Title: The zig-zag road to reality

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Abstract: The de Broglie-Bohm pilot-wave program is an attempt to formulate quantum theory (including quantum field theory) as a theory without observers, by assuming that the wave-function is not the complete description of a system, but must be supplemented by additional variables (beables). Although many progress has been made in order to extend the pilot-wave theory to quantum field theory, a compelling ontology for quantum field theory is still lacking and the choice of beable is likely to be relevant for the study of quantum non-equilibrium systems and their relaxation properties (Valentini).
The present work takes its root in the fact that in the standard model of particle physics, all fermions are fundamentally massless and acquire their bare mass when the Higgs field condenses. In our tentative to build a pilot-wave model for quantum field theory in which beables are attributed to massless fermions, we are naturally led to Weyl spinors and to Penrose's zig-zag picture of the electron.

In my talk, I will sketch this tentative and insist on some of its remarkable properties: namely that a positive-energy massive Dirac electron can be thought of as a superposition of positive and negative energy Weyl spinors of the same helicity, and that the massive Dirac electron can in principle move luminally at all times.
Based on a joint work with H. Wiseman.

The zig-zag road to reality

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3 December PIAF workshop

Overview

Summarize in one question:
How does the electron move?. . . in the pilot-wave theory

Plan

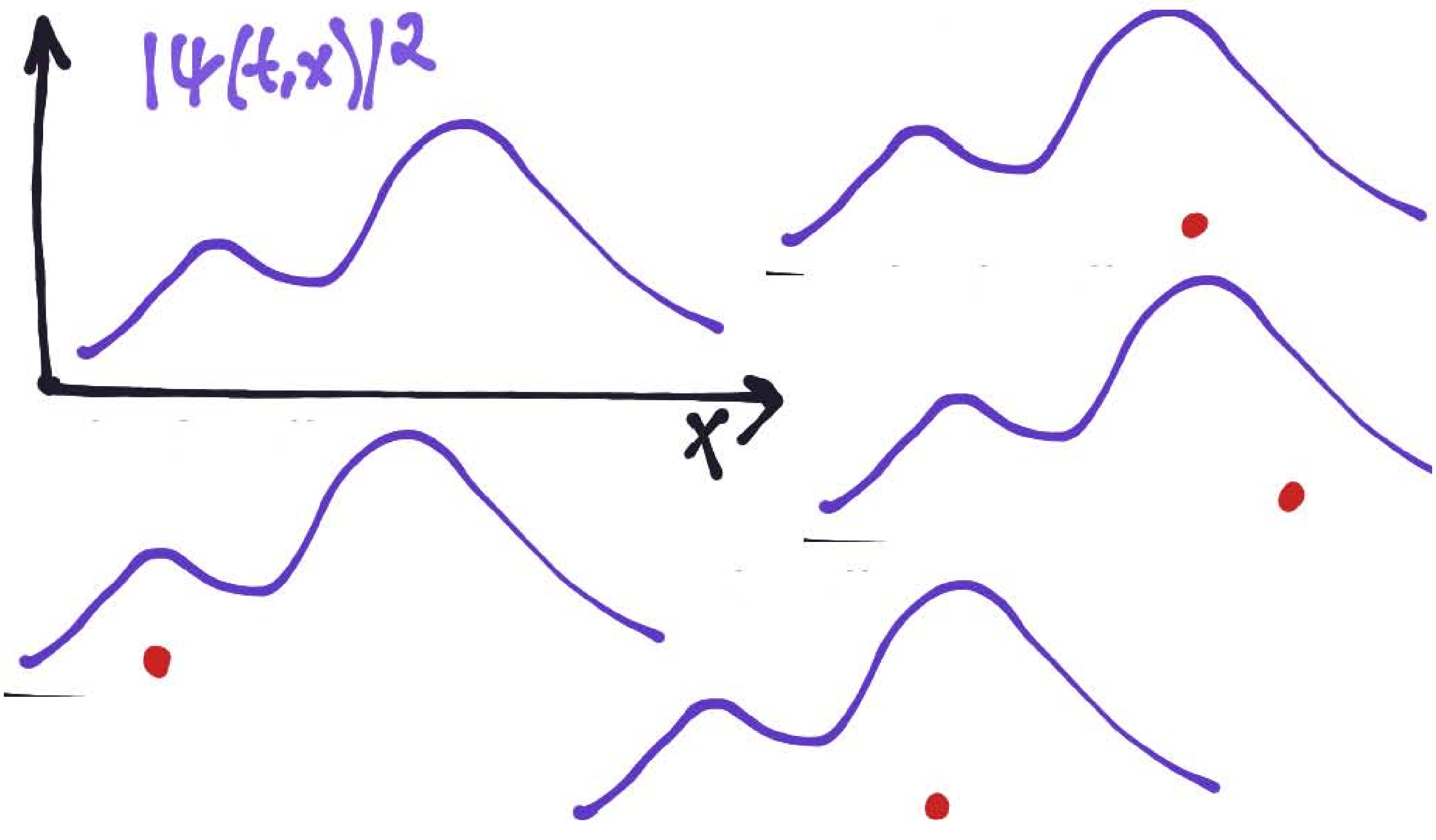
- The dBB pilot-wave theory
- Standard model of particle physics
- Weyl equations
- The zig-zag electron
- Relevance of all this

The non-relativistic de Broglie-Bohm pilot-wave theory

- Complete description for N particles: $(\psi(t,\vec{X}),\vec{X}(t))$
- Equations of motion:
  - Schrödinger equation.
  - Guidance equation.

