Title: The complementary contributions of free will, indeterminism and signalling to models of quantum correlations

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Abstract: To model statistical correlations that violate Bell inequalities (such as singlet state correlations), one must relax at least one of three physically plausible postulates: measurement independence (experimenters can freely choose measurement settings independently of any underlying variables describing the system); no-signalling (underlying marginal distributions for one observer cannot depend on the measurement setting of a distant observer), and determinism (all outcomes can be fully determined by the values of underlying variables).
It will be shown that, for any given model, one may quantify the degrees of measurement dependence, signalling and indeterminism, by three numbers M, S and I. It will further be shown how the Bell-CHSH inequality may be generalised to a "relaxed" Bell inequality, of the form <xy>+<xy'>+<x'y>-<x'y'><=B(I,S,M), where the upper bound is tight and ranges between 2 and 4. The usual Bell-CHSH inequality corresponds to I=S=M=0. More generally, the bound B(I,S,M) quantifies the necessary mutual tradeoff between I, S and M that is required to model a given violation of the Bell-CHSH inequality.
Some information-theoretic implications will be briefly described, as well as a no-signalling deterministic model of the singlet state that allows up to 86% experimental free will.</x'y'></x'y></xy'></xy>

Complementary contributions of free will, indeterminism & signalling to models of quantum correlations

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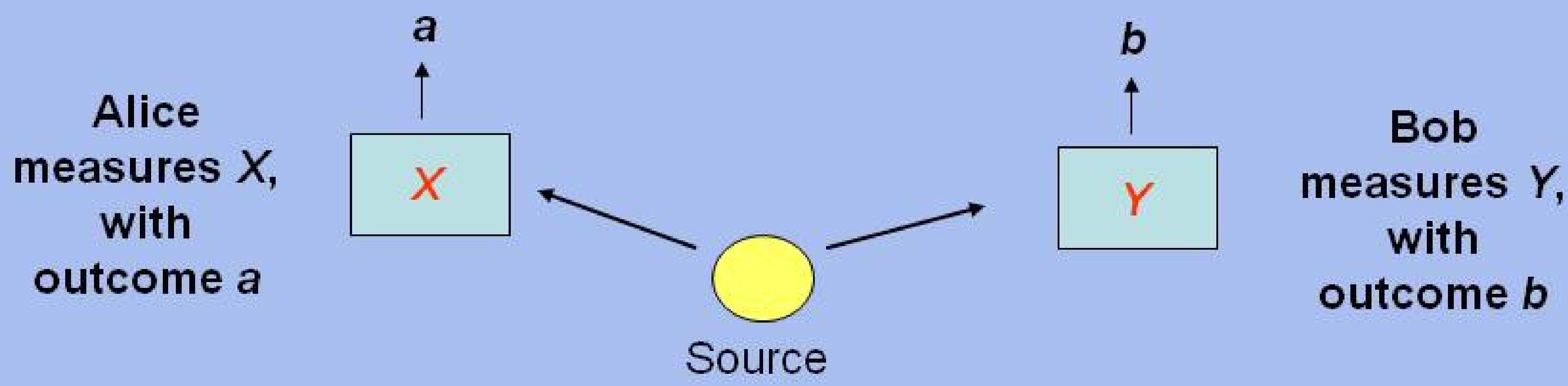
INTRODUCTION

- Correlation models
- Plausible assumptions lead to Bell inequalities
  - QM is not plausible !
- Relaxed assumptions lead to 'relaxed' Bell inequalities
  - how much relaxation to model QM correlations ??
- Singlet state models

Correlation models



- Collect joint measurement statistics for various measurement settings X and Y:

$$p(a, b \mid X, Y)$$

- Model the statistics by introducing underlying information (eg, about the source), in the form of a parameter λ:

$$p(a, b, \lambda \mid X, Y) = p(a, b \mid X, Y, \lambda)\ p(\lambda \mid X, Y) \qquad \text{(Bayes theorem)}$$

The measured statistics follow by summing over λ, i.e.,

$$p(a, b \mid X, Y) = \int d\lambda\ p(a, b \mid X, Y, \lambda)\ p(\lambda \mid X, Y)$$

Example: $\lambda \equiv \rho$ is a density operator, X and Y denote spin in the x and y directions,

$$p(\rho \mid X, Y) = \delta(\rho - \rho_S), \qquad p(a, b \mid X, Y, \rho) = \boldsymbol{tr}[\boldsymbol{\rho}\ \boldsymbol{E}_a^{(x)} \otimes \boldsymbol{E}_b^{(y)}]$$

