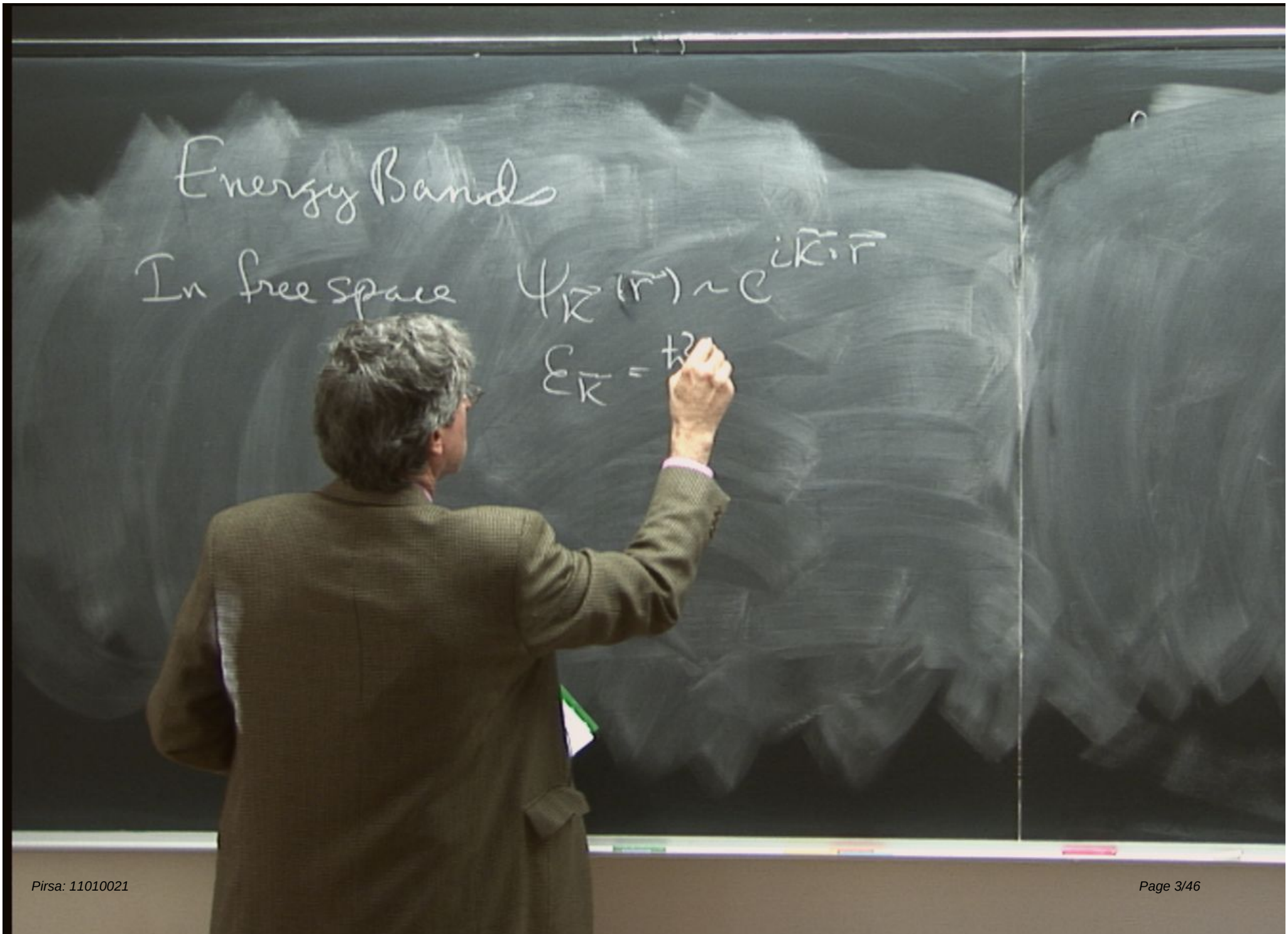
Title: Condensed Matter Review - Lecture 3

Date: Jan 05, 2011 10:15 AM

URL: http://pirsa.org/11010021

Abstract:

perimeter scholars international


Energy Bands
In free space $\psi_{\vec{k}}(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}}$
$\varepsilon_{\vec{k}} = \hbar^2$

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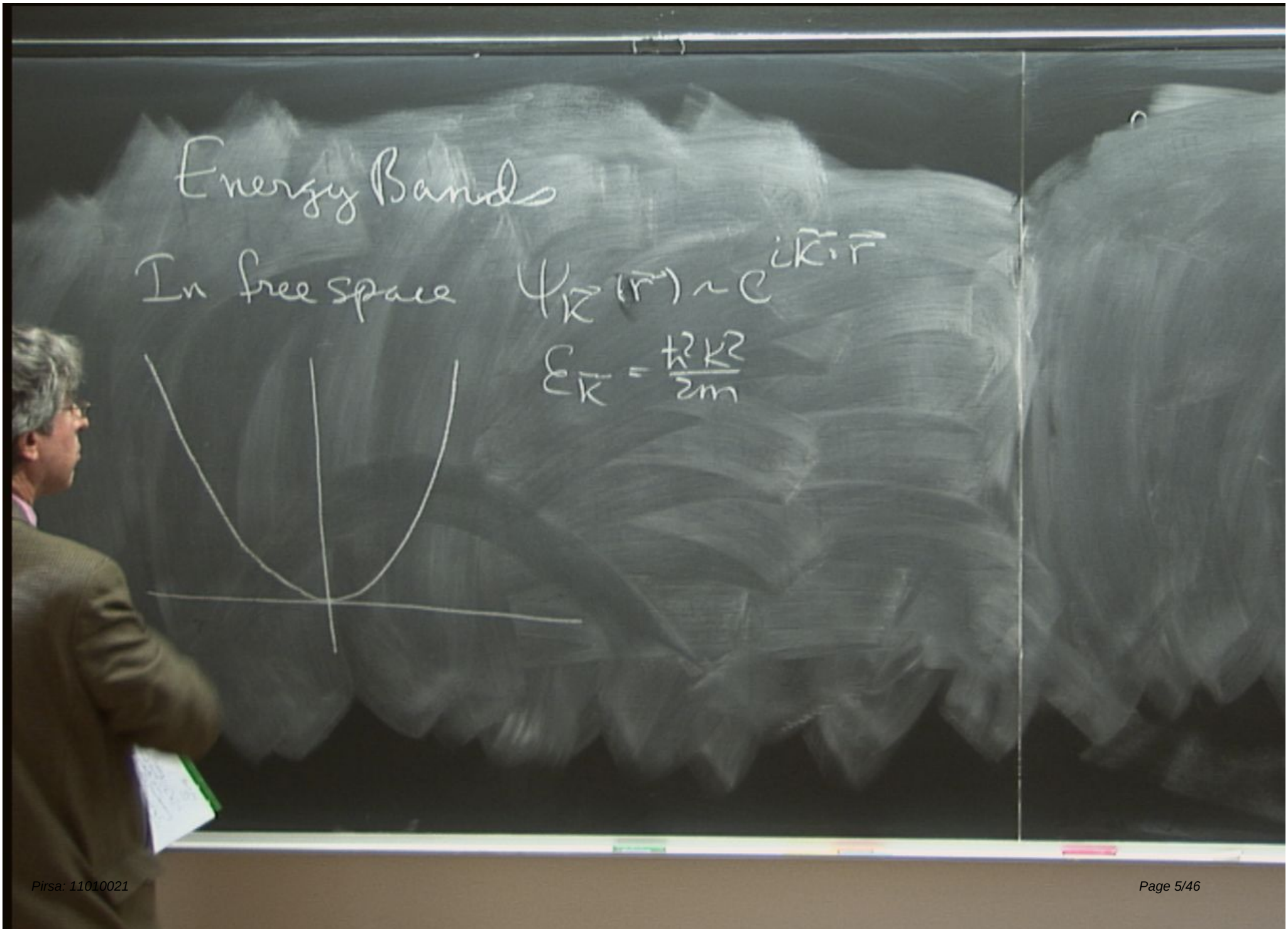
$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$$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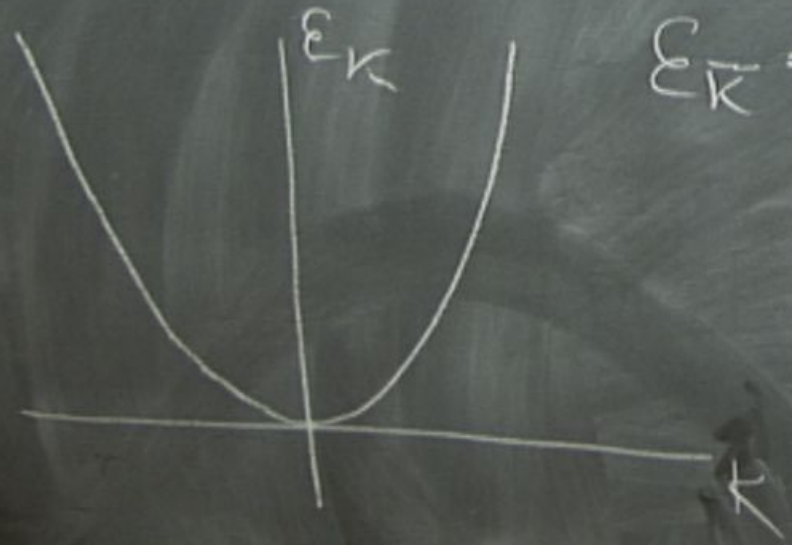

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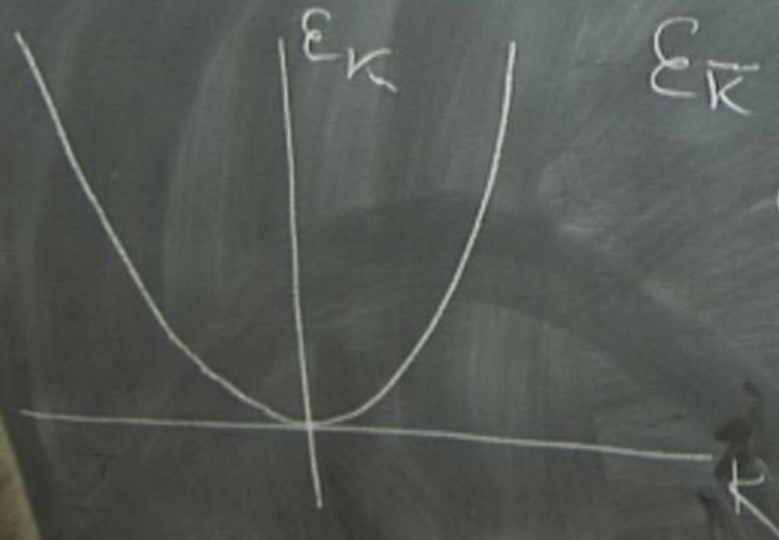

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$$

$$Q = \frac{2\pi}{a}$$

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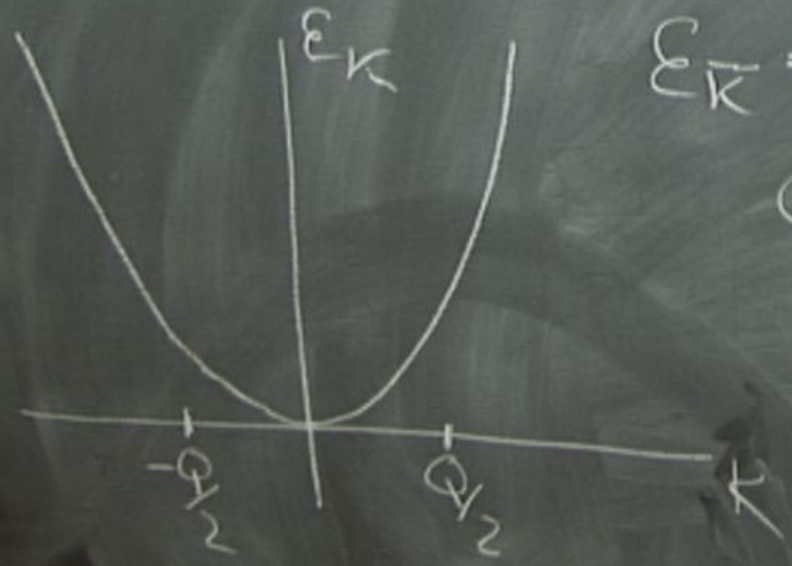

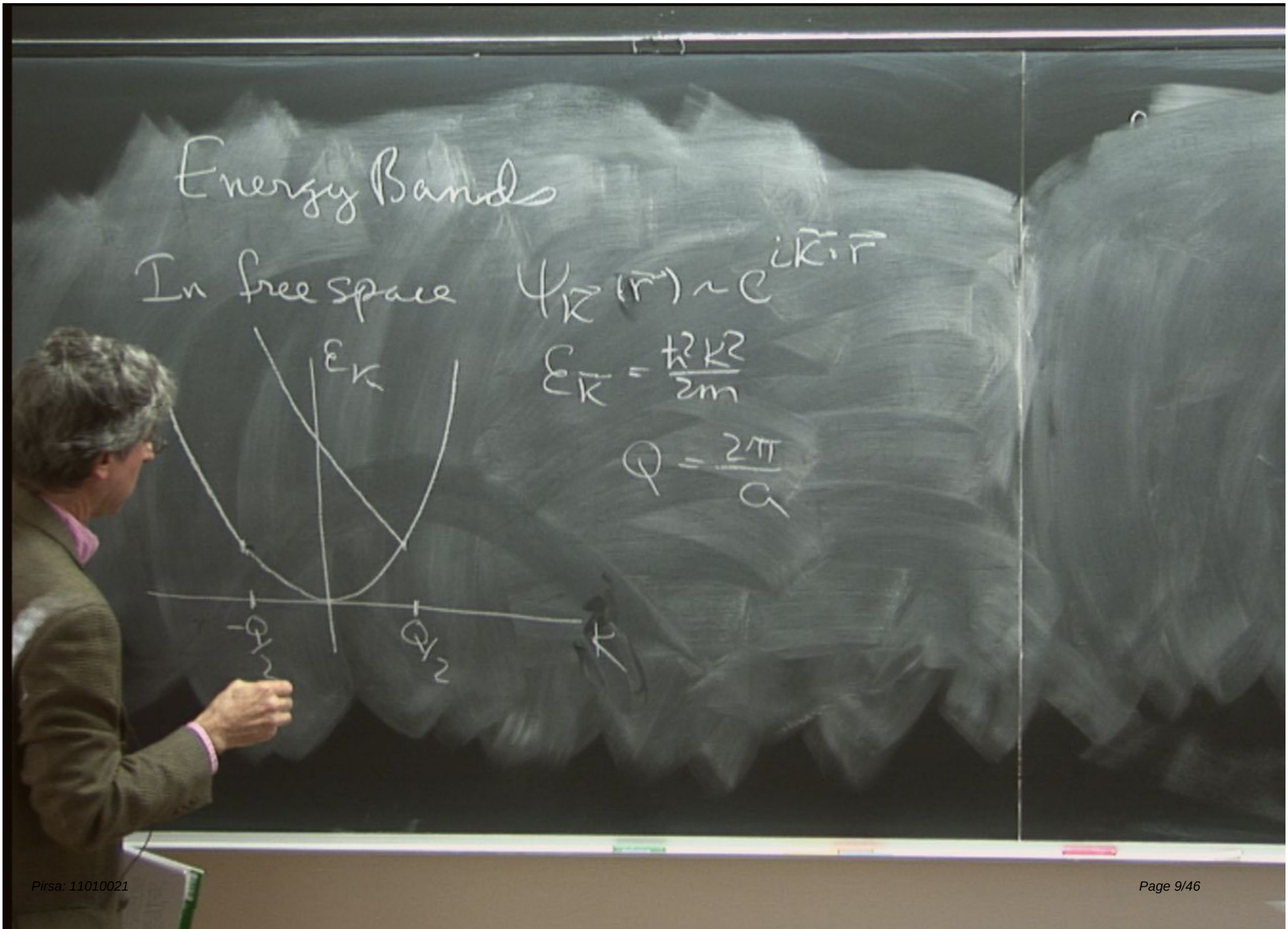
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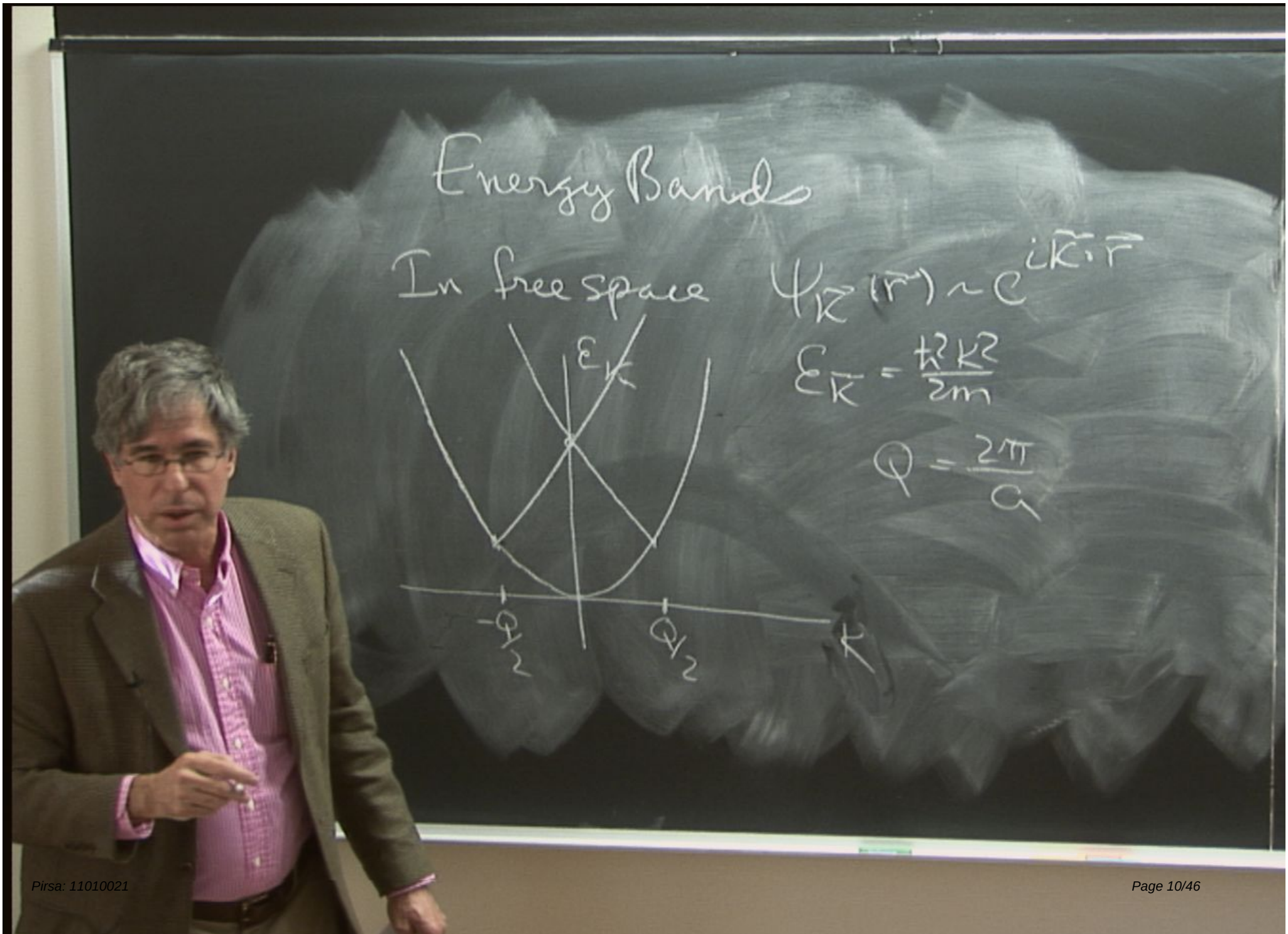
ε_k
$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$
$Q = \frac{2\pi}{a}$
$-Q/2$
$Q/2$
k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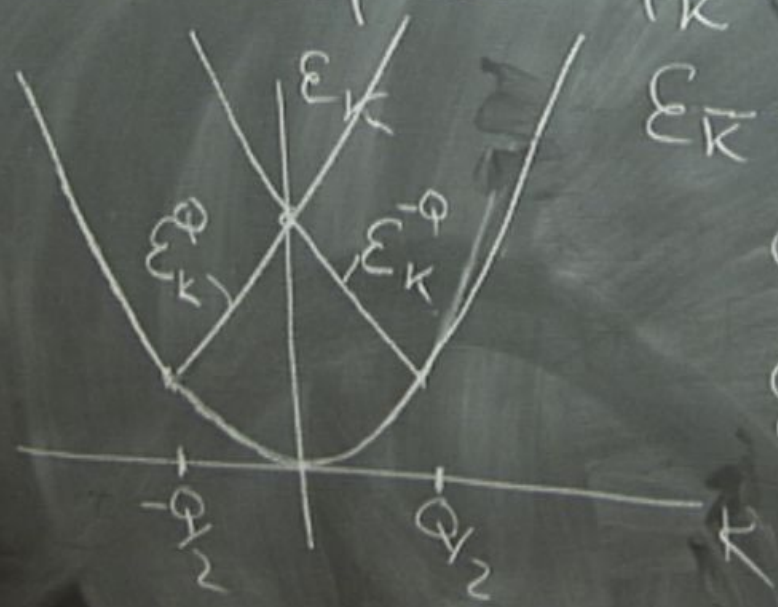