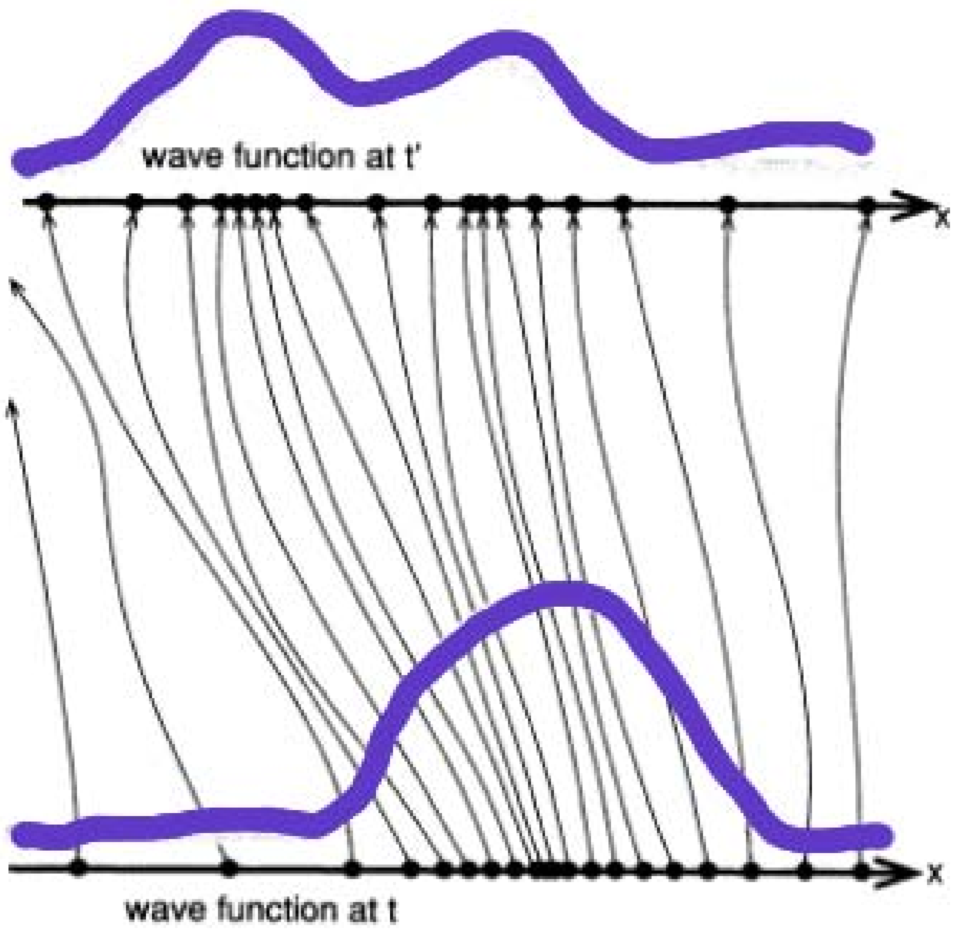
$$\vec{V}(t)=\left.\frac{\vec{J}(t,\vec{X})}{|\psi(t,\vec{X})|^2}\right|_{\vec{X}=\vec{X}(t)}$$

Ensemble



The distribution of positions $\rho(t,x)$ and $|\psi(t,x)|^2$ are two different things

