

Title: Condensed Matter Review - Lecture 4

Date: Jan 06, 2011 10:15 AM

URL: <http://pirsa.org/11010022>

Abstract:

Tight Binding Bands

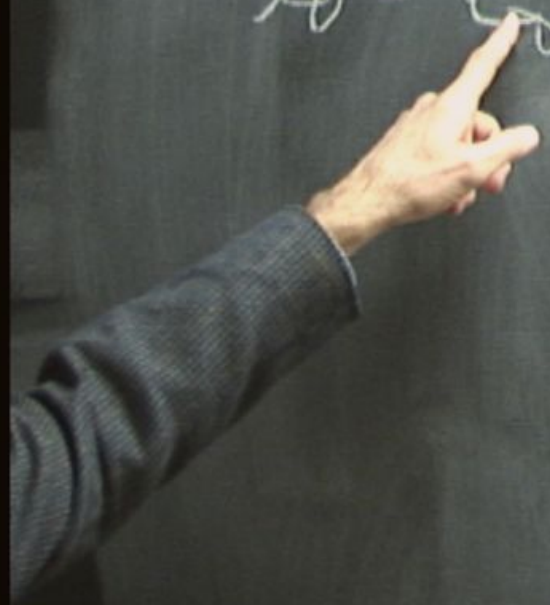
Tight Binding Bands

$$\mathcal{H} = 1$$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\delta} c_{i,\sigma} c_{i+\delta,\sigma}$$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,0} c_{i,0}^\dagger c_{i,0} - t \sum_{i,\delta} c_{i+\delta,0}^\dagger c_{i,0}$$


Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+8,\sigma}^\dagger c_{i,\sigma}$$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\delta} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

δ labels nearest neighbor site

Tight Binding Bands

$$H = E_0 \sum_{i,\sigma} C_{i,\sigma}^\dagger C_{i,\sigma} - t \sum_{i,\sigma} C_{i+\delta,\sigma}^\dagger C_{i,\sigma}$$

δ labels nearest neighbor site

i means $\sum_i$ is summed over latt. sites

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

δ labels nearest neighbor site

i means $\sum_i$ is summed over latt. sites
 $i+\delta$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

δ labels nearest neighbor site

i means $\vec{r}_i$ is summed over latt. sites

$i+\delta$ means $\vec{r}_i + \vec{\delta}$ e.g. for a square

Tight Binding Bands

$$H = E_0 \sum_{i,\sigma} C_{i,\sigma}^\dagger C_{i,\sigma} - t \sum_{i,\delta} C_{i+\delta,\sigma}^\dagger C_{i,\sigma}$$

δ labels nearest neighbor site

i means $\bar{r}_i$ is summed over latt. sites

$i+\delta$ means $\bar{r}_i + \vec{\delta}$ e.g. for a square

$$\vec{\delta} = \pm \hat{x}, \pm \hat{y}$$

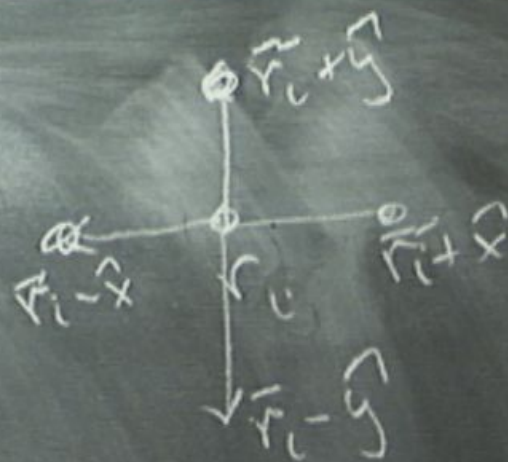
Then $\sum_{i,j} C_{i+j}^+ C_i$

$-C_{j,0}$

the
latt. sites
a square latt

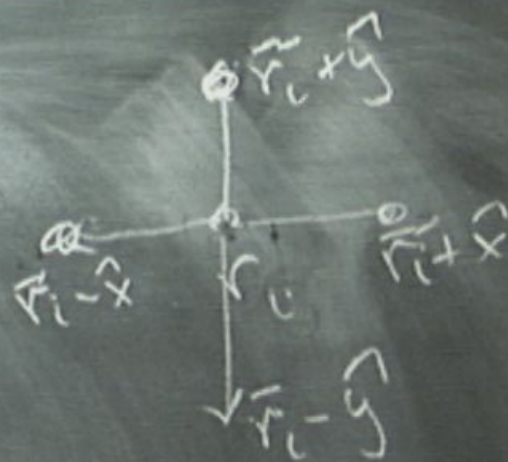
$\vec{x}, \vec{y}$

Then $\sum_{i,s} C_{i,s}^+ C_i$



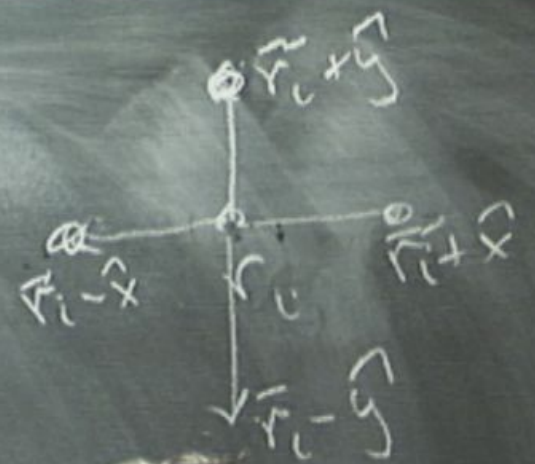
re latt

Then $\left[\sum_{i,s} C_{i+s}^+ C_i \right]$



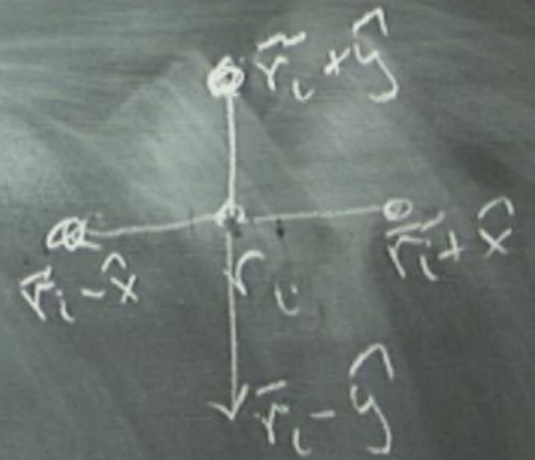
re latt

Then $\left[\sum_{i,s} C_{i+s}^+ C_i \right]$



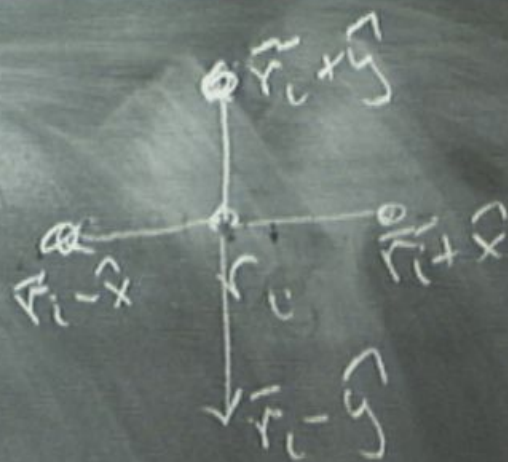
re latt

Then $\left[\sum_{l,s} C_{l+s}^+ C_l \right]$



Then $\left[\sum_{i,j} C_{i+j}^+ C_i \right]$

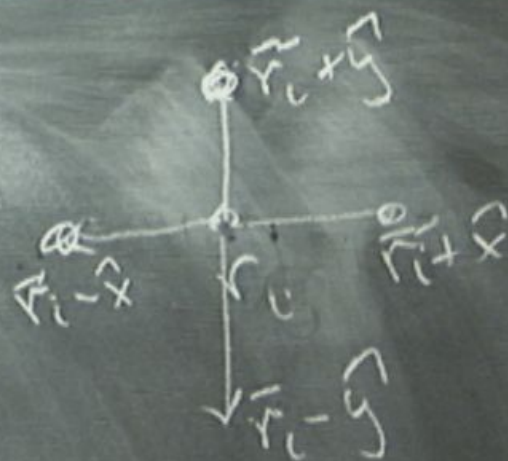
Slater has tabulated
overlap integrals for
atomic orbital



re latt

Then $\left[\sum_{i,j} C_{i+j}^+ C_i \right]$

Slater has tabulated
overlap integrals for
atomic orbital
(n, l)

