Title: Quantum non-locality: how much does it take to simulate quantum correlations?

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Abstract: Quantum correlations cannot be given any classical explanation that would satisfy Bell's local causality assumption. This quite intriguing feature of quantum theory, known as quantum non-locality, has fascinated physicists for years, and has more recently been proven to have interesting applications in quantum information processing.

To properly understand the power of quantum non-locality, it is important to be able to quantify it. One way for that is to compare it to other "non-local resources", such as classical communication or "non-local Popescu-Rohrlich (PR) boxes", and try to use these alternative resources to reproduce the quantum correlations. I will review known results on this subject, and present new simulations of multipartite non-local correlations.

Quantum non-locality: How much does it cost to simulate quantum correlations?

Cyril Branciard

University of Queensland

Dec 2, 2010 – PIAF workshop

Outline

1. Quantum Non-locality: basics
2. Simulation of singlet correlations
3. Simulation of GHZ correlations

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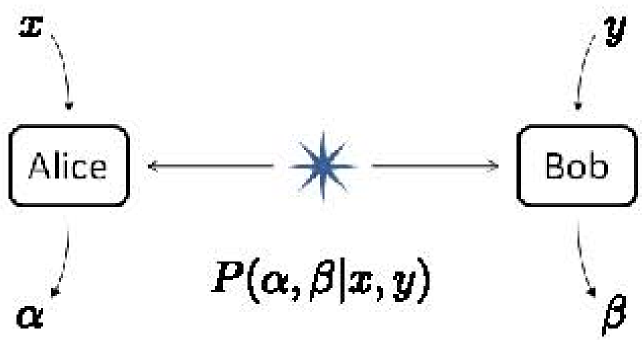
John Bell's locality assumption



- Bell's local causality assumption

$$P(\alpha, \beta | x, y) = \int d\lambda \rho(\lambda)\, P(\alpha | x, \lambda) P(\beta | y, \lambda)$$

- implies Bell inequalities, eg. CHSH inequality

$$I_{CHSH} = \langle a_0 b_0 + a_0 b_1 + a_1 b_0 - a_1 b_1 \rangle \leq 2$$

- Bell inequalities can be violated by quantum correlations
$\Longrightarrow$ Quantum Mechanics is "non-local"

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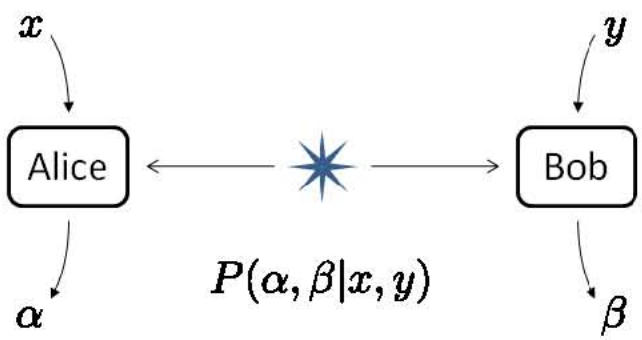

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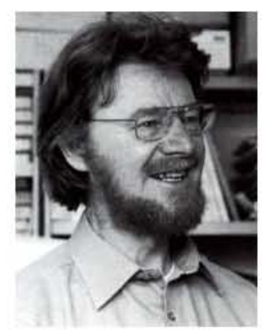

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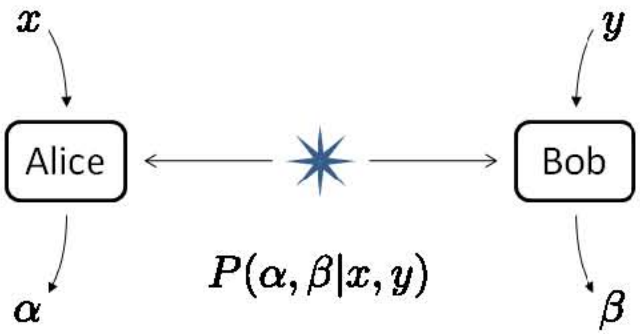

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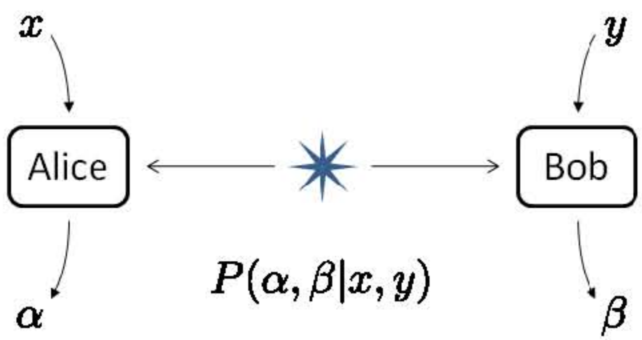

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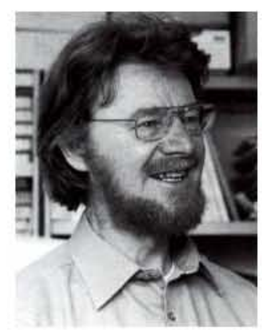

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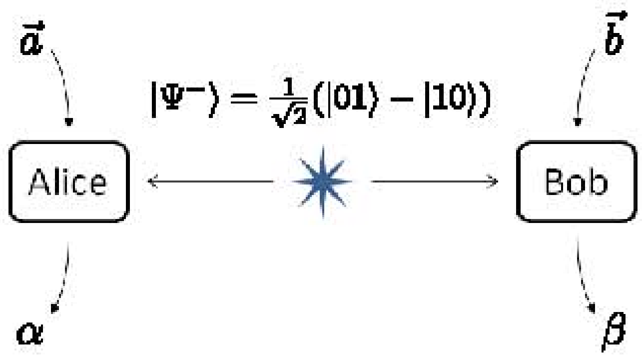

- Singlet correlations: $\langle \alpha\beta \rangle = -\vec{a} \cdot \vec{b}$
- For a proper choice of measurement settings:

$$I_{CHSH} = 2\sqrt{2} > 2 \quad \Rightarrow \quad Non-local\,!$$

- Non-locality of quantum correlations has been observed in many experiments (up to a few persistent loopholes) since the '80s...