Equivariance I



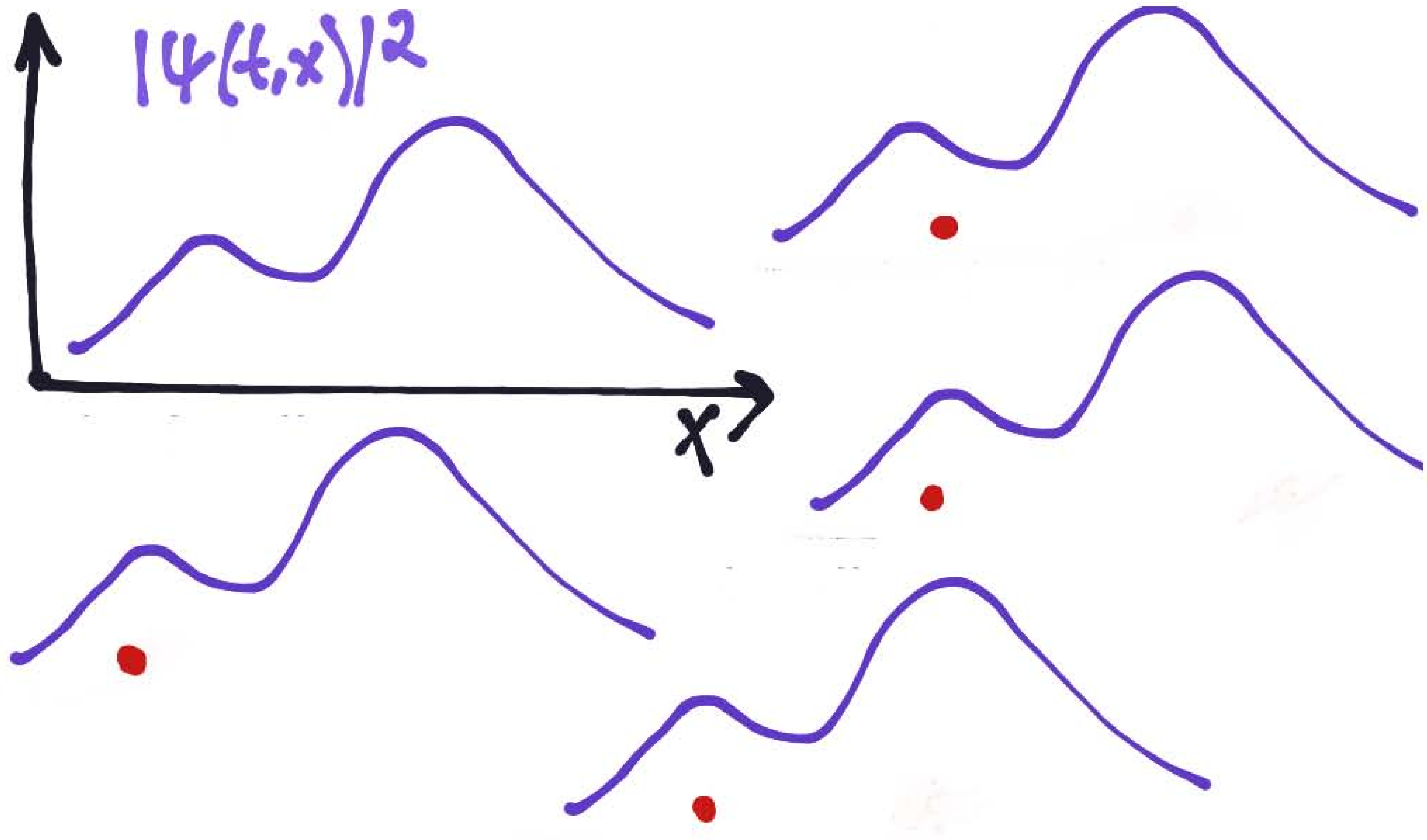
Equivariance II

- $\rho(t,\vec{X}) = |\psi(t,\vec{X})|^2$ if $\rho(t_0,\vec{X}) = |\psi(t_0,\vec{X})|^2$.
- Consequence of

$$\vec{v}_k = \frac{\vec{j}_k}{|\psi|^2} \quad \text{where} \quad \partial_t|\psi|^2 + \sum_k \vec{\nabla}_k \cdot \vec{j}_k = 0 \qquad (1)$$

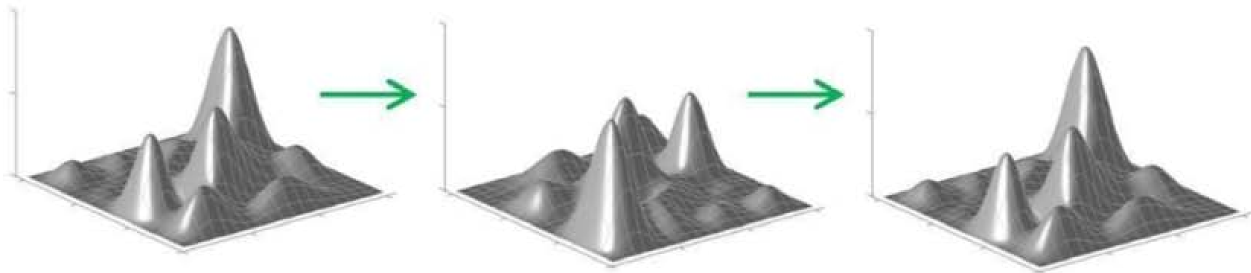
- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$: quantum equilibrium distribution

