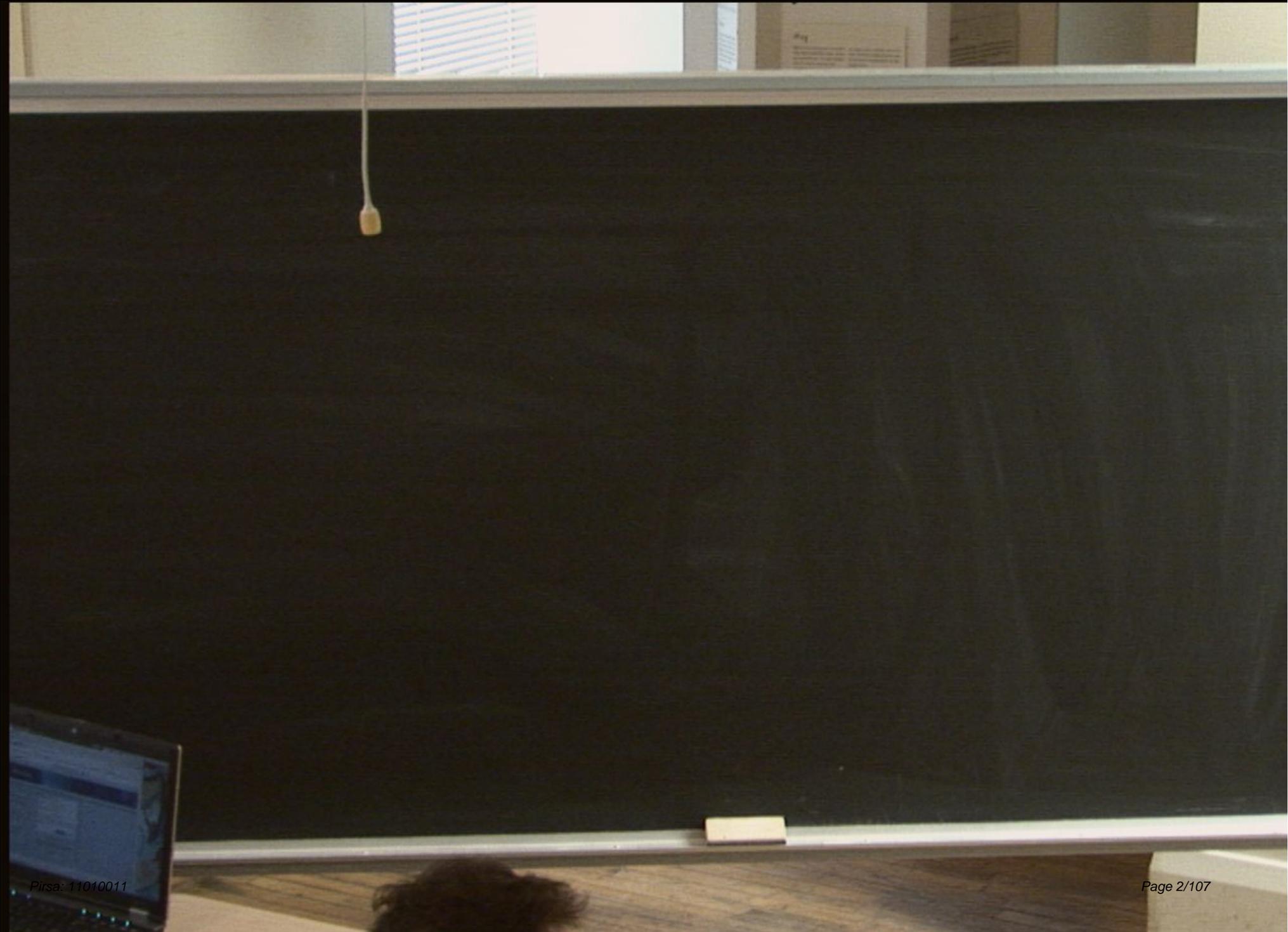


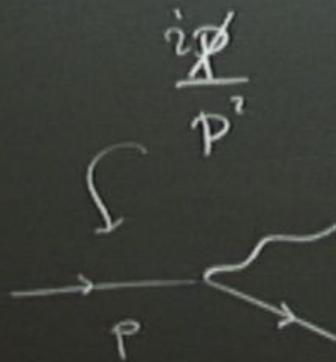
Title: Standard Model - Lecture 10

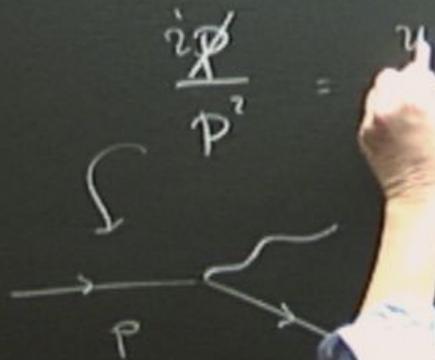
Date: Jan 14, 2011 09:00 AM

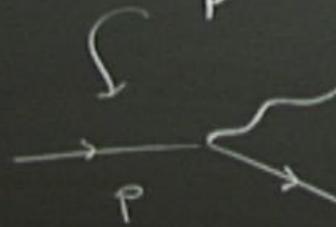
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Abstract:



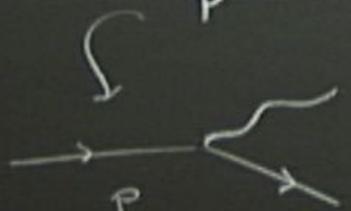




$$\frac{i\not{p}}{p^2} \hat{=} i \frac{u(p)\bar{u}(p)}{p^2}$$


\not{p} is on shell $p^2 = 0$

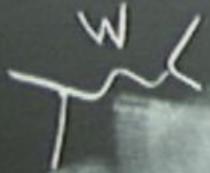
$$\not{p} = \sum_s$$

$$\frac{i\not{\partial}}{p^2} \hat{=} i \frac{u(p)\bar{u}(p)}{p^2}$$


$$(1, 0, 0, E - \mathcal{O}(k_1^2))$$

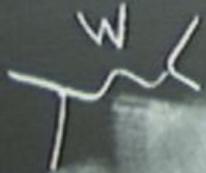
$\not{\partial}$ is on shell $p^2 = 0$

$$\not{\partial} = \sum_s u^{(s)}(p)\bar{u}^{(s)}(p)$$



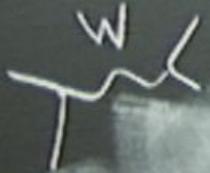
$$\frac{g^2}{q^2 - m_W^2} \gamma^\mu \gamma^\nu$$

$$-\frac{g^2}{m_W^2} \gamma^\mu \gamma^\nu$$



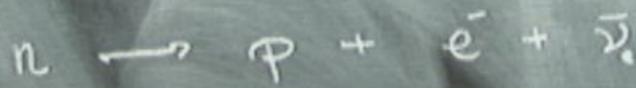
$$\frac{g^2}{q^2 - m_W^2} \bar{\psi} \psi$$

$$- \frac{g^2}{m_W^2} \bar{\psi} \psi$$



$$\frac{g^2}{q^2 - m_W^2} \bar{u}$$

$$- \frac{g^2}{m_W^2}$$





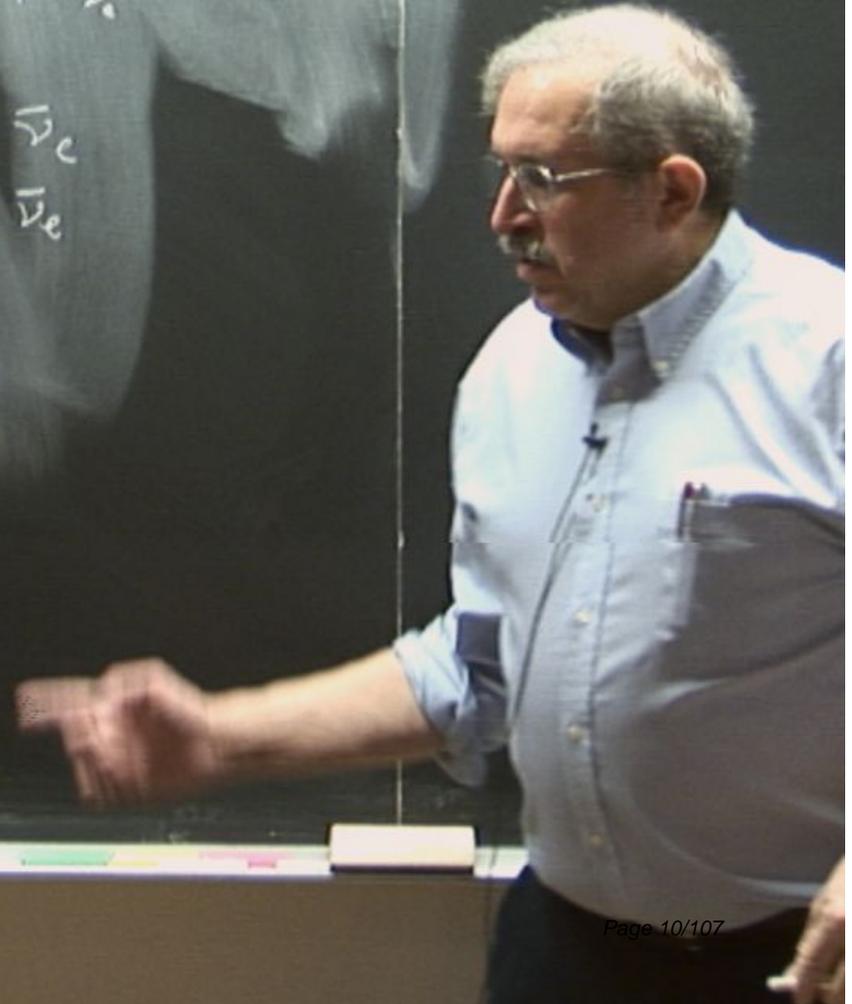
$$\frac{g}{g^2 - m_W^2} \approx$$

$$-\frac{g}{m_W^2}$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e$$

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$$



$$K^0 \rightarrow \pi^+ \pi^-$$

$$K^0 \rightarrow \pi^+ \pi^- \pi^0$$

$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \text{P} = -1 \quad \text{P} = +1 \cdot (-1) \\
 \text{S} = 0
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \text{P} = -1 \quad \text{P} = (-1)(-1) \\
 \text{L} = 0
 \end{array}$$



$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{} \\
 P = -1 \quad I = +1 \cdot (-1) \\
 S = 0
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{} \\
 P = -1 \quad I = (-1)(-1) \\
 \underbrace{} \\
 L = 0
 \end{array}$$

$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad P = +1 \cdot (-1) \\
 S = 0
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad P = (-1)(-1) \\
 \swarrow \quad \searrow \\
 L = 0
 \end{array}$$

Lec + Yang

$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad I = +1 \cdot (-1) \\
 S = 0
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad I = (-1)(-1) \\
 L = 0
 \end{array}$$

