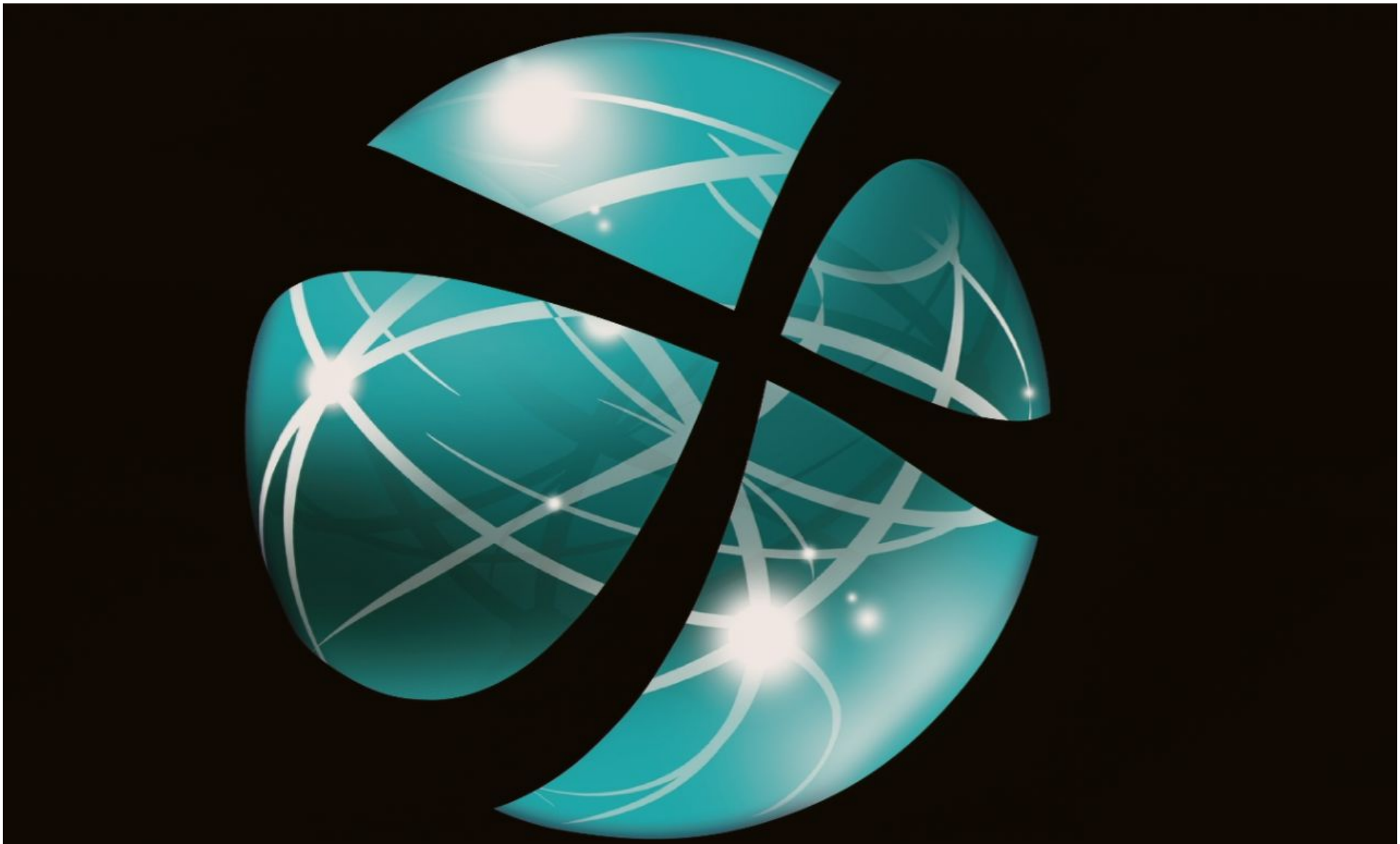
Title: Standard Model - Lecture 8

Date: Jan 12, 2011 09:00 AM

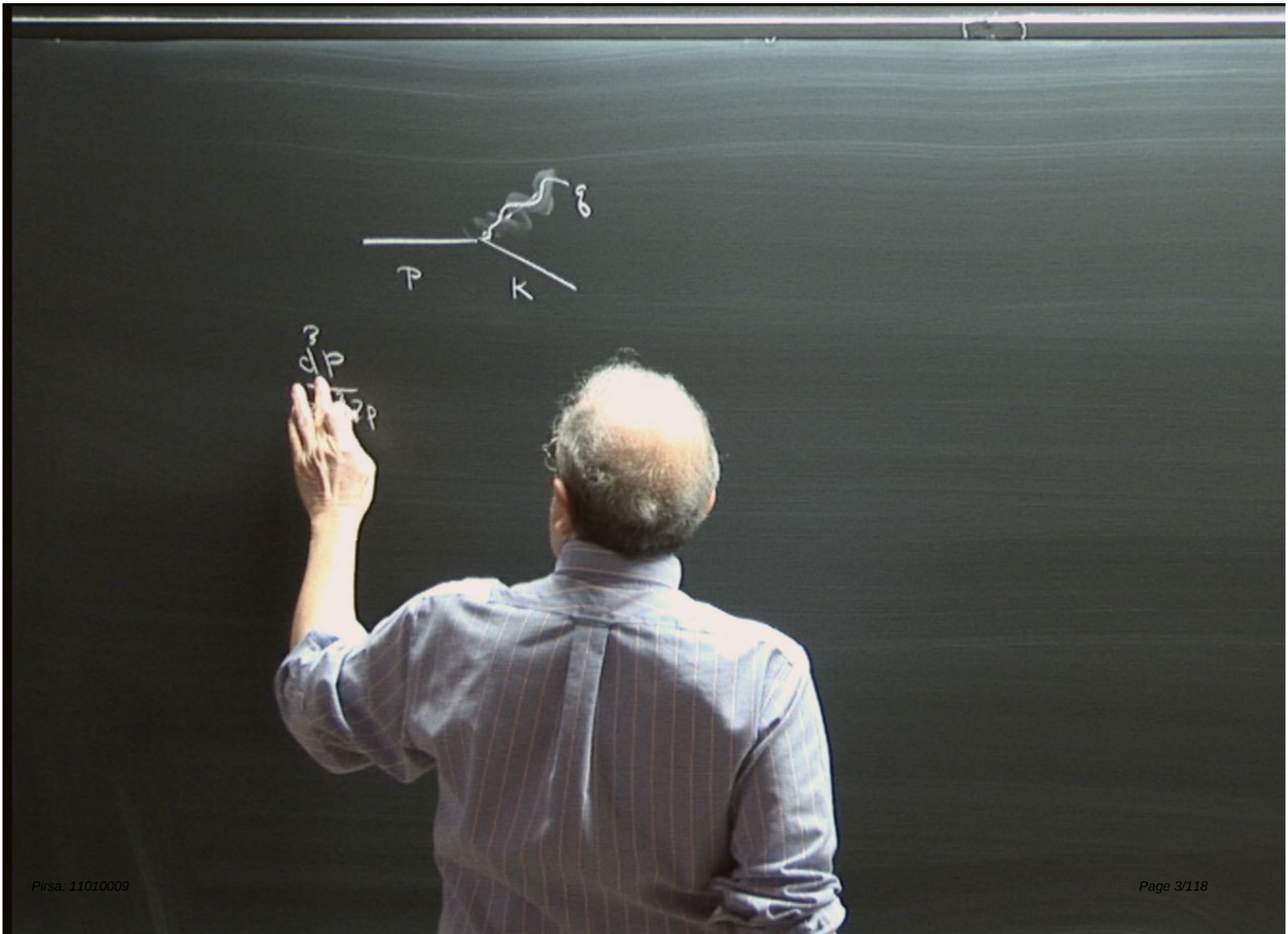
URL: http://pirsa.org/11010009

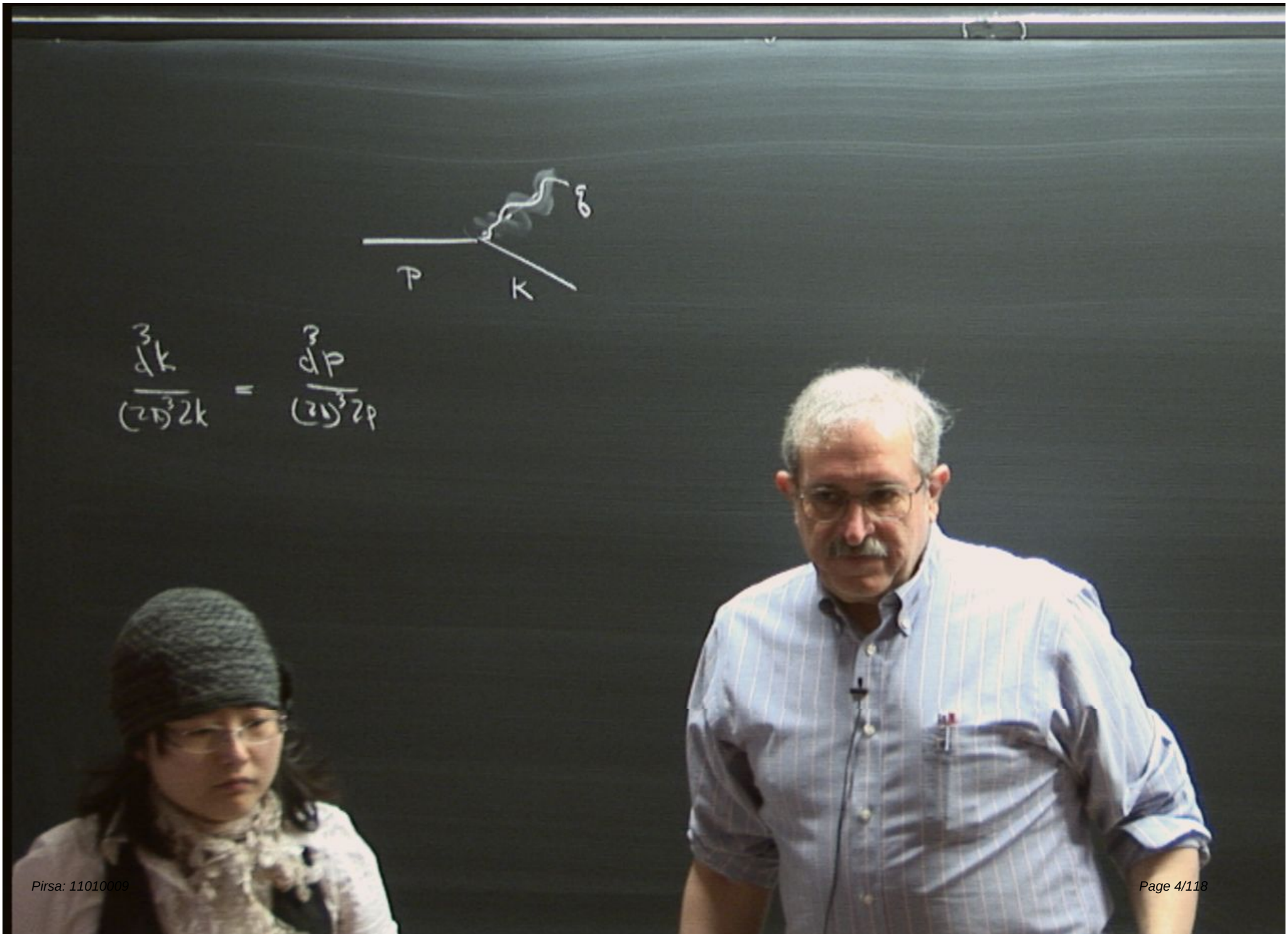
Abstract:

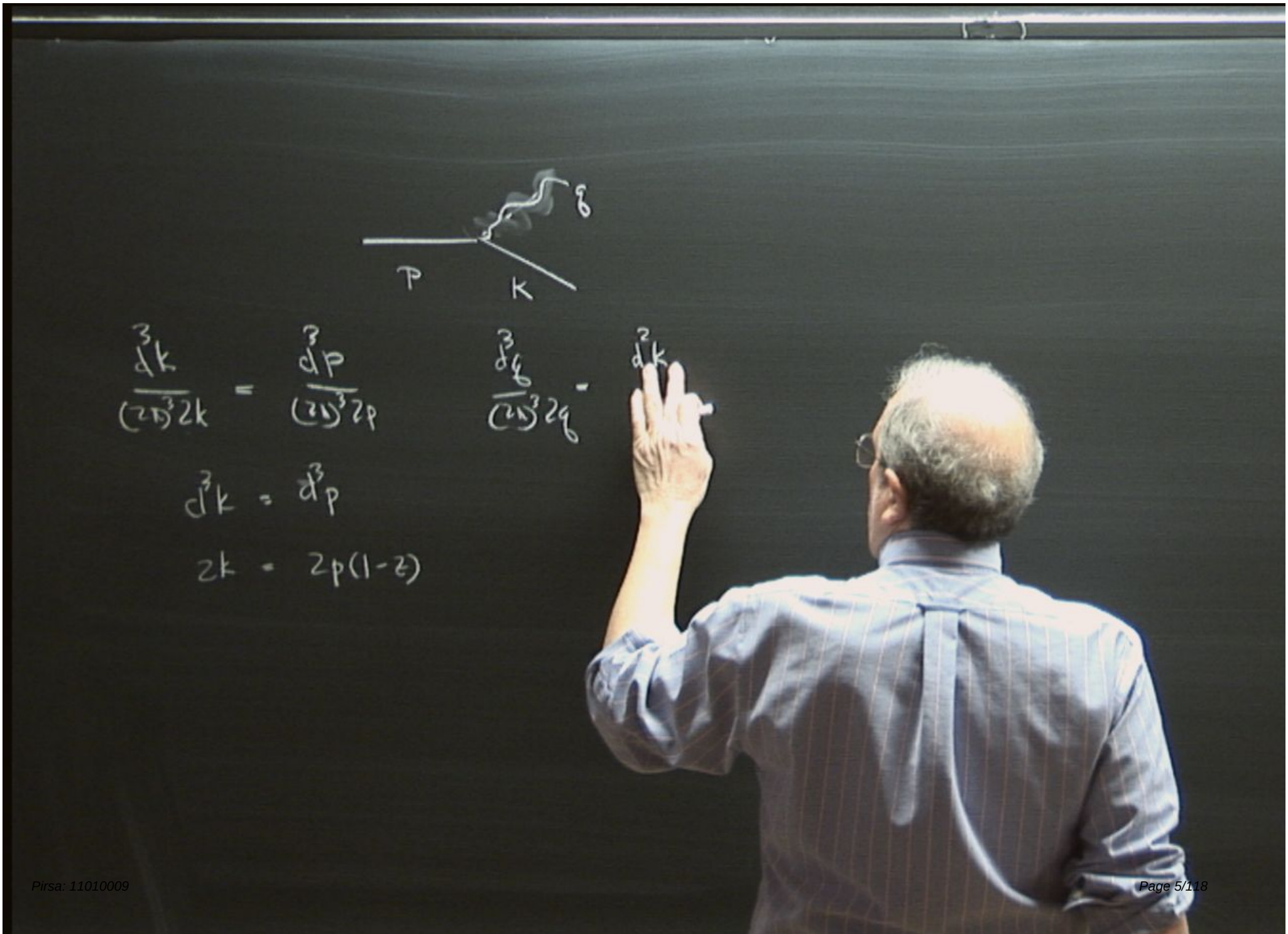


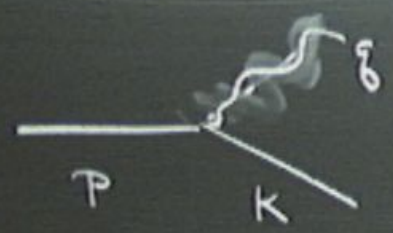
perimeter scholars

international


q
p
k


q
p
k
$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p}$

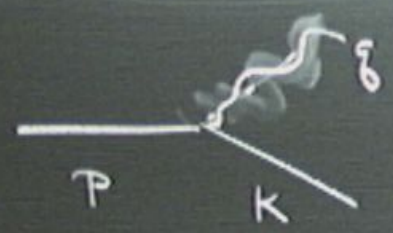

q
p
k
$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p} \frac{d^3q}{(2\pi)^3 2q} \cdot d^2k$
$d^3k = d^3p$
$2k = 2p(1-z)$



$$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p} \qquad \frac{d^3q}{(2\pi)^3 2q} = \frac{d^2k_\perp}{(2\pi)^2 2} \frac{dk_\parallel}{k}$$

$$d^3k = d^3p$$

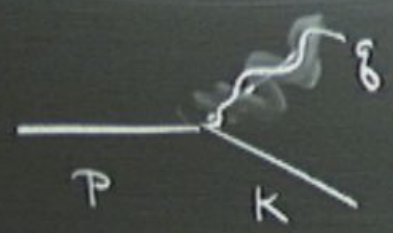
$$2k = 2p(1-z)$$



$$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p} \qquad \frac{d^3q}{(2\pi)^3 2q} = \frac{d^2k_T}{(2\pi)^3 2} \frac{dk_\parallel}{k} = \frac{d^2k_T}{16\pi^3} \frac{dz}{z}$$

$$d^3k = d^3p$$

$$2k = 2p(1-z)$$

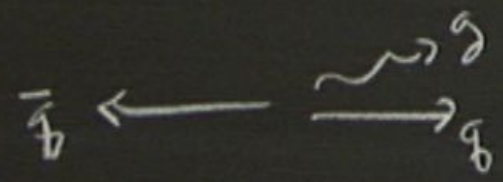


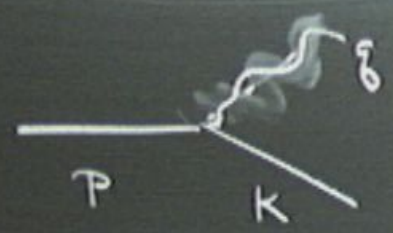
$$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p} \qquad \frac{d^3q}{(2\pi)^3 2q} = \frac{d^2k_T}{(2\pi)^3 2} \frac{dk_\parallel}{k} = \frac{d^2k_T}{16\pi^3} \frac{dz}{z}$$

$$d^3k = d^3p$$

$$2k = 2p(1-z)$$

$$\sigma(e^+e^- \to q\bar{q}g)$$



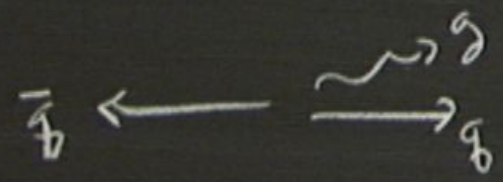


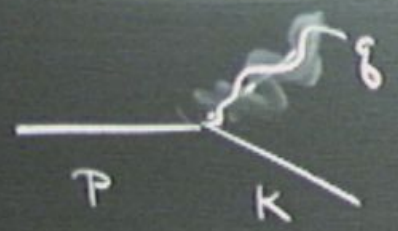
$$\frac{d^3k}{(2\pi)^3 2k} = \frac{d^3p}{(2\pi)^3 2p} \qquad \frac{d^3q}{(2\pi)^3 2q} = \frac{d^2k_T}{(2\pi)^3 2\,k} \cdot \frac{dk_\parallel}{} = \frac{d^2k_T}{16\pi^3}\frac{dz}{z}$$

$$d^3k = d^3p$$

$$2k = 2p(1-z)$$

$$\sigma(e^+e^- \to q\bar{q}g) = \sigma_0(e^+e^- \to q\bar{q}) \int \frac{dk_T^2}{k_T^2} \int dz \quad \frac{4}{}$$



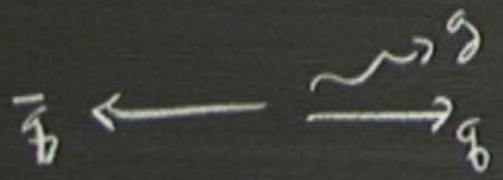


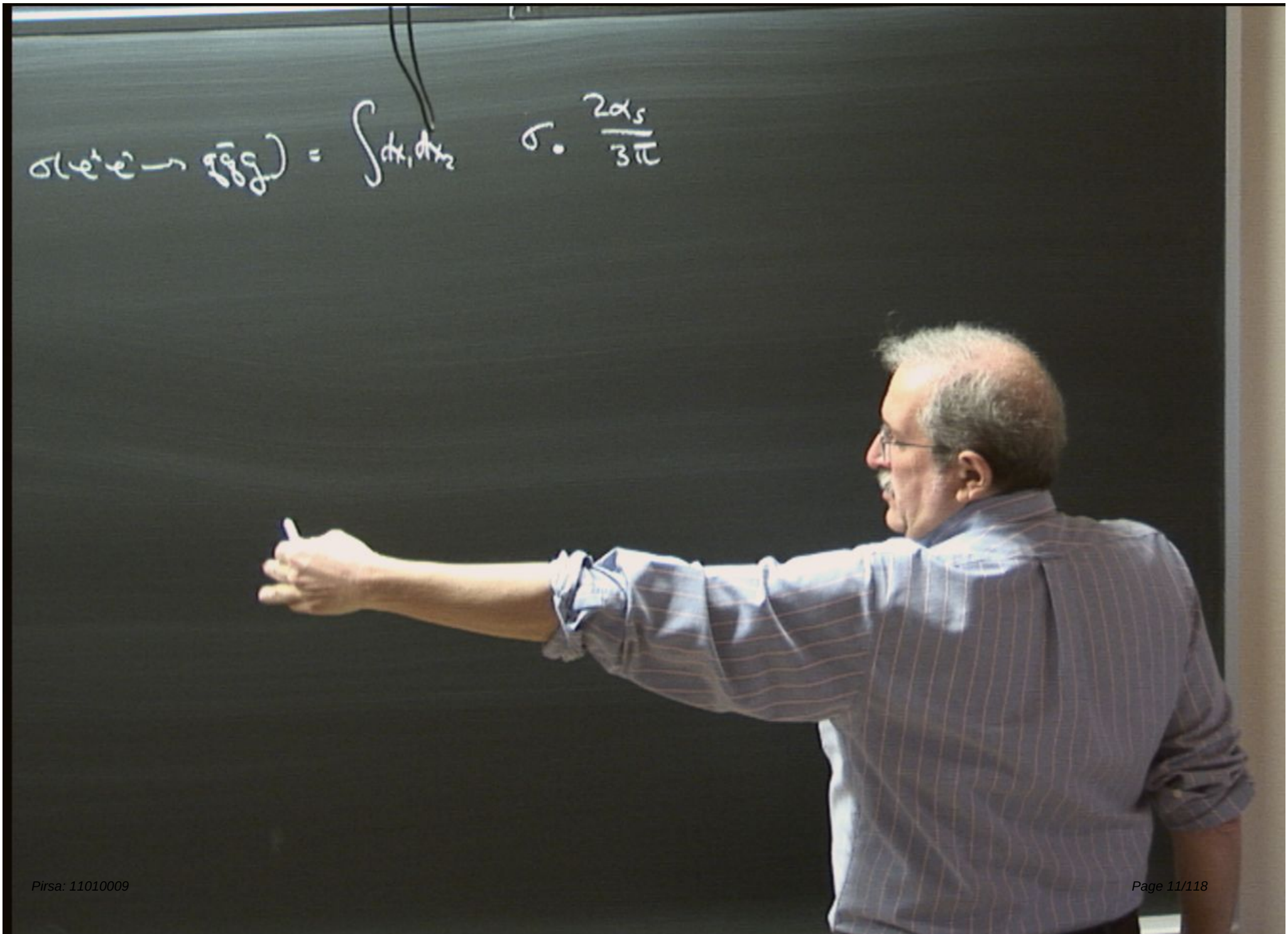
$$= \frac{d^3p}{(2\pi)^3 2p} \frac{d^3q}{(2\pi)^3 2q} = \frac{d^2k_T}{(2\pi)^3 2} \frac{dk_\parallel}{k} = \frac{d^2k_T}{16\pi^3} \frac{dz}{z}$$

$$= d^3p$$

$$= 2p(1-z)$$

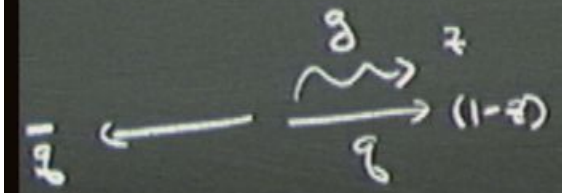
$$\sigma(e^+e^- \to q\bar{q}g) = \sigma_0(e^+e^- \to q\bar{q}) \int \frac{dk_T^2}{k_T^2} \int dz \; \frac{4}{3}\frac{\alpha_s}{\pi} \; \frac{1+(1-z)^2}{z}$$

