Title: The status of determinism in noncontextual models of quantum theory

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Abstract: In an ontological model of quantum theory that is Bell-local, one can assume without loss of generality that the outcomes of measurements are determined deterministically by the ontic states (i.e. the values of the local hidden variables). The question I address in this talk is whether such determinism can always be assumed in a noncontextual ontological model of quantum theory, in particular whether it can be assumed for nonprojective measurements. While it is true that one can always represent a measurement by a deterministic response function by incorporating ancillary degrees of freedom into one's description (for instance those of the apparatus), I show that in moving to such a representation, one typically loses the warrant to apply the assumption of measurement noncontextuality. The implications for experimental tests of measurement noncontextuality will be discussed.

The status of determinism in noncontextual models of quantum theory

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Perimeter Institute for Theoretical Physics,
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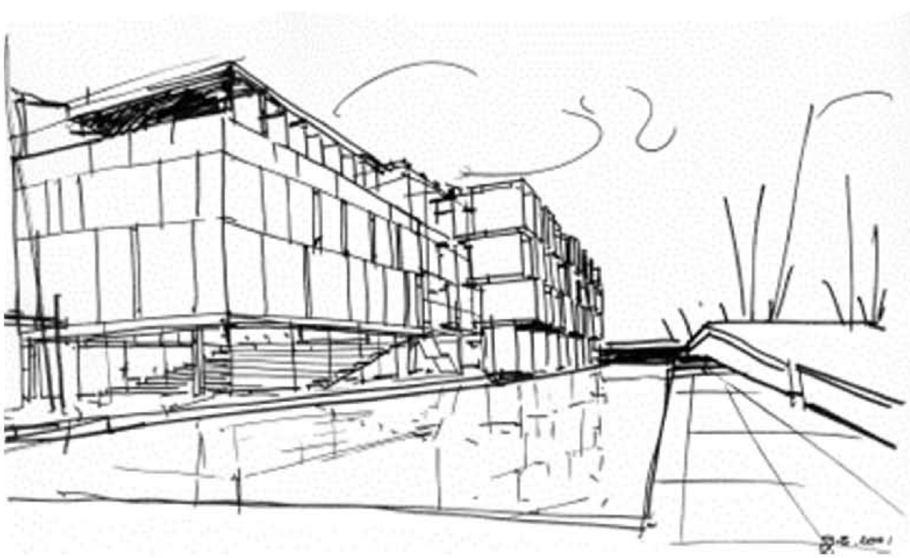
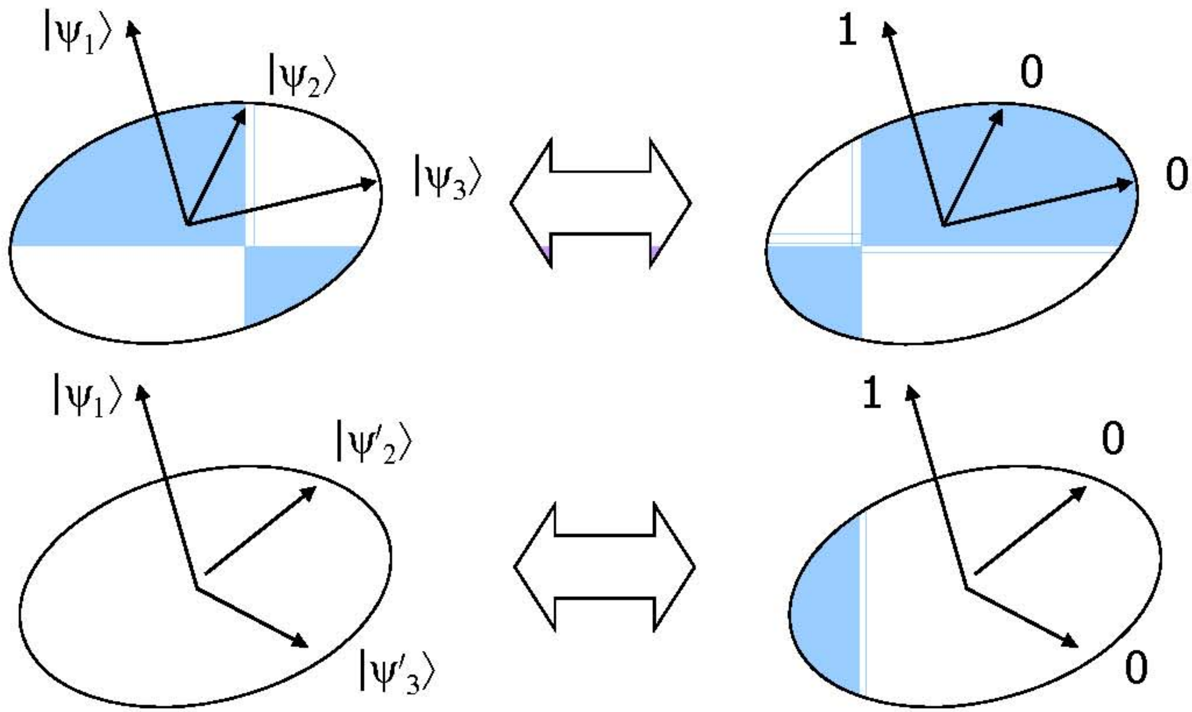


My view on the correct interpretation of quantum mechanics and how contextuality comes out as the key phenomenon to tackle in this research program

One understands something best when one can apply it to useful purposes. Therefore, we need to understand the technological applications of contextuality

The traditional notion of a noncontextual hidden variable model of quantum theory

Traditional notion of a noncontextual hidden variable model:
For every hidden state λ, every basis of vectors receives a 0-1 valuation, wherein exactly one element is assigned the value 1 (corresponding to the outcome that would occur for λ), and every vector is assigned the same value regardless of the basis it is considered a part (i.e. the context).



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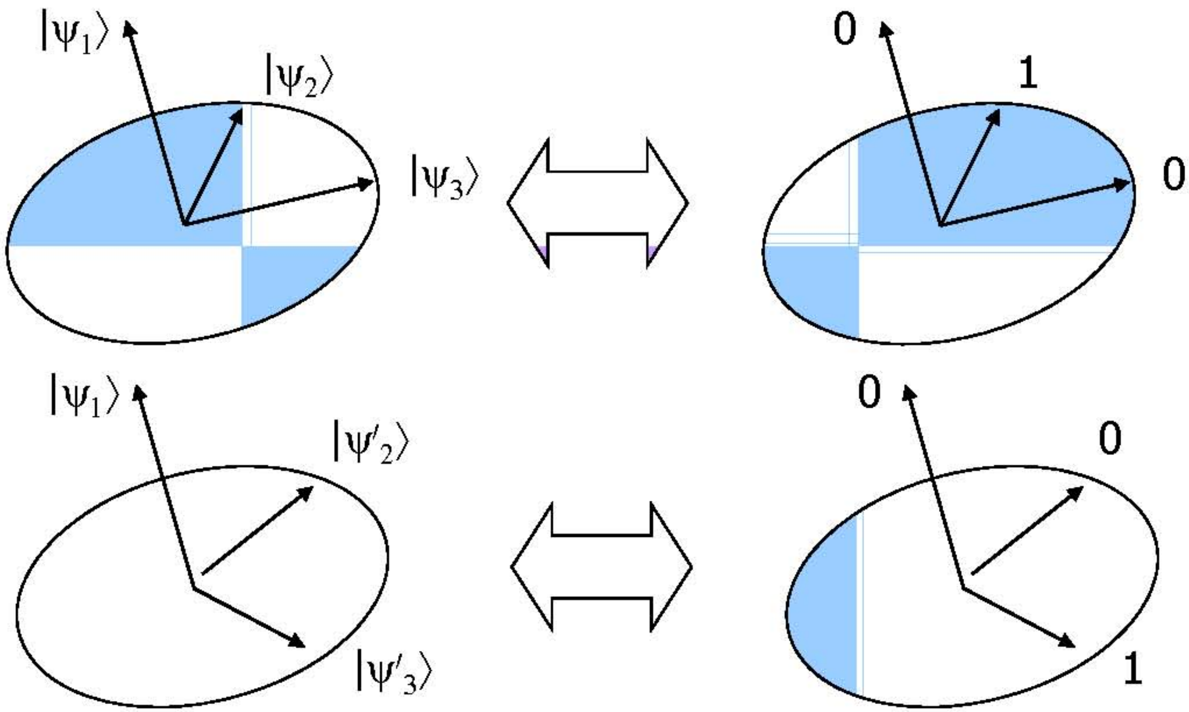
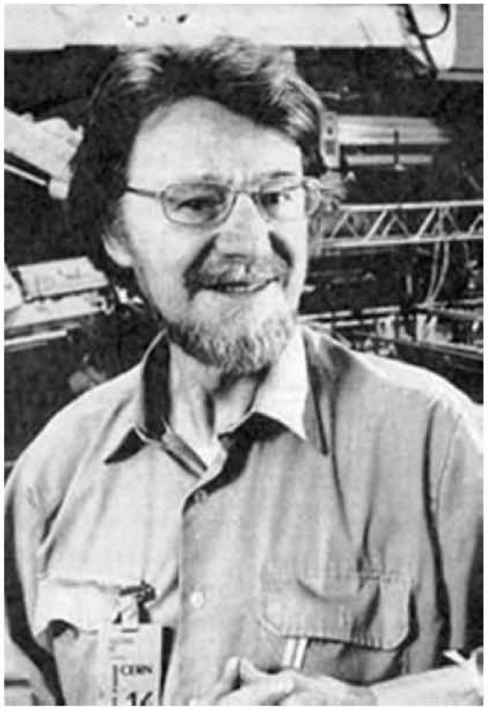


Traditional notion of a noncontextual hidden variable model:
For every λ, every projector Π is assigned a value 0 or 1 regardless of how it is measured (i.e. the context)

$$v(\Pi) = 0 \text{ or } 1 \quad \text{for all } \Pi$$

$$\{\Pi_1, \Pi_2, \Pi_3\} \qquad v(\Pi_1) = 1$$

$$\{\Pi_1, \Pi_2', \Pi_3'\} \qquad v(\Pi_1) = 1$$



John S. Bell

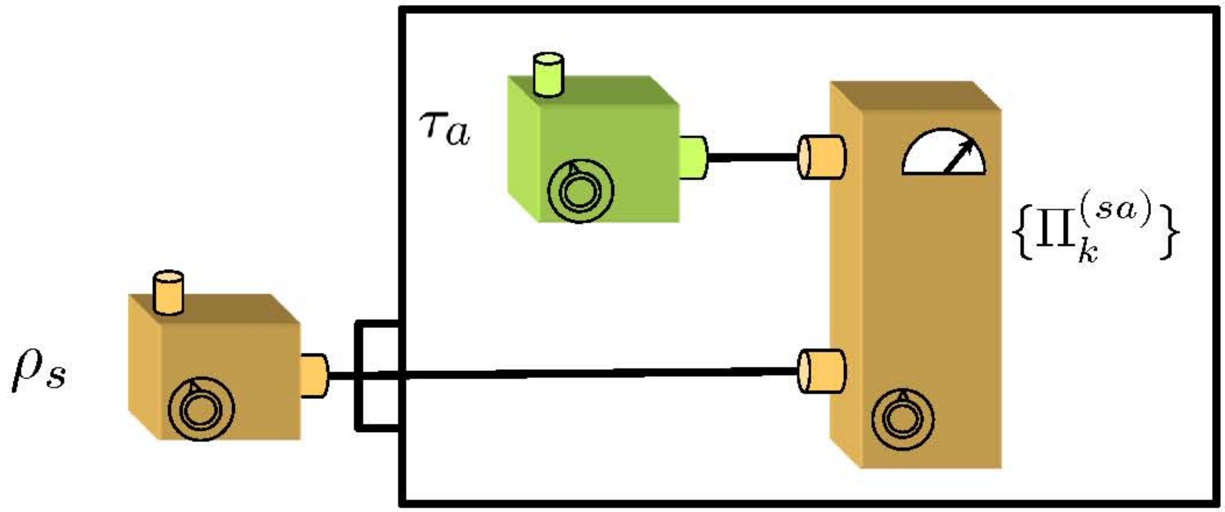
Ernst Specker (with son) and
Simon Kochen

Bell-Kochen-Specker theorem: A traditional noncontextual hidden variable model of quantum theory for Hilbert spaces of dimension 3 or greater is impossible.

The most general sort of measurement in quantum theory

Standard Measurements	**Generalized Measurements**
$\{\Pi_i\}$	$\{E_d\}$
$\langle\psi\|\Pi_i\|\psi\rangle \geq 0\,,\ \forall\|\psi\rangle$	$\langle\psi\|E_d\|\psi\rangle \geq 0\,,\ \forall\|\psi\rangle$
$\sum_i \Pi_i = I$	$\sum_d E_d = I$
$P(i) = \mathrm{tr}(\rho\Pi_i)$	$P(d) = \mathrm{tr}(\rho E_d)$
$\Pi_i\Pi_j = \delta_{ij}\,\Pi_i$	———

Measurement by coupling to an ancilla

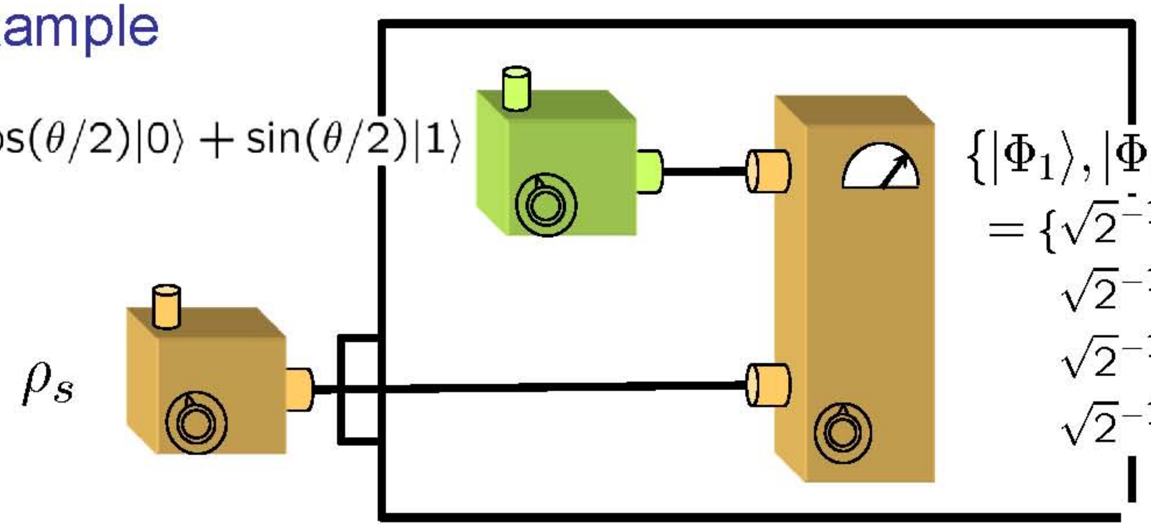


$$p(k) = \mathrm{Tr}_{sa}[\Pi_k^{(sa)}(\rho_s \otimes \tau_a)]$$
$$= \mathrm{Tr}_s[\underbrace{\mathrm{Tr}_a(\Pi_k^{(sa)}\tau_a)}_{E_k^{(s)}}\ \rho_s]$$

Naimark's theorem: Every POVM can be implemented by coupling to an ancilla and implementing a projective measurement

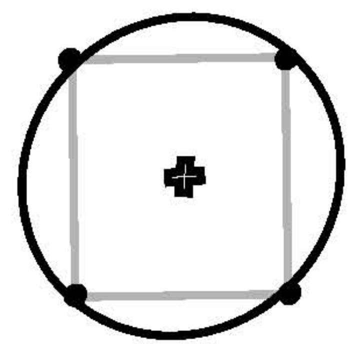
Example

$|\theta\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle$



$$\{|\Phi_1\rangle, |\Phi_2\rangle, |\Phi_3\rangle, |\Phi_4\rangle\}$$
$$= \{\sqrt{2}^{-1}(|0\rangle|0\rangle + |1\rangle|1\rangle),$$
$$\sqrt{2}^{-1}(|0\rangle|0\rangle - 1\rangle|1\rangle),$$
$$\sqrt{2}^{-1}(|0\rangle|1\rangle + |1\rangle|0\rangle),$$
$$\sqrt{2}^{-1}(|0\rangle|1\rangle - |1\rangle|0\rangle)\}$$

$$E_k^{(s)} = \mathrm{Tr}_a(\Pi_k^{(sa)}\tau_a)$$
$$= \langle\theta|_a|\Phi_k\rangle_{sa}\langle\Phi_k|_{sa}|\theta\rangle_a$$
$$\langle\theta|_a|\Phi_{1(2)}\rangle_{sa} = \sqrt{2}^{-1}[\cos(\theta/2)|0\rangle_s \pm \sin(\theta/2)|1\rangle_s] =$$
$$\langle\theta|_a|\Phi_{3(4)}\rangle_{sa} = \sqrt{2}^{-1}[\sin(\theta/2)|0\rangle_s \pm \cos(\theta/2)|1\rangle_s] =$$



$$\{E_k\} = \{\frac{1}{2}|\theta\rangle\langle\theta| \, , \, \frac{1}{2}|-\theta\rangle\langle-\theta| \, , \, \frac{1}{2}|\pi-\theta\rangle\langle\pi-\theta| \, , \, \frac{1}{2}|\pi+\theta\rangle\langle\pi+\theta|\}$$

$\theta = \pi/4$

Generalizing the notion of noncontextuality

From projective measurements to POVMs

So that we can model real experiments, where inevitable decoherence (coupling to the environment) implies that no measurement is truly projective

A popular proposal for how to generalize the notion of noncontextuality to POVMs

VOLUME 91, NUMBER 12 PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2003

Quantum States and Generalized Observables: A Simple Proof of Gleason's Theorem

P. Busch*
Department of Mathematics, University of Hull, Hull HU6 7RX, United Kingdom
(Received 29 May 2003; published 19 September 2003)

"An interpretation of valuations as truth value assignments would require the numbers v(E) to be either 1 or 0, indicating the occurrence or nonoccurrence of an outcome associated with E. Valuations with this property are referred to as dispersion-free. The above theorem entails immediately that dispersion-free effect valuations […] do not exist. It follows that noncontextual hidden variables, understood as dispersion-free, globally defined, valuations, are excluded in quantum mechanics."