Lee + Yang



$$K^0 \rightarrow \pi^+ \pi^-$$

$P = -1$
 $S = 0$
 $P = +1 \cdot (-1)$

$$K^0 \rightarrow \pi^+ \pi^- \pi^0$$

$P = -1$
 $P = (-1)(-1)$
 $L = 0$

Lee + Yang \rightarrow "Weak interaction" violates parity

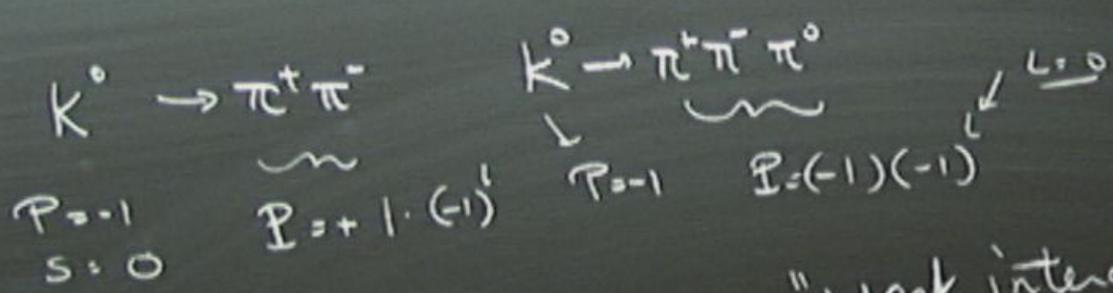
$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad S = 0 \\
 P = +1 \cdot (-1)
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad P = (-1)(-1) \\
 \swarrow \quad \searrow \\
 L = 0
 \end{array}$$

Lee + Yang \rightarrow "Weak interaction" violates parity
 Feynman + Gell-Mann \rightarrow Marshak Sudarshan

$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad S = 0 \quad P = +1 \cdot (-1)
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{\hspace{2cm}} \\
 P = -1 \quad P = (-1)(-1) \quad L = 0
 \end{array}$$

Lee + Yang \rightarrow "Weak interaction" violates parity
 Feynman + Gell-Mann
 Marshak Sudarshan

$$\delta \mathcal{L} = - \frac{4G_F}{\sqrt{2}} [$$



Lee + Yang \rightarrow "Weak interaction" violates parity
 Feynman + Gell-Mann
 Marshak Sudarshan

$$\delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}} [J_\mu^+ J^{\mu-}]$$

$$J_\mu^+ = [(j^{\mu 1} + i j^{\mu 2}) - (j^{\mu 5 1} + i j^{\mu 5 2})]$$

$$\begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \\
 \underbrace{\hspace{10em}} \\
 P = -1 \quad S = 0 \quad P = +1 \cdot (-1)
 \end{array}
 \qquad
 \begin{array}{l}
 K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{\hspace{10em}} \\
 P = -1 \quad P = (-1)(-1) \quad L = 0
 \end{array}$$

Lee + Yang \rightarrow "Weak interaction" violates parity
 Feynman + Gell-Mann \rightarrow Marshak Sudarshan

$$\delta \mathcal{L} = - \frac{4G_F}{\sqrt{2}} [J_\mu^+ J^{\mu -}]$$

$$J_\mu^+ = [(j^{\mu 1} + i j^{\mu 2}) - (j^{\mu 5 1} + i j^{\mu 5 2})]$$

$V \quad - \quad A$

$$J^{\mu+} = \bar{u}_L \gamma^{\mu} d_L$$

$$J^{\mu+} = (\bar{u}_L \gamma^{\mu} d_L + \bar{u}_L \gamma^{\mu} e_L)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_{eL} \gamma^{\mu} e_L)$$

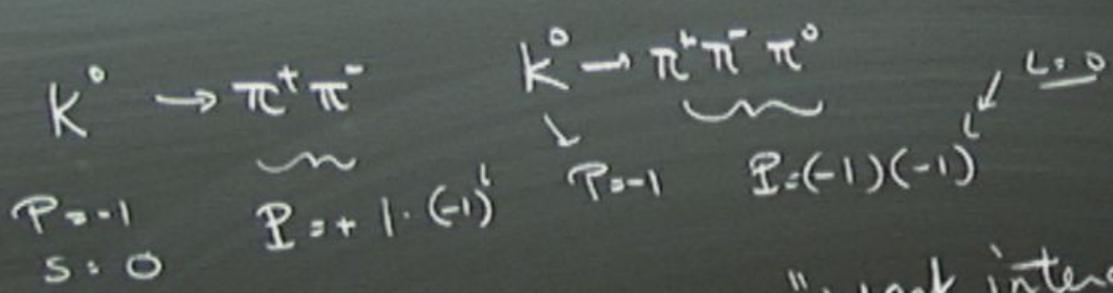
$$\begin{array}{ccc}
 K^0 \rightarrow \pi^+ \pi^- & & K^0 \rightarrow \pi^+ \pi^- \pi^0 \\
 \underbrace{}_{P=-1, S=0} & & \underbrace{}_{P=-1, L=0} \\
 & & \underbrace{}_{P=+1 \cdot (-1)} \quad \underbrace{}_{P=(-1)(-1)}
 \end{array}$$

Lee + Yang \rightarrow "Weak interaction" violates parity
 Feynman + Gell-Mann \rightarrow Marshak Sudarshan

$$\delta \mathcal{L} = - \frac{G_F}{\sqrt{2}} [J_\mu^+ J^{\mu -}]$$

$$J_\mu^+ = [(j^{\mu 1} + i j^{\mu 2}) - (j^{\mu 51} + i j^{\mu 52})]$$

$V \quad - \quad A$



Lee + Yang \rightarrow "Weak interaction" violates parity
 Marshak Sudarshan.

Feynman + Gell-Mann

$$\delta \mathcal{L} = - \frac{G_F}{\sqrt{2}} [J_\mu^+ J^{\mu-}] \quad G = 1.17 \times 10^{-5} / \alpha \bar{v}^2$$

$$J_\mu^+ = [(j^{\mu 1} + i j^{\mu 2}) - (j^{\mu 5 1} + i j^{\mu 5 2})]$$

V — Axial Vector

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

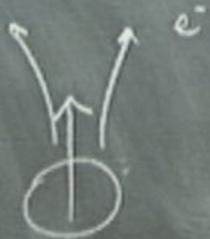
$$\frac{g}{\hbar} = -\frac{1}{2} \cdot \left(\frac{\sqrt{e}}{c}\right)$$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\frac{g}{\hbar} = -\frac{1}{2} \cdot \left(\frac{\sqrt{e}}{c}\right)$$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_{eL} \gamma^{\mu} e_L)$$

$$\frac{g}{2} = -\frac{1}{2} \left(\frac{v_e}{c} \right)$$

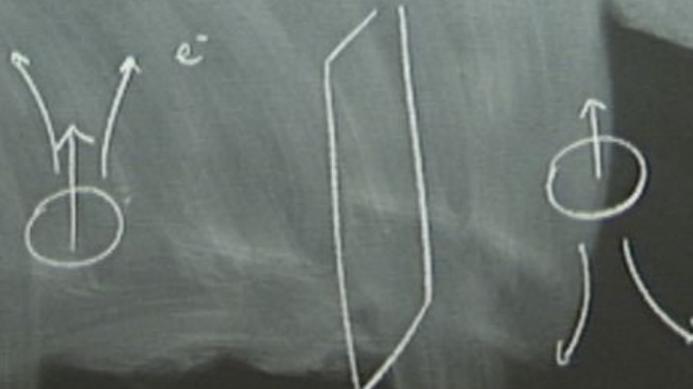


K^0
 $P = -1$
 $S = 0$

F_{e-}
 δ

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_{eL} \gamma^{\mu} e_L)$$

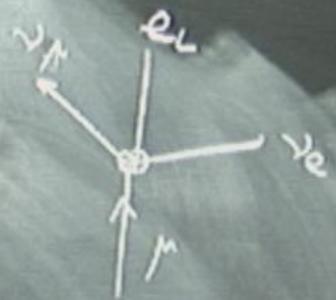
$$\vec{h} = -\frac{1}{2} \left(\frac{v_e}{c} \right)$$



K^0
 $P = -1$
 $S = 0$

$F = 1$
 $S = 0$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_{eL} \gamma^{\mu} e_L)$$



$$M =$$

$$\frac{4L G_F}{\sqrt{2}}$$



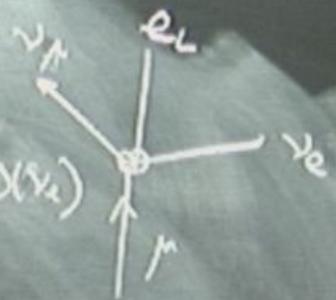
$$K^0$$

$$P = -1$$

$$S = 0$$

Fe
8

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

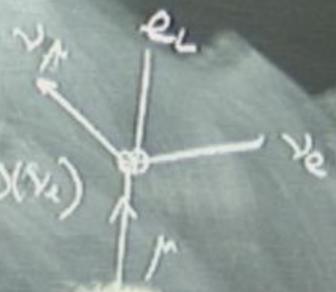


$M =$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_\mu) \gamma^\mu u_L(\mu) \bar{u}_L(e) \gamma_\mu u_L(\nu_e)$$

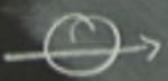
K^0
 $P = -1$
 $S = 0$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$$M =$$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_\mu) \gamma^\mu u_L(\mu) \bar{\nu}_L(e) \gamma_\mu \nu_L(e)$$



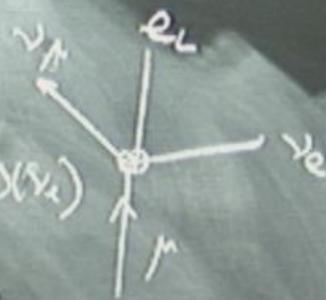
$\hat{3}$

$$u = \sqrt{\frac{2}{3}}$$

K^0
 $P = -1$
 $S = 0$

Fe
 δ

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$M =$

$$\frac{4L C_{IF}}{\sqrt{2}}$$

$$\bar{u}_L(\nu_\mu) \gamma^\mu u(\mu) \bar{u}_L(e) \gamma^\mu u(\nu_e)$$



$\hat{3}$

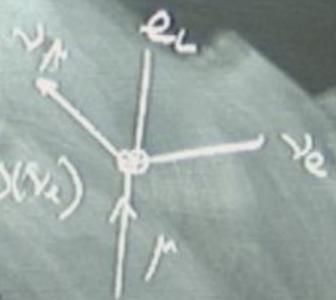
$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \hat{3}_L$$

K^0
 $P = -1$
 $S = 0$

Fe

δ

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$M =$

$$\frac{4iG_F}{\sqrt{2}}$$

$$\bar{u}_L(\nu_\mu) \gamma^\mu u_L(\mu) \bar{\nu}_{eL}(e) \gamma_\mu \nu_e$$



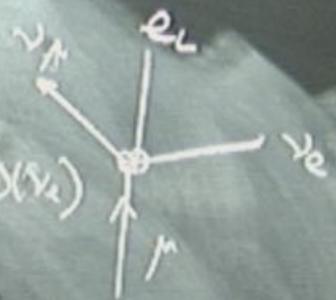
$\hat{3}$

$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

K^0
 $P = -1$
 $S = 0$

Fee
 δ

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$M =$



$$\frac{4iG_F}{\sqrt{2}}$$

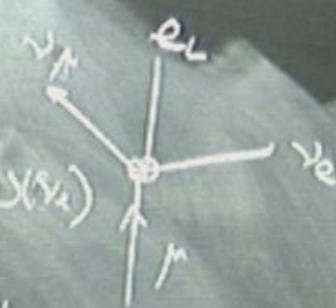
$$\bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{\nu}_{eL} \gamma_\mu u_L(\nu_e)$$

$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

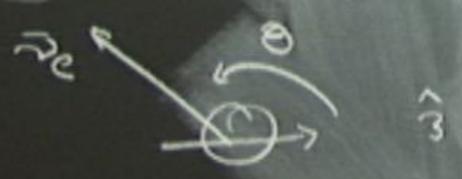
K^0
 $P = -1$
 $S = 0$

Fe
 δ

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$M_L =$



$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{\nu}_{eL}(\nu_e) \gamma_\mu u_L(\nu_e)$$

