Title: Condensed Matter Review - Lecture 1

Date: Jan 03, 2011 10:15 AM

URL: http://pirsa.org/11010019

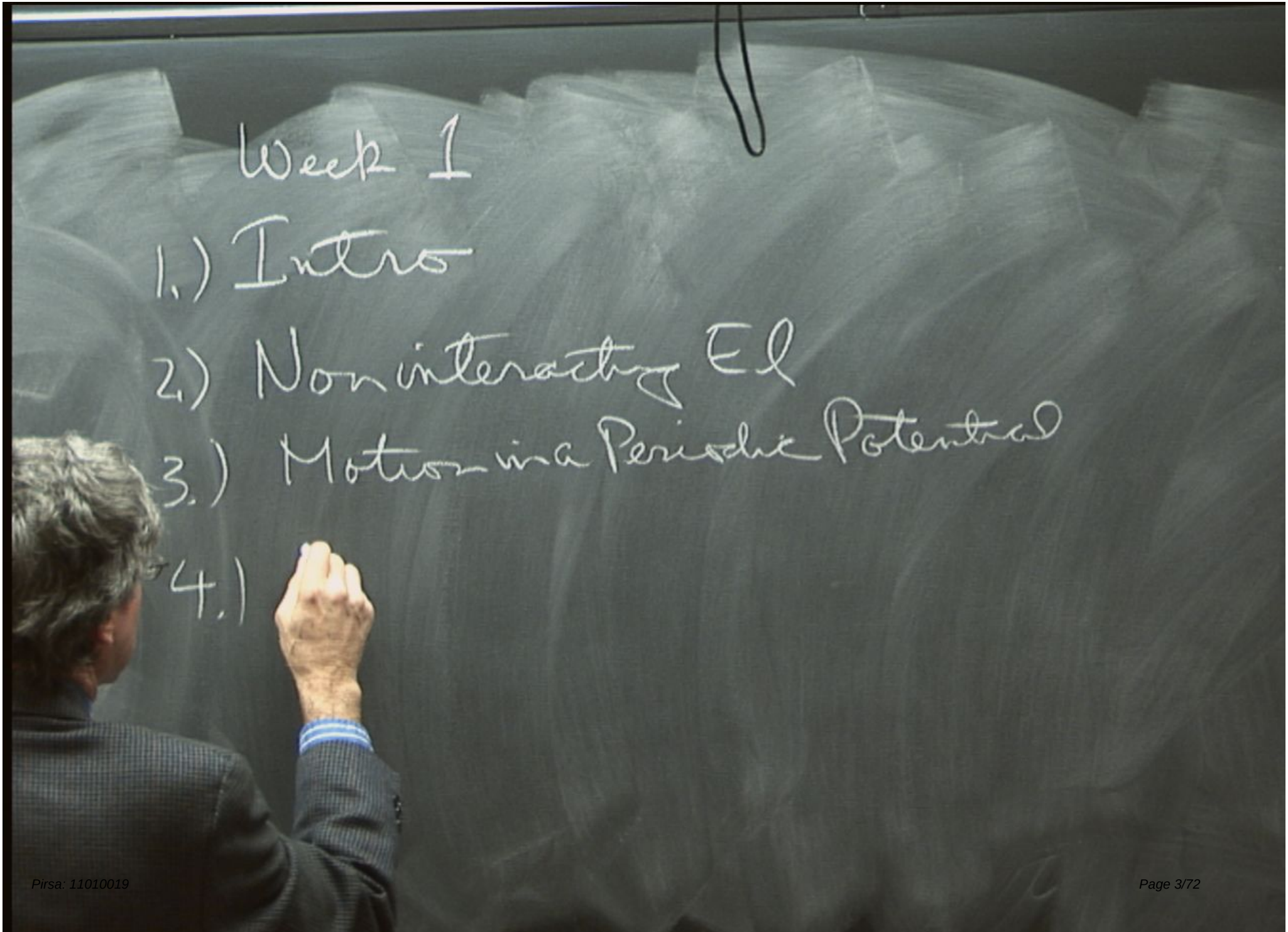
Abstract:

Course Plan

Week 1 — Normal Metals - Fermi Liquid

Week 2 - Conventional (BCS) Superconductors

Week 3 - Unconventional Metals + S.C.'s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NFE
Tight-Binding
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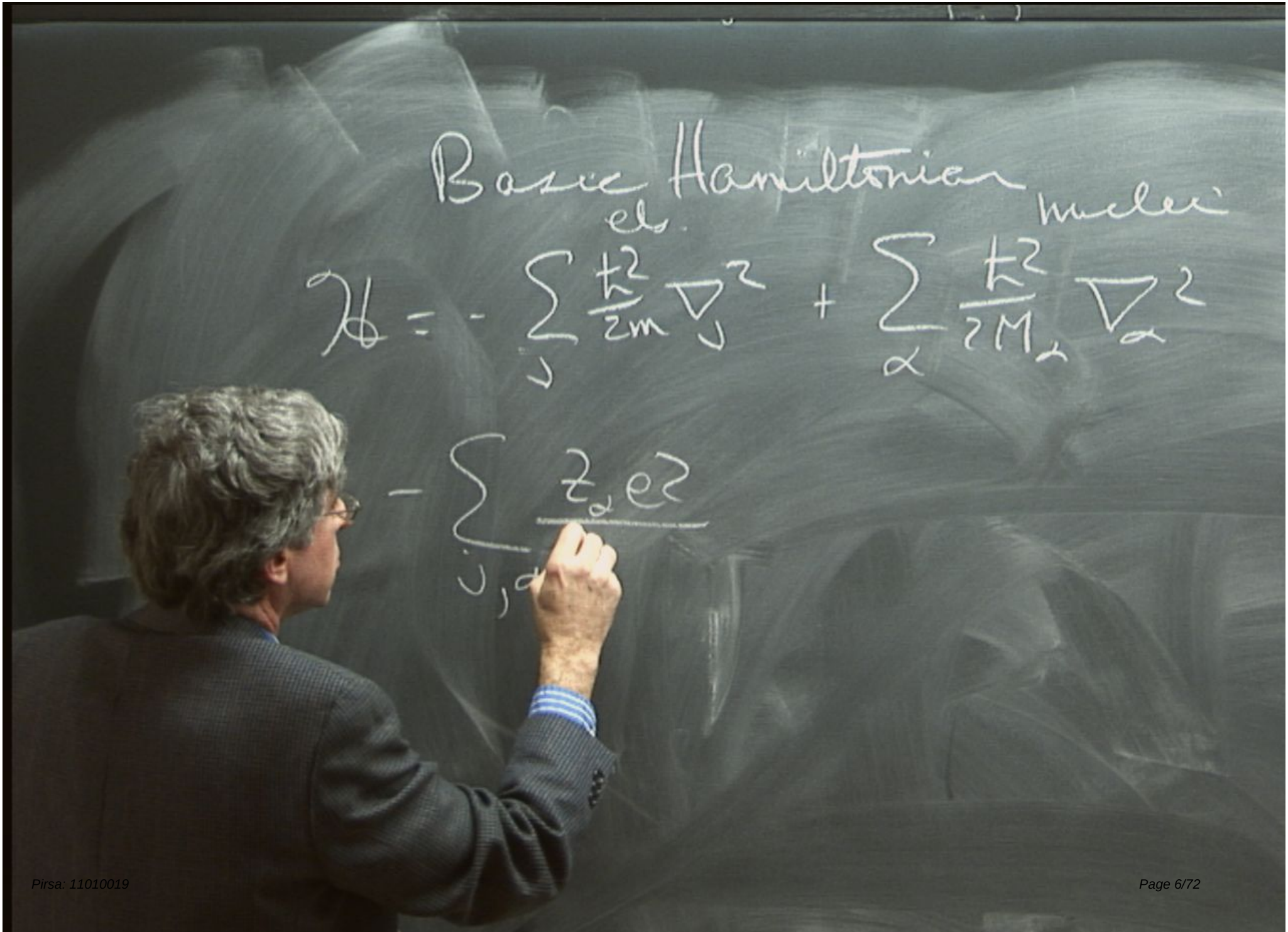
5. Hartree Fock for Fermions

6. Landaus Fermi Liquid Theory


Basic Hamiltonian
el.
nuclei
$\mathcal{H} = -\sum_j \frac{\hbar^2}{2m}\nabla_j^2 + \sum_\alpha \frac{\hbar^2}{2M_\alpha}\nabla_\alpha^2$
$-\sum_{j,\alpha} \frac{Z_\alpha e^2}{}$

