Title: Change of quantum reference frames; towards a quantum relativity principle?

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Abstract: Matt Palmer
An explicit description of a physical system is necessarily written with respect to a particular reference frame. It is important to know how to adapt the description when a different, equally valid, reference frame is chosen. In the case of classical frames there is a well-defined covariance of the description. The question we want to address is: How can we extend this description of change of reference frame to the case where the frames are quantum objects?

We study this problem within specific toy models, and approach it operationally. We define a procedure that will change the quantum reference frame with which a quantum system is described. We find this procedure induces decoherence in the system and is described by a non-unitary CP map, which is in interesting distinction to the reversible nature of the classical change of frame procedures.

Change of quantum reference frames: Towards a quantum relativity principle?

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Motivation

Why study quantum reference frames?

- Allows us to define observables within QM when symmetries are present.
- Study decoherence and degradation effects of using a quantised frame; analogous behaviour to that in quantised geometry?

Why study change of quantum reference frames?

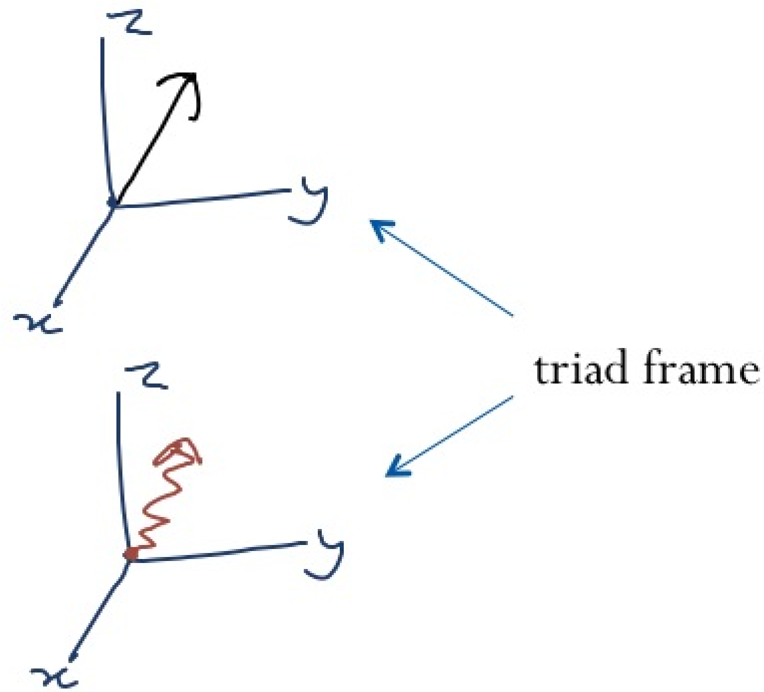
- Put constraints on replacing a quantum reference frame: Can Alice discard or decrease the decoherence from a decayed QRF by replacing it?
- Is there a quantum analogue to the relativity principle? This is the first step (kinematics).

Reference frames in physics

- Physical situations are always described in terms of reference frames:

Classical frames:

- Classical vectors:
- Quantum states:

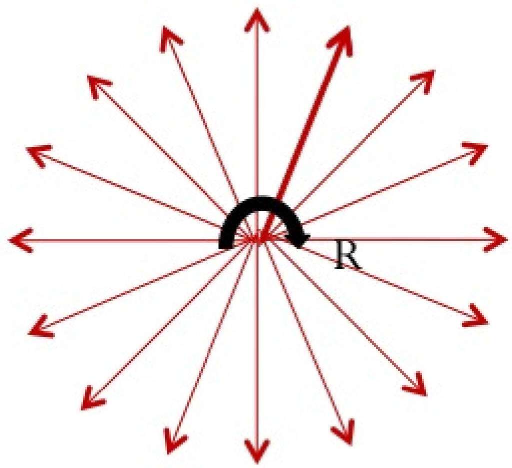


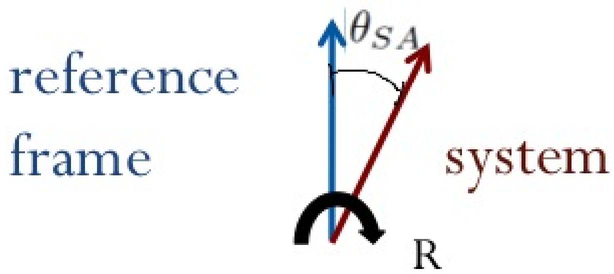
- Symmetries can be local or global, in the following we shall deal with global symmetries.

