Title: Algorithm for the shortest path through time

Date: Dec 03, 2010 09:00 AM

URL: http://pirsa.org/10120066

Abstract: Feynman showed that the path of least action is determined by quantum interference. The interference may be viewed as part of a quantum algorithm for minimising the action. In fact, Lloyd describes the Universe as a giant quantum computer whose purpose is to calculate its own state. Could the direction of time that the universe is apparently following be determined by a quantum algorithm? The answer lies in the violation of time reversal (T) invariance that is being observed in an increasing number of particle accelerator experiments. The violation signifies a fundamental asymmetry between the past and future and calls for a major shift in the way we think about time. Here we show that processes which violate T invariance induce destructive interference between different paths that the universe can take through time. The interference eliminates all paths except for two that represent continuously forwards and continuously backwards time evolution. This suggests that quantum interference from T violation processes gives rise to the phenomenological unidirectional nature of time. A path consisting exclusively of forward steps gives the shortest path to a point which is in the forwards direction. The quantum interference, therefore, underlies a quantum algorithm that determines shortest path through time.

Algorithm for the shortest path through time

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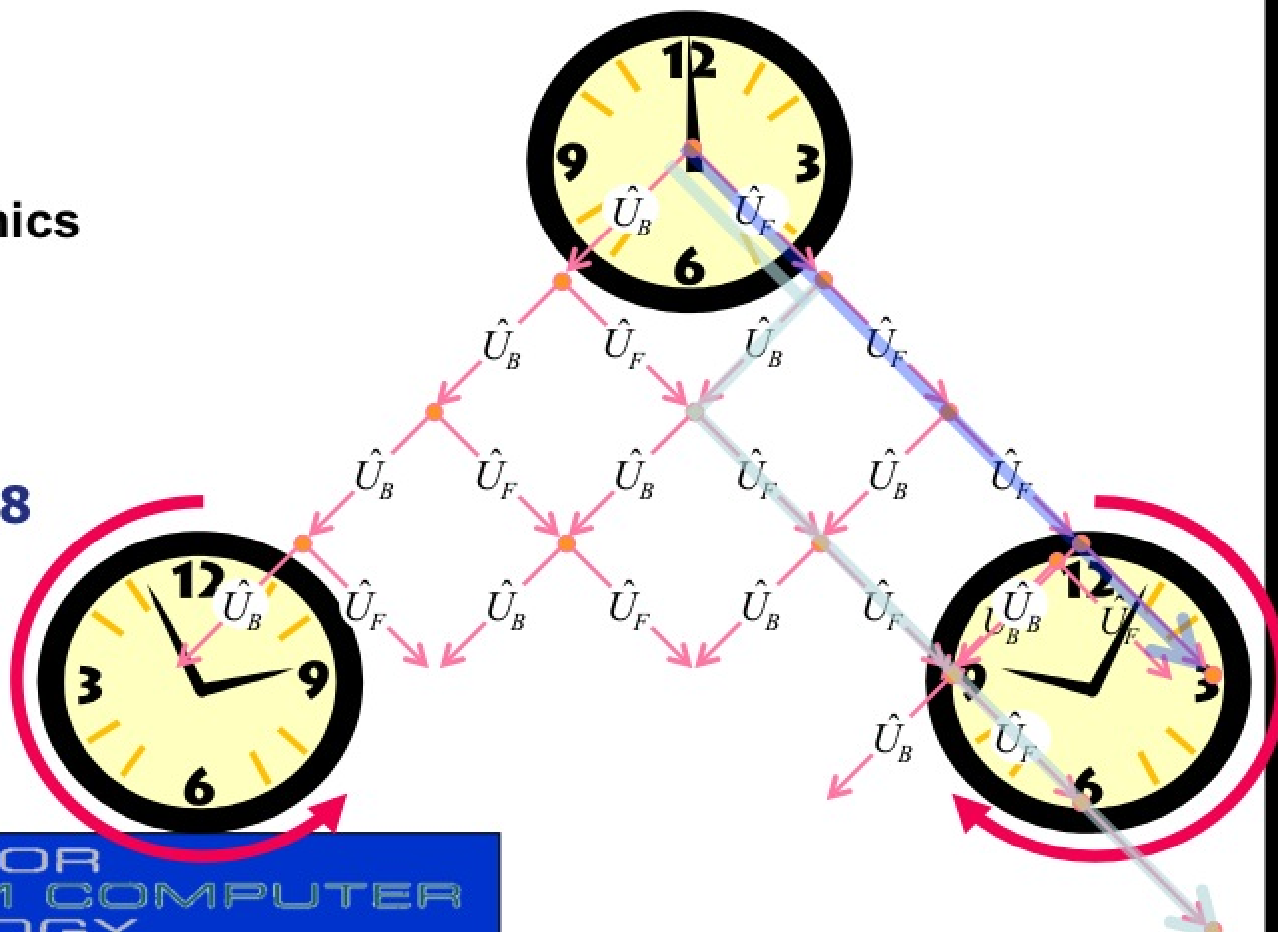
Preprint arXiv:0911.4528



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Introduction

Prob. 1: What is the origin of the "direction of time" ?

Quantum Gravity

needed for Planck scale energy densities

E.g. **Wheeler-DeWitt equation** (canonical approach)

- topological invariance implies the **Hamiltonian constraint** $\hat{H}\left|\psi\right\rangle = 0$
- $\left|\psi\right\rangle$ represents ***whole history of universe***
- **no preference for either direction of time**
- use WKB approx to introduce classical time variable (ad hoc)

In other theories the notion of time is less clear

[**Topological invariance:** physical predictions of a theory are unchanged by different choices of topologically- equivalent spacetime manifolds such as the redefinition of spacelike hypersurfaces]

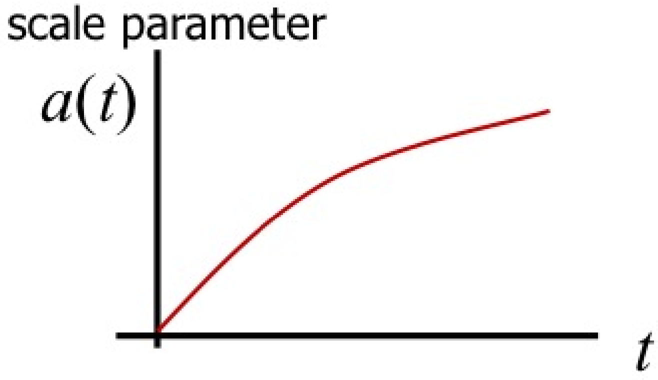
Misner, Rev. Mod. Phys. **29,** 497 (1957)

General Relativity – classical spacetime

assumes evolution in a fixed direction of time

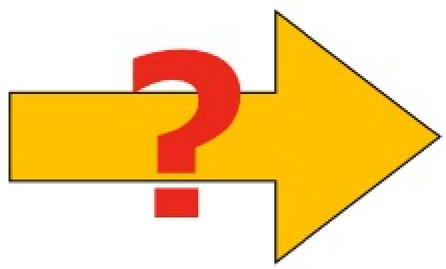
e.g. FRW universe

$$ds^2 = dt^2 + a(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$



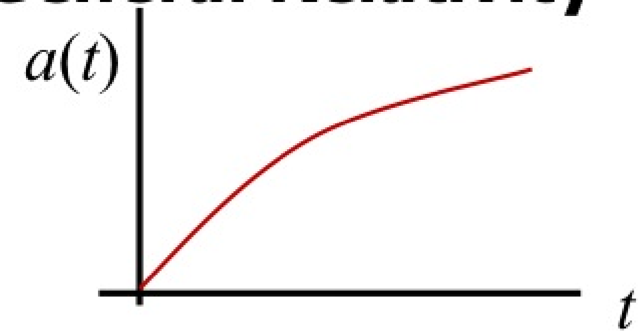
The **big** questions are...