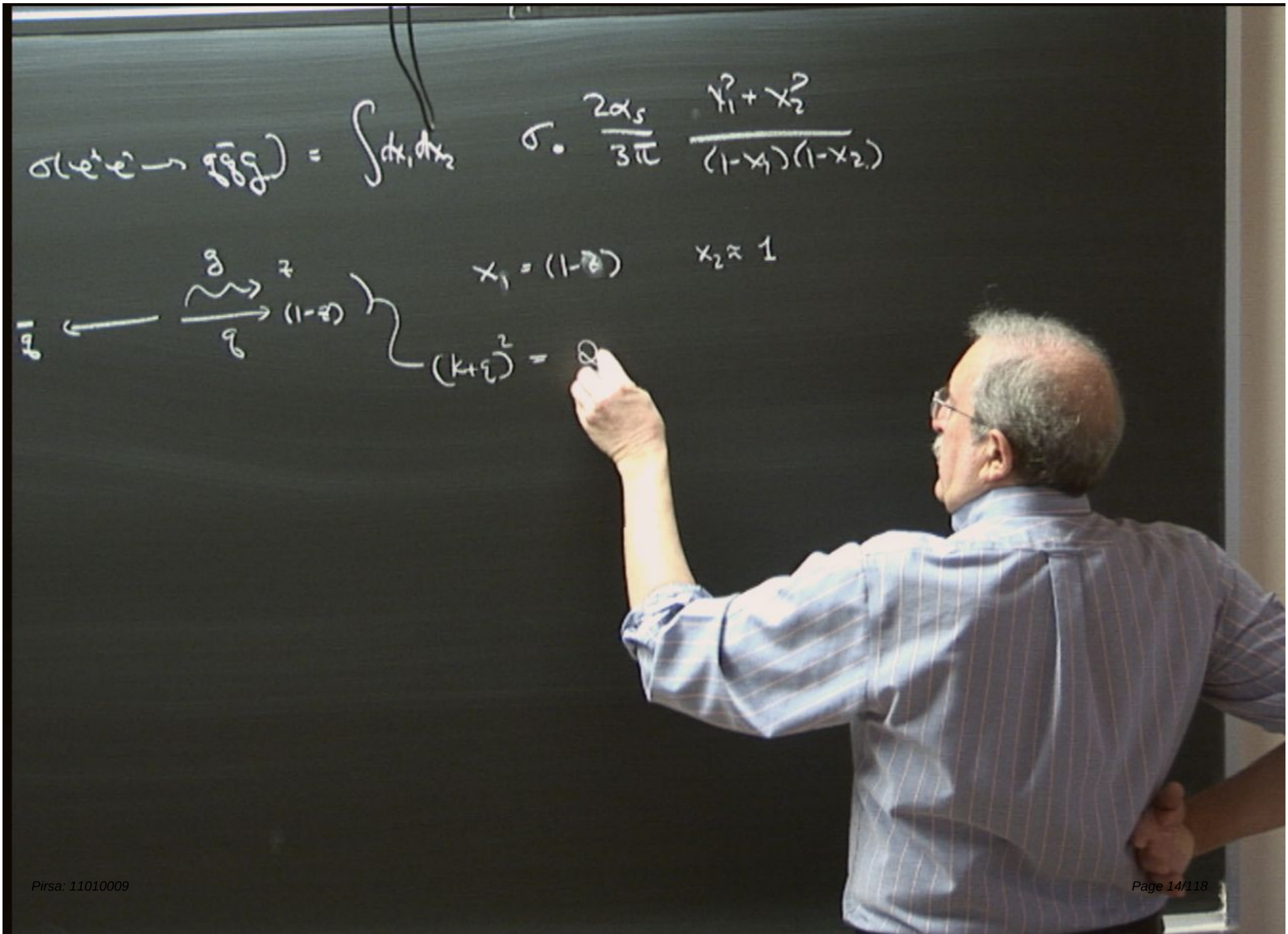


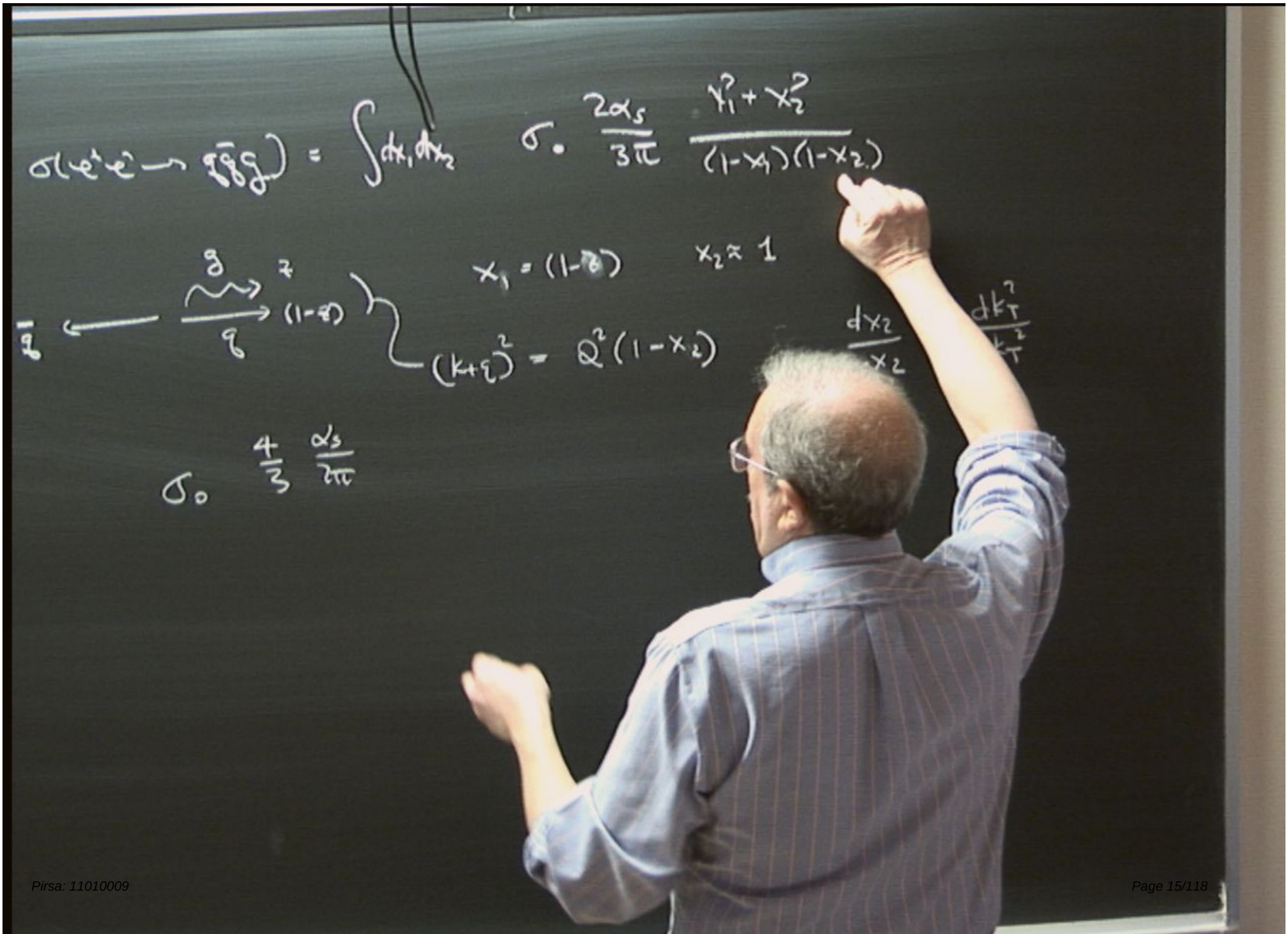

$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1 dx_2 \quad \sigma \cdot \frac{2\alpha_s}{3\pi}$

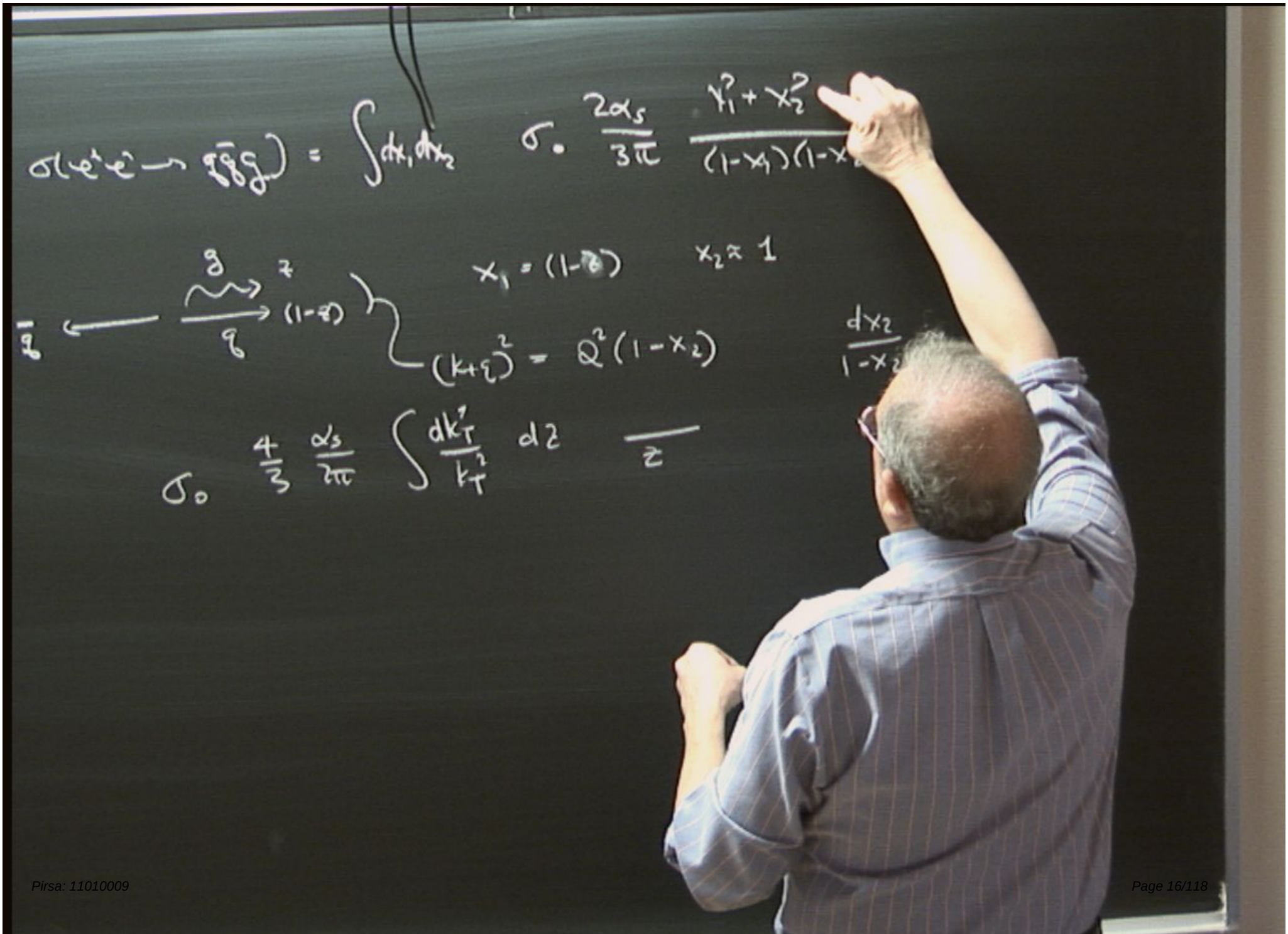
$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$




$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1 dx_2 \; \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$
$x_1 = (1-z)$
$x_2 \approx 1$
g
z
$\bar{q}$
q
$(1-z)$
$(k+q)^2 = Q$


$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1 dx_2 \; \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$
$x_1 = (1-z)$
$x_2 \approx 1$
$(k+q)^2 = Q^2(1-x_2)$
$\sigma_0 \; \frac{4}{3} \frac{\alpha_s}{2\pi}$


$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1 dx_2 \quad \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$
$x_1 = (1-z) \qquad x_2 \approx 1$
$\bar{q} \longleftarrow \xrightarrow{g \rightsquigarrow z} q \quad (1-z)$
$(k+q)^2 = Q^2(1-x_2)$
$\frac{dx_2}{1-x_2}$
$\sigma_0 \quad \frac{4}{3} \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \quad \frac{}{z}$

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\bar{q} \longleftarrow \xrightarrow[q]{g \rightsquigarrow z} (1-z)$$

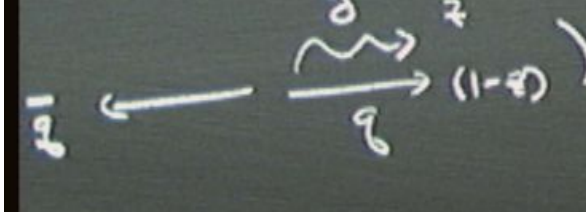
$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$

$$\frac{dx_2}{1-x_2} \sim \frac{dk_T^2}{k_T^2}$$

$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

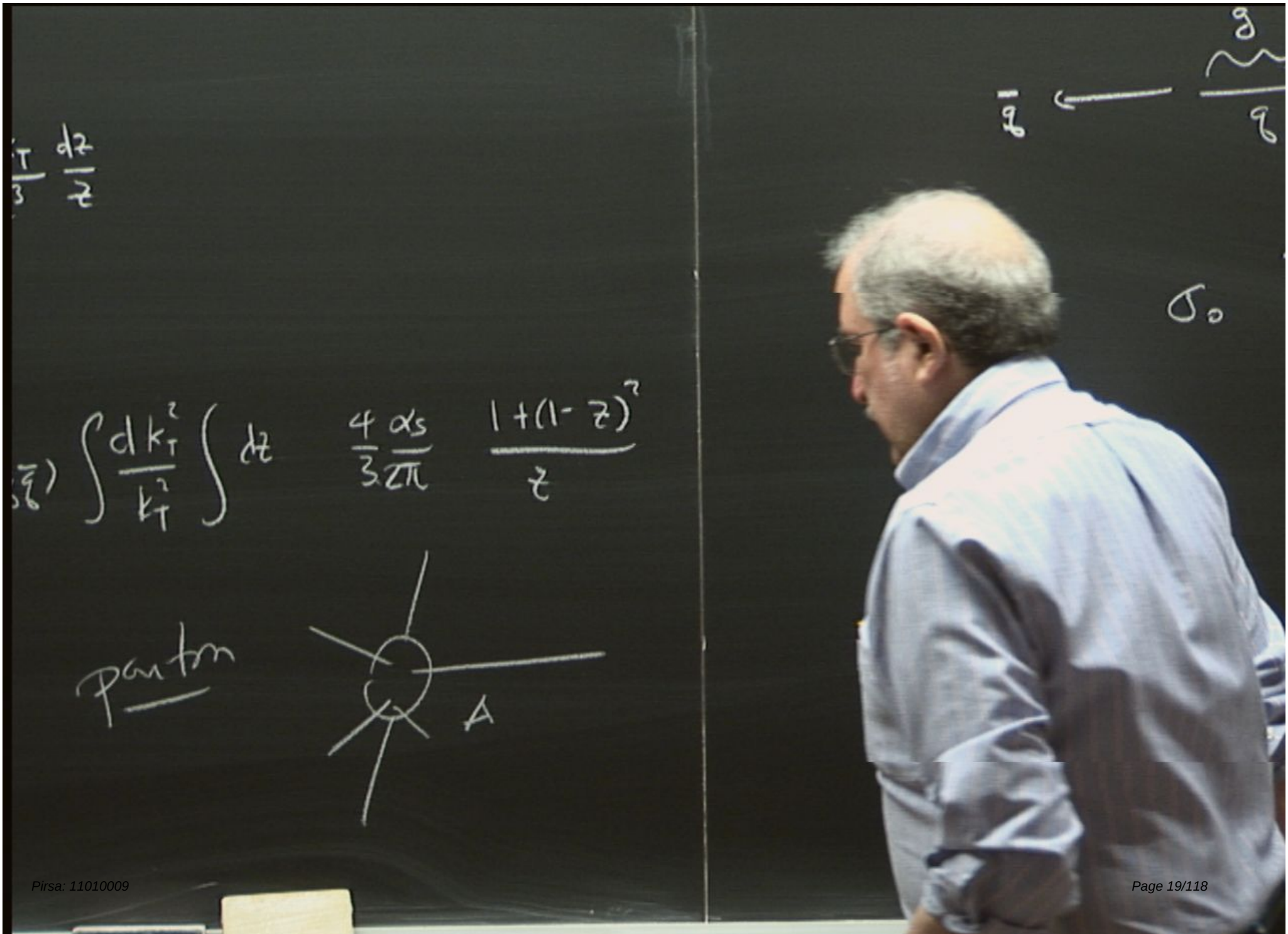


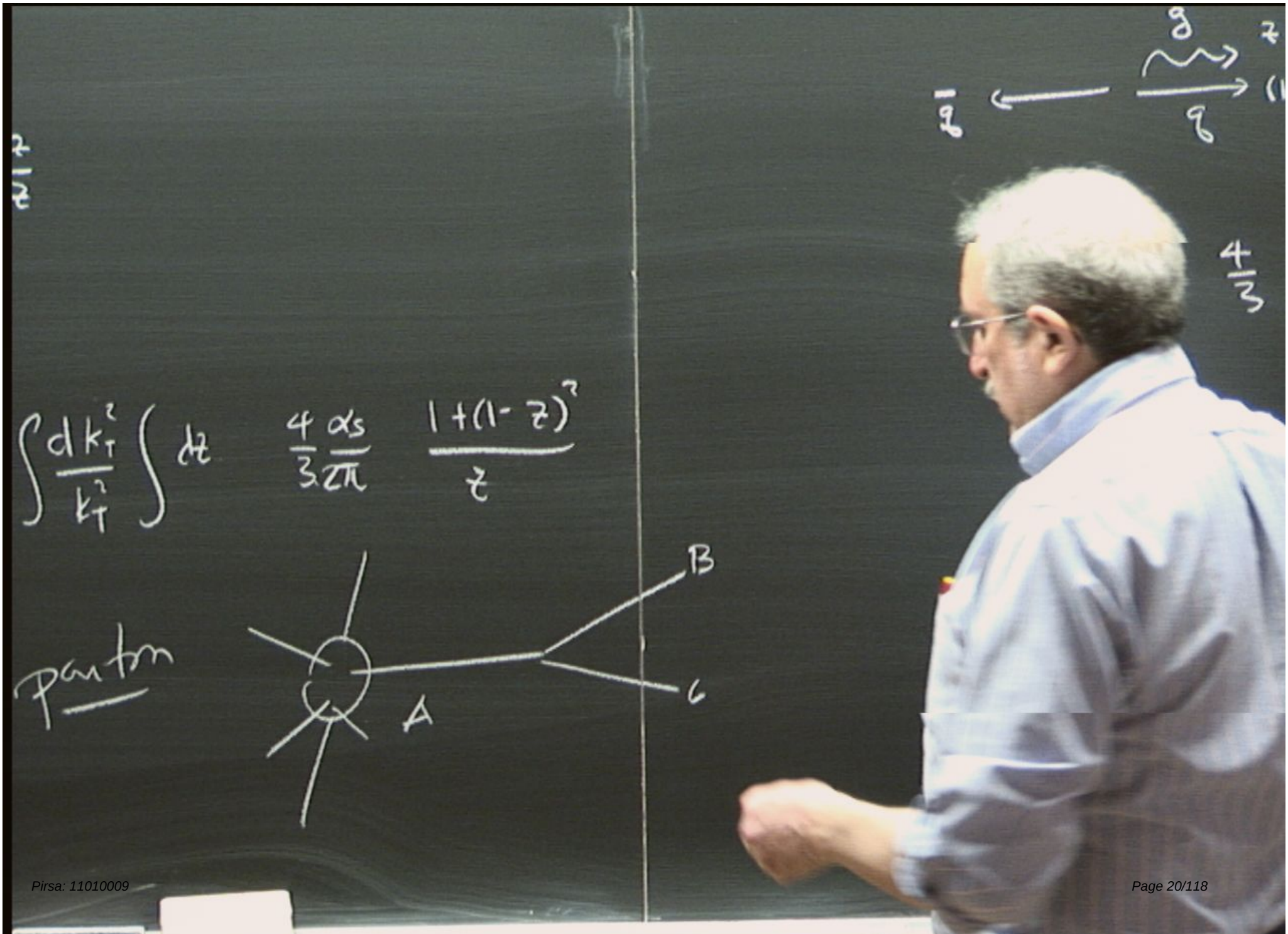
$x_1 = (1-z)$ $\quad x_2 \approx 1$

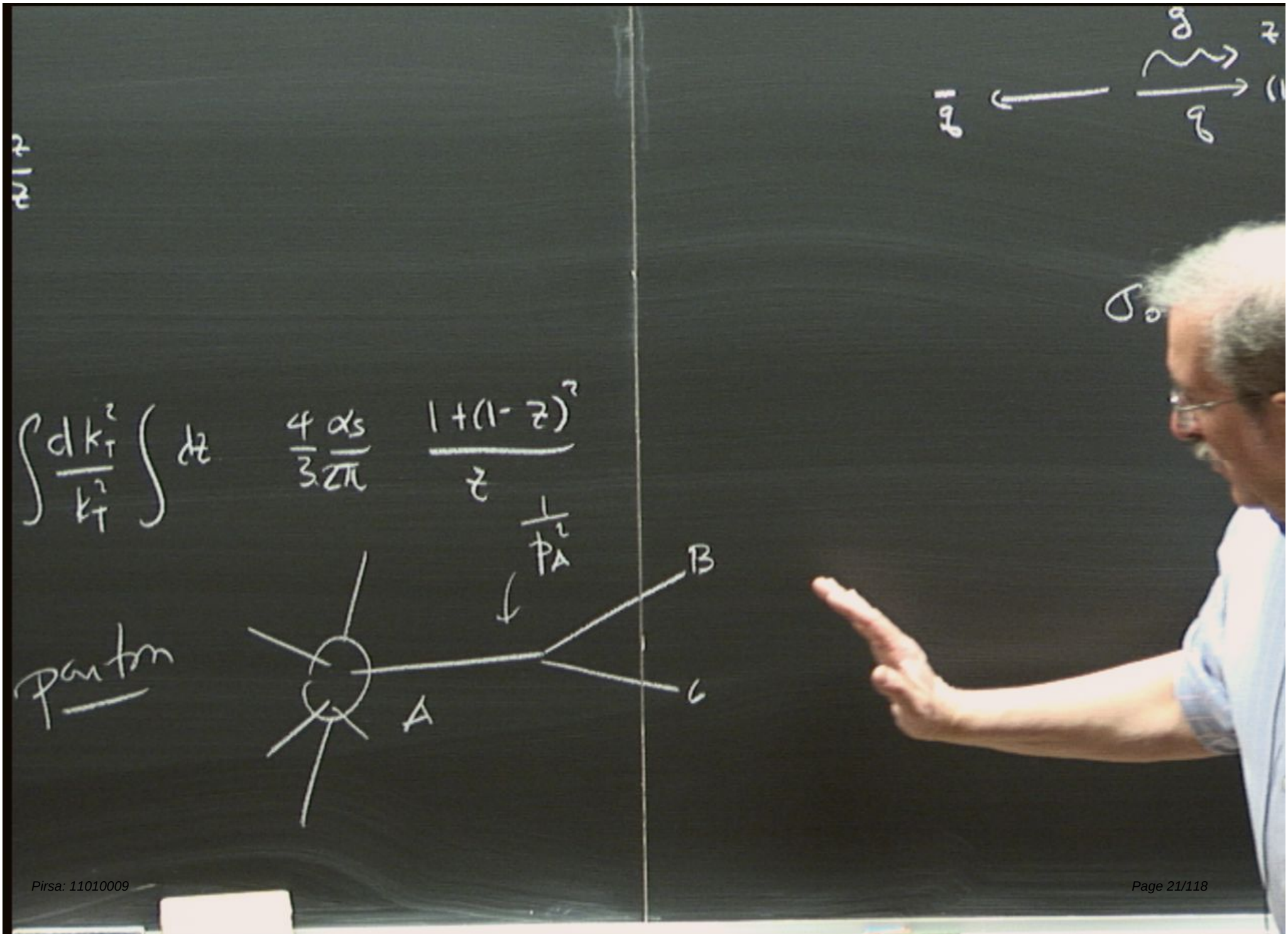
$(k+q)^2 = Q^2(1-x_2)$

$$\frac{dx_2}{1-x_2} \sim \frac{dk_T^2}{k_T^2}$$

$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$


$\frac{dz}{z}$
$\int \frac{dk_T^2}{k_T^2} \int dz \quad \frac{4}{3}\frac{\alpha_s}{2\pi} \quad \frac{1+(1-z)^2}{z}$
parton
A
$\bar{q}$
q
g
σ_0


$\int \frac{dk_T^2}{k_T^2} \int dz \quad \frac{4}{3}\frac{\alpha_s}{2\pi} \quad \frac{1+(1-z)^2}{z}$
$\frac{4}{3}$
parton
A
B
C


$\int \frac{dk_T^2}{k_T^2} \int dz \quad \frac{4}{3}\frac{\alpha_s}{2\pi} \quad \frac{1+(1-z)^2}{z}$
$\frac{1}{p_A^2}$
parton
A
B
C
$\bar{q}$
q
g
σ_0

$\frac{z}{\bar{z}}$

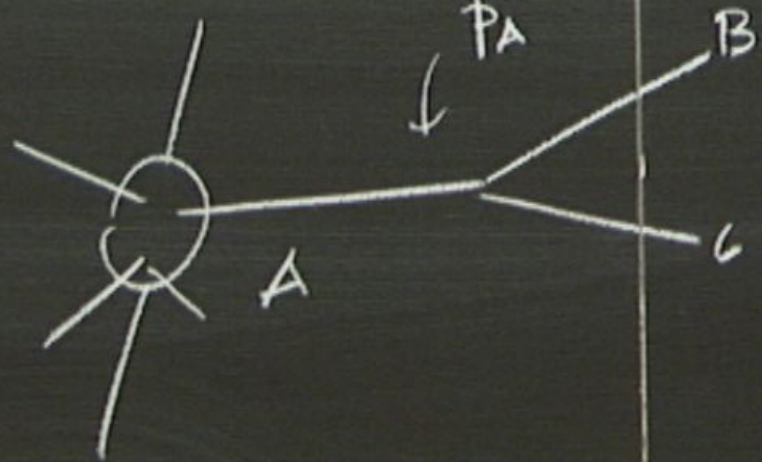
$\bar{q} \longleftarrow \xrightarrow{q} \; (\; \overset{g}{\rightsquigarrow} \; z$

$\sigma_0 \quad \frac{4}{3}$

$$\int \frac{dk_T^2}{k_T^2} \int dz \quad \frac{4}{3}\frac{\alpha_s}{2\pi} \quad \frac{1+(1-z)^2}{z}$$

$\sigma(\to BC)$

$\frac{1}{p_A^2}$



parton

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\bar{q} \longleftarrow \xrightarrow[q]{} (1-z) \overset{g}{\rightsquigarrow} z$$

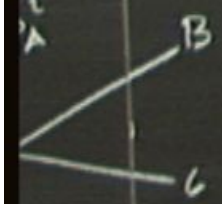
$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$

$$\frac{dx_2}{1-x_2} \sim \frac{dk_T^2}{k_T^2}$$

$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz$$



$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

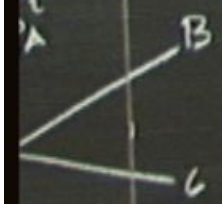
$$\bar{q} \longleftarrow \xrightarrow[q]{g \rightsquigarrow z} (1-z)$$

$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$ $\quad \dfrac{dx_2}{1-x_2} \sim \dfrac{dk_T^2}{k_T^2}$

$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \, \frac{\alpha_s}{2\pi} \, P_{A\to B}(z)$$



$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\bar{q} \longleftarrow \xrightarrow[q]{g \rightsquigarrow z} (1-z)$$

$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$ $\quad \frac{dx_2}{1-x_2} \sim \frac{dk_T^2}{k_T^2}$

$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \, \frac{\alpha_s}{2\pi} \, P_{A\to B}(z)$$

Altarelli +

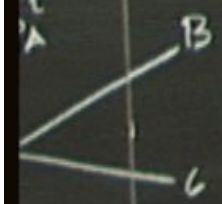
$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \, \frac{2\alpha_s}{3\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$\bar{q} \longleftarrow \xrightarrow[q]{g \rightsquigarrow z} (1-z) \qquad x_1 = (1-z) \qquad x_2 \approx 1$$

$$(k+q)^2 = Q^2(1-x_2) \qquad \frac{dx_2}{1-x_2} \sim \frac{dk_T^2}{k_T^2}$$

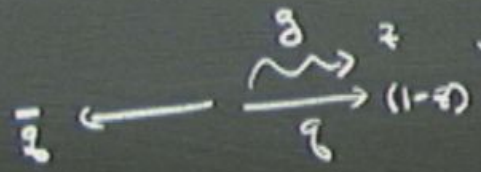
$$\sigma_0 \, \frac{4}{3} \, \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2} \, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \, \frac{\alpha_s}{2\pi} \, \underbrace{P_{A\to B}(z)}$$



Altarelli, Parisi
Splitting Functions.

