Title: ρ-ontism: thermodynamics and the interpretation of quantum theory

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Abstract: Can a density matrix be regarded as a description of the physically real properties of an individual system? If so, it may be possible to attribute the same objective significance to statistical mechanical properties, such as entropy or temperature, as to properties such as mass or energy. Non-linear modifications to the evolution of a density matrix can be proposed, based upon this idea, to account for thermodynamic irreversibility. Traditional approaches to interpreting quantum phenomena assume that an individual system is described by a pure state, with density matrices arising only through a statistical mixture or through tracing out entangled degrees of freedom. Treating the density matrix as fundamental can affect the viability of some of these interpretations, and introducting thermodynamically motivated non-linearities will not, in themselves, help in solving the quantum measurement problem.

ρ-ontism

Thermodynamics and the Interpretation of Quantum Theory

- Question: maybe the wavefunction is *epistemic*?
- A different question: maybe the density matrix is *real??*
  - Objective microscopic entropy
  - Non-unitary evolution
- But: standard solutions to the measurement problem are affected
  - Different kind of non-unitarity required for resolution of measurement problem
  - There is a tension between the two
    - Is unification possible?

Density Matrices and Entropy

- Motivation:
  - to attribute an entropy to an individual system, that is an objective property of that system.
  - Not of an ensemble, or an assembly, not epistemic, not subjective.
- If the density matrix is a representation of the actual, objective physical state of an individual system, then the Gibbs-von Neumann entropy can be an objective property of that system.

Epistemic: proper mixture $\rho_e = \sum_i p_i |i\rangle\langle i|$

Reduced: improper mixture $\rho_r = Tr_2\left[|\Psi_{12}\rangle\langle\Psi_{12}|\right]$

$$S[\rho] = -k\, Tr[\rho \ln \rho] > 0$$

Entropy: Subjective, perspectival

Ontic: not a mixture $\varrho = \sum_i w_i |i\rangle\langle i|$

$$S[\varrho] = -k\, Tr[\varrho \ln \varrho] > 0$$

Entropy: Objective, fundamental

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Rho-Ontism: thermodynamics

Quantum thermodynamics

Problem: $i\frac{d\varrho}{dt}=[H,\varrho]$ then $\frac{dS}{dt}=0$

- Ontic Density Matrix indistinguishable from a mixture
- Ontic Entropy does not increase in time
  - Entropy increase only due to inaccessible microscopic correlations
  - Inaccessibility is inaccessible to us
  - Entropy increase only apparent, subjective!

So: Non-Unitary Dynamics (NUD)

$$\frac{d}{dt}Tr[H\varrho]=0 \qquad \frac{d}{dt}Tr[\varrho]=0 \qquad \frac{dS}{dt}\geq 0$$

$$\frac{dS}{dt}=0$$ for pure states

Quantum thermodynamics

Beretta (1981,1984,2009)

$$\rho=\gamma\gamma^{\dagger} \qquad \frac{d\gamma}{dt}=\gamma\Delta H \qquad \Delta H=H-\langle H\rangle I \qquad \frac{d\langle H\rangle}{dt}=\frac{d\,Tr[\rho]}{dt}=0$$

$$\varrho=\gamma\gamma^{\dagger} \qquad \frac{d\gamma}{dt}=\gamma\Delta H+X$$

$\gamma(t)$ A parameterised path in square root density matrix space

$\dot{\gamma}(t)$ Tangent vector to the path

$A\cdot B=Tr[A B^{\dagger}+A^{\dagger} B]$ Inner product of operator space

$2\gamma A\cdot\dot{\gamma}(t)=\frac{d\langle A\rangle}{dt}$ Rate of change of A along path:

$2\gamma A\perp\dot{\gamma}(t)$ No change

$2\gamma A\|\dot{\gamma}(t)$ Max change

$$2\gamma S=S'\perp(\gamma,H)+S'\|(\gamma,H)$$

$S'\perp(\gamma,H)$ Direction of maximum change in S, with no change in normalisation or energy

Quantum thermodynamics

$$\frac{d\gamma}{dt} = \gamma \Delta H + \frac{1}{\tau} S'\perp(\gamma, H)$$

Bonus! For pure states: $S'\perp(\gamma, H) = 0$

$$\frac{d\varrho}{dt} = \frac{-i}{\hbar}[H, \varrho] - \frac{1}{\tau} \frac{\begin{vmatrix} \varrho \ln \varrho & \varrho & \frac{1}{2}\{H, \varrho\} \\ Tr[\varrho \ln \varrho] & 1 & Tr[H\varrho] \\ Tr[H\varrho \ln \varrho] & Tr[H\varrho] & Tr[\varrho H^2] \end{vmatrix}}{Tr[H^2 \varrho] - (Tr[H\varrho])^2}$$

