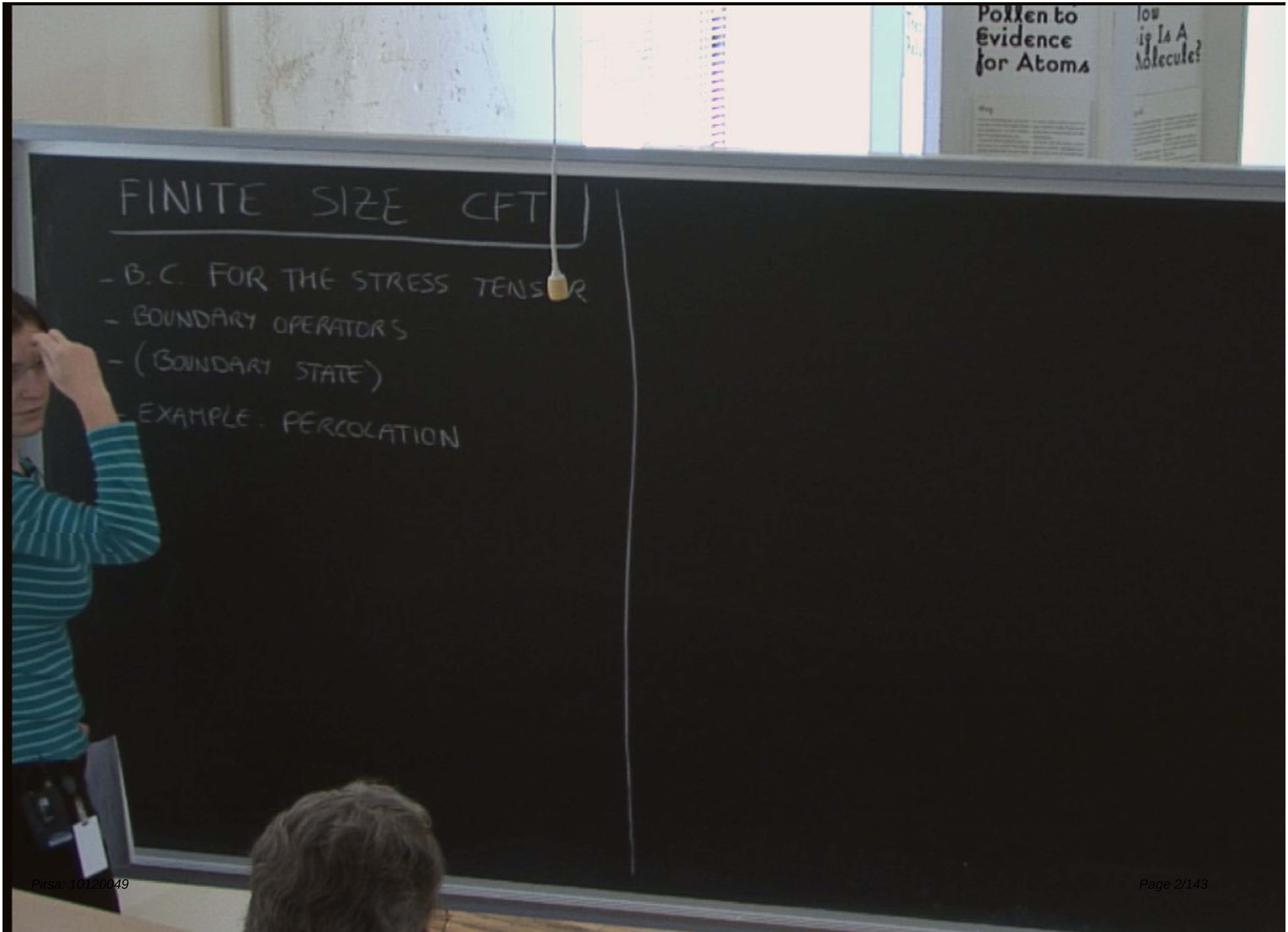
Title: Conformal Field Theory - Additional Lecture 2

Date: Dec 14, 2010 10:00 AM

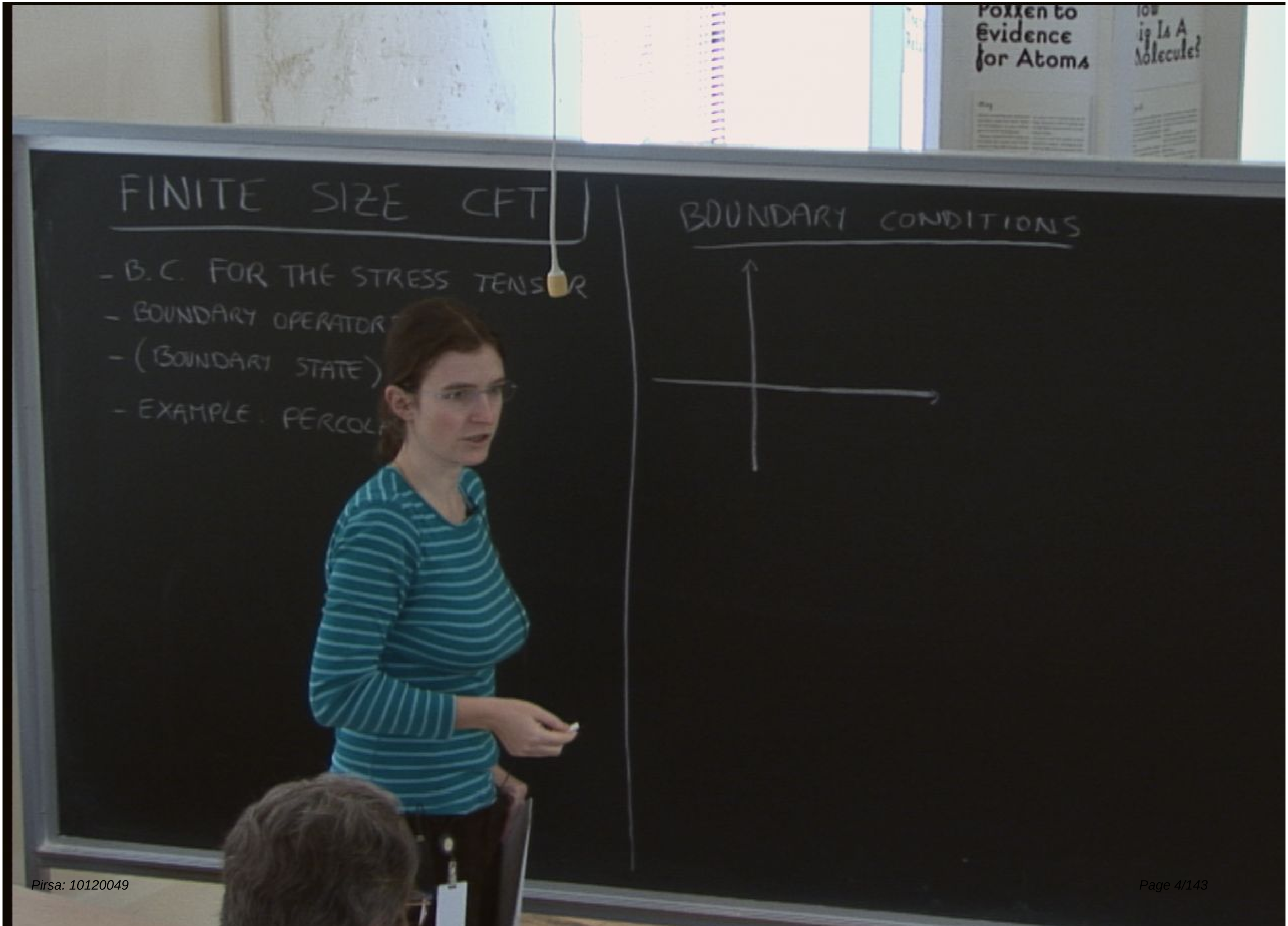
URL: http://pirsa.org/10120049

Abstract:

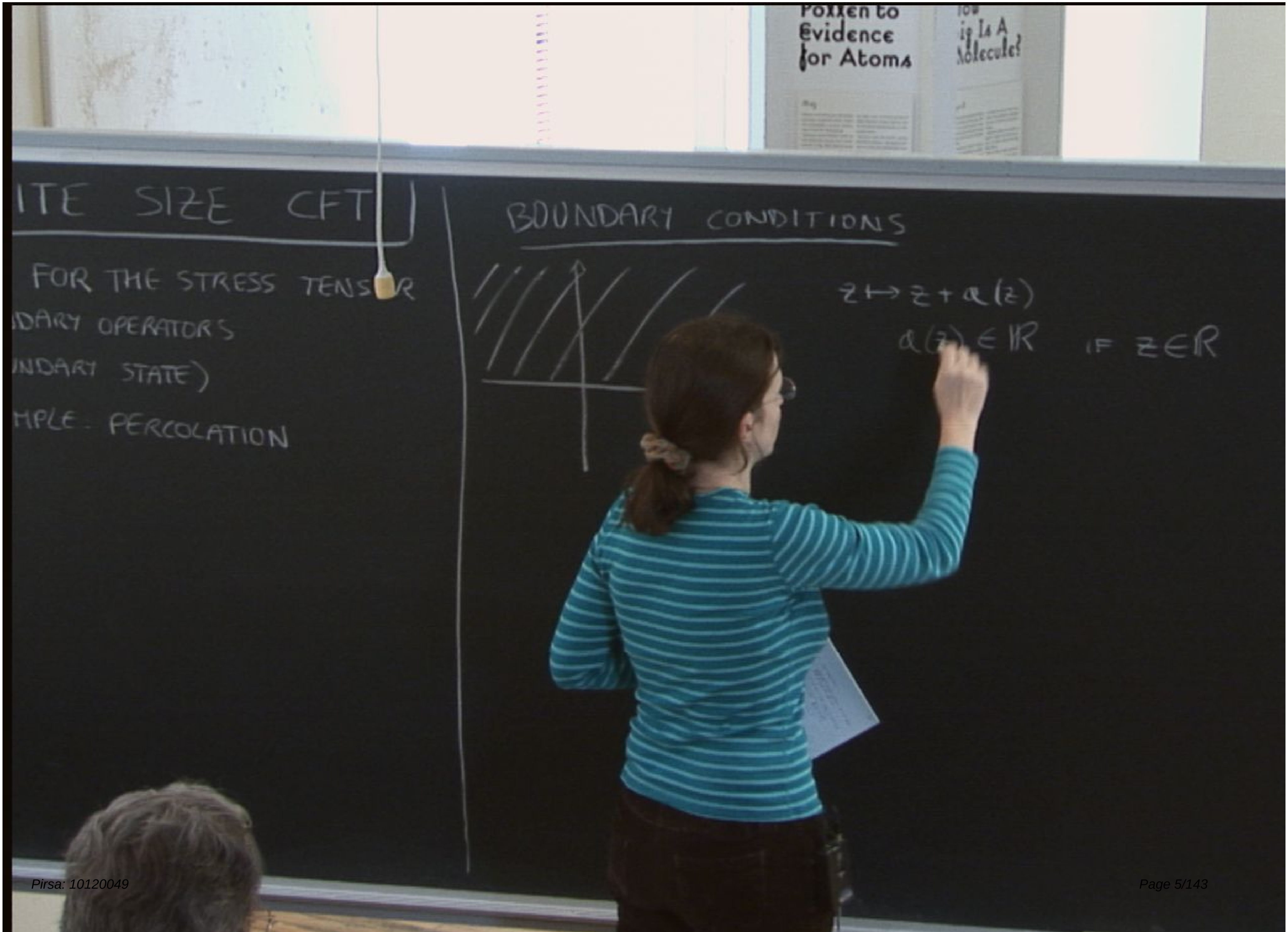

FINITE SIZE CFT
- B.C. FOR THE STRESS TENSOR
- BOUNDARY OPERATORS
- (BOUNDARY STATE)
- EXAMPLE. PERCOLATION
Pollen to
Evidence
for Atoms

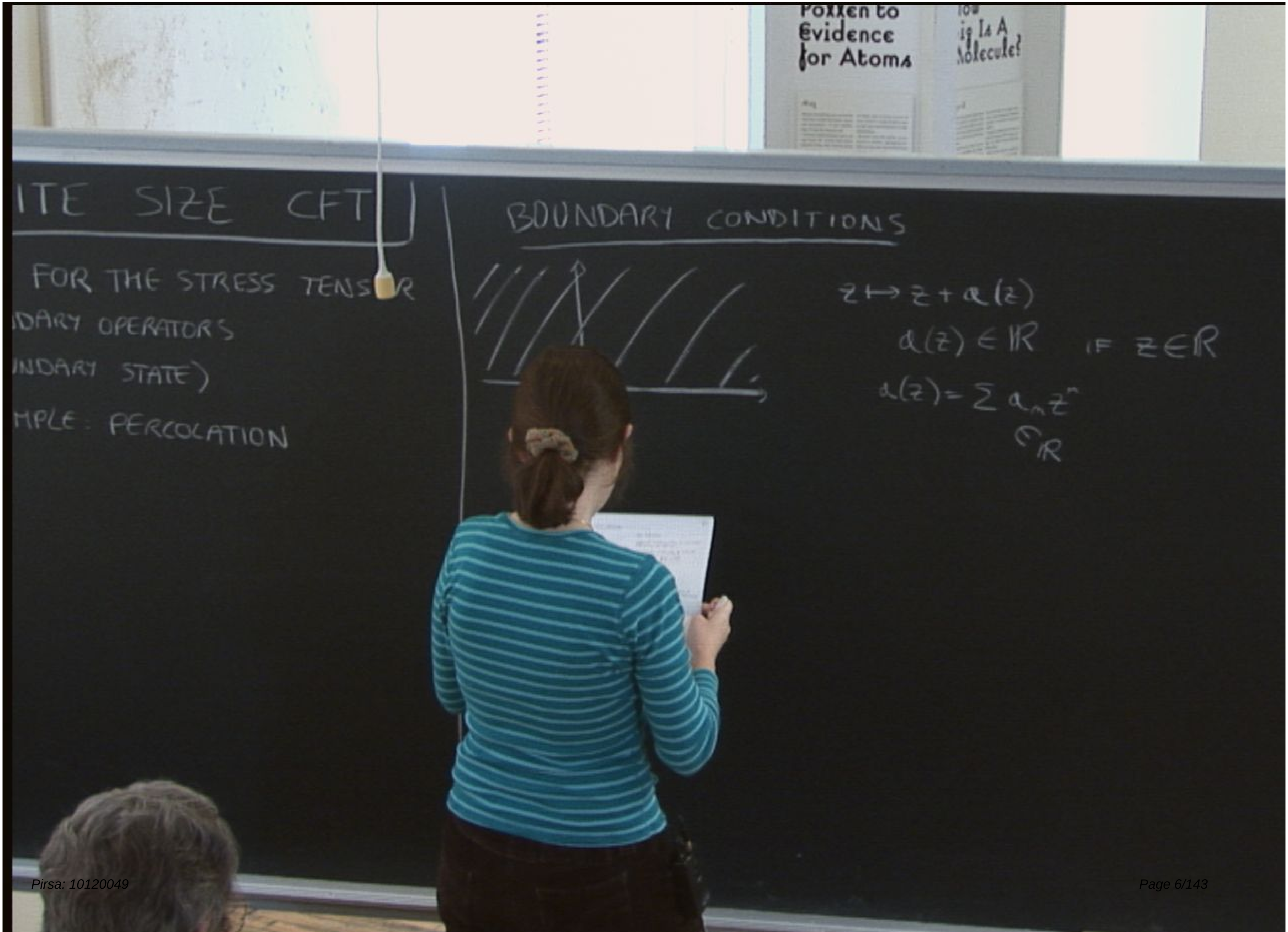
FINITE SIZE CFT

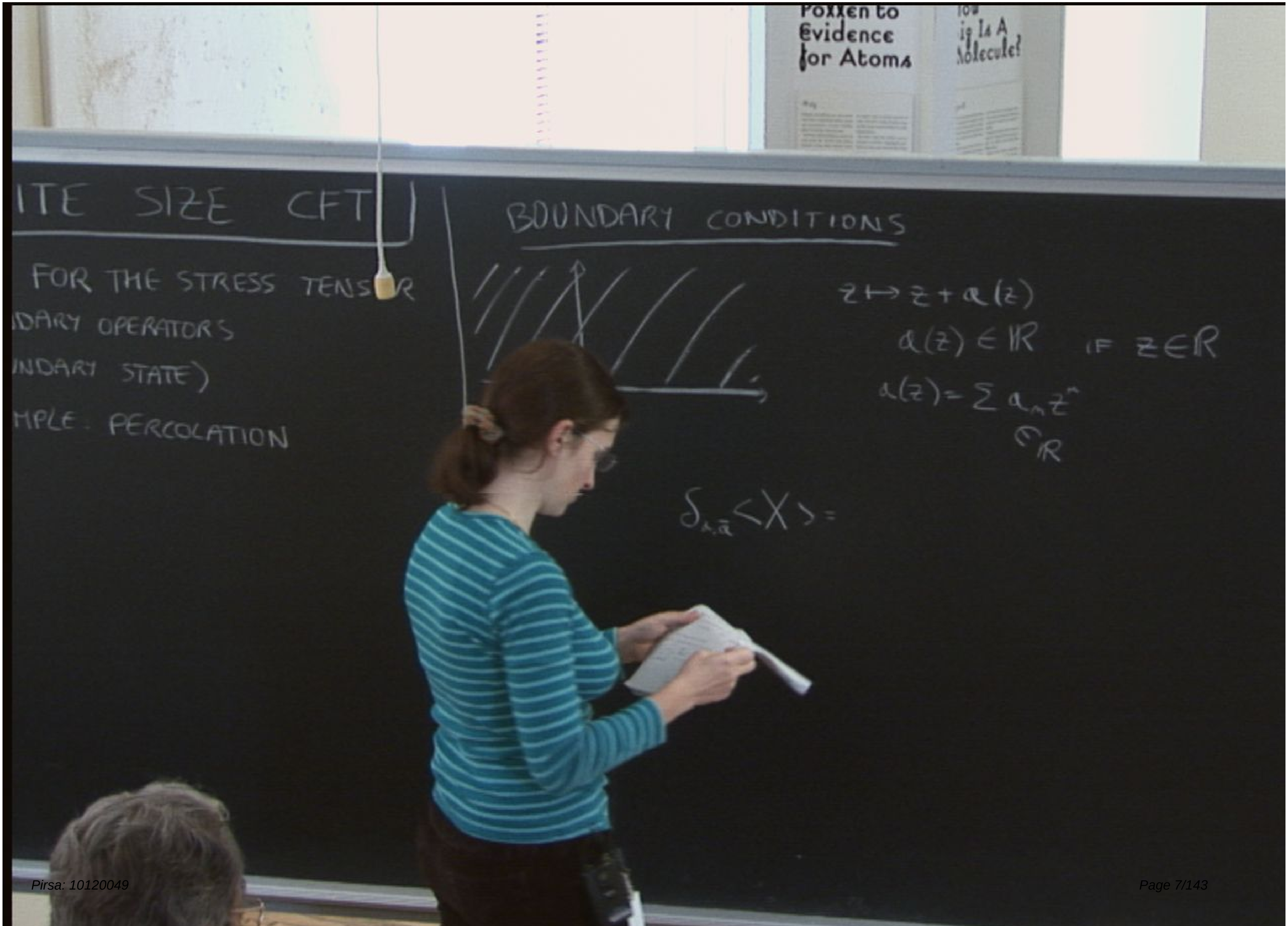
- B.C. FOR THE STRESS TENSOR
- BOUNDARY OPERATORS
- (BOUNDARY STATE)
- EXAMPLE: PERCOLATION

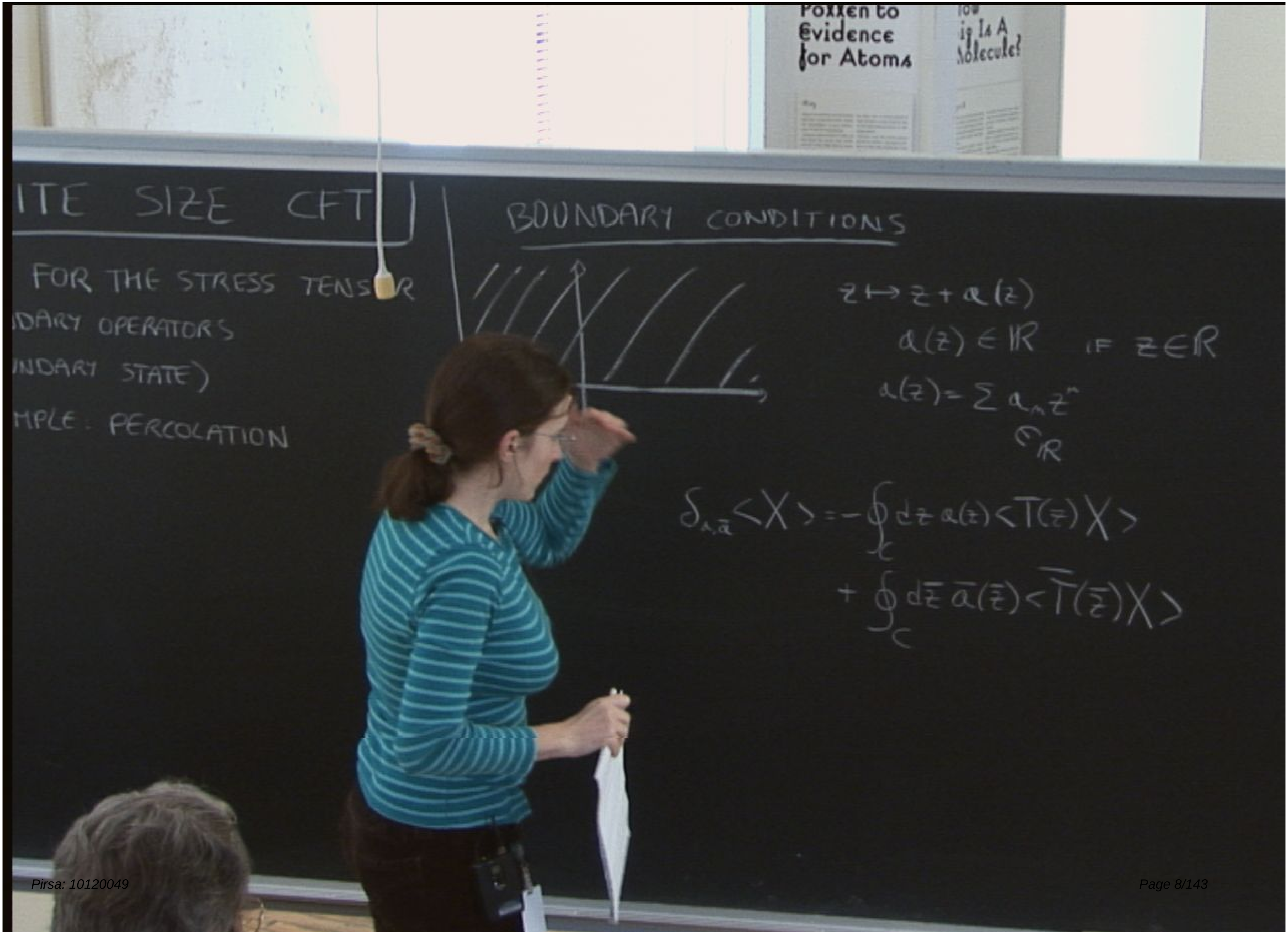


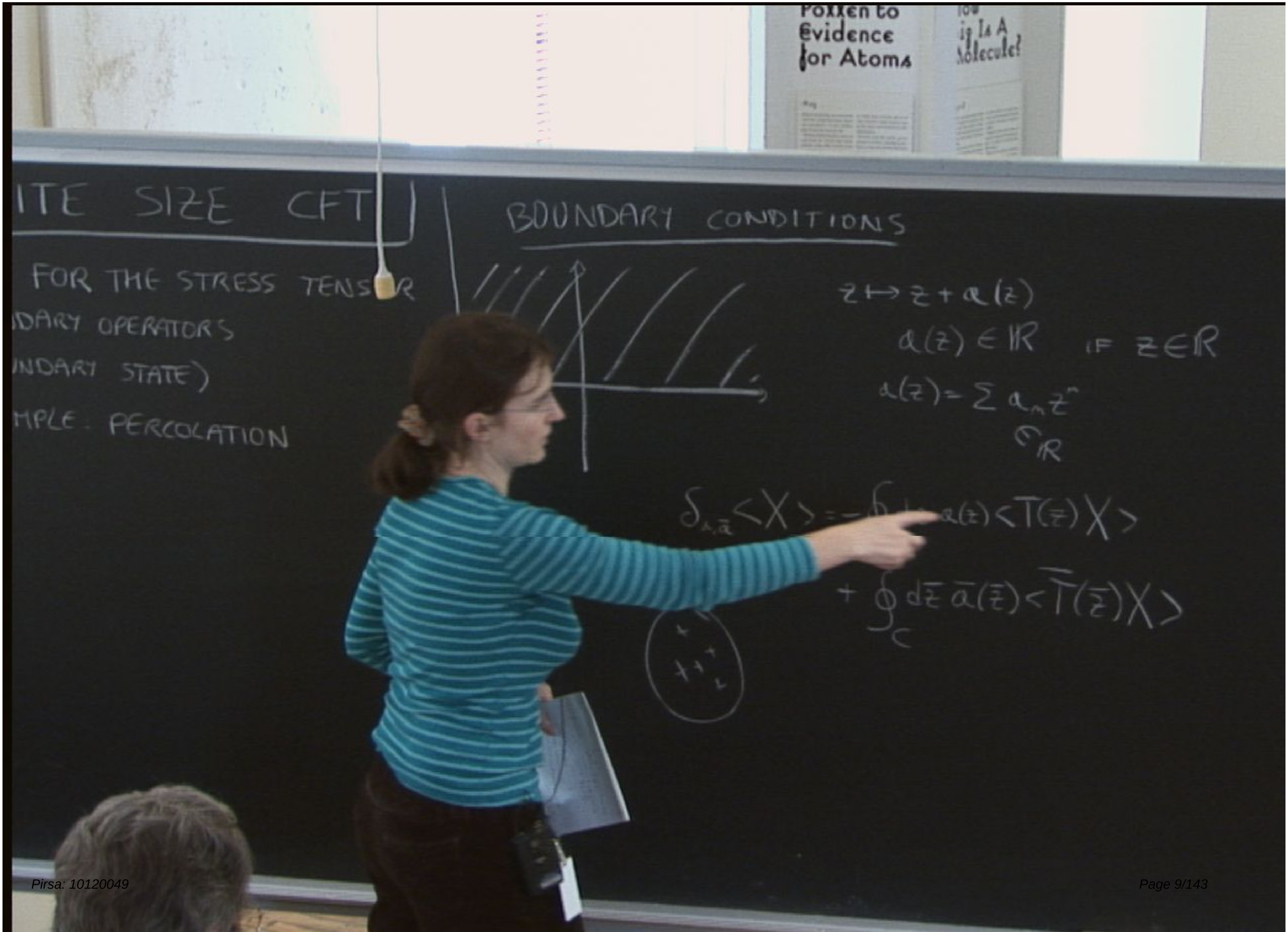
Evidence
for Atoms
FINITE SIZE CFT
- B.C. FOR THE STRESS TENS R
- BOUNDARY OPERATOR
- (BOUNDARY STATE)
- EXAMPLE: PERCOL
BOUNDARY CONDITIONS