Reference frames for global symmetry

- If a theory is invariant under some symmetries, then one way to construct observables is to introduce a reference frame.

Under rotations:





We have the 'relational observable' θ_{SA} (relative angle $\theta_S - \theta_A$).

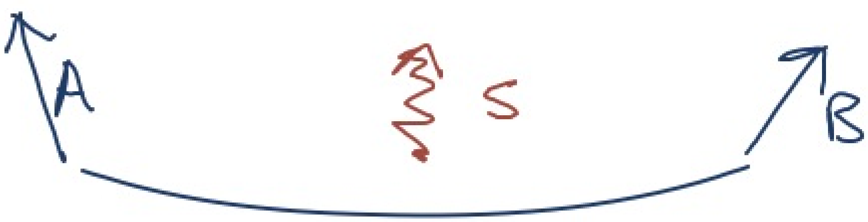
If the system is quantum the corresponding relational pure state is

$$|\theta_{SA}\rangle = |\theta_S - \theta_A\rangle$$

Change of classical frame

What if we want to change the reference frame that we use to define our relational observable $|\theta_{SA}$?

- We can relate the frames by an angle θ_{BA}



$$\theta_{BA} = \theta_B - \theta_A$$

- New description of the state $|\theta_{SB}\rangle$ related to the original $|\theta_{SA}\rangle$ by a unitary $U(-\theta_{BA}) = e^{-i\theta_{BA}\hat{J}}$, where $\hat{J}$ is the generator of rotations:

$$|\theta_{SB}\rangle = e^{-i\theta_{BA}\hat{J}}\,|\theta_{SA}\rangle$$

This just says $|\theta_S - \theta_B\rangle = |(\theta_S - \theta_A) - (\theta_B - \theta_A)\rangle$

Change of quantum frame?

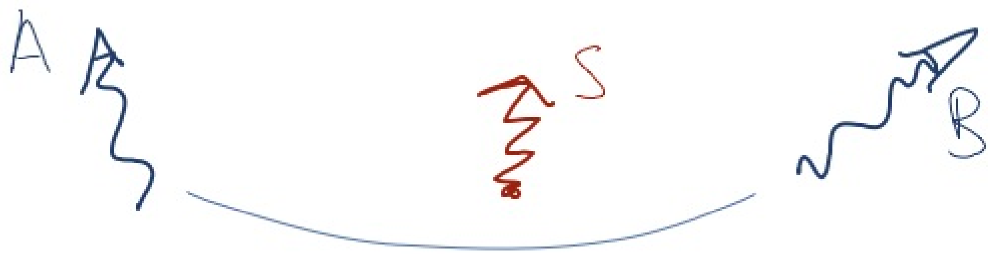
- What if the reference frame is a quantum object too?

quantum
reference
frame

quantum
system

What is a change of quantum reference frame?

Change of quantum frame?



$$\hat{\theta}_{BA}?$$

- The relative angle encoding the change of frame θ_{BA} is now quantised. Operationally we cannot determine the relationship without a measurement.
- How are the states related?? $|\theta_{SB}\rangle \longrightarrow |\theta_{SA}\rangle$??

Before: $|\theta_{SB}\rangle = U(\theta_{BA})\,|\theta_{SA}\rangle = e^{-i\theta_{BA}\hat{J}}\,|\theta_{SA}\rangle$

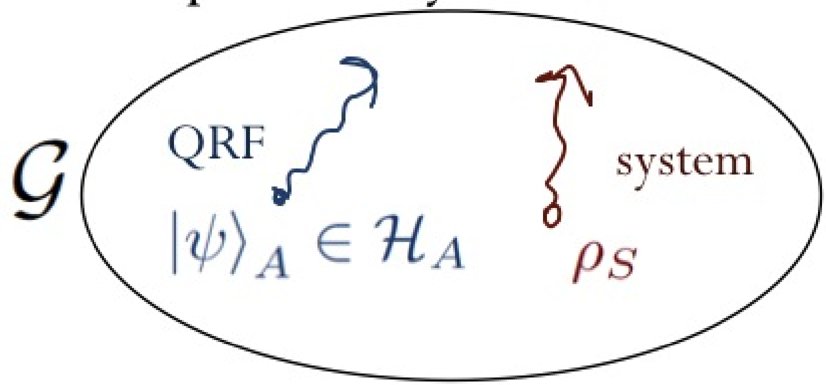
Now: $U(-\hat{\theta}_{BA}) = e^{-i\hat{\theta}_{BA}\hat{J}}?$

➢ An operator of this form is ill-defined. We need to define a change of (quantum) frame procedure in a different way.

