

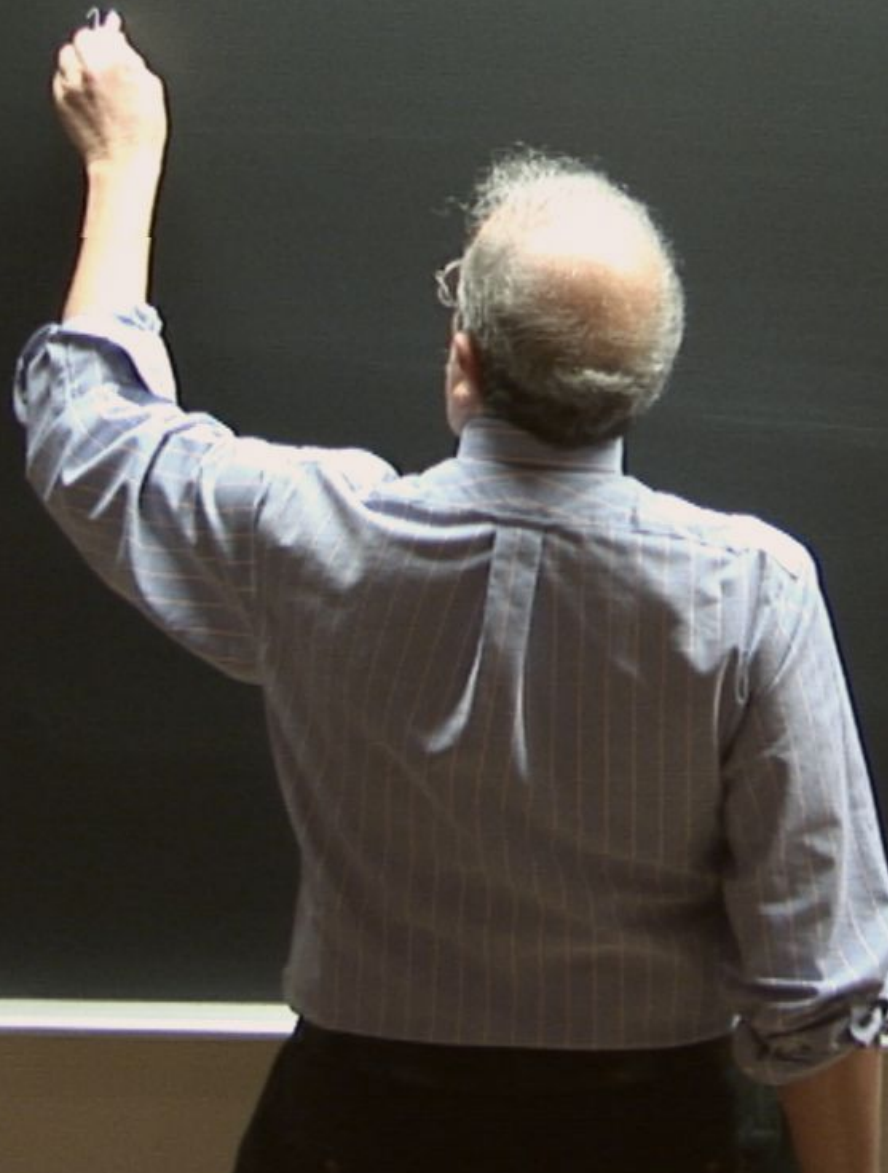
Title: Standard Model - Lecture 13

Date: Jan 19, 2011 09:00 AM

URL: <http://pirsa.org/11010016>

Abstract:

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$



$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ \mu \end{pmatrix}_L$$

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ \mu \end{pmatrix}_L$$

$$K^- \rightarrow \mu^- \bar{\nu}$$

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$\begin{array}{c} \bar{K}^- \rightarrow \bar{\mu} \nu \\ \hline s \bar{u} \end{array}$$

$$\Lambda^0 \rightarrow p \pi^-$$

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$\underline{K^-} \rightarrow \mu^- \bar{\nu}$$

$$s \bar{u}$$

$$\Lambda^0 \rightarrow p \pi^-$$

$$s \rightarrow u + W^-$$

$$s \not\rightarrow d + Z^0_{\mu^+ \mu^-}$$

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ \mu \end{pmatrix}_L$$

$$\underline{K^-} \rightarrow \mu^- \bar{\nu} \quad \Lambda^0 \rightarrow p \pi^-$$

$$s \bar{u}$$

$$s \rightarrow u + W^-$$

$$s \not\rightarrow d + Z^0_{\mu^+\mu^-}$$

$$BR(K^0 \rightarrow \pi^+ \mu^- \bar{\nu}) = 27\%$$

$$BR(K^0 \rightarrow \mu^+ \mu^-) \sim 10^{-8}!$$

$$\pi^- \rightarrow \mu^- \bar{\nu} \quad n \rightarrow p e^- \bar{\nu}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ \mu \end{pmatrix}_L$$

$$\overbrace{K^-} \rightarrow \mu^- \bar{\nu} \quad \Lambda^0 \rightarrow p \pi^-$$

$$s \bar{u}$$

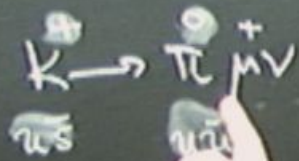
$$s \rightarrow u + W^-$$

$$s \not\rightarrow d + Z^0_{\mu^+ \mu^-}$$

$$BR(K^0 \rightarrow \pi^+ \mu^- \bar{\nu}) = 27\%$$

$$BR(K^0 \rightarrow \mu^+ \mu^-) \sim 10^{-8}!$$

$$K^- \rightarrow \pi^0 \mu^- \bar{\nu}$$



$$\begin{array}{ccc} \overset{+}{K} & \rightarrow & \overset{+}{\pi} \\ \bar{u}s & & u\bar{u} \end{array}$$

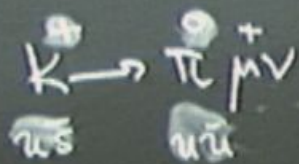
$$\langle \pi | \bar{u} \gamma^\mu s |$$

$$K \rightarrow \pi \mu \nu$$

$\bar{u}s$
 $\bar{u}d$

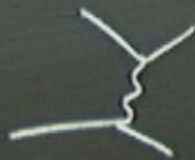
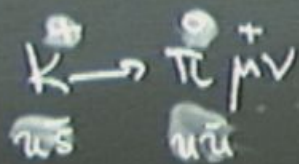
$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle$$





$$\langle \pi | \bar{u} \gamma^\mu \xi | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+ (q^2)}_{f_+} + (p_K - p_\pi)^\mu f_-$$

$$q = (p_K - p_\pi)$$



$$J_\mu^+$$

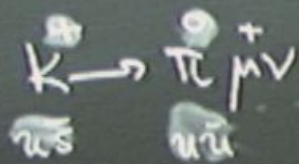
$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+(\vec{q})}_{\text{pointed to}} + (p_K - p_\pi)^\mu$$

$$q = (p_K - p_\pi)$$

$$f_+(0) = 1$$

$$q^2 = 0$$

$$\bar{u}_L \gamma^\mu s_L + \frac{1}{q} \bar{u}_L \gamma^\mu d_L$$



$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+(q^2)}_{f_+(0)} + (p_K - p_\pi)^\mu f_0(q^2)$$

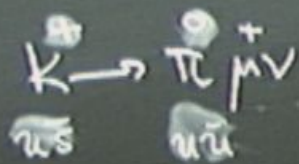
$$q = (p_K - p_\pi)$$

$$q^2 = 0$$

$$f_+(0) = 1$$

$$= \underbrace{\sqrt{\frac{2}{3}}}_{\uparrow} \bar{u}_L \gamma^\mu s_L + \frac{1}{\sqrt{6}} \bar{u}_L \gamma^\mu d_L$$

$$0.2237 \pm 0.013$$



$$J_\mu^+$$

$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+(q^2)} + (p_K - p_\pi)^\mu f_0(q^2)$$

$$q = (p_K - p_\pi)$$

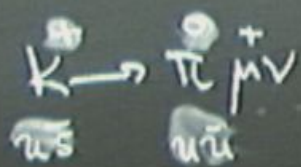
$$q^2 = 0$$

$$f_+(0) = 1$$

$$-\sqrt{\frac{2}{3}} \bar{u}_L \gamma^\mu s_L + \frac{1}{\sqrt{6}} \bar{u}_L \gamma^\mu d_L$$

$$\uparrow$$

$$0.2237 \pm 0.013$$



$$J_\mu^+$$

$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+(\vec{q})}_{f_+(0)} + (p_K - p_\pi)^\mu f_0(\vec{q})$$

$$\vec{q} = (p_K - p_\pi)$$

$$\vec{q}^2 = 0$$

$$f_+(0) = 1$$

$$= \underbrace{\sqrt{\frac{2}{3}}}_{0.2237 \pm 0.013} \bar{u}_L \gamma^\mu s_L + \frac{1}{2} \bar{u}_L \gamma^\mu d_L + \frac{1}{2} \bar{d}_L \gamma^\mu u_L + \frac{1}{2} \bar{u}_D \gamma^\mu u_D + \frac{1}{2} \bar{d}_D \gamma^\mu d_D + \frac{1}{2} \bar{e} \gamma^\mu e$$