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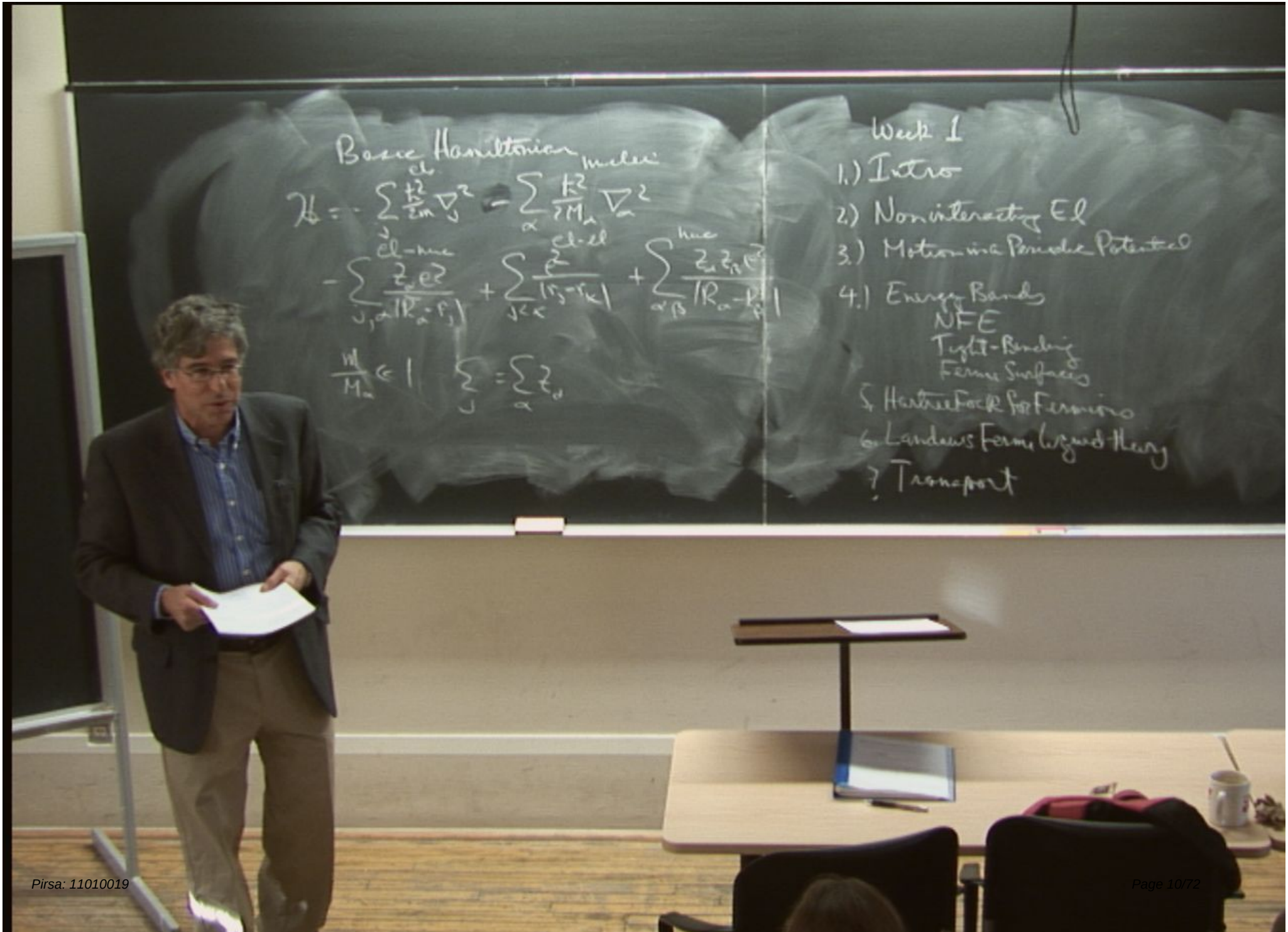
$$\frac{m}{M_\alpha} \ll 1 \qquad \sum_j = \sum_\alpha Z_\alpha$$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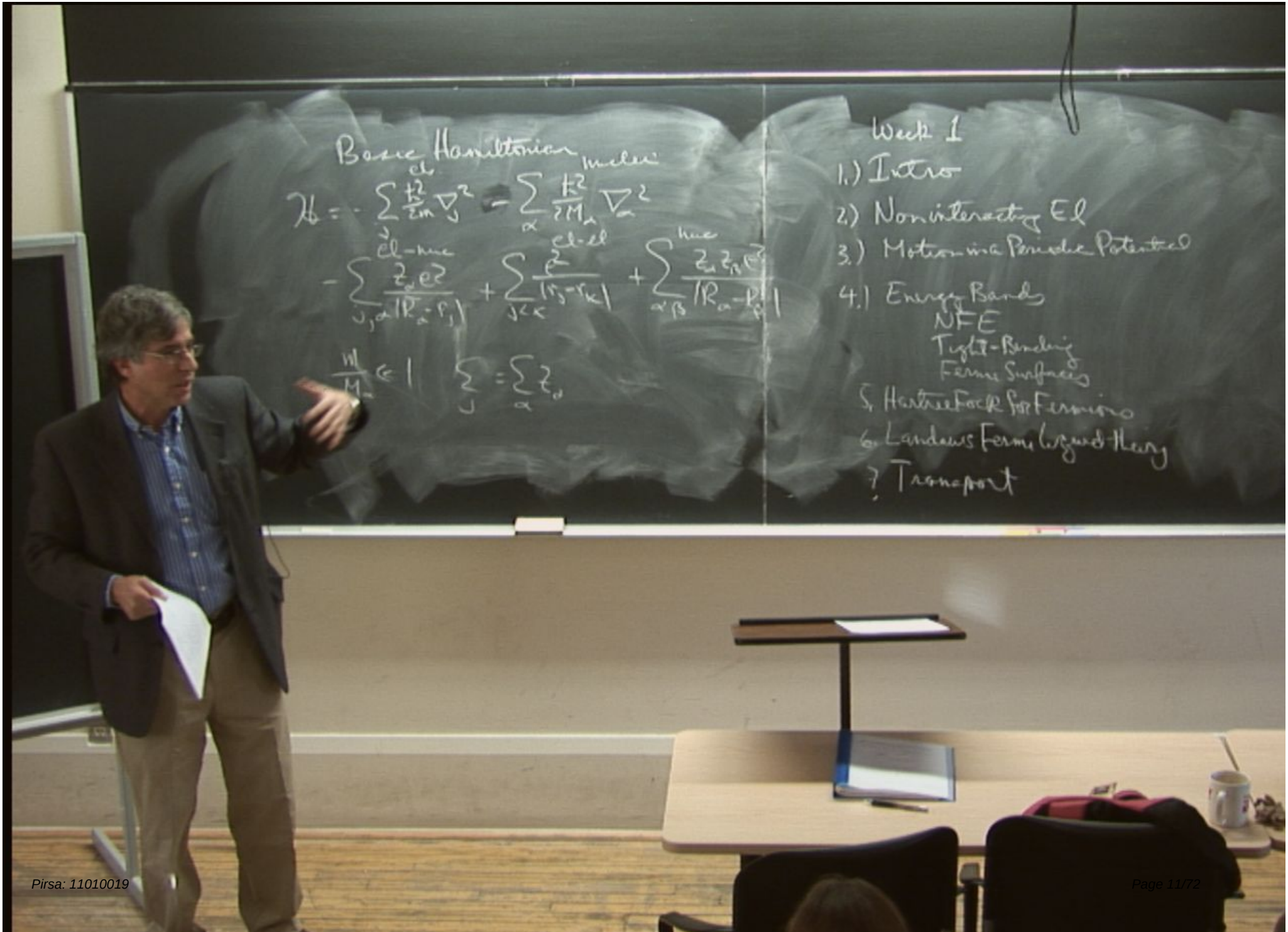
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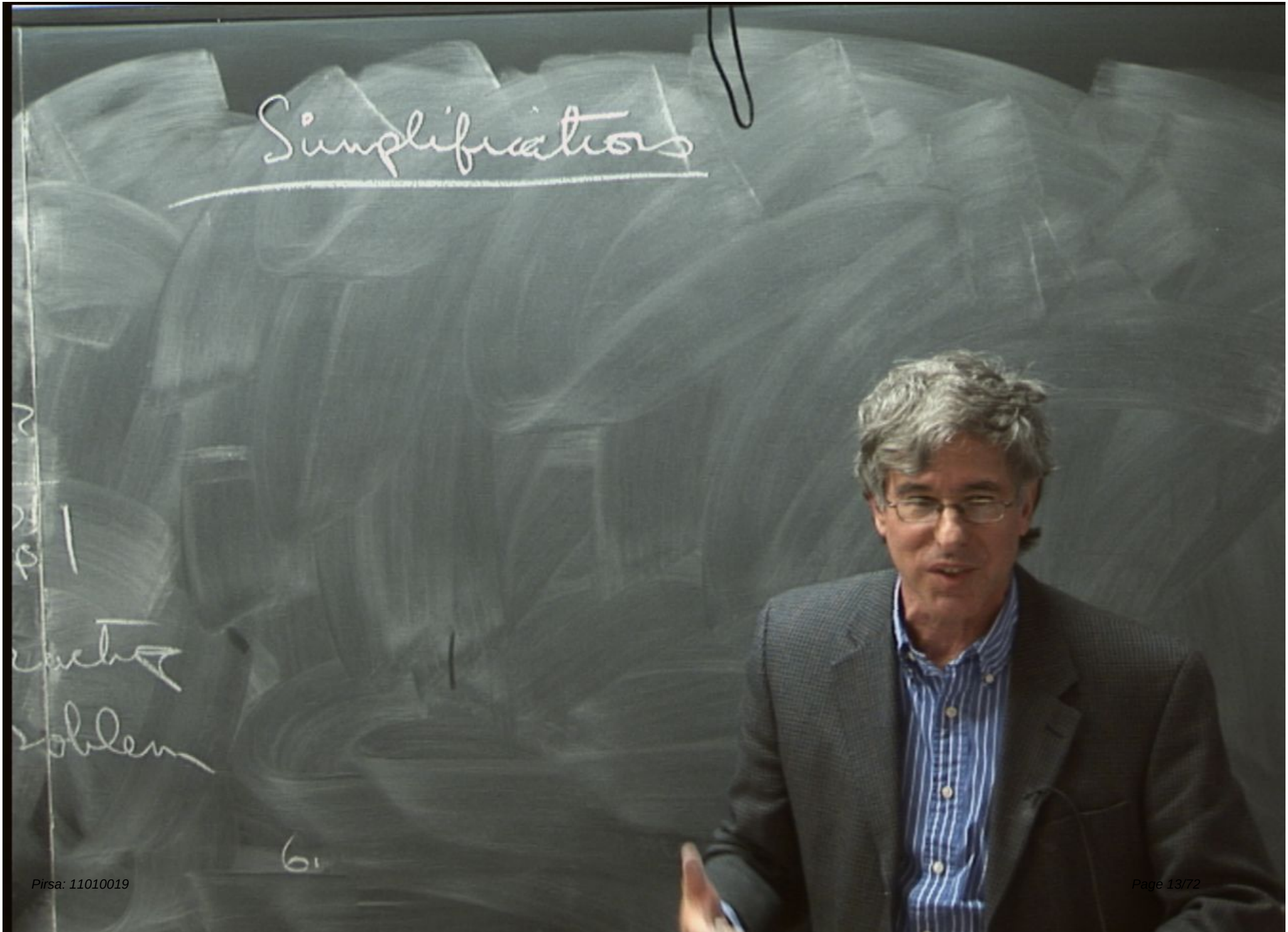
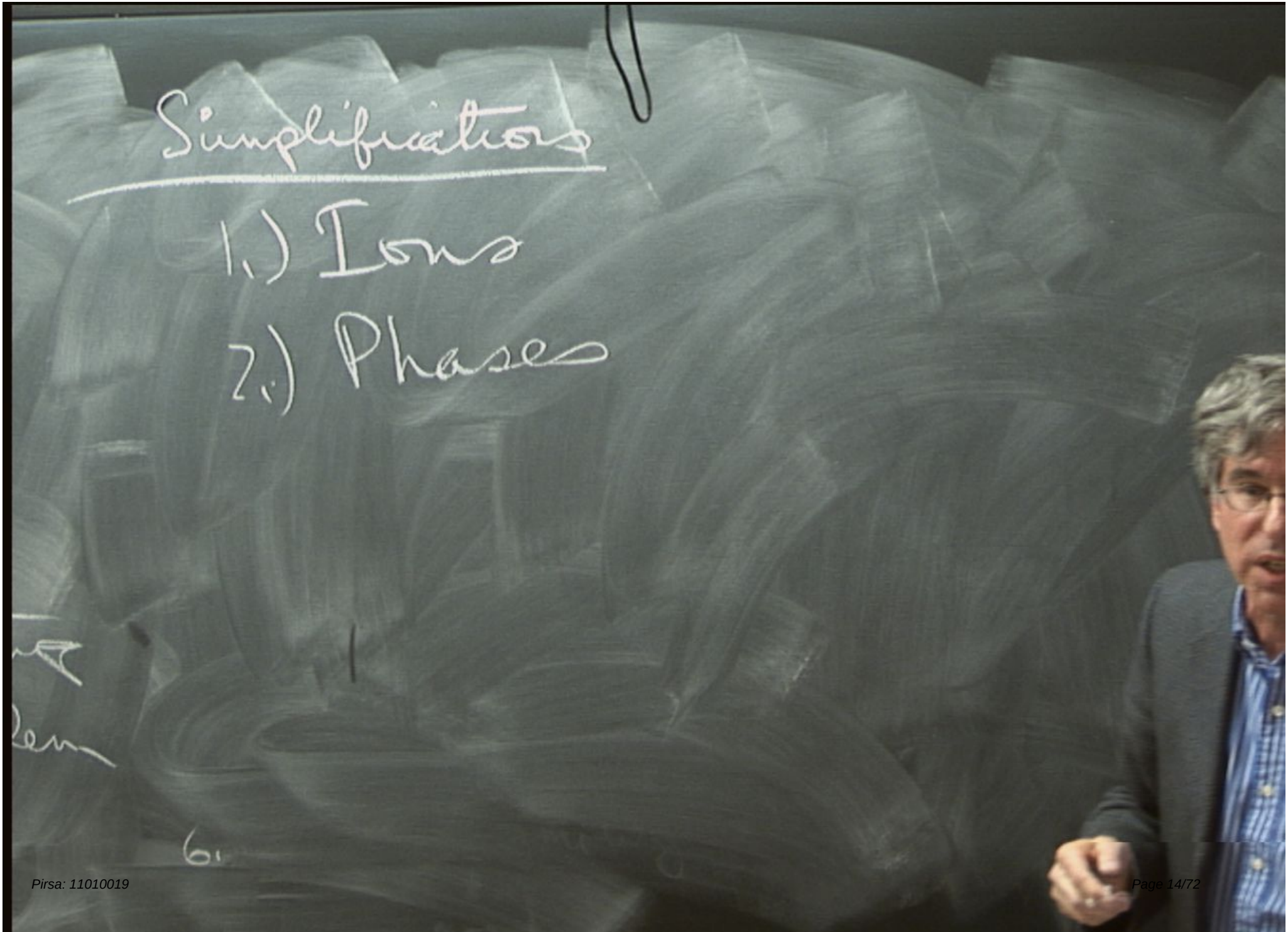
Strongly Interacting Many-Body Problem