Quantum non-equilibrium

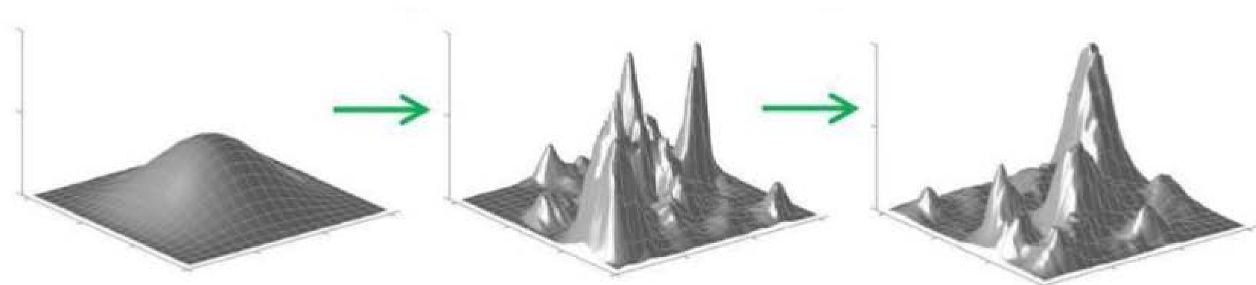


Relaxation to quantum equilibrium

Time evolution of quantum equilibrium



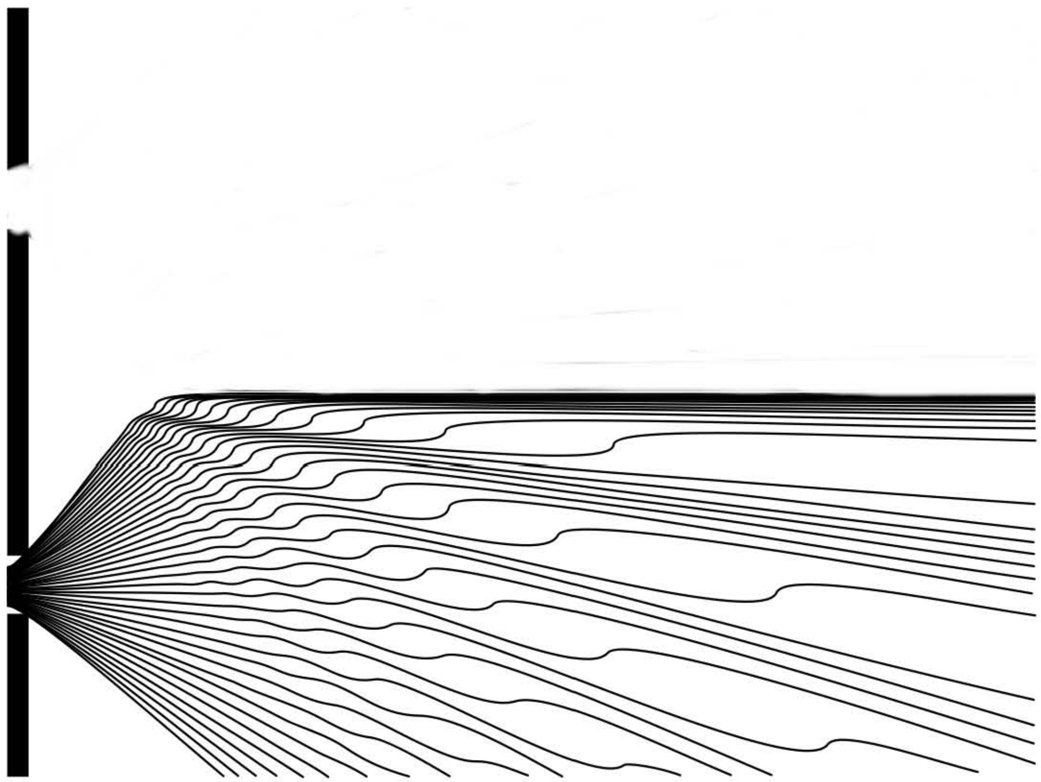
Non-equilibrium relaxes to equilibrium



A. Valentini and H. Westman *Proc. Roy. Soc. Lond. A* **461**: 253-272 (2005)

Why quantum non-equilibrium?

- Possibility to test PWT against standard QM.
- Non-equilibrium in the early universe $\rightarrow$ relaxation (except for exotic particles - relic non-equilibrium).
- Interference with relic:



The pilot-wave theory for the Dirac electron

- Dirac equation:

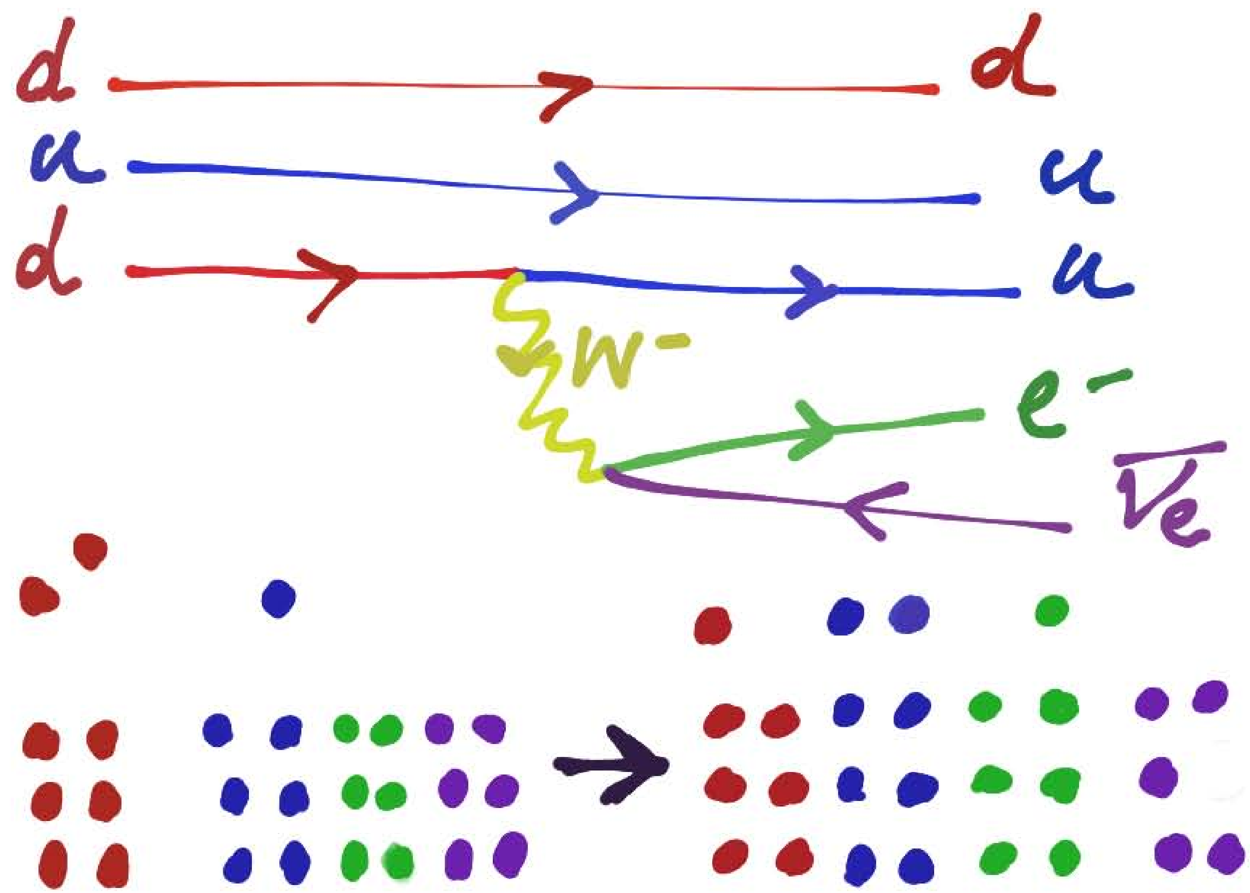
$$(i\gamma^{\mu}\partial_{\mu} - m)\psi(t,\vec{x}) = 0 \ .$$

- Conserved current $j^{\mu} = \bar{\psi}\gamma^{\mu}\psi$
- Velocity $\vec{v} = \frac{\vec{j}}{j^0}$.