τ A new physical constant(-ish), value to be determined by experiment

Epistemic: $\rho = \sum_i p_i |i\rangle\langle i| \rightarrow \rho$

Ontic: $\varrho = \sum_i w_i |i\rangle\langle i| \rightarrow \frac{1}{Z} e^{-H/kT}$

Mixed: $\zeta = \sum_i p_i \varrho_i \rightarrow \sum_i \frac{1}{Z_i} e^{-H_i/kT_i}$

An empirically testable signature!

The Measurement Problem

(in an oversimplified nutshell!)

Linear evolution gives: $$\frac{1}{\sqrt{2}}\left(\psi_u+\psi_d\right)\Phi_0 \rightarrow \frac{1}{\sqrt{2}}\left(\psi_u\Phi_u+\psi_d\Phi_d\right)$$

(which is *not* a simple statistical mix)

What occurs is $\psi_u\Phi_u$ or $\psi_d\Phi_d$

(which *is* a simple statistical mix)

- The wavefunction is real and does represent the state of a physical object.
  - Linear evolution is right and there is no extra structure to the world: both outcomes occur.
  - Linear evolution is right, but there is extra structure. These hidden variables determine which outcome has occurred.
  - Linear evolution is not right, so that the state of the world is actually U or D. The wavefunction collapses.
- The wavefunction does *not* represent the state of any part of the world, at all. The change from the superposition to a statistical mix is the expression of an invasive measurement interaction and/or an epistemic update.

The Measurement Problem

Density matrix realism

- Pure states evolve into pure states. Superposition still occurs

Can "a little bit" of impurity help?

$$\varrho^{(SM)} = \frac{1}{2}\left(\varrho_u^{(S)} + \varrho_d^{(S)}\right)\varrho_0^{(M)} \rightarrow \frac{1}{2}\left(\varrho_u^{(S)}\varrho_u^{(M)} + \varrho_d^{(S)}\varrho_d^{(M)}\right)$$

- NUD does not help: localisation does not occur.
  - NUD is only evolution, there is no extra structure to the world: both outcomes occur.
  - NUD is the only evolution, but there is extra structure. These hidden variables determine which outcome has occurred.
  - NUD needs an additional evolution, so that the state of the world is actually U or D. The density matrix collapses.

No Collapse

- Many worlds $\varrho^{(SM)} = \frac{1}{2}\left(\varrho_u^{(S)} + \varrho_d^{(S)}\right)\varrho_0^{(M)} \rightarrow \frac{1}{2}\left(\varrho_u^{(S)}\varrho_u^{(M)} + \varrho_d^{(S)}\varrho_d^{(M)}\right)$
  - The core interpretation would not appear to change
  - But NUD may make probability interpretation harder to sustain?

$$Tr\left[H\varrho_1\right] = Tr\left[H\varrho_2\right] \qquad \varrho = a\varrho_1 + b\varrho_2 \rightarrow \frac{1}{2}\varrho_1 + \frac{1}{2}\varrho_2$$

Branch specific entropy can go down:

$$\frac{1}{2}\left(\varrho_u^{(S)} + \varrho_d^{(S)}\right)\varrho_0^{(M)} \rightarrow \frac{1}{2}\left(\varrho_u^{(S)}\varrho_u^{(M)} + \varrho_d^{(S)}\varrho_d^{(M)}\right)$$

$$S\left[\frac{1}{2}\left(\varrho_u^{(S)} + \varrho_d^{(S)}\right)\varrho_0^{(M)}\right] \leq S\left[\frac{1}{2}\left(\varrho_u^{(S)}\varrho_u^{(M)} + \varrho_d^{(S)}\varrho_d^{(M)}\right)\right]$$

$$S\left[\frac{1}{2}\left(\varrho_u^{(S)} + \varrho_d^{(S)}\right)\varrho_0^{(M)}\right] \geq S\left[\varrho_u^{(S)}\varrho_u^{(M)}\right], S\left[\varrho_d^{(S)}\varrho_d^{(M)}\right]$$

Objective entropy increases but subjective entropy goes down?