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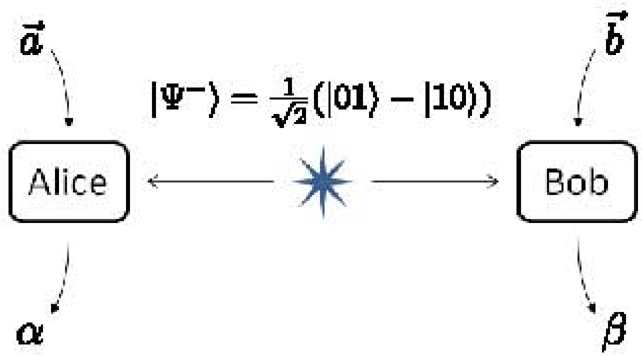

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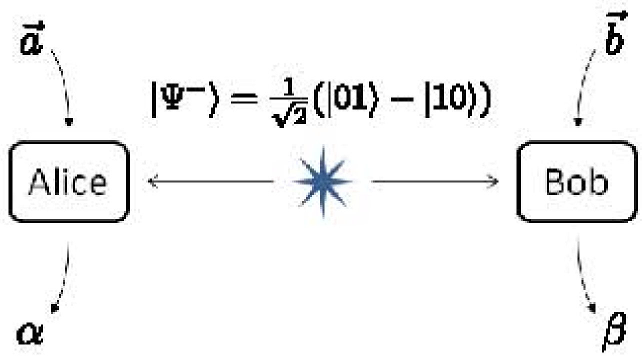

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Non-locality as a resource

- Non-locality is not only a laboratory curiosity!
  It has been realized that it could be useful for information processing applications
  - Non-local computation
  - Secret key distribution
  - Device-independent quantum state estimation
  - Randomness generation

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  - Amount of violation of a Bell inequality / Robustness to noise
  - More convenient: to other non-local resources that are easy to quantify

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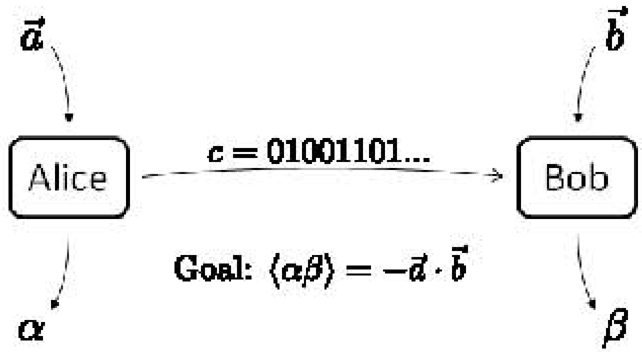
Simulation of singlet correlations – history



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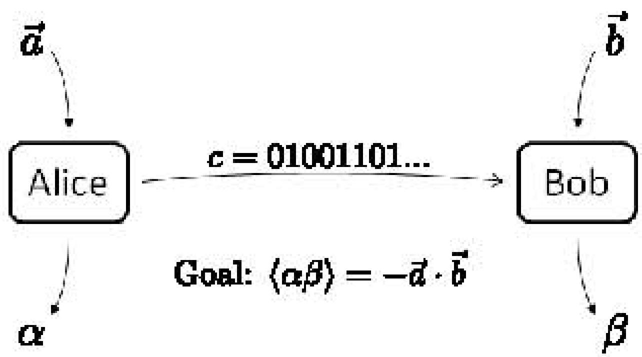

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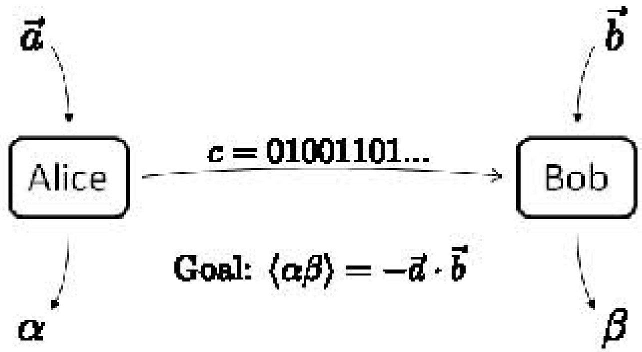

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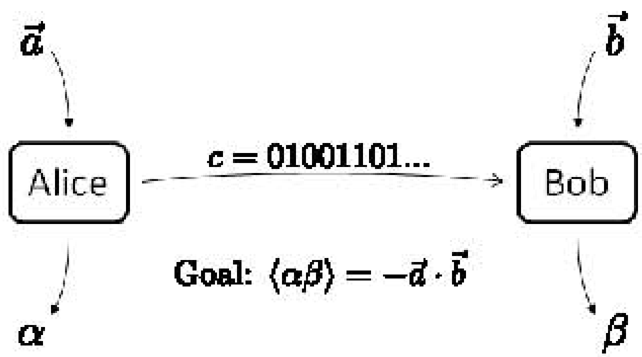