Plausible assumptions for models

$$p(a, b \mid X, Y) = \int d\lambda \; p(a, b \mid X, Y, \lambda) \; p(\lambda \mid X, Y)$$

Determinism: The underlying variable determines all measurement outcomes

$$p(a, b \mid X, Y, \lambda) = 0 \text{ or } 1$$

No-signalling: The underlying marginal distributions are independent of distant measurement settings

$$p(a \mid X, Y, \lambda) = p(a \mid X, Y', \lambda) , \qquad p(b \mid X, Y, \lambda) = p(b \mid X', Y, \lambda)$$

Measurement independence: The measurement settings can be chosen independently of the underlying variable

$$p(\lambda \mid X, Y) = p(\lambda \mid X', Y') = p(\lambda) \qquad [\Leftrightarrow \; p(X,Y,\lambda) = p(X,Y)\, p(\lambda)]$$

A Bell inequality

- If outcomes *a, a', b, b'* of *X, X', Y, Y'* are labelled by ±1, and there is an underlying model satisfying

  (a) determinism, (b) no-signalling, and (c) measurement independence,

  then the measured correlations must satisfy

  $$\langle ab\rangle + \langle ab'\rangle + \langle a'b\rangle - \langle a'b'\rangle \leq 2 .$$

  - but quantum correlations allow a LHS as much as $2\sqrt{2}$!!

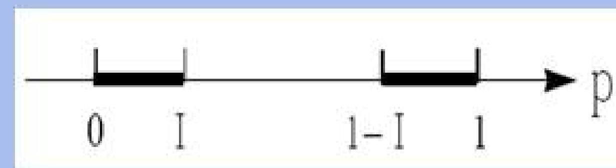
- Hence, to model Bell inequality violation by quantum systems, one ***has*** to give up at least one assumption.

  1. Which assumption should we give up ?
     - philosophy ....
  2. By how much ?
     - physics !

Quantifying indeterminism, signalling and measurement independence

- Indeterminism: I is the smallest number such that

$$p(a \mid X, Y, \lambda) \in [0, I] \text{ or } [1-I, 1]$$



$I=0$ if and only if determinism holds.

- Signalling: S is the smallest number such that

$$p(a \mid X, Y, \lambda) - p(a \mid X, Y', \lambda) \leq S$$

$S=0$ if and only if no-signalling holds

- Measurement dependence: M is the smallest number such that

$$\int d\lambda \; |p(\lambda \mid X, Y) - p(\lambda \mid X', Y')| \leq M$$

$M=0$ if and only if measurement independence holds

Note: if $M=2$, then there is no overlap:
at most ***one*** of the two joint settings
(X,Y), (X',Y'), can be chosen, for any λ.

A relaxed Bell inequality

- If an underlying model has indeterminism I, signalling S, and measurement dependence M, then

$$\langle ab\rangle + \langle ab'\rangle + \langle a'b\rangle - \langle a'b'\rangle \le B(I,S,M)$$

- This reduces to the usual Bell-CHSH inequality for $I=S=M=0$:

$$B(0,0,0) = 2.$$

- More generally, a violation of the Bell-CHSH inequality places constraints on the degrees of indeterminism, signalling and measurement dependence that must be present. These are determined by the function $B(I,S,M)$.

- For example, the maximum quantum violation, with LHS = $2\sqrt{2}$, implies the constraints

$$I \ge 20\%, \quad \text{and/or} \quad S \ge 60\%, \quad \text{and/or} \quad M \ge 0.28.$$

for any underlying model.

Local and deterministic model of an entangled state

- To model ***all*** spin correlations of a singlet state, it is known that:

  $S=M=0 \Rightarrow I = \frac{1}{2}$ (100% indeterminism required)

  $I=M=0 \Rightarrow S = 1$ (100% signalling required)

  (more generally, for $M=0$, conjecture need $S+2I \geq 1$)

- However, in contrast, one does ***not*** require maximum measurement dependence:

  $I=S=0 \Rightarrow M = 0.28$ (only 14% of the maximum value)

- Thus, one only has to give up 14% ‘experimental free will’ – measurement dependence is a relatively strong resource for modelling entanglement.