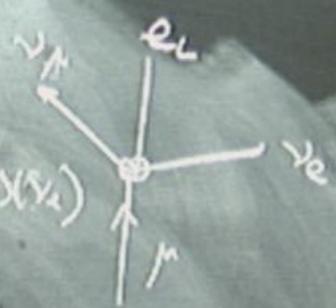
$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

$$v = \sqrt{2E_{\nu e}} \begin{pmatrix} \dots \\ \dots \\ \dots \\ \dots \end{pmatrix}$$

K^0
 $P = -1$
 $S = 0$

$F_{e\nu}$
 δ

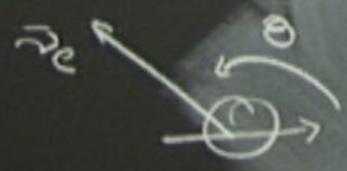
$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$K^0 \rightarrow$
 $P = -1$
 $S = 0$

$iM =$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_e) \bar{\nu}_e \gamma^\mu u(\mu) \bar{u}_L(e) \gamma^\mu u(e)$$



$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

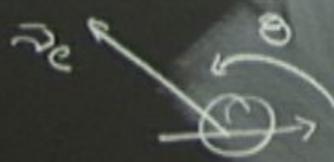
$$v = \sqrt{2F_{ye}} \begin{pmatrix} \sin \theta_W \\ \cos \theta_W \end{pmatrix}$$

$$(\bar{\sigma}^\mu)_{\alpha\beta} (\bar{\sigma}^\mu)_{\gamma\delta} = \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

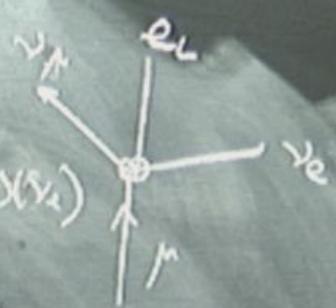
Feyn
 $\delta\mathcal{L}$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$iM =$



$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_\mu) \gamma^\mu u_L(\mu) \bar{u}_L(e^-) \gamma^\mu u_L(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \nu = \sqrt{2E_{\nu e}} \begin{pmatrix} \sin \theta_L \\ \cos \theta_L \end{pmatrix}$$

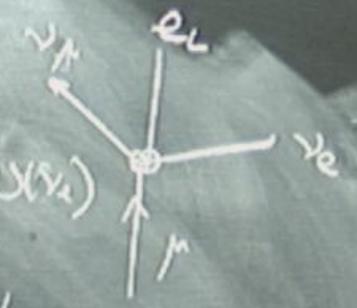
$$(\bar{\sigma}^\mu)_{\alpha\beta} (\sigma^\mu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

$$= \frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_\mu) \epsilon_{\alpha\delta} u_L(e^-) u_L(\mu) \epsilon_{\beta\gamma} u_L(\nu_e)$$

$K^0 \rightarrow$
 $P = 1$
 $S = 0$

Feyn
 $\delta \mathcal{L}$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$K^0 \rightarrow$
 $P = -1$
 $S = 0$

$iM =$

$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_e) \not{\epsilon}^+ u_L(\mu) \bar{u}_L(e) \not{\sigma} u_L(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \nu = \sqrt{2E_{\nu e}} \begin{pmatrix} \sin \theta_L \\ 0 \\ \cos \theta_L \end{pmatrix}$$

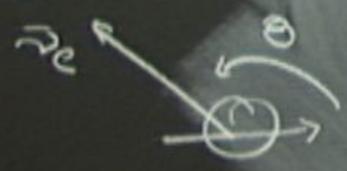
$$(\bar{\sigma}^+)_{\alpha\beta} (\bar{\sigma}^+)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

$$= \frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_e) \epsilon_{\alpha\delta} u_L(e) \underbrace{u_L(\mu) \epsilon_{\beta\gamma} u_L(\nu_e)}_{\text{}} \not{\sigma}$$

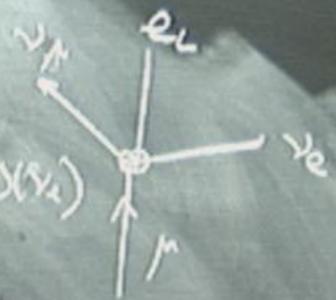
Feyn
 $\delta \mathcal{L}$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$iM =$



$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_e) \not{\epsilon}^+ u_L(\mu) \bar{u}_L(e) \not{\sigma}_\mu u_L(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \nu = \sqrt{2E_{\nu e}} \begin{pmatrix} \sin \theta_L \\ \cos \theta_L \end{pmatrix}$$

$$(\bar{\sigma}^\mu)_{\alpha\beta} (\sigma^\mu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

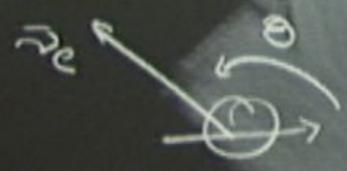
$$= \frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_e) \epsilon_{\alpha\delta} u_L(e) \underbrace{u_L(\mu) \epsilon_{\beta\gamma} u_L(\nu_e)}_{\sqrt{2E_{\nu e}} \sin \theta_L}$$

$K^0 \rightarrow$
 $P = -1$
 $S = 0$

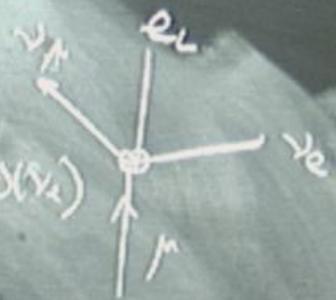
Feyn
 $\delta \mathcal{L}$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$iM =$



$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_\mu) \gamma^\mu u_L(\mu) \bar{u}_L(\bar{e}) \gamma^\mu u_L(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \nu = \sqrt{2E_\nu} \begin{pmatrix} \sin \theta/2 \\ 0 \\ \cos \theta/2 \end{pmatrix}$$

$$(\bar{\sigma}^\mu)_{\alpha\beta} (\sigma^\mu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

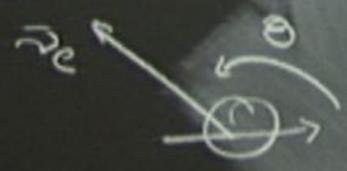
$$= \frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_\mu) \epsilon_{\alpha\delta} u_L(\bar{e}) u_L(\mu) \epsilon_{\beta\gamma} u_L(\nu_e) \sqrt{2E_\nu} m \sin \theta/2$$

$K^0 \rightarrow$
 $P = 1$
 $S = 0$

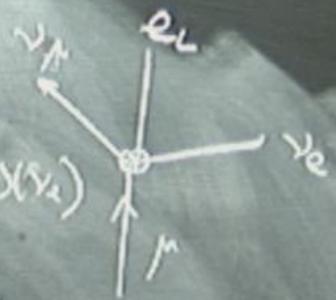
Feyn
 δL

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$iM =$



$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_\mu) \bar{e}^+ u(\mu) \bar{u}_L(e^-) \sigma_{\mu\nu} u(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \nu = \sqrt{2E_{\nu e}} \begin{pmatrix} \sin \theta_L \\ \cos \theta_L \end{pmatrix}$$

$$(\bar{\sigma}^\mu)_{\alpha\beta} (\bar{\sigma}^\nu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

$$= \frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_\mu) \epsilon_{\alpha\delta} u_\delta(e^-) \bar{u}_L(\mu) \epsilon_{\beta\gamma} u(\nu_e) \sqrt{2E_{\nu e}} \cos \theta_L$$

$$K^0 \rightarrow$$

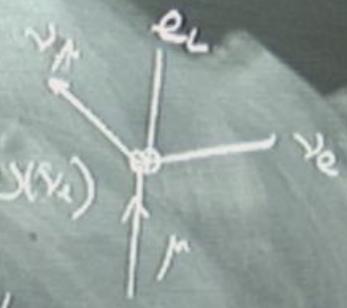
$$P = -1$$

$$S = 0$$

F_e

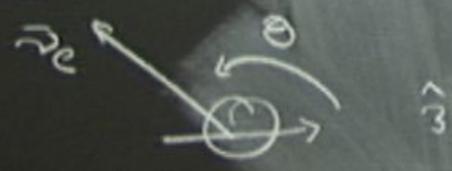
δ_c

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$iM =$

$$\frac{4i C_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{u}_L(e^-) \gamma_\mu u_L(\nu_e)$$



$$u = \sqrt{m} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \nu = \sqrt{2E_\nu} \begin{pmatrix} \sin \theta/2 \\ 0 \\ \cos \theta/2 \end{pmatrix}$$

$$(\bar{\sigma}^\mu)_{\alpha\beta} (\sigma^\mu)_{\gamma\delta} = 2 \epsilon_{\alpha\gamma} \epsilon_{\beta\delta}$$

$$\frac{8i}{\sqrt{2}} C_F \bar{u}_L(\nu_e) \epsilon_{\alpha\delta} u_L(e^-) \bar{u}_L(\mu) \epsilon_{\beta\gamma} u_L(\nu_e)$$

$$\sqrt{2E_\nu} m \cos^2 \theta/2$$

$\nu_e \longleftrightarrow e^-$

$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$
$$= (4E_e E_\nu)^{\frac{1}{2}} = (2p_e \cdot p_\nu)^{\frac{1}{2}}$$

small
cross



$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^{\frac{1}{2}} = (2p_e \cdot p_\nu)^{\frac{1}{2}}$$

$$= p_e$$

$$|M|^2 = 2E_\nu m_x \frac{1}{2}(1 + \cos\theta)$$

$\left(\begin{matrix} \sin\theta \\ \cos\theta \end{matrix} \right)$





$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^{\frac{1}{2}} = (2p_e \cdot p_\nu)^{\frac{1}{2}}$$

$$= \left[(p_e + p_\nu)^2 \right]^{\frac{1}{2}} = \left[(p_+ - p_-)^2 \right]^{\frac{1}{2}}$$

$$= m_\mu^2 = 2E_e$$

$$|m|^2 = 2E_\nu m_\mu^2 \frac{1}{2}(1 + \cos\theta)$$

$\left(\begin{matrix} \sin\theta \\ \cos\theta \end{matrix} \right)$

$\nu_e \longleftrightarrow e^-$

$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^{\frac{1}{2}} = (2p_e \cdot p_\nu)^{\frac{1}{2}} \\ = \left[(p_e + p_\nu)^2 \right]^{\frac{1}{2}} = \left[(p_e - p_\nu)^2 \right]^{\frac{1}{2}} \\ = m_\mu^2 - 2p_e \cdot p_\nu$$

$$|M|^2 = 2E_e m_\mu^2 \frac{1}{2}(1 + \cos\theta)$$

$\left(\begin{matrix} \sin\theta \\ \cos\theta \end{matrix} \right)$



$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^k = (2P_e \cdot P_\nu)^k$$

$$= [(P_e + P_\nu)^2]^{1/2} = [(P_e - P_\nu)^2]^{1/2}$$

$$|M|^2 = 2E_e m_\nu \frac{1}{2}(1 + \cos\theta)$$

$$X_U = \frac{2P_e \cdot P_e}{m^2} \quad X_V = \frac{2P_\nu \cdot P_\nu}{m^2}$$

$\left(\frac{\sin^2 \theta}{\cos^2 \theta} \right)$



$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^k = (2P_e \cdot P_\nu)^k$$

$$= [(P_e + P_\nu)^2]^{k/2} = [(P_e - P_\nu)^2]^{k/2}$$

$$= m_\mu^2 - 2P_e \cdot P_\nu$$

$$= m_\mu^2 (1 - x_{\bar{\nu}})$$

$$|M|^2 = \underbrace{(2E_e)}_{x_{\bar{\nu}}} m_\mu^2 \frac{1}{2} (1 + \cos\theta) m_\mu^2 (1 - x_{\bar{\nu}})$$

$$x_e = \frac{2P_e \cdot P_e}{m^2} \quad x_{\bar{\nu}} = \frac{2P_{\bar{\nu}} \cdot P_\mu}{m^2}$$

$$I = \frac{1}{2m_\mu} \frac{m_\mu^2}{128\pi^3} \int dx_e dx_{\bar{\nu}} \quad x_{\bar{\nu}} (1 - x_{\bar{\nu}})$$