$$K \rightarrow \pi \mu \nu$$

$\begin{matrix} \bar{u}s & & u\bar{u} \end{matrix}$



$$J_\mu^+$$

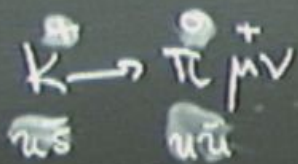
$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (p_K + p_\pi)^\mu \underbrace{f_+(q^2)}_{f_+(0)} + (p_K - p_\pi)^\mu f_0(q^2)$$

$$q = (p_K - p_\pi)$$

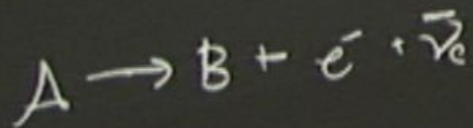
$$q^2 = 0$$

$$f_+(0) = 1$$

$$J_\mu^+ = \underbrace{\sum_{us} \bar{u}_L \gamma^\mu s_L}_{0.2237 \pm 0.013} + \frac{1}{2} \bar{u}_L \gamma^\mu d_L + \frac{1}{2} \bar{d}_L \gamma^\mu u_L + \frac{1}{2} \bar{\nu}_\mu \gamma^\mu \mu^- + \frac{1}{2} \bar{e} \gamma^\mu \nu_e$$



$$J_\mu^+$$



$$\langle I=0 | \bar{u} \gamma^\mu d | I=+1 \rangle = \sqrt{2} P_A^1$$

$P=+1$
 ρ

$P=+1$
 A

$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (P_K + P_\pi)^\mu \underline{f_+(q^2)} + (P_K - P_\pi)^\mu f_0(q^2)$$

$$q = (P_K - P_\pi)$$

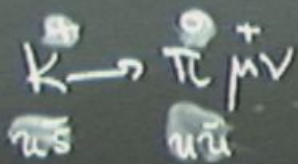
$$f_+(0) = 1$$

$$q^2 = 0$$

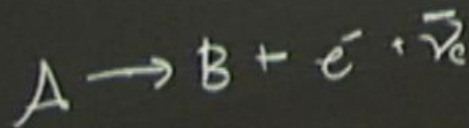
$$\bar{u}_L \gamma^\mu s_L$$

$$0.2237 \pm 0.0013$$

$$+ \frac{1}{2} \bar{u}_L \gamma^\mu d_L + \frac{1}{2} \bar{\nu}_L \gamma^\mu \mu^- + \frac{1}{2} \bar{\nu}_e \gamma^\mu e$$



$$J_\mu^+$$



$$\langle I=0 | \bar{u} \gamma^\mu d | I=+1 \rangle_{P=+1} = \sqrt{2} P_A^+$$

$P=+1$
 ρ

$P=+1$
 A

$$\langle \pi | \bar{u} \gamma^\mu s | K \rangle = (P_K + P_\pi)^\mu \underline{f_+(q^2)} + (P_K - P_\pi)^\mu$$

$$q = (P_K - P_\pi)$$

$$f_+(0) = 1$$

$$q^2 = 0$$

$$V_{us} \bar{u}_L \gamma^\mu s_L$$

$$0.2237 \pm 0.0013$$

$$+ V_{ud} \bar{u}_L \gamma^\mu d_L + \frac{1}{2} \bar{\nu}_\mu \gamma^\mu \mu^- + \frac{1}{2} \bar{\nu}_e \gamma^\mu e$$

$$V_{ud} = 0.97418 \pm 0.0026$$

Cabibbo

$$(V_{ud})^2 + (V_{uc})^2 = 1$$

Cabibbo

$$(V_{us})^2 + (V_{ud})^2 = 1 - .0004 \approx .9996$$

$$V_{us} = \sin \theta_c$$

$$V_{ud} = \cos \theta_c$$

$$J_\mu^+ = \bar{\nu}_e \gamma_\mu e_L + \bar{\nu}_\mu \gamma_\mu \mu_L + \bar{u} \gamma_\mu (\cos \theta_c d + \sin \theta_c s)$$

$$\begin{array}{ccc} K^0 & \rightarrow & \pi^+ \pi^- \\ d\bar{s} & & d\bar{u} \quad u\bar{d} \end{array} \leftarrow \begin{array}{c} \bar{K}^0 \\ s\bar{d} \end{array}$$

$$\begin{array}{ccc} K^+ & \rightarrow & \pi^+ \mu^+ \nu \\ u\bar{s} & & u\bar{u} \end{array}$$

$$A \rightarrow B + e^-$$

$$\begin{array}{l} \langle I=0 | \\ P=+1 \\ \beta \end{array}$$

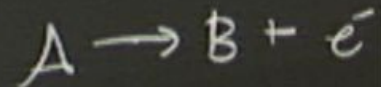
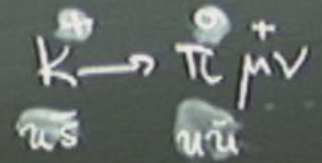
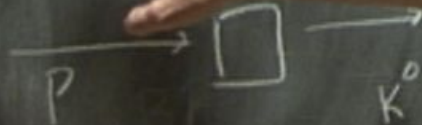
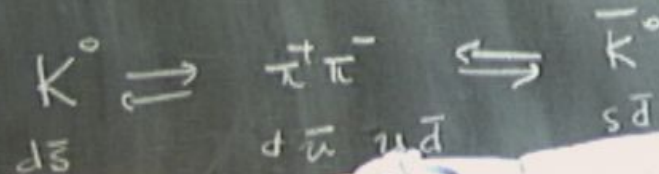
Cabibbo

$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004 = .9996$$

$$V_{us} = \sin \theta_c$$

$$V_{ud} = \cos \theta_c$$

$$J_\mu^+ = \bar{\nu}_e \gamma_\mu e_L + \bar{\nu}_\mu \gamma_\mu \mu_L + \bar{u} \gamma_\mu (\cos \theta_c d + \sin \theta_c s)$$



$$\langle I=0 | \bar{\nu} \gamma_\mu \nu \rangle$$

$P=+1$
 β

Cabibbo

$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004$$

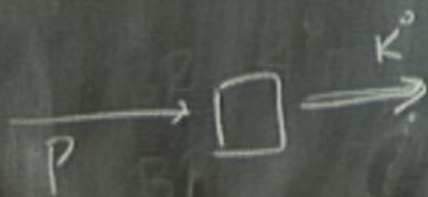
$$\approx .0007$$

$$V_{us} = \sin \theta_c$$

$$V_{ud} = \cos \theta_c$$

$$J_\mu^+ = \bar{\nu}_e \gamma_\mu e_L + \bar{\nu}_\mu \gamma_\mu \mu_L + \bar{u} \gamma_\mu (\cos \theta_c d + \sin \theta_c s)$$

$$K^0 \rightleftharpoons \begin{matrix} \pi^+ \pi^- \\ d\bar{u} \quad u\bar{d} \end{matrix} \rightleftharpoons \begin{matrix} \bar{K}^0 \\ s\bar{d} \end{matrix}$$



[mixture of K^0 & $\bar{K}^0$]

$$\begin{matrix} K^0 \\ u\bar{s} \end{matrix} \rightarrow \begin{matrix} \pi^+ \\ u\bar{u} \end{matrix} \mu^+ \nu$$

$$A \rightarrow B + e^-$$

$$\langle I=0 | \bar{\nu} \gamma_\mu \nu | P=+1 \rangle$$

Cabibbo

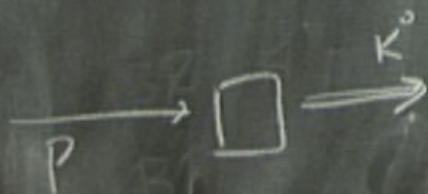
$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004 = .9996$$

$$V_{us} = \sin \theta_c$$

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$$\begin{array}{ccc} K^0 & \rightleftharpoons & \pi^+ \pi^- \\ d\bar{s} & & d\bar{u} \quad u\bar{d} \end{array} \rightleftharpoons \begin{array}{c} \bar{K}^0 \\ s\bar{d} \end{array}$$



mixture of
 K^0 & $\bar{K}^0$

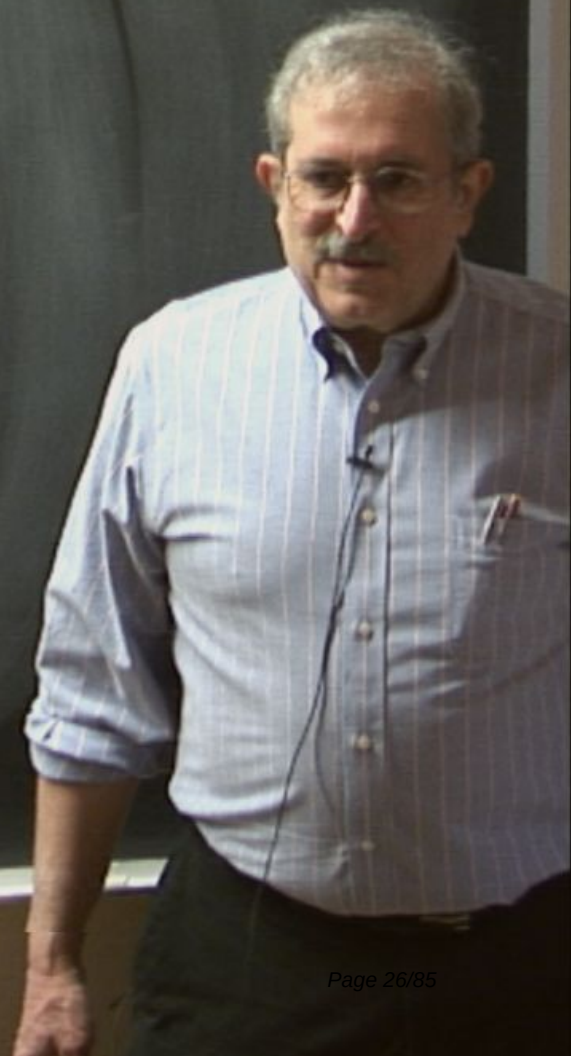
$$\frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$\frac{1}{\sqrt{2}}(1K^0 + 1\bar{K}^0)$$

$$\frac{1}{\sqrt{2}}(1K^0 - 1\bar{K}^0)$$

$$m_K = 498 \text{ MeV}$$

$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$



$$\frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$$\frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

$$m_K = 498 \text{ MeV}$$

$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$CP |K^0\rangle = -|\bar{K}^0\rangle$$

$$CP |\bar{K}^0\rangle = -|K^0\rangle$$

$$\frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad (P = -1) \quad m_K = 498 \text{ MeV}$$

$$\frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad CP = +1 \quad \Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