$$\varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m}$$

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$$\varepsilon_k^{Q} = \frac{\hbar^2 (Q+k)^2}{2m}$$

Effect of Periodic Potential

$$V(\vec{r}) = \sum_{\vec{Q}} V_{\vec{Q}} e^{i\vec{Q}\cdot\vec{r}}$$

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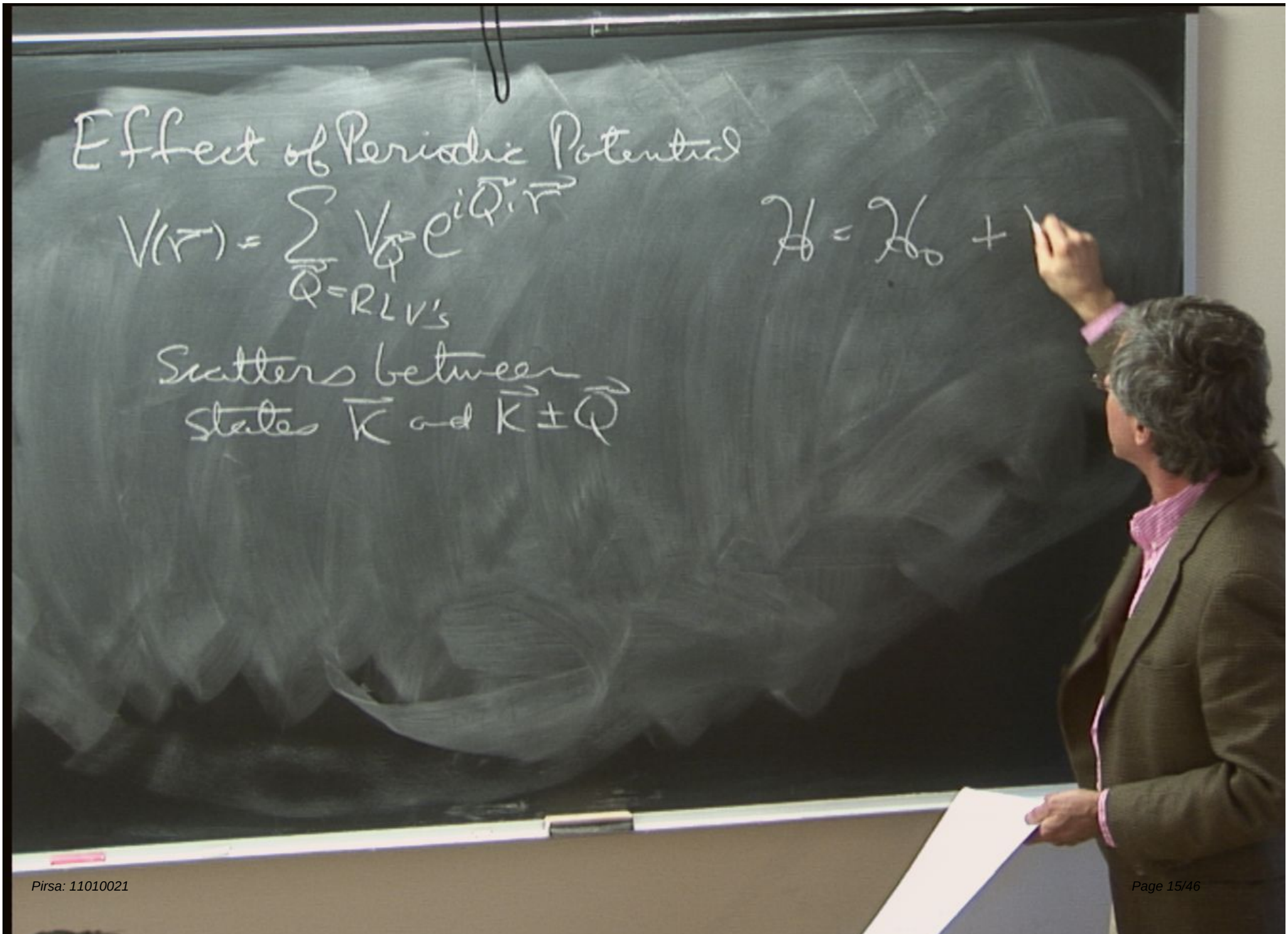
Scatters between states $\vec{K}$ and $\vec{K} \pm \vec{Q}$


Effect of Periodic Potential
$V(\vec{r}) = \sum_{\vec{Q}=RLV's} V_{\vec{Q}} e^{i\vec{Q}\cdot\vec{r}}$
$\mathcal{H} = \mathcal{H}_0 +$
Scatters between
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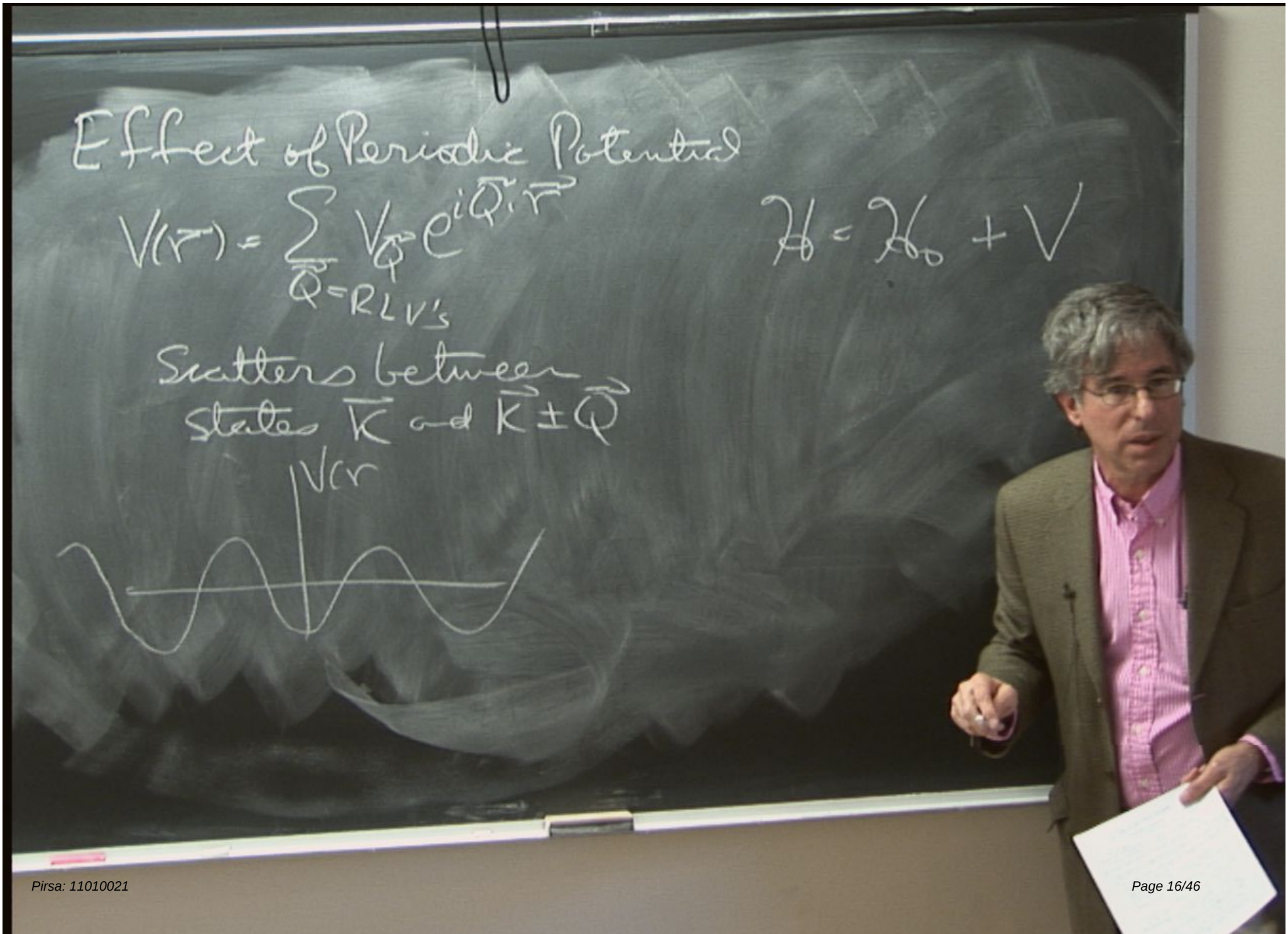
Effect of Periodic Potential
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Scatters between
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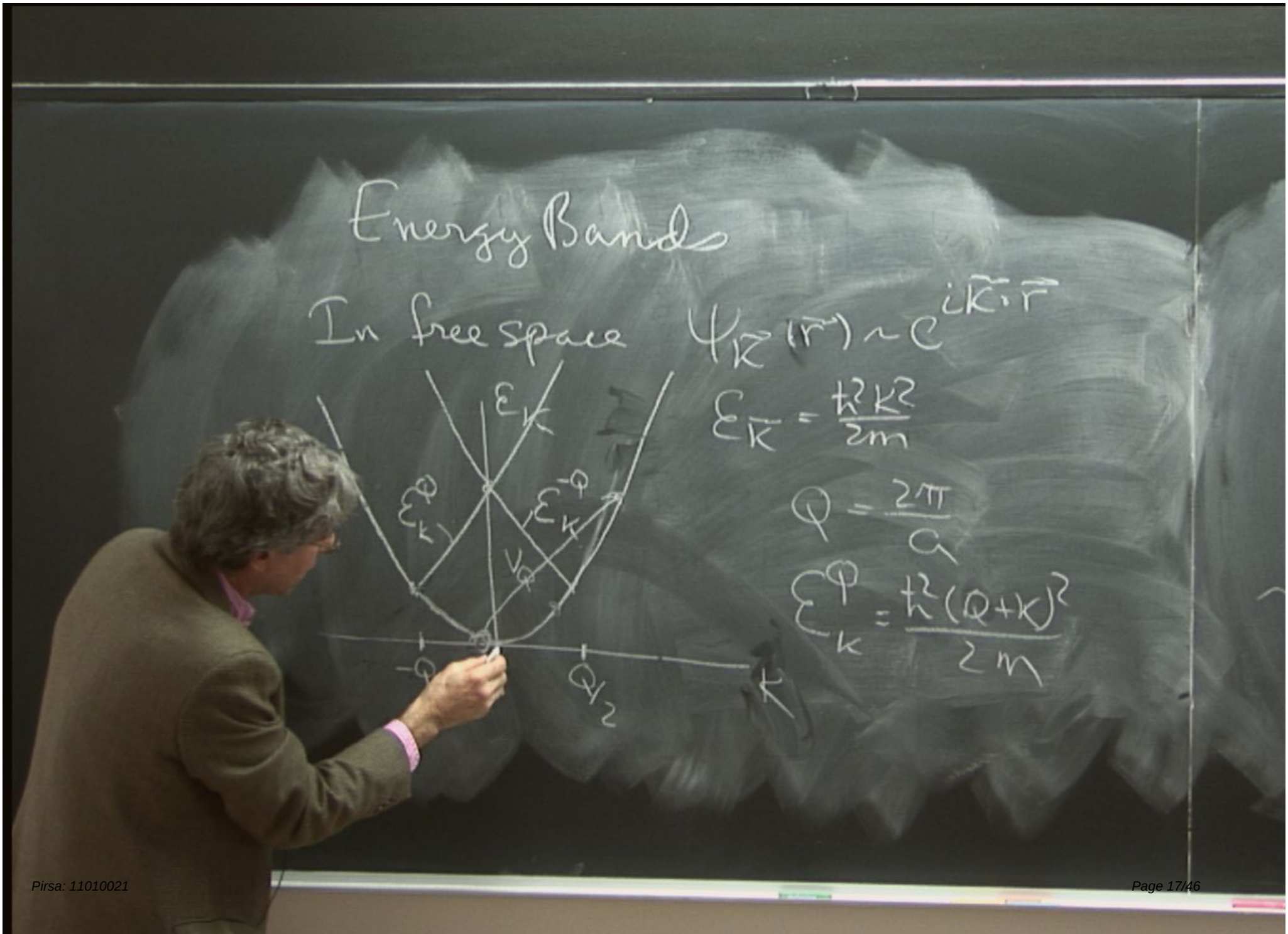
V(r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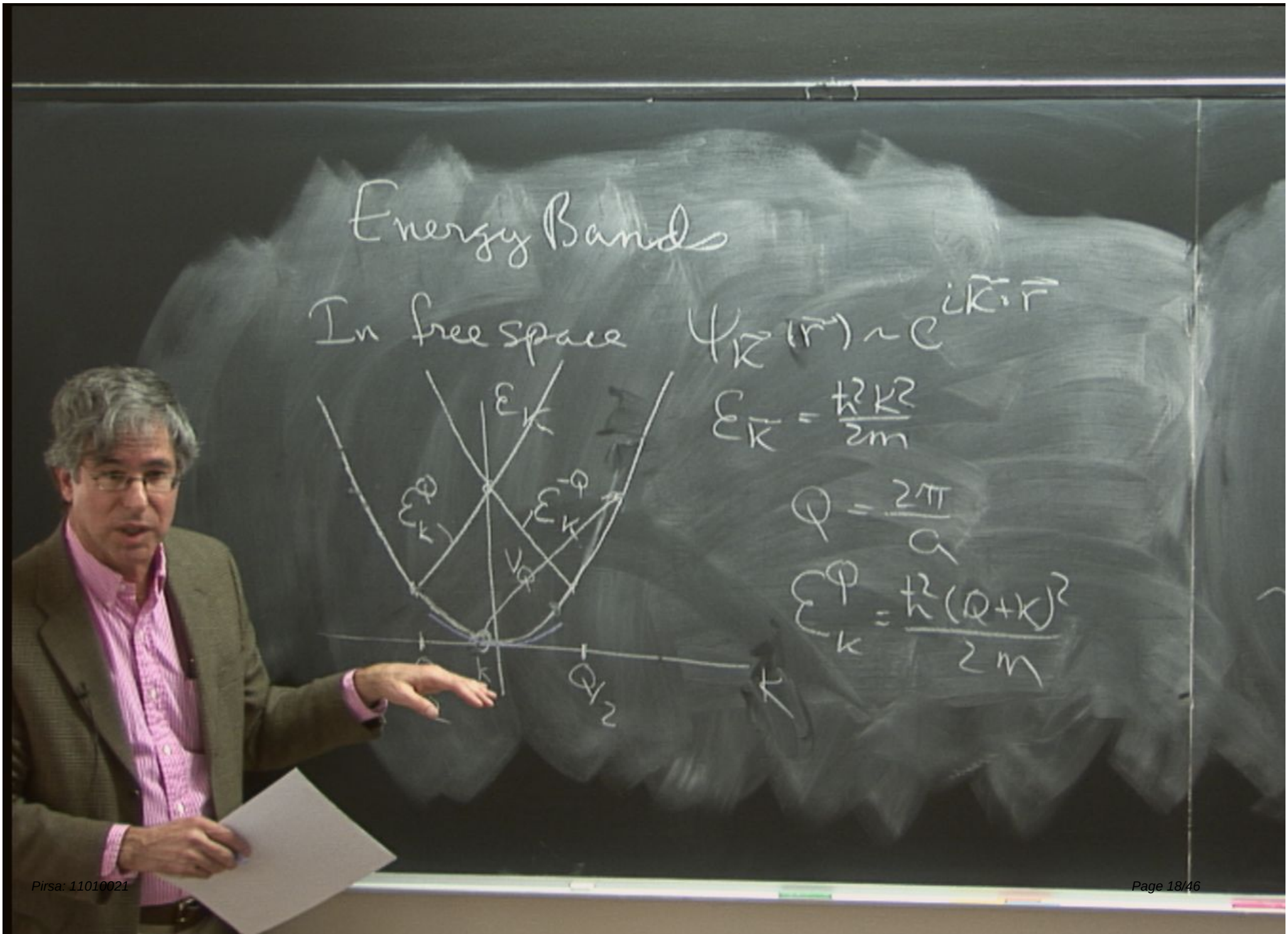
ε_k
ε_k^{Q}
ε_k^{-Q}
V_Q
$-Q$
$Q/2$
k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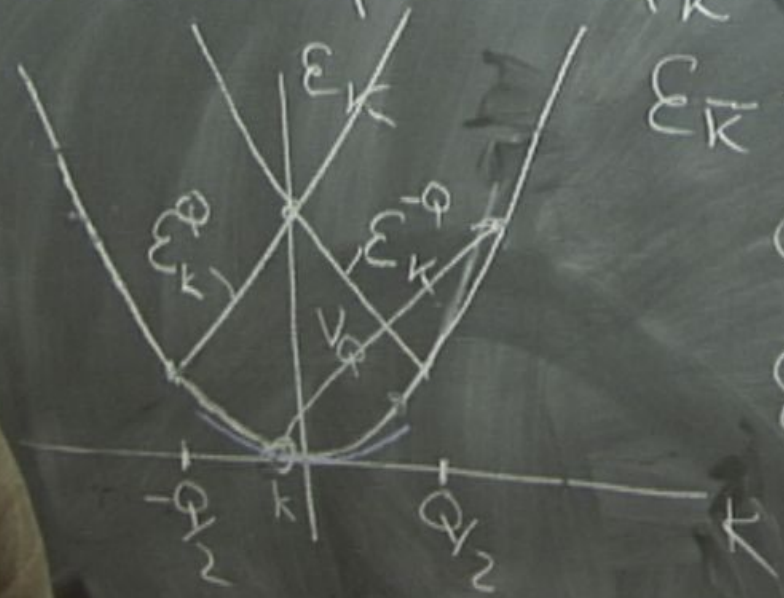