GS GF



$$u_e = \sqrt{2E_e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad u_\nu = \sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$= (4E_e E_\nu)^{\frac{1}{2}} = (2P_e \cdot P_\nu)^{\frac{1}{2}}$$

$$= \left[(P_e + P_\nu)^2 \right]^{\frac{1}{2}} - \left[(P_e - P_\nu)^2 \right]^{\frac{1}{2}}$$

$$= m_\mu^2 - 2P_e \cdot P_\nu$$

$$= m_\mu^2 (1 - x_\nu)$$

$$|M|^2 = \underbrace{2E_e}_{x_\nu} m_\mu^2 \frac{1}{2} (1 + \cos\theta) m_\mu^2 (1 - x_\nu)$$

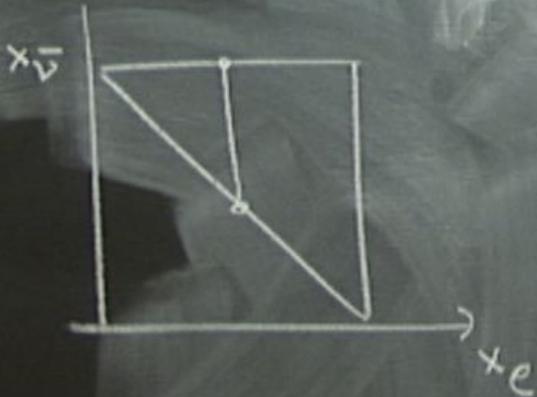
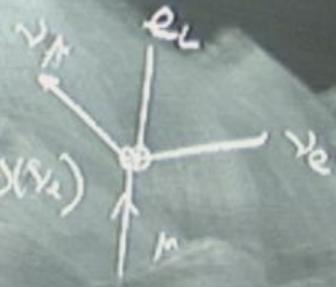
$$x_e = \frac{2P_e \cdot P_e}{m^2} \quad x_\nu = \frac{2P_\nu \cdot P_\mu}{m^2}$$

$$I = \frac{1}{2m_\mu} \frac{m_\mu^2}{128\pi^3} \int dx_e dx_\nu \quad x_\nu (1 - x_\nu) \quad \frac{64G_F^2 m_\mu^3}{2}$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$iM =$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{u}_L(e) \gamma_\mu u_L(\nu_e)$$

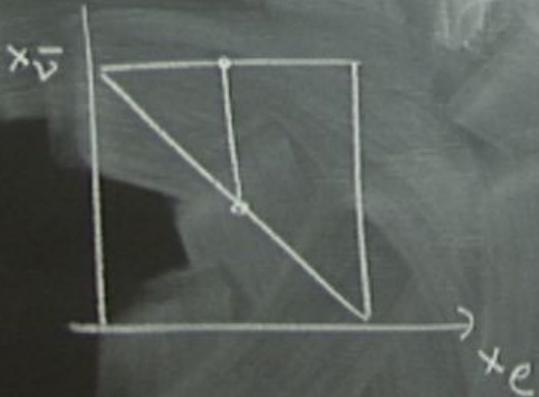
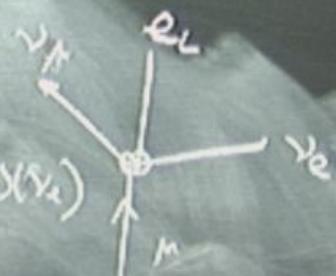


$$\int_0^1 dx_e \int_{1-x_e}^1 dx_nu \quad x_nu(1-x_nu)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$iM =$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{u}_L(e) \gamma_\mu u_L(\nu_e)$$



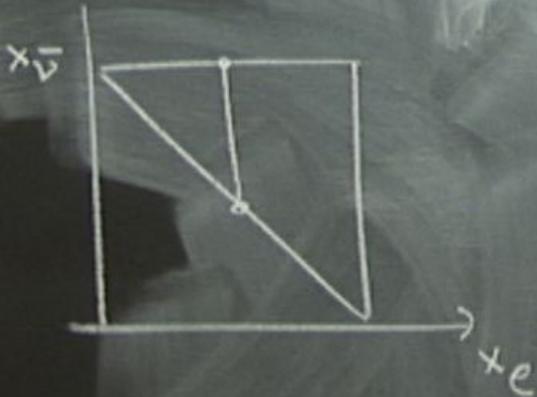
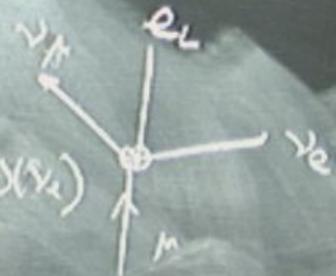
$$\int_0^1 dx_e \int_{1-x_e}^1 dx_\nu x_\nu (1-x_\nu)$$

$$\left(\frac{x_e^2}{2} - \frac{x_e^3}{3} \right)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$iM =$

$$\frac{4iG_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{u}_L(e) \gamma_\mu u_L(\nu_e)$$



$$\int_0^1 dx_e \int_{1-x_e}^1 dx_nu x_nu(1-x_nu)$$

$$\int_0^1 dx_e \left(\frac{x_e^2}{2} - \frac{x_e^3}{3} \right)$$

$$I = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$G_F = 1.166$$

$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= [(P_e + P_\nu)^2]^{\frac{1}{2}} = [(P_e - P_\nu)^2]^{\frac{1}{2}} \\ &= (m_\mu^2 - 2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= (m_\mu^2 (1 - x_\nu)) \end{aligned}$$

$$\frac{G_F m_\mu^3}{2}$$

$$I = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$G_F = 1.16637 \pm 0.00001 \times 10^{-5} / \text{GeV}^2$$



$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= [(P_e + P_\nu)^2]^{\frac{1}{2}} = [(P_\mu - P_\nu)^2]^{\frac{1}{2}} \\ &= (m_\mu^2 - 2E_\nu^2) \\ &= (m_\mu^2 (1 - x)) \end{aligned}$$

$$\int dx_e dx_{\bar{\nu}} x_{\bar{\nu}} (1 - x_{\bar{\nu}})$$

$$\frac{64 G_F^2 m_\mu^5}{2}$$



$$I = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$G_F = 1.16637 \times 10^{-5} / \text{GeV}^2$$

$$\frac{dI}{dx_e}$$



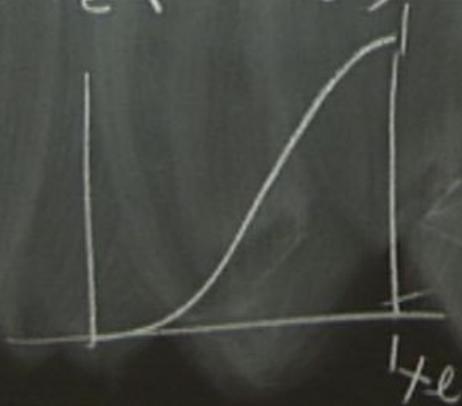
$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu) \\ &= [(P_e + P_\nu)^2] = [(P_\mu - P_\nu)] \\ &= (m_\mu^2 - 2P_\mu \cdot P_\nu) \\ &= (m^2 (1 - x_e)) \end{aligned}$$

$$I \sim \frac{G_F^2 m_\mu^5}{192\pi^3}$$

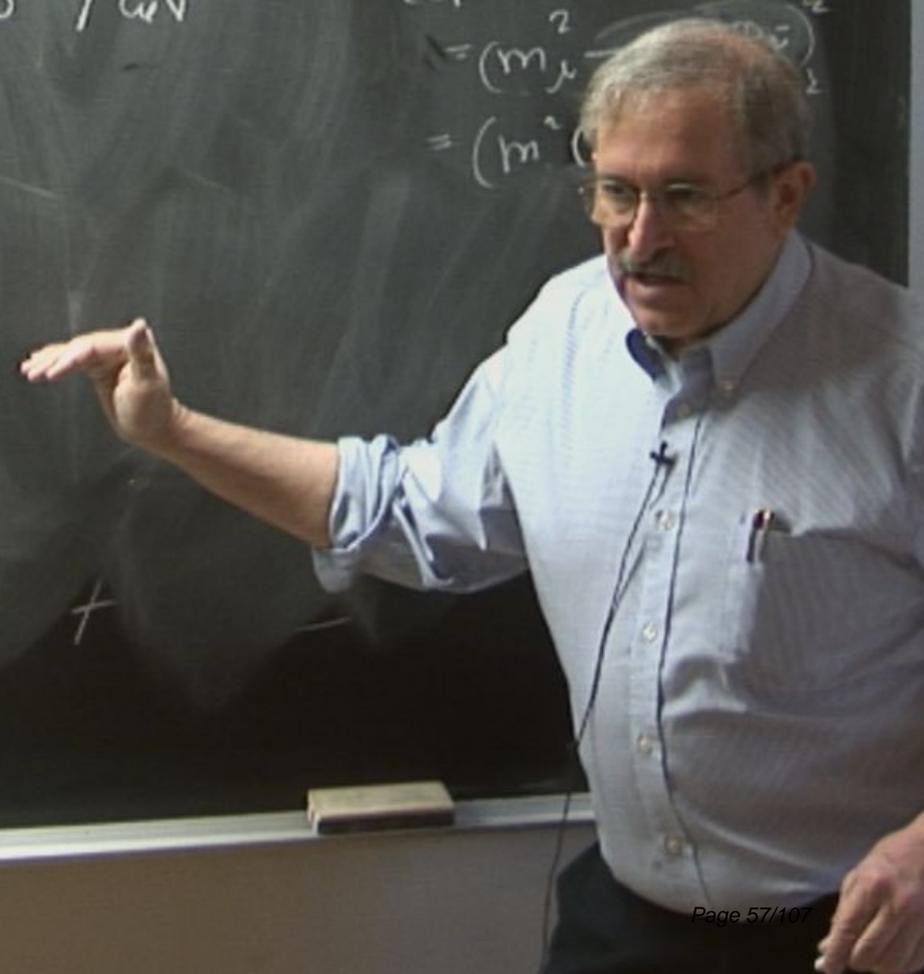
$$G_F = 1.16637 \pm 0.00001 \times 10^{-5} / \text{GeV}^2$$

$$\frac{dI}{dx_e} \sim x_e^2 \left(1 - \frac{2x_e}{3}\right)$$



$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

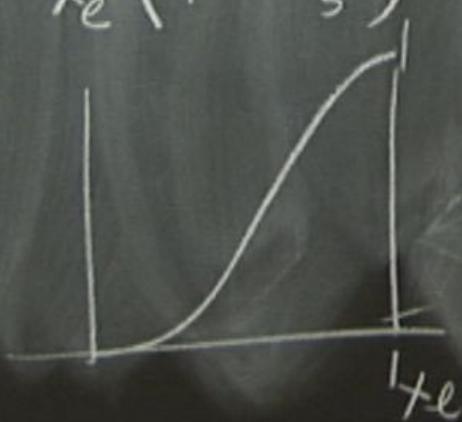
$$\begin{aligned} & (2P_e \cdot P_\nu) \\ &= [(P_e + P_\nu)^2] - [(P_e - P_\nu)^2] \\ &= (m_e^2 - m_\nu^2) - (m_e^2 - m_\nu^2) \end{aligned}$$