Quantum reference frames

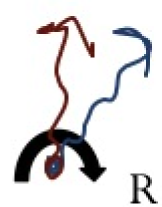
- In QRFs we use another quantum system in a state $|\psi\rangle_A \in \mathcal{H}_A$ to act as a reference orientation for the quantum system ρ_S

Total system $\rho_S \otimes \psi_A$



- Discard the external classical frame by averaging total state over the symmetry group: 'Relational encoding' and 'physical state':

$$\sigma_{SA} = \mathcal{E}_A(\rho_S) = \mathcal{G}(\rho_S \otimes \psi_A)$$



'G-twirl': $\mathcal{G}(\rho_S \otimes \psi_A) := \int \mathrm{d}\,\theta\, \mathcal{U}_{SA}(\theta)[\rho_S \otimes \psi_A]$

•From Bartlett, Rudolph, Spekkens, Turner, 2009 (see also B.R.S. Review 2007)

Decoherence and noise

- We now possess only the information in the physical state

$$\sigma_{SA} = \mathcal{E}_A(\rho_S) = \mathcal{G}(\rho_S \otimes \psi_A)$$

- The QRF is a finite, limited resource and is imprecise: Cannot perfectly distinguish orientations.
- We can 'recover' the qubit $\rho_S' = \mathcal{R}(\sigma_{SA})$ by switching back to a classical frame (Bartlett, Rudolph, Spekkens, Turner 2009). This introduces noise.

➢ Encoding then recovering the quantum state ρ_S with a finite size QRF is seen as decoherence on the state:

$$\rho_S' = \mathcal{R} \otimes \mathcal{E}_A(\rho_S) = \int \mathrm{d}\,\theta f(\theta)\mathcal{U}(\theta)[\rho_S]$$

Example: Phase quantum RF

U(1) symmetry (as in BRST2009). Reference frame is a harmonic oscillator with a finite maximum photon number A.

$$\sum_{n=0}^{A} e^{in\theta} |n\rangle_{\text{Fock}} = |\theta_A\rangle \qquad\qquad |\psi\rangle_S = \alpha|0\rangle + \beta|1\rangle$$

reference frame — system

Physical state (how the information is encoded):

$$\sigma_{SA} = \mathcal{G}(\psi_S \otimes \theta_A) = \int \frac{\mathrm{d}\,\phi}{2\pi} \mathcal{U}_{SA}(\phi)[\psi_S \otimes \theta_A] = \sum_{N=0}^{A} \psi_{SA}^{(N)}$$

$$\left|\psi_{SA}^{(N)}\right\rangle = \alpha|0N\rangle + \beta|1(N-1)\rangle$$

- Each subspace of total photon number N has a representation of the qubit
- except for N=0, A+1

- Decoherence is a function of QRF dimension A:

$$\rho'_S = \mathcal{R} \circ \mathcal{E}(\rho_S) = \begin{bmatrix} \rho_{00} & \frac{A}{A+1}\rho_{01} \\ \frac{A}{A+1}\rho_{10} & \rho_{11} \end{bmatrix}$$

