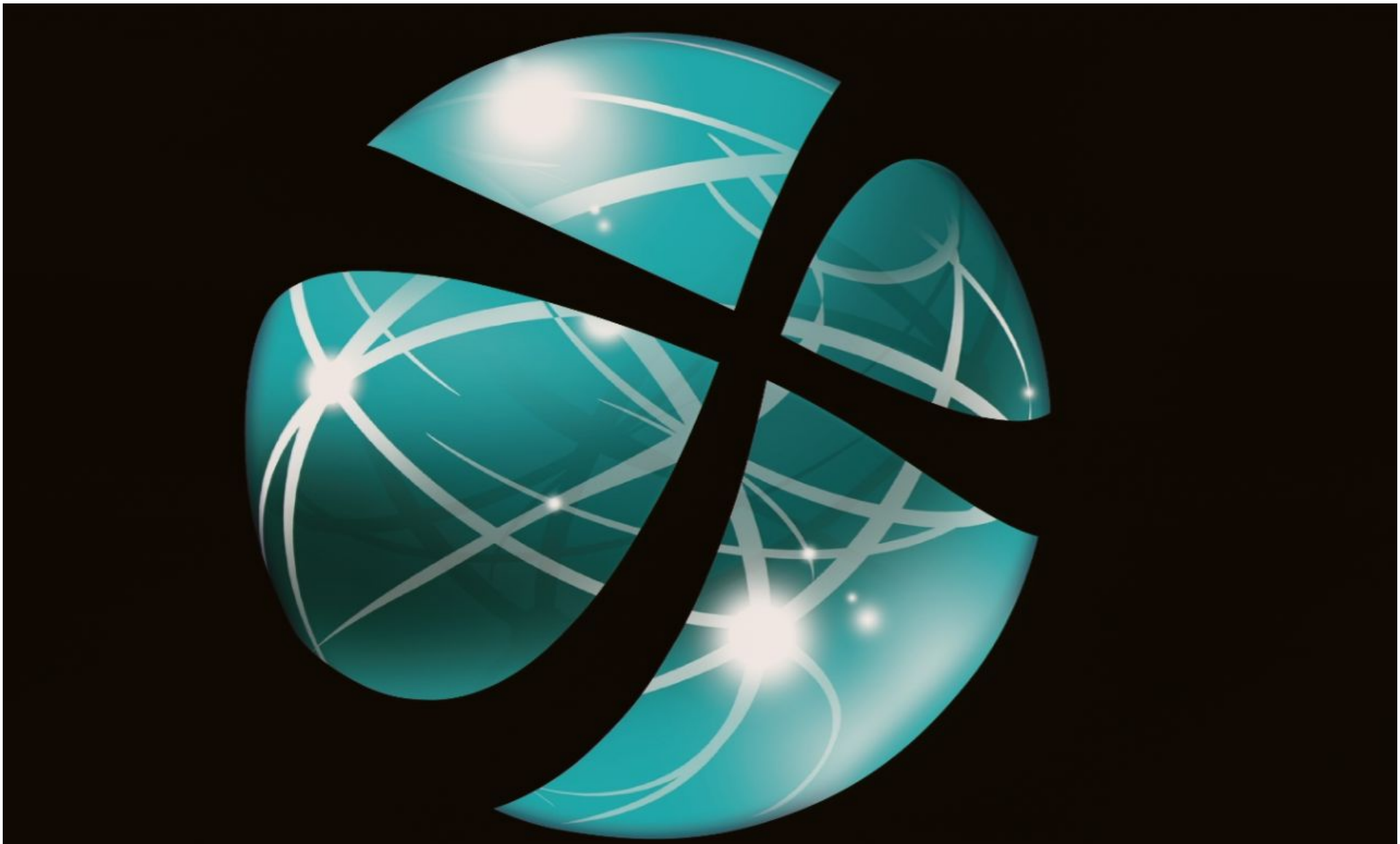
Title: Condensed Matter Review - Lecture 2

Date: Jan 04, 2011 10:15 AM

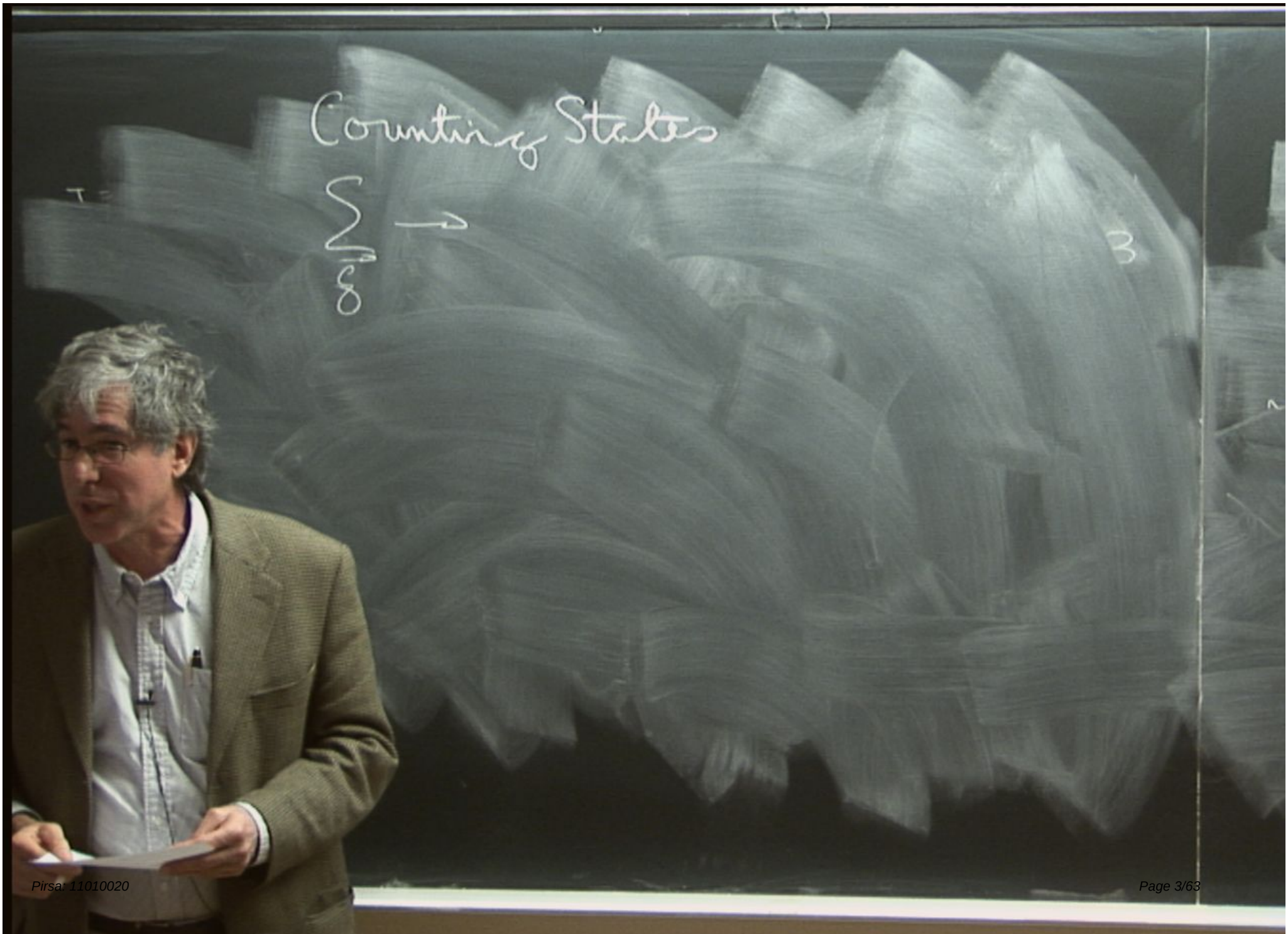
URL: http://pirsa.org/11010020

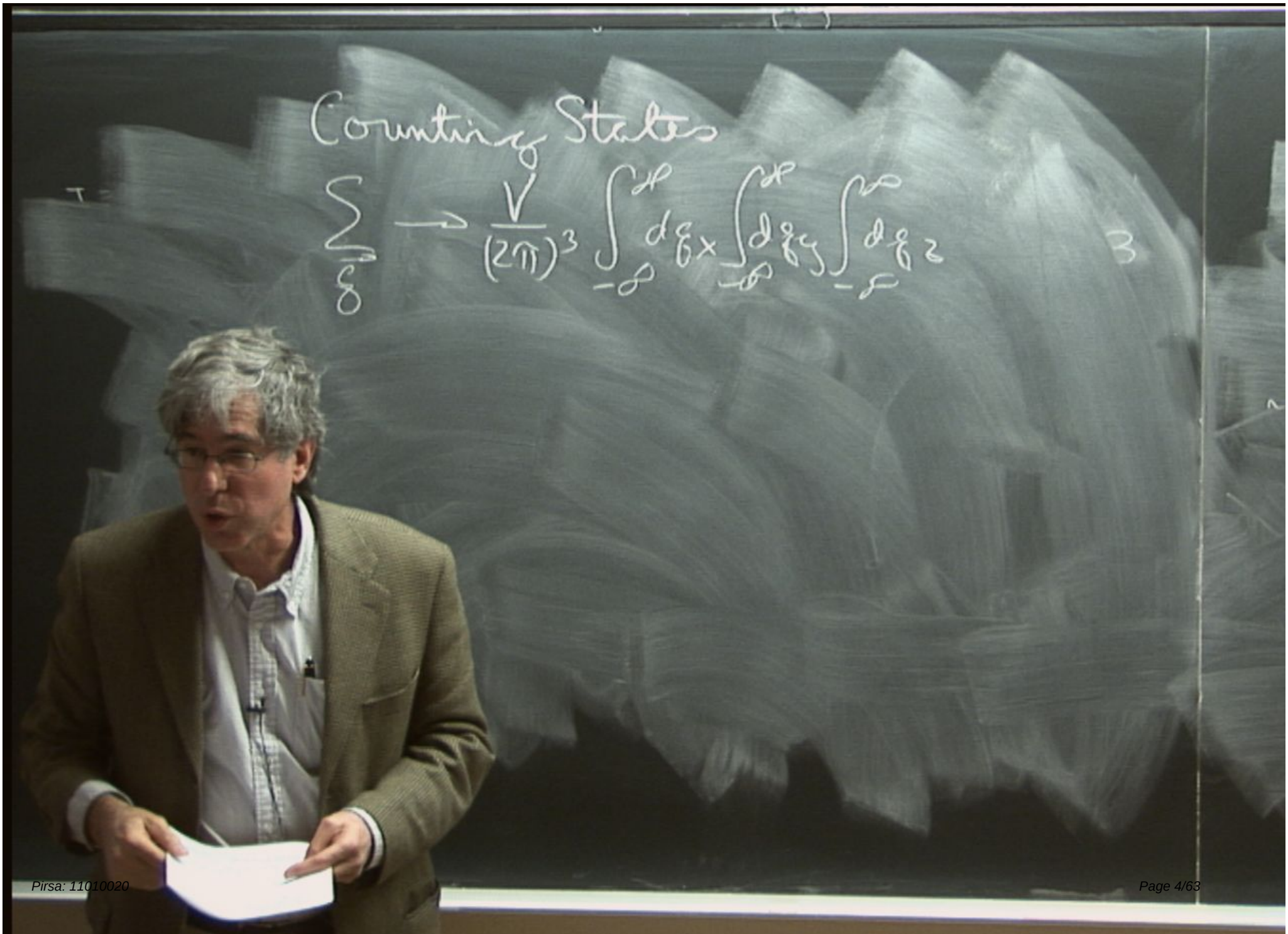
Abstract:

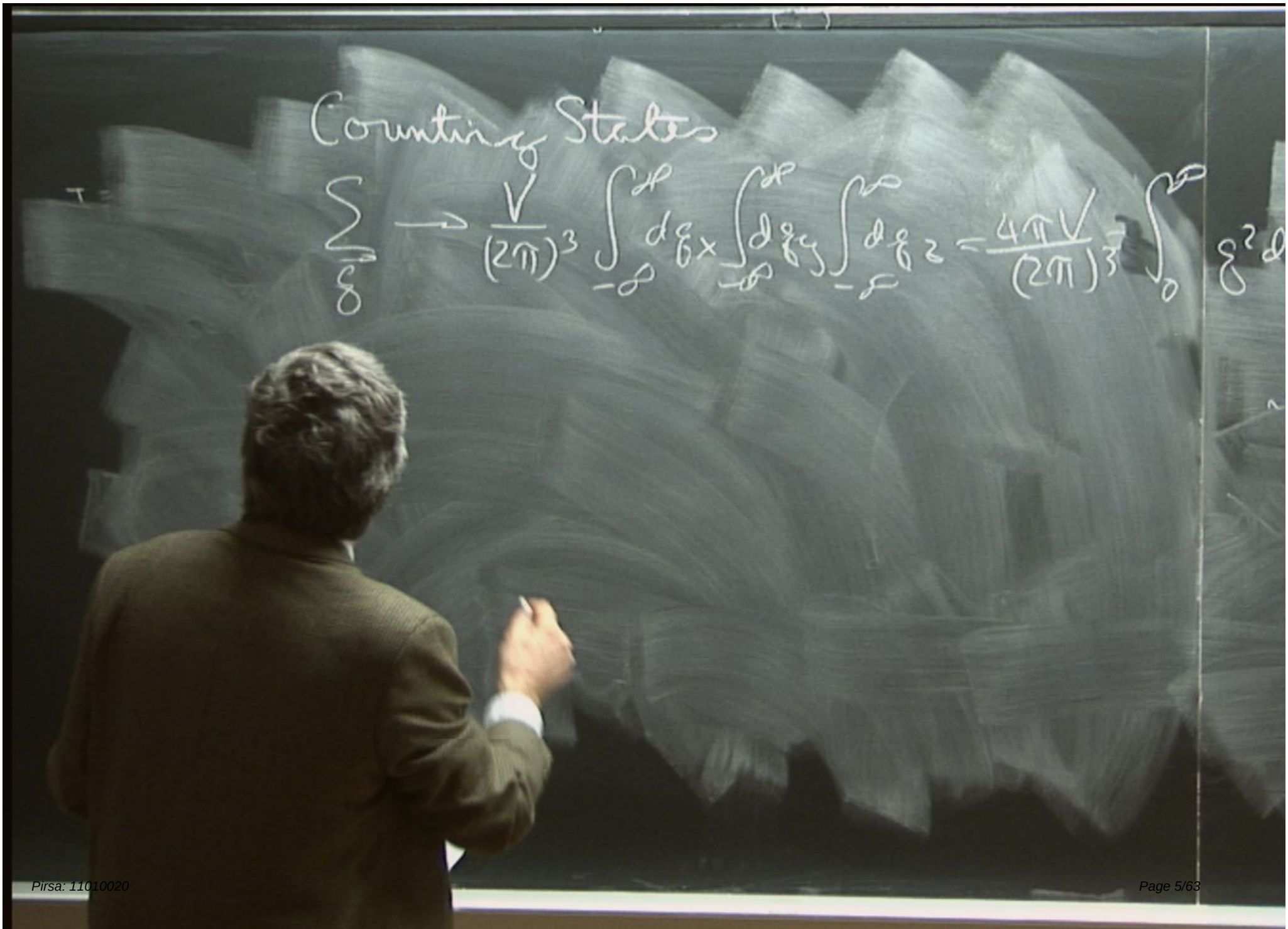


perimeter scholars

international


Counting of States


Counting States
$\sum_{\vec{q}} \longrightarrow \frac{V}{(2\pi)^3} \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \int_{-\infty}^{\infty} dq_z$


Counting States
$\sum_{\vec{q}} \longrightarrow \frac{V}{(2\pi)^3} \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \int_{-\infty}^{\infty} dq_z = \frac{4\pi V}{(2\pi)^3} \int_0^{\infty} q^2 d$