VOLUME 90, NUMBER 19 PHYSICAL REVIEW LETTERS week ending 16 MAY 2003

Kochen-Specker Theorem for a Single Qubit using Positive Operator-Valued Measures

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(Received 2 October 2002; published 12 May 2003)

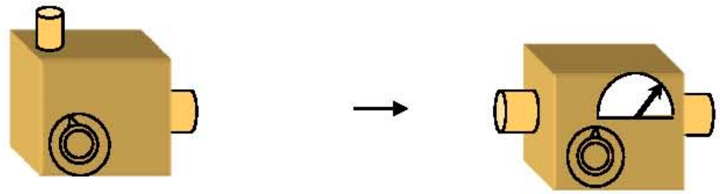
"Each equation contains eight positive-semidefinite operators whose sum is the identity. Therefore, a noncontextual hidden-variable theory must assign the answer yes to one and only one of these eight operators."

An alternative proposal for how to generalize the notion of noncontextuality to POVMs

And for how to generalize it from quantum theory to any operational theory

So that for any given experimental data, we can say whether it can be explained by a noncontextual model regardless of the empirical status of quantum theory

Operational theories



Preparation P

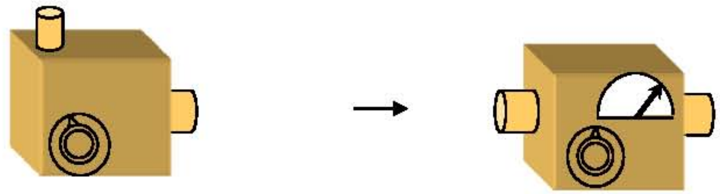
Measurement M

These are defined as lists of instructions

An operational theory specifies

$p(k|\mathrm{P}, \mathrm{M}) \equiv$ The probability of outcome k of M given P

Operational Quantum Mechanics



Preparation

P

Measurement

M

Density operator

ρ

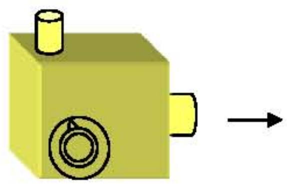
Positive operator-valued measure (POVM)

$\{E_k\}$

$$p(k|\mathsf{P},\mathsf{M}) = \mathrm{Tr}[E_k \rho]$$

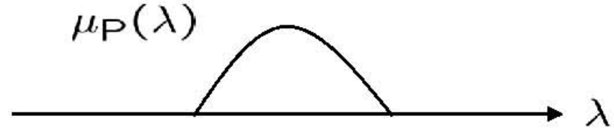
A hidden variable model of an operational theory

Preparation
P

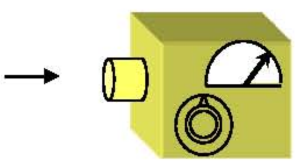


$\int \mu_{\mathsf{P}}(\lambda) d\lambda = 1$

$\mu_{\mathsf{P}}(\lambda)$



Measurement
M



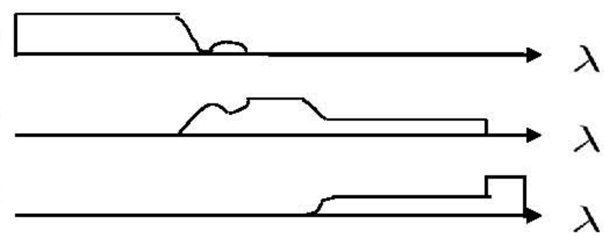
$0 \leq \xi_{\mathsf{M},k} \leq 1$

$\sum_k \xi_{\mathsf{M},k}(\lambda) = 1$ for all λ

$\xi_{\mathsf{M},1}(\lambda)$

$\xi_{\mathsf{M},2}(\lambda)$

$\xi_{\mathsf{M},3}(\lambda)$

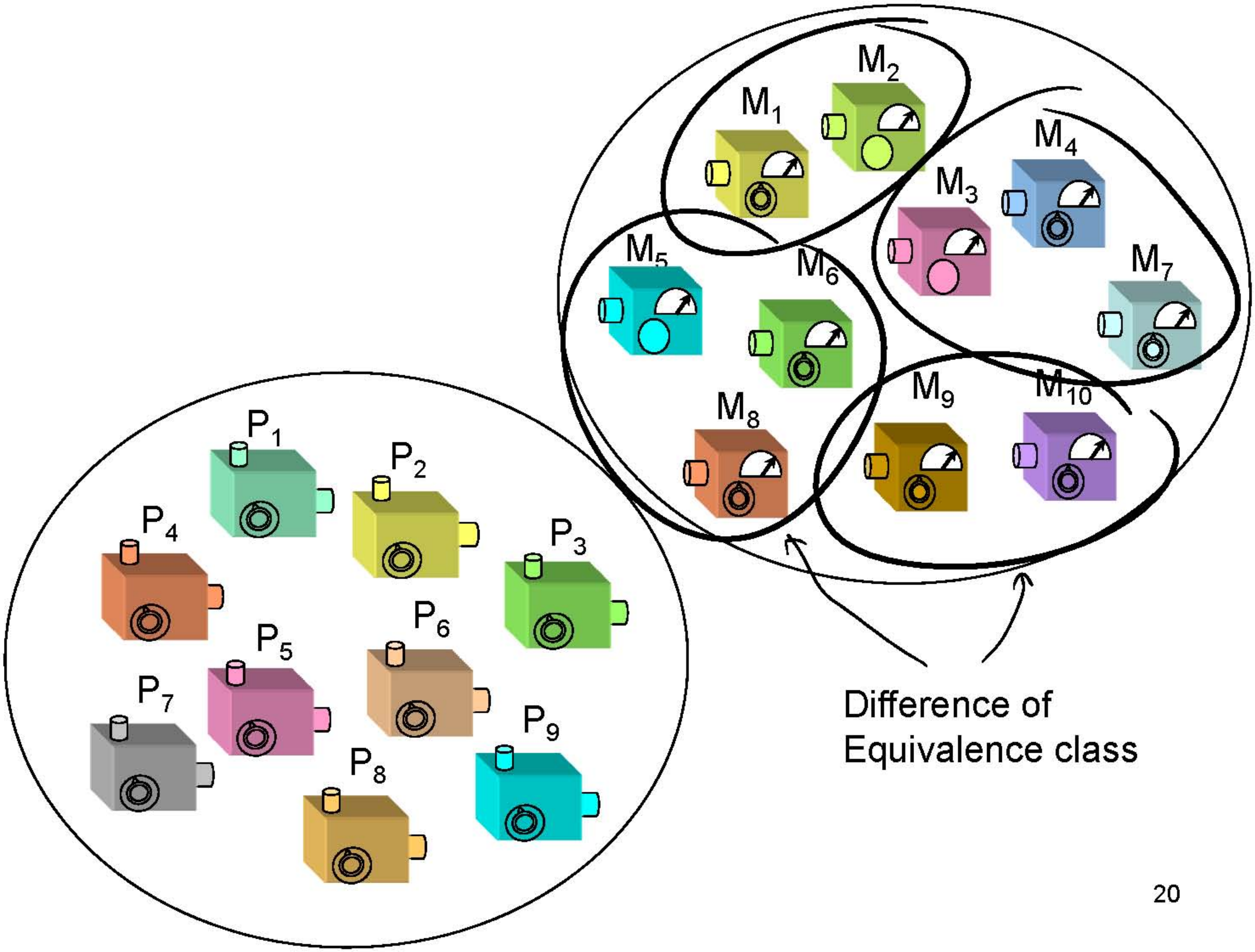


$$p(k|\mathsf{P},\mathsf{M}) = \int d\lambda\, \xi_{\mathsf{M},k}(\lambda)\, \mu_{\mathsf{P}}(\lambda)$$

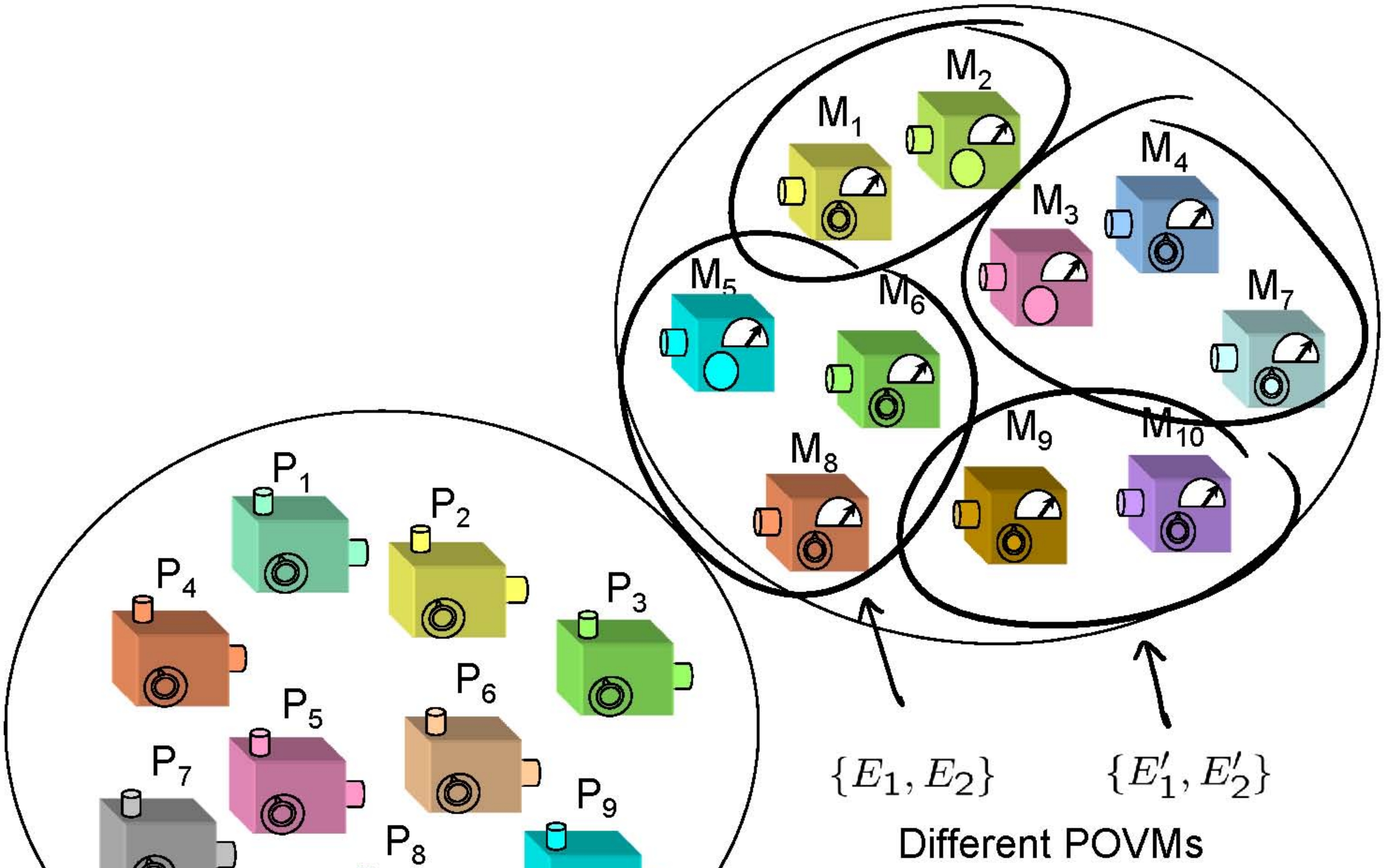
Generalized definition of noncontextuality:

A hidden variable model of an operational theory is noncontextual if

Operational equivalence of two experimental procedures → Equivalent representations in the realist model

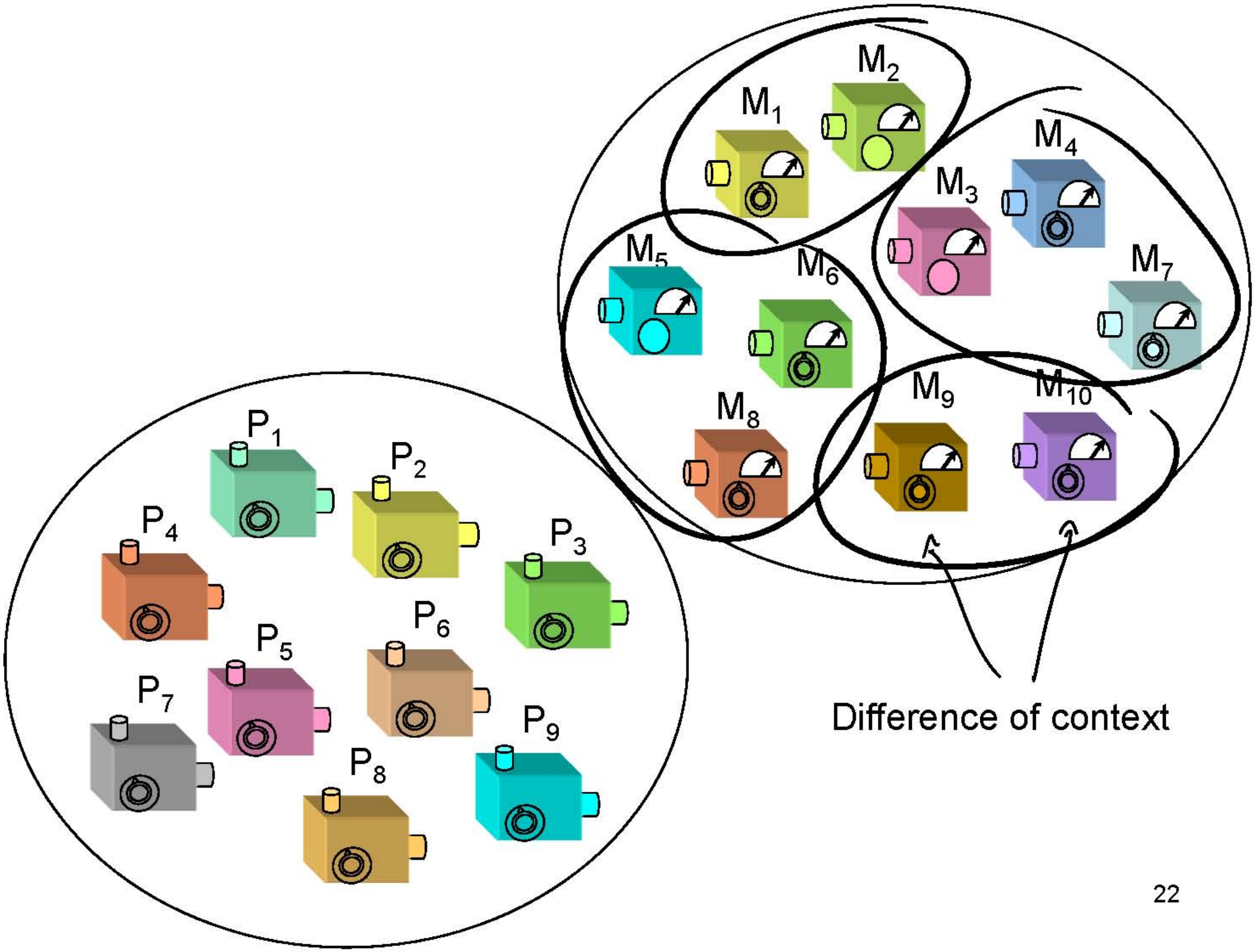


M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
Difference of
Equivalence class