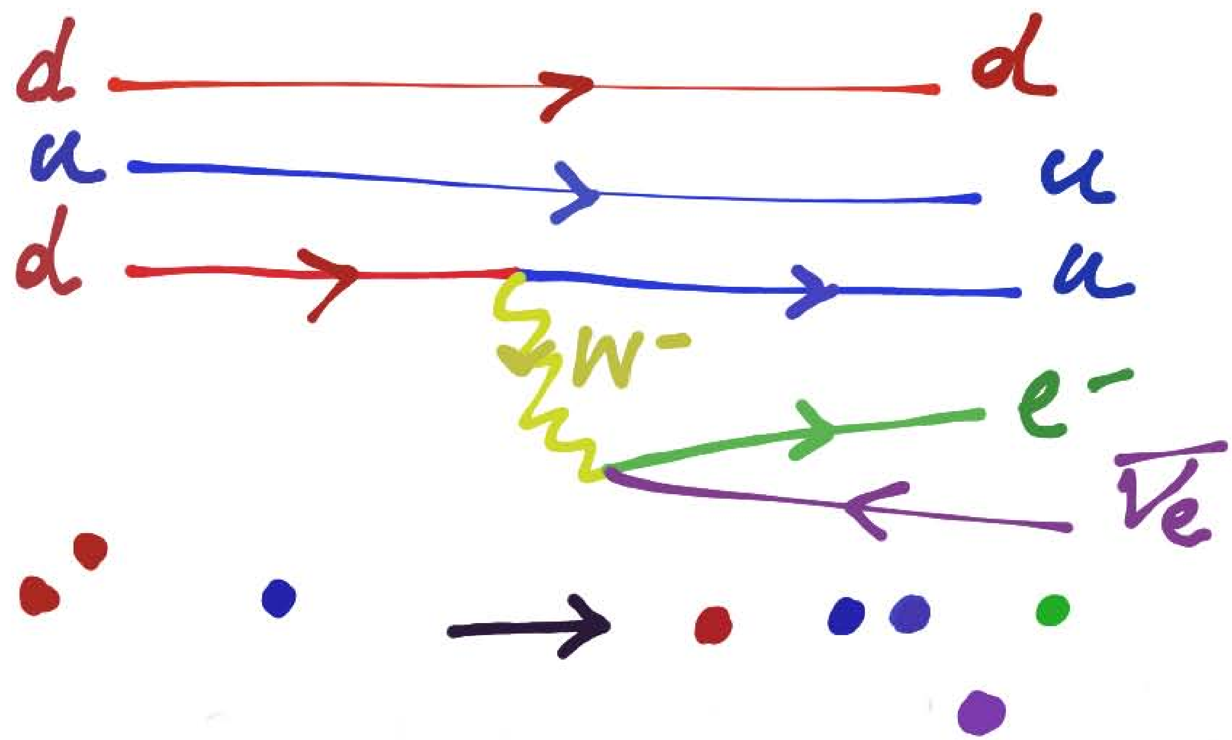
Pilot-wave for quantum field theory

- Dirac-sea approach
- particle and anti-particle approach
- field approach

Beta decay in the Dirac-sea pilot-wave theory



Beta decay in the DGTZ model



But things are not easy as they seem…

Bare particle ontology

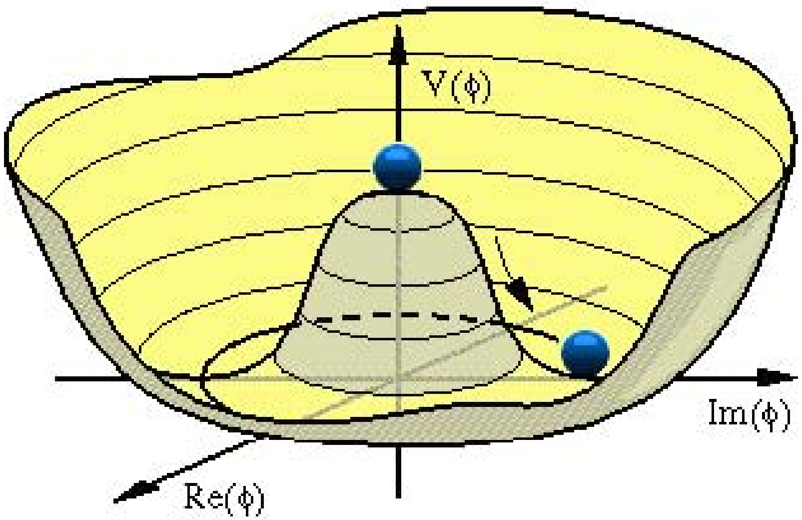
- Schrödinger equation:

$$i\frac{d|\Psi(t)\rangle}{dt} = \hat{H}|\Psi(t)\rangle \ .$$

- Hamiltonian operator $\hat{H}$ involves bare particle creators and annihilators.
- Bare particle ontology.
- Bare particle might have negative mass (cf classical model of self-energy).

Standard model of particle physics

- The Higgs as the origin of the electron mass.



- Massless ontology for fermions seems fundamental.
- Time at which the Higgs condensation takes place.

Weyl equations I

- Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi(t, \vec{x}) = 0 \ .$$

- Gamma matrices in the Weyl representation:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{I}_2 \\ \mathbf{I}_2 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}.$$

- $m = 0$ and $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $\rightarrow$ two Weyl equations:

$$\partial_t \psi_L = \vec{\sigma} \cdot \vec{\nabla} \psi_L \ (\tilde{\sigma}^\mu \partial_\mu \psi_L = 0),$$
$$\partial_t \psi_R = -\vec{\sigma} \cdot \vec{\nabla} \psi_R \ (\sigma^\mu \partial_\mu \psi_R = 0).$$

Weyl equations II

Helicity

$$\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}u_R(\vec{p}) = u_R(\vec{p}) \text{ and } \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}u_L(\vec{p}) = -u_L(\vec{p}) \ .$$

$\sigma^\mu \partial_\mu \psi_R = 0$

- pos-E=right-handed
- neg-E=left-handed
- 4-current $j_R^\mu = \psi_R^\dagger \sigma^\mu \psi_R$
- $\frac{1}{\sqrt{(2\pi)^3}} u_R(\vec{p}) e^{-i|\vec{p}|t} e^{i\vec{p}\cdot\vec{x}}$
- $\frac{1}{\sqrt{(2\pi)^3}} u_L(\vec{p}) e^{i|\vec{p}|t} e^{i\vec{p}\cdot\vec{x}}$