Defining a change of quantum frame

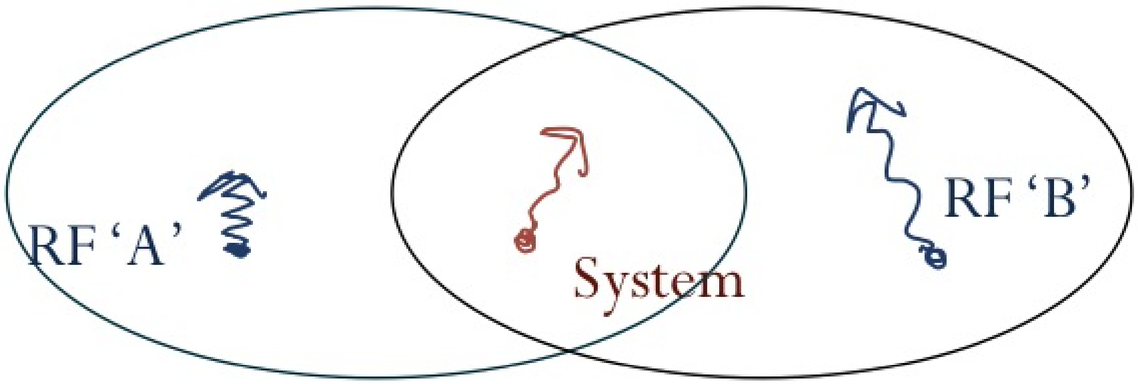
Since the quantum reference frames are quantum systems there are far more defining parameters:

- Hilbert space dimensions
- Type of state
- Noise? (mixed?)
- as well as orientation

Implications:

- ➢ We can no longer change frames simply by a rotation.
- ➢ Many options: Need to define a procedure

Change of QRF procedure:



The change of the quantum reference frame procedure should:

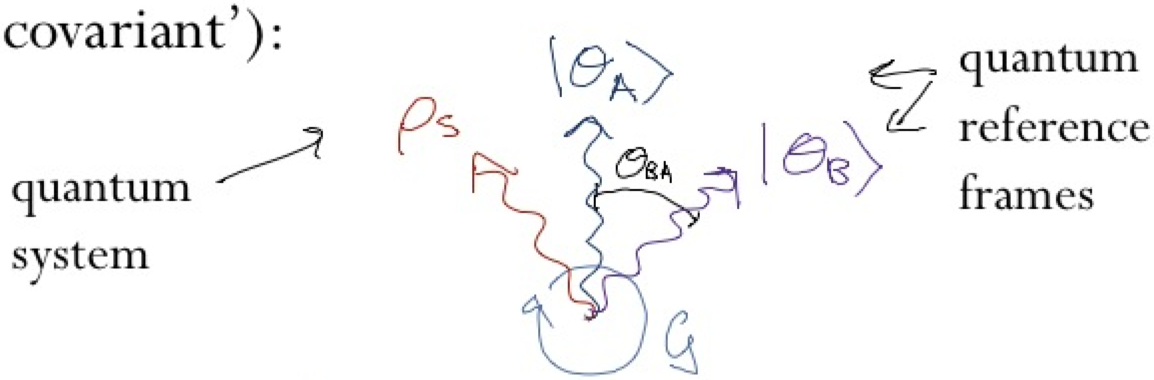
- Be relational: Only the physical states are known. The procedure will then be a map from σ_{SA} to σ_{SB} .
- Reproduce the classical case in the limit of large QRFs: Unitary operator: $\sigma_{SB} = \mathcal{U}(-\theta_{BA})[\sigma_{SA}]$

➢ We will see that the change of quantum RF is noisy: We have 'quantised' the relationship between RFs.

Candidate (illustrated for U(1))

Projective measurement on RFs A and B to determine relative orientation θ_{BA}. Projector $\Pi_{AB}^{\phi,\theta_{BA}} = |\phi\rangle\langle\phi|_A \otimes |\phi+\theta_{BA}\rangle\langle\phi+\theta_{BA}|_B$

- The measurement cannot have a preferred direction: Average over direction ('G-covariant'):



Measurement with outcome θ_{BA}:

$$\mathcal{M}_{AB}^{\theta_{BA}}[\sigma_{ASB}] \propto \int \frac{\mathrm{d}\,\phi}{2\pi} \Pi_{AB}^{\phi,\theta_{BA}} \sigma_{ASB} \Pi_{AB}^{\phi,\theta_{BA}}$$

- Trace over RF A to obtain a post measurement state $\sigma_{BS}^{\theta_{BA}}$

$$\mathrm{Tr}_A\left[\mathcal{M}_{AB}^{\theta_{BA}}[\sigma_{ASB}]\right] \propto \sigma_{SB}^{\theta_{BA}} = \int \frac{\mathrm{d}\,\phi}{2\pi} f(\phi)\left(\mathcal{U}_B(\phi)\otimes\mathcal{I}_S\right)[\sigma_{SB}]$$

Interpreting

Alice's point of view:

- Starts with information about A and S only: $\sigma_{SAB} = \sigma_{SA} \otimes I_B$
  Performs change of frame to obtain $\sigma_{BS}^{\theta_{BA}}$.

