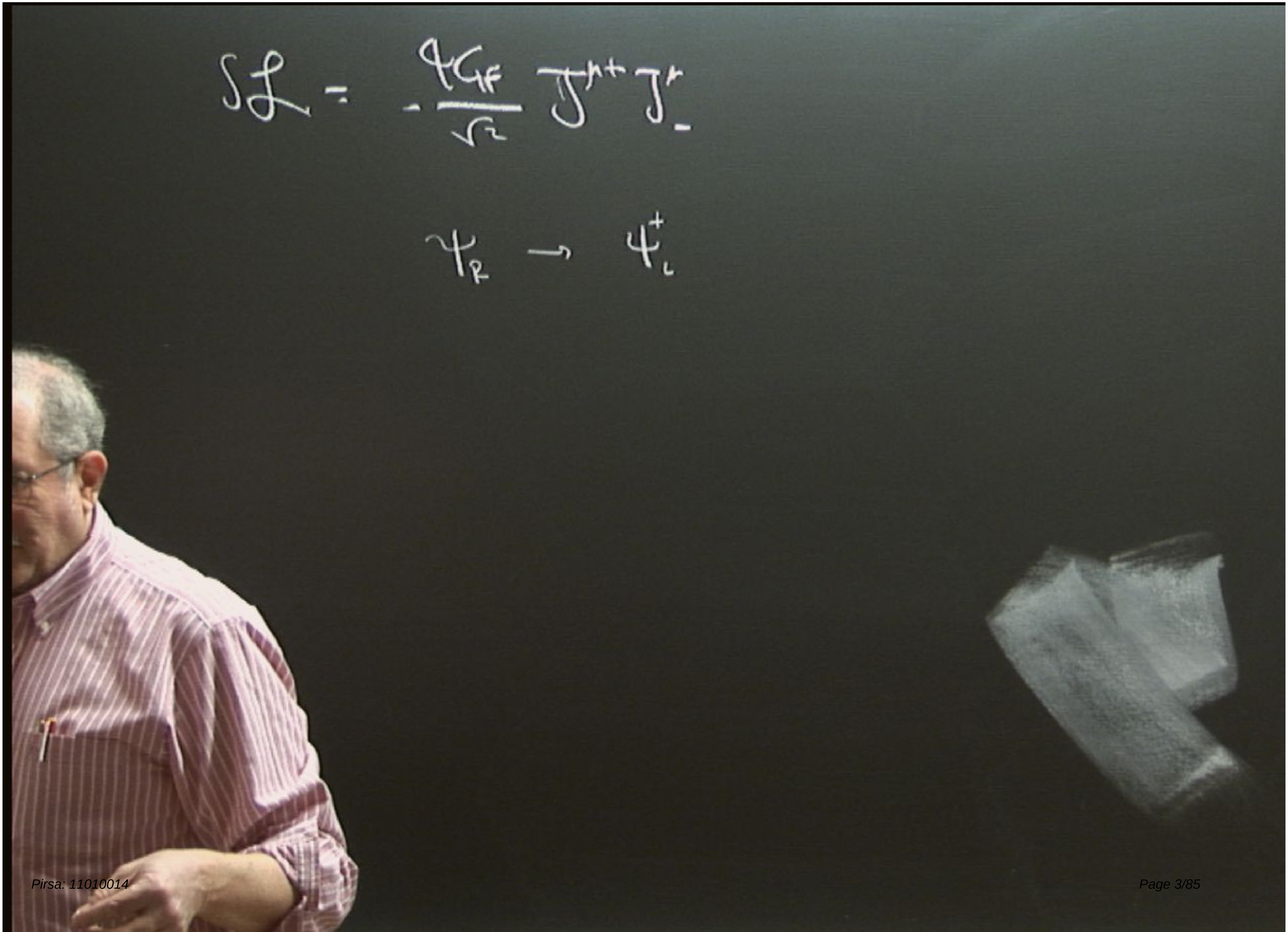
Title: Standard Model - Lecture 11

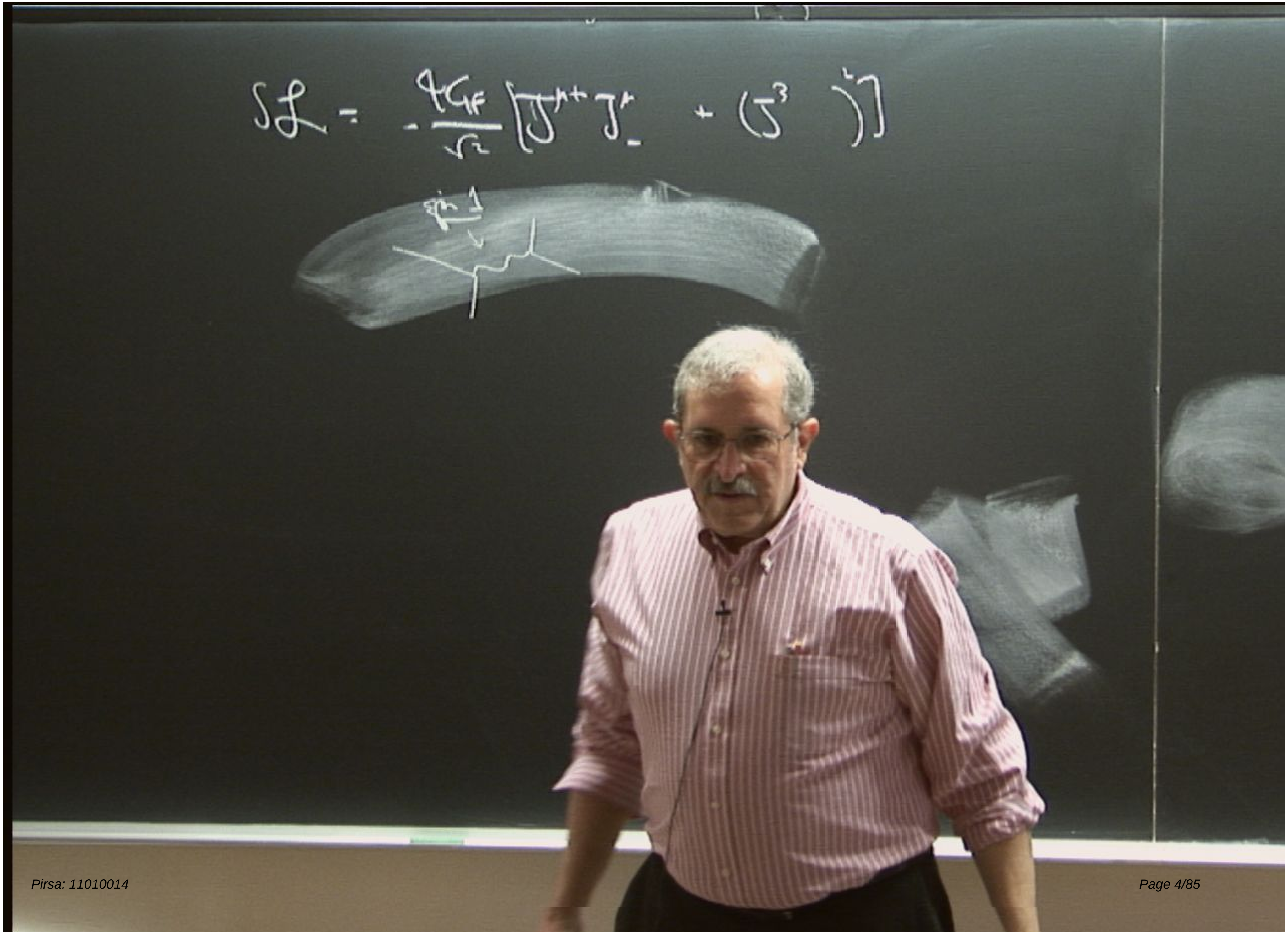
Date: Jan 17, 2011 09:00 AM

URL: http://pirsa.org/11010014

Abstract:

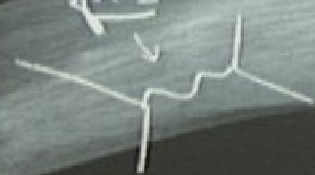
$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}} J^{\mu +} J^{\mu}_{-}$$


$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}} J^{\mu +} J^{\mu}_{-}$
$\psi_R \rightarrow \psi^{+}_{L}$


$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ )^2\right]$
spin 1

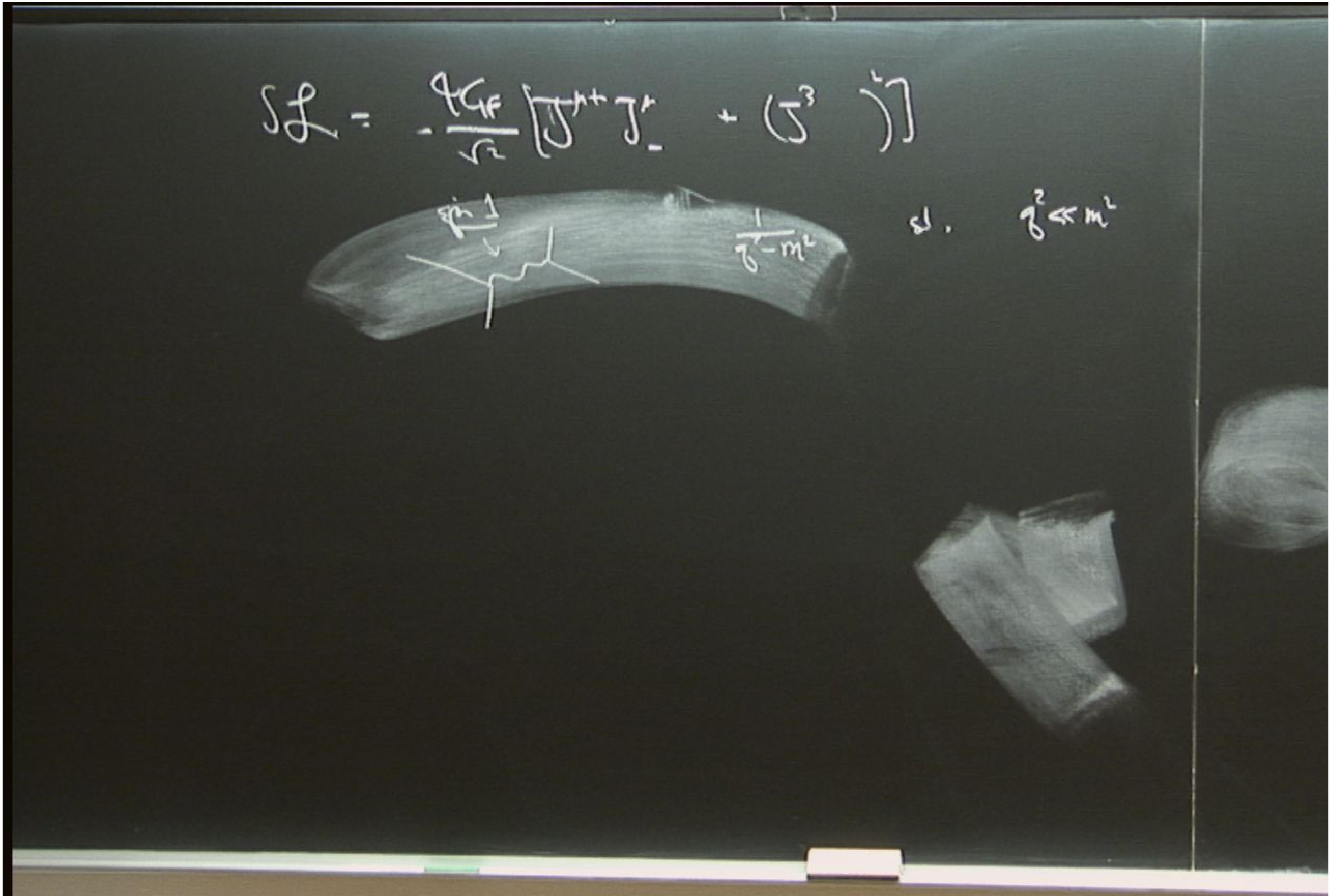
$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3 \quad)^2\right]$$

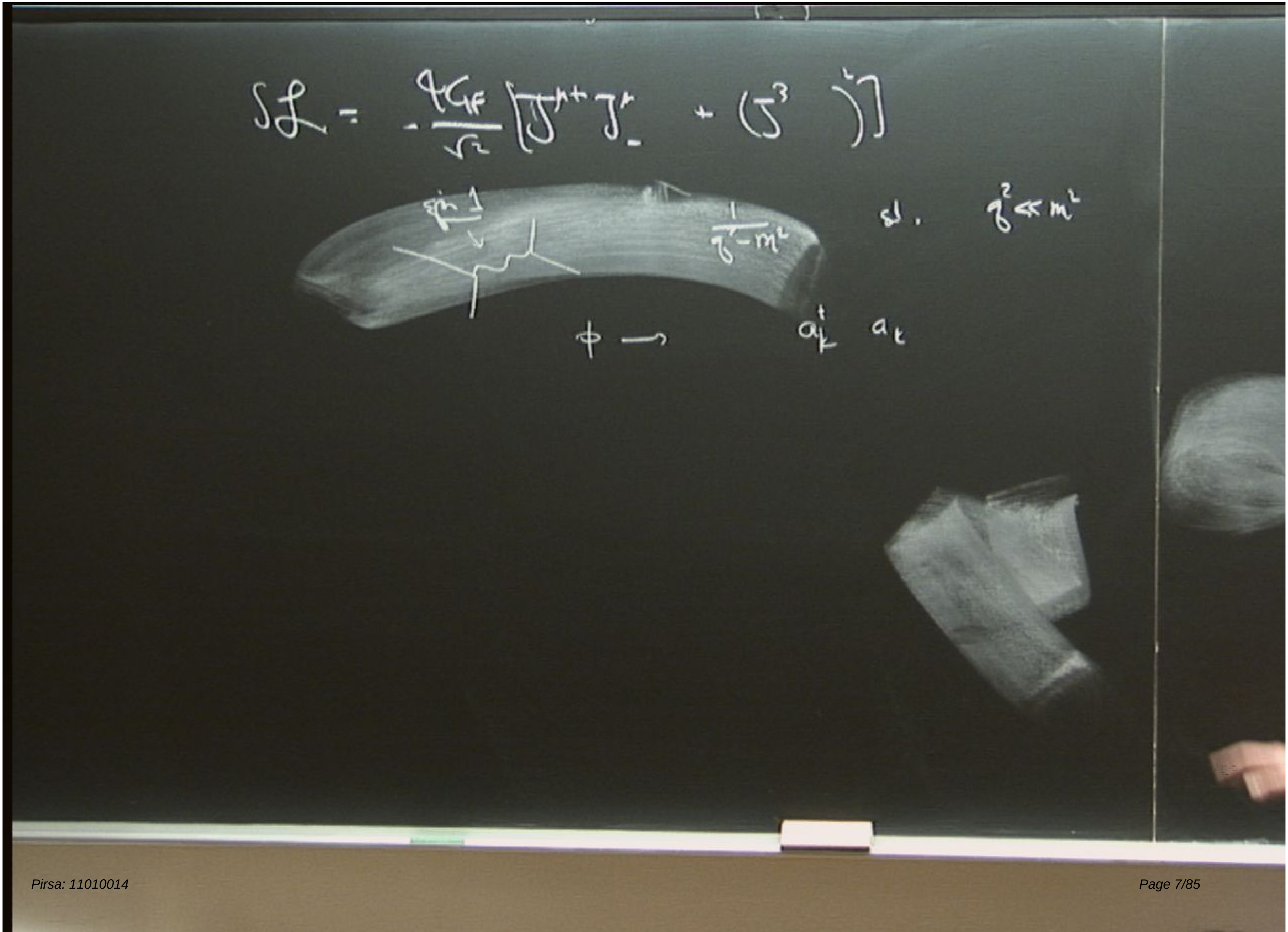
spin 1



$\frac{1}{q^2 - m^2}$

s.t. $q^2 \ll m^2$


$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu+}J^{\mu}_{-} + (J^{3}\ )^{2}\right]$
spin 1
$\frac{1}{q^2-m^2}$
$q^2 \ll m^2$


$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}} \left[J^{\mu+} J^{\mu}_{-} + (J^3 \quad)^2 \right]$
spin 1
$\frac{1}{q^2 - m^2}$
sd. $q^2 \ll m^2$
$\phi \longrightarrow \quad a_k^\dagger \quad a_k$

$$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ \ )^2\right]$$

spin 1

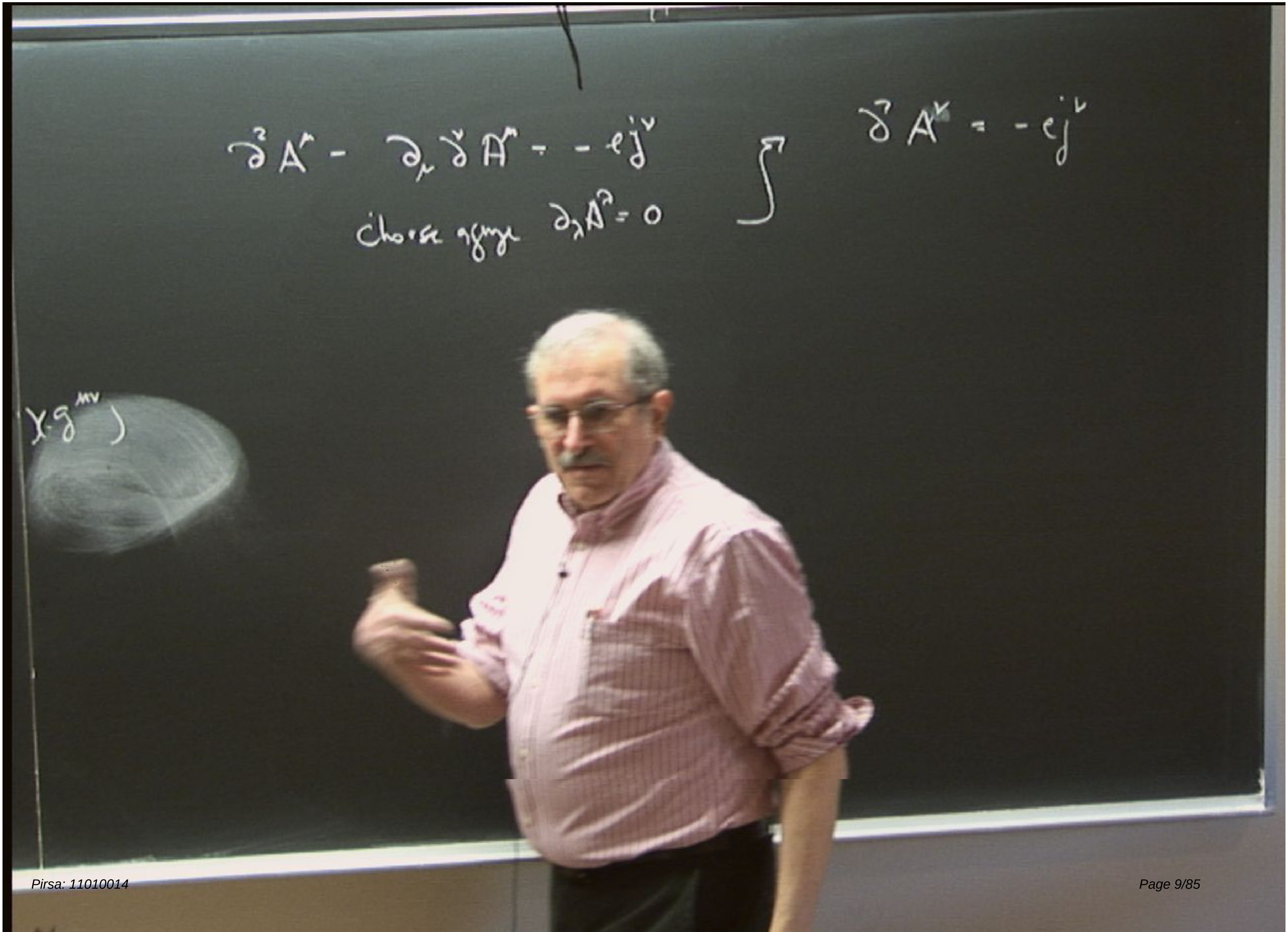
$\frac{1}{q^2 - m^2}$

s.l. $q^2 \ll m^2$

$\phi \longrightarrow$

$[a_k, a^{\dagger}_{k'}]\ (2\pi)^3 \delta(k - k')$

$[a^{\mu}_{k}, a^{\dagger\nu}_{k'}] = (2\pi)^3 \delta(\vec{k} - \vec{k}')(-g^{\mu\nu})$


$\partial^2 A^\nu - \partial_\mu \partial^\nu A^\mu = -e j^\nu$
choose gauge $\partial_\lambda A^\lambda = 0$
$\partial^2 A^\nu = -e j^\nu$

$$\partial^2 A^\mu - \partial_\mu \partial^\nu A^\mu = -e j^\nu \qquad \partial^2 A^\nu = -e j^\nu$$

choose gauge $\partial_\lambda A^\lambda = 0$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^* D^\mu \phi \qquad D_\mu \phi = \partial_\mu \phi + ie A_\mu \phi$$

$\ldots g^{\mu\nu})$

$$j^\mu = \frac{1}{e}\frac{\delta \mathcal{L}}{\delta A_\mu} = -i(\phi^* D^\mu \phi - (D^\mu \phi)^* \phi)$$

$$= i(\partial^\mu \phi^* \phi - \phi^* \partial^\mu \phi) + 2e^2$$

$$\partial^2 A^\mu - \partial_\mu \partial^\nu A^\mu = -e j^\nu$$

$$\partial^2 A^\nu = -e j^\nu$$

choose gauge $\partial_\lambda A^\lambda = 0$

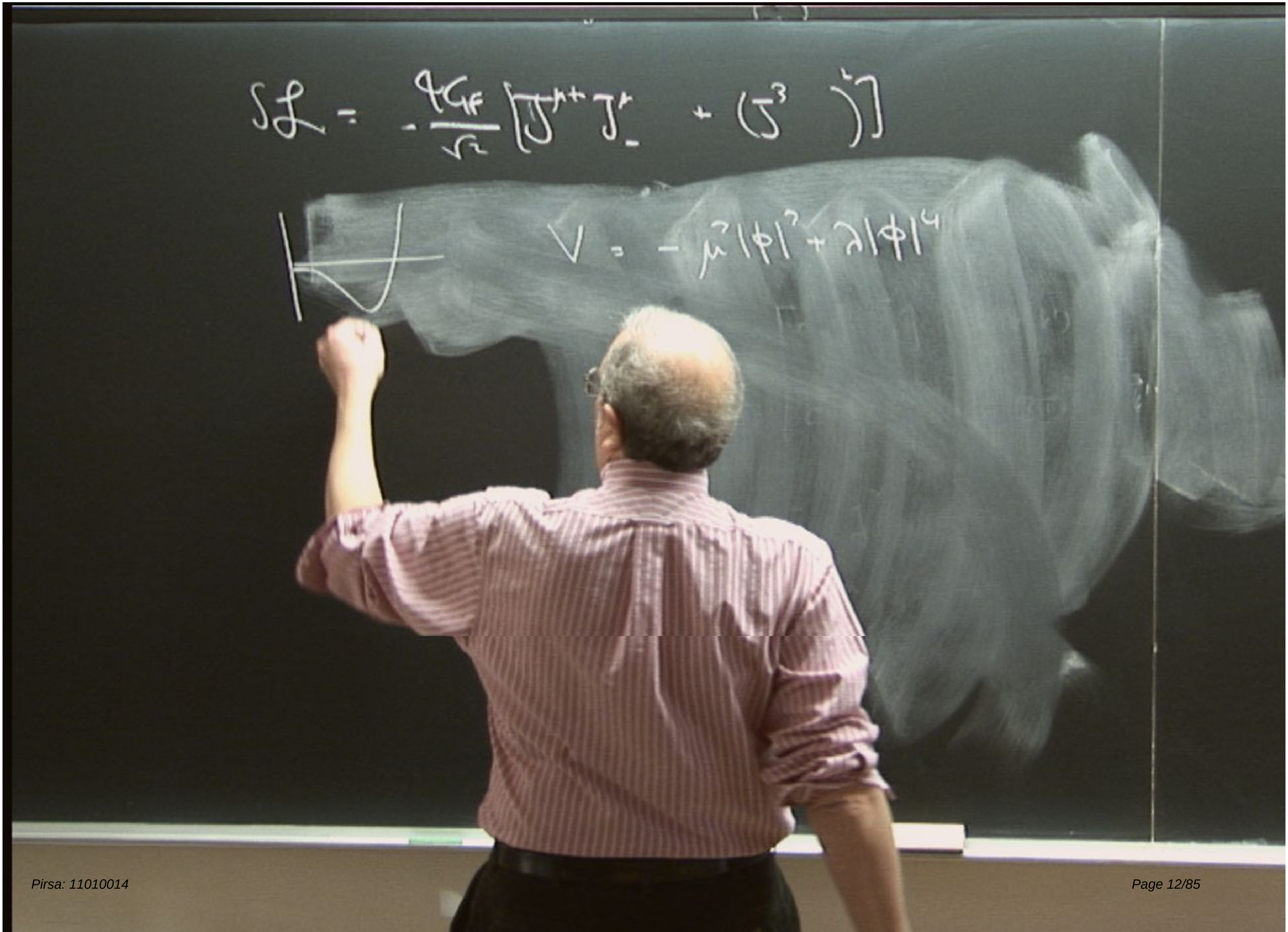
$$- V(\phi)$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^* D^\mu \phi \quad , \quad D_\mu \phi = \partial_\mu \phi + i e A_\mu \phi$$

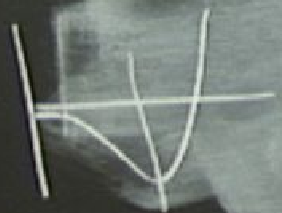
$X g^{\mu\nu})$

$$j^\mu = \frac{1}{e}\frac{\delta \mathcal{L}}{\delta A_\mu} = -i(\phi^* D^\mu \phi - (D^\mu \phi)^* \phi)$$

$$= i(\partial^\mu \phi^* \phi - \phi^* \partial^\mu \phi) + 2e^2 \phi^* \phi A_\mu$$

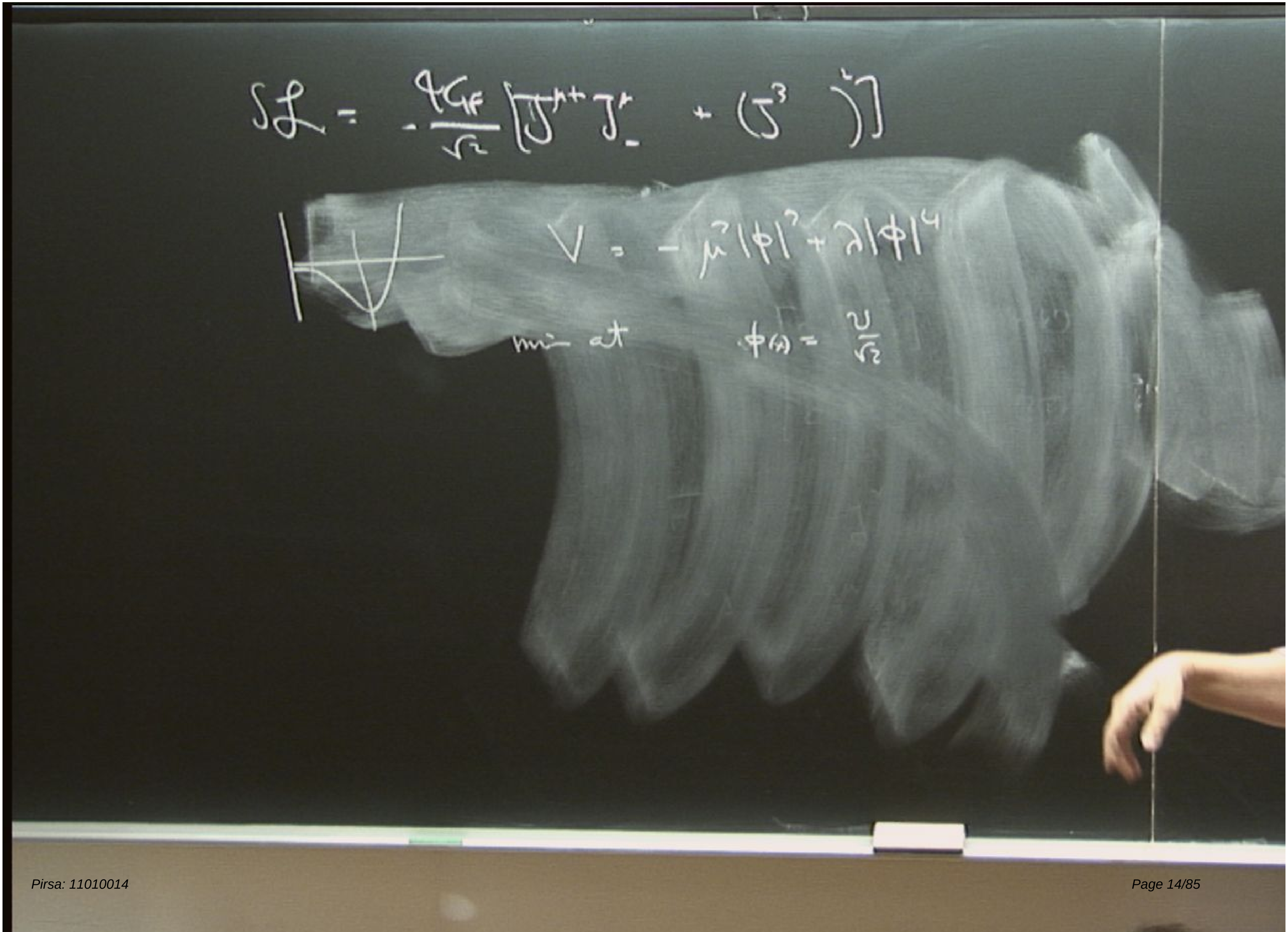

δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
V = - μ^2|φ|^2 + λ|φ|^4

$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3 \quad)^2\right]$$

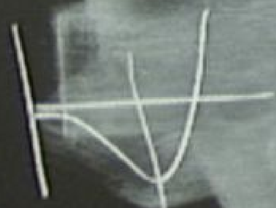


$$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

min at $\phi = \frac{v}{\sqrt{2}}$


$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu+} J^{\mu}_{-} + (J^3 \;)^2\right]$
$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$
min at $\phi(x) = \frac{v}{\sqrt{2}}$