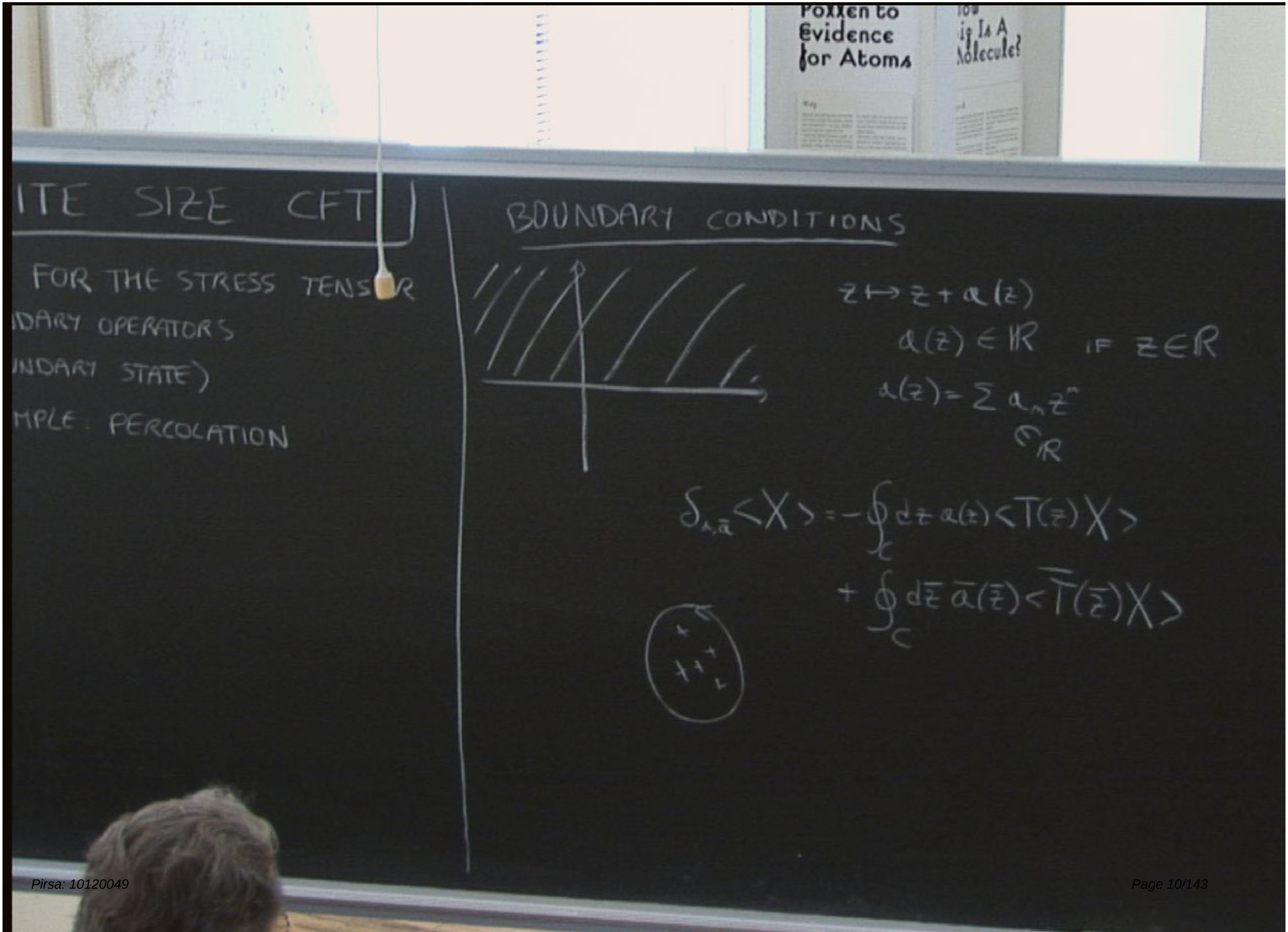

Pollen to Evidence for Atoms
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
UNDARY STATE)
MPLE. PERCOLATION
BOUNDARY CONDITIONS
$z \mapsto z + \alpha(z)$
$\alpha(z) \in \mathbb{R}$ IF $z \in \mathbb{R}$

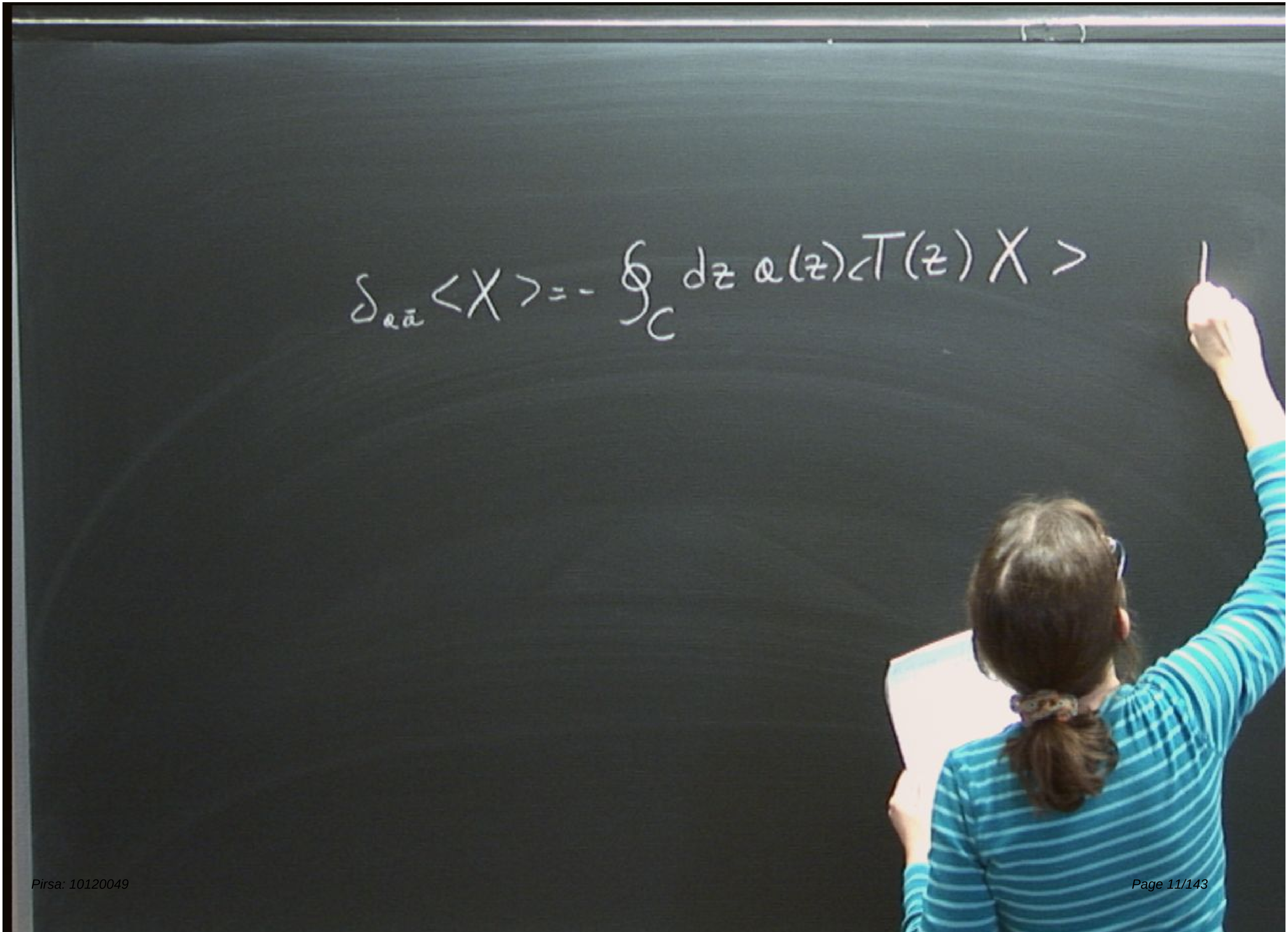

Evidence
for Atoms
Molecule?
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
NDARY STATE)
MPLE: PERCOLATION
BOUNDARY CONDITIONS
$z \mapsto z + \alpha(z)$
$\alpha(z) \in \mathbb{R}$ IF $z \in \mathbb{R}$
$\alpha(z) = \sum a_n z^n$
$\in \mathbb{R}$

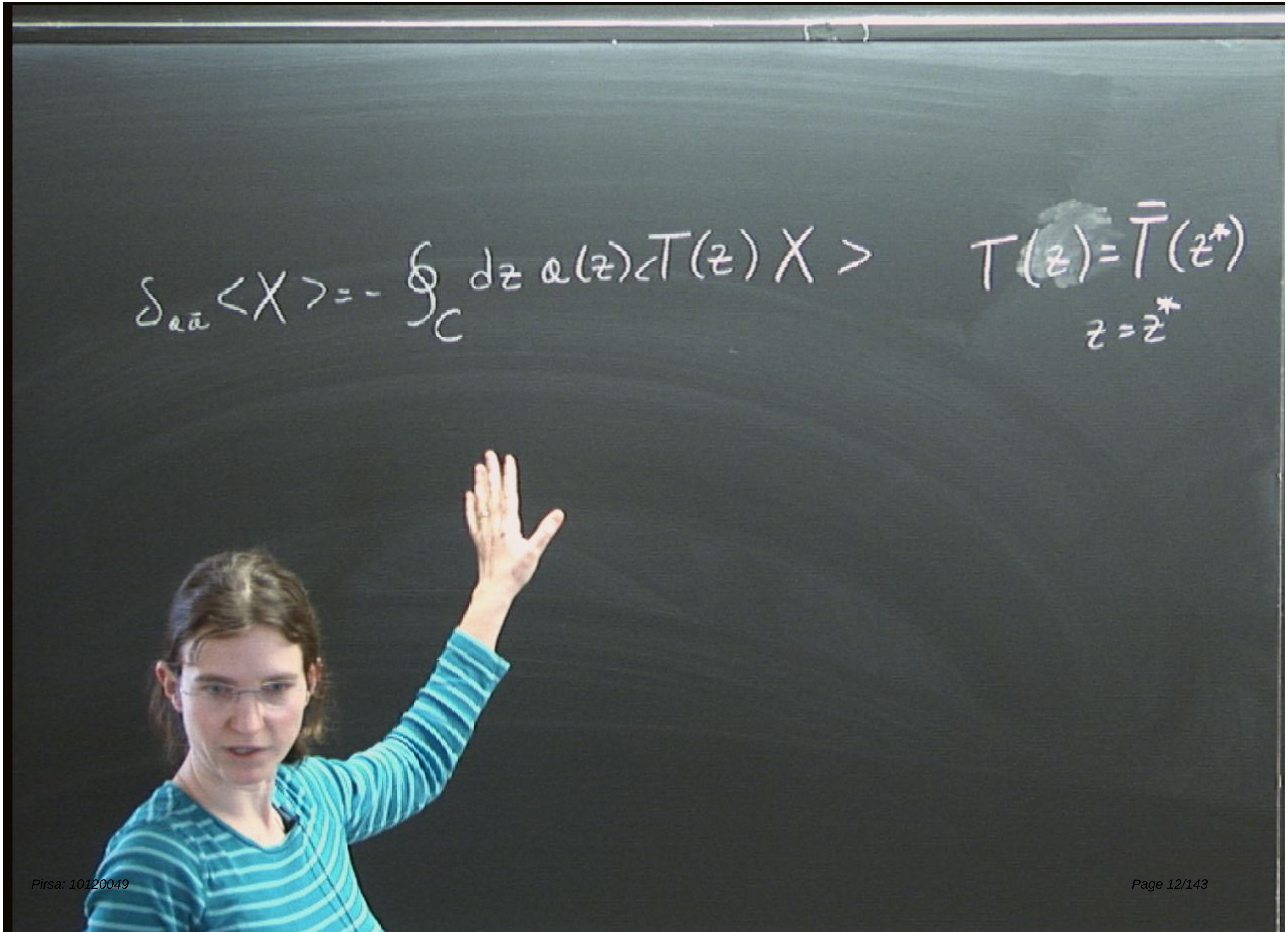

Evidence
for Atoms
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
NDARY STATE)
MPLE. PERCOLATION
BOUNDARY CONDITIONS
z ↦ z + α(z)
α(z) ∈ ℝ IF z ∈ ℝ
α(z) = Σ a_n z^n
∈ ℝ
δ_{α,ᾱ}⟨X⟩ =


Poxxen to
Evidence
for Atoms
Is A
Molecule?
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
NDARY STATE)
MPLE: PERCOLATION
BOUNDARY CONDITIONS
$z \mapsto z + \alpha(z)$
$\alpha(z) \in \mathbb{R}$ IF $z \in \mathbb{R}$
$\alpha(z) = \sum a_n z^n$
$\in \mathbb{R}$
$\delta_{\alpha,\bar{\alpha}} \langle X \rangle = -\oint_C dz\, \alpha(z) \langle T(z) X \rangle$
$+ \oint_C d\bar{z}\, \bar{\alpha}(\bar{z}) \langle \bar{T}(\bar{z}) X \rangle$


Poxxen to
Evidence
for Atoms
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
NDARY STATE)
MPLE: PERCOLATION
BOUNDARY CONDITIONS
$z \mapsto z + a(z)$
$a(z) \in \mathbb{R}$ IF $z \in \mathbb{R}$
$a(z) = \sum a_n z^n$
$\in \mathbb{R}$
$+ \oint_C d\bar{z}\, \bar{a}(\bar{z}) \langle \bar{T}(\bar{z}) X \rangle$


Poxxen to Evidence for Atoms
ITE SIZE CFT
FOR THE STRESS TENSOR
DARY OPERATORS
UNDARY STATE)
MPLE: PERCOLATION
BOUNDARY CONDITIONS
z ↦ z + α(z)
α(z) ∈ ℝ if z ∈ ℝ
α(z) = Σ α_n z^n
∈ ℝ
δ_{α,ᾱ} <X> = − ∮_C dz α(z) <T(z) X>
+ ∮_C dz̄ ᾱ(z̄) <T̄(z̄) X>

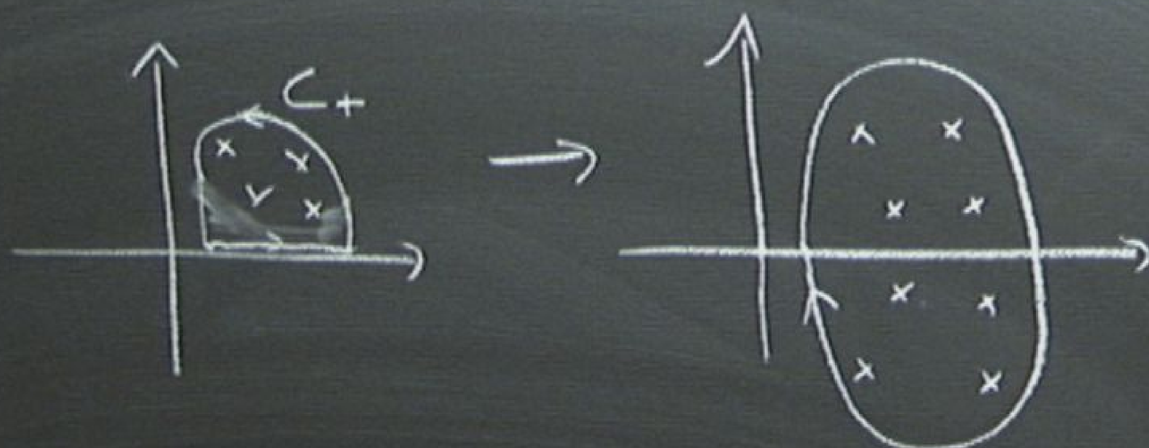

$\delta_{a\bar{a}} \langle X \rangle = - \oint_C dz\, a(z) \langle T(z)\, X \rangle$

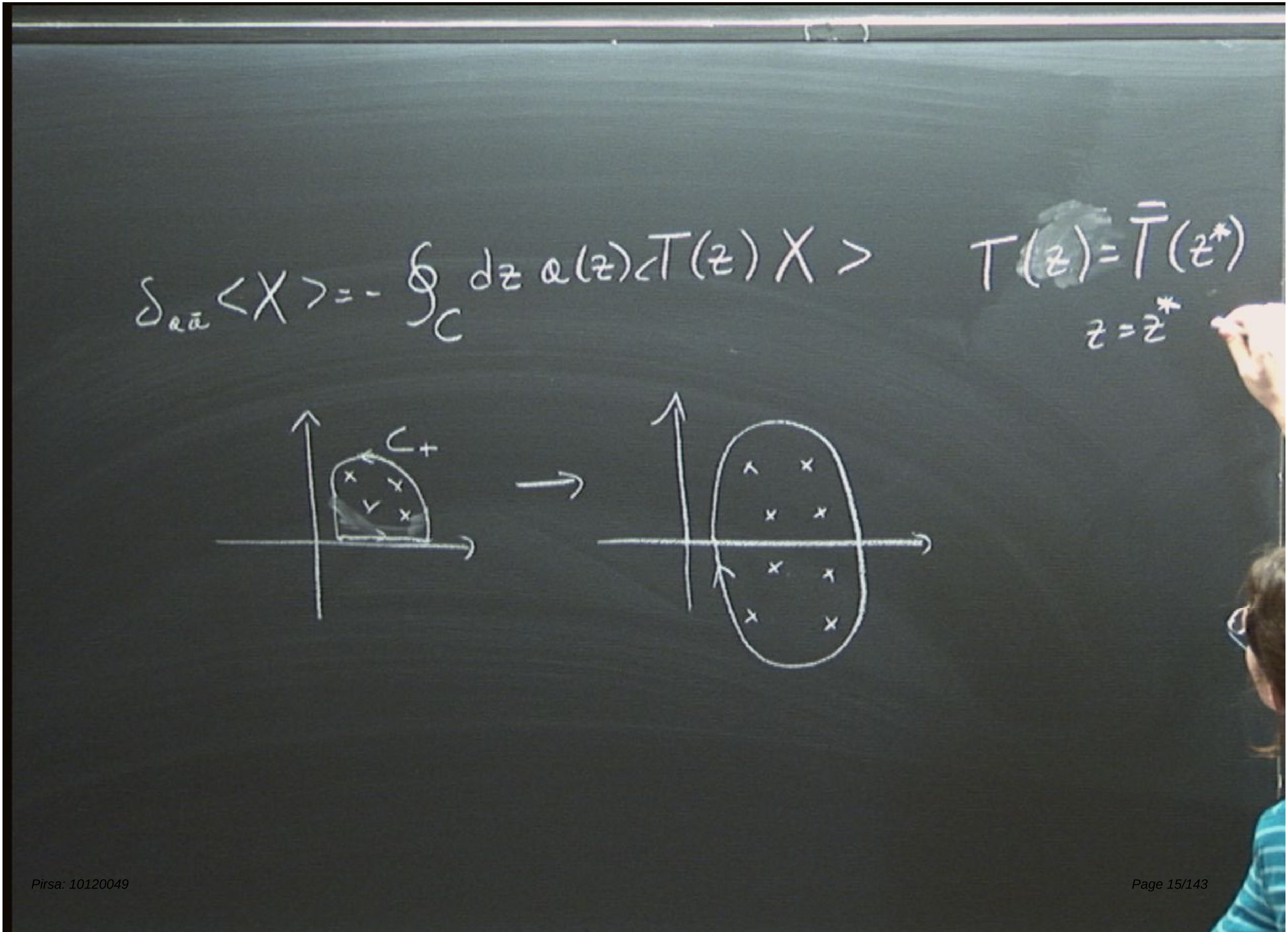

$\delta_{a\bar{a}} \langle X \rangle = -\oint_C dz\, a(z) \langle T(z)\, X \rangle$
$T(z) = \bar{T}(z^*)$
$z = z^*$

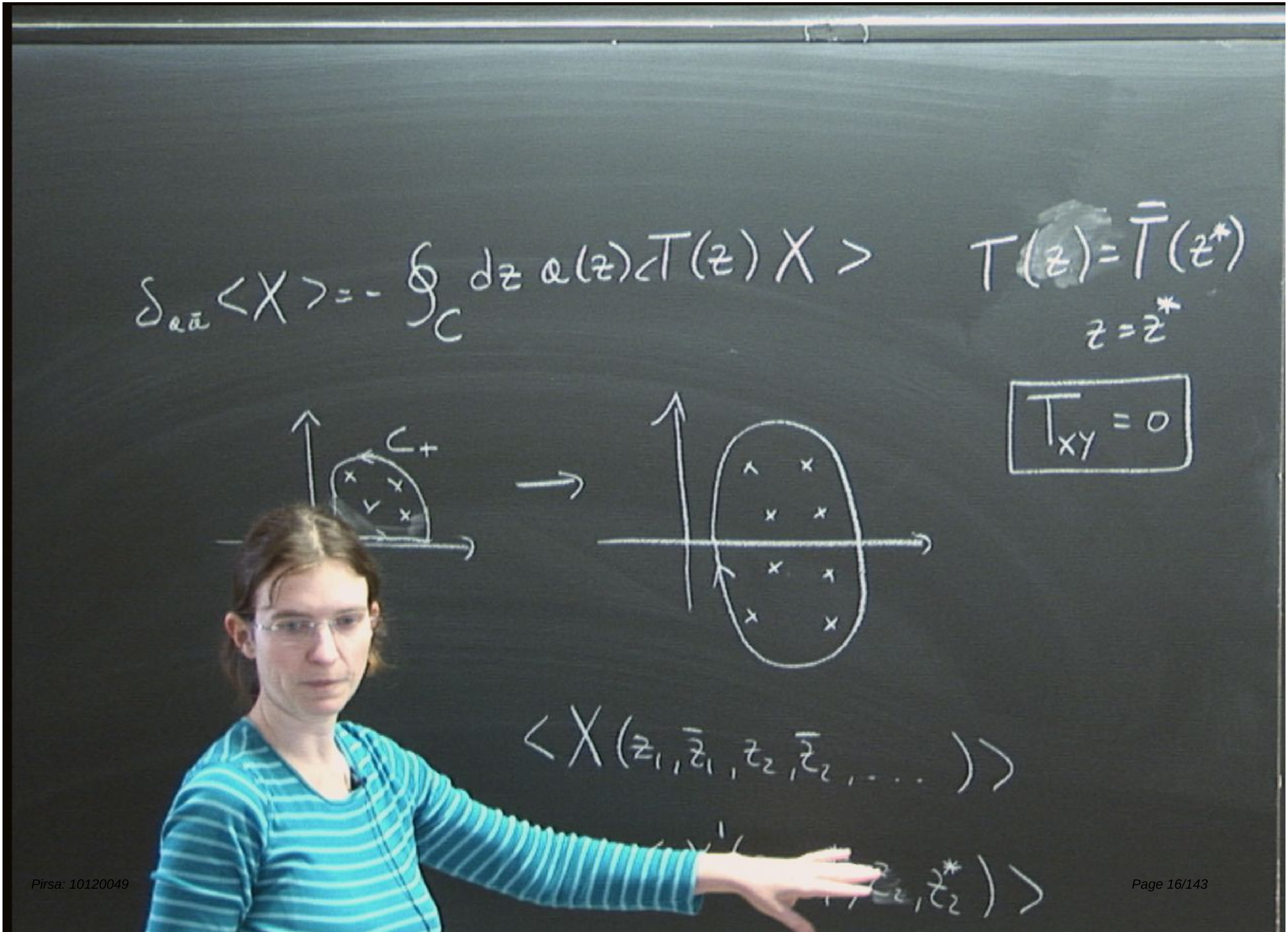

$\delta_{a\bar{a}} \langle X \rangle = -\oint_C dz\, a(z) \langle T(z) X \rangle$
$T(z) = \bar{T}(z^*)$
$z = z^*$
C_+

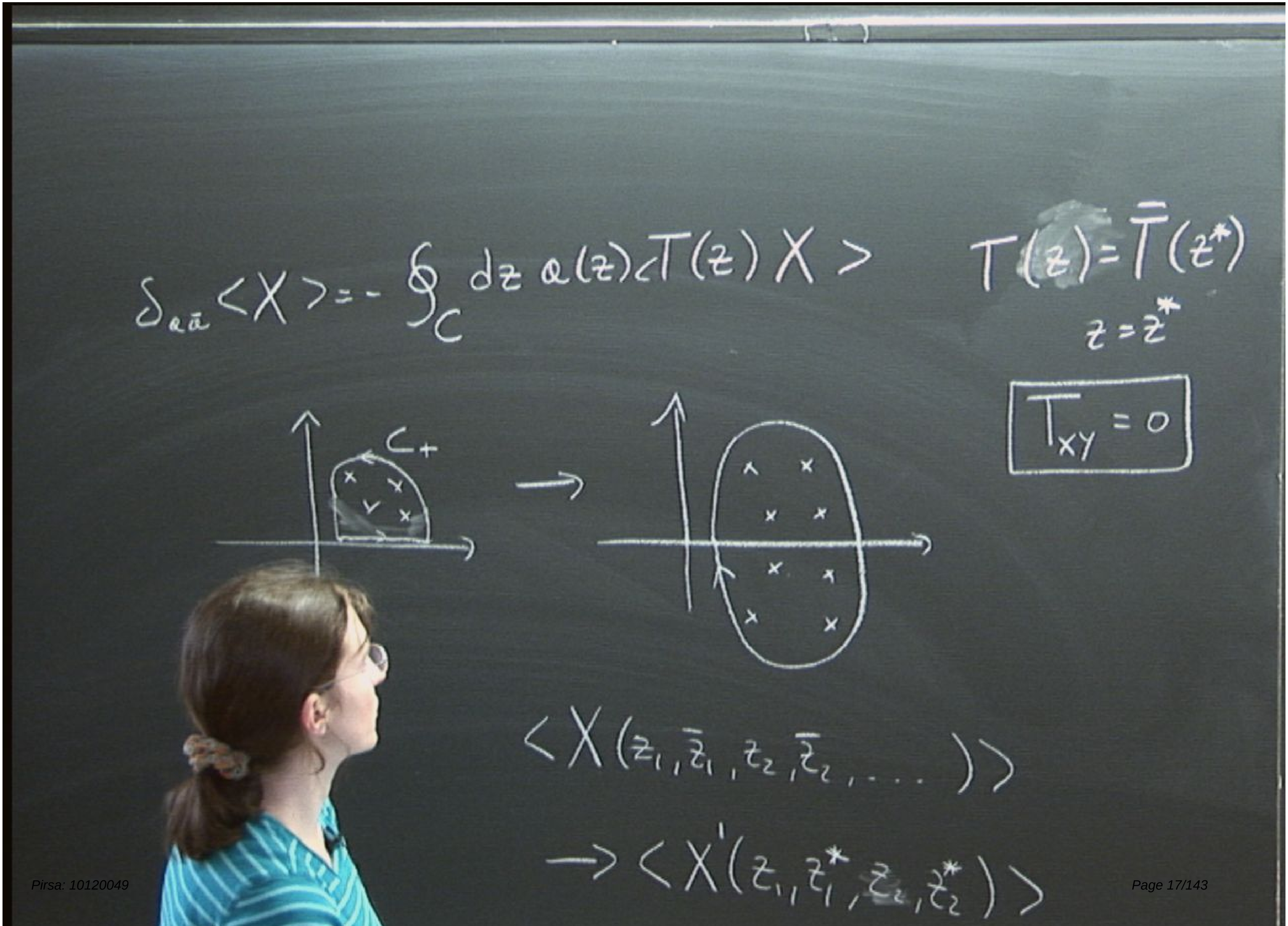
$$\delta_{a\bar{a}}\langle X\rangle = -\oint_C dz\, a(z)\langle T(z)\, X\rangle \qquad T(z) = \bar{T}(z^*)$$

$$z = z^*$$




$\delta_{a\bar{a}} \langle X \rangle = - \oint_C dz\, a(z) \langle T(z) X \rangle$
$T(z) = \bar{T}(z^*)$
$z = z^*$
C_+


δ_{aā} <X> = - ∮_C dz a(z) <T(z) X >
T(z) = T̄(z*)
z = z*
T_xy = 0
C+
<X(z₁, z̄₁, z₂, z̄₂, . . .)>