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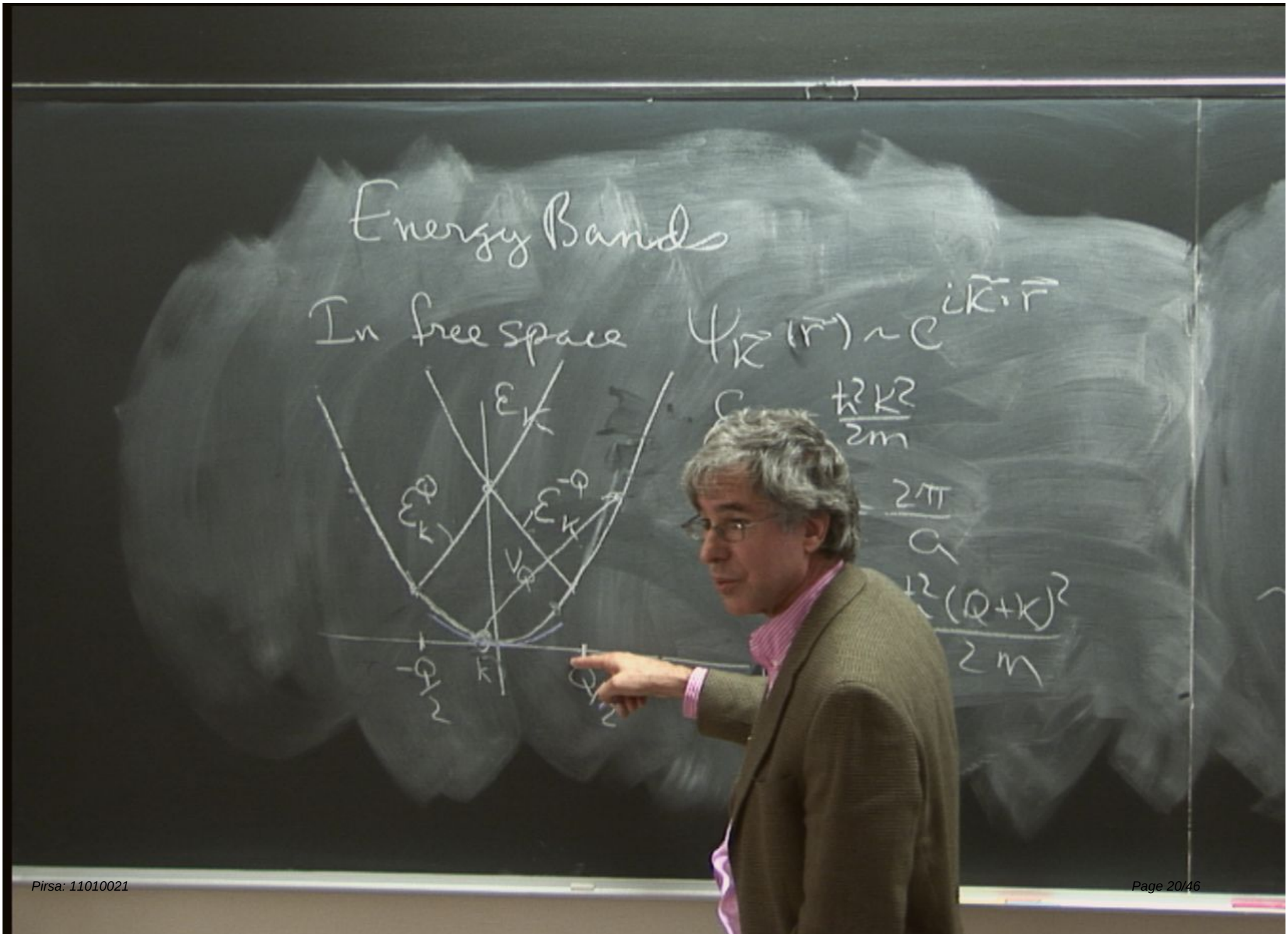
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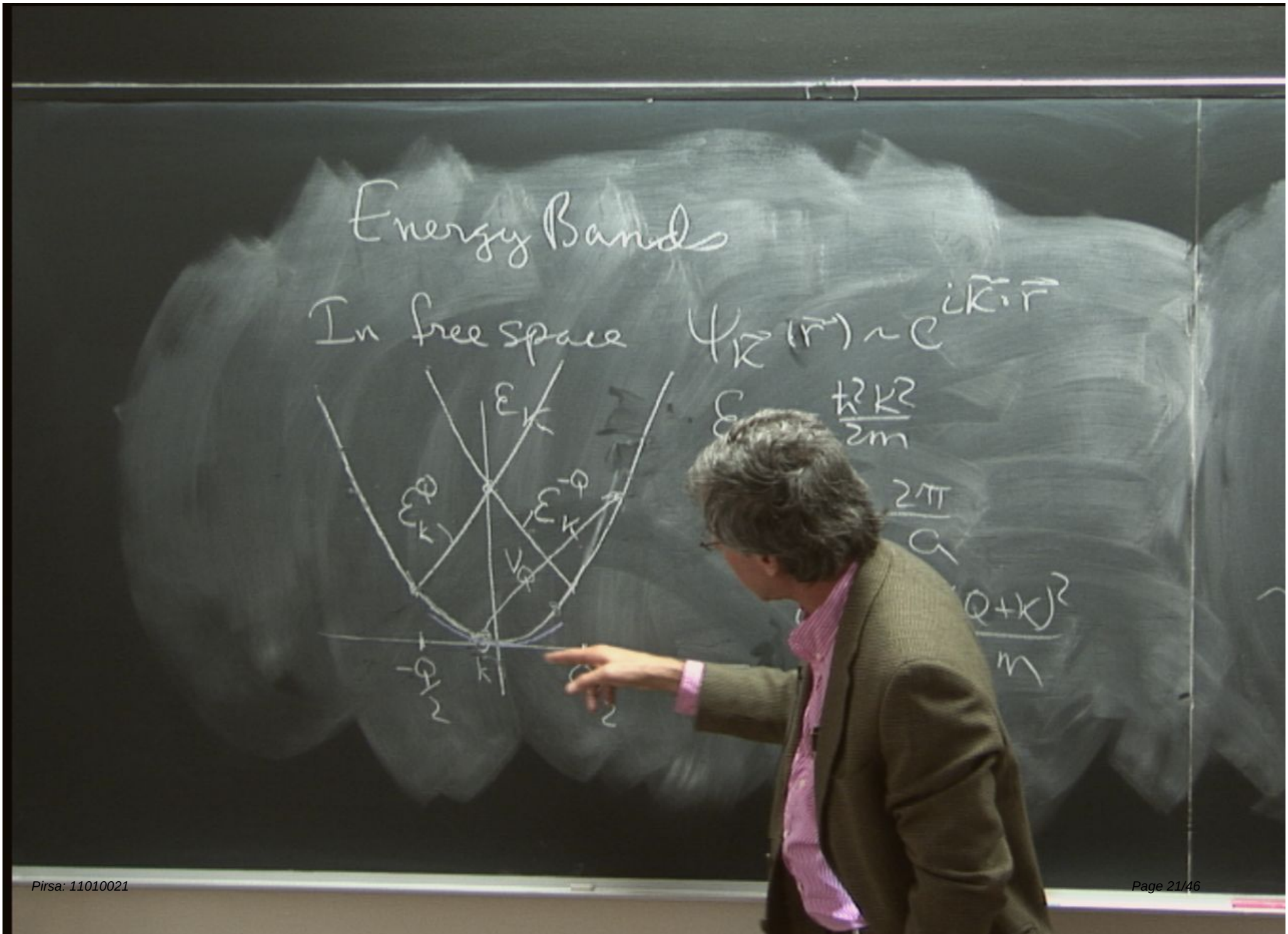
ε_k
ε_k^{Q}
ε_k^{-Q}
V_Q
$-Q/2$
k
$Q/2$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ε_k
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ε_k^Q
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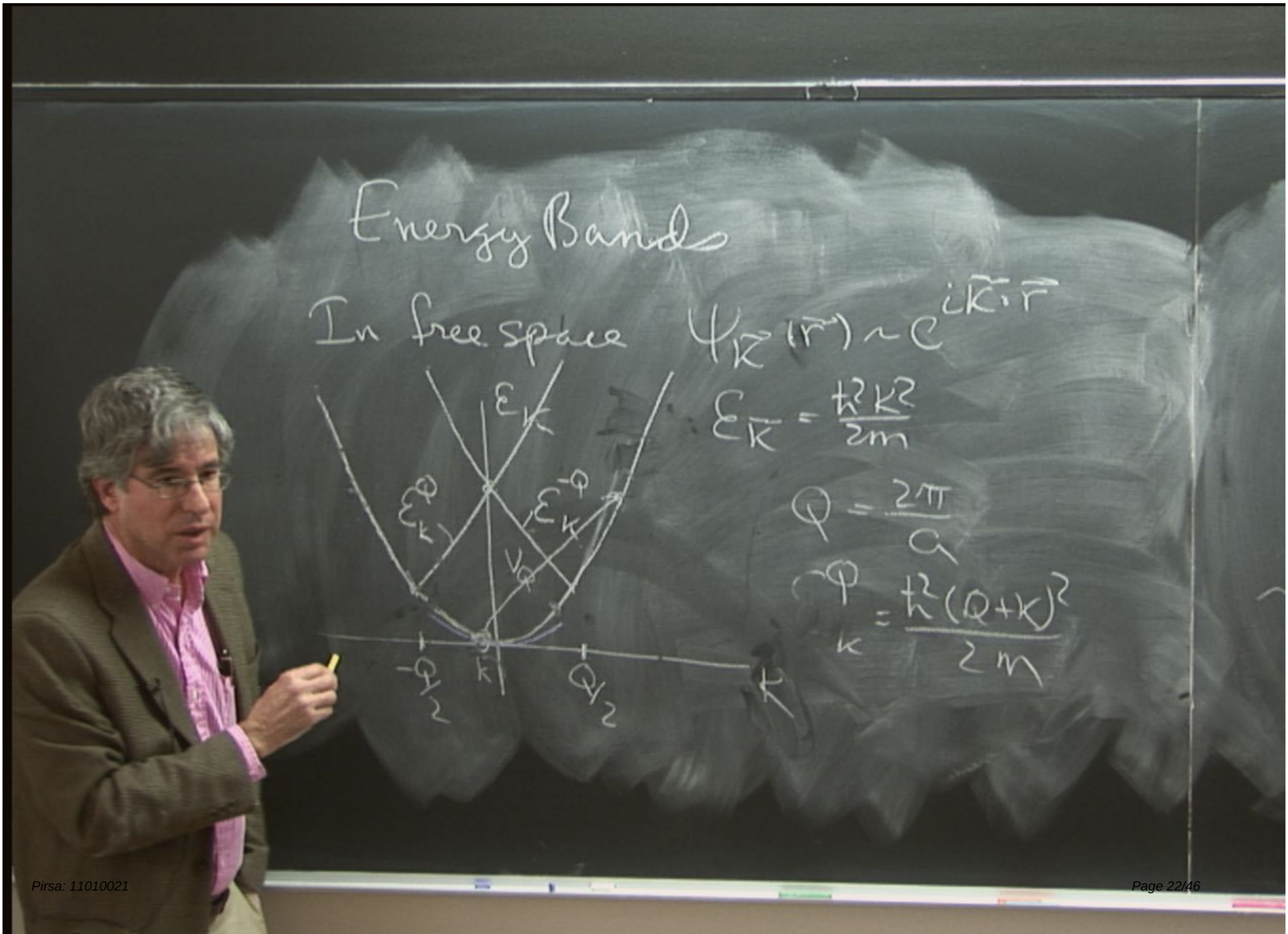