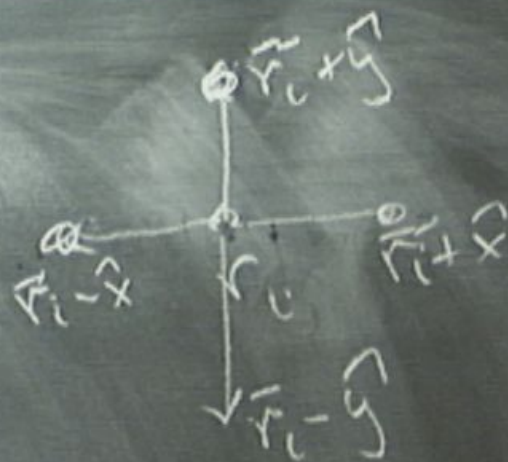


re latt

Then $\left[\sum_{i,j} C_{i+j}^+ C_i \right]$

Slater has tabulated
overlap integrals for
atomic orbital

$(n, l) \quad n=1, 2, \dots \quad l=s, p, d, f, \dots$

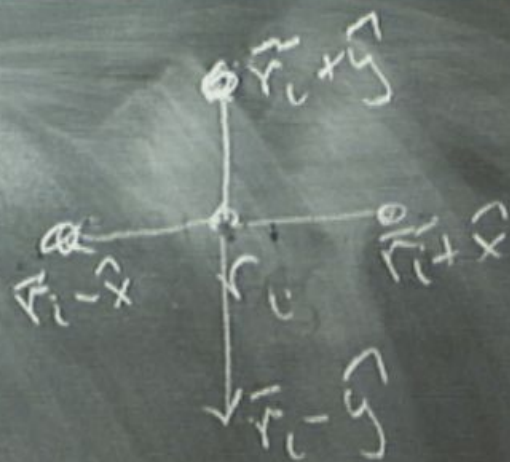


re latt

Then $\left[\sum_{i,j} C_{i+j}^+ C_i \right]$

Slater has tabulated
overlap integrals for
atomic orbital

$(n, l) \quad n=1, 2, \dots \quad l=s, p, d, f, \dots$



re latt

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

Diagonalize by F.T.

$$c_{i,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\vec{k},\sigma} c_{\vec{k},\sigma}^\dagger e^{i\vec{k} \cdot \vec{r}_i} \quad k \in \text{B.Z.}$$

Tight Binding Bands

$$H = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

Diagonalize by F.T.

$$c_{i,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\vec{k},\sigma} c_{\vec{k}}^\dagger e^{i\vec{k} \cdot \vec{r}_i}$$

$$H = \sum_{i,\sigma} \frac{1}{N} \sum_{\vec{k},\vec{k}'} E_0 e^{i\vec{k} \cdot \vec{r}_i}$$

$\in \text{B.Z.}$

$$e^{i\vec{k} \cdot (\vec{r}_i + \delta)} - e^{i\vec{k}' \cdot \vec{r}_i}$$

long Bands

$$+ \epsilon_{i,\sigma} c_{i,\sigma} - t \sum_{i,\delta} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

F.T.

$$\sum_{\vec{k},\sigma} c_{\vec{k},\sigma}^\dagger e^{i\vec{k} \cdot \vec{r}_i} \quad \vec{k} \in \text{B.Z.}$$

$$+ \epsilon_{\vec{k}} e^{-i(\vec{k}-\vec{k}') \cdot \vec{r}_i} - t \sum_{\delta} e^{i\vec{k} \cdot (\vec{r}_i + \vec{\delta}) - i\vec{k}' \cdot \vec{r}_i} c_{\vec{k},\sigma}^\dagger c_{\vec{k}',\sigma}$$

Then

Slater
over
atoms
(v)

Tight Binding Bands

$$H = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\sigma} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

diagonalize by F.T.

$$c_{i,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\vec{k},\sigma} c_{\vec{k},\sigma}^\dagger e^{i\vec{k} \cdot \vec{r}_i} \quad \vec{k} \in \text{B.Z.}$$

$$H = \sum_{i,\sigma} \frac{1}{N} \sum_{\vec{k},\vec{k}'} E_0 e^{-i(\vec{k}-\vec{k}') \cdot \vec{r}_i} - t \sum_{\delta} e^{i\vec{k} \cdot (\vec{r}_i + \vec{\delta})} e^{-i\vec{k}' \cdot \vec{r}_i} c_{\vec{k},\sigma}^\dagger c_{\vec{k}'}$$

$$\sum_i e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_i} = N$$



$$\sum_i e^{i(\vec{k} - \vec{k}') \cdot \vec{r}_i} = N \delta_{\vec{k}, \vec{k}'}$$

$$\sum_{\vec{l}} e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_l} = N \delta_{\vec{k}, \vec{k}'}$$

$$\vec{k} = 2\pi \left(\frac{l}{L}, \frac{m}{L}, \frac{n}{L} \right)$$

for $L \times L \times L$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\delta} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

diagonalize by F.T.

$$c_{i,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\vec{k},\sigma} c_{\vec{k},\sigma}^\dagger e^{i\vec{k} \cdot \vec{r}_i} \quad \vec{k} \in \text{B.Z.}$$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{\vec{k},\vec{k}'} \left(E_0 e^{-i(\vec{k}-\vec{k}') \cdot \vec{r}_i} - t \sum_{\delta} e^{i\vec{k} \cdot (\vec{r}_i + \delta)} e^{-i\vec{k}' \cdot \vec{r}_i} \right) c_{\vec{k},\sigma}^\dagger c_{\vec{k}',\sigma}$$

Tight Binding Bands

$$\mathcal{H} = E_0 \sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} - t \sum_{i,\delta} c_{i+\delta,\sigma}^\dagger c_{i,\sigma}$$

Diagonalize by F.T.

$$c_{i,\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_{\vec{k},\sigma} c_{\vec{k},\sigma}^\dagger e^{i\vec{k} \cdot \vec{r}_i} \quad \vec{k} \in \text{B.Z.}$$

$$\mathcal{H} = \sum_{i,\sigma} \frac{1}{N} \sum_{\vec{k},\vec{k}'} \left(E_0 e^{-i(\vec{k}-\vec{k}') \cdot \vec{r}_i} - t \sum_{\delta} e^{i\vec{k} \cdot (\vec{r}_i + \vec{\delta})} e^{-i\vec{k}' \cdot \vec{r}_i} \right) c_{\vec{k},\sigma}^\dagger c_{\vec{k}',\sigma}$$

$$\sum_{\vec{r}_L} e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_L} = N \delta_{\vec{k}, \vec{k}'}$$

$$-t \sum_{\vec{\delta}} e^{i\vec{k} \cdot \vec{\delta}}$$

$$\vec{k} = 2\pi \left(\frac{l}{L}, \frac{m}{L}, \frac{n}{L} \right)$$

for $L \times L \times L$

For square

$$= -2t(\cos k_x + \cos k_y)$$

$$+ \sum_{\vec{k}, \sigma} \sum_{\vec{k}', \sigma'}$$

$$\sum_{\vec{r}_L} e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_L} = N \delta_{\vec{k}, \vec{k}'}$$

$$-t \sum_{\vec{s}} e^{i\vec{k} \cdot \vec{s}}$$

$$\vec{k} = 2\pi \left(\frac{q}{L}, \frac{m}{L}, \frac{n}{L} \right)$$

for $L \times L \times L$

For square

$$= -2t(\cos k_x + \cos k_y)$$

$$\mathcal{H} = \sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_{\text{square}}$$

$$c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}', \sigma}$$

$$\sum_i e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_i} = N \delta_{\vec{k}, \vec{k}'}$$

$$-t \sum_{\vec{\delta}} e^{i\vec{k} \cdot \vec{\delta}}$$

$$\vec{k} = 2\pi \left(\frac{l}{L}, \frac{m}{L}, \frac{n}{L} \right)$$

for $L \times L \times L$

For square

$$= -2t(\cos k_x + \cos k_y)$$

$$\mathcal{H} = \sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

Square

$$c_{\vec{k}, \sigma} c_{\vec{k}', \sigma}$$

$$\sum_i e^{i(\vec{k}-\vec{k}') \cdot \vec{r}_i} = N \delta_{\vec{k}, \vec{k}'}$$

$$-t \sum_{\vec{s}} e^{i\vec{k} \cdot \vec{s}}$$

$$\vec{k} = 2\pi \left(\frac{l}{L}, \frac{m}{L}, \frac{n}{L} \right)$$

for $L \times L \times L$

For square

$$\Rightarrow -2t(\cos k_x + \cos k_y)$$

$$\mathcal{H} = \sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^\dagger c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