Simplifications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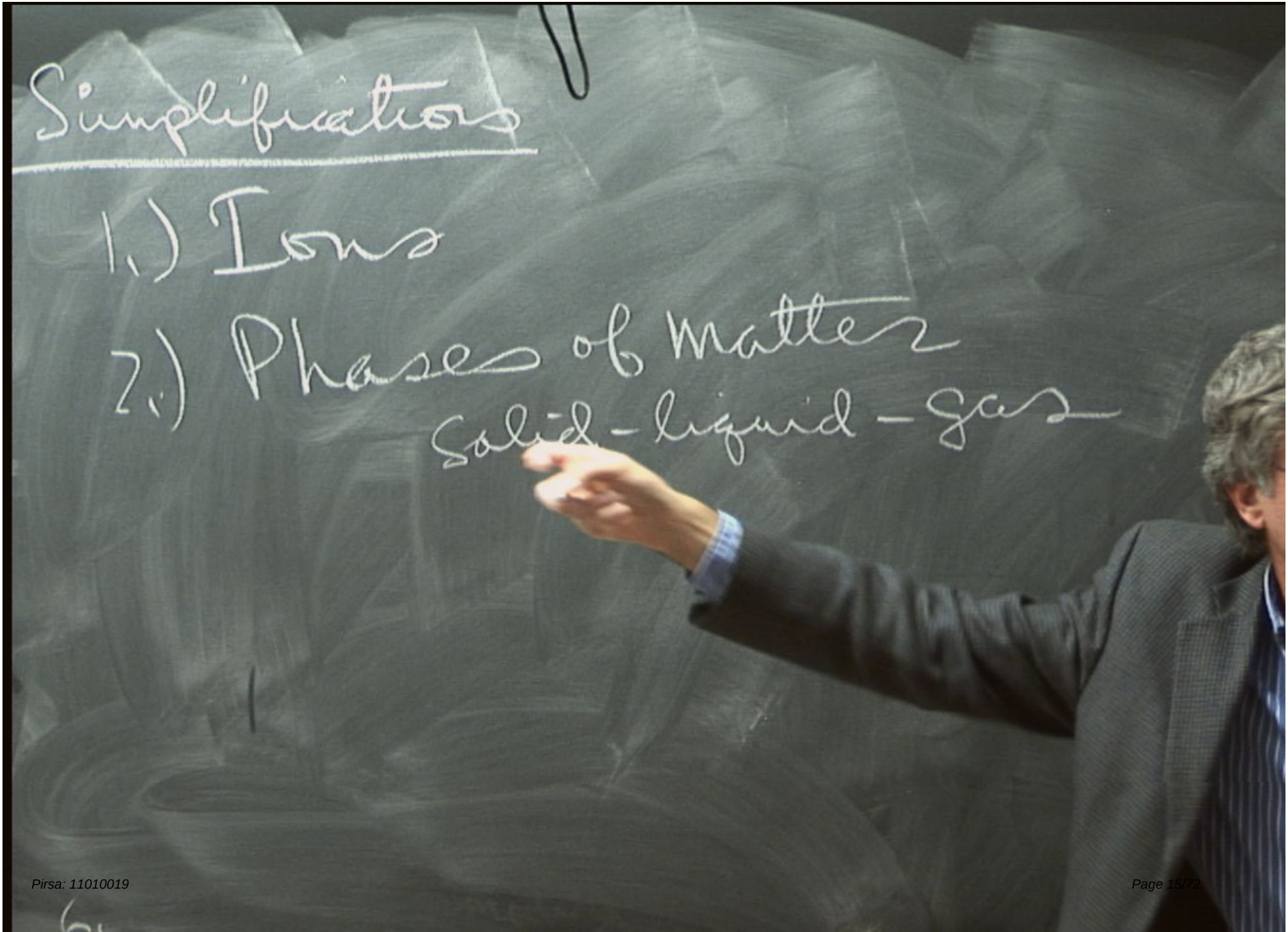
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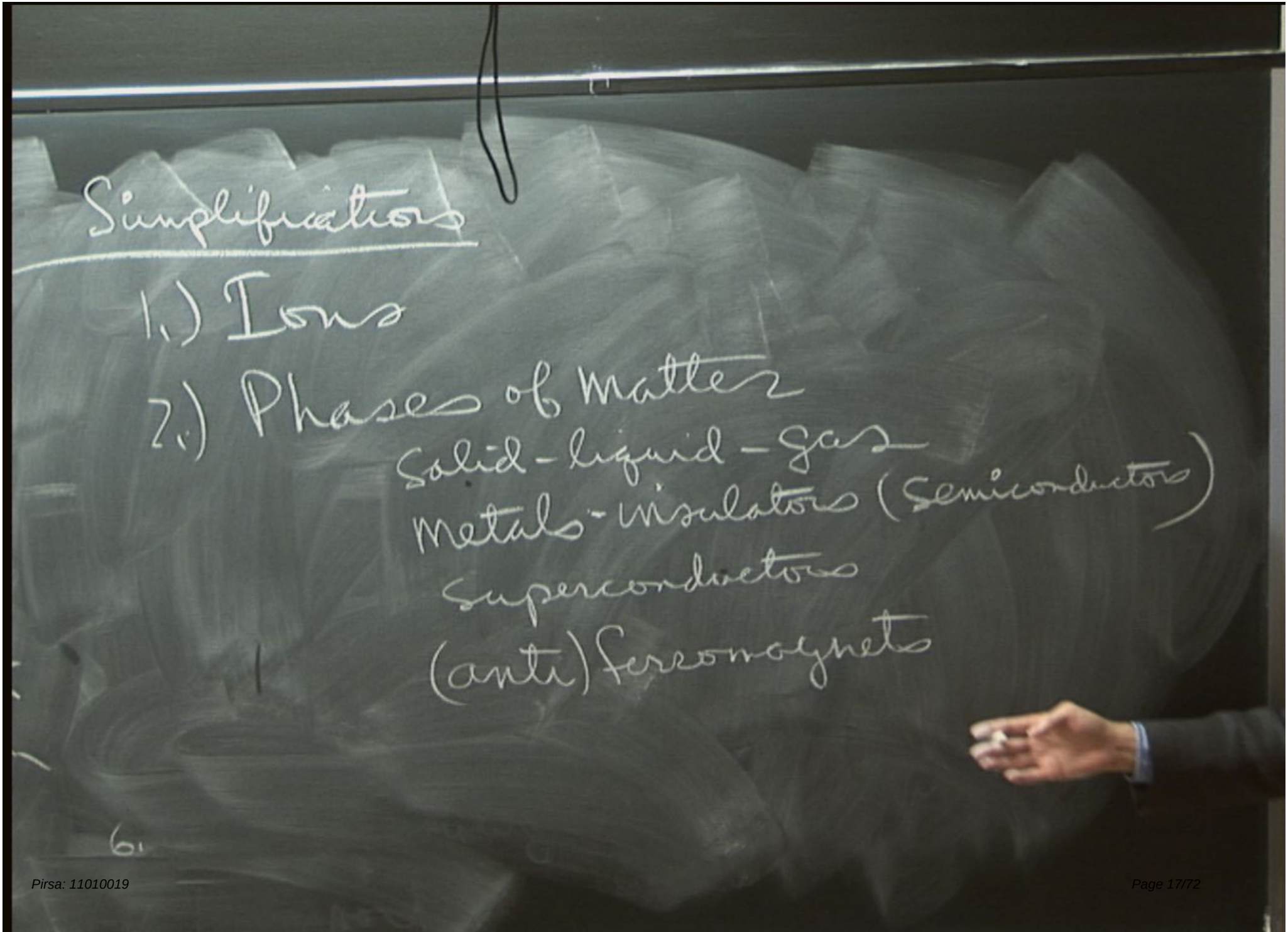
Solid - liquid - gas

Metals - insulators (semiconductors)

superconductors

6.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Metals - insulators (semiconductors)
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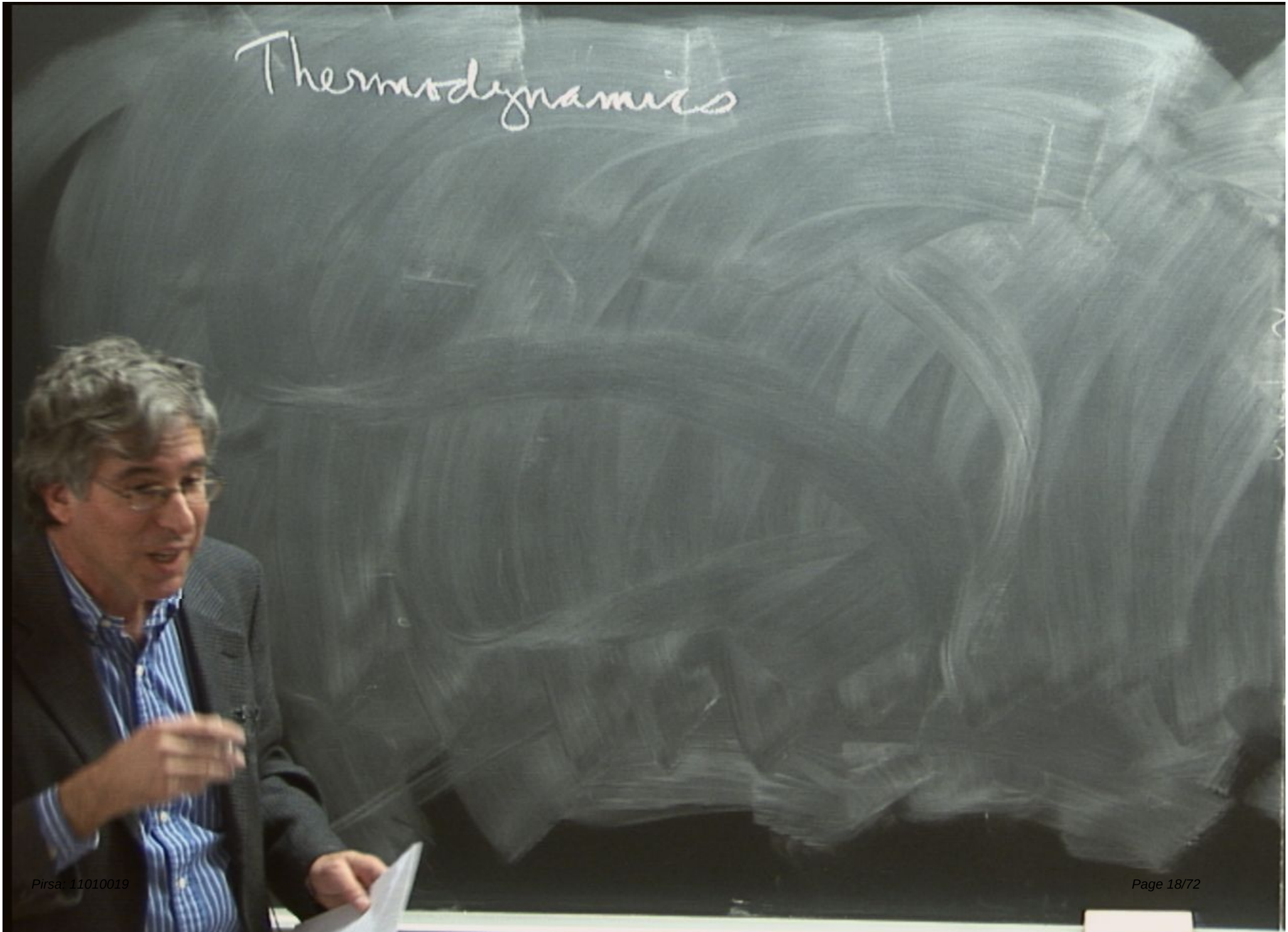
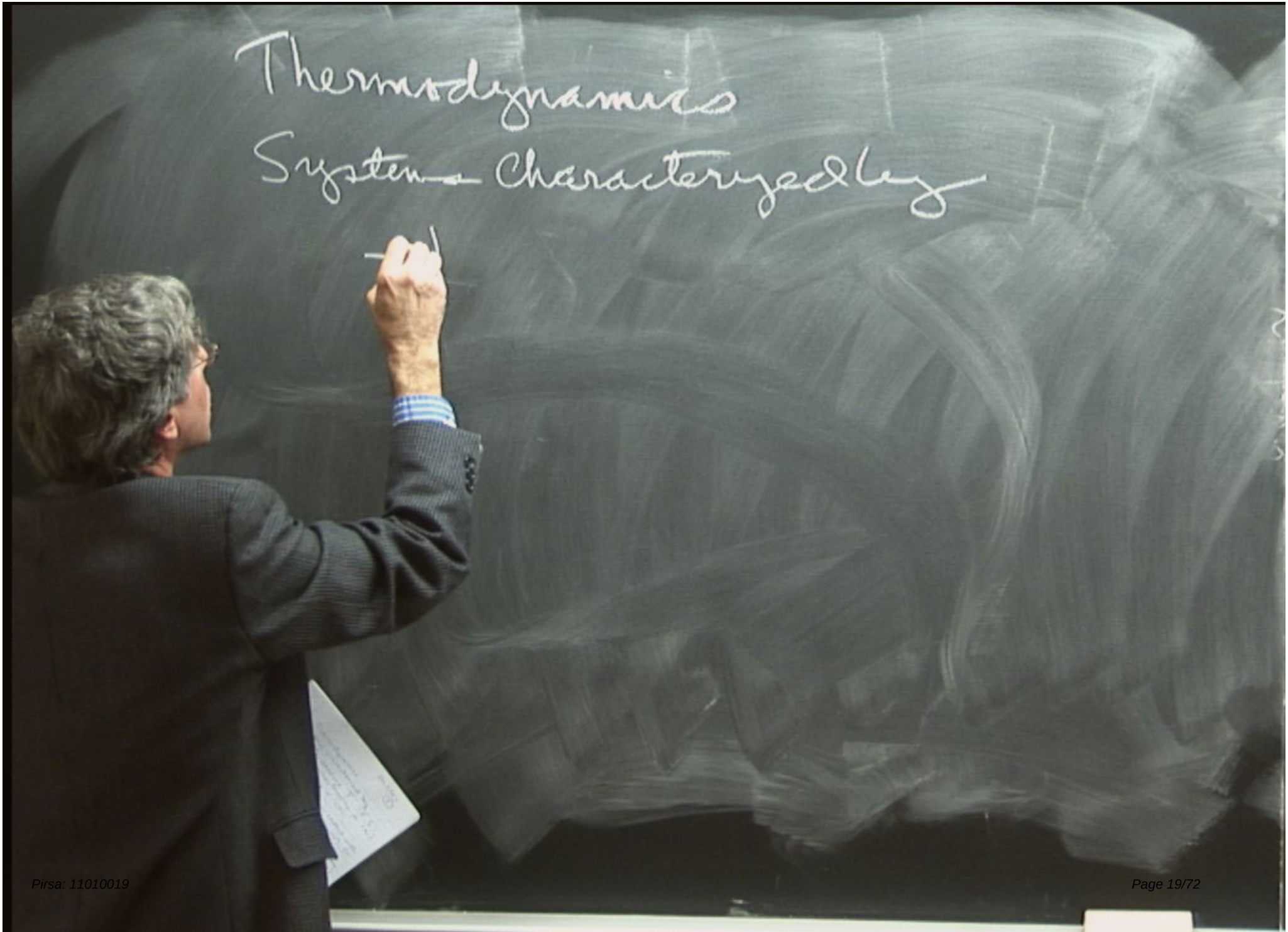
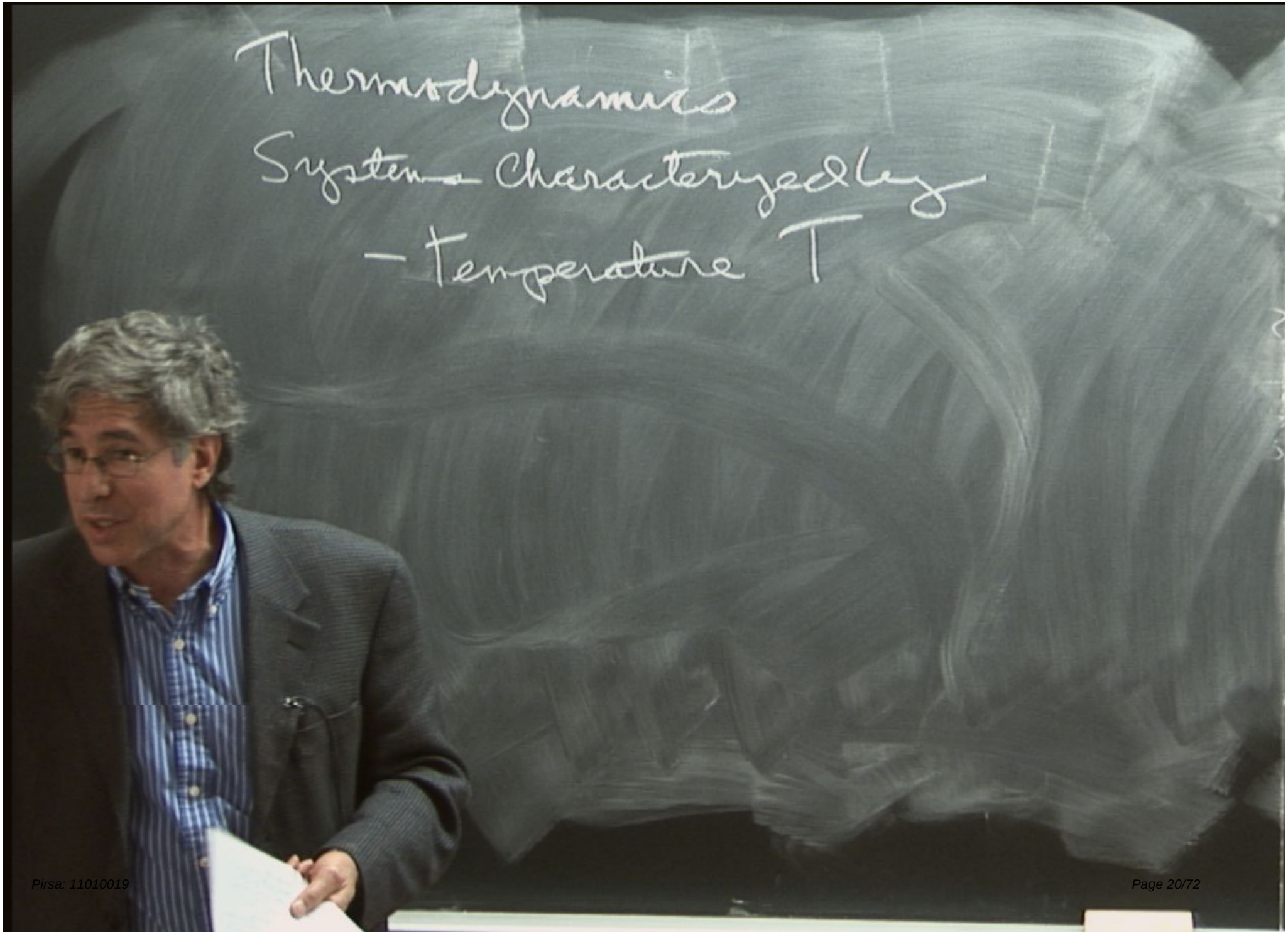
(anti) ferromagnets