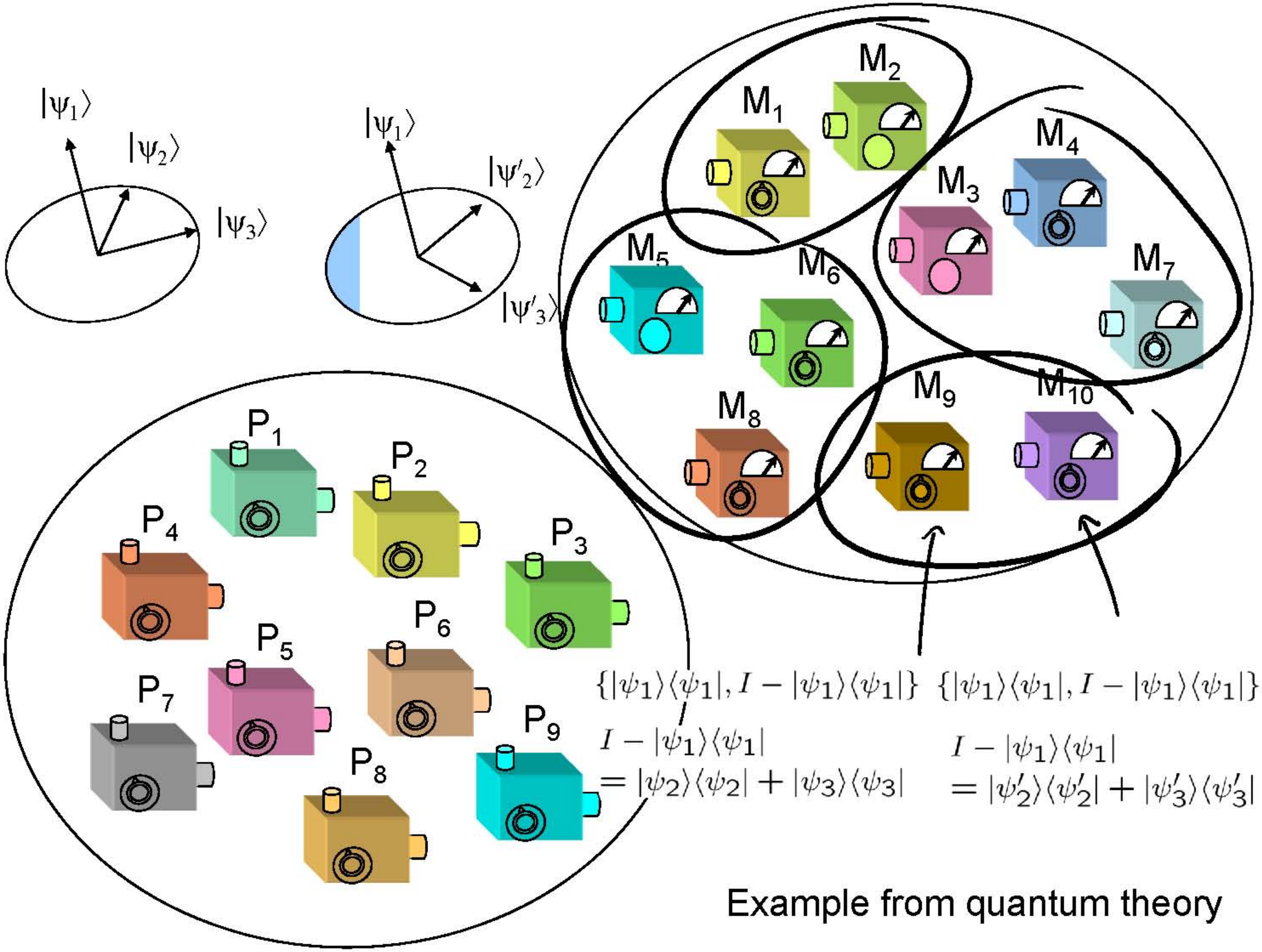
Example from quantum theory

21

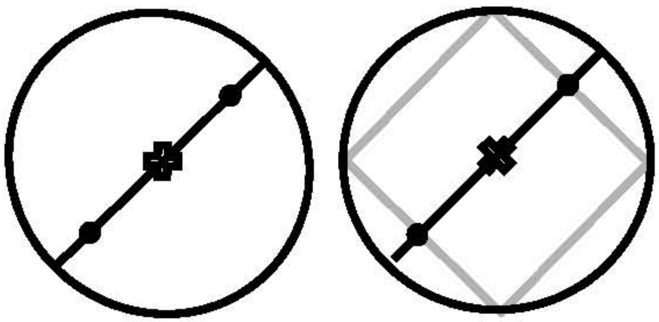


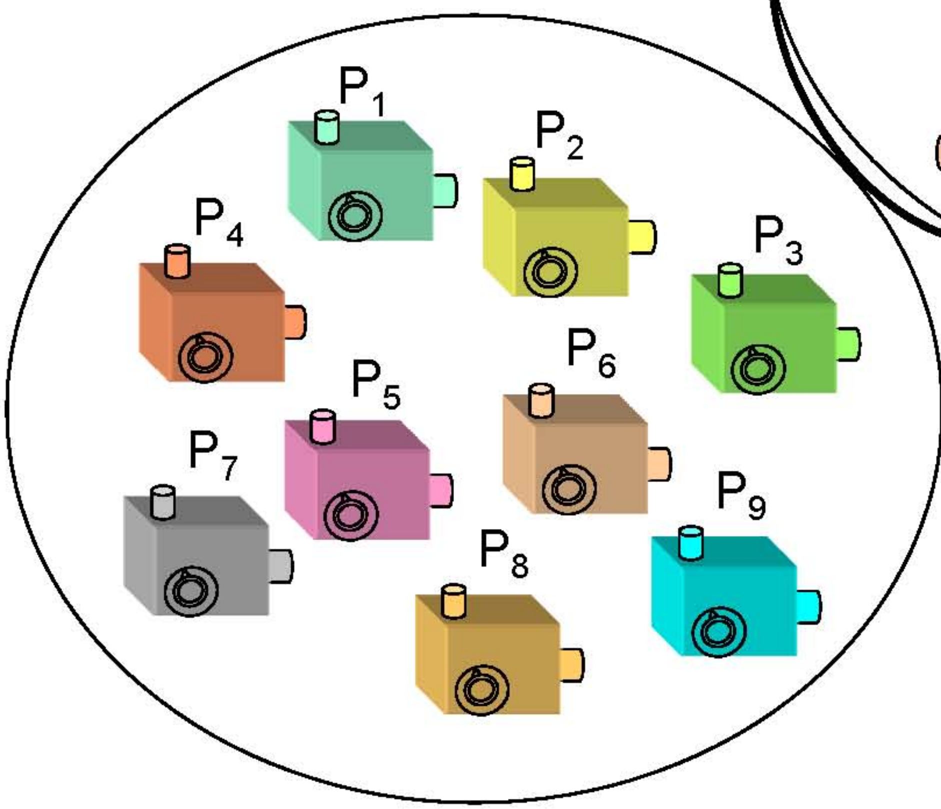
P1
P2
P3
P4
P5
P6
P7
P8
P9
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
Difference of context

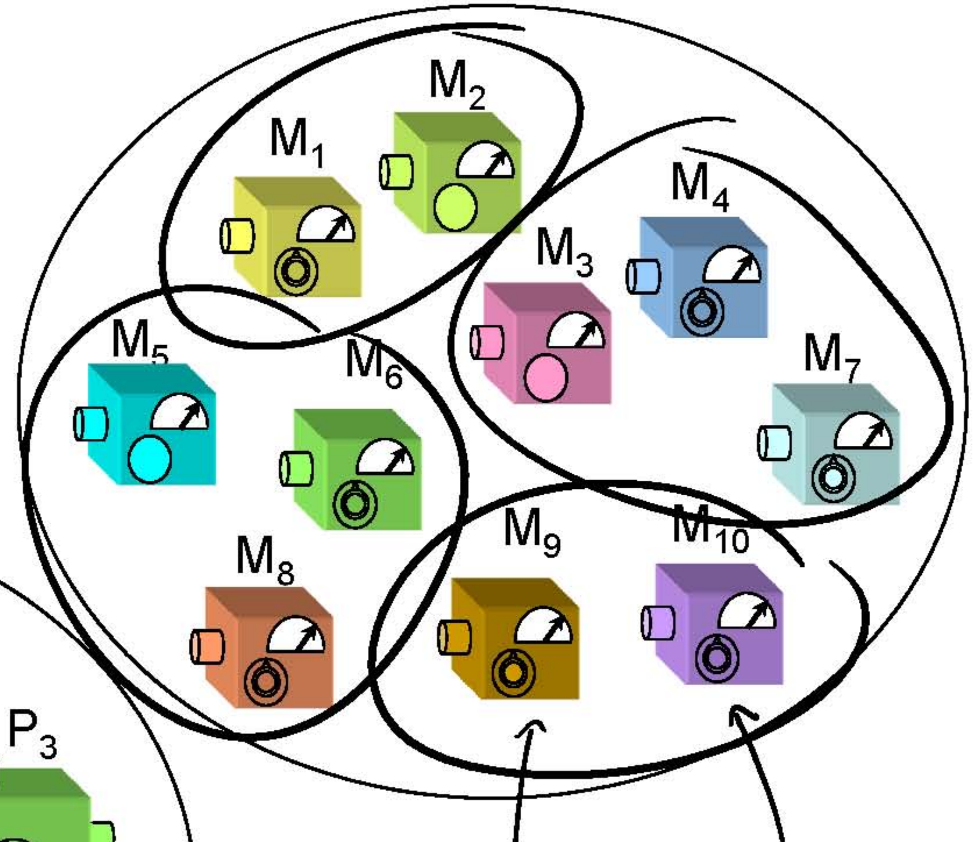
22



Example from quantum theory



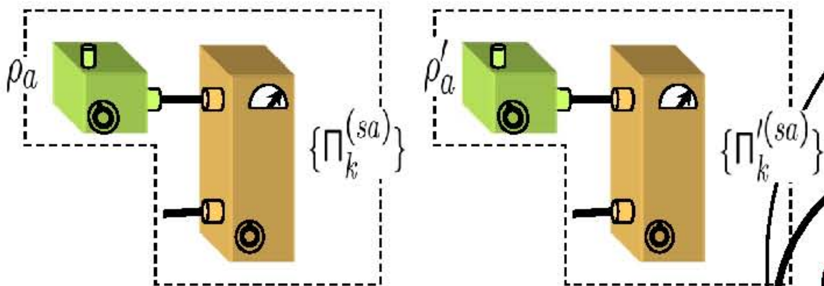


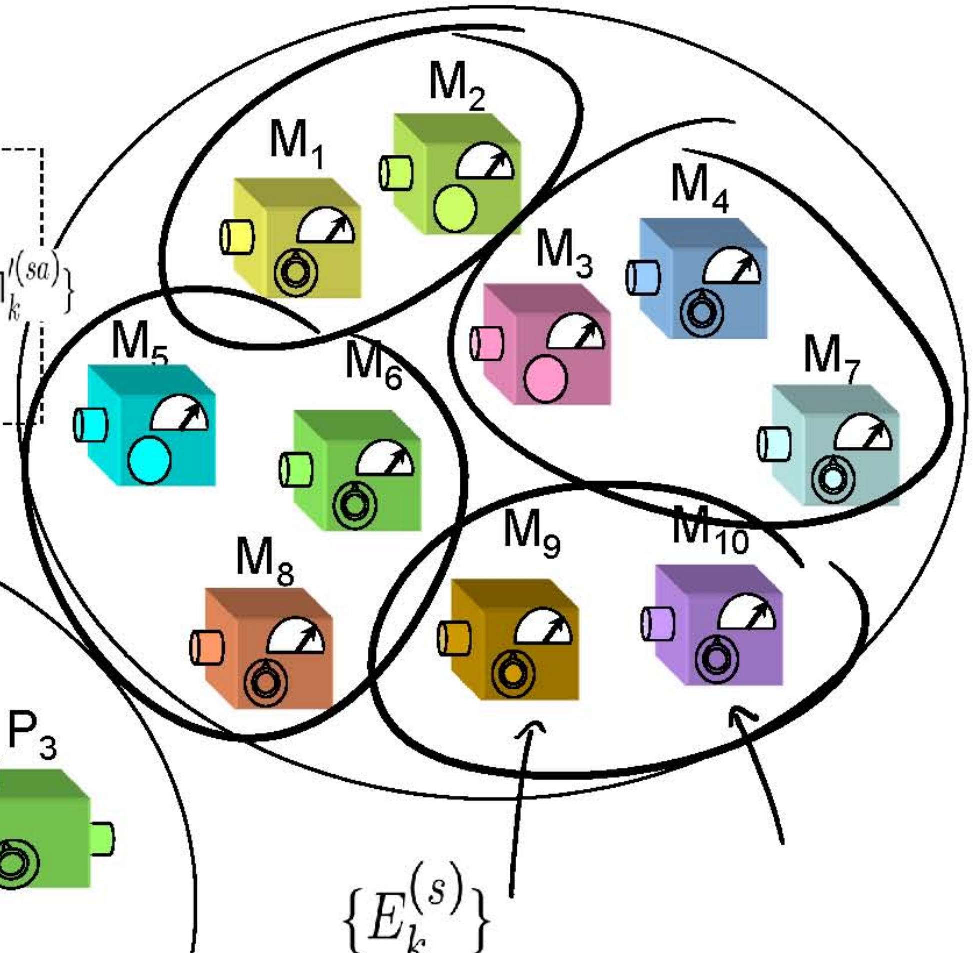


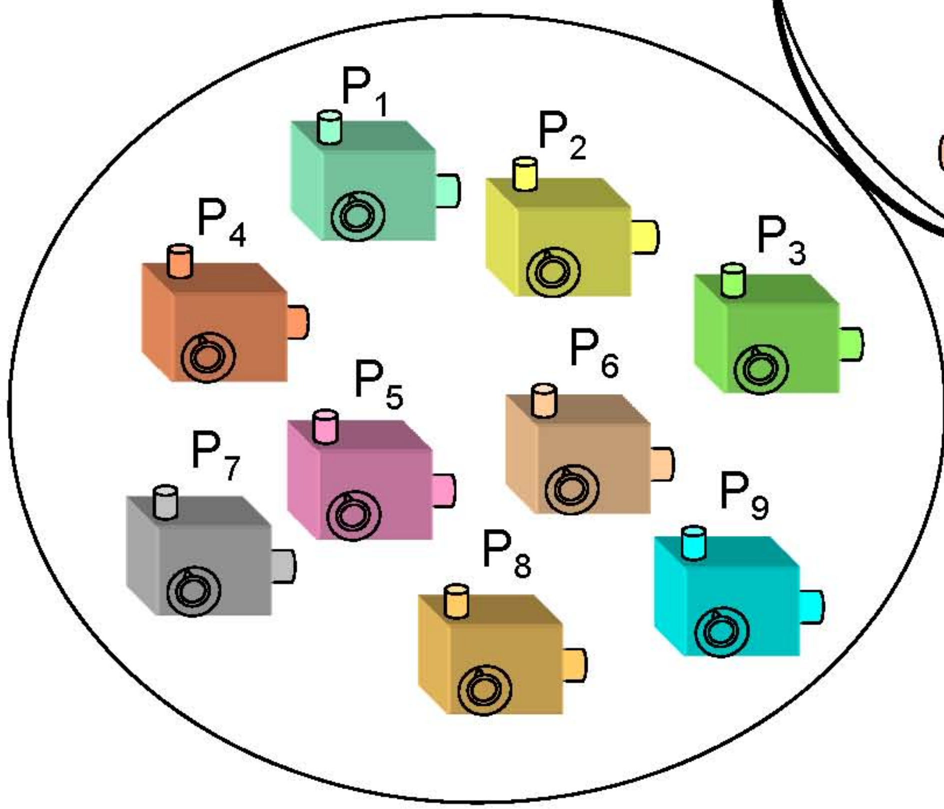
$$\{E, I - E\}$$
$$E = q|\frac{\pi}{4}\rangle\langle\frac{\pi}{4}| + (1-q)\frac{1}{2}I$$

$$\{E, I - E\}$$
$$E = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|+\rangle\langle +|$$