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

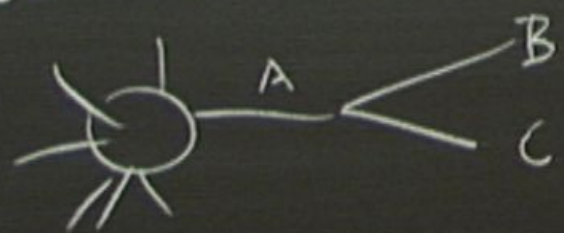


$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$ $\qquad \dfrac{dx_2}{1-x_2} \sim \dfrac{dk_T^2}{k_T^2}$

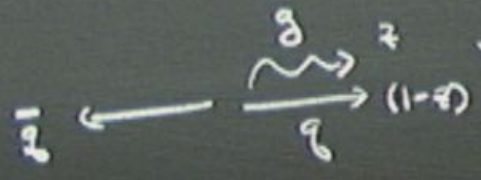
$$\sigma_0 \frac{4}{3} \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2}\, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \, \frac{\alpha_s}{2\pi} \underbrace{P_{A\to B}(z)}$$



Altarelli, Parisi
Splitting Functions.

$$\sigma(e^+e^- \to q\bar{q}g) = \int dx_1\, dx_2 \quad \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

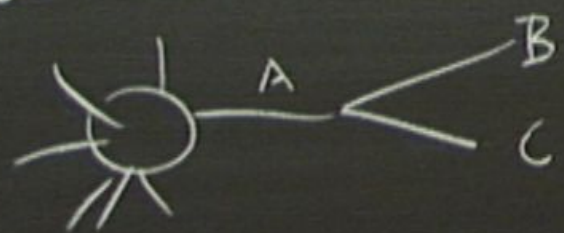


$x_1 = (1-z)$ $\quad x_2 \approx 1$

$(k+q)^2 = Q^2(1-x_2)$ $\quad \dfrac{dx_2}{1-x_2} \sim \dfrac{dk_T^2}{k_T^2}$

$$\sigma_0 \frac{4}{3} \frac{\alpha_s}{2\pi} \int \frac{dk_T^2}{k_T^2}\, dz \, \frac{1+(1-z)^2}{z} \quad \checkmark$$

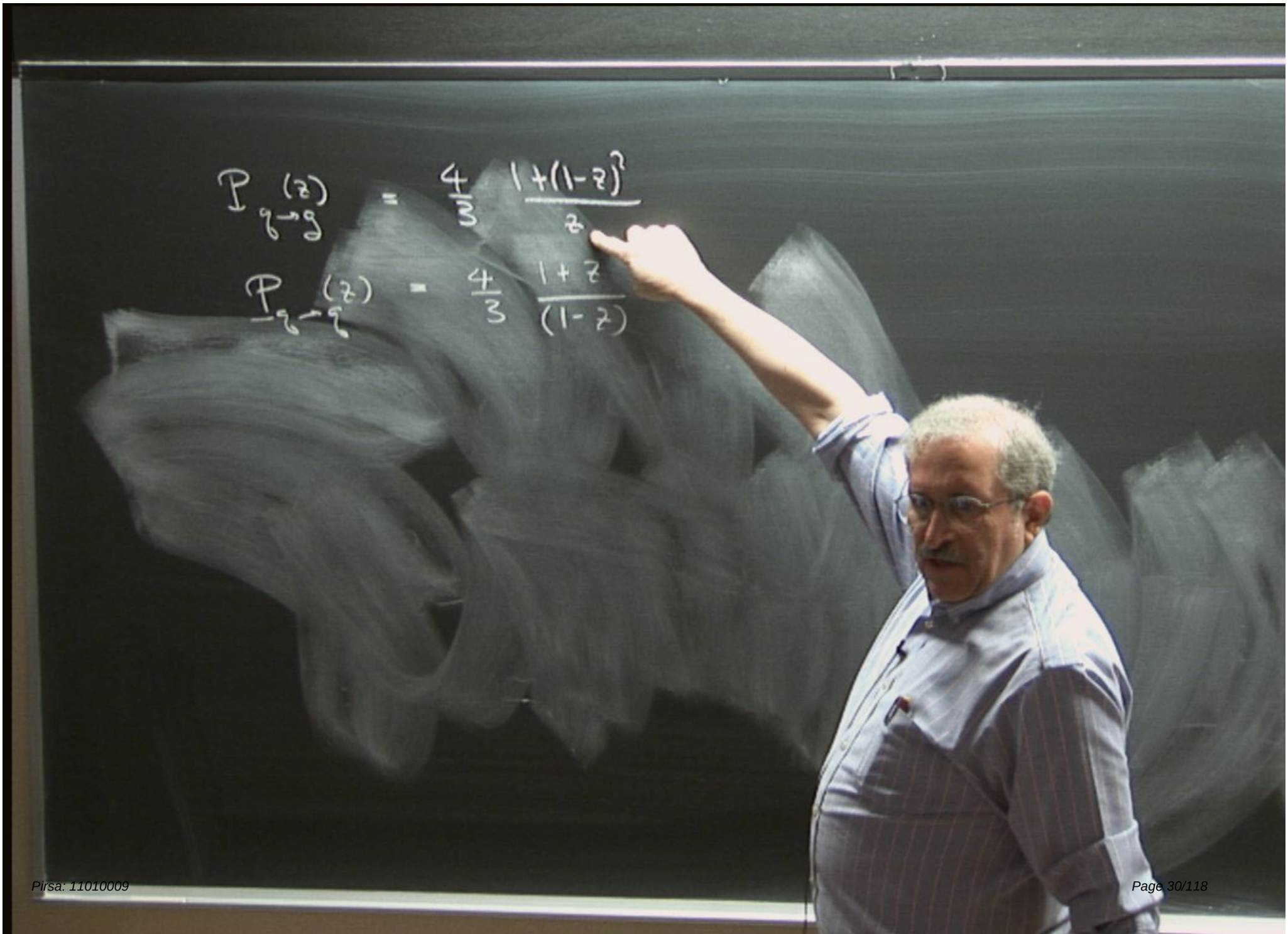
$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \, \frac{\alpha_s}{2\pi} \underbrace{P_{A\to B}(z)}$$



Altarelli, Parisi
Splitting Functions.

$$P_{q\to g}(z) = \frac{4}{3}\,\frac{1+(1-z)^2}{z}$$

$$P_{q\to q}(z) =$$


$P_{q\to g}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$
$P_{q\to q}(z) = \frac{4}{3}\frac{1+z}{(1-z)}$

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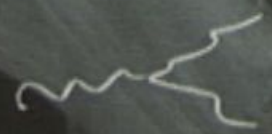
$P_{g\to q}$

$P_{g\to g}$



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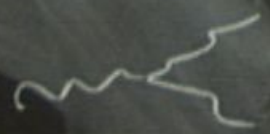
$$P_{g\to q} = P_{g\to \bar{q}} = \frac{1}{2}\left[z+(1-z)^2\right]$$

$$P_{g\to g}$$



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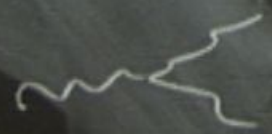
$$P_{g\to g}$$

$$P_{g\to g} = 3\,[$$



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$P_{g\to g}$

$$P_{g\to g} = 3\left[\frac{1}{z(1-z)} + \frac{z^3}{1-z} + \frac{(1-z)^3}{z} - B\,\delta(z-1)\right]$$

$$P_{q\to q}(z) = \frac{4}{3}\,\frac{1+(1-z)^2}{z}$$

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$q \to g$

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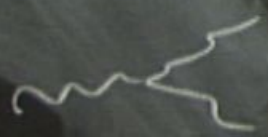
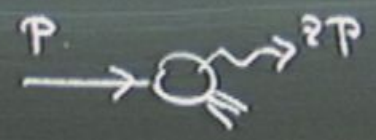


$$P_{q\to q}(z) = \frac{4}{3}\,\frac{1+(1-z)^2}{z}$$



$q \to g$

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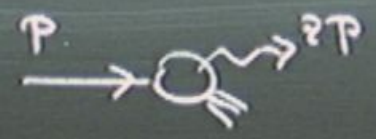
$(\quad)^4$

$$P_{q\to g}(z) = \frac{4}{3}\,\frac{1+(1-z)^2}{z}$$



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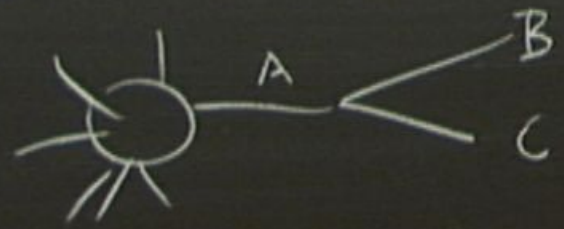
MHV

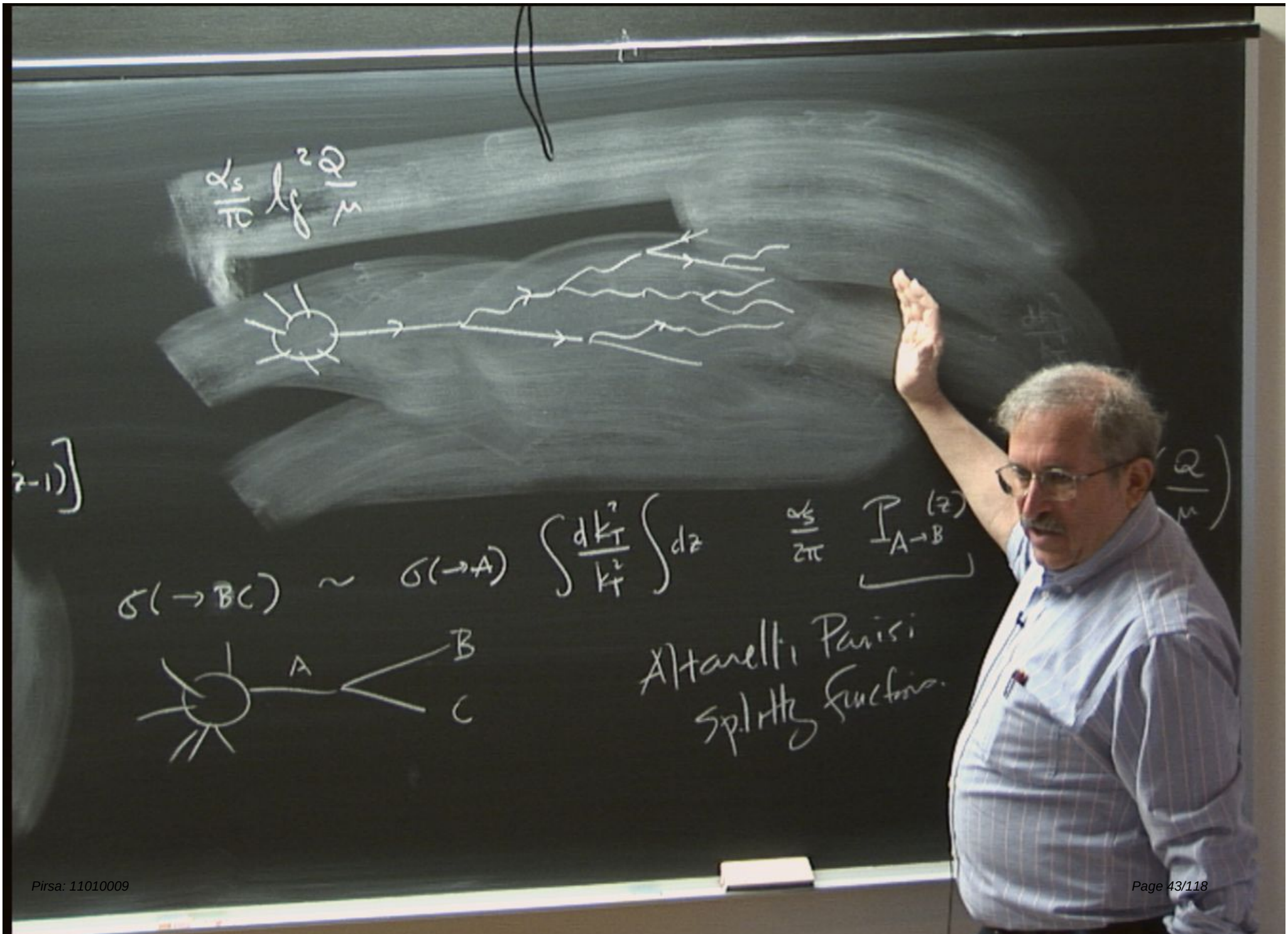
$(\quad)^4$

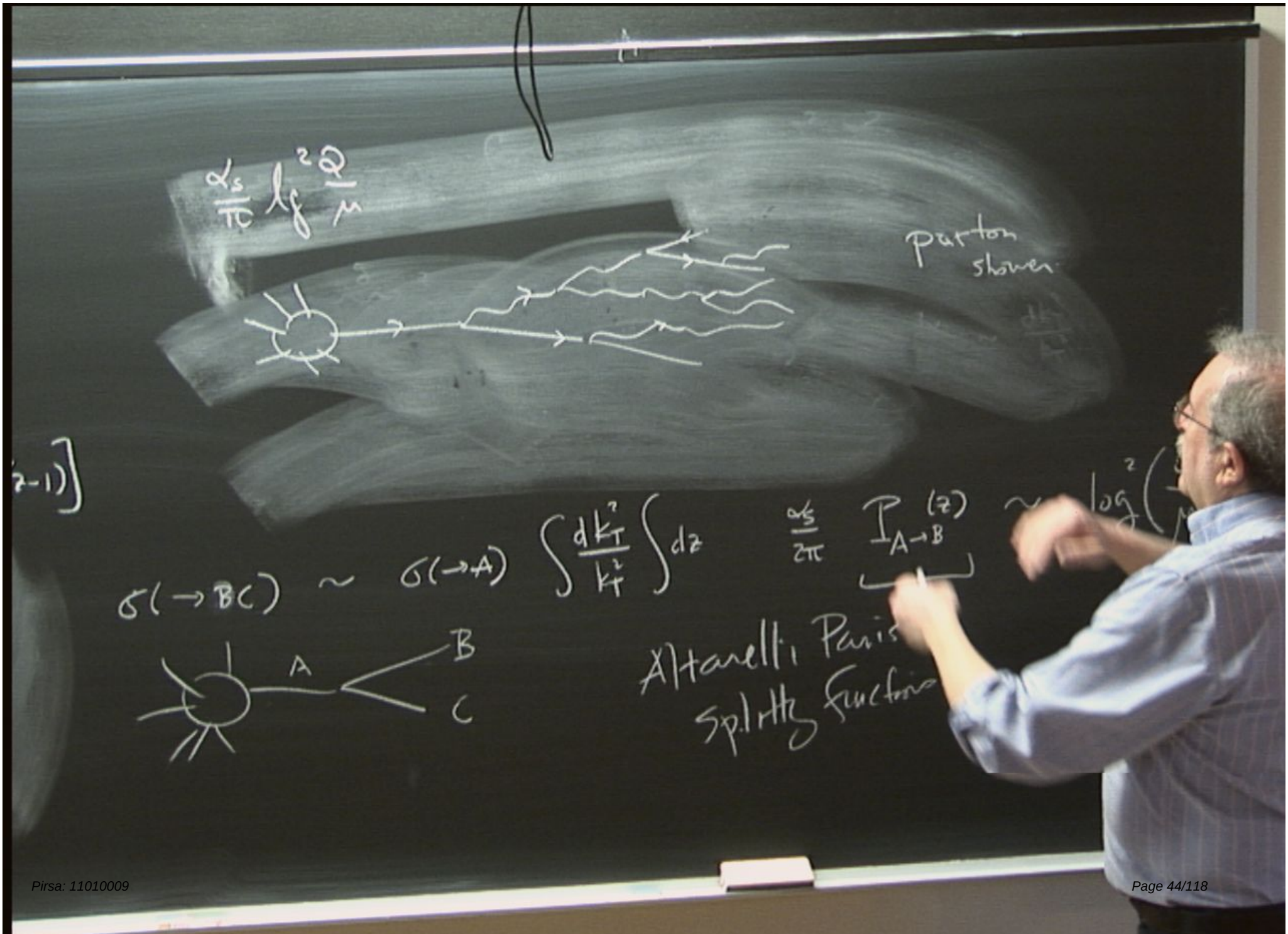
$$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{\mu}$$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \; \frac{\alpha_s}{2\pi} \underbrace{P_{A\to B}(z)} \sim \log^2\left(\frac{Q}{\mu}\right)$$



Altarelli, Parisi
splitting function.


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \frac{\alpha_s}{2\pi} P_{A\to B}(z)$
A
B
C
Altarelli, Parisi
splitting function.


$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{\mu}$
parton shower
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \frac{\alpha_s}{2\pi} P_{A\to B}(z) \sim \log^2$
A
B
C
Altarelli, Paris
splitting function

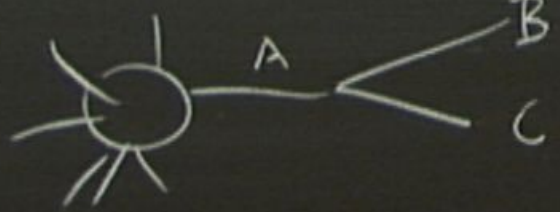
$\frac{\alpha_s}{\pi} \ln^2 \frac{Q}{\mu}$

$\alpha_s \sim 1$

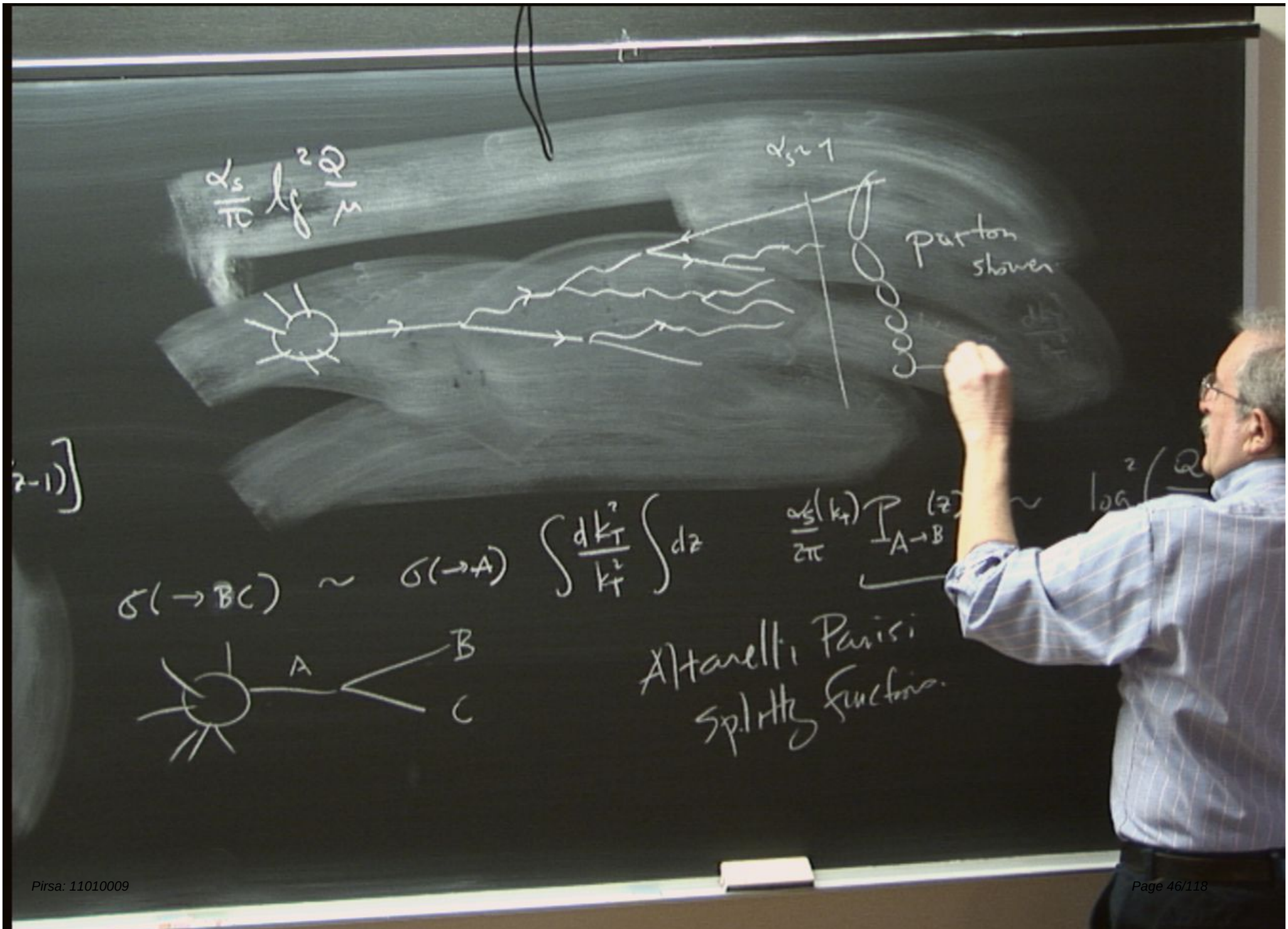
parton shower

$z-1)]$

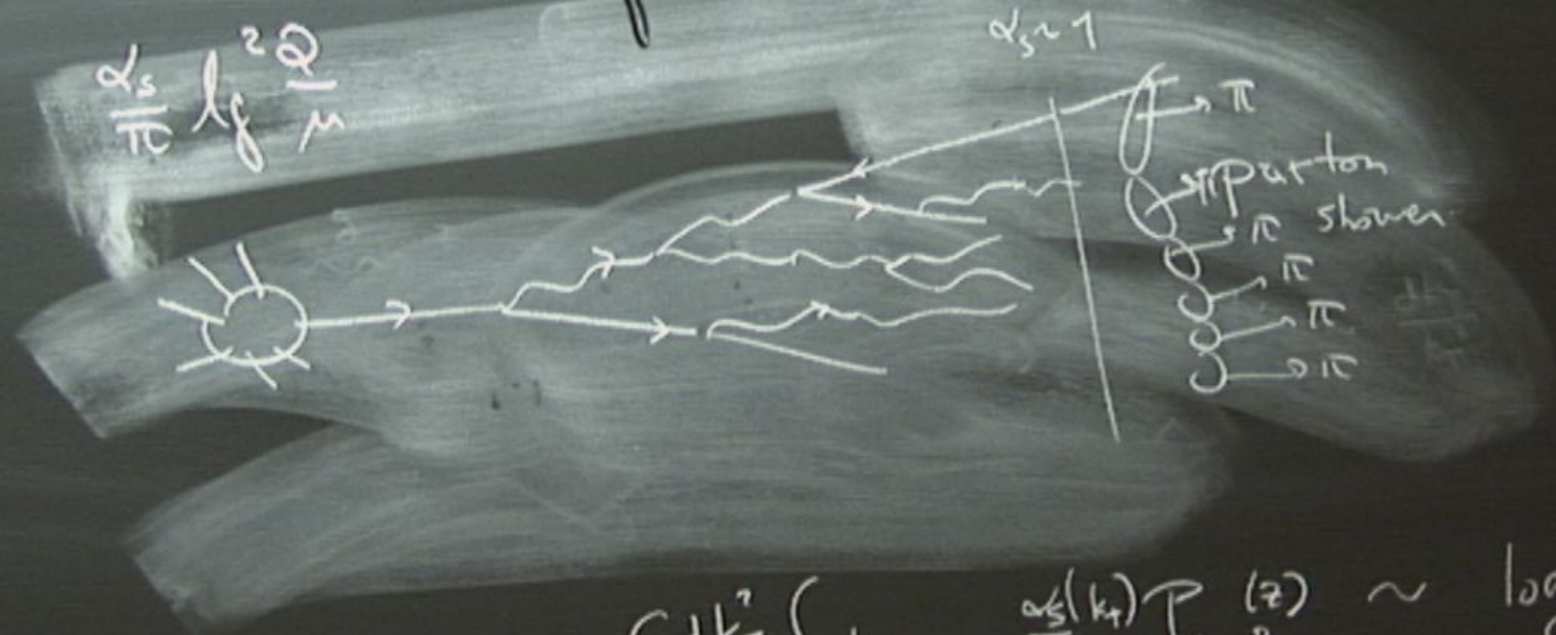
$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \; \frac{\alpha_s}{2\pi} \underbrace{P_{A\to B}(z)} \sim \log^2\left(\frac{Q}{\mu}\right)$$



Altarelli, Parisi
splitting function.

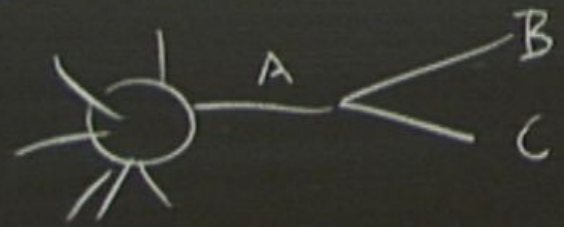

$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
parton shower
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \frac{\alpha_s(k_T)}{2\pi} P_{A\to B}(z) \sim \log^2$
A
B
C
Altarelli Parisi
splitting function

$\frac{\alpha_s}{\pi} \ln^2 \frac{Q}{\mu}$

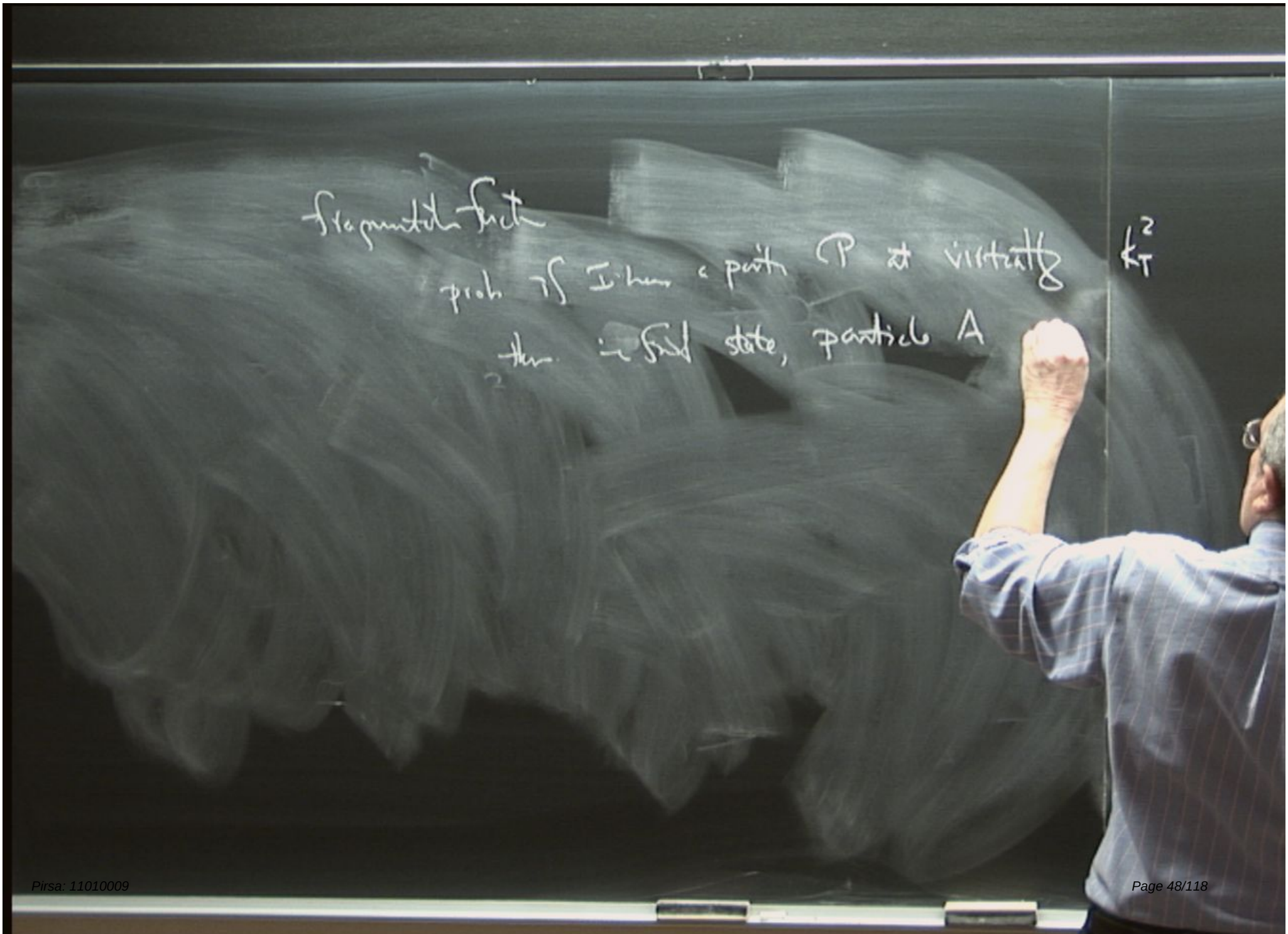


$z-1)]$

$$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \; \frac{\alpha_s(k_T)}{2\pi} \underbrace{P_{A\to B}(z)} \sim \log^2\left(\frac{Q}{\mu}\right)$$



Altarelli Parisi
splitting function.