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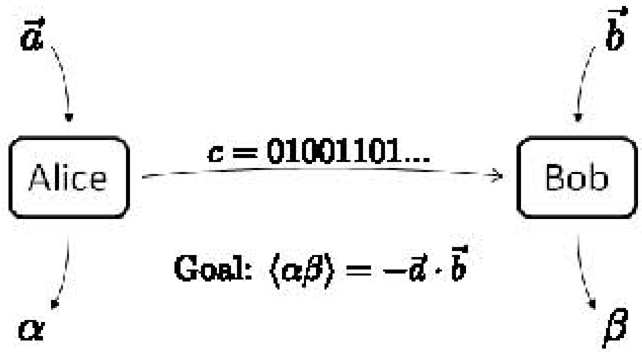

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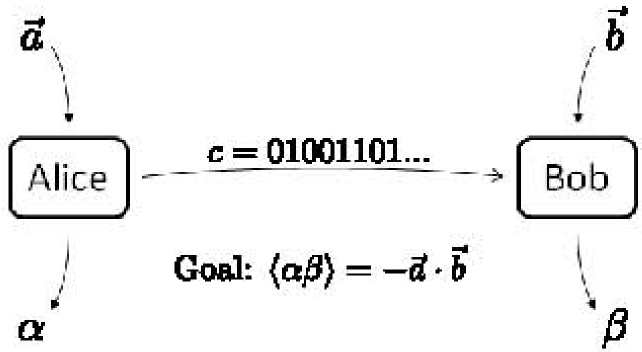

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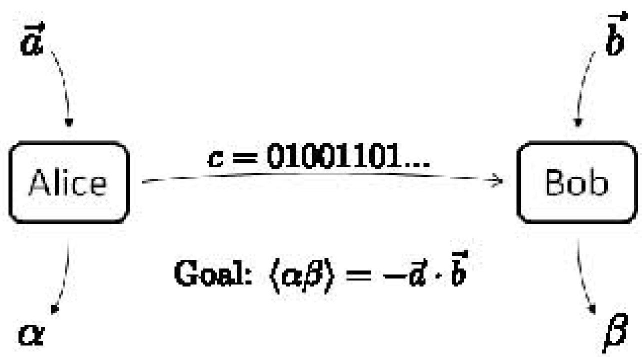

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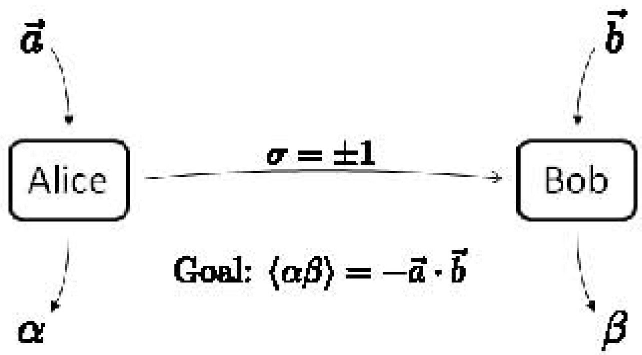

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- Bob outputs $\beta = \text{sign}[\vec{b} \cdot (\vec{\lambda}_1 + \sigma\vec{\lambda}_2)]$
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$$\langle \alpha\beta \rangle = -\vec{a} \cdot \vec{b}$$

B. Toner, D. Bacon, Phys. Rev. Lett. **91**, 187904 (2003)

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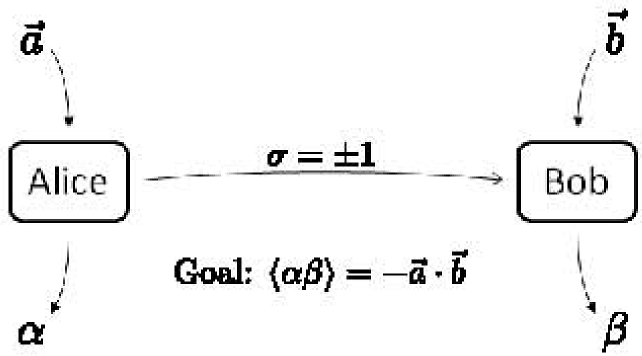

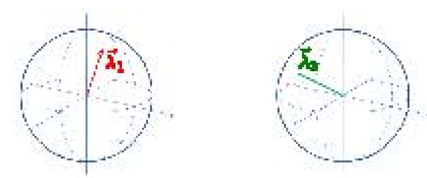

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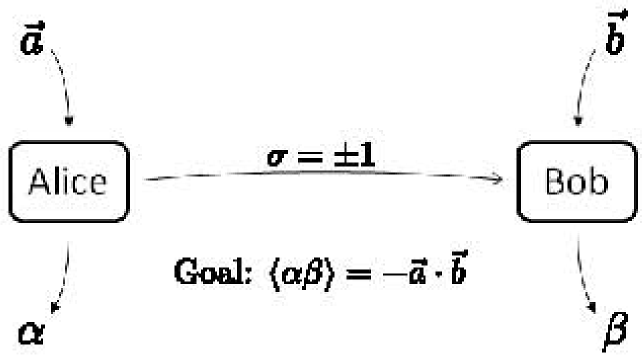