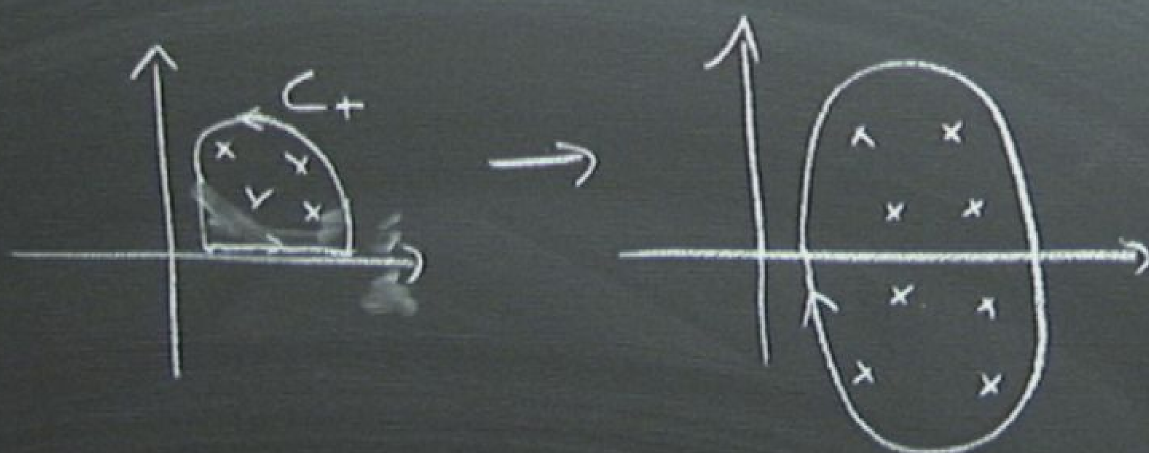

$\delta_{a\bar{a}} \langle X \rangle = - \oint_C dz\, a(z) \langle T(z) X \rangle$
$T(z) = \bar{T}(z^*)$
$z = z^*$
$T_{xy} = 0$
C_+
$\langle X(z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots) \rangle$
$\rightarrow \langle X'(z_1, z_1^*, z_2, z_2^*) \rangle$

$$\delta_{a\bar{a}}\langle X \rangle = -\oint_C dz\, a(z) \langle T(z) X \rangle$$

$$T(z) = \bar{T}(z^*)$$

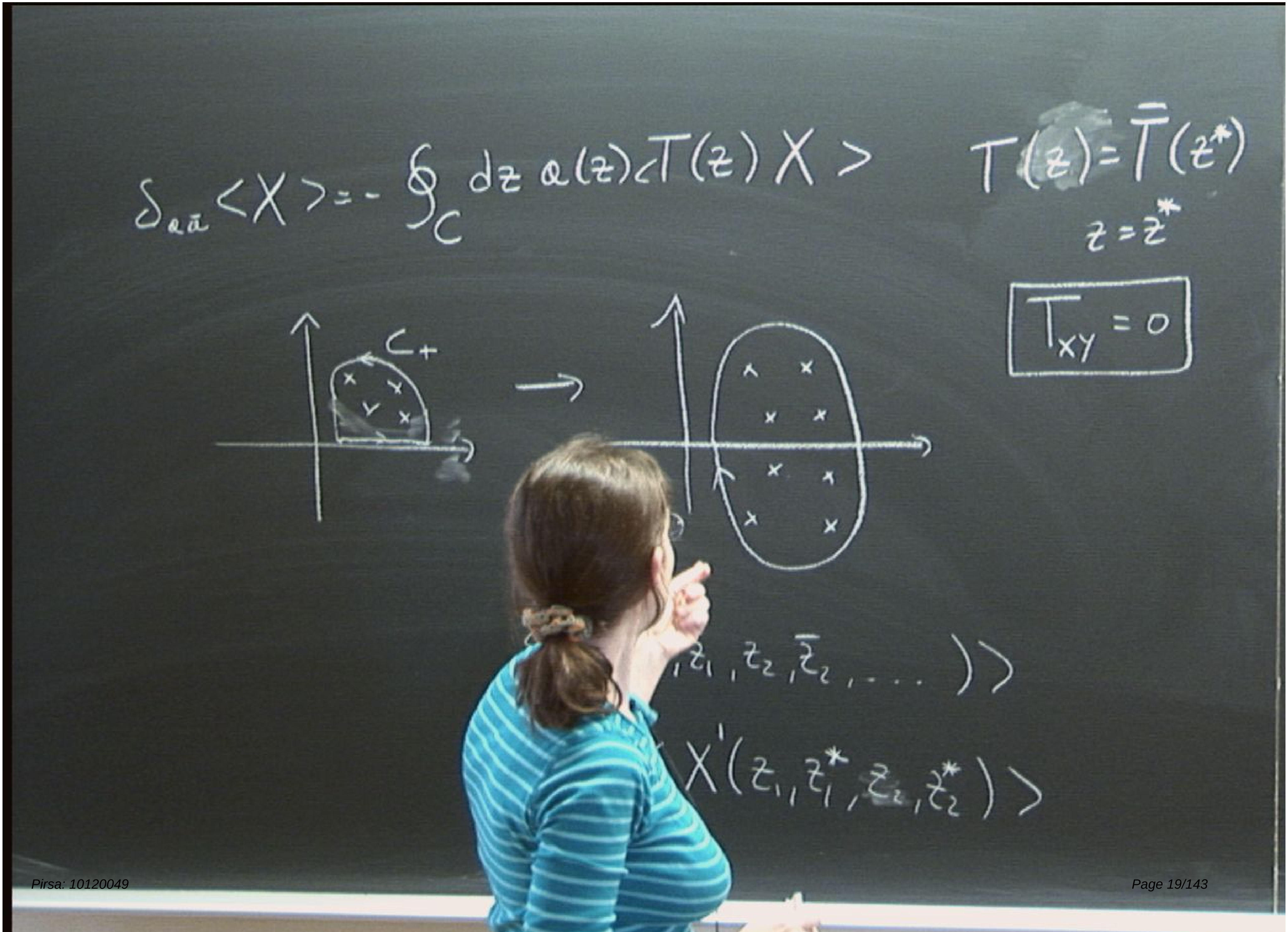
$$z = z^*$$

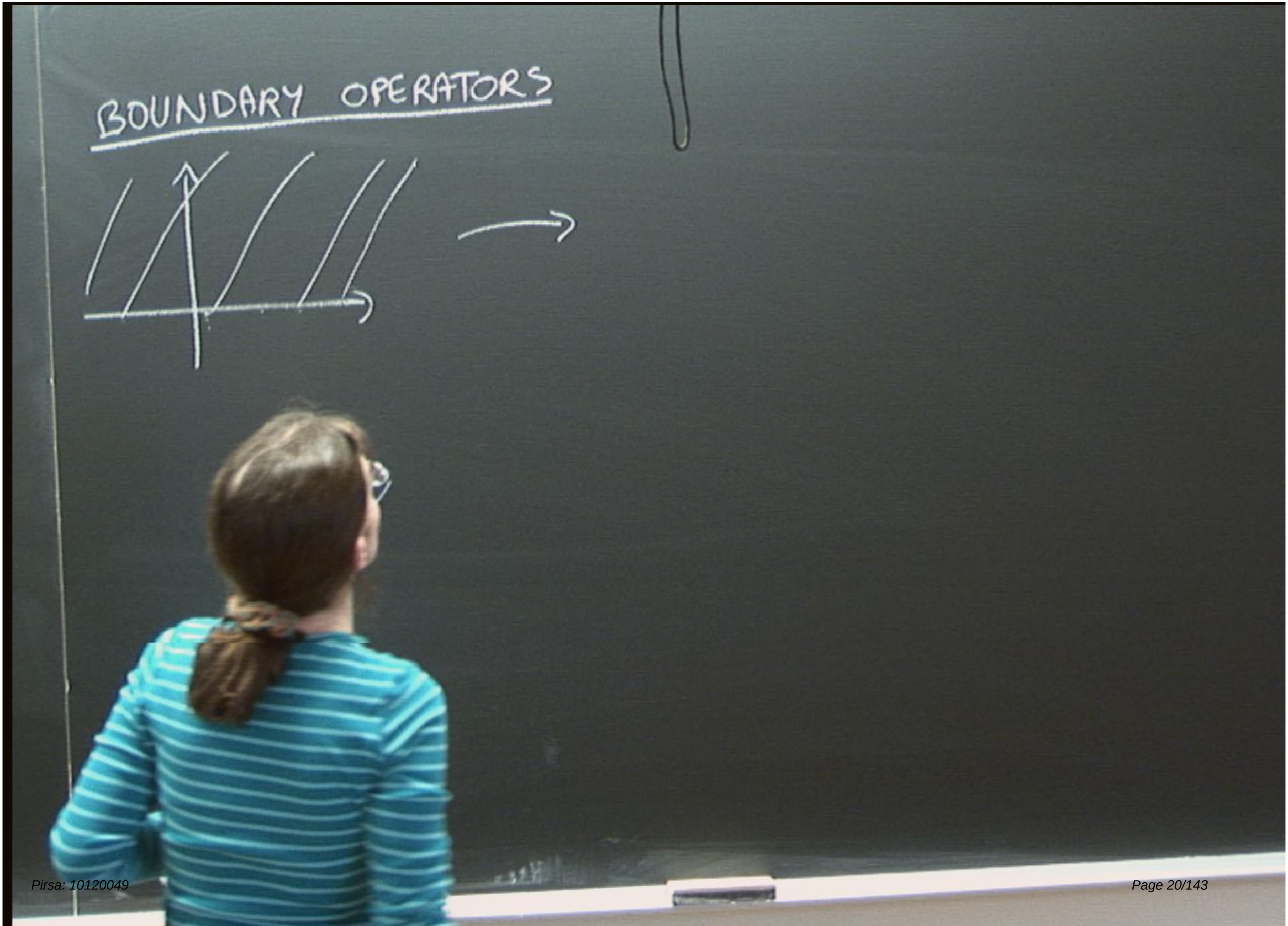
$$\boxed{T_{xy} = 0}$$

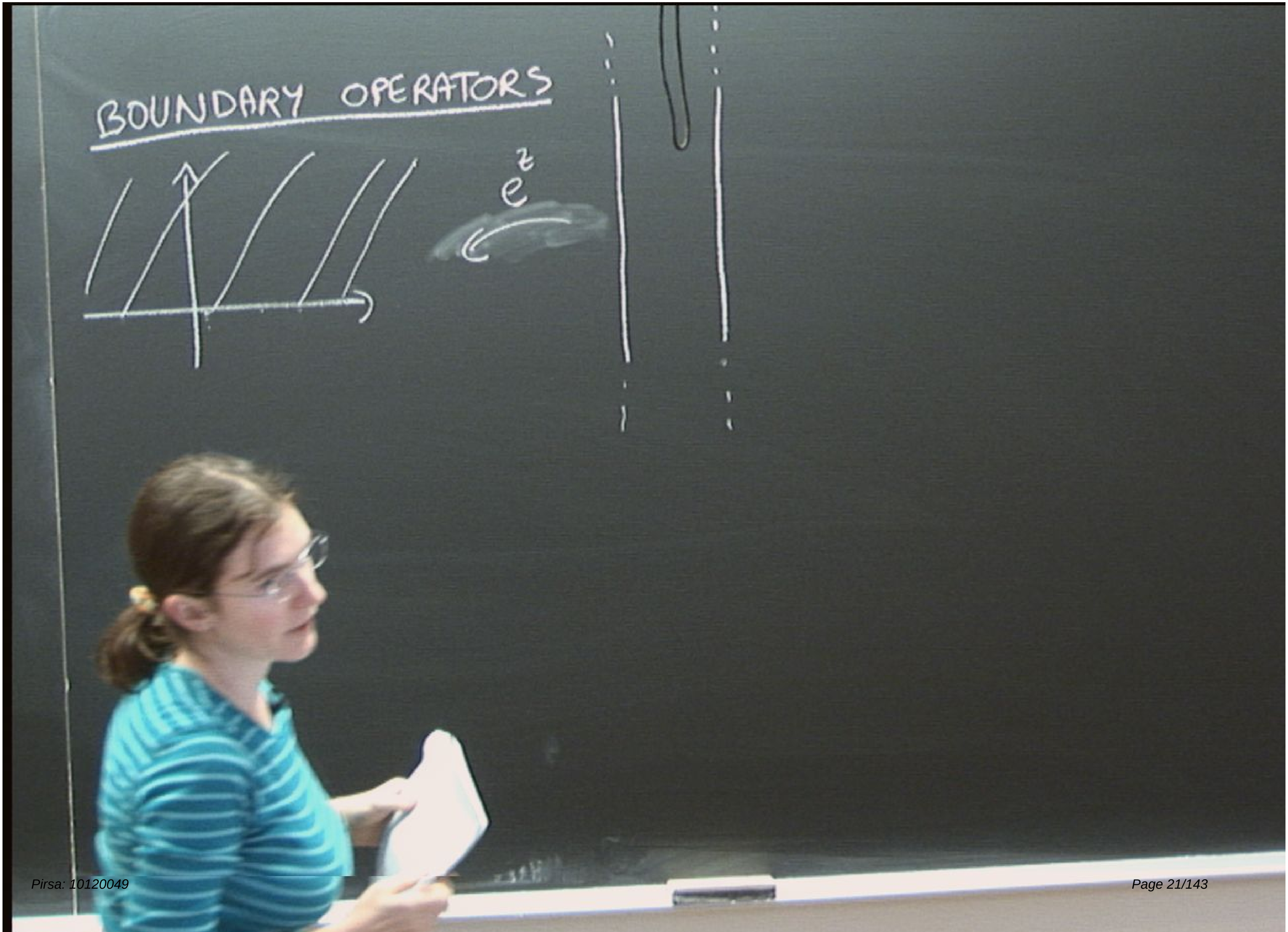


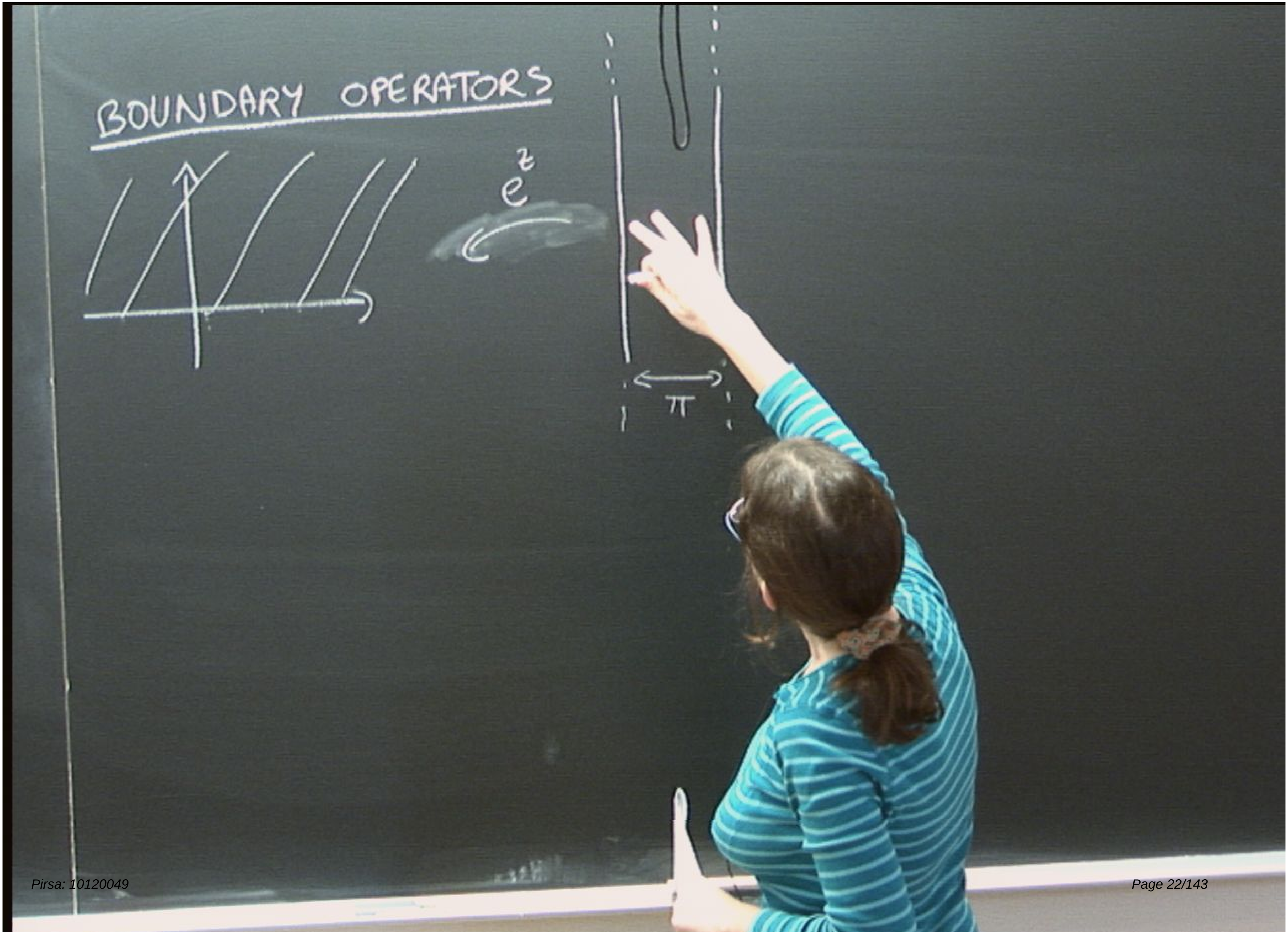
$$\langle X(z_1, \bar{z}_1, z_2, \bar{z}_2, \dots) \rangle$$

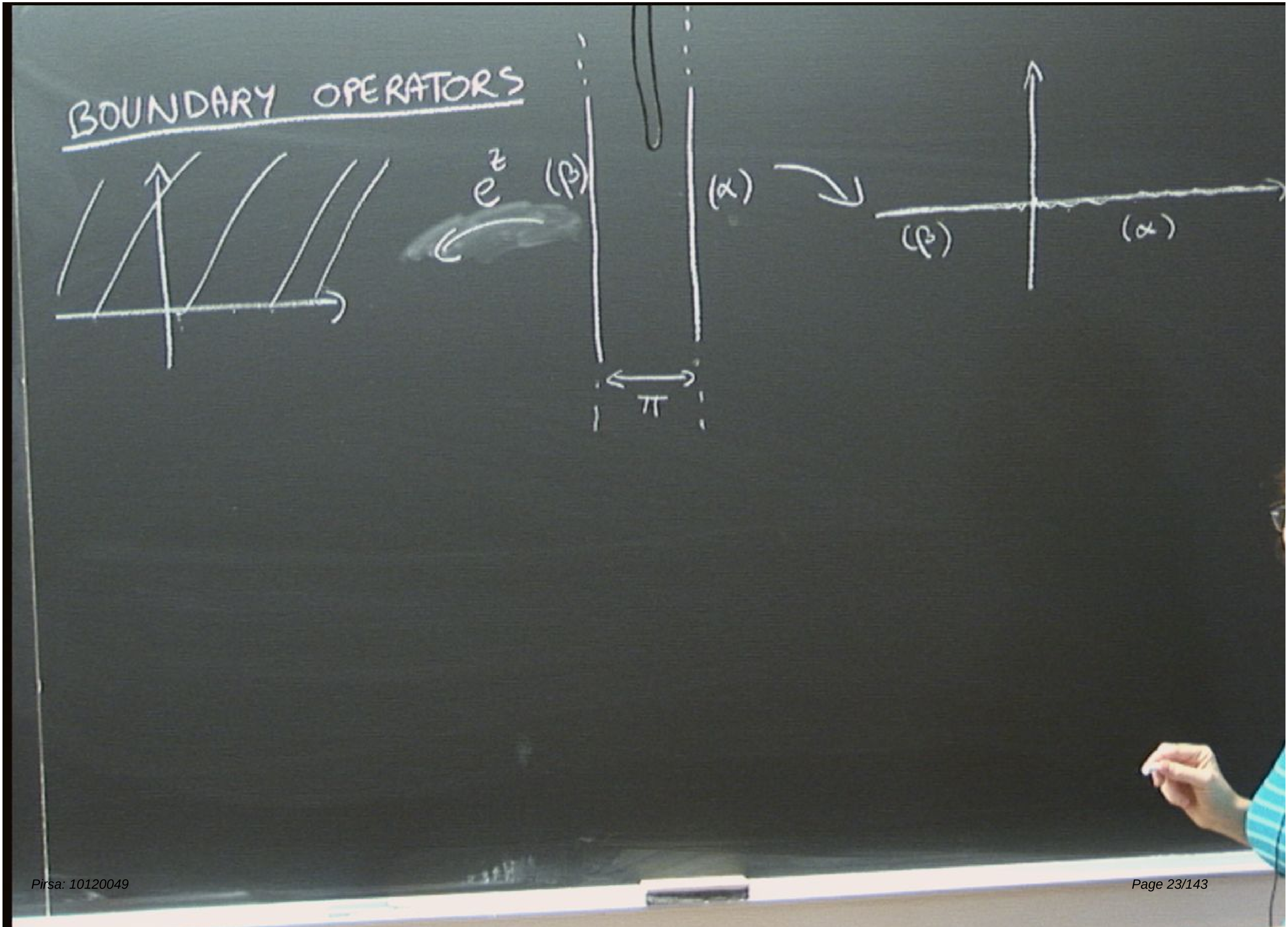
$$\to \langle X'(z_1, z_1^*, z_2, z_2^*) \rangle$$


δ_{aā} <X> = - ∮_C dz a(z) <T(z) X>
T(z) = T̄(z*)
z = z*
T_{xy} = 0
C_+
z_1, z_2, z̄_2, ...))>
X'(z_1, z_1*, z_2, z_2*)>