Thermodynamics


Thermodynamics
System Characterized by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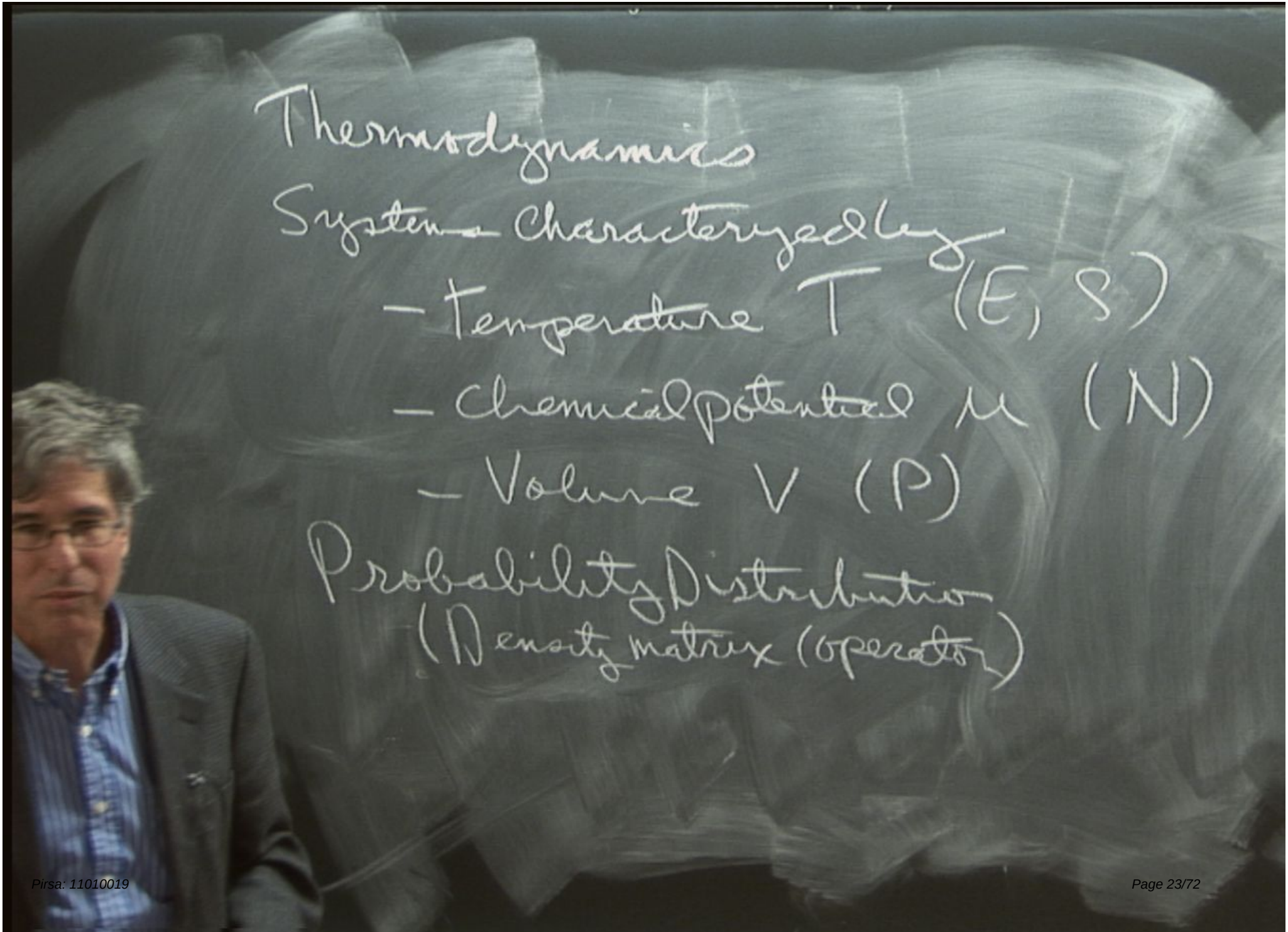
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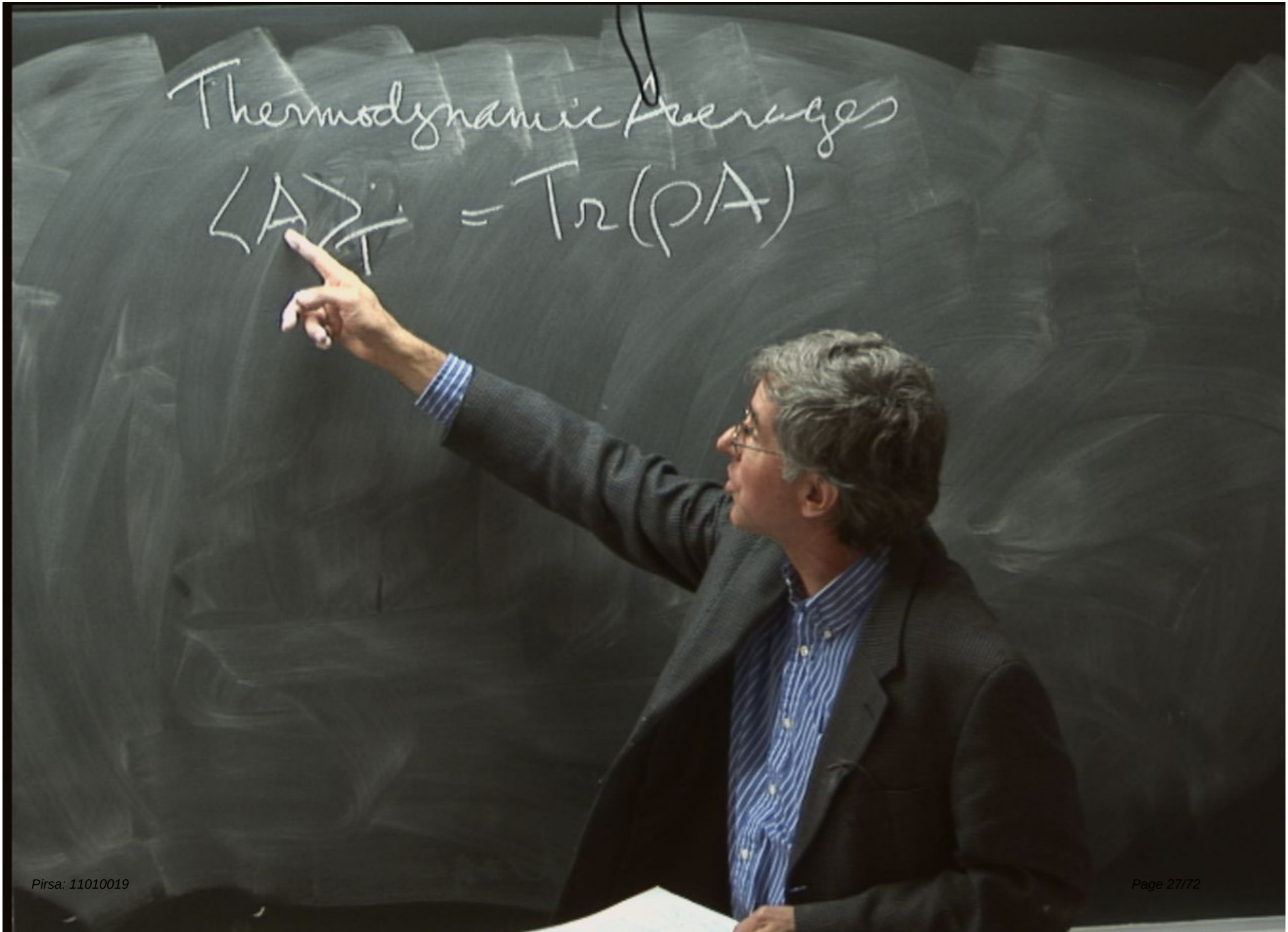
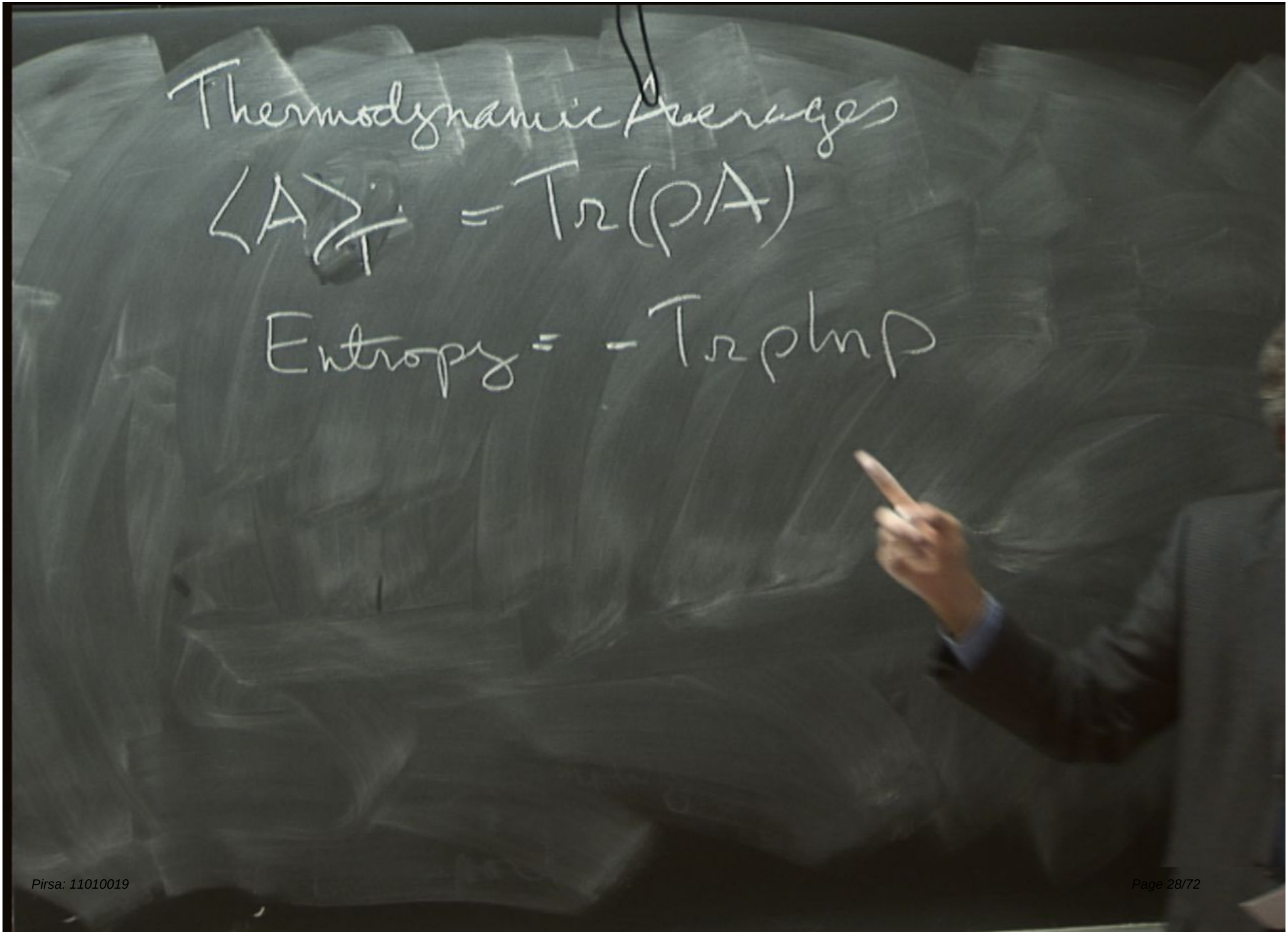

