Title: Local scale invariance as an alternative to Lorentz invariance

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Abstract: I will present a recent result showing that general relativity admits a dual description in terms of a 3D scale invariant theory. The dual theory was discovered by starting with the basic observation that, fundamentally, all observations can be broken down into local comparisons of spatial configurations. Thus, absolute local spatial size is unobservable. Inspired by this principle of "relativity of size", I will motivate a procedure that allows the refoliation invariance of general relativity to be traded for 3D local scale invariance. This trade does away with "many fingered time" and offers a new possibility for dealing with the many technical and conceptual difficulties associated with the Wheeler-DeWitt equation.

The Dynamics of Shape

A Dual Description of Gravity

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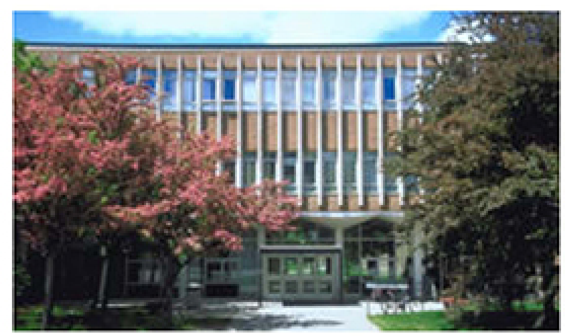


December 1, 2010

Introduction: Local 3D conformal symmetry

Observation

All measurments are *local* comparisons of configurations.

$\longrightarrow$ Eg, Length measurements, pointers, eye, clock, etc...

"...in physics the only observations we must consider are position observations, if only the positions of instrument pointers. (...) If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency."

– John Bell, 1982

$\therefore$ local scale is *un*observable.

$\Rightarrow$ the symmetry $g^{ab}(x) \longrightarrow e^{4\phi(x)} g^{ab}(x)$.

(3 + 1) general relativity (ADM)

Split the 4–metric

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^a N_a & N_a \\ N_a & g_{ab} \end{pmatrix}$$

$N(x) \rightarrow$ *lapse*, $N_a(x) \rightarrow$ *shift*, and $g_{ab} \rightarrow$ 3–metric.

Legendre Transform: $(g_{ab}, \dot{g}_{ab}) \longrightarrow (g_{ab}, \pi^{ab})$

$$S_{\text{EH}} \longrightarrow H_{\text{ADM}} = \int d^3x \, [N(x,t)S(x,t) + N_a(x,t)H^a(x,t)]$$

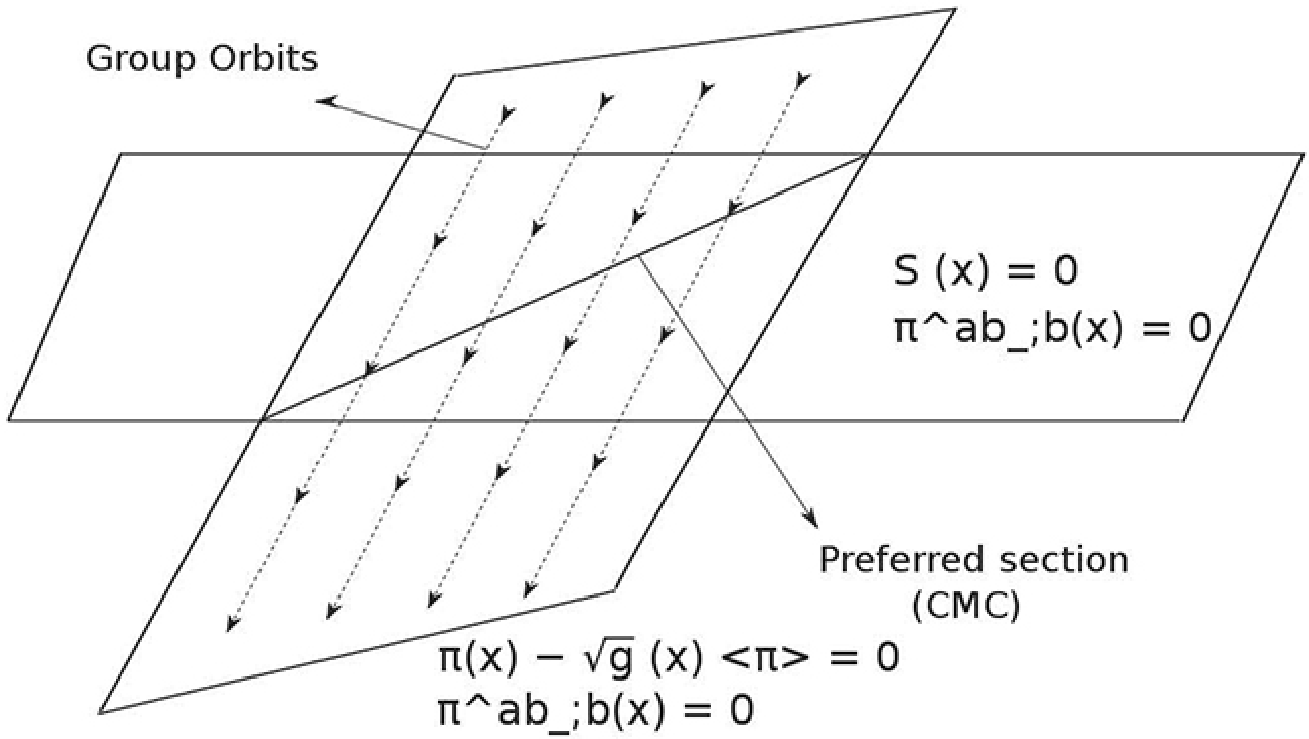
- Diff constraint: $H^a = \nabla_b \pi^{ab} \approx 0 \Rightarrow$ Generates 3–diffeos
- Hamiltonian/Scalar constraint: $S = \frac{G_{abcd}\pi^{ab}\pi^{cd}}{\sqrt{g}} + \sqrt{g}R \approx 0 \Rightarrow$ Generates refoliations and reparametrizations. (Hard!)