Example from quantum theory





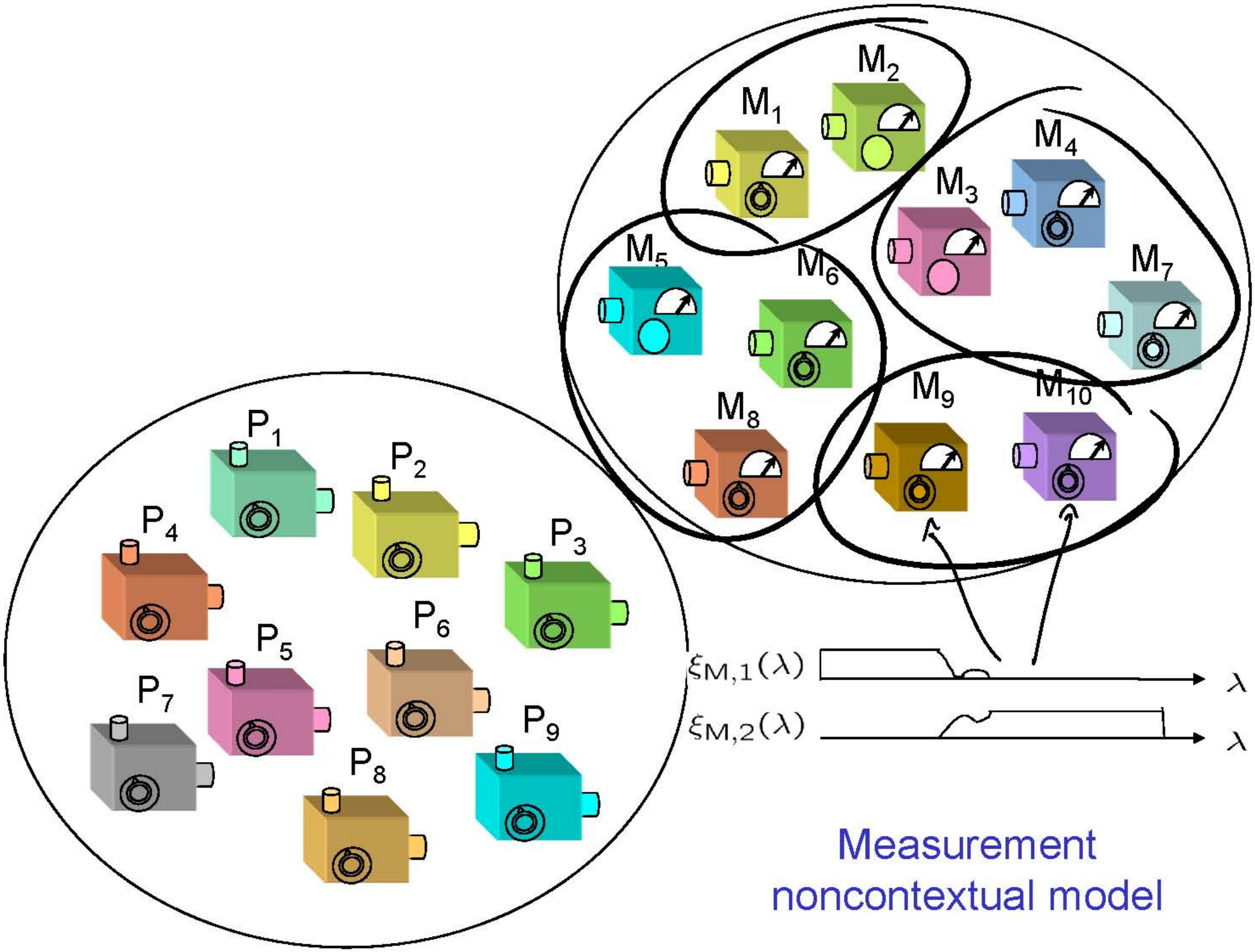


$$E_k^s = \mathrm{Tr}_a(\Pi_k^{(sa)} \rho_a)$$

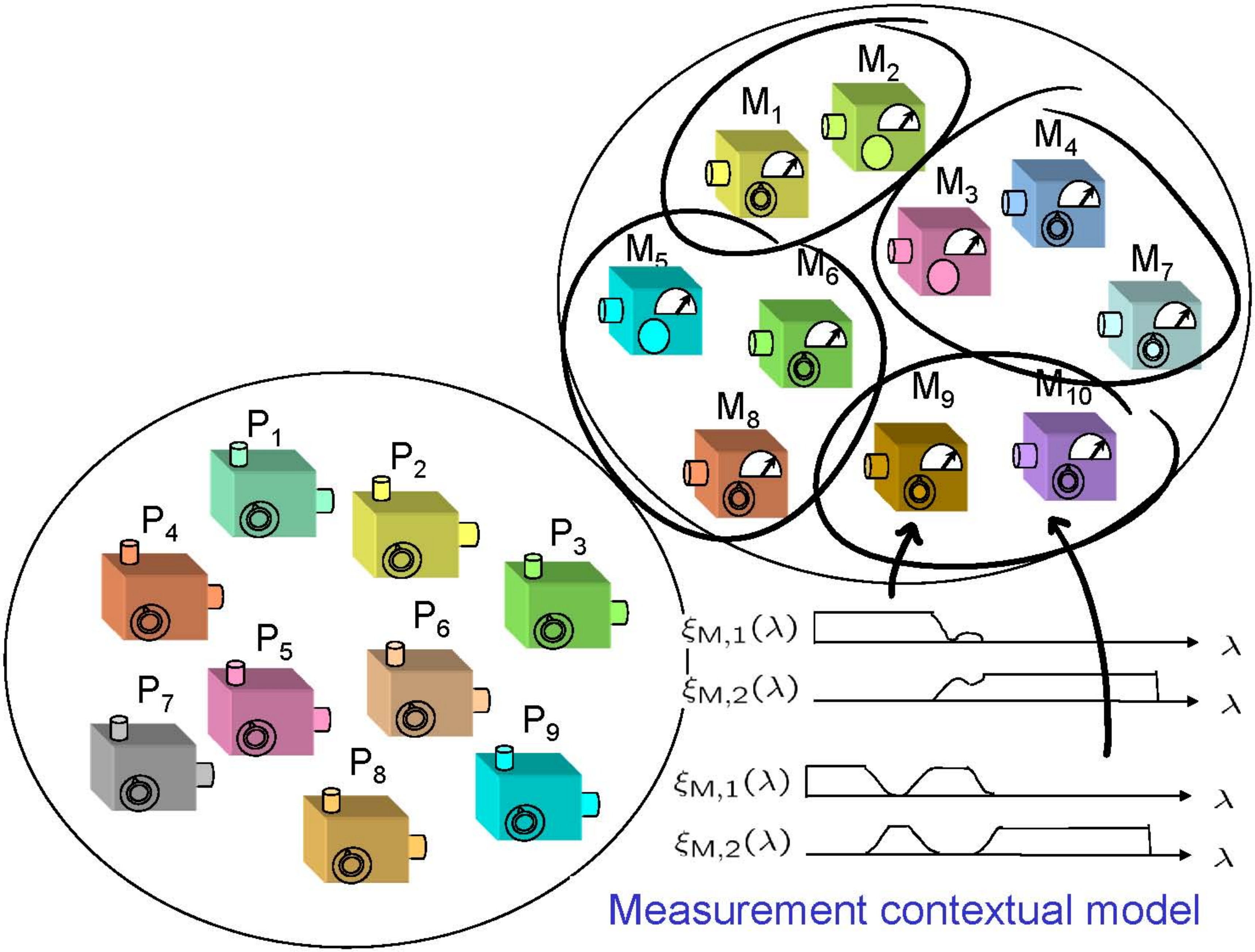
$\{E_k^{(s)}\}$

$$E_k^s = \mathrm{Tr}_a(\Pi_k'^{(sa)} \rho'_a)$$

Example from quantum theory

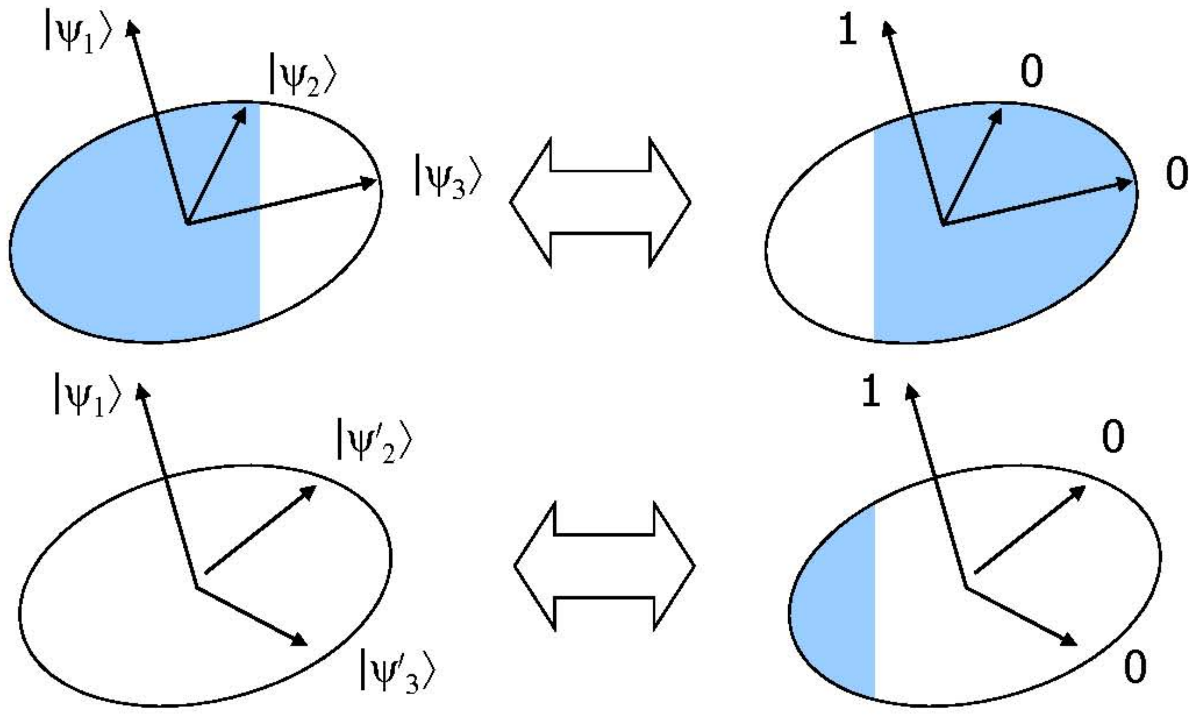


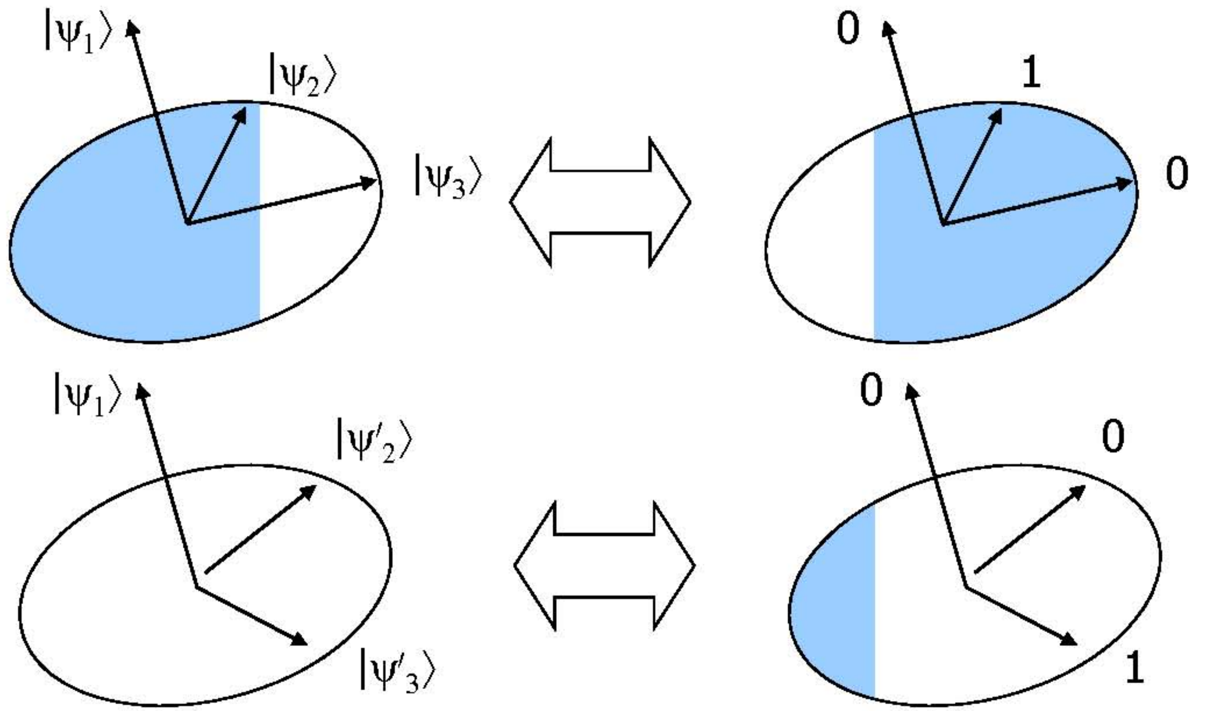
Measurement noncontextual model



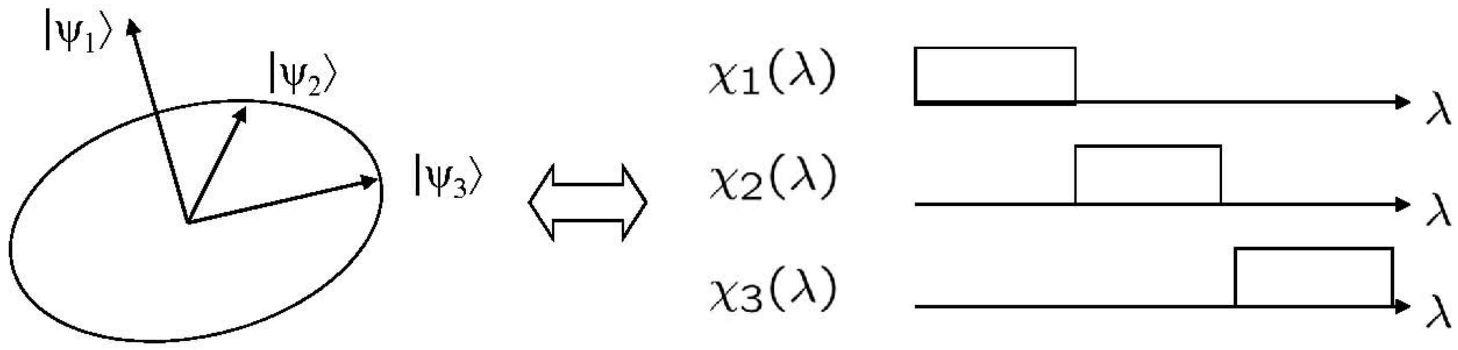
Measurement contextual model

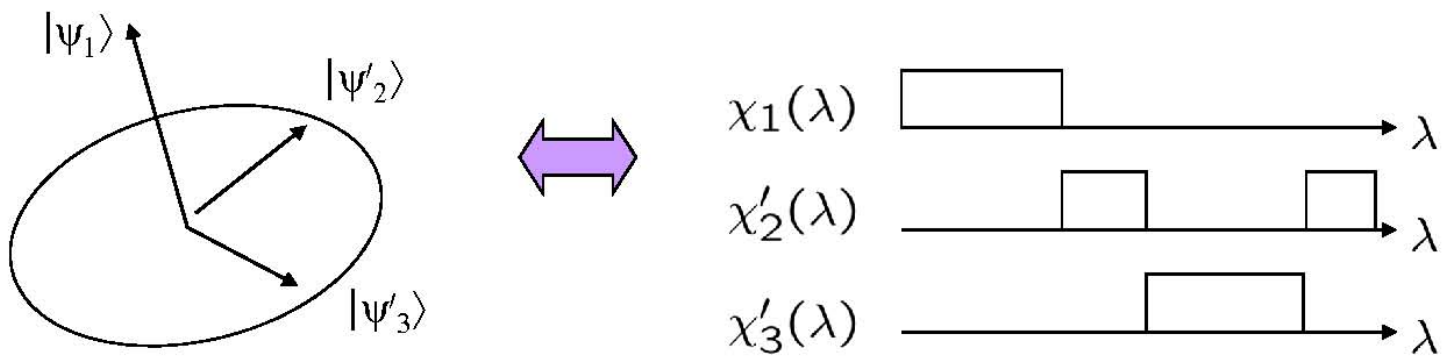
New notion versus traditional notion for representation of projective measurements


$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
1
0
0
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
1
0
0

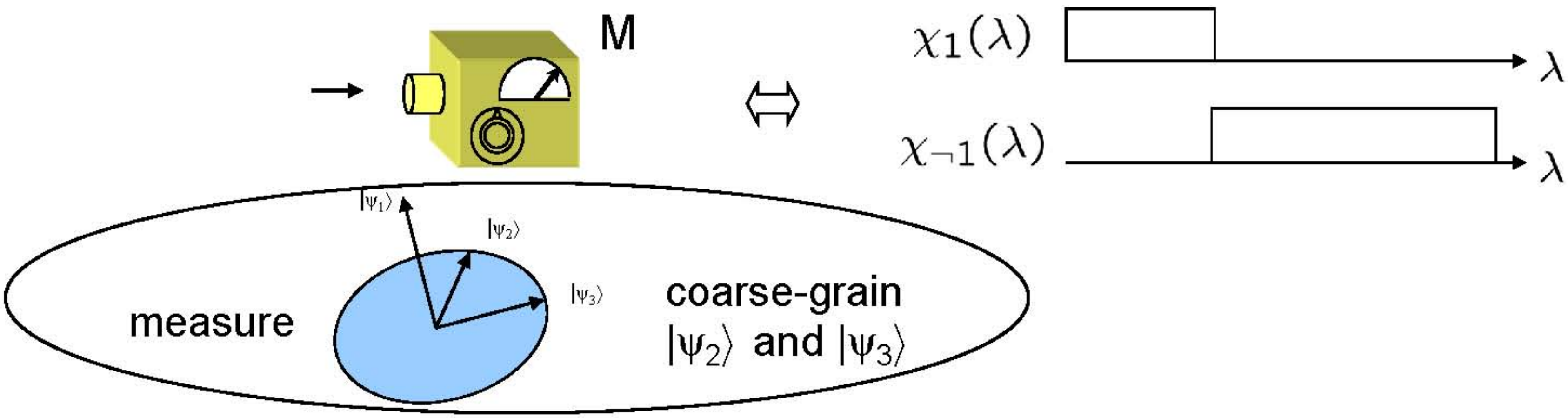

$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
0
1
0
$|\psi_1\rangle$
$|\psi'_2\rangle$
$|\psi'_3\rangle$
0
0
1

How to formulate the traditional notion of noncontextuality:

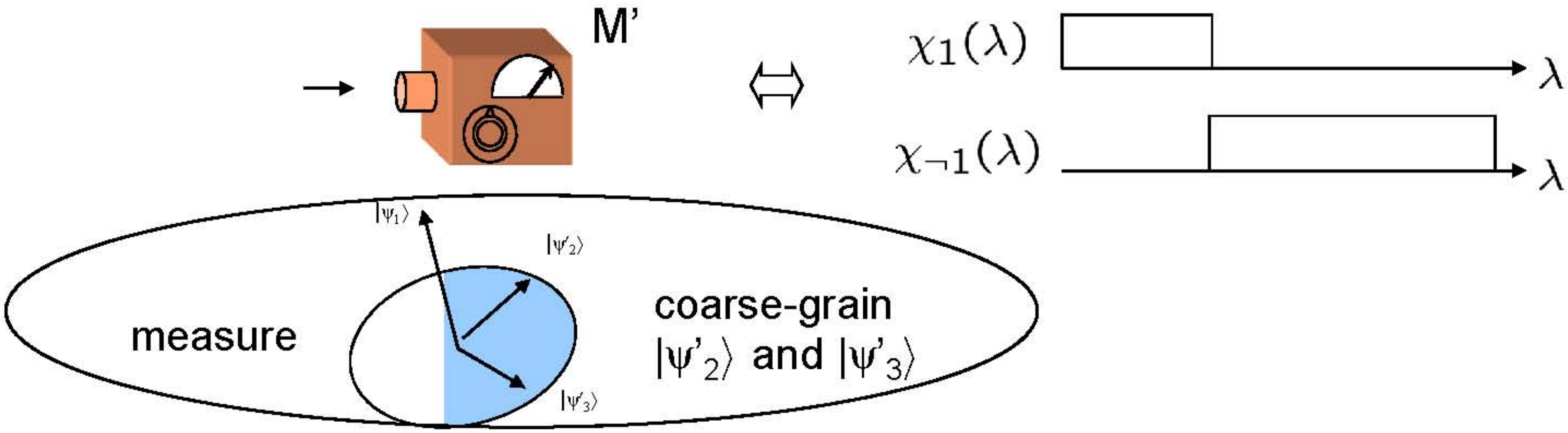




This is equivalent to assuming:



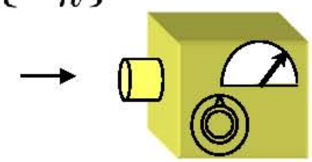
$$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$$



$$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$$

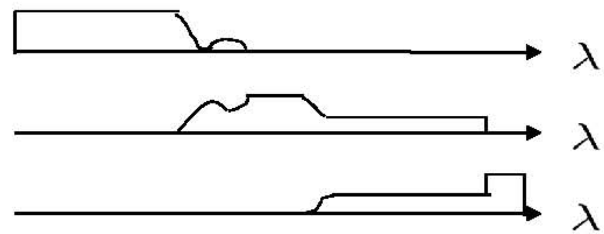
But recall that the most general representation was

$\{P_k\}$ M



$\Longleftrightarrow$

$\xi_{P_1}(\lambda)$
$\xi_{P_2}(\lambda)$
$\xi_{P_3}(\lambda)$



Therefore:

traditional notion of noncontextuality for projective mmts = revised notion of noncontextuality for projective mmts and outcome determinism for projective mmts

So, the new definition of noncontextuality is not simply a generalization of the traditional notion

For projective measurements, it is a revision of the traditional notion

Local determinism:

We ask: Does the outcome depend on space-like separated events (in addition to local settings and λ)?

Local causality:

We ask: Does the probability of the outcome depend on space-like separated events (in addition to local settings and λ)?

Traditional notion of measurement noncontextuality:

We ask: Does the outcome depend on the measurement context (in addition to the observable and λ)?

The revised notion of measurement noncontextuality:

We ask: Does the probability of the outcome depend on the measurement context (in addition to the observable and λ)?

Noncontextuality and determinism are separate issues traditional notion of noncontextuality for projective mmts = revised notion of noncontextuality for projective mmts

and

outcome determinism for projective mmts

No-go theorems for previous notion of noncontextuality are not necessarily no-go theorems for the new notion!

In face of contradiction, could give up determinism!