Square

$$c_{\vec{k}, \sigma} c_{\vec{k}', \sigma}$$

x.8

$$K = 2^{11} \left(\frac{1}{L} \right) \frac{1}{L} \frac{1}{L}$$

for $L \times L \times L$

$$2t(\cos k_x + \cos k_y)$$

C_{10}

$$E(\bar{K}) = E_0 - 2t(\cos k_x + \cos k_y)$$

Square

$\kappa \cdot \delta$

$$\bar{K} = 2^{11} \left(\frac{\pi}{L} \right) \bar{L} \bar{L} \bar{L}$$

for $L \times L \times L$

$$2t(\cos \kappa_x + \cos \kappa_y)$$

$\sigma_{\bar{K},0}$

$$E(\bar{K}) = E_0 - 2t(\cos \kappa_x + \cos \kappa_y)$$

Square

For small $|\bar{K}|$

$$-t \sum_{\delta} e^{i\vec{k} \cdot \delta}$$

$$\vec{k} = \frac{2\pi}{L} \left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L} \right)$$

for $L \times L \times L$

$$= -2t(\cos k_x + \cos k_y)$$

square

$$\sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

square

For small $|\vec{k}|$

$$\mathcal{E}(\vec{k}) \approx E_0 - 4t$$

s_g

$$+ t(k_x^2 + k_y^2)$$

$$\mathcal{E}(\vec{k}) \approx E_0 - 2t \left(1 - \frac{k_x^2}{2} + 1 - \frac{k_y^2}{2} \right)$$

s_g

$$-t \sum_{\delta} e^{i\vec{k} \cdot \delta}$$

$$\vec{k} = 2\pi \left(\frac{u}{L}, \frac{v}{L} \right)$$

for $L \times L \times$

For square

$$= -2t(\cos k_x + \cos k_y)$$

$$\mathcal{H} = \sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

Square

cos

$$e^{i\vec{k}' \cdot \vec{r}_c} c_{\vec{k}', \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) \approx E_0 - 4t \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2)$$

For small $|\vec{k}|$

$$\mathcal{E}(\vec{k}) \approx E_0 - 2t(1 - \frac{k_x^2}{2} - \frac{k_y^2}{2})$$

$$-t \sum_{\delta} e^{i\vec{k} \cdot \delta}$$

$$\vec{k} = 2\pi \left(\frac{u}{L}, \frac{v}{L} \right)$$

for $L \times L \times$

For square

$$= -2t(\cos k_x + \cos k_y)$$

$$\mathcal{H} = \sum_{\vec{k}, \sigma} \mathcal{E}(\vec{k}) c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma}$$

$$\mathcal{E}(\vec{k}) = E_0 - 2t(\cos k_x + \cos k_y)$$

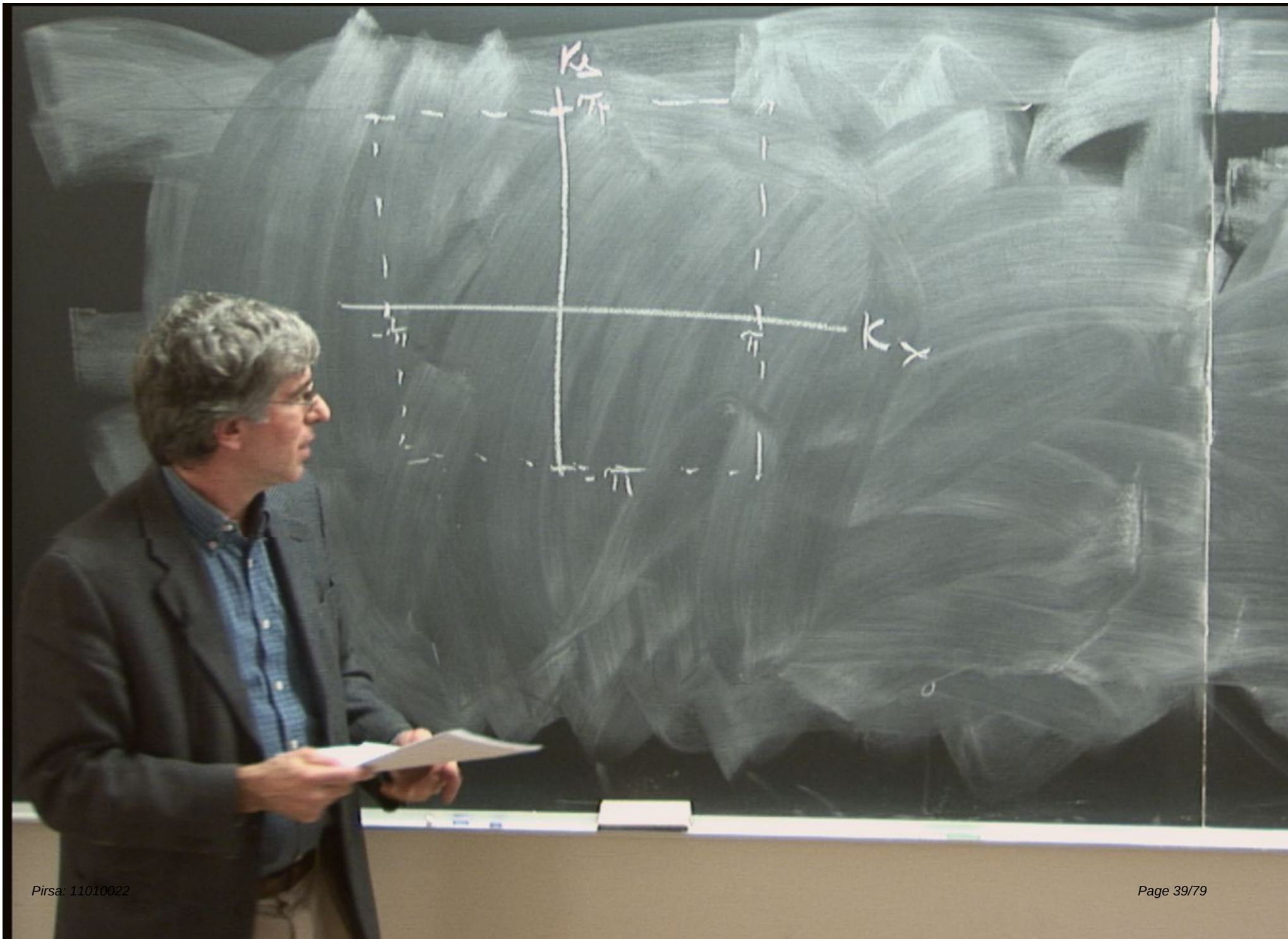
Square

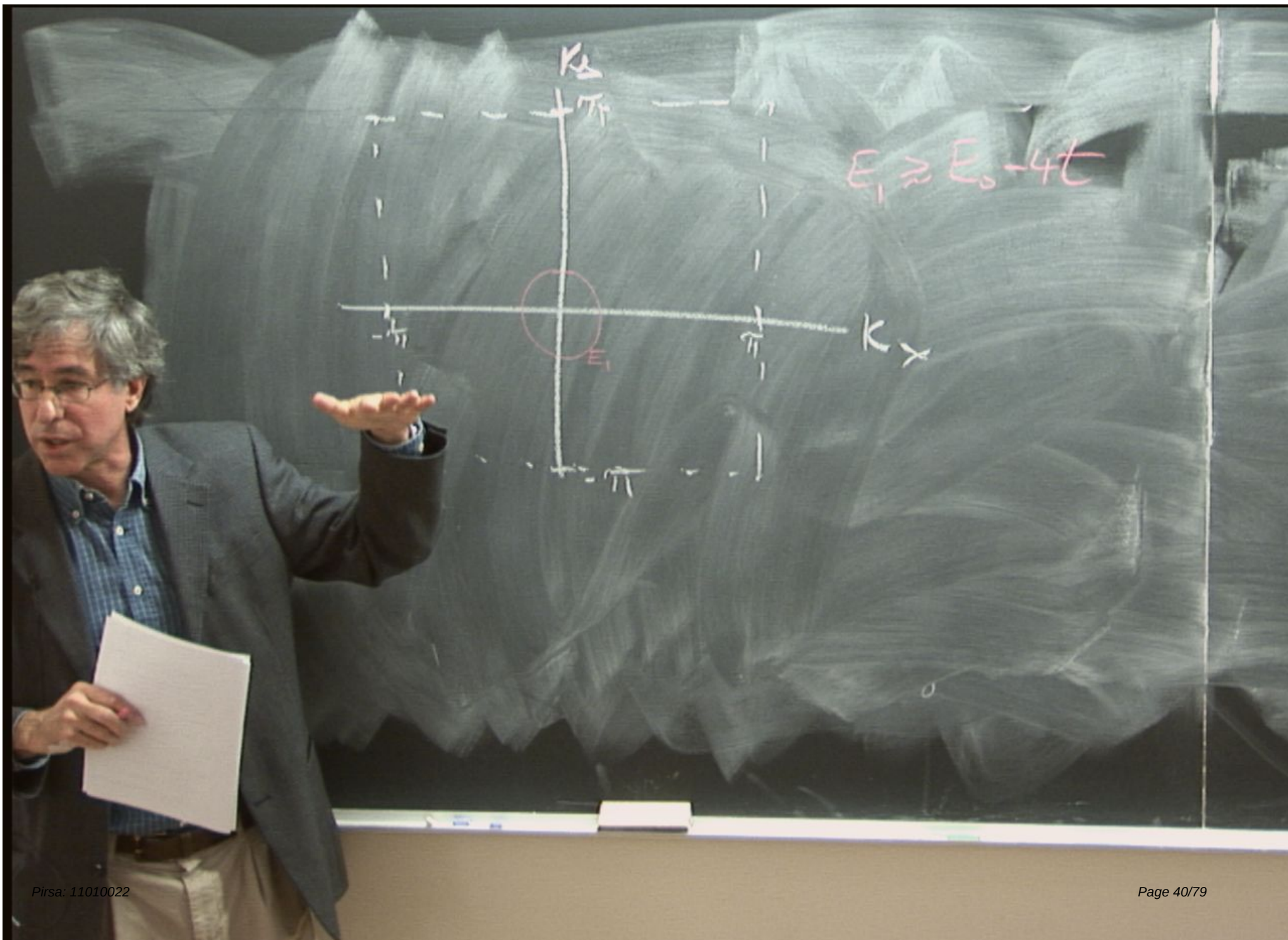
$$\mathcal{E}(\vec{k}) \approx E_0 - 4t \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2)$$