Quantum Gravity



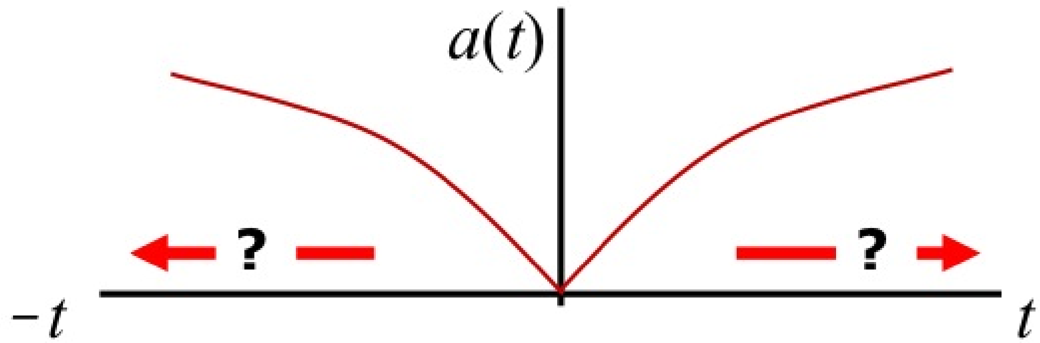
General Relativity

$$\hat{H}\left|\psi\right\rangle = 0$$



no time

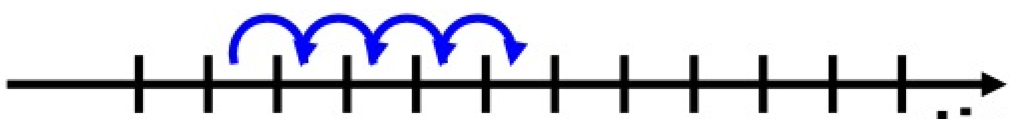
classical time

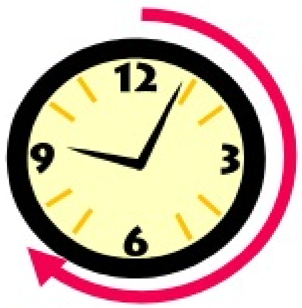


c.f. Sakharov, Sov. Phys. Usp. **34**, 404 (1991).

Are the directions of time distinguishable?

How does universe know which direction to pick?





Is there some kind of spontaneous symmetry breaking?

This is the

direction of time problem

Arrows of time (Eddington)

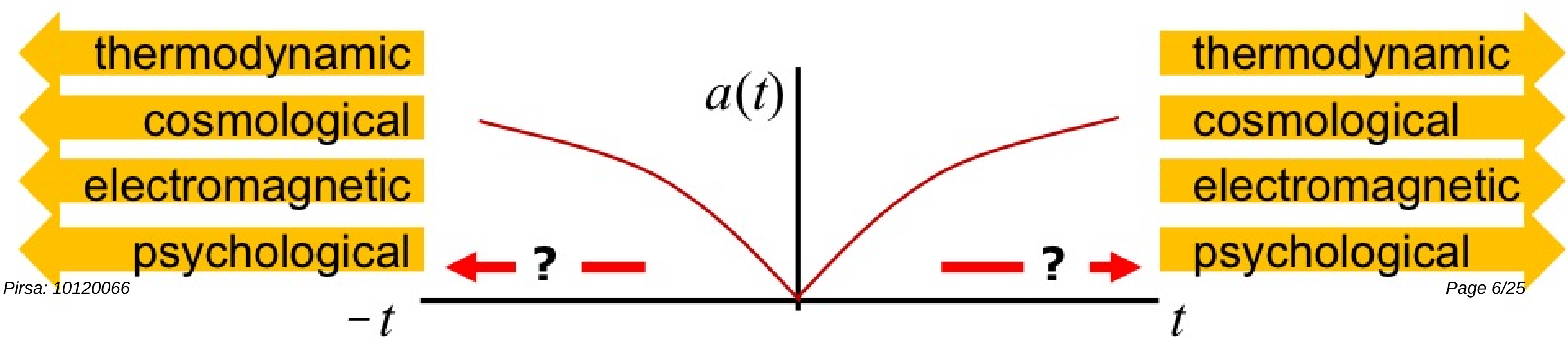
[The Nature Of The Physical World, 1928]

thermodynamic	(entropy doesn't decrease)
cosmological	(expanding universe)
electromagnetic	(spontaneous emission not absorption)
psychological…	(remember past not future)
matter-antimatter	(CP violation favours matter)

But the first set (✳) relies on time-symmetric dynamical laws

$$\frac{d^2x}{dt^2} = \frac{F}{m} \qquad \frac{d|\psi\rangle}{dt} = -\frac{i}{\hbar}\hat{H}|\psi\rangle$$

and so they don't give an *objective direction of time* :



Only possibility for breaking the symmetry in time is given by

matter-antimatter arrow (CP violation favours matter)

We need to look at how time reversal symmetry is violated in a dynamical law

Time reversal operator

[Wigner, *Group theory* (1959)]

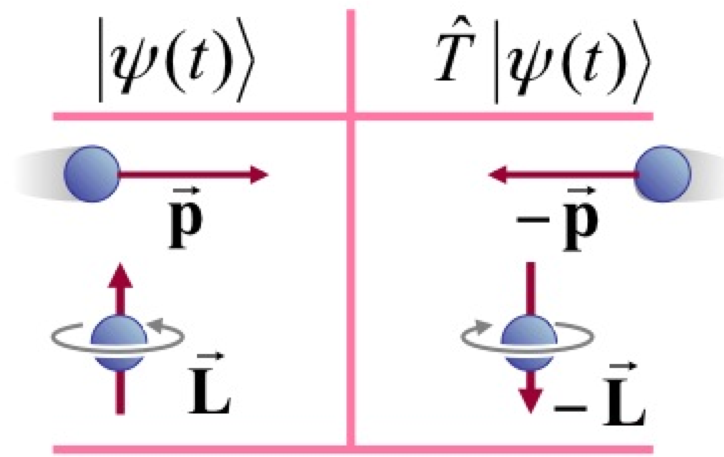
$$\hat{T} = \hat{U}\,\hat{K}$$

unitary operator ($\hat{U}$)

anti-unitary operator ($\hat{K}$)
- action is complex conjugation

$$\hat{K}\left(a|0\rangle + b|1\rangle\right) = a^{*}\hat{K}|0\rangle + b^{*}\hat{K}|1\rangle$$

$$\hat{K}\hat{K}^{\dagger} = \hat{K}^{\dagger}\hat{K} = \hat{1}$$



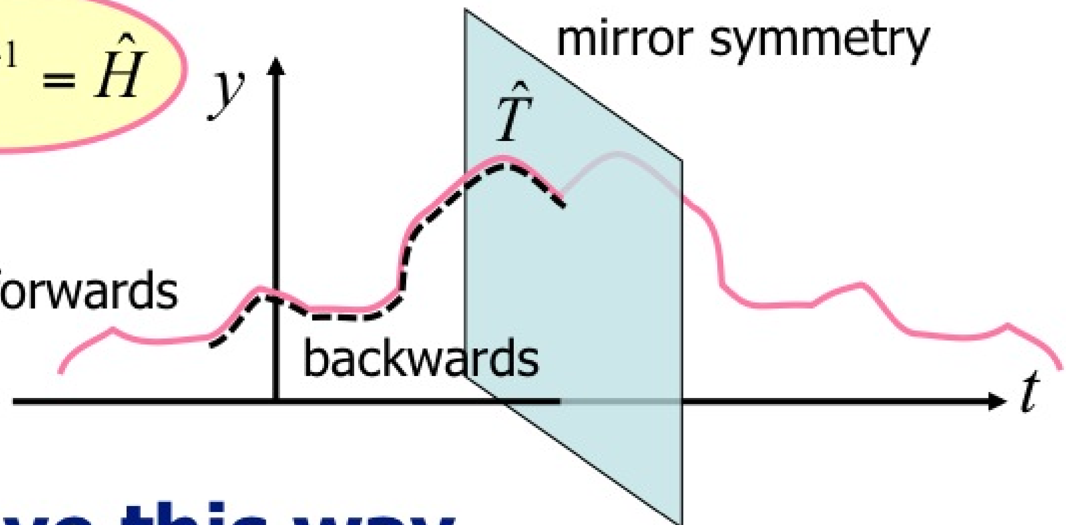
Typical Schrodinger equation

Backwards evolution is simply <u>backtracking</u> the forwards evolution

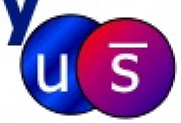
$$-i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$$

$$\hat{T}\hat{H}\hat{T}^{-1} = \hat{H}$$

$$i\hbar\frac{\partial}{\partial t}\hat{T}|\psi\rangle = \hat{T}\hat{H}\hat{T}^{-1}\hat{T}|\psi\rangle$$
$$= \hat{H}\hat{T}|\psi\rangle$$