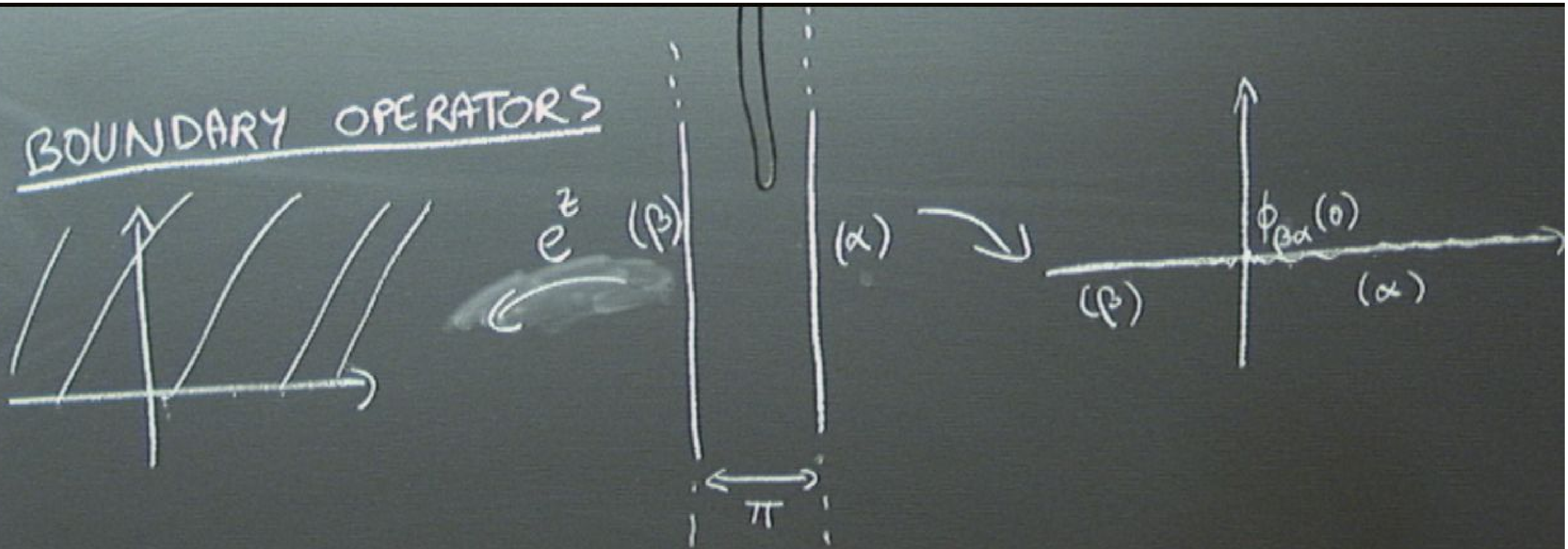

BOUNDARY OPERATORS

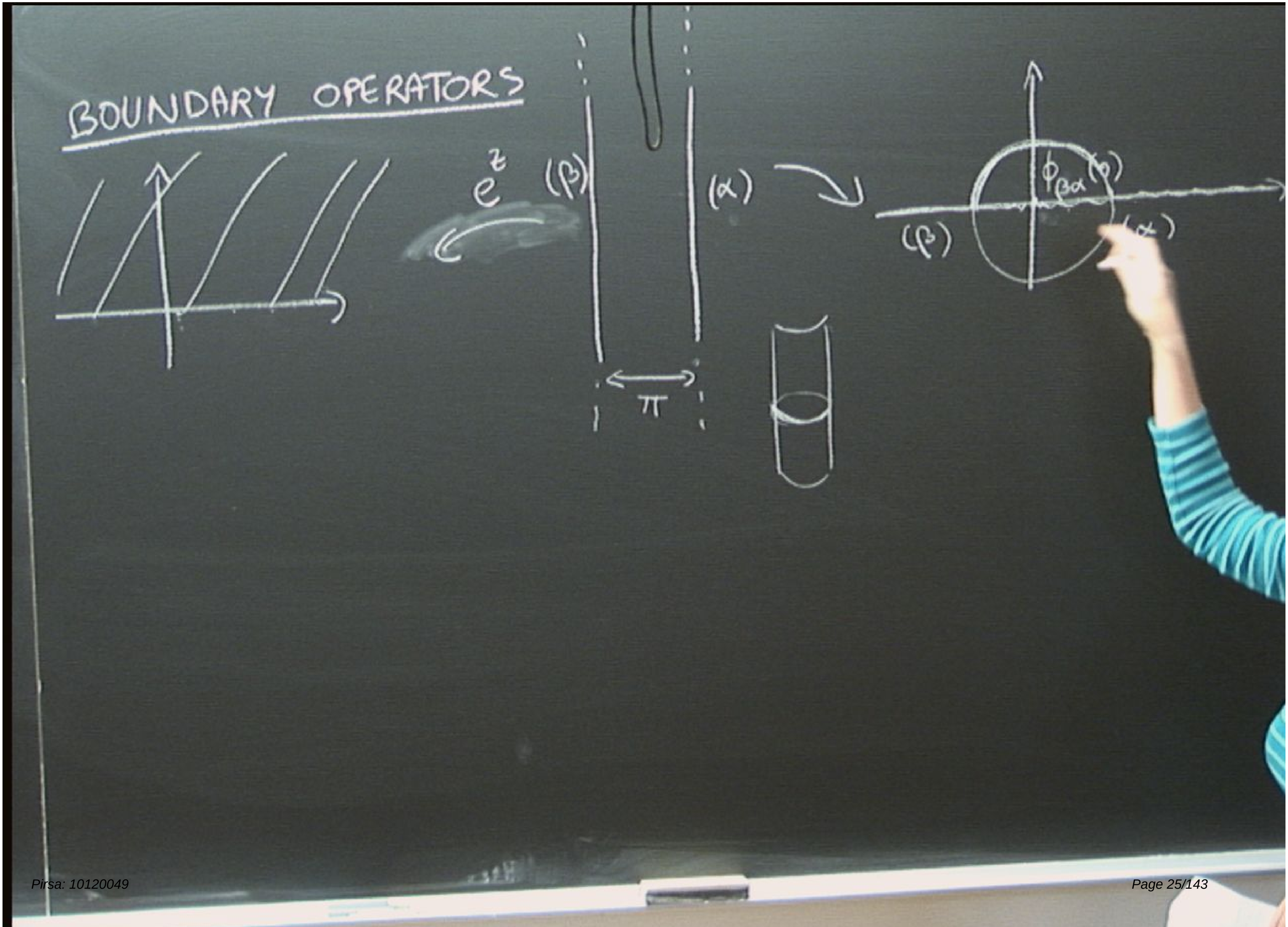

BOUNDARY OPERATORS
e^z

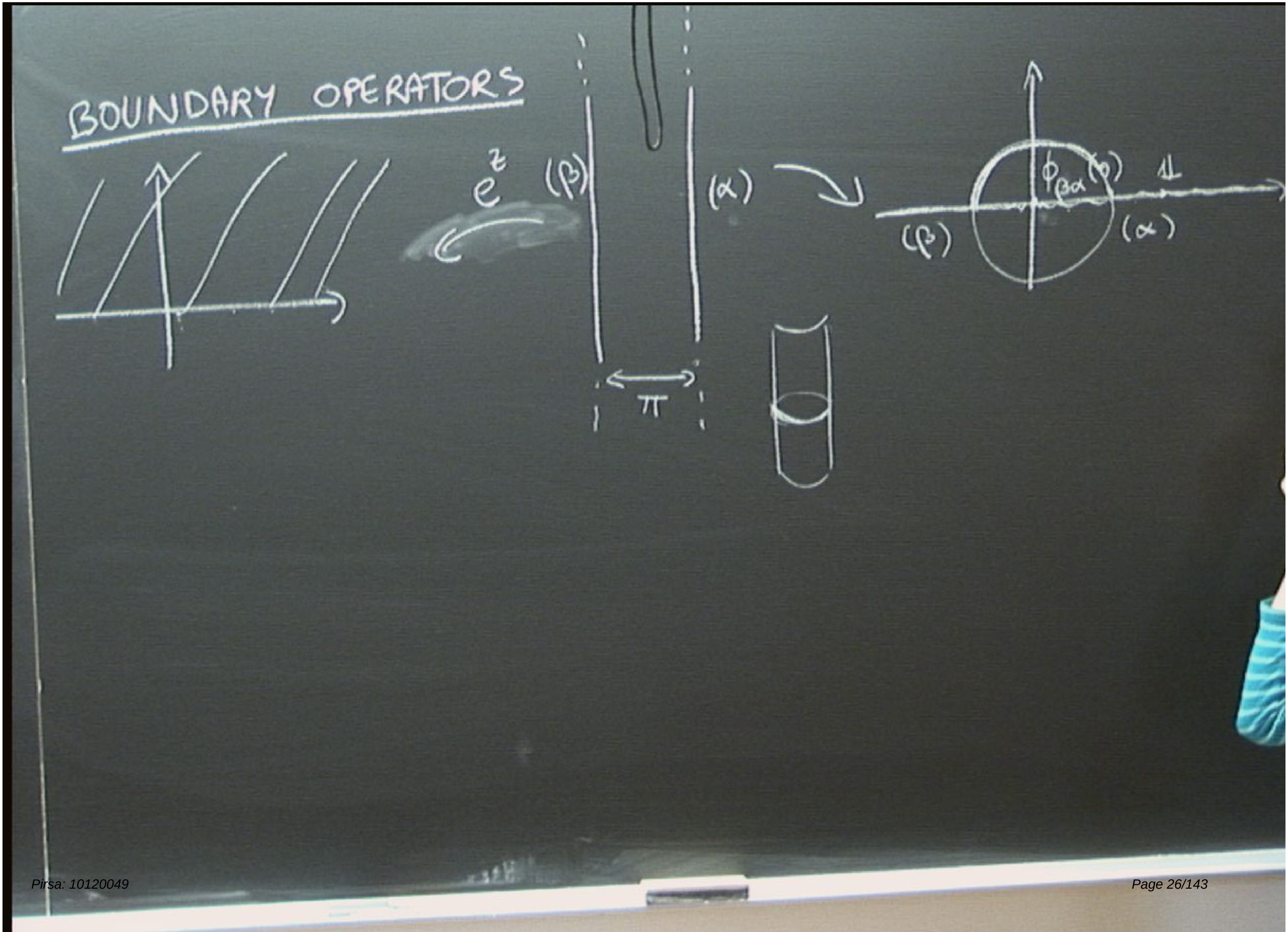

BOUNDARY OPERATORS
e^z
π

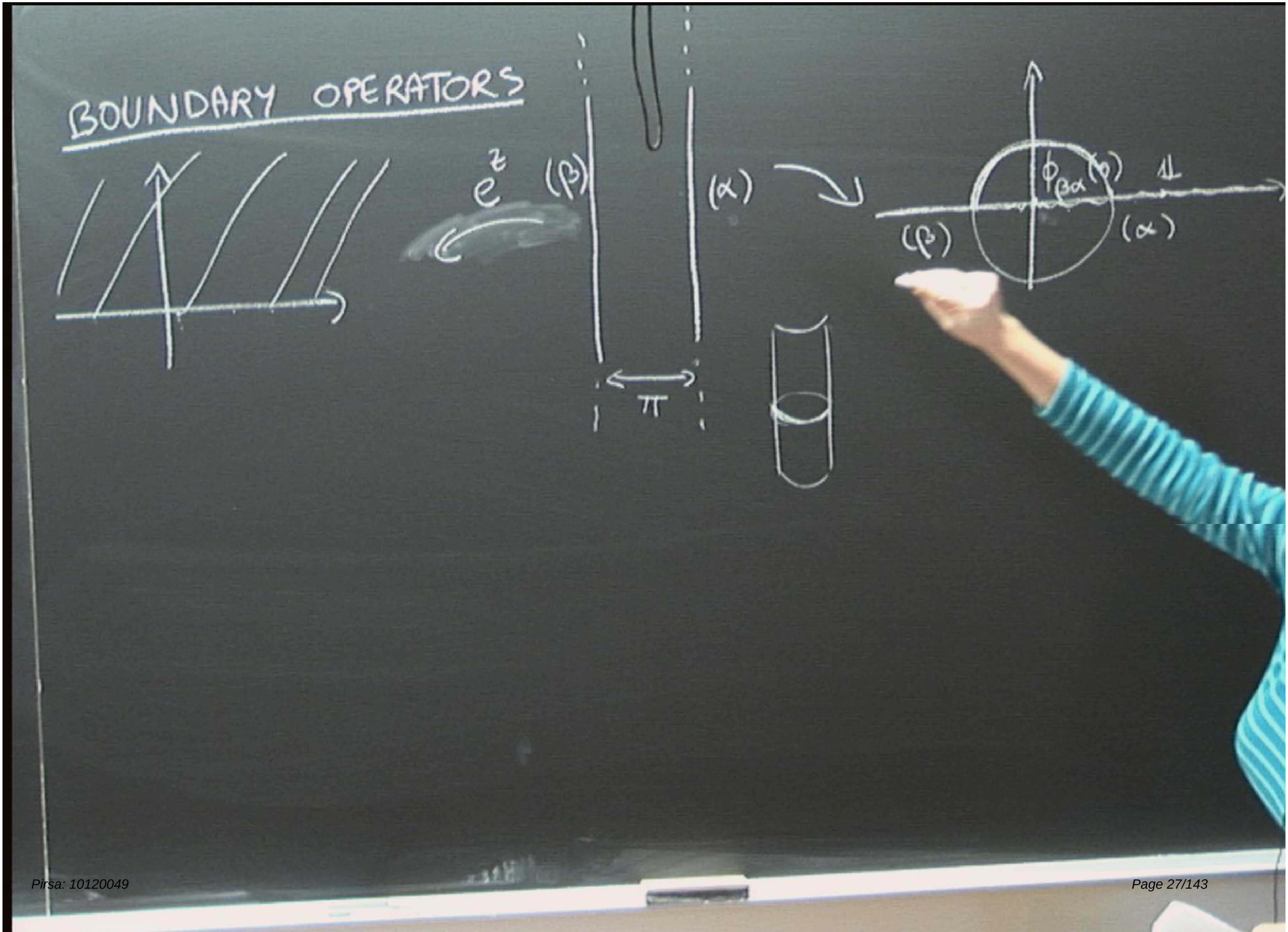

BOUNDARY OPERATORS
e^z
(β)
(α)
π
(β)
(α)

BOUNDARY OPERATORS

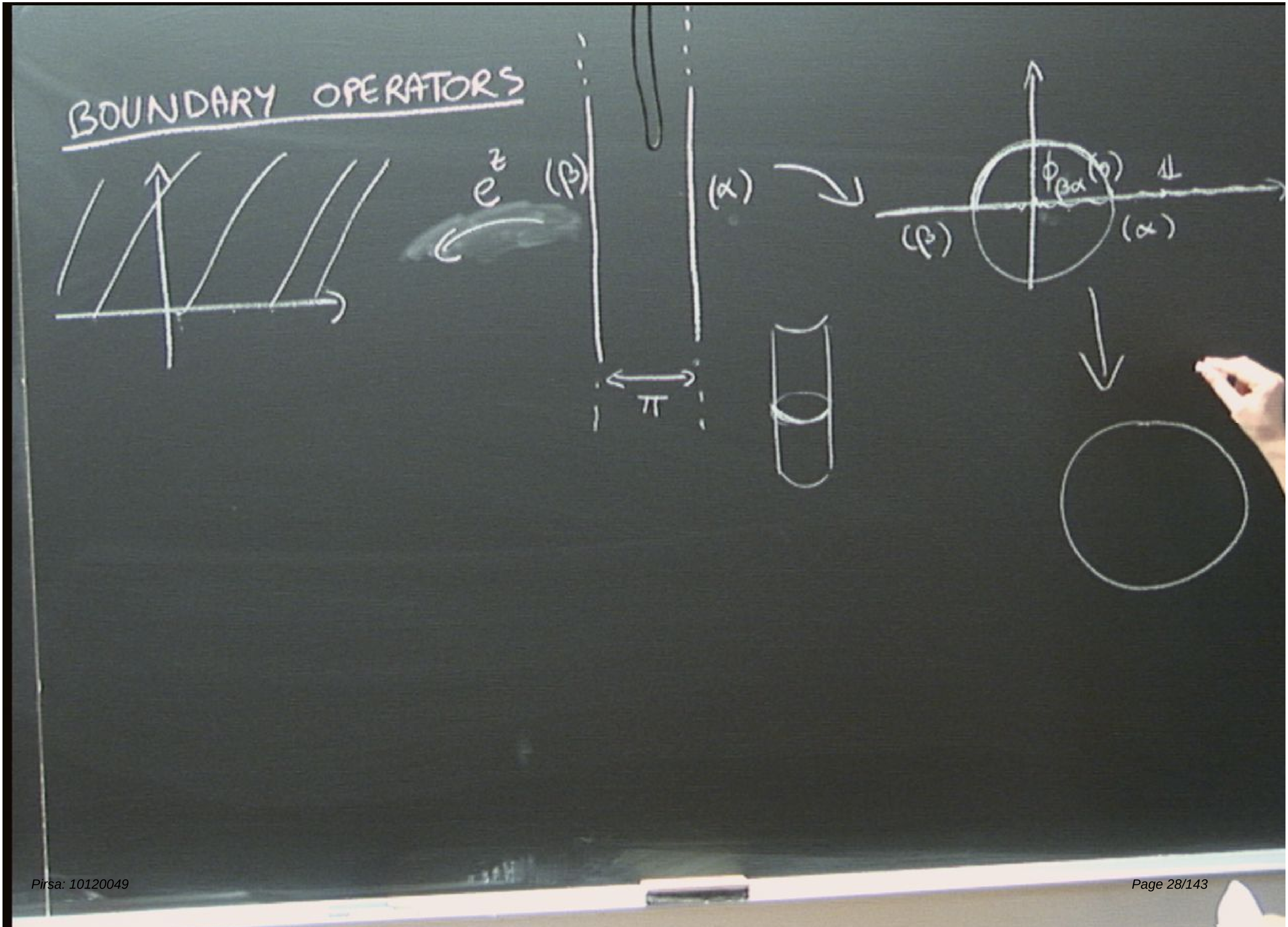


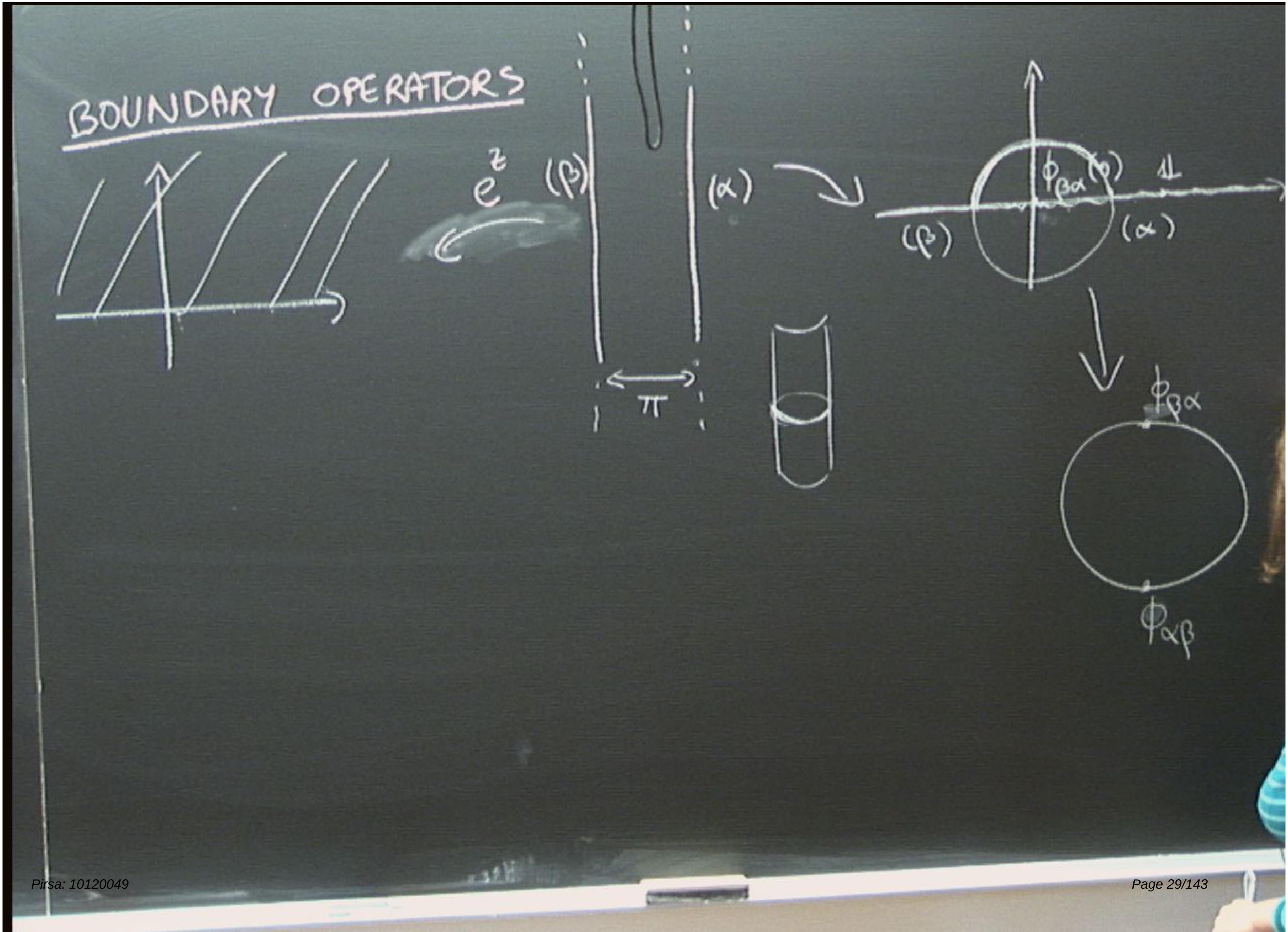

BOUNDARY OPERATORS
e^z
(β)
(α)
π
φ_βα(0)
(β)
(α)

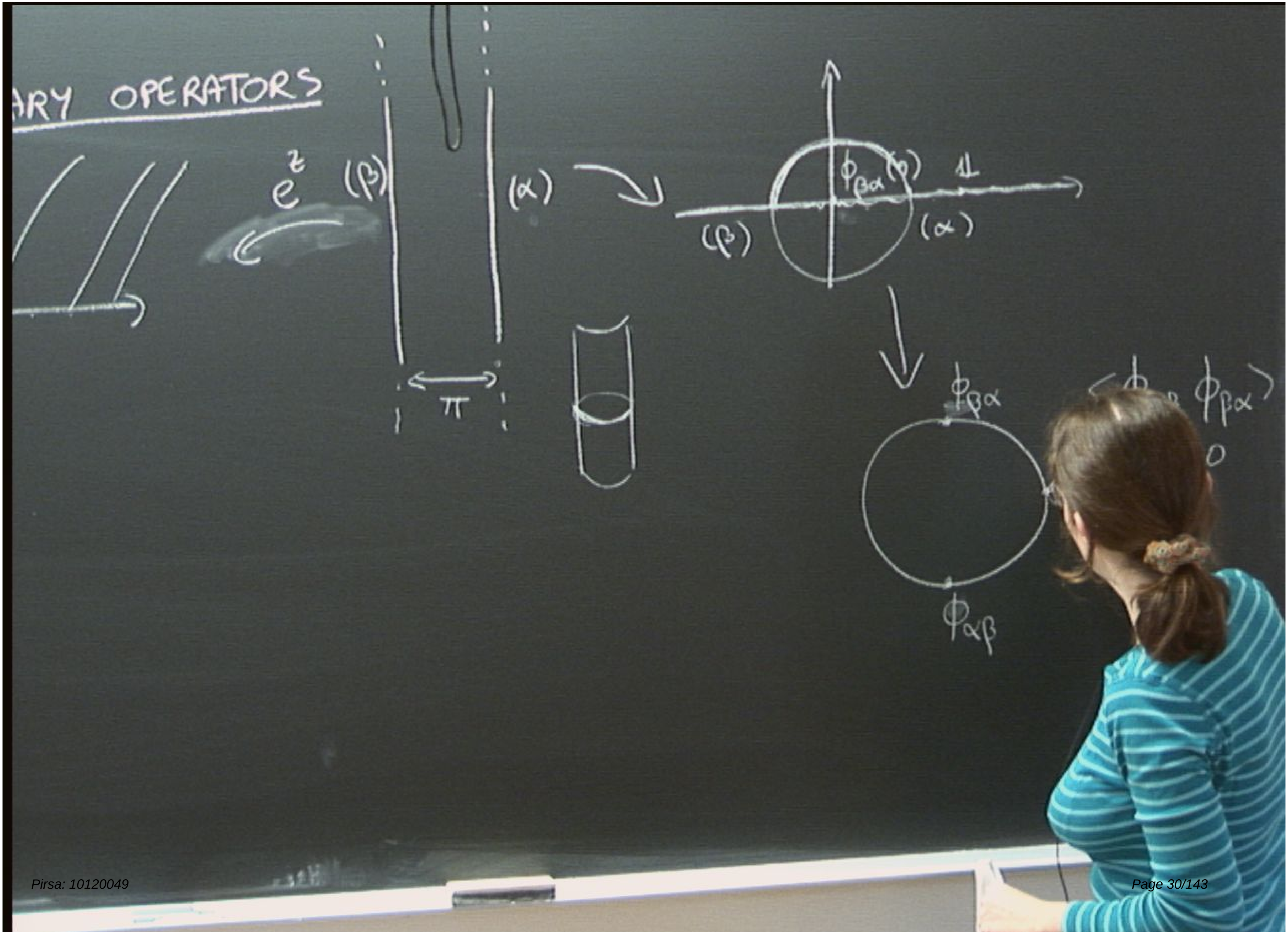

BOUNDARY OPERATORS
e^z
(β)
(α)
π
$\phi_{\beta\alpha}(0)$
𝟙
(β)
(α)

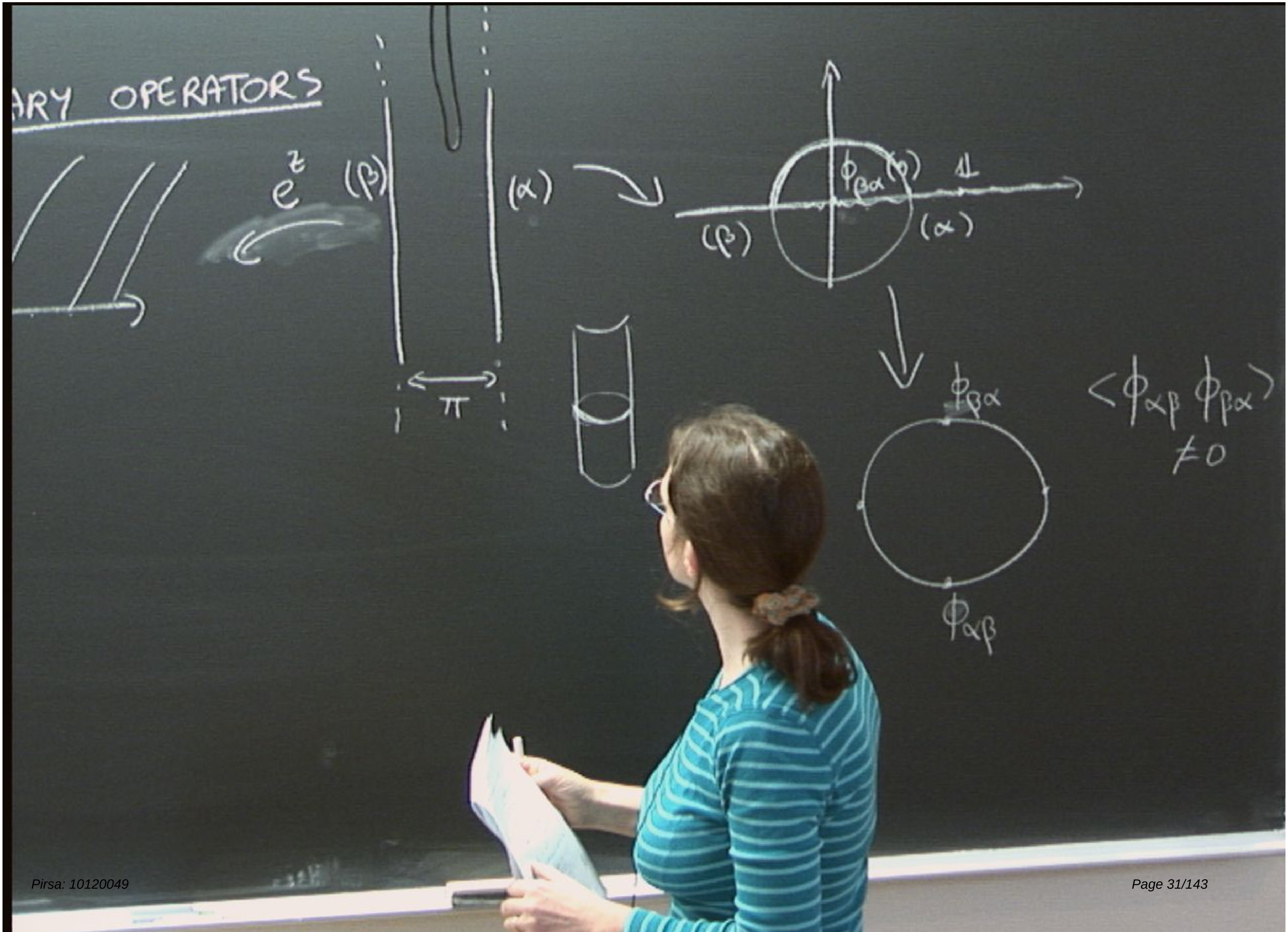


BOUNDARY OPERATORS
(β)
(α)
π
(β)
(α)

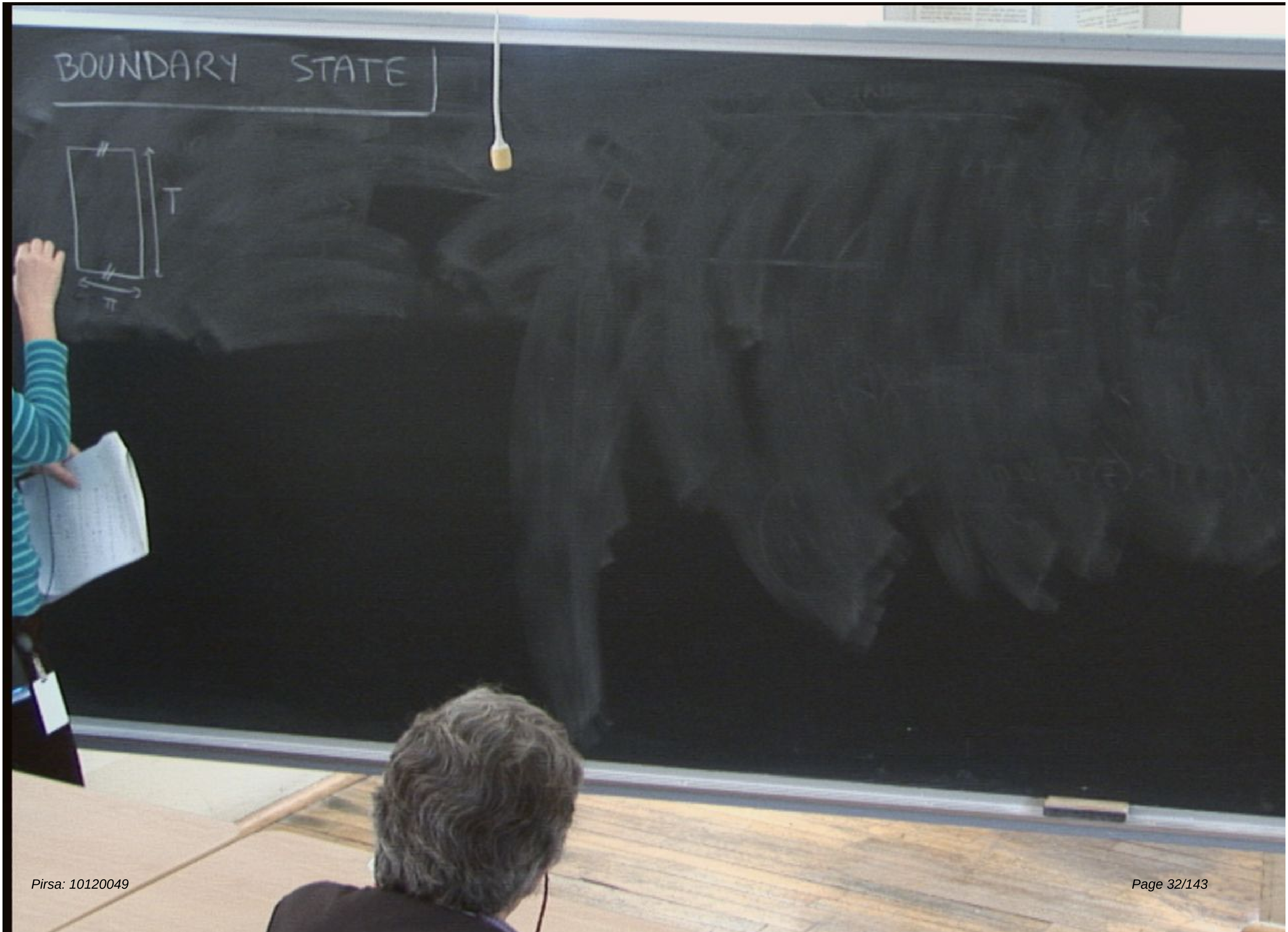

BOUNDARY OPERATORS
e^z
(β)
(α)
π
$\phi_{\beta\alpha}(0)$
$\mathbb{1}$
(β)
(α)

