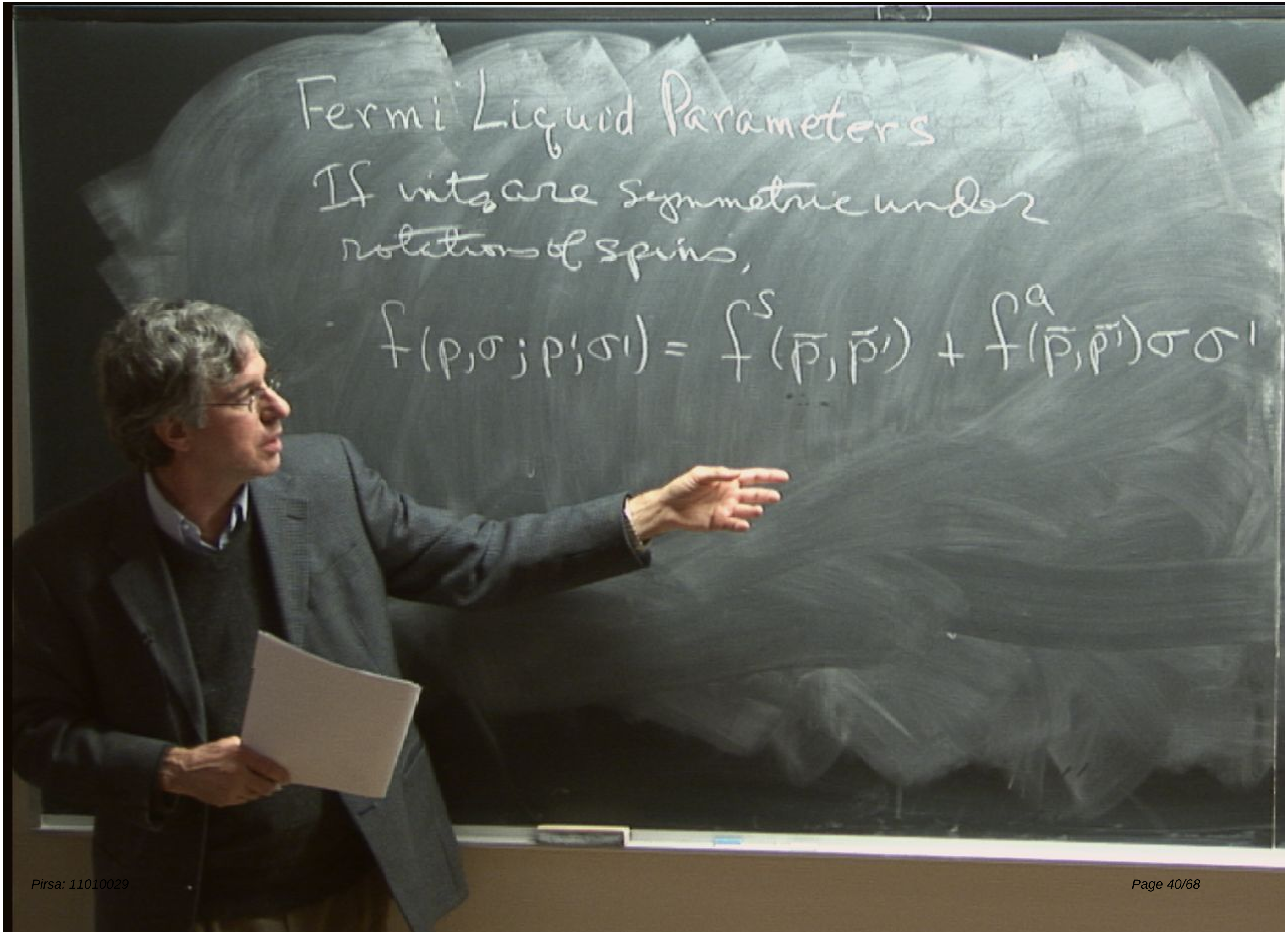
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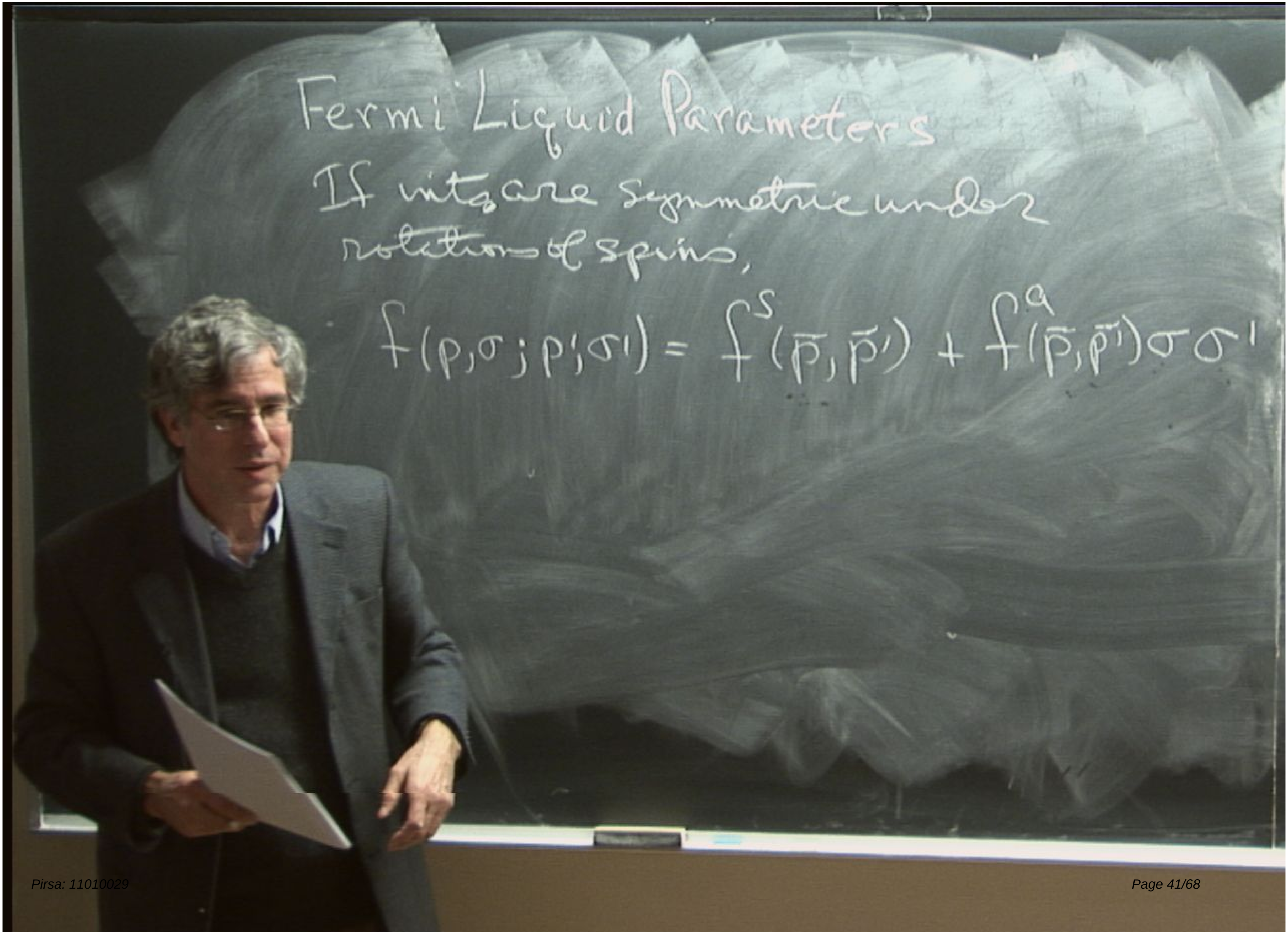
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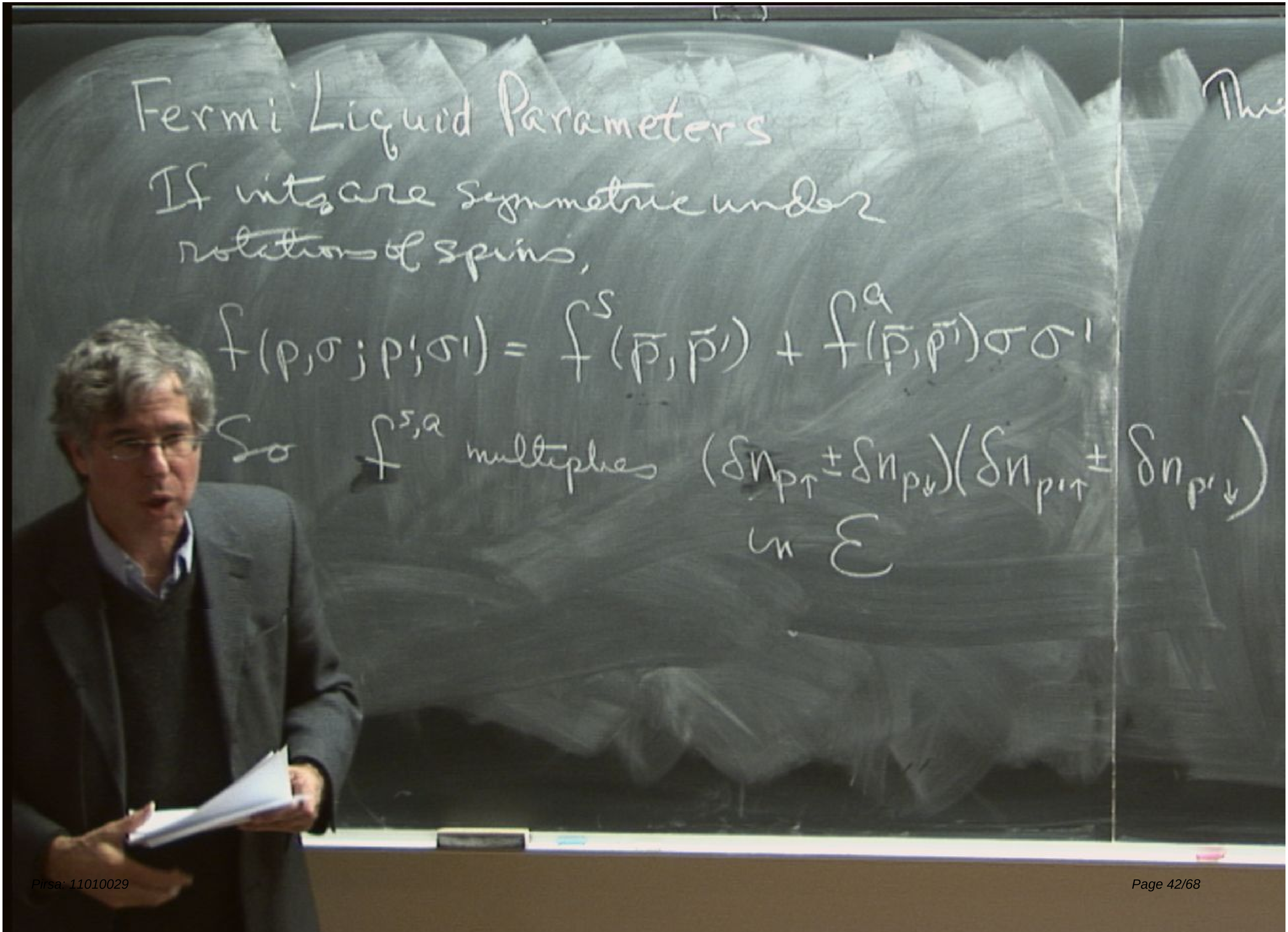
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Fermi Liquid Parameters