Bob's point of view (control case):

- Already has the encoding $\sigma_{SB} = \mathcal{G}(\rho_S \otimes \theta_B)$

➢ Compare Alice and Bob's descriptions $\sigma_{BS}^{\theta_{BA}}$ and σ_{SB}.

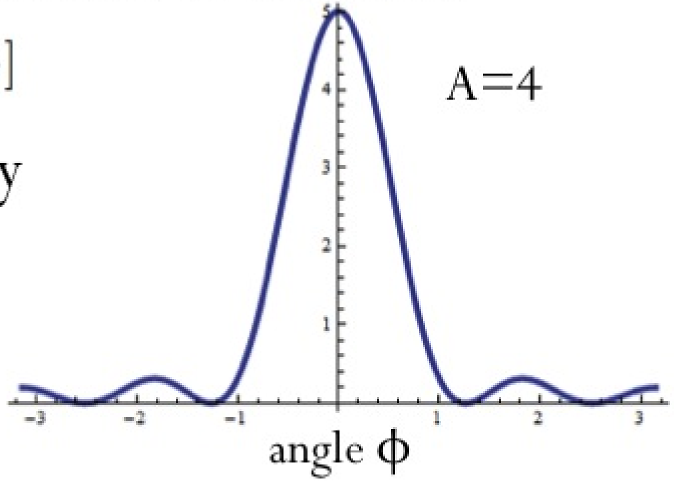
Results

- Alice estimates what the description is in Bob's frame:

$$\sigma_{SB}^{\theta_{BA}} = \int \frac{\mathrm{d}\,\phi}{2\pi} f(\phi)\,(\mathcal{I}_S \otimes \mathcal{U}_B(\phi))\,[\sigma_{SB}]$$

She mixes over directions of the RF by

$$f(\phi) = (A+1)\,|\langle\phi|0\rangle_A|^2$$



- She sees noise is added to Bob's $\sigma_{SB} = \mathcal{G}(\rho_S \otimes \theta_B)$
- Alice observes a flat probability distribution over measurement outcomes θ_{BA}. (Since she has no prior knowledge of ρ_B)
- She identifies decoherence:

$$\rho'_S = \mathcal{U}(-\theta_A - \theta_{BA})[\begin{bmatrix} \rho_{00} & \frac{A}{A+1}\frac{B}{B+1}\rho_{01} \\ \frac{A}{A+1}\frac{B}{B+1}\rho_{10} & \rho_{11} \end{bmatrix}]$$

Intrinsic decoherence (Milburn)

$$\sigma_{SB}^{\theta_{BA}} = \int \frac{\mathrm{d}\,\phi}{2\pi} f(\phi)\,(\mathcal{I}_S \otimes \mathcal{U}_B(\phi))\,[\sigma_{SB}]$$

- The final state of the change of frame procedure has the form of 'intrinsic decoherence' resulting from imperfect knowledge of evolution parameters (Milburn 2006),

$$\rho' = \sum_{n=0}^{\infty} p_n(\varepsilon)\mathcal{U}(n\varepsilon)[\rho]$$

- We obtain this in a different context (and with a continuum of evolution parameters)

➢ Change of QRF is a physical example of a process giving intrinsic decoherence (in the abelian case)

Results from the change procedure

- Our change of QRF procedure gives intrinsic decoherence in the abelian case
- Alternatively it can be interpreted as encoding with a mixed frame
- Decoherence always increases (except in the classical limit)
- In the classical limit the effect of change of QRF becomes the unitary relationship for classical frame (tick).
- Note we have worked without dynamics.

> This procedure is non-reversible; forms a semigroup structure:
> One cannot decrease the noise of a state by changing QRF, and in fact one always increases noise (except in the classical limit).

Discussion

- In the quantum regime the non-abelian symmetry group case is more complicated. Noncommutativity means the change of QRF procedure does not reproduce a form of intrinsic decoherence. Can we recover this scheme in a semi-classical limit??
- Noether's theorem indicates that if we have symmetries, we have conserved quantities. What are the conserved quantities when the symmetry is fuzzy?
- How is the quantum implementation of the symmetries compatible with the dynamics?

Is there a quantum relativity principle?