$\tilde{\sigma}^\mu \partial_\mu \psi_L = 0$

- pos-E=left-handed
- neg-E=right-handed
- 4-current $j_L^\mu = \psi_L^\dagger \tilde{\sigma}^\mu \psi_L$
- $\frac{1}{\sqrt{(2\pi)^3}} u_L(\vec{p}) e^{-i|\vec{p}|t} e^{i\vec{p}\cdot\vec{x}}$
- $\frac{1}{\sqrt{(2\pi)^3}} u_R(\vec{p}) e^{i|\vec{p}|t} e^{i\vec{p}\cdot\vec{x}}$

Weyl equations II

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Weyl equations II

Helicity

$$\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}u_R(\vec{p}) = u_R(\vec{p}) \text{ and } \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}u_L(\vec{p}) = -u_L(\vec{p}) \ .$$

$\sigma^\mu\partial_\mu\psi_R = 0$

- pos-E=right-handed
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- 4-current $j_R^\mu = \psi_R^\dagger\sigma^\mu\psi_R$
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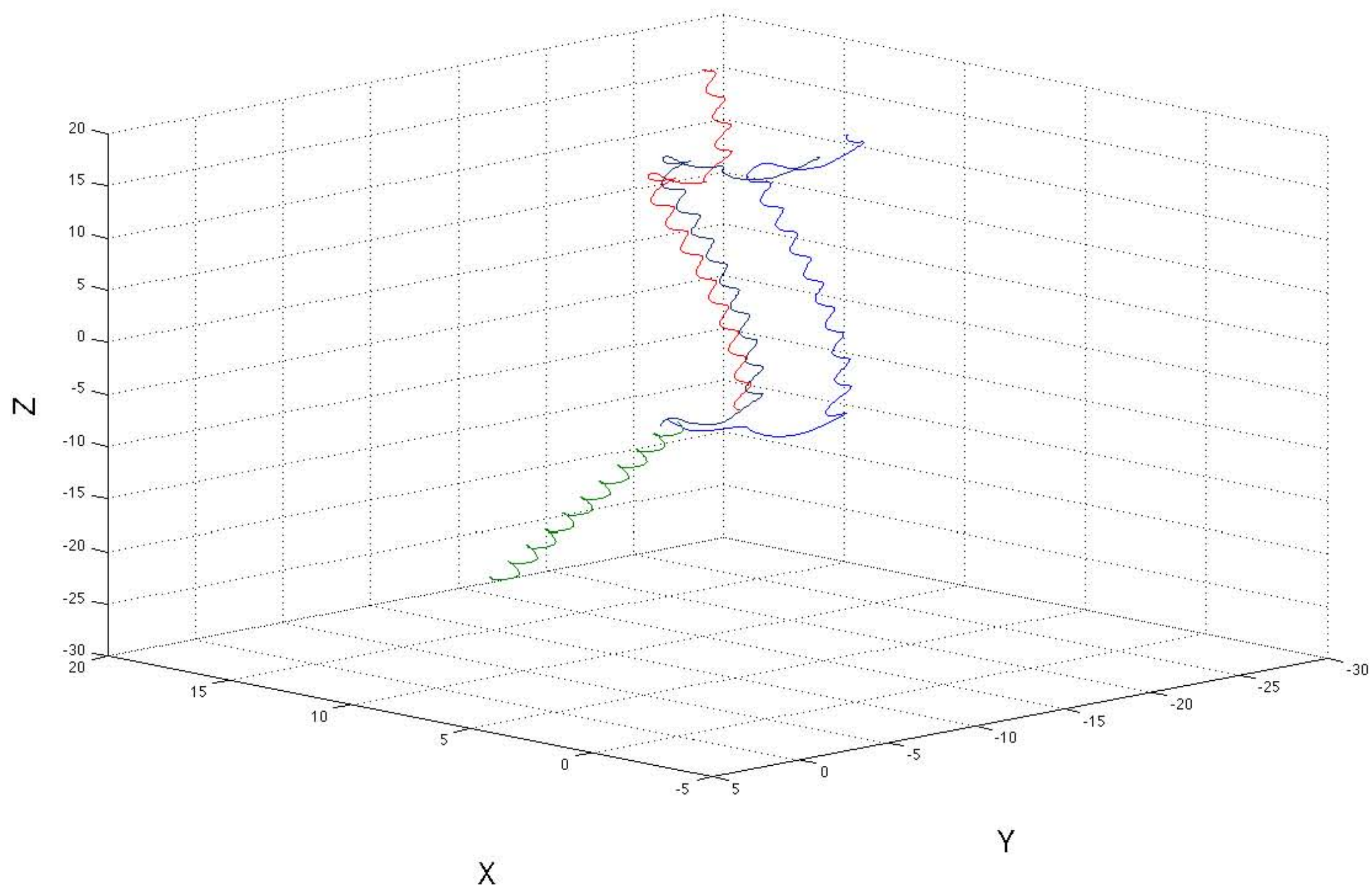
$\tilde{\sigma}^\mu\partial_\mu\psi_L = 0$

- pos-E=left-handed
- neg-E=right-handed
- 4-current $j_L^\mu = \psi_L^\dagger\tilde{\sigma}^\mu\psi_L$
- $\frac{1}{\sqrt{(2\pi)^3}}u_L(\vec{p})e^{-i|\vec{p}|t}e^{i\vec{p}\cdot\vec{x}}$
- $\frac{1}{\sqrt{(2\pi)^3}}u_R(\vec{p})e^{i|\vec{p}|t}e^{i\vec{p}\cdot\vec{x}}$

Pilot-wave theory for a single Weyl fermion

- Let us consider the R case for instance.
- $j_R^\mu = \psi_R^\dagger \sigma^\mu \psi_R$
- $\vec{v} = \vec{j}_R / j_R^0$ with $|\vec{v}| = 1$
- Weyl fermions always move luminally
- Not the case for the massless Dirac electron!