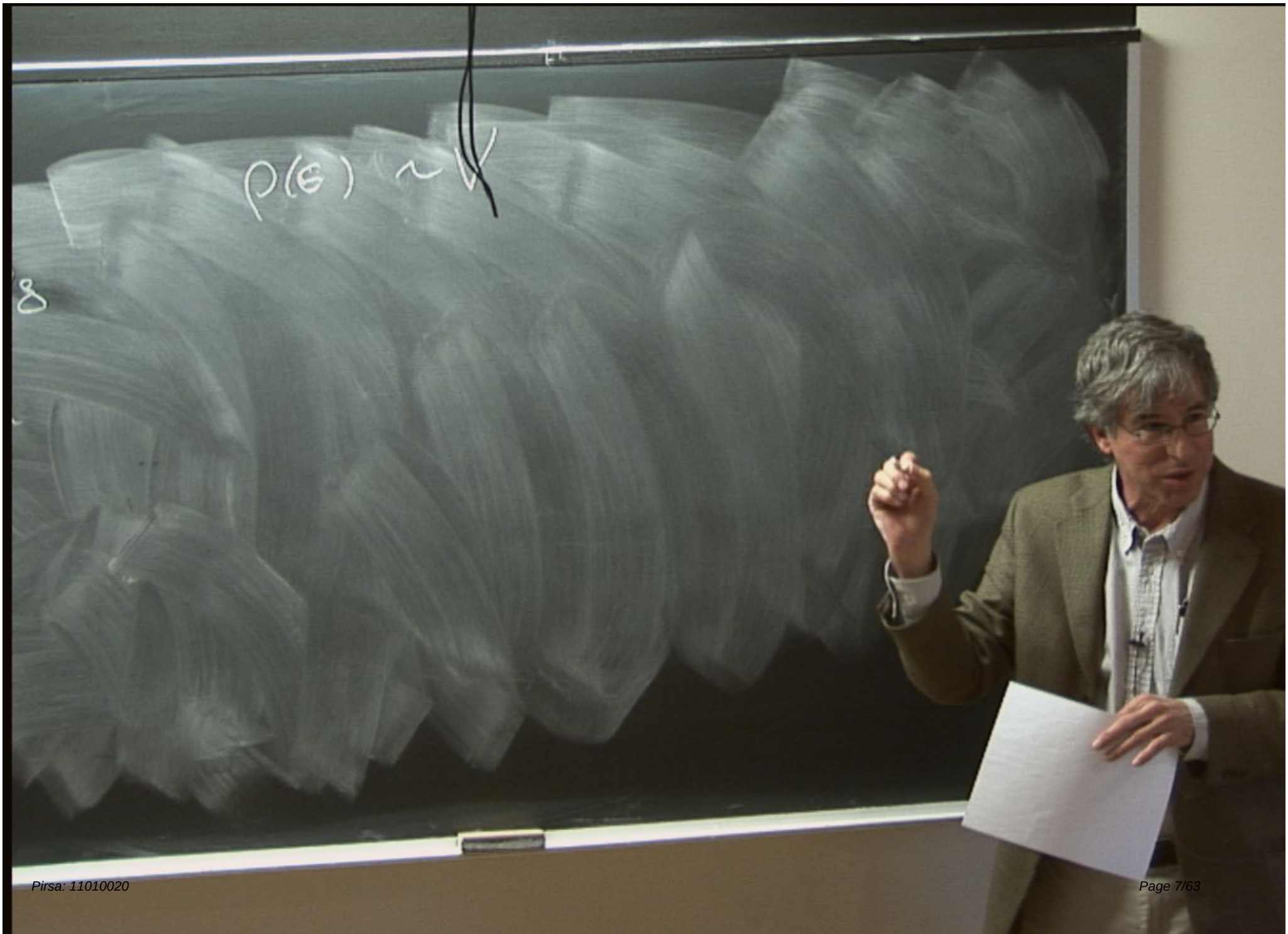
Counting States

$$\sum_{\vec{q}} \longrightarrow \frac{V}{(2\pi)^3}\int_{-\infty}^{\infty} dq_x \times \int_{-\infty}^{\infty} dq_y \int_{-\infty}^{\infty} dq_z = \frac{4\pi V}{(2\pi)^3}\int_0^{\infty} q^2 dq$$

$$\varepsilon_q = \frac{\hbar^2 q^2}{2m} \rightarrow q = \sqrt{\frac{2m\varepsilon_q}{\hbar^2}} \quad dq \cdots$$

$$\sum_{q} \longrightarrow \frac{4\pi V}{(2\pi)^3}\int_0^{\infty} \frac{2m\varepsilon_q}{\hbar^2}\sqrt{\frac{m}{2\hbar^2\varepsilon_q}}\, d\varepsilon_q$$

$$= \underbrace{\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}}_{\text{Density of States } \rho(\epsilon)} \int_0^{\infty} \sqrt{\epsilon}\, d\epsilon$$


ρ(E) ~ V

Counting States

$$\sum_{\vec{k}} \longrightarrow \frac{V}{(2\pi)^3} \int_{-\infty}^{\infty} dk_x \times \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z = \frac{4\pi V}{(2\pi)^3} \int_0^{\infty} k^2 dk$$

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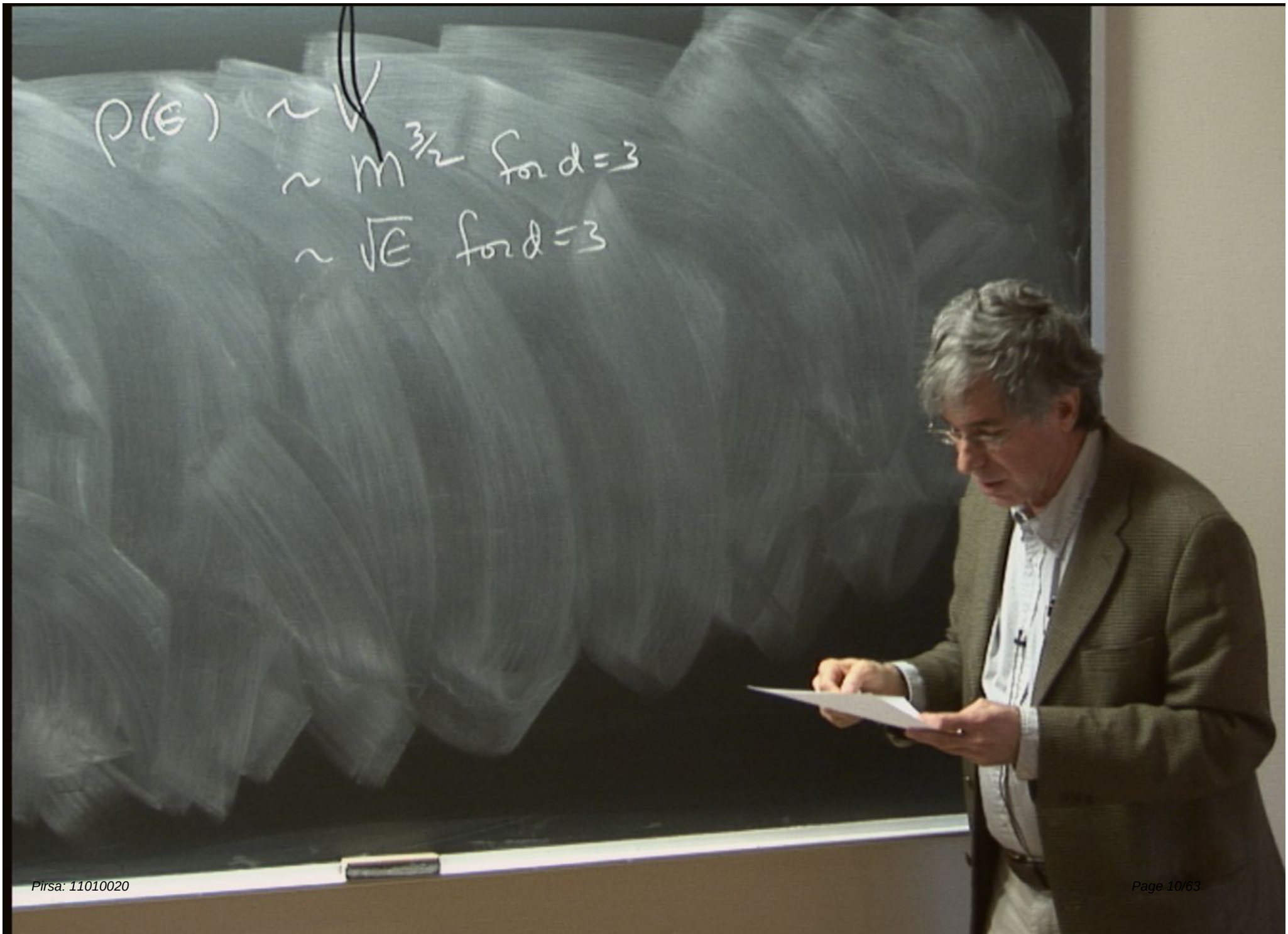
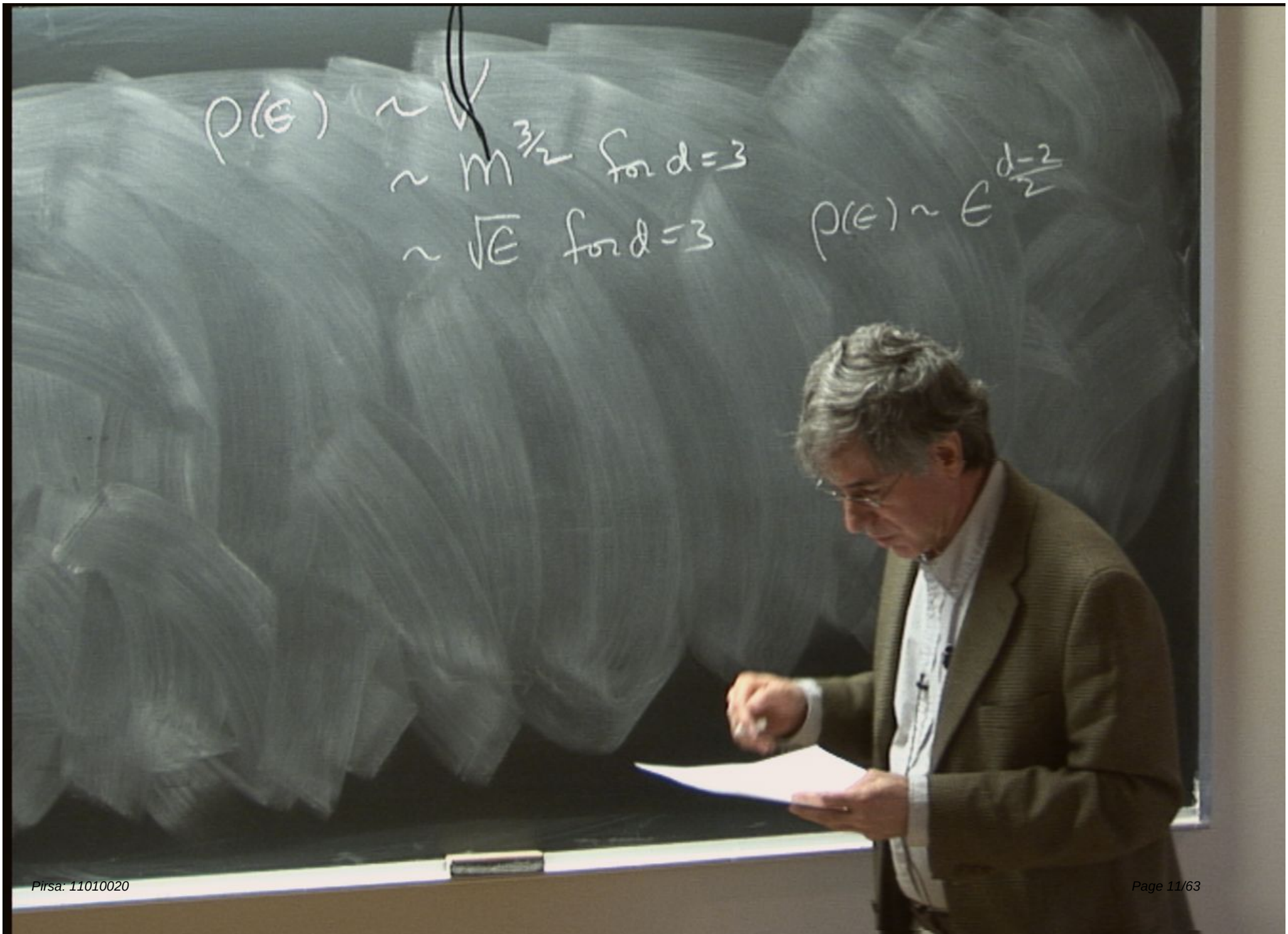
$$= \underbrace{\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^{\infty} \sqrt{\epsilon}\, d\epsilon}_{\text{Density of States } \rho(\epsilon)}$$

$$\rho(\epsilon) \sim V$$
$$\sim m^{3/2} \text{ for } d=3$$

$$g^2 dg$$


$\rho(E) \sim V$
$\sim m^{3/2}$ for $d=3$
$\sim \sqrt{E}$ for $d=3$

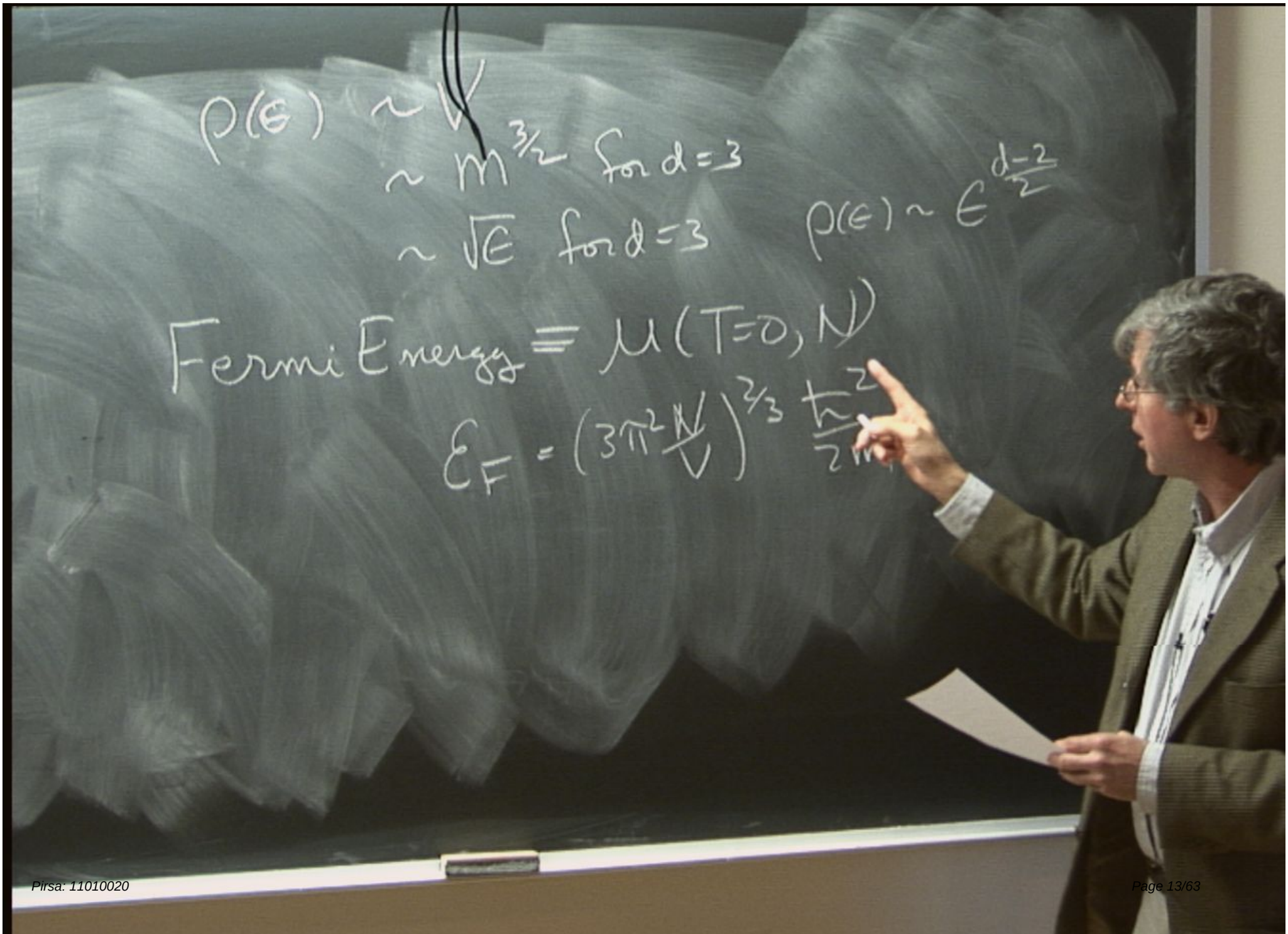

$\rho(\epsilon) \sim V$
$\sim m^{3/2}$ for $d=3$
$\sim \sqrt{\epsilon}$ for $d=3$
$\rho(\epsilon) \sim \epsilon^{\frac{d-2}{2}}$

$$\rho(\epsilon) \sim V$$

$$\sim m^{3/2} \text{ for } d=3$$

$$\sim \sqrt{\epsilon} \text{ for } d=3 \qquad \rho(\epsilon) \sim \epsilon^{\frac{d-2}{2}}$$

$$\text{Fermi Energy} \equiv \mu(T=0, N)$$


$\rho(\epsilon) \sim V$
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$\sim \sqrt{\epsilon}$ for $d=3$
$\rho(\epsilon) \sim \epsilon^{\frac{d-2}{2}}$
Fermi Energy $\equiv \mu(T=0, N)$
$\varepsilon_F = \left(3\pi^2 \frac{N}{V}\right)^{2/3} \frac{\hbar^2}{2m}$