$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ )^2\right]$$

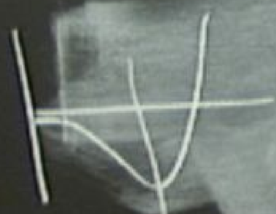


$$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

min at $\phi(x) = \frac{v}{\sqrt{2}}$

$$\langle\Omega|\,\phi(x)\,|\Omega\rangle = \frac{v}{\sqrt{2}}$$

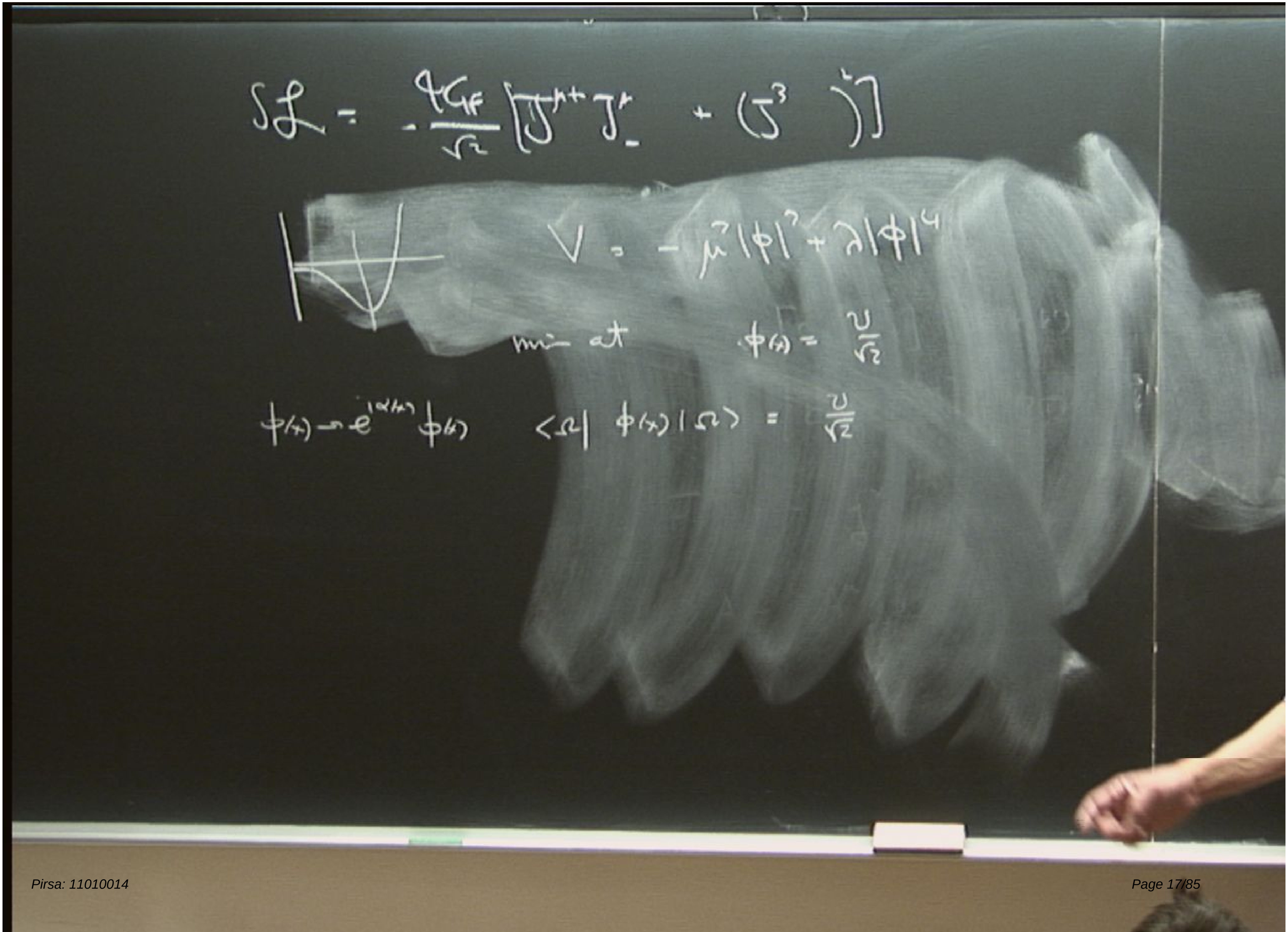
$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ )^2\right]$$

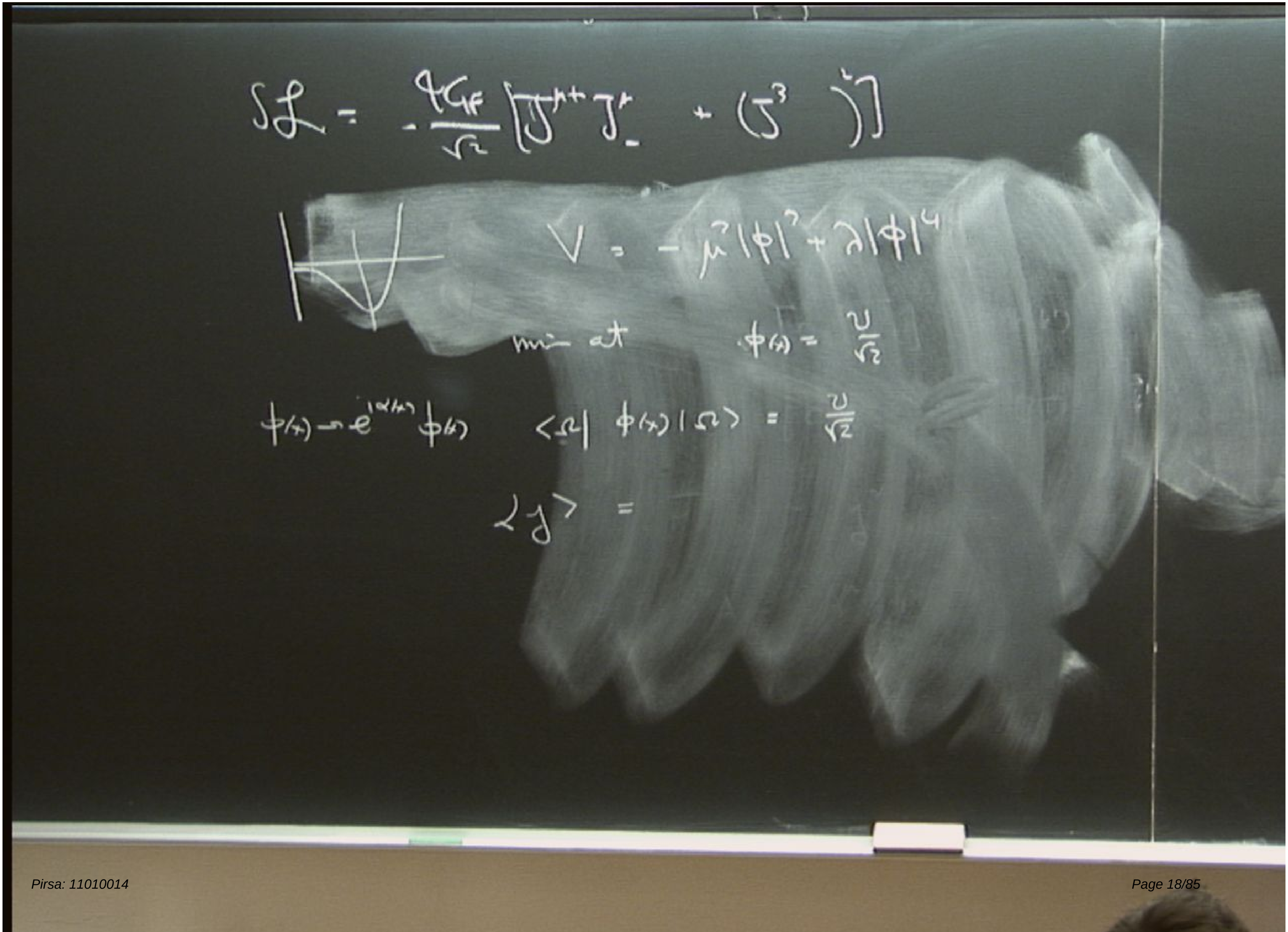


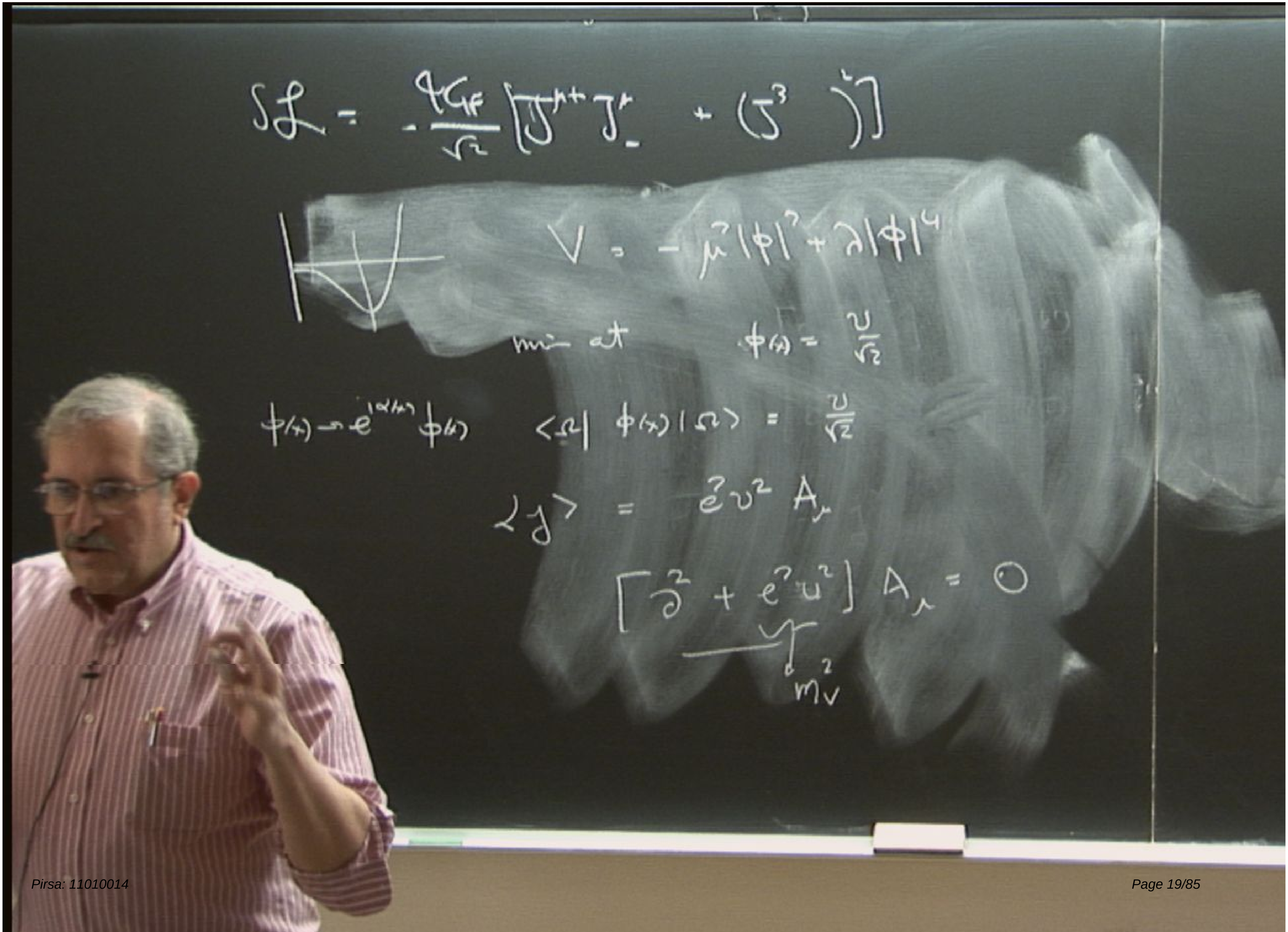
$$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

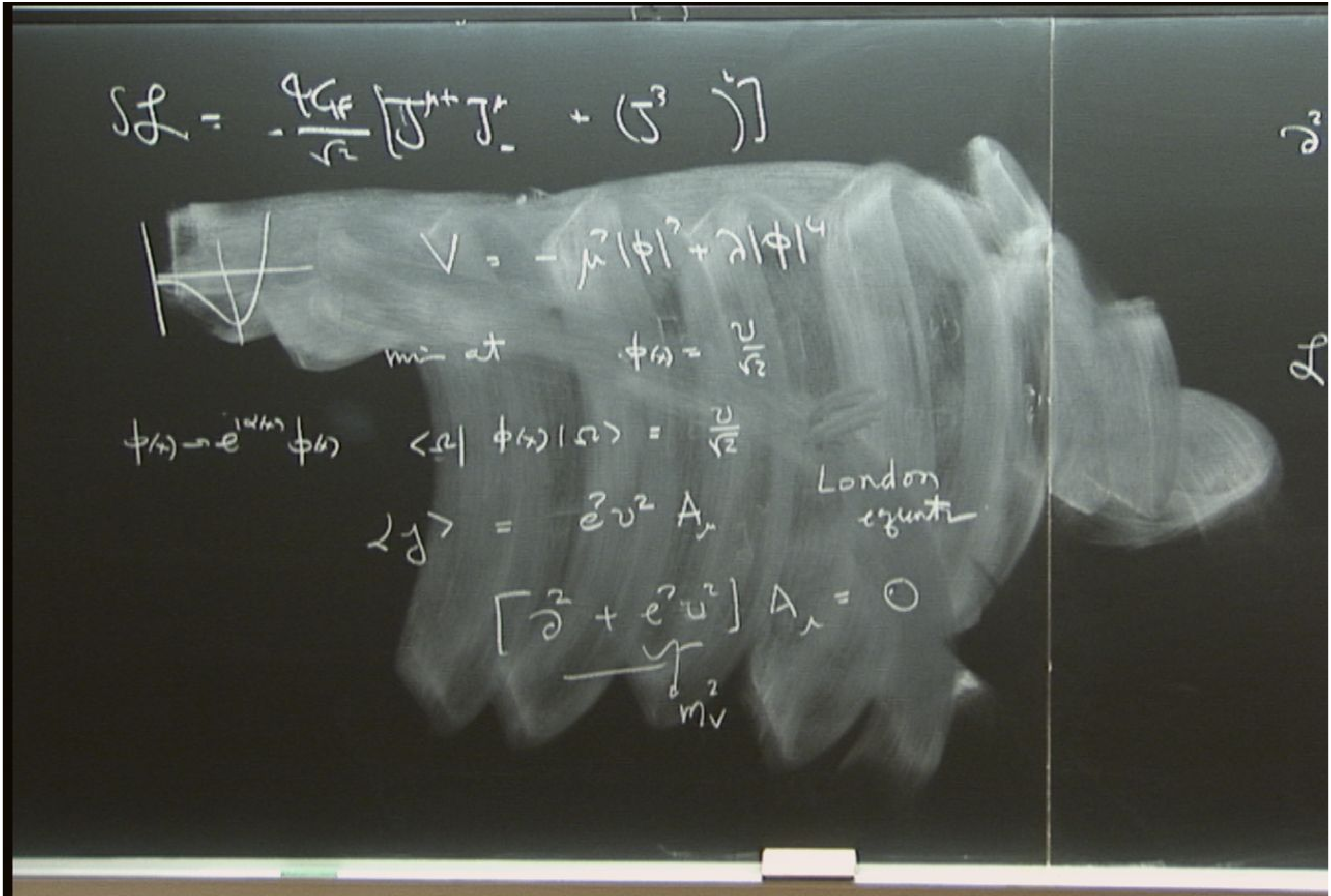
min at $\phi(x) = \frac{v}{\sqrt{2}}$

$\phi(x) \to e^{i\alpha(x)}\phi(x)$ $\quad \langle\Omega|\ \phi(x)\ |\Omega\rangle = \frac{v}{\sqrt{2}}$


δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
V = - μ^2 |φ|^2 + λ|φ|^4
min at φ(x) = v/√2
φ(x) → e^{iα(x)} φ(x)
⟨Ω| φ(x) |Ω⟩ = v/√2


δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
V = - μ^2 |φ|^2 + λ|φ|^4
min at φ(x) = v/√2
φ(x) → e^{iα(x)} φ(x)
⟨Ω| φ(x) |Ω⟩ = v/√2
⟨J⟩ =


δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
V = -μ^2|φ|^2 + λ|φ|^4
min at φ(x) = v/√2
φ(x) → e^{iα(x)} φ(x)
<Ω| φ(x) |Ω> = v/√2
<J> = e^2 v^2 A_μ
[∂^2 + e^2 v^2] A_μ = 0
m_V^2



$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}[J^{\mu +} J^{\mu}_{-} + (J^3 \;)^2]$
$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$
min at $\phi(x) = \frac{v}{\sqrt{2}}$
$\phi(x) \to e^{i\alpha(x)}\phi(x)$
$\langle\Omega|\phi(x)|\Omega\rangle = \frac{v}{\sqrt{2}}$
$\langle J \rangle = e^2 v^2 A_\mu$
London equation
$[\partial^2 + e^2 v^2] A_\mu = 0$
m_V^2

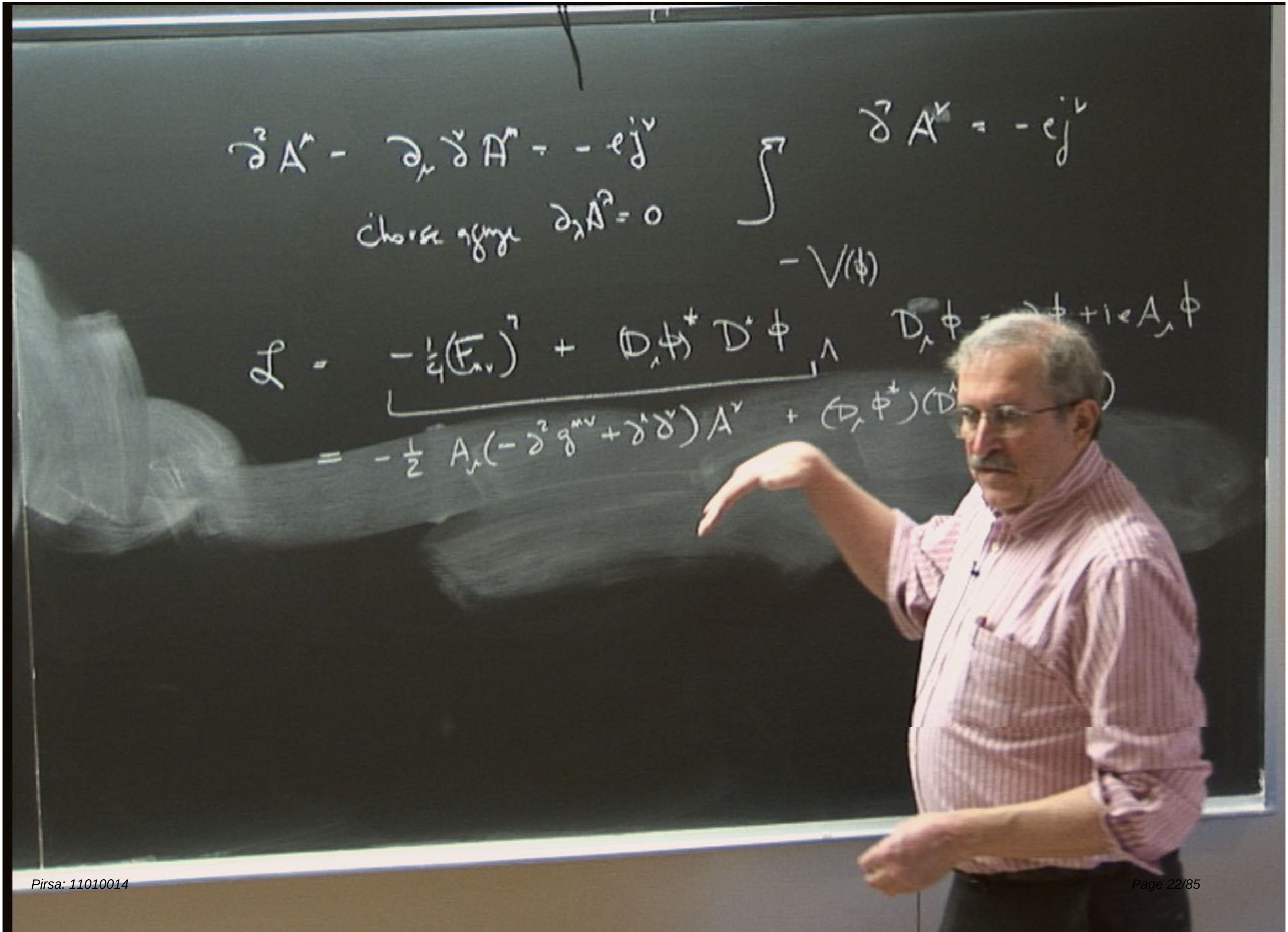
$$\partial^2 A^\nu - \partial_\mu \partial^\nu A^\mu = -e j^\nu$$

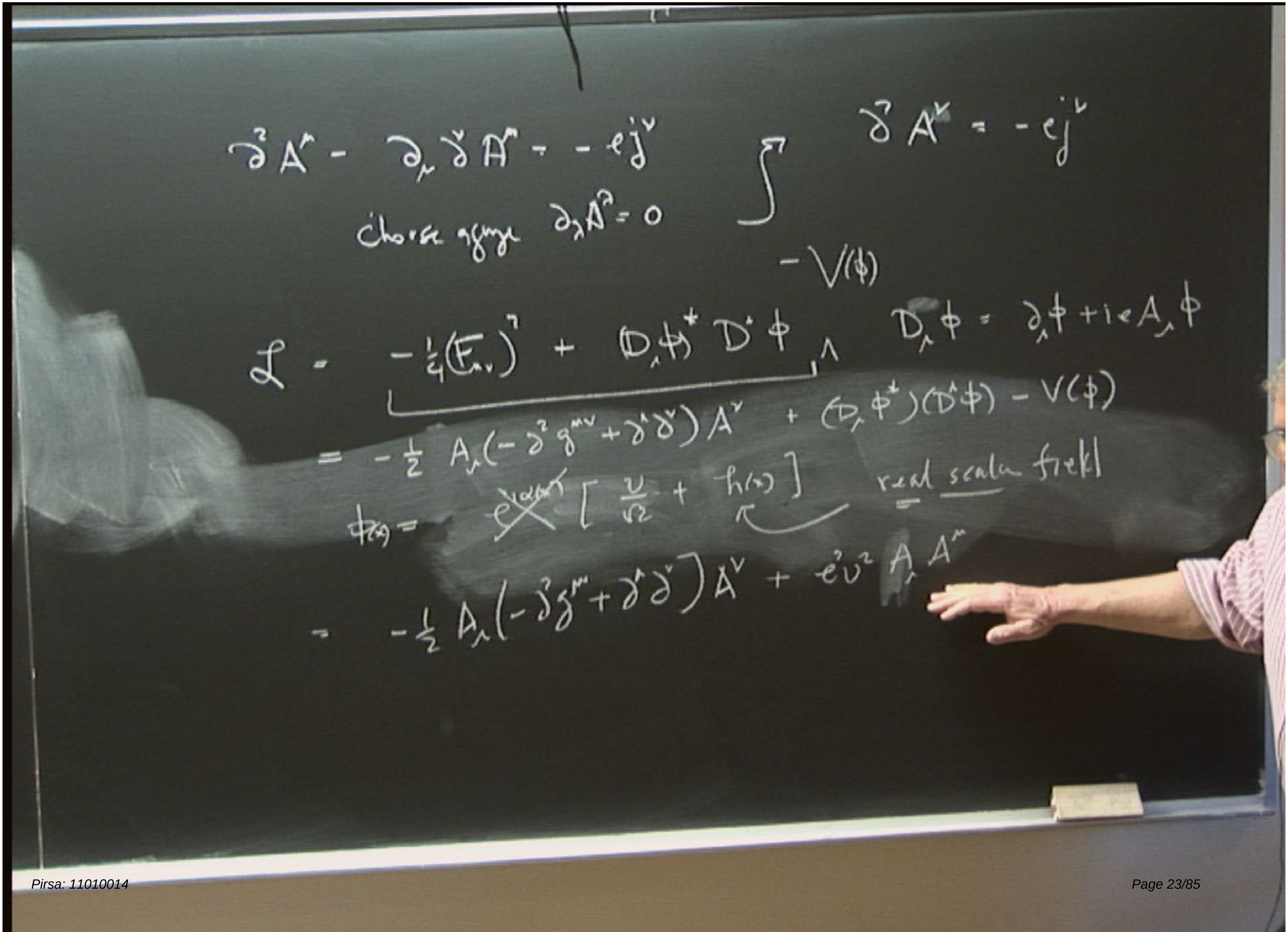
choose gauge $\partial_\lambda A^\lambda = 0$ ⇒ $\partial^2 A^\nu = -e j^\nu$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\lambda \phi)^* D^\lambda \phi - V(\phi) \qquad D_\lambda \phi = \partial_\lambda \phi + ieA_\lambda \phi$$

$$j^\mu = \frac{1}{e}\frac{\delta \mathcal{L}}{\delta A_\mu} = -i(\phi^* D^\mu \phi - (D^\mu \phi)^* \phi)$$

$$= i(\partial^\mu \phi^* \phi - \phi^* \partial^\mu \phi) + 2e\, \phi^* \phi A_\mu$$


$\partial^2 A^\mu - \partial_\mu \partial^\nu A^\mu = -e j^\nu$
choose gauge $\partial_\lambda A^\lambda = 0$
$\partial^2 A^\nu = -e j^\nu$
$-V(\phi)$
$D_\mu \phi$
$+ ie A_\mu \phi$
$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^* D^\mu \phi$
$= -\frac{1}{2} A_\mu(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) A^\nu + (D_\mu \phi^*)(D^\mu$