Thermodynamic Averages
⟨A⟩ = Tr(ρA)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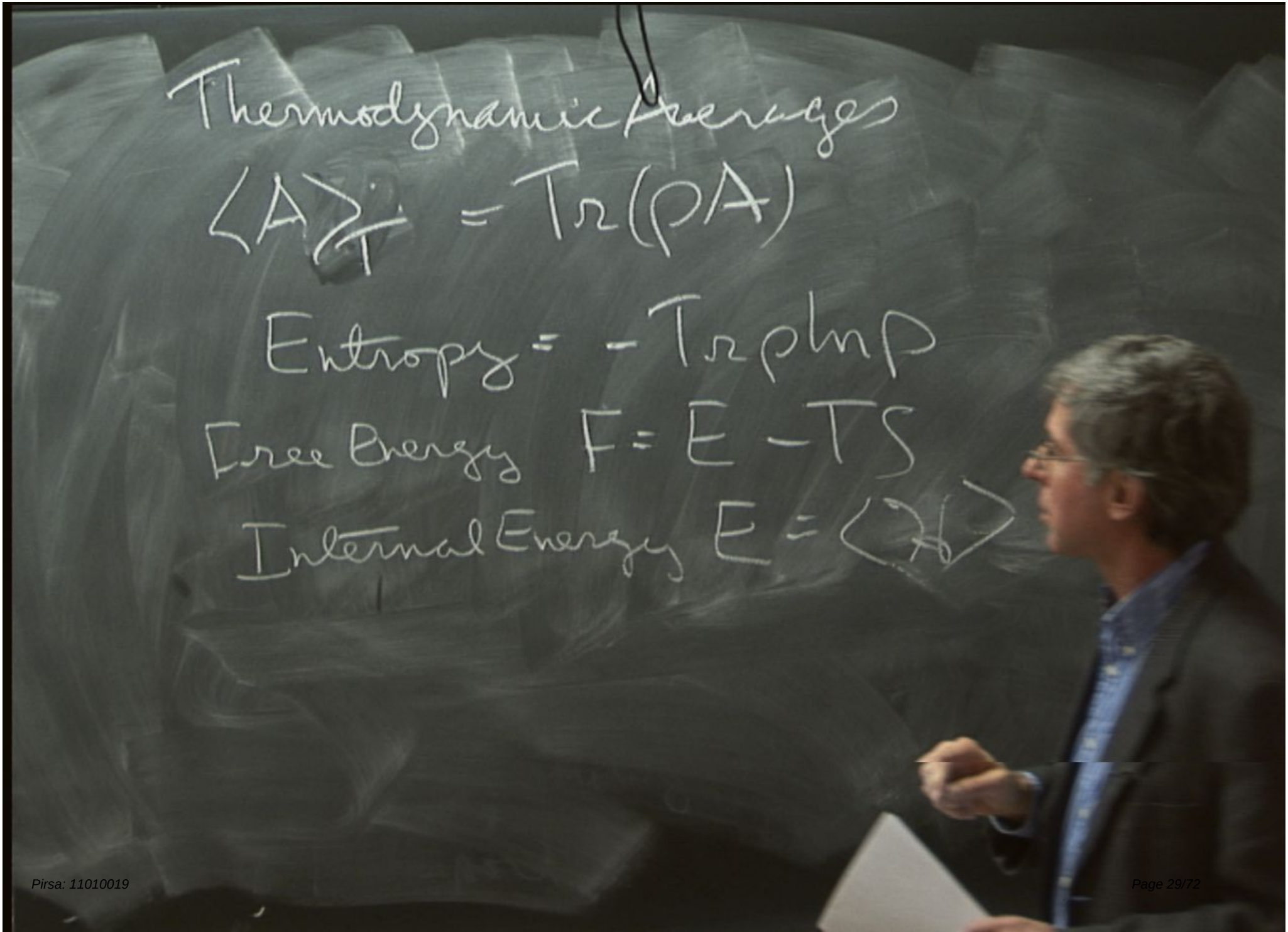
$\langle A \rangle_T = Tr(\rho A)$
Entropy = $-Tr\rho\ln\rho$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Free Energy $F = E - TS$
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Entropy $= -\mathrm{Tr}\,\rho \ln \rho$

Free Energy $F = E - TS$ Helmholtz

Internal Energy $E = \langle \mathcal{H} \rangle$

Partition Function $Z = \mathrm{Tr}\, e^{-\beta \mathcal{H}}$

$F = -T \ln Z$ [Show from defs of S, P, E]

$= F(T, V, N)$

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allows calc of thermal props

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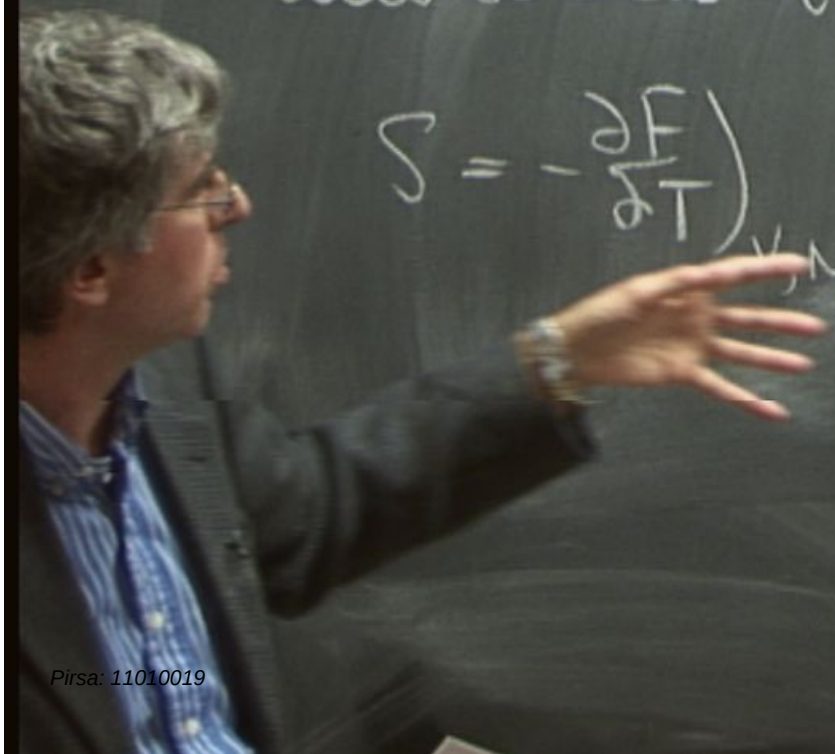