$$I \sim \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$G_F = 1.16637 \pm 0.00001 \times 10^{-5} / \text{GeV}^2$$

$$\frac{dI}{dx_e} \sim x_e^2 \left(1 - \frac{2x_e}{3}\right)$$



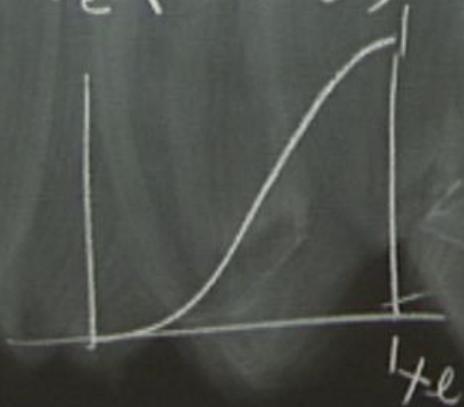
$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^\frac{1}{2} \\ &= [(P_e + P_\nu)^2]^\frac{1}{2} = [(P_\mu - P_\nu)^2]^\frac{1}{2} \\ &= (m_\mu^2 - 2P_\mu \cdot P_\nu)^\frac{1}{2} \\ &= (m^2 (1 - x_\nu))^\frac{1}{2} \end{aligned}$$

$$I \sim \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$G_F = 1.16637 \pm 0.00001 \times 10^{-5} / \text{GeV}^2$$

$$\frac{d\Gamma}{dx_e} \sim x_e^2 \left(1 - \frac{2x_e}{3}\right)$$

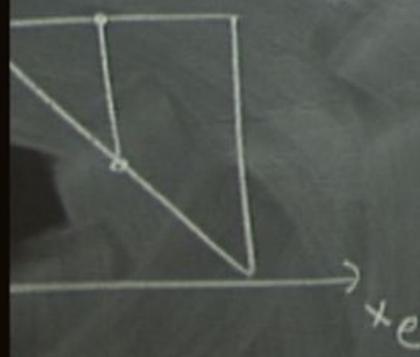
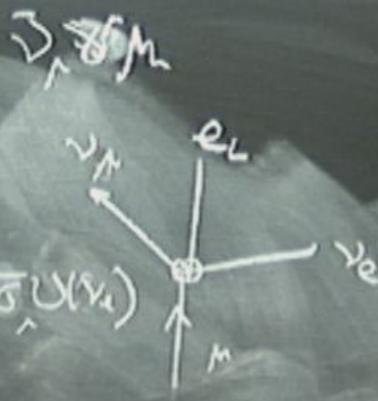


$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= [(P_e + P_\nu)^2]^{\frac{1}{2}} = [(P_\mu - P_{\nu_e})^2]^{\frac{1}{2}} \\ &= (m_\mu^2 - 2P_\mu \cdot P_{\nu_e})^{\frac{1}{2}} \\ &= (m_\mu^2 (1 - x_\nu))^{\frac{1}{2}} \end{aligned}$$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\frac{4i G_F}{\sqrt{2}} \bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) \bar{u}_L(e) \gamma^\mu u_L(\nu_e)$$

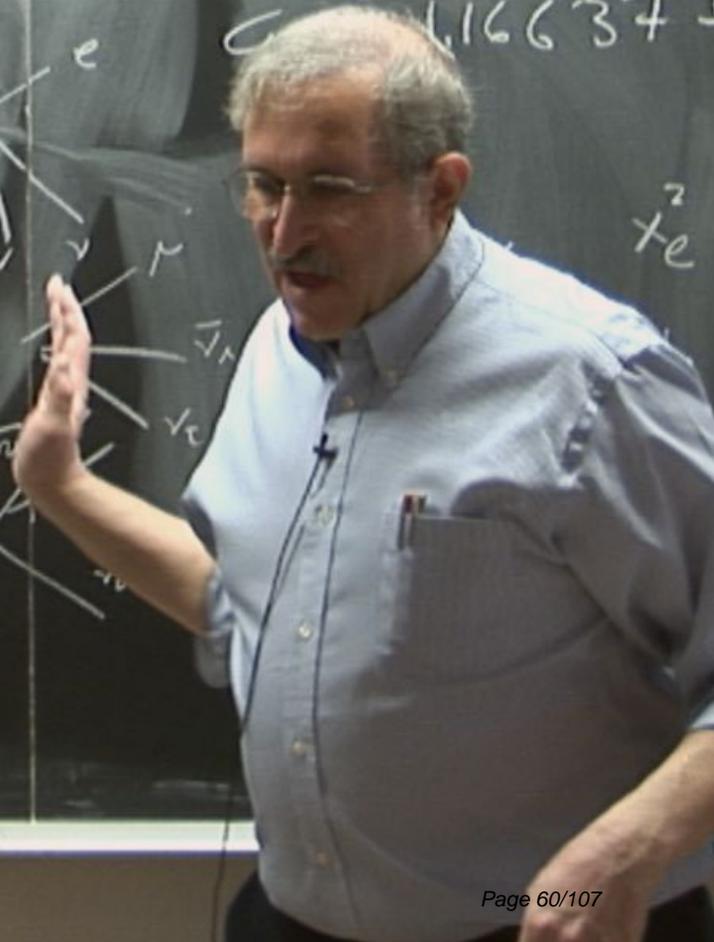
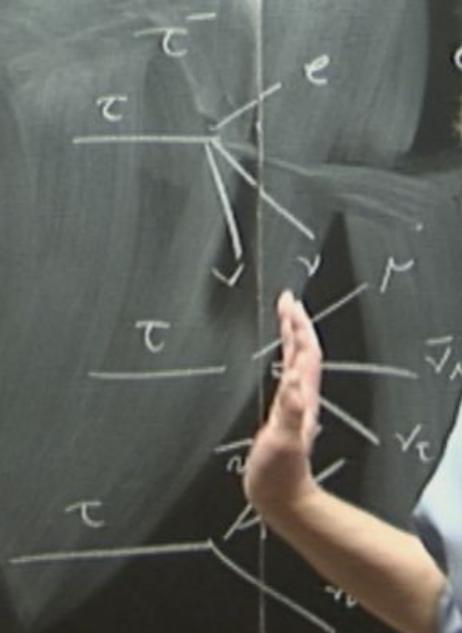


$$\int_0^1 dx_e \int_{1-x_e}^1 dv_{\bar{\nu}} x_{\bar{\nu}}(1-x_{\bar{\nu}})$$

$$\int_0^1 dx_e \left(\frac{x_e^2}{2} - \frac{x_e^3}{3} \right)$$

$$I = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

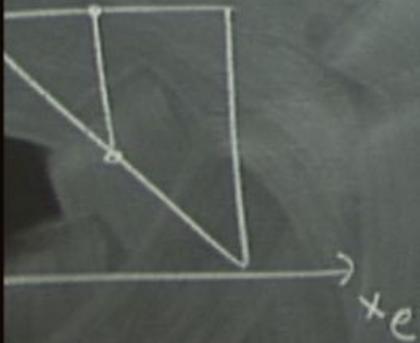
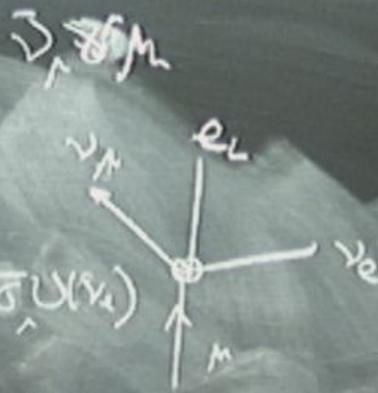
$G = 1.16637 \dots$



$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

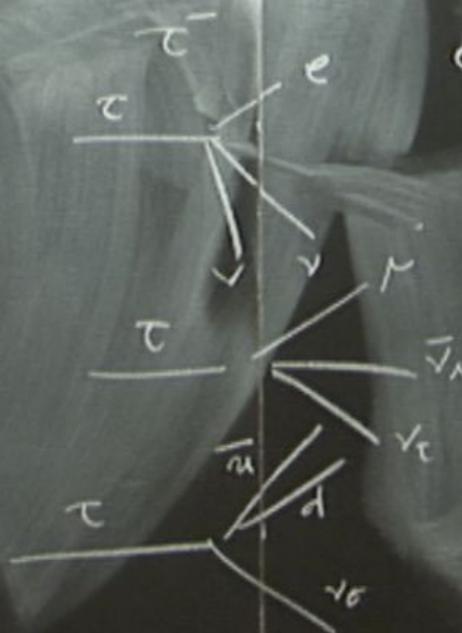
$$\frac{4 G_F}{\sqrt{2}}$$

$$\bar{u}_L(\nu_e) \gamma^\mu u_L(\mu) + \bar{u}_L(e) \gamma^\mu u_L(\nu_e)$$



$$\int_0^1 dx_e \int_{1-x_e}^1 dv_{\bar{\nu}} x_{\bar{\nu}}(1-x_{\bar{\nu}})$$

$$\int_0^1 dx_e \left(\frac{x_e^2}{2} - \frac{x_e^3}{3} \right)$$

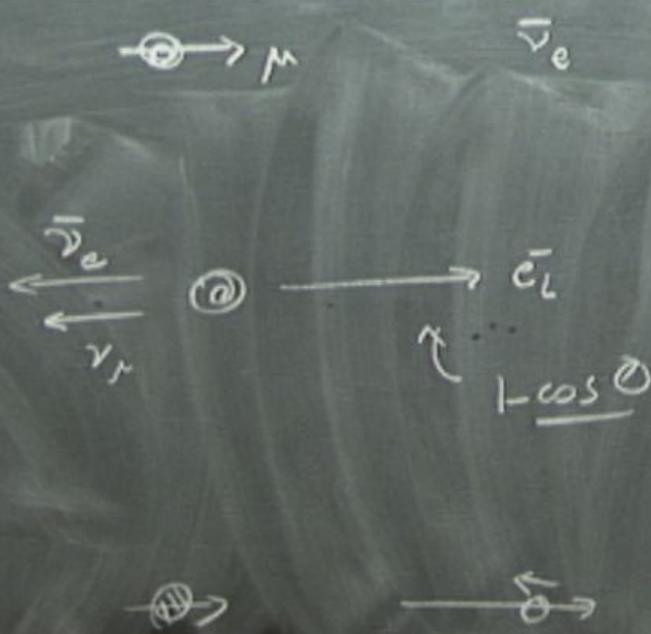


$$I = \frac{G_F^2 m_p^5}{192 \pi^3}$$

$$G_F = 1.16637 \times 10^{-5}$$

$$\frac{dI}{dx_e} \sim x_e^2$$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$