∂²A^μ − ∂_μ ∂^ν A^μ = − e j^ν
∂² A^ν = − e j^ν
choose gauge ∂_λ A^λ = 0
− V(φ)
L = − 1/4 (F_μν)² + (D_μ φ)* D^μ φ
D_μ φ = ∂_μ φ + i e A_μ φ
= − 1/2 A_μ (− ∂² g^μν + ∂^μ ∂^ν) A^ν + (D_μ φ*)(D^μ φ) − V(φ)
φ(x) = e^{iα(x)} [v/√2 + h(x)]
real scalar field
= − 1/2 A_μ (− ∂² g^μν + ∂^μ ∂^ν) A^ν + e² v² A_μ A^μ

$$\partial^2 A^\mu - \partial_\mu \partial^\nu A^\mu = -e j^\nu \qquad \partial^2 A^\nu = -e j^\nu$$

choose gauge $\partial_\lambda A^\lambda = 0$

$$-V(\phi)$$

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + (D_\lambda \phi)^* D^\lambda \phi \qquad D_\lambda \phi = \partial_\lambda \phi + ieA_\lambda \phi$$

$$= -\tfrac{1}{2} A_\mu(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) A^\nu + (D_\lambda \phi^*)(D^\lambda \phi) - V(\phi)$$

$$\phi(x) = e^{i\alpha(x)} \left[\frac{v}{\sqrt{2}} + \frac{h(x)}{\sqrt{2}} \right]$$

real scalar field

$$= -\tfrac{1}{2} A_\lambda(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) A^\nu + e^2 v^2 A_\mu A^\mu + \tfrac{1}{2}(\partial h)^2 - m_h^2 h^2$$

$$+ h^3 + h^4$$

mechanism

$$\partial^2 A^\nu - \partial_\nu \partial^\mu A^\mu = -e j^\nu$$

choose gauge $\partial_\lambda A^\lambda = 0$ $\Rightarrow$ $\partial^2 A^\nu = -e j^\nu$

$$-V(\phi)$$

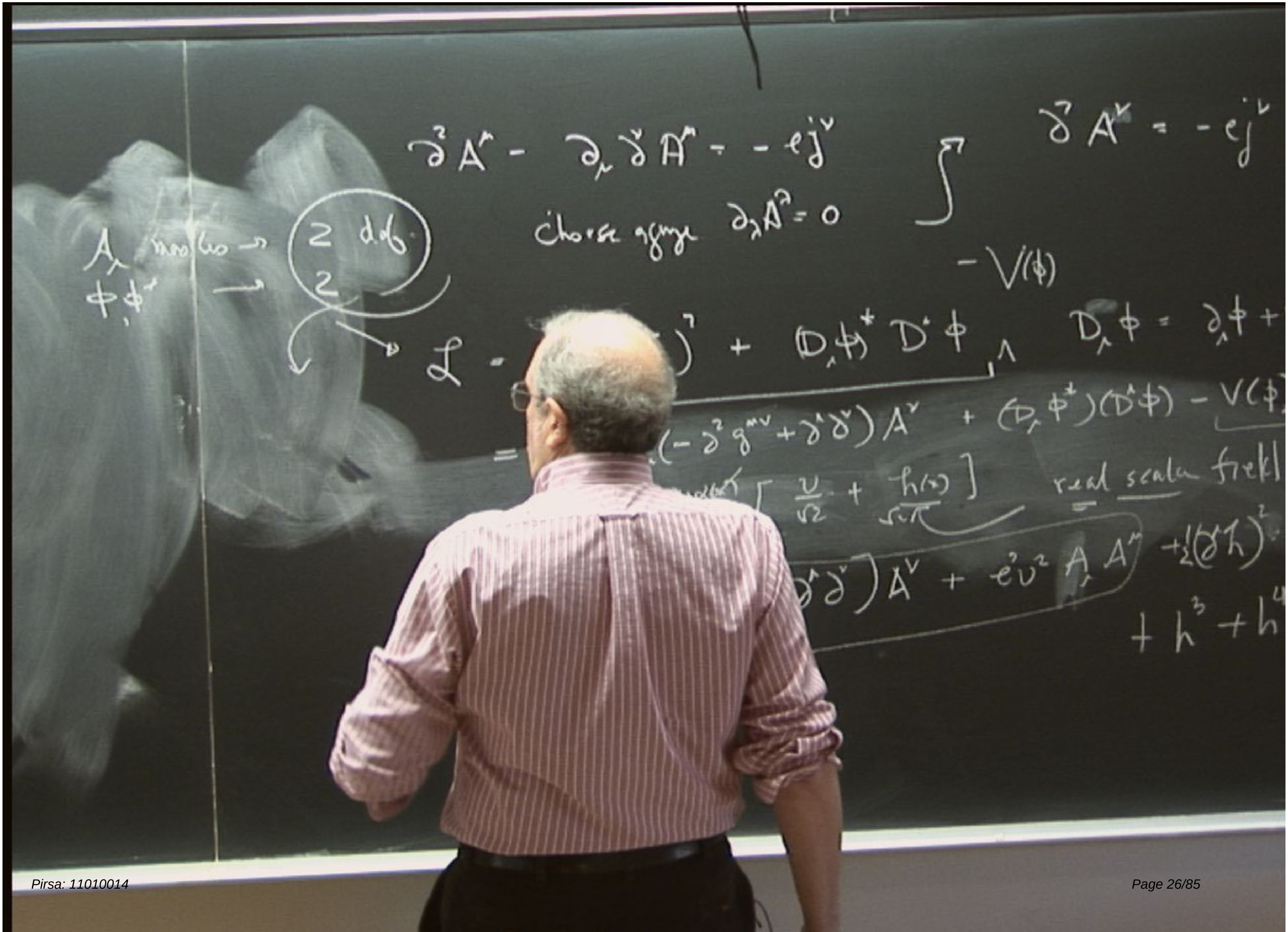
$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D_\mu \phi)^* D^\mu \phi \qquad D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

$$= -\frac{1}{2} A_\mu \left(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu\right) A^\nu + (D_\mu \phi^*)(D^\mu \phi) - V(\phi)$$

real scalar field

$$\phi(x) = e^{i\alpha(x)} \left[\frac{v}{\sqrt{2}} + \frac{h(x)}{\sqrt{2}} \right]$$

$$= -\frac{1}{2} A_\mu \left(-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu\right) A^\nu + e^2 v^2 A_\mu A^\mu + \frac{1}{2}(\partial h)^2 - m_h^2 h^2 + h^3 + h^4$$


choose gauge $\partial_\lambda A^\lambda = 0$
$- V(\phi)$
real scalar field

$$\partial^2 A^\mu - \partial_\mu \partial^\nu A^\mu = -e j^\nu \qquad \partial^2 A^\nu = -e j^\nu$$

choose gauge $\partial_\lambda A^\lambda = 0$

$-V(\phi)$

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + (D_\mu\phi)^* D^\mu \phi \qquad D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

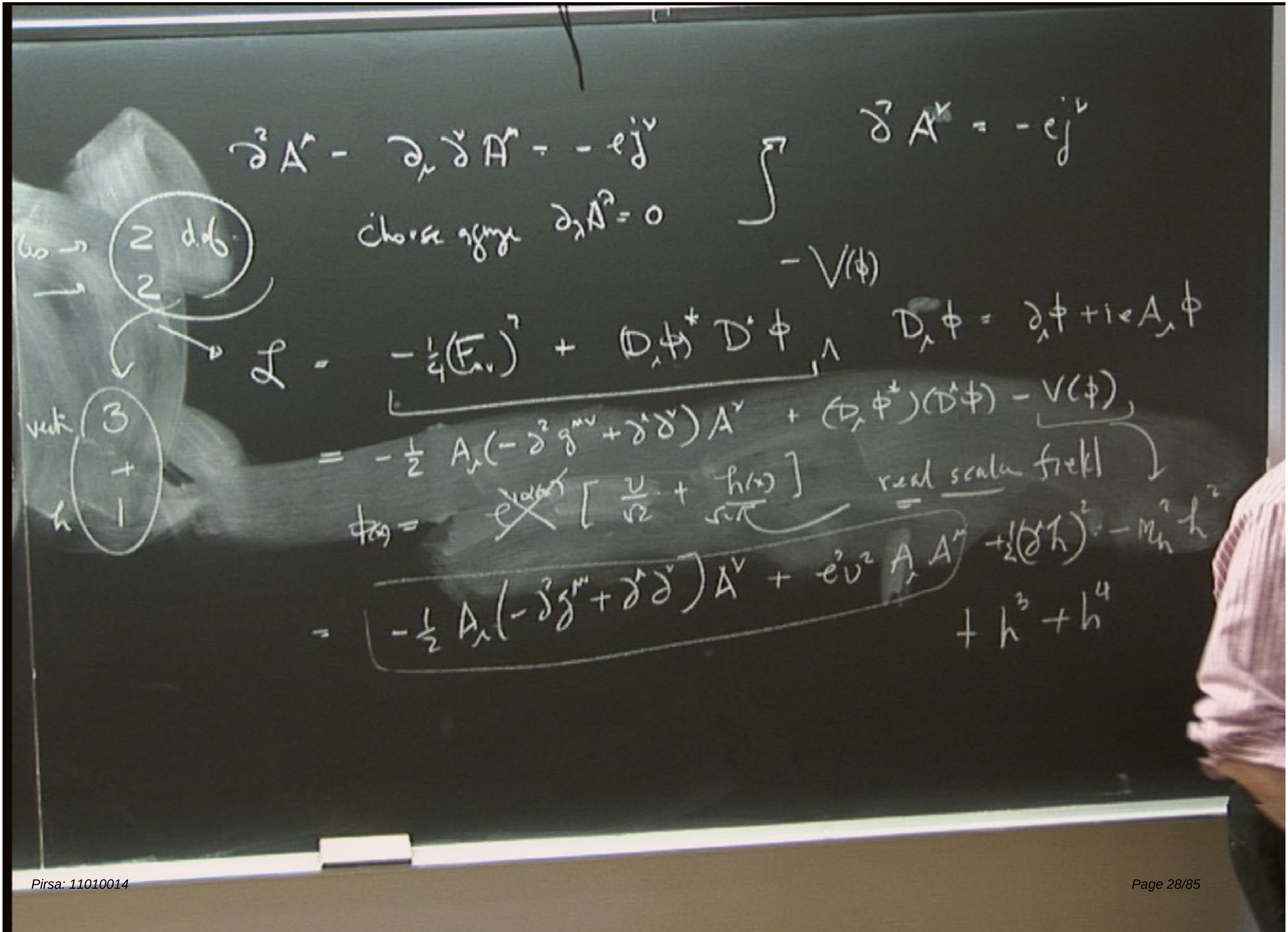
$$= -\tfrac{1}{2} A_\mu(-\partial^2 g^{\mu\nu} + \partial^\mu\partial^\nu) A^\nu + (D_\mu\phi^*)(D^\mu\phi) - V(\phi)$$

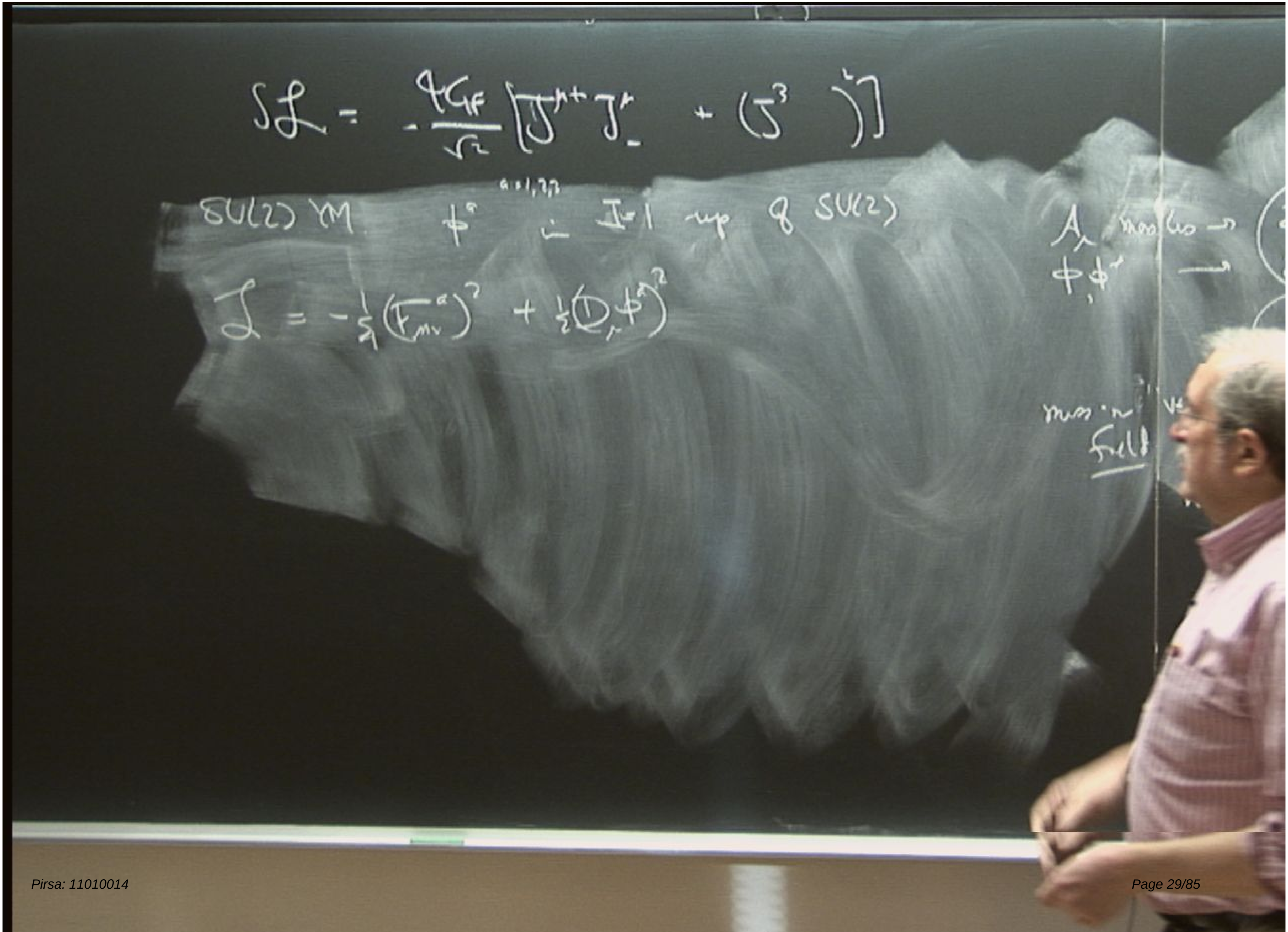
$$\phi(x) = e^{i\alpha(x)} \left[\frac{v}{\sqrt{2}} + \frac{h(x)}{\sqrt{2}} \right]$$

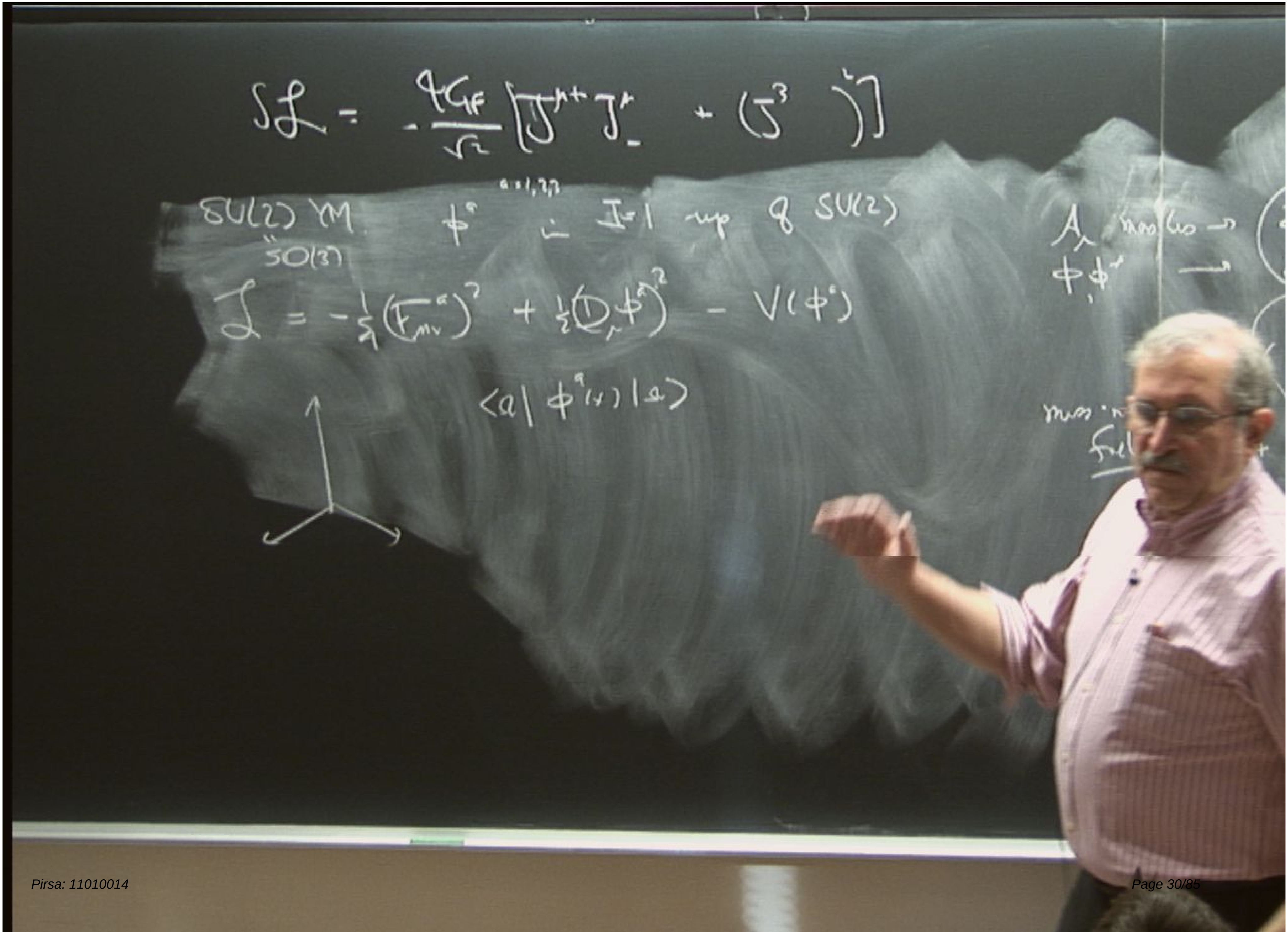
real scalar field

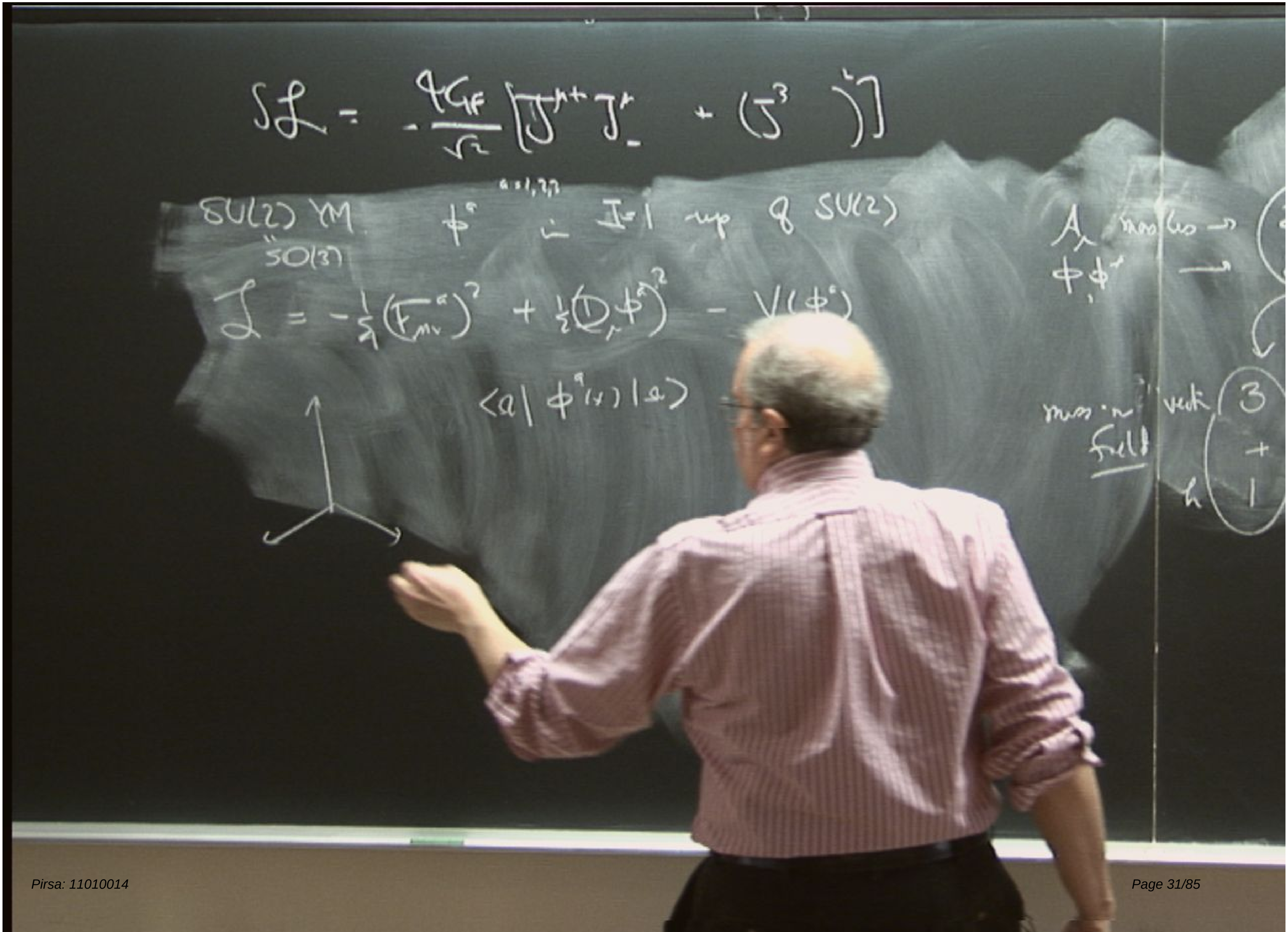
$$= -\tfrac{1}{2} A_\mu(-\partial^2 g^{\mu\nu} + \partial^\mu\partial^\nu) A^\nu + e^2 v^2 A_\mu A^\mu + \tfrac{1}{2}(\partial h)^2 - m_h^2 h^2 + h^3 + h^4$$

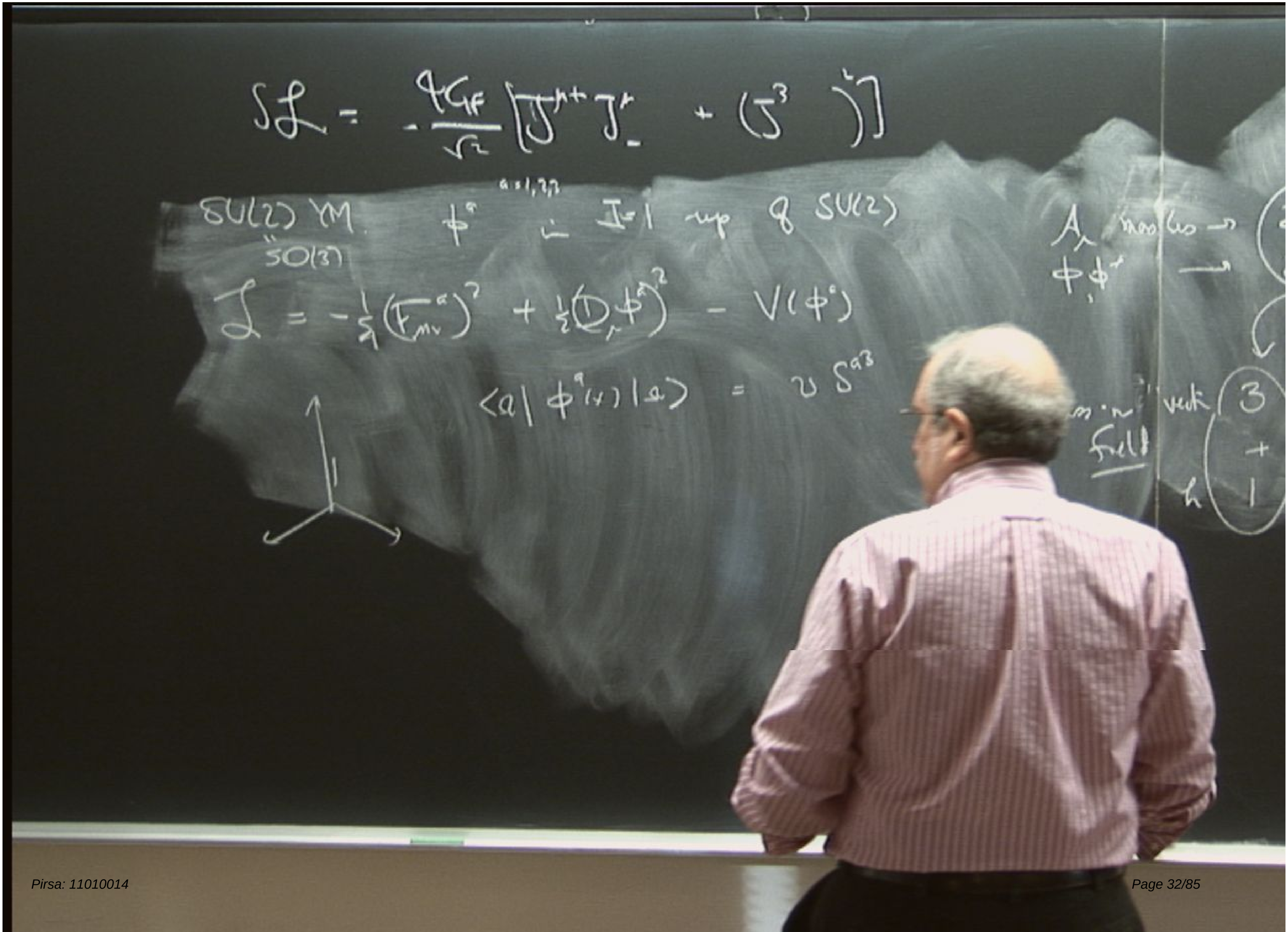
vector: 3 = 2 d.o.f. + 1; h: 1

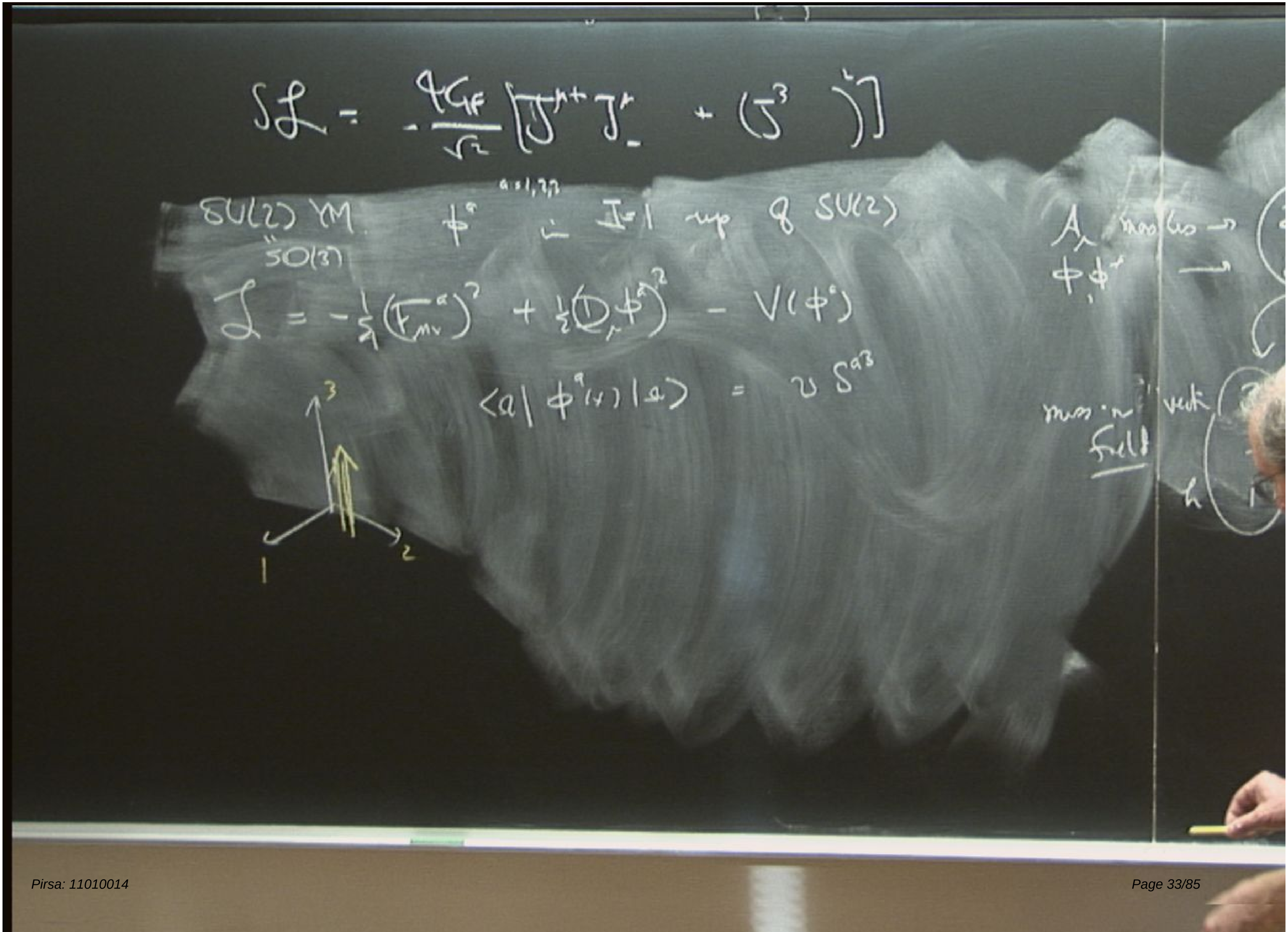

∂²A^μ − ∂_μ ∂^ν A^μ = − e j^ν
∂² A^ν = − e j^ν
choose gauge ∂_λ A^λ = 0
2 d.o.f.
2
− V(φ)
D_λ φ = ∂_λ φ + i e A_λ φ
𝓛 = − ¼ (F_μν)² + (D_λ φ)* D^λ φ
vector 3
+
h 1
= − ½ A_μ (− ∂² g^μν + ∂^μ ∂^ν) A^ν + (D_λ φ*)(D^λ φ) − V(φ)
real scalar field
φ(x) = [v/√2 + h(x)/√2]
= − ½ A_μ (− ∂² g^μν + ∂^μ ∂^ν) A^ν + e² v² A_μ A^μ + ½ (∂h)² − m_h² h²
+ h³ + h⁴