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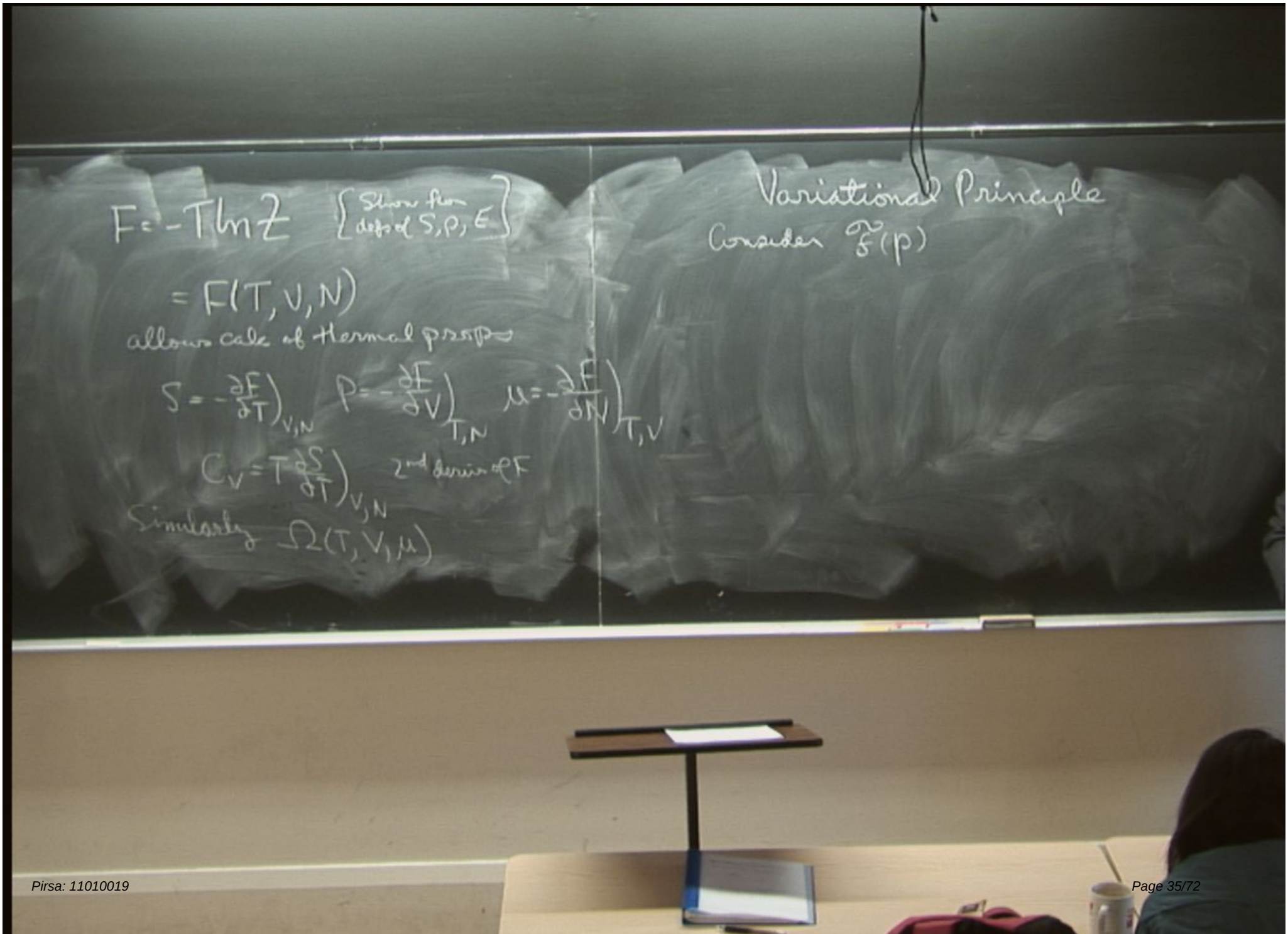
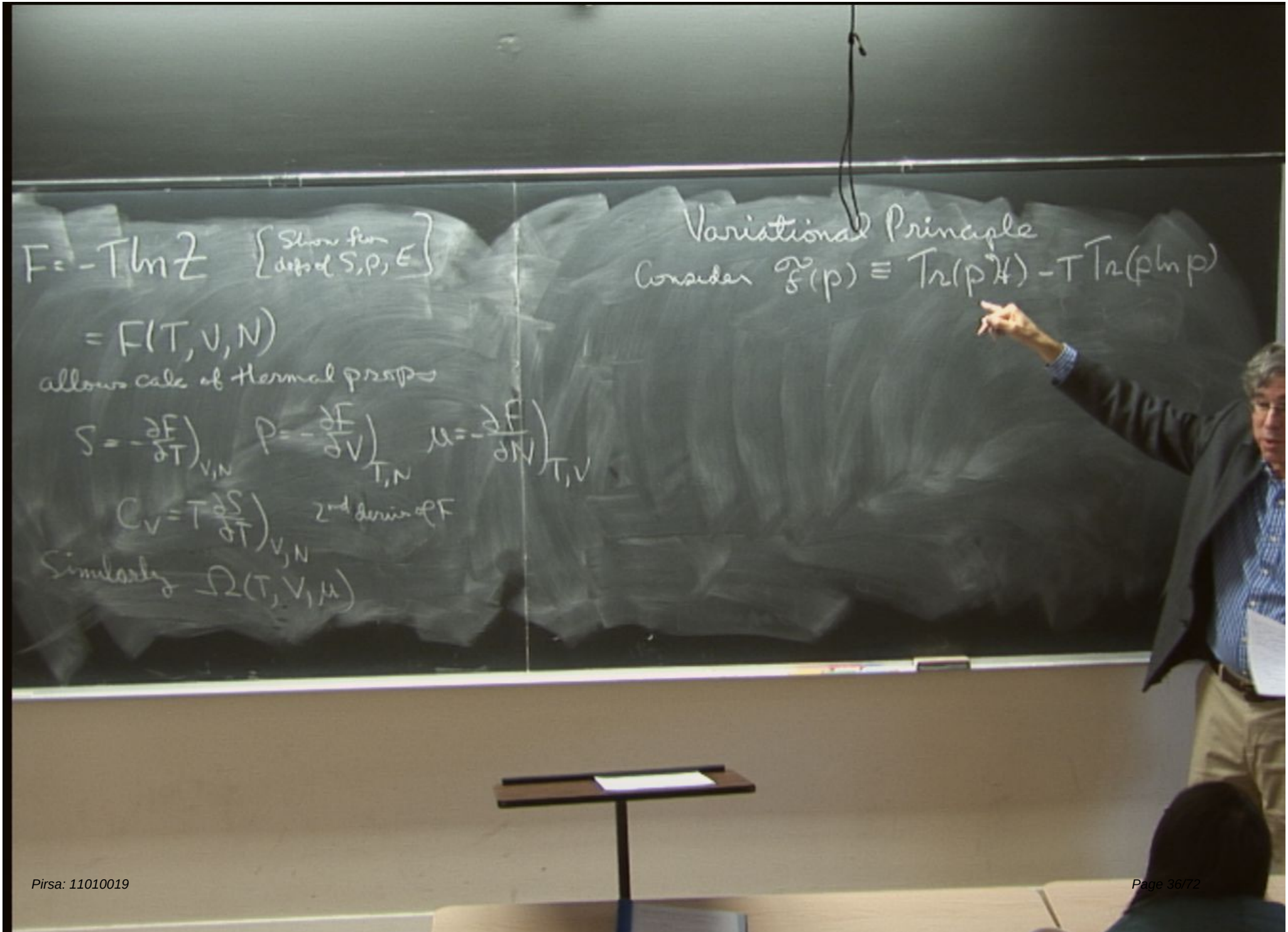
Variational Principle

Consider $\delta(p)$

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where $\rho = \frac{e^{-\beta\mathcal{H}}}{Z}$

$\left(\frac{\partial F}{\partial N}\right)_{T,V}$

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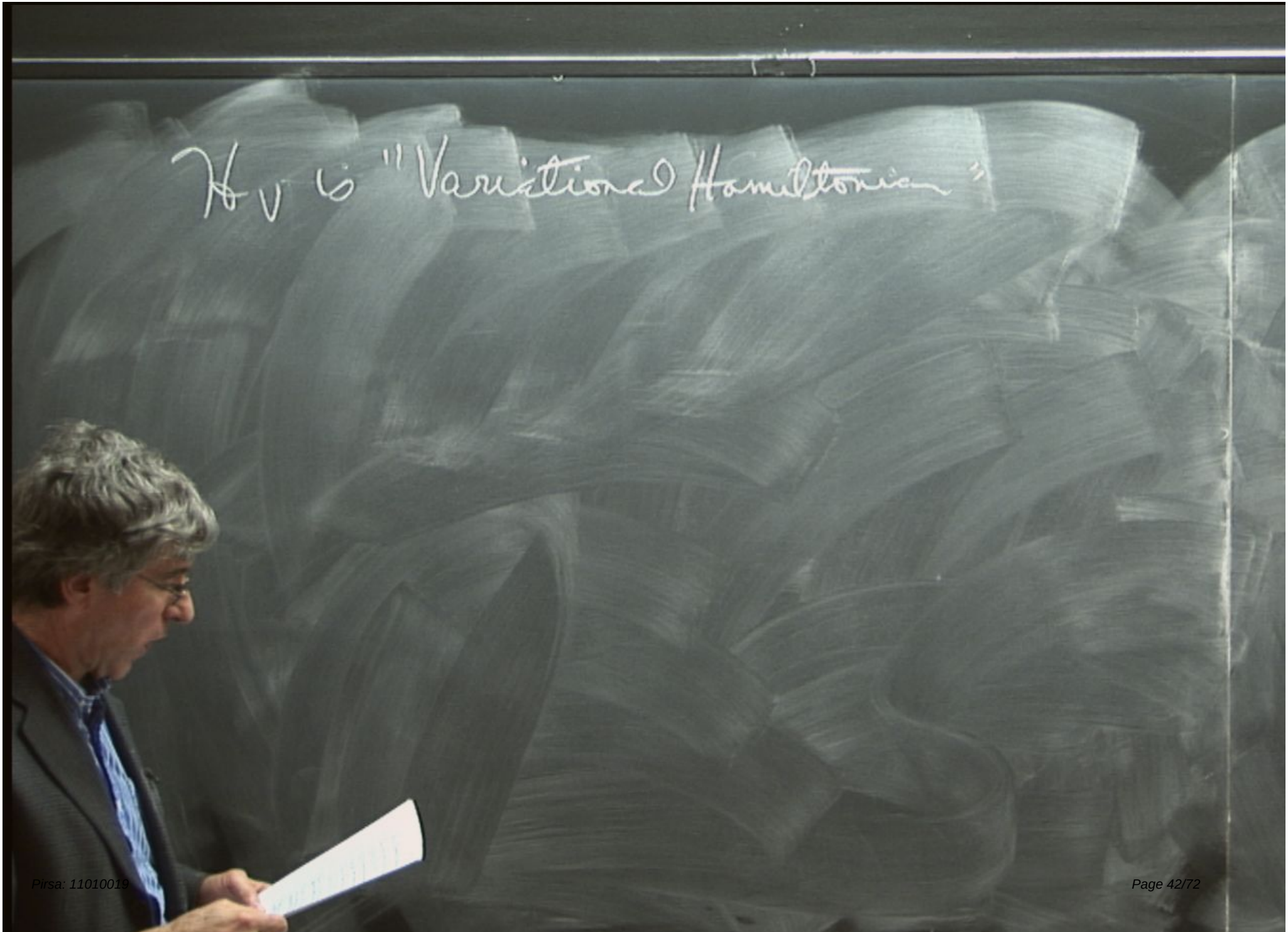
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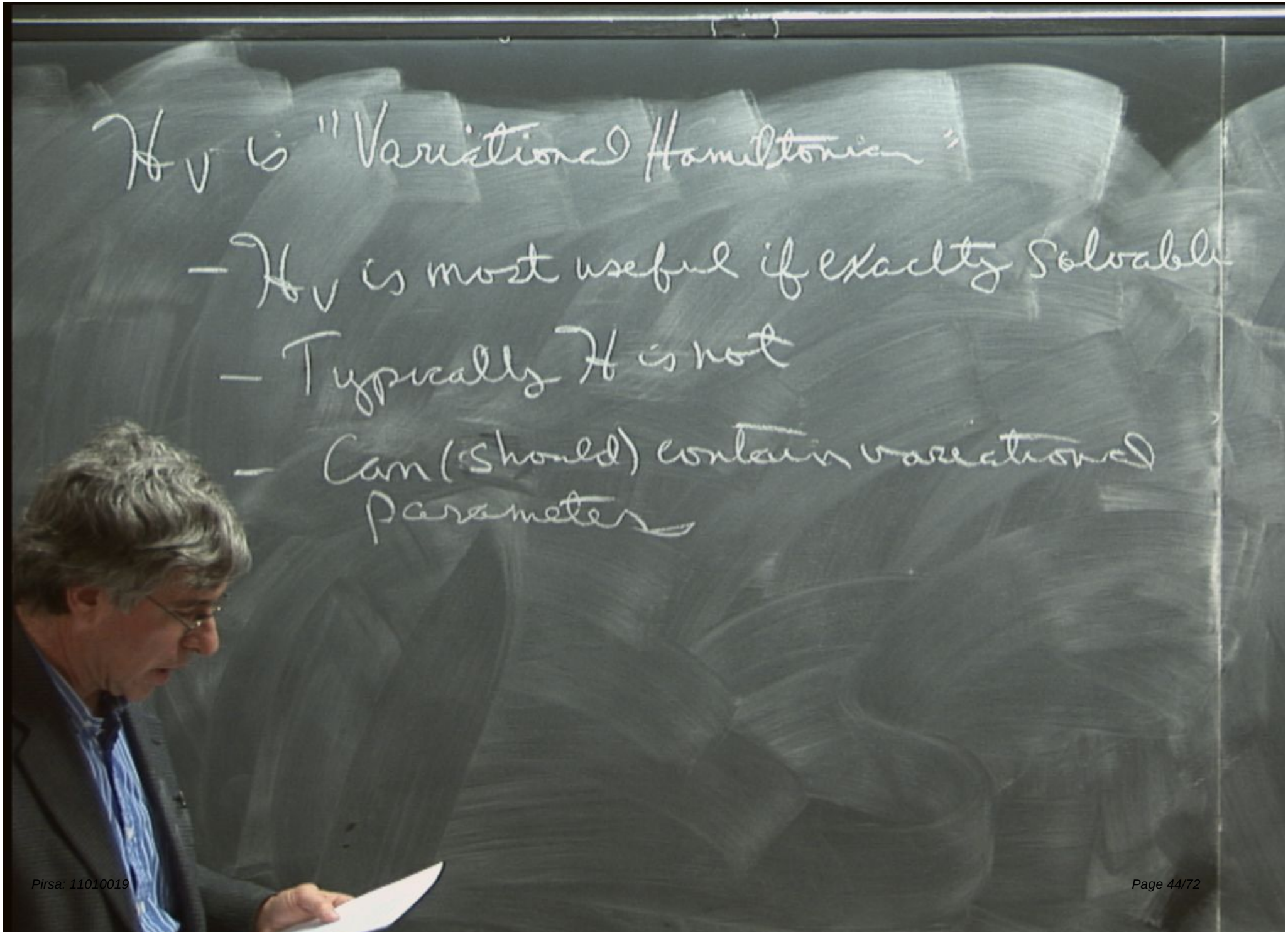
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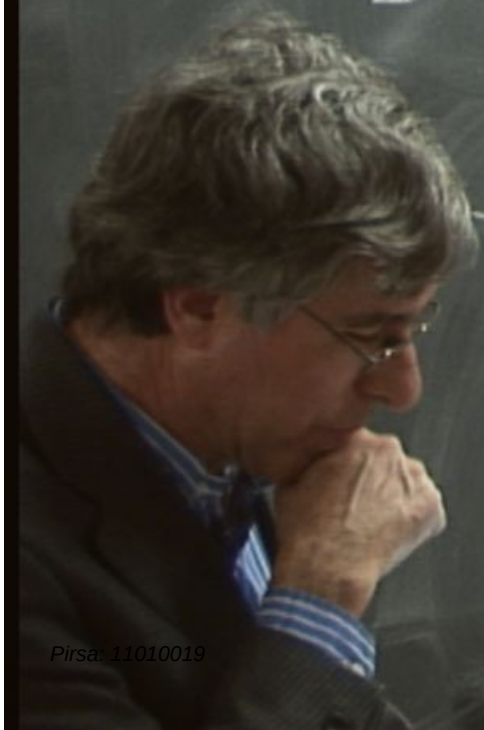