But kaons don't behave this way



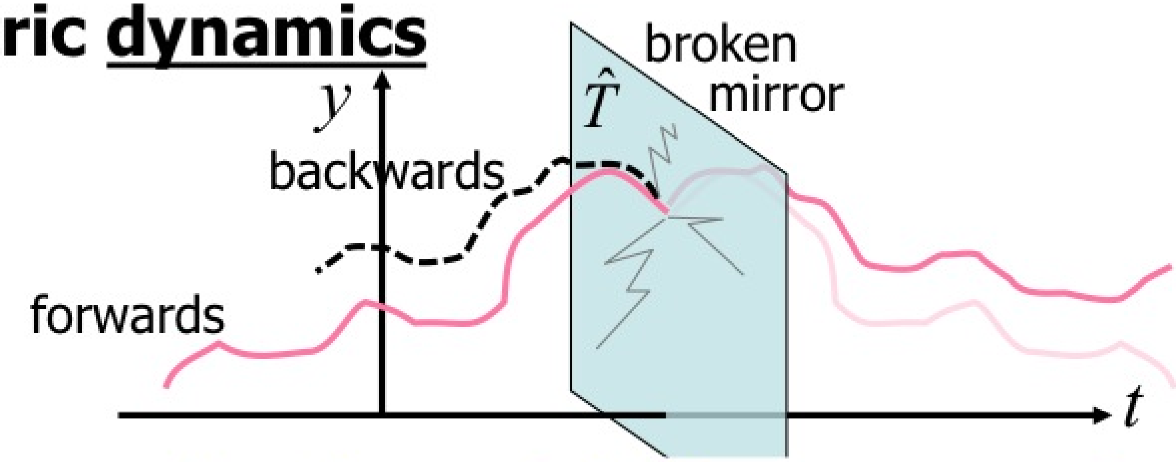
boson, neutral,
½ m_p, lifetime 10^{-8}s

Violation of time reversal invariance

- a small (0.2%) violation of CP & T invariance in neutral ***kaon*** decay
- discovered in 1964 by Cronin & Fitch (Nobel Prize 1980)
- partially accounts for observed dominance of matter over antimatter
- direct T violation in K meson decay detected (independent of CPT invariance). [CPLEAR, Phys. Lett. B **444**, 43-51 (1998)]

Gives time asymmetric dynamics

$$\hat{T}\hat{H}\hat{T}^{-1} \neq \hat{H}$$



a fundamental time asymmetry

Huw Price :

concept of an *objective direction of time* not incoherent
but there is a *lack of evidence* for it

"The Flow of Time", Oxford Handbook of Philosophy of Time, (2011).

I agree :

T violating processes are **relatively rare** & magnitudes are **relatively small** and no large scale effects have **previously** been shown

But I will show that T violation processes can have *large-scale physical effects* and can determine the direction of time

Prob. 2: Dynamical Equation with 2 versions of $\hat{H}$

To model T violating processes:

- **choose** the direction of time evolution (→) and a version of $\hat{H}$:

$$-i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$$

- **apply the time reversal operation** $\hat{T}$ to get evolution in the opposite direction of time (←) according to $\hat{T}\hat{H}\hat{T}^{-1}$:

$$i\hbar\frac{\partial}{\partial t}\hat{T}|\psi\rangle = \hat{T}\hat{H}\hat{T}^{-1}\hat{T}|\psi\rangle \quad \Big\} \quad i\hbar\frac{\partial}{\partial t}|\phi\rangle = \hat{T}\hat{H}\hat{T}^{-1}|\phi\rangle$$

BUT when the direction of time evolution cannot be specified:

- we have no argument for favouring $\hat{H}$ over $\hat{T}\hat{H}\hat{T}^{-1}$
- so both Hamiltonians must appear in the equation of motion **!!!!**
- we don't have such an equation

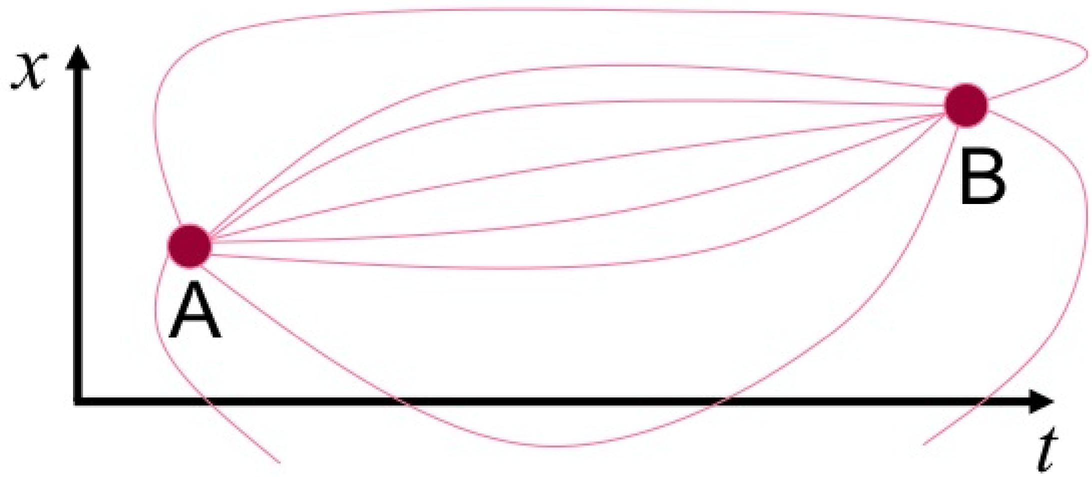
The universe has no reference for direction of time.

Therefore, there is no satisfactory quantum formalism for describing a universe that exhibits T violation.

Paths through time

- no *á priori* direction of time evolution
- so must allow for both directions
- i.e. multiple paths through time

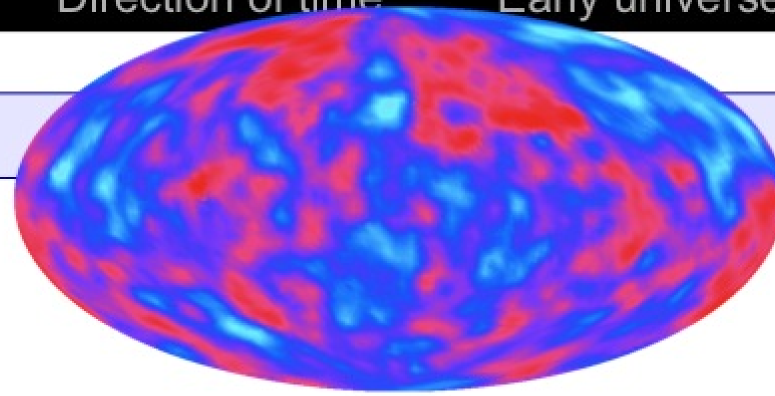
Recall Feynman's sum over histories: [Rev. Mod. Phys. **20**, 367 (1948)]



total transition amplitude from A to B
=
sum the amplitudes of all possible paths from A to B

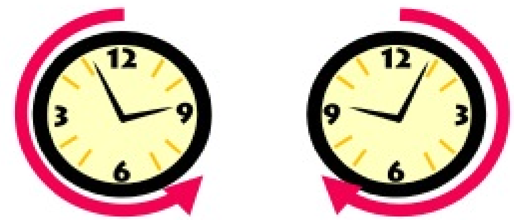
$$\langle B, t_b | A, t_a \rangle = \langle B | U(t_b, t_N) U(t_N, t_{N-1}) \mathrm{L} \; U(t_1, t_a) | A \rangle$$

$$= \int \mathrm{L} \int dx_1 \mathrm{L} \; dx_N \prod_n \langle x_n, t_n | x_{n-1}, t_{n-1} \rangle$$



Model of the universe:

- **it is closed** in the sense that it does not interact with any other physical system
- **it has no external clocks** and so analysis needs to be unbiased with respect to the direction of time
- **both versions of the Hamiltonian** should appear in the dynamical equation of motion