fragmentation function
prob. of finding a parton P at virtuality k_T^2
then in final state, particle A

fragmentation function

prob. of finding a parton P at virtuality k_T^2

then in final state, particle A w.

non fract z q

$$\int dz \, f^{(q)}_{P\to A}$$

$\sigma($

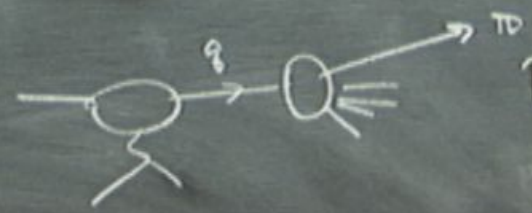
fragmentation fnctn

prob. of finding a parton P at virtuality k_T^2

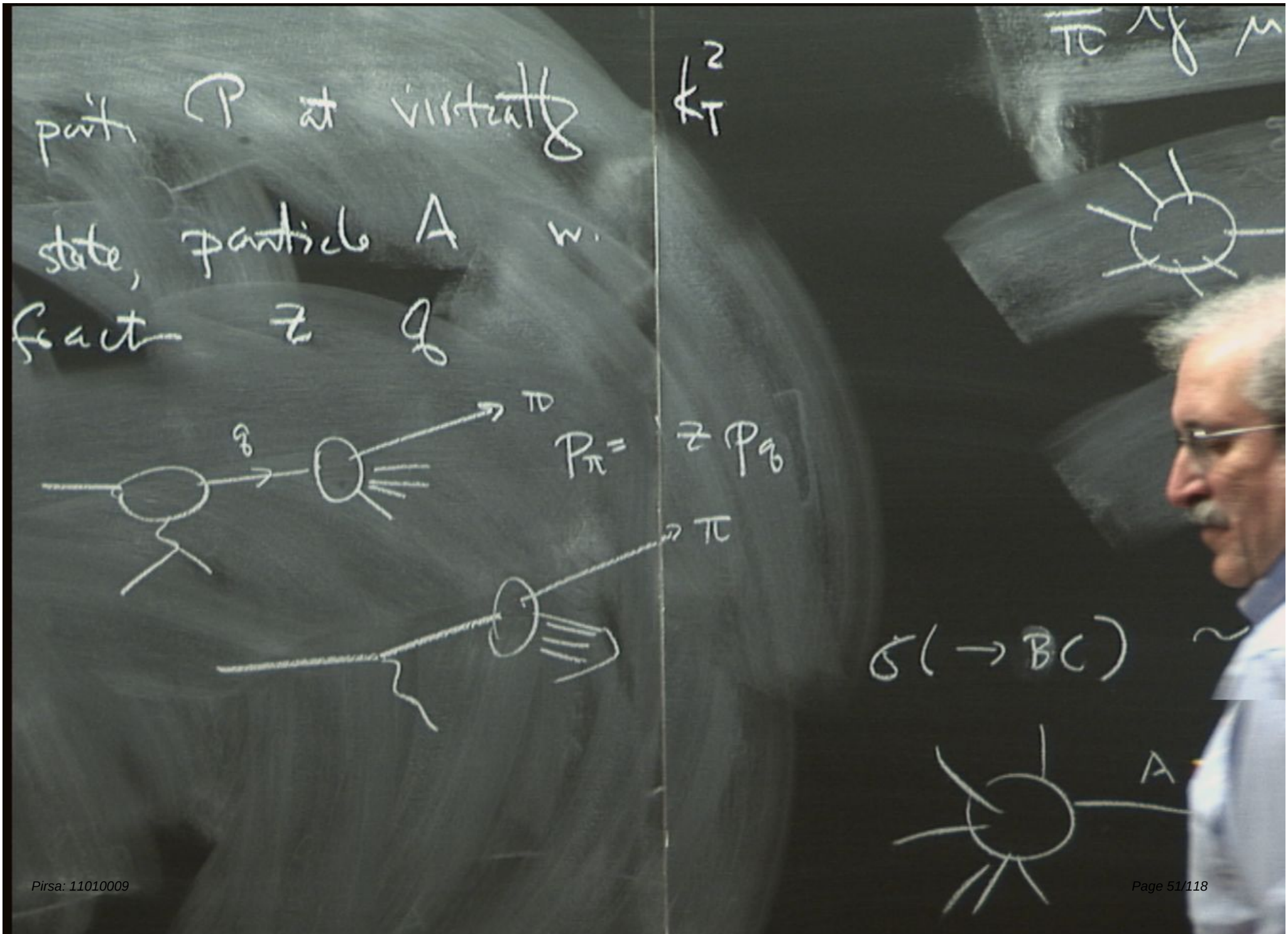
then in final state, particle A w.

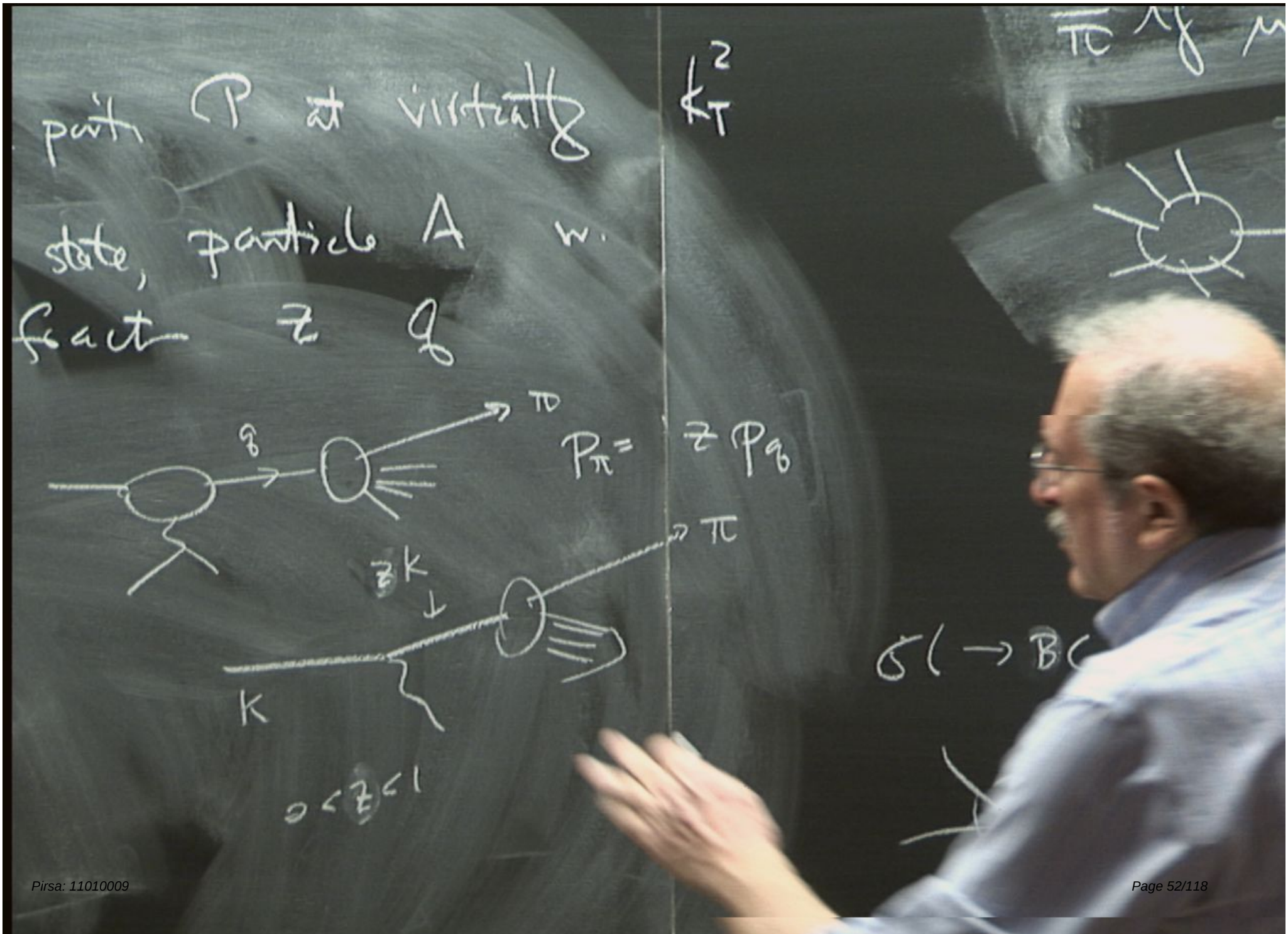
non fract. z q

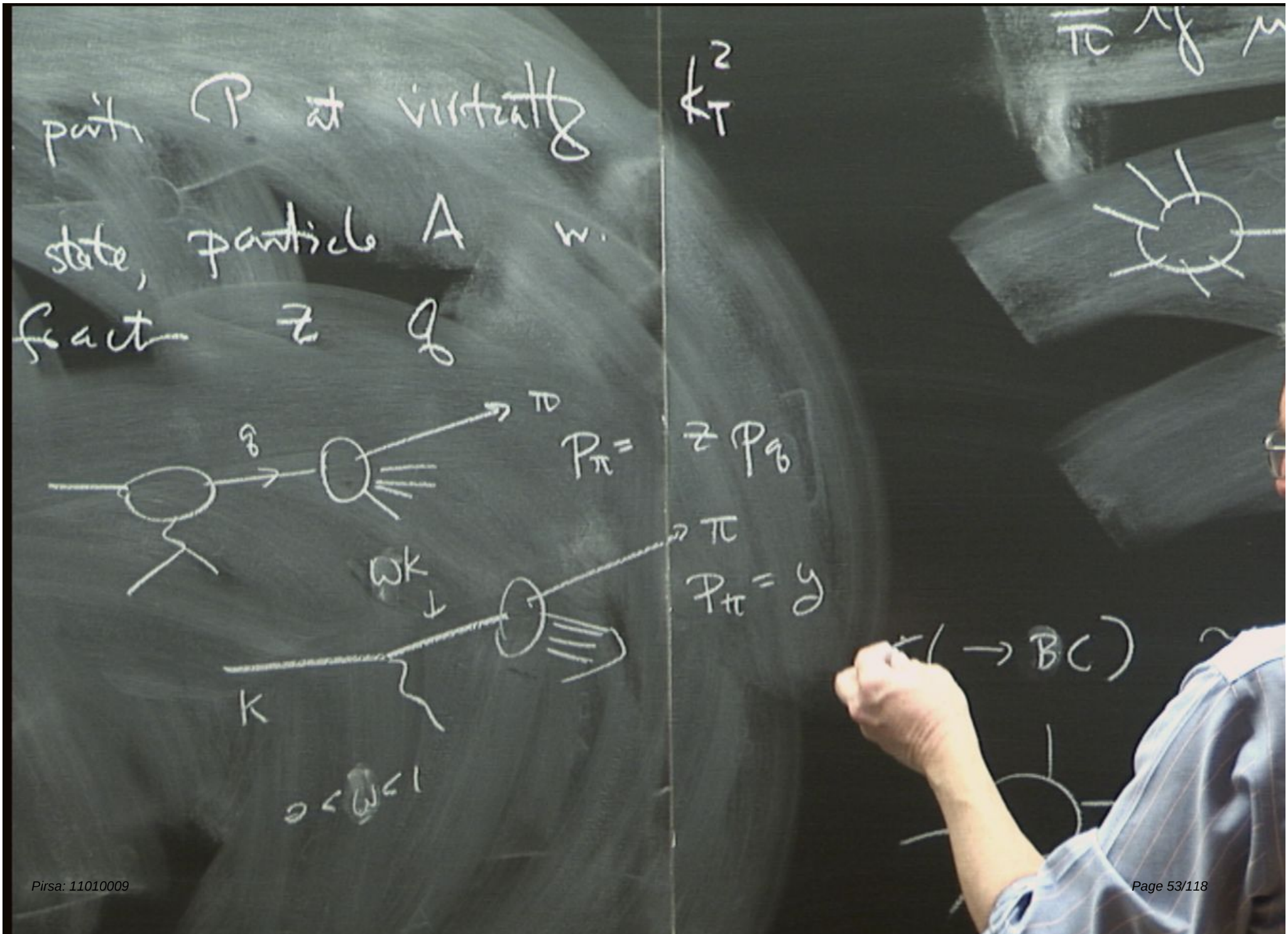
$\int dz \, f^{(q)}_{P\to A}$



$p_\pi = z\, p_q$


k_T^2
π
q
π
$P_\pi = z P_q$
π
$\sigma(\to BC) \sim$
A

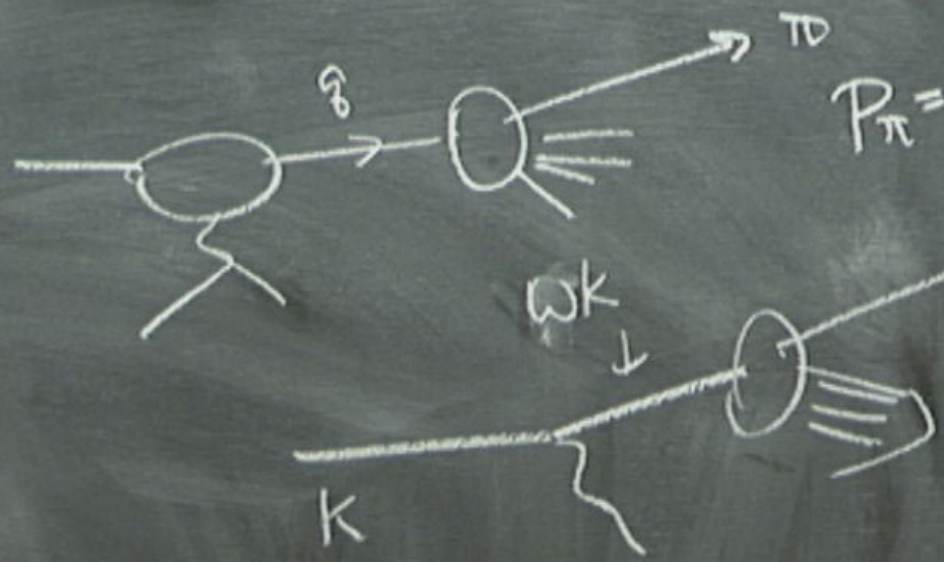

path P at virtuality k_T^2
state, particle A w.
fract z q
q
π
$P_\pi = z P_q$
π
zK
K
$0 < z < 1$
$\sigma(\to BC$


path P at virtuality k_T^2
state, particle A w.
fract z q
q
π
$P_\pi = z P_q$
π
$P_\pi = y$
ωk
K
$0 < \omega < 1$
π
$\to BC)$

path P at virtuality k_T^2

state, particle A w.

fract z q

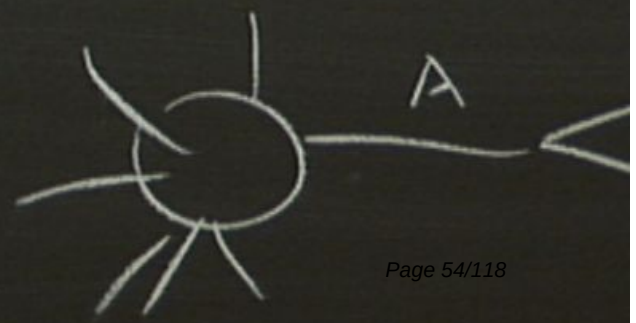


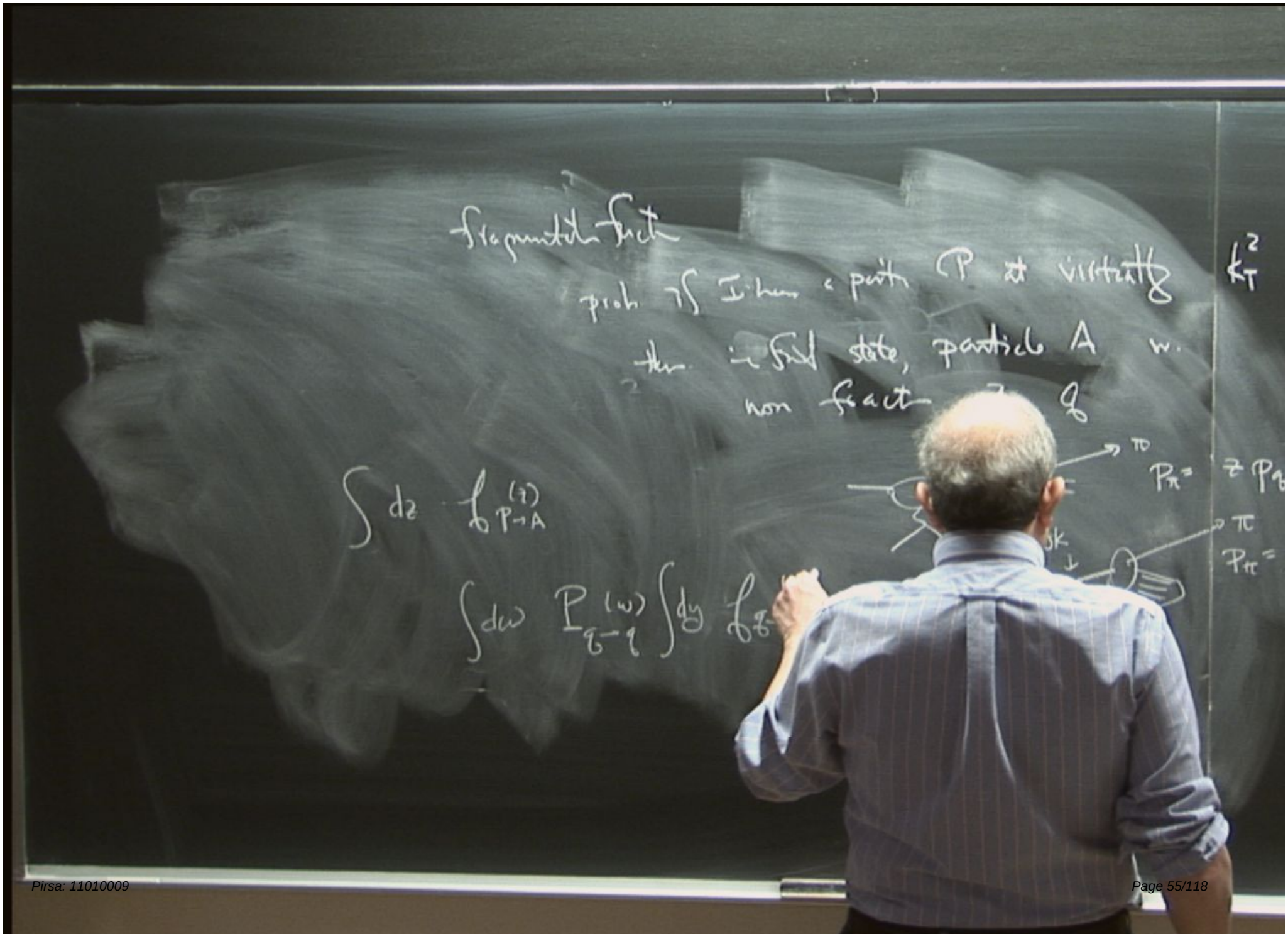
$P_\pi = z P_q$

$P_\pi = y P_q = \omega y k$

$0 < \omega < 1$

$\sigma(\to BC) \sim$




fragmentation function
prob. of I. hum a parton P at virtuality
the in final state, particle A w.
non fract
k_T^2
$\int dz \, f^{(q)}_{P\to A}$
$\int dw \, P^{(w)}_{q\to q} \int dy \, f_{q}$
π^0
$P_\pi =$
$z P_q$
π
$P_\pi =$

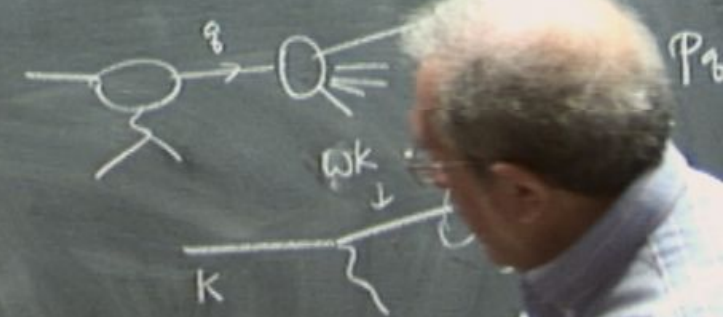
fragmentation function

prob of I hum a parton P at virtuality k_T^2

then in final state, particle A w.

non fract z q

$$\int dz \; f^{(q)}_{P\to A}$$



$$\int d\omega \; P^{(\omega)}_{q\to q} \int dy \; f^{(y)}_{q\to\pi}$$

$$\int dz \; \delta(z-\omega y) \qquad 0<\omega<1$$

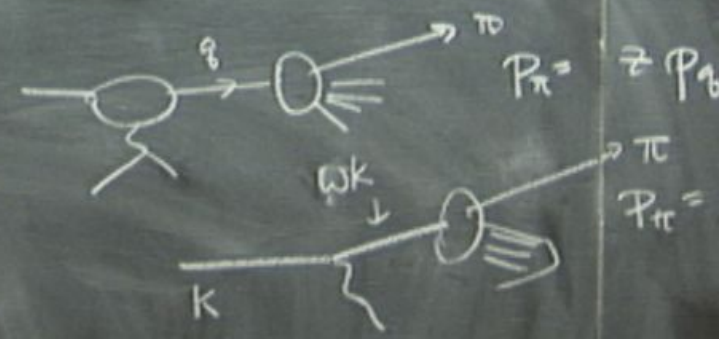
fragmentation function

prob of I. hum a path $\mathcal{P}$ at virtuality k_T^2

then in final state, particle A w.

non fract z of

$$\int dz \; f^{(q)}_{\mathcal{P}\to A}$$



$$\int d\omega \; P^{(\omega)}_{q\to q} \int dy \; f^{(y)}_{q\to\pi} \int dz \; \delta(z-\omega y) \qquad 0<\omega<1$$

$$= \int dz \int \frac{d\omega}{\omega} \; P^{(\omega)}_{q\to q} \; f_{q\to\pi}\left(\frac{z}{\omega}\right)$$

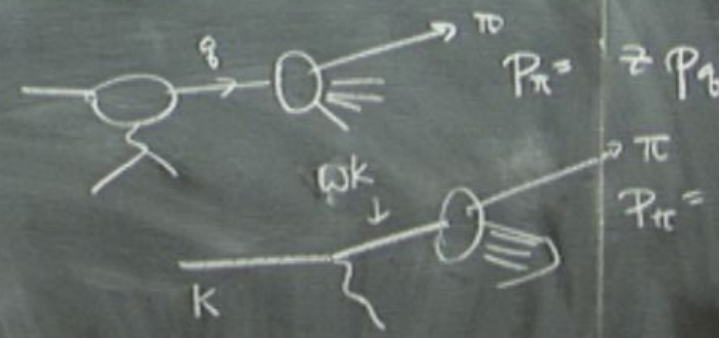
fragmentation function

prob. of I. hum a parton P at virtuality k_T^2

then in final state, particle A w.

mom fraction z of

$$\int dz \; f^{(z)}_{P\to A}$$



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fragmentation function

prob. of I. hum a parton $\mathcal{P}$ at virtuality k_T^2

then in final state, particle A w.