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$$CP |K^0\rangle = -|\bar{K}^0\rangle$$

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$$\pi^+\pi^-$$

$$CP = +1$$

$$\frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad P = -1 \quad m_K = 498 \text{ MeV}$$

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$$CP |\bar{K}^0\rangle = -|K^0\rangle$$

$$\pi^+\pi^- \quad CP = +1$$

$$\pi^+\pi^-$$

$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P = -1) \quad m_K = 498 \text{ MeV}$$

$$7 \quad |K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP = +1 \quad \Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$CP |K^0\rangle = -|\bar{K}^0\rangle$$

$$CP |\bar{K}^0\rangle = -|K^0\rangle$$

$$\pi^+\pi^- \quad CP = +1$$

$$\pi^+\pi^-\pi^0 \quad CP = -1$$

$$|K_S^0\rangle \rightarrow \pi^+\pi^-$$

$$|K_L^0\rangle \rightarrow \pi^+\pi^-\pi^0$$

$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P = -1)$$

$$m_K = 498 \text{ MeV}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP = +1$$

$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$CP |K^0\rangle = -|\bar{K}^0\rangle$$

$$\pi^+\pi^- \quad CP = +1$$

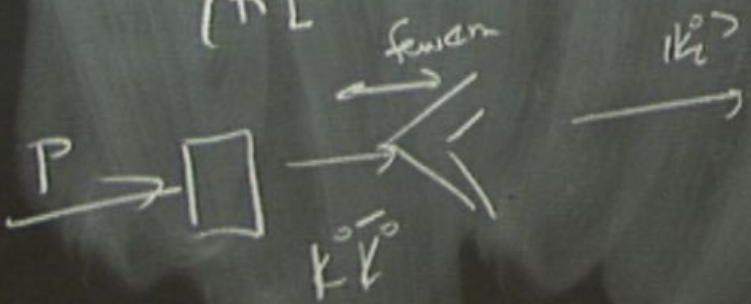
$$CP |\bar{K}^0\rangle = -|K^0\rangle$$

$$\pi^+\pi^-\pi^0 \quad CP = -1$$

$$|K_S^0\rangle \rightarrow \pi^+\pi^-$$

$$|K_L^0\rangle \rightarrow \pi^+\pi^-\pi^0$$

10^3 times longer



Cabibbo

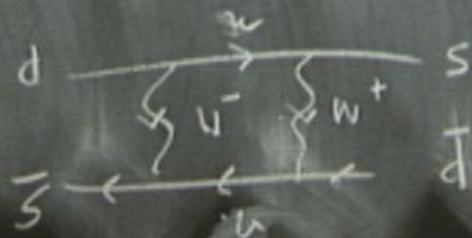
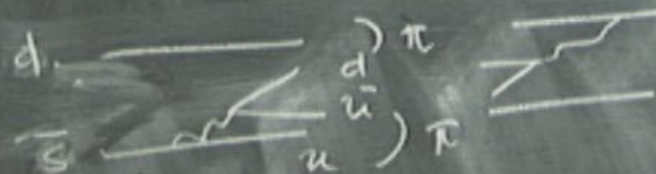
$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004$$

$$\approx .9997$$

$$V_{us} = \sin \theta_c$$

$$V_{ud} = \cos \theta_c$$

$$J_\mu^+ = \bar{\nu}_e \gamma_\mu e_L + \bar{\nu}_\mu \gamma_\mu \mu_L + \bar{u} \gamma_\mu (\cos \theta_c d + \sin \theta_c s)$$



Cabibbo

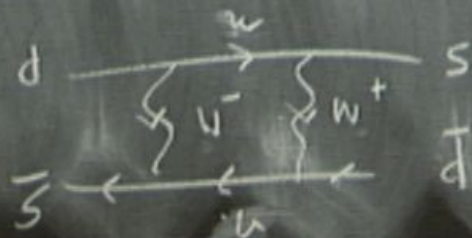
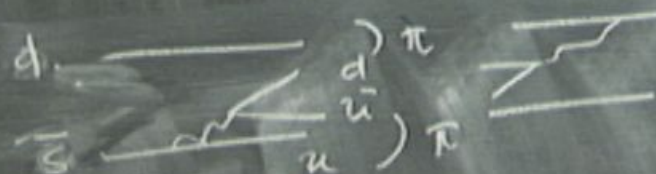
$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004$$

$$\approx .0007$$

$$V_{us} = \sin \theta_c$$

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$$J_\mu^+ = \bar{\nu}_e \gamma_\mu e_L + \bar{\nu}_\mu \gamma_\mu \mu_L + \bar{u} \gamma_\mu (\cos \theta_c d + \sin \theta_c s)$$



$$\int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{k^2 - m^2} \right) \frac{k}{k^2} \frac{k}{k^2}$$

Cabibbo

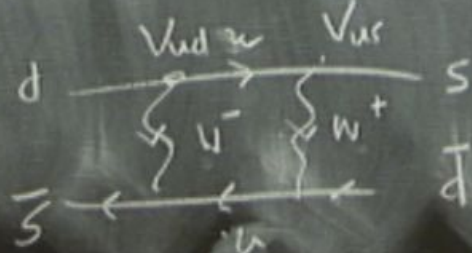
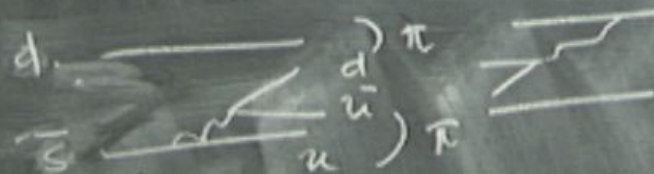
$$(V_{ud})^2 + (V_{us})^2 = 1 - .0004$$

$$\approx .0007$$

$$V_{us} = \sin \theta_c$$

$$V_{ud} = \cos \theta_c$$

$$J_\mu^+ = \bar{\nu}_{eL} \gamma_\mu e_L + \bar{\nu}_{\mu L} \gamma_\mu \mu_L + \bar{u}_L \gamma_\mu (\cos \theta_c d_L + \sin \theta_c s_L)$$



$$\int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{k^2 - m^2} \right)^2 \frac{k}{k^2} \frac{k}{k^2}$$

$$g^2 \frac{1}{m_W^2} V_{ud}^2 V_{us}^2$$

$$\alpha_W G_F$$

$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P=-1)$$

$$m_K = 498 \text{ MeV}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP=+1$$

$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$\sim \frac{G_F^2}{2}$$

GIM

$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P=-1)$$

$$m_K = 498 \text{ MeV}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP=+1$$

$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$\sim \frac{G_F^2}{2}$$

$$\bar{c}_L \gamma_\mu (-\sin \theta_c d + \cos \theta_c s)_L$$

GIM

$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P=-1)$$

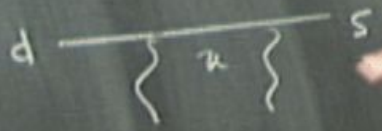
$$m_K = 498 \text{ MeV}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP=+1 \quad \boxed{\Delta m = 3.5 \times 10^{-12} \text{ MeV}}$$

$$\sim \frac{G_F^2}{2}$$

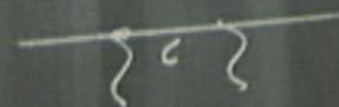
$$\bar{c}_L \gamma_\mu (-\sin\theta_c d + \cos\theta_c s)_L$$

GIM



$$\cos\theta_c \sin\theta_c$$

$$-\sin\theta_c$$



$$= \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle) \quad (P=-1)$$

$$m_K = 498 \text{ MeV}$$

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP=+1$$

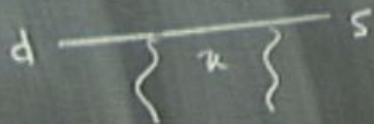
$$\Delta m = 3.5 \times 10^{-12} \text{ MeV}$$

$$\sim \frac{G_F^2}{m_W^2}$$

$$\bar{c}_L \gamma_\mu (-\sin\theta_c d + \cos\theta_c s)_L$$

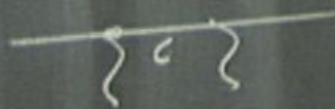
$$\alpha_W G_F^2 V_{ud}^2 V_{us}^2 \left(\frac{m_c}{m_W}\right)^2$$