$$\frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$1.16637 \dots$$

$$\sim x_e^2$$



$$\frac{G_F^2 m_\mu^5}{192\pi^3}$$

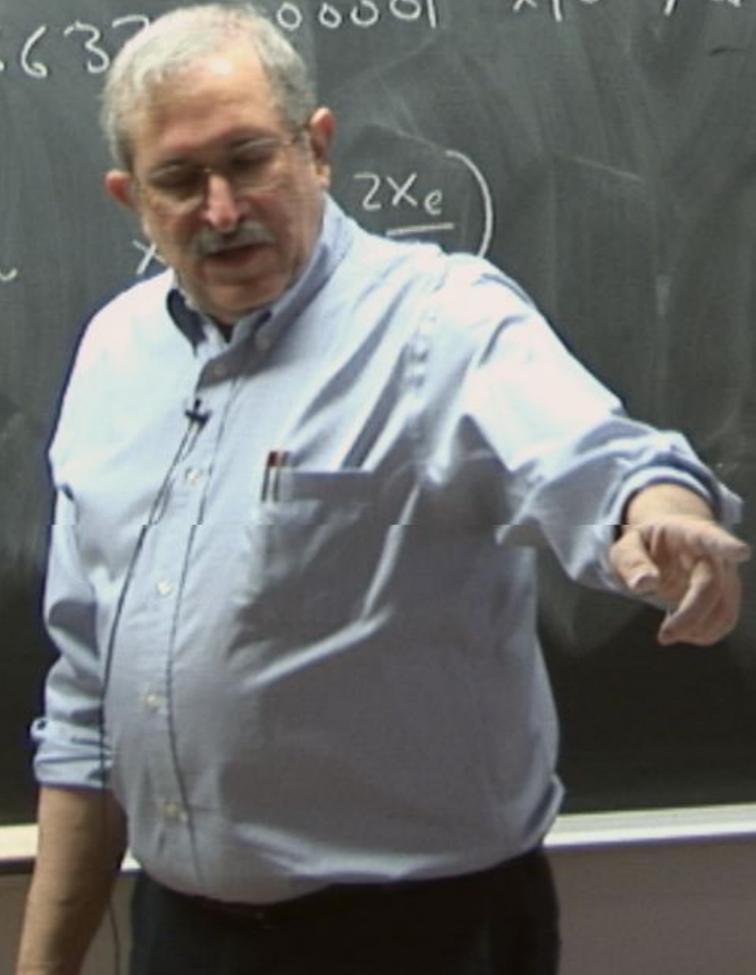
$$1.16637 \times 10^{-5} / \text{GeV}^2$$



$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= [(P_e + P_\nu)^2]^{\frac{1}{2}} = [(P_\mu - P_\nu)^2]^{\frac{1}{2}} \\ &= (m_\mu^2 - 2P_\mu \cdot P_\nu)^{\frac{1}{2}} \\ &= (m_\mu^2 (1 - x_\nu)) \end{aligned}$$

$$\sim x \left(\frac{2x_e}{m_\mu} \right)$$



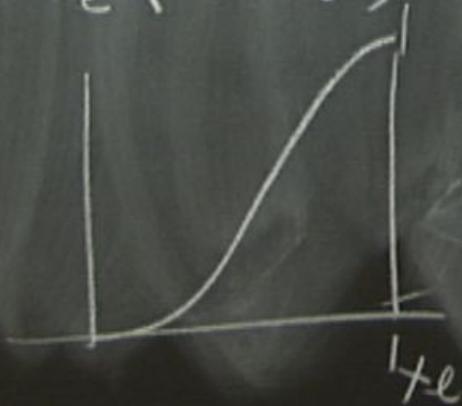


$$\frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$1.16637 \pm 0.00001 \times 10^{-5} / \text{GeV}^2$$

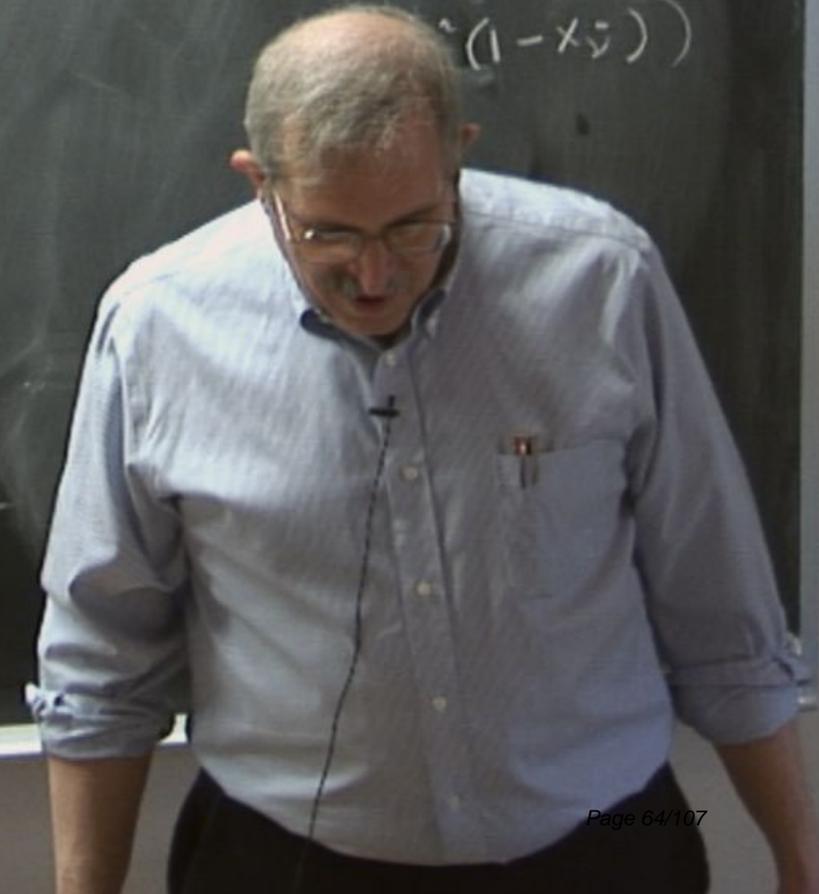


$$\sim x_e^2 \left(1 - \frac{2x_e}{3}\right)$$



$$\sqrt{2E_\nu} \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} & (2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= [(P_e + P_\nu)^2]^{\frac{1}{2}} = [(P_e - P_\nu)^2]^{\frac{1}{2}} \\ &= (m_\mu^2 - 2P_e \cdot P_\nu)^{\frac{1}{2}} \\ &= (m_\mu^2 (1 - x_e))^{\frac{1}{2}} \end{aligned}$$



$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \underbrace{\bar{\nu}_L \gamma^\mu e_L})$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \underbrace{\bar{\nu}_L \gamma^\mu e_L})$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$iM = i \frac{4G_F}{\sqrt{2}}$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$iM = i \frac{4G_F}{\sqrt{2}} (\bar{u}_L \sigma^\mu \nu_{eL}) (\bar{l} \gamma_\mu u)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$iM = i \frac{4G_F}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L) (\bar{\nu}_{eL} \gamma_\mu e_L)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$iM = i \frac{4G_F}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L) (\bar{\nu}_{eL} \gamma_\mu e_L)$$

$$\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$$

$$= -\frac{1}{2} \langle \pi^+ | \bar{d}_L \gamma_\mu u_L | \pi^+ \rangle$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$iM = i \frac{4G_F}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L) (\bar{\nu}_{eL} \gamma_\mu e_L)$$

$$\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$$

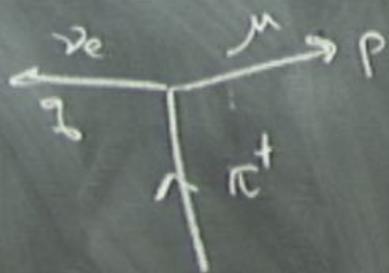
$$= -\frac{1}{2} \langle 0 | \bar{d} \gamma_\mu \gamma^5 u | \pi^+ \rangle = i \frac{1}{2} f_\pi k^\mu \sqrt{2}$$

$$P_L = \frac{1 - \gamma^5}{2}$$

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_{\pi} k^{\mu} (\bar{u}_i \gamma_{\mu} v)$$

$$|\pi^+\rangle = i \frac{1}{2} f_{\pi} k^{\mu} \sqrt{2}$$

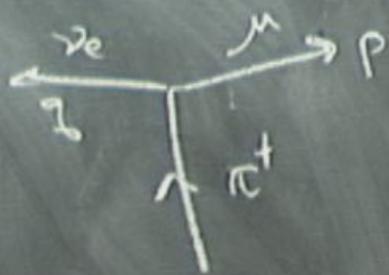
$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_\pi k^\mu (\bar{u}_l \gamma_\mu \nu)$$



$$\bar{u}_l \not{k} \nu = \bar{u}_l \not{\not{p}} \nu$$

$$|\pi^+\rangle = i \frac{1}{\sqrt{2}} f_\pi k^\mu \sqrt{2}$$

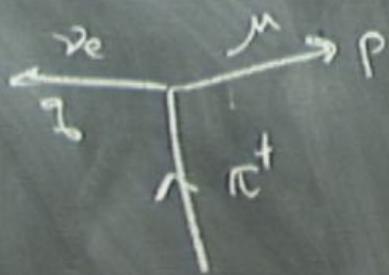
$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_\pi k^\mu (\bar{u}_L \gamma_\mu \nu)$$



$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} \nu - m_\nu \bar{u}_L \nu_L$$

$$|\pi^+\rangle = \frac{1}{\sqrt{2}} f_\pi k^\mu \sqrt{2}$$

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_{\pi} k^{\mu} (\bar{u}_L \gamma_{\mu} \nu)$$

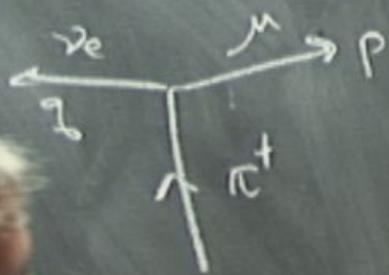


$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} \nu - m_{\mu} \bar{u}_L \nu_L$$

$$I(\pi^+ \rightarrow \mu^+ \nu) = \frac{G_F^2 f_{\pi}^2 m_{\mu}^2}{4\pi} m_{\mu}^2$$

$$|\pi^+\rangle = i \frac{1}{\sqrt{2}} f_{\pi} k^{\mu} \sqrt{2}$$

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_{\pi} k^{\mu} (\bar{u}_L \gamma_{\mu} \nu)$$



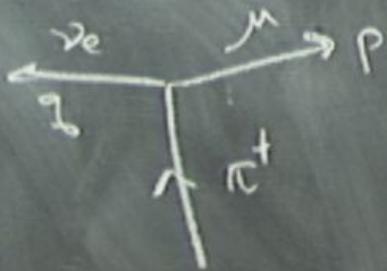
$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} + \cancel{0} \nu$$

$$= m_{\nu} \bar{u}_L \nu_L$$

$$I(\pi^+ \rightarrow \rho^+ \pi^0) = \frac{G_F^2 f_{\pi}^2 m_{\pi}^2}{4\pi} m_{\nu}^2 \left(1 - \frac{m_{\nu}^2}{m_{\pi}^2}\right)^2$$

$$= i \frac{1}{2} f_{\pi} k^{\mu} \sqrt{2}$$

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_\pi k^\mu (\bar{u}_L \gamma_\mu \nu)$$

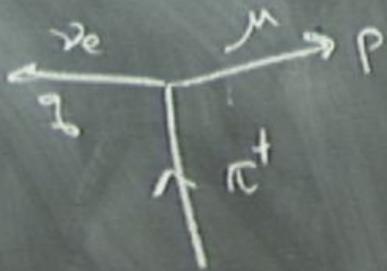


$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} \nu - m_\nu \bar{u}_L \nu_L$$

$$I(\pi^+ \rightarrow \mu^+ \nu) = \frac{G_F^2 f_\pi^2 m_\pi^2}{4\pi} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$|\pi^+\rangle = i \frac{1}{\sqrt{2}} f_\pi k^\mu \sqrt{2}$$

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_\pi k^\mu (\bar{u}_L \gamma_\mu \nu)$$



$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} \nu - m_\nu \bar{u}_L \nu_L$$