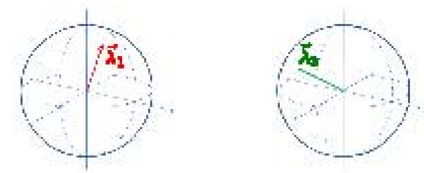

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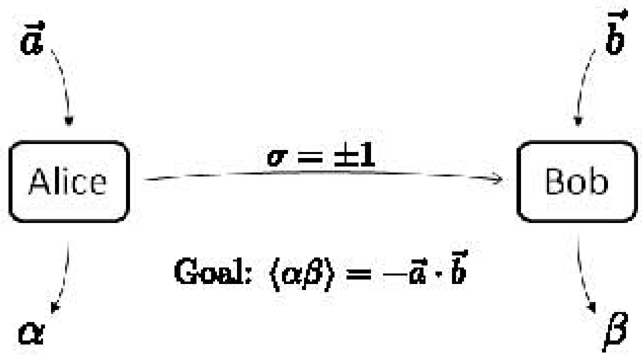

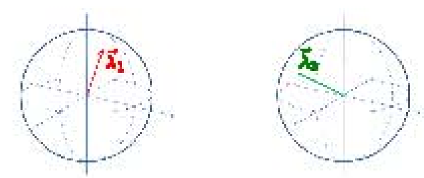

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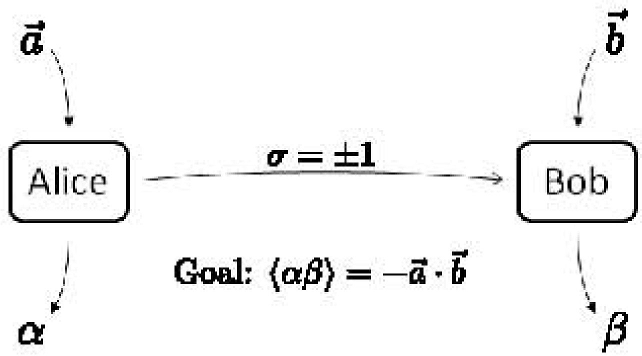

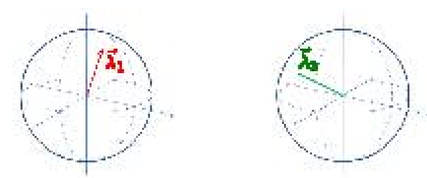

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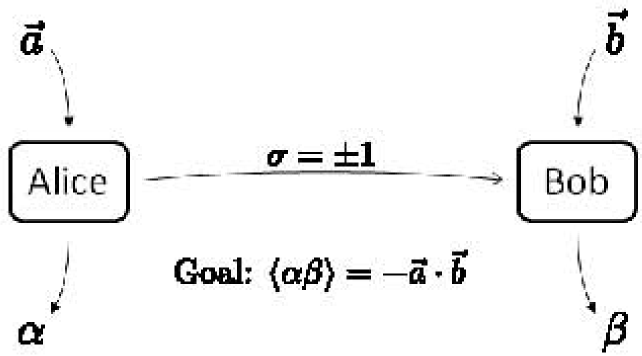

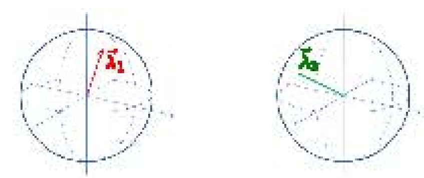

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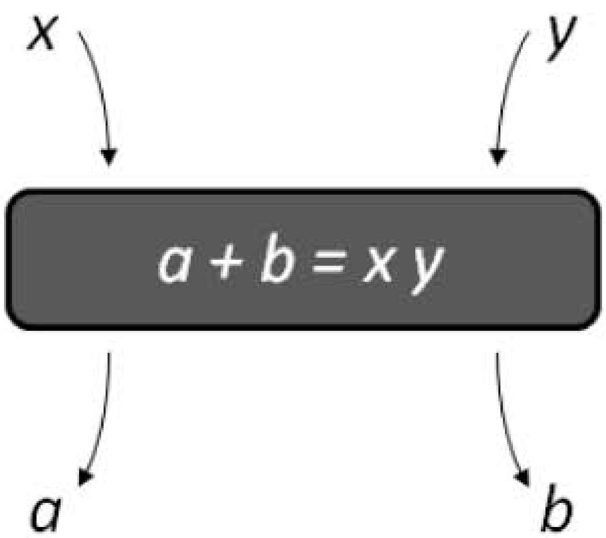
Simulation of singlet correlations with PR boxes



"PR box": S. Popescu, D. Rohrlich, Found. Phys. **24**, 379 (1994)

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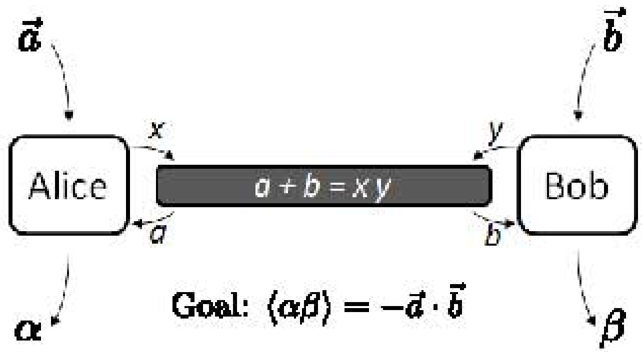