GIM



$$\cos\theta_c \sin\theta_c$$

$$-\sin\theta_c \cos\theta_c$$



$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$$I = \frac{1}{2}$$

$$\bar{e}_R$$

$$I = 0$$

$$S\mathcal{L} = m_e \bar{e} e$$

$$= m_e (\bar{e}_L e_R + \bar{e}_R e_L)$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$$I = \frac{1}{2}$$

$$e^-_R$$

$$I = 0$$

~~$$S\mathcal{L} = m_e \bar{e} e$$~~
~~$$= m_e (\bar{e}_L e_R + \bar{e}_R e_L)$$~~

$$S\mathcal{L} = \lambda_e \frac{\bar{L} \cdot \phi}{\sqrt{2}} e_R + h.c.$$

$$\lambda_e (\bar{\nu} \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$$

$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$

$$\frac{\lambda_e}{\sqrt{2}} \bar{e}_L v e_R$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

$$I = \frac{1}{2}$$

$$\bar{e}_R$$

$$I = 0$$

~~$$S\mathcal{L} = m_e \bar{e} e$$~~
~~$$= m_e (\bar{e}_R e_L + \bar{e}_L e_R)$$~~

$$S\mathcal{L} = \lambda_e \frac{\bar{L} \cdot \phi e_R + h.c.}{\lambda (\bar{\nu} \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R}$$

$\uparrow \langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$

$$\frac{\lambda_e}{\sqrt{2}} \bar{e}_L \nu e_R \quad 10^{-5}$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$I = \frac{1}{2}$$

$$\bar{e}_R$$

$$I = 0$$

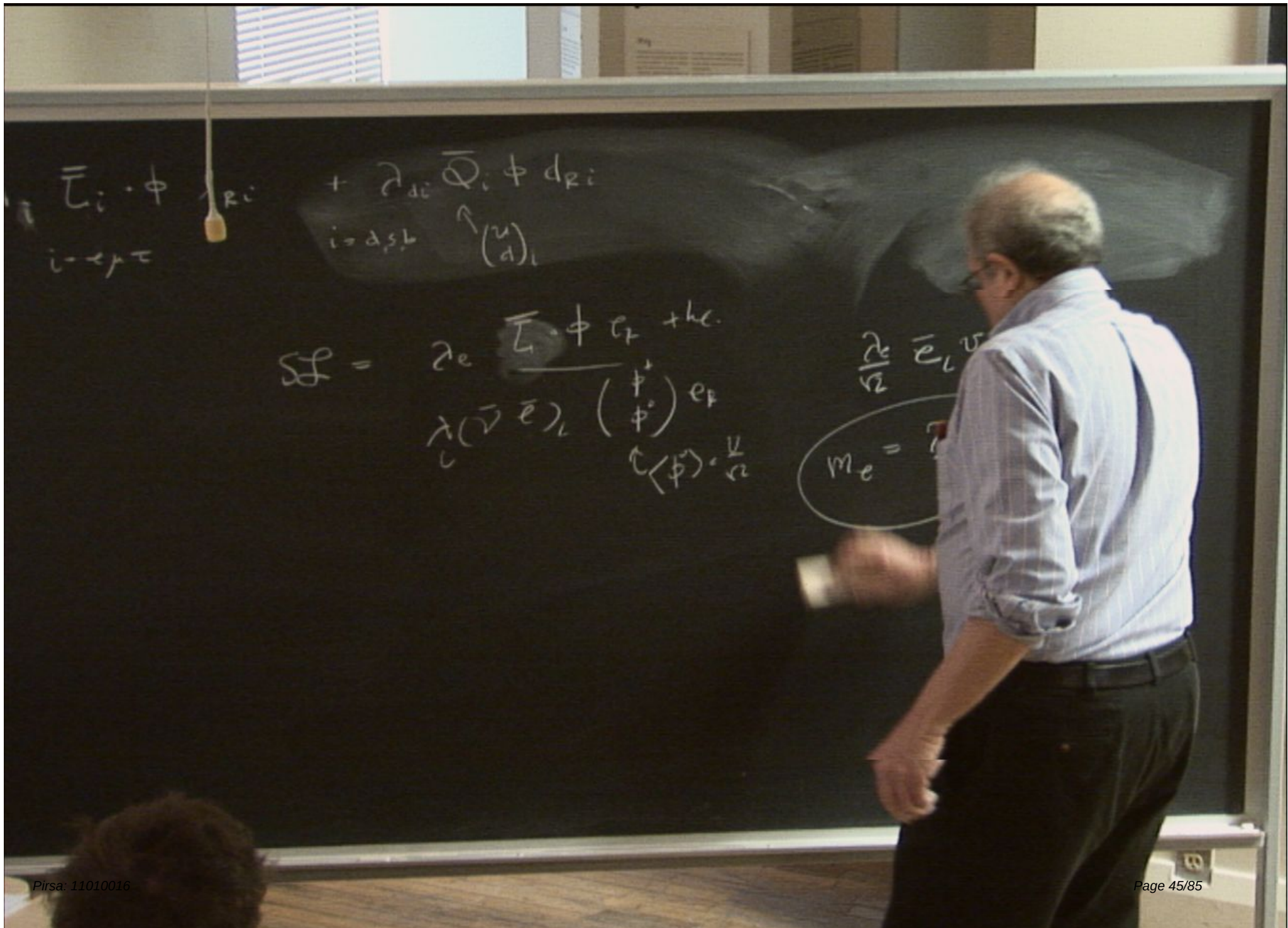
~~$$SL = m_e \bar{e} e - m_e (\bar{e}_R e_R + \bar{e}_L e_L)$$~~

$$SL = \frac{\lambda_e \bar{L} \cdot \phi e_R + h.c.}{\lambda (\bar{\nu} \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R}$$

$\uparrow \langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$

$$\frac{\lambda_e}{\sqrt{2}} \bar{e}_L \nu e_R \quad 10^{-5}$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$



$$\bar{L}_i \cdot \phi$$

$$i = e, \mu, \tau$$

$$+ \lambda_{di} \bar{Q}_i \phi d_{Ri}$$

$$i = d, s, b \quad \uparrow \begin{pmatrix} u \\ d \end{pmatrix}_L$$

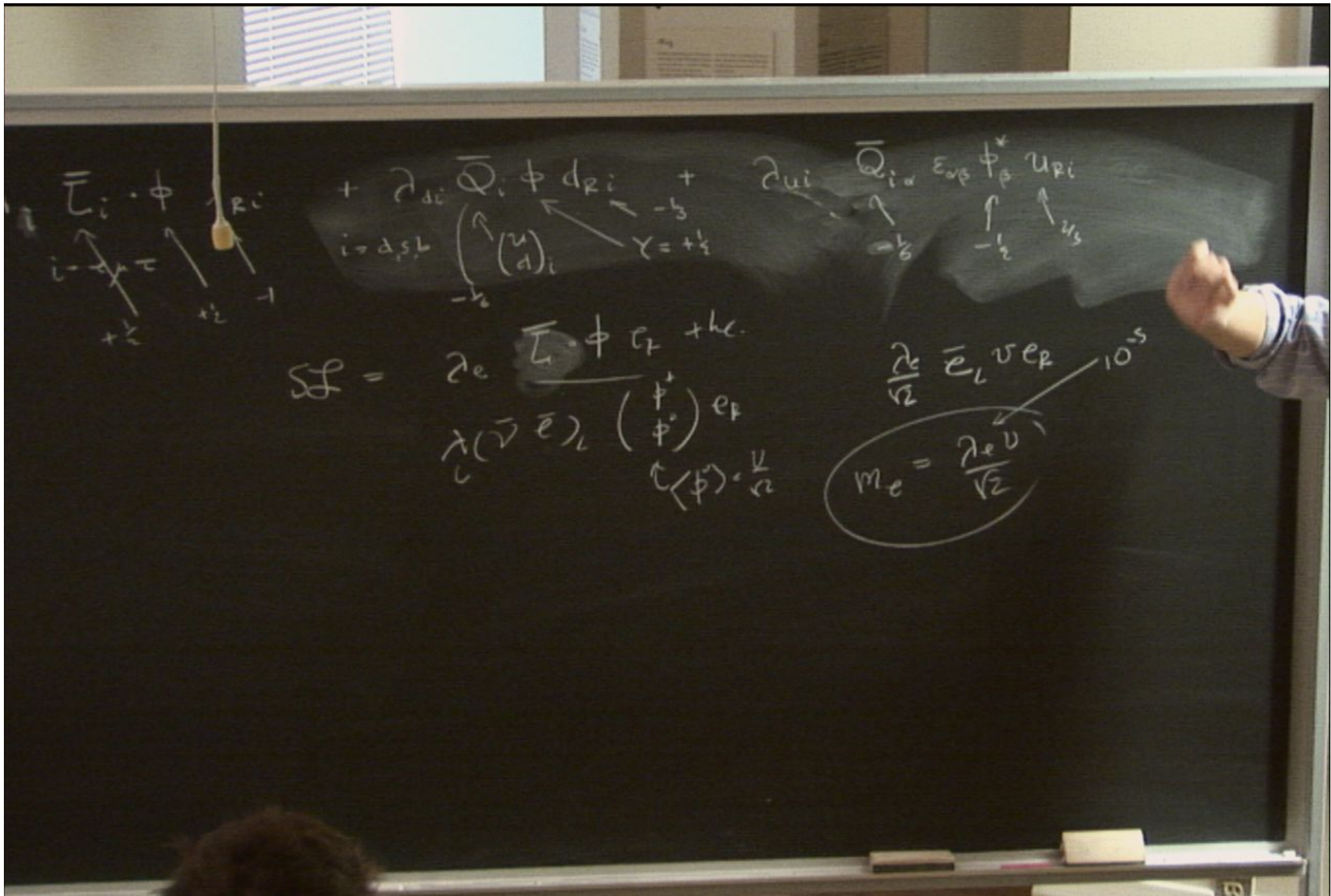
$$\mathcal{L} = \lambda_e \bar{L}_i \cdot \phi e_{Ri} + h.c.$$