mom fract z q

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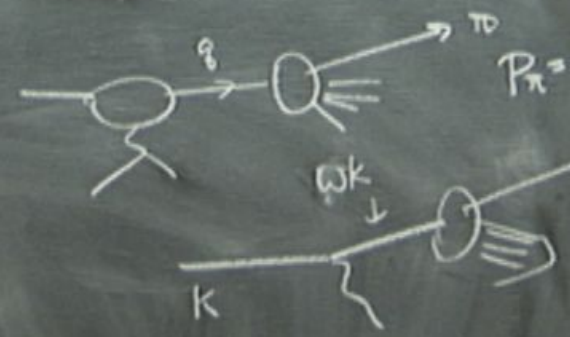
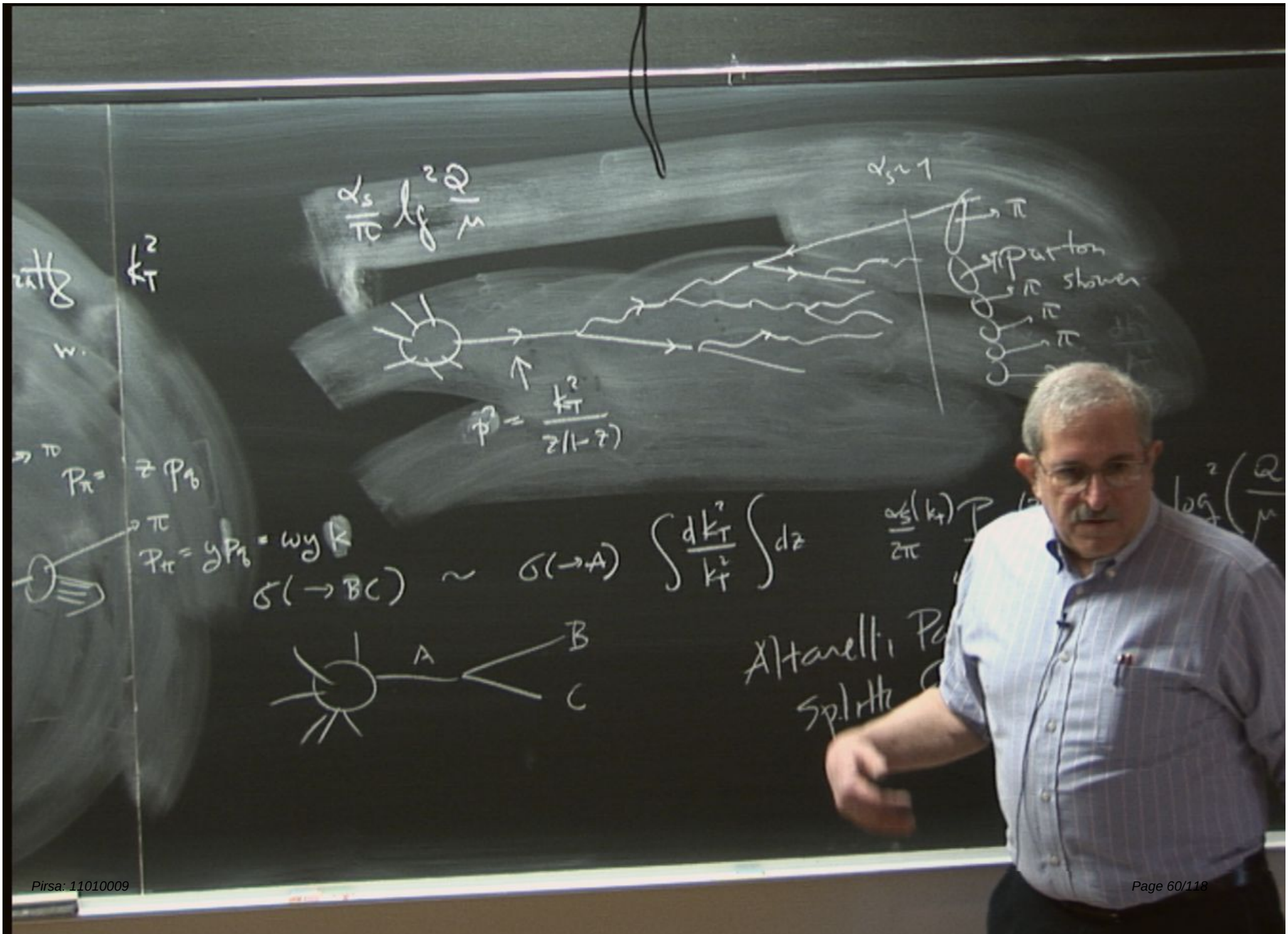
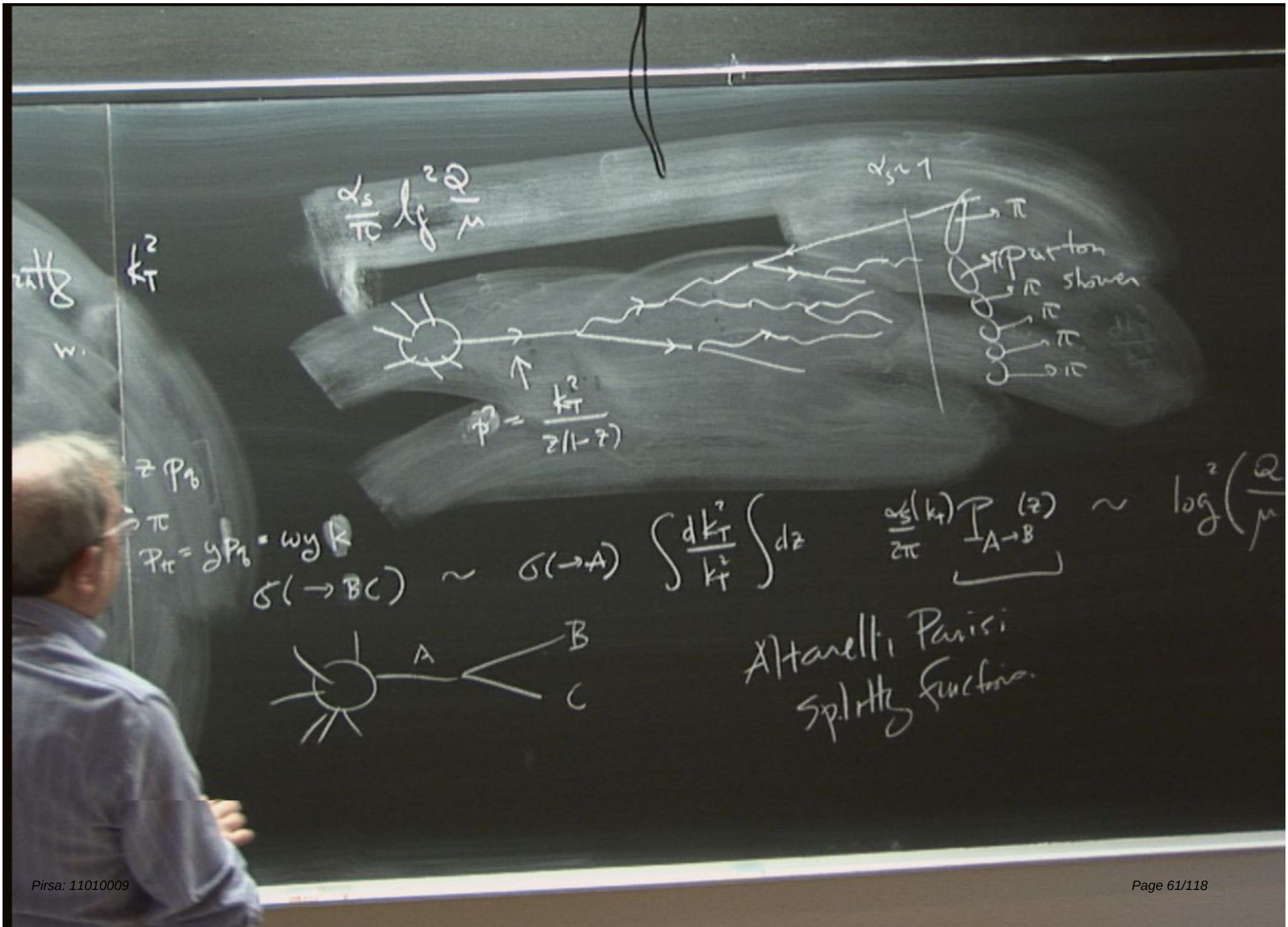
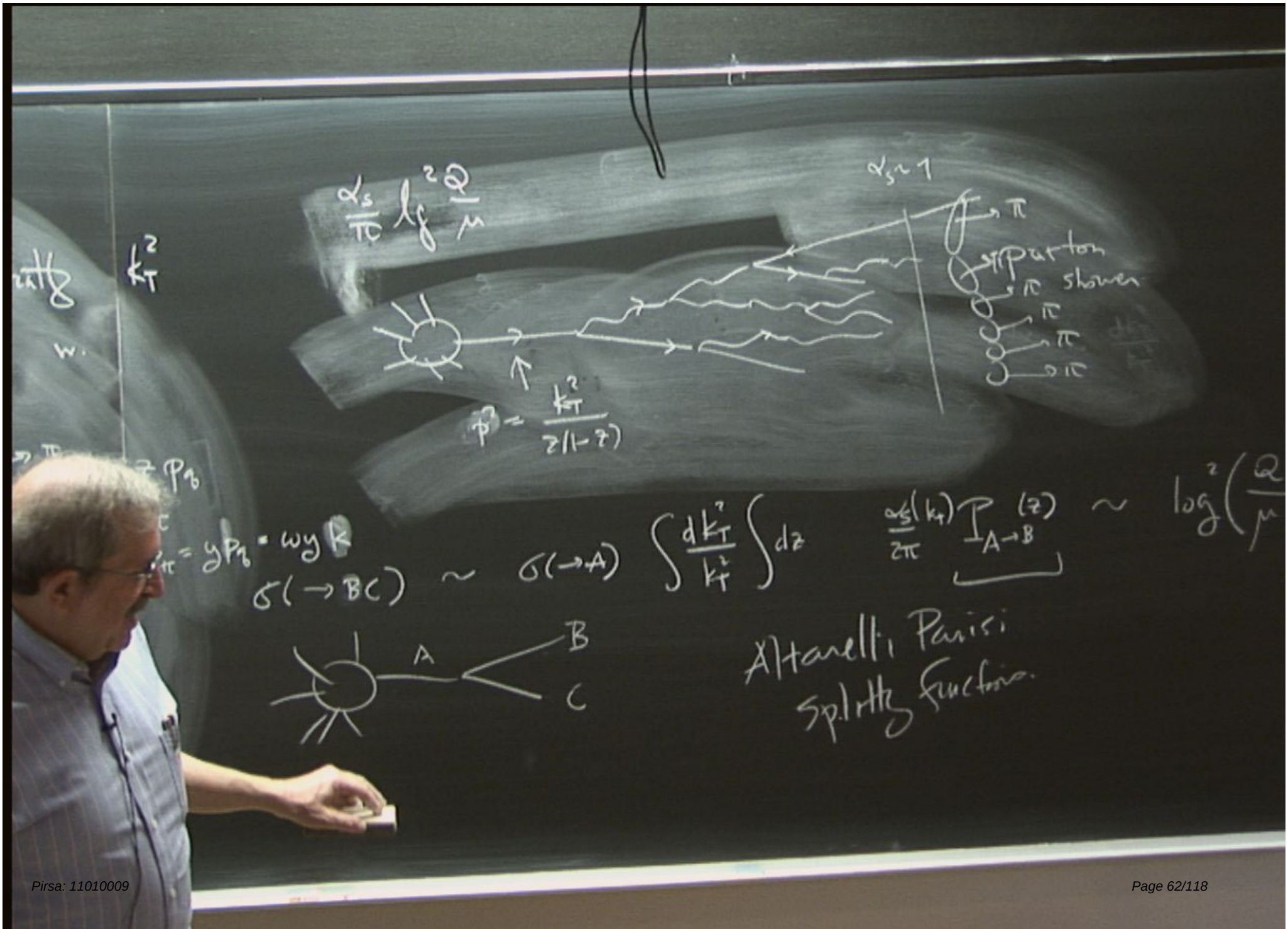
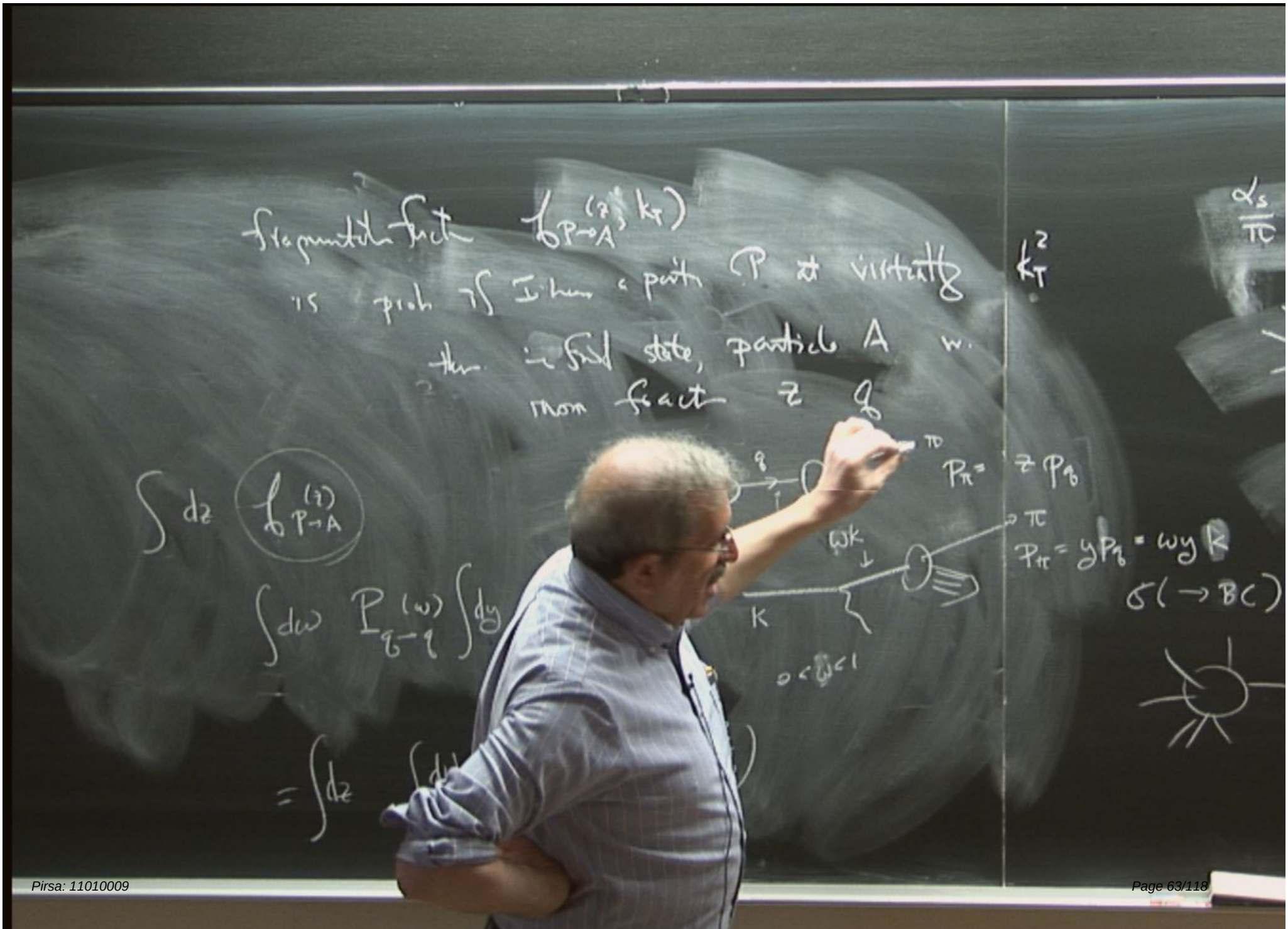



$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
k_T^2
parton shower
$p^2 = \frac{k_T^2}{z(1-z)}$
$p_\pi = z\, p_q$
$p_\pi = y\, p_q = \omega y\, k$
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz$
$\frac{\alpha_s(k_T)}{2\pi}$
A
B
C
Altarelli, Pa
Splitt


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
k_T^2
π
parton shower
π
π
π
π
$p^2 = \frac{k_T^2}{z(1-z)}$
$z\, p_B$
π
$p_\pi = y p_B = \omega y k$
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \; \frac{\alpha_s(k_T)}{2\pi} P_{A\to B}(z) \sim \log^2\left(\frac{Q}{\mu}\right.$
A
B
C
Altarelli, Parisi
splitting functions


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
k_T^2
π
parton shower
π
π
π
π
$p^2 = \frac{k_T^2}{z(1-z)}$
$\sigma(\to BC) \sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz \frac{\alpha_s(k_T)}{2\pi} P_{A\to B}(z) \sim \log^2\left(\frac{Q}{\mu}\right.$
A
B
C
Altarelli Parisi
splitting functions


fragmentation fnctn
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$P_\pi = z P_q$
$P_\pi = y P_q = \omega y k$
$\sigma(\to BC)$
$0 < \omega < 1$

Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega) f_{P'\to}$$

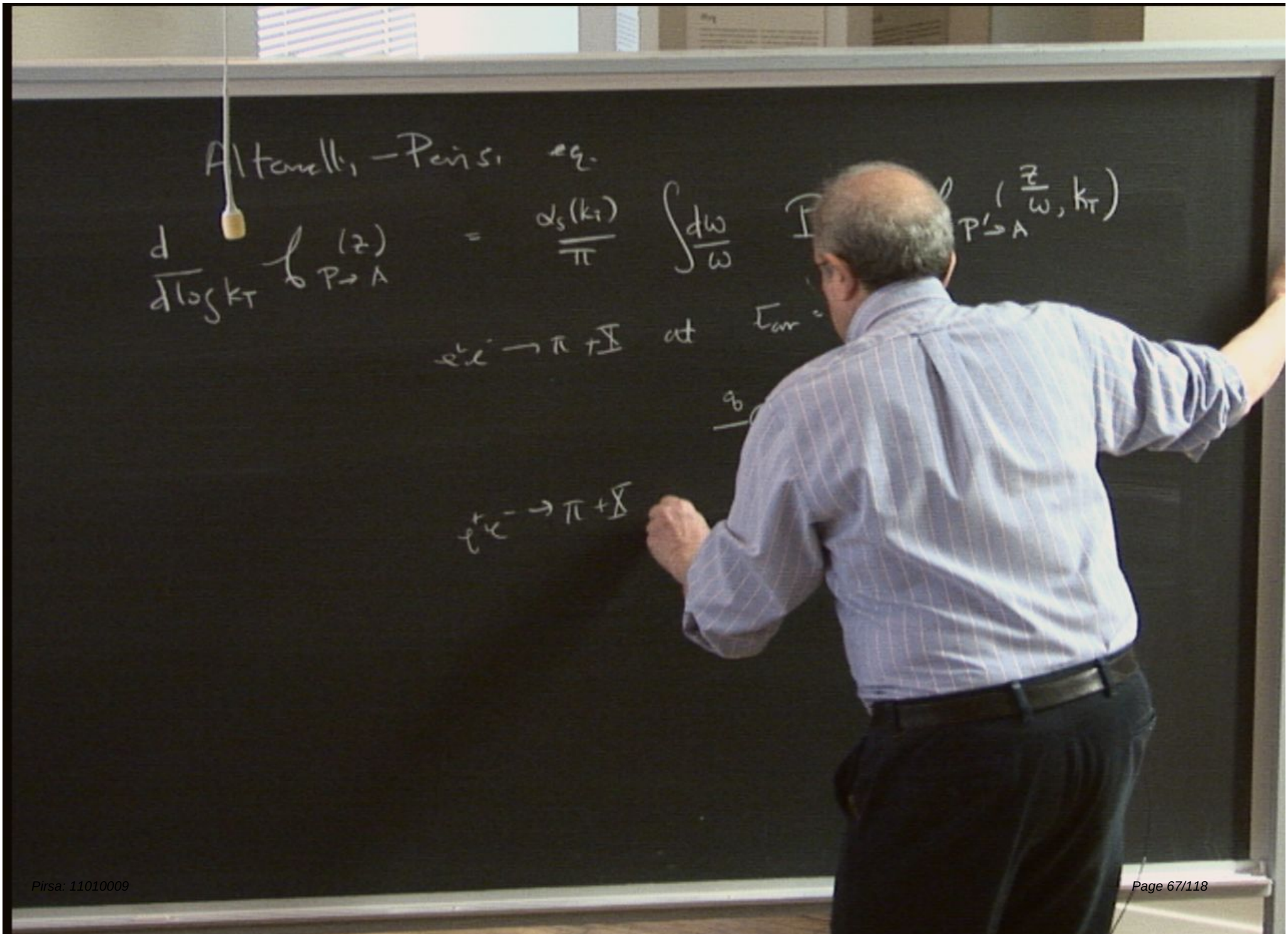
Altarelli, – Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

Altarelli-Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

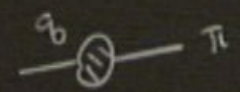
$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$


Altarelli, – Parisi eq.
$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega}$
$e^+e^- \to \pi + X$ at
$e^+e^- \to \pi + X$

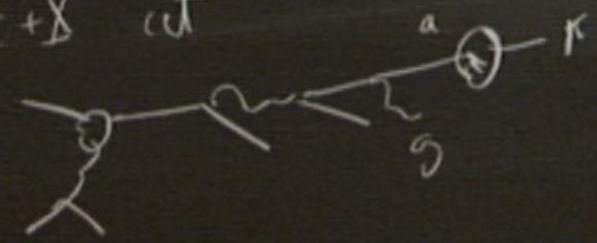
Altarelli – Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



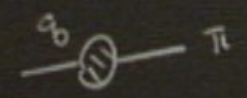
$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$



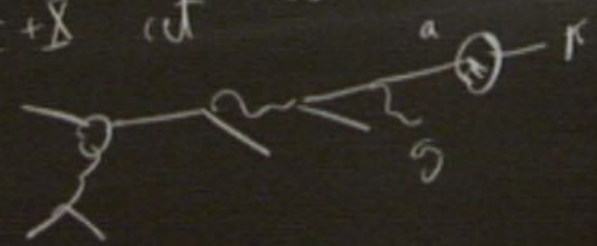
Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



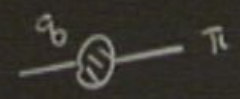
$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$



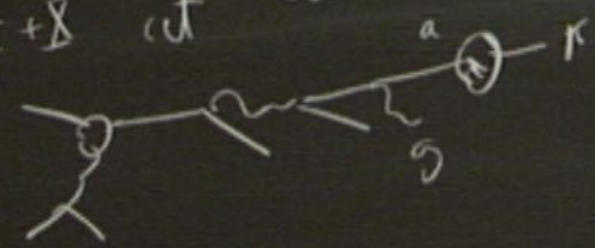
Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



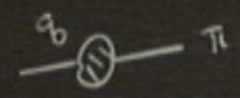
$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$



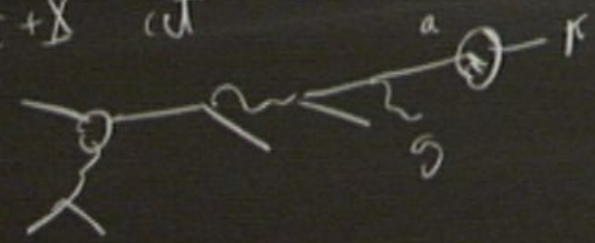
Altarelli–Parisi eq.

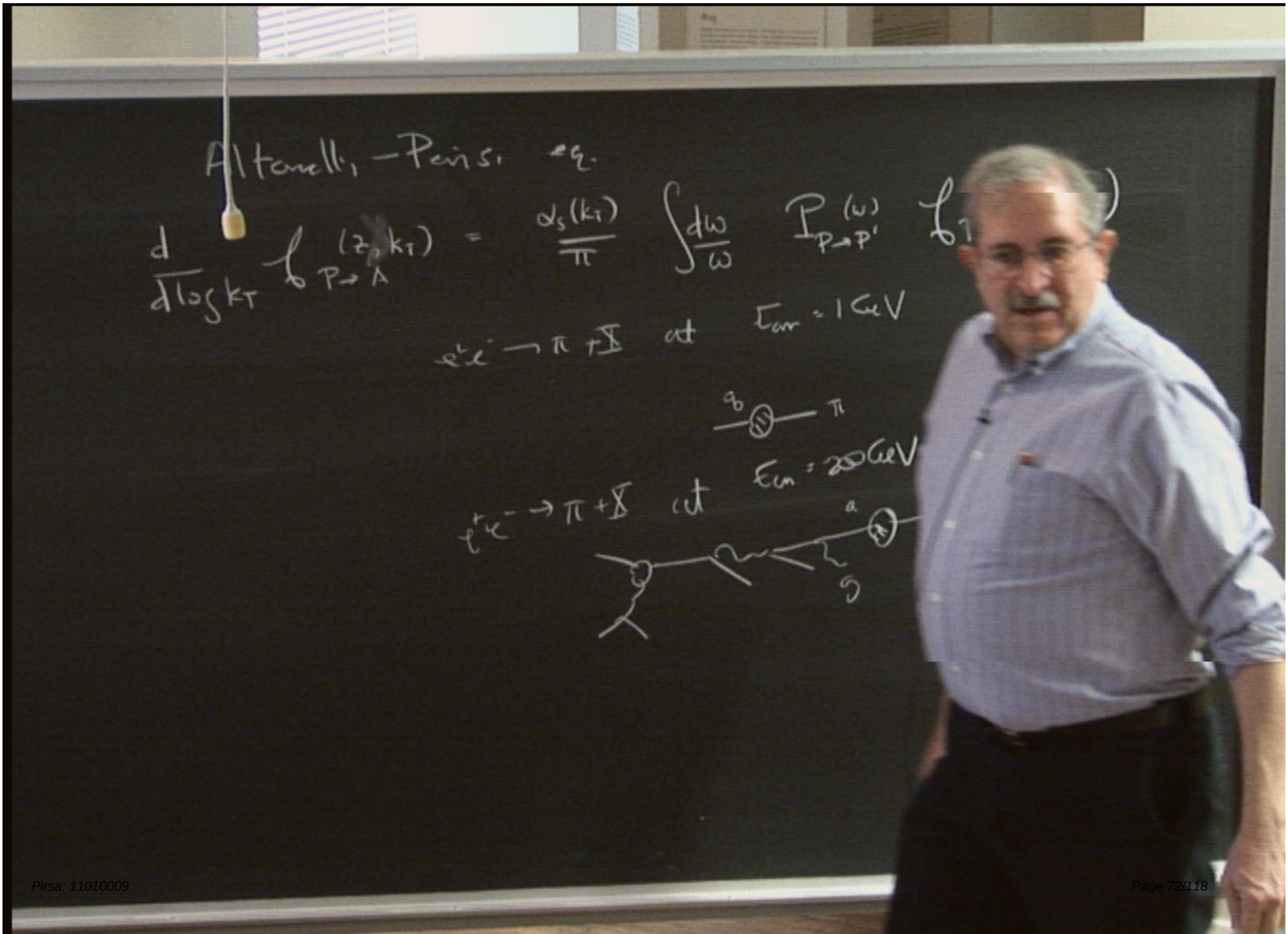
$$\frac{d}{d\log k_T} f_{P\to A}(z) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$

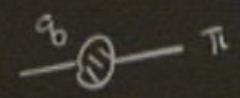



Altarelli – Parisi eq.
$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega) f$
$e^+e^- \to \pi + X$ at $E_{cm} = 1$ GeV
q_b
π
$E_{cm} = 200$ GeV
$e^+e^- \to \pi + X$ at
a
g

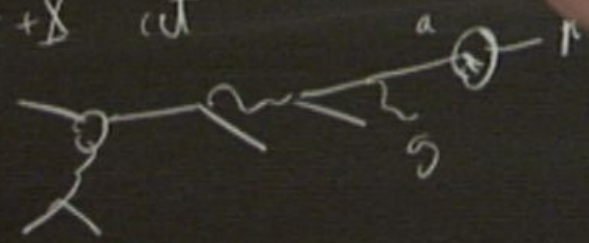
Altarelli – Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$

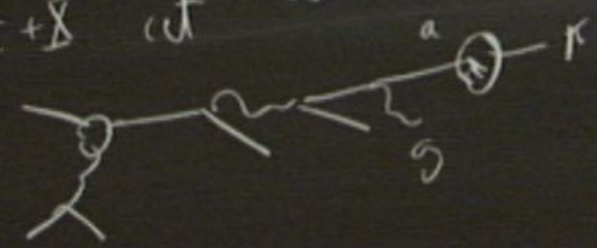


Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$

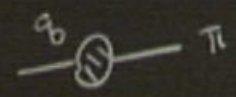
$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$



Altarelli – Parisi eq.

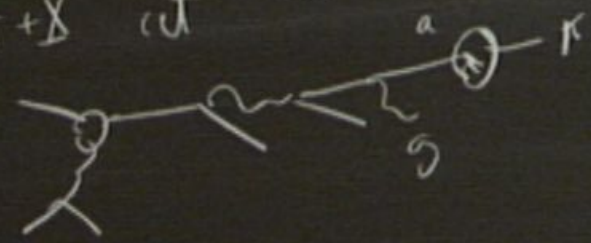
$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$



$E_{cm} = 200\,\mathrm{GeV}$

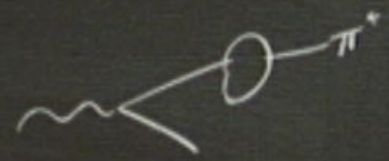
$e^+e^- \to \pi + X$ at

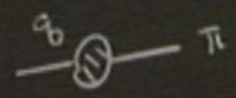


Altarelli – Parisi eq.