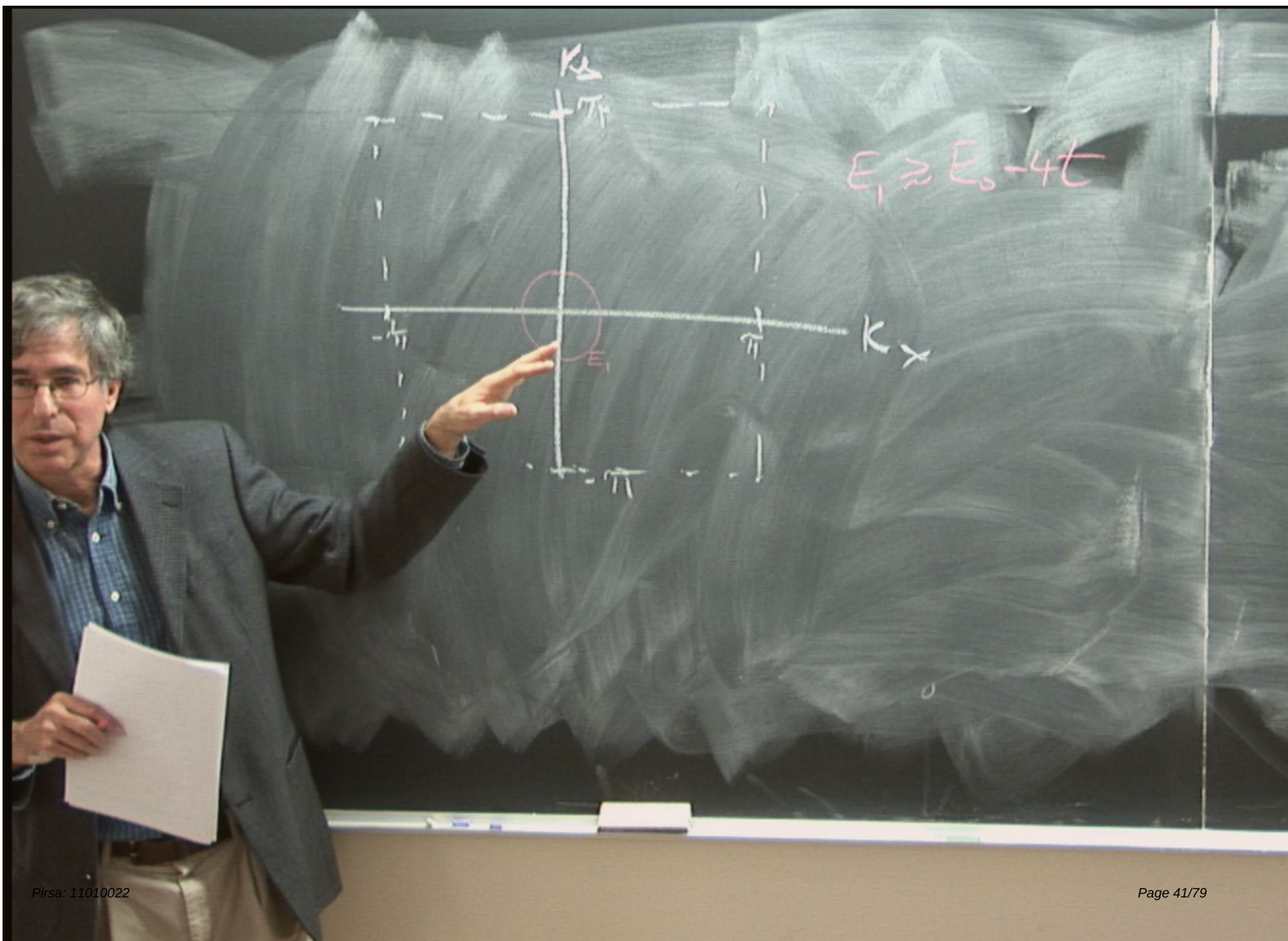
For small $|\vec{k}|$

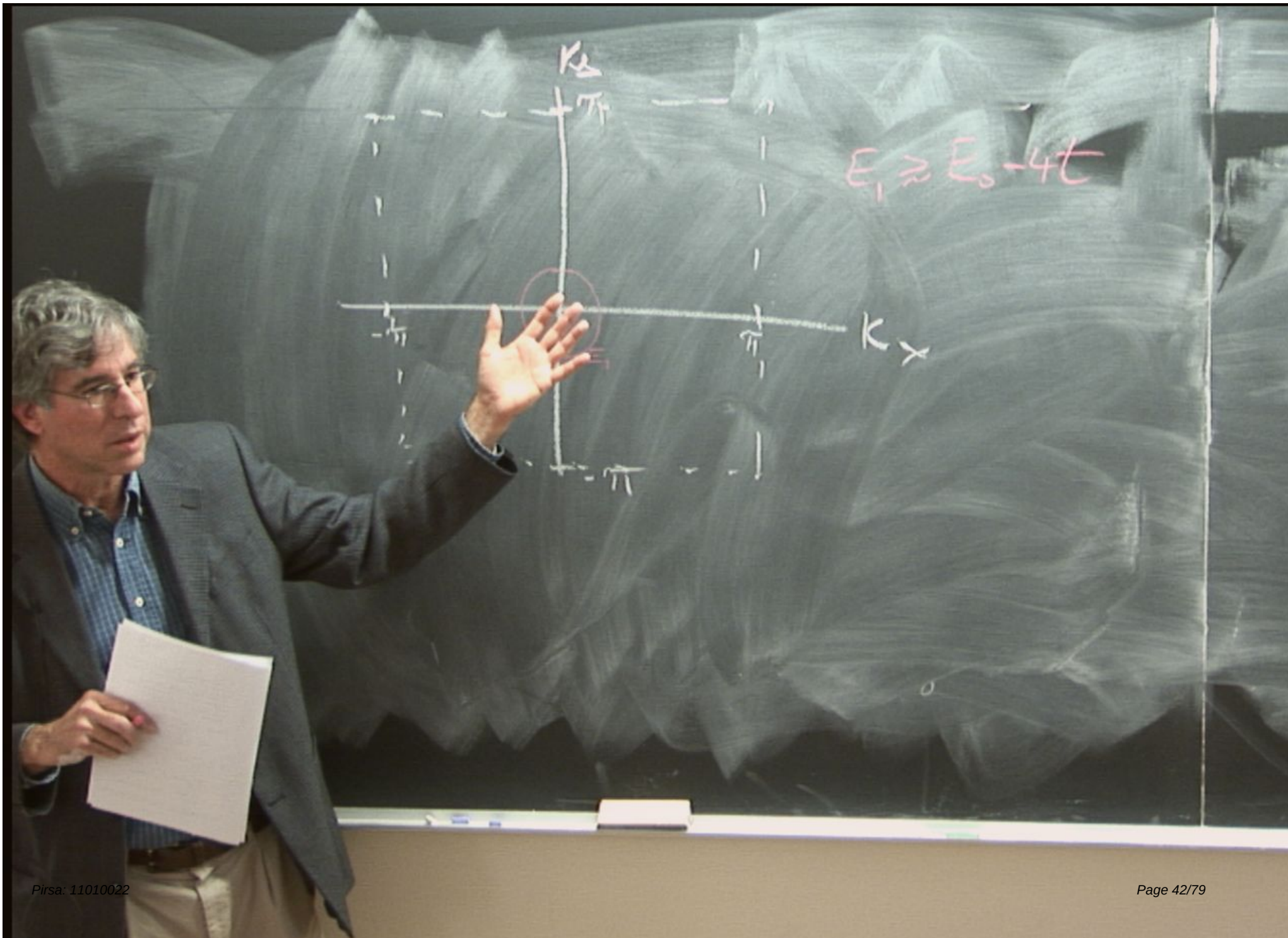
$$\mathcal{E}(\vec{k}) \approx E_0 - 2t(1 - \frac{k_x^2}{2} - \frac{k_y^2}{2})$$