$$\lambda (\bar{\nu} \bar{e})_L \begin{pmatrix} \bar{\nu} \\ \bar{e} \end{pmatrix}_R e_R$$

$$\uparrow \langle \bar{\nu} \rangle = \frac{v}{\sqrt{2}}$$

$$\frac{1}{2} \bar{e}_L \nu$$

$$m_e = ?$$



$$\bar{L}_i \cdot \phi \quad \begin{matrix} \nearrow R_i \\ \nwarrow i = d, s, b \\ \nearrow +\frac{1}{2} \\ \nwarrow +\frac{1}{2} \end{matrix}$$

$$+ \lambda_{di} \bar{Q}_i \phi d_{Ri} + \lambda_{ui} \bar{Q}_i \epsilon_{\alpha\beta} \phi_\beta^* u_{Ri}$$

$i = d, s, b$ $\begin{pmatrix} \uparrow (u) \\ \uparrow (d) \end{pmatrix}_i$ $\begin{matrix} \nwarrow -\frac{1}{3} \\ \nearrow \gamma = +\frac{1}{3} \end{matrix}$ $\begin{matrix} \nwarrow -\frac{1}{6} \\ \nearrow -\frac{1}{2} \end{matrix}$ $\begin{matrix} \nwarrow -\frac{1}{6} \\ \nearrow u_s \end{matrix}$

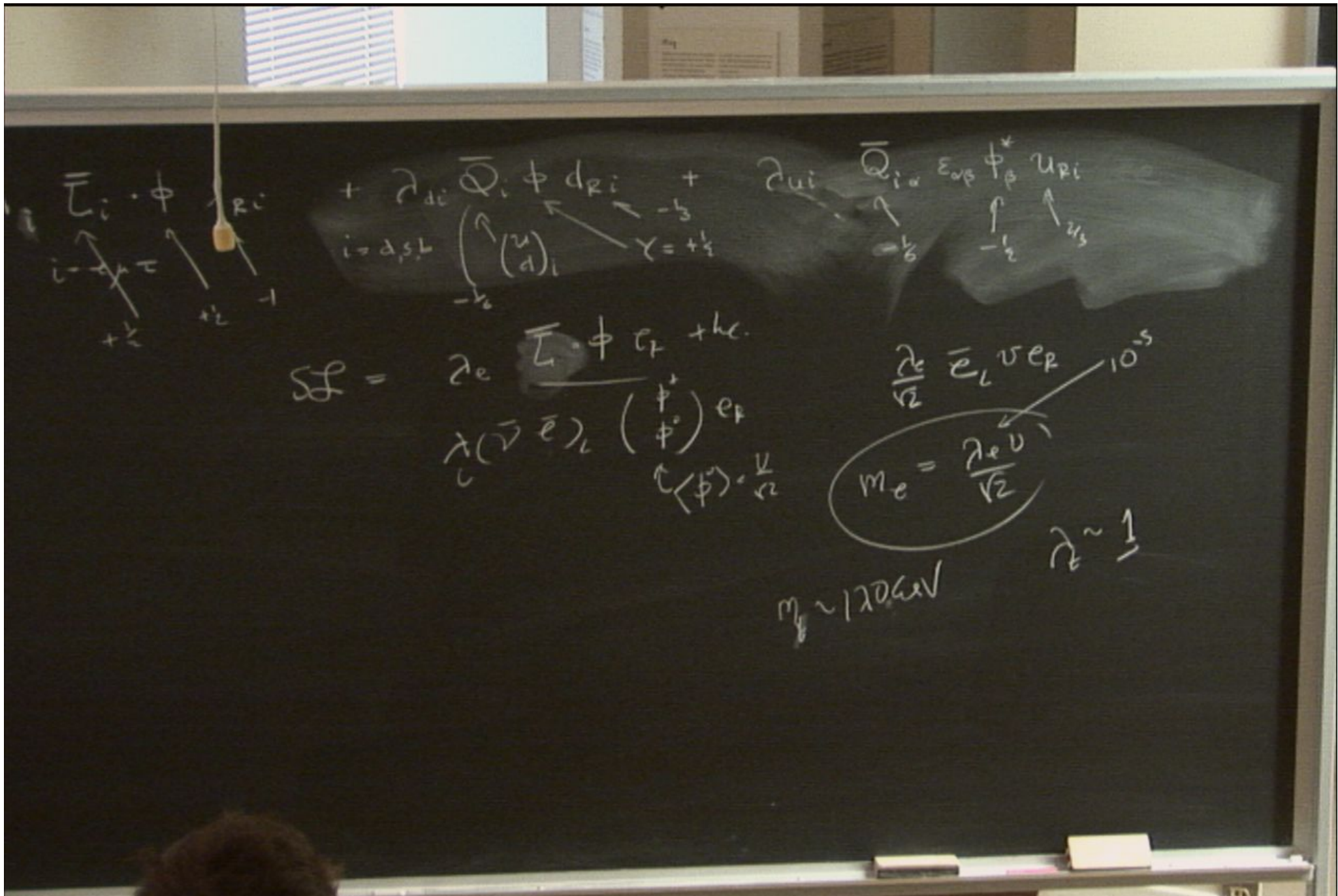
$$S\mathcal{L} = \lambda_e \bar{L}_i \cdot \phi e_{Ri} + h.c.$$

$$\lambda (\bar{\nu} \bar{e})_L \begin{pmatrix} \uparrow \phi^+ \\ \uparrow \phi^0 \end{pmatrix} e_R$$

$\uparrow \langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$

$$\frac{\lambda_e}{\sqrt{2}} \bar{e}_L v e_R \quad 10^{-5}$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$



$$\bar{L}_i \cdot \phi$$

Diagram showing hypercharge assignments for $\bar{L}_i$ and ϕ . Arrows point from $\bar{L}_i$ to $+\frac{1}{2}$ and from ϕ to -1 . A diagonal arrow from $\bar{L}_i$ to ϕ is labeled $i \rightarrow d, s, b$ and $+\frac{1}{6}$.

$$+ \lambda_{di} \bar{Q}_i \phi d_{Ri}$$

Diagram showing hypercharge assignments for $\bar{Q}_i$, ϕ , and d_{Ri} . Arrows point from $\bar{Q}_i$ to $-\frac{1}{6}$, from ϕ to $-\frac{1}{3}$, and from d_{Ri} to $+\frac{1}{3}$. A diagonal arrow from $\bar{Q}_i$ to d_{Ri} is labeled $i \rightarrow d, s, b$ and $-\frac{1}{6}$.

$$+ \lambda_{ui} \bar{Q}_i \epsilon_{\alpha\beta} \phi_\beta^* u_{Ri}$$

Diagram showing hypercharge assignments for $\bar{Q}_i$, ϕ_β^* , and u_{Ri} . Arrows point from $\bar{Q}_i$ to $-\frac{1}{6}$, from ϕ_β^* to $+\frac{1}{2}$, and from u_{Ri} to $+\frac{1}{3}$. A diagonal arrow from $\bar{Q}_i$ to ϕ_β^* is labeled $i \rightarrow d, s, b$ and $-\frac{1}{6}$.

$$S\mathcal{L} = \lambda_e \bar{L}_i \cdot \phi e_{Ri} + h.c.$$

$$\lambda (\bar{\nu} e)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$$

Diagram showing hypercharge assignments for $\bar{\nu}$, e , and $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$. Arrows point from $\bar{\nu}$ to $+\frac{1}{2}$, from e to $-\frac{1}{2}$, and from $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ to $+\frac{1}{2}$. A diagonal arrow from $\bar{\nu}$ to $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is labeled $i \rightarrow d, s, b$ and $+\frac{1}{2}$.

$$\frac{\lambda_e}{\sqrt{2}} \bar{e}_L v e_R$$

$$m_e = \frac{\lambda_e v}{\sqrt{2}}$$

$$m_f \sim 170 \text{ GeV}$$

$$\lambda_e \sim 1$$

$\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} \in \mathbb{C} \oplus \mathbb{C} \cong \mathbb{C}^2$
General 3×3 complex matrix

$$\lambda_{\ell}^{ij} \bar{L}_i^j \phi \ell_R^i + \lambda_d^{ij} \bar{Q}_i^j \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^j \phi$$