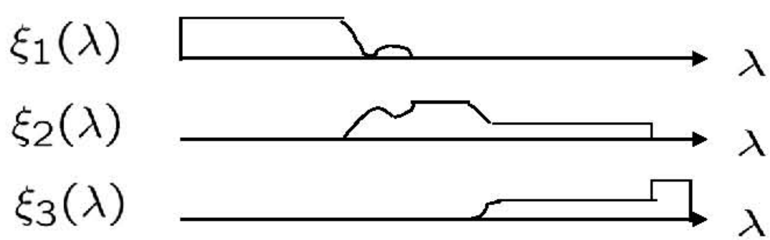
Can we justify an assumption of outcome determinism in some way?

Many people have a strong intuition that allowing outcome *in*determinism does not add any generality and that consequently we may as well assume outcome determinism. Recall Fine's theorem for instance

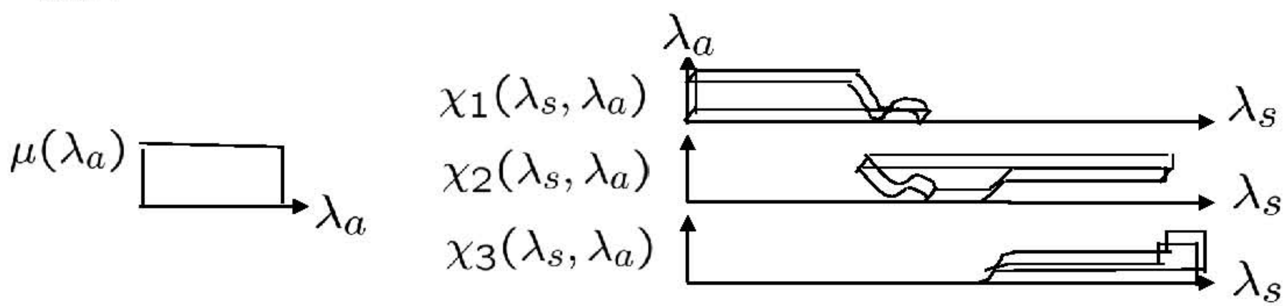
Outcome-deterministic ontic extension

For any given model, we can always build one that is outcome-deterministic on a larger system

Example: Replace



with



The argument in favor of the popular proposal

Premiss: If two measurements have the same statistics for all preparations, then they should be represented by the same response function in the hidden variable model

Operational equivalence→ Ontic equivalence

The assumption of measurement noncontextuality

Premiss: Every measurement can be represented by an outcome-deterministic response function on a larger system

Operational unsharpness is consistent with ontic sharpness

Outcome-deterministic ontic extensions of measurements

Purported conclusion: If two measurements have the same statistics for all preparations, then they should be represented by the same outcome-deterministic response functions

A reason to be suspicious

Premiss: If two measurements have the same statistics for all preparations, then they can be represented in the quantum formalism by the same POVM

An assumption of quantum theory

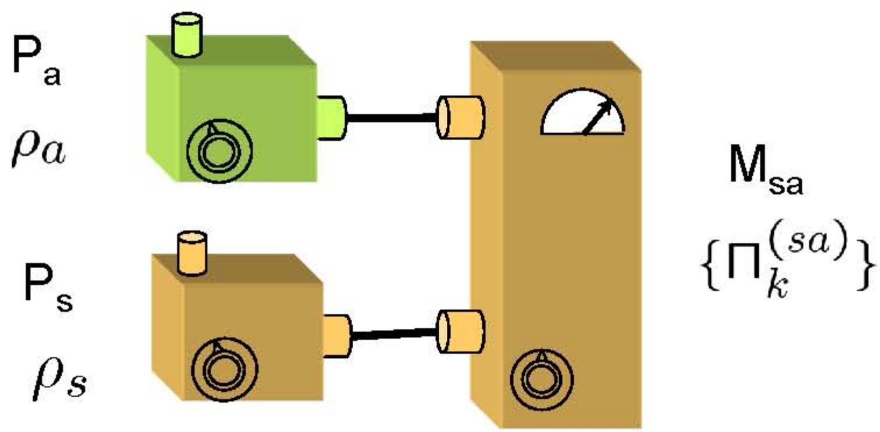
Premiss: Every measurement can be represented by a projective-valued measure on a larger system

Naimark's theorem

Purported conclusion: If two measurements have the same statistics for all preparations, then they can be represented by the same projector-valued measure

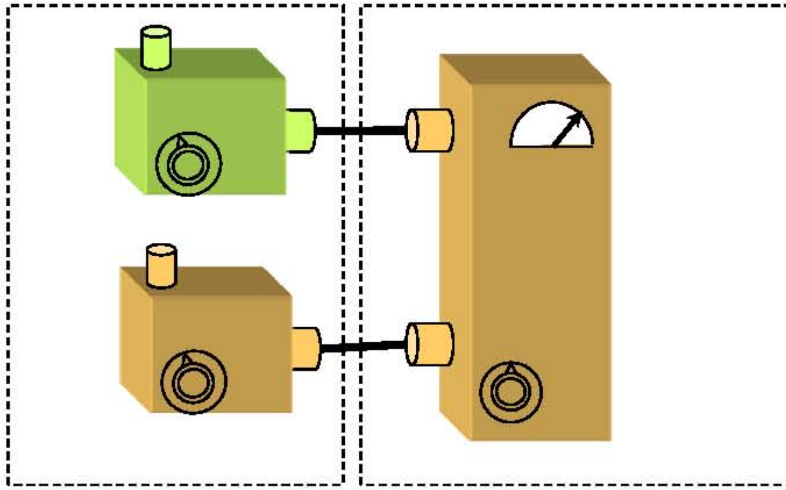
FALSE

Two partitions of an experiment



$P_{sa} = (P_s, P_a)$

$\rho_{sa} = \rho_s \otimes \rho_a$



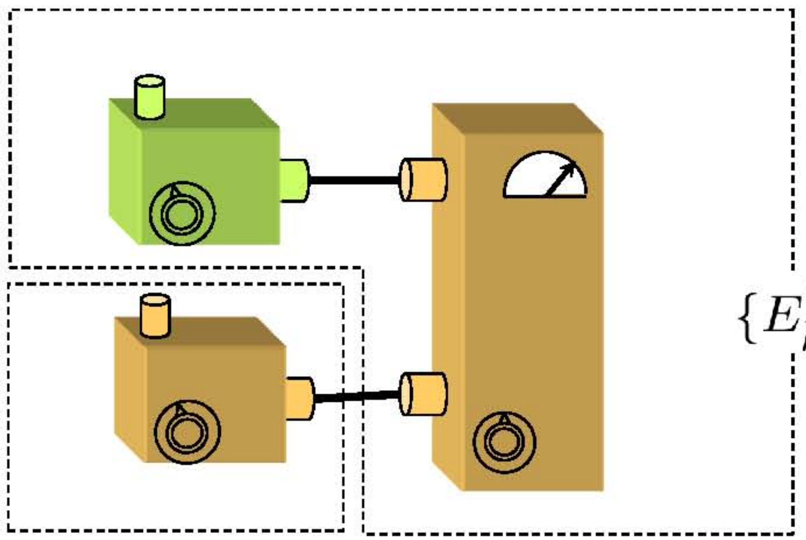
M_{sa}

$\{\Pi_k^{(sa)}\}$

$$p(k|\mathsf{P}_{sa}, \mathsf{M}_{sa}) = \mathrm{Tr}_{sa}(\rho_{sa}\Pi_k^{(sa)})$$

P_s

ρ_s



$M_s = (P_a, M_{sa})$

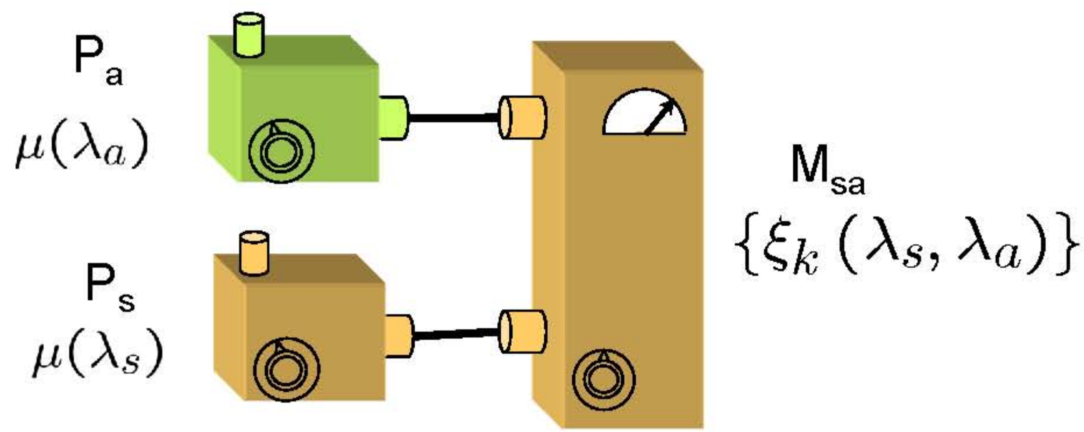
$\{E_k^{(s)} = \mathrm{Tr}(\rho_a \Pi_k^{(sa)})\}$

$$p(k|\mathsf{P}_s, \mathsf{M}_s) = \mathrm{Tr}_s(\rho_s E_k^{(s)})$$

Note:

It is not the case that a single measurement procedure can be represented either by a POVM or by a projector-valued measure

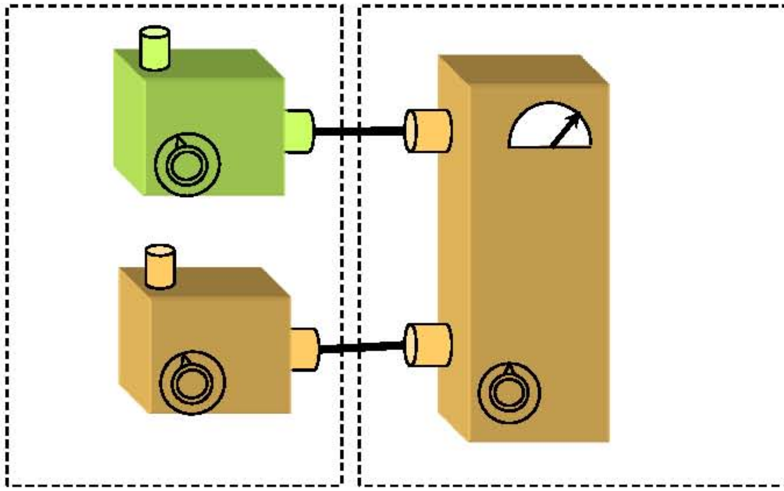
There are many Neumark extensions of a given POVM



P_a
$\mu(\lambda_a)$
P_s
$\mu(\lambda_s)$
M_{sa}
$\{\xi_k(\lambda_s, \lambda_a)\}$

$P_{sa} = (P_s, P_a)$

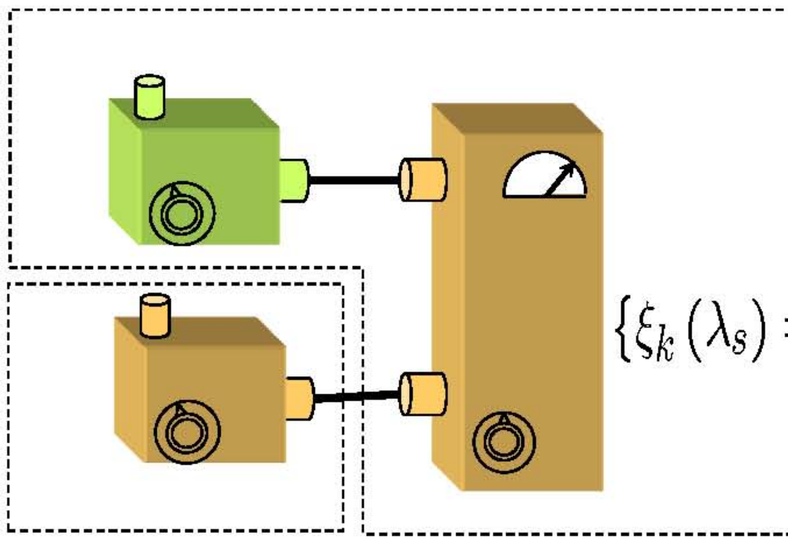
$\mu(\lambda_s, \lambda_a) = \mu(\lambda_s)\mu(\lambda_a)$



M_{sa}

$\{\xi_k(\lambda_s, \lambda_a)\}$

$$p(k|\mathsf{P}_{sa}, \mathsf{M}_{sa}) = \int d\lambda_s d\lambda_a \mu(\lambda_s, \lambda_a)\xi_k(\lambda_s, \lambda_a)$$



$M_s = (P_a, M_{sa})$

$\{\xi_k(\lambda_s) = \sum_{\lambda_a} \xi_k(\lambda_s, \lambda_a)\mu(\lambda_a)\}$

P_s

$\mu(\lambda_s)$

$$p(k|\mathsf{P}_s, \mathsf{M}_s) = \int d\lambda_s \mu(\lambda_s)\xi_k(\lambda_s)$$

Even if the response function is sharp on the composite space, it may not be sharp on the system space

Do we always have a sharp ontic extension of a set of unsharp response functions?

A refinement of the argument in favor of the popular proposal

Premiss: If two measurements on *s* have the same statistics for all preparations on *s*, then they should be represented by the same response functions on *s*

Operational equivalence on *s* → Ontic equivalence on *s*

The assumption of measurement noncontextuality

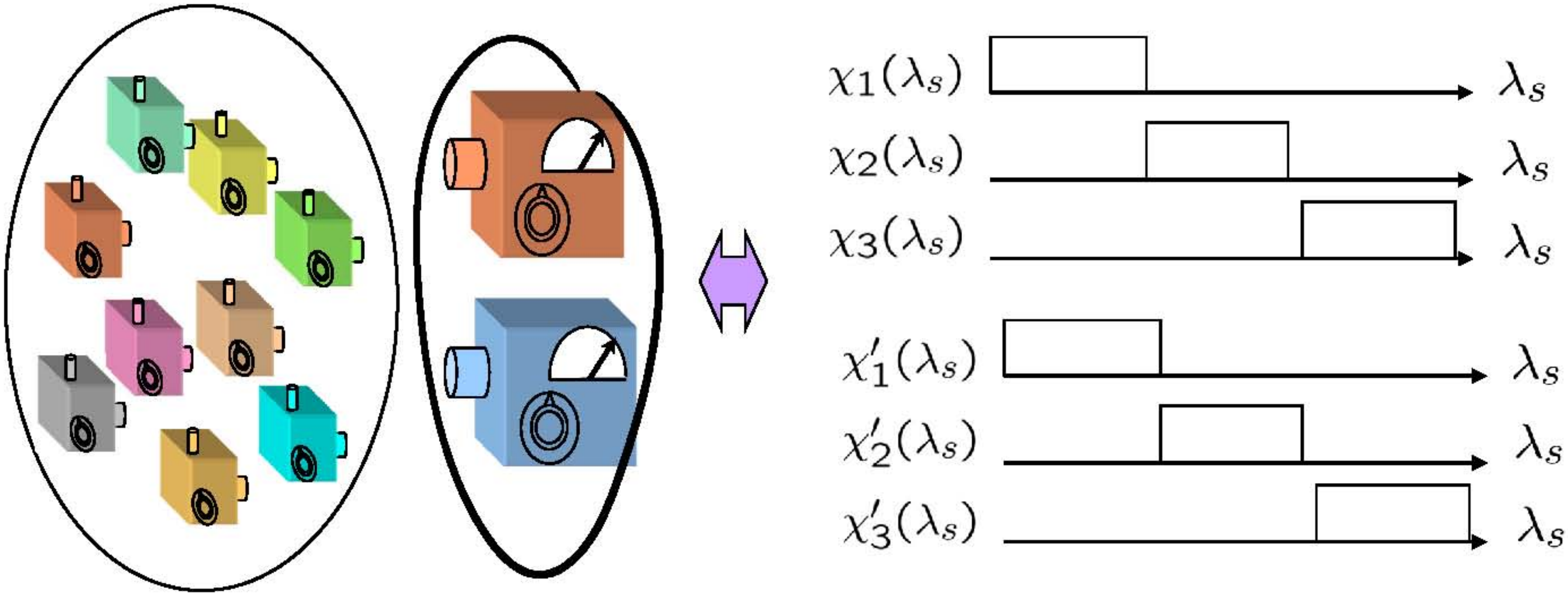
Premiss: Every measurement on *s* can be represented by an outcome-deterministic response function on *sa* (and a distribution on a)

Operational unsharpness is consistent with ontic sharpness

Outcome-deterministic ontic extensions of measurements

Purported conclusion: If two measurements on *s* have the same statistics for all preparations on *s*, then they should be represented by the same outcome-deterministic response functions on some system (on *s* or *on sa*).

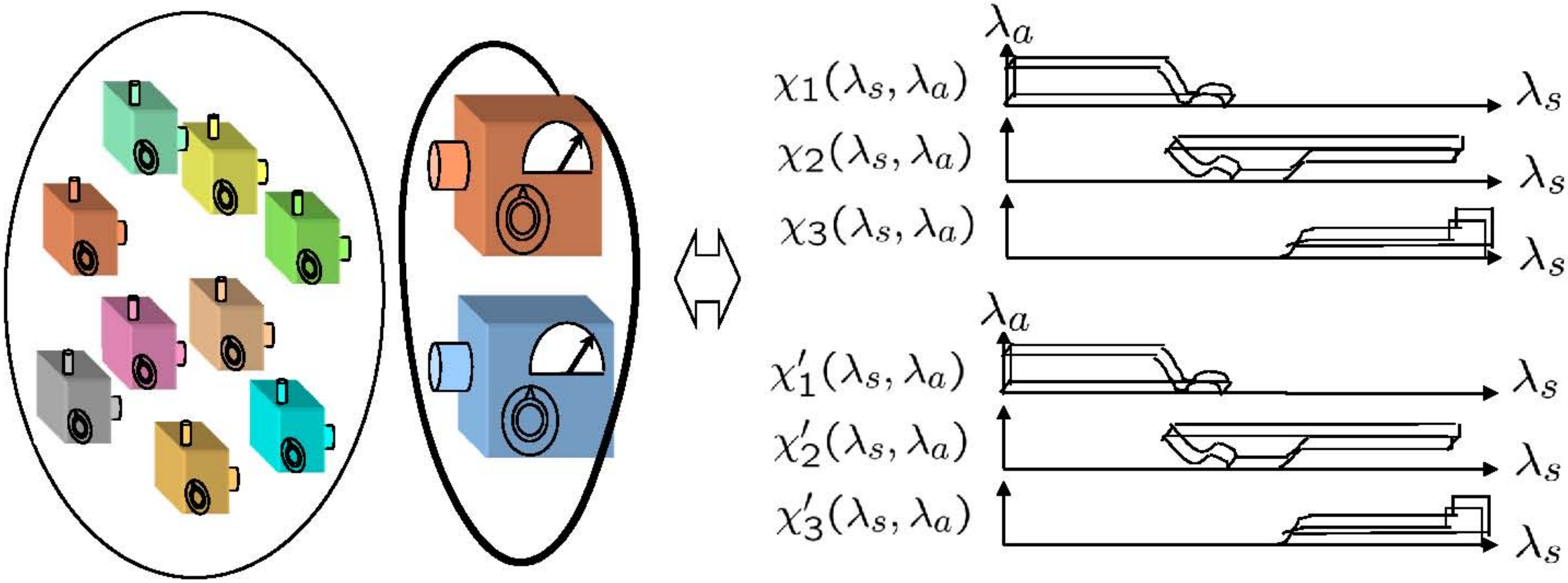
What needs to be shown to justify the popular proposal



Popular proposal **on *s***: If two measurements on *s* have the same statistics for all preparations on *s*, then they should be represented by the same outcome-deterministic response functions **on *s***.