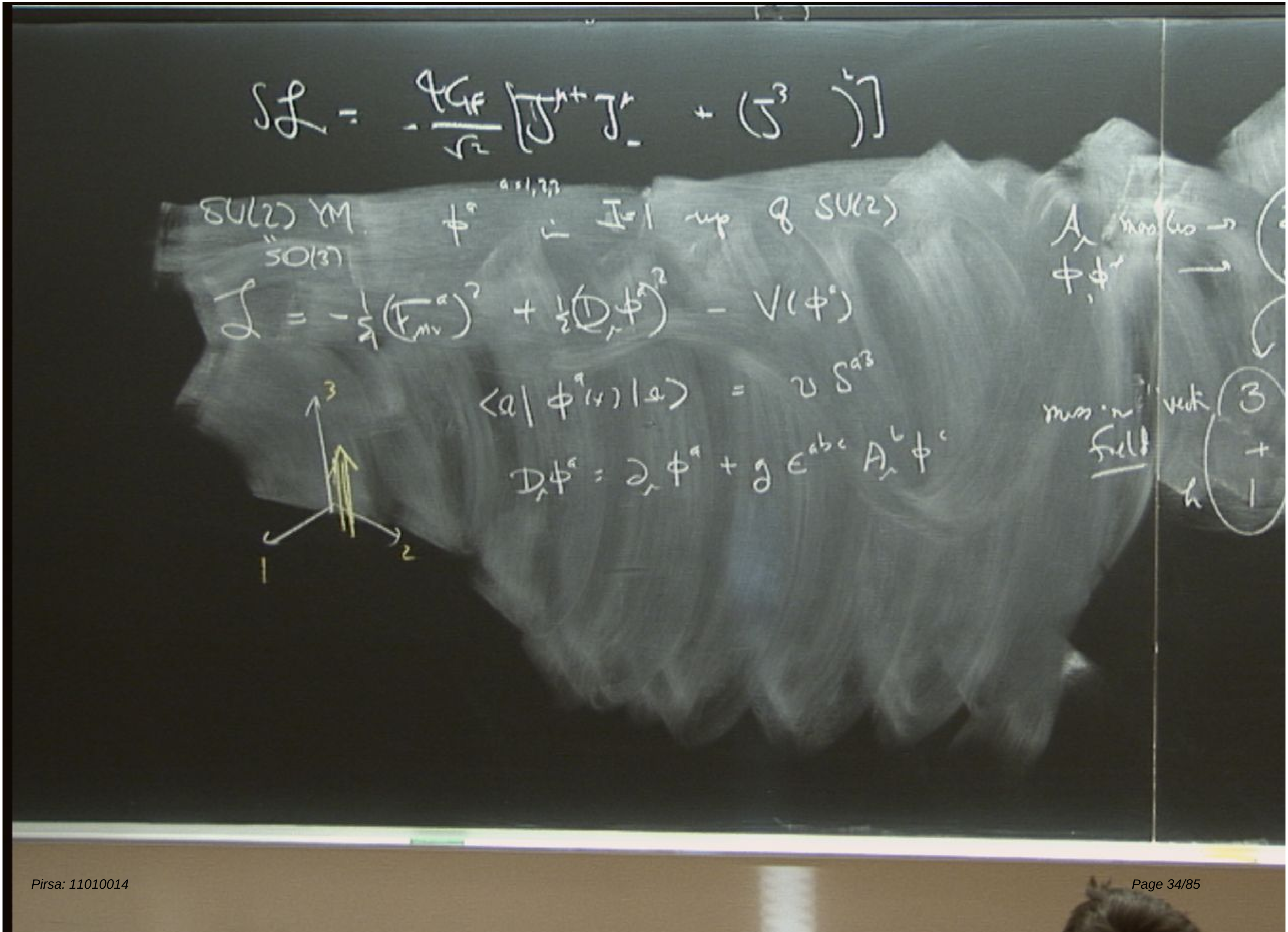

δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
SU(2) YM   φ^a a=1,2,3 : I=1 rep of SU(2)
L = -1/4 (F_μν^a)^2 + 1/2 (D_μ φ^a)^2
A_μ massless →
φ, φ^+ →

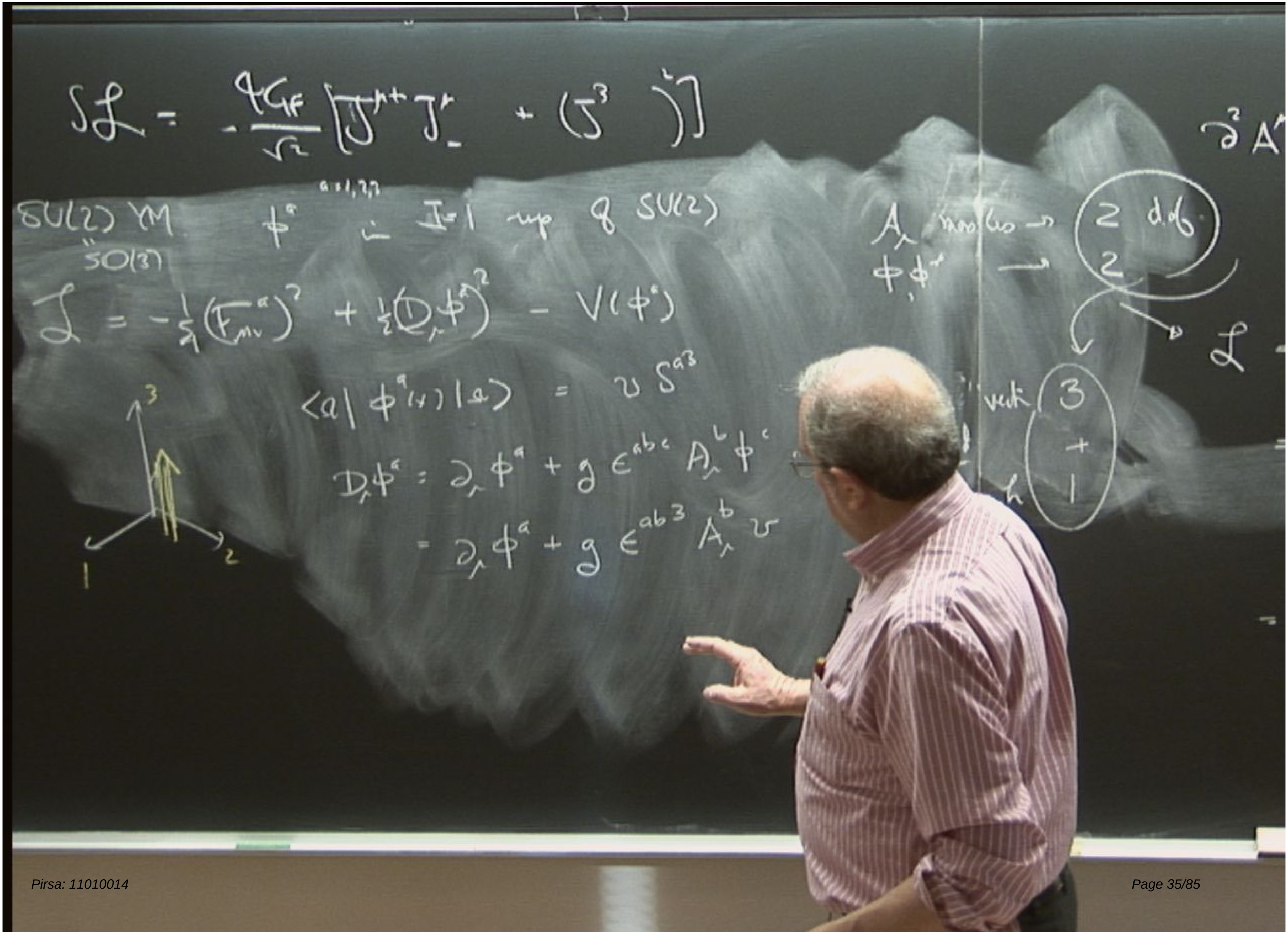

SU(2) YM
SO(3)
a=1,2,3
I=1
SU(2)
V(ϕ^a)
⟨a| ϕ^a(x) |a⟩

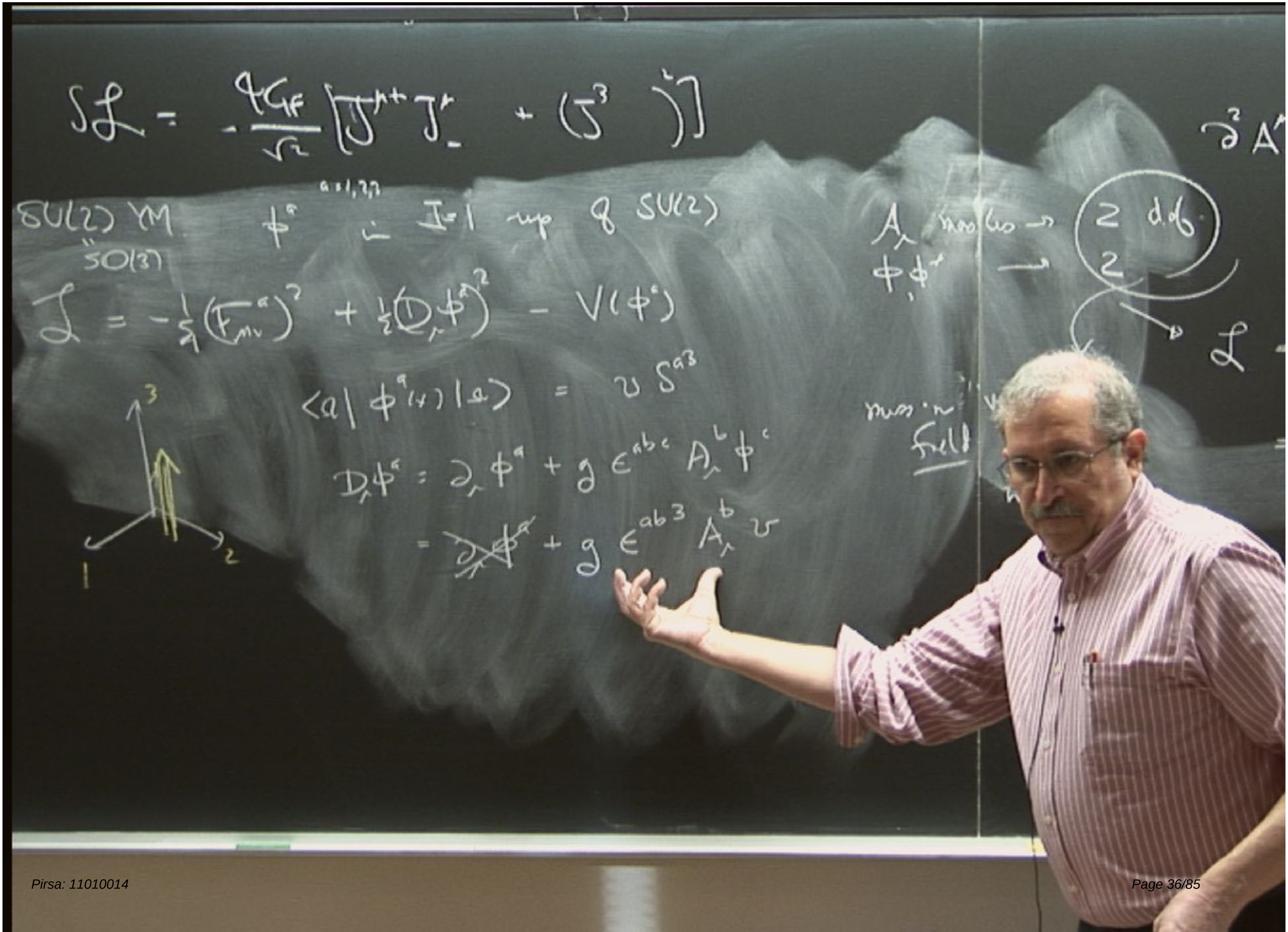

SU(2) YM
SO(3)
a = 1, 2, 3
I = 1 rep of SU(2)


SU(2) YM
SO(3)
I=1 rep of SU(2)
field


δL = - 4G_F/√2 [J^{μ+} J^μ_- + (J^3)^2]
SU(2) YM   φ^a   a=1,2,3   I=1 rep of SU(2)
SO(3)
L = -1/4 (F_{μν}^a)^2 + 1/2 (D_μ φ^a)^2 - V(φ^a)
<Ω| φ^a(x) |Ω> = v δ^{a3}
3
1
2
A_μ massless →
φ_1 φ^2 →
mass in vector field
h


δℒ = − 4G_F/√2 [J^{μ+} J^μ_− + (J^3)^2]
a=1,2,3
SU(2) YM = SO(3)
φ^a : I=1 rep of SU(2)
ℒ = −1/4 (F_{μν}^a)^2 + 1/2 (D_μ φ^a)^2 − V(φ^a)
⟨Ω| φ^a(x) |Ω⟩ = v δ^{a3}
D_μ φ^a = ∂_μ φ^a + g ε^{abc} A_μ^b φ^c
3
1
2
A_μ massless →
φ, φ^a →
massive vect
field
3
+
h
1


SU(2) YM
SO(3)
I = 1 rep of SU(2)
A modes → 2 d.o.f.
vect 3 + 1

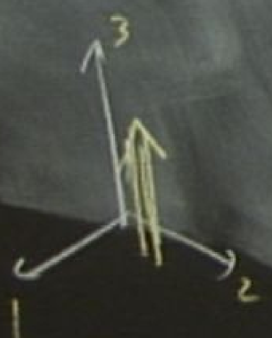

SU(2) YM
SO(3)
I=1 rep of SU(2)
field
2 d.o.f.

$$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ )^2\right]$$

$SU(2)$ YM $\quad \phi^a$ ($a=1,2,3$) : $I=1$ rep of $SU(2)$

" $SO(3)$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi^a)$$



$$\langle \Omega | \phi^a(x) | \Omega \rangle = v\,\delta^{a3}$$

$$D_\mu \phi^a = \partial_\mu \phi^a + g\,\epsilon^{abc} A_\mu^b \phi^c$$

$$= \cancel{\partial_\mu \phi^a} + g\,\epsilon^{ab3} A_\mu^b v$$

$$\frac{1}{2}\ g^2 \epsilon^{ab3} \epsilon^{ac3} A_\mu^b A_\mu^c v^2$$

A_μ massless $\rightarrow$ 2 d.o.f.

$\phi, \phi^a \longrightarrow$ 2

"massive" vector field: 3 + 1 (h)

$\partial^2 A^\mu$

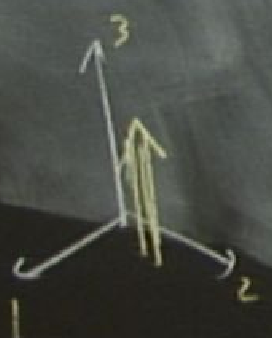
$\mathcal{L} =$

$$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3 \;)^2\right]$$

$SU(2)$ YM $\quad \phi^a$ ($a=1,2,3$) in $I=1$ rep of $SU(2)$

" $SO(3)$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi^a)$$



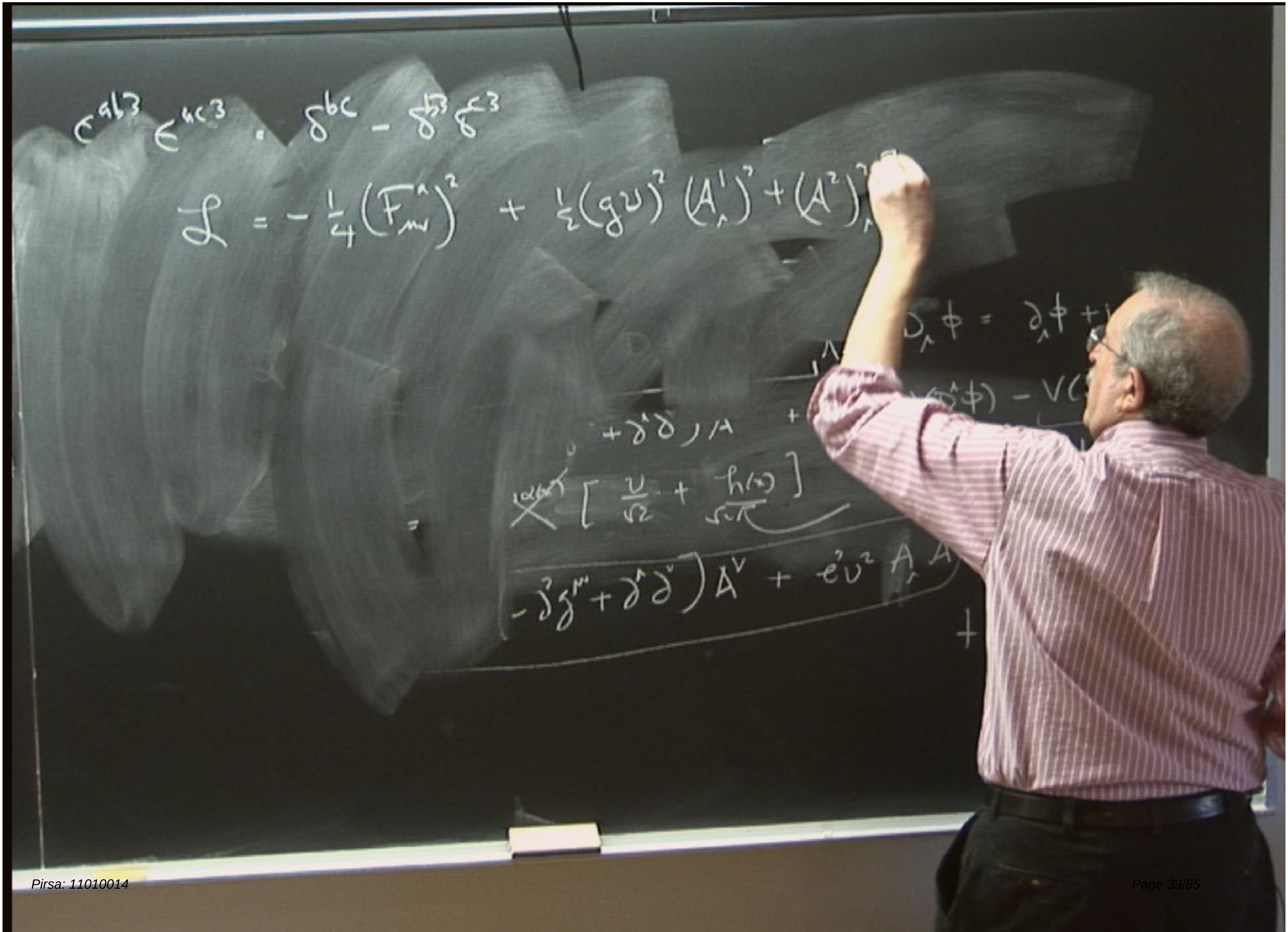
$$\langle \Omega | \phi^a(x) | \Omega \rangle = v\,\delta^{a3}$$

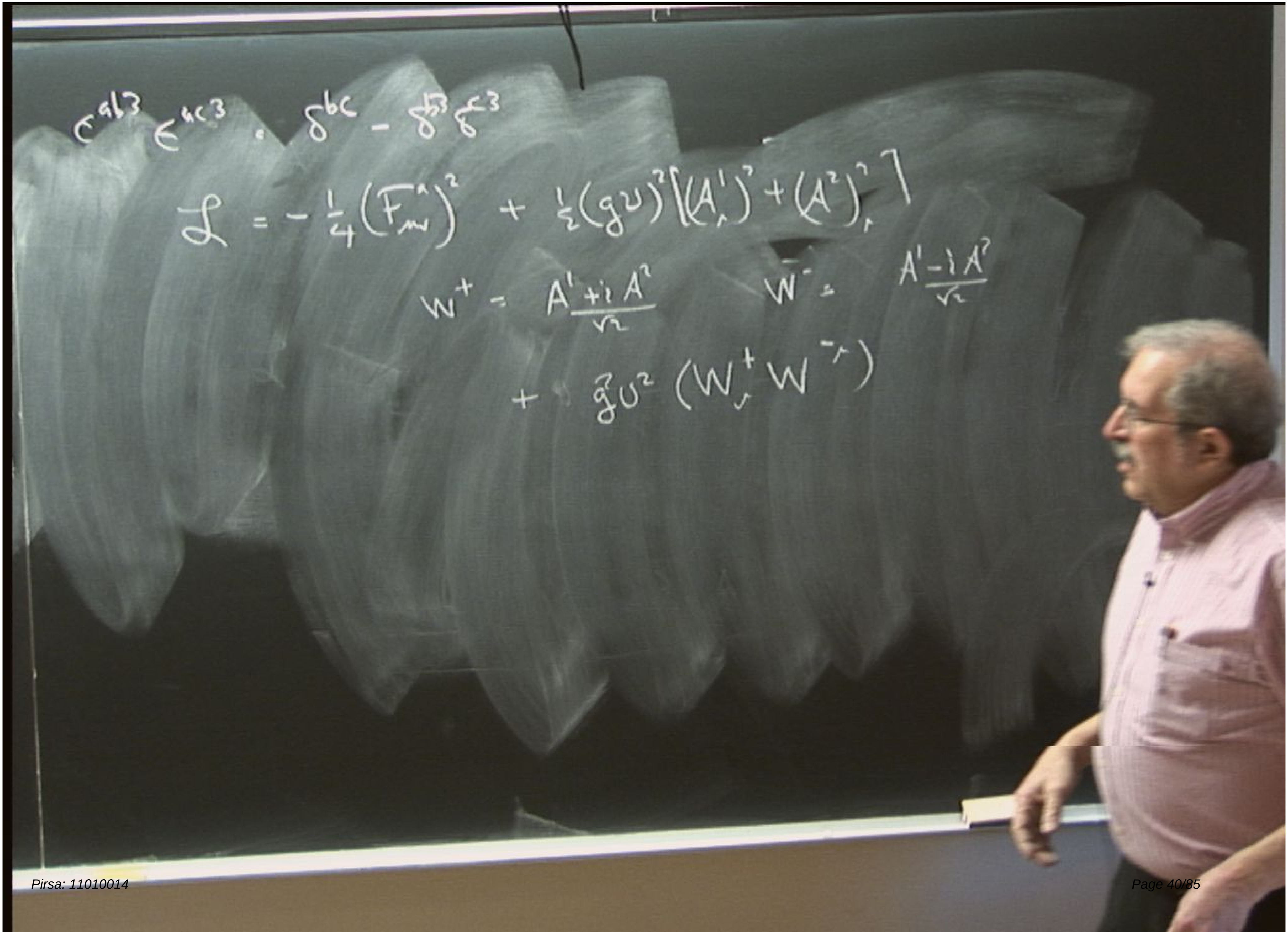
$$D_\mu \phi^a = \partial_\mu \phi^a + g\,\epsilon^{abc} A_\mu^b \phi^c$$

$$= \cancel{\partial_\mu \phi^a} + g\,\epsilon^{ab3} A_\mu^b v$$

$$\frac{1}{2} g^2 \epsilon^{ab3} \epsilon^{ac3} A_\mu^b A_\mu^c v^2$$

$$\epsilon^{ab3} \epsilon^{ac3}$$


$\epsilon^{ab3}\epsilon^{ac3} = \delta^{bc} - \delta^{b3}\delta^{c3}$
$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(gv)^2 (A^1_\mu)^2 + (A^2_\mu)^2$



$\epsilon^{ab3}\epsilon^{ac3} = \delta^{bc} - \delta^{b3}\delta^{c3}$
$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(gv)^2[(A^1_\mu)^2 + (A^2_\mu)^2]$
$W^+ = \frac{A^1 + iA^2}{\sqrt{2}}$ $W^- = \frac{A^1 - iA^2}{\sqrt{2}}$
$+ g^2 v^2 (W^+_\mu W^{-\mu})$

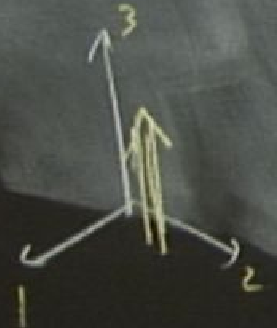
$$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu +} J^{\mu}_{-} + (J^3\ )^2\right]$$

SU(2) YM $\quad \phi^a$ ($a=1,2,3$) in $I=1$ rep of SU(2)