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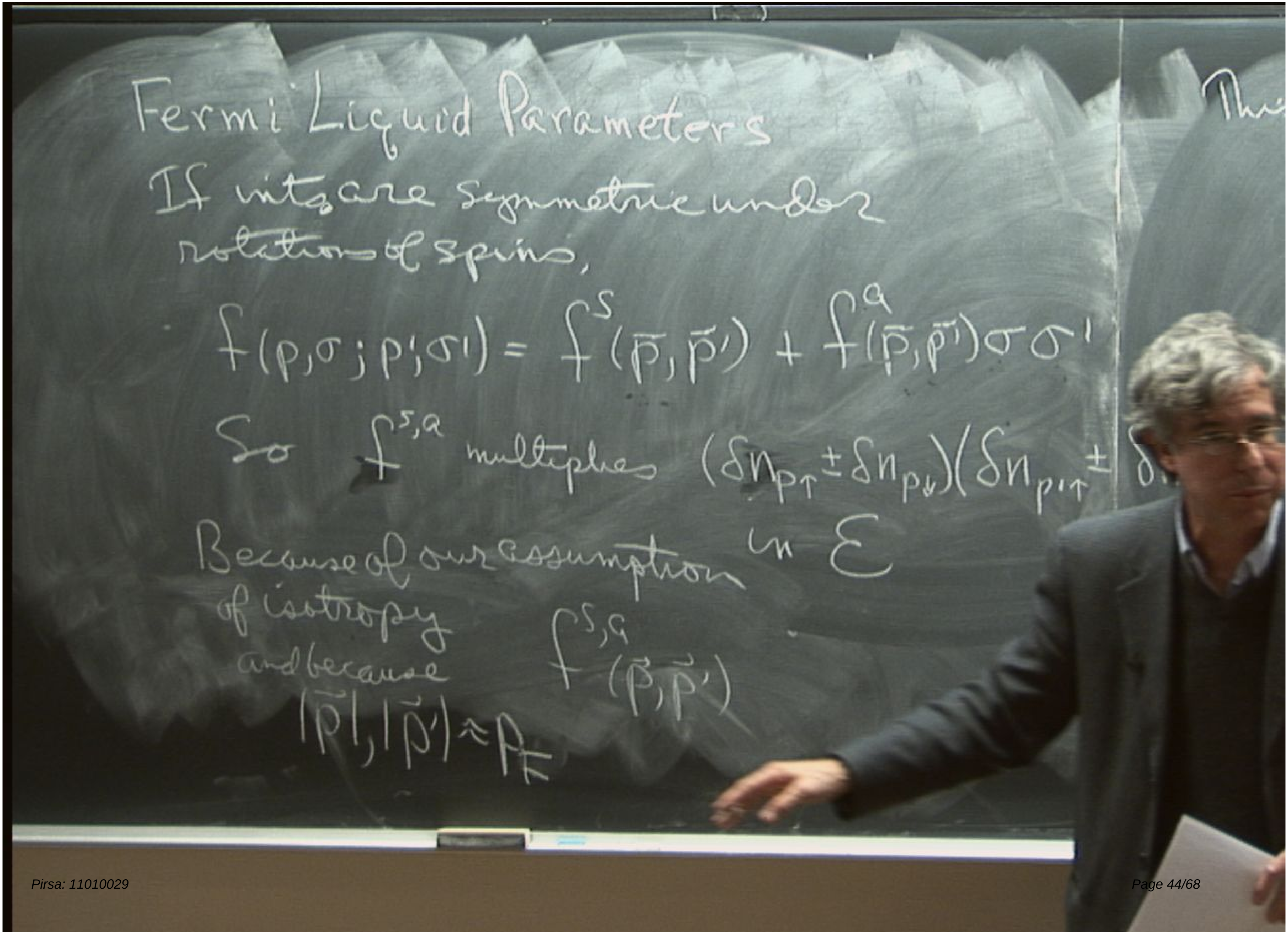
Because of our assumption of isotropy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Because of our assumption
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and because
f^{s,a}(p̄, p̄')
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Fermi Liquid Parameters