General 3×3 complex matrix

$$\sum_i \bar{\lambda}_i^j \bar{L}_i^i \phi \ell_R^j + \sum_d \bar{\lambda}_d^j \bar{Q}_d^i \phi d_R^j + \sum_u \bar{\lambda}_u^j \bar{Q}_u^i \epsilon_{\alpha\beta} \phi_p^* u_L^j + h.c.$$

General 3×3 complex matrix

$$\bar{L}^i \phi \ell_R^j + \lambda_d^{ij} \bar{Q}^i \phi d_R^j + \lambda_u^{ij} \bar{Q}^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

(3x3 complex matrix)

$$\begin{aligned}
 & \bar{L}_L^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_L^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_L^i \epsilon_{\alpha\beta} \phi_\beta^* u_R^j + h.c. \\
 & 3 \times 3 \text{ complex matrix} \quad + \quad \bar{L}_L^i \not{D} L_L^i + \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_R^i \not{D} u_R^i
 \end{aligned}$$



$$\bar{L}_L^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_L^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_L^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

3×3 complex matrix

$$+ \bar{L}_L^i \not{D} L_L^i + \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$$\lambda_l = \underbrace{U D V^\dagger}_{\substack{\text{real} \\ \text{phase} \\ \text{diagonal}}}$$

$$\lambda_l \lambda_l^\dagger$$

$$\lambda_l^\dagger \lambda_l$$

$$V^\dagger l_R = l'_R$$

$$\bar{L}_i^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_i^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

3x3 complex matrix

$$+ \bar{L}_i^i \not{D} L^i + \bar{Q}_i^i \not{D} Q^i + \bar{u}_R^i \not{D} u_R^i$$

$$\lambda_l = \frac{U Y_l V^\dagger}{\text{real part diagonal}}$$

$$\lambda_l \lambda_l^\dagger$$

$$\lambda_l^\dagger \lambda_l$$

$$V^\dagger l_R = l'_R$$

$$U^\dagger L = L'$$

$$Y_l \bar{L}'_i \phi l'^i_R$$

$$\bar{L}_L^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_L^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_L^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

3x3 complex matrix

$$+ \bar{L}_L^i \not{D} L_L^i + \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_R^i \not{D} u_R^i$$

$$\lambda_l = \frac{U Y_l V^\dagger}{\text{real part diagonal}}$$

$$\lambda_l \lambda_l^\dagger$$

$$\lambda_l^\dagger \lambda_l$$

$$V^\dagger l_R = l'_R$$

$$U^\dagger L_L = L'_L$$

$$Y_l \bar{L}_L^i$$

$$\bar{L}_i^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_i^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

3x3 complex matrix

$$+ \bar{L}_i^i i \not{D} L^i + \bar{Q}_i^i i \not{D} Q^i + \bar{u}_R^i i \not{D} u_R^i$$

$$\lambda_l = \frac{U Y_l V^\dagger}{\text{real part diagonal}}$$

$$\lambda_l \lambda_l^\dagger$$

$$\lambda_l^\dagger \lambda_l$$

$$V^\dagger l_R = l'_R$$

$$U^\dagger L = L'$$

$$Y_l \bar{L}'_i l'_R$$

$$\bar{L}'_i i A_l \frac{\sigma^a}{2} L'_i$$

$$\bar{L}_i^i \phi l_R^i + \lambda_d^{ij} \bar{Q}_i^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

3x3 complex matrix

$$+ \bar{L}_i^i \not{D} L^i + \bar{Q}_i^i \not{D} Q^i + \bar{u}_R^i \not{D} u_R^i$$

$$\lambda_l = \frac{U Y_l V^\dagger}{\text{real part diagonal}}$$

$$\lambda_l \lambda_l^\dagger$$

$$\lambda_l^\dagger \lambda_l$$

$$V^\dagger l_R = l'_R \quad U^\dagger L = L'$$

$$Y_l \bar{L}'_i \phi l'^i_R$$

$$\bar{L}_i A_{ij}^a \sigma_a^i L = \bar{L}'_i A'_{ij} \frac{\sigma_a^i}{2} L'$$

$$\phi d_R^j + \lambda_{ij}^{\nu} \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \bar{\phi}_\beta^* u_L^j + h.c.$$

$$\tau_L^i + \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_R^i \not{D} u_R^i$$

$$V^\dagger l_R = l'_R \quad U^\dagger L = L'$$

$$\forall_i \bar{L}'_i \phi l'_{iR}$$

$$\bar{L}_i A_{ij}^a \not{L}_j = \bar{L}'_i A'_{ij} \frac{\sigma^a}{2} L'_j$$

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

μ

$$\phi d_R^j + \lambda_{ij}^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_R^* u_L^j + h.c.$$

$$L^i + \bar{Q}_i^i \not{D} Q^i + \pi_R^i \not{D} u_R^i$$

$$V^\dagger l_R = l'_R \quad U^\dagger L = L'$$

$$Y_i \bar{L}'_i \phi l'_{iR}$$

$$\bar{L}_i A_{ij}^a \not{D} L_j = \bar{L}'_i A_{ij}^a \frac{\sigma^a}{2} L'_j$$

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

$$\mu^- \not\rightarrow e^- \gamma$$

$$\mathcal{O}_d = U_d Y_d V_d^\dagger$$

$$\mathcal{O}_u = U_u Y_u V_u^\dagger$$

$$\sum_i \lambda_{ij}^i \bar{\ell}_i \cdot \phi \ell_j^i + \sum_d \lambda_d^{ij} \bar{Q}^i \cdot \phi d_j^i + \dots$$

General 3×3 complex matrix

$$+ \bar{\ell}_i i \not{D} \ell_i + \bar{Q}^i i \not{D} Q^i$$

$$Z_1 \gamma^\mu u_L + \bar{u}_R \underbrace{Q_{2R}}_{(-\frac{2}{3} \tilde{g} \tilde{\omega})} \gamma^\mu u_R$$

$$V^T \ell$$

$$\bar{\ell}_i A^a_i$$

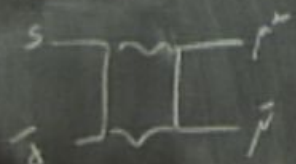
$$\sum_i \lambda_{ij}^i \bar{L}_i \cdot \Phi l_j^i + \lambda_{id}^i \bar{Q}_i \cdot \Phi d_j^i + \lambda_{iu}^i \bar{Q}_i \cdot \Phi e_j^i$$

General 3×3 complex matrix

$$+ \bar{L}_i i \not{D} L_i + \bar{Q}_i i \not{D} Q_i +$$

$$\sum_f \bar{u}_f' \underbrace{Q_{2L}}_{\left(\frac{1}{2} - \frac{2}{3} s_w^2\right)} \gamma^\mu u_f' + \bar{u}_f \underbrace{Q_{2R}}_{\left(-\frac{2}{3} s_w^2\right)} \gamma^\mu u_f$$

$V_u^\dagger V_u = 1$
 $U_u^\dagger U_u = 1$



$$s \rightarrow d + Z^0 \rightarrow p^+ p^-$$

$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + \text{h.c.}$$

$$\bar{Q}_i^i \not{D} Q_i + \bar{u}_L^i \not{D} u_L^i$$

$$W^+ \bar{u}_{iL} \gamma^\mu d_{iL} \\ = W^+ \bar{u}_{iL} \underbrace{V_{ui}^+ \gamma^\mu V_{di}}_{\text{CKM}} d_{iL}$$

V_{CKM}

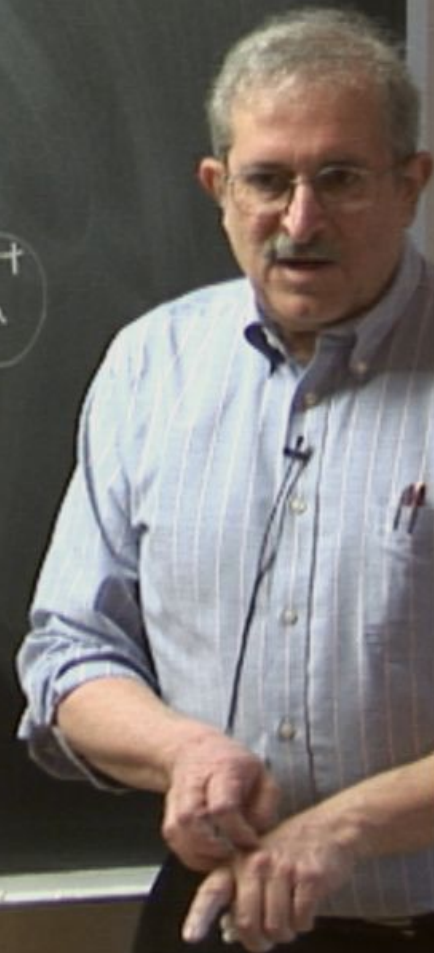
$\underbrace{\quad}_{SU(3) \text{ unitary matrix}}$

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

$$\mu^- \not\rightarrow e^- \gamma$$

$$\gamma_d = U_d \gamma_d V_d^+$$

$$\gamma_u = U_u \gamma_u \underbrace{V_u^+}_{\text{circled}}$$



$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

$$\bar{Q}_i^i \not{D} Q_i + \bar{u}_L^i \not{D} u_L^i$$

$$W^+ \bar{u}_{iL} \gamma^\mu d_{iL} \\ = W^+ \bar{u}_{iL} \underbrace{V_{uV_d}^\dagger \gamma^\mu V_d}_{V_{CKM}} d_{iL}$$