"=" SO(3)

$$D_\mu \phi = \partial_\mu - i g t^b A^b_\mu \phi$$

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu \phi^a)^2 - V(\phi^a)$$

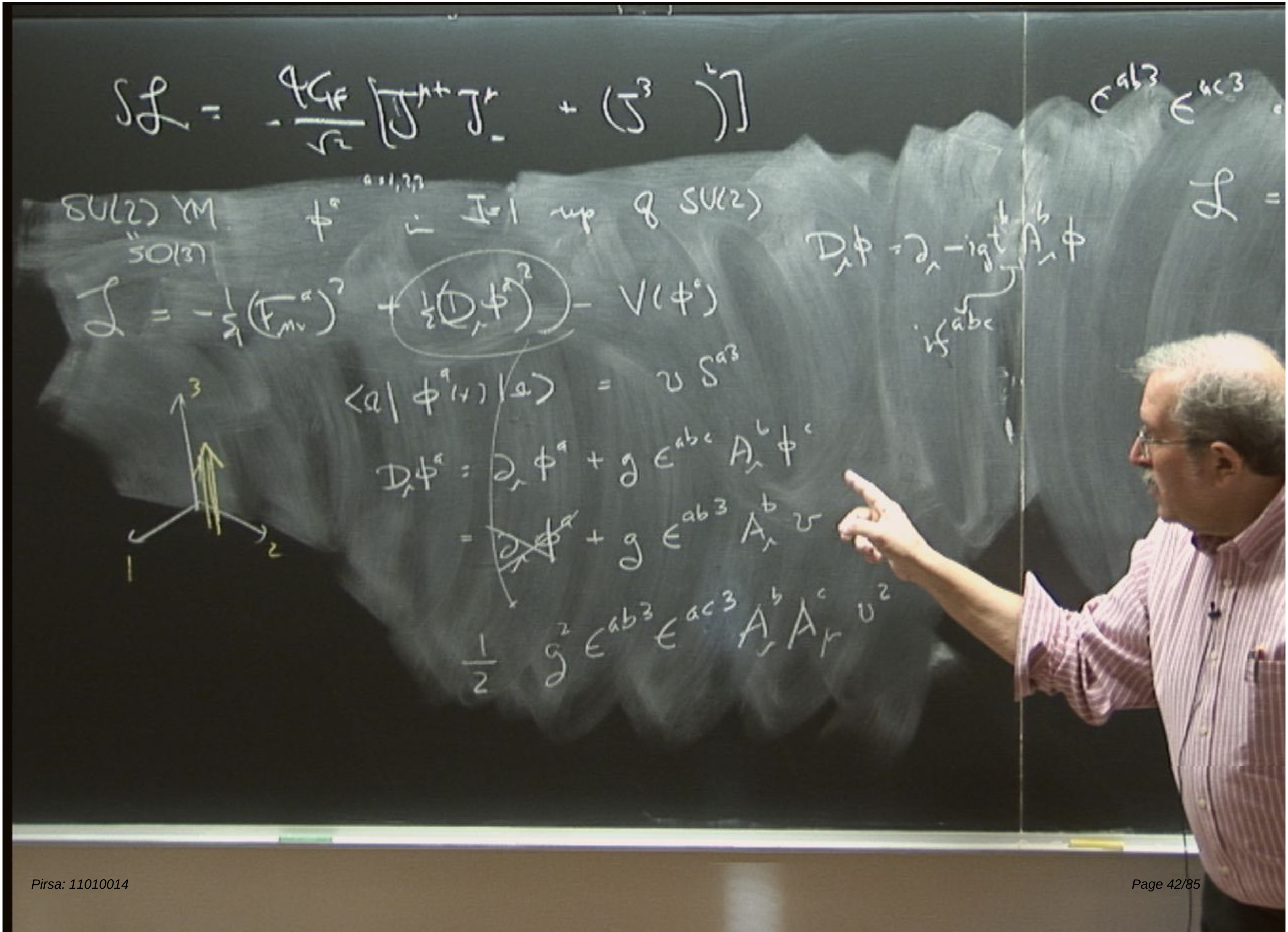


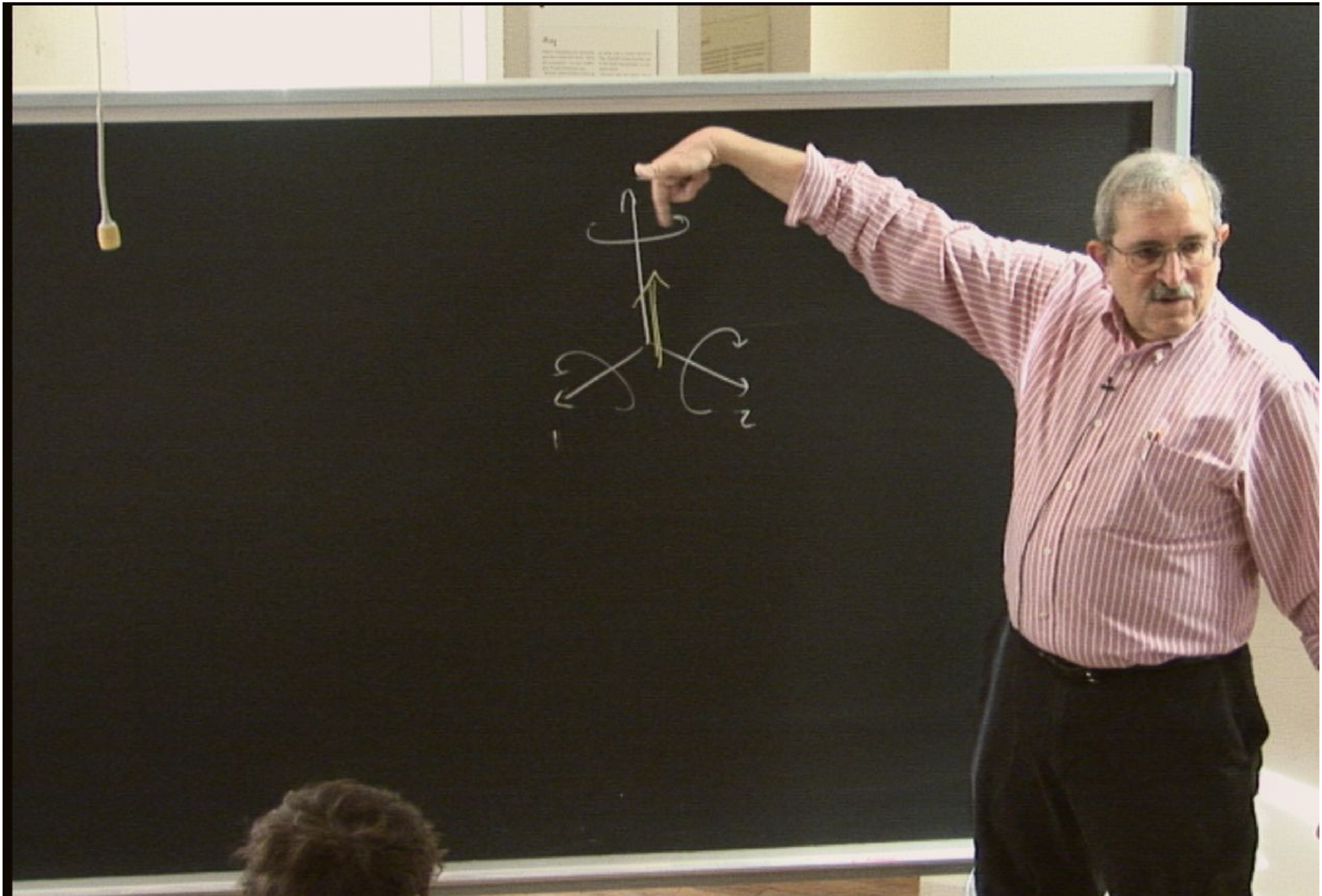
$$\langle \Omega | \phi^a(x) | \Omega \rangle = v\,\delta^{a3}$$

$$D_\mu \phi^a = \partial_\mu \phi^a + g\,\epsilon^{abc} A^b_\mu \phi^c$$

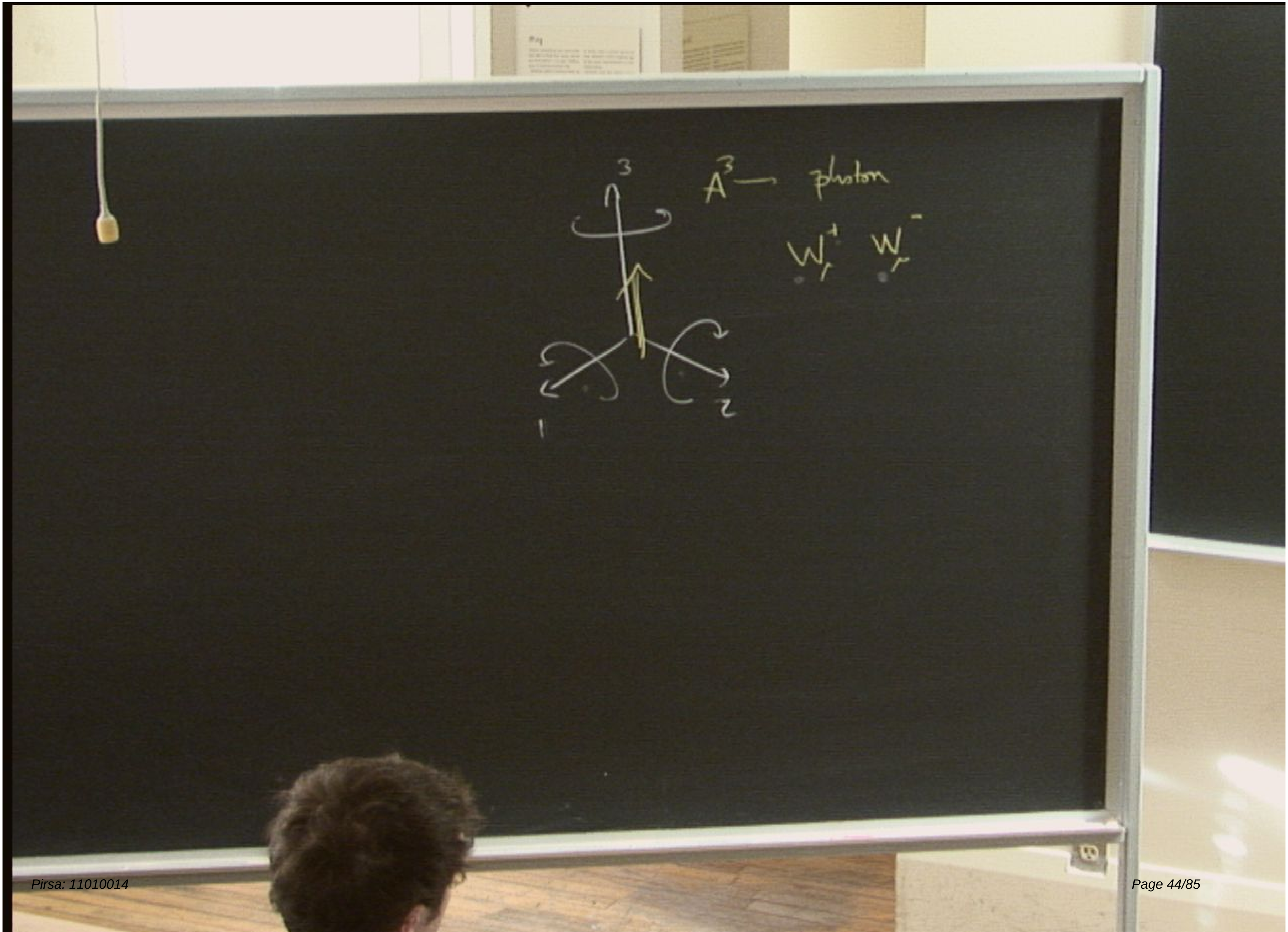
$$= \partial_\mu \phi^a + g\,\epsilon^{ab3} A^b_\mu v$$

$$\frac{1}{2}\, g^2 \epsilon^{ab3} \epsilon^{ac3} A^b_\mu A^c_\mu v^2$$

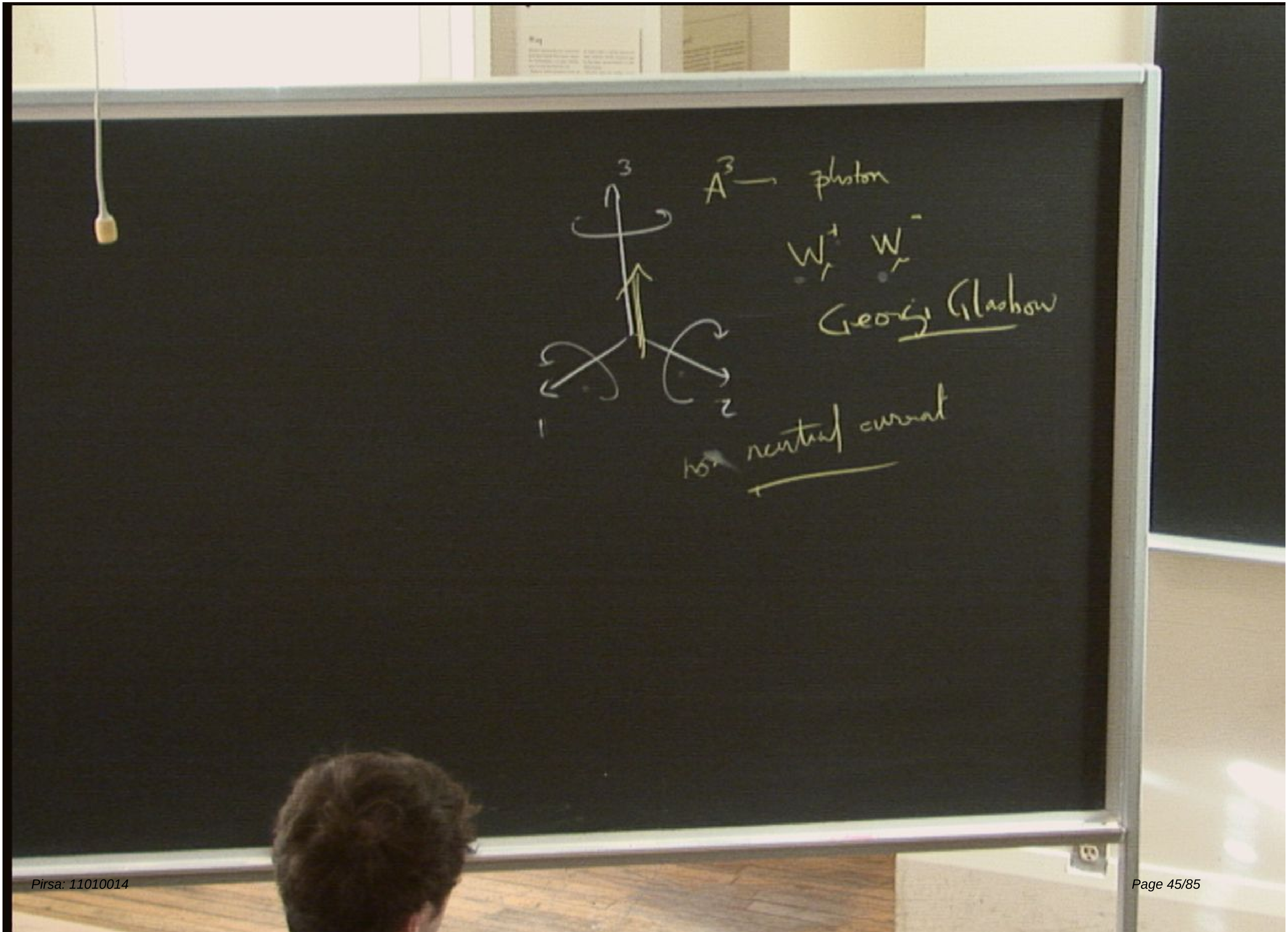

$\delta\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[J^{\mu+} J^{\mu}_{-} + (J^3)^2\right]$
SU(2) YM = SO(3)
ϕ^a a = 1, 2, 3 I = 1 rep of SU(2)
$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu\phi^a)^2 - V(\phi^c)$
$\langle\Omega|\phi^a(x)|\Omega\rangle = v\,\delta^{a3}$
$D_\mu\phi^a = \partial_\mu\phi^a + g\,\epsilon^{abc}A^b_\mu\phi^c$
$= \partial_\mu\phi^a + g\,\epsilon^{ab3}A^b_\mu v$
$\frac{1}{2}g^2\epsilon^{ab3}\epsilon^{ac3}A^b_\mu A^c_\mu v^2$
$D_\mu\phi = \partial_\mu - igt^b A^b_\mu\phi$
if^{abc}
$\epsilon^{ab3}\,\epsilon^{ac3}$
$\mathcal{L} =$
3
2
1

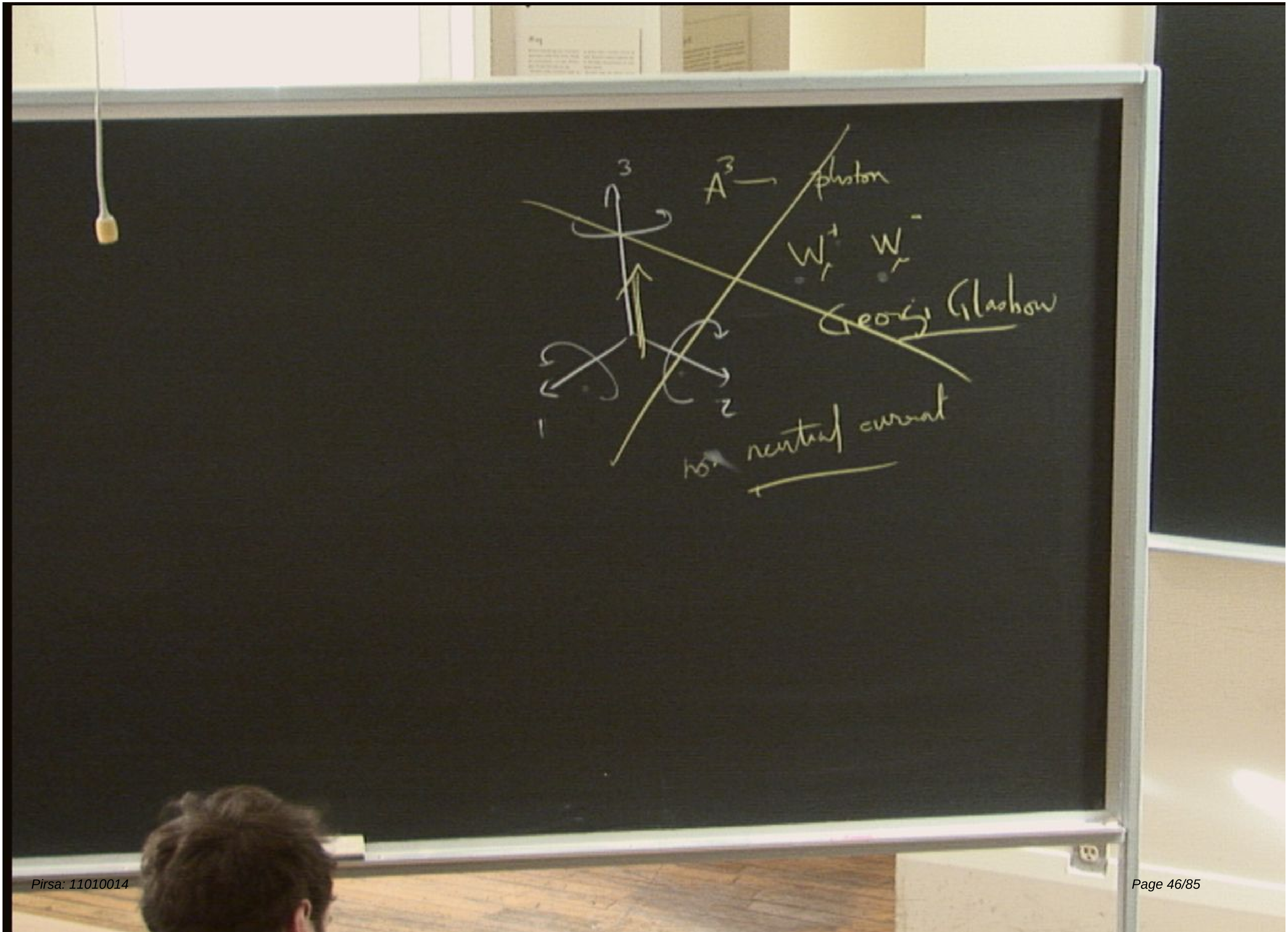


1
2



3
A^3 — photon
W^+_μ W^-_μ
1
2


3
A^3 → photon
W^+_μ W^-_μ
Georgi Glashow
2
1
neutral current


3
A^3 → photon
W^+ W^-
Georgi Glashow
1
2
neutral current

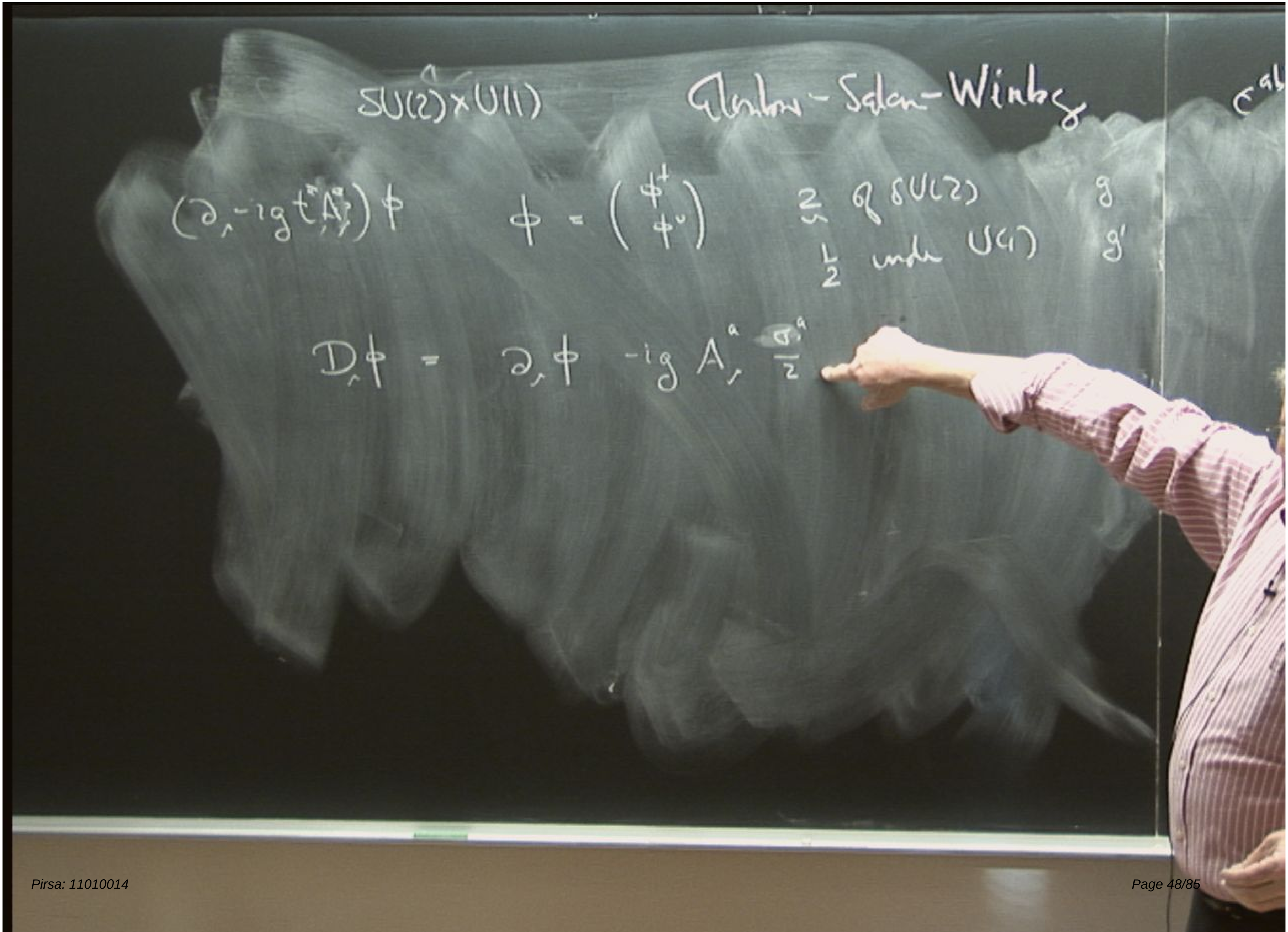
SU(2) × U(1)   Glashow – Salam – Weinberg

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

2 of SU(2) g

$\frac{1}{2}$ under U(1) g'

ϵ^{ab}


SU(2)×U(1)
Glashow-Salam-Weinberg
(∂_μ - ig t^a A^a_μ) φ
φ = (φ^+ φ^0)
2 of SU(2) g
1/2 under U(1) g'
D_μ φ = ∂_μ φ - ig A^a_μ σ^a/2

$SU(2) \times U(1)$ Glashow – Salam – Weinberg

$(\partial_\mu - i g t^a A^a_\mu)\phi$ $\quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

2 of $SU(2)$ $\quad g$

$\frac{1}{2}$ under $U(1)$ $\quad g'$

$$D_\mu \phi = \left(\partial_\mu \phi - i g A^a_\mu \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu \right) \phi$$

SU(2) × U(1)   Glashow – Salam – Weinberg

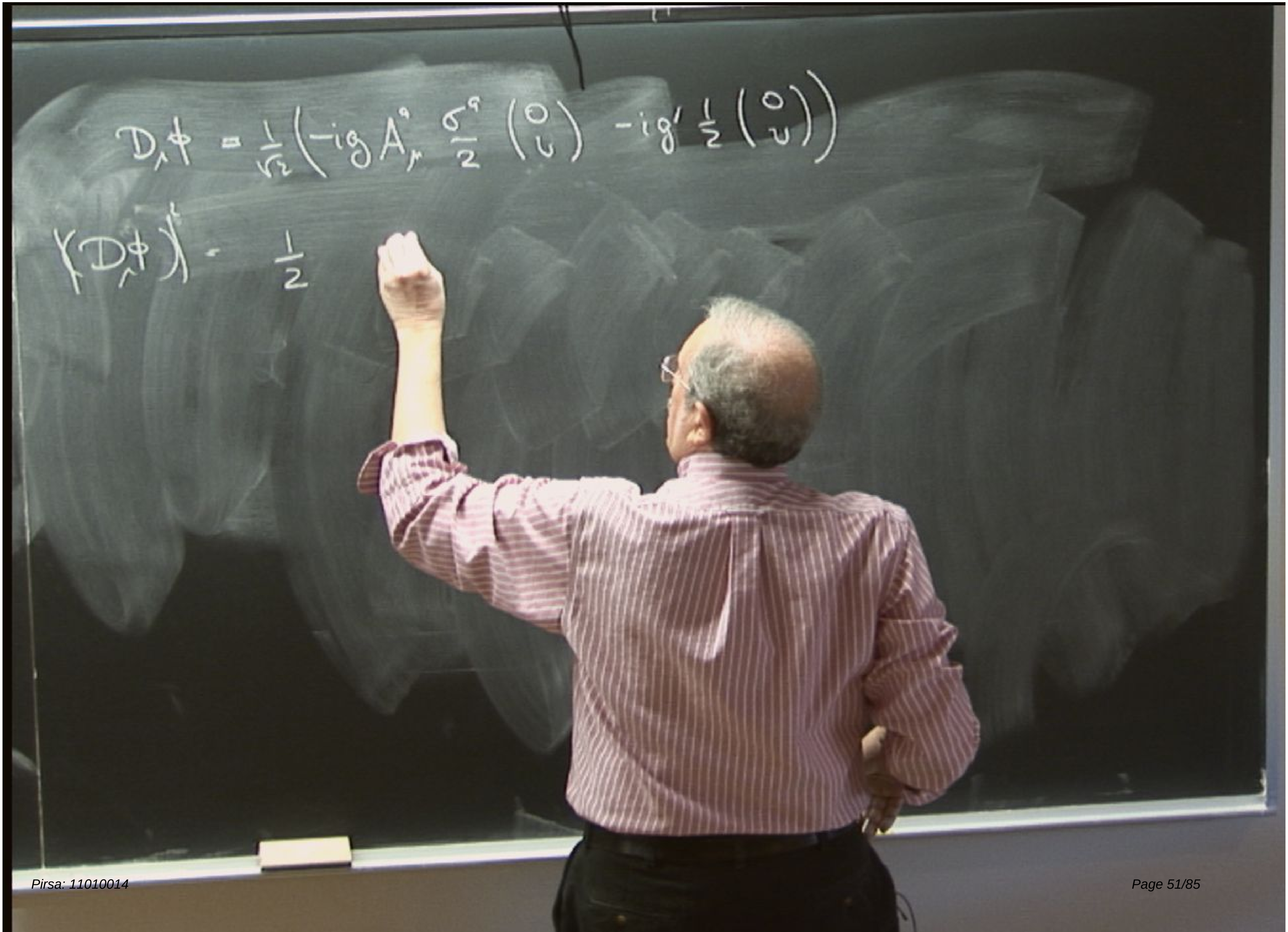
$(\partial_\mu - i g t^a A^a_\mu)\phi$ $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ spin $\frac{1}{2}$ of SU(2) g

Higgs field $\frac{1}{2}$ under U(1) g'

$$D_\mu \phi = \left(\partial_\mu - i g A^a_\mu \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu \right)\phi$$

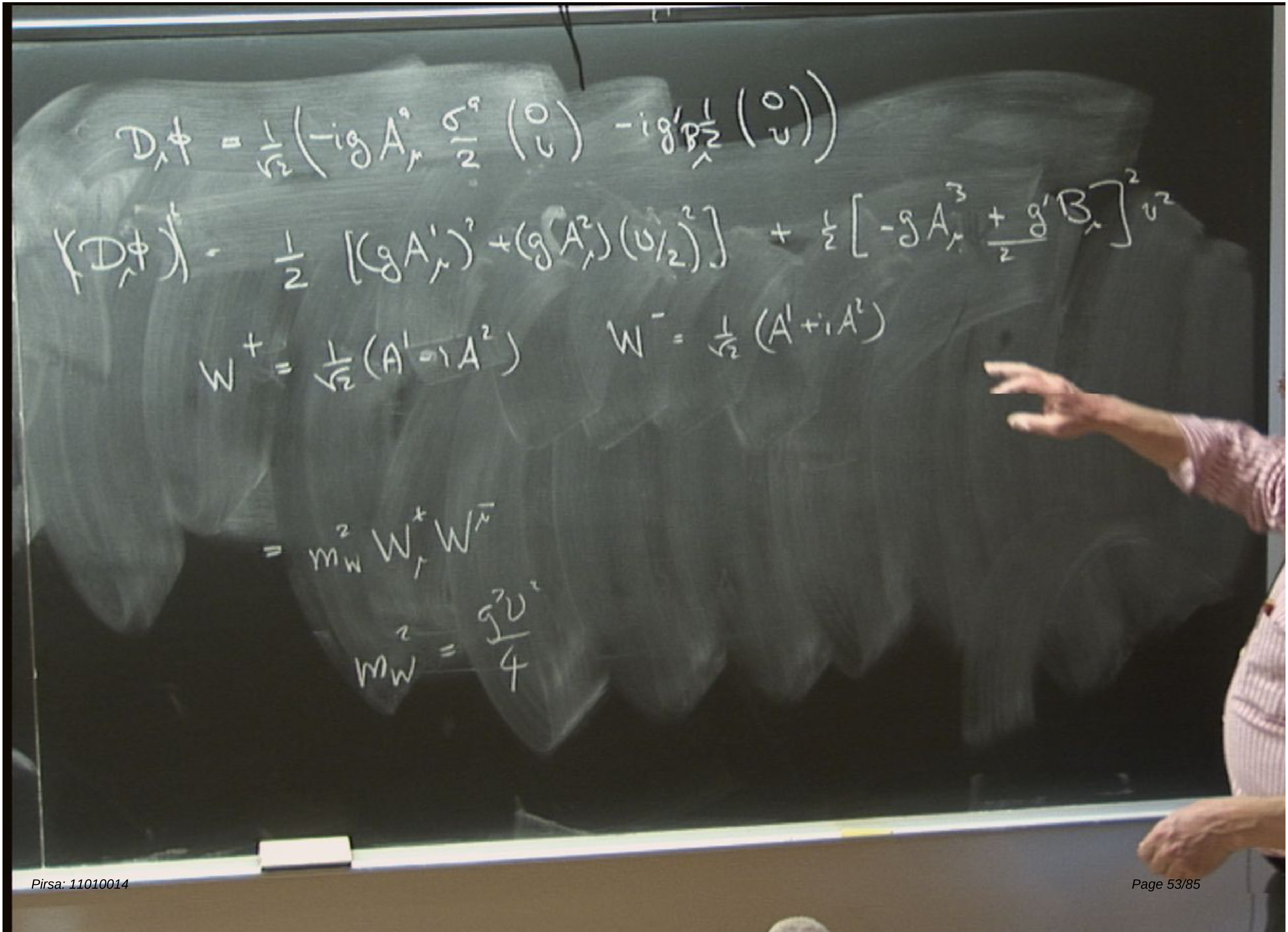
$$\langle \Omega | \phi(x) | \Omega \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\phi(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$