$\frac{2\pi}{a}$
V_Q
$-Q/2$
k
$Q/2$
$(Q+k)^2$
m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ε_k
ε_k^{Q}
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V_Q
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k
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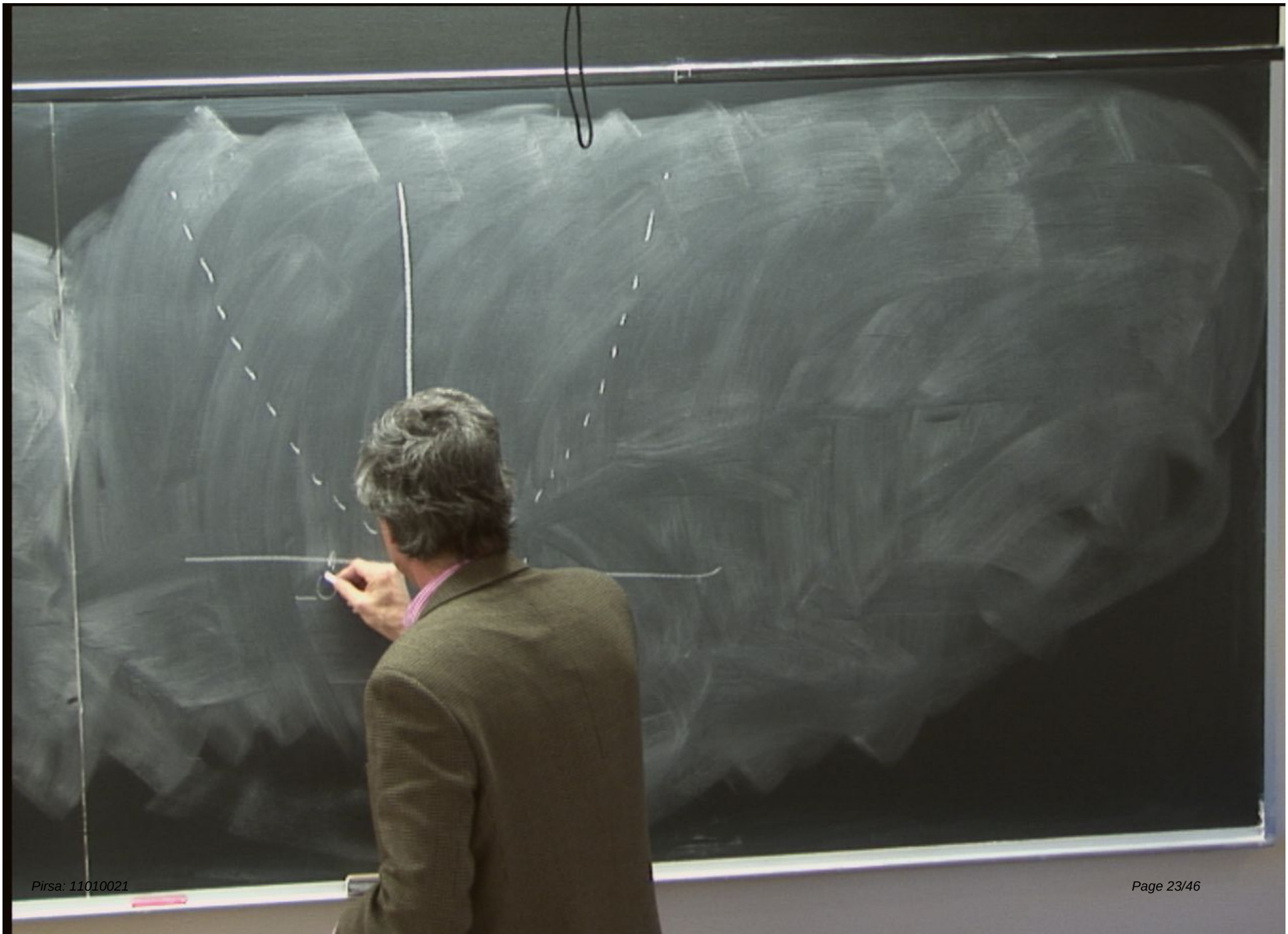
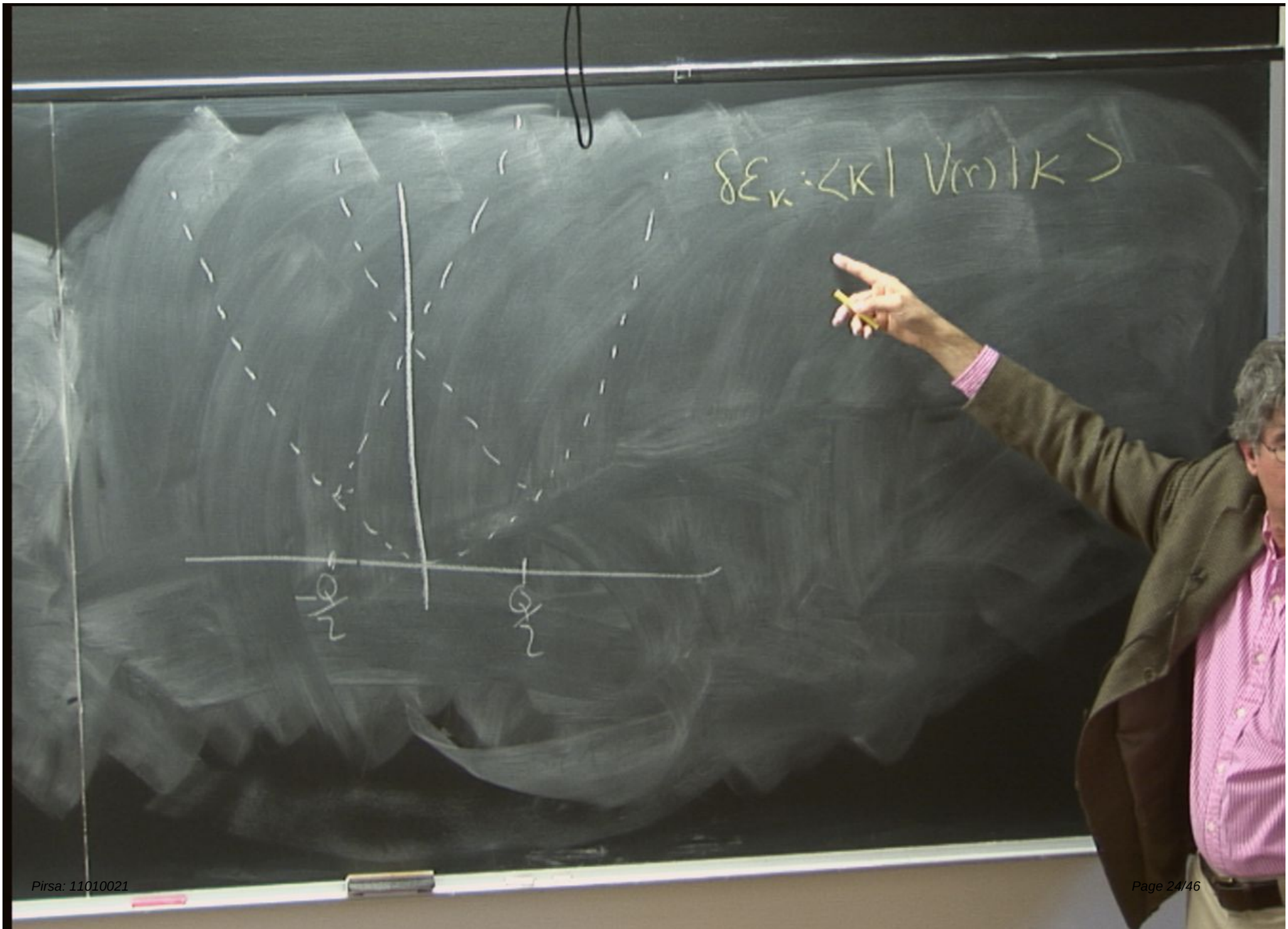
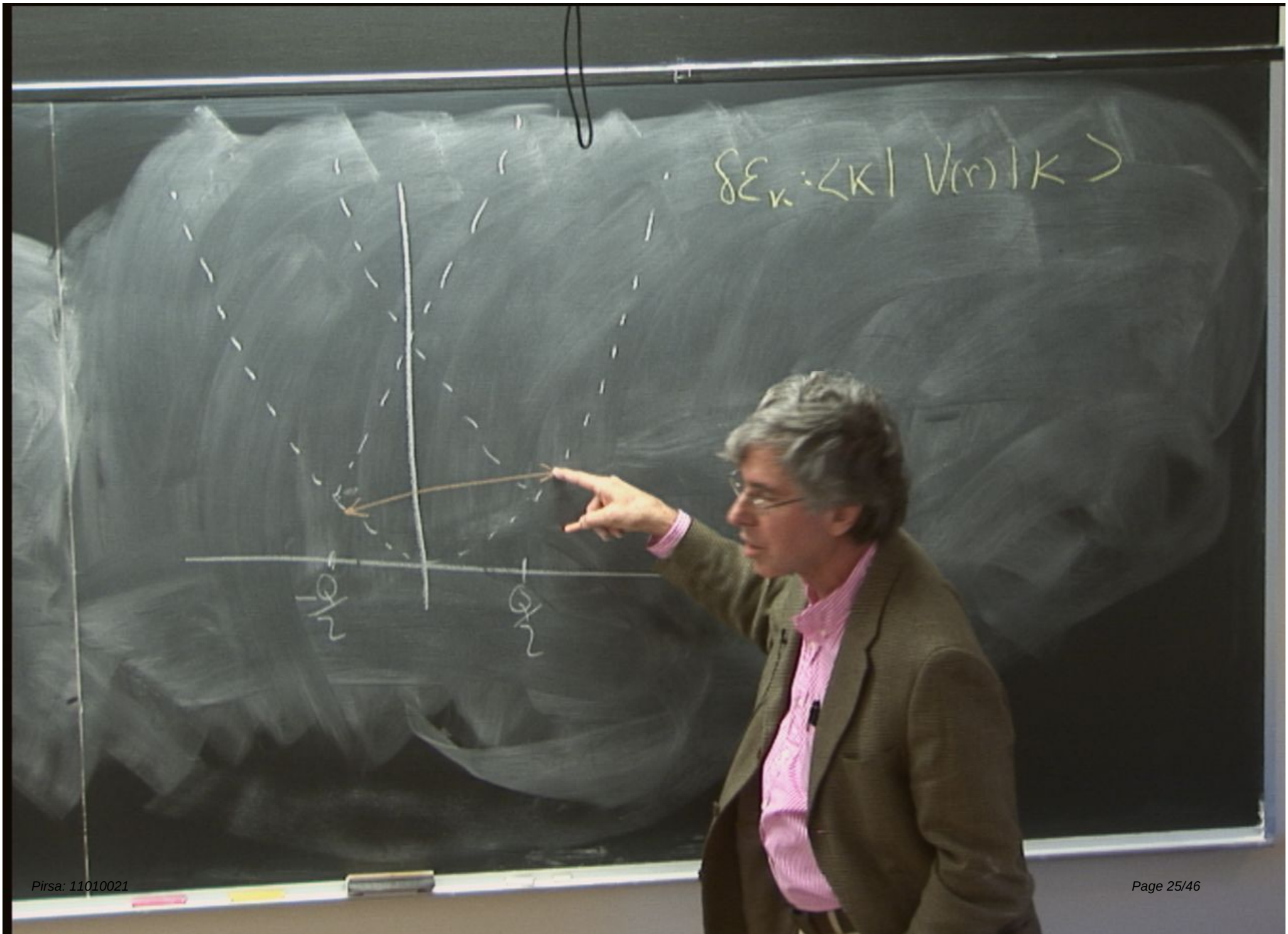
k