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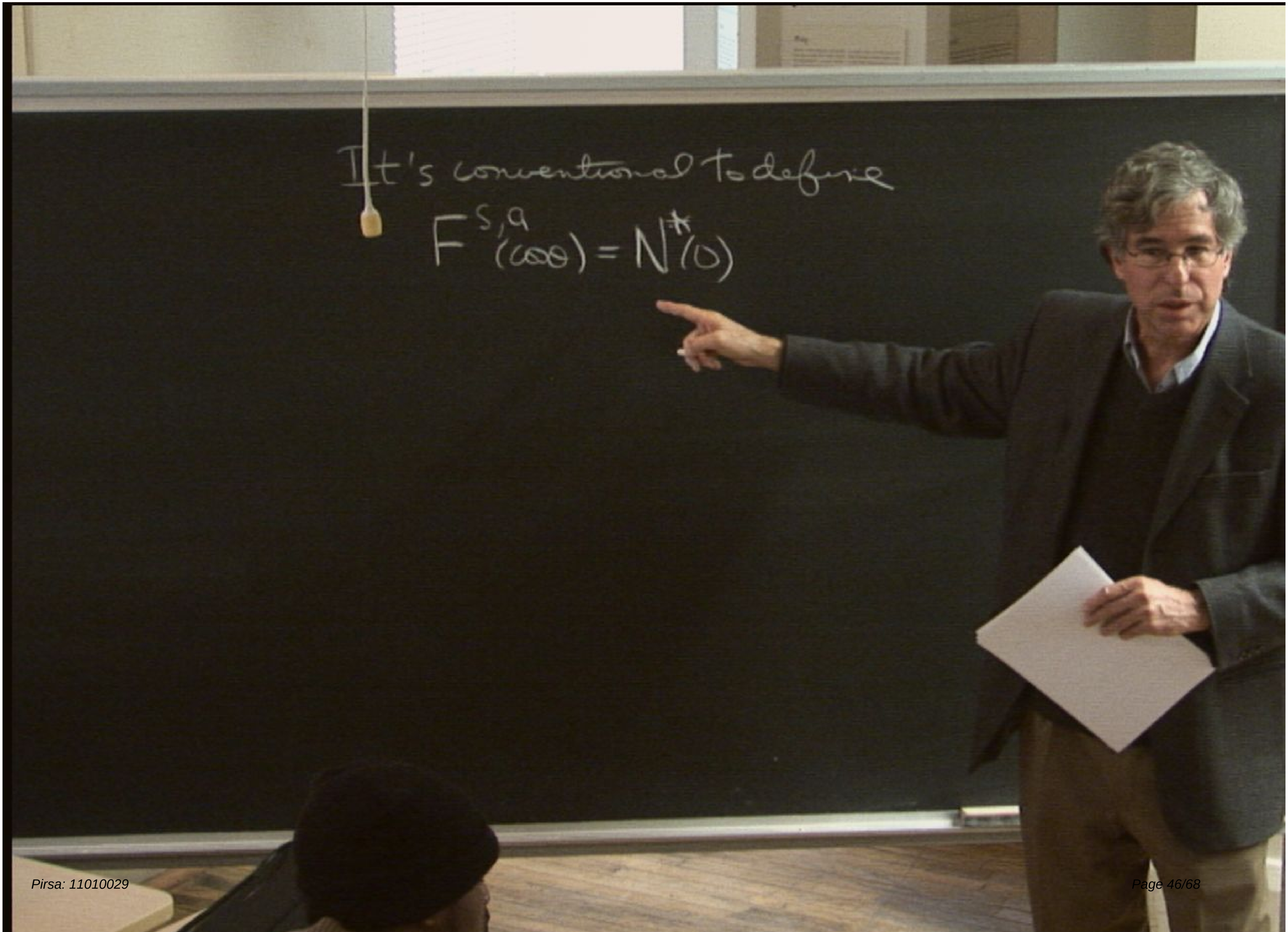
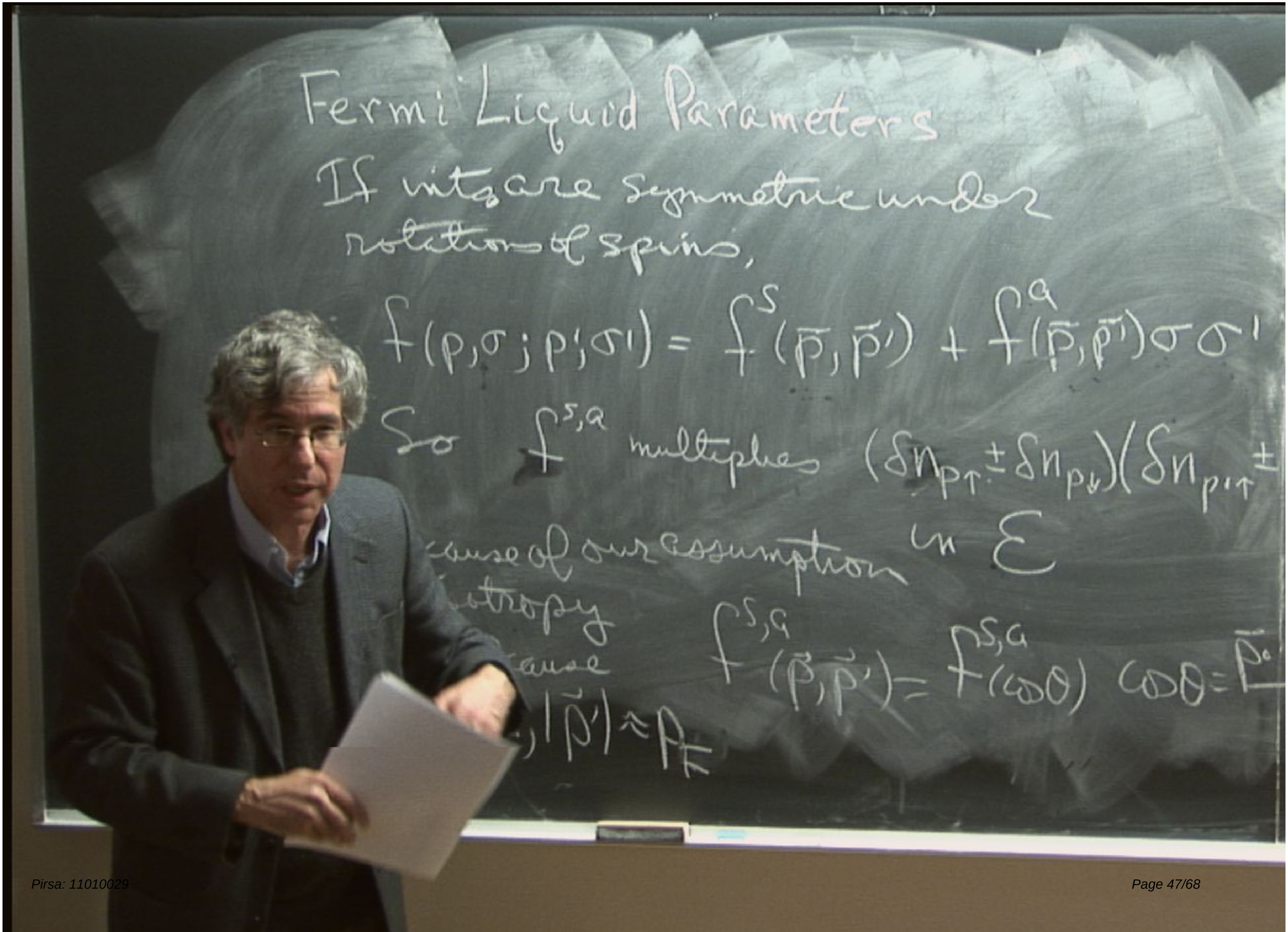
Because of our assumption of isotropy and because $|\vec{p}|, |\vec{p}'| \approx p_F$