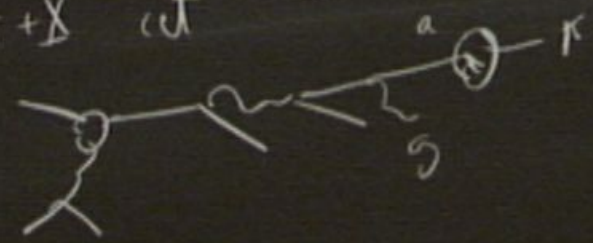
$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

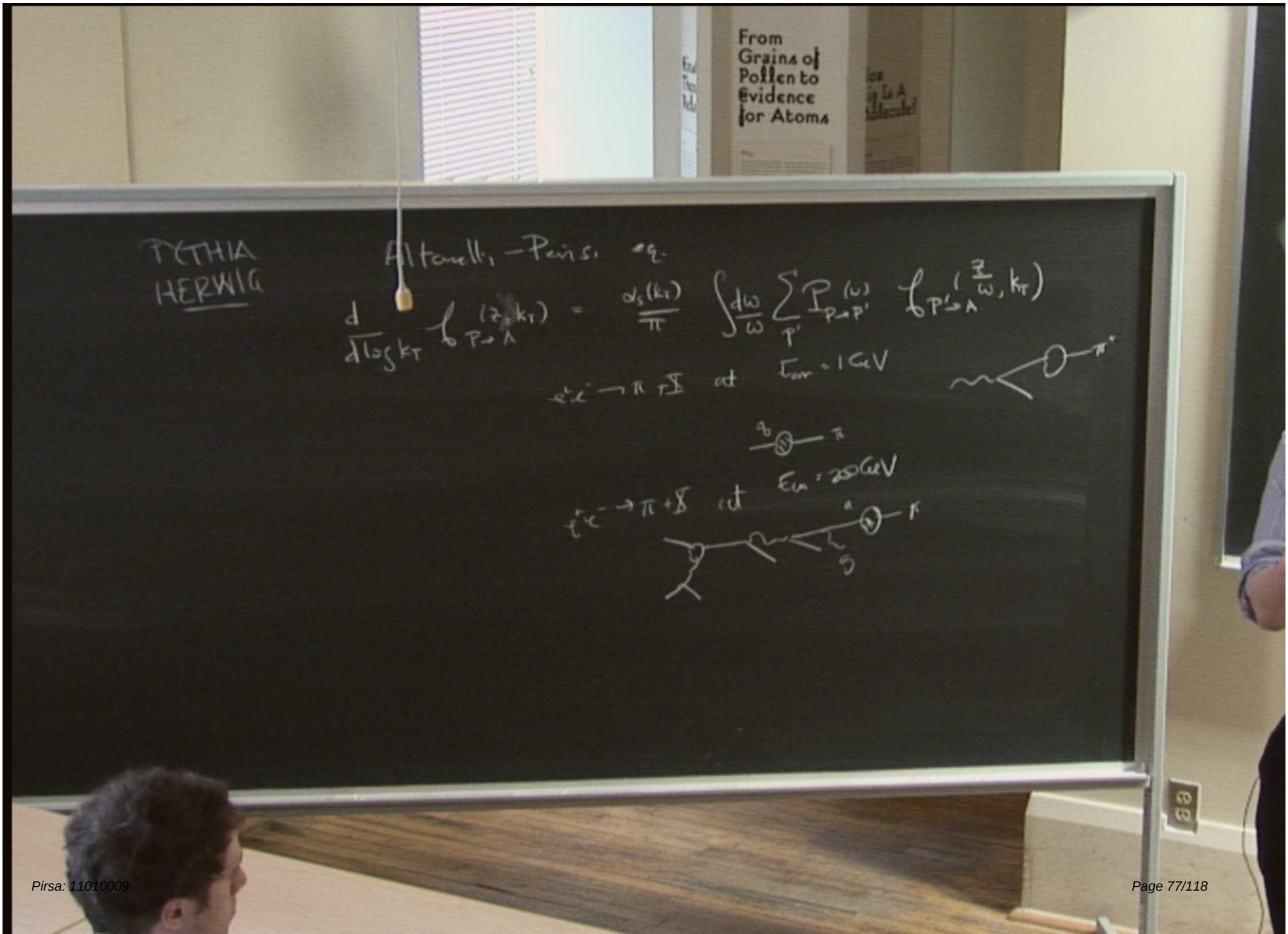
$e^+e^- \to \pi + X$ at $E_{cm} = 1\,\mathrm{GeV}$

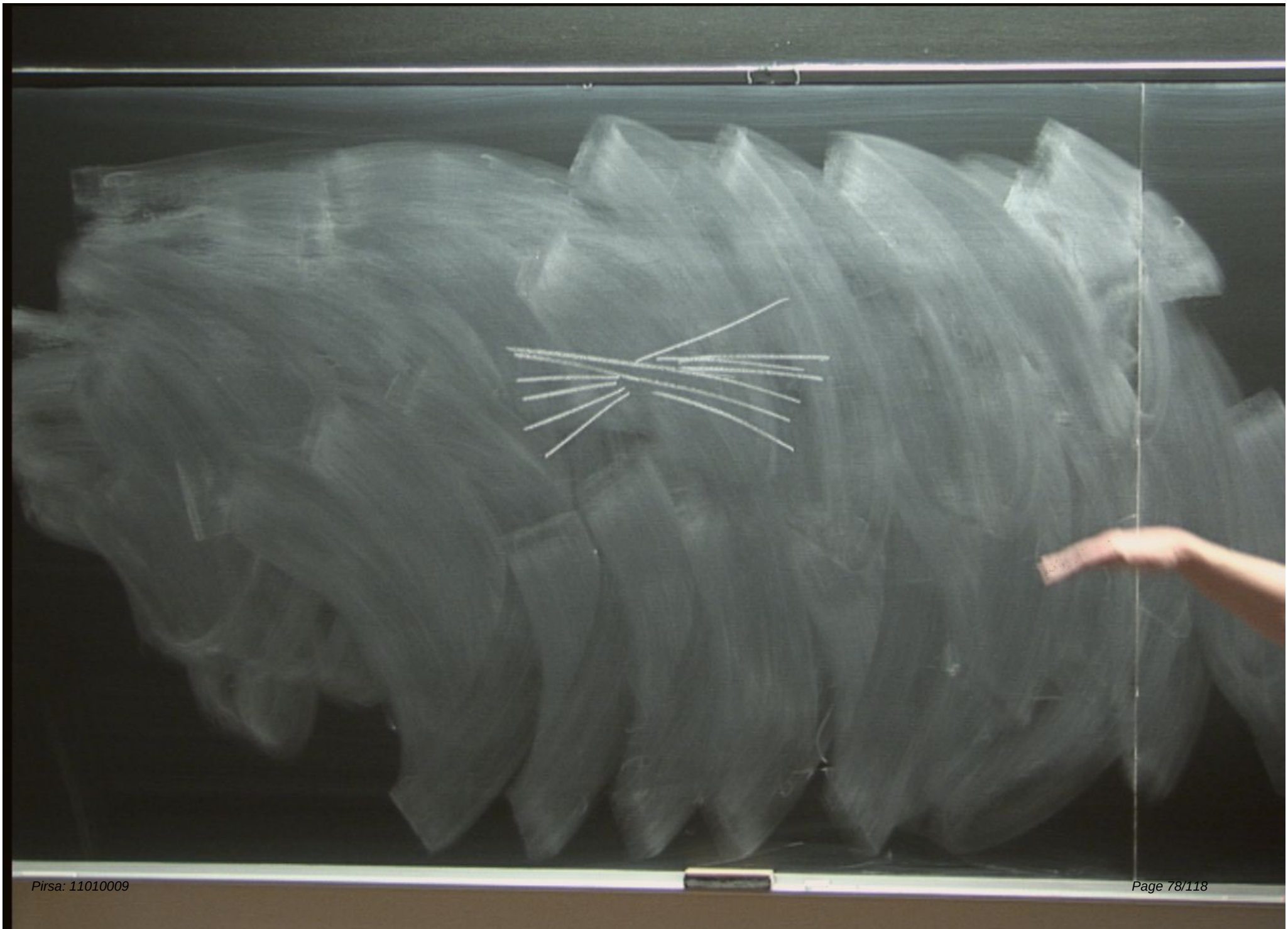


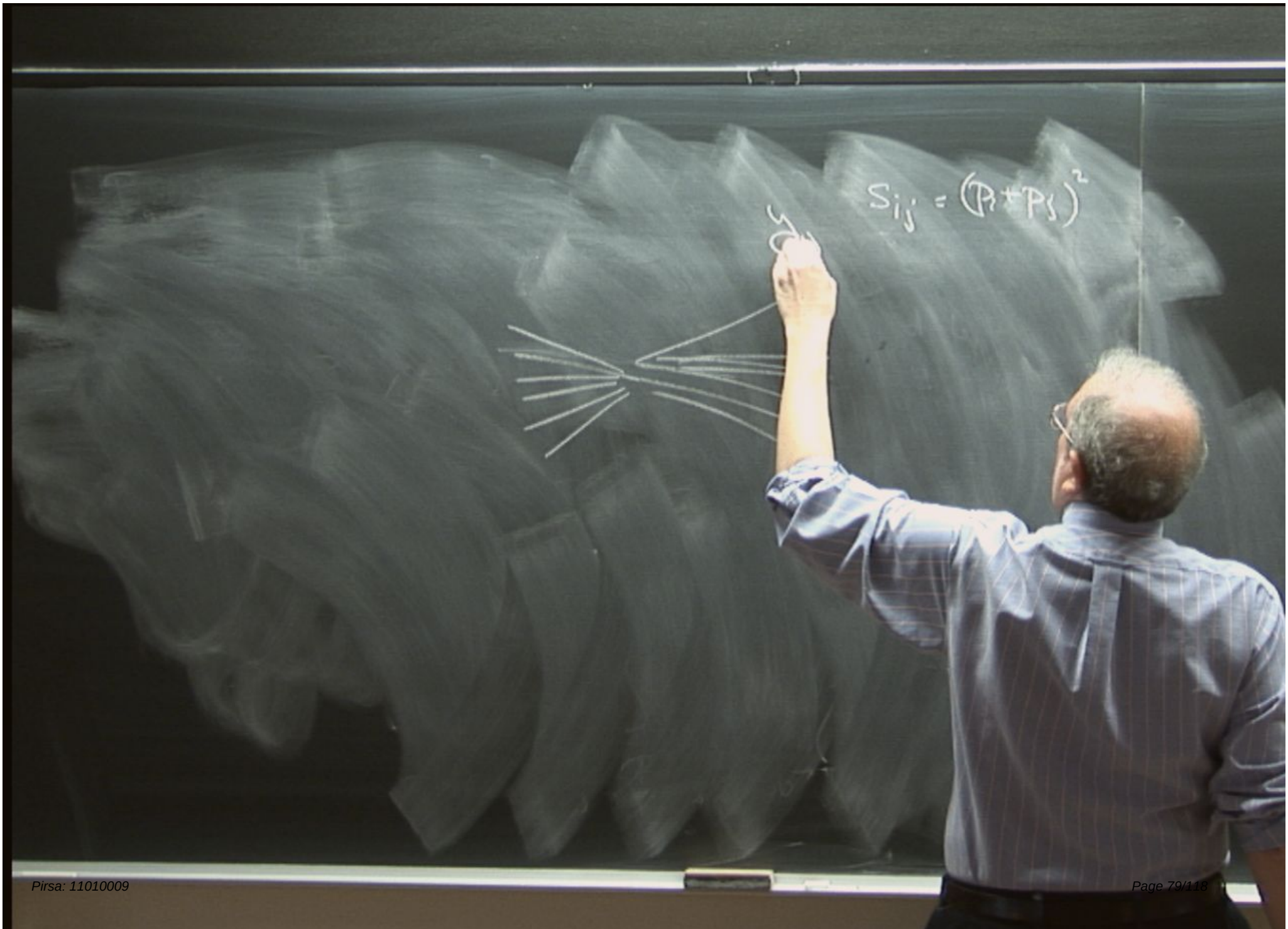


$e^+e^- \to \pi + X$ at $E_{cm} = 200\,\mathrm{GeV}$

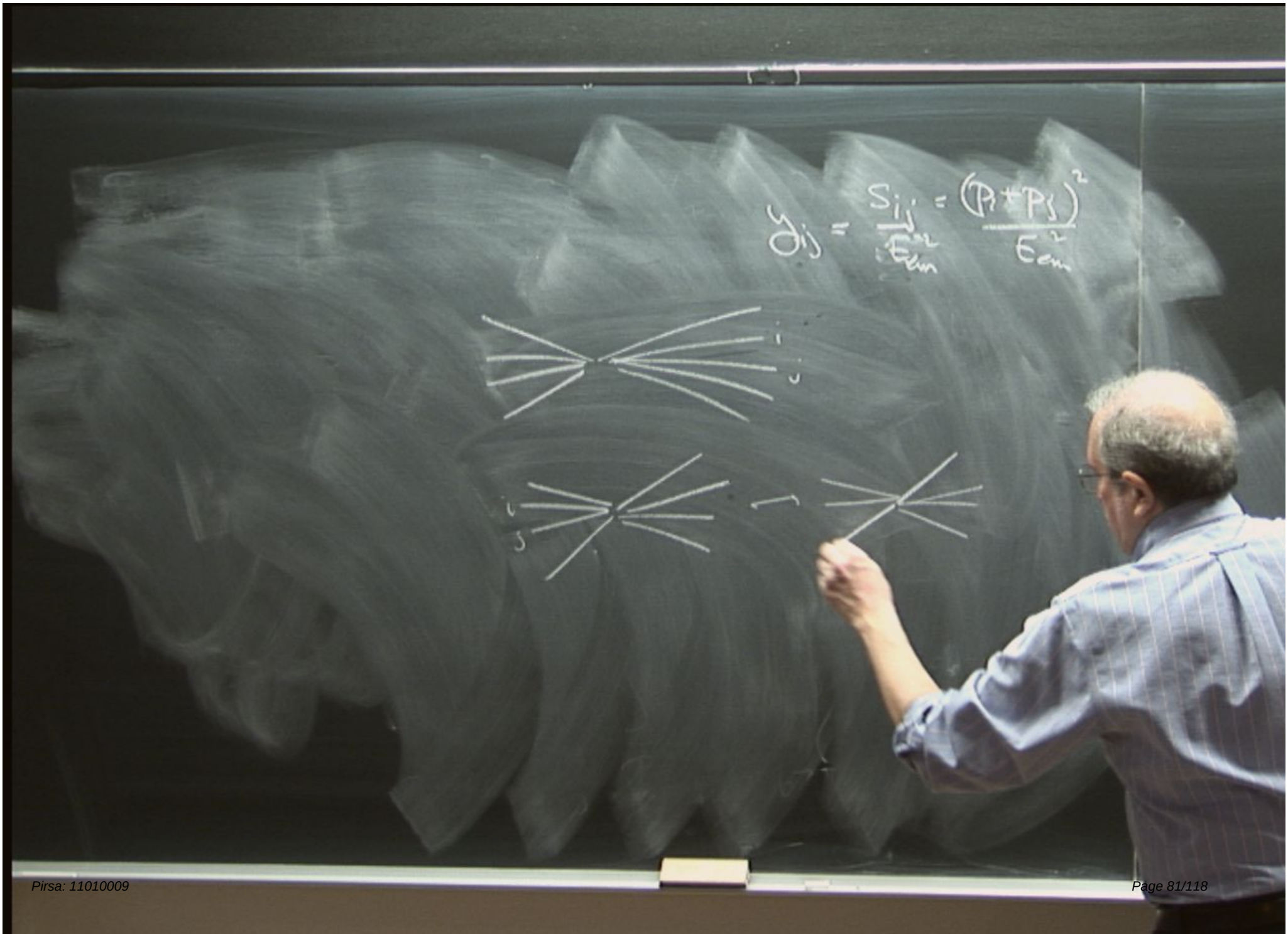



From Grains of Pollen to Evidence for Atoms
PYTHIA
HERWIG
Altarelli-Parisi eq.
$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'} P_{P\to P'}(\omega)\, f_{P'\to A}(\frac{z}{\omega}, k_T)$
$e^+e^- \to \pi + \gamma$ cut $E_{cm} = 1$ GeV
$e^+e^- \to \pi + \gamma$ cut $E_{cm} = 20$ GeV

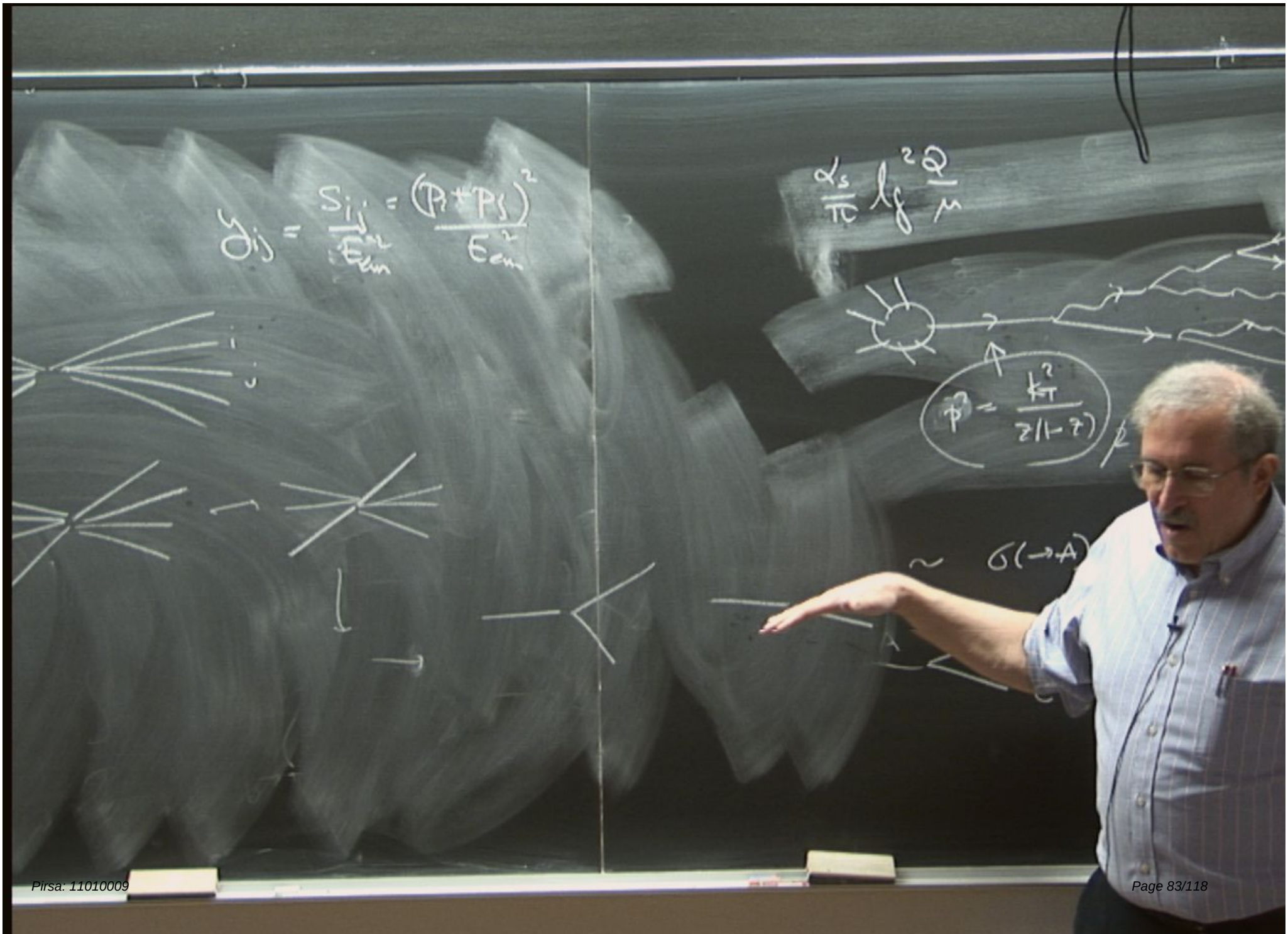



$S_{ij} = (p_i + p_j)^2$

$$y_{ij} = \frac{s_{ij}}{E_{lab}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$


$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i+p_j)^2}{E_{cm}^2}$

$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$


$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i+p_j)^2}{E_{cm}^2}$
$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$p^2 = \frac{k_T^2}{z(1-z)}$
$\sim \sigma(\to A)$

$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

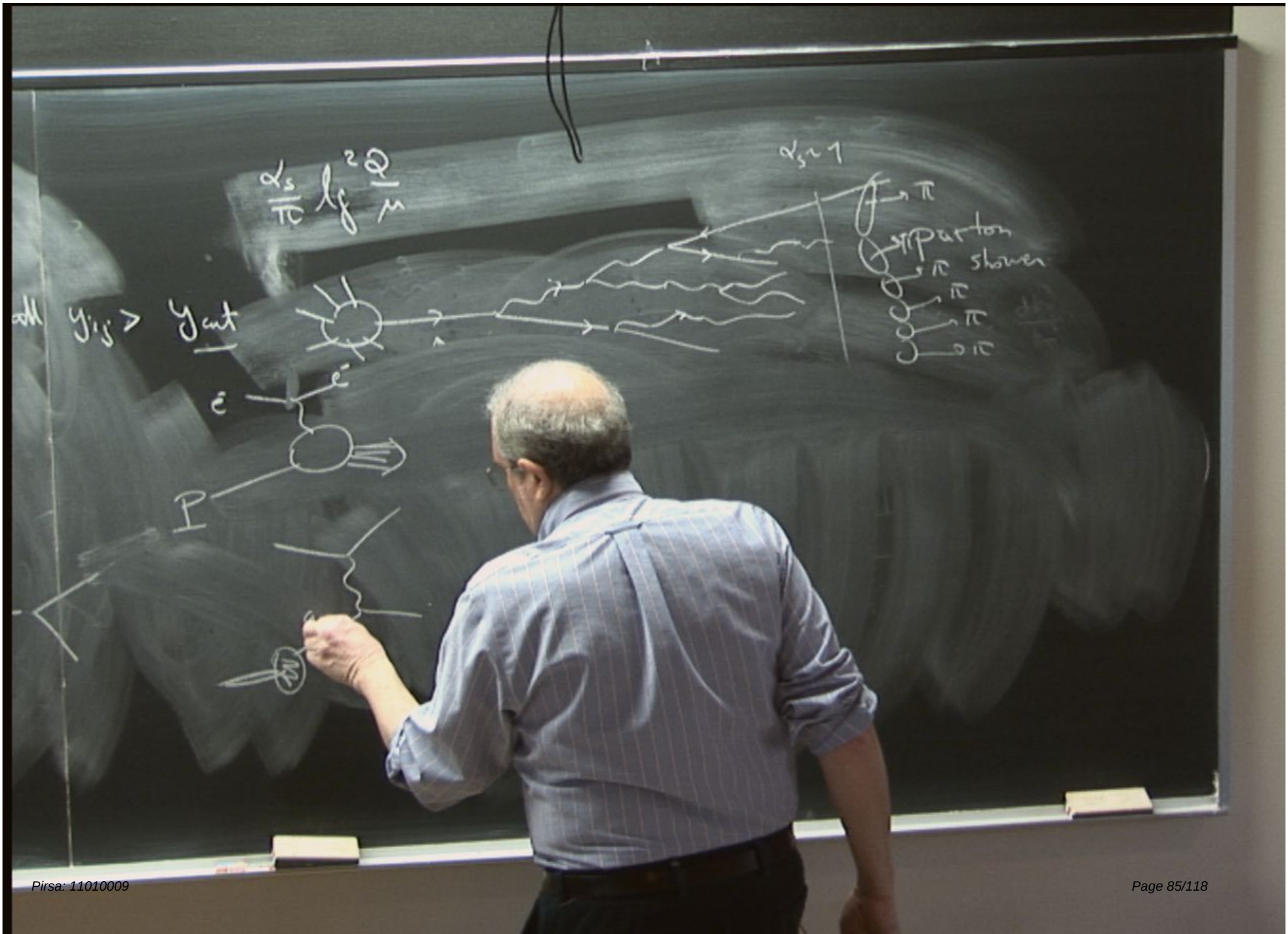
stop when all $y_{ij} > y_{cut}$

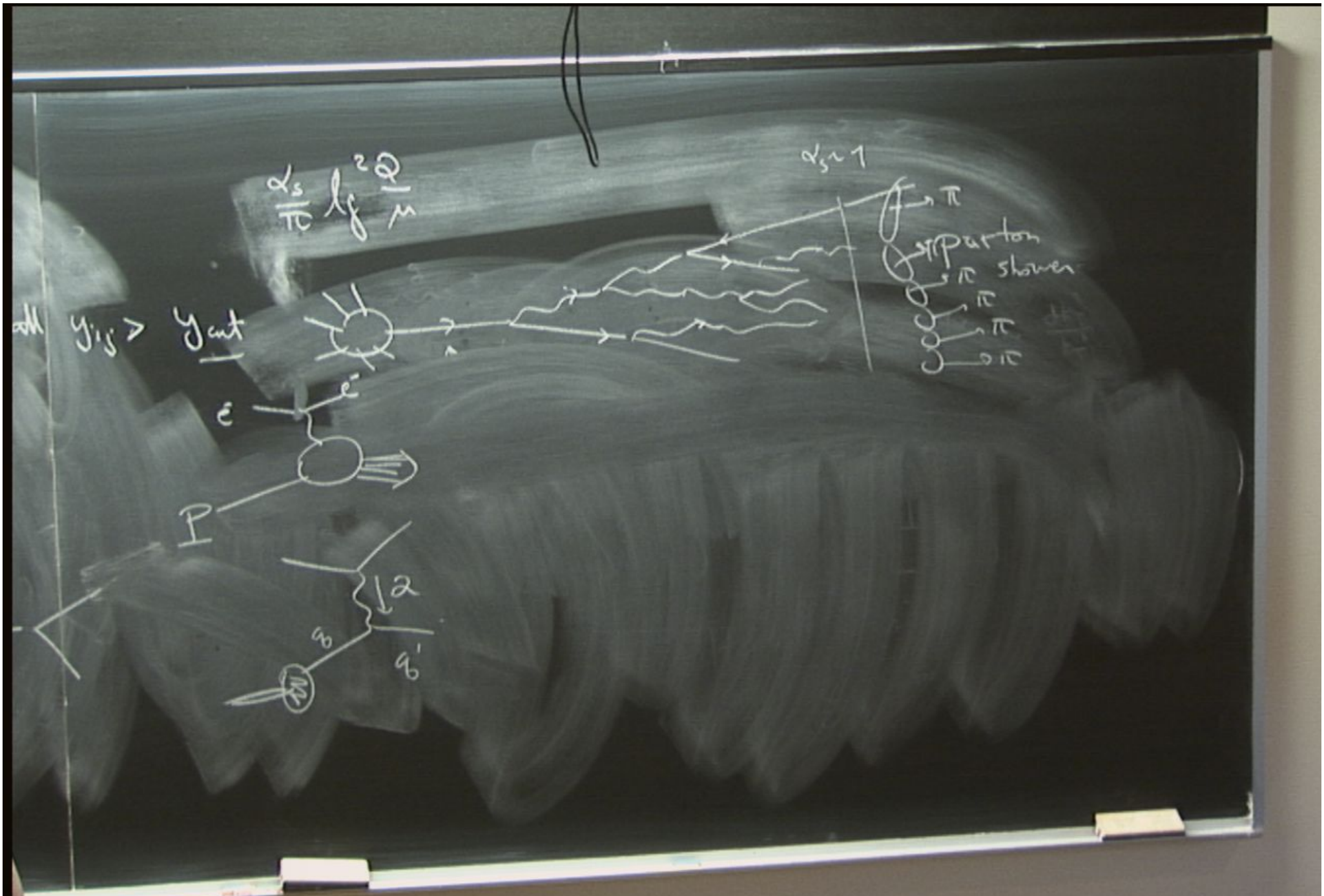
$$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$$

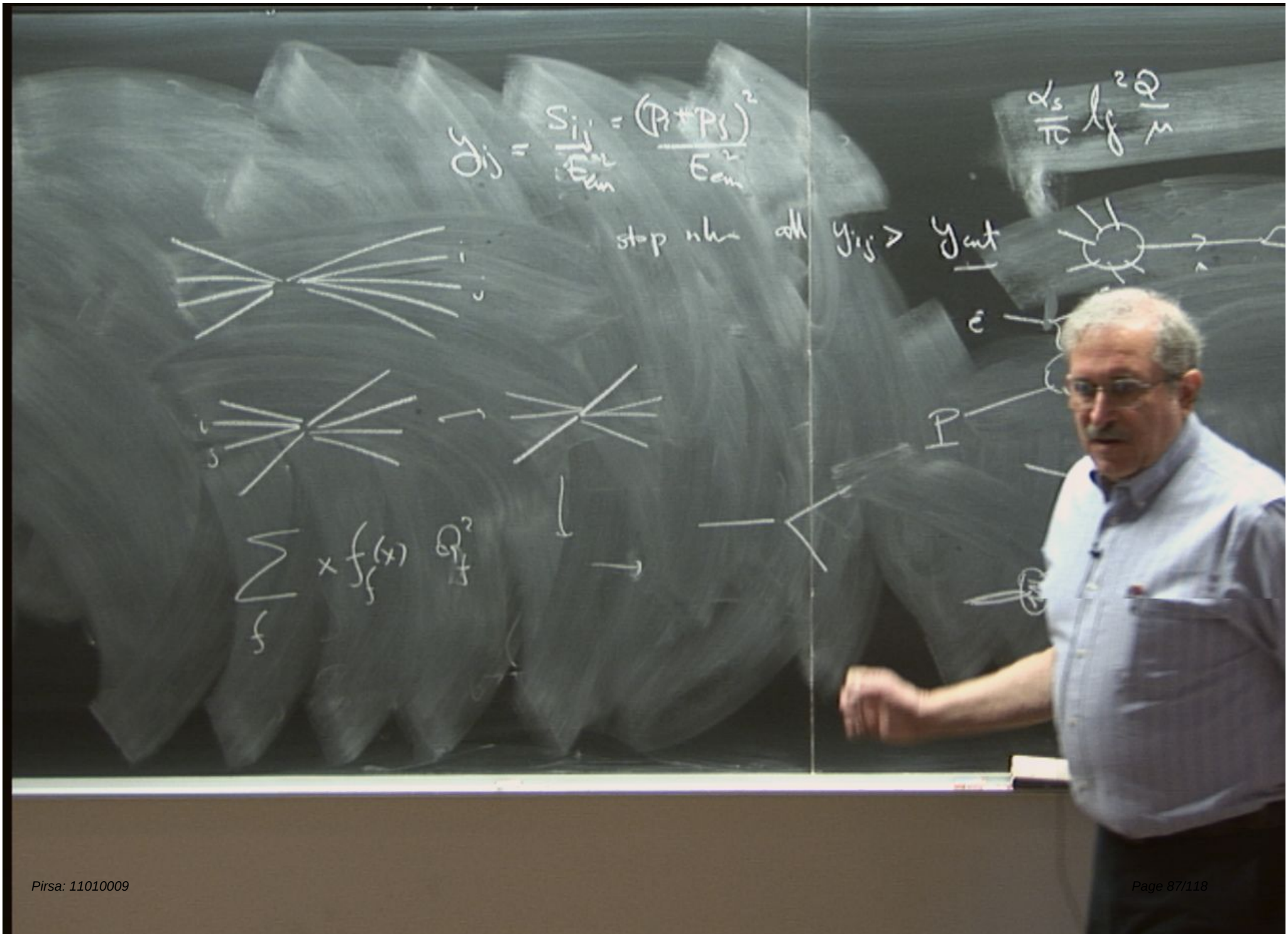
$$p = \frac{k_T^2}{z(1-z)} \propto \log k_T$$

$$\sim \sigma(\to A) \int \frac{dk_T^2}{k_T^2} \int dz$$

Alta


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
all $y_{ij} > y_{cut}$
parton shower
π
e^-
e^-
P