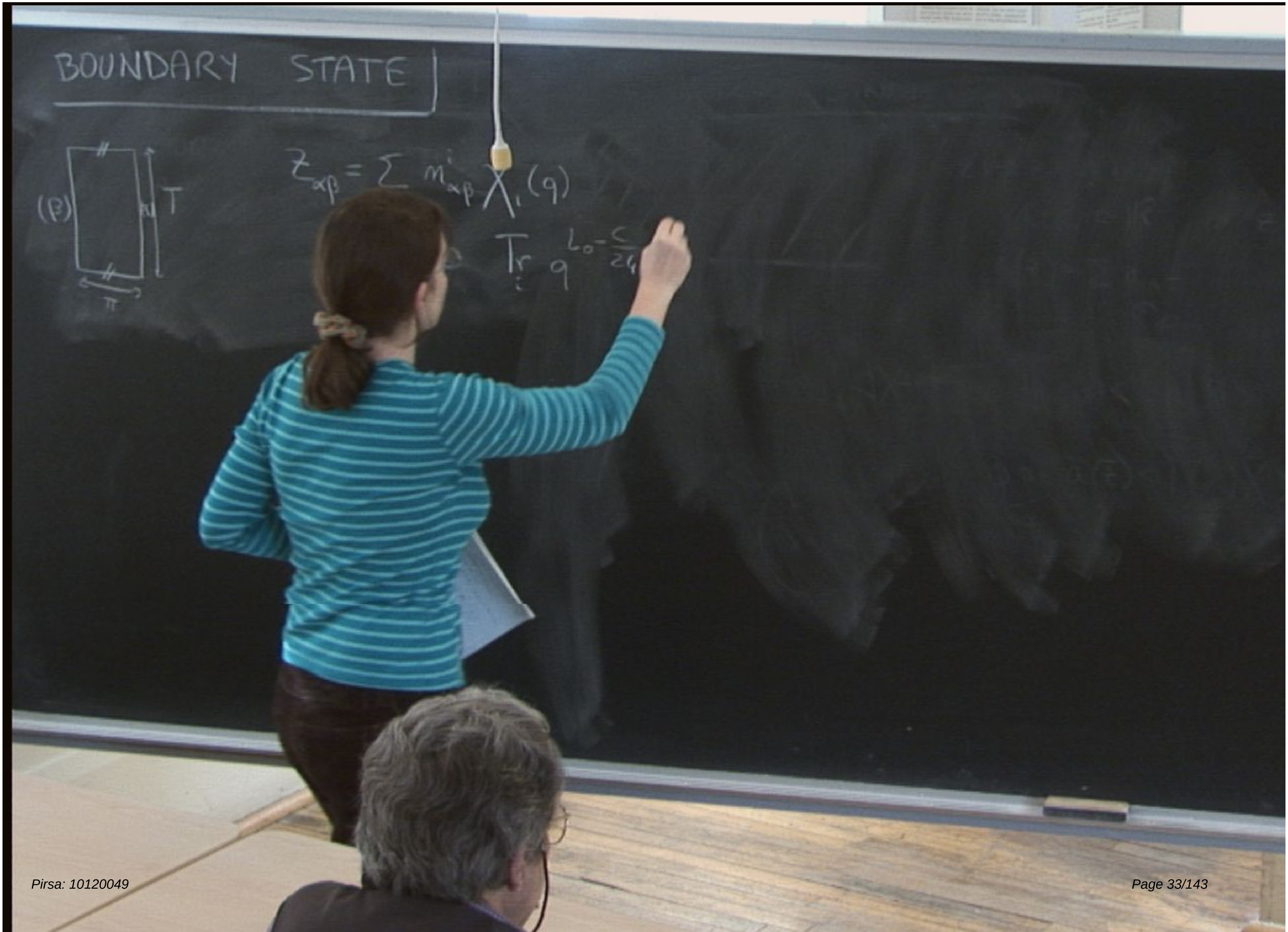

BOUNDARY OPERATORS
e^z
(β)
(α)
π
$\phi_{\beta\alpha}(0)$
𝟙
(β)
(α)
$\phi_{\beta\alpha}$
$\phi_{\alpha\beta}$

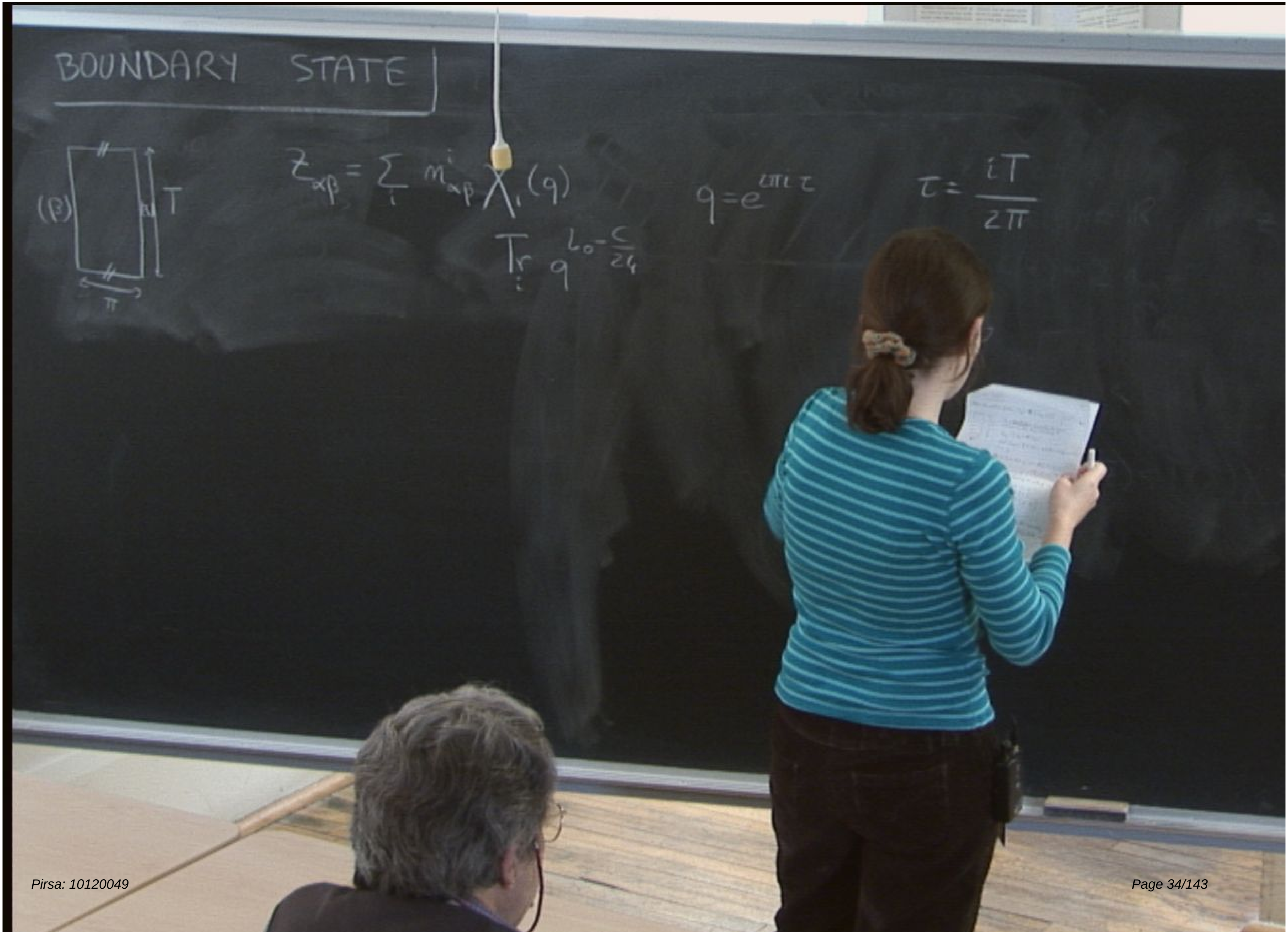

ARY OPERATORS
e^z
(β)
(α)
π
φ_βα(p)
𝟙
(β)
(α)
φ_βα
φ_αβ
⟨φ φ_βα⟩
0

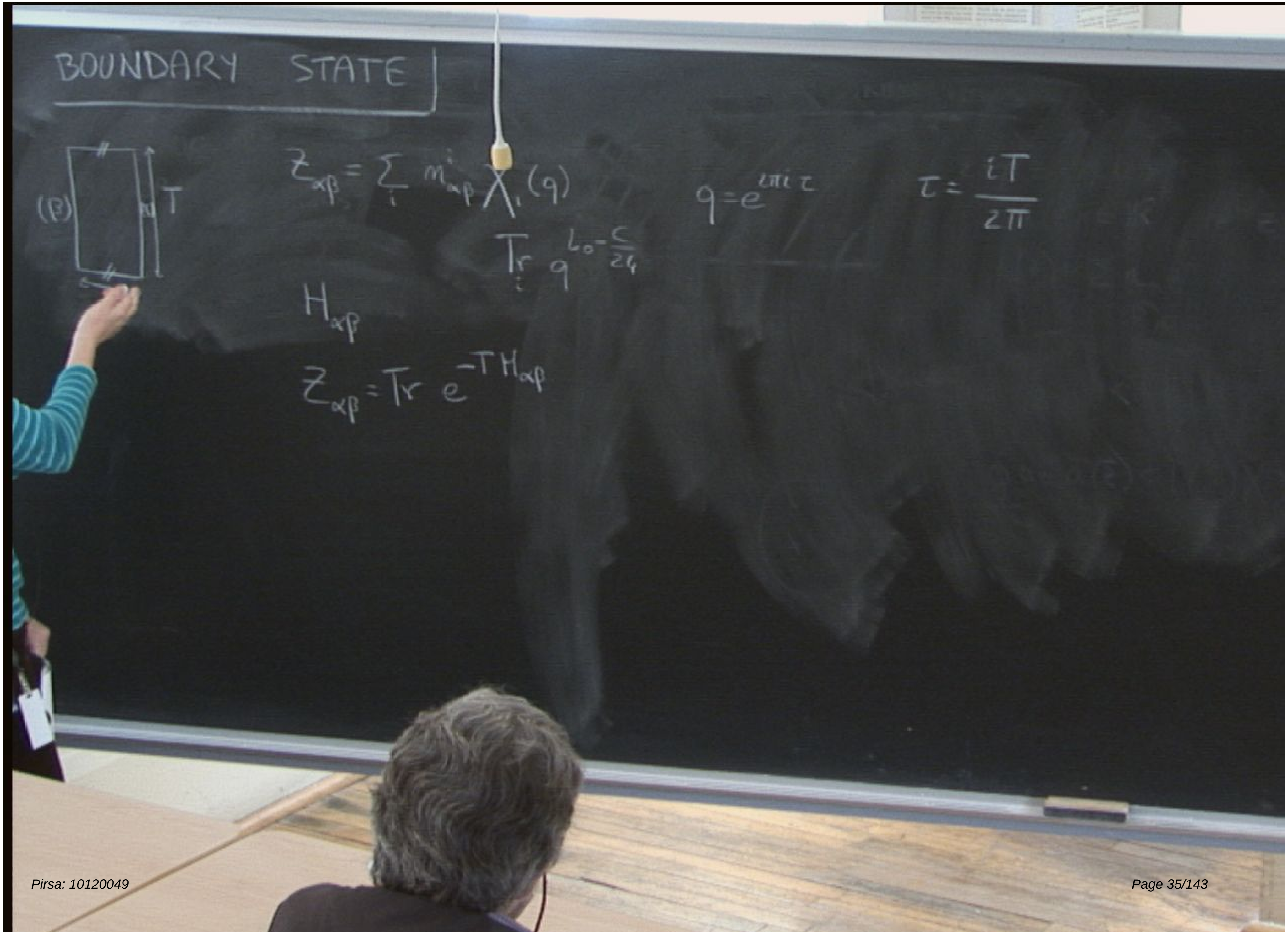


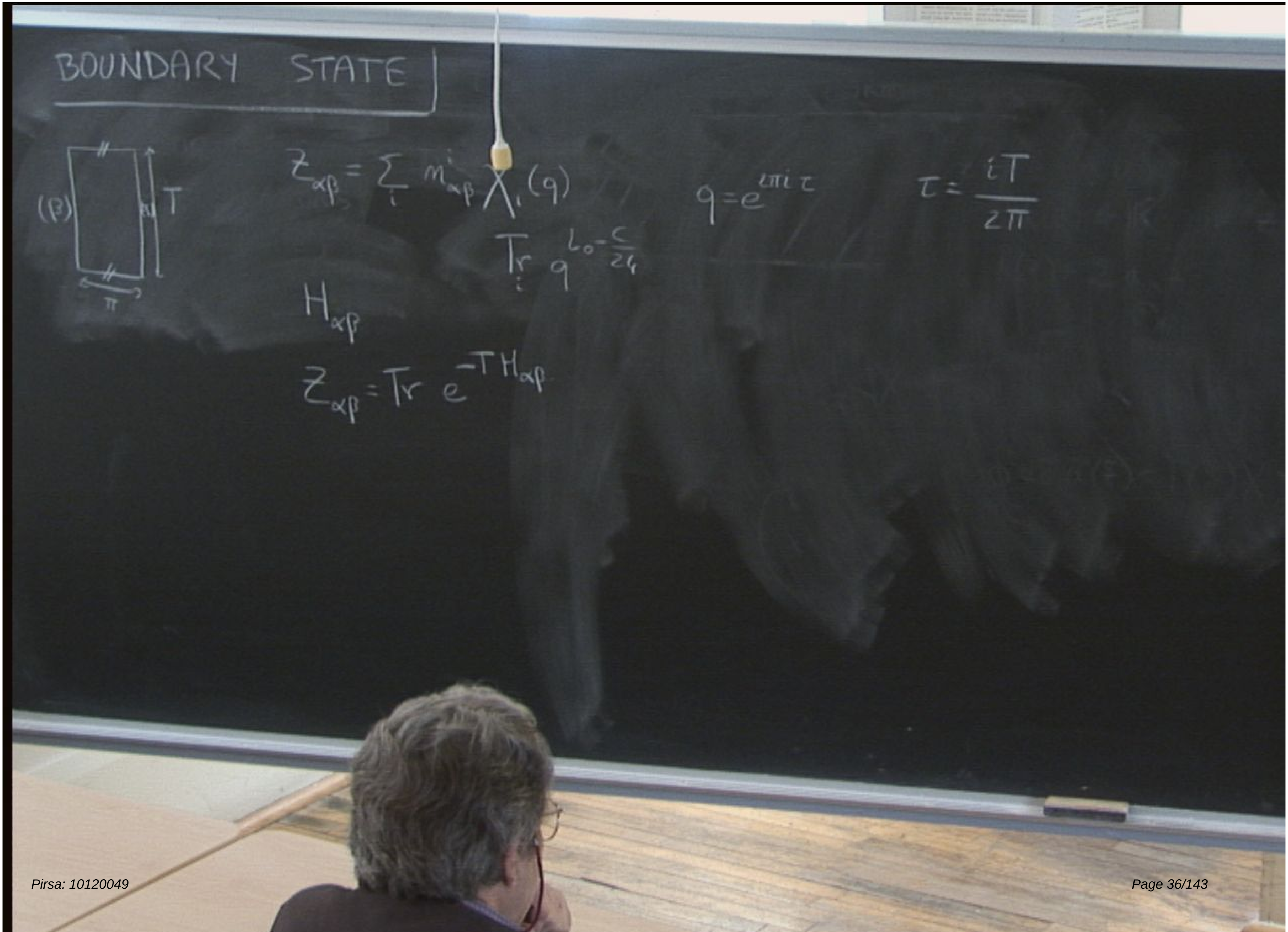
ARY OPERATORS
e^z
(β)
(α)
π
$\phi_{\beta\alpha}$(φ)
(β)
(α)
$\phi_{\beta\alpha}$
$\phi_{\alpha\beta}$
$\langle \phi_{\alpha\beta}\, \phi_{\beta\alpha} \rangle$
$\neq 0$

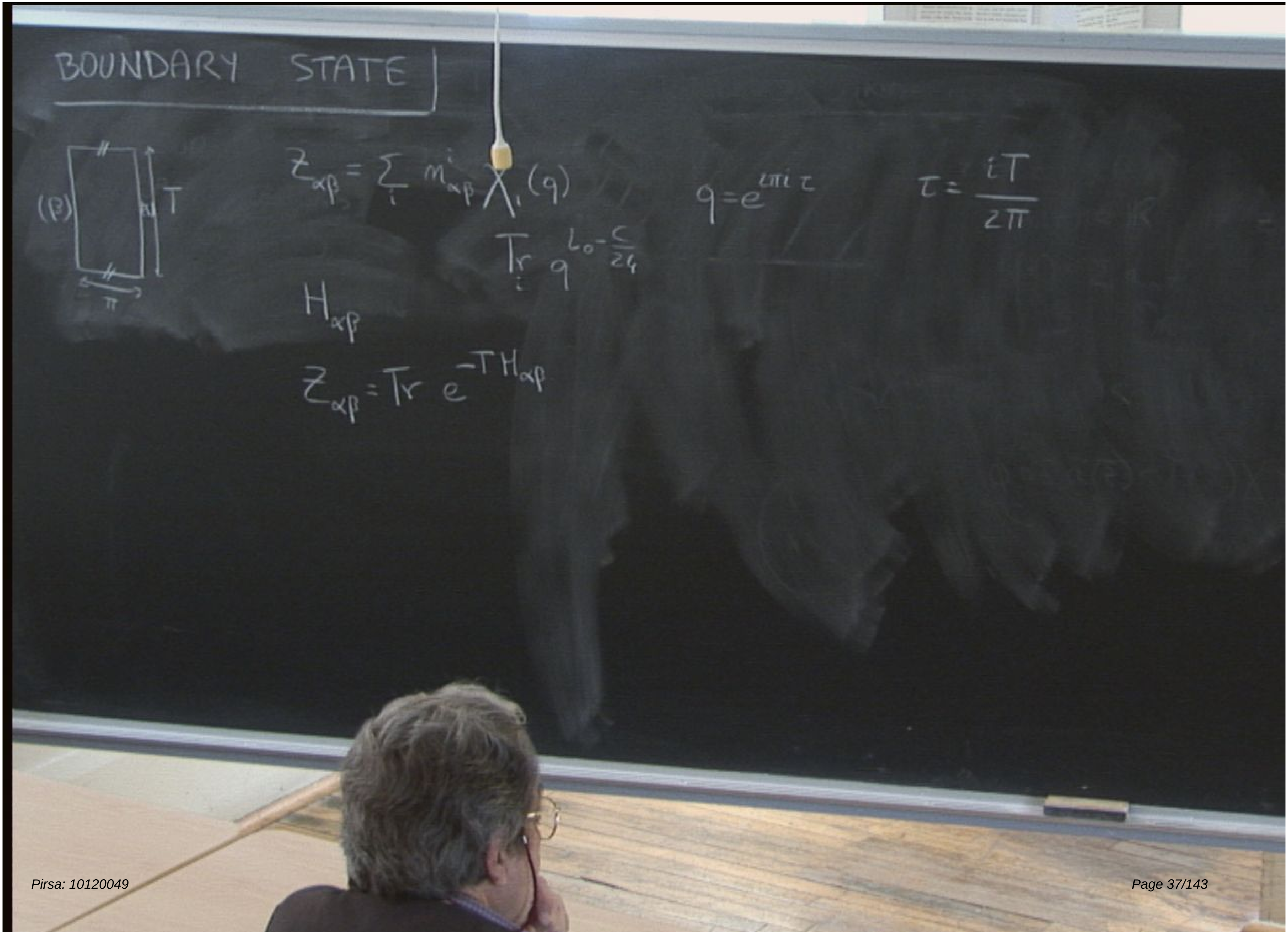

BOUNDARY STATE
T
π


BOUNDARY STATE

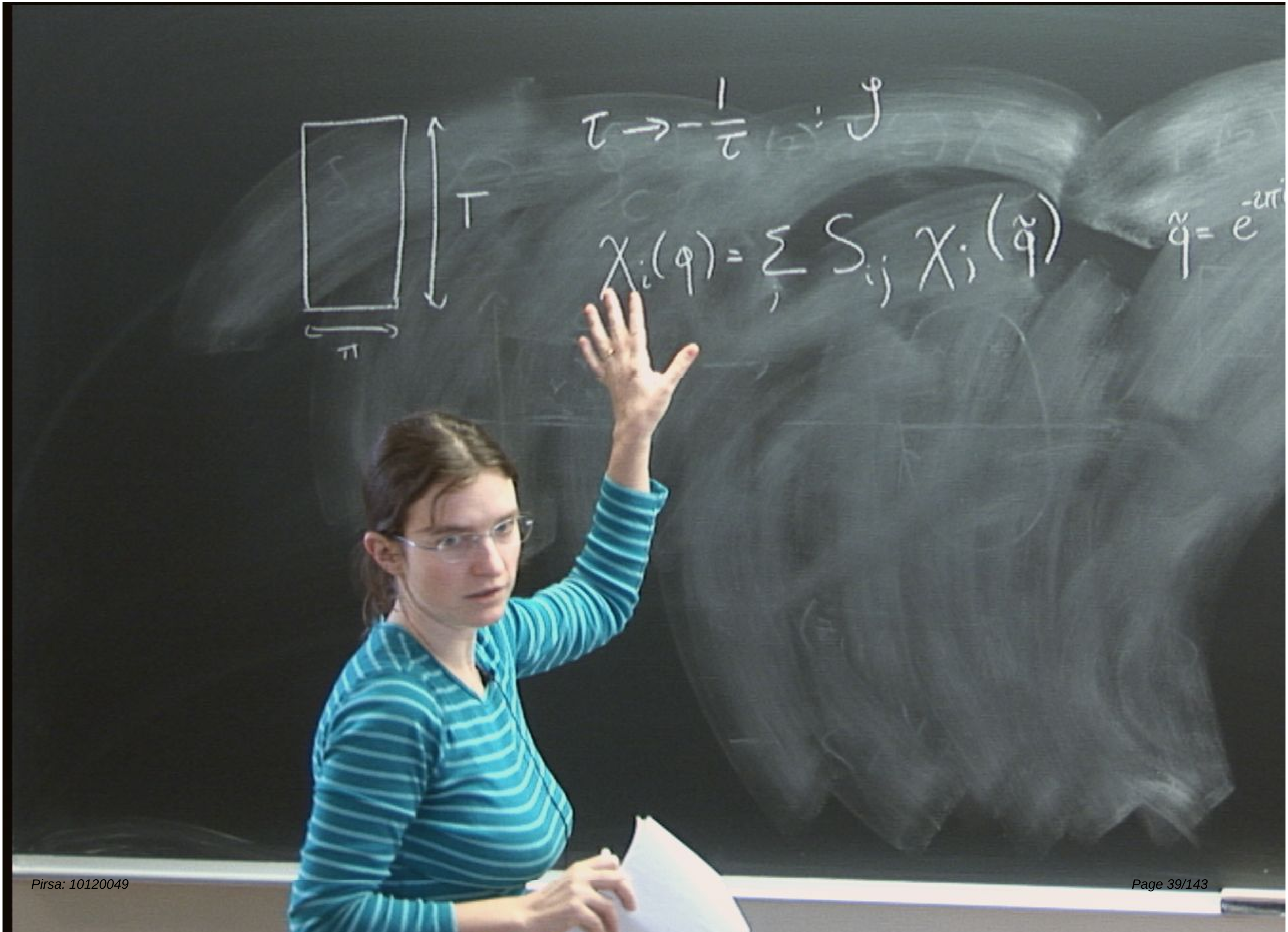

BOUNDARY STATE
(β)
T
π
$Z_{\alpha\beta} = \sum_i n^i_{\alpha\beta} \chi_i(q)$
$\mathrm{Tr}_i\, q^{L_0 - \frac{c}{24}}$
$q = e^{2\pi i \tau}$
$\tau = \frac{iT}{2\pi}$

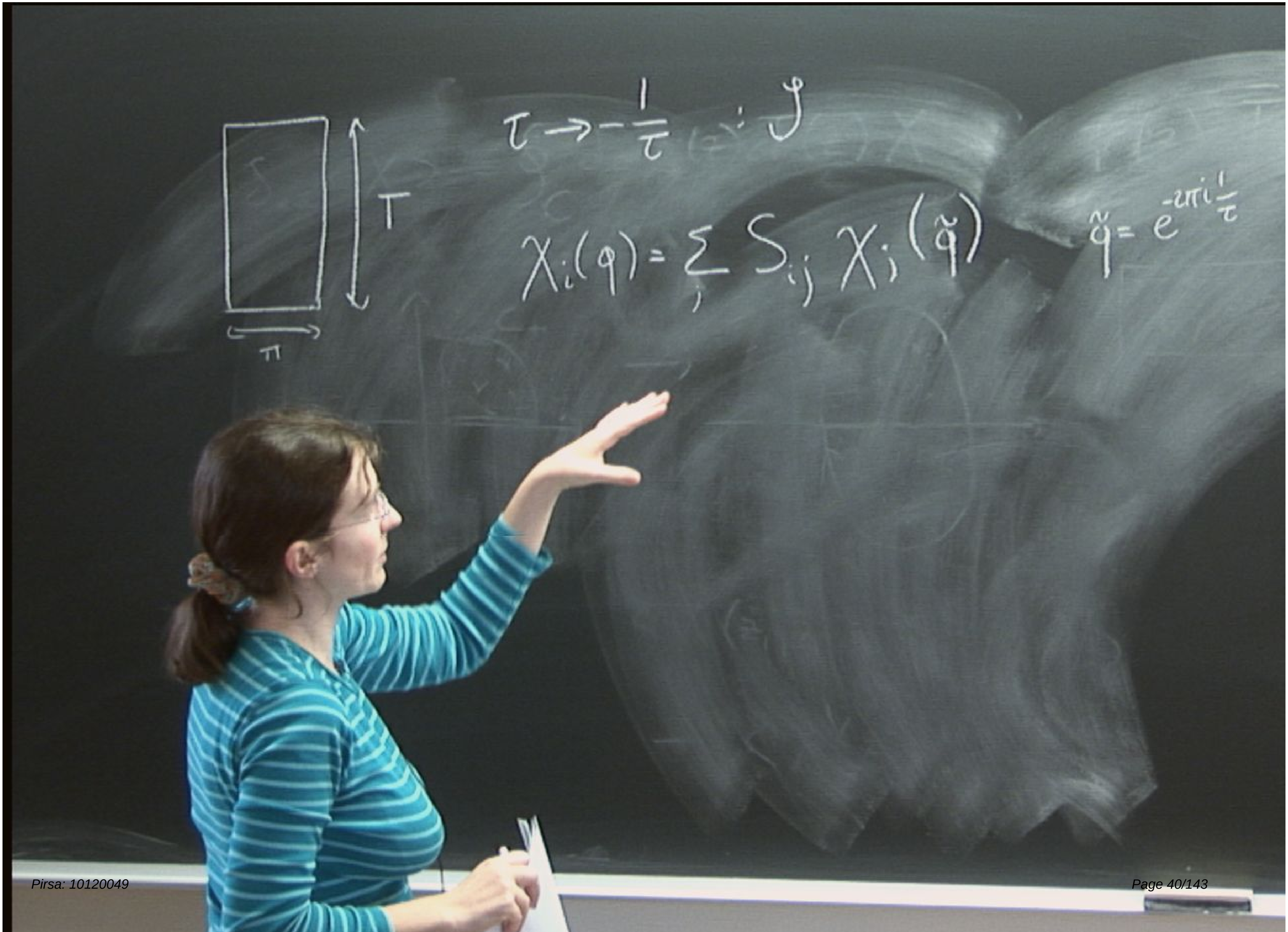

BOUNDARY STATE
(β)
T
$Z_{\alpha\beta} = \sum_i n^i_{\alpha\beta} \chi_i(q)$
$q = e^{2\pi i \tau}$
$\tau = \frac{iT}{2\pi}$
$\mathrm{Tr}_i\, q^{L_0 - \frac{c}{24}}$
$H_{\alpha\beta}$
$Z_{\alpha\beta} = \mathrm{Tr}\, e^{-T H_{\alpha\beta}}$