Dμφ = 1/√2 (−ig Aaμ σa/2 (0 v) − ig' 1/2 (0 v))
(Dμφ)² = 1/2

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-ig A^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\ v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\ v\end{pmatrix}\right)$$

$$(D_\mu\phi)^2 = \frac{1}{2}\left[(gA^1_\mu)^2 + (gA^2_\mu)(v/2)^2\right] + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$$


$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-ig A^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\v\end{pmatrix}\right)$
$(D_\mu\phi)^2 = \frac{1}{2}\left[(gA^1_\mu)^2 + (gA^2_\mu)^2\right](v/2)^2 + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$
$W^+ = \frac{1}{\sqrt{2}}(A^1 - iA^2)$
$W^- = \frac{1}{\sqrt{2}}(A^1 + iA^2)$
$= m_W^2 W^+_\mu W^{-\mu}$
$m_W^2 = \frac{g^2 v^2}{4}$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-ig A^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\v\end{pmatrix}\right)$$

$$(D_\mu \phi)^2 = \frac{1}{2}\left[(g A^1_\mu)^2 + (g A^2_\mu)(v/2)^2\right] + \frac{1}{2}\left[\frac{-g A^3_\mu + g' B_\mu}{2}\right]^2 v^2$$

$$W^+ = \frac{1}{\sqrt{2}}(A^1 - i A^2) \qquad W^- = \frac{1}{\sqrt{2}}(A^1 + i A^2)$$

$$Z_\mu =$$

$$= m_W^2 W^+_\mu W^{-\mu}$$

$$m_W^2 = \frac{g^2 v^2}{4}$$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-igA^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\ v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\ v\end{pmatrix}\right)$$

$$|D_\mu \phi|^2 = \frac{1}{2}\left[(gA^1_\mu)^2 + (gA^2_\mu)(v/2)^2\right] + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$$

$$W^+ = \frac{1}{\sqrt{2}}(A^1 - iA^2) \qquad W^- = \frac{1}{\sqrt{2}}(A^1 + iA^2)$$

$$Z_\mu = \frac{gA^3_\mu - g'B}{\sqrt{g^2 + (g')^2}} \qquad A_\mu = \frac{g'A^3 + gB}{\sqrt{g^2 + g'^2}}$$

$$= m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z Z_\mu Z^\mu$$

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z = (g^2 + g'^2)\frac{v^2}{4}$$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-ig A^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\ v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\ v\end{pmatrix}\right)$$

$$(D_\mu\phi)^2 = \frac{1}{2}\left[(gA^1_\mu)^2 + (gA^2_\mu)(v/2)^2\right] + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$$

$$W^+ = \frac{1}{\sqrt{2}}(A^1 - iA^2) \qquad W^- = \frac{1}{\sqrt{2}}(A^1 + iA^2)$$

$$Z_\mu = \frac{gA^3_\mu - g'B}{\sqrt{g^2 + (g')^2}} \qquad A_\mu = \frac{g'A^3 + gB}{\sqrt{g^2 + g'^2}}$$

$$= m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z Z_\mu Z^\mu$$

$$m_W^2 = \frac{g^2 v^2}{4}, \qquad m_Z = (g^2 + g'^2)\frac{v^2}{4}, \qquad m_A = 0$$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-igA^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\ v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\ v\end{pmatrix}\right)$$

$$|D_\mu\phi|^2 = \frac{1}{2}\left[(gA^1_\mu)^2 + (gA^2_\mu)(v/2)^2\right] + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$$

$$W^+ = \frac{1}{\sqrt{2}}(A^1 - iA^2) \qquad W^- = \frac{1}{\sqrt{2}}(A^1 + iA^2)$$

$$Z_\mu = \frac{gA^3_\mu - g'B}{\sqrt{g^2 + (g')^2}} \qquad A_\mu = \frac{g'A^3 + gB}{\sqrt{g'^2 + g^2}}$$

$$= m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z Z_\mu Z^\mu$$

$$m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z = (g^2 + g'^2)\frac{v^2}{4}, \quad m_A = 0$$

$$D_\mu \phi = \frac{1}{\sqrt{2}}\left(-ig A^a_\mu \frac{\sigma^a}{2}\begin{pmatrix}0\\ v\end{pmatrix} - ig' B_\mu \frac{1}{2}\begin{pmatrix}0\\ v\end{pmatrix}\right)$$

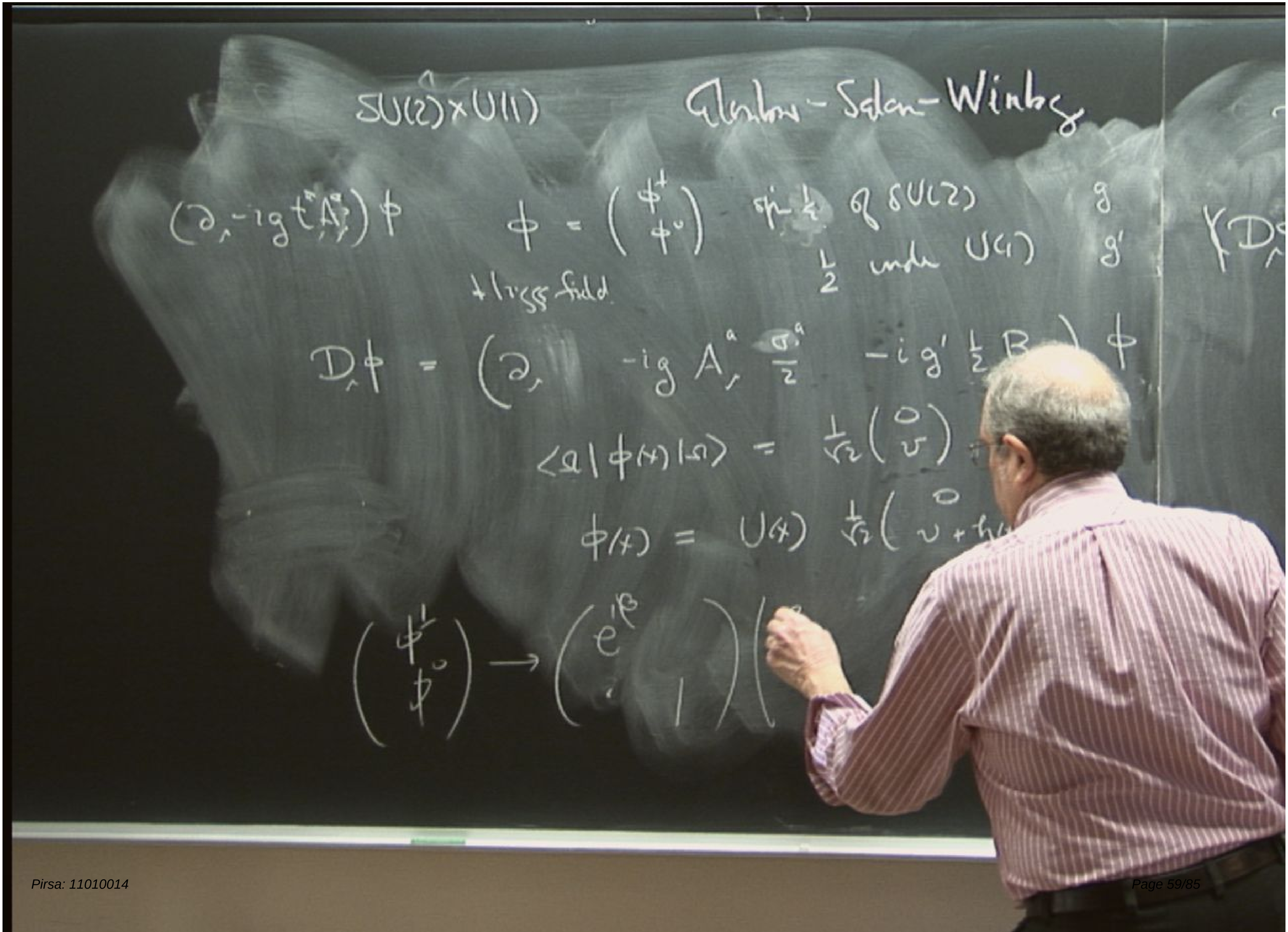
$$|D_\mu\phi|^2 = \frac{1}{2}\left([(gA^1_\mu)^2 + (gA^2_\mu)](v/2)^2\right) + \frac{1}{2}\left[\frac{-gA^3_\mu + g'B_\mu}{2}\right]^2 v^2$$

$$W^+ = \frac{1}{\sqrt{2}}(A^1 - iA^2) \qquad W^- = \frac{1}{\sqrt{2}}(A^1 + iA^2)$$

$$Z_\mu = \frac{gA^3_\mu - g'B}{\sqrt{g^2+(g')^2}} \qquad A_\mu = \frac{g'A^3 + gB}{\sqrt{g^2+g'^2}}$$

$$= m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} m_Z Z_\mu Z^\mu$$

$$m_W^2 = \frac{g^2 v^2}{4}, \qquad m_Z = (g^2+g'^2)\frac{v^2}{4}, \qquad m_A = 0$$


SU(2)×U(1)
Higgs field
under U(1)

$SU(2) \times U(1)$ Glashow – Salam – Weinberg

$(\partial_\mu - i g t^a A^a_\mu)\phi$

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ spin $\frac{1}{2}$ of $SU(2)$ g

$\frac{1}{2}$ under $U(1)$ g'

Higgs field

$D_\mu \phi = \left(\partial_\mu - i g A^a_\mu \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu\right)\phi$

$\langle \Omega | \phi(x) | \Omega \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$\phi(x) = U(x) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\beta} & \\ & 1 \end{pmatrix}\begin{pmatrix} \phi' \\ \phi' \end{pmatrix}$

SU(2) × U(1) Glashow – Salam – Weinberg

isospin hypercharge

$(\partial_\mu - i g t^a A^a_\mu)\phi$

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ spin $\frac{1}{2}$ of SU(2) g

$\frac{1}{2}$ under U(1) g'

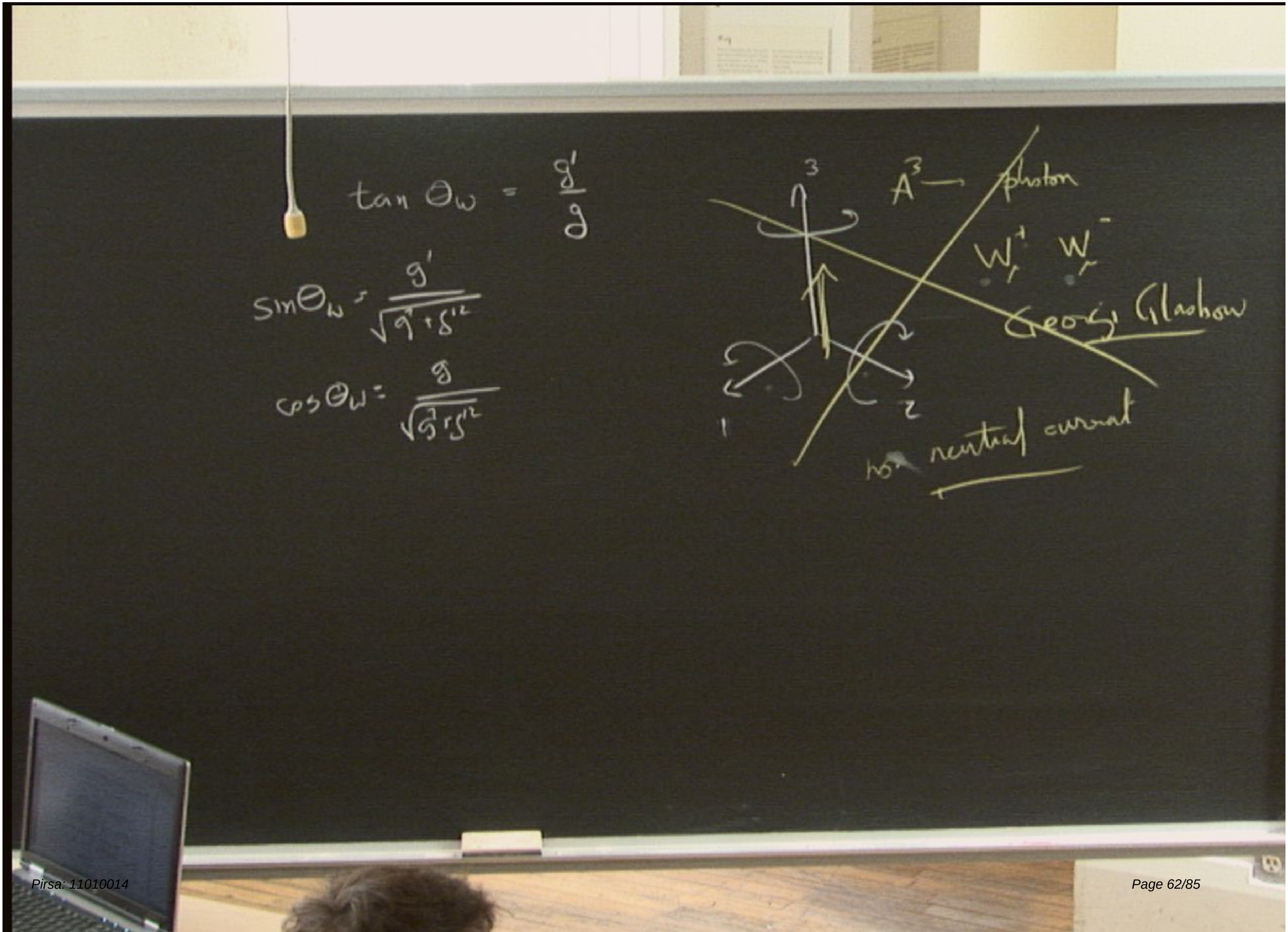
Higgs field

$D_\mu \phi = \left(\partial_\mu - i g A^a_\mu \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu \right)\phi$

$\langle \Omega | \phi(x) | \Omega \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$\phi(x) = U(x) \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

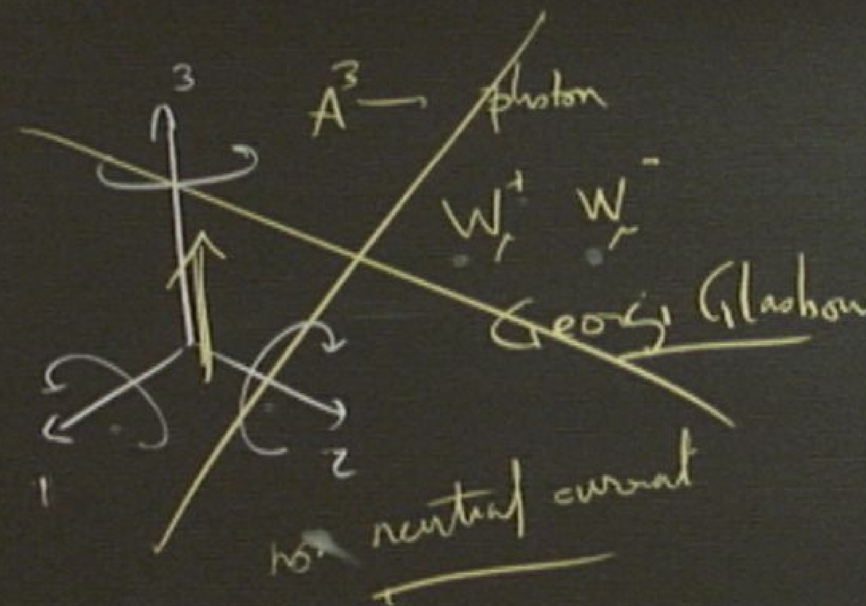
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \begin{pmatrix} e^{i\beta} & \\ & 1 \end{pmatrix}\begin{pmatrix} \phi' \\ \phi' \end{pmatrix}$


$\tan\theta_W = \frac{g'}{g}$
$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$
$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$
3
1
2
$A^3 \longrightarrow$ photon
W^+_μ W^-_μ
Georgi Glashow
neutral current

$$\tan\Theta_W = \frac{g'}{g}$$

$$\sin\Theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\cos\Theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$



SU(2) × U(1)   Glashow – Salam – Weinberg

isospin   hypercharge

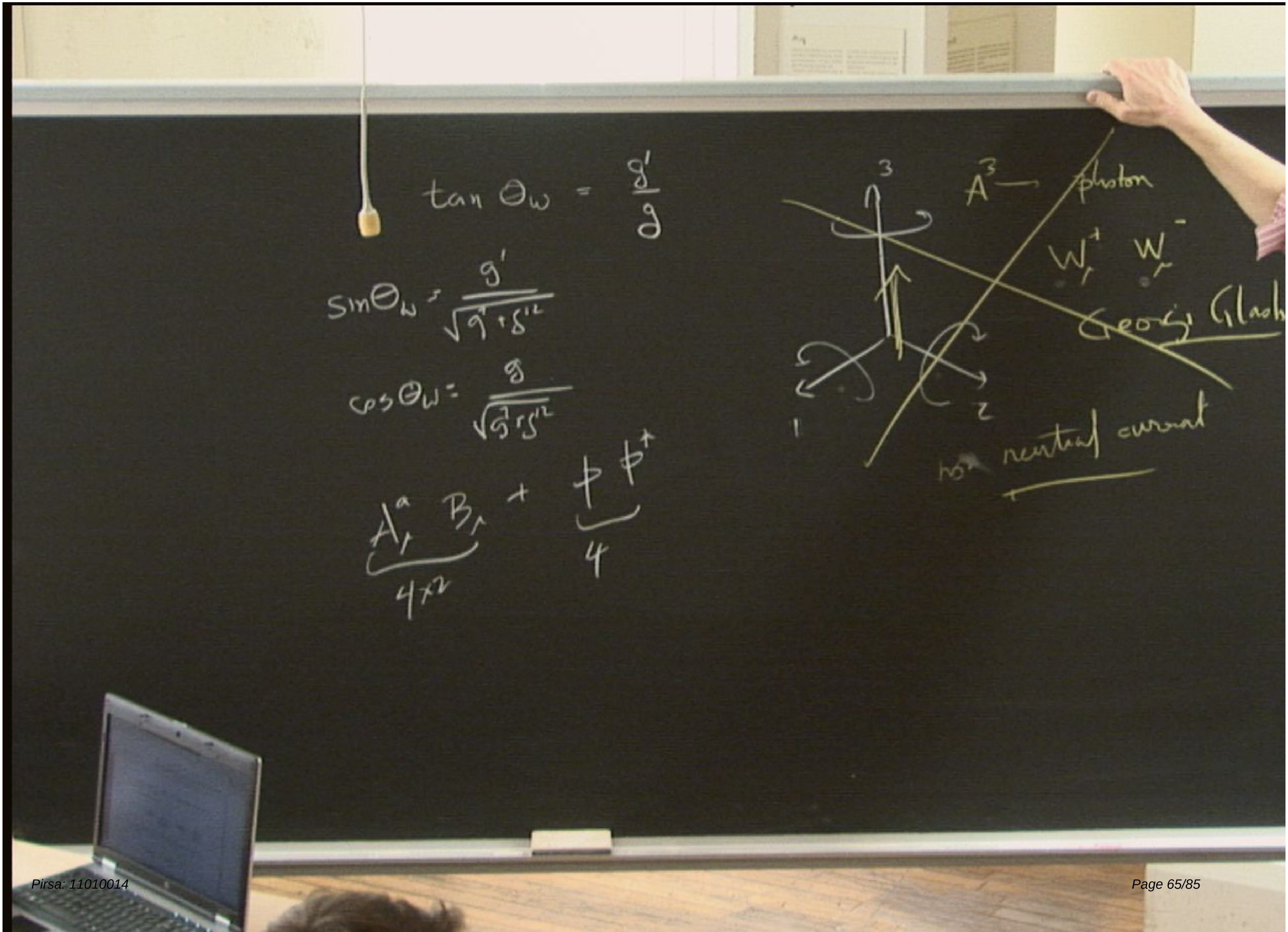
$(\partial_\mu - i g t^a A^a_\mu)\phi$ $\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$ spin $\frac{1}{2}$ of SU(2) g

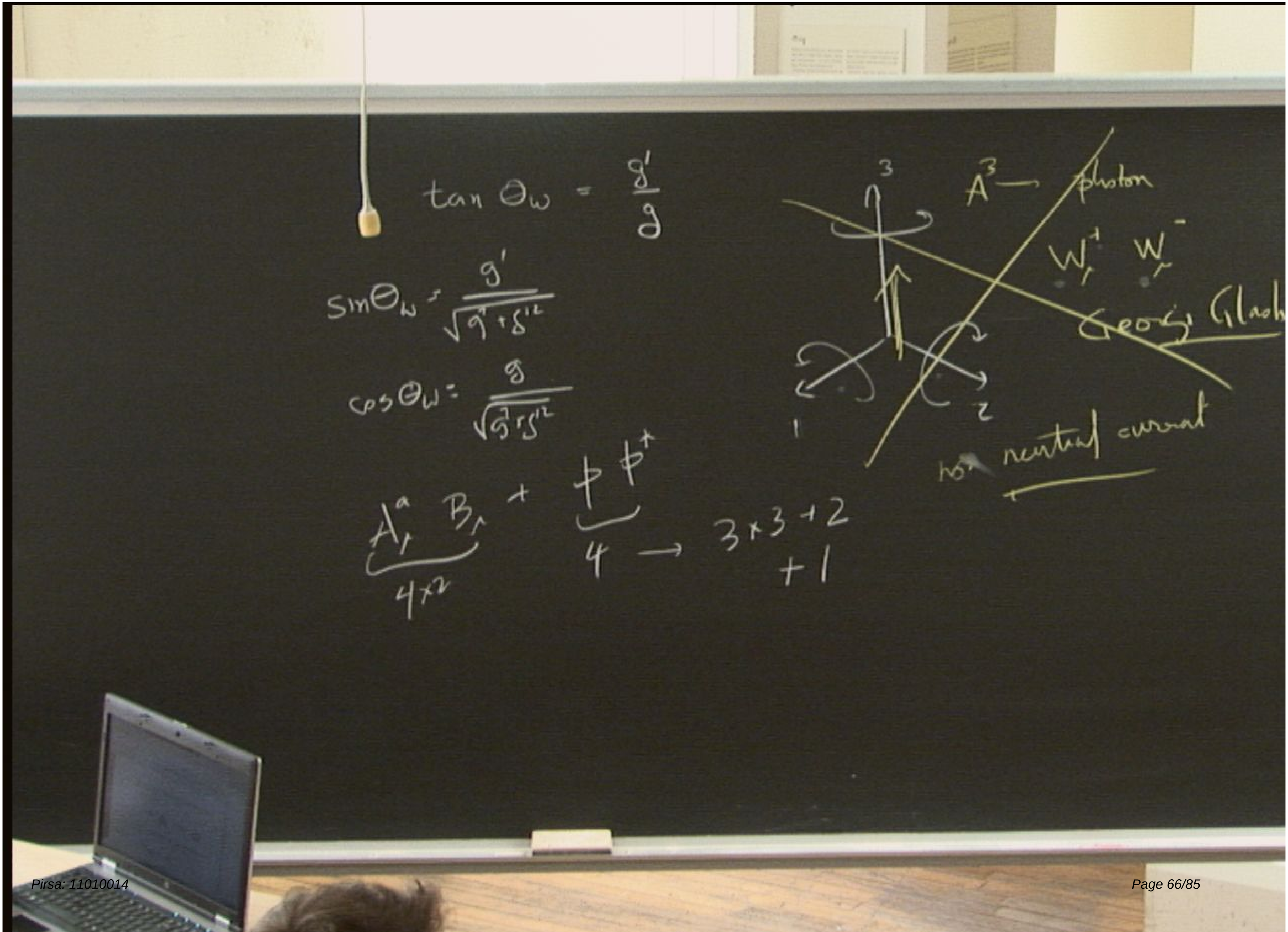
$\frac{1}{2}$ under U(1) g'

Higgs field

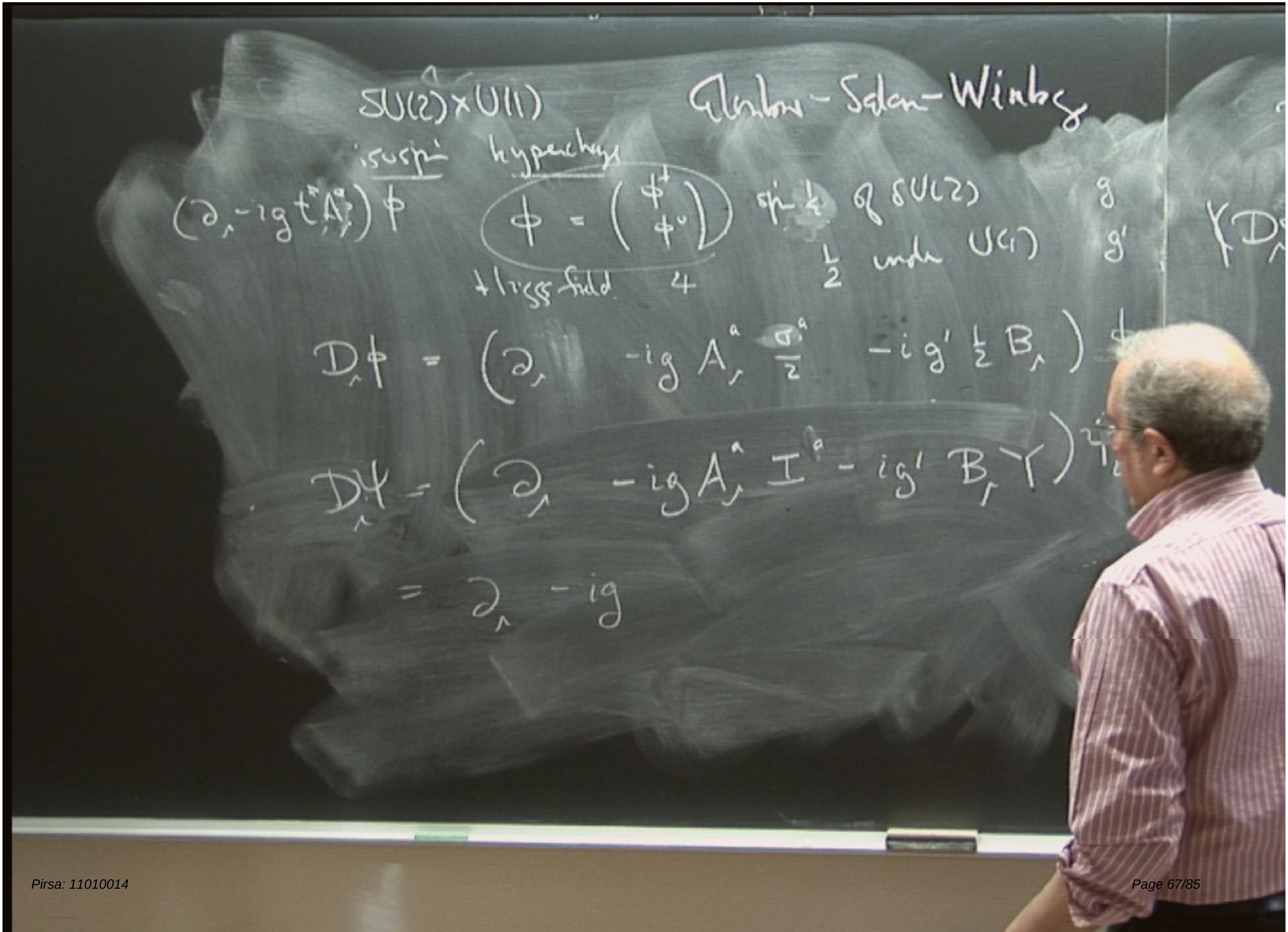
$$D_\mu \phi = \left(\partial_\mu - i g A^a_\mu \frac{\sigma^a}{2} - i g' \frac{1}{2} B_\mu\right)\phi$$