Trajectories for Weyl fermions



Free Dirac QFT=Weyl QFT with interactions

- Start from the massive Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\ .$$

- Split ψ into ψ_L and ψ_R.
- Hamiltonian formalism.
- Quantize the theory.
- Massive electron? Eigenstates of the Hamiltonian.

QFT Hamiltonian

- Fields:

$$\psi_R(\vec{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int d^3p [u_R(\vec{p}) e^{-i|\vec{p}|t + i\vec{p}\cdot\vec{x}} c_R(\vec{p}) + u_L(\vec{p}) e^{i|\vec{p}|t + i\vec{p}\cdot\vec{x}} \zeta_L(\vec{p})]$$

$$\psi_L(\vec{x}) = \frac{1}{\sqrt{(2\pi)^3}} \int d^3p [u_L(\vec{p}) e^{-i|\vec{p}|t + i\vec{p}\cdot\vec{x}} c_L(\vec{p}) + u_R(\vec{p}) e^{i|\vec{p}|t + i\vec{p}\cdot\vec{x}} \zeta_R(\vec{p})] \quad .$$

- Free Hamiltonian:

$$\widehat{H}_0 = \sum_{\chi \in \{L,R\}} \int d^3p |\vec{p}| [c^\dagger_\chi(\vec{p}) c_\chi(\vec{p}) - \zeta^\dagger_\chi(\vec{p}) \zeta_\chi(\vec{p})]$$

- Interaction Hamiltonian ($m\psi^\dagger_R \psi_L + m\psi^\dagger_L \psi_R$):

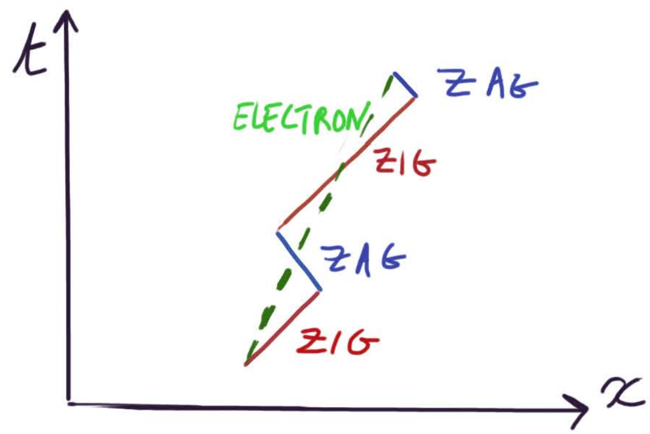
$$\widehat{H}_I = m \sum_{\chi \in \{L,R\}} \int d^3p [\alpha(\vec{p}) \zeta^\dagger_\chi(\vec{p}) c_\chi(\vec{p}) + \alpha^*(\vec{p}) c^\dagger_\chi(\vec{p}) \zeta_\chi(\vec{p})]$$

The zig-zag electron

- Superposition of positive and negative energy Weyl spinors:

$$|e_R^-(\vec{p})\rangle = \beta_1(\vec{p})c_R^\dagger(\vec{p})|0_W\rangle + \beta_2(\vec{p})\zeta_R^\dagger(\vec{p})|0_W\rangle \ .$$

- Jump between the zig and the zag motion (jump-rate à la DGTZ).
- Spacetime representation:



Penrose: The road to reality

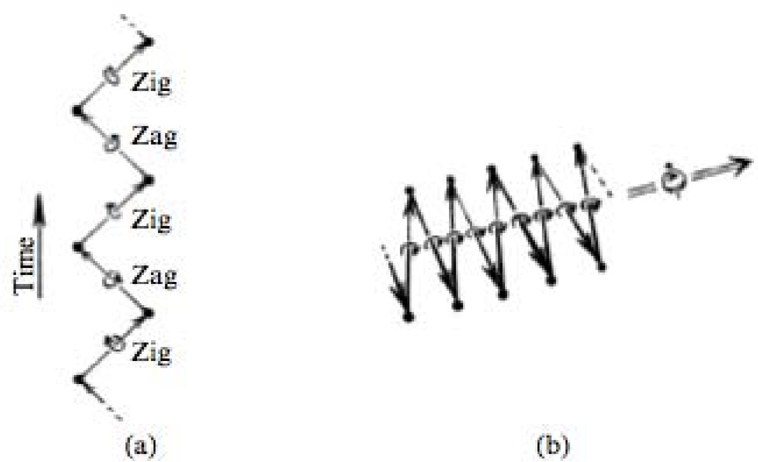
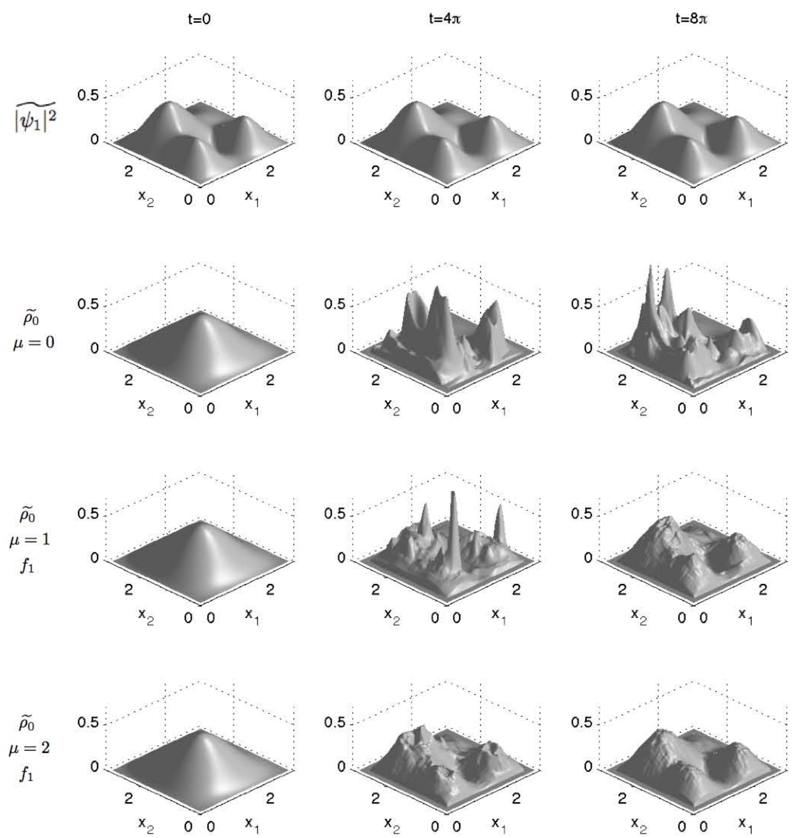
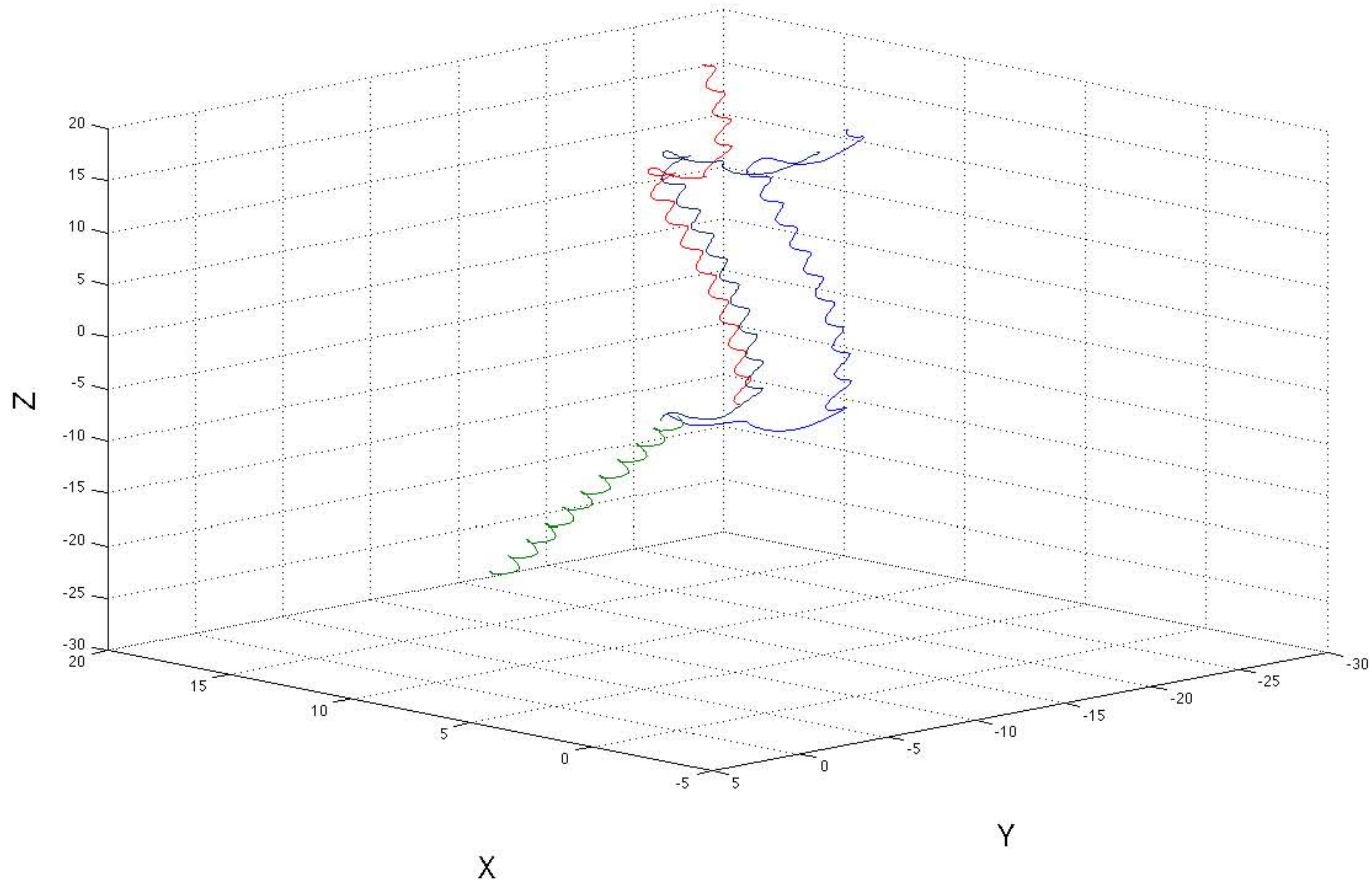


Fig. 25.1 Zigzag picture of the electron. (a) The electron (or other massive particle of spin $\frac{1}{2}$) can be viewed, in spacetime, as oscillating between a left-handed massless zig particle (helicity $-\frac{1}{2}$, as described by the unprimed 2-spinor α_A or, in the more usual physicist's notation, by the part projected out by $\frac{1}{2}(1-\boldsymbol{\gamma}_5)$) and a right-handed massless zag particle (helicity $+\frac{1}{2}$, as described by the primed 2-spinor $\beta_{B'}$, the part projected out by $\frac{1}{2}(1+\boldsymbol{\gamma}_5)$). Each is the source for the other, with the rest-mass as coupling constant. (b) From a 3-space perspective, in the 'rest-frame' of the electron, there is a continual reversal of the velocity (always the speed of light), but the direction of spin remains constant. (For reasons of clarity, the figure is drawn not quite in the electron's rest-frame, the electron drifting slowly off to the right.)

Relevance I [1]



Relevance II



Summary

- Bare particle ontology in pilot-wave QFTs
- Massless fermions as fundamental
- massless Weyl or massless Dirac ontology
- Relaxation simulations for massless fermions (Weyl or Dirac)