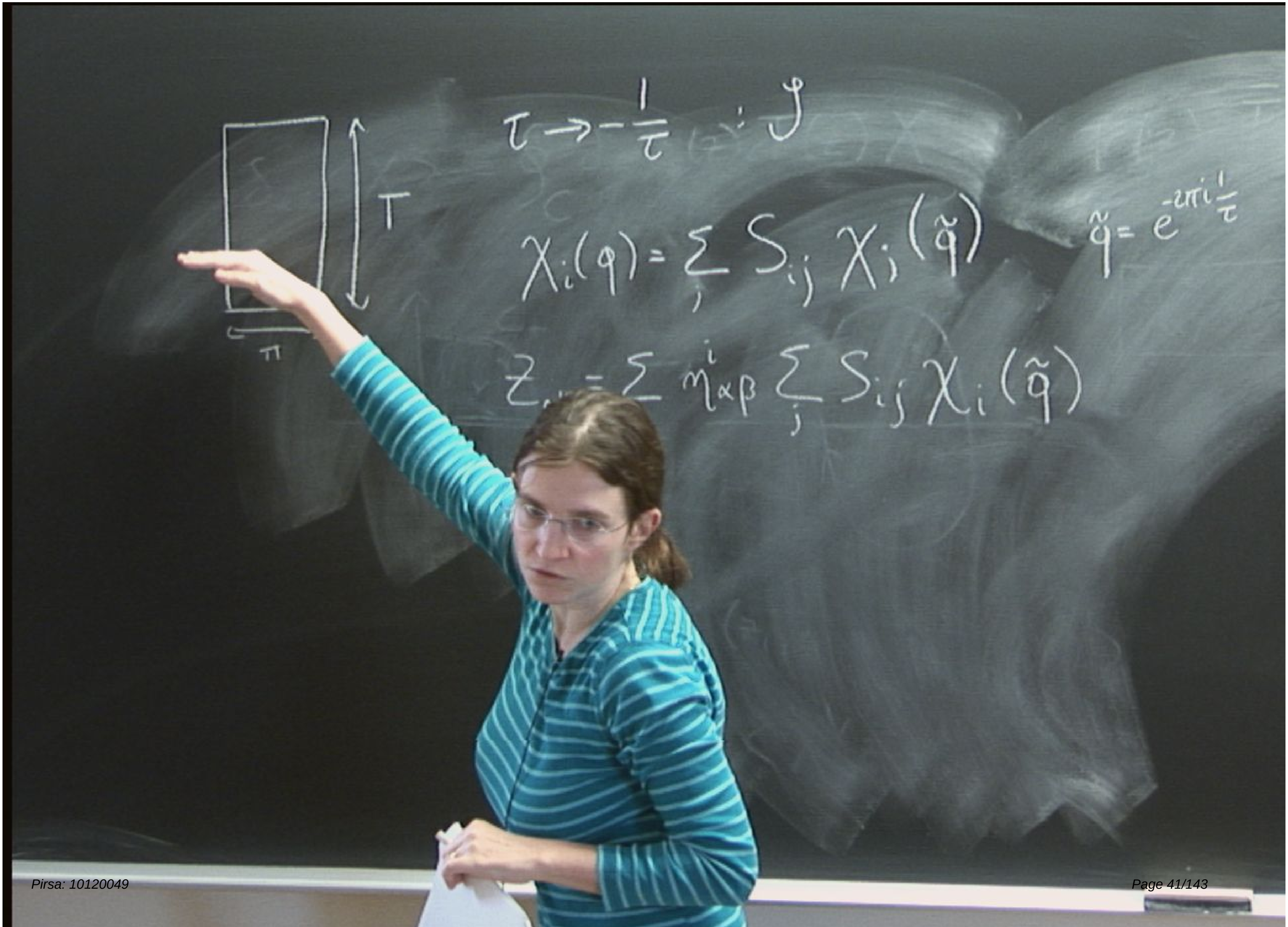

BOUNDARY STATE
(β)
T
π
$Z_{\alpha\beta} = \sum_i n^i_{\alpha\beta} \chi_i(q)$
$\mathrm{Tr}_i\, q^{L_0 - \frac{c}{24}}$
$q = e^{2\pi i \tau}$
$\tau = \frac{iT}{2\pi}$
$H_{\alpha\beta}$
$Z_{\alpha\beta} = \mathrm{Tr}\, e^{-T H_{\alpha\beta}}$

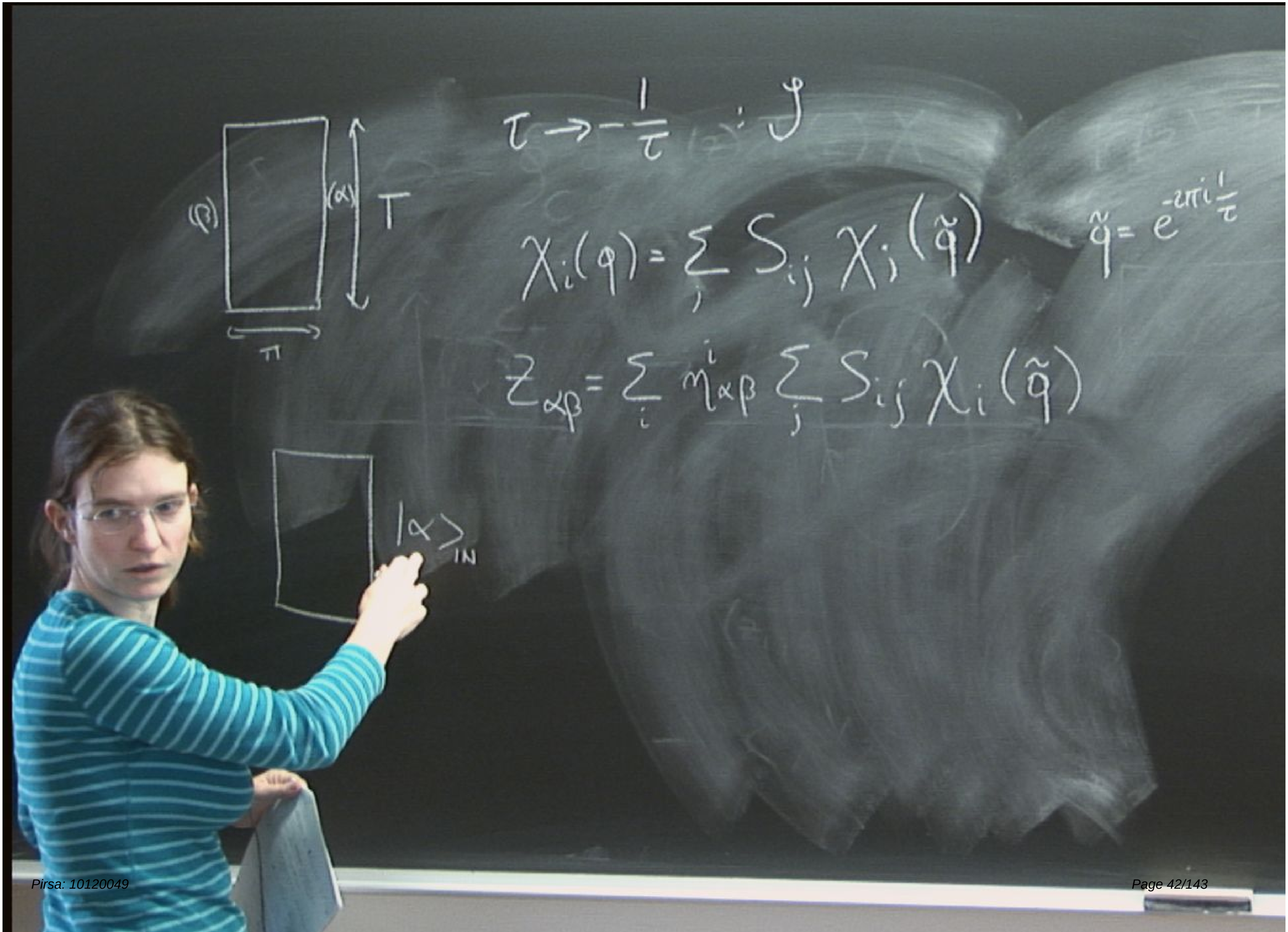

BOUNDARY STATE
(β)
T
π
$Z_{\alpha\beta} = \sum_i n^i_{\alpha\beta} \chi_i(q)$
$\mathrm{Tr}_i \, q^{L_0 - \frac{c}{24}}$
$q = e^{2\pi i \tau}$
$\tau = \frac{iT}{2\pi}$
$H_{\alpha\beta}$
$Z_{\alpha\beta} = \mathrm{Tr} \, e^{-T H_{\alpha\beta}}$


τ → −1/τ
T
π

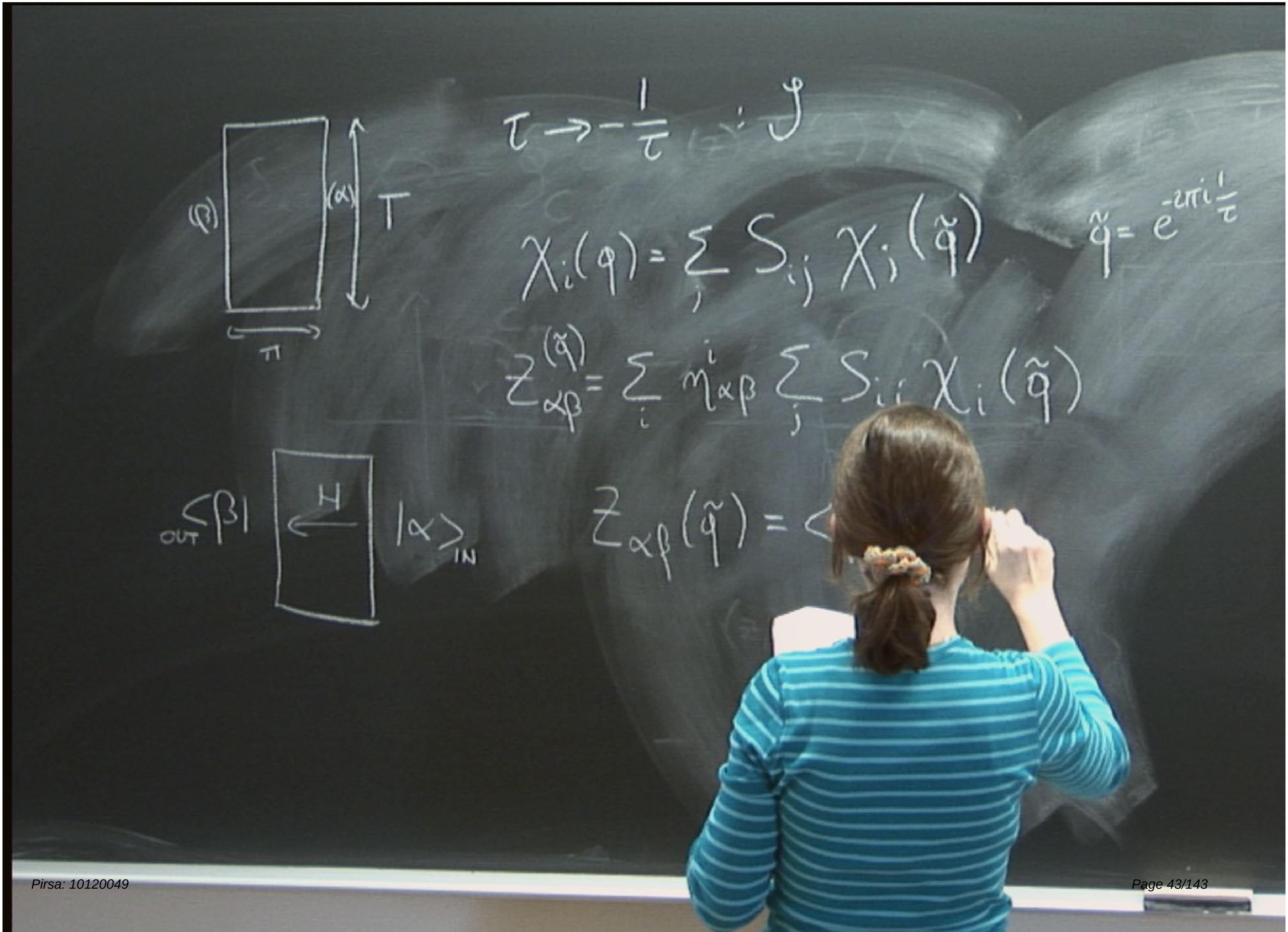

τ → -1/τ
T
π
χ_i(q) = Σ_j S_ij χ_j(q̃)
q̃ = e^-2πi


τ → -1/τ : S
T
π
χ_i(q) = Σ_j S_ij χ_j(q̃)
q̃ = e^{-2πi 1/τ}

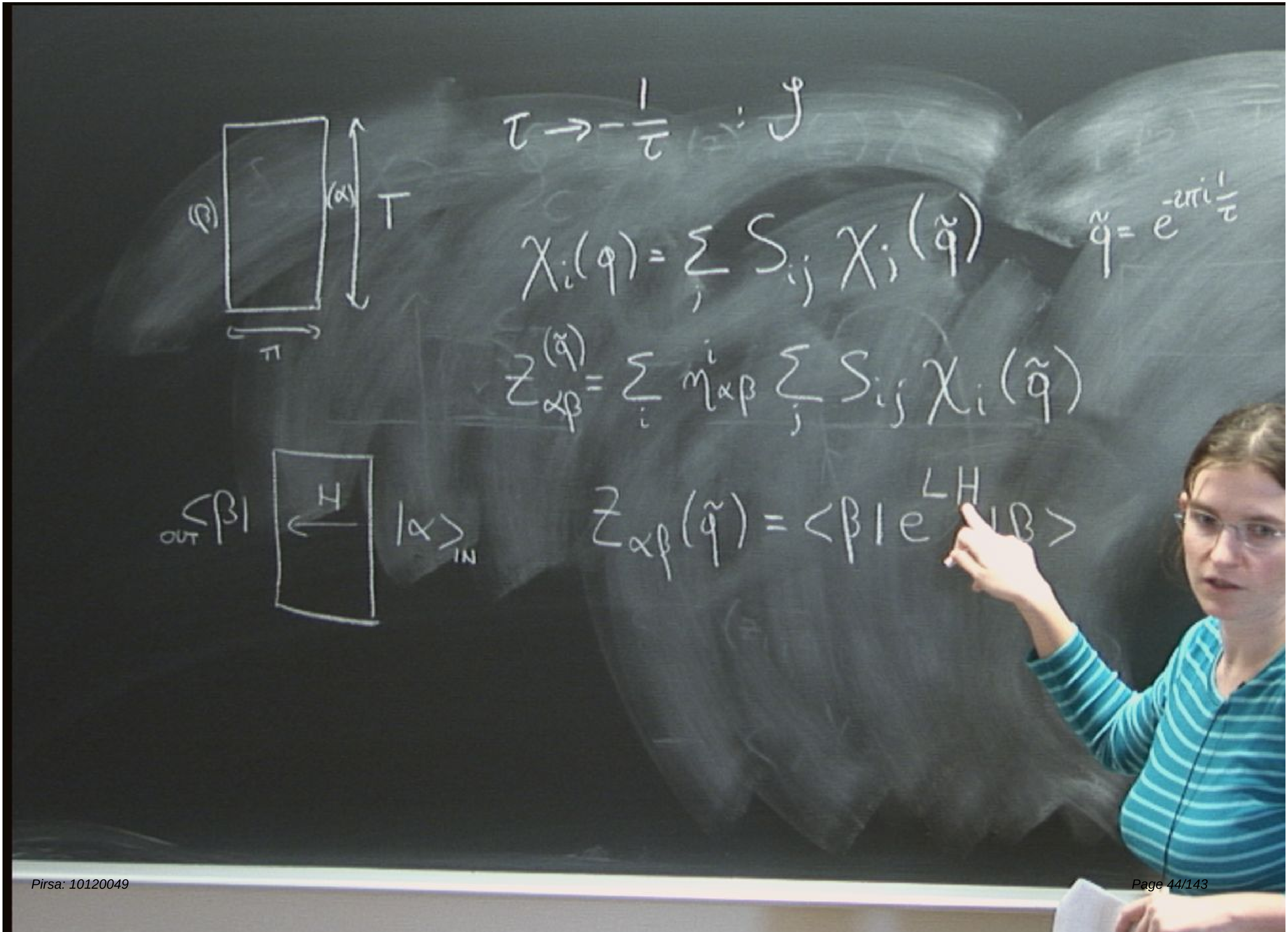

$\tau \to -\frac{1}{\tau}$: S
T
π
$\chi_i(q) = \sum_j S_{ij} \chi_j(\tilde{q})$
$\tilde{q} = e^{-2\pi i \frac{1}{\tau}}$

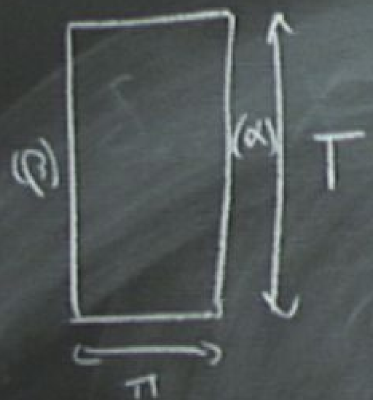


$\tau \to -\frac{1}{\tau}$: $\mathcal{S}$
(β) (α) T
π
$\chi_i(q) = \sum_j S_{ij}\, \chi_j(\tilde{q})$
$\tilde{q} = e^{-2\pi i \frac{1}{\tau}}$
$Z_{\alpha\beta} = \sum_i \eta^i_{\alpha\beta} \sum_j S_{ij}\, \chi_i(\tilde{q})$
$|\alpha\rangle_{IN}$



$\tau \to -\frac{1}{\tau}$: $\mathcal{S}$
(β) (α) T
π
$\chi_i(q) = \sum_j S_{ij}\, \chi_j(\tilde{q})$
$\tilde{q} = e^{-2\pi i \frac{1}{\tau}}$
$Z^{(\tilde{q})}_{\alpha\beta} = \sum_i \eta^i_{\alpha\beta} \sum_j S_{ij}\, \chi_i(\tilde{q})$
${}_{OUT}\langle\beta|$ H $|\alpha\rangle_{IN}$
$Z_{\alpha\beta}(\tilde{q}) = \langle$

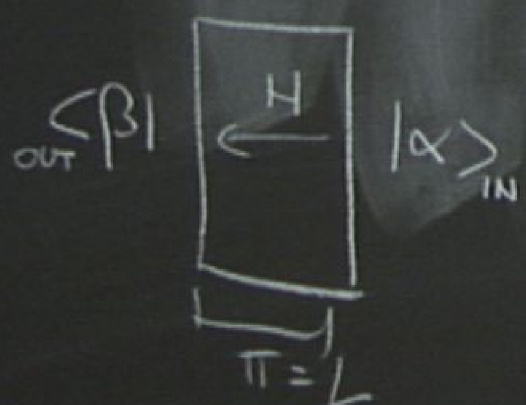

τ → −1/τ : 𝒮
(β) (α) T
π
χ_i(q) = Σ_j S_ij χ_j(q̃)
q̃ = e^{-2πi 1/τ}
Z_αβ(q̃) = Σ_i η^i_αβ Σ_j S_ij χ_i(q̃)
OUT ⟨β| H |α⟩ IN
Z_αβ(q̃) = ⟨β| e^{LH} |β⟩



$$\tau \to -\frac{1}{\tau} : S$$

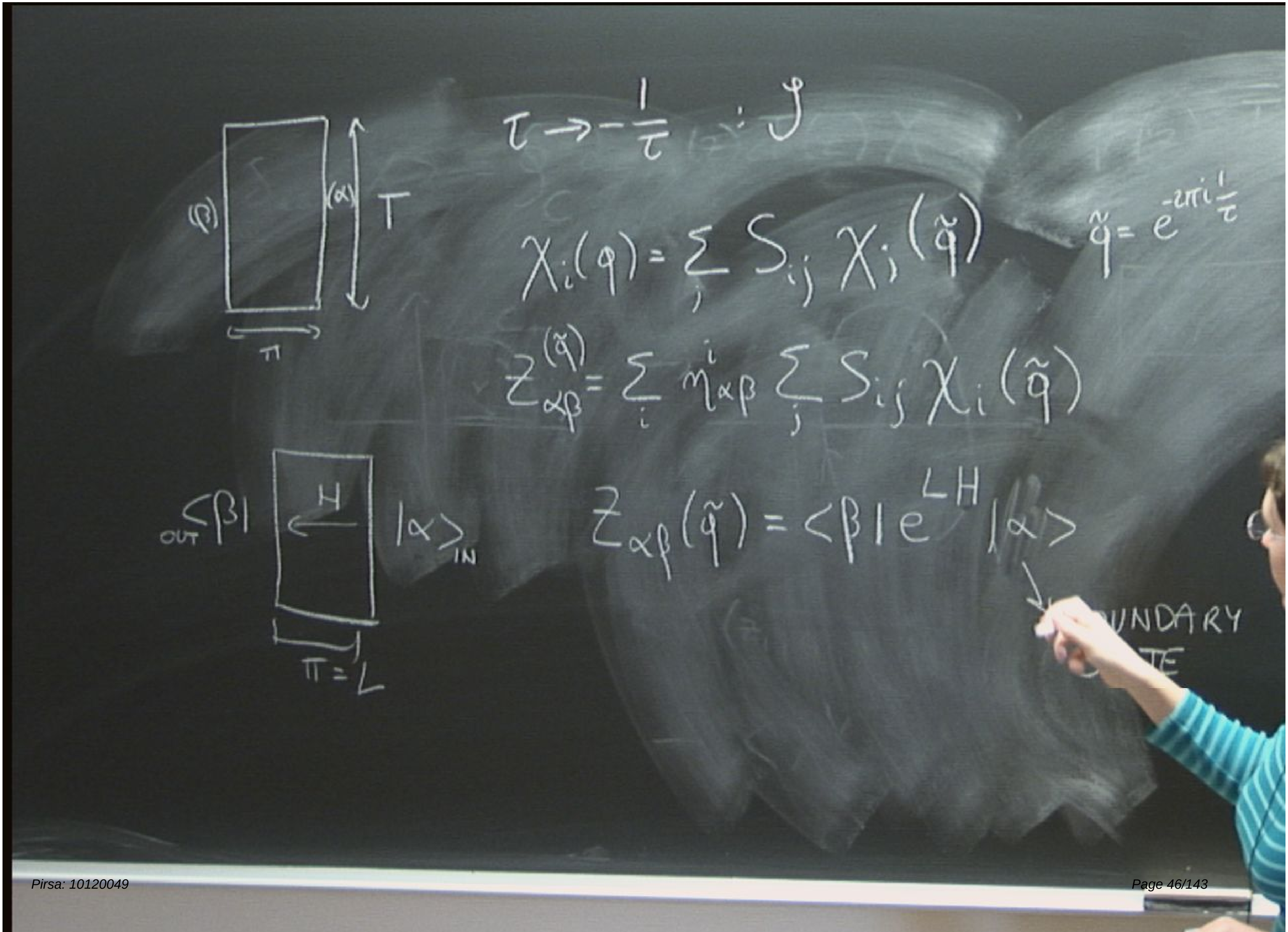
$$\chi_i(q) = \sum_j S_{ij}\, \chi_j(\tilde{q}) \qquad \tilde{q} = e^{-2\pi i \frac{1}{\tau}}$$

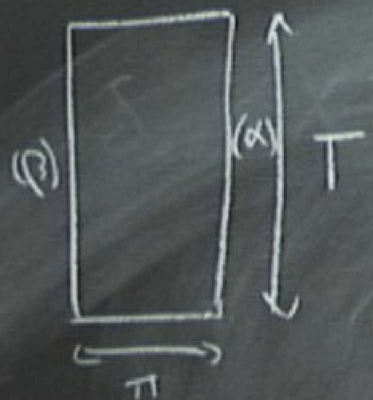
$$Z^{(\tilde{q})}_{\alpha\beta} = \sum_i \eta^i_{\alpha\beta} \sum_j S_{ij}\, \chi_i(\tilde{q})$$



$$Z_{\alpha\beta}(\tilde{q}) = \langle\beta| e \ldots$$

BOUN… STATE

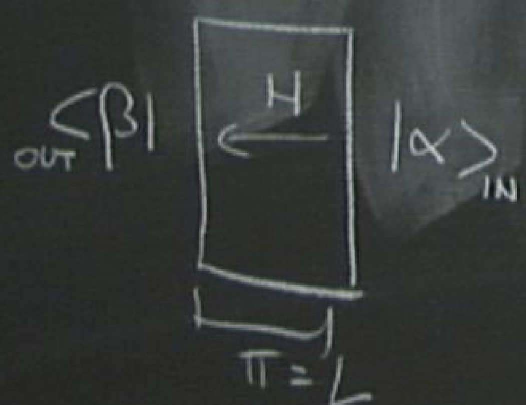

τ → −1/τ : S
(β)
(α)
T
π
χ_i(q) = Σ_j S_ij χ_j(q̃)
q̃ = e^{-2πi 1/τ}
Z^(q̃)_αβ = Σ_i η^i_αβ Σ_j S_ij χ_j(q̃)
out⟨β|
H
|α⟩ in
π = L
Z_αβ(q̃) = ⟨β| e^{LH} |α⟩
UNDARY



$$\tau \to -\frac{1}{\tau} : \mathcal{S}$$

$$\chi_i(q) = \sum_j S_{ij}\, \chi_j(\tilde{q}) \qquad \tilde{q} = e^{-2\pi i \frac{1}{\tau}}$$

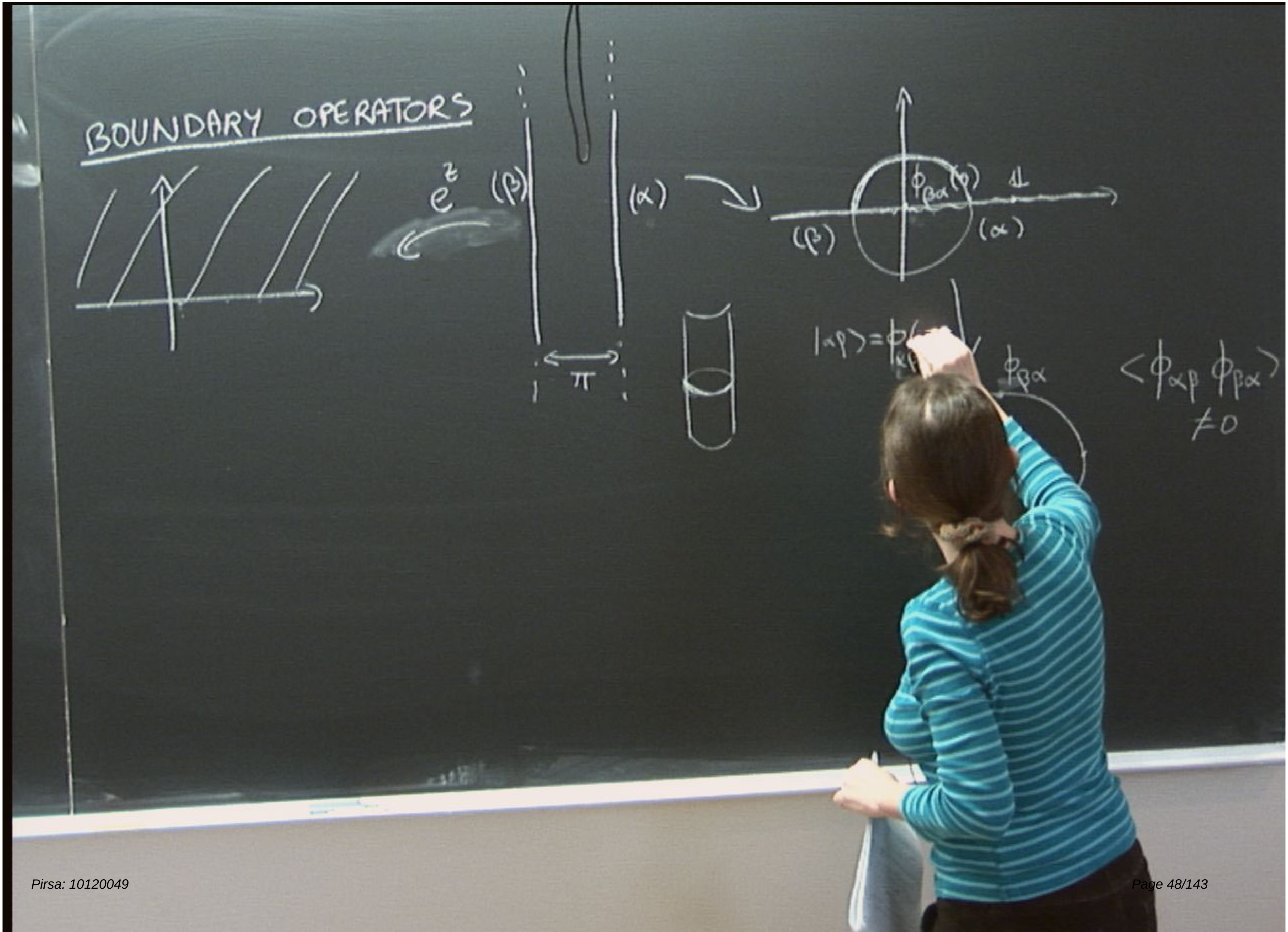
$$Z^{(\tilde{q})}_{\alpha\beta} = \sum_i \eta^i_{\alpha\beta} \sum_j S_{ij}\, \chi_i(\tilde{q})$$