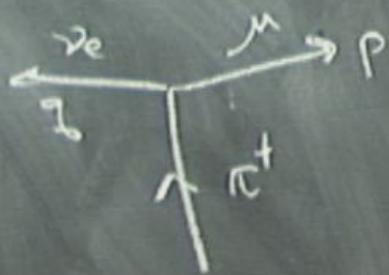
$$I(\pi^+ \rightarrow \mu^+ \nu) = \frac{2 \sqrt{2} G_F^2 m_\pi^2}{4\pi} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$|\pi^+\rangle = i \frac{1}{\sqrt{2}} f_\pi k^\mu \sqrt{2}$$

$$\left(\frac{I \rightarrow e^+}{I \rightarrow \mu^+} \right) \sim \left(\frac{m_e}{m_\mu} \right)^2 \sim \left(\frac{0.511}{1.057} \right)^2 \sim 0.23$$

(up to 1.23 × 10⁻⁴)

$$iM = \frac{4G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} f_\pi k^\mu (\bar{u}_L \gamma_\mu \nu)$$



$$\bar{u}_L \not{k} \nu = \bar{u} \not{k} \nu = m_\nu \bar{u}_L \nu_L$$

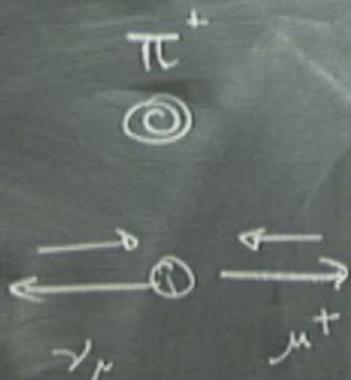
$$I(\pi^+ \rightarrow \mu^+ \nu) = \frac{2 \int_0^1 dx \frac{x^2 m_\pi^2}{4\pi} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$|\pi^+\rangle = i \frac{1}{\sqrt{2}} f_\pi k^\mu \sqrt{2}$$

$$\left(\frac{I \rightarrow e^+}{I \rightarrow \mu^+} \right) \sim \left(\frac{m_e}{m_\mu} \right)^2 \sim \left(\frac{0.511}{1.057} \right)^2 \sim 0.23$$

(uptd 1.28×10^{-4}
 1.23×10^{-4})

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$



$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$

$$i \frac{4G_F}{\sqrt{2}} (\bar{\mu}_L \gamma^\mu \nu_L) (\bar{d}_L \gamma_\mu u_L)$$

$$\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$$

$$= -\frac{1}{2} \langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi^+ \rangle$$

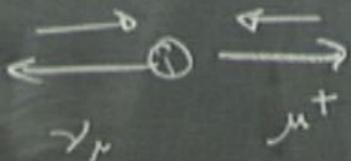
$$P_L = \frac{1-\gamma_5}{2}$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

π^+
 \odot

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$



$$i \frac{4G_F}{\sqrt{2}} (\bar{\mu}_L \sigma^\mu \nu_{\mu L}) (\bar{d}_L \gamma_\mu u_L)$$

helicity = ν dab $\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$

$$= -\frac{1}{2} \langle 0 | \bar{d} \gamma^\mu \gamma^5 u$$

$$- p_L = \frac{1 - \gamma^5}{2}$$

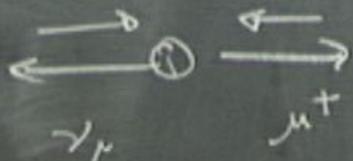
$$f \left(\frac{m_\mu}{E_\mu} \right)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

π^+


$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$



$$i \frac{4G_F}{\sqrt{2}} (\bar{\mu}_L \gamma^\mu \nu_{\mu L}) (\bar{d}_L \gamma_\mu u_L)$$

helicity = $\frac{m_\mu}{E_\mu}$ $\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$

$$= -\frac{1}{2} \langle 0 | \bar{d} \gamma^\mu \gamma^5 u$$

$$P_L = \frac{1 - \gamma^5}{2}$$

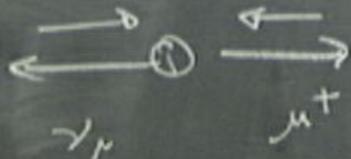
$$\left(\frac{m_\mu}{E_\mu} \right)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

π^+


$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$



helicity = v/c

$$i \frac{4G_F}{\sqrt{2}} (\bar{\mu}_L \sigma^\mu \nu_{\mu L}) (\bar{d}_L \gamma_\mu u_L)$$

$$\langle 0 | (\bar{d}_L \gamma_\mu u_L) | \pi^+ \rangle$$

$$= -\frac{1}{2} \langle 0 | \bar{d} \gamma^\mu \gamma^5 u$$

$$- p_L = \frac{1 - \gamma^5}{2}$$

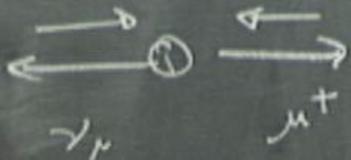
$$\left(\frac{m_\mu}{E_\mu} \right)$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

π^+

$$\pi^+ \rightarrow e^+ \nu$$

$$\pi^+ \rightarrow \mu^+ \nu$$



helicity = 0

$$\left(\frac{m_\mu}{E_\mu} \right)$$

$$4 \left(\bar{\nu}_L \gamma^\mu \nu_L \right) \left(\bar{l}_L \gamma_\mu u_L \right)$$

$$\langle \pi^+ | \bar{l}_L \gamma_\mu u_L | 0 \rangle$$

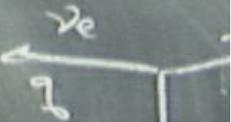
$$= \frac{1 - \gamma^5}{2}$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{\mu L} \gamma^\mu e_L)$$

Dirac e- scaly

$$\frac{ds}{dx dy_0} = (\text{const}) \left(\frac{1}{Q^2}\right)^2 (1 + (1-y)^2)$$

$$iM =$$



$$= i \frac{1}{2} \frac{1}{\pi}$$

$$J^{\mu\nu} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

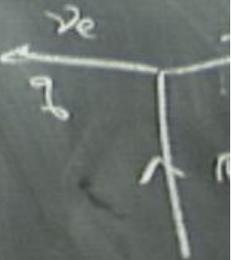
$$Q_f^2 f_f(x)$$

DI e scaly

$$\frac{d\sigma}{dx dy}$$

$$= (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$$

$$\frac{d\sigma}{dx dy}$$



$$= i \frac{1}{2} \int_{-\pi}^{\pi}$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

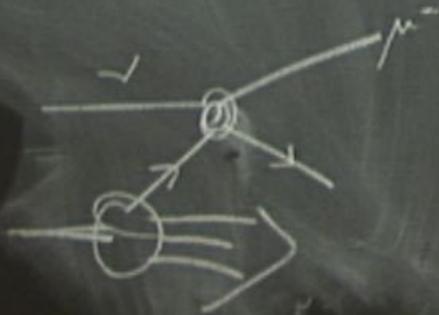
$$Q_f^2 f_f(x)$$

DI e scatt

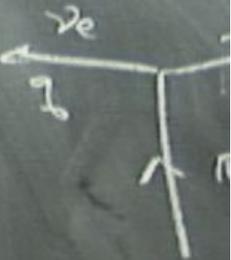
$$\frac{d\sigma}{dx dy}$$

$$= (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$$

$$\frac{d\sigma}{dx dy} (\nu p \rightarrow \mu^- X)$$



$$iM =$$



$$= i \frac{1}{2} f_\pi$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{\mu L} \gamma^\mu e_L)$$

$$Q_f^2 f_f^2$$

Die e scutg

$$\frac{d\sigma}{dx dy}$$

$$= (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$$

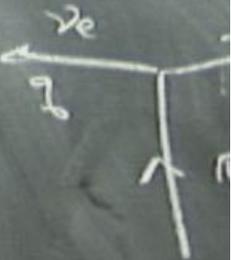
$$\frac{d\sigma}{dx dy}$$

$$(\nu p \rightarrow \mu^- \gamma)$$

$$= \frac{C_{IF} S}{\pi}$$



$$iM =$$



$$= i \frac{1}{2} f_{\pi}$$

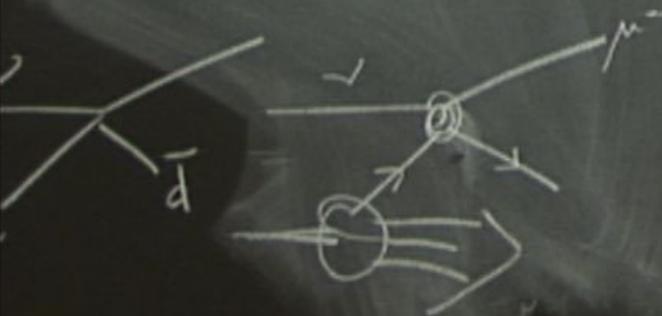
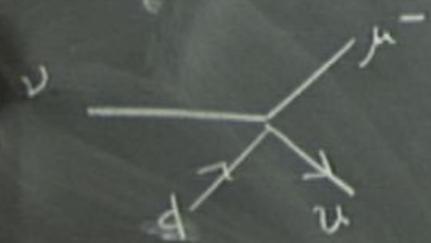
$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_{eL} \gamma^\mu e_L)$$

$$Q_{Feyn}^2 f(x)$$

DI e- scattg

$$\frac{d\sigma}{dx dy} = (\text{const}) \left(\frac{1}{Q^2}\right)^2 (1 + (1-y)^2)$$

$$\frac{d\sigma}{dx dy} (\nu p \rightarrow \mu^- X) = \frac{G_{IF} S}{\pi} f(x)$$



$$iM = \dots$$

$$= i \frac{1}{2} \int_{-\pi}^{\pi} \dots$$

$$J^{\mu+} = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

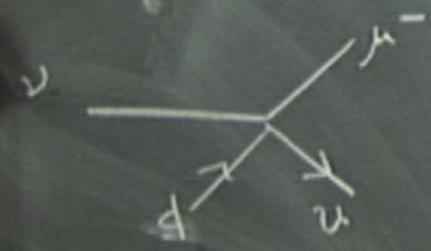
$Q_F^2 f_f(x)$

DI e- scats

$$\frac{d\sigma}{dx dy} =$$

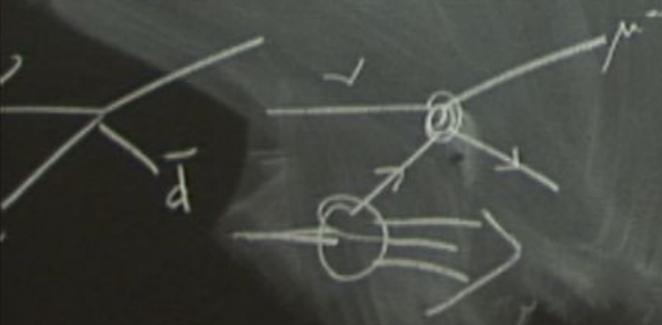
$$(\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$$