$$f^{s,a}(\vec{p},\vec{p}') = f^{s,a}(\cos\theta) \quad \cos\theta = \frac{\vec{p}\cdot\vec{p}'}{p_F^2}$$

Th


It's conventional to define
F^{S,a}(cosθ) = N^*(0)


Fermi Liquid Parameters
If ints are symmetric under
rotations of spins,
$f(p,\sigma;p',\sigma') = f^{s}(\vec{p},\vec{p}') + f^{a}(\vec{p},\vec{p}')\sigma\sigma'$
So $f^{s,a}$ multiplies $(\delta n_{p\uparrow} \pm \delta n_{p\downarrow})(\delta n_{p'\uparrow} \pm$
our assumption in $\mathcal{E}$
$f^{s,a}(\vec{p},\vec{p}') = f^{s,a}(\cos\theta)$ $\cos\theta =$
$|\vec{p}'| \approx p_F$

It's conventional to define

$$F^{s,a}(\cos\theta) = N^{*}(0)\, f^{s,a}(\cos\theta) = \sum_{\ell=0} (2\ell+1)\, F^{s,a}_{\ell}\, P_{\ell}(\cos\theta)$$

Then $F^{s,a}_{\ell} = \langle F^{s,a}(\cos\theta)\, P_{\ell}(\cos\theta) \rangle$

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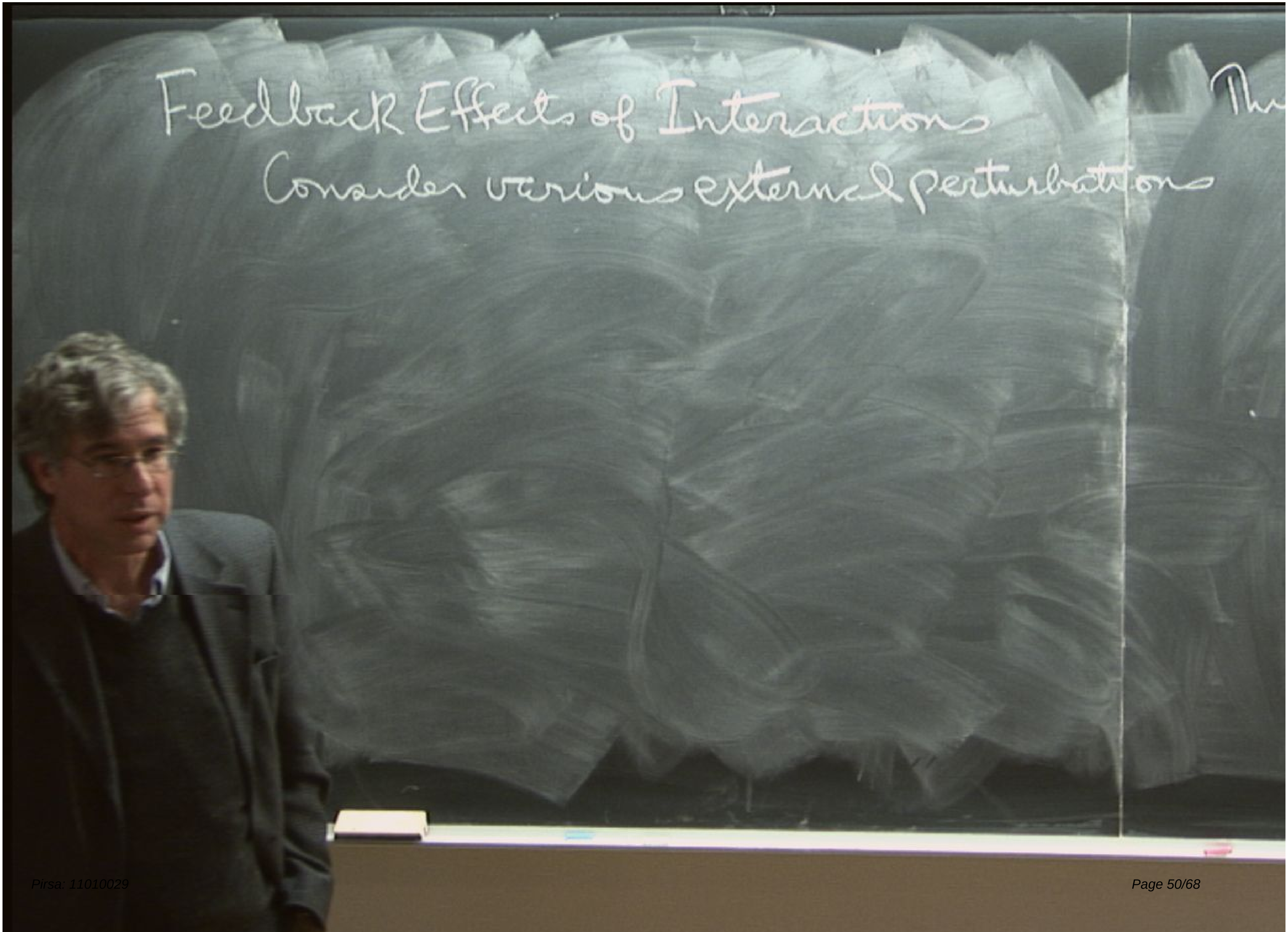
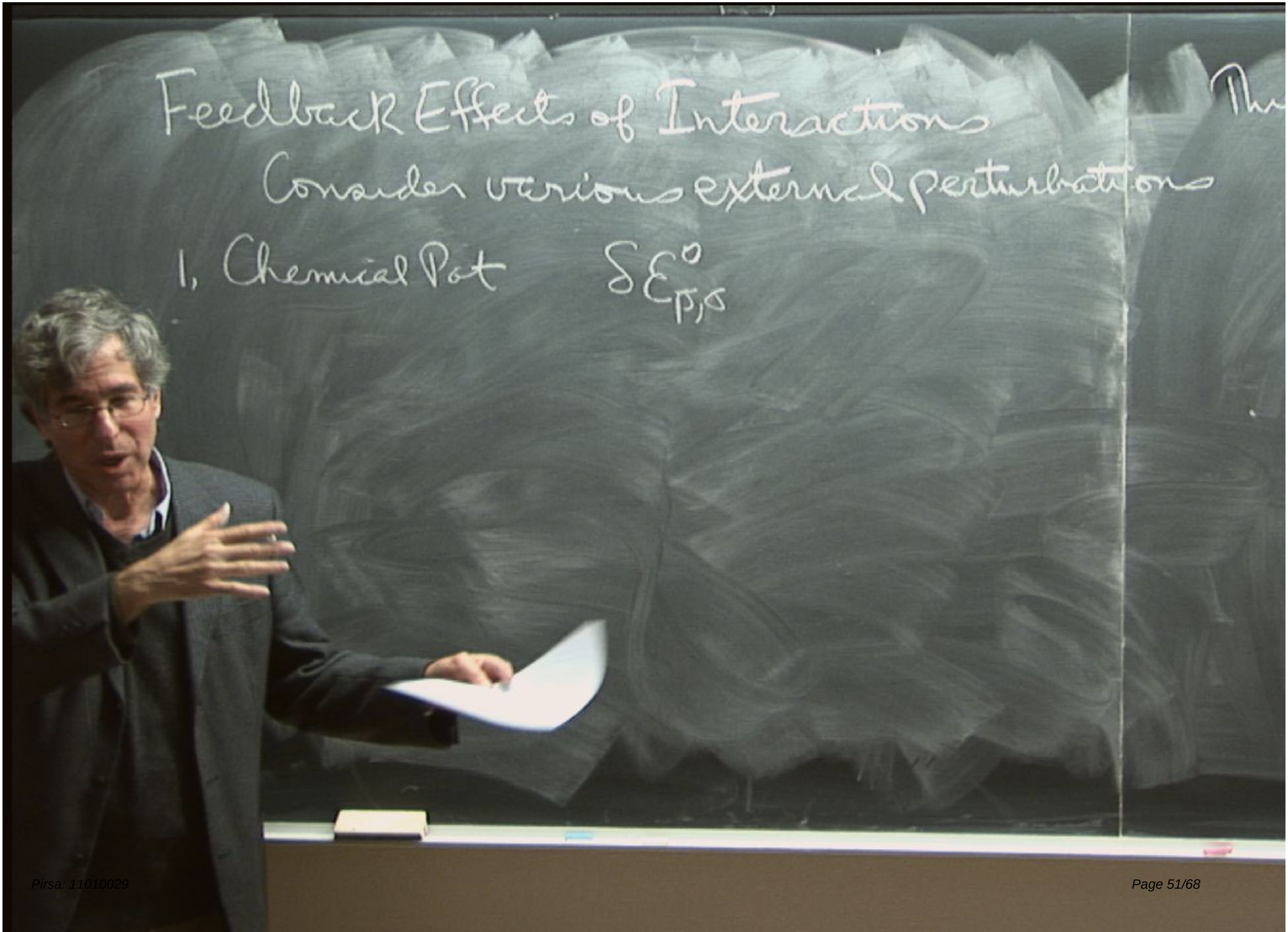
$$F^{s,a}(\cos\theta) = N^*(0)\, f^{s,a}(\cos\theta) = \sum_{l=0} (2l+1)\, F_l^{s,a}\, P_l(\cos\theta)$$