- Alice and Bob share 2 vectors $\vec{\lambda}_1, \vec{\lambda}_2$ on S^2
- Alice calculates $c = \text{sign}(\vec{a}\cdot\vec{\lambda}_1)\text{sign}(\vec{a}\cdot\vec{\lambda}_2)$;
  she inputs $x = \frac{1-c}{2}$ into the PR box, and outputs $\alpha = \text{sign}(\vec{a}\cdot\vec{\lambda}_1)(-1)^a$
- Bob calculates $d = \text{sign}(\vec{b}\cdot\vec{\lambda}_+)\text{sign}(\vec{b}\cdot\vec{\lambda}_-)$, where $\vec{\lambda}_\pm = \vec{\lambda}_1 \pm \vec{\lambda}_2$;
  he inputs $y = \frac{1-d}{2}$ into the PR box, and outputs $\beta = -\text{sign}(\vec{b}\cdot\vec{\lambda}_+)(-1)^b$
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N. Cerf, N. Gisin, S. Massar, S. Popescu, Phys. Rev. Lett. **94**, 220403 (2005)

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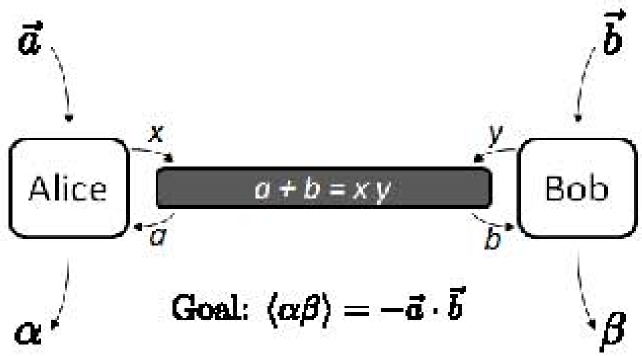

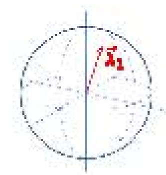

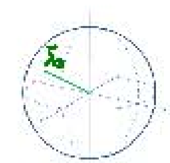

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N. Cerf, N. Gisin, S. Massar, S. Popescu, Phys. Rev. Lett. **94**, 220403 (2005)

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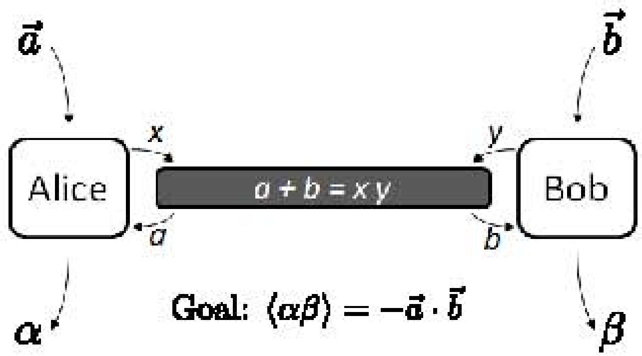

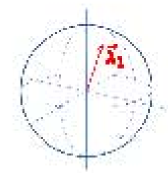

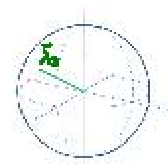

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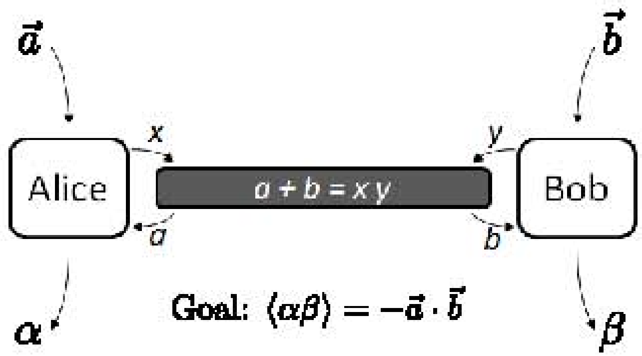

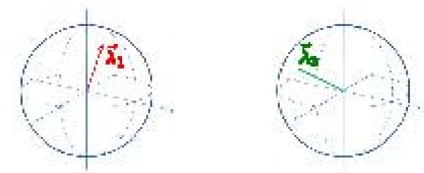

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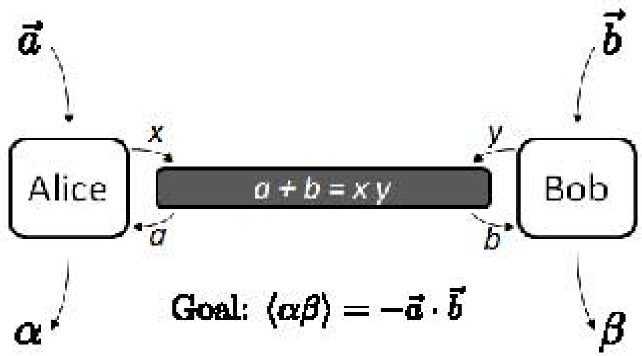

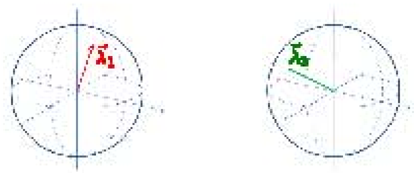

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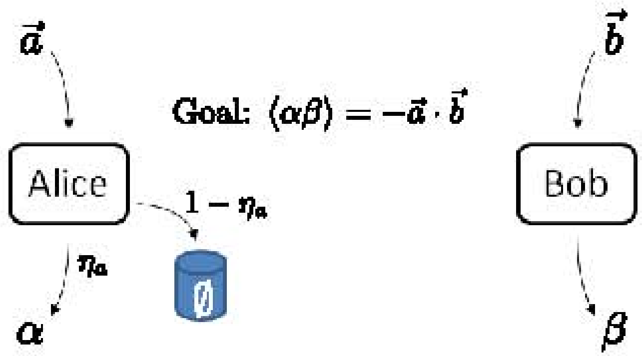