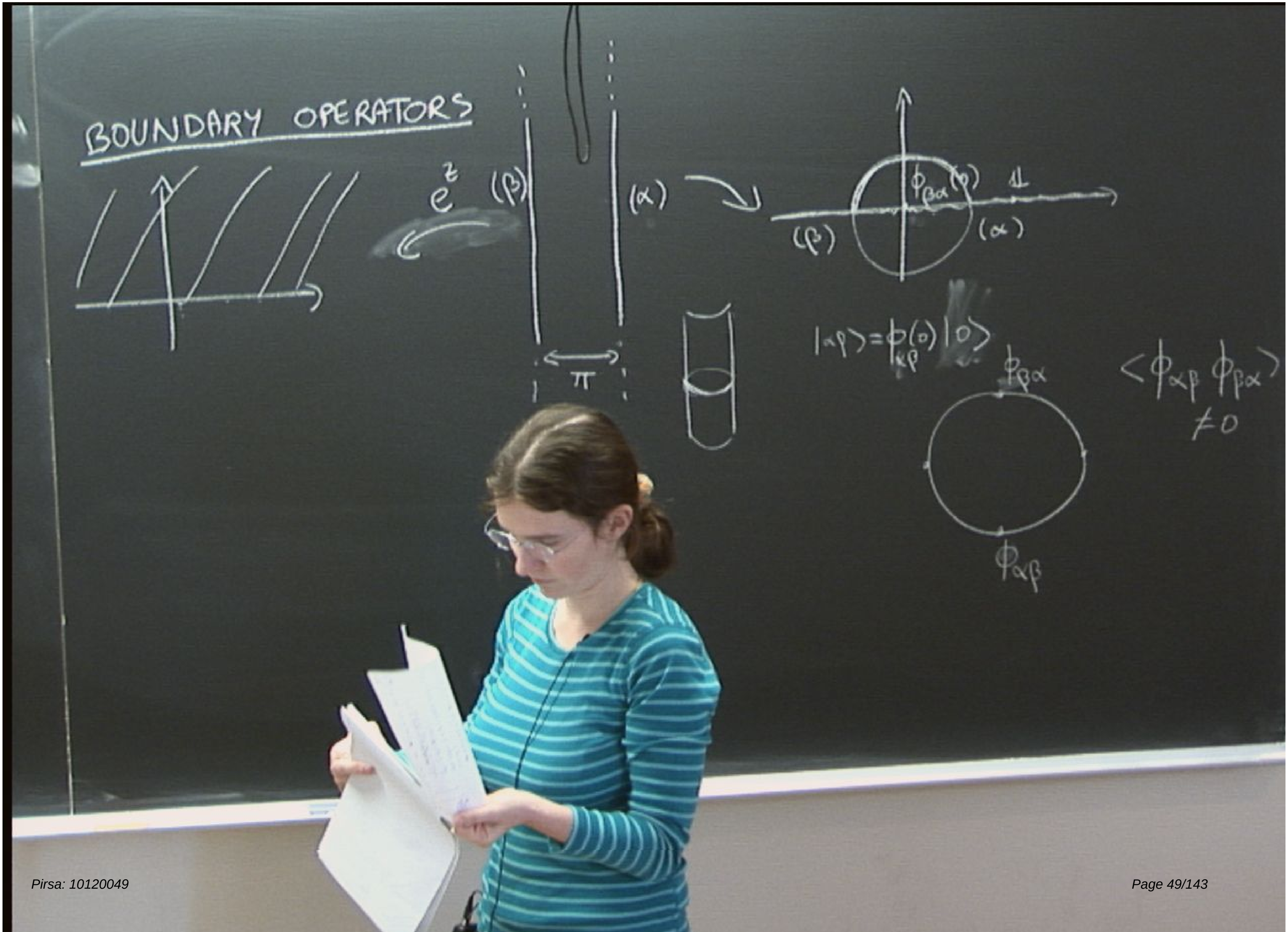


$$Z_{\alpha\beta}(\tilde{q}) = \langle\beta| e^{LH} |\alpha\rangle$$

$|\alpha\rangle$ → BOUNDARY STATE

$$(T + \bar{T})|\alpha\rangle = 0$$


BOUNDARY OPERATORS
e^z
(β)
(α)
π
(β)
(α)
|αβ>=φ
φβα
<φαβ φβα>
≠0

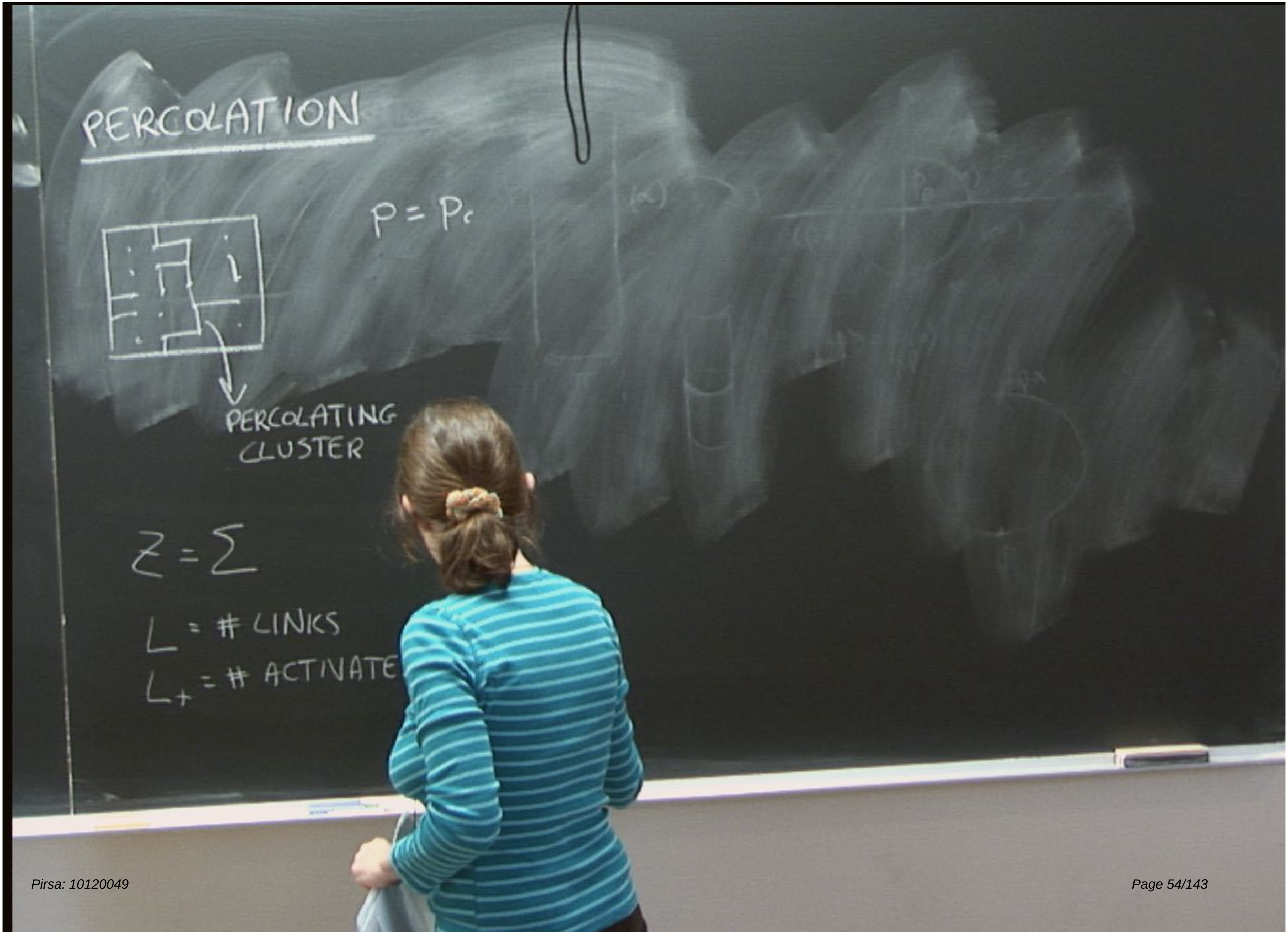

BOUNDARY OPERATORS
e^z
(β)
(α)
π
φβα(0)
1
(β)
(α)
|αβ> = φαβ(0)|0>
φβα
φαβ
<φαβ φβα>
≠ 0