Then $F_l^{s,a} = \langle F^{s,a}(\cos\theta)\, P_l(\cos\theta) \rangle$

$$= 2 \sum_{p,p'} f^{s,a}(\vec{p}, \vec{p}')\, P_l(\cos\theta_{p,p'})\, \delta(\varepsilon_{p'})$$


Feedback Effects of Interactions
Consider various external perturbations

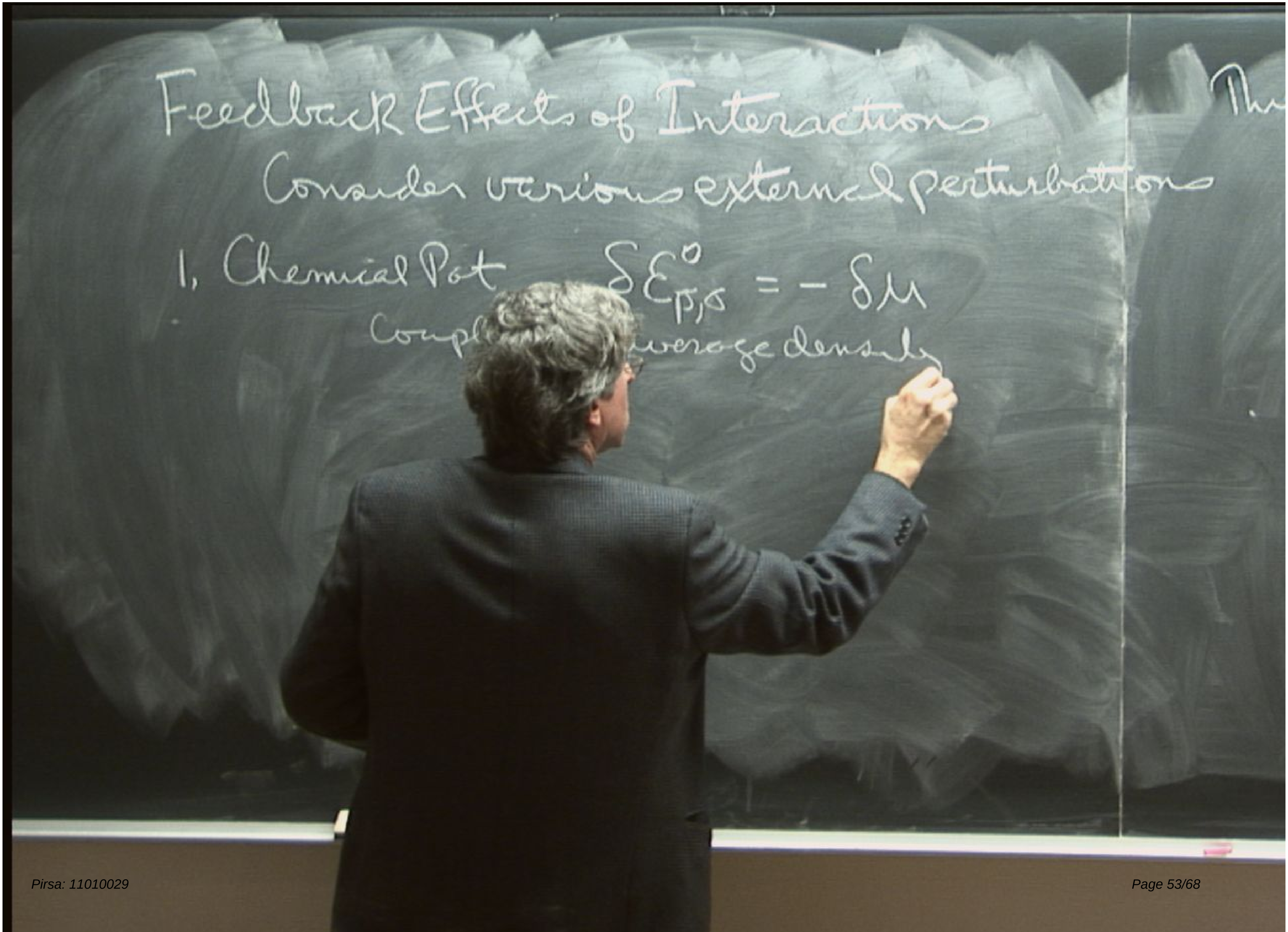

Feedback Effects of Interactions
Consider various external perturbations
1, Chemical Pot
$\delta \mathcal{E}^0_{p,\sigma}$