- Post-selection is a non-local resource:
  cf. detection loophole in Bell experiments
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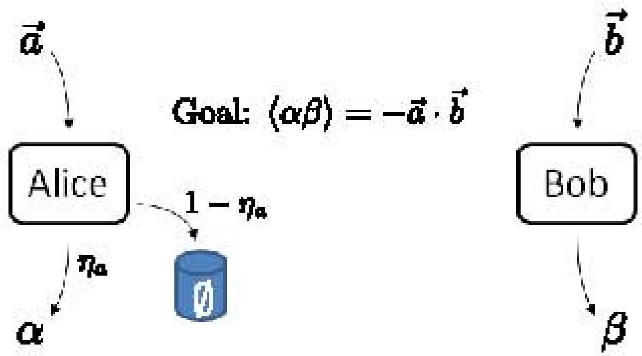

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Open questions in the bipartite case

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  - *The marginals are problematic!*
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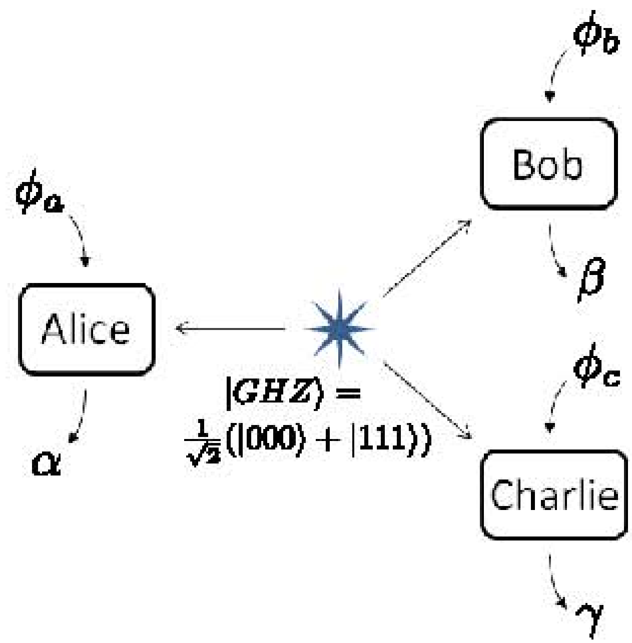
Outline

1. Quantum Non-locality: basics
2. Simulation of singlet correlations
3. Simulation of GHZ correlations

GHZ correlations



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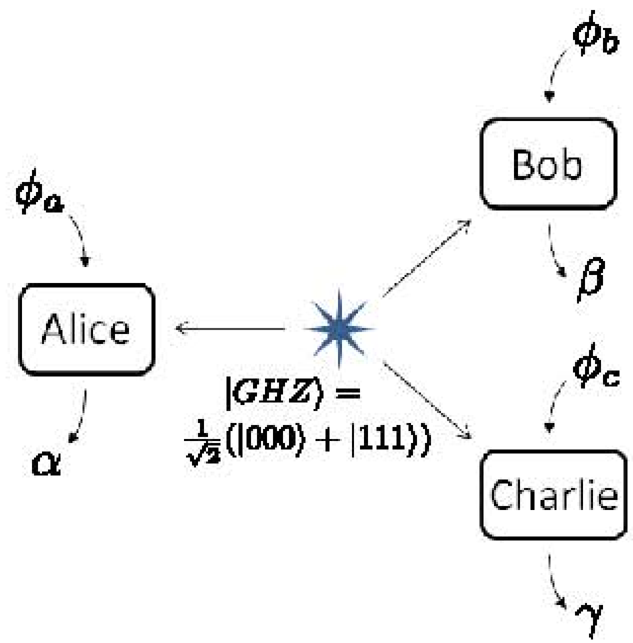

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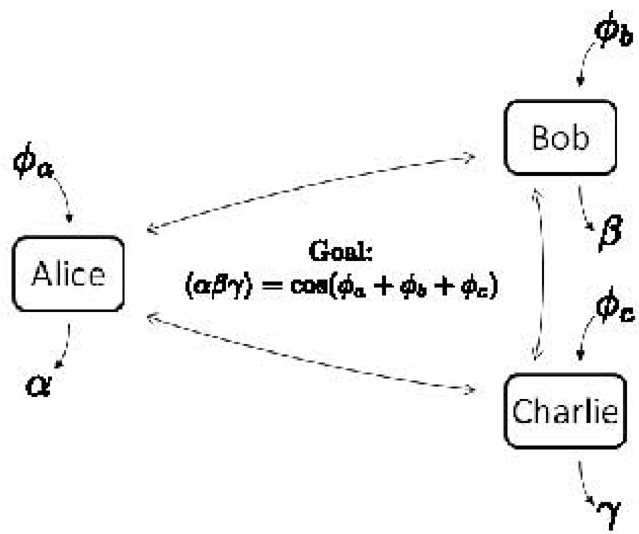