$i\eta =$



$$\frac{d\sigma}{dx dy} (ep \rightarrow \mu^- X)$$

$$= \frac{C_{IF} S}{\pi} \left[x f_d(x) + x f_{\frac{u}{2}}(x) (1-y)^2 \right]$$



$$+ = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$$Q_{f_i}^2 f_f(x)$$

scatg $\frac{d\sigma}{dx dy} = (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$

$$\frac{d\sigma}{dx dy} (\nu p \rightarrow \mu^- X) = \frac{C_{IF} S}{\pi} [x f_d(x) + x f_u(x) (1-y)^2]$$

$$\frac{d\sigma}{dx dy} (\bar{\nu} p \rightarrow \mu^+ X) = \frac{C_{IF} S}{\pi} [x f_u(x) (1-y)^2 + x f_d(x)]$$

$$iM = \frac{4G_F}{\sqrt{2}} \bar{l} f_\pi k^\mu$$

$$+ = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$$Q_{f_i}^2 f_f(x)$$

scatg

$$\frac{d\sigma}{dx dy}$$

$$= (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$$

$$\frac{d\sigma}{dx dy} (\nu p \rightarrow \mu^- X) = \frac{G_F^2 S}{\pi} \left[x f_d(x) + x f_{\bar{u}}(x) (1-y)^2 \right]$$

$$\frac{d\sigma}{dx dy} (\bar{\nu} p \rightarrow \mu^+ X) = \frac{G_F^2 S}{\pi} \left[x f_u(x) (1-y)^2 + x f_{\bar{d}}(x) \right]$$

$$iM = \frac{4G_F}{\sqrt{2}} \bar{l} \gamma^\mu f_\pi k^\mu$$

$$+ = 2(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L)$$

$$Q_{f, f}^2$$

scatg $\frac{d\sigma}{dx dy} = (\text{const}) \frac{1}{(Q^2)^2} (1 + (1-y)^2)$

$$\frac{d\sigma}{dx dy} (\nu p \rightarrow \mu^- X) = \frac{C_{IF} S}{\pi} \left[x f_d(x) + x f_u(x) (1-y)^2 \right]$$

$$\frac{d\sigma}{dx dy} (\bar{\nu} p \rightarrow \mu^+ X) = \frac{C_{IF} S}{\pi} \left[x f_u(x) (1-y)^2 + x f_d(x) \right]$$

y^2

$$\left[x f_{\bar{u}}^{(x)} (1-y)' \right]$$

$$\left[-y)^2 + x f_{\bar{d}}^{(x)} \right]$$



3)



$$x f_{\frac{x}{u}}(x) (1-y)']$$

$$-y)^2 + x f_{\frac{x}{d}}(x)']$$

y^2)

$$\times f_{\bar{u}}^{(x)} (1-y)^2]$$

$$-y)^2 + x f_{\bar{d}}^{(x)}]$$



$$\delta \mathcal{L} = - \frac{4G_F}{\sqrt{2}} \left[J_{\mu} \right]$$

J^2

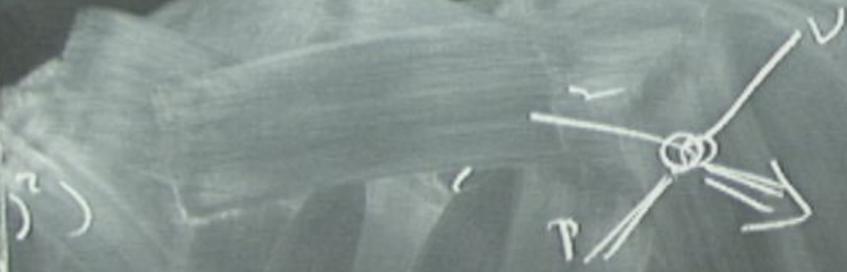
$x f_{\bar{u}}^{(x)} (1-y)']$

$(1-y)^2 + x f_{\bar{d}}^{(x)}]$



$$J_{\gamma}^{\mu} = J^1 + i J^2$$

$$\delta\mathcal{L} = -\frac{4C_F}{\sqrt{2}} \left[(J_{\gamma}^1)^2 + (J_{\gamma}^2)^2 \right]$$



$$J_{1,1}^k = J^1 + iJ^2$$

$$S\mathcal{L} = -\frac{4C_F}{\sqrt{2}} \left[(J_{1,1}^1)^2 + (J_{1,1}^2)^2 \right. \\ \left. + (J_{1,1}^3 - s_w^2 J_{1,1}^0)^2 \right]$$

$$\times f_{\frac{u}{d}}^{(x)} (1-y)']$$

$$-y)^2 + x f_{\frac{d}{u}}^{(x)}]$$



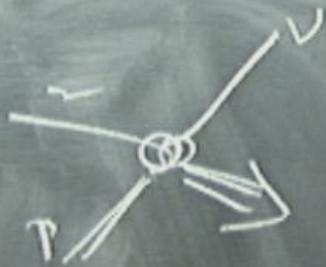
$$J_{\uparrow}^{\mu} = J^1 + iJ^2$$

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[(J_{\uparrow}^1)^2 + (J_{\uparrow}^2)^2 + (J^3 - s_w^2 J_{EM}^{\uparrow})^2 \right]$$

$$\times f_{\bar{u}}(x) (1-y)']$$

$$-y)^2 + x f_{\bar{d}}(x)']$$

$$J_{\uparrow}^{\mu} - s_w^2 J_{EM}^{\uparrow} = \bar{f} \gamma^{\mu} (P_L I_3^3 - s_w^2 Q_f) f$$



$$J_{\uparrow}^{\mu} = J^1 + iJ^2$$

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[(J_{\uparrow}^1)^2 + (J_{\uparrow}^2)^2 + (J^3 - s_w^2 J_{EM})^2 \right]$$

$$\times f_{\bar{u}}(x) (1-y)']$$

$$-y)^2 + x f_{\bar{d}}(x)']$$

$$J_{\uparrow}^{\mu} - s_w^2 J_{EM}^{\mu} = \bar{f} \gamma^{\mu} (I_L I_3^3 - s_w^2 Q_f) f$$

$$u_L = \left(\frac{1}{2} - s_w^2 \frac{2}{3} \right)$$

$$u_R = -s_w^2 \frac{2}{3}$$



$$J_{\mu}^{\nu} = J^1 + iJ^2$$

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[(J_{\mu}^1)^2 + (J_{\mu}^2)^2 + (J_{\mu}^3 - s_w^2 J_{EM}^{\mu})^2 \right]$$

$$\times f_{\bar{u}}(x) (1-y)']$$

$$-y)^2 + x f_{\bar{d}}(x)']$$

$$J_{\mu}^3 - s_w^2 J_{EM}^{\mu} = \bar{f} \gamma_{\mu} \left(\frac{1}{2} I_3^3 - \dots \right)$$

$$u_L = \left(\frac{1}{2} - s_w^2 \frac{2}{3} \right)$$

$$21_R = -s_w^2 \frac{2}{3}$$

"New hat current"



$$J_{\mu}^{\nu} = J^{\nu 1} + i J^{\nu 2}$$

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[(J_{\mu}^{\nu})^2 + (J_{\mu}^{\nu})^2 + (J^{\nu 1} - s_w^2 J_{EM}^{\nu})^2 \right]$$

$$\left[x f_{\bar{u}}(x) (1-y)^2 \right]$$

$$\left[-y)^2 + x f_{\bar{d}}(x) \right]$$

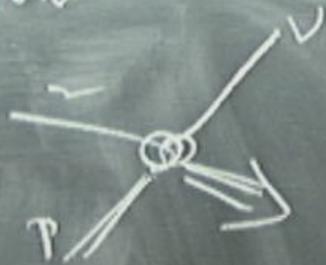
$$J_{\mu}^{\nu} - s_w^2 J_{EM}^{\nu} = \bar{f} \gamma^{\mu} (P_L I_{3f}^3 - s_w^2 Q_f) f$$

$$u_L = \left(\frac{1}{2} - s_w^2 \frac{2}{3} \right)$$

$$u_R = -s_w^2 \frac{2}{3}$$

$N_p \approx N_n$

"Neutral current"



$$g_{\mu\nu} \quad J_{\mu}^{\nu} = J^1 + iJ^2$$

$$(g_{\mu\nu} \frac{\sigma_{\mu}^{\nu}}{2} q_L)$$

$$S_{\mathcal{L}} = -\frac{4G_F}{\sqrt{2}} \left[(J_{\mu}^{\nu})^2 + (J_{\mu}^2)^2 \right]$$

$$+ (J^3 - s_w^2 J_{EM})$$

$$J_{EM}^{\mu} = \bar{f} \gamma^{\mu} (I_L I_f^3 - s_w^2 Q_f) f$$

$$q_L = \begin{pmatrix} \frac{1}{2} \\ -s_w^2 \frac{2}{3} \end{pmatrix}$$

$$q_R = -s_w^2 \frac{2}{3}$$

$N_p \approx N_n$

"Neutral current"



$$q_\mu(a) \quad J_\mu^r = J^1 + iJ^2$$

$$(q_L^\mu \gamma_\mu^r \frac{\sigma_1}{2} q_L)$$

$$S\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \left[(J_\mu^r)^2 + (J_\mu^2)^2 + (J^{3\mu} - s_w^2 J_{EM}^\mu)^2 \right]$$

$$J_{EM}^\mu = \int f \gamma^\mu (I_L I_3^3 - s_w^2 Q_f) f$$

$$q_L = \begin{pmatrix} \frac{1}{2} \\ -s_w^2 \frac{2}{3} \end{pmatrix}$$

$$U_R = -s_w^2 \frac{2}{3}$$

et $\epsilon N_p \approx N_n$

$$\frac{f_{\beta}^2(1-y)^2 + f_{\beta}^2(x)}{f_{\beta}^2(x) + f_{\beta}^2(1-y)^2}$$

"New hat current"



$$g_{\beta}^2(x) \quad \vec{J}_{\beta}^{\mu} = \vec{J}^1 + i \vec{J}^2$$

$$(g_{\beta}^2 \gamma^{\mu} \frac{\sigma_{\mu}^i}{2} g_{\beta}^2)$$

$$\delta \mathcal{L} = - \frac{4G_F}{\sqrt{2}} \left[(\vec{J}_{\beta}^1)^2 + (\vec{J}_{\beta}^2)^2 + (\vec{J}^3 - s_w^2 \vec{Q})^2 \right]$$

$$\vec{J}_{EM}^{\mu} = \bar{f} \gamma^{\mu} (I_L I_3^3 - s_w^2 Q)$$

$$u_L = \begin{pmatrix} \frac{1}{2} & -s_w^2 \frac{2}{3} \end{pmatrix} \quad u_R =$$

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$N_p \approx N_n$

"Neutral-current"

$$r = \frac{\sigma(\bar{\nu}, CC)}{\sigma(\nu, CC)} = \frac{f_{\bar{\nu}}(1-y)^2 + f_{\bar{\nu}}(y)}{f_{\nu}(y) + f_{\nu}(1-y)^2}$$

$$R_{\nu} = \frac{\sigma(\nu NC)}{\sigma(\nu CC)} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 (1+r)$$

$$R_{\bar{\nu}} = \frac{\sigma(\bar{\nu} NC)}{\sigma(\bar{\nu} CC)} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 (1 + \frac{1}{r})$$



$$S_{FS} = -\frac{4G_F}{\sqrt{2}}$$

$$J_{em}^{\mu} = \bar{\psi} \gamma^{\mu} (I_L)$$

$$u_L = \left(\frac{1}{2} - s_w \right)$$

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$N_p \approx N_n$

"New hat current"

$$r = \frac{\sigma(\bar{v}, CC)}{\sigma(v, CC)} = \frac{f_{\bar{g}}(1-y)^2 + f_{\bar{g}}(y)}{f_{\bar{g}}(v) + f_{\bar{g}}(1-y)^2}$$



$$S\mathcal{L} = -\frac{4G}{\sqrt{2}}$$

$$R_v = \frac{\sigma(v, NC)}{\sigma(v, CC)} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 (1+r)$$

$$R_{\bar{v}} = \frac{\sigma(\bar{v}, NC)}{\sigma(\bar{v}, CC)} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 (1 + \frac{1}{r})$$

$$J_{em} = f \gamma^u (P_L)$$

$$u_L = \left(\frac{1}{2} - s_w^2 \frac{2}{3} \right)$$