Forwards and Backwards evolution

Evolution of state $|\psi_0\rangle$ over time interval τ in the **forward** direction

$$|\psi_F(\tau)\rangle = \hat{U}_F(\tau)|\psi_0\rangle$$

where $\hat{U}_F(\tau) = \exp\left(-i\hat{H}_F\tau\right)$ $(\mathrm{h}=1)$

and $\hat{H}_F$ = Hamiltonian for **forward time evolution.**

Evolution of state $|\psi_0\rangle$ over time interval τ in the **backward** direction

$$|\psi_B(\tau)\rangle = \hat{U}_B(\tau)|\psi_0\rangle$$

$(h = 1)$

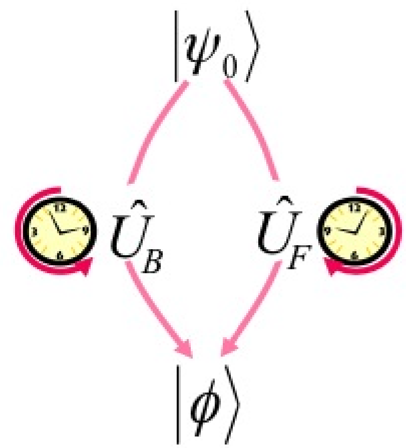
where $\hat{U}_B(\tau) = \exp\left(i\hat{H}_B\tau\right)$

and $\hat{H}_B = \hat{T}\hat{H}_F\hat{T}^{-1}$ = Hamiltonian for **backward time evolution.**

Constructing paths:

- $\langle\phi|\hat{U}_F(\tau)|\psi_0\rangle$ and $\langle\phi|\hat{U}_B(\tau)|\psi_0\rangle$ are probability amplitudes for the system to evolve from $|\psi_0\rangle$ to $|\phi\rangle$ **via two paths in** time

- we have no basis for favouring one path over the other so **assign an equal statistical weighting to each using Feynman's sum over histories**



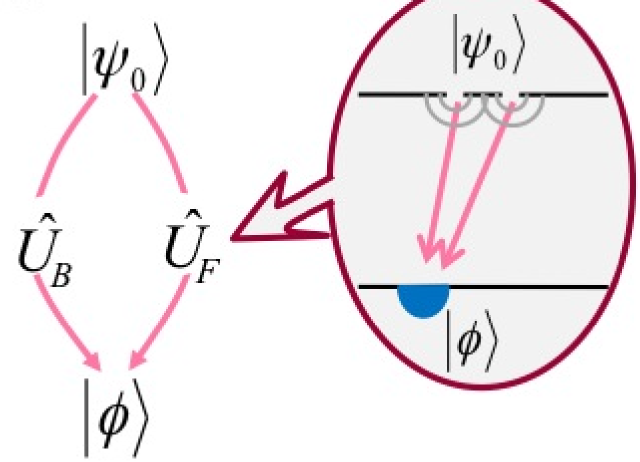
[Feynman Rev. Mod. Phys. **20**, 367 (1948)]

The total amplitude for $|\psi_0\rangle \rightarrow |\phi\rangle$ is proportional to

$$\langle\phi|\hat{U}_F(\tau)|\psi_0\rangle + \langle\phi|\hat{U}_B(\tau)|\psi_0\rangle$$

$$= \langle\phi|\hat{U}_F(\tau) + \hat{U}_B(\tau)|\psi_0\rangle$$

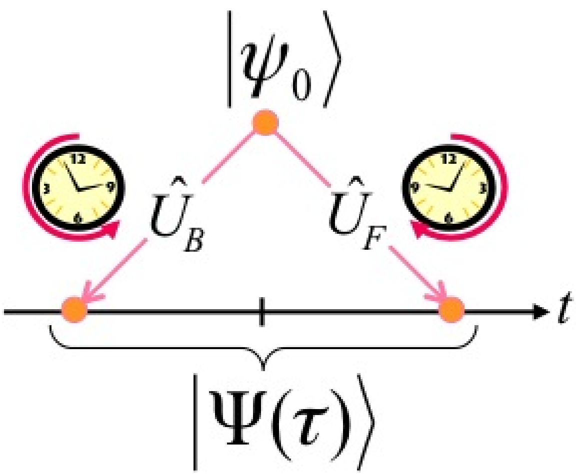
c.f. double slit:



This is true for all states $|\phi\rangle$, so

$$|\Psi(\tau)\rangle \propto \left[\hat{U}_F(\tau) + \hat{U}_B(\tau)\right]|\psi_0\rangle$$

which we call **time-symmetric evolution**.

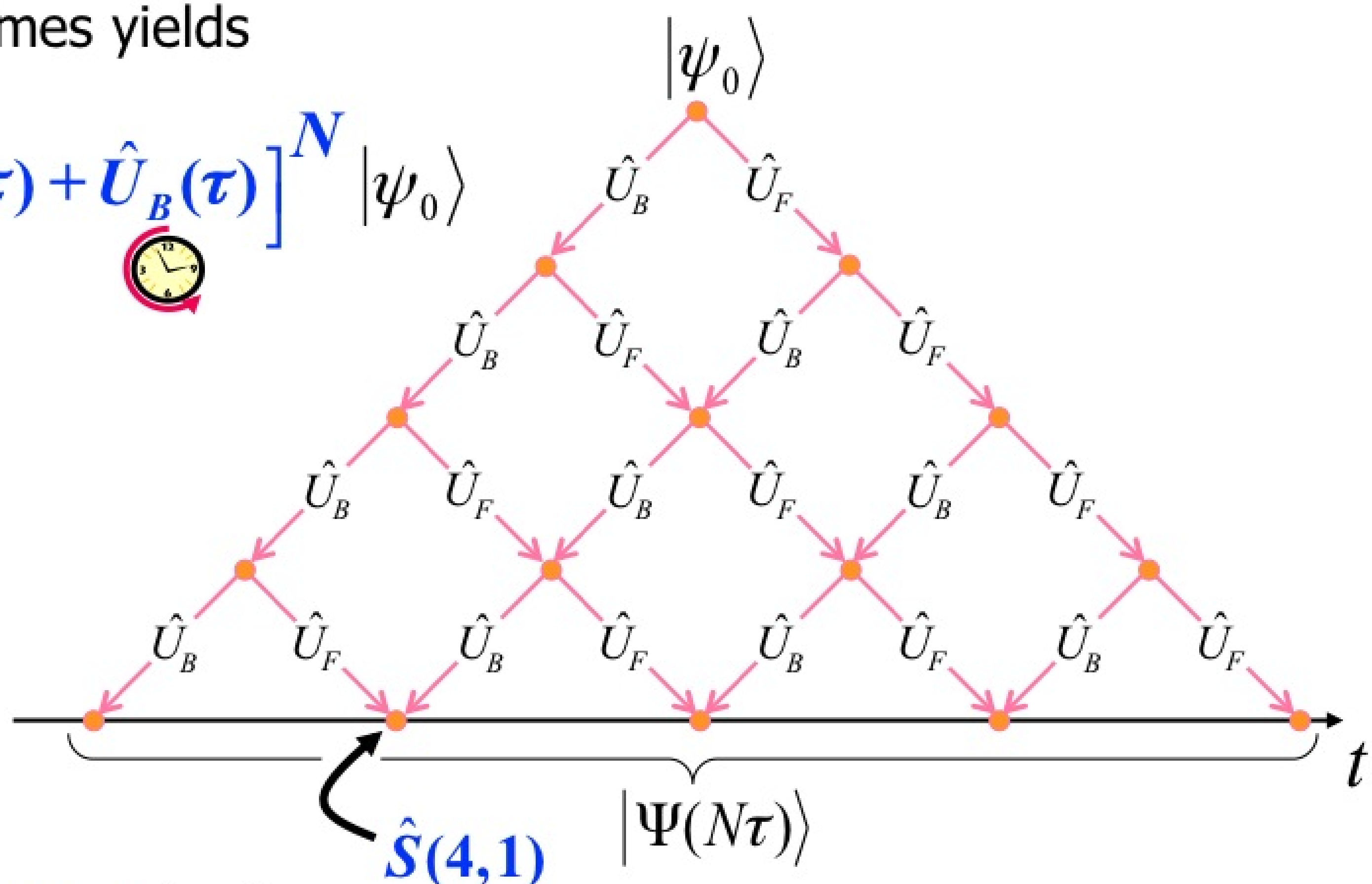


Time-symmetric evolution over an **additional time interval** of τ is given by

$$|\Psi(2\tau)\rangle \propto \left[\hat{U}_F(\tau) + \hat{U}_B(\tau)\right]|\Psi(\tau)\rangle = \left[\hat{U}_F(\tau) + \hat{U}_B(\tau)\right]^2|\psi_0\rangle$$