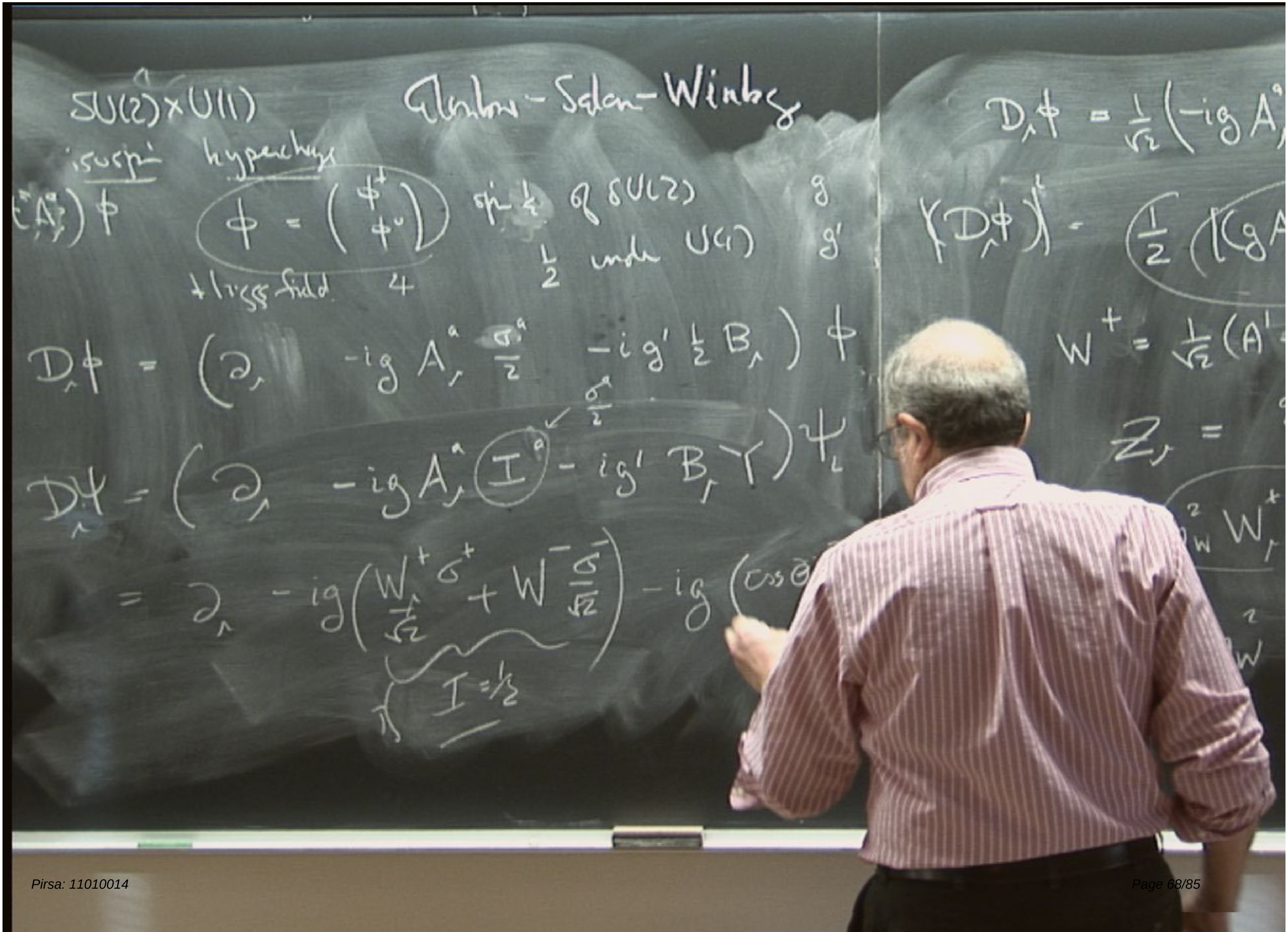
$$D_\mu \psi = \left(\partial_\mu - i g A^a_\mu I^a - i g' B_\mu\right)\psi_L$$

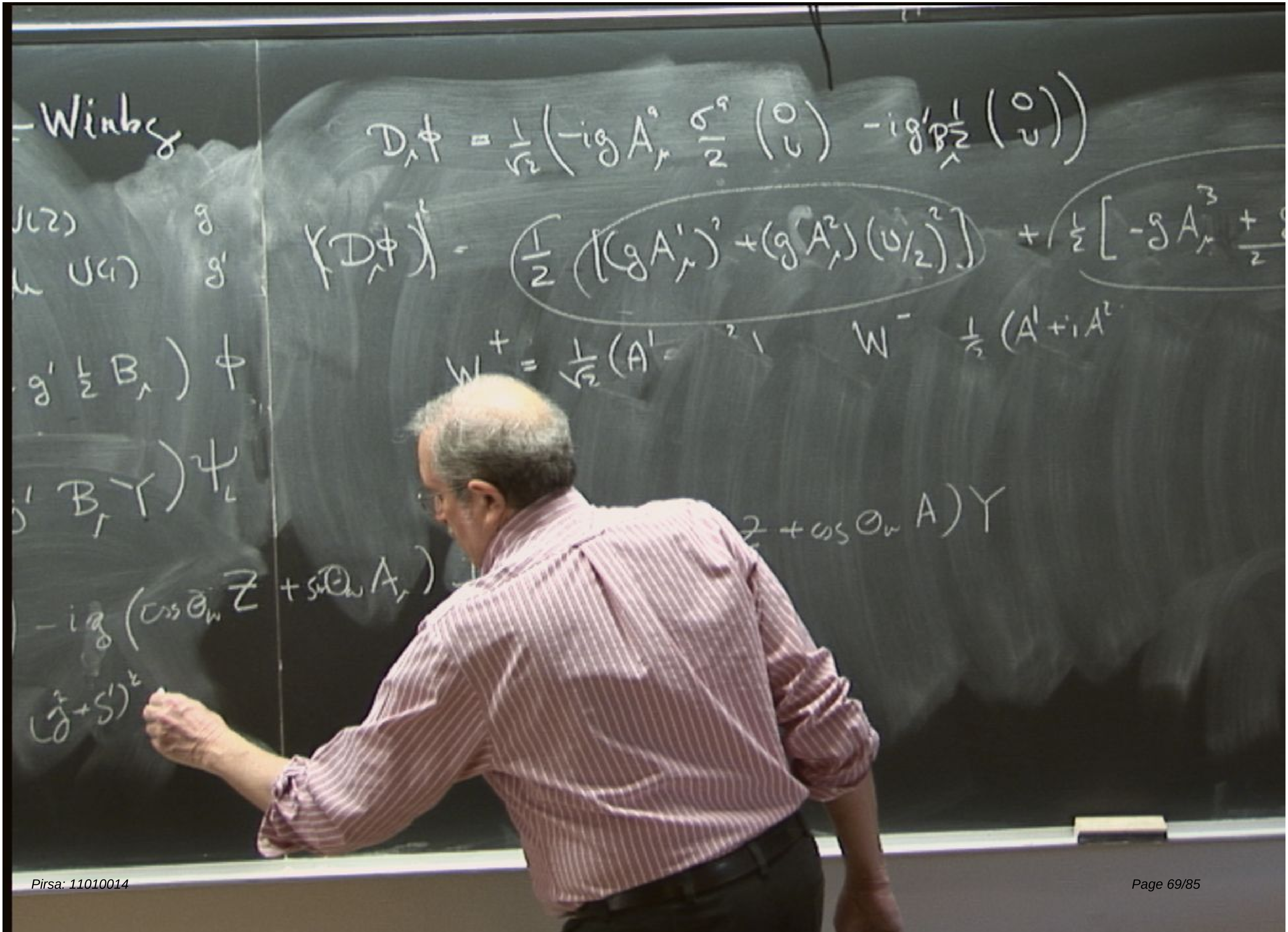

tan Θ_W = g'/g
sinΘ_W = g'/√(g²+g'²)
cosΘ_W = g/√(g²+g'²)
A^a_μ B_μ + φ φ*
4×2
4
3
1
2
A³ → photon
W⁺_μ W⁻_μ
Georgi Glash
no neutral current

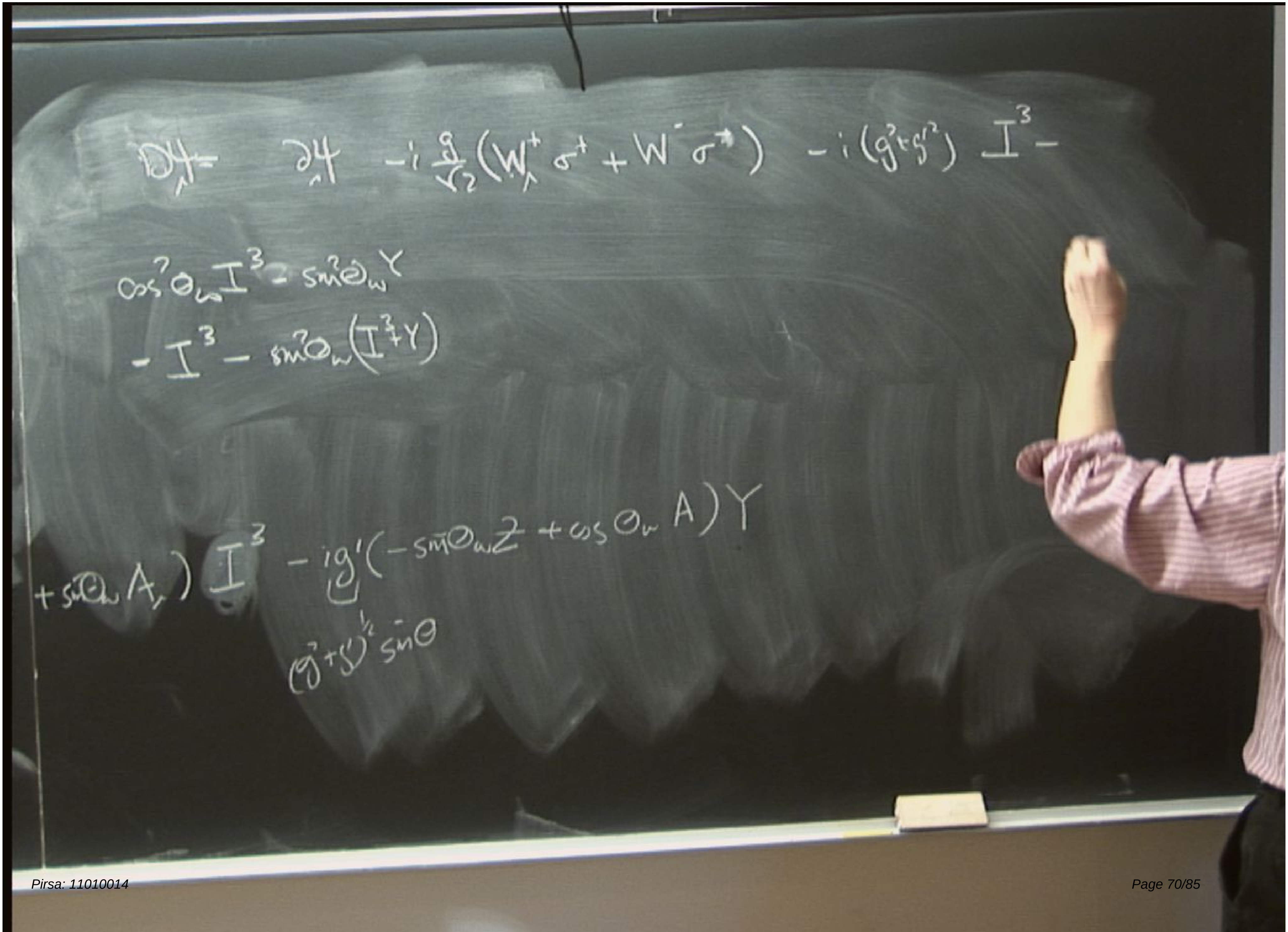


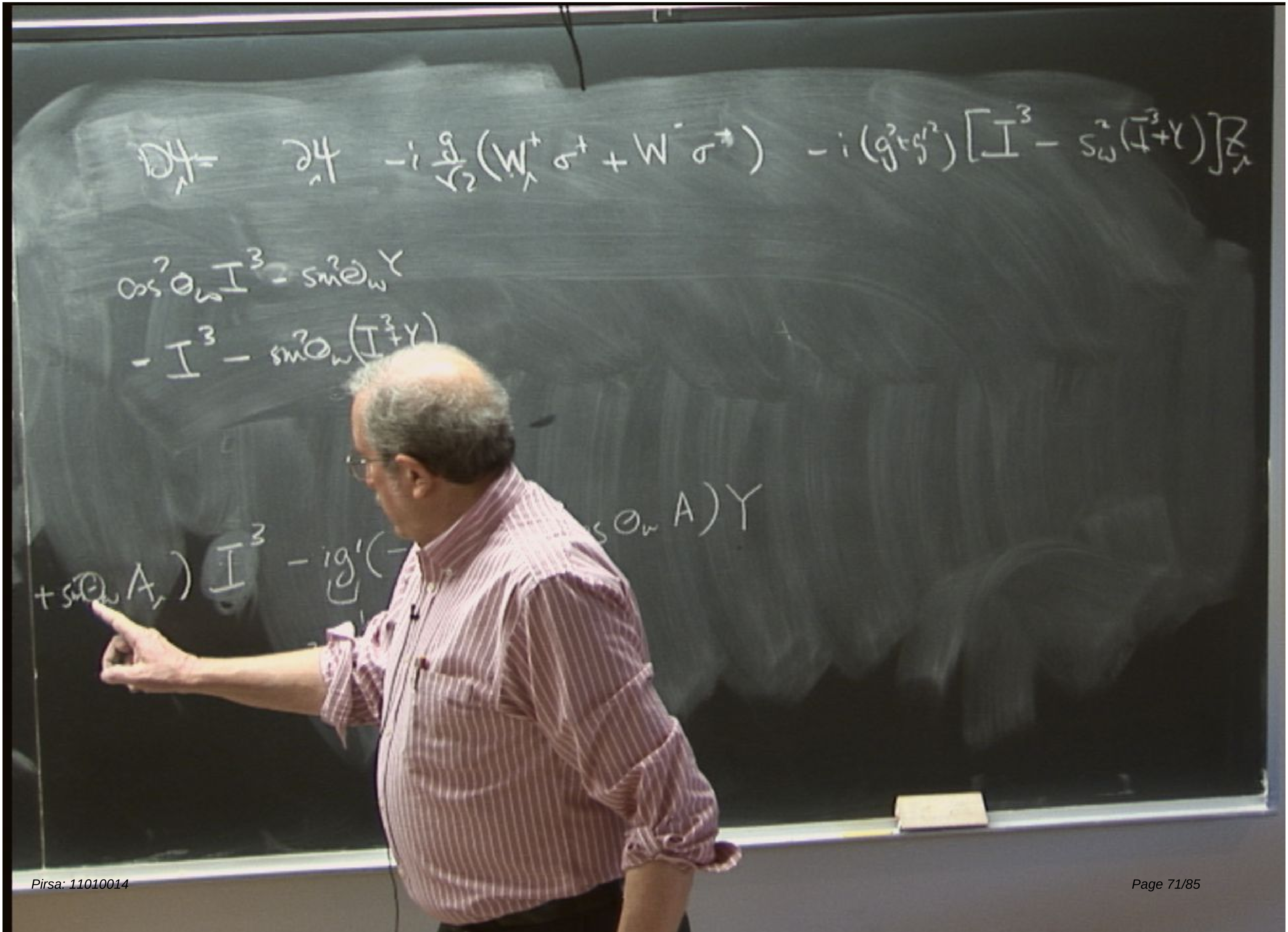
$\tan\theta_W = \frac{g'}{g}$
$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$
$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$
$A^a_\mu \; B_\mu + \phi\,\phi^*$
4×2
$4 \rightarrow 3\times 3 + 2$
$+1$
3
1
2
$A^3 \rightarrow$ photon
$W^+_\mu \; W^-_\mu$
Georgi Glash
no neutral current

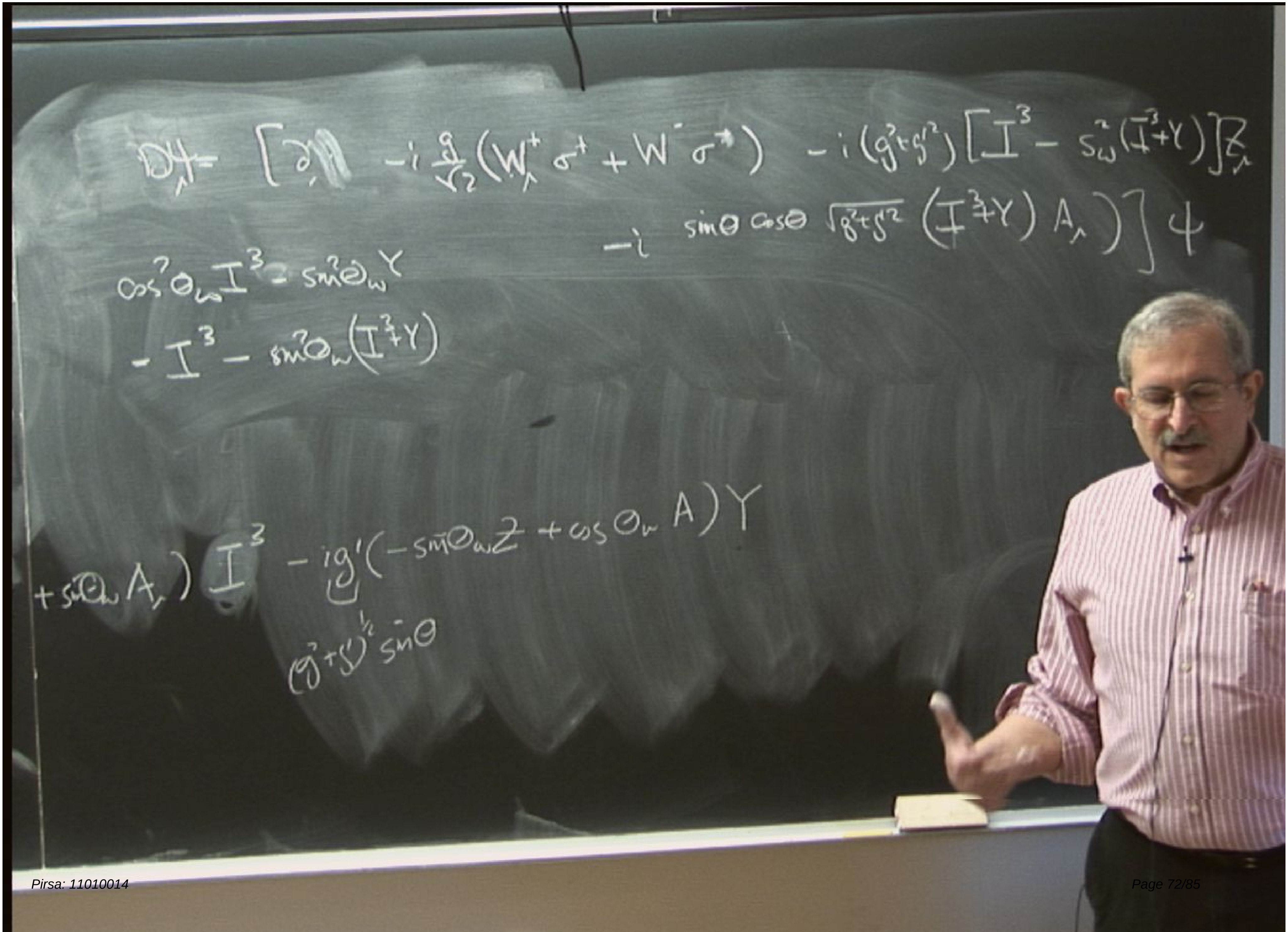

SU(2)×U(1)
Glashow-Salam-Weinberg
isospin hypercharge
$(\partial_\mu - ig t^a A^a_\mu)\phi$
$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$
Higgs field 4
spin 1/2 of SU(2) g
1/2 under U(1) g'
$D_\mu \phi = (\partial_\mu - ig A^a_\mu \frac{\sigma^a}{2} - ig' \frac{1}{2} B_\mu)\phi$
$D_\mu \psi = (\partial_\mu - ig A^a_\mu I^a - ig' B_\mu Y)\psi$
$= \partial_\mu - ig$

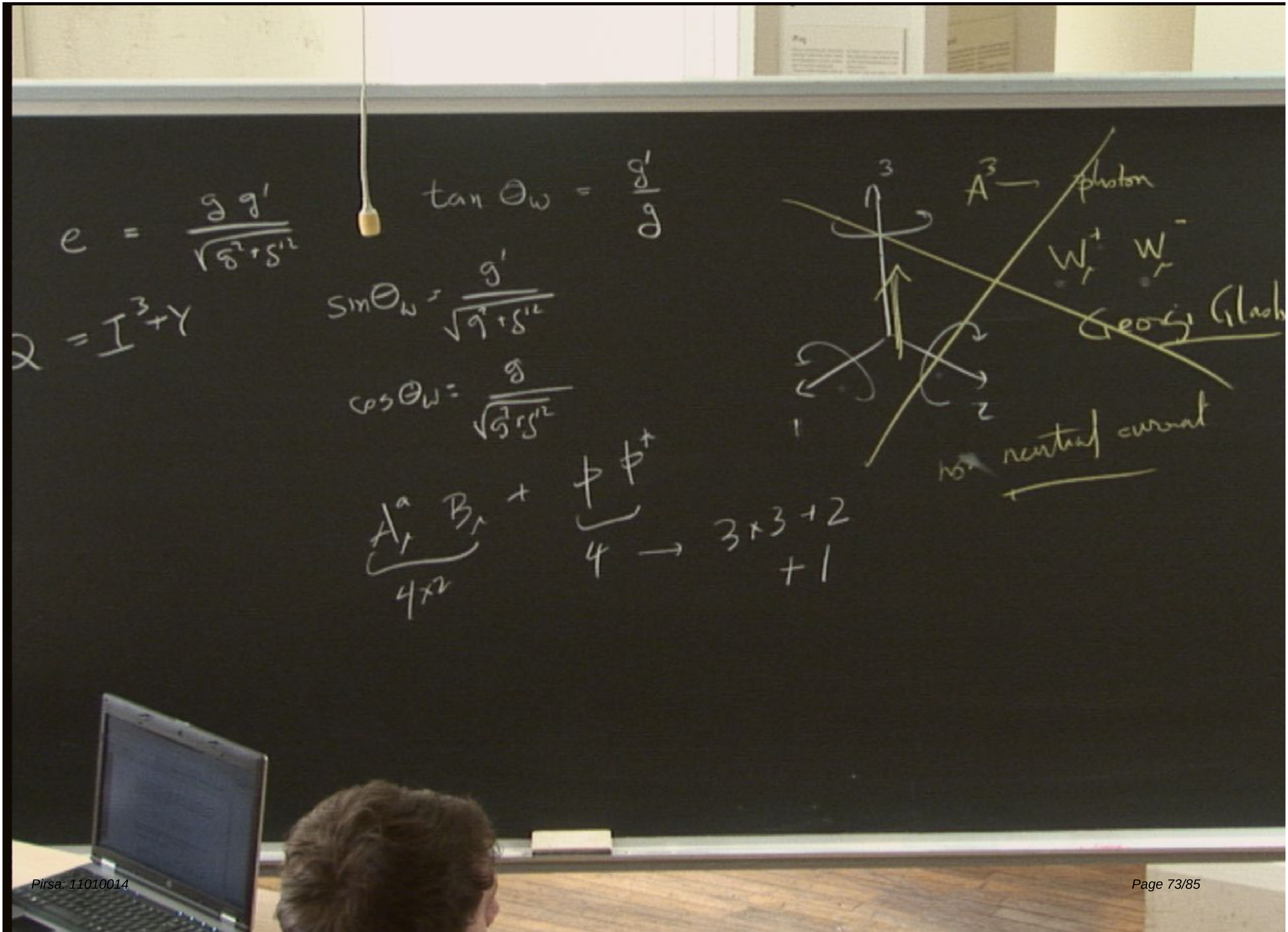

SU(2)×U(1)
hypercharge
Higgs field
4
under U(1)
g
g'

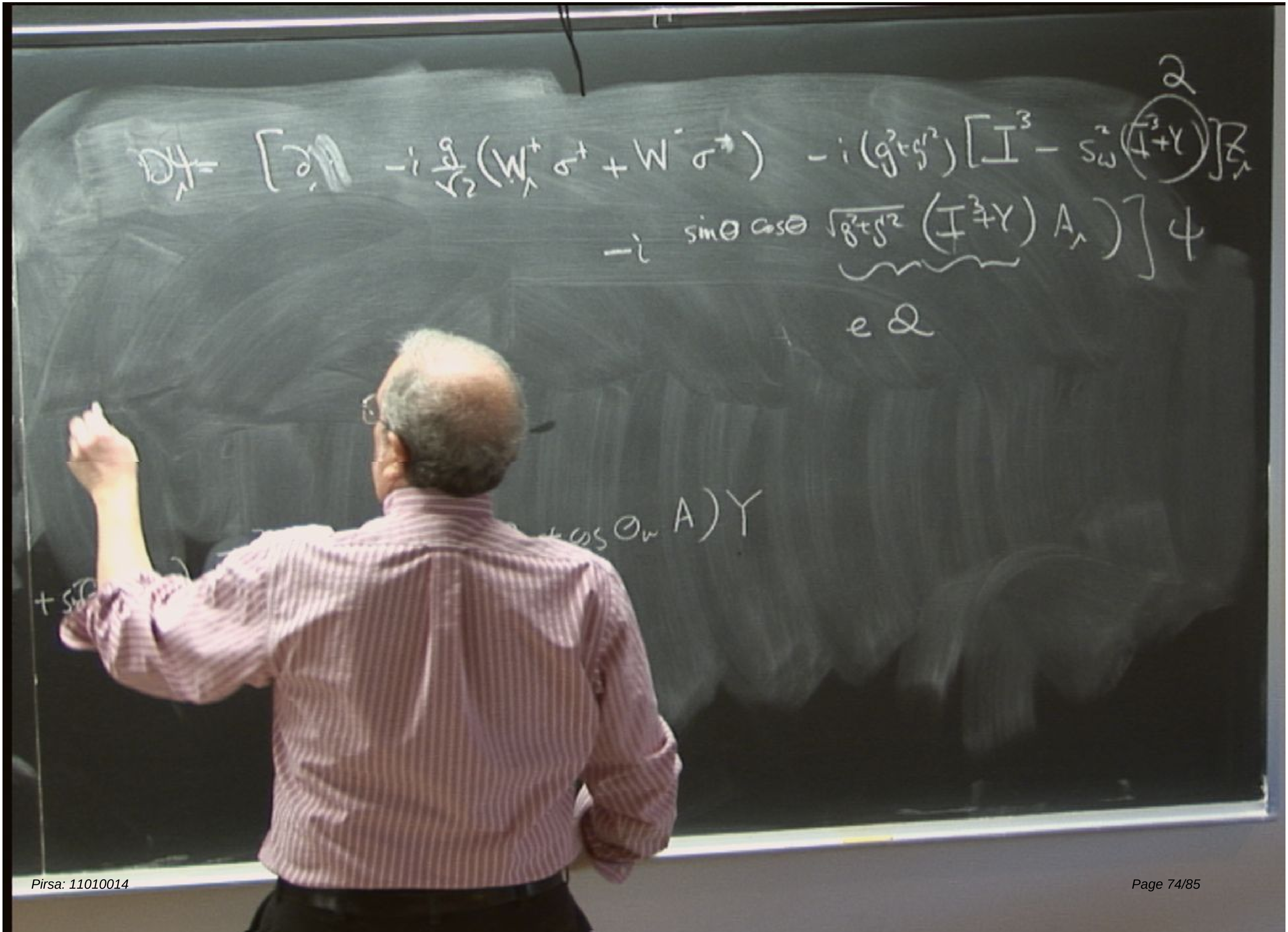


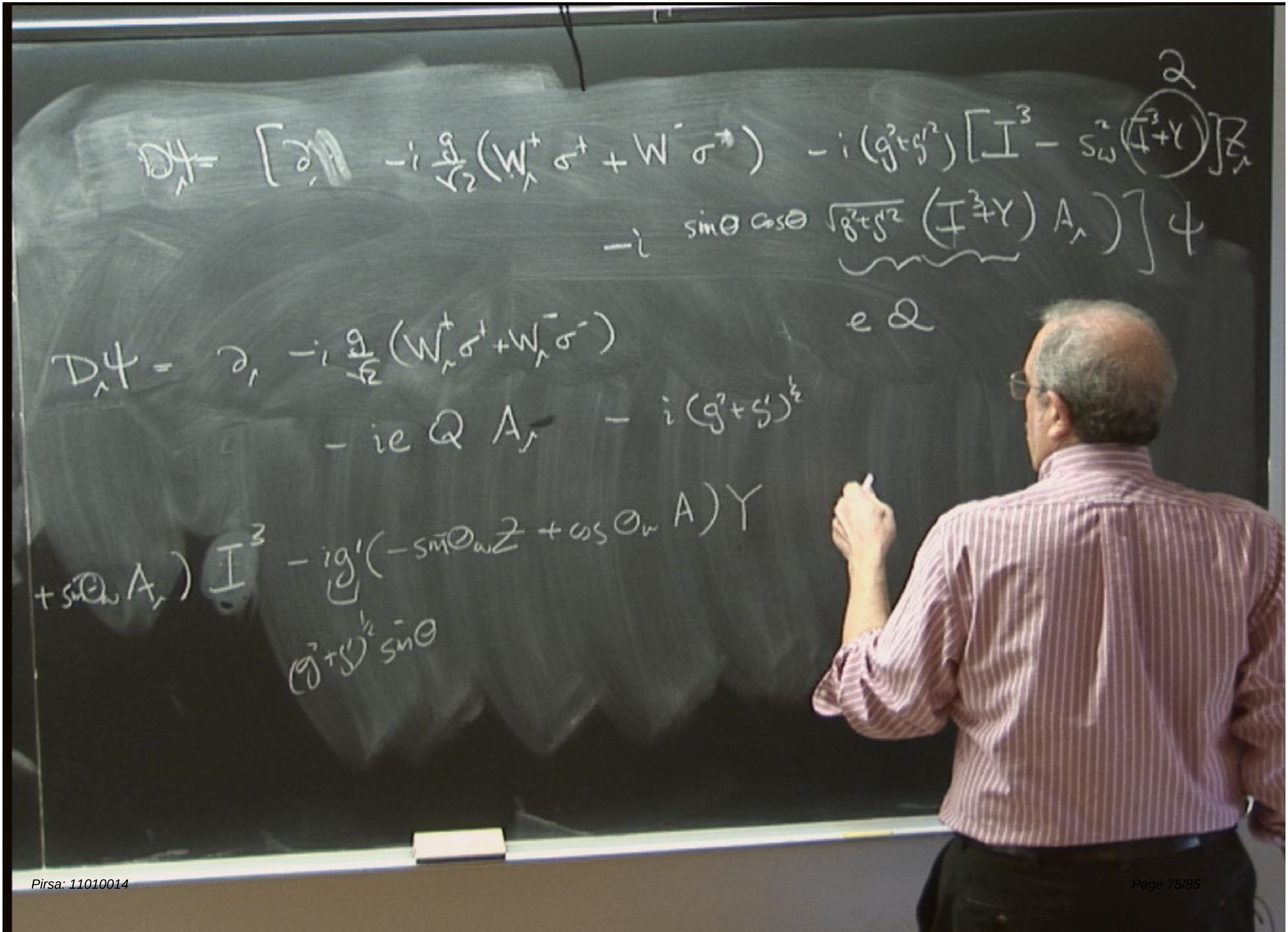

Dψ = ∂ψ − i g/√2 (W+ σ+ + W− σ−) − i(g²+g'²) I³ −
cos²θw I³ − sin²θw Y
= I³ − sin²θw (I³+Y)
+ sinθw Aμ) I³ − ig'(−sinθw Z + cosθw A) Y
(g²+g'²)^½ sinθ


cos²θw I³ − sin²θw Y
− I³ − sin²θw (I³ + Y)