No Collapse

Pilot wave theory $\langle x|\psi\rangle = R(x)e^{iS(x)}$

$$\frac{dX}{dt} = \frac{1}{m}\Im\left[\frac{\nabla_x \langle x|\psi\rangle}{\langle x|\psi\rangle}\right]_{x=X(t)} = \frac{1}{m}\nabla_x S(x)_{x=X(t)}$$

Pilot wave theory for ontic density matrix? (Maroney 2005)

$$\varrho = \sum_i w_i |\psi_i\rangle\langle\psi_i|$$

$$\varrho(x) = \langle x|\varrho|x\rangle = \sum_i w_i R_i^2(x)$$

$$J(x) = \sum_i w_i R_i^2(x)\nabla_x S_i(x)$$

$$\frac{dX}{dt} = \frac{J(x)}{\varrho(x)}_{x=X(t)}$$

Will this work with NUD? Need a locally conserved probability current

Many body no-signalling constraint leads to

$$\frac{d\gamma}{dt} = \gamma\Delta H + \frac{1}{\tau}\left(S'\perp(\gamma_\alpha, H_\alpha)\otimes\gamma_\beta + S'\perp(\gamma_\beta, H_\beta)\otimes\gamma_\alpha\right)$$

Better: a spatially localised entropy steepest ascent $S'\perp(\gamma(x), H(x))$

But: Branch specific entropy still can go down

Dynamic collapse

Bassi, Ghirardi (2003) http://pirsa.org/07060000

CSL equation:

$$\frac{d\left|\Psi(t)\right\rangle}{dt}=\left[-\frac{i}{\hbar}H+M\cdot V-\frac{\gamma}{2}M^{\dagger}\cdot M\right]\left|\Psi(t)\right\rangle$$

The mass density matter field becomes localised $m(x,t)=\left\langle\Psi(t)\right|M\left|\Psi(t)\right\rangle$

Master equation:

$$\frac{d\rho}{dt}=-\frac{i}{\hbar}\left[H,\rho\right]+\gamma M\rho\cdot M^{\dagger}-\frac{\gamma}{2}\left\{M^{\dagger}\cdot M,\rho\right\}$$

Substituting in the ontological density matrix

$$\frac{d\varrho}{dt}=-\frac{i}{\hbar}\left[H,\varrho\right]+\gamma M\varrho\cdot M^{\dagger}-\frac{\gamma}{2}\left\{M^{\dagger}\cdot M,\varrho\right\}$$

The mass density $m(x,t)=Tr\left[M\varrho\right]$ does not localise.

The CSL equation no longer solves the measurement problem!

Dynamic collapse

GRW Collapse

Localisation operator $$L(x')=\left(\frac{2a}{\pi}\right)^{3/4}\int e^{-a(x'-x'')^2}|x''\rangle\langle x'|dx''$$

Random chance of a spontaneous collapse taking place, with a probability $1/\tau$ per unit time

at location x' with probability $P(x')=Tr\left[L(x')\varrho L(x')\right]dx'$

Density matrix becomes localised about x' $$\varrho\rightarrow\frac{L(x')\varrho L(x')}{Tr\left[L(x')\varrho L(x')\right]}$$

But note: there is no equation for: $\zeta=\sum_i p_i\varrho_i$

$$\frac{d\zeta}{dt}\neq-\frac{i}{\hbar}[H,\zeta]+\gamma M\zeta\cdot M^{\dagger}-\frac{\gamma}{2}\{M^{\dagger}\cdot M,\zeta\}$$

Entropy decreases *objectively*: $$S\left[\frac{1}{2}\left(\varrho_u^{(S)}+\varrho_d^{(S)}\right)\varrho_0^{(M)}\right]\geq S\left[\varrho_u^{(S)}\varrho_u^{(M)}\right],S\left[\varrho_d^{(S)}\varrho_d^{(M)}\right]$$

Conclusion

- Modified Schrodinger dynamics required to address macroscopic time asymmetry (thermodynamics) and to address macroscopically distinct outcomes (quantum) are not only different, but may even contradict each other.
  - The ontological density matrix in CSL equation prevents localisation occurring.
  - GRW collapses create absolute decreases in density matrix entropy.
  - No collapse theories still allow perspectival decreases in density matrix entropy.
    - NUD may undermine arguments on how to understand probabilities in Many Worlds.
- Density matrix realism, as an attempt to remove subjectivity from thermodynamics, cannot succeed without also addressing issues of interpretation of quantum theory.
  - Requires further development of the proposed non-unitary dynamics.
- Likewise, questions of interpretation could be radically affected by any empirical observation of rho-ontic based non-unitary dynamics.
- As NUD, like CSL, makes novel empirical predictions
    - Ultimately it is experiment that must be the decider!