Repeating this N times yields

$$\left|\Psi(N\tau)\right\rangle \propto \left[\hat{U}_F(\tau)+\hat{U}_B(\tau)\right]^N \left|\psi_0\right\rangle$$



Let

$$\left|\Psi(N\tau)\right\rangle \propto \sum_{m=0}^{N} \hat{S}(N,m)\left|\psi_0\right\rangle$$

- $\hat{S}(N,m)$ is a sum containing $\binom{N}{m}$ different terms
- $\left\langle\phi\right|\hat{S}(N,m)\left|\psi_0\right\rangle$ is a sum over a set of paths each comprising m **forwards** steps and $N-m$ **backwards** steps

The limit τ◊ 0 ?

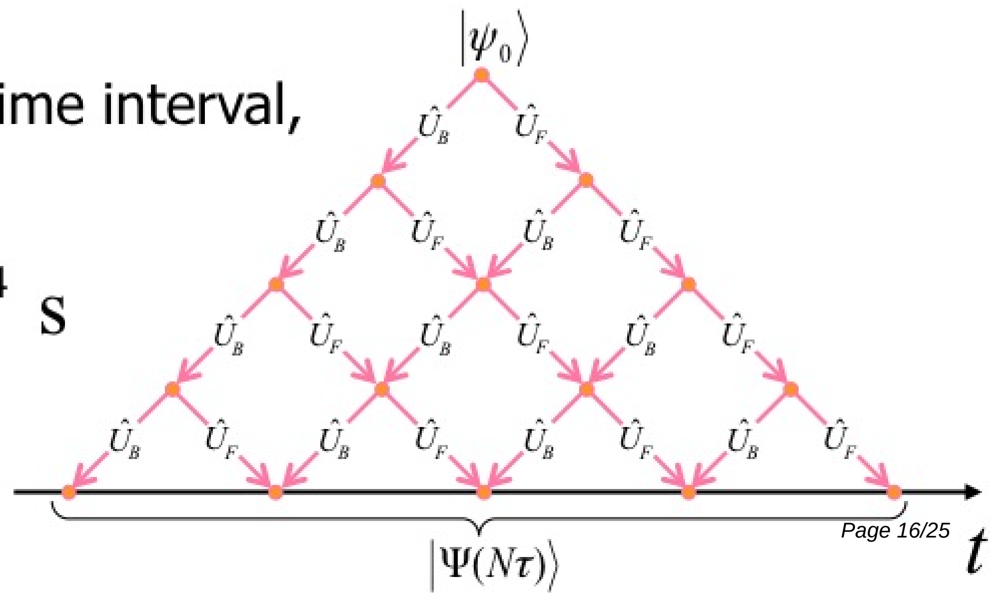
- fix total time t_{tot} and set $\tau = \dfrac{t_{\text{tot}}}{N}$. Take limit as $N \to \infty$.
- we find $\dfrac{1}{2^N}\left[\hat{U}_F(\tau) + \hat{U}_B(\tau)\right]^N = \left[\exp[-i\frac{1}{2}(\hat{H}_F - \hat{H}_B)\tau] + O(\tau^2)\right]^N$

$$\to \exp[-i\frac{1}{2}\underbrace{(\hat{H}_F - \hat{H}_B)}_{\text{effective Hamiltonian}} t_{\text{tot}}] \quad \text{as } N \to \infty$$

effective Hamiltonian
=0 for conventional clock device
∴ no time in conventional sense

- Set τ to be a small physical time interval, **Planck time**

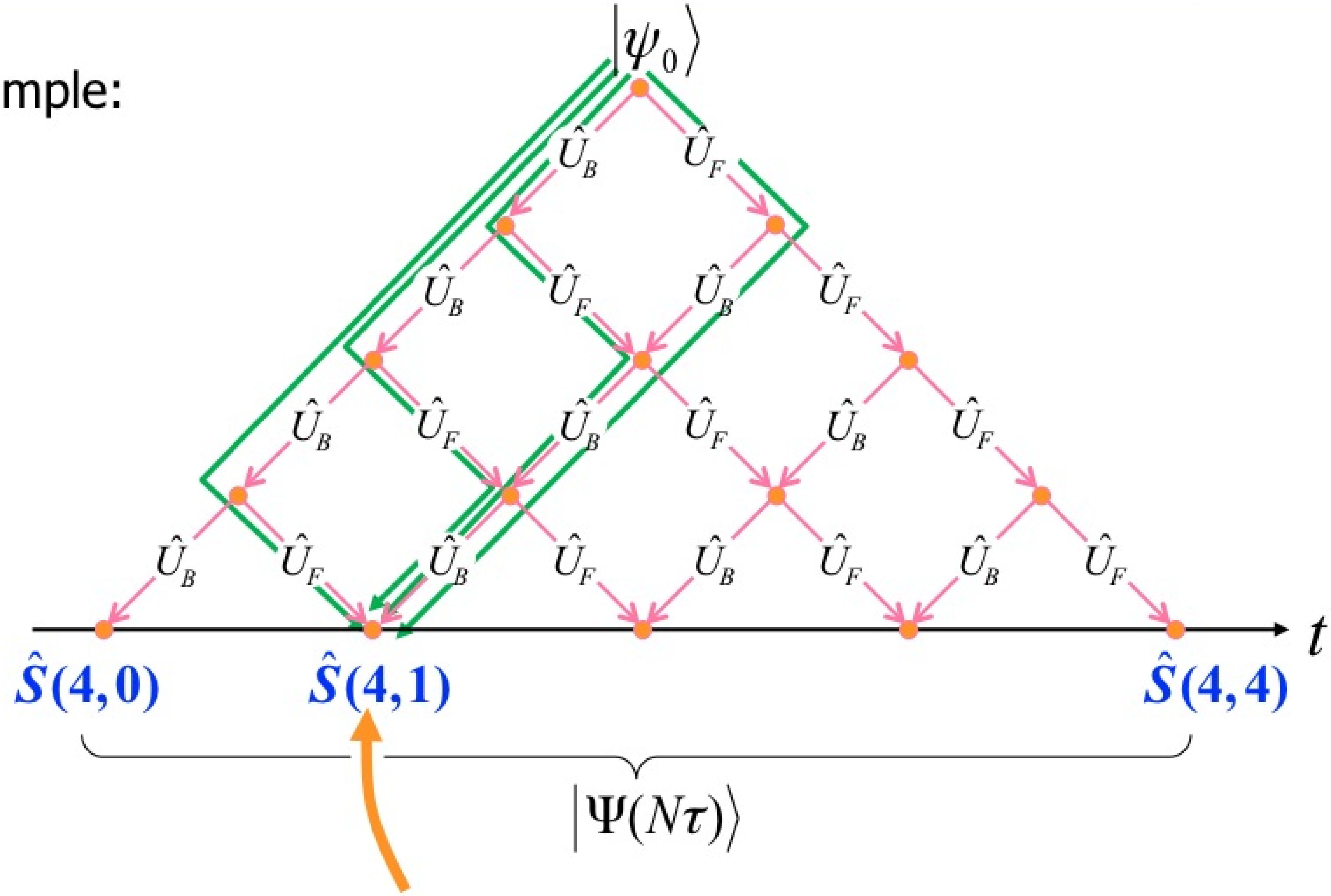
$$\tau \approx 5 \times 10^{-44}\ \text{s}$$



Interference

Multiple paths

Example:



4 terms interfere

$$\left[\hat{U}_F(\tau)+\hat{U}_B(\tau)\right]^N |\psi_0\rangle = \sum_{n=0}^{N} \hat{S}(N,m)|\psi_0\rangle$$