Square



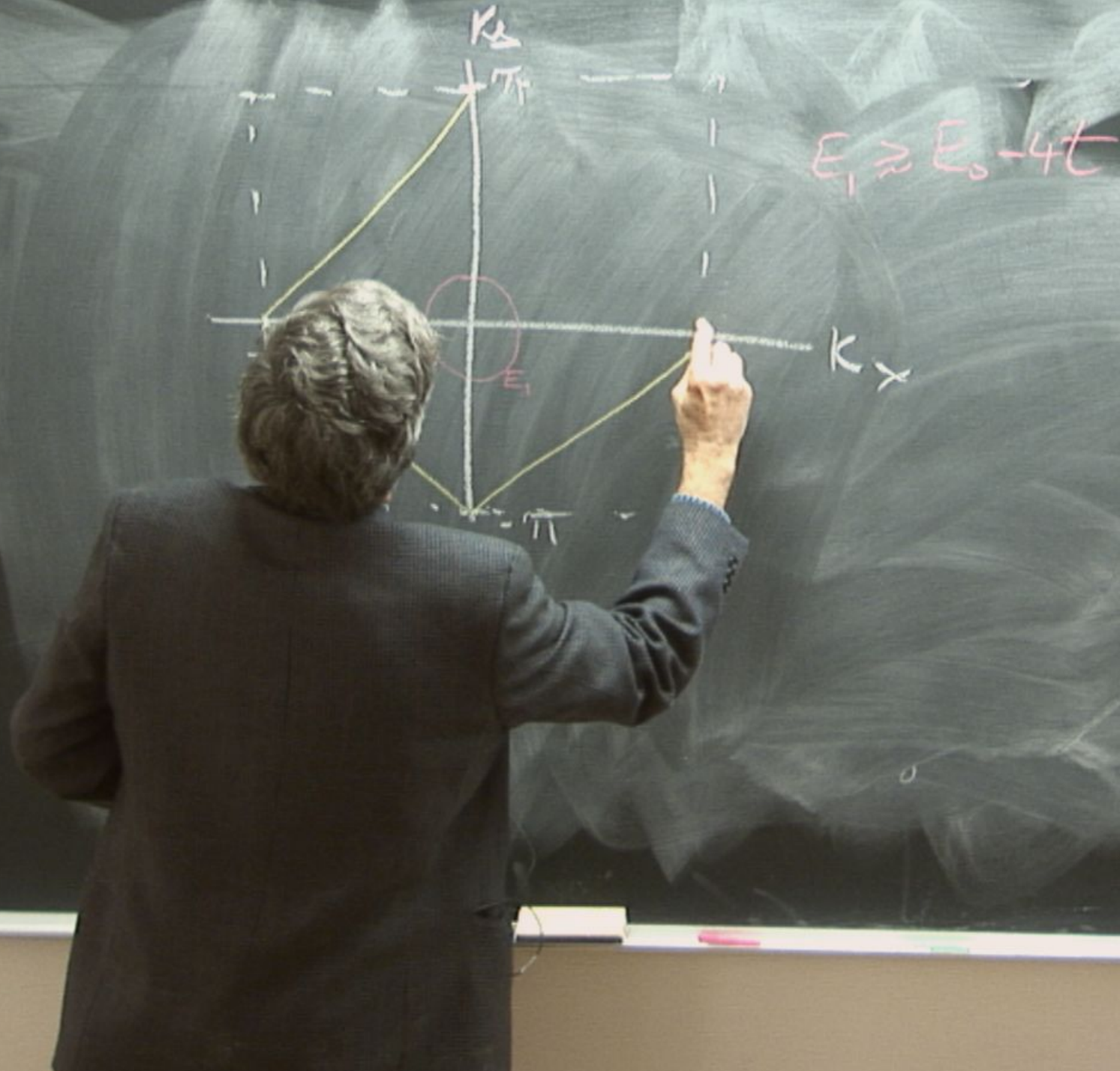


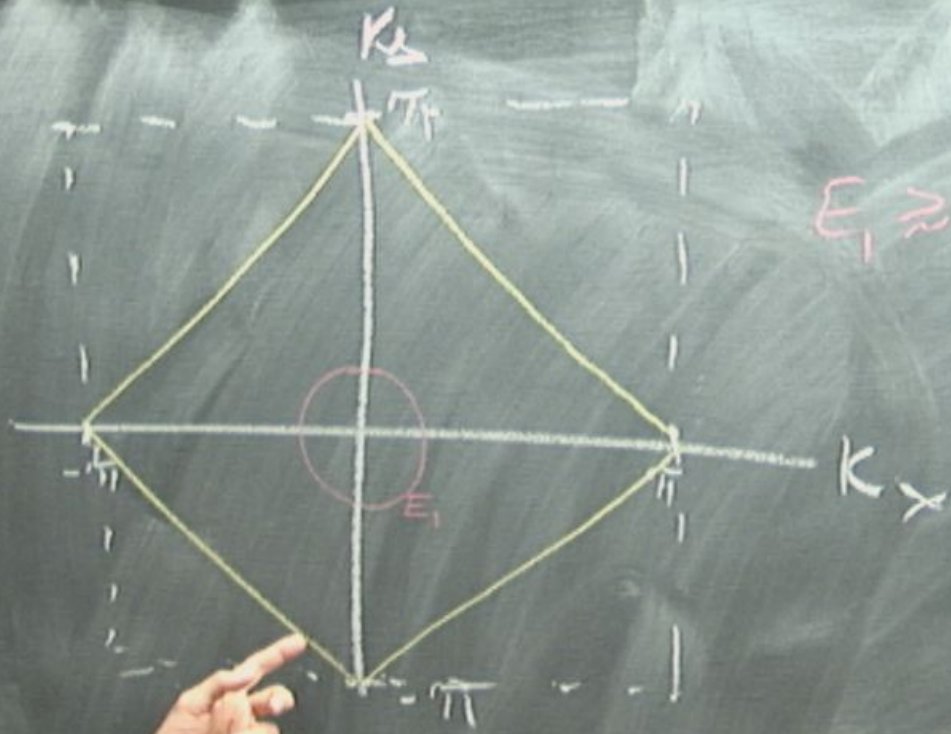




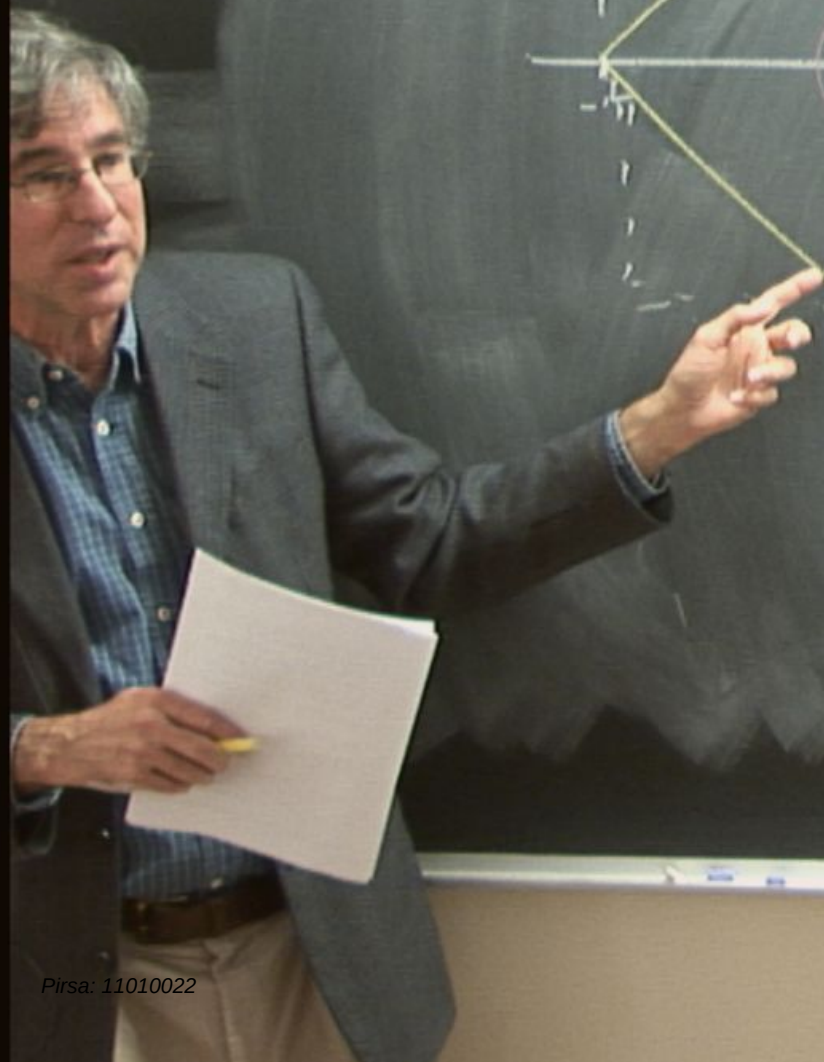


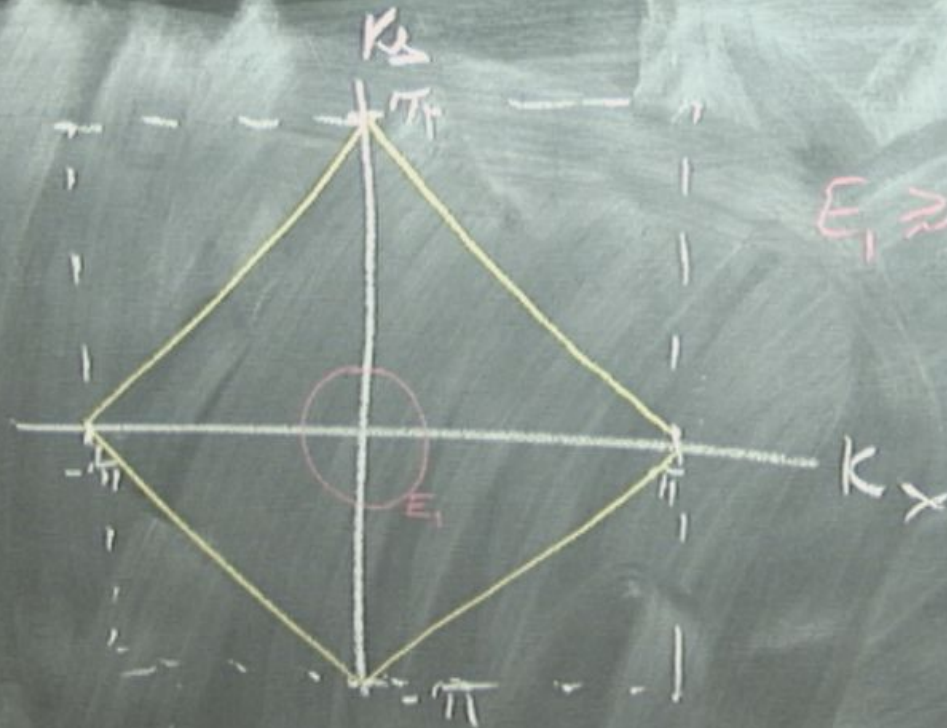
$$E_1 \approx E_0 - 4t$$

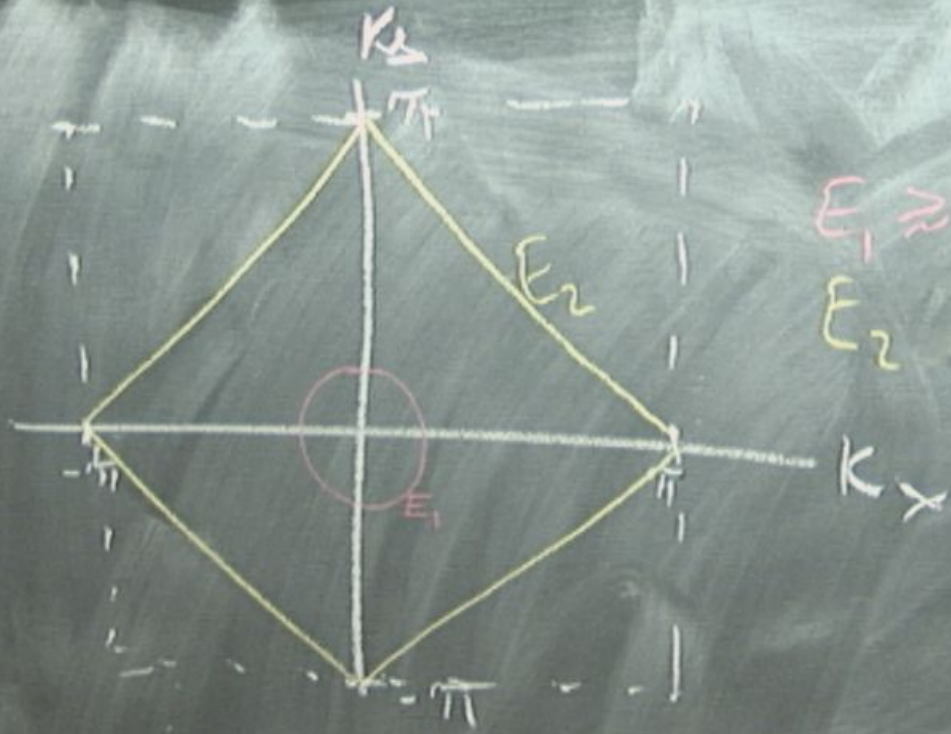