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Fermi Energy $\equiv \mu(T=0, N)$

$$\varepsilon_F = \underbrace{\left(3\pi^2 \frac{N}{V}\right)}_{K_F^2}{}^{2/3} \frac{\hbar^2}{2m}$$

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Fermi Surface

Surface in

$$\rho(\epsilon) \sim V$$

$$\sim m^{3/2} \text{ for } d=3$$

$$\sim \sqrt{\epsilon} \text{ for } d=3 \qquad \rho(\epsilon) \sim \epsilon^{\frac{d-2}{2}}$$

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<u>Fermi Surface</u>

Surface in $\vec{k}$-space $\varepsilon(\vec{k}) = \varepsilon_F$

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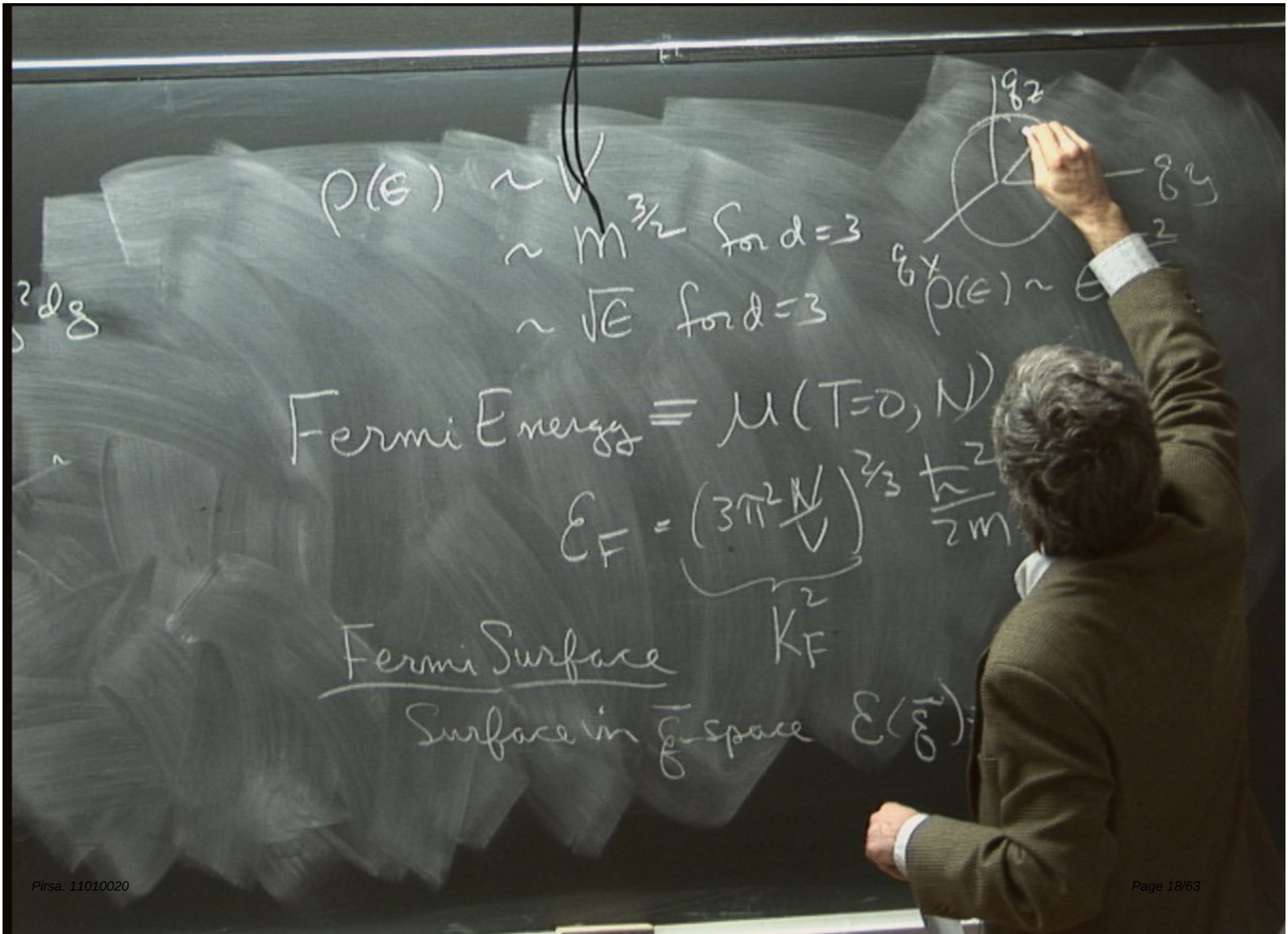
<u>Fermi Surface</u>

Surface in $\vec{k}$-space $\varepsilon(\vec{k}) = \varepsilon_F$


$\rho(\epsilon) \sim V$
$\sim m^{3/2}$ for $d=3$
$\sim \sqrt{\epsilon}$ for $d=3$
q_z
q_y
q_x
$\rho(\epsilon) \sim \epsilon^2$
Fermi Energy $\equiv \mu(T=0, N)$
$\varepsilon_F = \underbrace{\left(3\pi^2 \frac{N}{V}\right)}_{K_F^2}{}^{2/3} \frac{\hbar^2}{2m}$
Fermi Surface
Surface in $\vec{q}$-space $\varepsilon(\vec{q})$

$\rho(\epsilon) \sim V$

$\sim m^{3/2}$ for $d=3$

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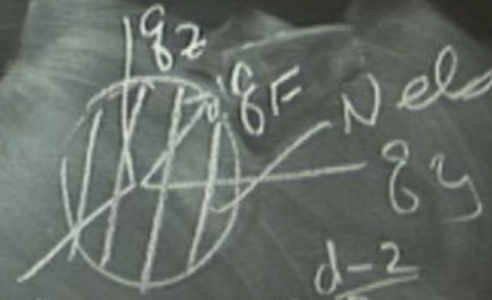

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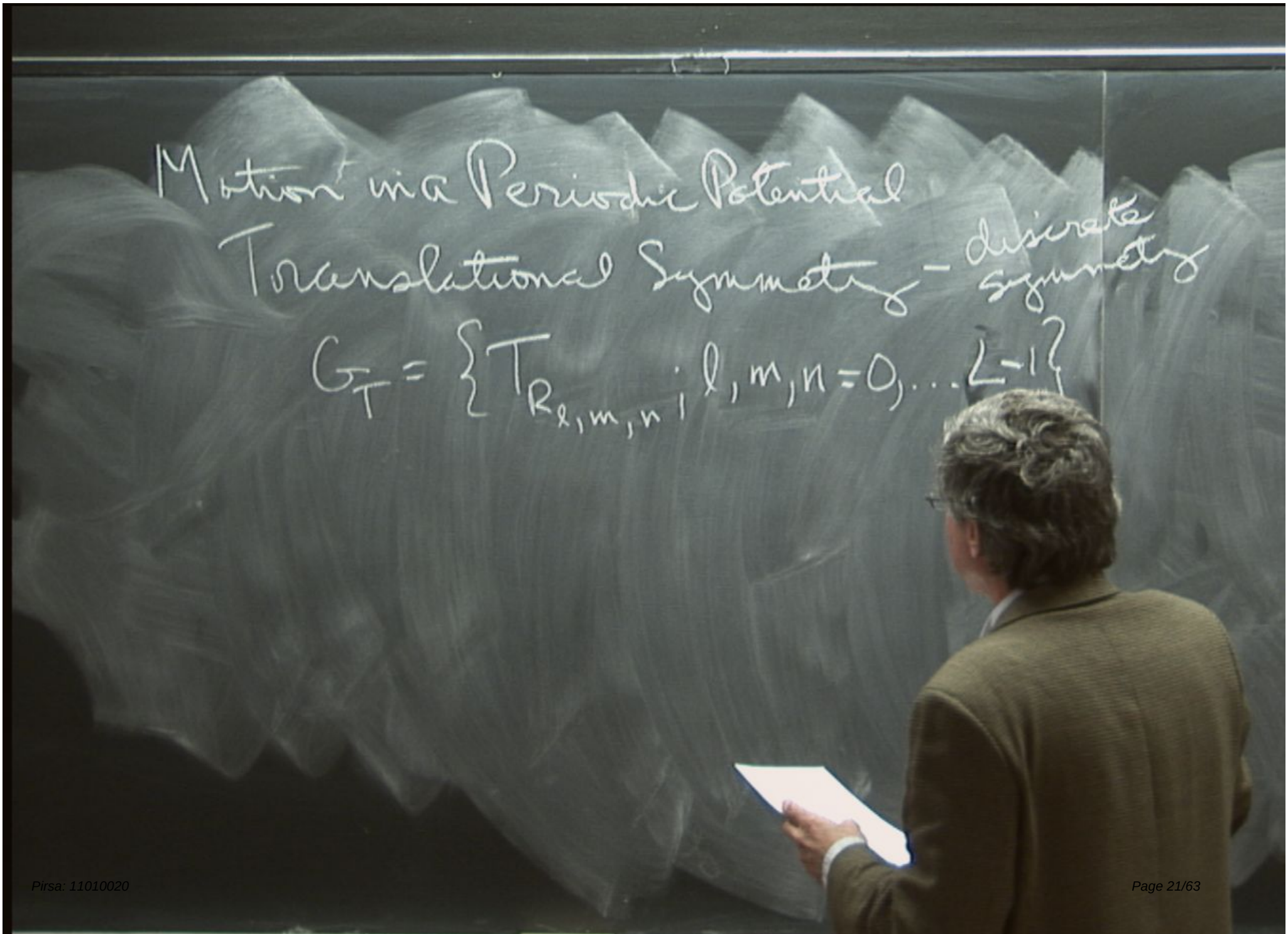
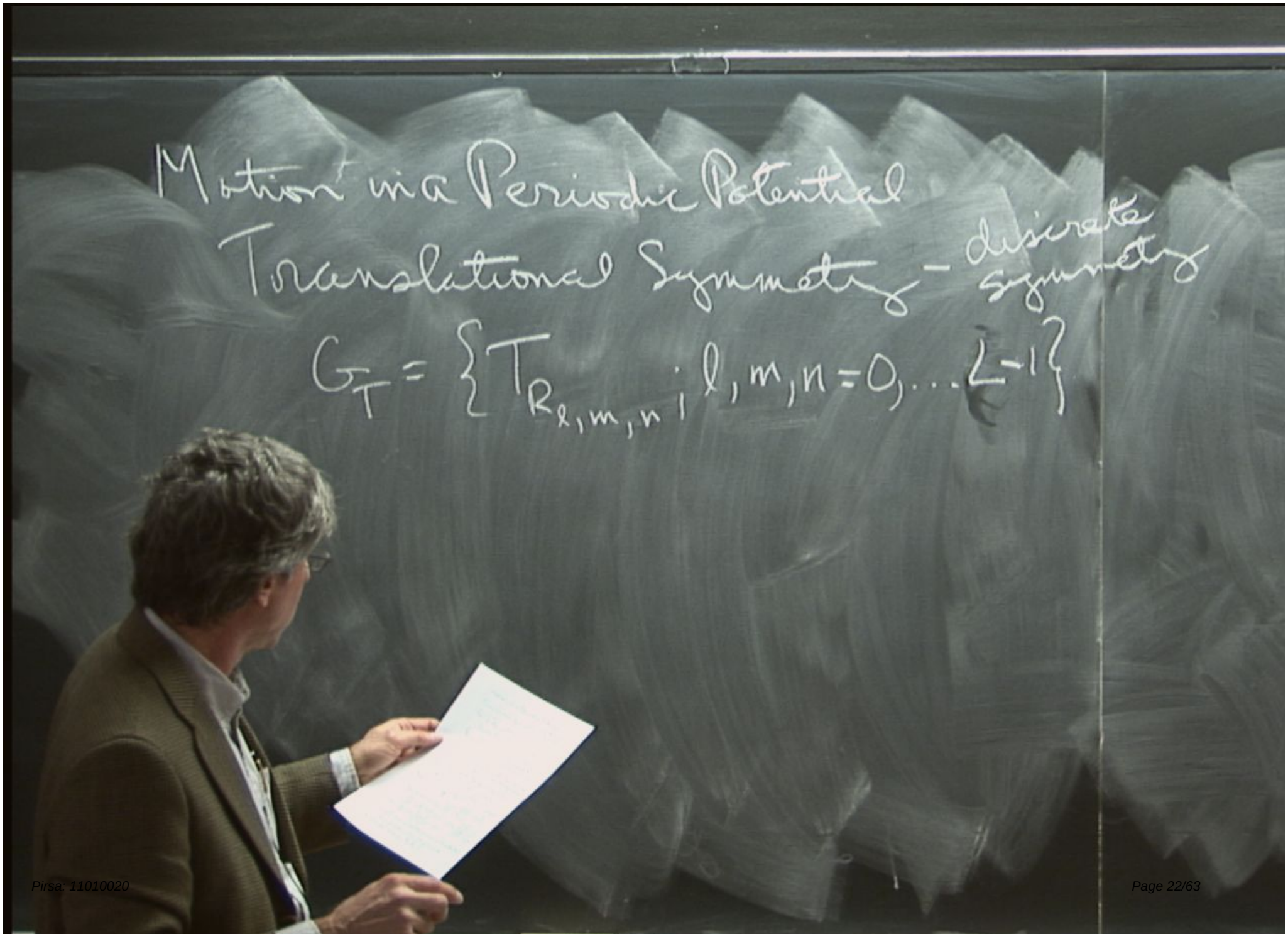
<u>Fermi Surface</u>

Surface in $\vec{k}$-space $\varepsilon(\vec{k}) = \varepsilon_F$

Motion in a Periodic Potential


Motion in a Periodic Potential
Translational Symmetry – discrete symmetry
$G_T = \{T_{R_{l,m,n}} ; l, m, n = 0, \ldots L-1\}$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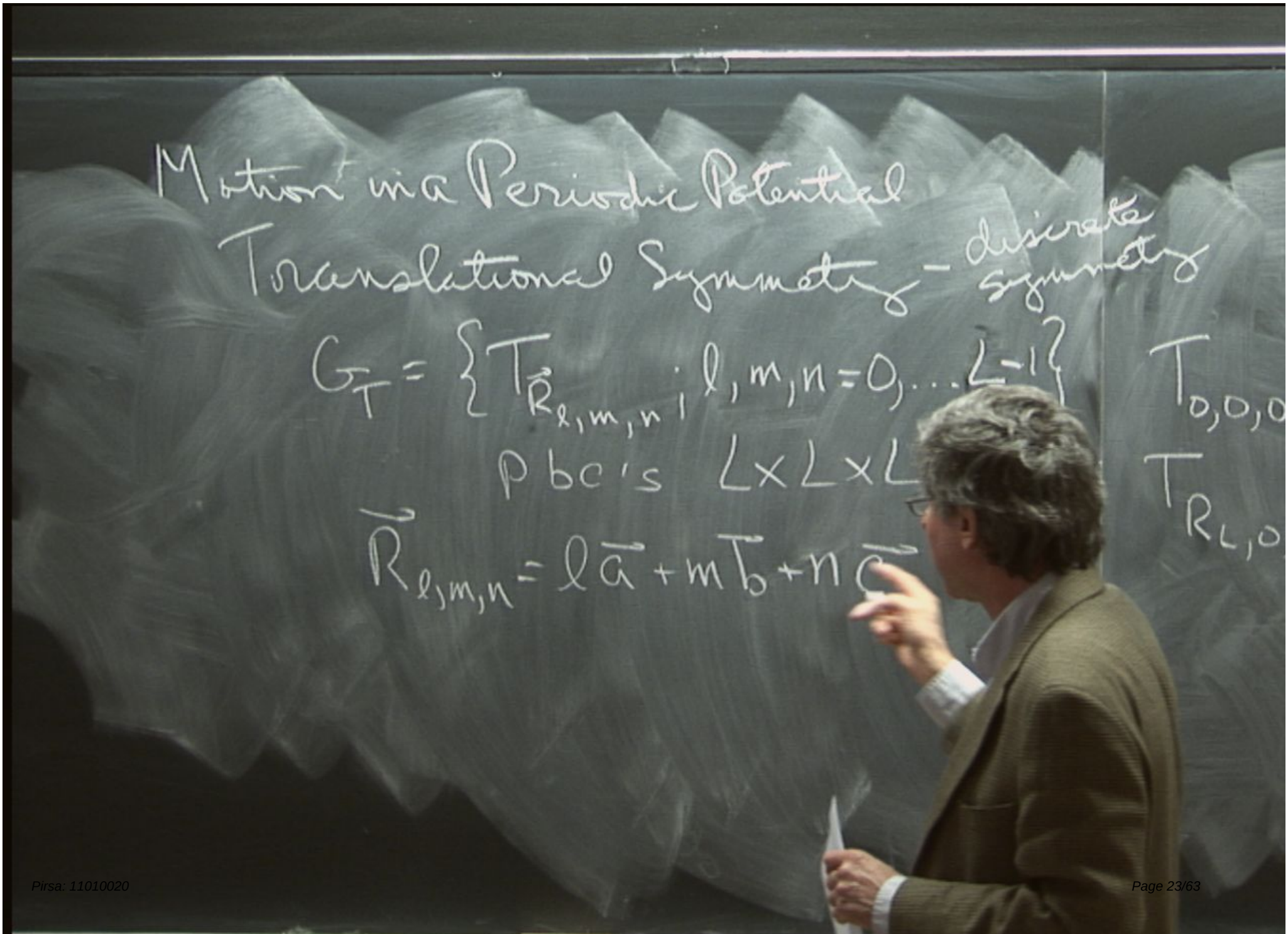