What needs to be shown to justify the popular proposal

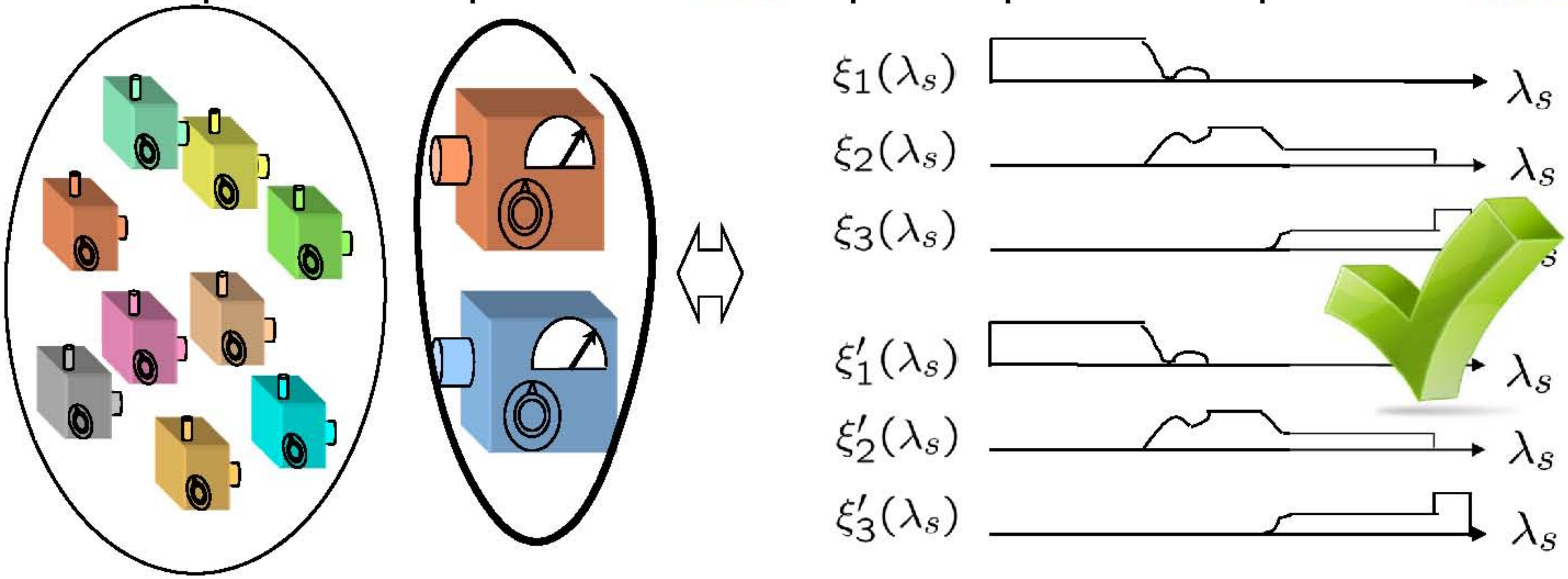
Or



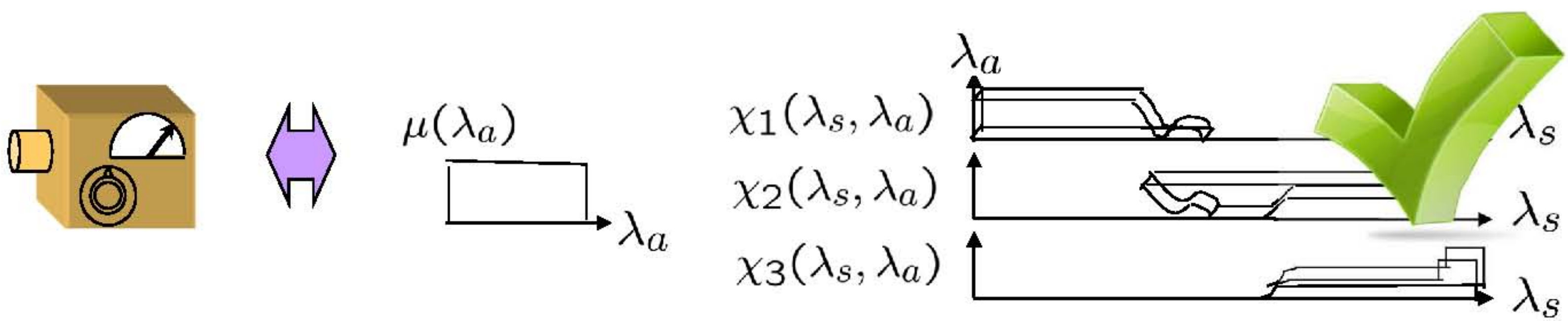
Popular proposal **on *sa***: If two measurements on *s* have the same statistics for all preparations on *s*, then they should be represented by the same outcome-deterministic response functions **on *sa***.

The assumptions we can justify are…

P: Operational equivalence on *s* implies equivalent response fns on *s*

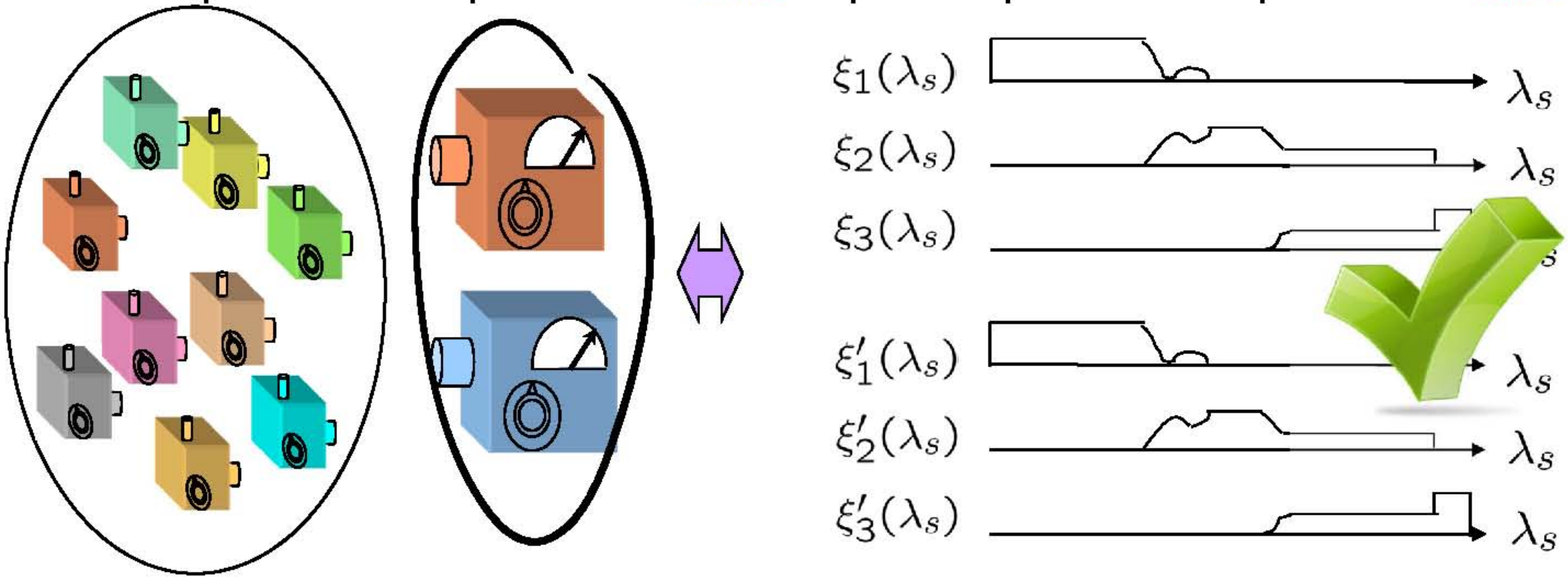


P: Every POVM on *s* can be represented by a set of outcome-deterministic response functions on *sa* (and a distribution on *a*)

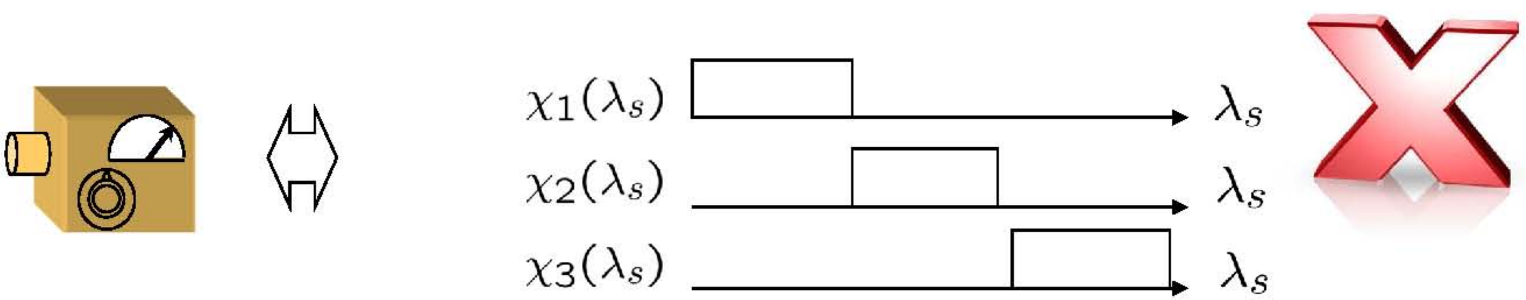


The assumptions we need to get the popular proposal on s are…

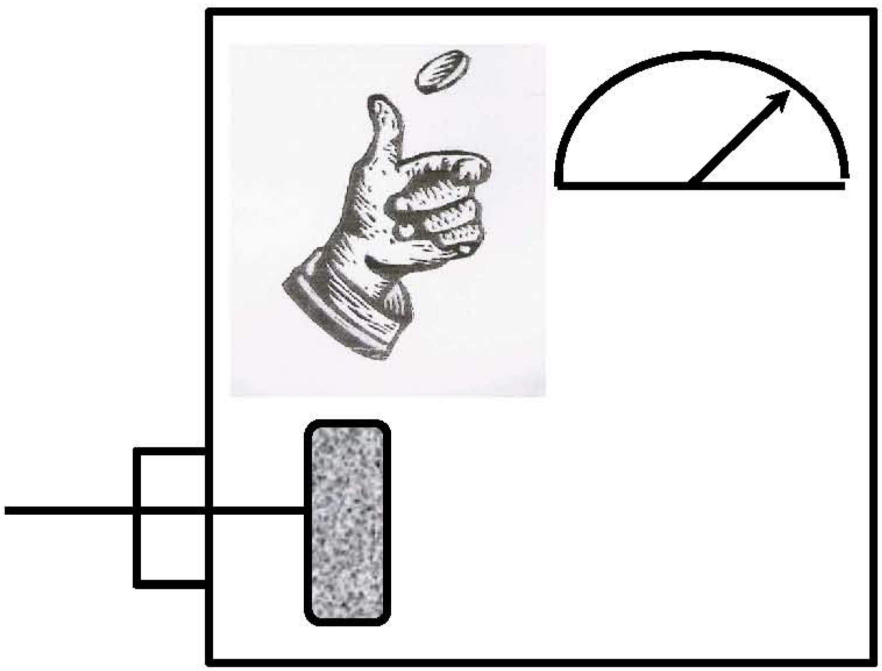
P: Operational equivalence on *s* implies equivalent response fns on *s*



P: Every POVMs on *s* can be represented by a set of outcome-deterministic response functions on *s*



Why a POVM on *s cannot* be represented by an outcome-deterministic response function on *s*