Feedback Effects of Interactions

Consider various external perturbations

1. Chemical Pot $\qquad \delta \varepsilon^{0}_{p,\sigma} = -\,\delta\mu$

Th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Feedback Effects of Interactions

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   Couples to the average density

2. Magnetic field: $\delta\varepsilon^{0}_{\bar{p},\sigma} = -\,\sigma\,\mu_F B$

3. Vector Potential: $\delta\varepsilon^{0}_{\bar{p},\sigma} = -\,\vec{A}\cdot\frac{e\bar{p}}{m}$

In each case

$$\delta\varepsilon_{\bar{p},\sigma} = \delta\varepsilon^{0}_{p,\sigma} + \sum_{p',\sigma'} f(p,\sigma;p',\sigma')\,\delta n_{p',\sigma'}$$

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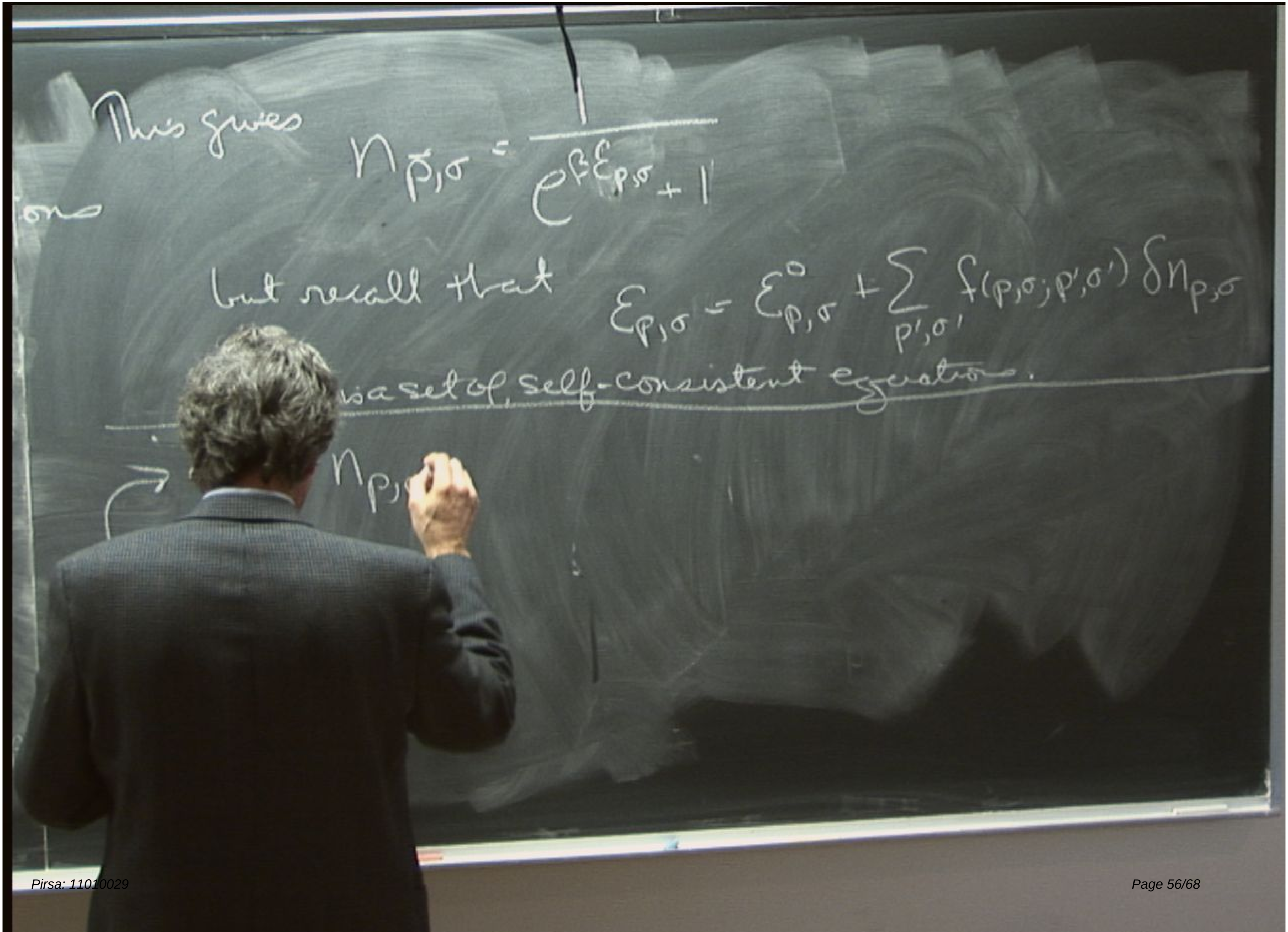
This gives $n_{p,\sigma} = \frac{1}{e^{\beta \varepsilon_{p,\sigma}}+1}$
ons
but recall that $\varepsilon_{p,\sigma} = \varepsilon^{0}_{p,\sigma} + \sum_{p',\sigma'} f(p,\sigma;p',\sigma')\,\delta n_{p,\sigma}$
is a set of self-consistent equations.
$n_{p,}$

This gives $n_{p,\sigma} = \dfrac{1}{e^{\beta \varepsilon_{p,\sigma}} + 1} = f(\varepsilon_{p,\sigma})$