Motion in a Periodic Potential
Translational Symmetry - discrete symmetry
$G_T = \{T_{\vec{R}_{l,m,n}} ; l,m,n = 0,\ldots L-1\}$
pbc's $L \times L \times L$
$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$
$T_{0,0,0}$
$T_{R_{L,0}}$

Motion in a Periodic Potential

Translational Symmetry – discrete symmetry

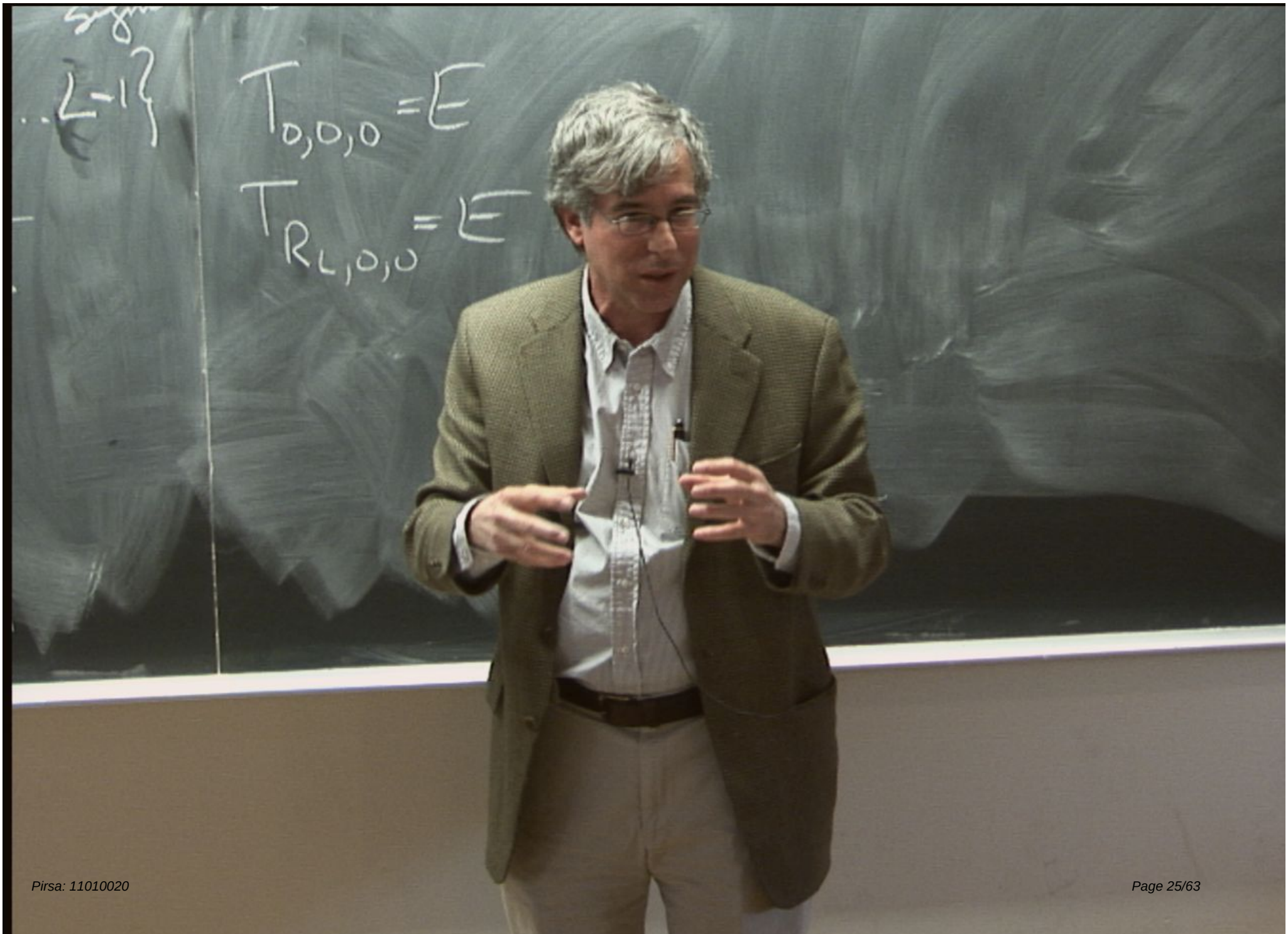
$$G_T = \{ T_{\vec{R}_{l,m,n}} ; l,m,n = 0, \ldots L-1 \}$$

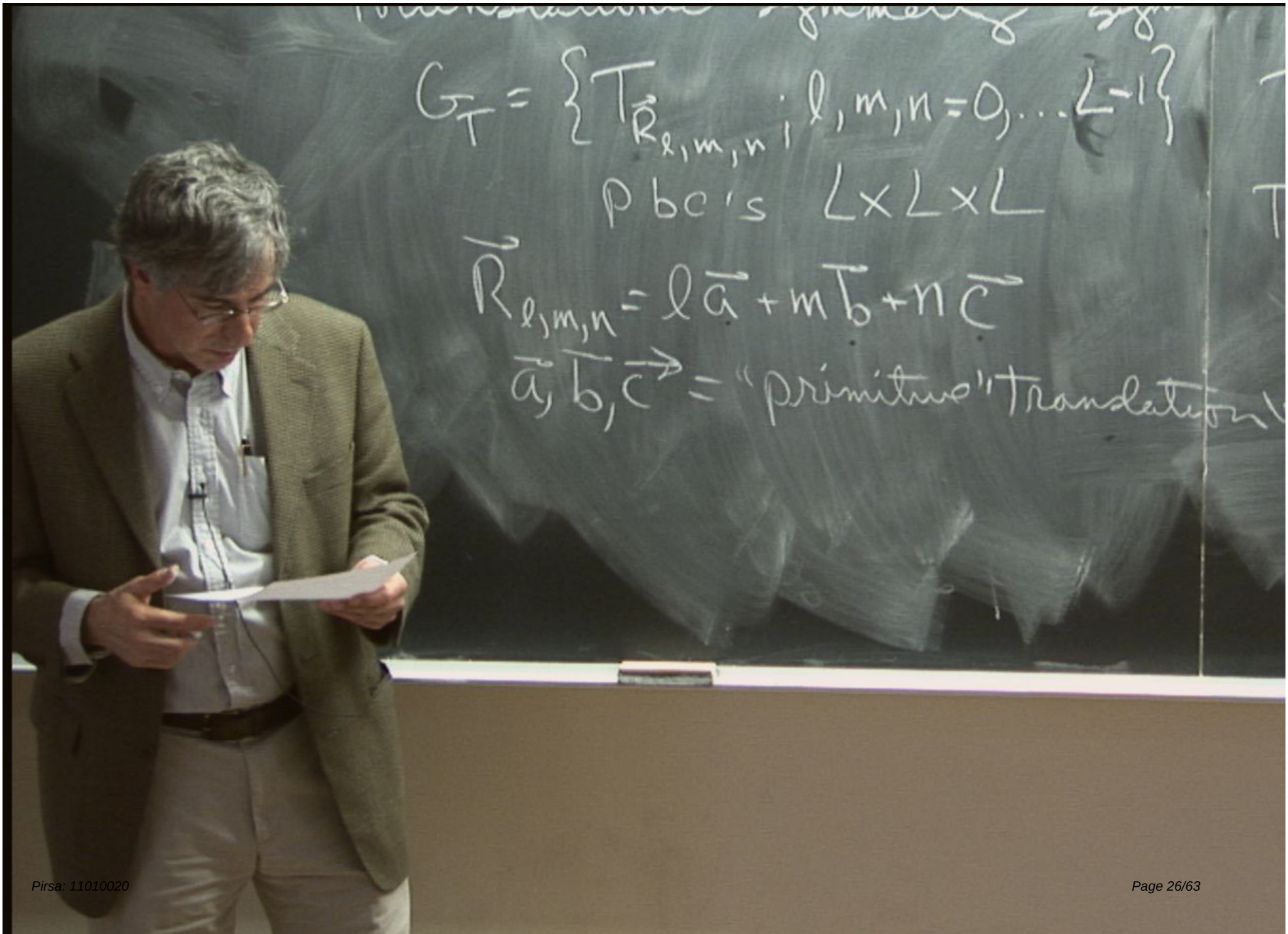
pbc's $L \times L \times L$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$T_{0,0,0} =$

$T_{R_{L,0,0}} =$


$T_{0,0,0} = E$
$T_{R_L,0,0} = E$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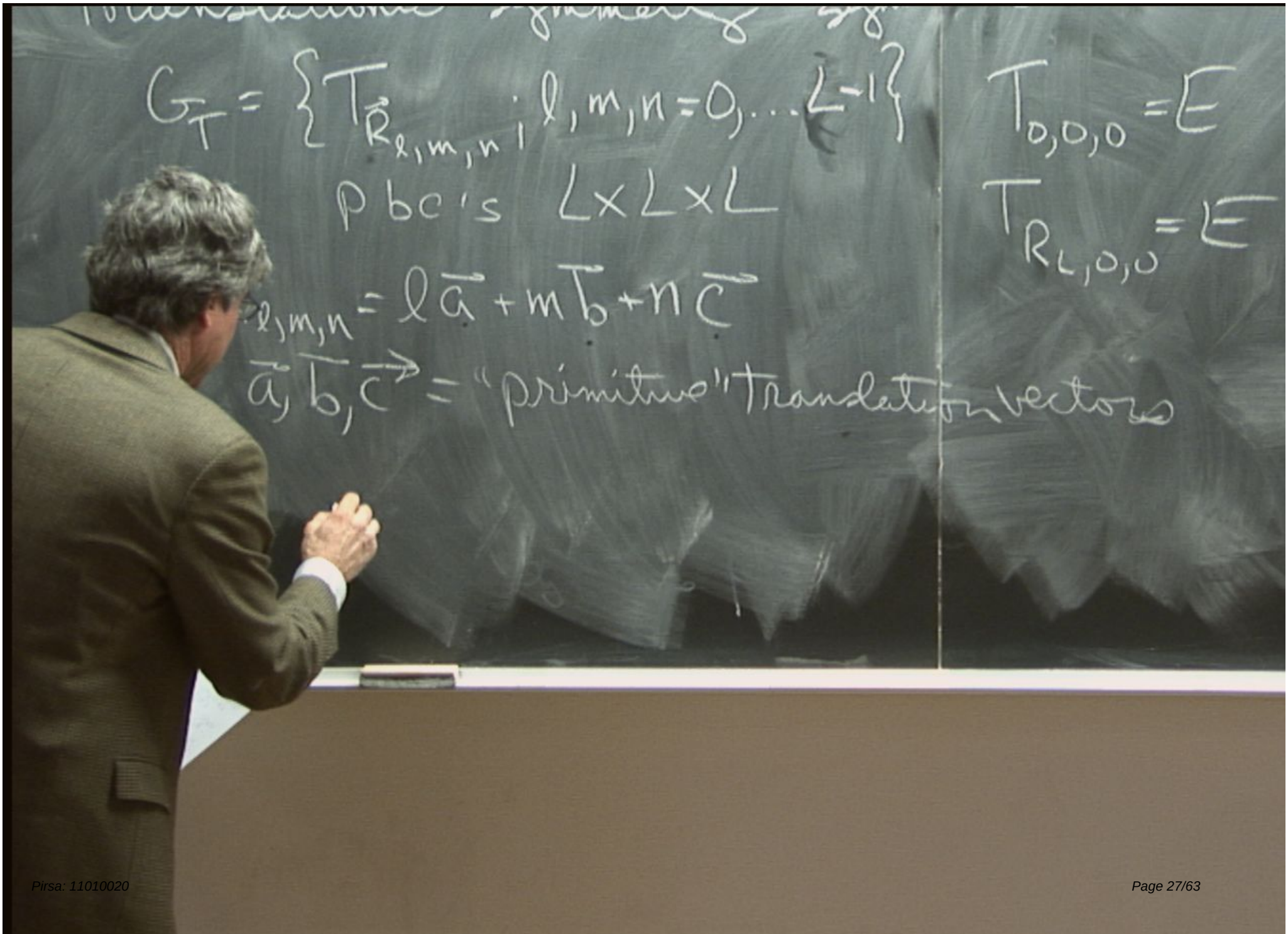
$\vec{a}, \vec{b}, \vec{c}$ = "primitive" translation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$\vec{a}, \vec{b}, \vec{c}$ = "primitive" translation vectors
$T_{0,0,0} = E$
$T_{R_{L,0,0}} = E$

$$G_T = \{ T_{\vec{R}_{l,m,n}} ; l,m,n = 0, \ldots L-1 \}$$

pbc's $L \times L \times L$

$$\vec{R}_{l,m,n} = l\vec{a} + m\vec{b} + n\vec{c}$$

$\vec{a}, \vec{b}, \vec{c}$ = "primitive" translation vectors

$$\underbrace{|\vec{a} \cdot \vec{b} \times \vec{c}|}_{V_c} \neq 0$$

$$T_{0,0,0} = E$$

$$T_{R_{L,0,0}} = E$$

1.) Functions with the symmetry of G_T

$$T_{\vec{R}}\, f(\vec{r}) = f(\vec{r}+\vec{R}) = f(\vec{r})$$

$$T_{0,0,0} = E$$

$$T_{R_L,0,0} = E$$

vectors

1.) Functions with the symmetry of G_T

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vectors

1.) Functions with the symmetry of G_T

$$T_{\vec{R}}\, f(\vec{r}) = f(\vec{r}+\vec{R}) = f(\vec{r})$$

$$f(\vec{r}) = \sum_{\vec{Q}} F(\vec{Q})\, e^{i\vec{Q}\cdot\vec{r}}$$

$$T_{0,0,0} = E$$

$$T_{R_L,0,0} = E$$

vectors

γ

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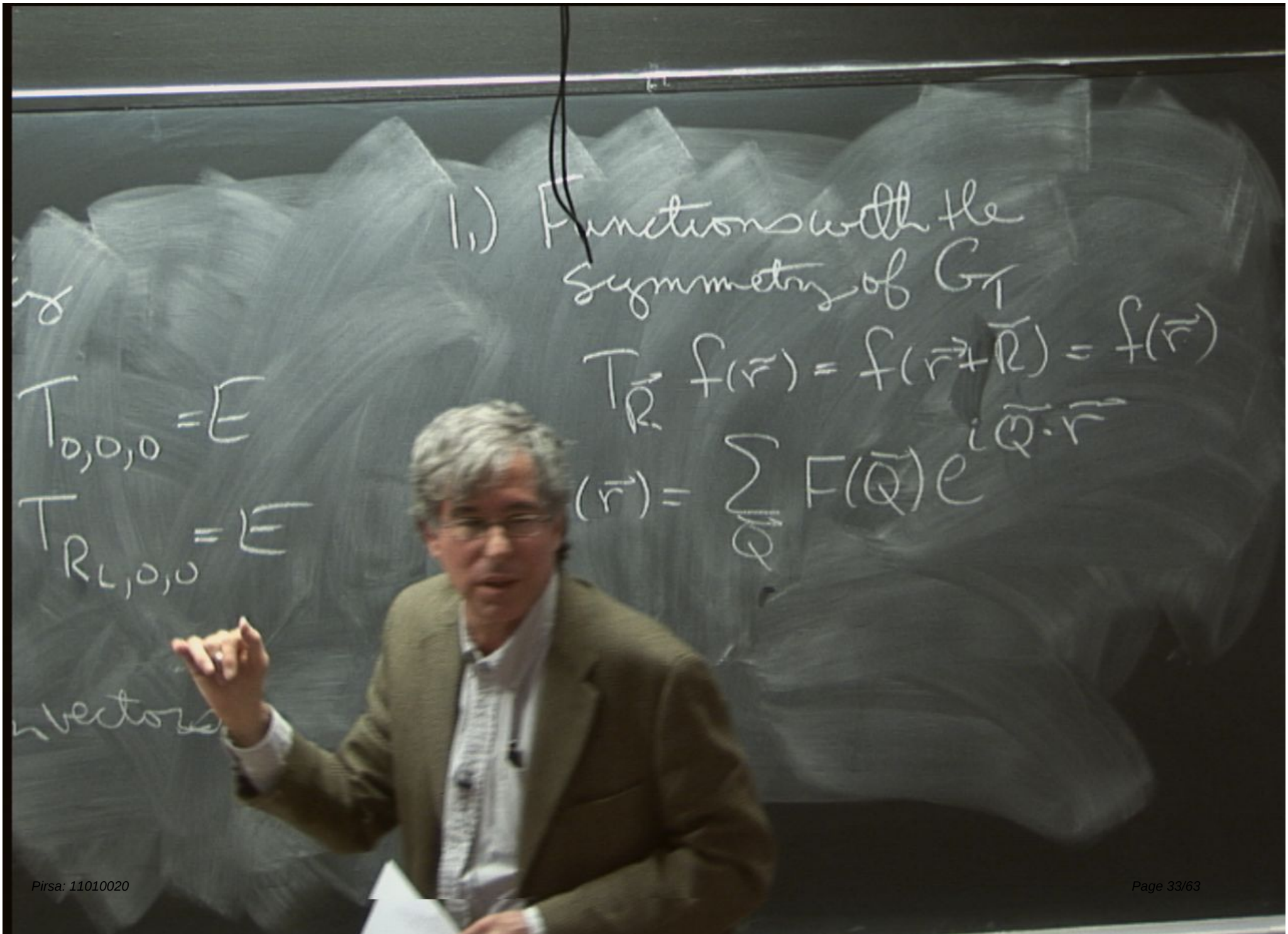
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$T_{\vec{R}} f(\vec{r}) = f(\vec{r}+\vec{R}) = f(\vec{r})$
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$T_{0,0,0} = E$
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$T_{0,0,0} = E$