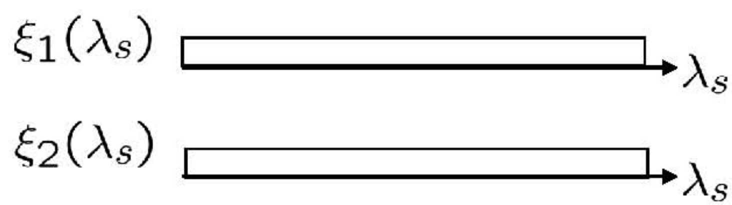


POVM

$$\{\frac{I}{2}, \frac{I}{2}\}$$

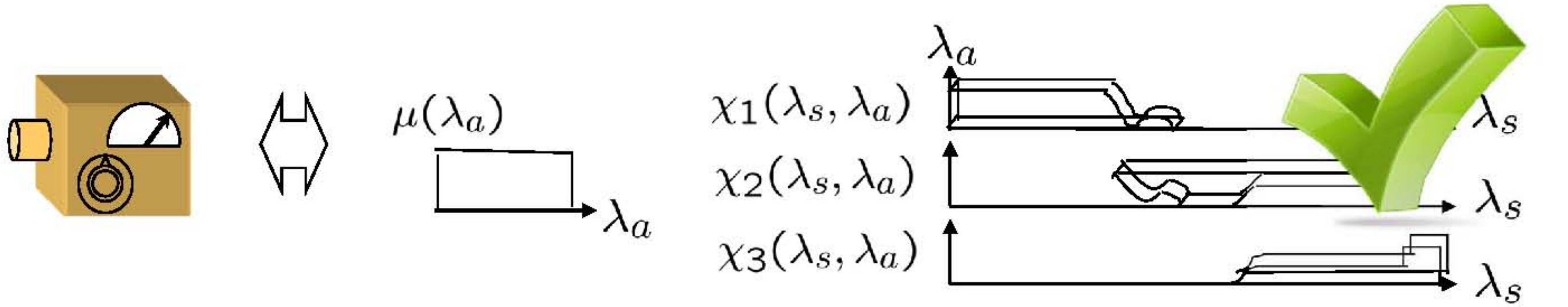
Response functions

$$\{\frac{1}{2}, \frac{1}{2}\}$$

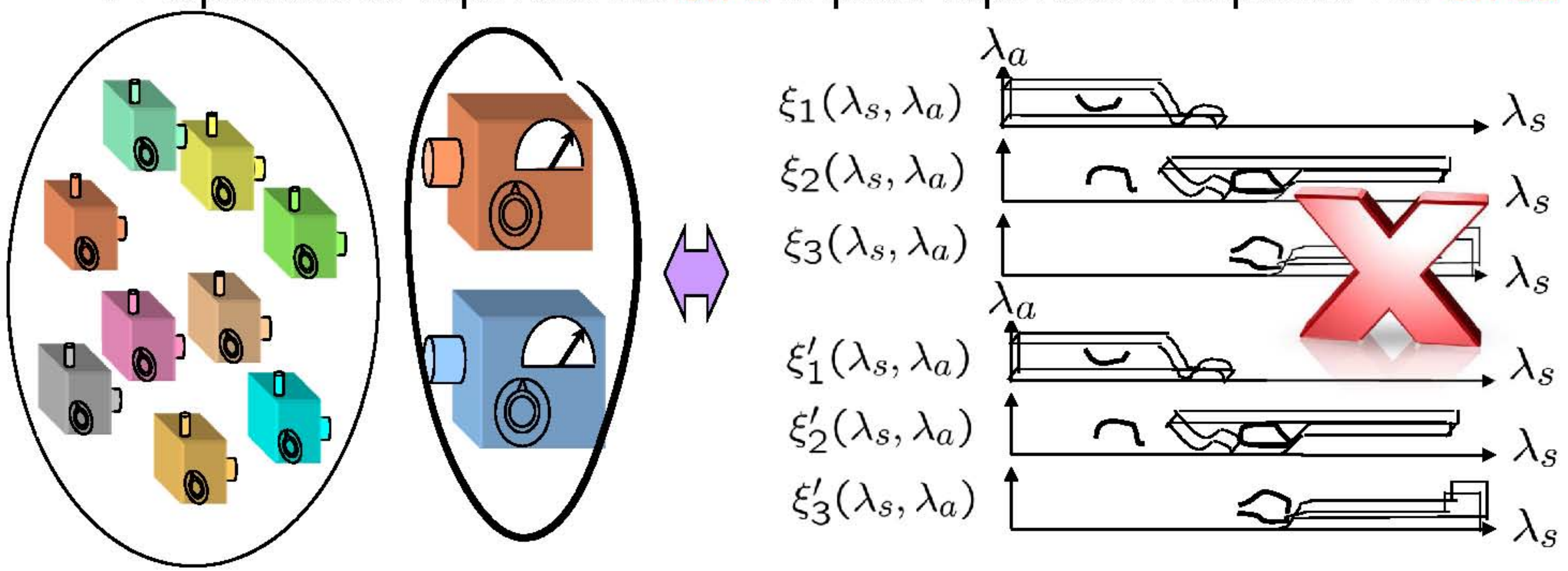


The assumptions we need to get the popular proposal on *sa* are…

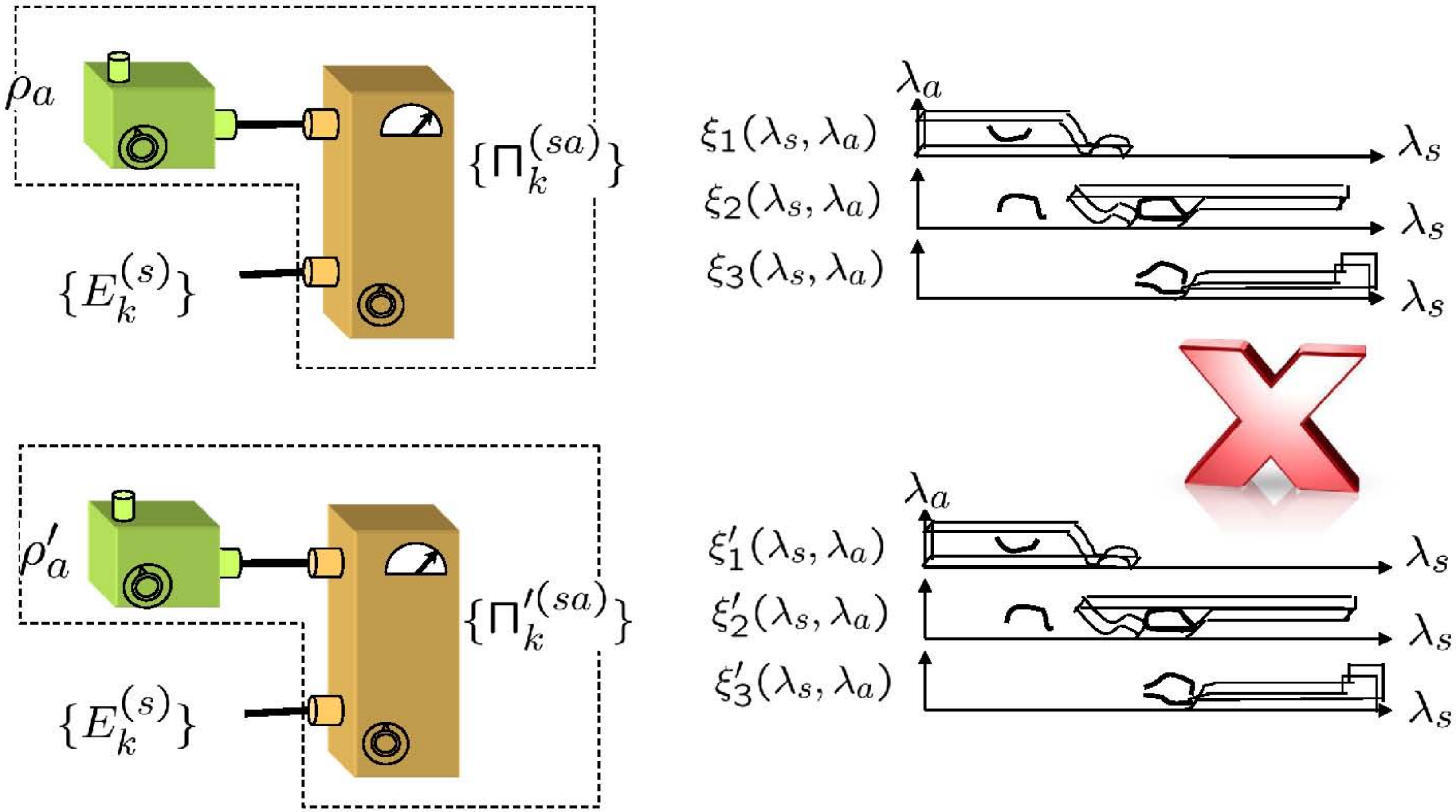
P: Every POVMs on *s* can be represented by a set of outcome-deterministic response functions on *sa* (and a distribution on *a*)



P: Operational equivalence on *s* implies equivalent response fns on *sa*



Why operational equivalence on *s cannot* imply equivalent response functions on *sa*

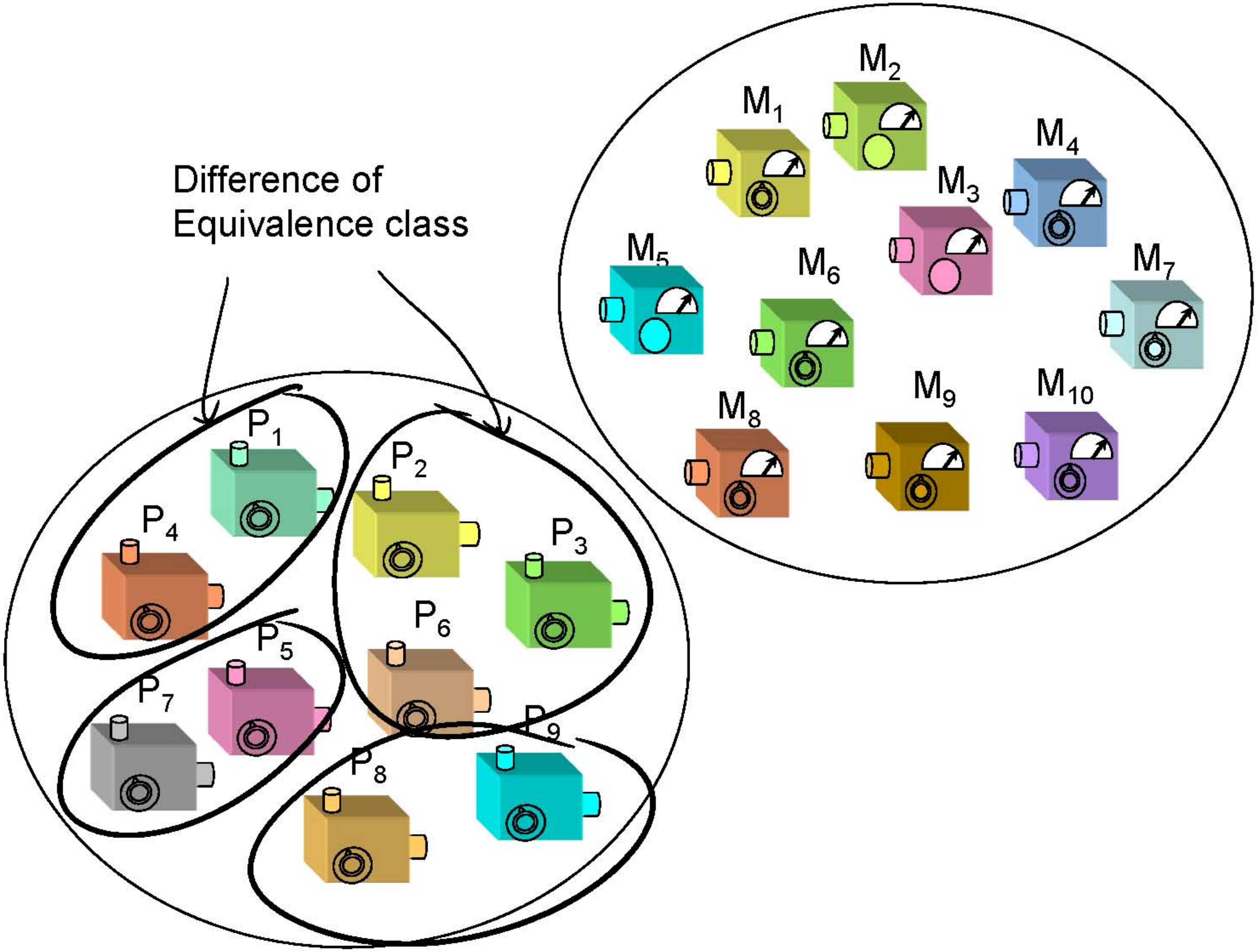


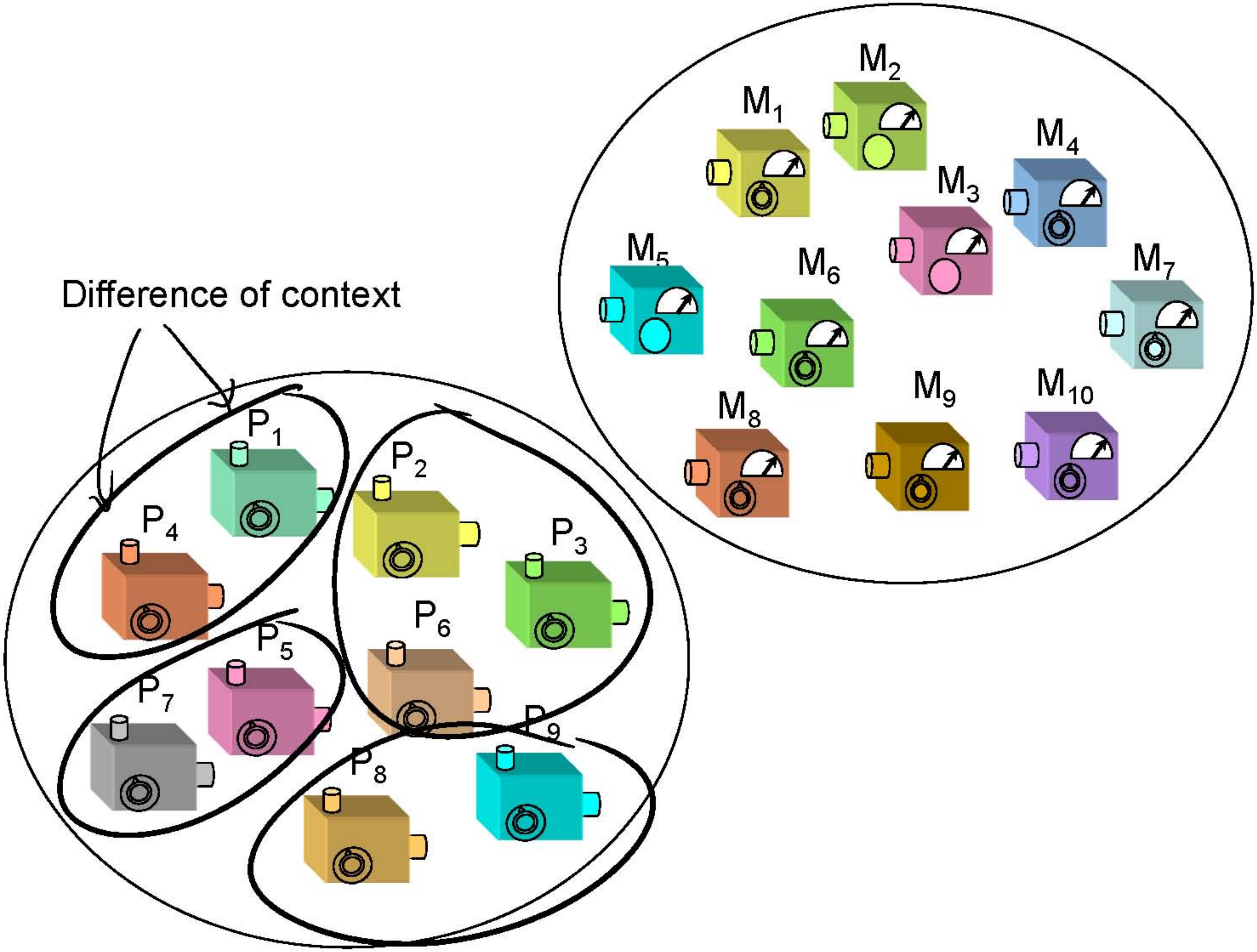
Can we justify an assumption of outcome determinism in some way?

Yes, from an assumption of noncontextuality for preparations but only for projective measurements