$$E_1 \approx E_0 - 4t$$



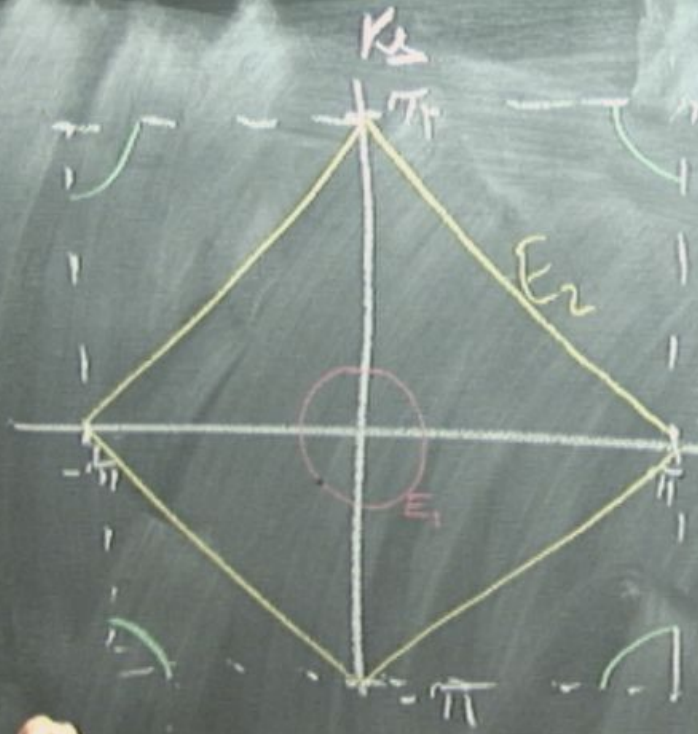




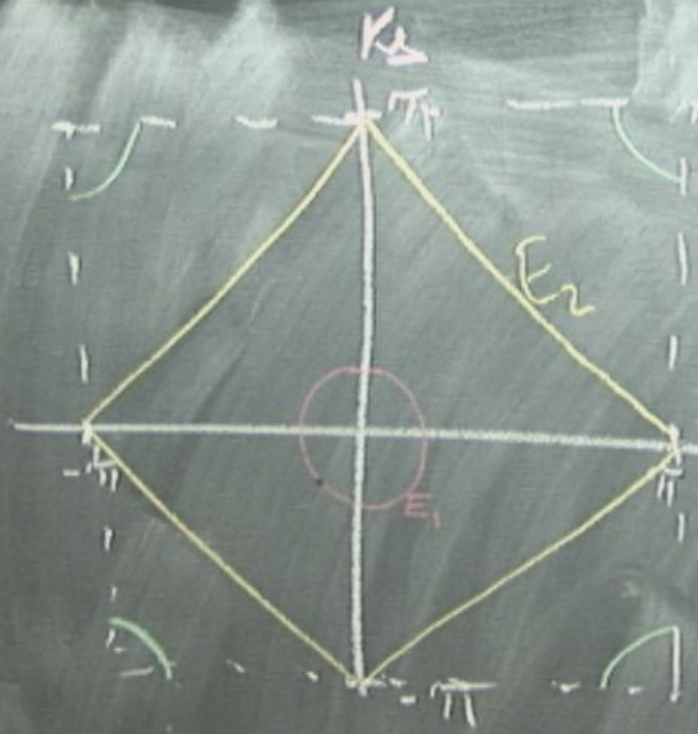
$$E_1 \approx E_0 - 4t$$

$$E_2$$





$$\begin{aligned}
 E_1 &\approx E_0 - 4t \\
 E_2 &= E_0 \\
 E_3 &\approx E_0 + 4t \\
 k_x
 \end{aligned}$$