V_{CKM}

$\underbrace{\quad}_{SU(3) \text{ unitary matrix}}$

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

$$\mu^- \not\rightarrow e^- \gamma$$

$$\mathcal{L}_d = U_d \gamma_d V_d^\dagger \\ \mathcal{L}_u = U_u \gamma_u \underbrace{V_u^\dagger}_{\text{circled}}$$

$$\sum_{ij} \lambda_{ij}^{\dagger} \bar{L}^i \cdot \phi \ell_R^j + \lambda_d^{\dagger} \bar{Q}^i \cdot \phi d_R^j + \lambda_u^{\dagger} \bar{Q}^i \cdot \phi u_R^j$$

general 3x3 complex matrix

$$+ \bar{L}^i \not{D} L^i + \bar{Q}^i \not{D} Q^i +$$

$$V_{CKM} = \begin{pmatrix} \cos \theta e^{i\alpha} & \sin \theta e^{i\beta} \\ \sin \theta e^{i\delta} & \cos \theta \end{pmatrix}$$

$$W^+ \frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu$$

$$= W^+ \bar{u}_L'$$

$$\sum_{ij} \lambda_{ij}^{\dagger} \bar{L}_i^i \phi l_R^j + \lambda_d^{\dagger} \bar{Q}_i^i \phi d_R^j + \lambda_u^{\dagger} \bar{Q}_i^i \phi u_R^j$$

general 3x3 complex matrix

$$+ \bar{L}_i^i \not{D} L_i^i + \bar{Q}_i^i \not{D} Q_i^i +$$

$$V_{KM} = \begin{pmatrix} \cos \theta e^{i\alpha} & \sin \theta e^{i\beta} \\ -\sin \theta e^{i\gamma} & \cos \theta \end{pmatrix} e^{i(\beta - \alpha + \gamma)}$$

$$W^+ \frac{1}{\sqrt{2}} \bar{u}_i \gamma^\mu$$

$$= W^+ \bar{u}_i$$

$$\sum_{ij} \lambda_{ij}^u \bar{L}_i^j \phi l_R^j + \lambda_{ij}^d \bar{Q}_i^j \phi d_R^j + \lambda_{ij}^e \bar{Q}_i^j \phi e_R^j$$

general 3x3 complex matrix

$$+ \bar{L}_i^j \not{D} L_i^j + \bar{Q}_i^j \not{D} Q_i^j +$$

$$V_{CKM} = \begin{pmatrix} \bar{u}' & \bar{c}' \end{pmatrix} \begin{pmatrix} \cos \theta e^{i\alpha} & \sin \theta e^{i\beta} \\ -\sin \theta e^{i\delta} & \cos \theta \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix} e^{i(\beta - \alpha + \delta)}$$

$$u' \rightarrow \bar{u}'' e^{-i\alpha}$$

$$\bar{c}' \rightarrow \bar{c}'' e^{i\delta}$$

$i=(u,c,t)$

$$\lambda_{ij}^e \bar{Q}_i^j \phi e_R^j$$

W

$$\sum_i \lambda_{ij}^i \bar{L}_i^i \phi l_R^j + \lambda_d^{ij} \bar{Q}_i^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

real 3×3 complex matrix

$$+ \bar{L}_i^i \not{D} L_i^i + \bar{Q}_i^i \not{D} Q_i^i + \bar{u}_L^i \not{D} u_L^i$$

$$\begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\beta} \\ \sin\theta e^{i\delta} & \cos\theta \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix} = e^{i(\beta-\alpha+\delta)} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$u' \rightarrow \bar{u}'' e^{-i\alpha}$$

$$\bar{c}' \rightarrow \bar{c}'' e^{i\delta}$$

$$d' \rightarrow e^{i(\beta-\alpha)} d = W^+ \bar{u}_L' V_{ud} d_L'$$

$$W^+ \frac{1}{\sqrt{2}} \gamma^\mu d_{iL}$$

$$\sum_{ij} \lambda_{ij}^d \bar{L}_i^j \phi d_R^j + \lambda_d^i \bar{Q}_i^j \phi d_R^j + \lambda_u^i \bar{Q}_i^j \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

real 3×3 complex matrix

$$\begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\beta} \\ \sin\theta e^{i\delta} & \cos\theta \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix} = e^{i(\beta-\alpha+\delta)} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\begin{aligned} u' &\rightarrow \bar{u}' e^{-i\alpha} \\ \bar{c}' &\rightarrow \bar{c}' e^{i\delta} \end{aligned}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$d' \rightarrow e^{i\beta} d$$

$$W^+ \frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L = W^+ \bar{u}_L \underbrace{V_u^\dagger \gamma^\mu V_d}_{V_{CKM}} d_L$$

V_{CKM}

$U(3)$ unitary matrix

$$\sum_i \lambda_{ij}^i \bar{L}_i^i \phi l_R^j + \lambda_d^{ij} \bar{Q}_i^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_i^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

real 3×3 complex matrix

$$+ \bar{L}_i^i \not{D} L_i^i + \bar{Q}_i^i \not{D} Q_i^i + \bar{u}_L^i \not{D} u_L^i$$

$$\begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\beta} \\ -\sin\theta e^{i\delta} & \cos\theta \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix} = e^{i(\beta-\alpha+\delta)} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\left. \begin{aligned} u' &\rightarrow \bar{u}' e^{-i\alpha} \\ \bar{c}' &\rightarrow \bar{c}' e^{i\delta} \end{aligned} \right\}$$

$$\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$i=(u,c,t)$

$$W^+ \frac{1}{\sqrt{2}} \gamma^\mu d_{iL} = W^+ \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu V_{ui}^+ V_d$$

$$d' \rightarrow e^{i\beta} d$$

∇

$$\sum_{ij} \lambda_{ij}^L \bar{L}_L^i \phi L_R^j + \sum_{ij} \lambda_{ij}^D \bar{Q}_L^i \phi d_R^j + \sum_{ij} \lambda_{ij}^u \bar{Q}_L^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

real 3×3 complex matrix

$$+ \bar{L}_L^i \not{D} L_L^i + \bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$$\begin{pmatrix} \cos \theta e^{i\alpha} & \sin \theta e^{i\beta} \\ \sin \theta e^{i\delta} & \cos \theta \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix} = e^{i(\beta - \alpha + \delta)} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\left. \begin{aligned} u' &\rightarrow \bar{u}' e^{-i\alpha} \\ \bar{c}' &\rightarrow \bar{c}' e^{i\delta} \end{aligned} \right\}$$

$$\begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$i = (u, r, t)$$

$$+ \partial_u^i \bar{Q}_\alpha^i \varepsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

$$\bar{Q}^i i \not{D} Q^i + \bar{u}_2^i i \not{D} u_2^i$$

$$C: \quad f_L \rightarrow \bar{f}_L$$

$$P: \quad \bar{f}_L \rightarrow \bar{f}_R$$

$$\bar{f}_L i \not{D} f_L$$

$$f_R \leftarrow \bar{f}_L$$

$i=(u,r,t)$

$$+ \lambda_u^i \bar{Q}_\alpha^i \varepsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

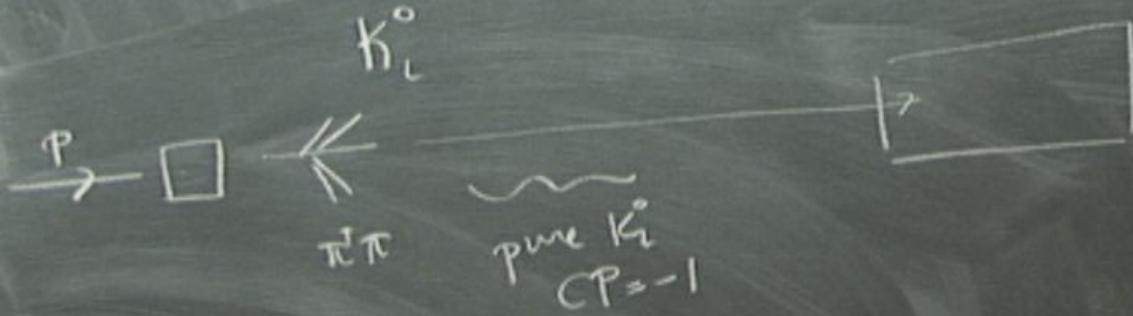
$$\bar{Q}_i^i \psi Q_i^i + \bar{u}_L^i \psi u_R^i$$

k_i^0

$i = (u, c, t)$

$$+ \lambda_u^i \bar{Q}_\alpha^i \varepsilon_{\alpha\beta} \phi_\beta^* u_L^i + \text{h.c.}$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$



$i = (u, c, t)$

$$+ \lambda_{ij}^u \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^j u_L^i + \text{h.c.}$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i i \not{D} u_L^i$$

K_L^0



pure K_L^0
 $CP = -1$



10^{-3} of the time

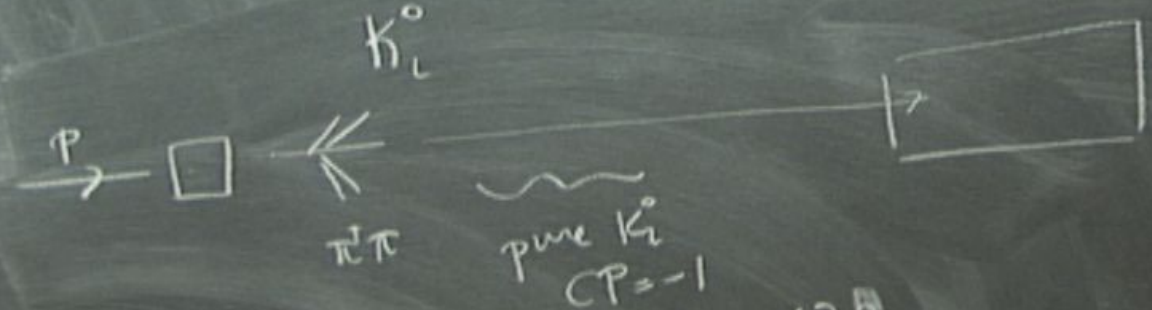
$K_L^0 \rightarrow \pi^+ \pi^-$!