$= E$

$L,0,0$

vectors

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Sum over all $\vec{Q}$ for which $\vec{Q}\cdot\vec{R} = 2\pi n$

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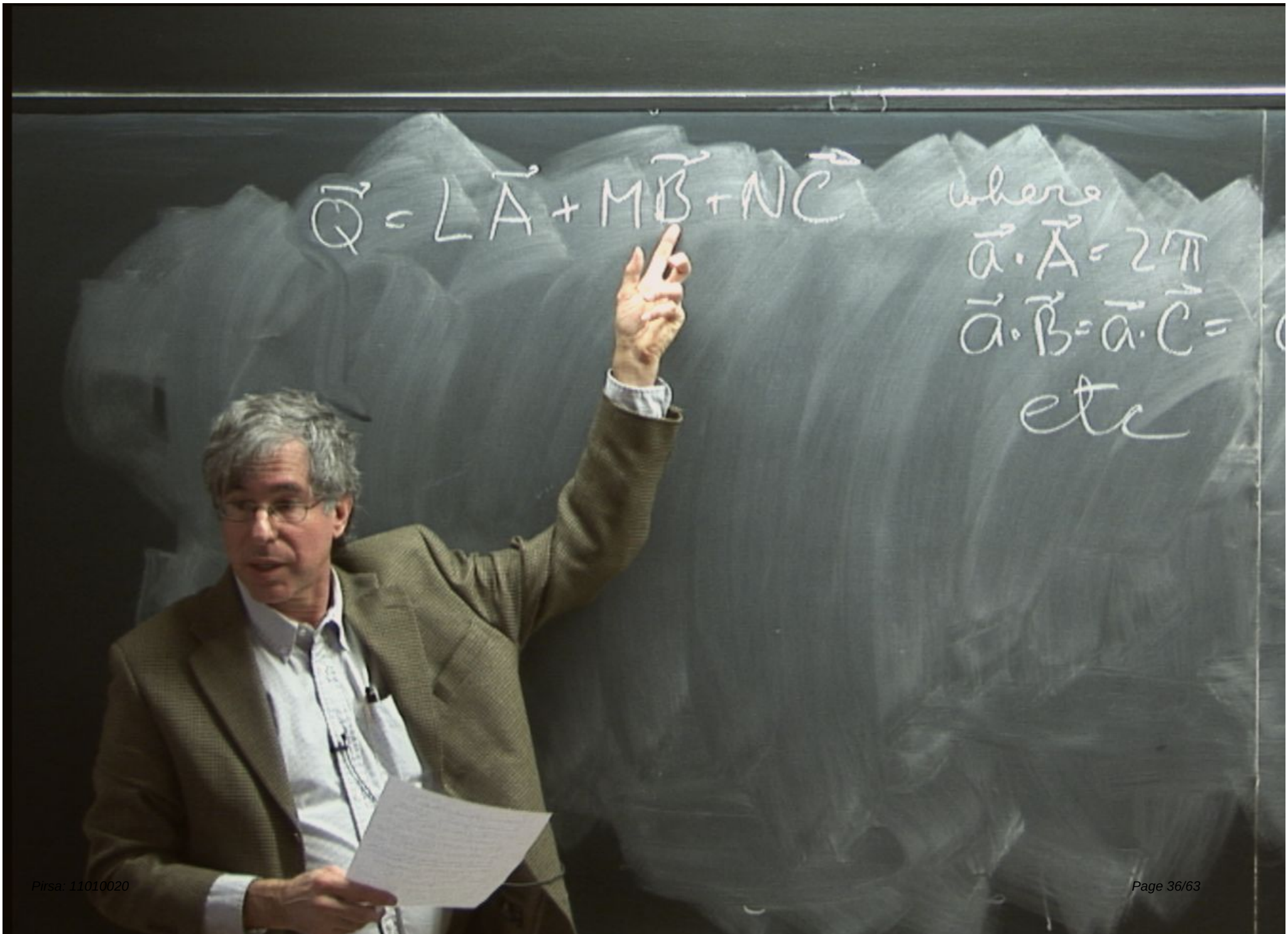
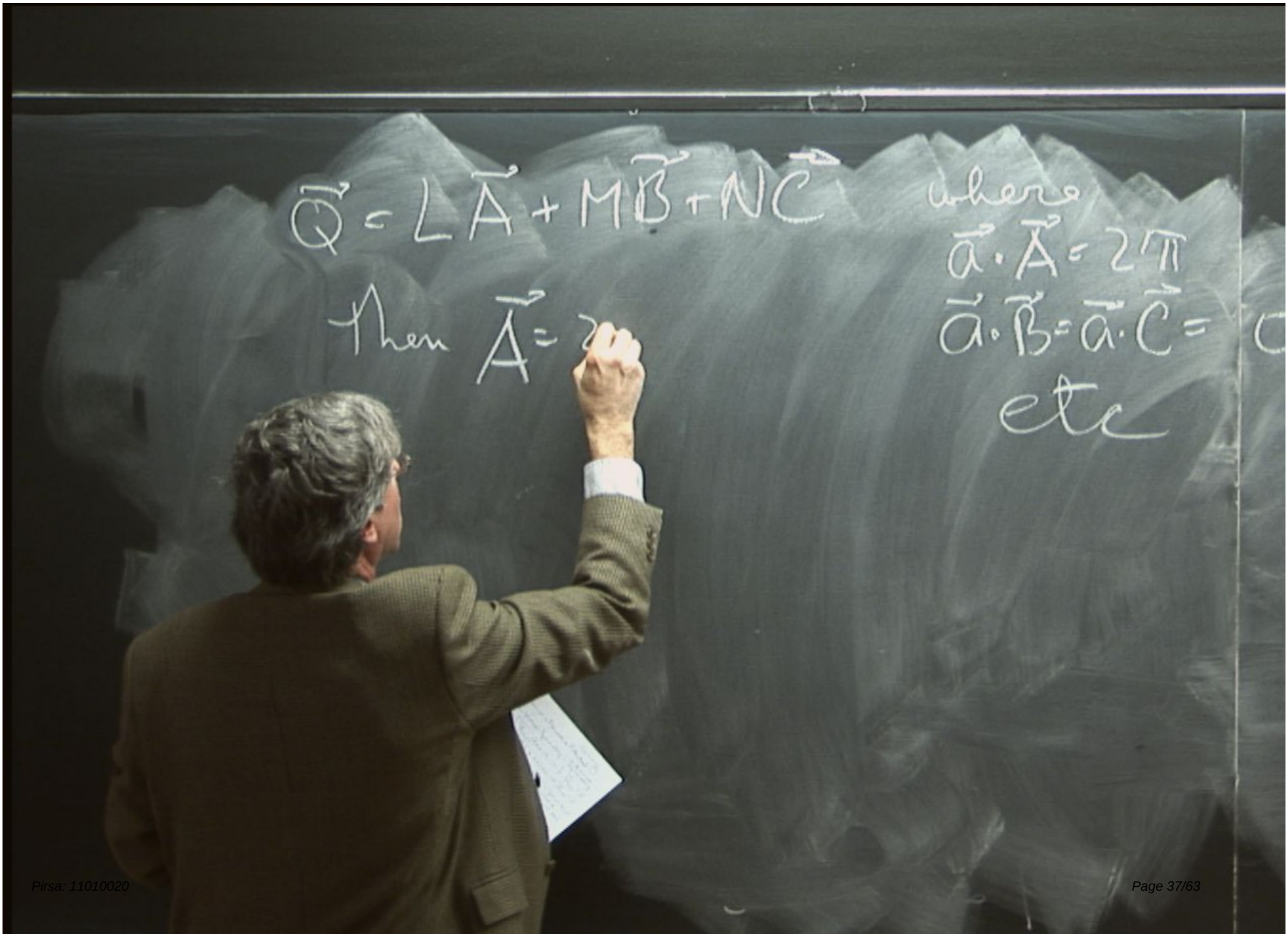
Sum over all $\vec{Q}$ for which $\vec{Q}\cdot\vec{R} = 2\pi n$


$\vec{Q} = L\vec{A} + M\vec{B} + N\vec{C}$
where
$\vec{a}\cdot\vec{A} = 2\pi$
$\vec{a}\cdot\vec{B} = \vec{a}\cdot\vec{C} =$
etc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where
$\vec{a}\cdot\vec{A} = 2\pi$
$\vec{a}\cdot\vec{B} = \vec{a}\cdot\vec{C} =$
then $\vec{A} =$
etc

$$\vec{Q} = L\vec{A} + M\vec{B} + N\vec{C}$$

where

$$\vec{a}\cdot\vec{A} = 2\pi$$

$$\vec{a}\cdot\vec{B} = \vec{a}\cdot\vec{C} = 0$$

etc

Then $\vec{A} = \dfrac{2\pi\, \vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$

$$\vec{Q} = L\vec{A} + M\vec{B} + N\vec{C}$$

Then $$\vec{A} = \frac{2\pi\, \vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

where

$$\vec{a}\cdot\vec{A} = 2\pi$$

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etc

$$\vec{A}\cdot\vec{b} = \vec{A}\cdot\vec{c} = 0$$

$\vec{Q} = L\vec{A} + M\vec{B} + N\vec{C}$ where

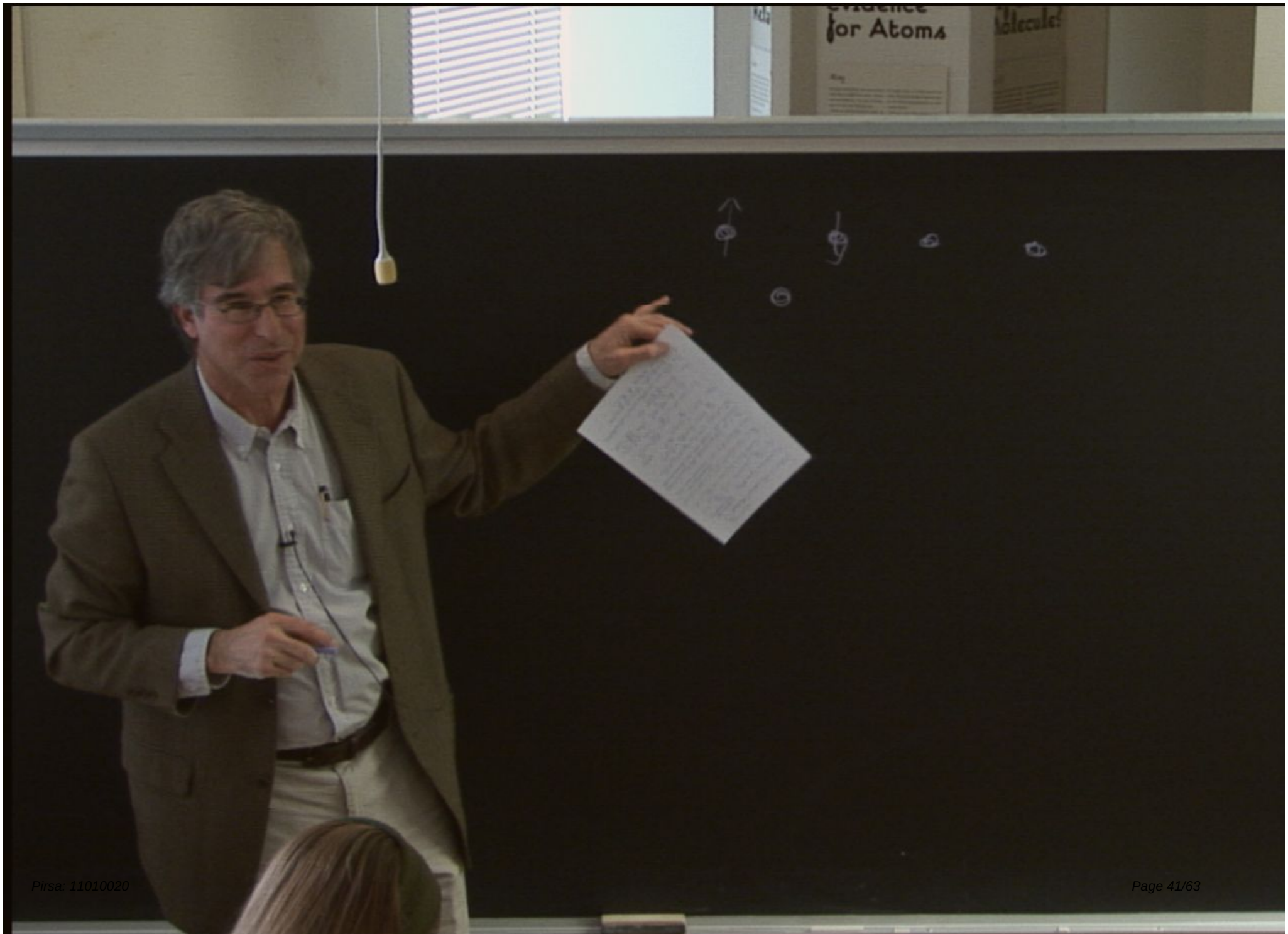
$\vec{a}\cdot\vec{A} = 2\pi$

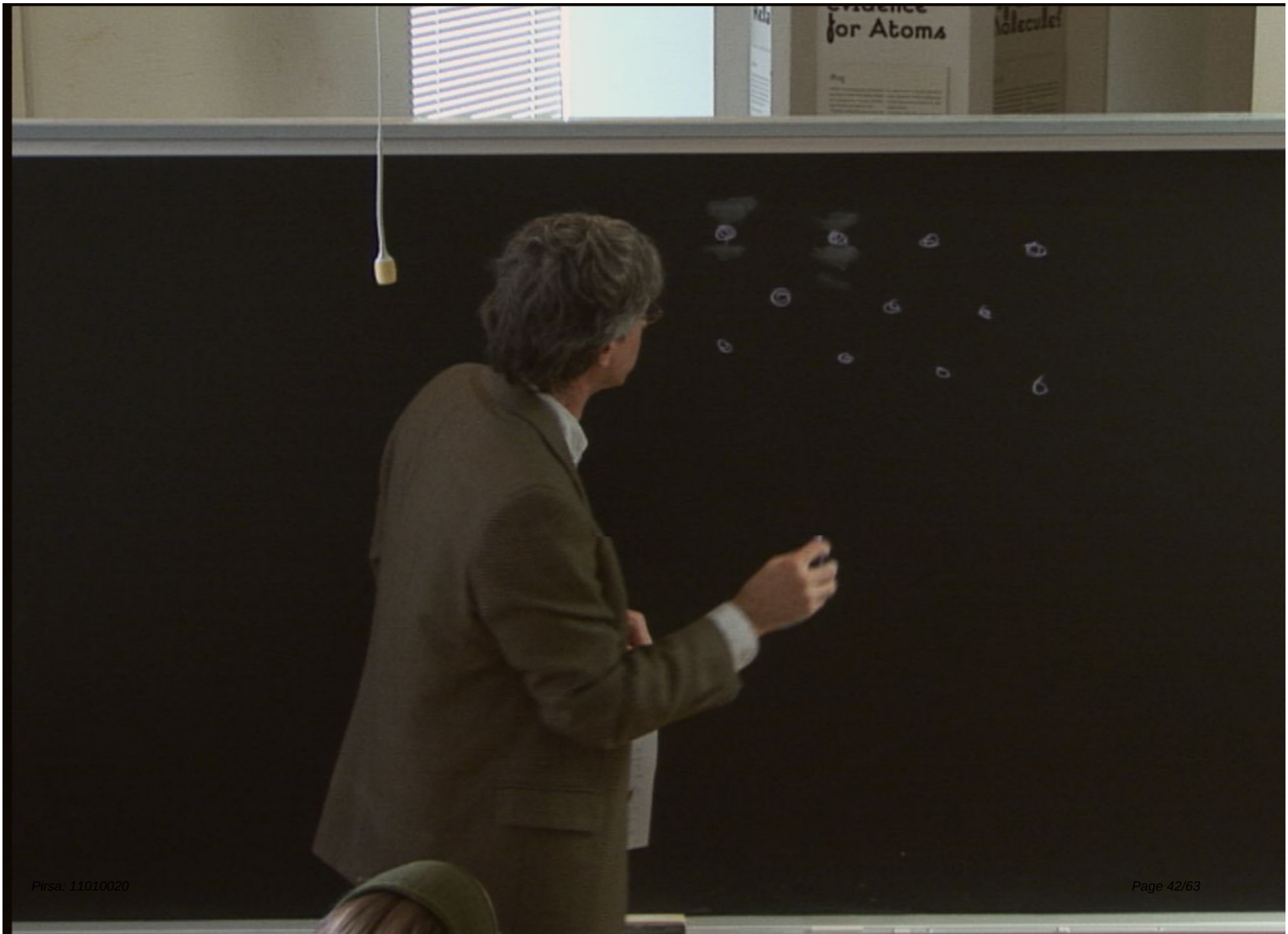
Then $\vec{A} = \dfrac{2\pi\, \vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$ $\quad \vec{a}\cdot\vec{B} = \vec{a}\cdot\vec{C} = 0$