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Special Case - Mean Field Theory

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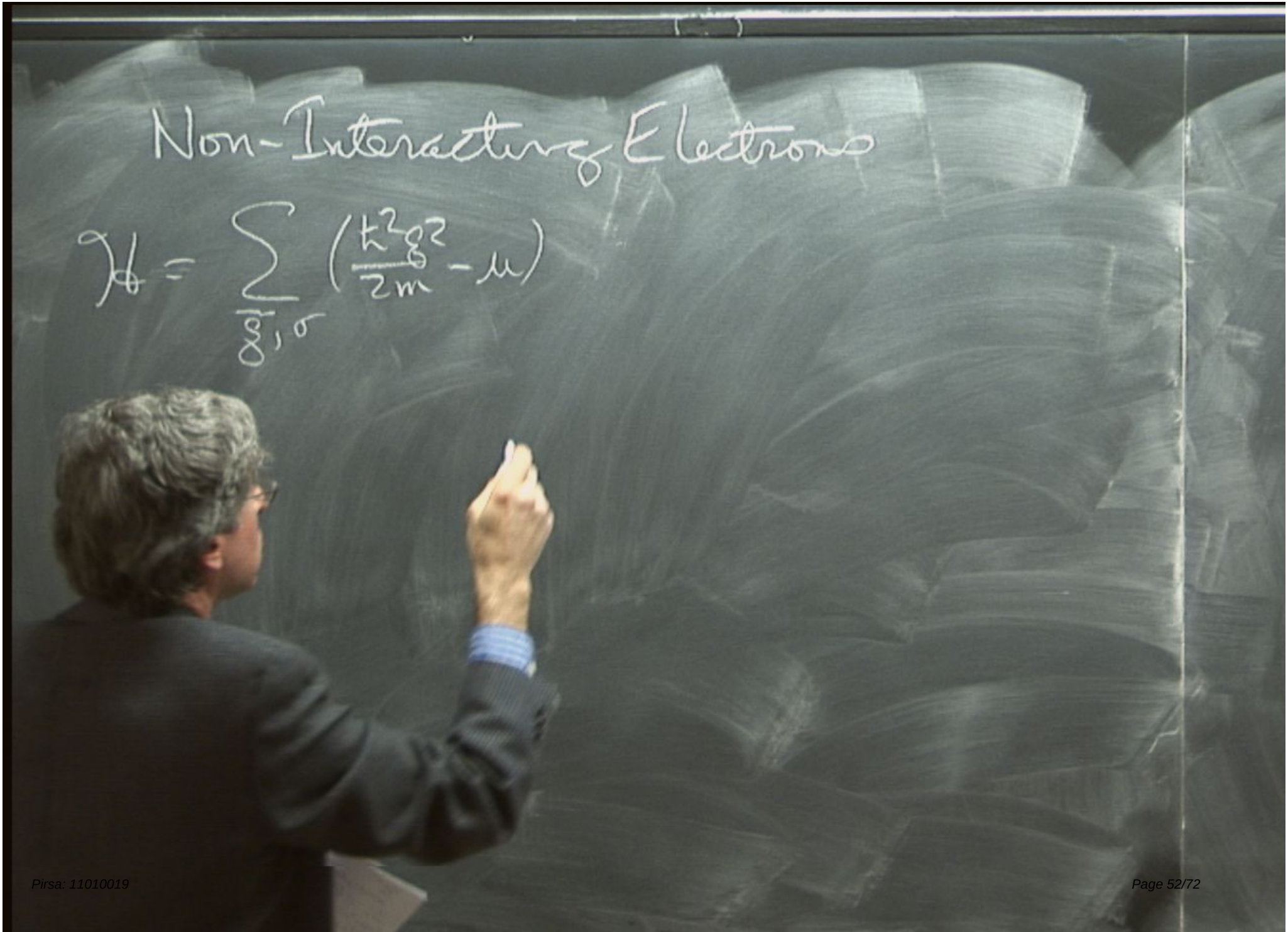
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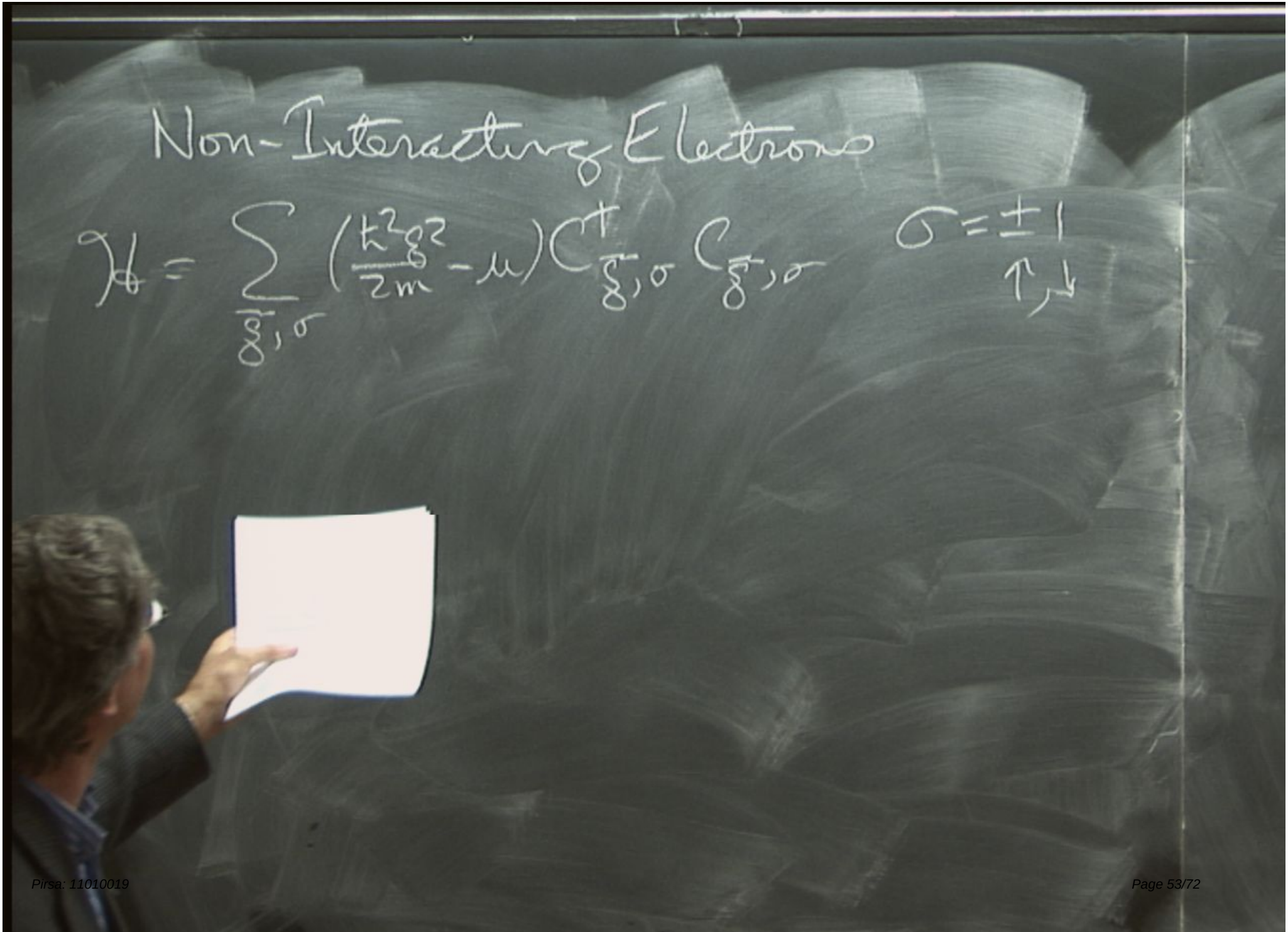
$\mathcal{H} = \sum_{\vec{k},\sigma} \left(\frac{\hbar^2 k^2}{2m} - \mu \right)$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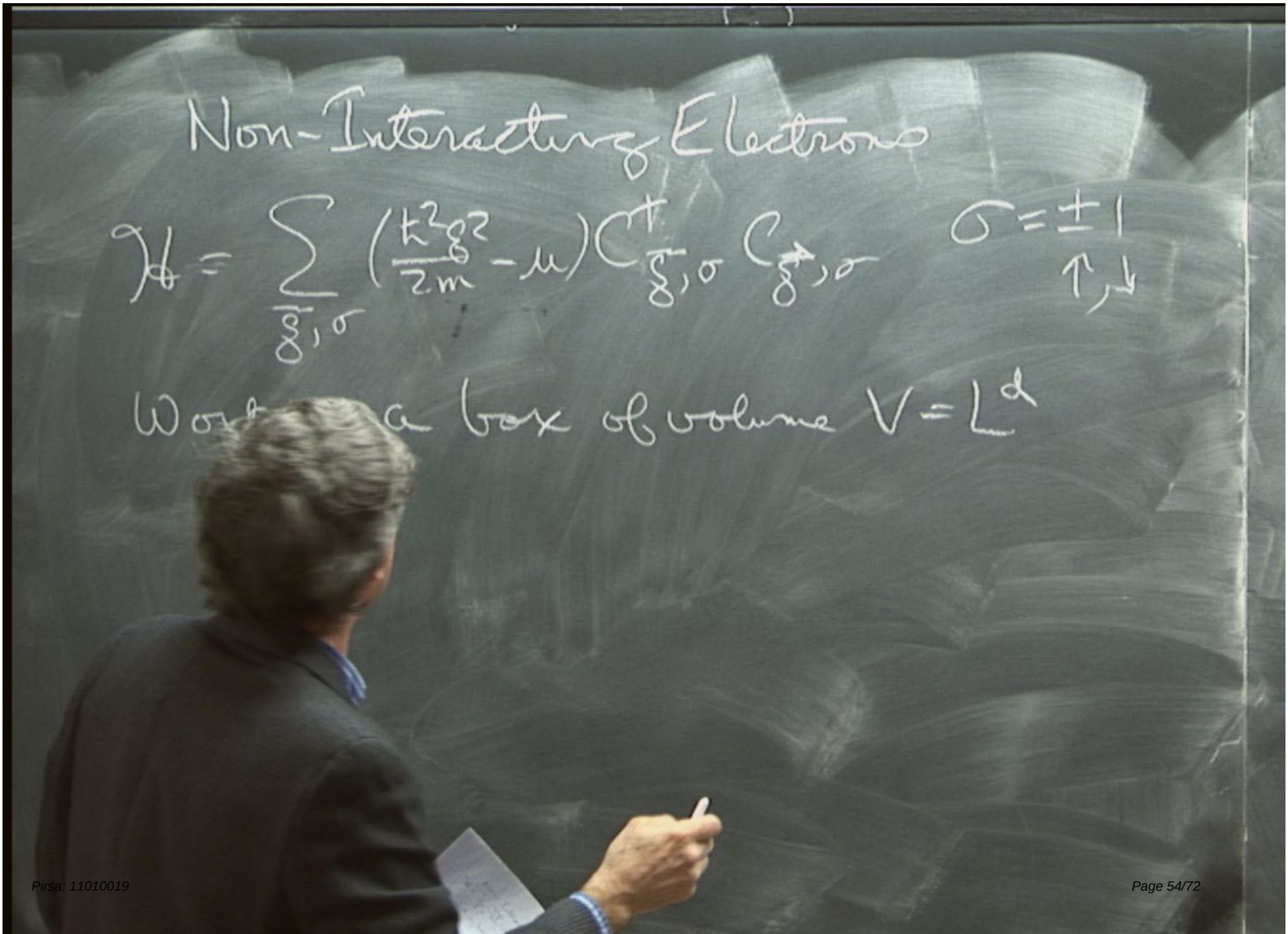
$\mathcal{H} = \sum_{\vec{k},\sigma} \left(\frac{\hbar^2 k^2}{2m} - \mu\right) c^{\dagger}_{\vec{k},\sigma} c_{\vec{k},\sigma}$
$\sigma = \pm 1$
$\uparrow, \downarrow$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Operators obey $\{ c_{\vec{k},\sigma}, c^\dagger_{\vec{k}',\sigma'} \} = \delta_{\vec{k},\vec{k}'} \delta_{\sigma,\sigma'}$

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Operators obey $\{c_{\vec{k},\sigma}, c^\dagger_{\vec{k}',\sigma'}\} = \delta_{\vec{k}\vec{k}'}\delta_{\sigma\sigma'}$

$$\{c, c\} = \{c^\dagger, c^\dagger\} = 0$$

Non-Interacting Electrons

$$\mathcal{H} = \sum_{\vec{k},\sigma} \left(\frac{\hbar^2 k^2}{2m} - \mu \right) c^\dagger_{\vec{k},\sigma} c_{\vec{k},\sigma} \qquad \sigma = \pm 1 \;\; \uparrow, \downarrow$$