$\delta\mathcal{E}_k = \langle k | V(r) | k \rangle$
$-\frac{Q}{2}$
$\frac{Q}{2}$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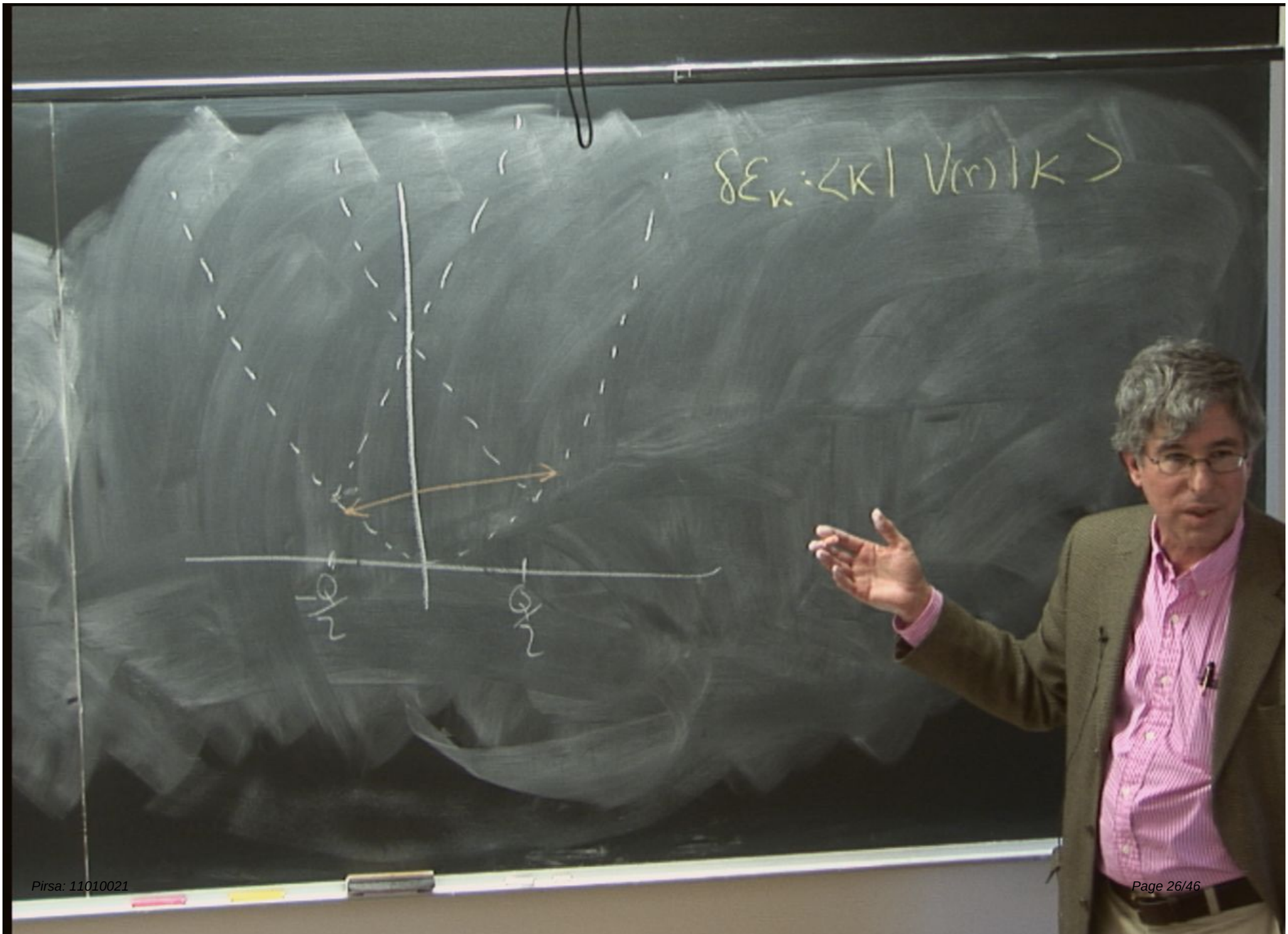
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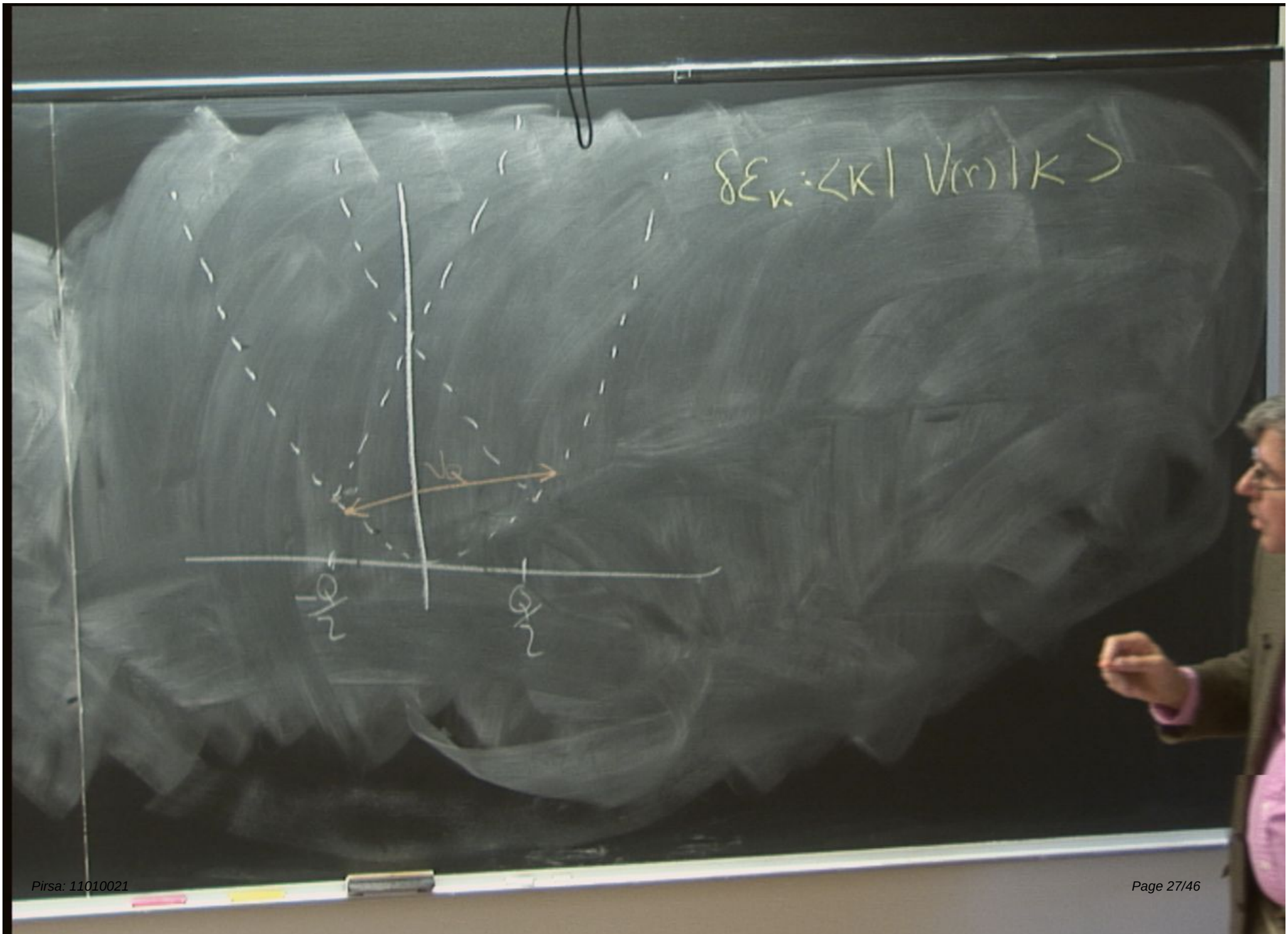
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λ_α
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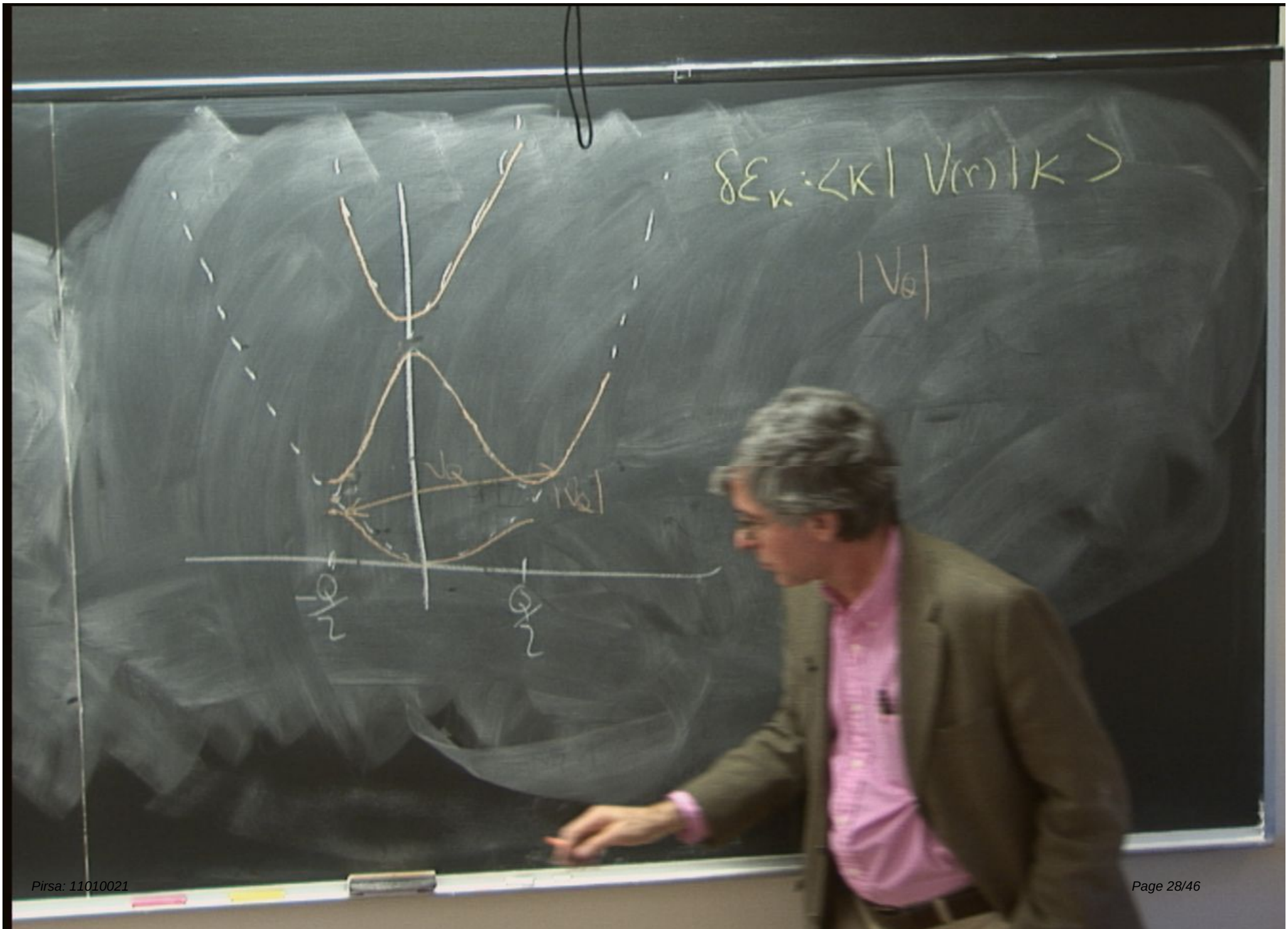
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$|V_Q|$
V_Q
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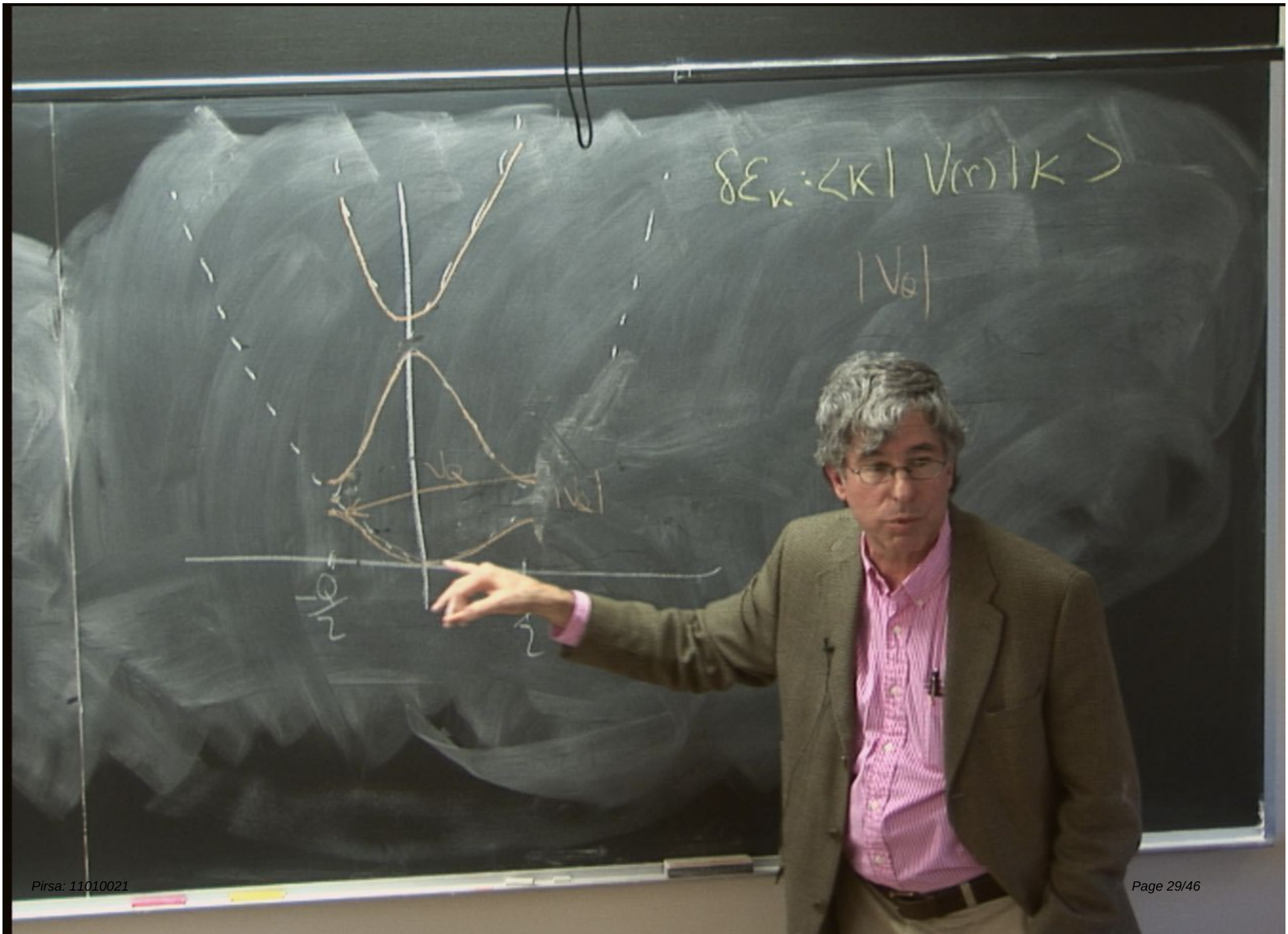
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$|V_Q|$
V_Q
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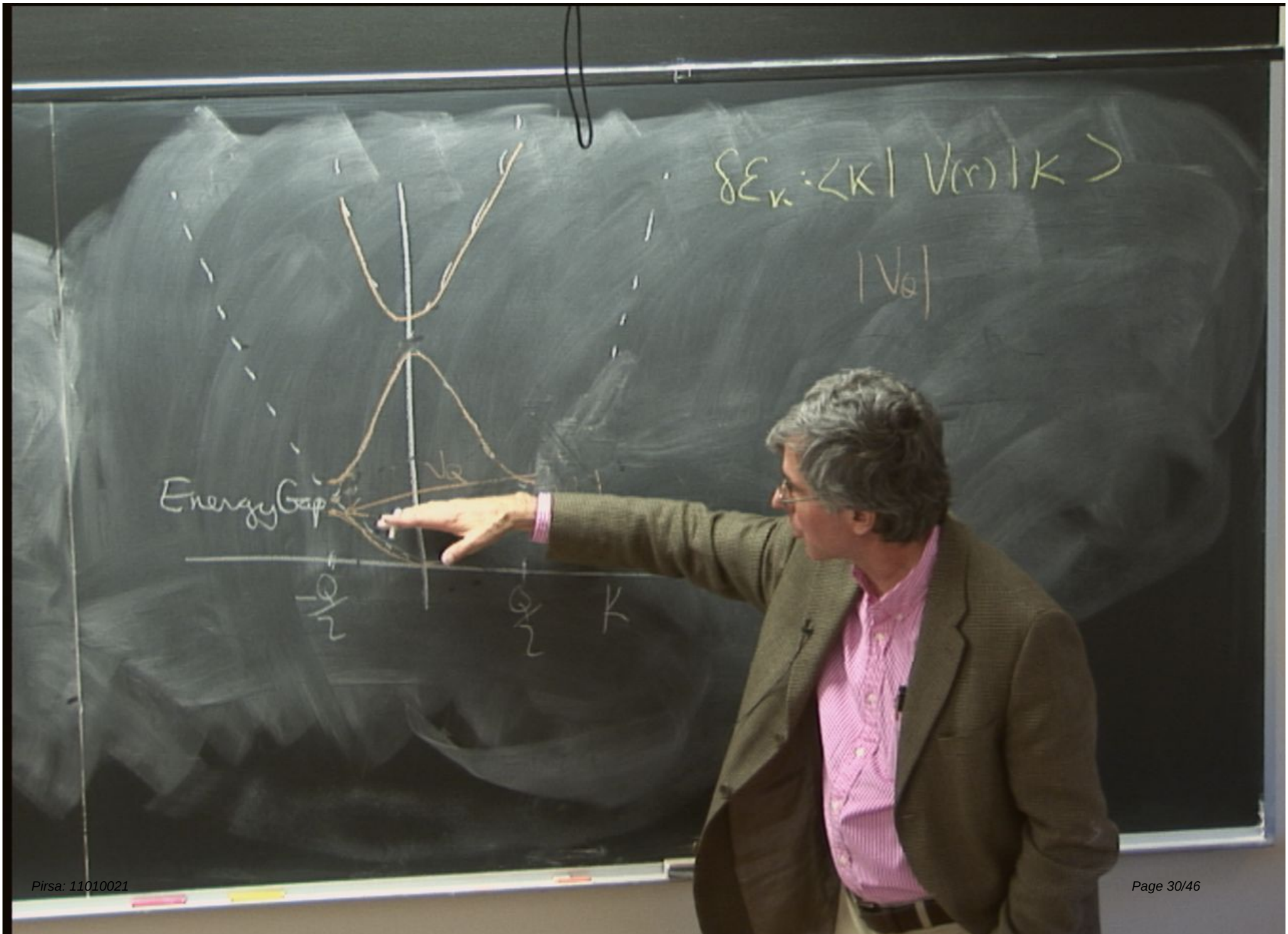
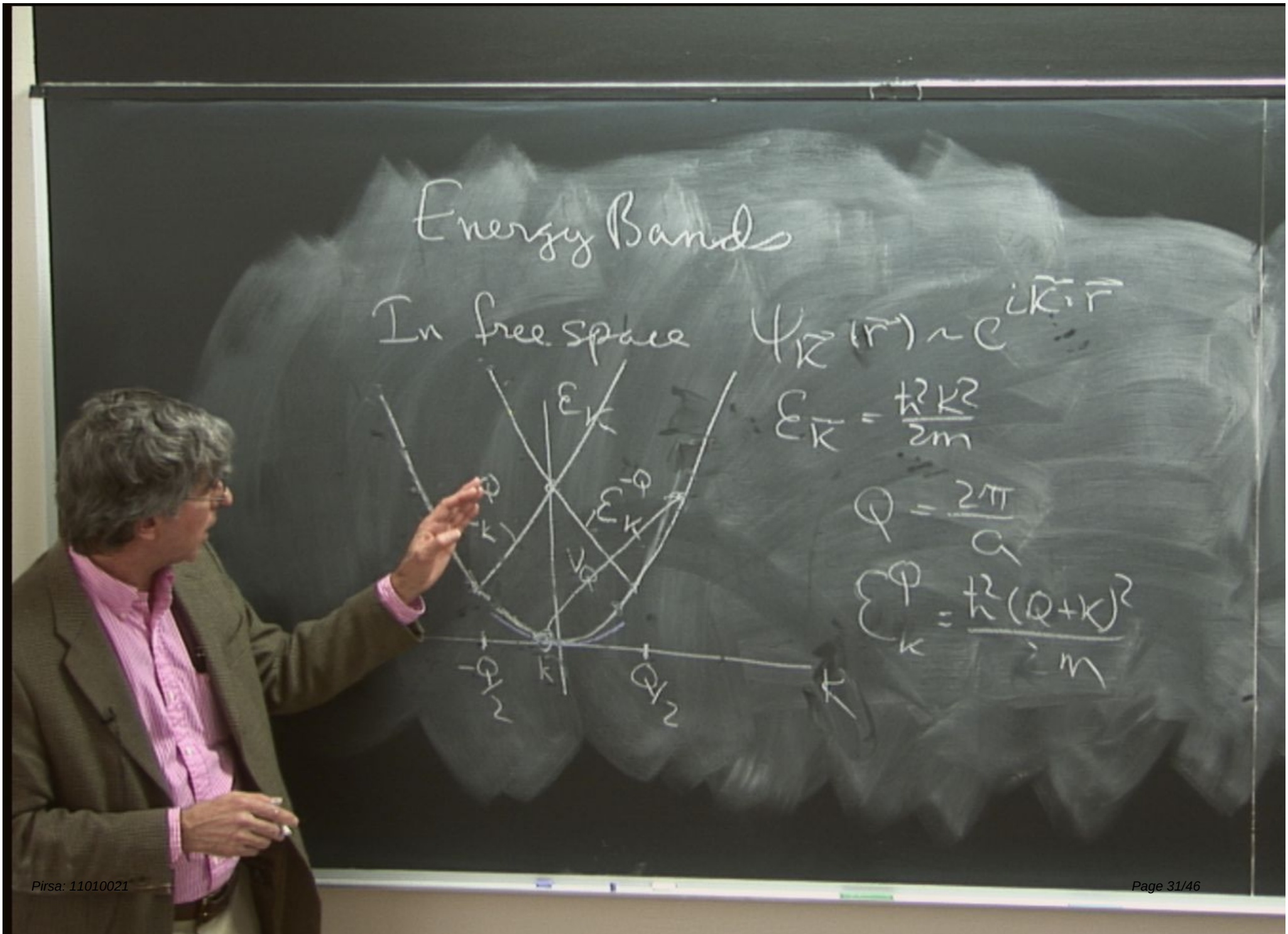
$\frac{Q}{2}$