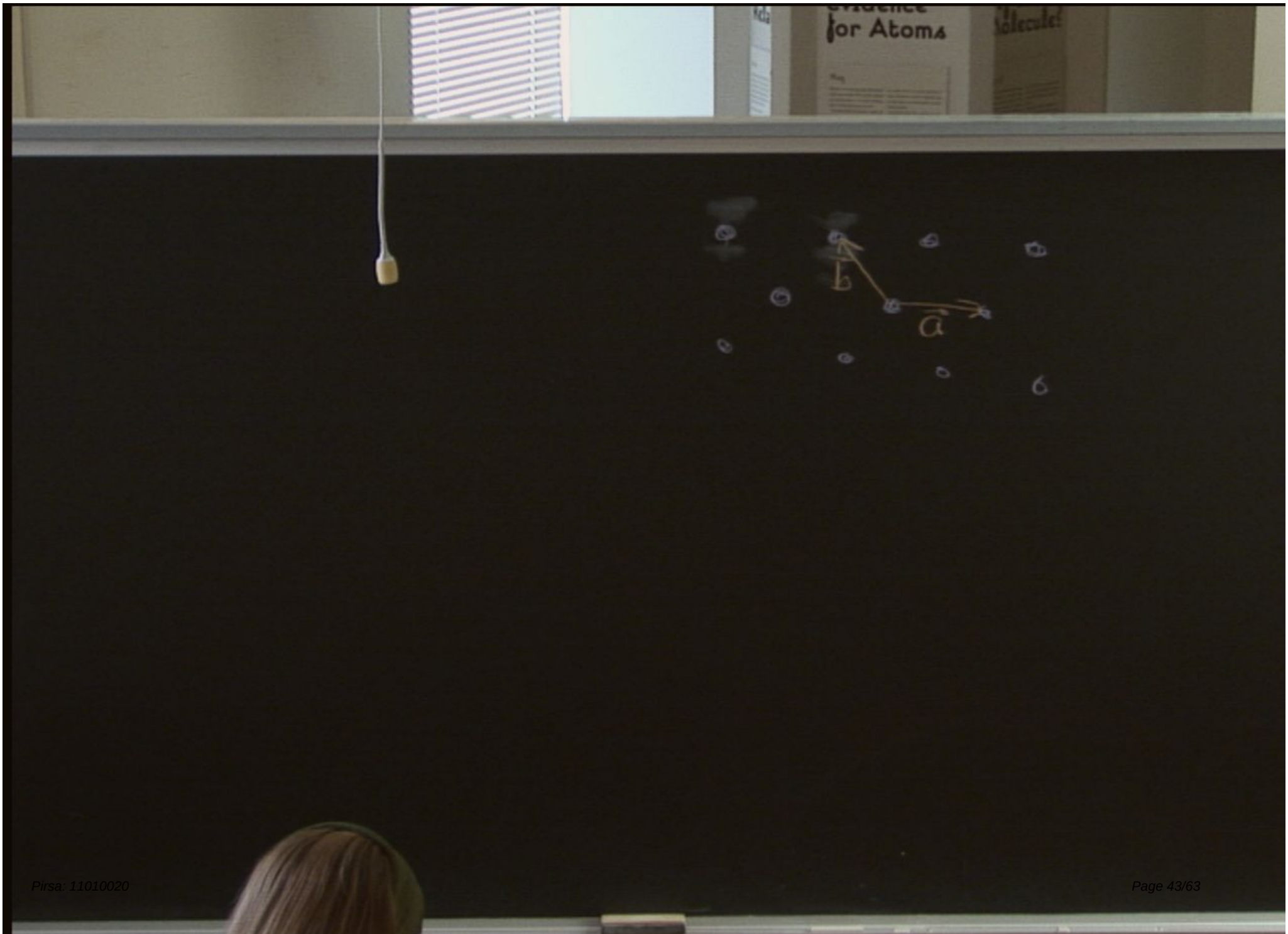
etc

etc $\quad \vec{A}\cdot\vec{b} = \vec{A}\cdot\vec{c} = 0$

$\vec{Q}$ are called "reciprocal lattice vectors"


for Atoms


for Atoms


for Atoms
b
a

$$\vec{Q} = L\vec{A} + M\vec{B} + N\vec{C}$$

where

$$\vec{a}\cdot\vec{A} = 2\pi$$

$$\vec{a}\cdot\vec{B} = \vec{a}\cdot\vec{C} = 0$$

etc

$$\vec{A}\cdot\vec{b} = \vec{A}\cdot\vec{C} = 0$$

Then $\vec{A} = \dfrac{2\pi\, \vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$

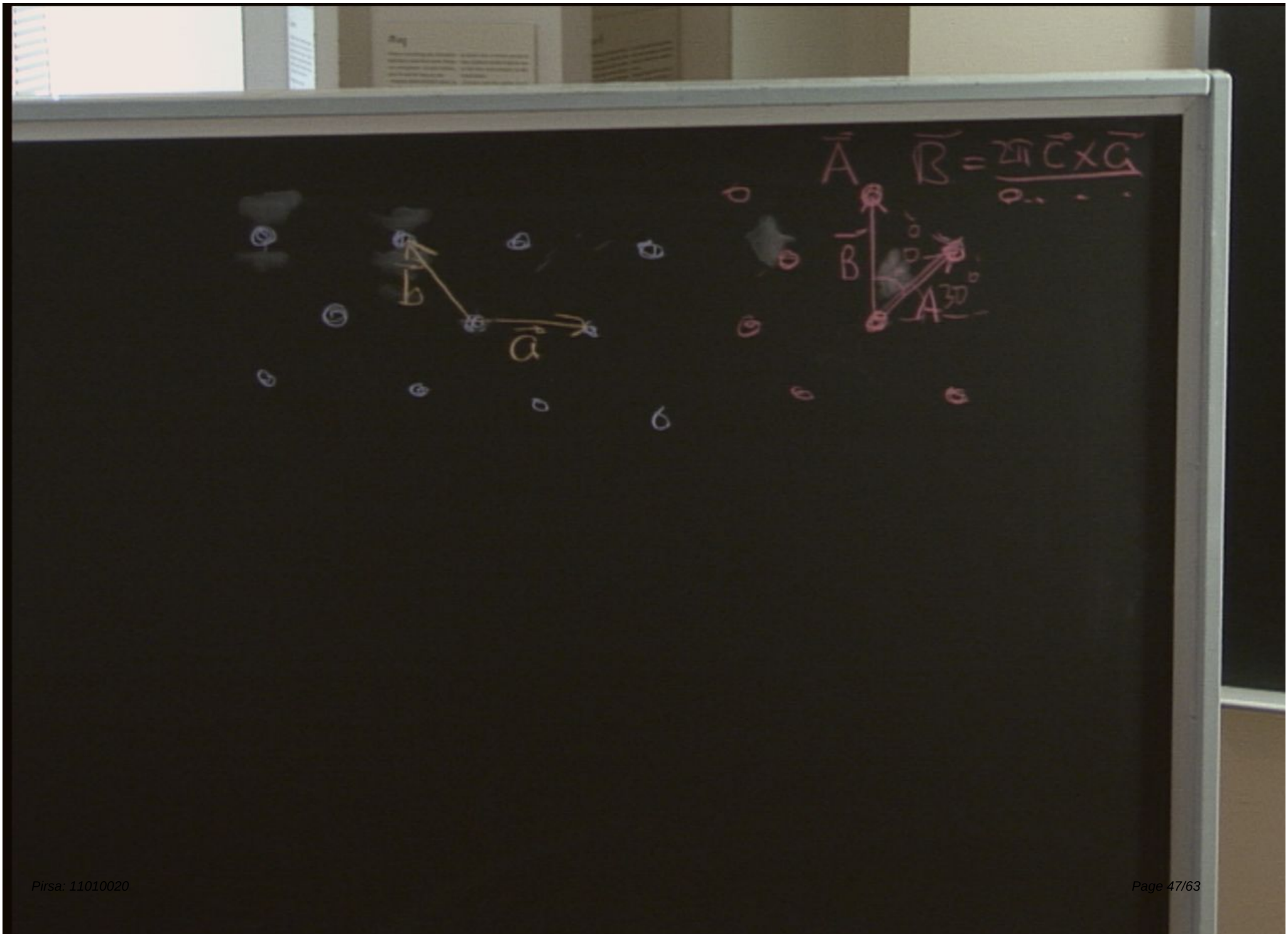
etc

$\vec{Q}$ are called "reciprocal lattice vectors"

RLV's form a periodic lattice with primitive vectors $\vec{A}, \vec{B}, \vec{C}$


$\vec{A}$
$\vec{B} = \frac{2\pi \vec{C} \times \vec{G}}{\ldots}$
$\vec{B}$
A
$\vec{b}$
$\vec{a}$


$\vec{A}$ $\vec{B} = \frac{2\pi \vec{C} \times \vec{G}}{\ldots}$
$\vec{b}$
$\vec{a}$
$\vec{B}$
$\vec{A}$

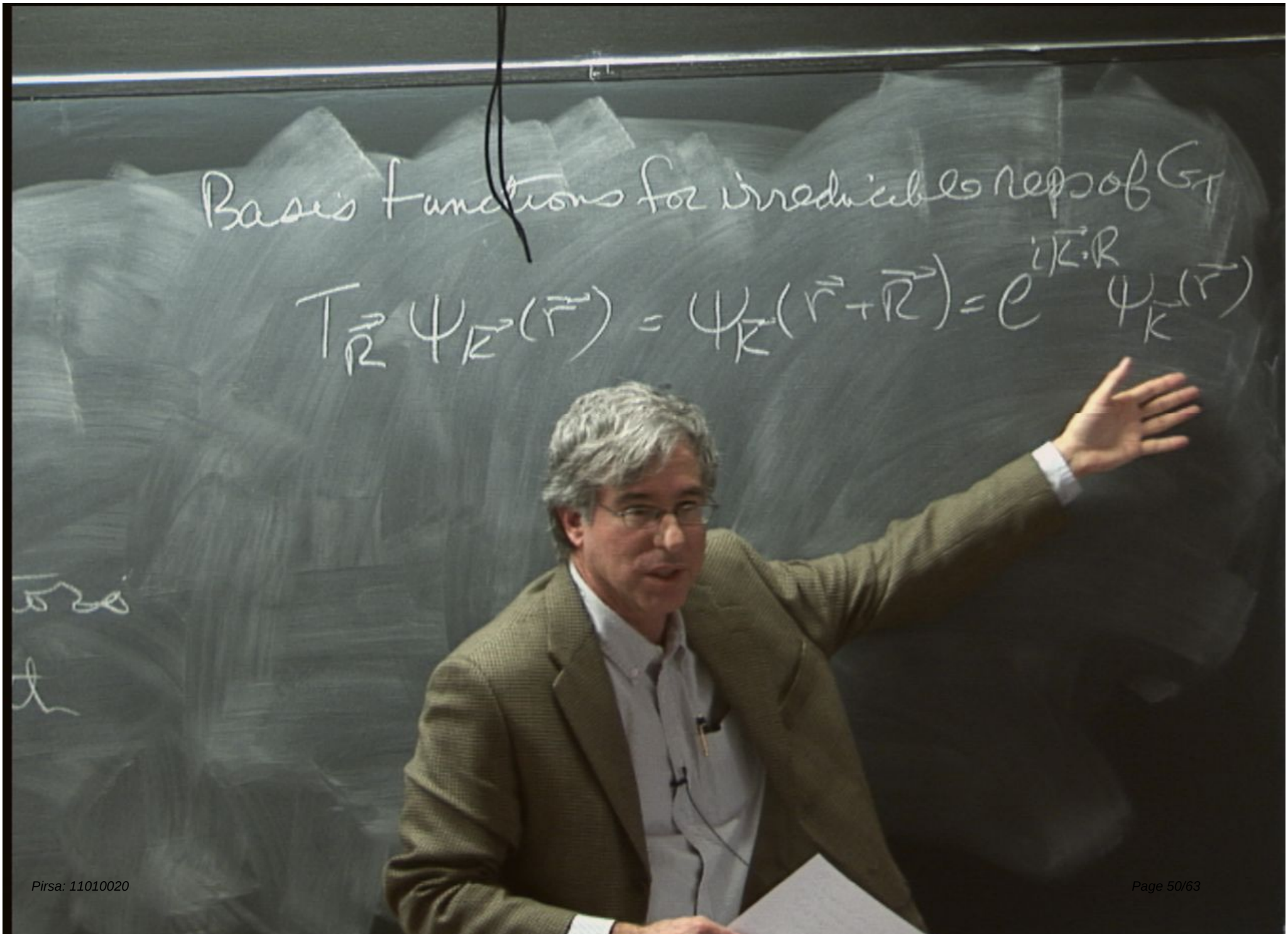

$\vec{A}$ $\vec{B} = \frac{2\pi \vec{C} \times \vec{G}}{\ldots}$
$\vec{b}$
$\vec{a}$
$\vec{B}$
$\vec{A}$
30°

Basis functions for irreducible reps of G_T

$$T_R \Psi_{\vec{k}}(\vec{r})$$

Basis functions for irreducible reps of G_T

$$T_{\vec{R}}\,\psi_{\vec{K}}(\vec{r}) = \psi_{\vec{K}}(\vec{r}+\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\,\psi_{\vec{K}}(\vec{r})$$


Basis functions for irreducible reps of G_T
$T_{\vec{R}}\psi_{\vec{k}}(\vec{r}) = \psi_{\vec{k}}(\vec{r}+\vec{R}) = e^{i\vec{k}\cdot\vec{R}}\psi_{\vec{k}}(\vec{r})$

Basis functions for irreducible reps of G_T

$$T_{\vec{R}}\,\psi_{\vec{K}}(\vec{r}) = \psi_{\vec{K}}(\vec{r}+\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\,\psi_{\vec{K}}(\vec{r})$$

For pbc's

$$\vec{K} = \frac{l}{L}\vec{A} + \frac{m}{L}\vec{B} + \frac{n}{L}\vec{C}$$

Basis functions for irreducible reps of G_T

$$T_{\vec{R}}\,\psi_{\vec{K}}(\vec{r}) = \psi_{\vec{K}}(\vec{r}+\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\,\psi_{\vec{K}}(\vec{r})$$

For pbc's

$$\vec{K} = \frac{l}{L}\vec{A} + \frac{m}{L}\vec{B} + \frac{n}{L}\vec{C}$$

$$\psi_{\vec{K}}(\vec{r}+L\vec{a}) = e^{i\vec{K}\cdot L\vec{a}}\,\psi_{\vec{K}}(\vec{r})$$

$$L\vec{A}\cdot\vec{a}$$

Basis functions for irreducible reps of G_T

Bloch's Theorem $T_{\vec{R}}\,\psi_{\vec{K}}(\vec{r}) = \psi_{\vec{K}}(\vec{r}+\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\,\psi_{\vec{K}}(\vec{r})$

For pbc's

$$\vec{K} = \frac{l}{L}\vec{A} + \frac{m}{L}\vec{B} + \frac{n}{L}\vec{C}$$

$$\psi_{\vec{K}}(\vec{r}+L\vec{a}) = e^{i\vec{K}\cdot L\vec{a}}\,\psi_{\vec{K}}(\vec{r})$$

$$l\,\vec{A}\cdot\vec{a} = 2\pi l$$

$, n = 0, \ldots L-1$

ed modulo a RLV.