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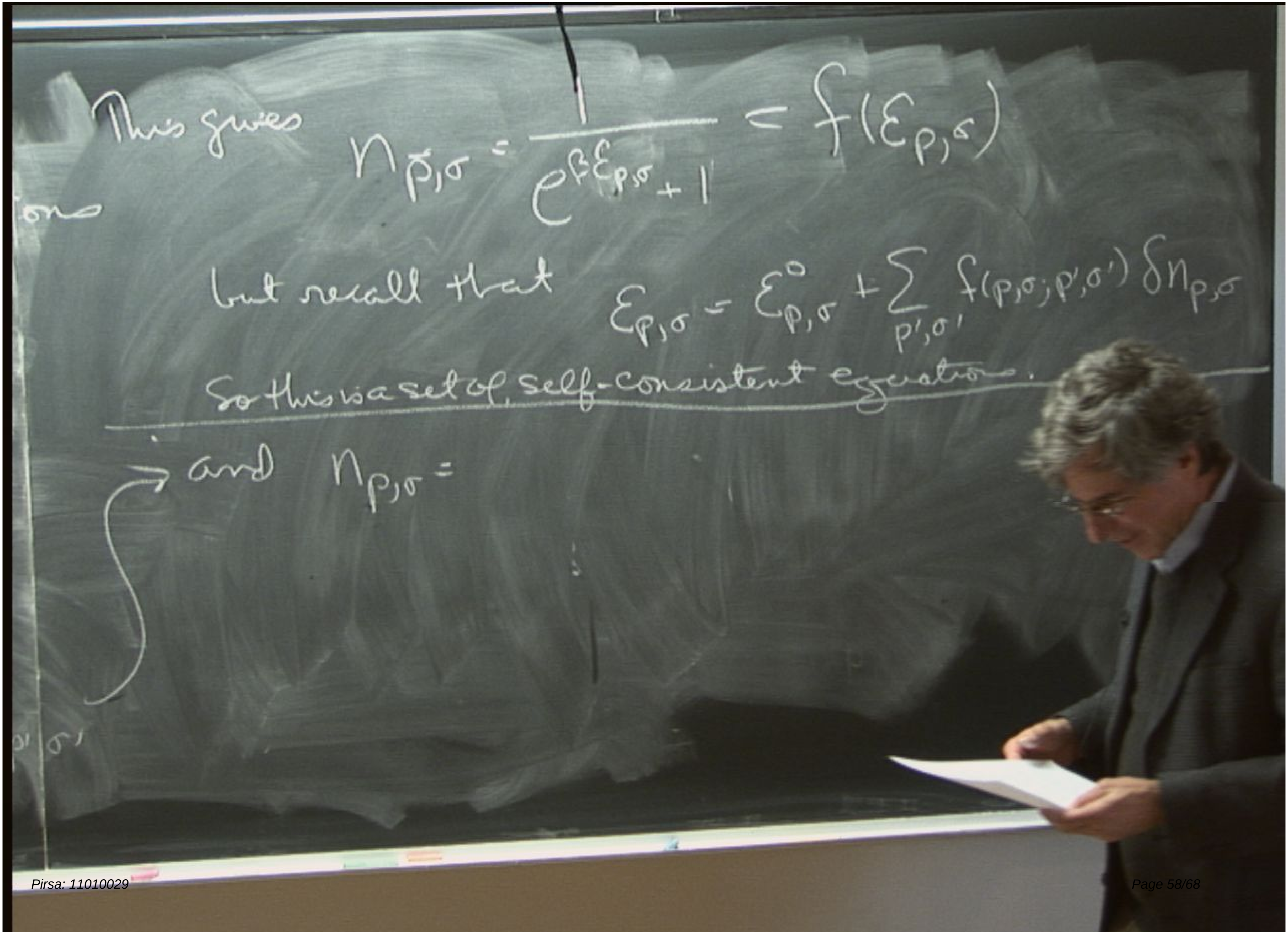
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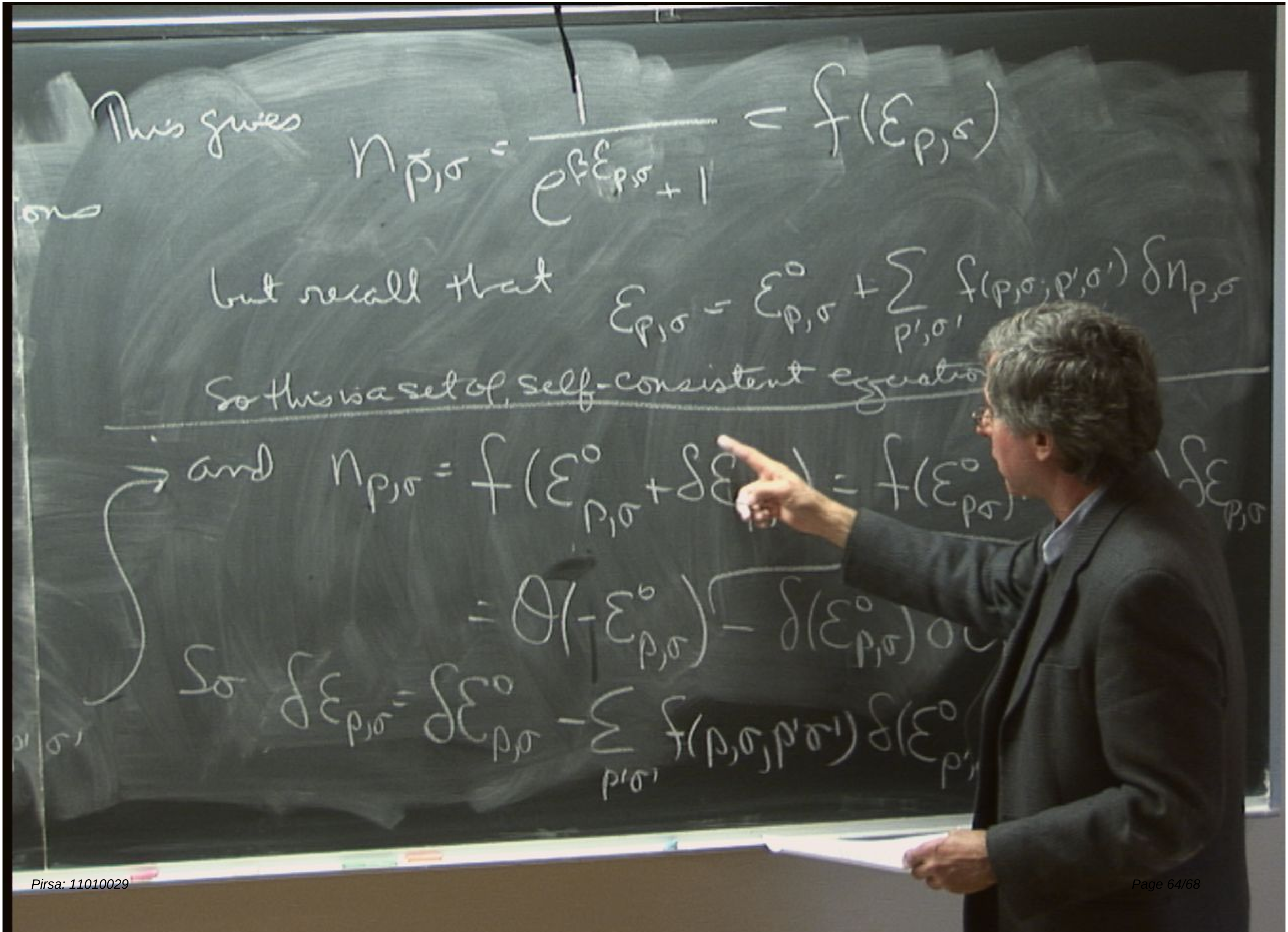