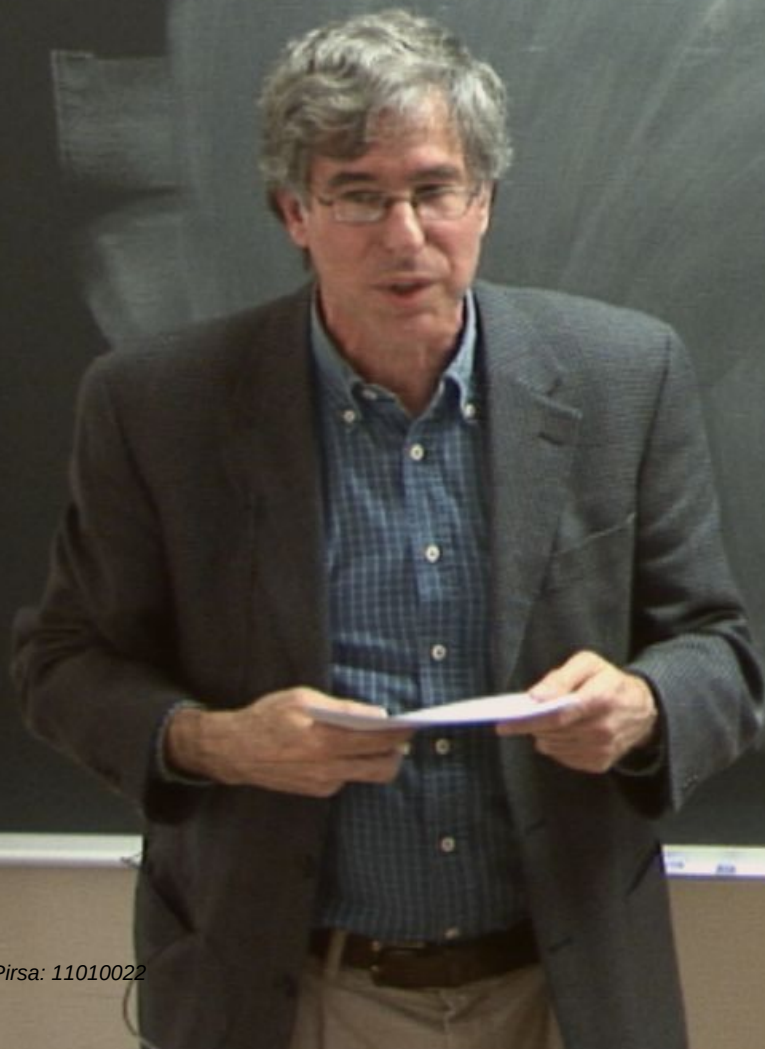
$$E_1 \approx E_0 - 4t$$

$$E_2 = E_0$$

$$E_3 \approx E_0 + 4t$$

k_x

Hartree Fock for Electrons in a Crystal



Hartree Fock for Electrons in a Crystal
Refs: X.-G. Wen QFT of Many-Body Systems

Antisymmetric Fock for Electrons in a Crystal
Refs: X.-G. Wen QFT of Many-Body Systems See 5.2

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.2
Bruus + Flensberg See 4.3

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.2

Bruus + Flensberg See 4.3

Hart =

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.2

Bruus + Flensberg See 4.3

$H_{\text{int}} =$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.2

Brune & Flensberg See 4.3

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\vec{R}_i - \vec{R}_j) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.2

Bruus + Flensberg See 4.3

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\vec{R}_i - \vec{R}_j) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.

Brueus + Flensberg See 4.3

$$H_{\text{int}} = \frac{1}{2} \sum_{\vec{R}} \sum_{i,j,\alpha,\beta} V(\vec{R}) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.

Bruus + Flensberg See 4.3

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\vec{R}_i - \vec{R}_j) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.

Brueckner + Flensberg See 4.3

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{i,j} V(\vec{R}_i - \vec{R}_j) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

$$\mathcal{H} = \sum_{\vec{k}, \alpha} (\epsilon_{\vec{k}} - \mu) C_{\vec{k}, \alpha}^\dagger C_{\vec{k}, \alpha}$$

Hartree Fock for Electrons in a Crystal

Refs: X.-G. Wen QFT of Many-Body Systems See 5.

Brueckner + Flensberg See 4.3

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} V(\vec{R}_i - \vec{R}_j) C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

$$\mathcal{H} = \sum_{\vec{k},\alpha} (E(\vec{k}) - \mu) C_{\vec{k},\alpha}^\dagger C_{\vec{k},\alpha} + \mathcal{H}_{\text{int}}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^{\dagger} C_{\vec{\xi}, \beta}^{\dagger} C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \sigma, \sigma'}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^{\dagger} C_{\vec{\xi}, \beta}^{\dagger} C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{R}, \vec{R}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} \equiv \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P} \quad V(\vec{R}_i - \vec{R}_j) C_{\vec{R}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{\substack{\vec{R}, \vec{R}' \\ \alpha, \beta}} \frac{1}{N} \sum_{\vec{R}, \vec{R}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{R} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R} =$$

$$(\vec{R}_i - \vec{R}_j) C_{\vec{R}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{R}', \alpha}$$

$$\mathcal{H}_{\text{int}} = \frac{1}{2N^2}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{K}, \vec{K}' \\ \alpha}} \sum_{\substack{\vec{\xi}, \vec{\xi}' \\ \beta}} \sum_{\vec{L}'} e^{i(\vec{K}-\vec{K}'+\vec{\xi}-\vec{\xi}')} \sum_{\vec{P}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{P}} V(\vec{P})$$

$$C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{K}, \vec{K}' \\ \alpha}} \sum_{\substack{\vec{\xi}, \vec{\xi}' \\ \beta}} \left[\sum_{\vec{l}'} e^{i(\vec{K}-\vec{K}'+\vec{\xi}-\vec{\xi}') \cdot \vec{l}'} \right] \sum_{\vec{P}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{P}} V(\vec{P})$$

$$C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{K}, \vec{K}' \\ \alpha}} \sum_{\substack{\vec{\xi}, \vec{\xi}' \\ \beta}} \left[\sum_{\vec{L}} e^{i(\vec{K}-\vec{K}'+\vec{\xi}-\vec{\xi}') \cdot \vec{R}_i} \right] \sum_{\vec{P}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{P}} V(\vec{P}) C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{K}, \vec{K}' \\ \alpha}} \sum_{\substack{\vec{\xi}, \vec{\xi}' \\ \beta}} \left[\sum_{i'} e^{i(\vec{K}-\vec{K}'+\vec{\xi}-\vec{\xi}') \cdot \vec{R}_{i'}} \right] \sum_{\vec{P}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{P}} \frac{V(\vec{P})}{V(\vec{P})}$$

$$C_{\vec{K}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{i,j \in B} \frac{1}{N} \sum_{\vec{R}, \vec{R}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{R} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{R}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{R}, \alpha}^+ C_{\vec{\xi} \beta}^+ C_{\vec{\xi}' \beta} C_{\vec{R}' \alpha}$$

$$H_{\text{int}} = \sum_{\vec{k}, \vec{k}'} \sum_{\alpha, \alpha'} \left[\sum_{\vec{R}_i} e^{i(\vec{k}-\vec{k}'+\vec{\xi}-\vec{\xi}') \cdot \vec{R}_i} \right] \sum_{\vec{p}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{p}} V(\vec{p}) C_{\vec{k}, \alpha}^+ C_{\vec{\xi} \beta}^+ C_{\vec{\xi}' \beta} C_{\vec{k}' \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{R}, \vec{R}'} \frac{1}{N} \sum_{\vec{\xi}, \vec{\xi}'} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i(\vec{\xi}-\vec{\xi}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{R}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{k}, \vec{k}' \\ \alpha}} \sum_{\substack{\vec{\xi}, \vec{\xi}' \\ \beta}} \left[\sum_{\vec{l}'} e^{i(\vec{k}-\vec{k}'+\vec{\xi}-\vec{\xi}') \cdot \vec{R}_i} \right] \sum_{\vec{P}} e^{-i(\vec{\xi}-\vec{\xi}') \cdot \vec{P}} \frac{V(\vec{P})}{V(\vec{P})}$$

$$C_{\vec{k}, \alpha}^+ C_{\vec{\xi}, \beta}^+ C_{\vec{\xi}', \beta} C_{\vec{k}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\substack{i,j \\ \alpha,\beta}} \frac{1}{N} \sum_{\vec{K}, \vec{K}'} \frac{1}{N} \sum_{\vec{g}, \vec{g}'} e^{i(\vec{K}-\vec{K}') \cdot \vec{R}_i} e^{i(\vec{g}-\vec{g}') \cdot \vec{R}_j}$$