Basis functions for irreducible reps of G_T

Bloch's Theorem (Floquet's Thm)

$$T_{\vec{R}}\,\psi_{\vec{K}}(\vec{r}) = \psi_{\vec{K}}(\vec{r}+\vec{R}) = e^{i\vec{K}\cdot\vec{R}}\,\psi_{\vec{K}}(\vec{r})$$

For pbc's

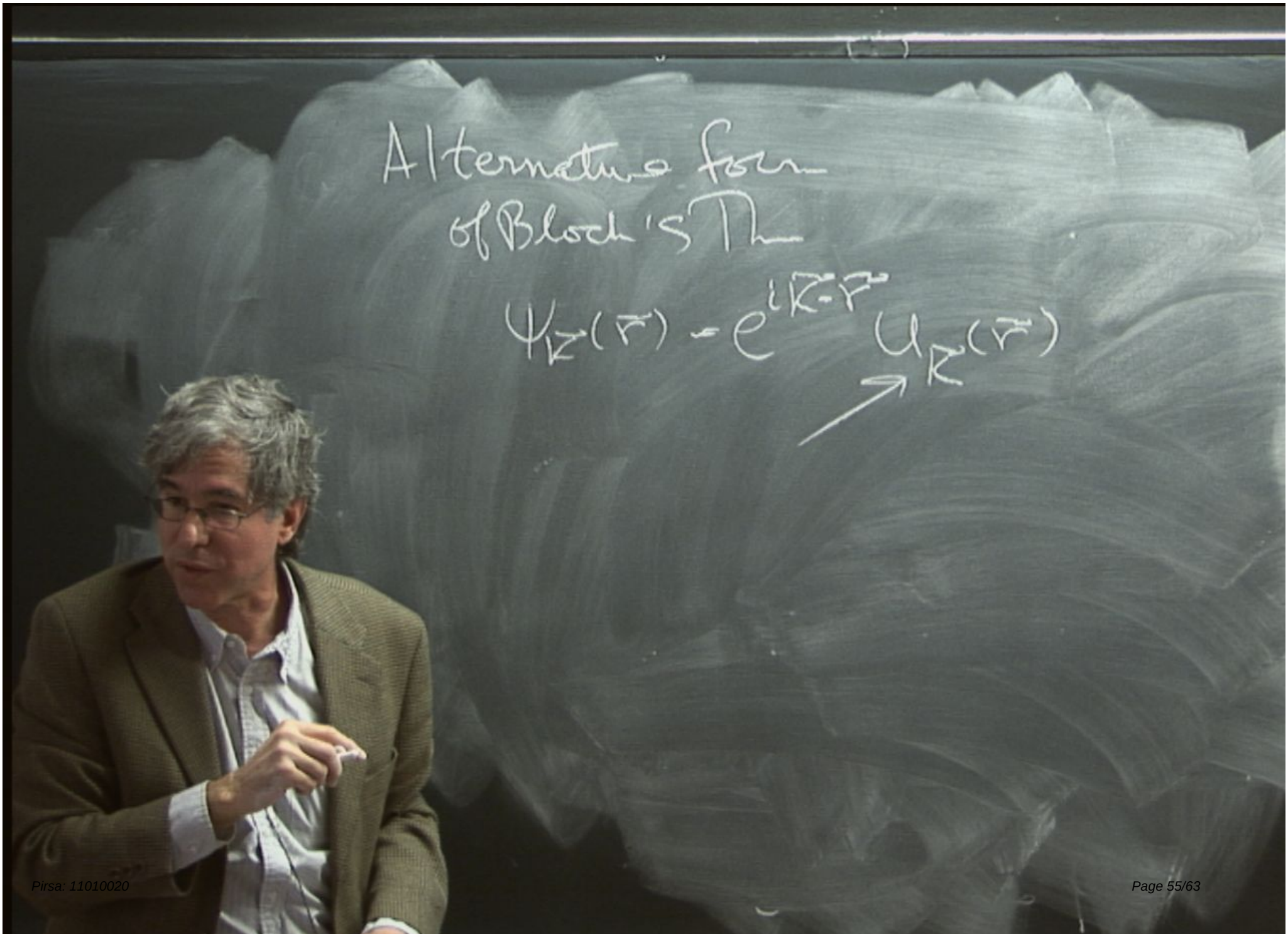
$$\vec{K} = \frac{l}{L}\vec{A} + \frac{m}{L}\vec{B} + \frac{n}{L}\vec{C}$$

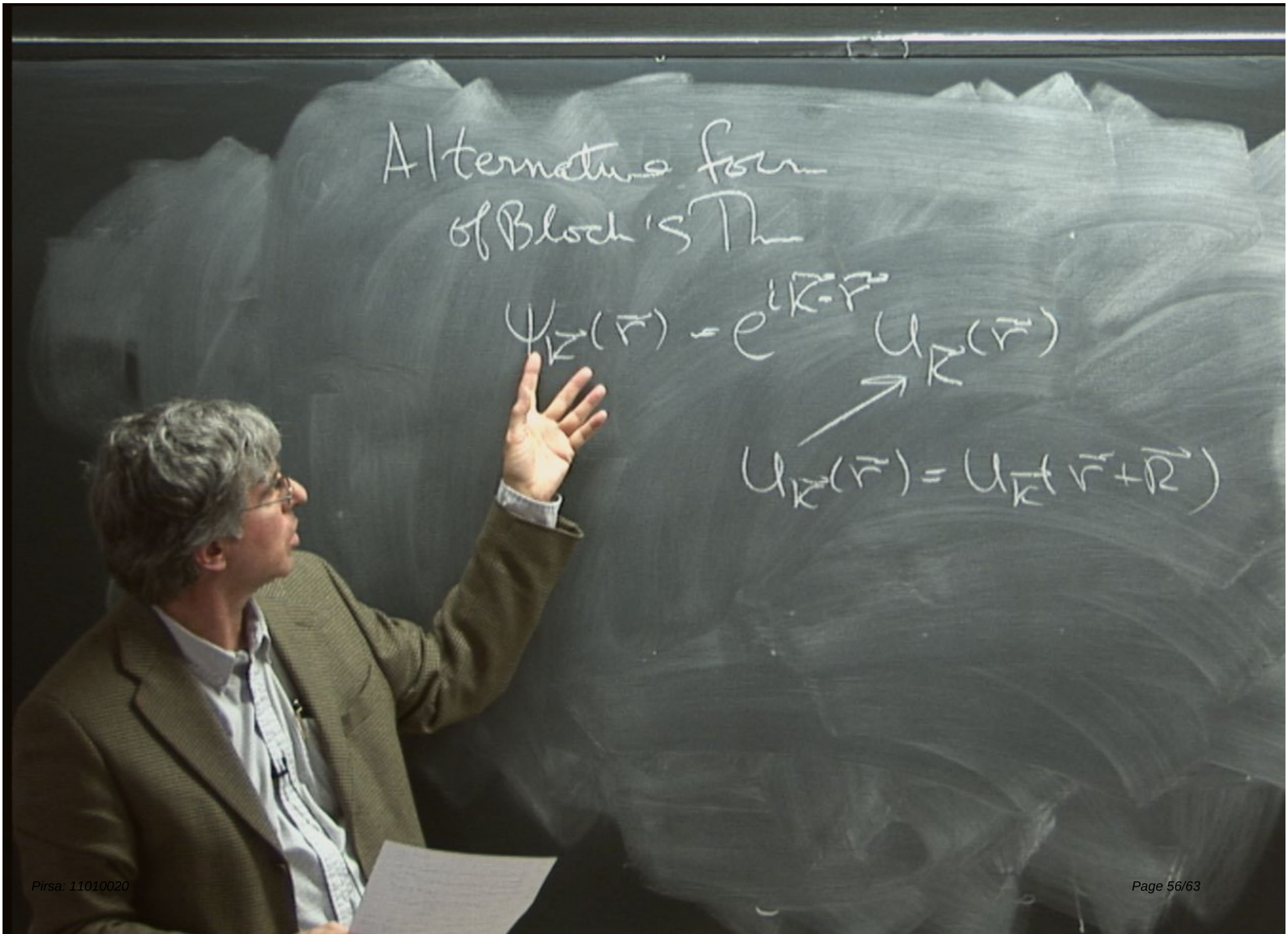
Then $\psi_{\vec{K}}(\vec{r}+L\vec{a}) = e^{i\vec{K}\cdot L\vec{a}}\,\psi_{\vec{K}}(\vec{r})$