$i=u,c,t$

$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$10^{-3} \text{ of the time}$
 $\swarrow K_L^0 \rightarrow \pi^+ \pi^- !$



Kobayashi + Maskawa 1972

$$i = (u, c, t)$$

$$+ \lambda_{ij}^u \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^j u_L^i + h.c.$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$$K_L^0$$



pure K_L^0
 $CP = -1$

Kobayashi + Maskawa

1972

10^{-3} of the time

$$\checkmark K_L^0 \rightarrow \pi^+ \pi^- !$$

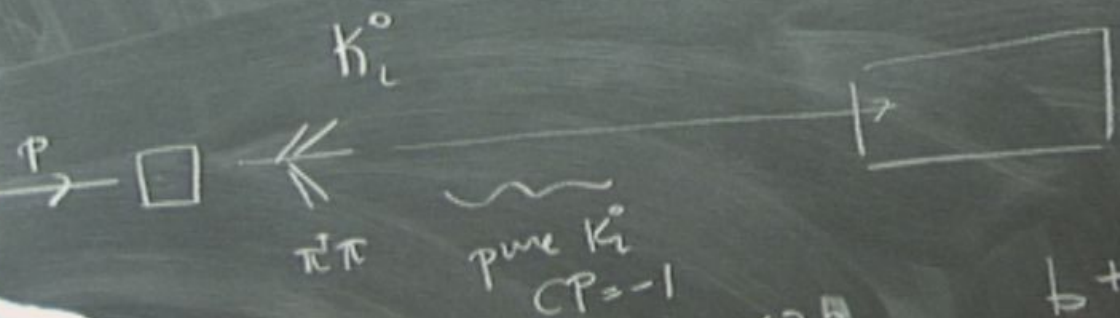
$i = (u, c, t)$

$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^j + h.c.$$

$$\bar{Q}_i^i \psi Q_i^i + \bar{u}_L^i \psi u_R^i$$

$10^{-3} Q_{th}^{true}$

$$\checkmark K_L^0 \rightarrow \pi^+ \pi^- !$$



+ Maskawa

1972

$q = 3 \text{ angles} + 6 \text{ phases.}$

$6 \text{ gauge} \rightarrow \text{remove 5 phase} \rightarrow \text{fixed reality}$

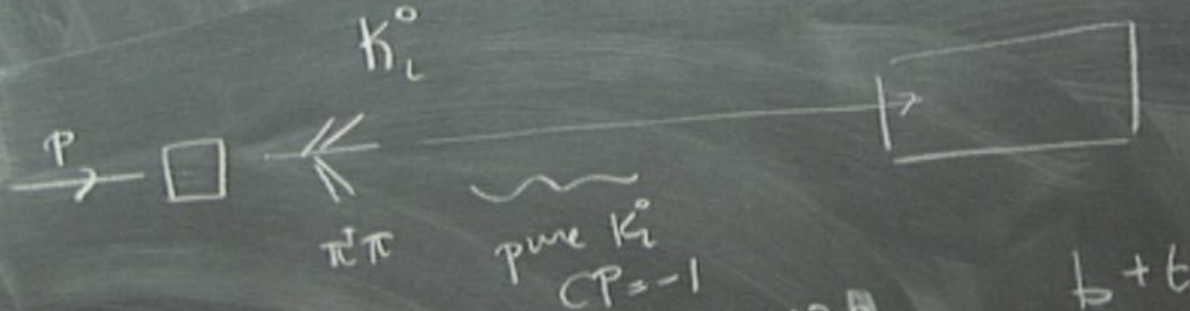
$i = (u, c, t)$

$$+ \lambda_{ij}^u \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^j u_L^i + h.c.$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$10^{-3} \text{ of the time}$

$$\checkmark K_L^0 \rightarrow \pi^+ \pi^- !$$



Kobayashi + Maskawa

1972

$$U(3) \rightarrow 9 = 3 \text{ angles} + 6 \text{ phases.}$$

6 gauge → remove 5 phases → fixed reality

$$\sum_i \bar{\ell}_L^i \not{D} \ell_R^i + \sum_d \bar{Q}_L^i \not{D} q_R^i + \sum_u \bar{Q}_L^i \not{D} u_R^i + \text{genet } 3 \times 3 \text{ complex matrix}$$

$$+ \bar{\ell}_L^i \not{D} \ell_L^i + \bar{Q}_L^i \not{D} Q_L^i +$$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\rho \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ -A\lambda^2 & 0 & 1 \end{pmatrix}$$

$$\lambda = \sin \theta_c$$

$\vec{p} \rightarrow \vec{p}'$

$$\sum_{ij} \bar{L}_i^i \phi L_R^j$$

general 3x3 complex matrix

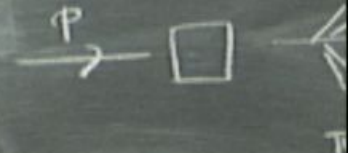
$$+ \sum_{ij} \bar{Q}_i^i \phi d_R^j$$

$$+ \sum_{ij} \bar{Q}_i^i \phi d_R^j \quad i=(u, c, t)$$

$$+ \bar{L}_i^i \not{D} L_i^i + \bar{Q}_i^i \not{D} Q_i^i +$$

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(e^{-i\eta}) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \lambda & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \sin \theta_c \quad A\lambda^3(1-e^{-i\eta})$$



Kobayashi +

U(3)

$$\sum_i \bar{\ell}_L^i \not{D} \ell_R^i + \sum_d \bar{Q}_L^i \not{D} q_R^i + \sum_u \bar{Q}_L^i \not{D} \ell_R^i + \text{genet } 3 \times 3 \text{ complex matrix}$$

$$+ \bar{\ell}_L^i \not{D} \ell_L^i + \bar{Q}_L^i \not{D} Q_L^i +$$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(e^{-i\eta}) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ \lambda & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \sin \theta_c \quad A\lambda^3(1 - e^{-i\eta})$$

$\xrightarrow{P} \square$

Kobayashi

$$\sum_i \bar{\ell}_L^i \phi \ell_R^i$$

general 3x3 complex matrix

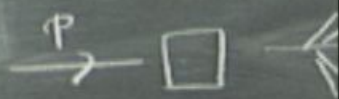
$$+ \sum_i \bar{d}_L^i \phi d_R^i$$

$$+ \sum_i \bar{u}_L^i \phi u_R^i$$

$$+ \bar{\ell}_L^i \not{D} \ell_L^i + \bar{d}_L^i \not{D} d_L^i + \bar{u}_L^i \not{D} u_L^i$$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(e^{-i\eta}) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ \lambda A^2 & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \sin \theta_c \quad A\lambda^3(1 - e^{-i\eta})$$



Kobayashi +

U(3)

$$i = (u, c, t)$$

$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \Phi_\beta^* u_L^j + h.c.$$

$$\bar{Q}_L^i \not{D} Q_L^i + \bar{u}_L^i \not{D} u_L^i$$

$$B \rightarrow \pi_4 + K_S^0$$

$$10^{-3} \text{ of the time}$$

$$K_L^0 \rightarrow \pi^+ \pi^-$$

$$K_L^0$$



pure K_L^0
 $CP = -1$

Kobayashi + Maskawa

1972

$$U(3) \rightarrow 9 = 3 \text{ angles} + 6 \text{ phases}$$

G gauge $\rightarrow$ remove 5 phases $\rightarrow$ free

$i = (u, c, t)$

$$+ \lambda_u^i \bar{Q}_\alpha^i \epsilon_{\alpha\beta} \phi_\beta^* u_L^i + h.c.$$

$$\bar{Q}_i^i \psi Q_i^i + \bar{u}_L^i \psi u_L^i$$

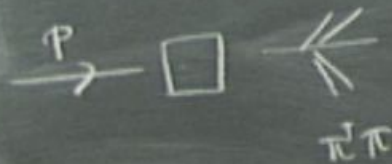
$$B^0 \rightarrow \pi_4^0 + K_S^0$$

10^{-3} of the time

$$B^0 \rightarrow \pi_4^0 + K^0$$

$$K_L^0 \rightarrow \pi^+ \pi^-$$

$$\sim O(1)$$



pure K_L^0
 $CP = -1$

Kobayashi + Maskawa

1972

$$U(3) \rightarrow 9 = 3 \text{ angles} + 6$$

G gauge $\rightarrow$ remove 5 phases

$b + \bar{b}$