Simplifying $\hat{S}(N,m)$

The **Zassenhaus** (*Baker-Campbell-Hausdorff*) formula

$$e^{-i\hat{A}\delta}\, e^{-i\hat{B}\delta} = e^{-i\hat{B}\delta}\, e^{-i\hat{A}\delta}\, e^{\delta^2[\hat{A},\hat{B}]+O(\delta^3)}$$

gives

$$\hat{S}(N,m) = \hat{U}_B(N-m)\hat{U}_F(m)$$

$$\times \sum_{v=0}^{N-m}\sum_{u=0}^{v} \cdots \sum_{k=0}^{1}\sum_{j=0}^{k} \exp\left[-(v+u+\cdots+k+j)\tau^2 \underbrace{[\hat{H}_F,\hat{H}_B]}_{-i\lambda} + O(\tau^3)\right]$$

Eigenvalue equation for commutator

$$i[\hat{H}_F,\hat{H}_B]|\lambda\rangle = \lambda|\lambda\rangle$$

$$\int \rho(\lambda)\hat{\Pi}(\lambda)d\lambda = \hat{1}$$

eigenvalue

degeneracy

trace 1 projection op.

$$\hat{S}(N,m) = \hat{U}_B[(N-m)\tau]\hat{U}_F(m\tau)\int \boldsymbol{I(N,m,\lambda)}\rho(\lambda)\hat{\Pi}(\lambda)d\lambda$$

eigenvalue

trace 1 projection op.

degeneracy

where

$$\boldsymbol{I(N,m,\lambda)} = \frac{\prod_{k=0}^{m-1}\{\exp[-i(N-k)\tau^2\lambda]-1\}}{\prod_{k=1}^{m}[\exp(-ik\tau^2\lambda)-1]}$$

Estimating eigenvalues λ

$$i[\hat{H}_F,\hat{H}_B]|\lambda\rangle = \lambda|\lambda\rangle$$

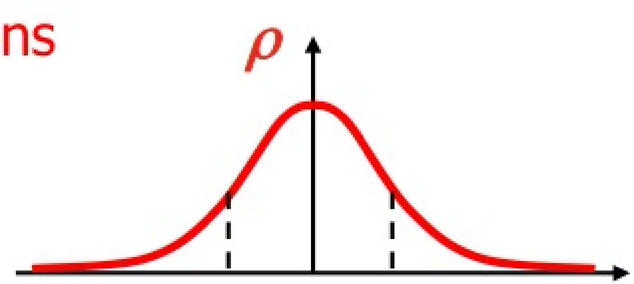
$$\frac{d\psi}{dt} = -i(\mathbf{M}-i\Gamma)\psi, \quad \psi = \begin{bmatrix} a \\ b \end{bmatrix} \begin{matrix} \leftarrow K^0 \\ \leftarrow \bar{K}^0 \end{matrix}$$

phenomenological model [Lee, PR 138, B1490 (1965)].

Eigenvalues for j th kaon $\quad \lambda_j \approx \pm 10^{17}\ \mathrm{s}^{-2}$

Eigenvalues for M kaons

$$\lambda_{SD} = \sqrt{M}\times 10^{17}\ \mathrm{s}^{-2}$$



Let $M = f\times 10^{80}$

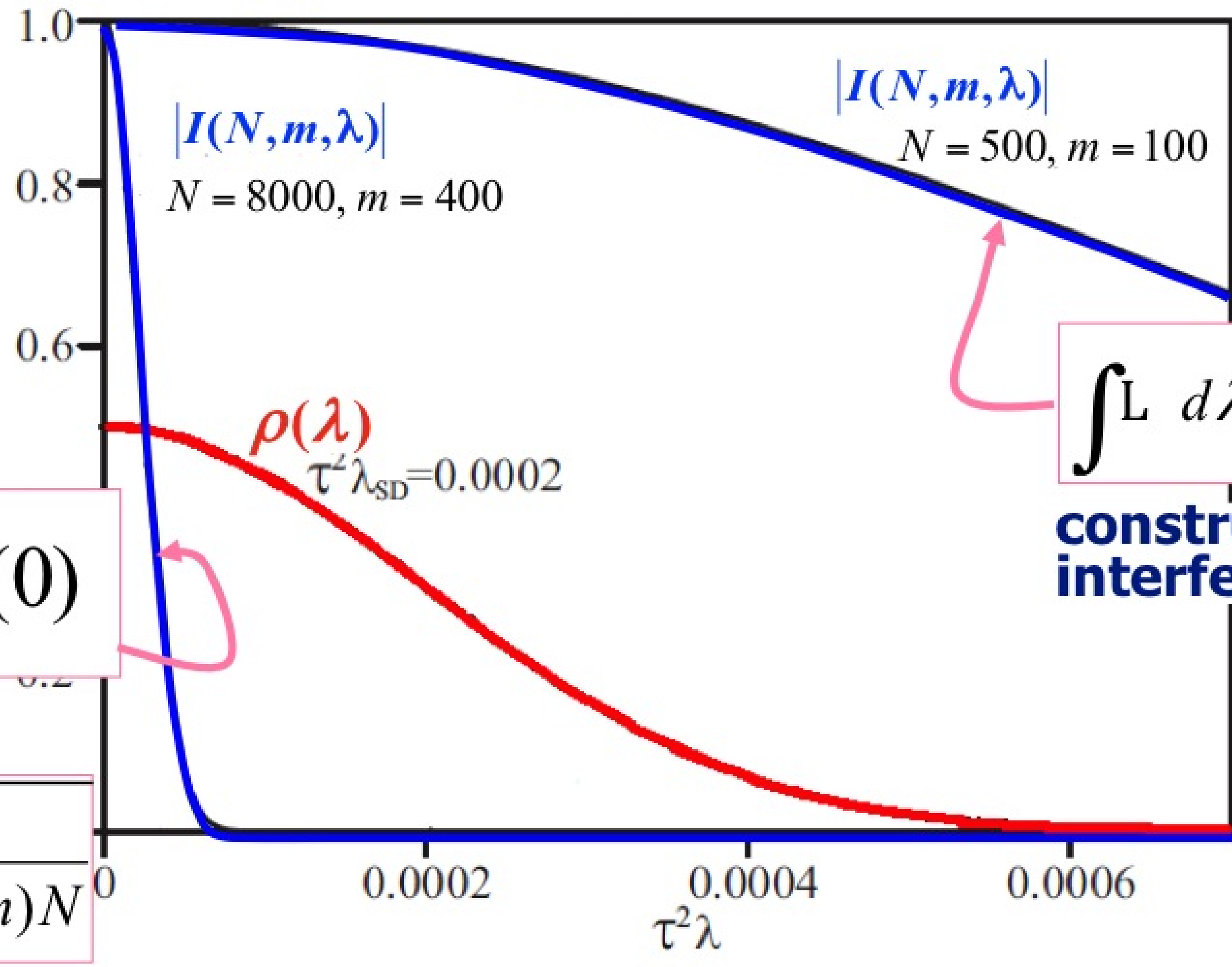
total # of particles

fraction

$$\lambda_{SD} = \sqrt{f}\times 10^{57}\ \mathrm{s}^{-2}$$

$$\hat{S}(N-n,n) = \hat{U}_B[(N-n)\tau]\hat{U}_F(n\tau)\int I(N,m,\lambda)\rho(\lambda)\hat{\Pi}(\lambda)d\lambda$$