$\delta\mathcal{E}_k = \langle k | V(r) | k \rangle$
$|V_Q|$
Energy Gap
V_Q
$-\frac{Q}{2}$
$\frac{Q}{2}$
k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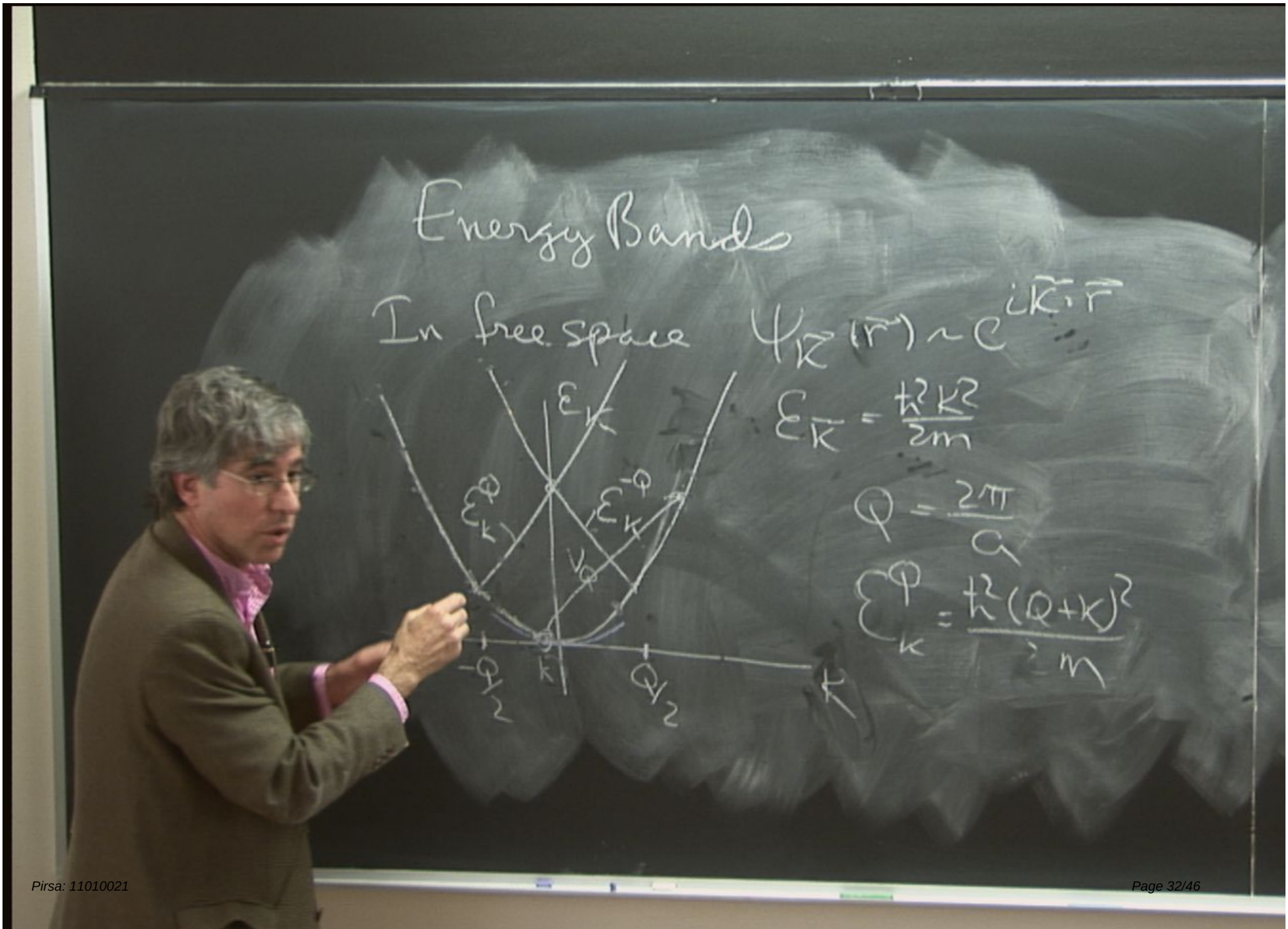
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ε_k^{-Q}
V_Q
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k
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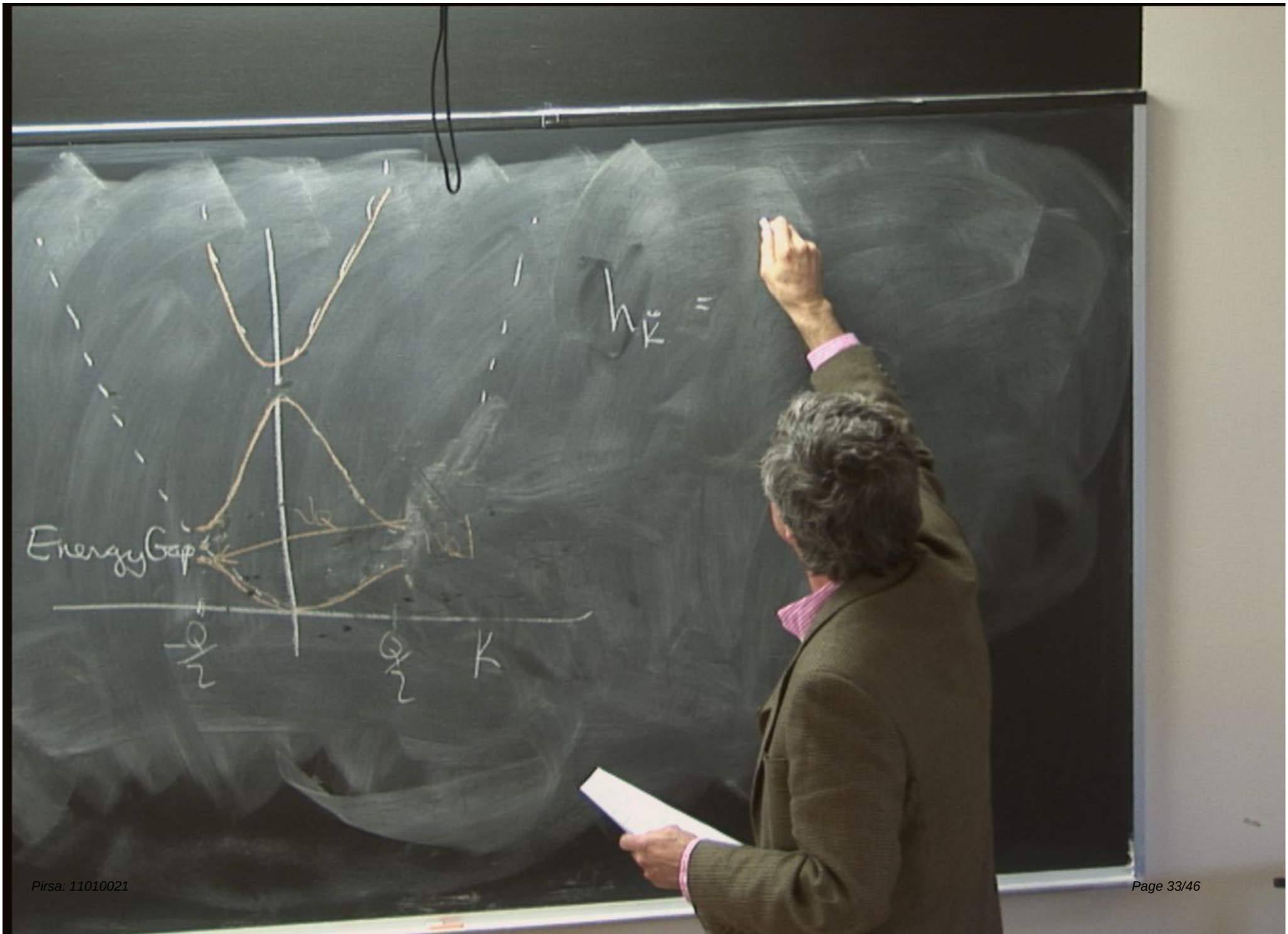
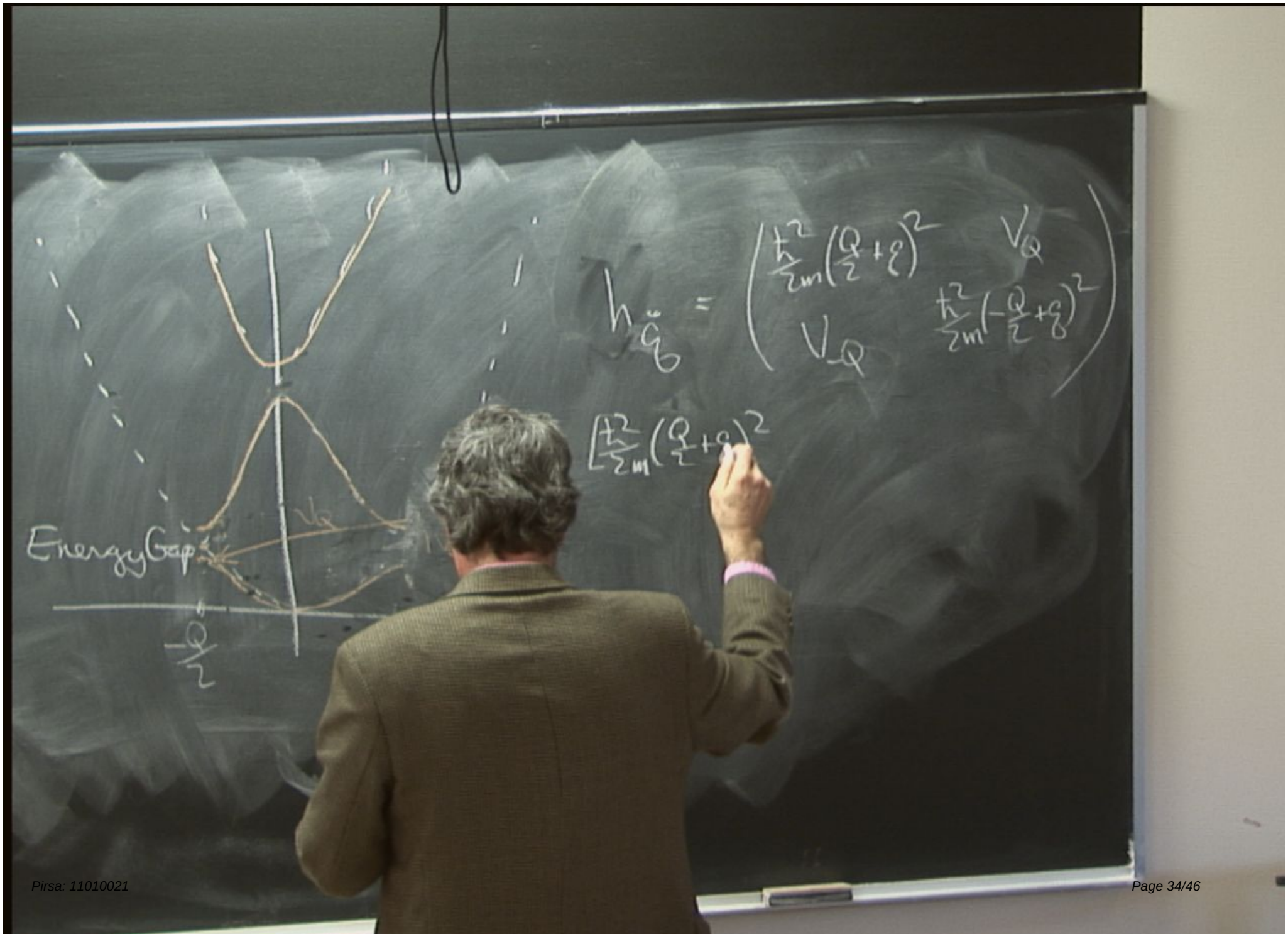
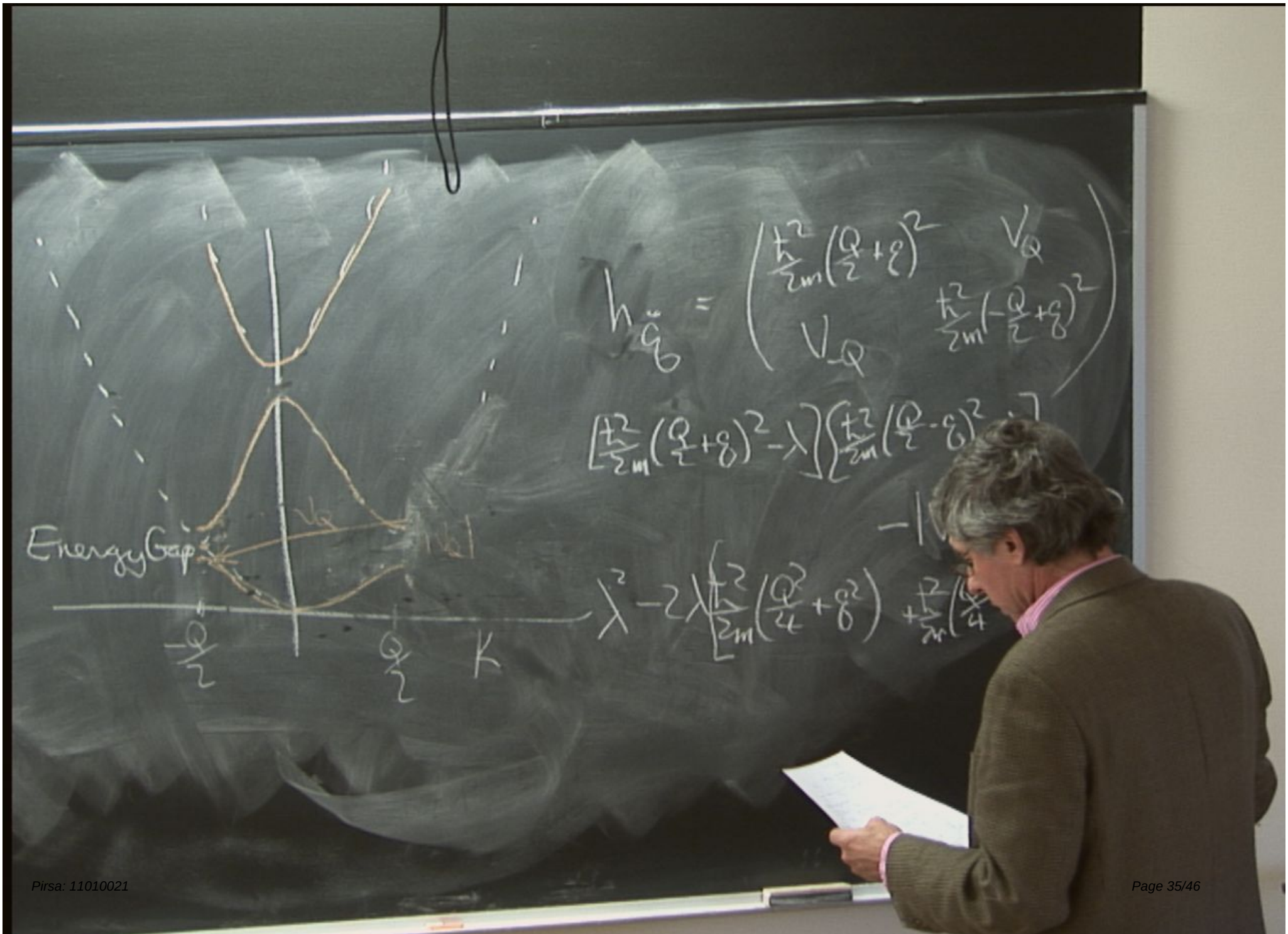
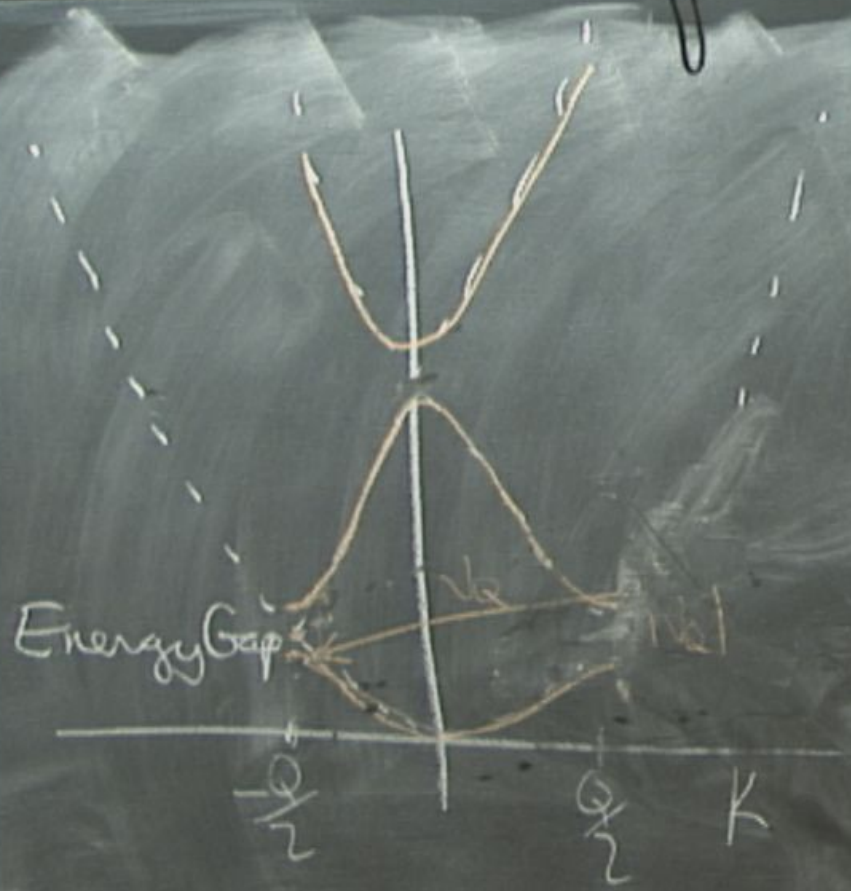
k