D_μψ = [∂_μ − i g/√2 (W_μ^+ σ^+ + W^- σ^-) − i (g^2+g'^2) [I^3 − s_w^2 (I^3+Y)] Z_μ
− i sinθ cosθ √(g^2+g'^2) (I^3+Y) A_μ)] ψ
cos^2θ_w I^3 − sin^2θ_w Y
= I^3 − sin^2θ_w (I^3+Y)
+ sinθ_w A_μ) I^3 − ig'(− sinθ_w Z + cos θ_w A) Y
(g^2+g'^2)^{-1/2} sinθ


e = gg' / √(g² + g'²)
tan θ_W = g'/g
sin θ_W = g' / √(g² + g'²)
cos θ_W = g / √(g² + g'²)
Q = I³ + Y
A³ → photon
W⁺ W⁻
Georgi Glashow
neutral current
A^a_μ B_μ + φ φ⁺
4×2
4 → 3×3 + 2
+1
3
1
2


e Q
cos Θw A) Y


e Q
- ie Q A

$$D_\mu\psi = \Big[\partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu\sigma^+ + W^-\sigma^-) - i(g^2+g'^2)\big[I^3 - s_\omega^2(I^3+Y)\big]Z_\mu$$

$$-i\,\sin\theta\cos\theta\,\sqrt{g^2+g'^2}\,(I^3+Y)\,A_\mu\Big)\Big]\psi$$

2

eQ

$$D_\mu\psi = \partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu\sigma^+ + W^-_\mu\sigma^-)$$

$$- ieQA_\mu - i(g^2+g'^2)^{\frac{1}{2}} Z_\mu \left(I^3 - s_\omega^2 Q\right)$$

$$+\sin\theta_w A_\mu)\,I^3 - ig'(-\sin\theta_w Z + \cos\theta_w A)\,Y$$

$$(g^2+g'^2)^{\frac{1}{2}}\sin\theta$$

$$2$$

$$D_\mu \psi = \Big[\partial_\mu - i\frac{g}{\sqrt{2}}\left(W^+_\mu \sigma^+ + W^- \sigma^-\right) - i(g^2+g'^2)\left[I^3 - s_\omega^2 (I^3+Y)\right] Z_\mu$$

$$- i \sin\theta \cos\theta \sqrt{g^2+g'^2}\,(I^3+Y)\,A_\mu \Big)\Big]\psi$$

$$eQ$$

$$D_\mu \psi = \Big[\partial_\mu - i\frac{g}{\sqrt{2}}\left(W^+_\mu \sigma^+ + W^-_\mu \sigma^-\right)$$

$$- ieQA_\mu - i(g^2+g'^2)^{\frac{1}{2}} Z_\mu \left(I^3 - s_\omega^2 Q\right)\Big]\psi$$

$$+ \sin\theta_w A_\mu) I^3 - ig'(-\sin\theta_w Z + \cos\theta_w A) Y$$

$$(g^2+g'^2)^{\frac{1}{2}} \sin\theta$$

$$D_\mu \psi = \Big[\partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu \sigma^+ + W^- \sigma^-) - i(g^2+g'^2)\big[I^3 - s_\omega^2 (I^3+Y)\big] Z_\mu$$

2

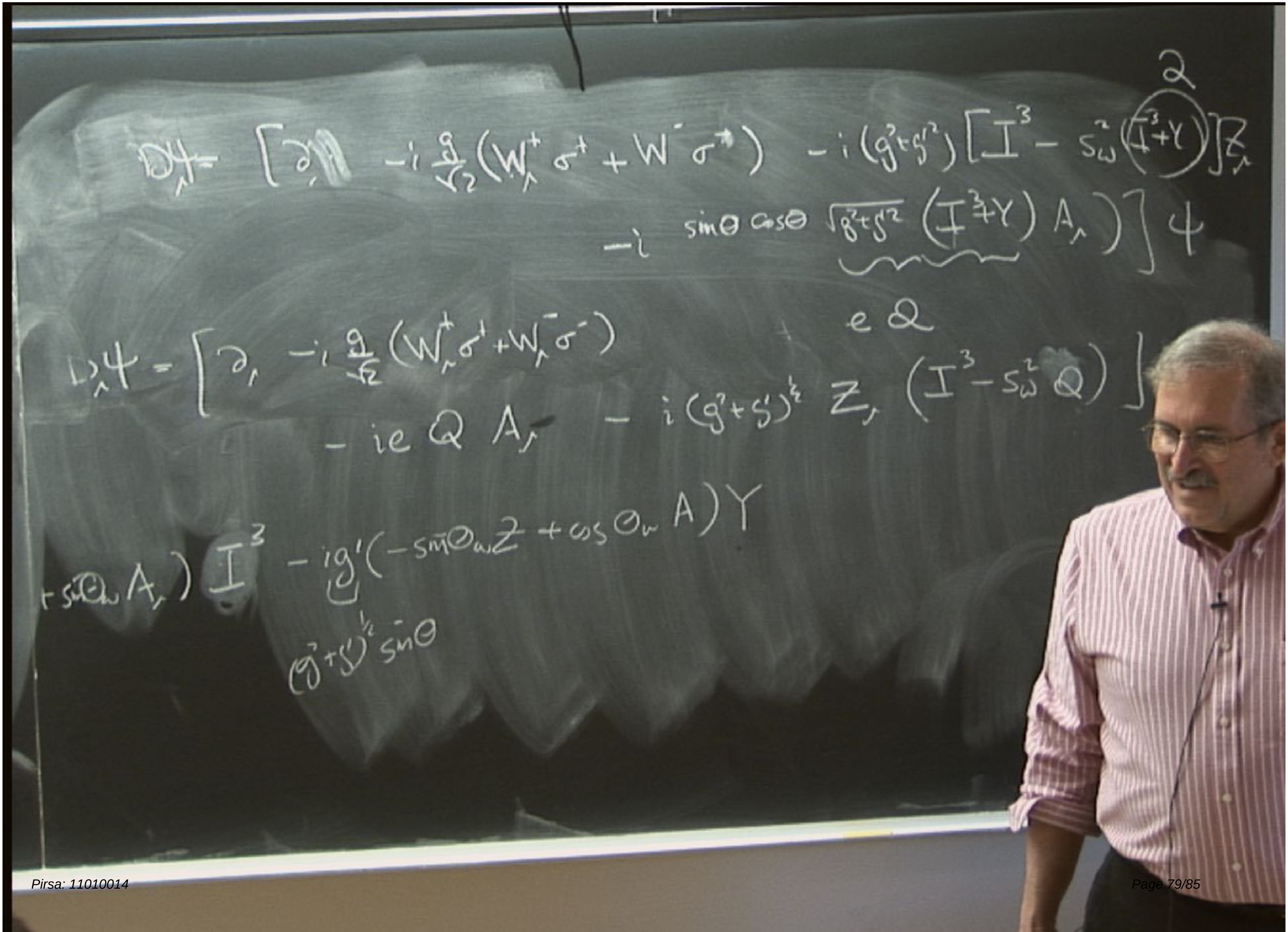
$$-i \sin\theta\cos\theta \sqrt{g^2+g'^2}\,(I^3+Y)\,A_\mu\Big)\Big]\psi$$

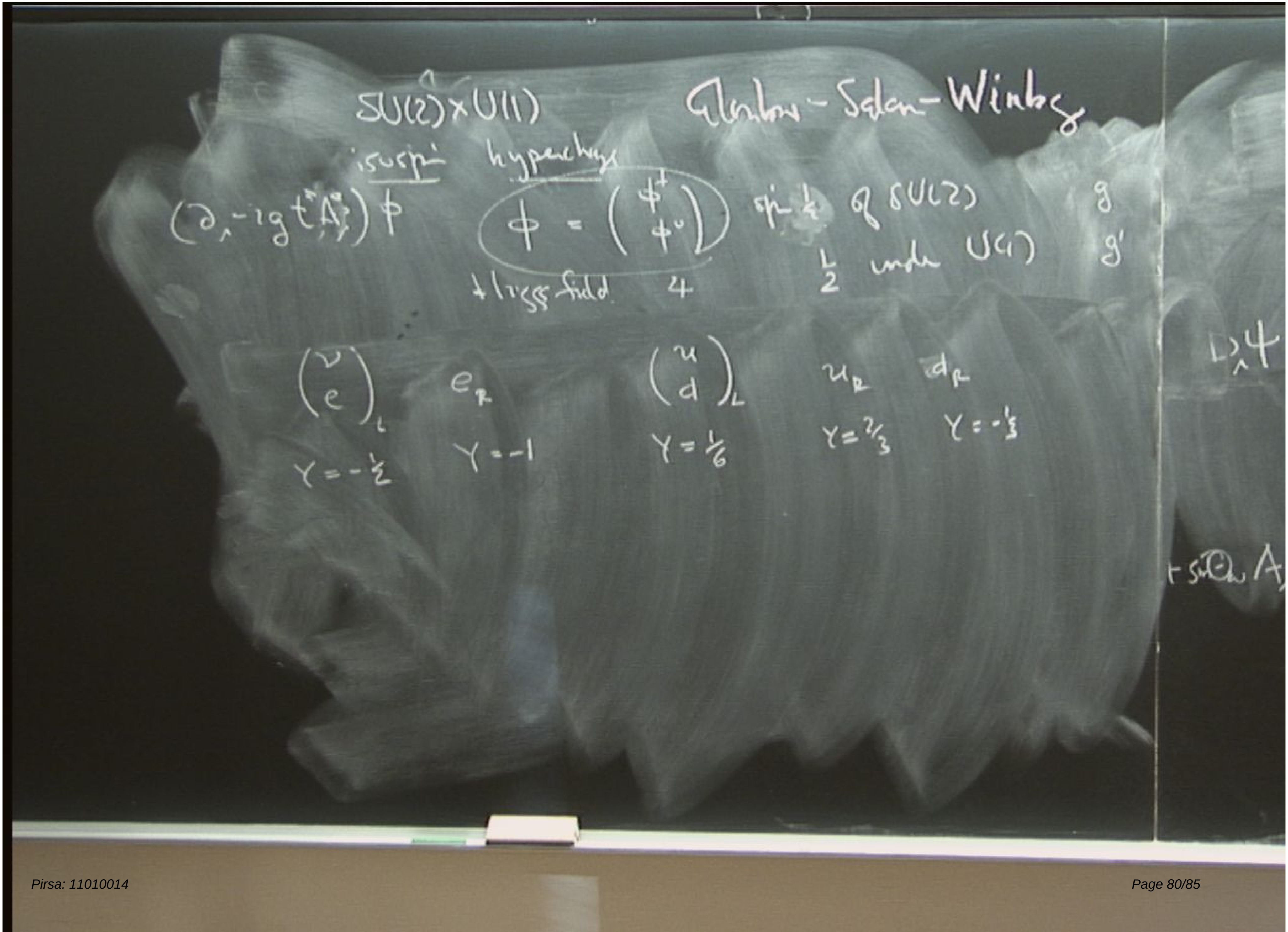
eQ

$$D_\mu \psi = \Big[\partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu \sigma^+ + W^-_\mu \sigma^-) - ieQA_\mu - i(g^2+g'^2)^{1/2} Z_\mu (I^3 - s_\omega^2 Q)\Big]\psi$$

$$+\sin\theta_w A_\mu)\, I^3 - ig'(-\sin\theta_w Z + \cos\theta_w A)\,Y$$

$$g' = (g^2+g'^2)^{1/2}\sin\theta$$


D_λψ = [∂_λ − i g/√2 (W_λ⁺ σ⁺ + W⁻ σ⁻) − i (g² + g'²) [I³ − s_w² (I³ + Y)] Z_λ
− i sinθ cosθ √(g² + g'²) (I³ + Y) A_λ)] ψ
e Q
D_λψ = [∂_λ − i g/√2 (W_λ⁺ σ⁺ + W_λ⁻ σ⁻)
− i (g² + g'²)^½ Z_λ (I³ − s_w² Q)]
− ie Q A_λ
+ sinθ_w A_λ) I³ − ig'(− sinθ_w Z + cos θ_w A) Y
(g² + g'²)^½ sinθ



SU(2) × U(1)
Glashow - Salam - Weinberg
isospin
hypercharge
$(\partial_\mu - i g t^a A^a_\mu)\phi$
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
spin 1/2 of SU(2)
g
1/2 under U(1)
g'
Higgs field
4
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$
e_R
$\begin{pmatrix} u \\ d \end{pmatrix}_L$
u_R
d_R
$Y = -\frac{1}{2}$
$Y = -1$
$Y = \frac{1}{6}$
$Y = \frac{2}{3}$
$Y = -\frac{1}{3}$
$D_\mu \psi$
$+ \sin\theta_W A$

$SU(2) \times U(1)$ Glashow - Salam - Weinberg

isospin hypercharge

$(\partial_\mu - i g t^a A^a_\mu)\phi$

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ spin $\frac{1}{2}$ of $SU(2)$ g

$\frac{1}{2}$ under $U(1)$ g'

Higgs field 4

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ e_R $\begin{pmatrix} u \\ d \end{pmatrix}_L$ u_R d_R

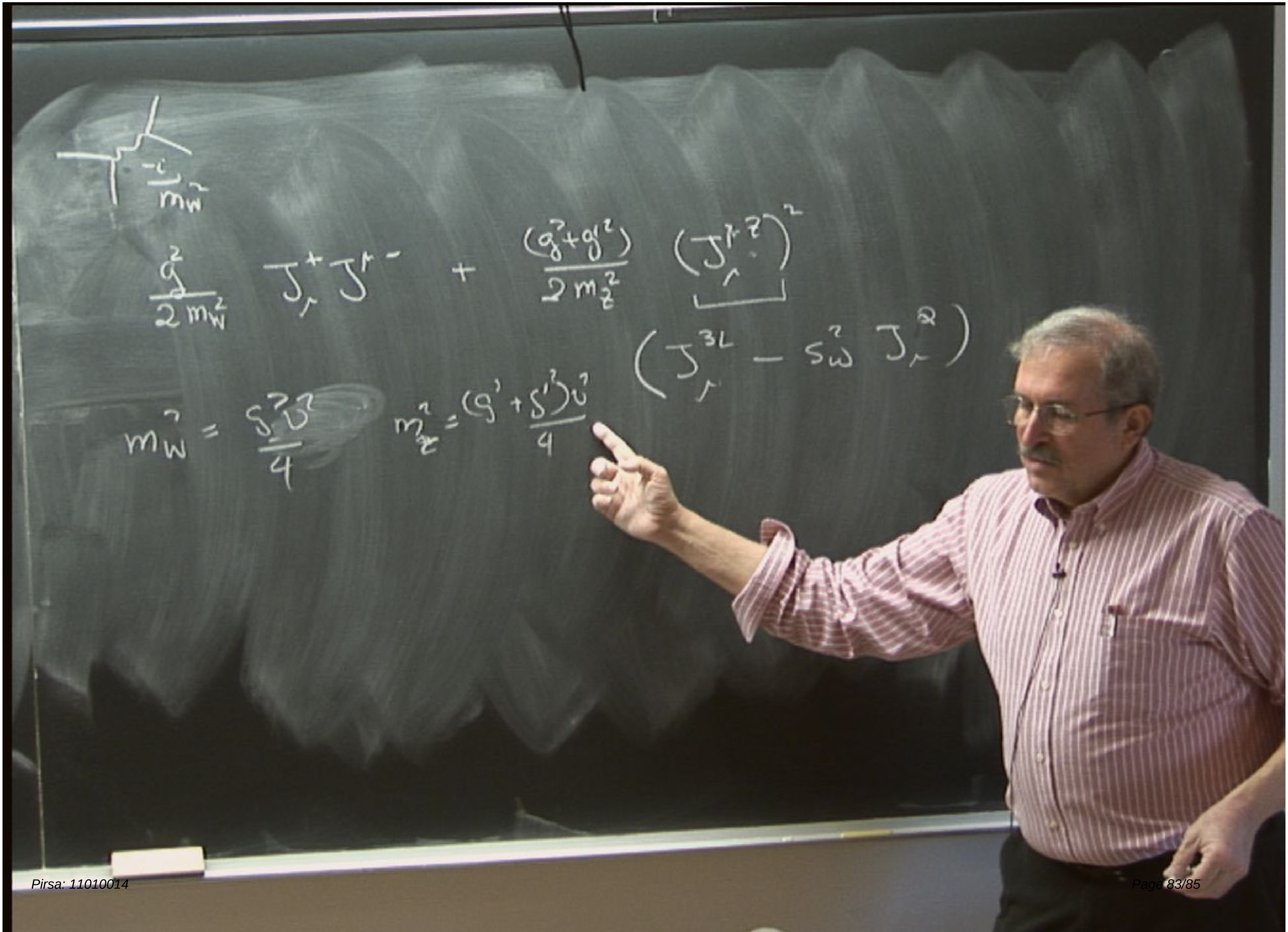
$Y = -\frac{1}{2}$ $Y = -1$ $Y = \frac{1}{6}$ $Y = \frac{2}{3}$ $Y = -\frac{1}{3}$

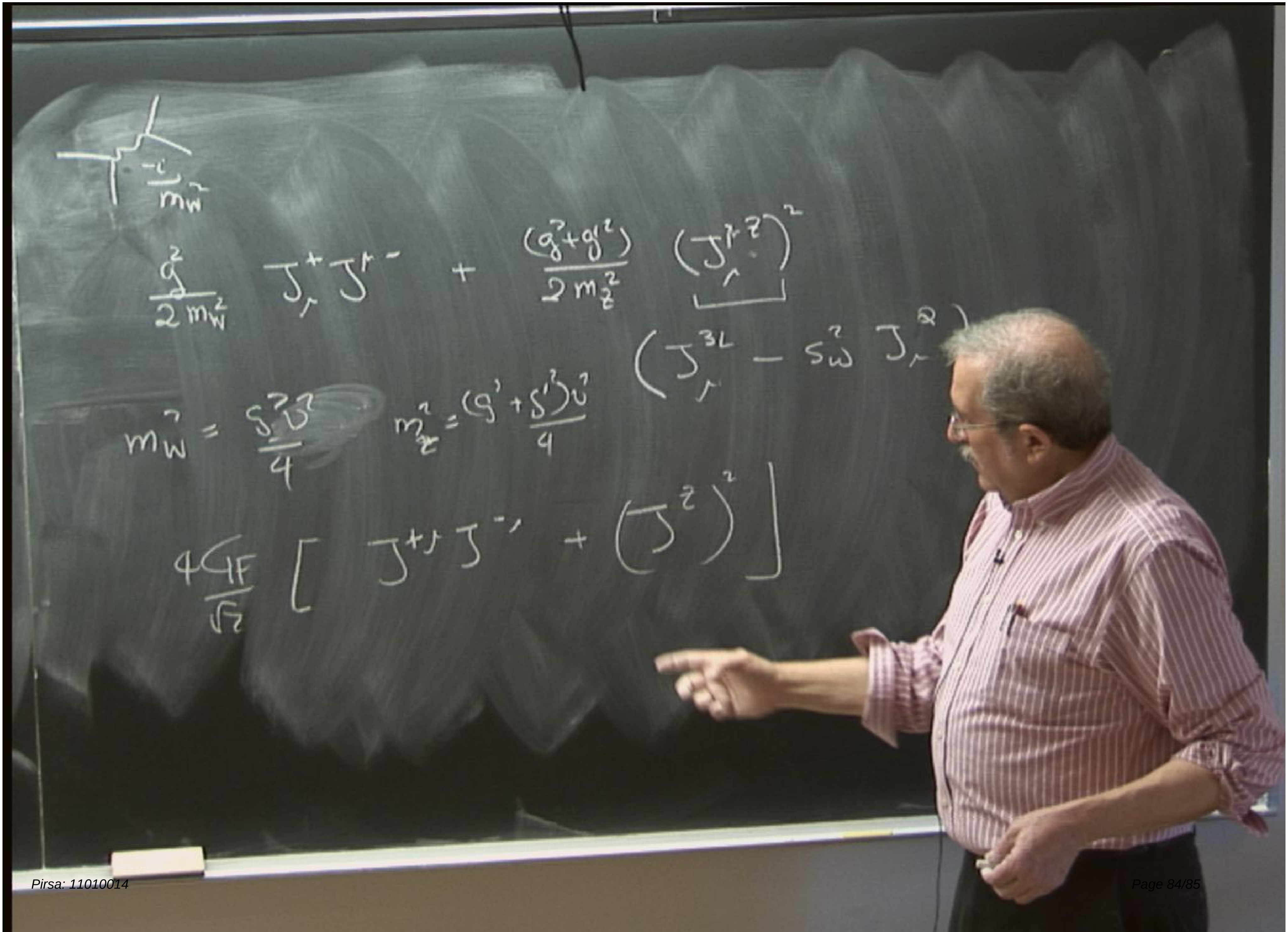
$$J^+_\mu = \bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L$$

$$J^{3Z}_\mu = \bar{\nu}_L \gamma^\mu \nu_L \tfrac{1}{2} + \bar{e}_L \gamma^\mu e_L \left(-\tfrac{1}{2} + s_W^2\right) + \bar{e}_R \gamma^\mu e_R \left(+ s_W^2\right)$$

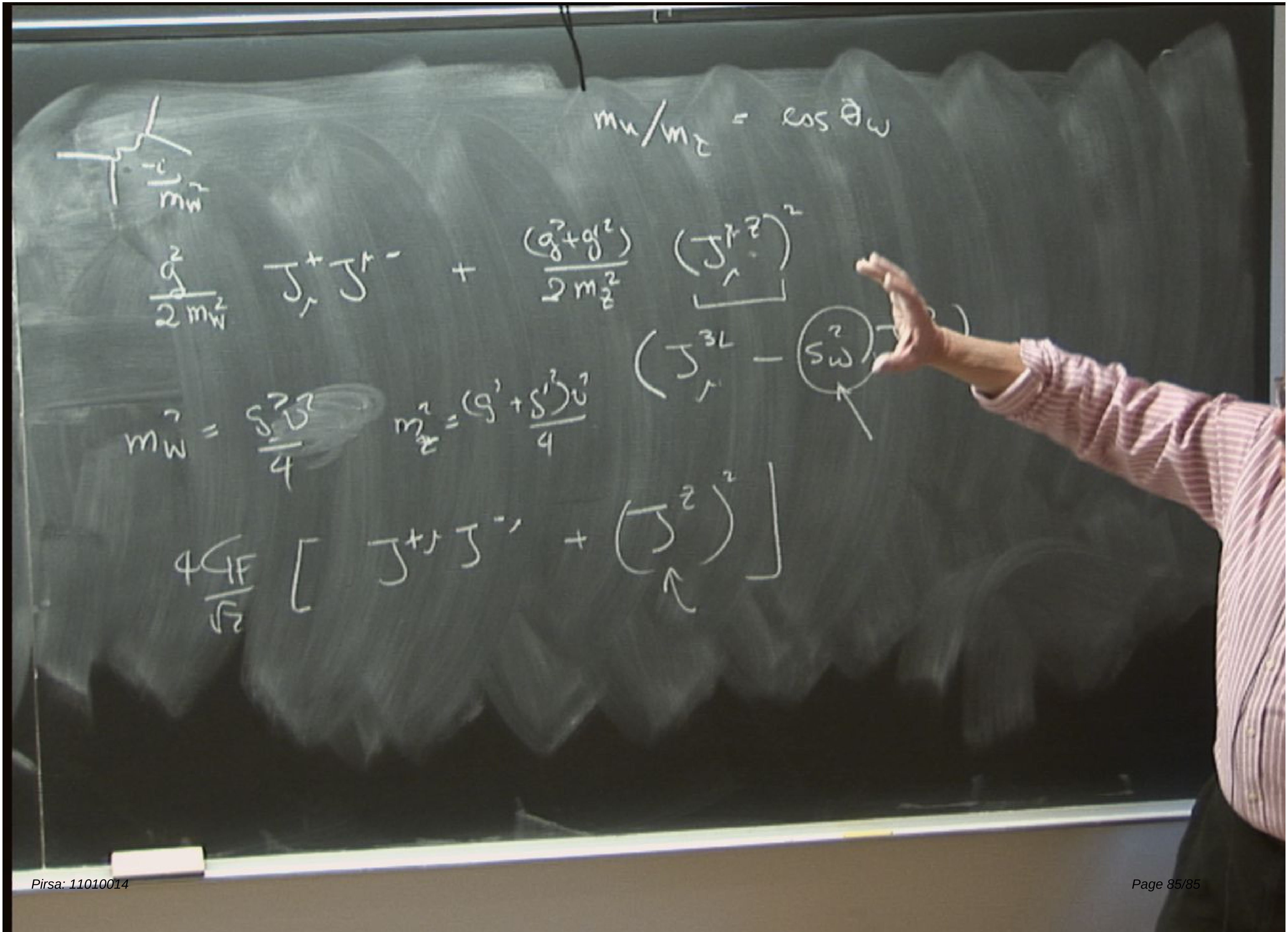
$+ \sin\theta_W A$

$$\frac{g^2}{2m_W^2} J_\mu^+ J^{\mu -} + \frac{(g^2+g'^2)}{2m_Z^2} J_\mu^{\mu Z}$$


-i / m_W^2
g^2 / 2 m_W^2 J_μ^+ J^μ- + (g^2+g'^2) / 2 m_Z^2 (J_μ^Z)^2
(J_μ^3L - s_W^2 J_μ^Q)
m_W^2 = g^2 v^2 / 4
m_Z^2 = (g^2 + g'^2) v^2 / 4



$\frac{-i}{m_W^2}$
$\frac{g^2}{2 m_W^2} J_\mu^+ J^{\mu -} + \frac{(g^2+g'^2)}{2 m_Z^2} (J^{\mu Z})^2$
$(J_\mu^{3L} - s_W^2 J_\mu^Q)$
$m_W^2 = \frac{g^2 v^2}{4}$
$m_Z^2 = \frac{(g^2+g'^2) v^2}{4}$
$\frac{4 G_F}{\sqrt{2}} [J^{+\mu} J^{-}_\mu + (J^Z)^2]$


$m_W / m_Z = \cos\theta_W$
$\frac{-i}{m_W^2}$
$\frac{g^2}{2m_W^2} J_\mu^+ J^{\mu -} + \frac{(g^2+g'^2)}{2m_Z^2} (J_\mu^Z)^2$
$(J_\mu^{3L} - s_W^2$
$m_W^2 = \frac{g^2 v^2}{4}$ $m_Z^2 = \frac{(g^2+g'^2)v^2}{4}$
$\frac{4G_F}{\sqrt{2}} [J^{+\mu} J^{-\mu} + (J^Z)^2]$