Comparison of $|I(N,m,\lambda)|$ with $\rho(\lambda)$



destructive interference

$$\int \mathrm{L}\ d\lambda \to \hat{\Pi}(0)$$

constructive interference

$$\int \mathrm{L}\ d\lambda \to \hat{1}$$

width

$$\tau^2\Delta\lambda \approx \sqrt{\frac{24}{m(N-m)N}}$$

Destructive interference

Only two paths survive if

$$N\tau \ ? \ f^{-1/2} \times 10^{-13}\ \text{s}$$

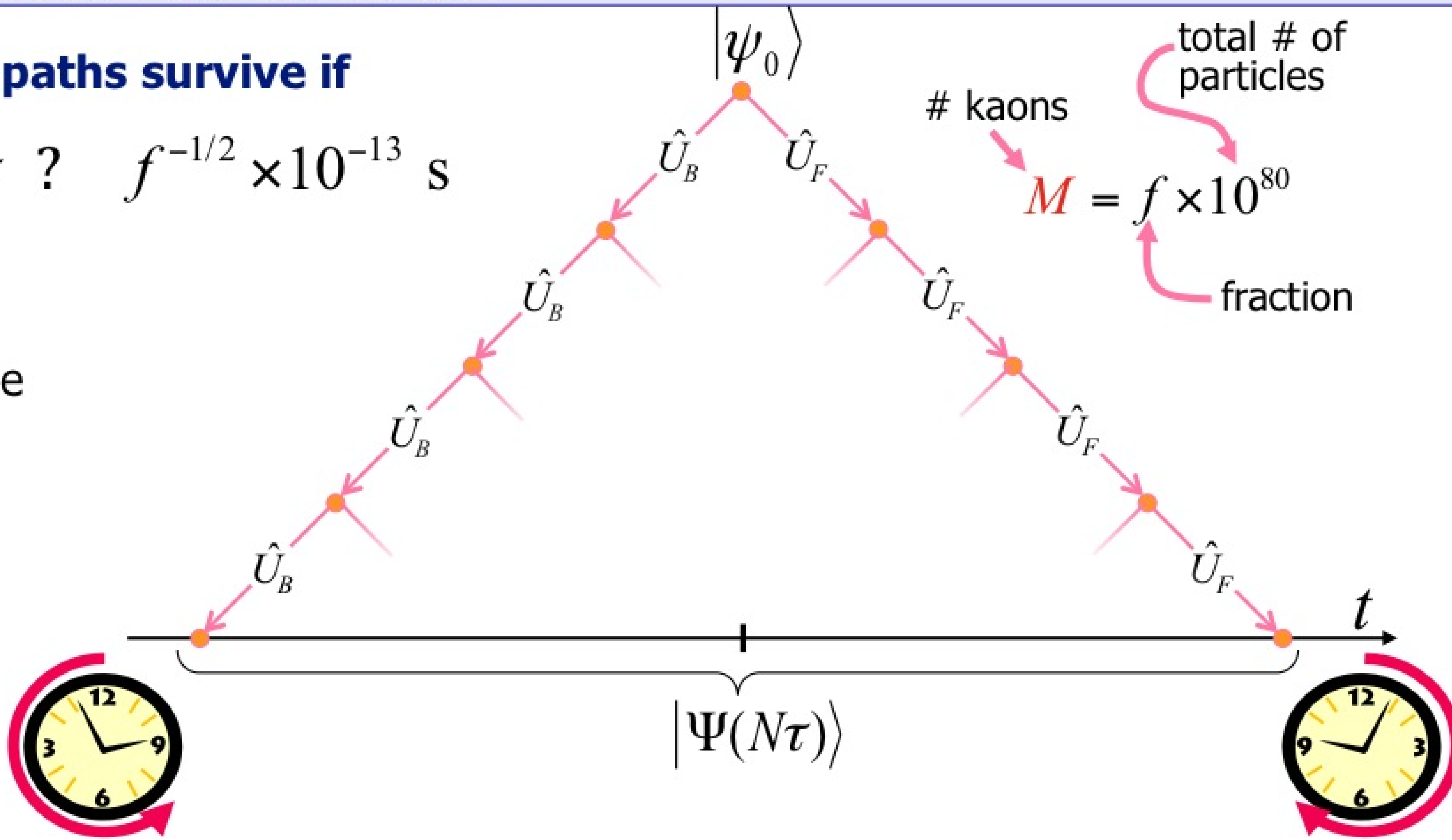
total time

kaons

total # of particles

$$M = f \times 10^{80}$$

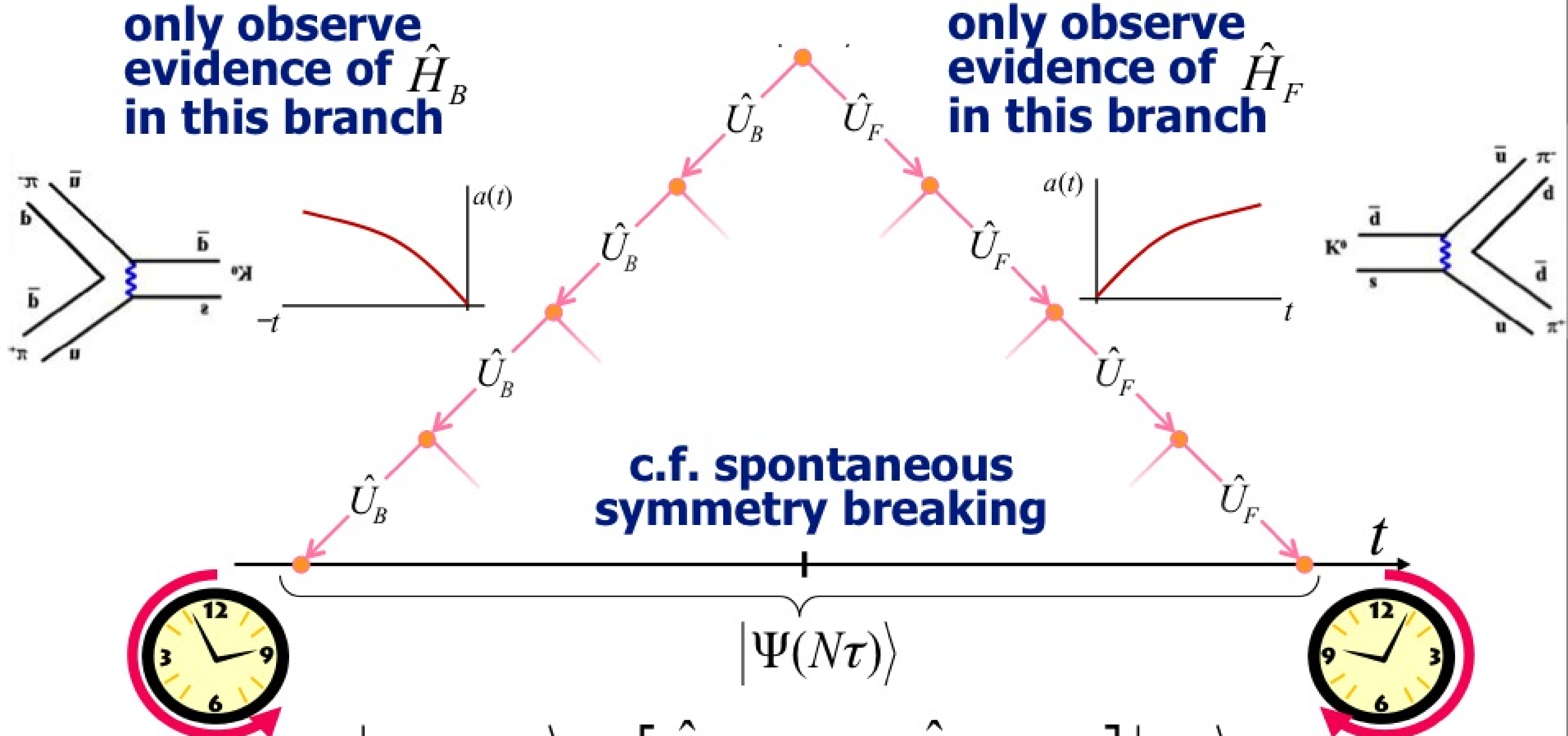
fraction



Bi-evolution equation of motion

$$|\Psi(N\tau)\rangle \propto \left[\hat{U}_F(N\tau) + \hat{U}_B(N\tau)\right]|\psi_0\rangle$$