Idea: Split $S \rightarrow S'$ (refoliations) $+ S_{\text{rep}}$ (reparametrizations)

$\Rightarrow$ Trade S' for *volume preserving* conformal constraints.

Basic geometric picture

Gauge fix scalar constraint, S', using the *integrable* surface $\pi(x) - \sqrt{g}(x)\langle\pi\rangle = 0$. ($\pi \equiv \pi^{ab} g_{ab}$ and $\langle\pi\rangle \equiv \frac{1}{V}\int dx \sqrt{g}\pi$)



- $\pi(x) - \sqrt{g}(x)\langle\pi\rangle = 0$ must be an *integrable* gauge fixing.
- Then, symmetries can be traded.
- Dual theory → treat $S'(x)$ as a gauge fixing of $\pi(x) - \sqrt{g}(x)\langle\pi\rangle$!

Dictionary → solution of GR map to solutions of Shape Dynamics.

General algorithm

Start with first class constraints, ie, $S'(x) \approx 0$.

1. Perform canonical transformation T_ϕ generated by

$$F[\phi] = \int d^3x\, g_{ab}(x) \exp\{\hat{\phi}(x)\} \Pi^{ab}(x)$$

   Then, $(\phi, \pi_\phi) \Rightarrow$ constraints $C = D - \pi_\phi$ (where, $D = \pi - \sqrt{g}\,\langle\pi\rangle$).

2. Impose $\pi_\phi \approx 0$. Check that N_0 exists such that

$$\{T_\phi S(N_0), \pi_\phi\} = 0$$

   If so, π_ϕ is a proper gauge fixing of $T_\phi S'$. ($N_0 = N_{\text{CMC}}$)

3. Define Dirac bracket. Then,

$$T_\phi S' \approx 0, \pi_\phi \approx 0 \quad \rightarrow \quad T_\phi S' = 0, \pi_\phi = 0.$$

   $\therefore C \rightarrow D$.

   Impose $T_\phi S' = 0$ and $\pi_\phi = 0$ by inserting appropriate ϕ and π_ϕ into Hamiltonian. (Dirac's trick)

4. Read off dictionary from dual Hamiltonian.

Construct dictionary

Compare ADM and dual Hamiltonians:

$$H_{\text{ADM}} = \int_\Sigma d^3x \,(N(x)S(x) + N^a(x)H_a(x))$$
$$H_{\text{dual}} = \int_\Sigma d^3x \left(\mathcal{N} N_{\text{CMC}} T_{\phi_0} S + \lambda\,(\pi - \langle\pi\rangle \sqrt{g}) + N^a(x) T_{\phi_0} \nabla_b \pi^{ab}\right). \quad (1)$$

(Note: $S_{\text{rep}} = \int d^3x\, N_{\text{CMC}} T_{\phi_0} S$)

Dictionary

In classical theory, must solve initial value constraints. Then $\phi_0 = 0$. Thus, eoms are identical in the gauges

$$N(x) = \mathcal{N} N_{\text{CMC}}(x)$$
$$\lambda(x) = 0. \quad (2)$$

Summary and Outlook

What we have done:

- Shown that GR is dual to a scale invariant theory.
- Local scale invariance is an alternative to Lorentz invariance.
- Found a *reduced phase space* for gravity (modulo reparametrizations).

To Do:

- Understand $S_{\text{rep}} \rightarrow$ can it be computed?! When?!
- Simple cases: $(2+1)$, symmetry reduced, cosmology...
- Special asymptotics: (A)dS. H_{dual} holographic RG flow eq'n??
- Renormalization: theory space and Hořava.

Conceptual issues:

- Boosts in dual theory: the meaning of scale invariance.
- Dual of causal structure.
- Gravity without 4–metric.