See 5.2

$$\vec{P} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{K}, \alpha}^+ C_{\vec{g}, \beta}^+ C_{\vec{g}', \beta} C_{\vec{K}', \alpha}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\substack{\vec{K}, \vec{K}' \\ \alpha}} \sum_{\substack{\vec{g}, \vec{g}' \\ \beta}} \left[\sum_{\vec{L}} e^{i(\vec{K}-\vec{K}'+\vec{g}-\vec{g}') \cdot \vec{R}_i} \right] \sum_{\vec{P}} e^{-i(\vec{g}-\vec{g}') \cdot \vec{P}} \frac{V(\vec{P})}{V(\vec{P})} C_{\vec{K}, \alpha}^+ C_{\vec{g}, \beta}^+ C_{\vec{g}', \beta} C_{\vec{K}', \alpha}$$

$$\sum_i \sqrt{(\vec{R}_i - \vec{R})} C_{i,\alpha}^\dagger C_{j,\beta}^\dagger C_{j,\beta} C_{i,\alpha}$$

Hint

$$u) C_{\vec{k},\alpha}^\dagger C_{\vec{k},\alpha} + \text{Hint}$$

$$\sum_i e^{i(\vec{k} - \vec{k}') \cdot \vec{R}_i}$$

ex 5.2

$$H_{\text{int}} = \sum_{\alpha, \beta} \sum_{\vec{k}, \vec{k}'} \sum_{\vec{g}, \vec{g}'} N \langle \dots \rangle$$

$$V(\vec{R}_i - \vec{R}_j) C_{\vec{k}, \alpha}^{\dagger} C_{\vec{g}, \beta}^{\dagger} C_{\vec{g}', \beta} C_{\vec{k}', \alpha}$$

$$\vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{P}$$

$$H_{\text{int}} = \sum_{\vec{R}_i} \left[\sum_{\vec{k}, \vec{k}'} e^{i(\vec{k} - \vec{k}' + \vec{g} - \vec{g}') \cdot \vec{R}_i} \right] \sum_{\vec{P}} e^{-i(\vec{g} - \vec{g}') \cdot \vec{P}} V(\vec{P})$$

$$-\vec{k}' + \vec{g} - \vec{g}' = 0 \quad C_{\vec{k}, \alpha}^{\dagger} C_{\vec{g}, \beta}^{\dagger} C_{\vec{g}', \beta} C_{\vec{k}', \alpha}$$

ex 5.2

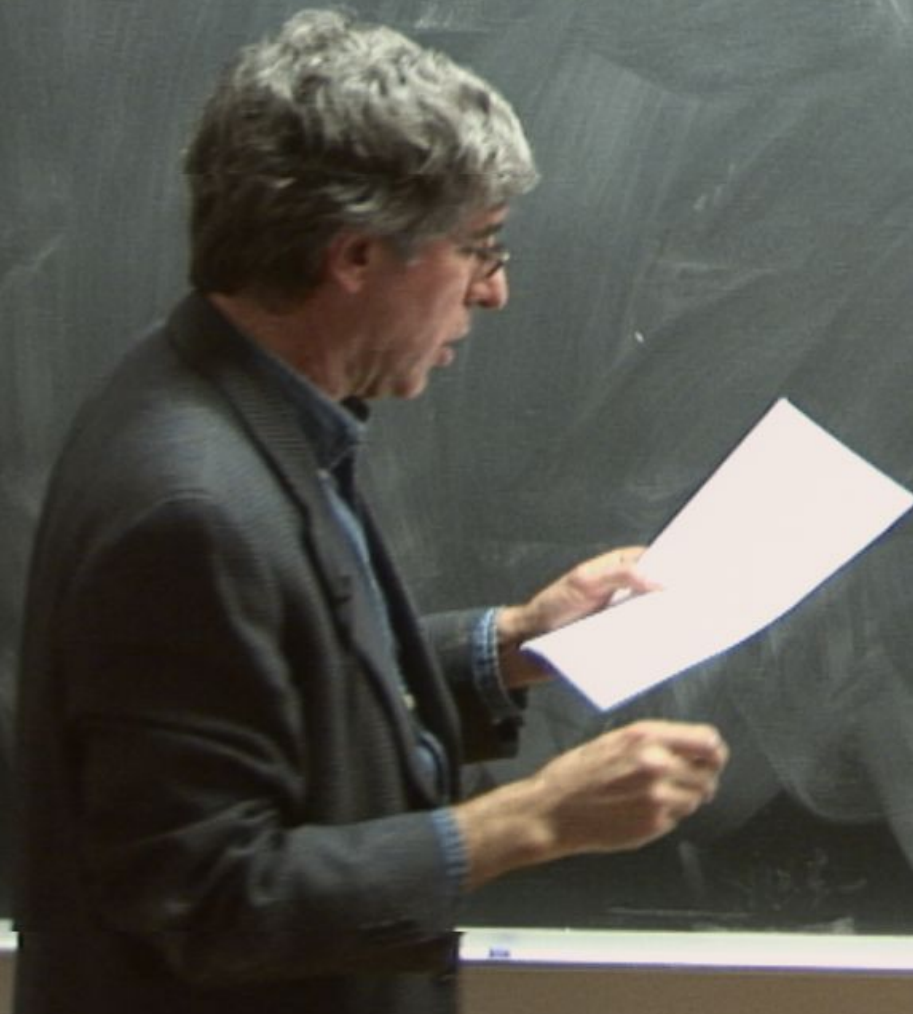
$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\alpha\beta} \sum_{\alpha'\beta'} \sum_{\vec{k}, \vec{k}'} \sum_{\vec{g}, \vec{g}'} V(\vec{R}_i - \vec{R}_j) C_{\vec{k}, \alpha}^{\dagger} C_{\vec{g}, \beta} C_{\vec{g}', \beta'} C_{\vec{k}', \alpha'}$$

$$\vec{p} = \vec{R}_i - \vec{R}_j \rightarrow \vec{R}_j = \vec{R}_i - \vec{p}$$

$$H_{\text{int}} = \frac{1}{2N^2} \sum_{\alpha\beta} \sum_{\alpha'\beta'} \left[\sum_{\vec{k}, \vec{k}'} \sum_{\vec{g}, \vec{g}'} e^{i(\vec{k} - \vec{k}' + \vec{g} - \vec{g}') \cdot \vec{R}_i} \right] \sum_{\vec{p}} e^{-i(\vec{g} - \vec{g}') \cdot \vec{p}} V(\vec{p}) C_{\vec{k}, \alpha}^{\dagger} C_{\vec{g}, \beta} C_{\vec{g}', \beta'} C_{\vec{k}', \alpha'}$$

$$\vec{k} - \vec{k}' + \vec{g} - \vec{g}' = 0$$

$$S_0 - (\bar{\xi} - \bar{\xi}') = \bar{K} - \bar{K}' \equiv \bar{Q}$$



$$S_0 - (\vec{\xi} - \vec{\xi}') = \vec{k} - \vec{k}' = \vec{Q}$$

$$\vec{\xi}' = \vec{\xi} + \vec{Q}$$

$$V(\vec{Q}) = \sum_{\vec{p}} e^{i\vec{Q} \cdot \vec{p}} V(\vec{p}) \quad \vec{k}' = \vec{k} - \vec{Q}$$

$$S_0 - (\bar{\epsilon} - \bar{\epsilon}') = \bar{k} - \bar{k}' = \bar{Q}$$

$$\bar{\epsilon}' = \bar{\epsilon} + \bar{Q}$$

$$V(\bar{Q}) = \sum_{\vec{p}} e^{i\bar{Q} \cdot \vec{p}} V(\vec{p})$$

$$\bar{k}' = \bar{k} - \bar{Q}$$

$$H_{\text{int}} = \frac{1}{2N} \sum_{\bar{Q}} V(\bar{Q}) \sum_{\substack{\vec{k}, \vec{\ell} \\ \vec{k} = \vec{\ell}}} c_{\vec{k}\alpha}^{\dagger} c_{\vec{\ell}\beta}^{\dagger} c_{\vec{\ell}+\bar{Q},\beta}$$

$$-(\vec{\epsilon} - \vec{\epsilon}') = \vec{k} - \vec{k}' = \vec{Q}$$

$$\vec{\epsilon}' = \vec{\epsilon} + \vec{Q}$$

$$V(\vec{Q}) = \sum_{\vec{p}} e^{i\vec{Q} \cdot \vec{p}} V(\vec{p}) \quad \vec{k}' = \vec{k} - \vec{Q}$$

$$H_{\text{int}} = \frac{1}{2N} \sum_{\vec{Q}} V(\vec{Q}) \sum_{\substack{\vec{k}, \alpha \\ \vec{\epsilon}, \beta}} C_{\vec{k}, \alpha}^{\dagger} C_{\vec{\epsilon}, \beta}^{\dagger} C_{\vec{\epsilon} + \vec{Q}, \beta} C_{\vec{k} - \vec{Q}, \alpha}$$

$$H_{\text{int}} =$$