Direction of time

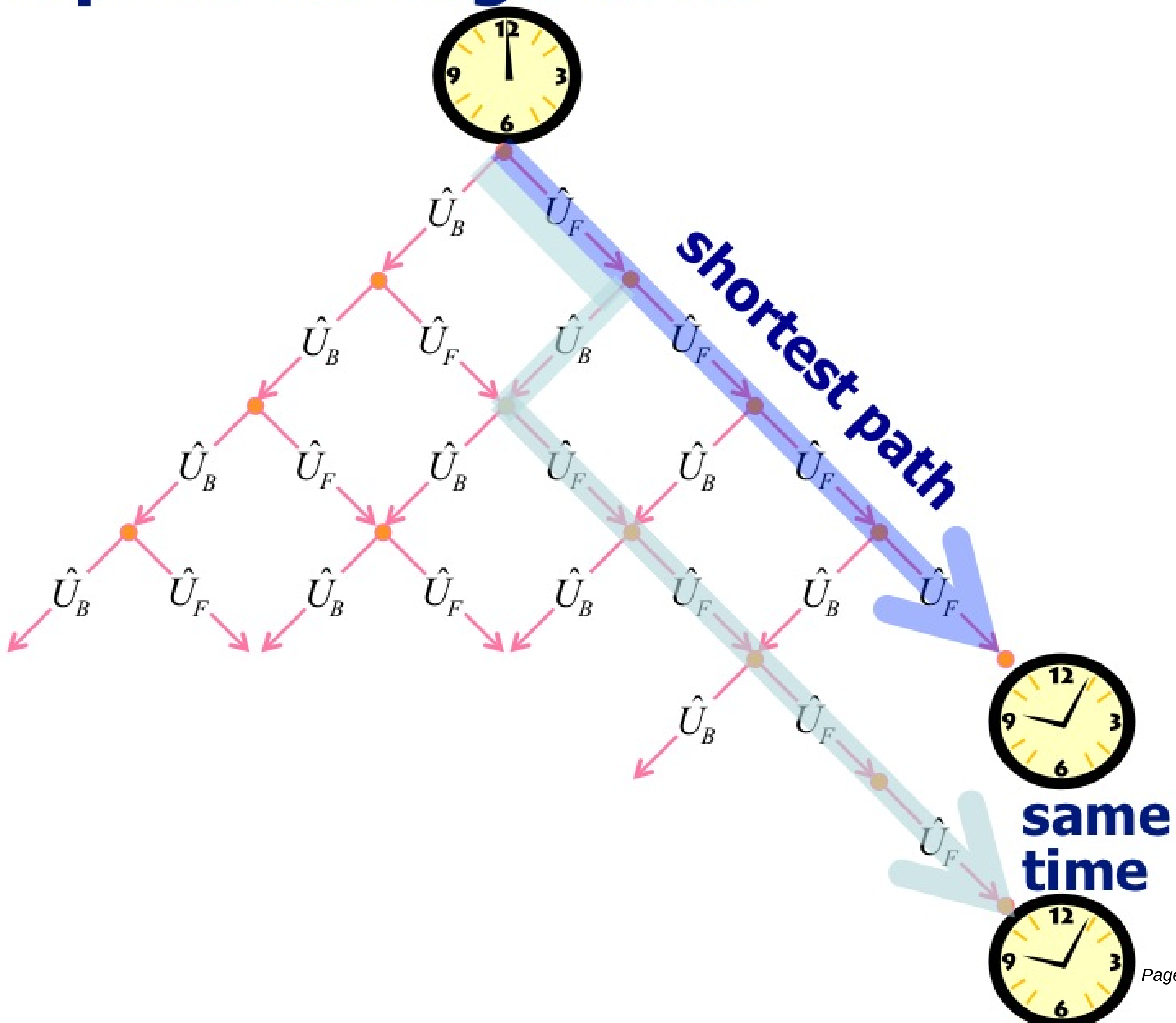


$$|\Psi(N\tau)\rangle \propto \left[\hat{U}_F(N\tau) + \hat{U}_B(N\tau)\right]|\psi_0\rangle$$

we observe only <u>one</u> of these terms

→ ***phenomenological*** **unidirectionality of time**

Shortest path through time



Early universe

T violation would be relatively rare, so ***no* interference:**

$$|\Psi(N\tau)\rangle \propto \left[\hat{U}_F(\tau)+\hat{U}_B(\tau)\right]^N |\psi_0\rangle, \qquad \hat{H}_F = \hat{H}_B$$

$$= \left[\exp(-i\hat{H}\tau)+\exp(i\hat{H}\tau)\right]^N |\psi_0\rangle$$

$$\propto \cos^N(\hat{H}\tau)|\psi_0\rangle$$

$\rightarrow$ eigenstate of $\cos(\hat{H}\tau)$

with largest eigenvalue (i.e. 1)

= zero eigenvalue of $\hat{H}$

Power method

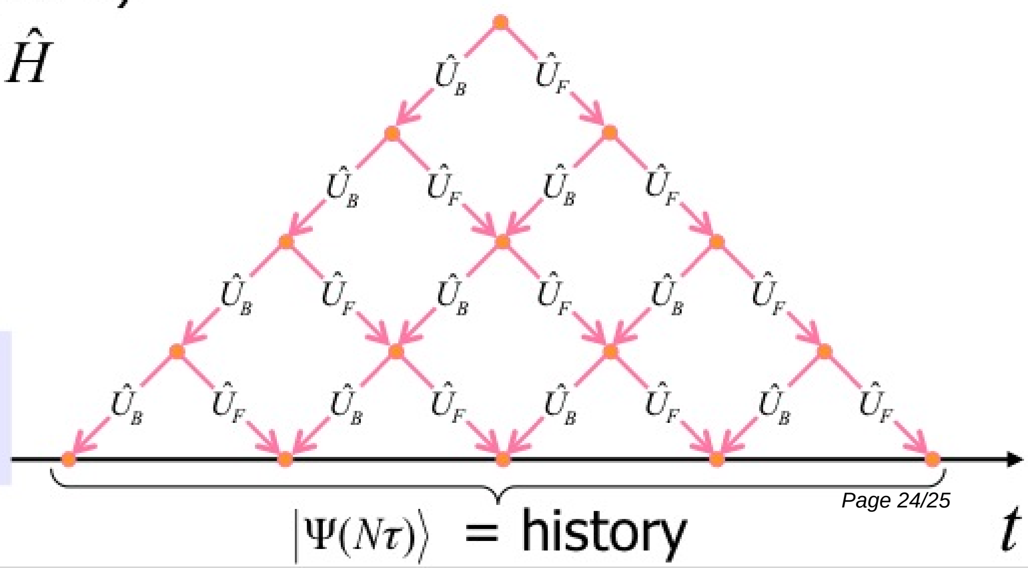
$$\hat{A}^N|\psi\rangle \approx \lambda_{\max}^N |\lambda_{\max}\rangle\langle\lambda_{\max}|\psi\rangle$$

largest eigenvalue

Hence

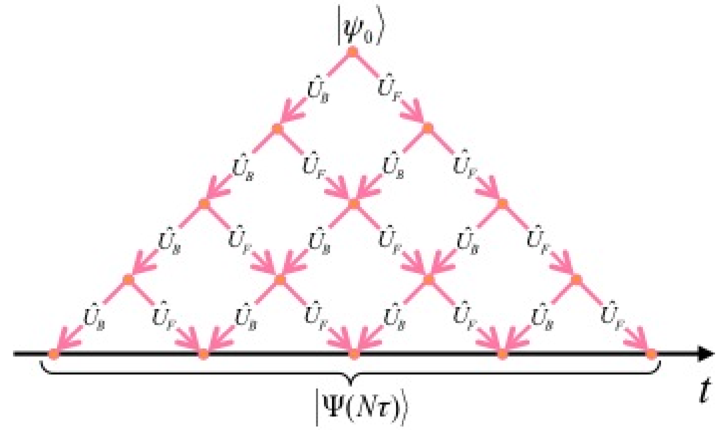
$$\hat{H}|\Psi(N\tau)\rangle \rightarrow 0$$

~ Hamiltonian constraint of the Wheeler-DeWitt eqn.



Summary

Prob : dynamical equation with 2 Hamiltonians



- use **Feynman's sum over histories** to account for both directions
- $$\left|\Psi(N\tau)\right\rangle \propto \left[\hat{U}_F(\tau)+\hat{U}_B(\tau)\right]^N\left|\psi_0\right\rangle$$

Prob : origin of direction of time

- **destructive interference** leaves only 2 paths

$$\left|\Psi(N\tau)\right\rangle \propto \left[\hat{U}_F(N\tau)+\hat{U}_B(N\tau)\right]\left|\psi_0\right\rangle$$

- **physical evidence** shows which path we experience
- quantum algorithm **for the shortest path to the "future"...**