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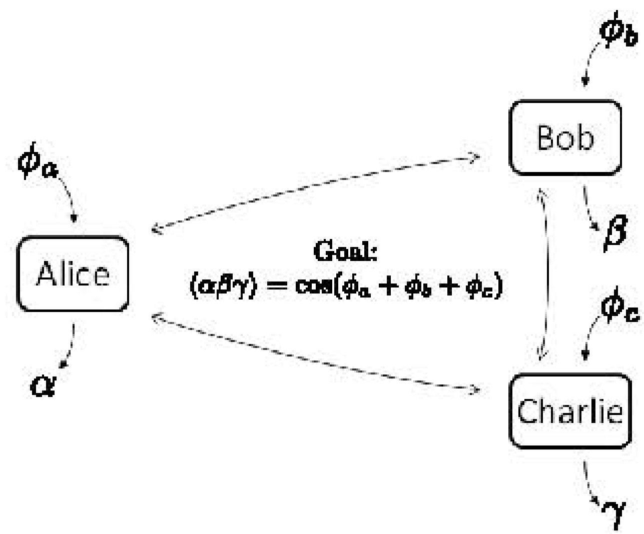

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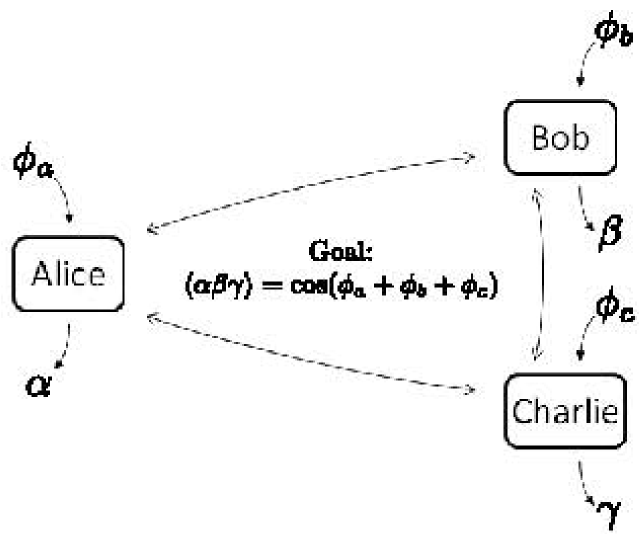

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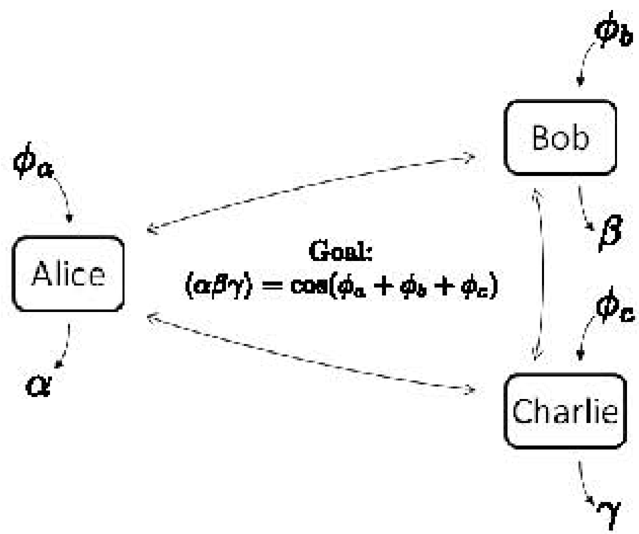

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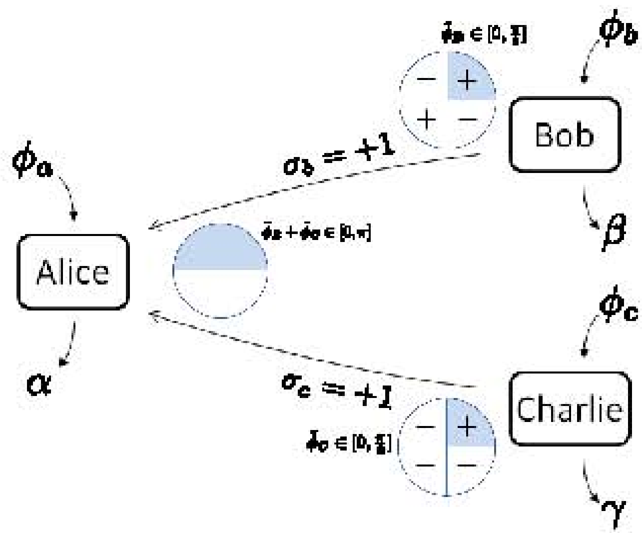

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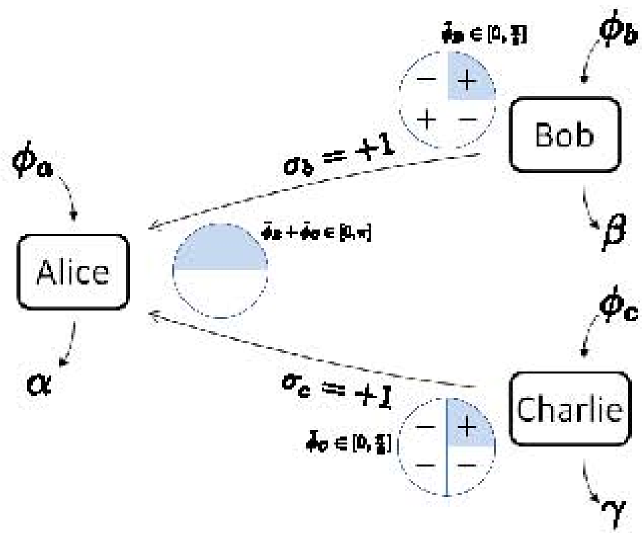