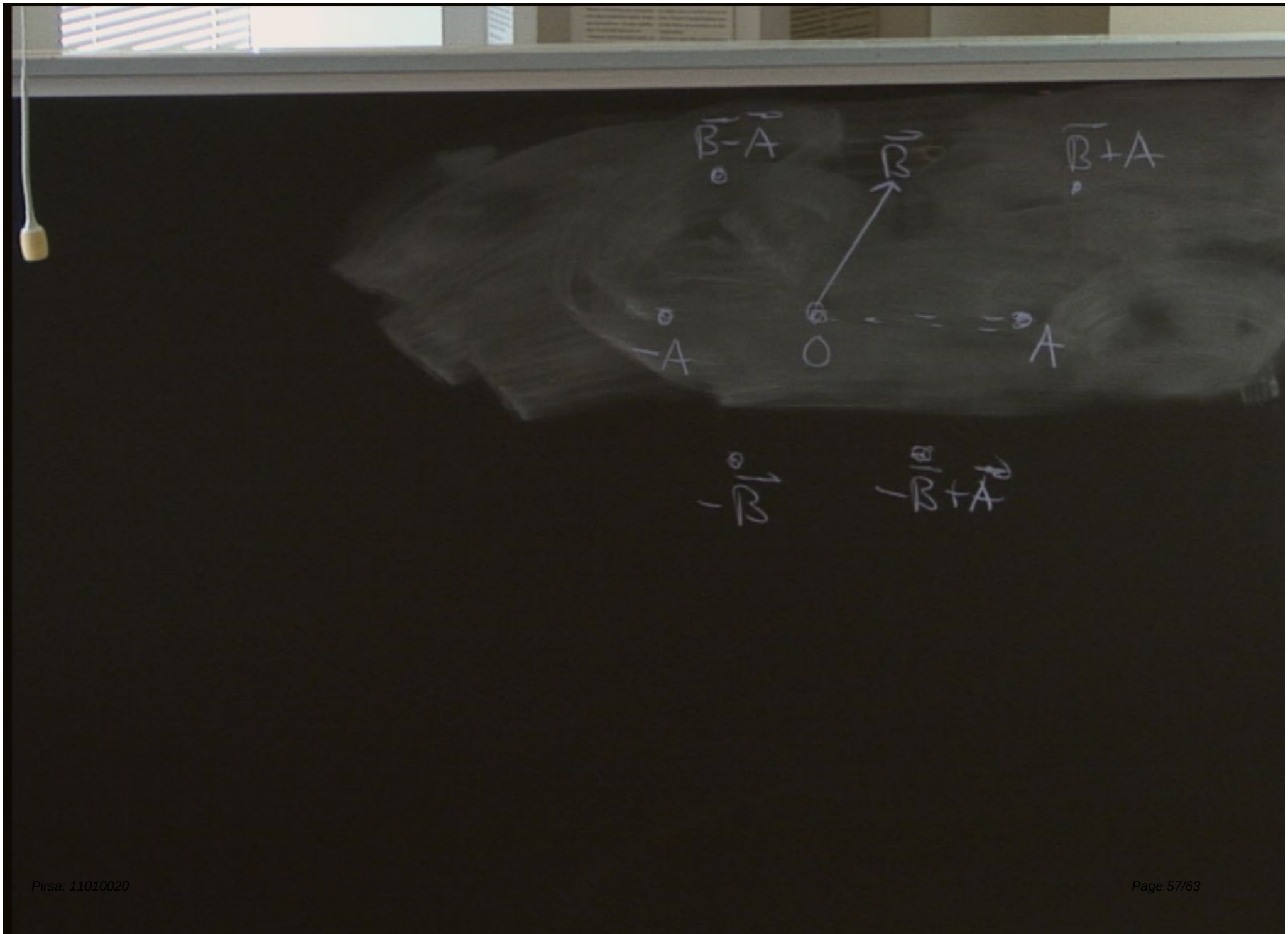
$$l\,\vec{A}\cdot\vec{a} = 2\pi l$$

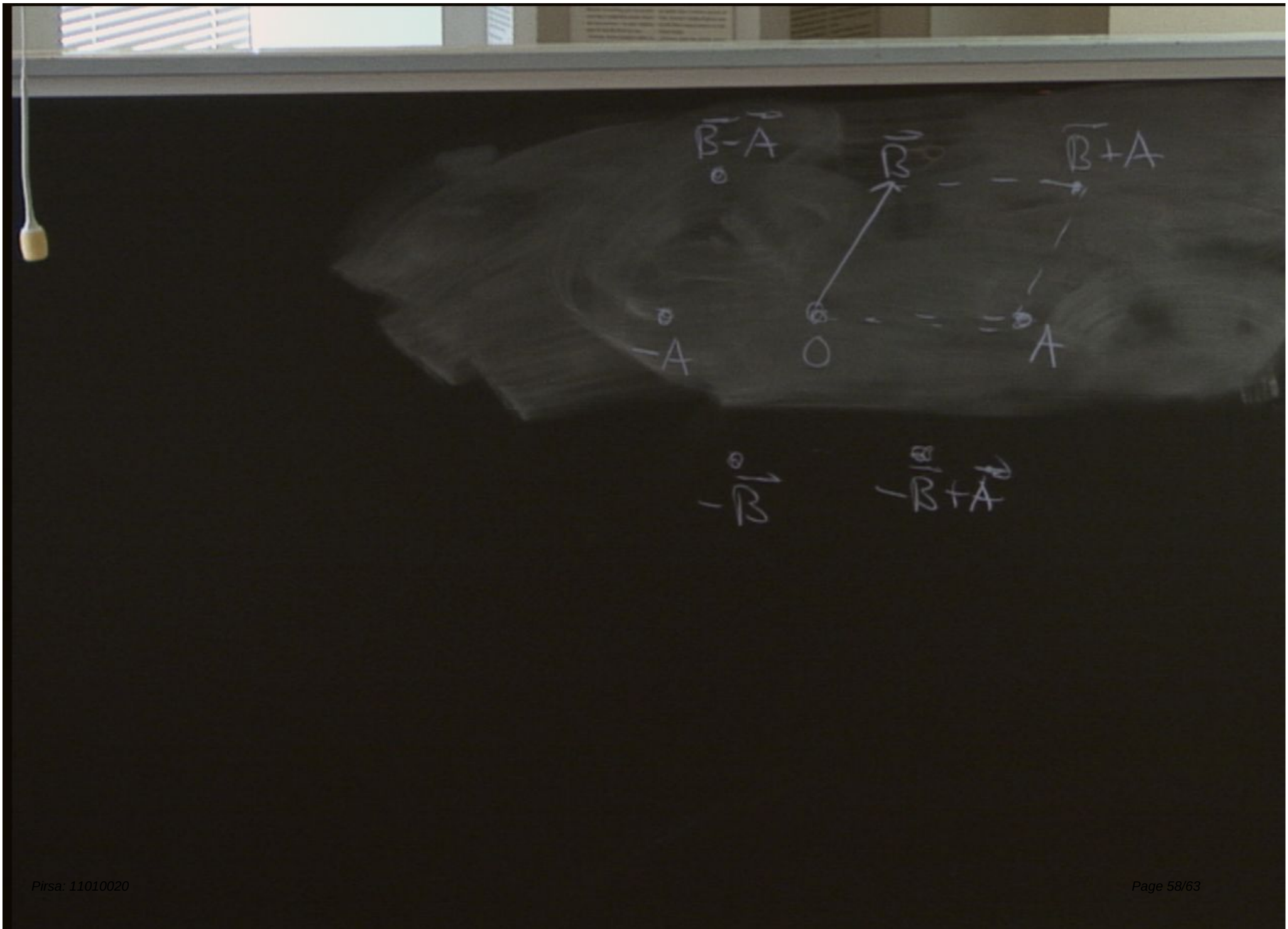
$l, m, n = 0, \ldots L-1$

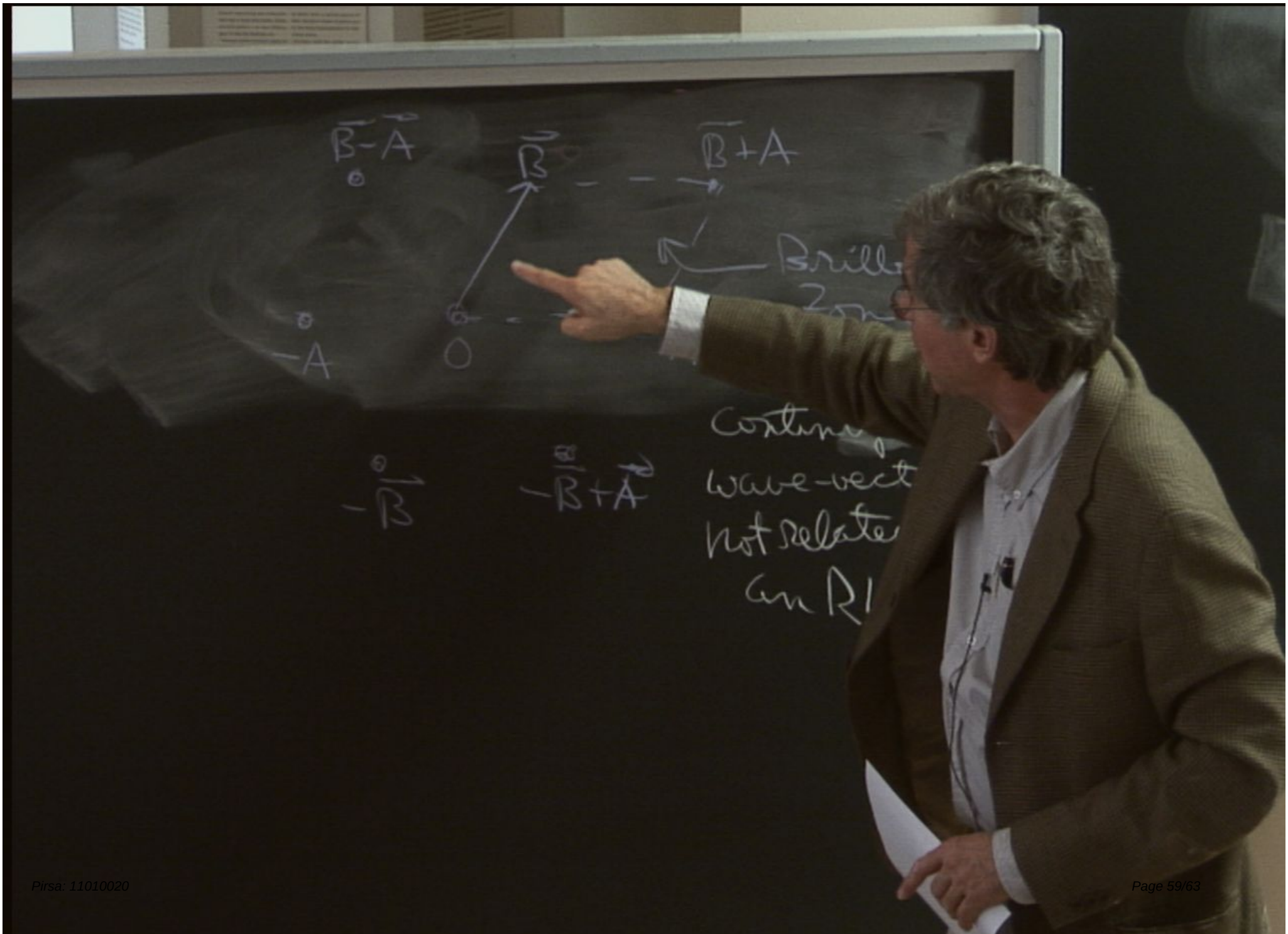
Defined modulo a RLV.

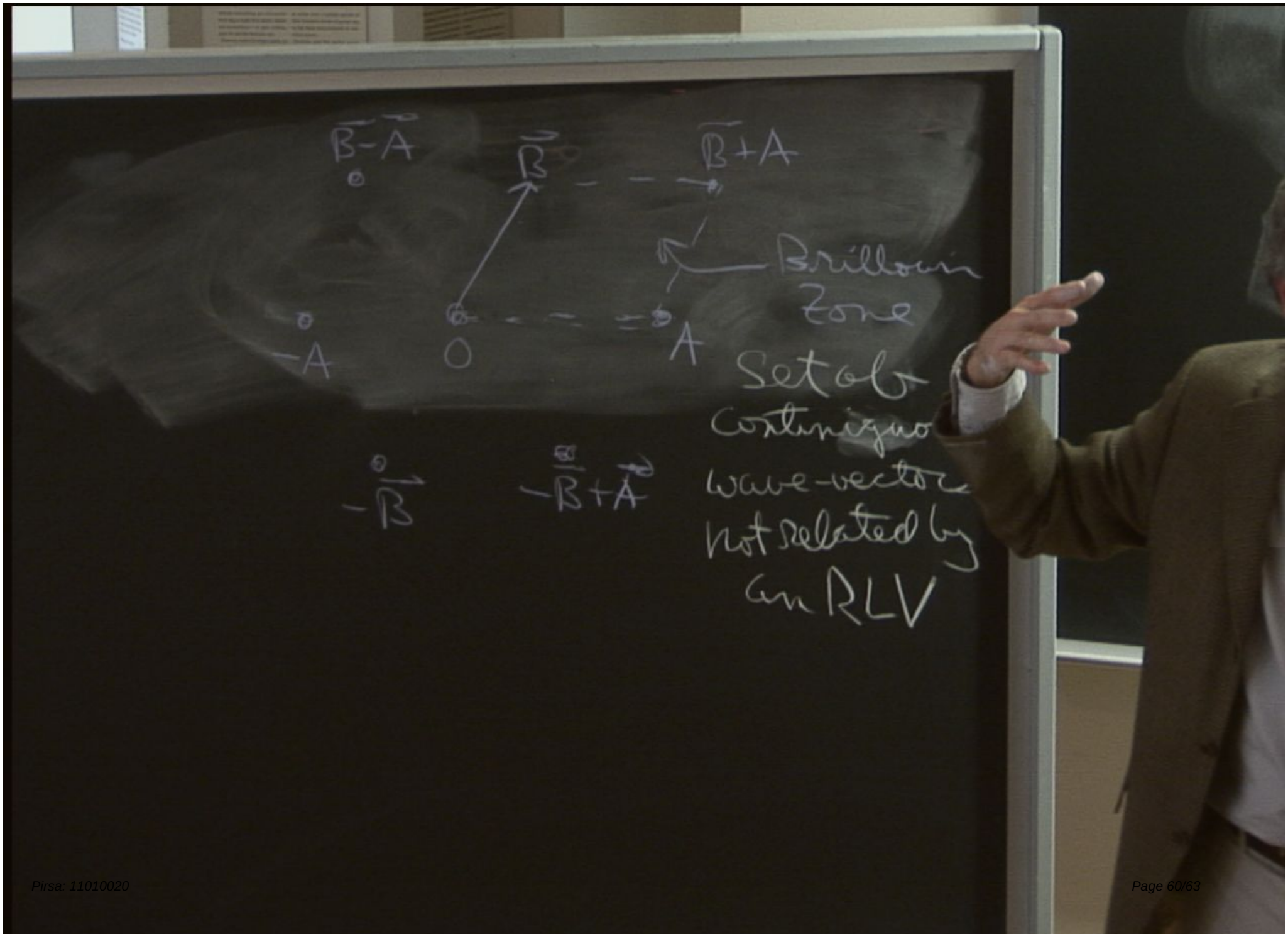

Alternative form
of Bloch's Th
$\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_k(\vec{r})$

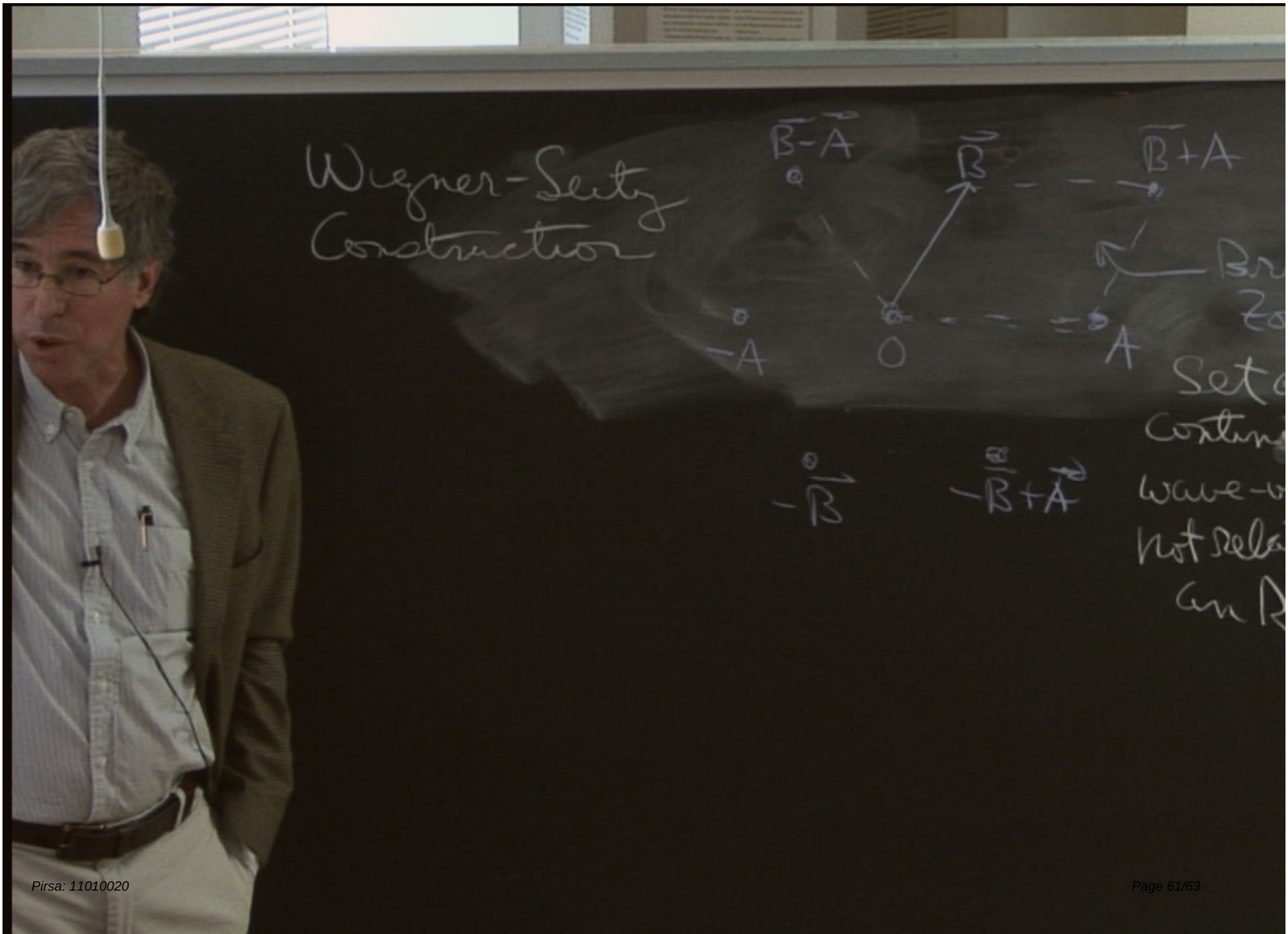

Alternative form
of Bloch's Th
$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r})$
$u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}+\vec{R})$


$\vec{B}-\vec{A}$
$\vec{B}$
$\vec{B}+A$
$-A$
O
A
$-\vec{B}$
$-\vec{B}+\vec{A}$

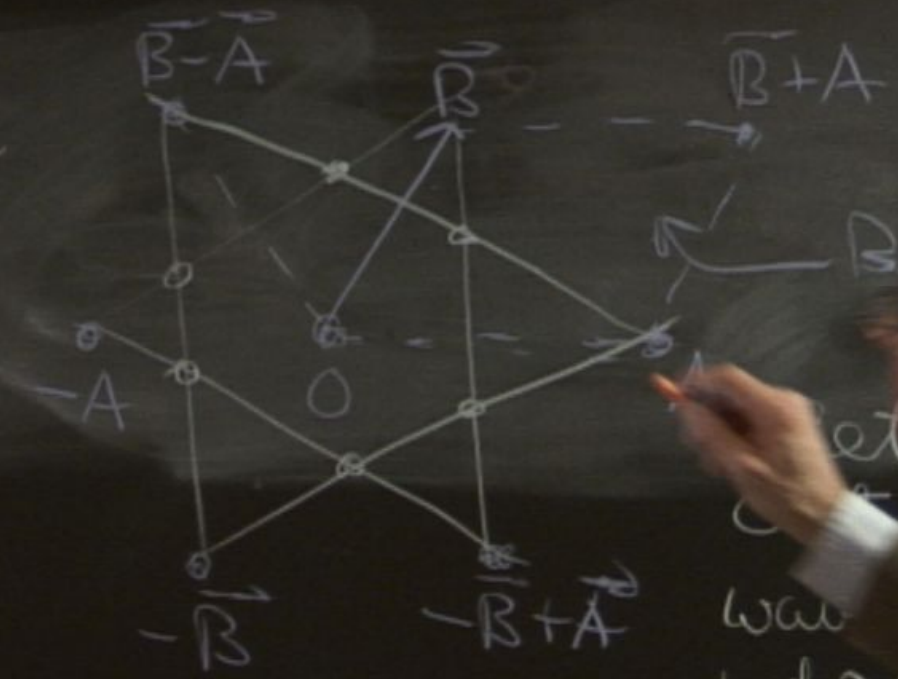

$\vec{B}-\vec{A}$
$\vec{B}$
$\vec{B}+A$
$-A$
O
A
$-\vec{B}$
$-\vec{B}+\vec{A}$


$\vec{B}-\vec{A}$
$\vec{B}$
$\vec{B}+A$
$-A$
O
$-\vec{B}$
$-\vec{B}+\vec{A}$

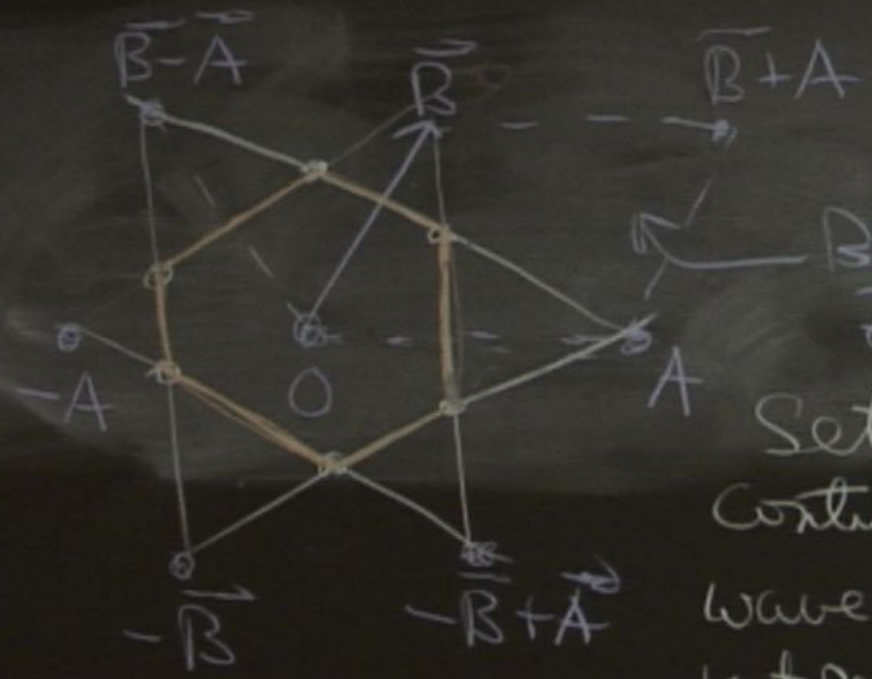

$\vec{B}-\vec{A}$
$\vec{B}$
$\vec{B}+A$
Brillouin Zone
$-A$
O
A
Set of continuous wave-vectors not related by an RLV
$-\vec{B}$
$-\vec{B}+\vec{A}$


Wigner-Seitz
Construction
$\vec{B}-\vec{A}$
$\vec{B}$
$\vec{B}+A$
$-A$
O
A
$-\vec{B}$
$-\vec{B}+\vec{A}$

Wigner-Seitz Construction



Wigner-Seitz Construction



Brillouin Zone

Set of contiguous wave-vectors not related by an RLV