What the model looks like

- The underlying variable λ is a unit 3-vector

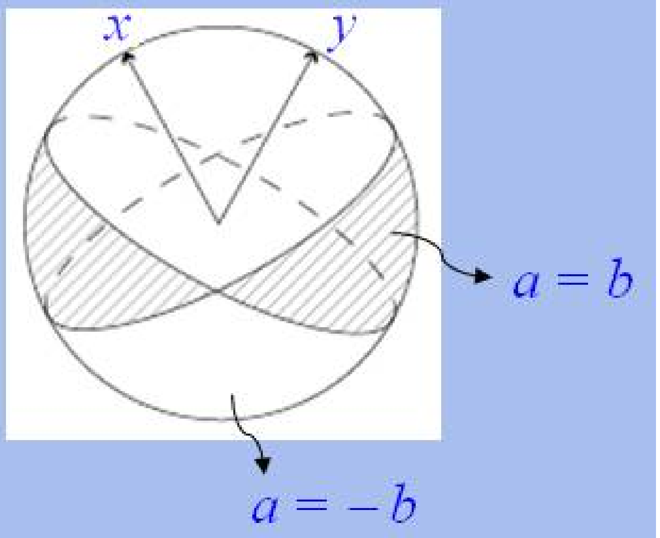
- The outcomes corresponding to measuring spin in the x and y directions are given by

$$a = \text{sign}\ \lambda . x\ , \quad b = -\text{sign}\ \lambda . y$$

- The correlation between the underlying variable and the measurement directions is given by

$$p(\lambda\,|x, y) = \begin{cases} \dfrac{1+\mathbf{x}\cdot\mathbf{y}}{8(\pi-\theta_{xy})} & \textit{for sign } \boldsymbol{\lambda}\cdot\mathbf{x} = \textit{sign } \boldsymbol{\lambda}\cdot\mathbf{y} \\ \dfrac{1-\mathbf{x}\cdot\mathbf{y}}{8\ \theta_{xy}} & \textit{for sign } \boldsymbol{\lambda}\cdot\mathbf{x} = -\textit{sign } \boldsymbol{\lambda}\cdot\mathbf{y} \end{cases}$$

$\approx\ 1/(4\pi)$ for information purposes



Information properties

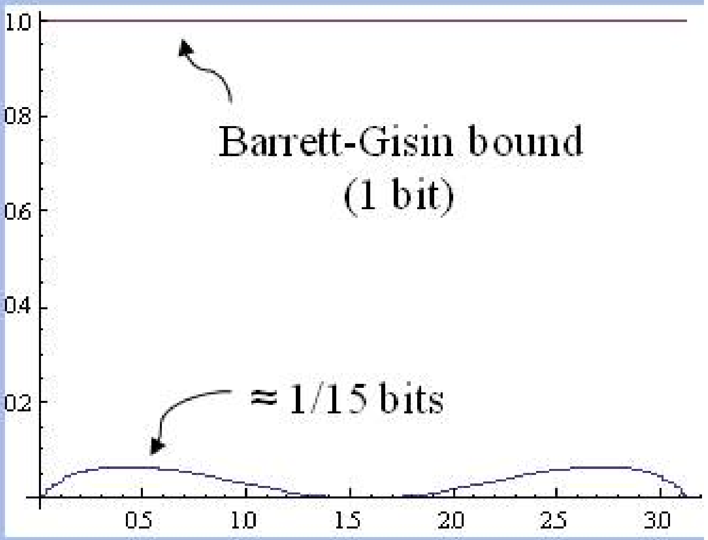
- The correlation between the underlying variable λ and the joint measurement directions (x,y) can be quantified by the Shannon mutual information, $I(\Lambda{:}X,Y)$.

- Barrett and Gisin have very recently shown, via mapping the Toner-Bacon communication model of the singlet state to a local deterministic model, that there are models with

$$I(\Lambda{:}X,Y) \leq 1\ bit$$

- The model from the previous slide improves this to

$$I(\Lambda{:}X,Y) \leq \log 4\pi - H_{min}[\, p(\lambda\,|x,y)\,] \approx 1/15\ bits.$$



Conclusions

- Relaxed Bell inequalities quantify the degrees of indeterminism, no-signalling and measurement dependence required for modelling violations of standard Bell inequalities

- There is a local deterministic model of the singlet state, via relaxing measurement independence by just 14% - corresponding to a correlation between the underlying variable and the joint measurement settings of no more than $\approx$ *1/15 bits*

- Further work includes: relaxing other Bell inequalities; relaxing EPR-Kochen-Specker theorems; exploring the singlet state model; relaxing 'outcome independence'; examining conjectures such as whether any quantum state has a local deterministic model via giving up no more than 33% measurement independence.