$h_{\vec{k}} =$
Energy Gap
$-\frac{Q}{2}$
$\frac{Q}{2}$
K


Energy Gap
V_Q
$-\frac{Q}{2}$
$h_{\vec{q}} = \begin{pmatrix} \frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+q\right)^2 \end{pmatrix}$
$\left[\frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2\right.$

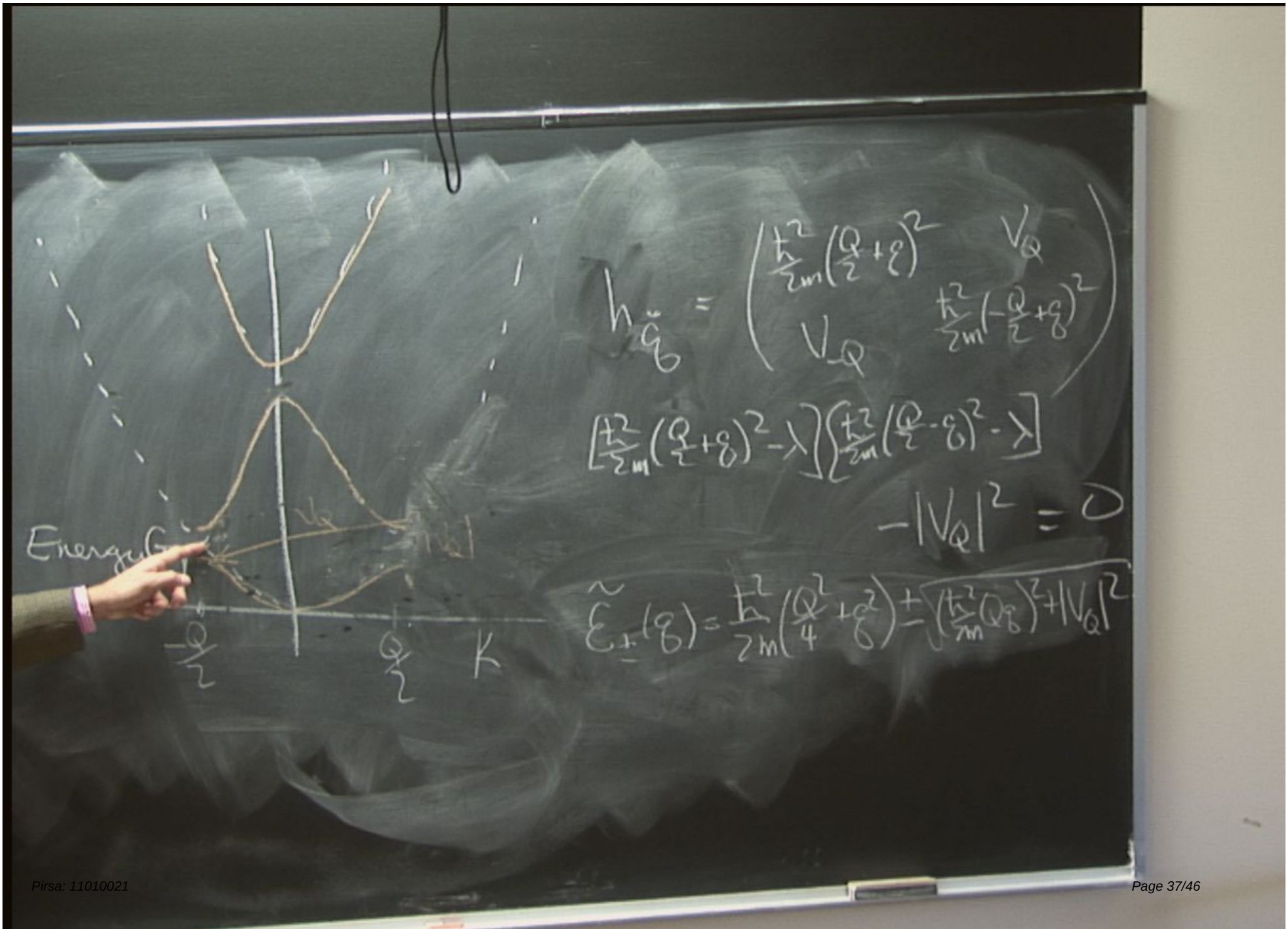

Energy Gap
$-\frac{Q}{2}$
$\frac{Q}{2}$
K
$h_{\vec{q}} = \begin{pmatrix} \frac{\hbar^2}{2m}(\frac{Q}{2}+q)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}(-\frac{Q}{2}+q)^2 \end{pmatrix}$
$\left[\frac{\hbar^2}{2m}(\frac{Q}{2}+q)^2-\lambda\right]\left[\frac{\hbar^2}{2m}(\frac{Q}{2}-q)^2\right.$
$\lambda^2-2\lambda\left[\frac{\hbar^2}{2m}(\frac{Q^2}{4}+q^2)\right.$

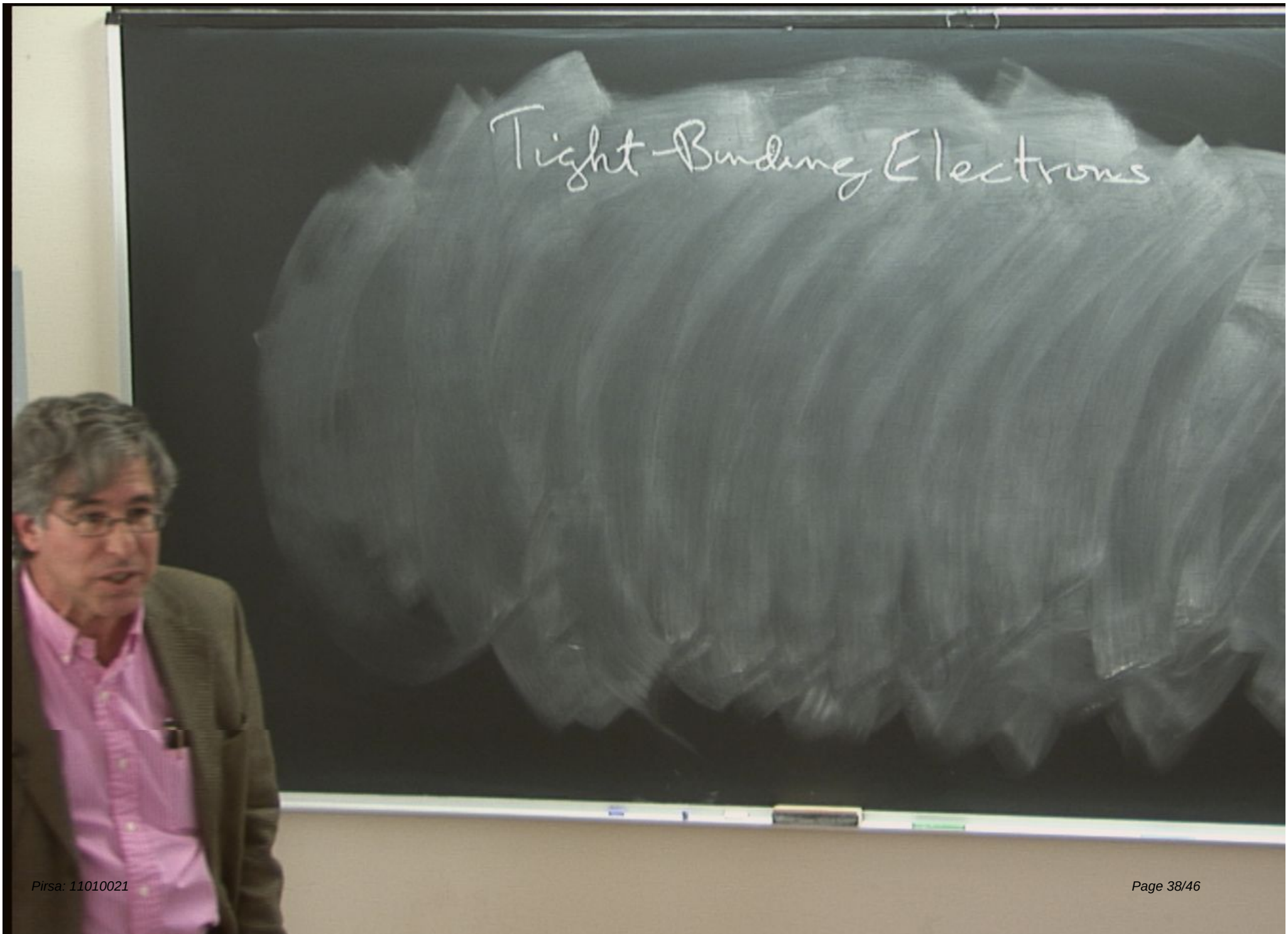


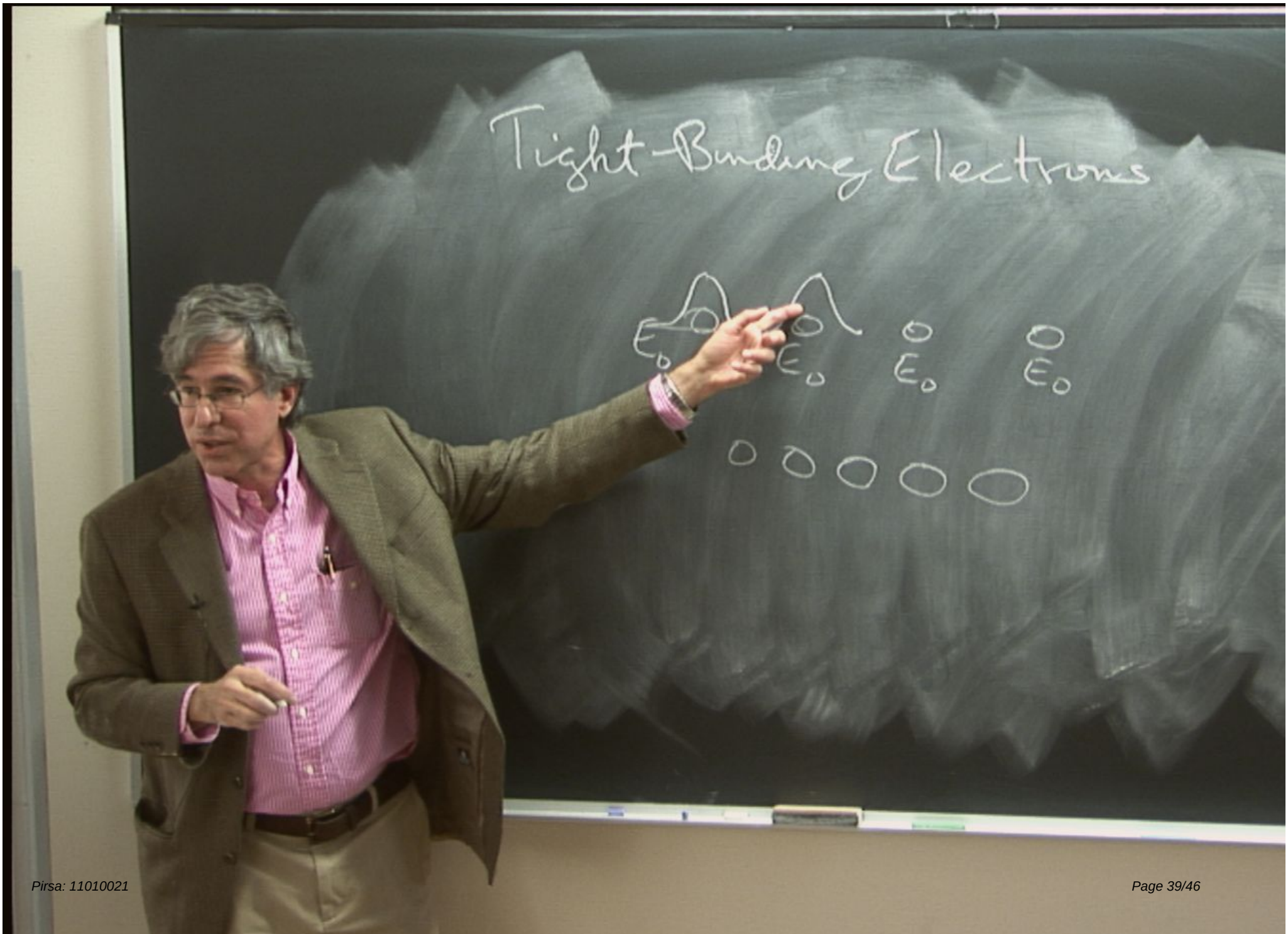
$$h_{\vec{q}} = \begin{pmatrix} \frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+q\right)^2 \end{pmatrix}$$

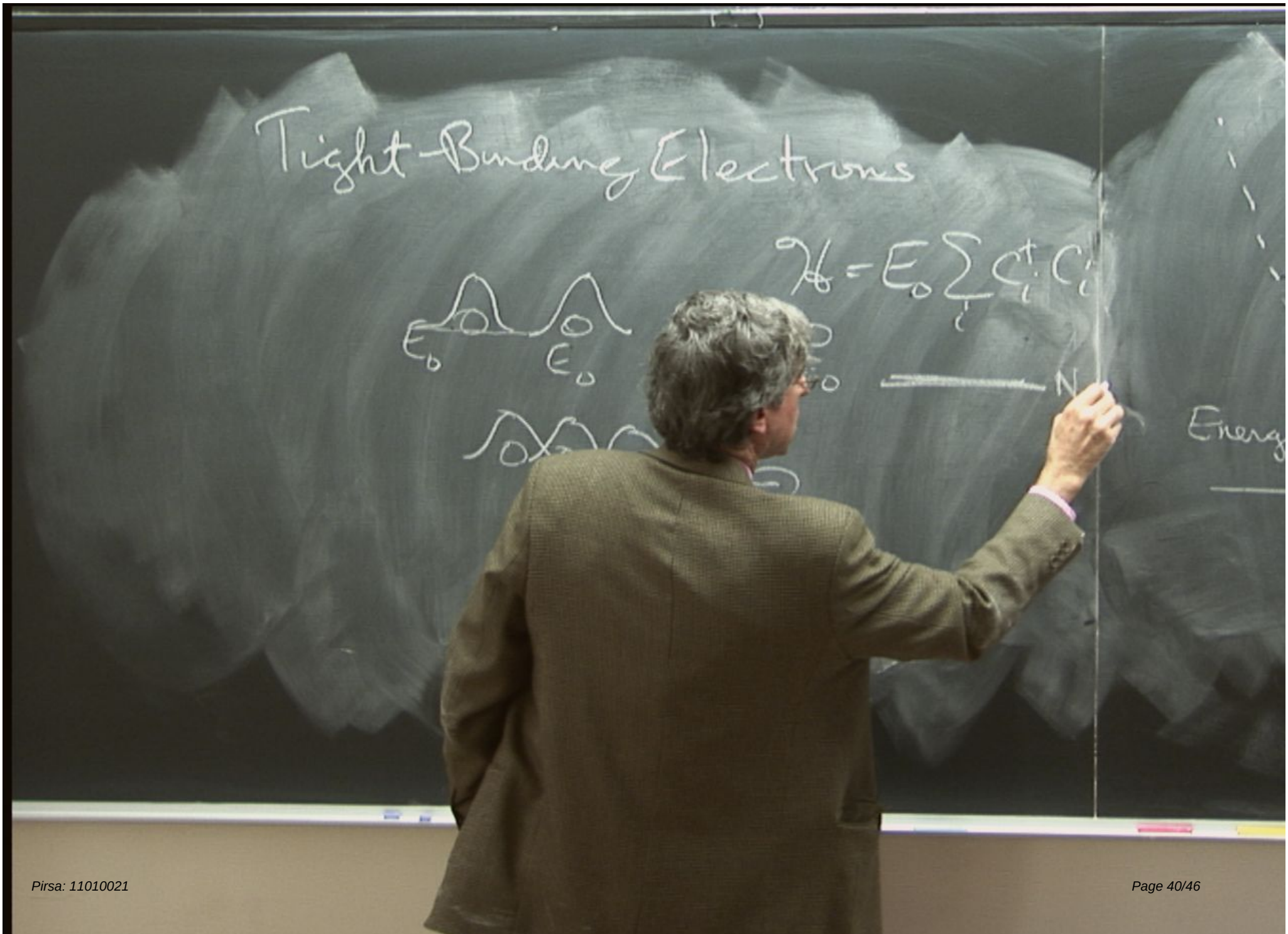
$$\left[\frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2-\lambda\right]\left[\frac{\hbar^2}{2m}\left(\frac{Q}{2}-q\right)^2-\lambda\right] - |V_Q|^2 = 0$$