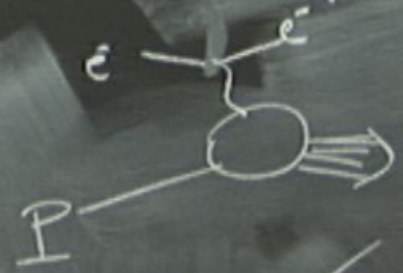

$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
$y_{ij} > y_{cut}$
π
parton
shower
e
e'
P
q
q'


$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$
$\frac{\alpha_s}{\pi} \ln^2 \frac{Q}{\mu}$
stop when all $y_{ij} > y_{cut}$
$\sum_f x f_f(x) Q_f^2$
P

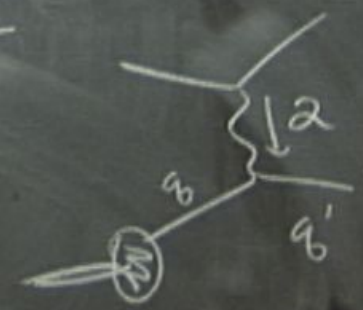
$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

$$\frac{\alpha_s}{\pi} \log^2 \frac{Q}{\mu}$$

stop when all $y_{ij} > y_{cut}$



$$\sum_f x f_f(x) \, Q_f^2$$

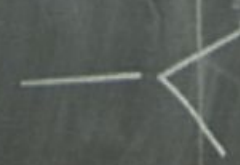


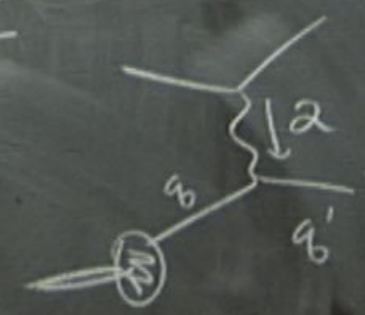
$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

$$\frac{\alpha_s}{\pi} \ln^2 \frac{Q}{\mu}$$

stop when all $y_{ij} > y_{cut}$

$$F_2(x) = \sum_f x f_f(x) Q_f^2$$



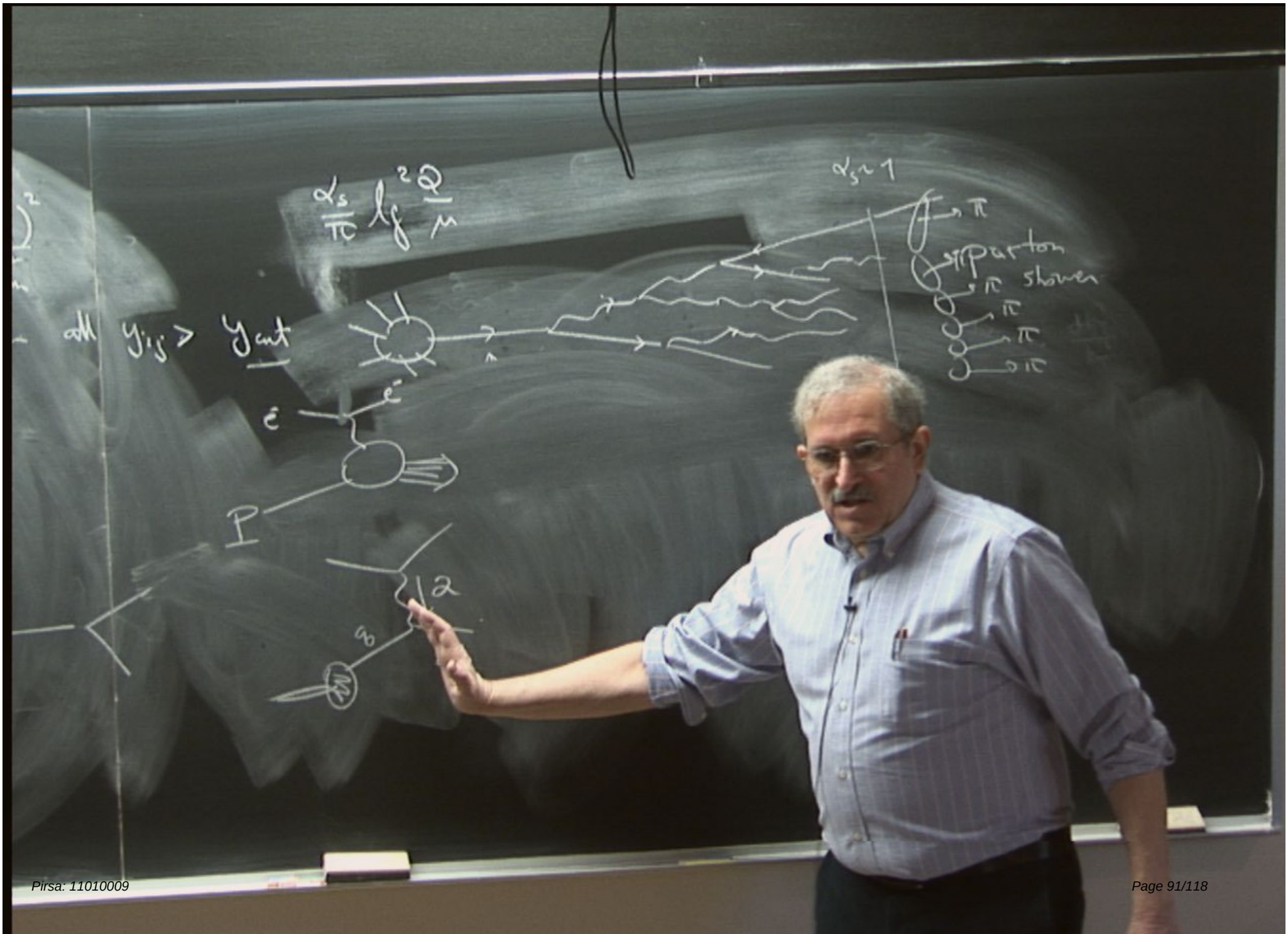


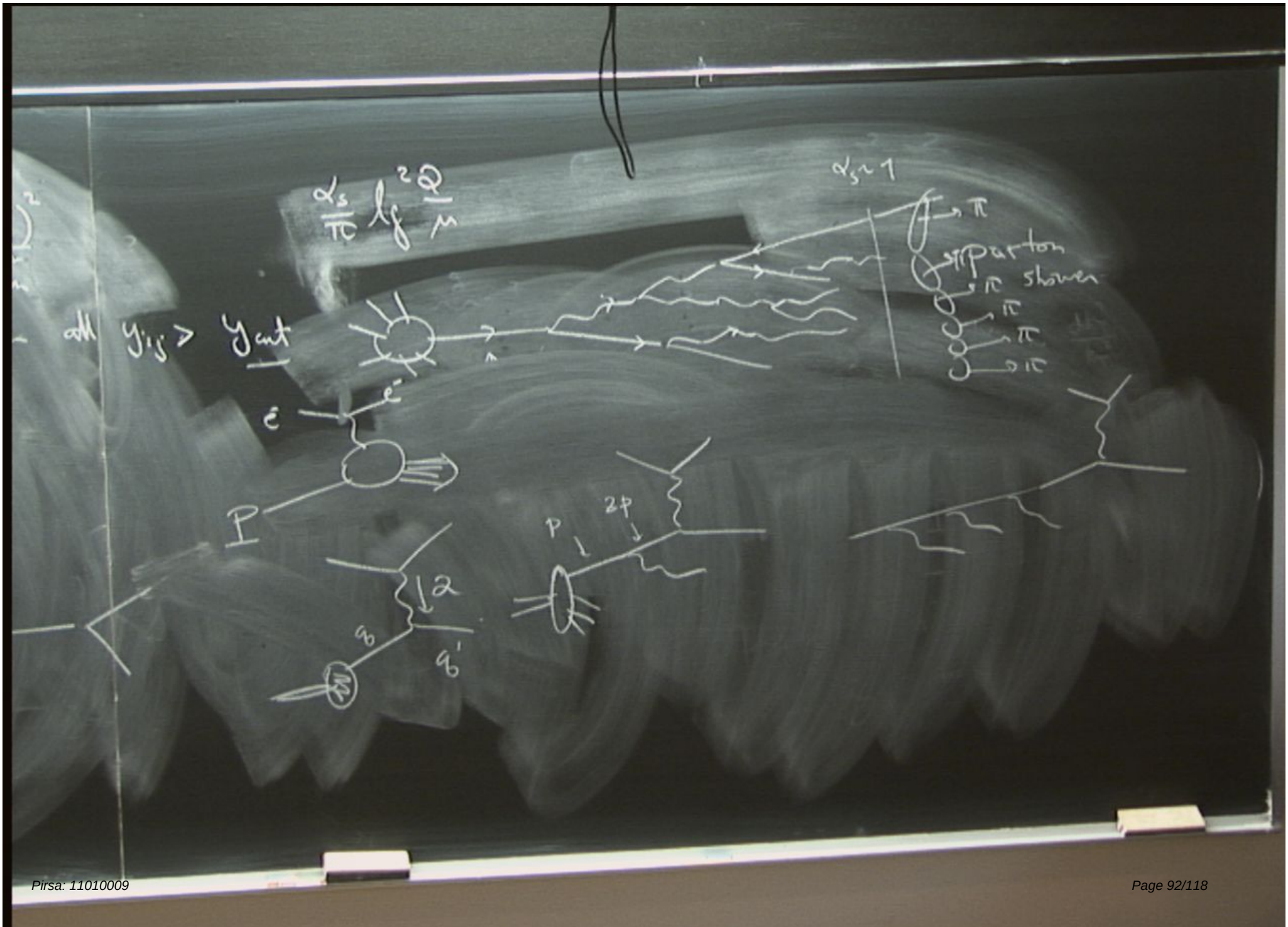
$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

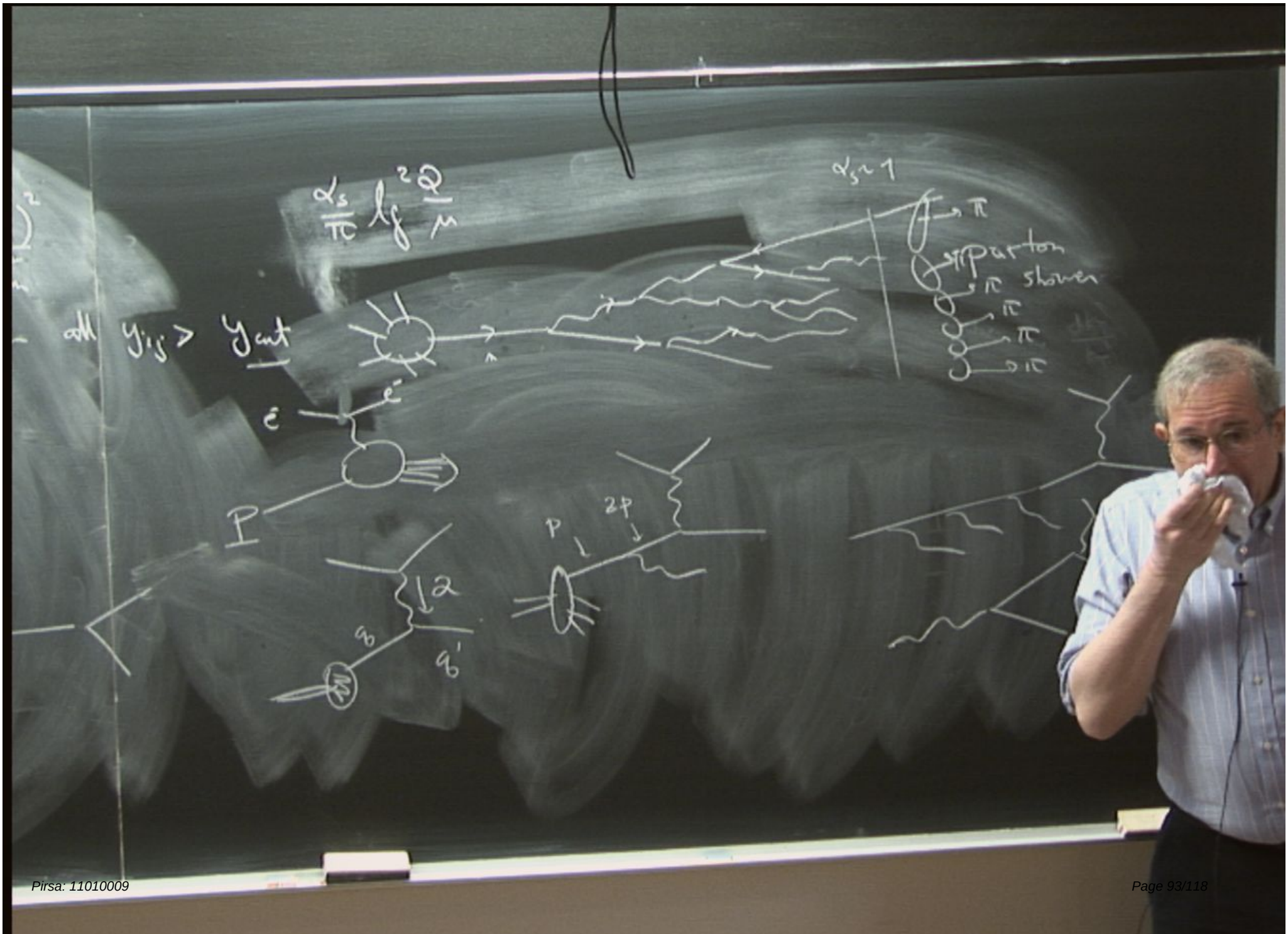
$$\frac{\alpha_s}{\pi} \ln^2 \frac{Q}{\mu}$$

stop when all $y_{ij} > y_{cut}$

$$F_2(x) = \sum_f x f_f(x) Q_f^2$$


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
all $y_{ij} > y_{cut}$
parton shower
π
π
π
π
e^-
e^-
P
2
a


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
all $y_{ij} > y_{cut}$
parton shower
π
e
e'
P
p
$2p$
q
q'


$\frac{\alpha_s}{\pi} \lg^2 \frac{Q}{\mu}$
$\alpha_s \sim 1$
all $y_{ij} > y_{cut}$
parton shower
π
e^-
e^-
P
p
2p
q
q'
2

$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

Fermilab $f_u(x)$

today

$Q \sim 1$ GeV

$Q \sim 10$ GeV

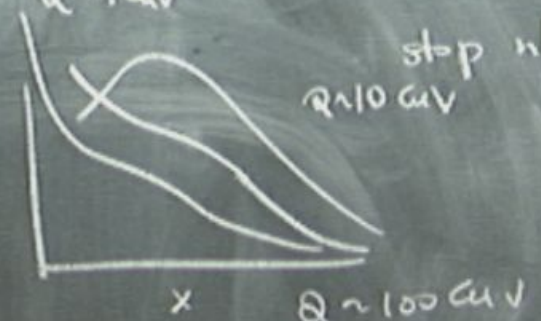
stop when all

$$F_2(x) = \sum_f x f_f(x) Q_f^2 \rightarrow$$

$$y_{ij} = \frac{S_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

Fridy $f_u(x)$

today



stop nh

$$F_2(x) = \sum_f x f_f(x) Q_f^2$$

Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'} P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$$\frac{d}{d\log Q} f_f(x, Q) = \frac{\alpha_s}{\pi} \int \frac{d\omega}{\omega} \sum_{f'}$$

Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'}{}' P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$$\frac{d}{d\log Q} f_f(x, Q) = \frac{\alpha_s}{\pi} \int \frac{d\omega}{\omega} \sum_{f'}{}' P_{f\to f'}(\omega)$$

Altarelli-Parisi eq.

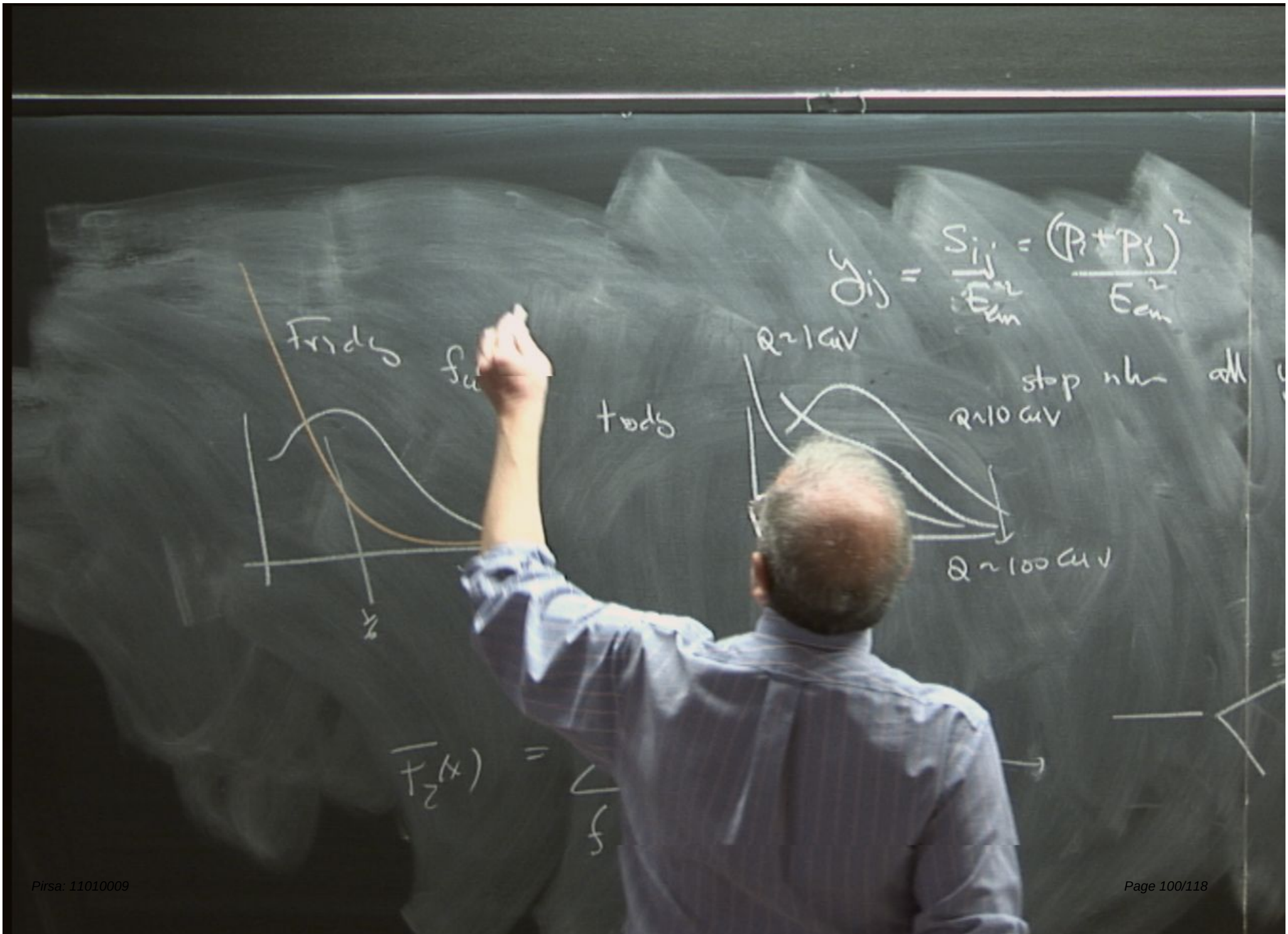
$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'}{}' P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

$$\frac{d}{d\log Q} f_f(x, Q) = \frac{\alpha_s}{\pi} \int \frac{d\omega}{\omega} \sum_{f'}{}' P_{f\to f'}(\omega)\, f_{f'}\left(\frac{x}{\omega}, Q\right)$$

Altarelli–Parisi eq.

$$\frac{d}{d\log k_T} f_{P\to A}(z, k_T) = \frac{\alpha_s(k_T)}{\pi} \int \frac{d\omega}{\omega} \sum_{P'}{}' P_{P\to P'}(\omega)\, f_{P'\to A}\left(\frac{z}{\omega}, k_T\right)$$

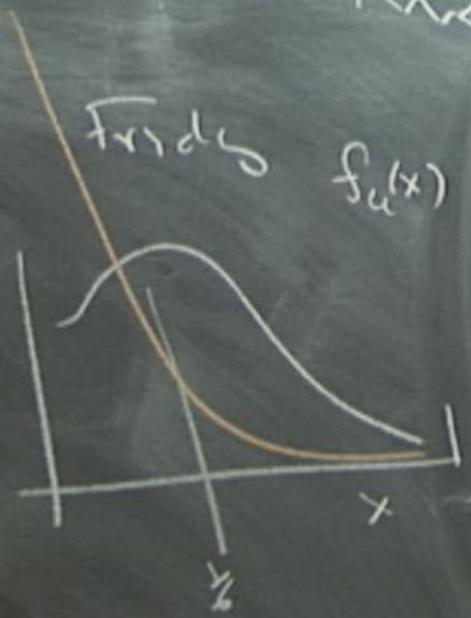
$$\frac{d}{d\log Q} f_f(x, Q) = \frac{\alpha_s}{\pi} \int \frac{d\omega}{\omega} \sum_{f'}{}' P_{f\to f'}(\omega)\, f_{f'}\left(\frac{x}{\omega}, Q\right)$$


$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$
Q~1 GeV
Q~10 GeV
Q~100 GeV
stop
today
$F_2(x) =$