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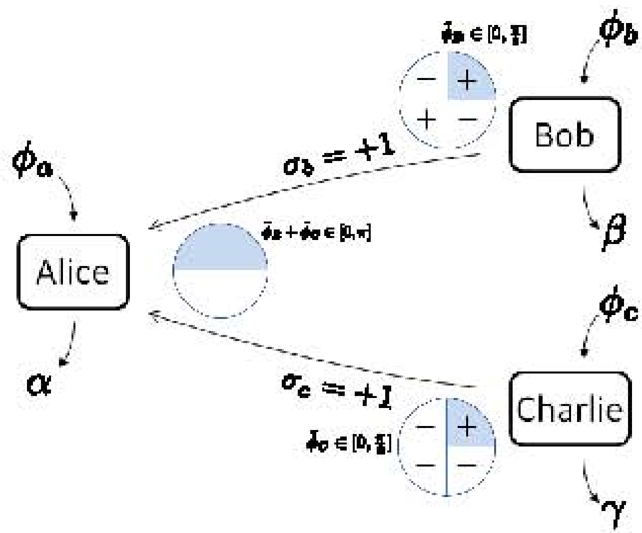

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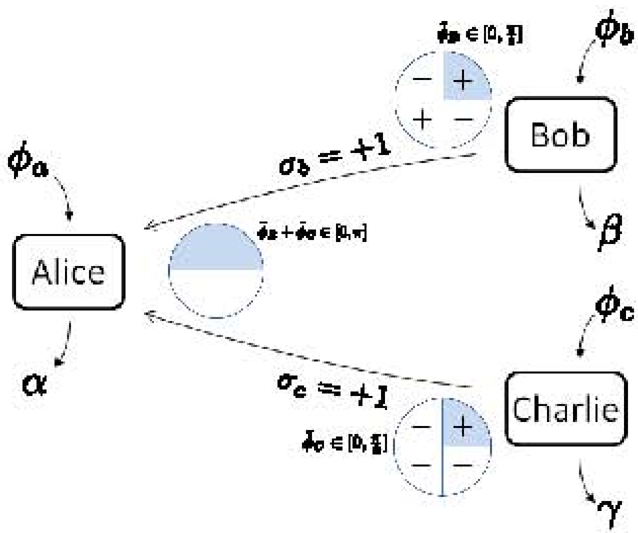

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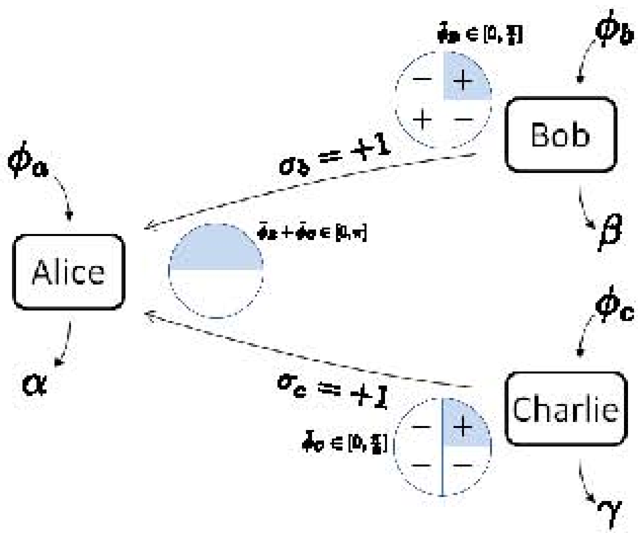

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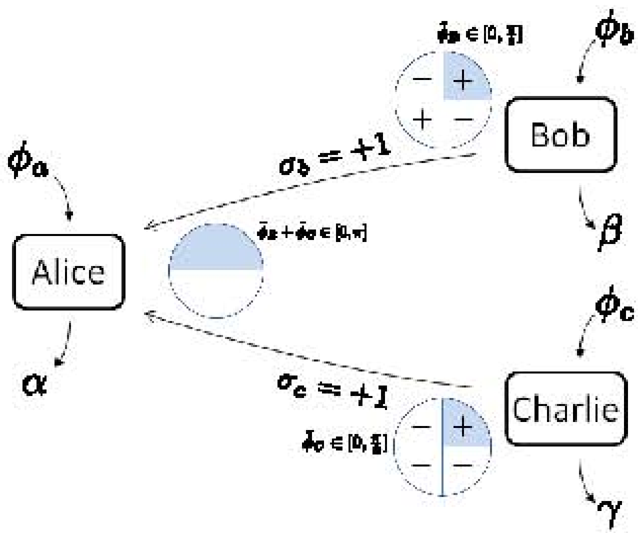

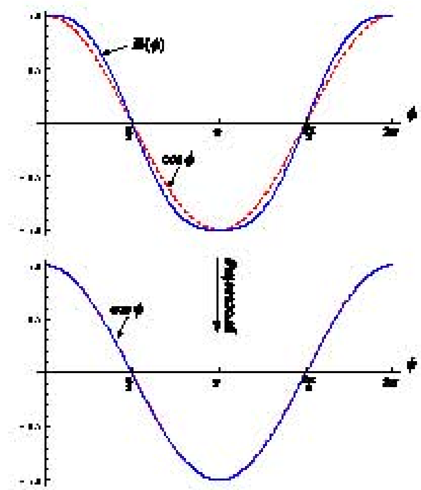

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Thanks for your attention