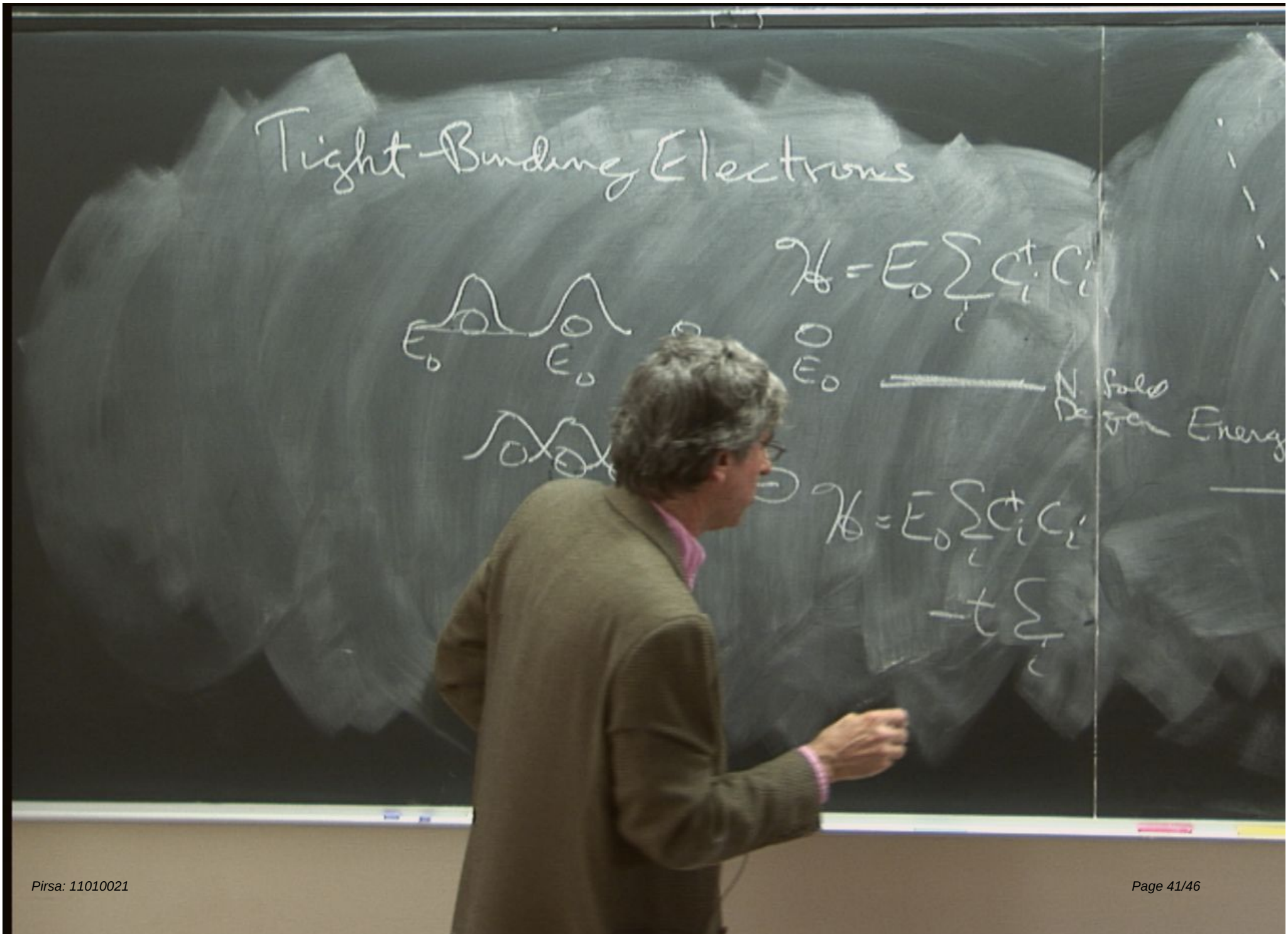
$$\tilde{\varepsilon}_{\pm}(q) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+q^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}Qq\right)^2+|V_Q|^2}$$

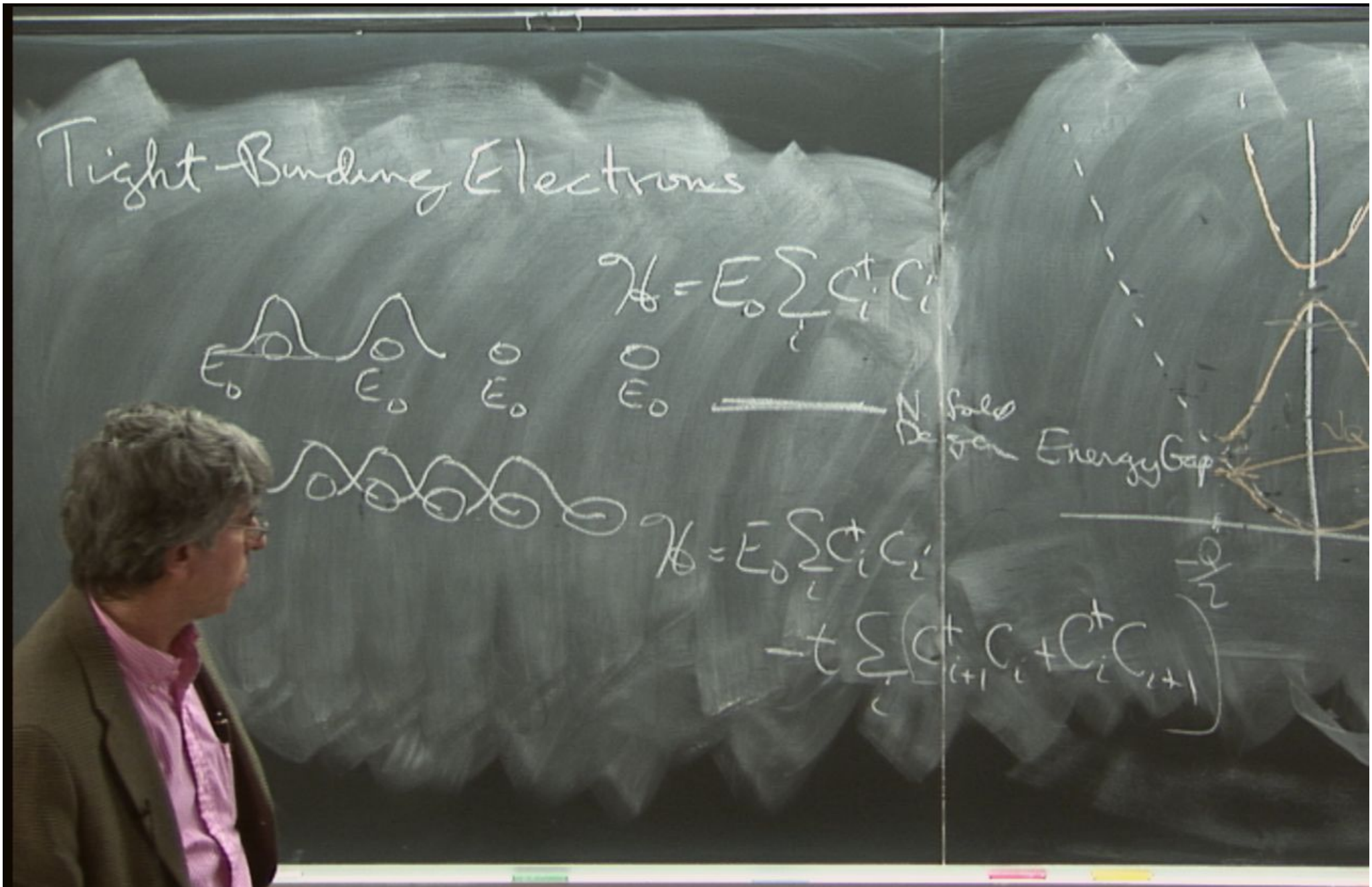

Energy
V_Q
$|V_Q|$
$-\frac{Q}{2}$
$\frac{Q}{2}$
K
$h_{\vec{q}} = \begin{pmatrix} \frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2 & V_Q \\ V_{-Q} & \frac{\hbar^2}{2m}\left(-\frac{Q}{2}+q\right)^2 \end{pmatrix}$
$\left[\frac{\hbar^2}{2m}\left(\frac{Q}{2}+q\right)^2-\lambda\right]\left[\frac{\hbar^2}{2m}\left(\frac{Q}{2}-q\right)^2-\lambda\right]$
$-|V_Q|^2 = 0$
$\tilde{\varepsilon}_{\pm}(q) = \frac{\hbar^2}{2m}\left(\frac{Q^2}{4}+q^2\right) \pm \sqrt{\left(\frac{\hbar^2}{2m}Qq\right)^2+|V_Q|^2}$


Tight-Binding Electrons


Tight-Binding Electrons
E_0
E_0
E_0
E_0

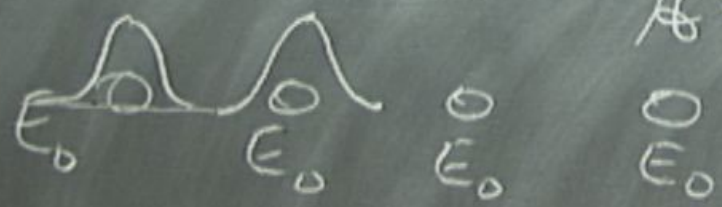

Tight-Binding Electrons
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
E_0
E_0
N


Tight-Binding Electrons
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
E_0
E_0
E_0
N fold
Degen Energy
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
$-t \sum_i$

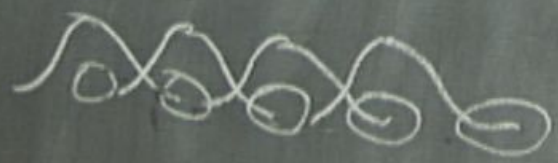

Tight-Binding Electrons
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
E_0 E_0 E_0 E_0
N fold Degen
Energy Gap
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
$-t \sum_i \left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} \right)$
$-\frac{Q}{2}$

Tight-Binding Electrons

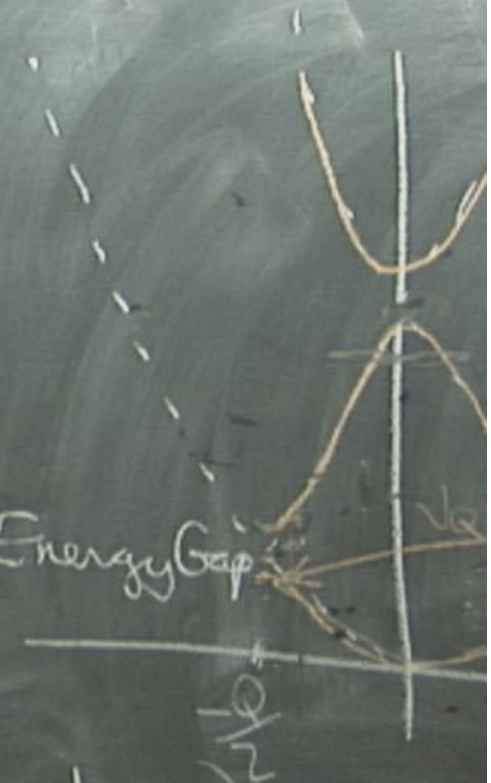
$$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$$



N fold Degen



$$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i - t \sum_i \left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} \right)$$



Tight-Binding Electrons

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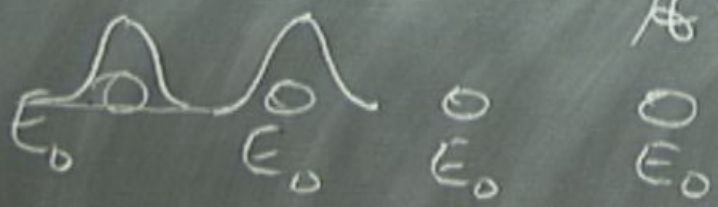

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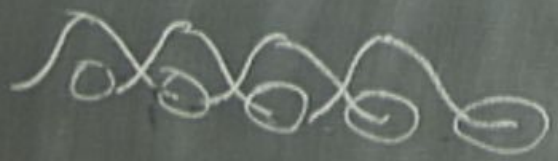
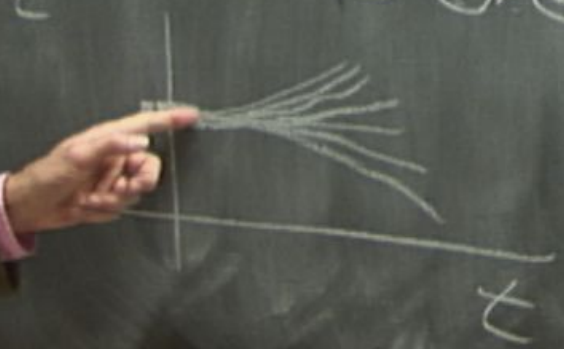
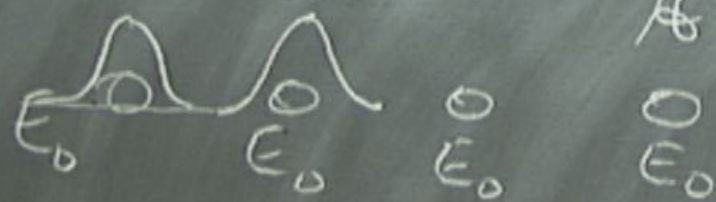




$$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i - t \sum_i \left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} \right)$$

Tight-Binding Electrons

$$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$$



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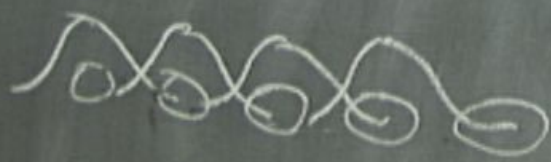
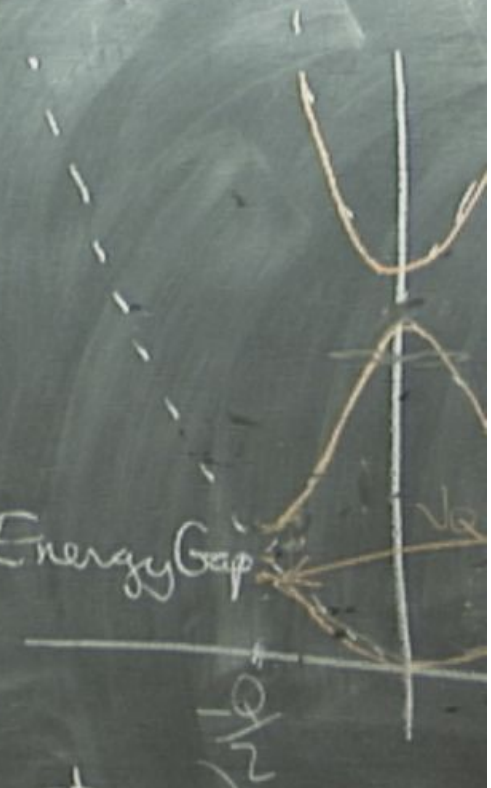
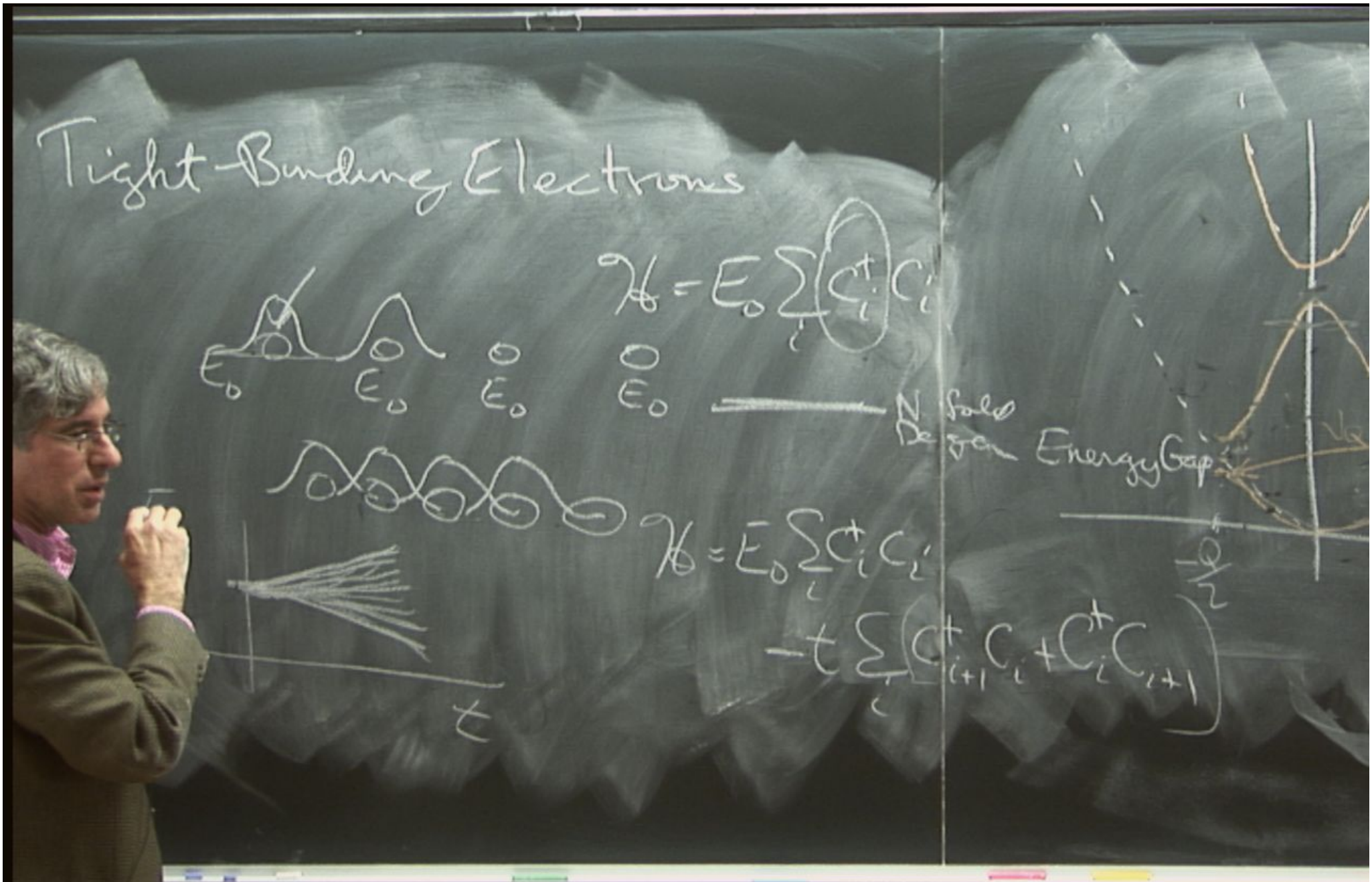


$$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i - t \sum_i \left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} \right)$$




Tight-Binding Electrons
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i$
E_0 E_0 E_0 E_0
N fold Degen
Energy Gap
$\mathcal{H} = E_0 \sum_i c_i^\dagger c_i - t \sum_i \left(c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} \right)$
t
$-\frac{\pi}{2}$