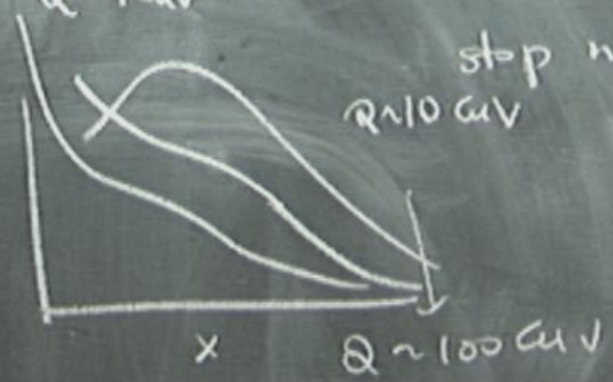
$\frac{\alpha_s}{\alpha}$

$$y_{ij} = \frac{s_{ij}}{E_{cm}^2} = \frac{(p_i + p_j)^2}{E_{cm}^2}$$

Friday $f_u(x)$

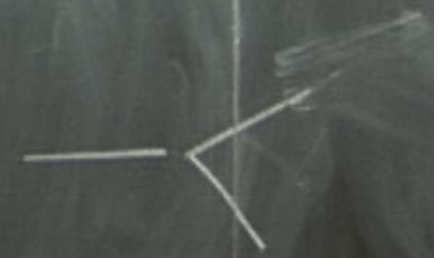


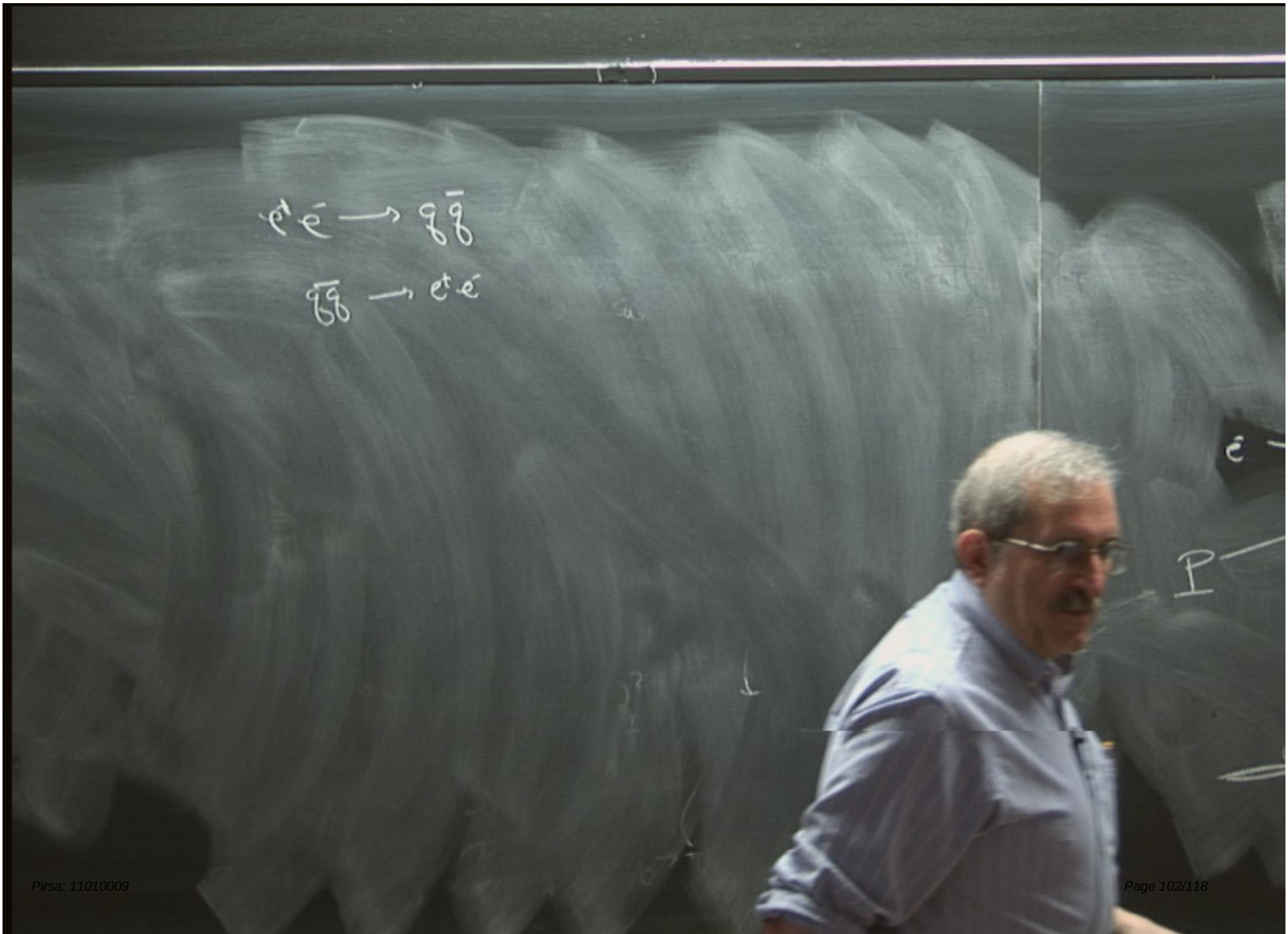
today

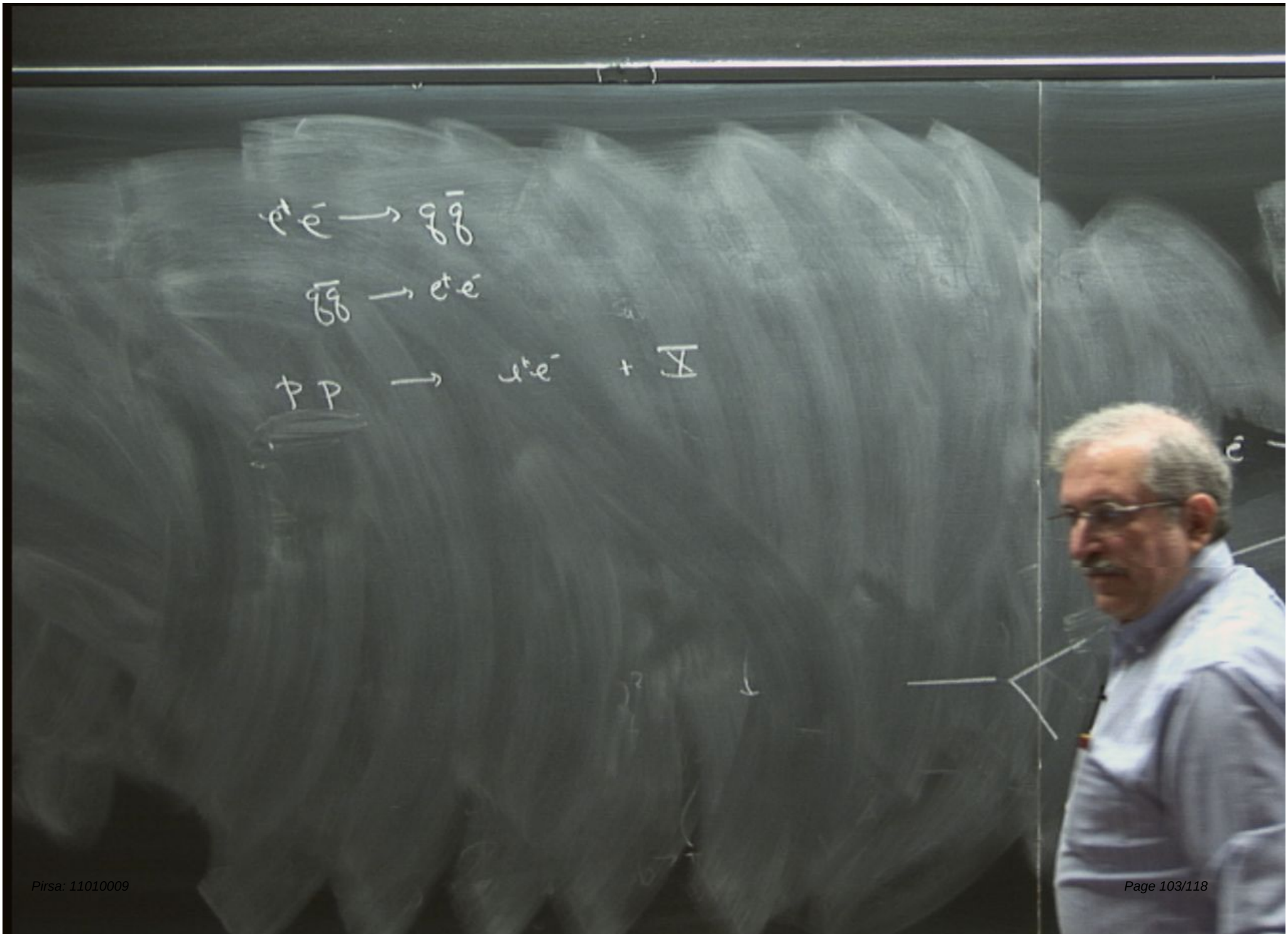


stop when all $y_{ij} >$

$$F_2(x) = \sum_f x f_f(x) Q_f^2$$




$e^+e^- \to q\bar{q}$
$q\bar{q} \to e^+e^-$

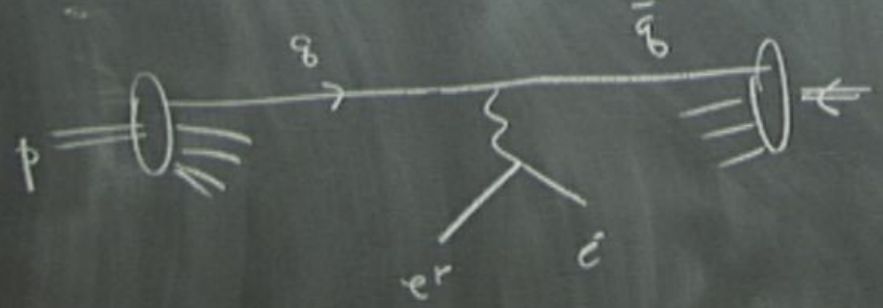

$e^+e^- \to q\bar{q}$
$q\bar{q} \to e^+e^-$
$pp \to \mu^+\mu^- + X$

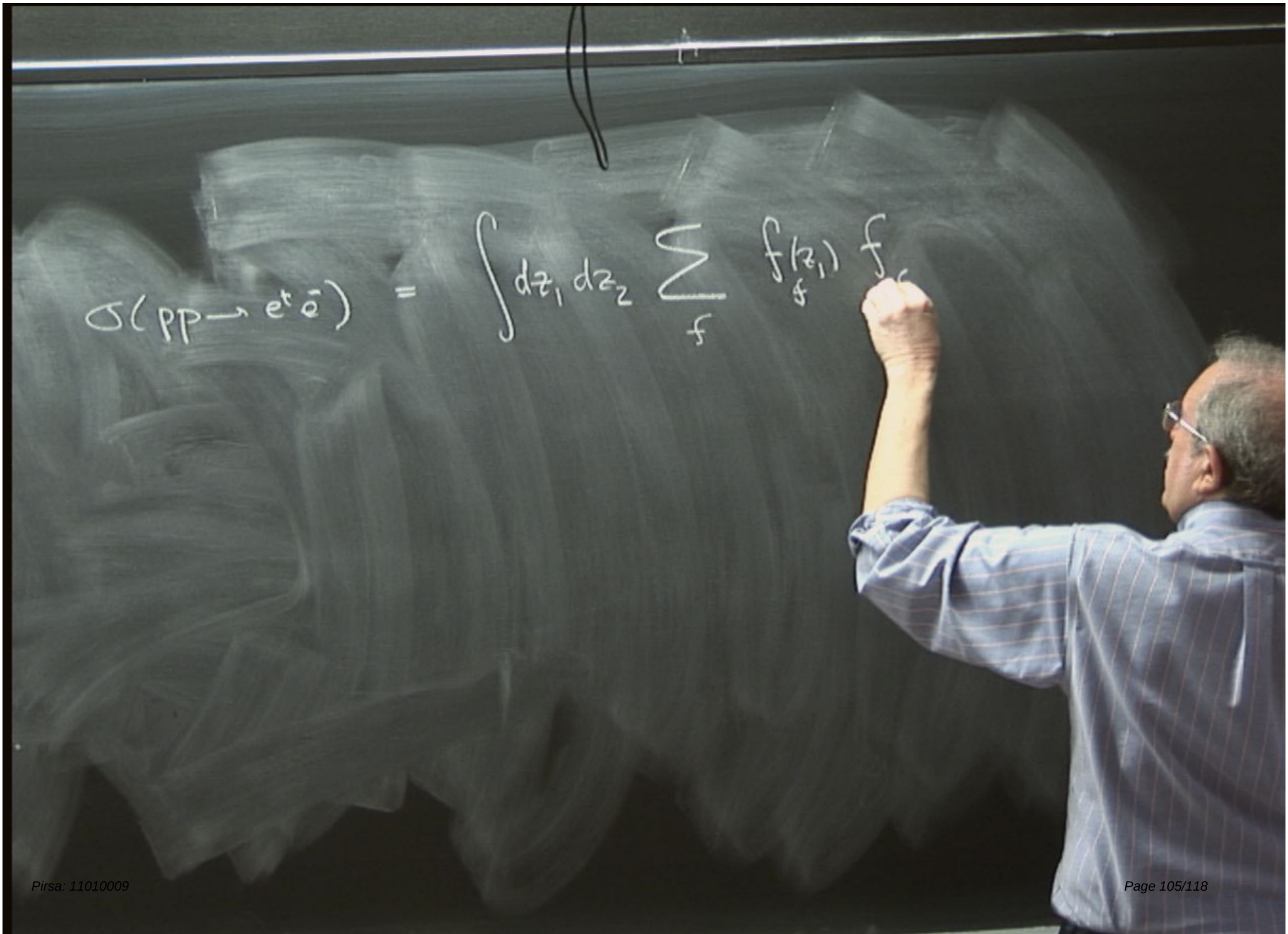
$e^+e^- \to q\bar{q}$

$q\bar{q} \to e^+e^-$

$p\,p \to \mu^+\mu^- + X$

Drell-Yan




$\sigma(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1)\, f$

$$\sigma(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1)\, f_{\bar{f}}(z_2)\, \sigma(q_f \bar{q}_f \to e^+ e^-$$

$$\sigma(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1)\, f_{\bar{f}}(z_2)\, \sigma(q_f \bar{q}_f \to e^+ e^-)$$

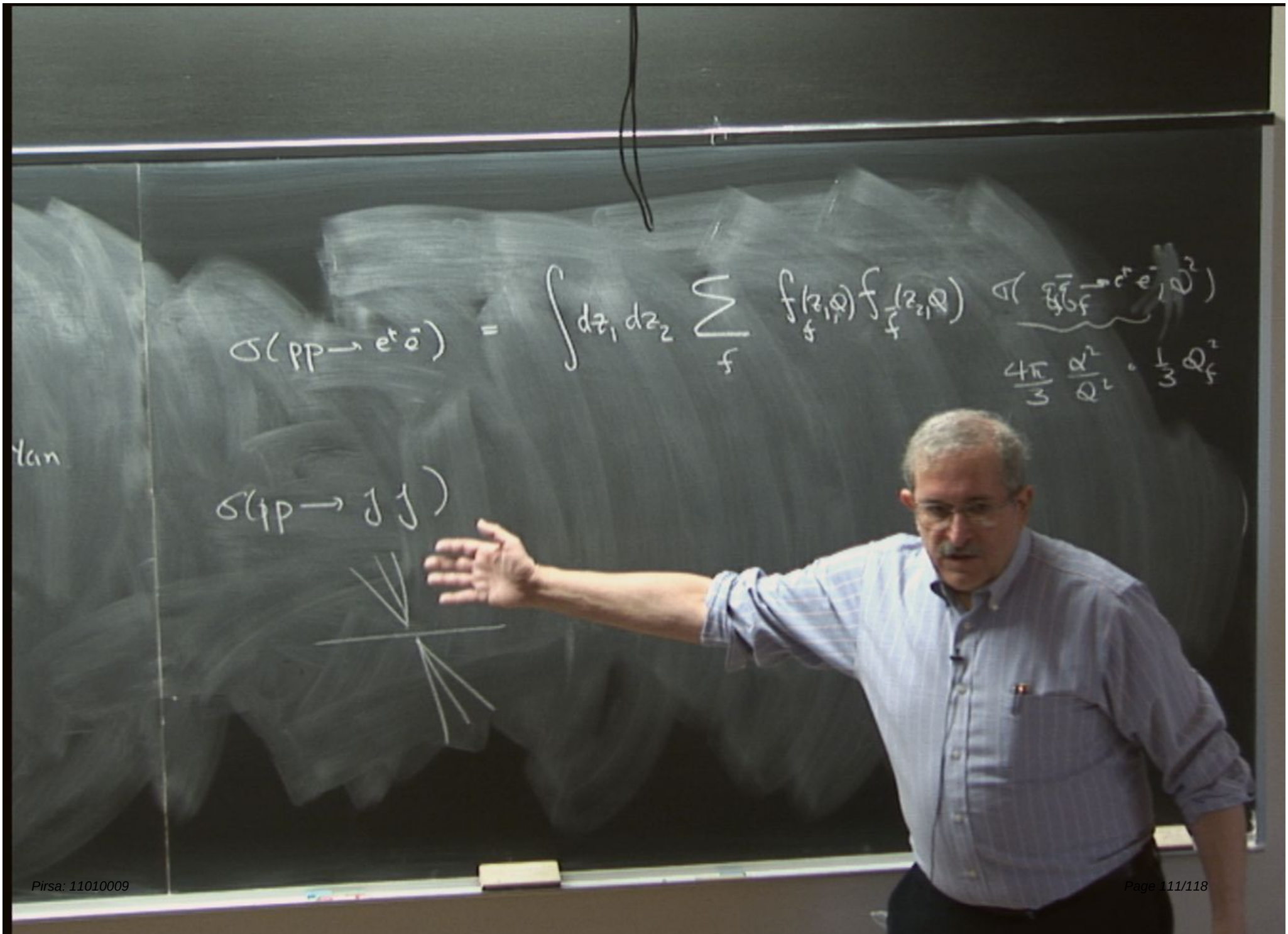
$$\sigma(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1)\, f_{\bar{f}}(z_2)\, \sigma(q_f \bar{q}_f \to e^+ e^-, Q^2)$$

$$(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1)\, f_{\bar{f}}(z_2)\, \sigma(q\bar{q}_f \to e^+ e^-, Q^2)$$

$$\frac{4\pi}{3}\alpha$$

$$(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1, Q)\, f_{\bar{f}}(z_2, Q)\, \sigma(\underbrace{q_f \bar{q}_f \to e^+ e^-}, Q^2)$$

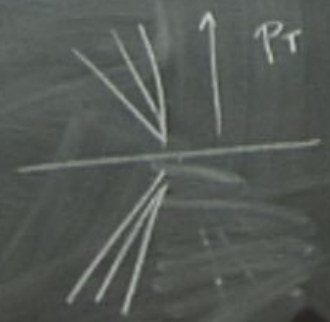
$$\frac{4\pi}{3} \frac{\alpha^2}{Q^2} \cdot \frac{1}{3} Q_f^2$$


$\sigma(pp \to e^+e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1,Q)\, f_{\bar f}(z_2,Q)\, \sigma(q_f\bar q_f \to e^+e^-, Q^2)$
$\frac{4\pi}{3}\frac{\alpha^2}{Q^2} \cdot \frac{1}{3} Q_f^2$
Yan
$\sigma(pp \to jj)$

$$\sigma(pp \to e^+ e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1, Q)\, f_{\bar{f}}(z_2, Q)\, \underbrace{\sigma(q_f \bar{q}_f \to e^+ e^-, Q^2)}_{\frac{4\pi}{3}\frac{\alpha^2}{Q^2} \cdot \frac{1}{3} Q_f^2}$$

Yan

$$\sigma(pp \to j\, j) = \int dz_1\, dz_2 \sum_{f_1 f_2} f_{f_1}(z_1, Q)\, f_{f_2}(z_2, Q)$$



$$\sum_{f_3 f_4} \frac{d\sigma(f_1 f_2 \to f_3 f_4)}{dp_T}$$

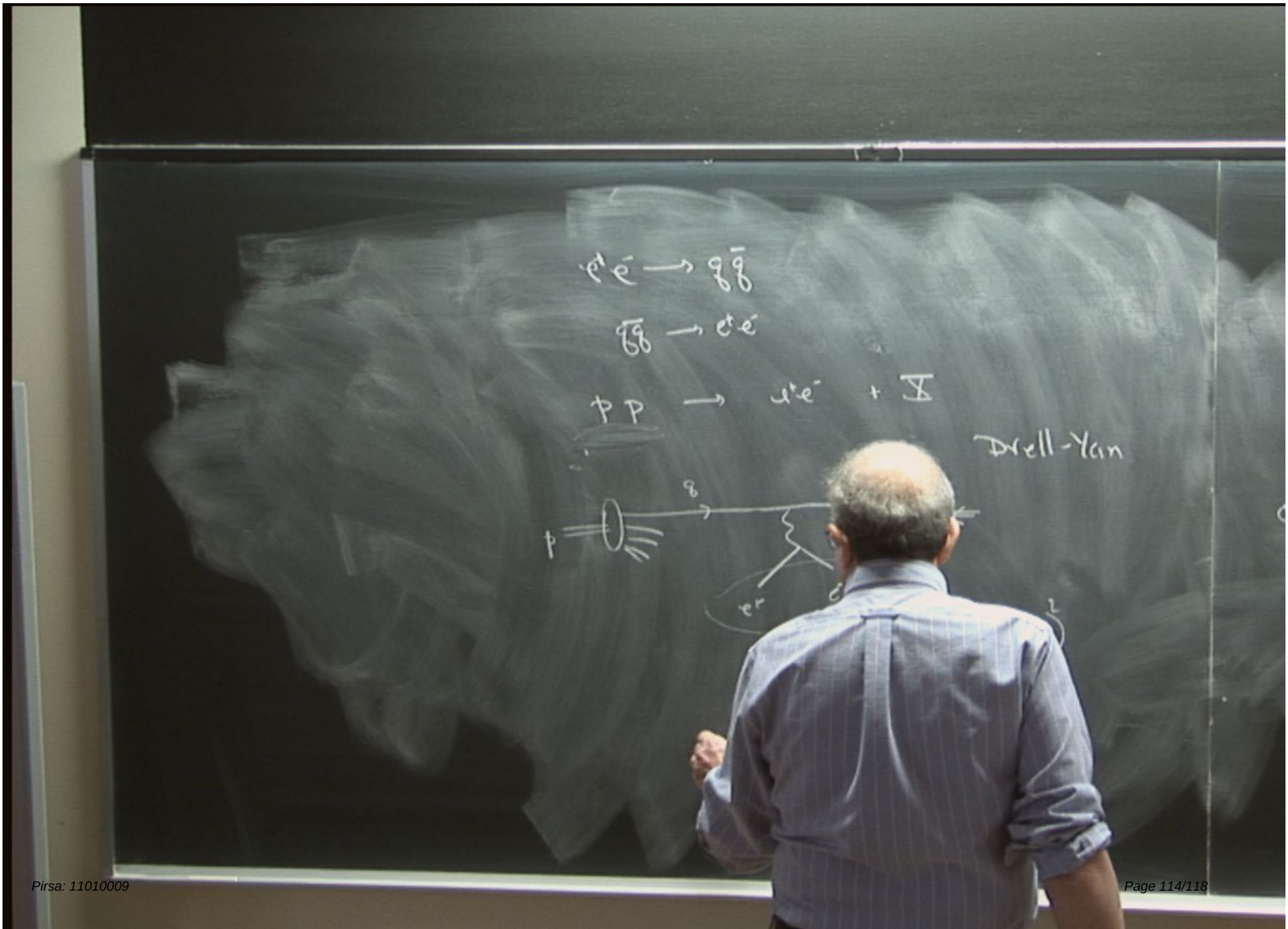
$$\sigma(pp \to e^+e^-) = \int dz_1\, dz_2 \sum_f f_f(z_1,Q)\, f_{\bar f}(z_2,Q)\, \underbrace{\sigma(q_f\bar q_f \to e^+e^-, Q^2)}_{\frac{4\pi}{3}\frac{\alpha^2}{Q^2}\cdot\frac{1}{3}Q_f^2}$$

Yan

$$\sigma(pp \to jj) = \int dz_1\, dz_2 \sum_{f_1 f_2} f_{f_1}(z_1,Q)\, f_{f_2}(z_2,Q)\, \frac{d\sigma(f_1 f_2 \to f_3 f_4)}{dp_T}$$

$f_3 f_4$

$gg \to gg$

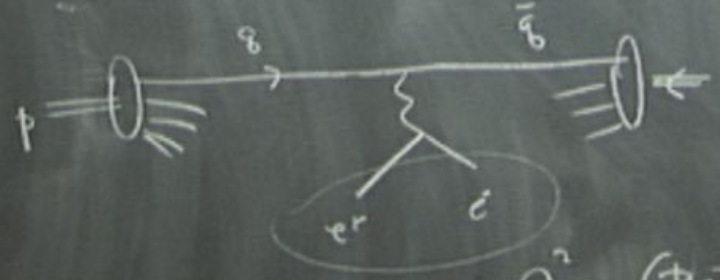

$e^+e^- \to q\bar{q}$
$q\bar{q} \to e^+e^-$
$pp \to \mu^+\mu^- + X$
Drell-Yan
p
q

$e^+e^- \to q\bar{q}$

$q\bar{q} \to e^+e^-$

$pp \to \mu^+\mu^- + X$

Drell-Yan



$Q^2 = (p_{e^-} + p_{e^+})^2$

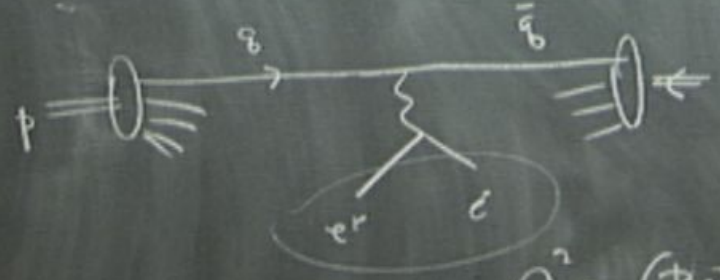
$Q = Q^+(1,0,0,1) + Q^-($

$e^+e^- \to q\bar{q}$

$q\bar{q} \to e^+e^-$

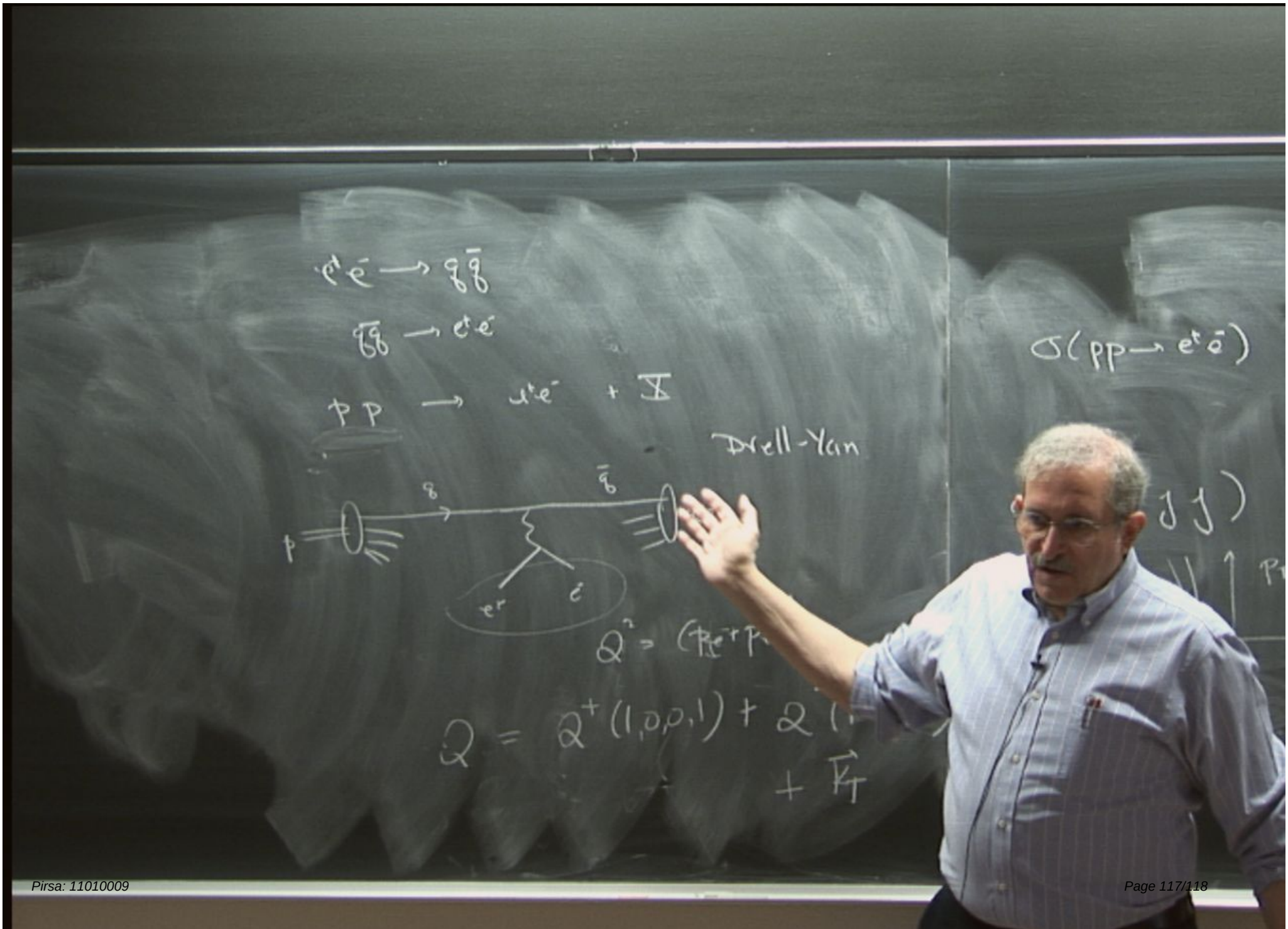
$pp \to \mu^+\mu^- + X$

Drell-Yan



$Q^2 = (p_{e^-} + p_{e^+})^2$

$Q = Q^+(1,0,0,1) + Q^-(1,0,0,-1) + \vec{Q}_T$

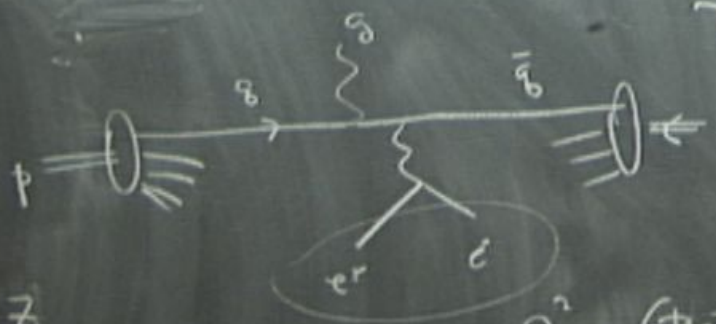

$e^+e^- \to q\bar{q}$
$q\bar{q} \to e^+e^-$
$pp \to \mu^+\mu^- + X$
Drell-Yan
$\sigma(pp \to e^+e^-)$
$Q = Q^+(1,0,0,1) + Q(\ldots)$
$+ \vec{Q}_T$

$e^+e^- \to q\bar{q}$

$q\bar{q} \to e^+e^-$

$pp \to \mu^+\mu^- + X$

Drell-Yan



$\frac{Q^+}{p_p} = x_1$

$Q^2 = (p_{e^-} + p_{e^+})^2$

$Q = Q^+(1,0,0,1) + Q^-(1,0,0,-1) + \vec{Q}_T$

$\sigma(pp \to e^+e^-)$

$\sigma(pp \to jj)$