The notion of preparation noncontextuality

Difference of
Equivalence class

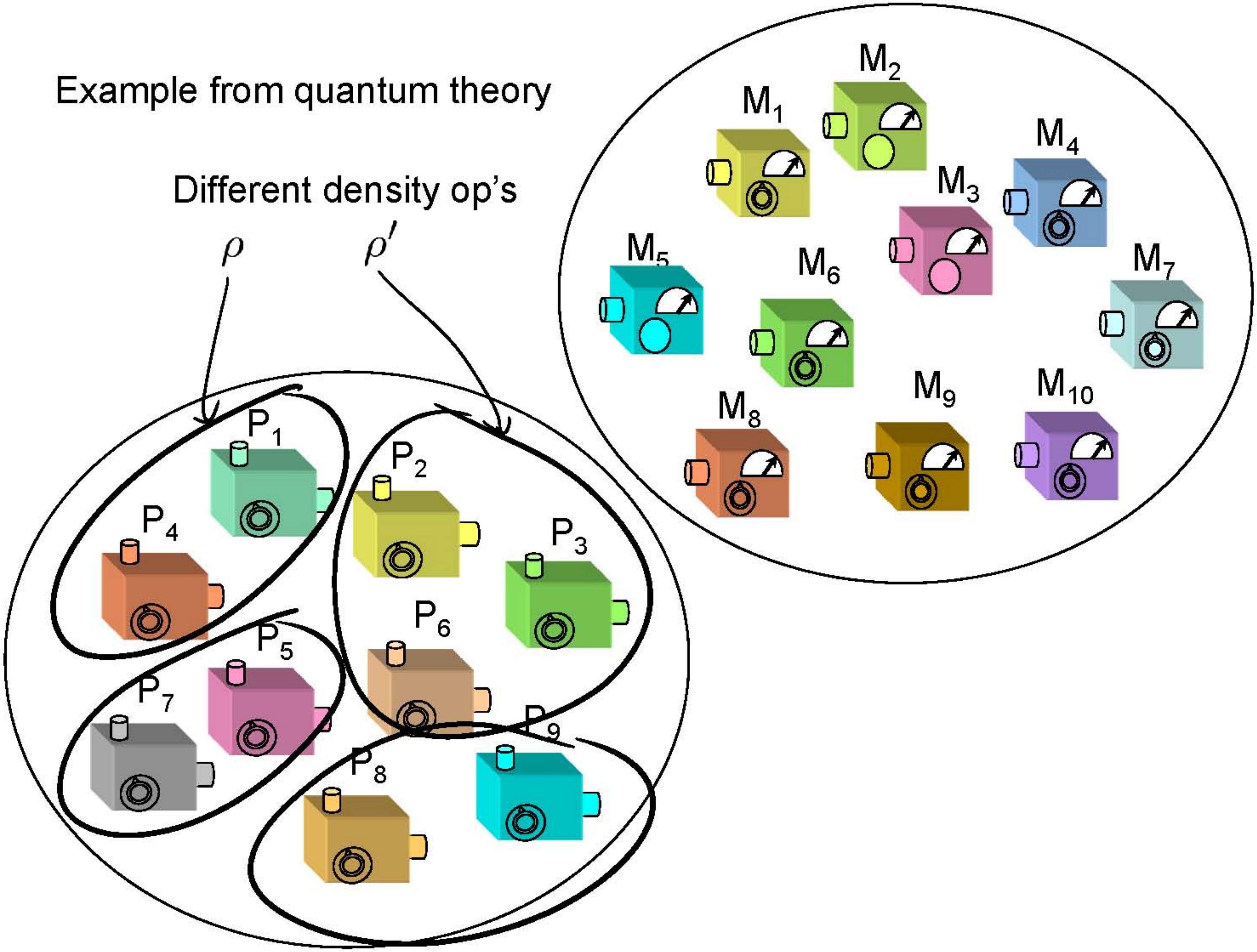




Difference of context
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}

Example from quantum theory

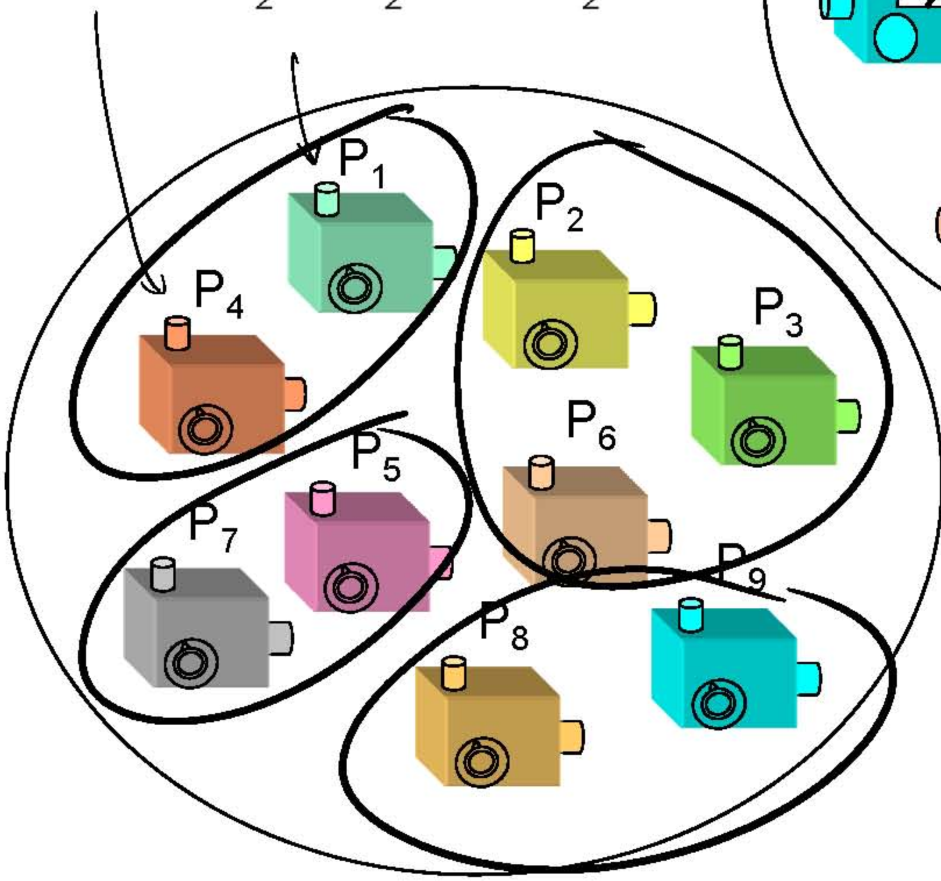
Different density op's

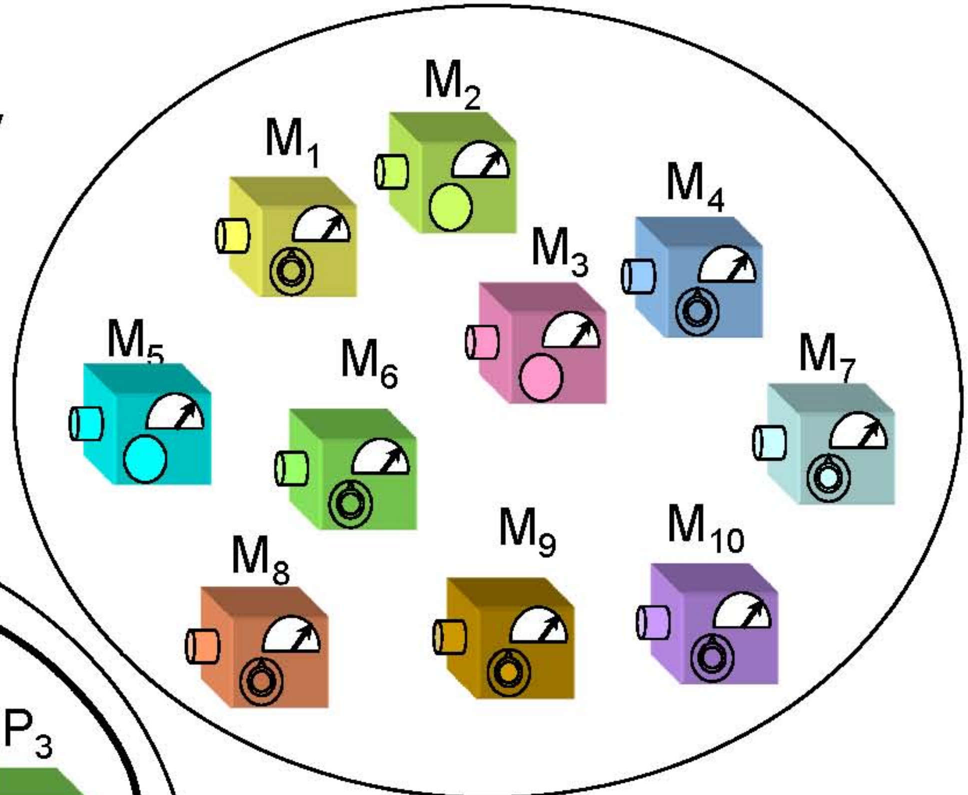


Example from quantum theory

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

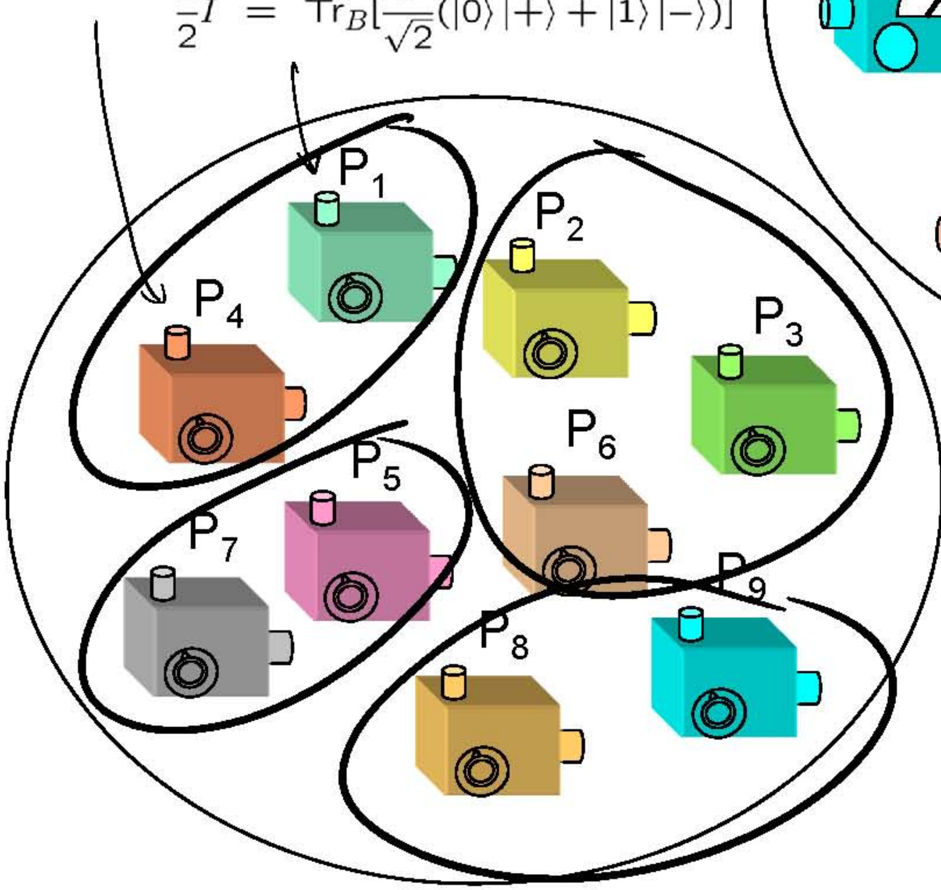


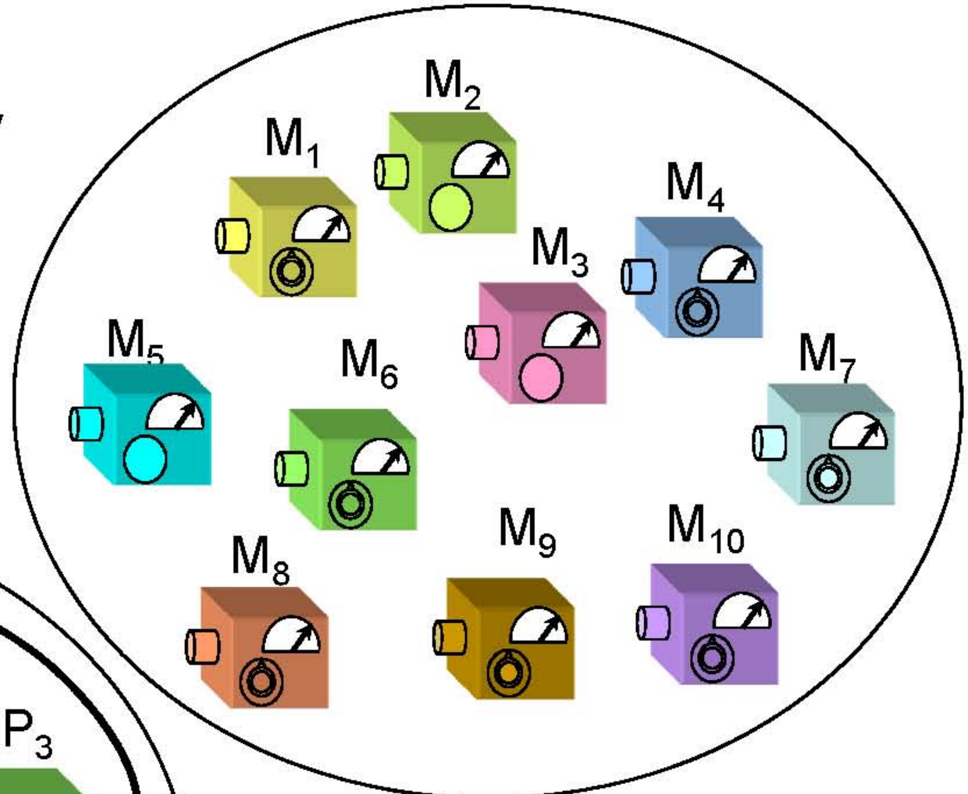


Example from quantum theory

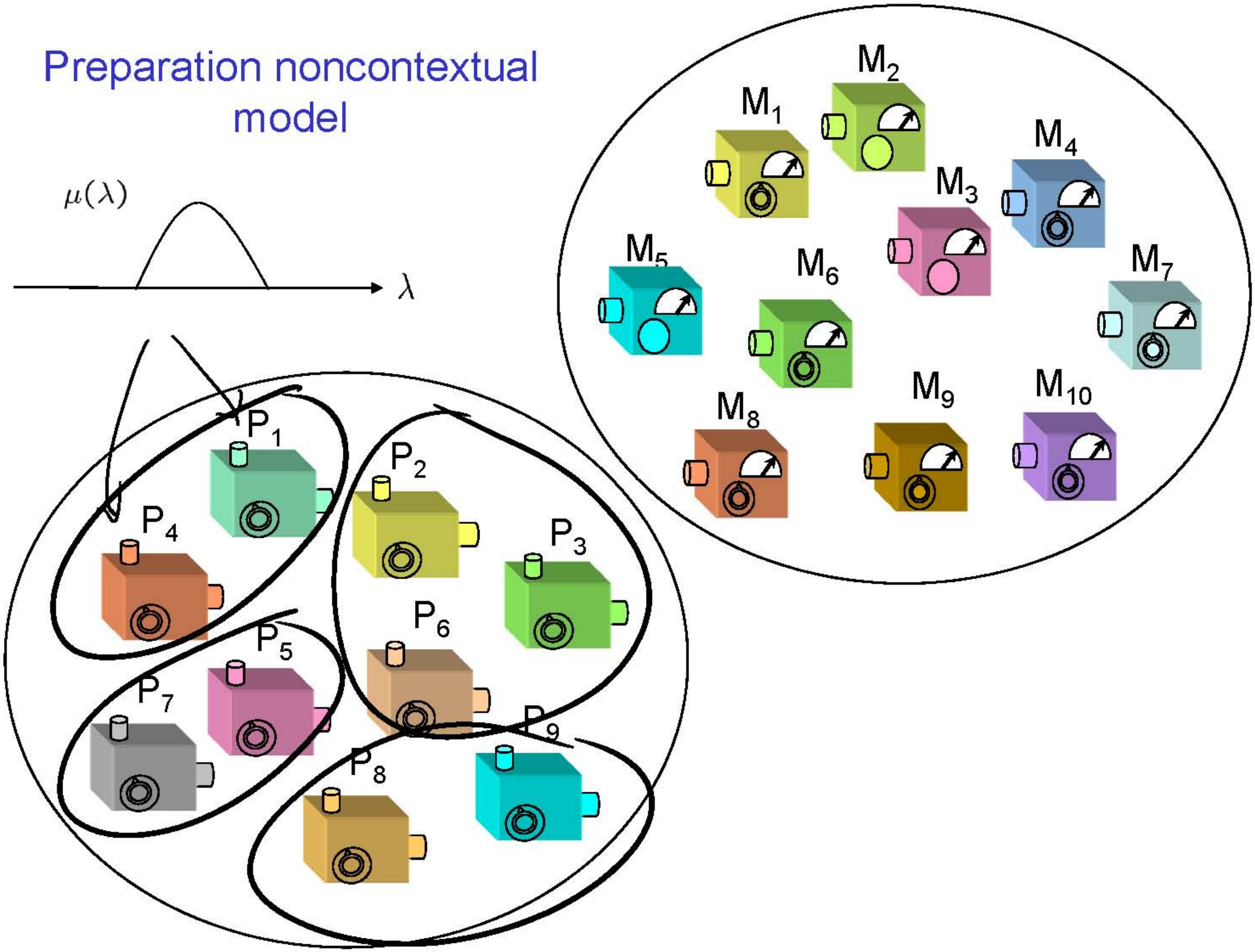
$$\frac{1}{2}I = \mathrm{Tr}_B[\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)]$$

$$\frac{1}{2}I = \mathrm{Tr}_B[\frac{1}{\sqrt{2}}(|0\rangle|+\rangle + |1\rangle|-\rangle)]$$

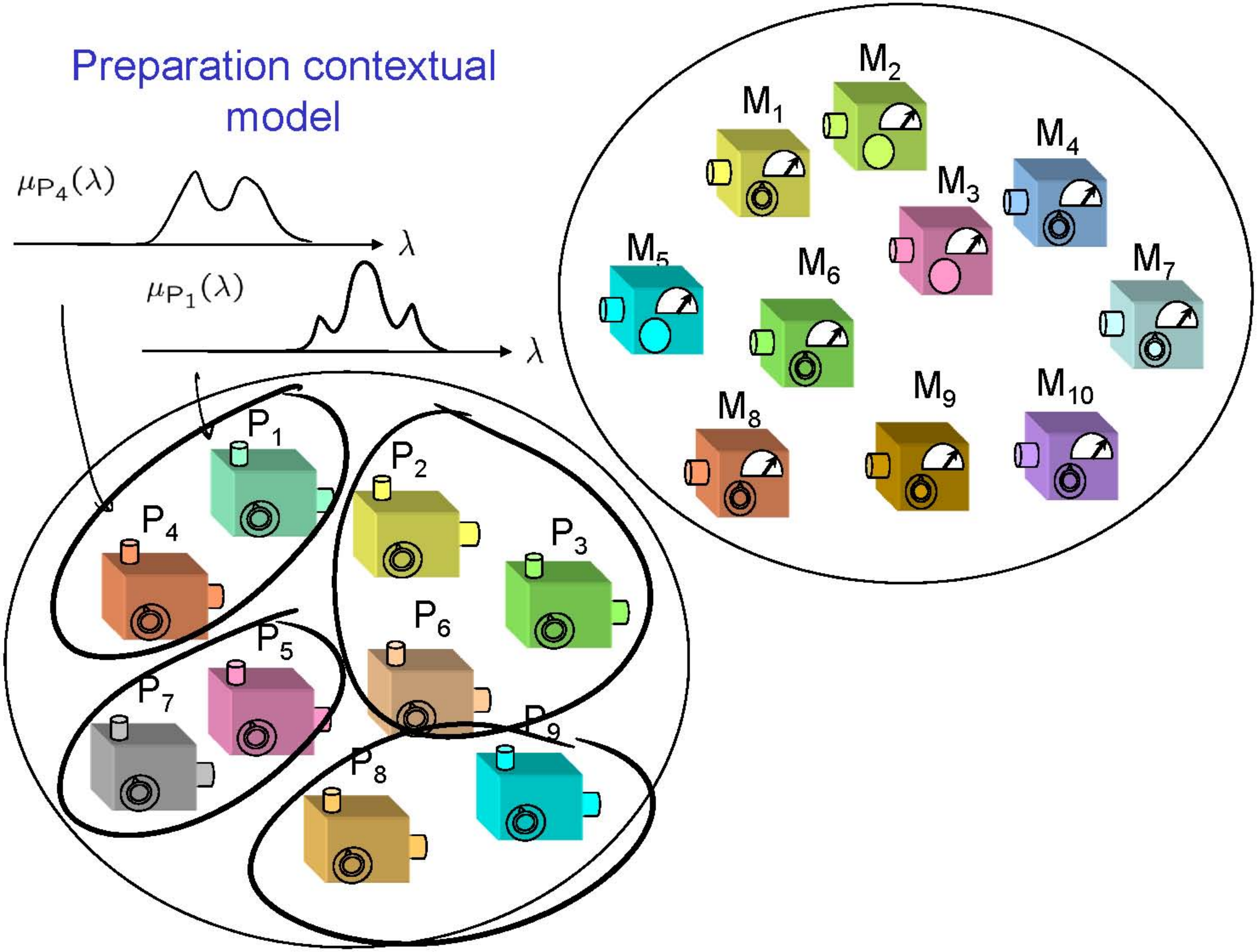




Preparation noncontextual model



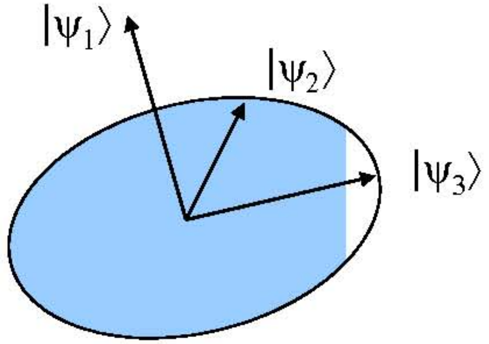
Preparation contextual model

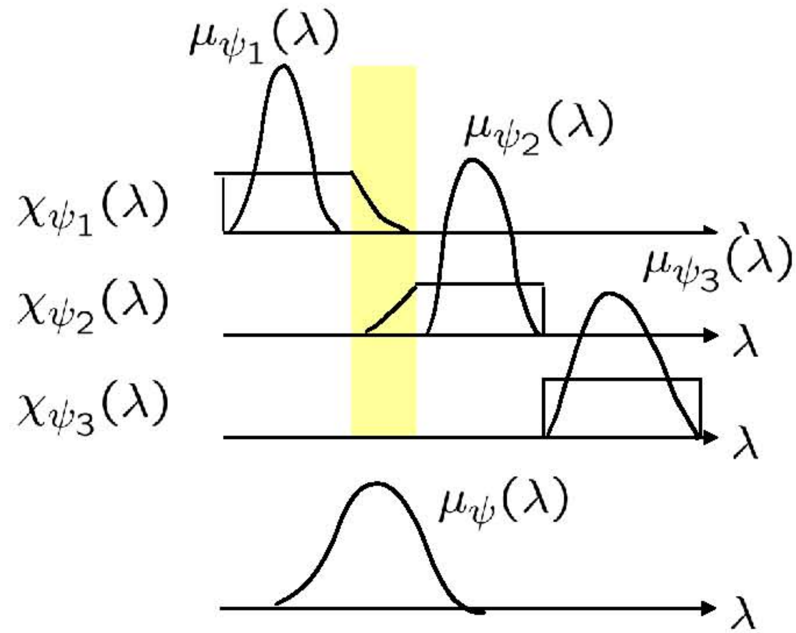


One can prove that

preparation noncontextuality ⟶ outcome determinism for projective measurements

Proof





$$\mu_{I/3}(\lambda) = \tfrac{1}{3}\mu_{\psi_1}(\lambda) + \tfrac{1}{3}\mu_{\psi_2}(\lambda) + \tfrac{1}{3}\mu_{\psi_3}(\lambda)$$
$$\mu_{I/3}(\lambda) = p\mu_{\psi}(\lambda) + \ldots$$

We've established that

preparation noncontextuality → outcome determinism for projective measurements

Therefore:

measurement noncontextuality and preparation noncontextuality → measurement noncontextuality and outcome determinism for projective measurements

We've established that

preparation noncontextuality → outcome determinism for projective measurements

Therefore:

measurement noncontextuality and preparation noncontextuality → Traditional notion of noncontextuality

no-go theorems for the traditional notion of noncontextuality can be salvaged as no-go theorems for the generalized notion

… and there are many new proofs

What needs to be done to obtain convincing experimental tests of universal noncontextuality (featuring measurements)

Theory side:
- Determine whether the implication from preparation noncontextuality to outcome determinism for sharp measurements holds for other operational theories
- Define robust notion of noncontextuality (operational closeness implies ontic closeness)

Experimental side:
Test operational equivalence of measurements and of preparations (the latter to justify outcome-determinism for projective measurements)