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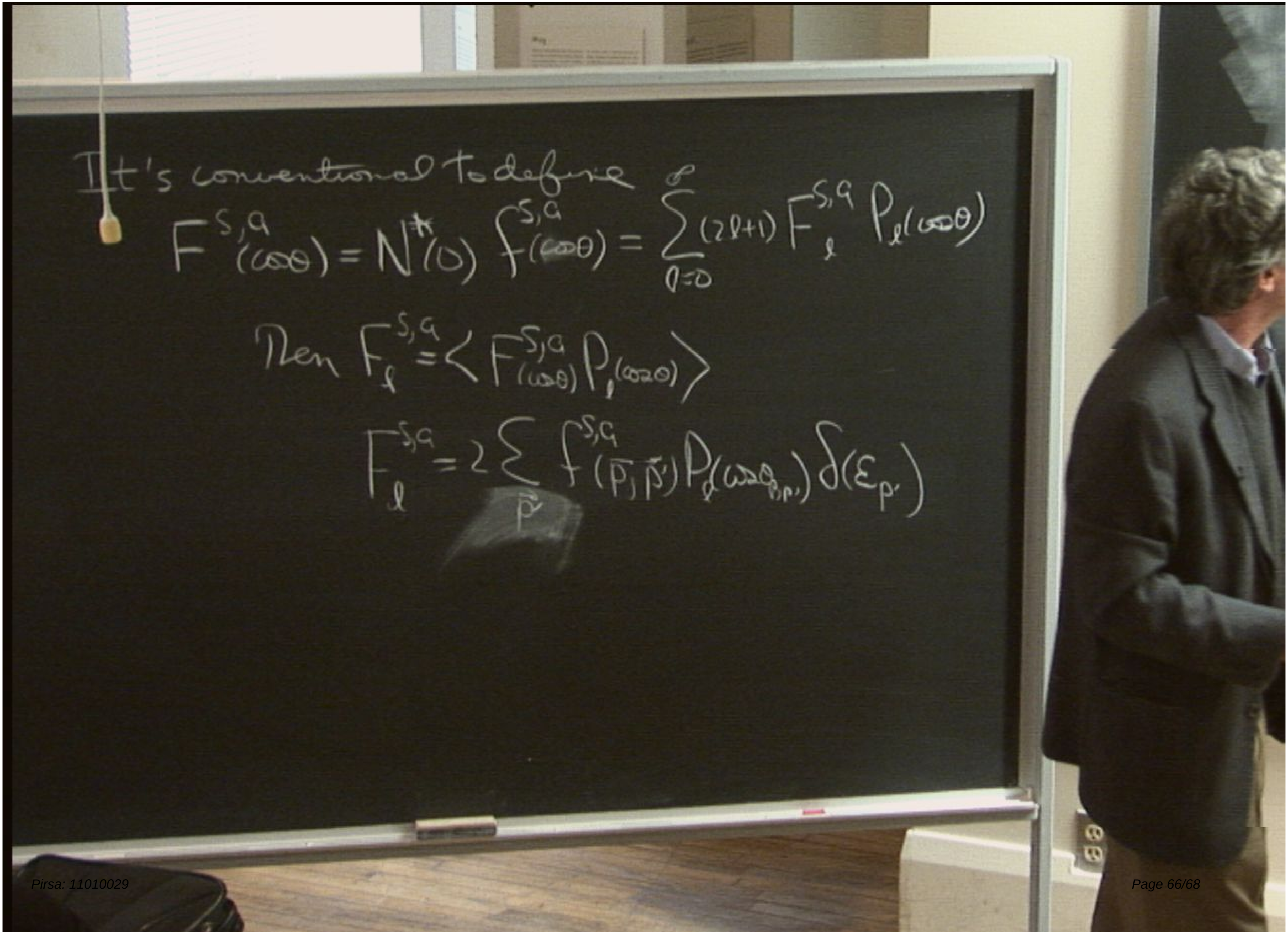
$p' \sigma'$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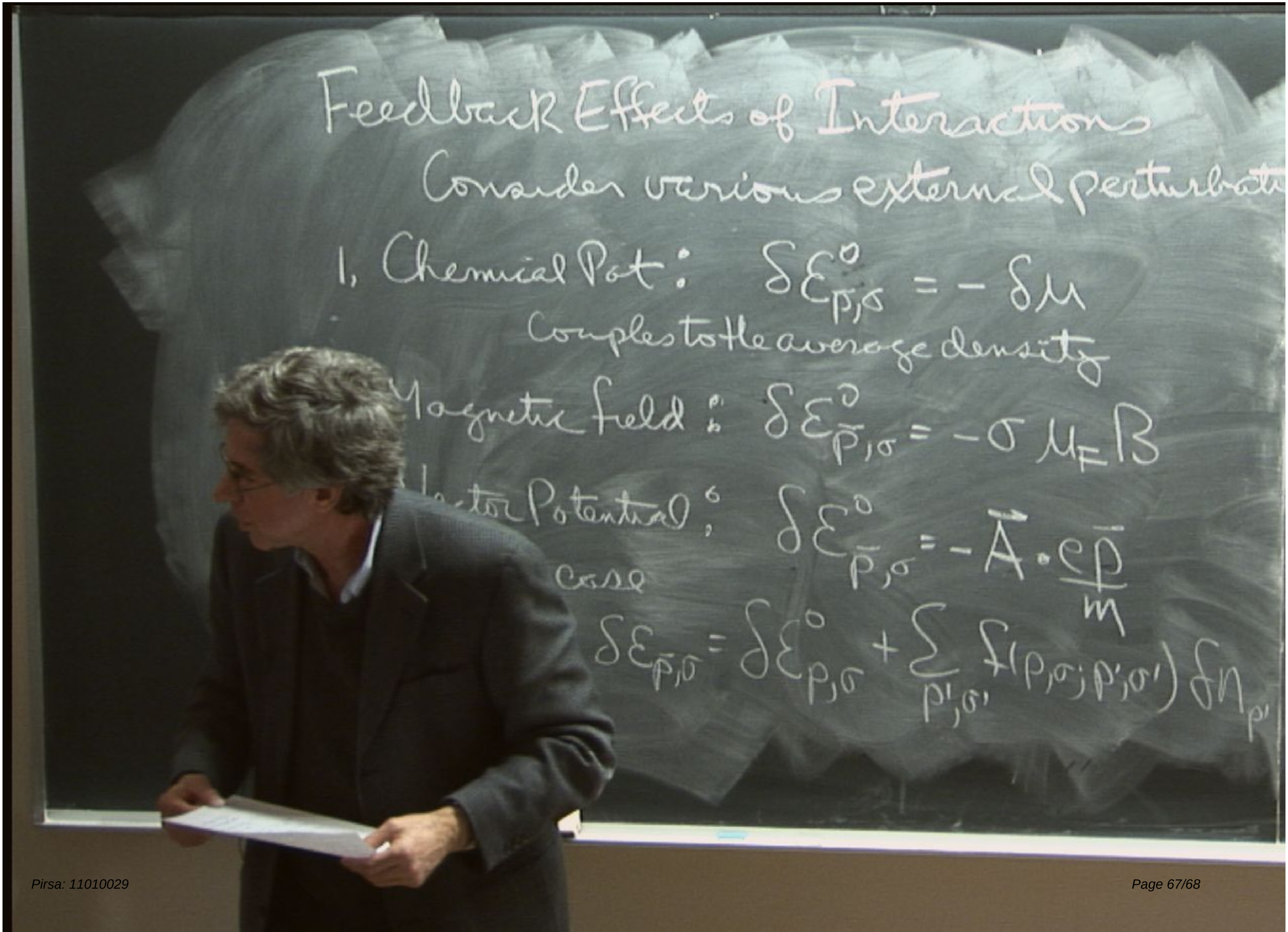
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