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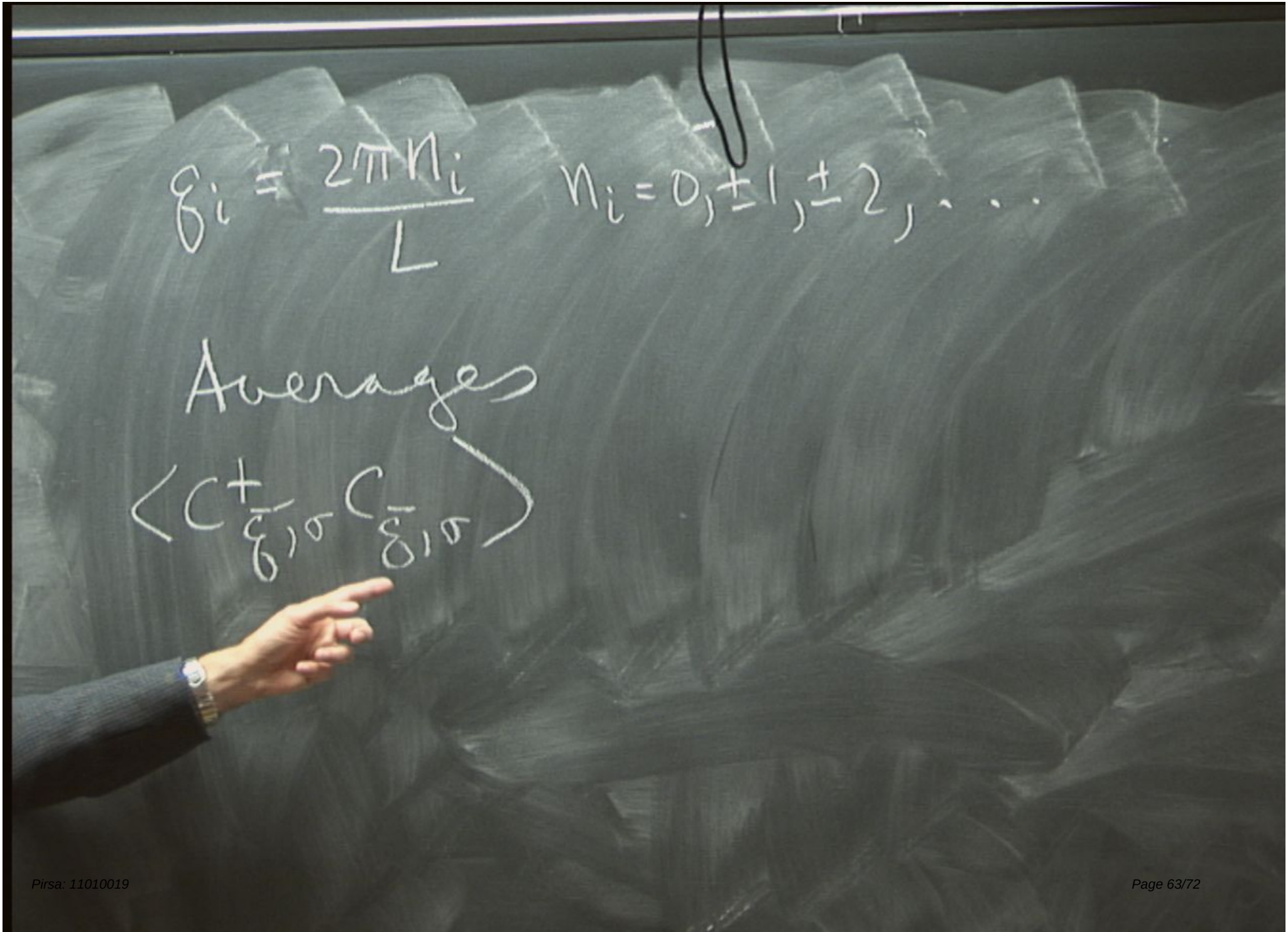
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Averages
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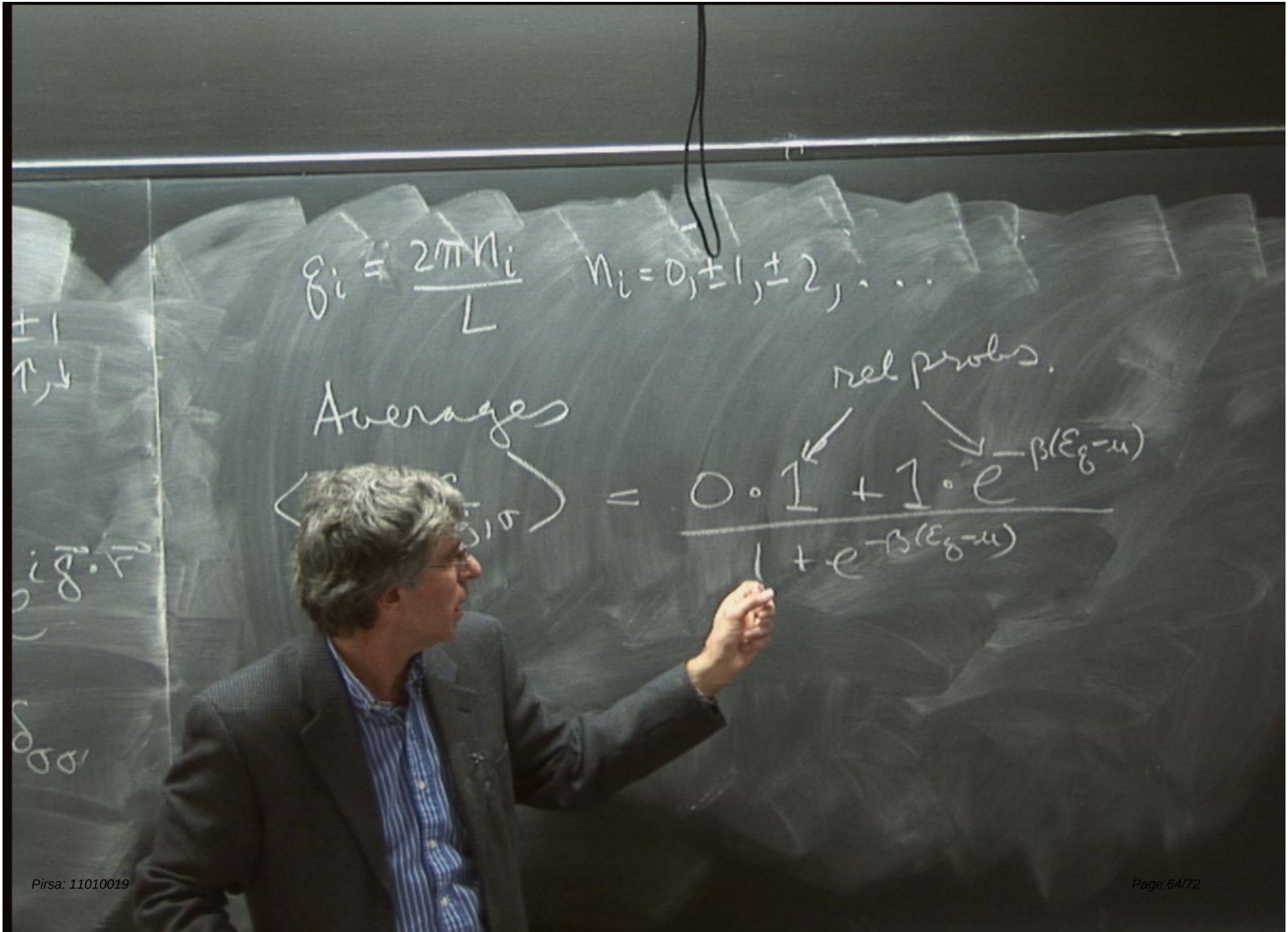
$\varepsilon_i = \frac{2\pi n_i}{L}$ $n_i = 0, \pm 1, \pm 2, \ldots$
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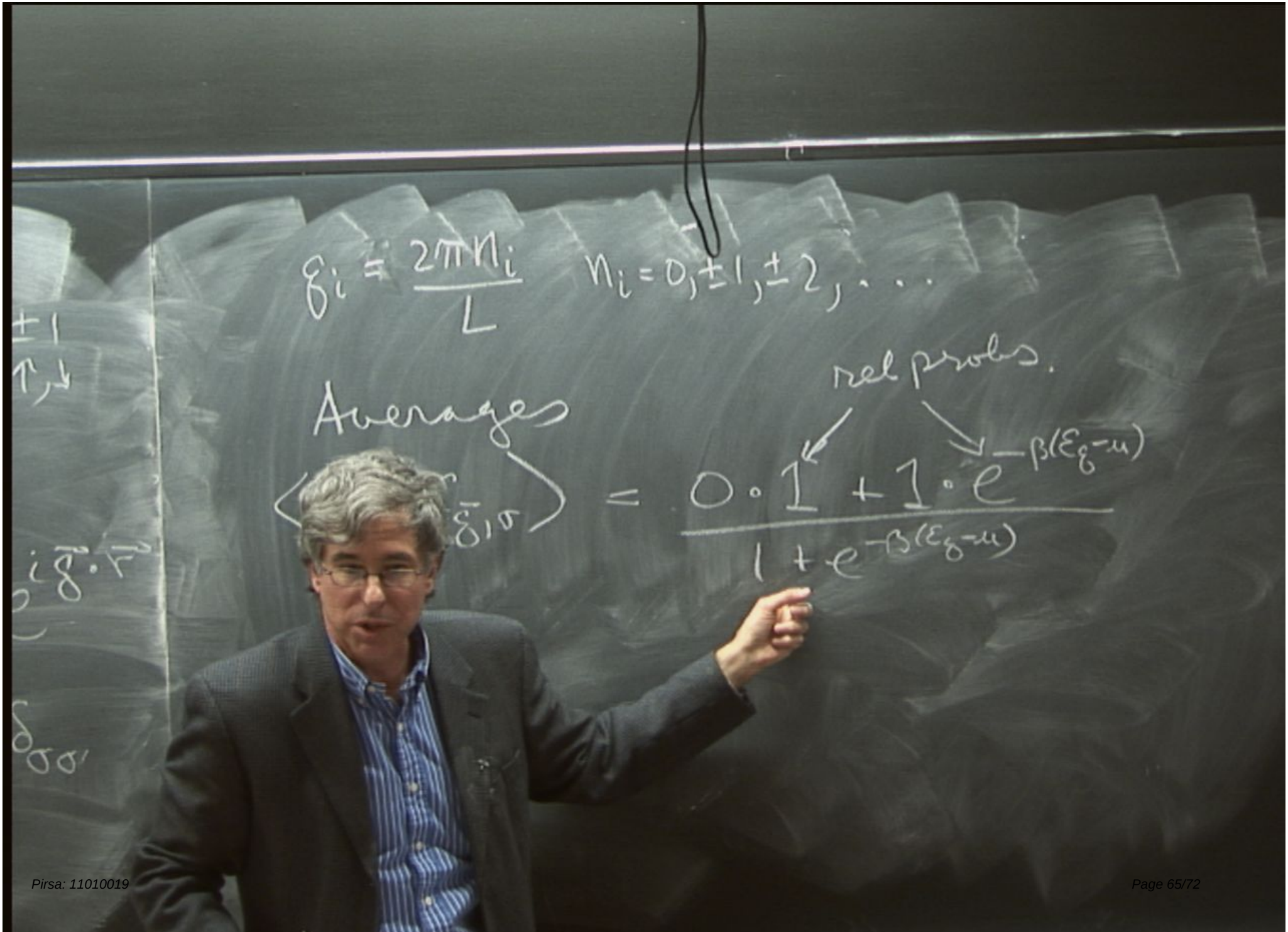
$\langle \quad \rangle = \frac{0 \cdot 1 + 1 \cdot e^{-\beta(\varepsilon_\beta - \mu)}}{1 + e^{-\beta(\varepsilon_\beta - \mu)}}$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Averages

occ

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Averages — occ, rel probs.

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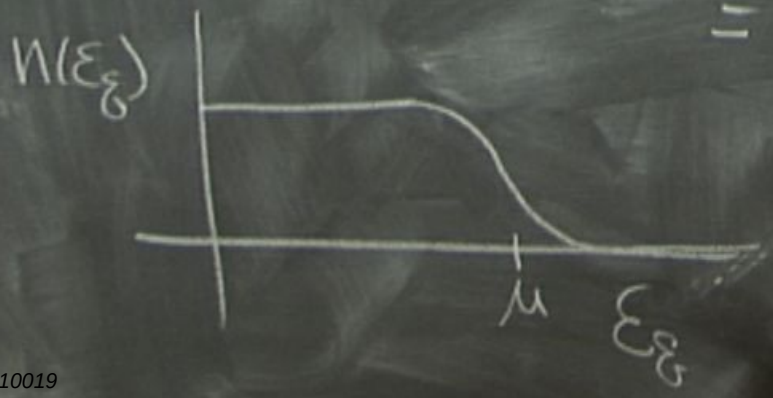

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