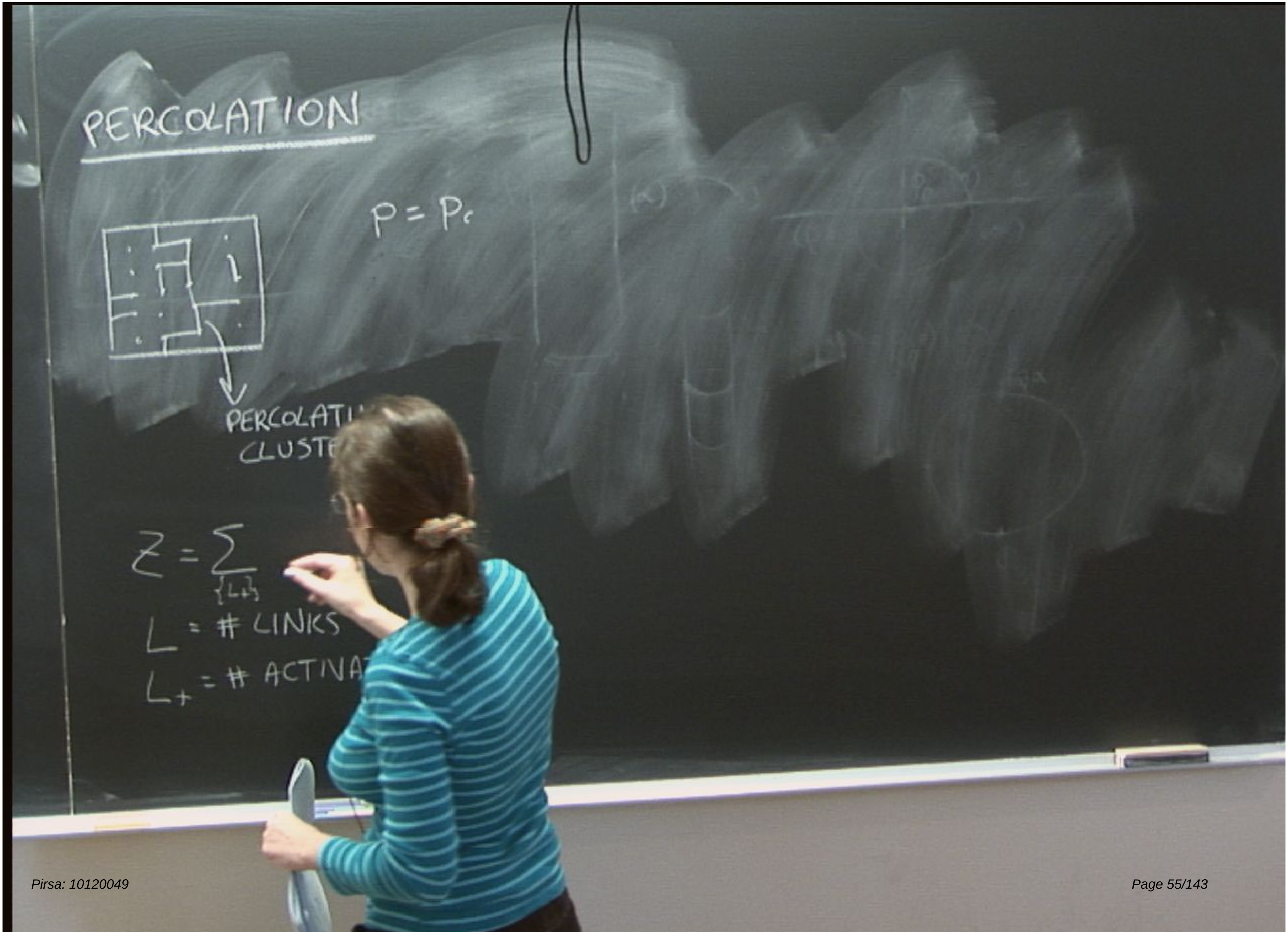

PERCOLATION

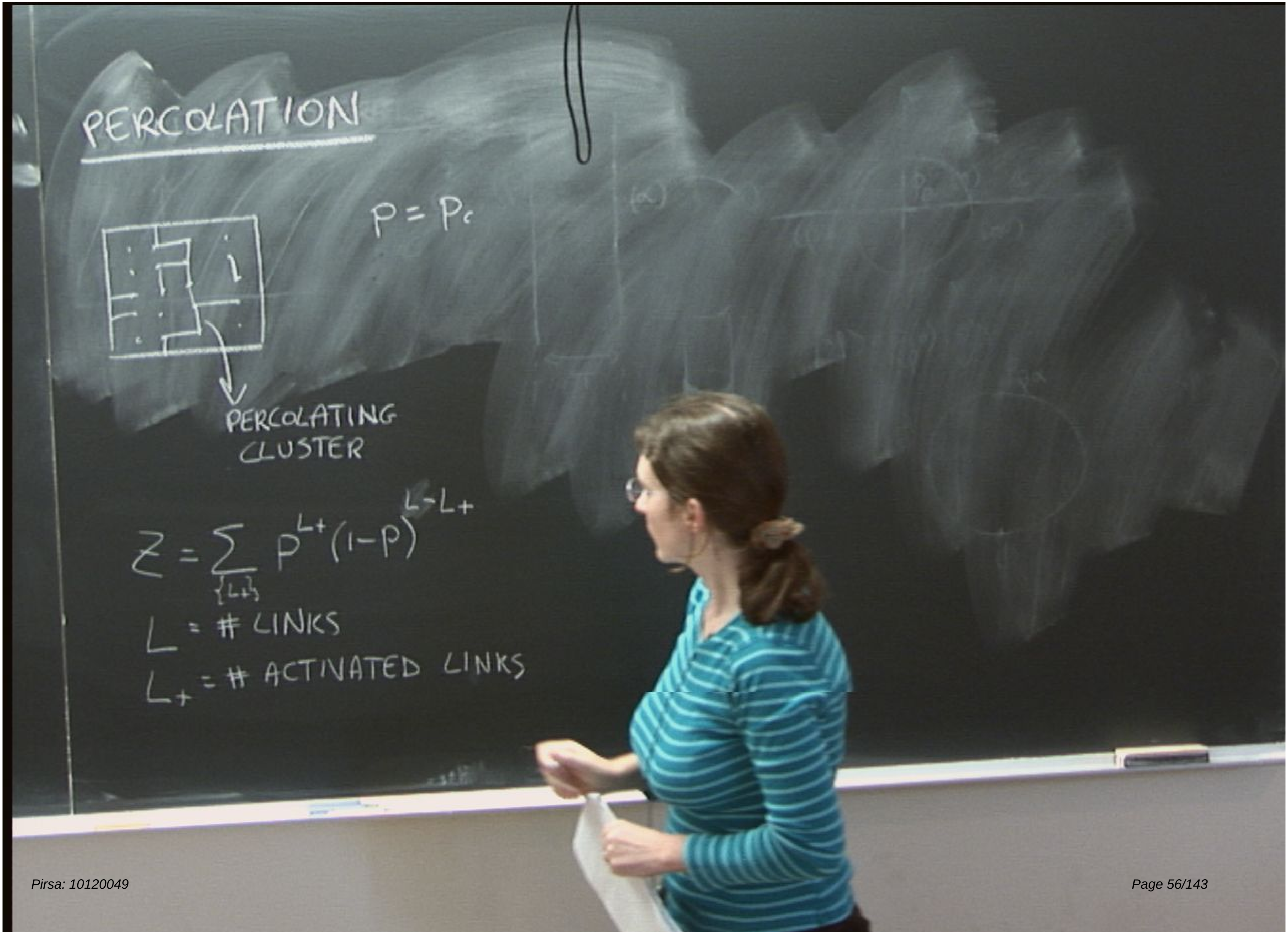

PERCOLATION
P

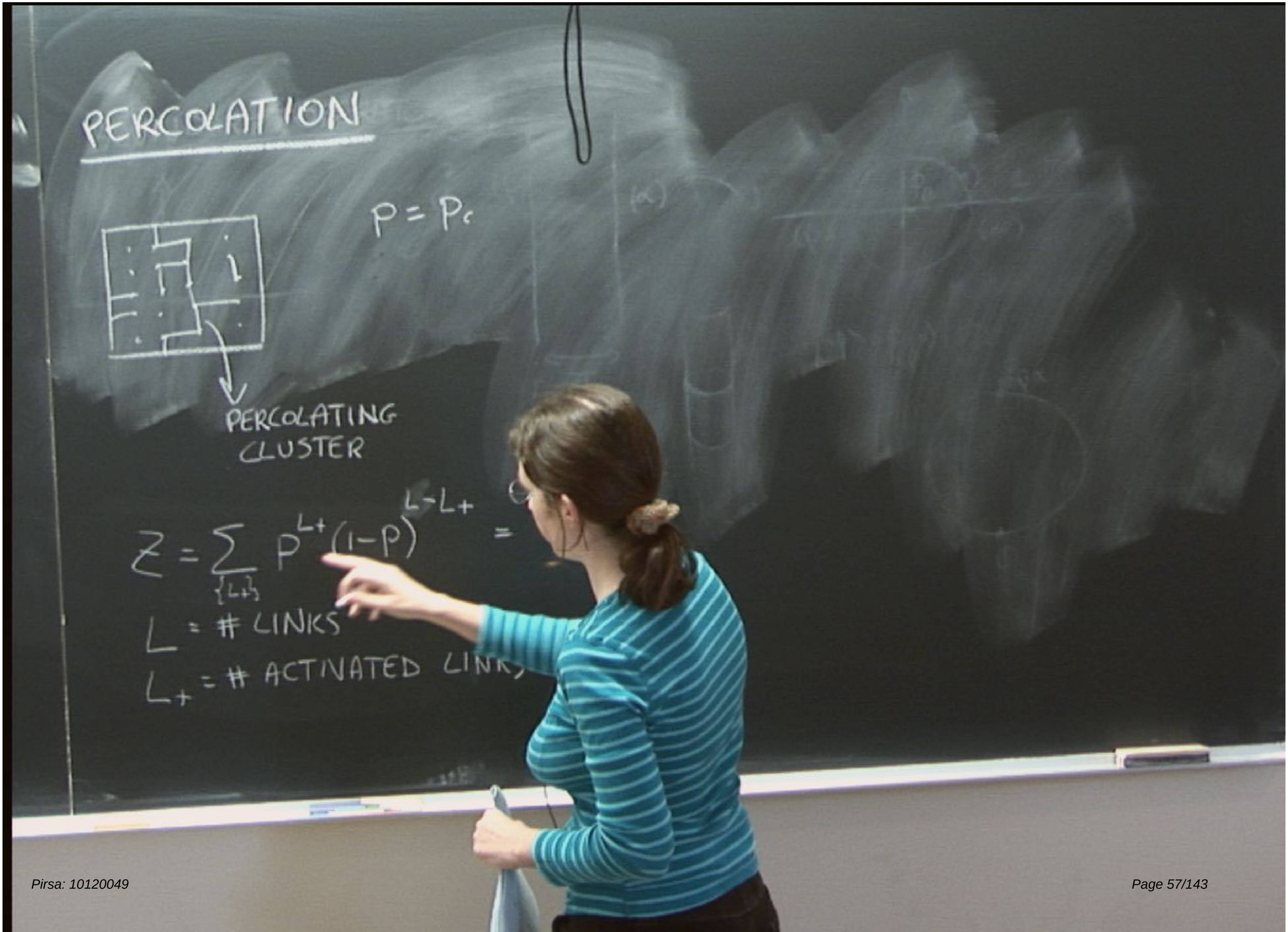

PERCOLATION
P = Pc
PERCOLATING
CLUSTER


PERCOLATION
P = Pc
PERCOLATING
CLUSTER

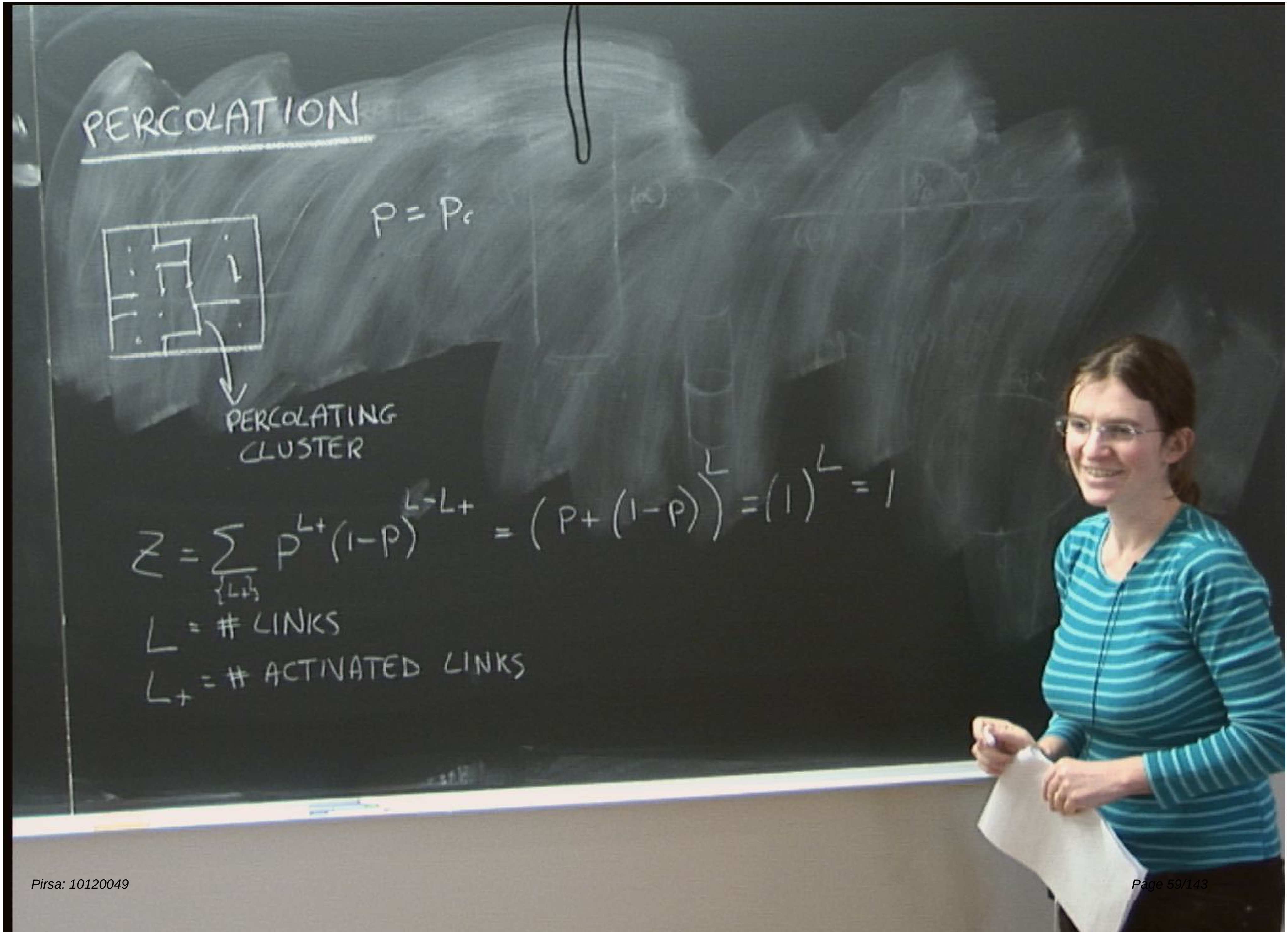

PERCOLATION
P = Pc
PERCOLATING
CLUSTER
Z = Σ
L = # LINKS
L+ = # ACTIVATE

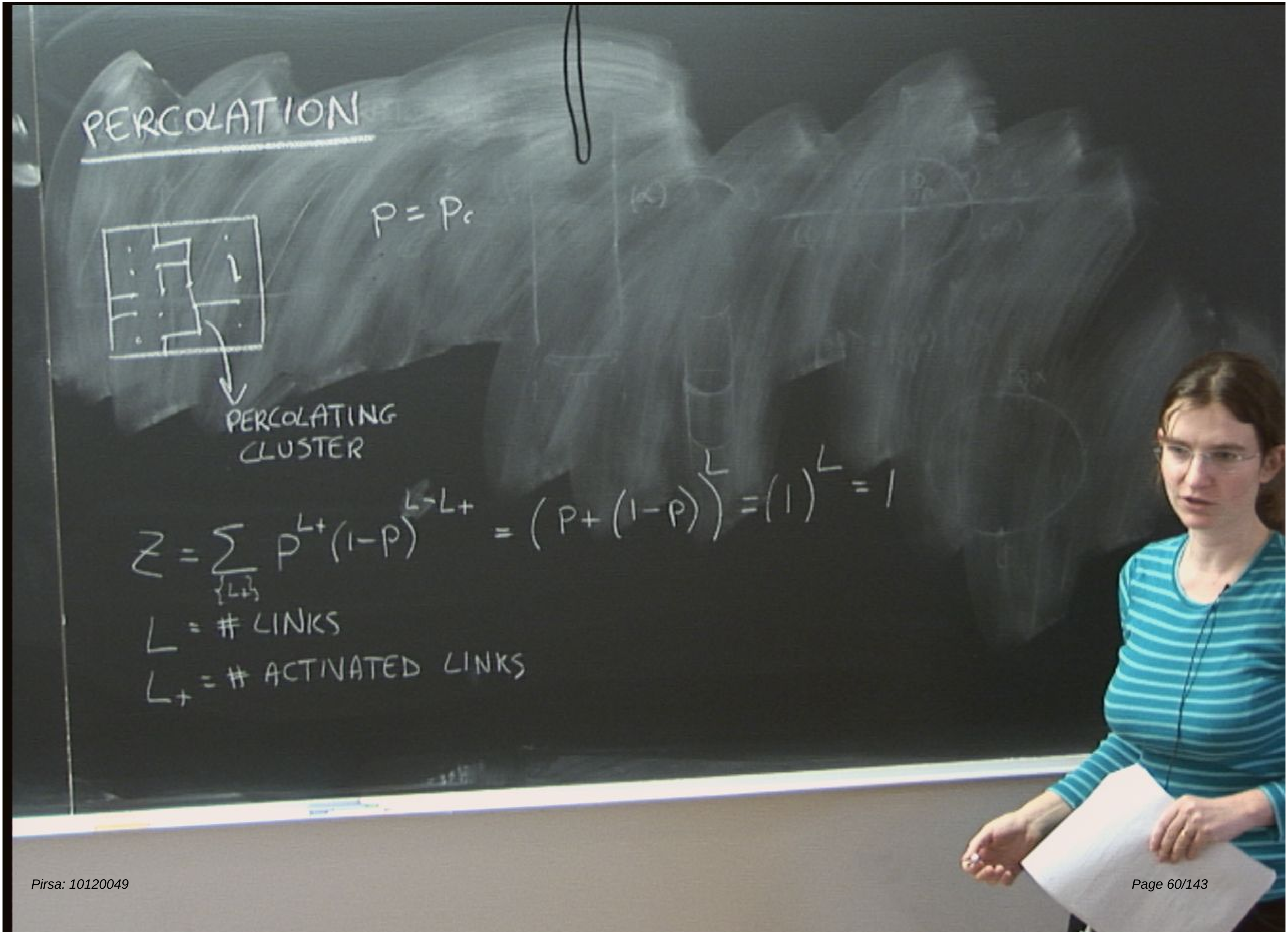

PERCOLATION
P = Pc
PERCOLATI
CLUSTE
Z = Σ {L+}
L = # LINKS
L+ = # ACTIVA

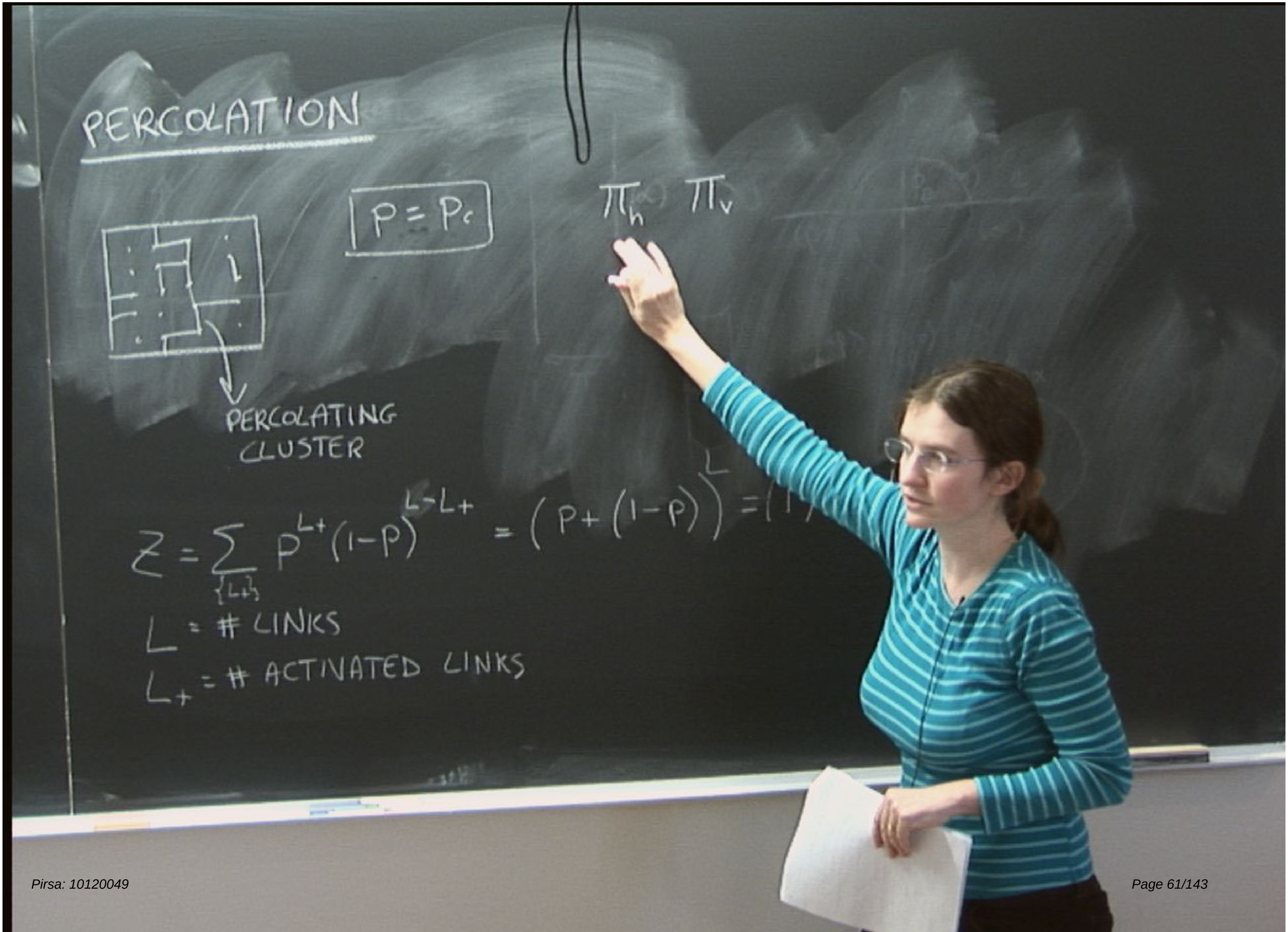

PERCOLATION
P = Pc
i
PERCOLATING CLUSTER
Z = ∑_{L+} p^{L+} (1−p)^{L−L+}
L = # LINKS
L+ = # ACTIVATED LINKS

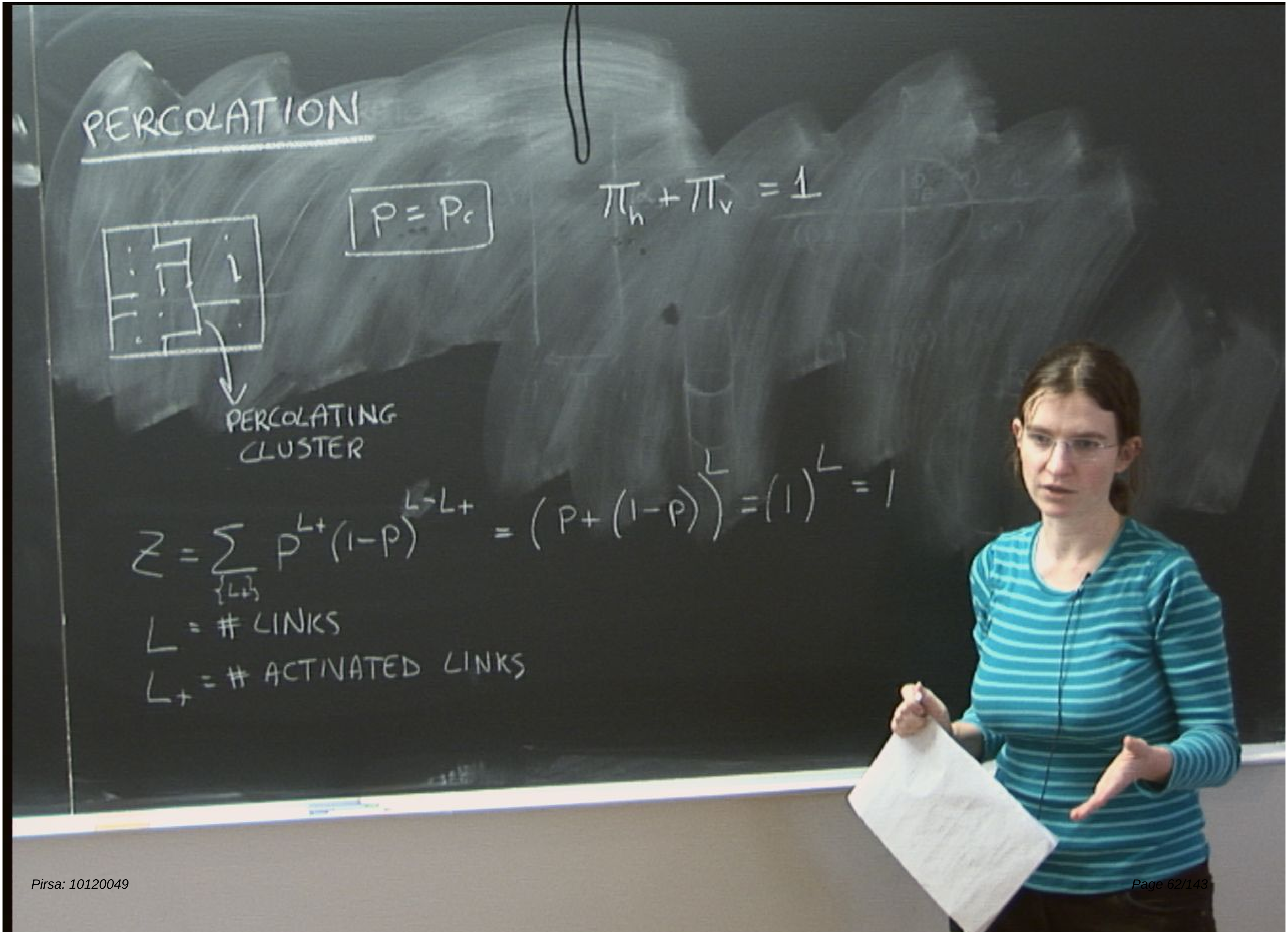

PERCOLATION
p = pc
PERCOLATING
CLUSTER
Z = Σ{L+} p^L+ (1-p)^L-L+ =
L = # LINKS
L+ = # ACTIVATED LINK

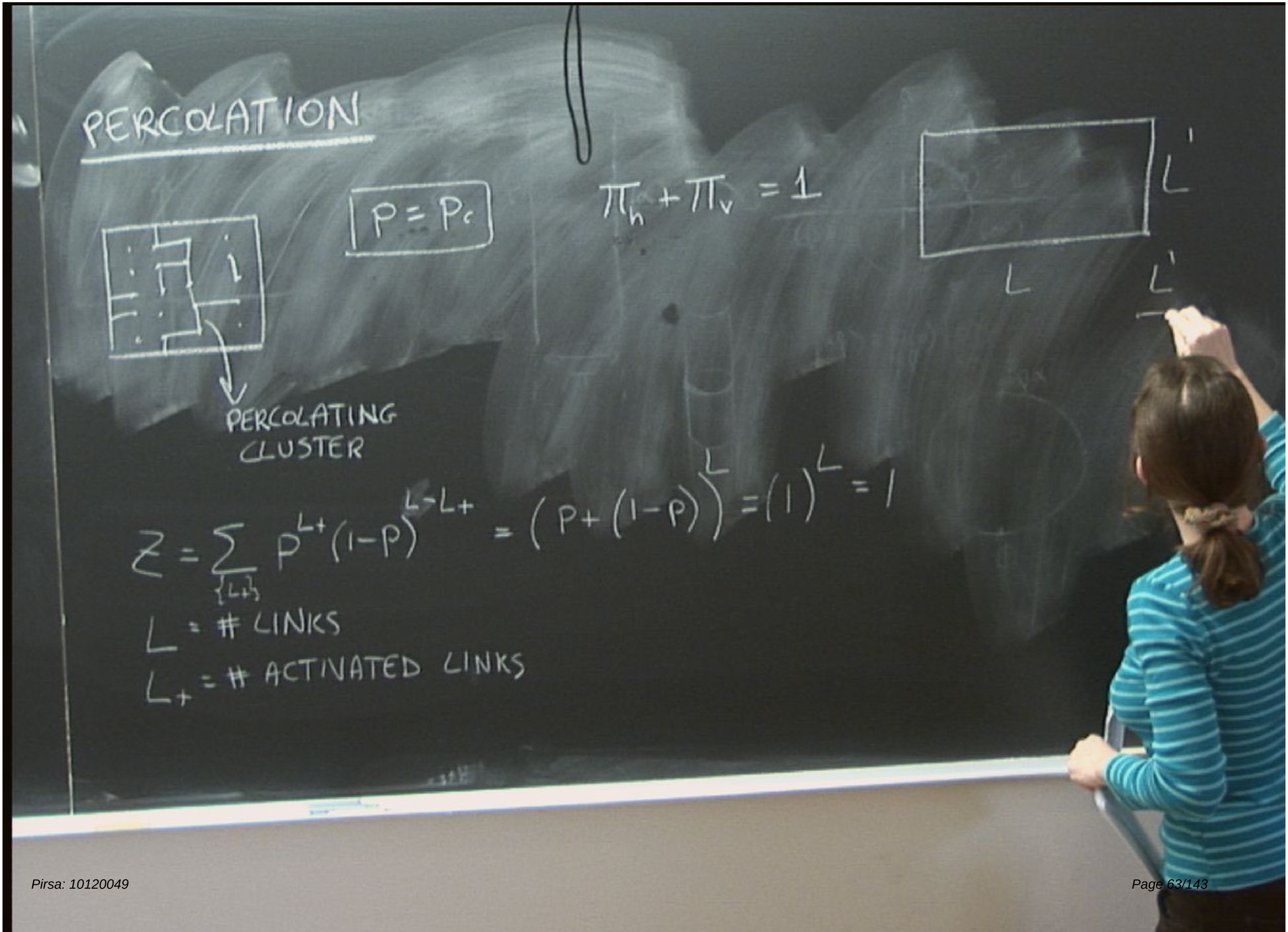

PERCOLATION
p = p_c
PERCOLATING CLUSTER
Z = Σ_{L+} p^{L+} (1−p)^{L−L+} = (p + (1−p))^L =
L = # LINKS
L+ = # ACTIVATED LINKS

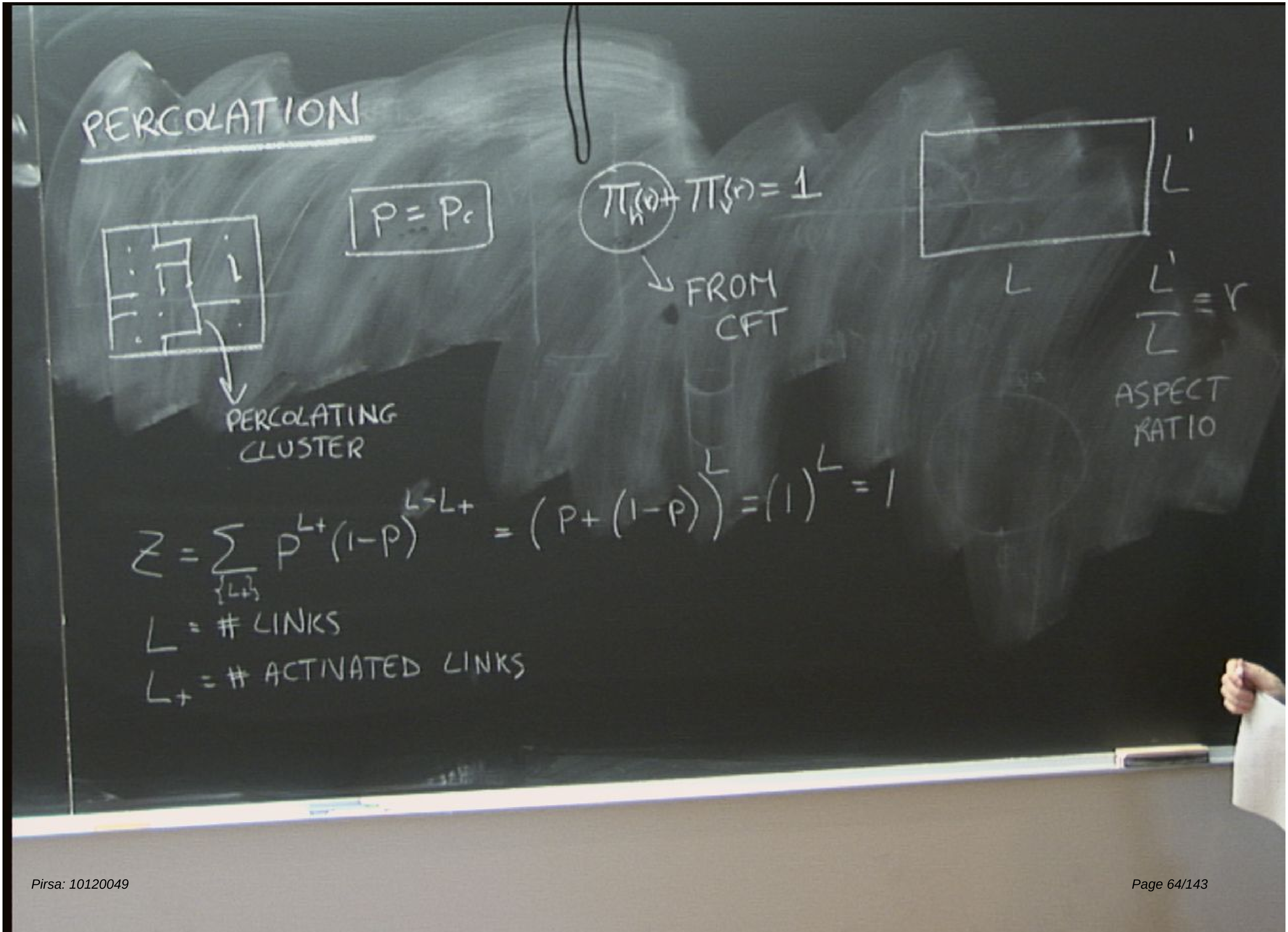

PERCOLATION
P = Pc
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1−p)^(L−L+) = (p + (1−p))^L = (1)^L = 1
L = # LINKS
L+ = # ACTIVATED LINKS

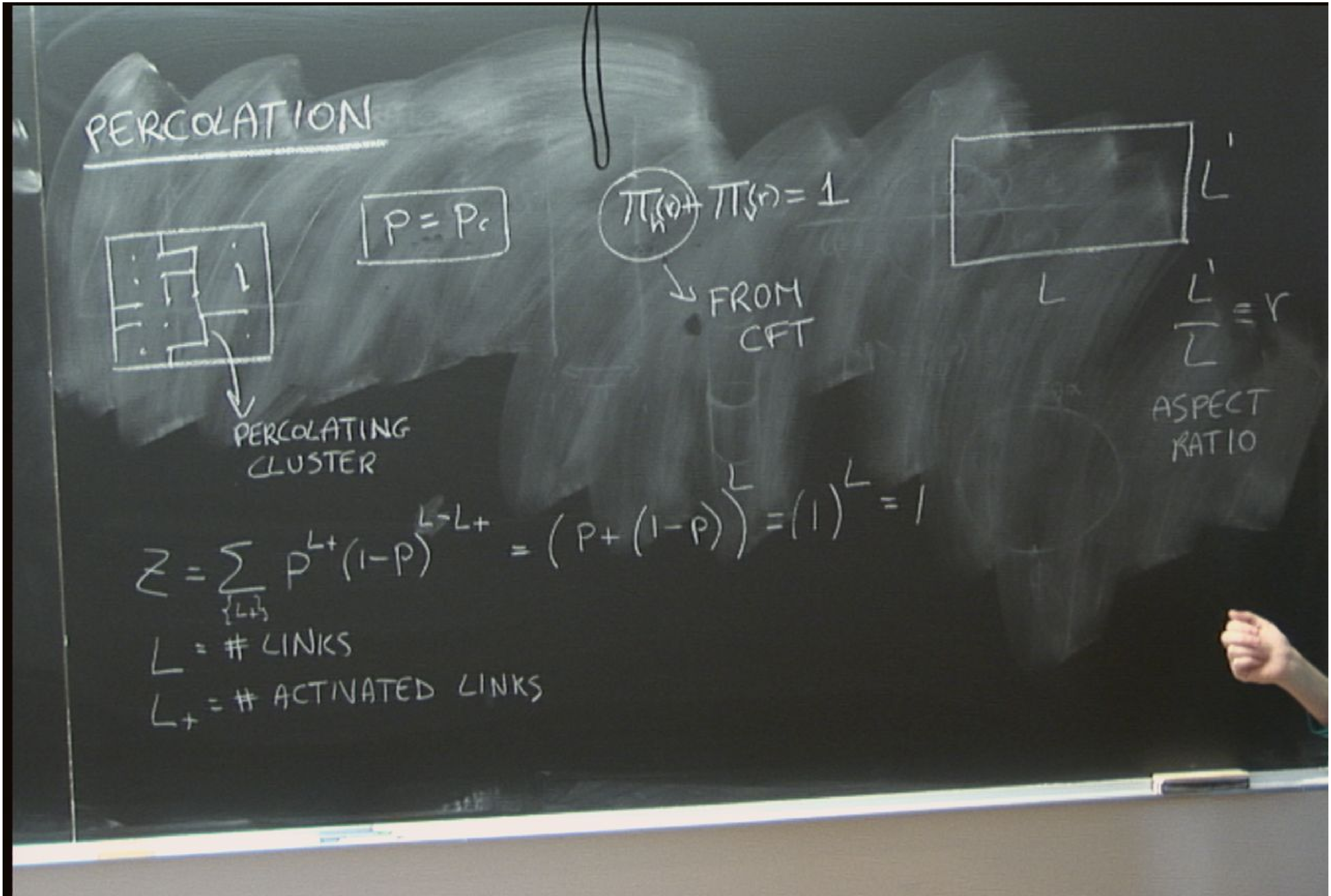

PERCOLATION
P = Pc
PERCOLATING CLUSTER
Z = ∑ p^L+ (1−p)^L−L+ = (p + (1−p))^L = (1)^L = 1
{L+}
L = # LINKS
L+ = # ACTIVATED LINKS

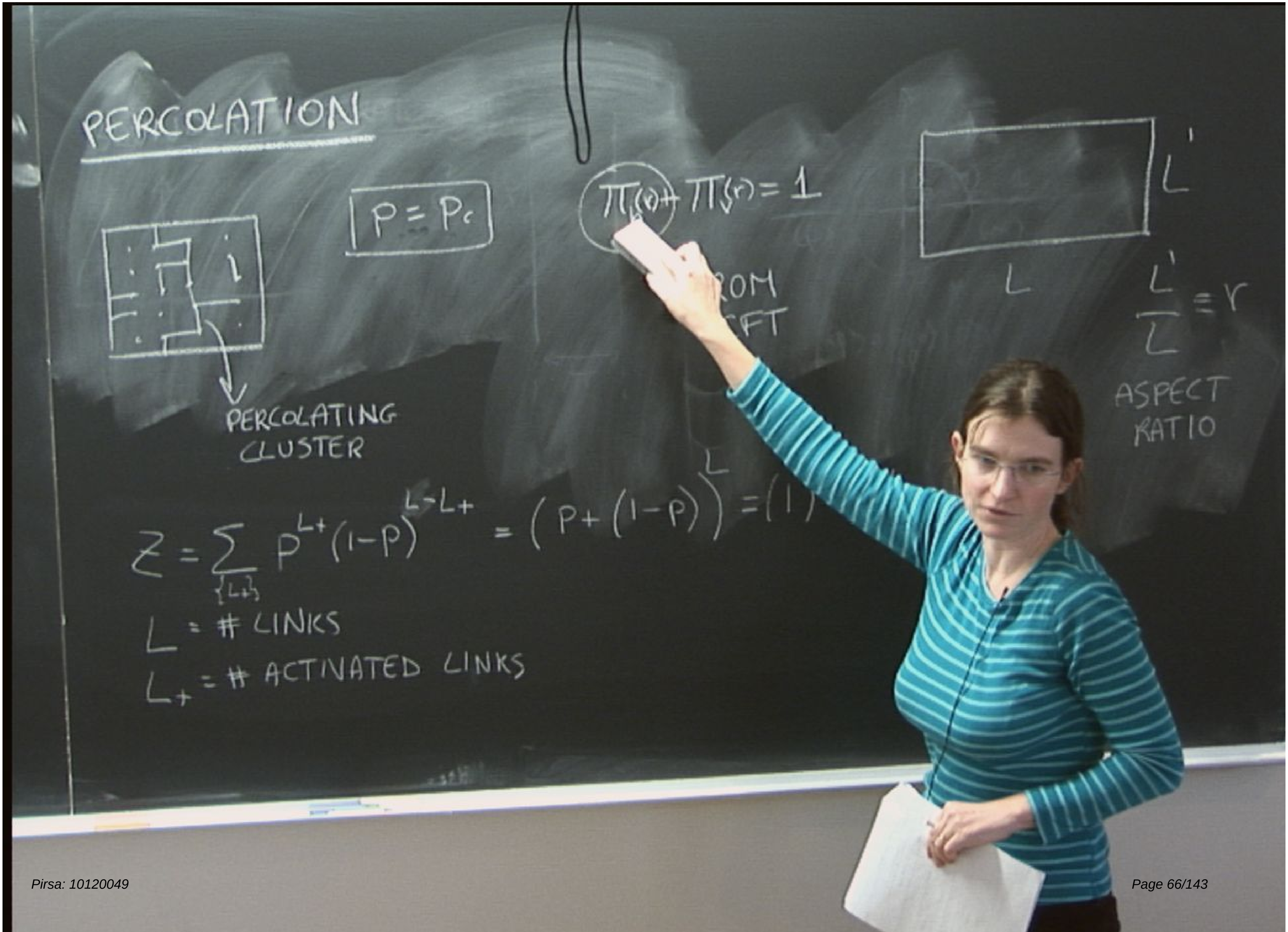

PERCOLATION
P = Pc
πh πv
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1−p)^(L−L+) = (p + (1−p))^L =
L = # LINKS
L+ = # ACTIVATED LINKS

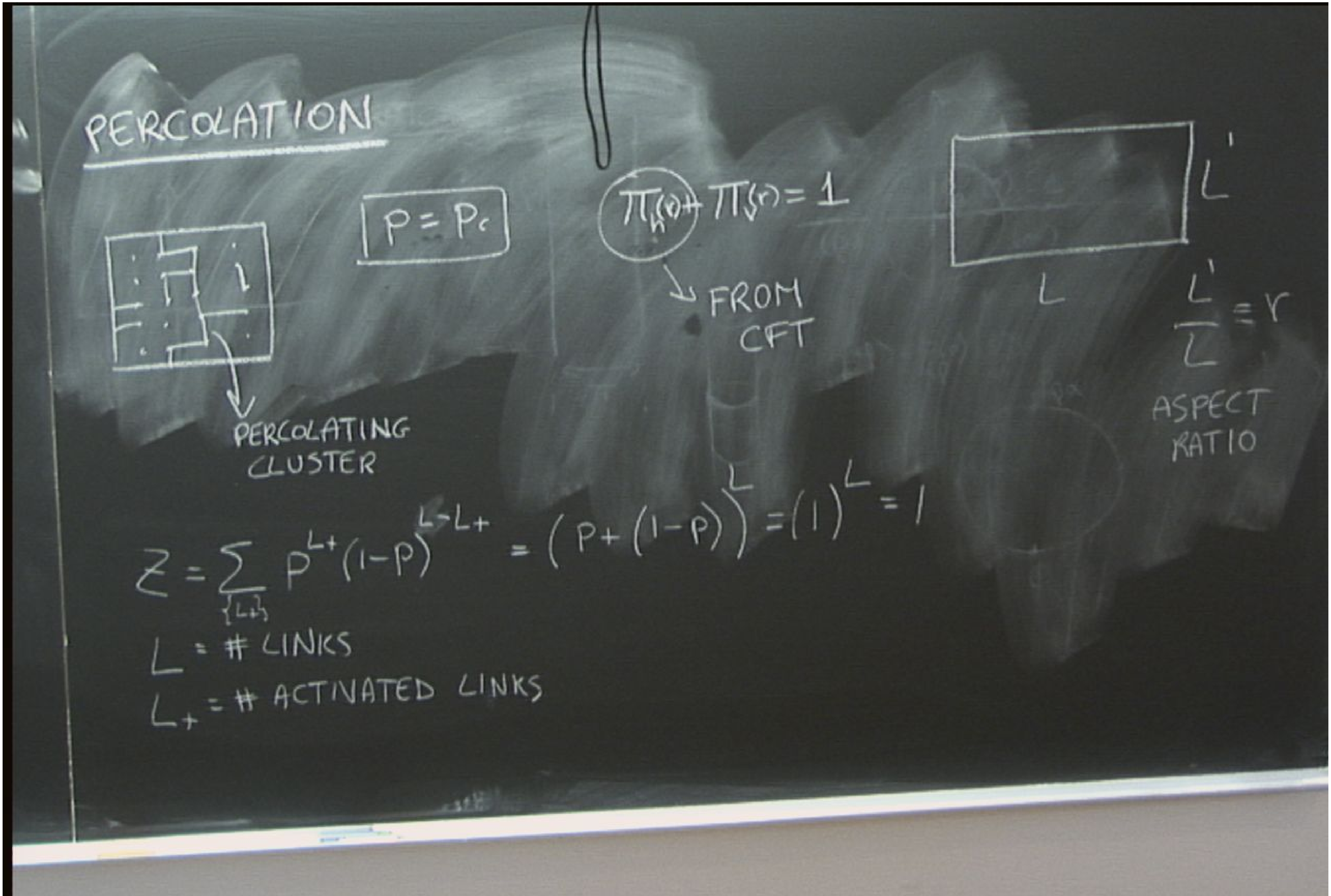

PERCOLATION
p = pc
πh + πv = 1
i
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1-p)^L-L+ = (p+(1-p))^L = (1)^L = 1
L = # LINKS
L+ = # ACTIVATED LINKS

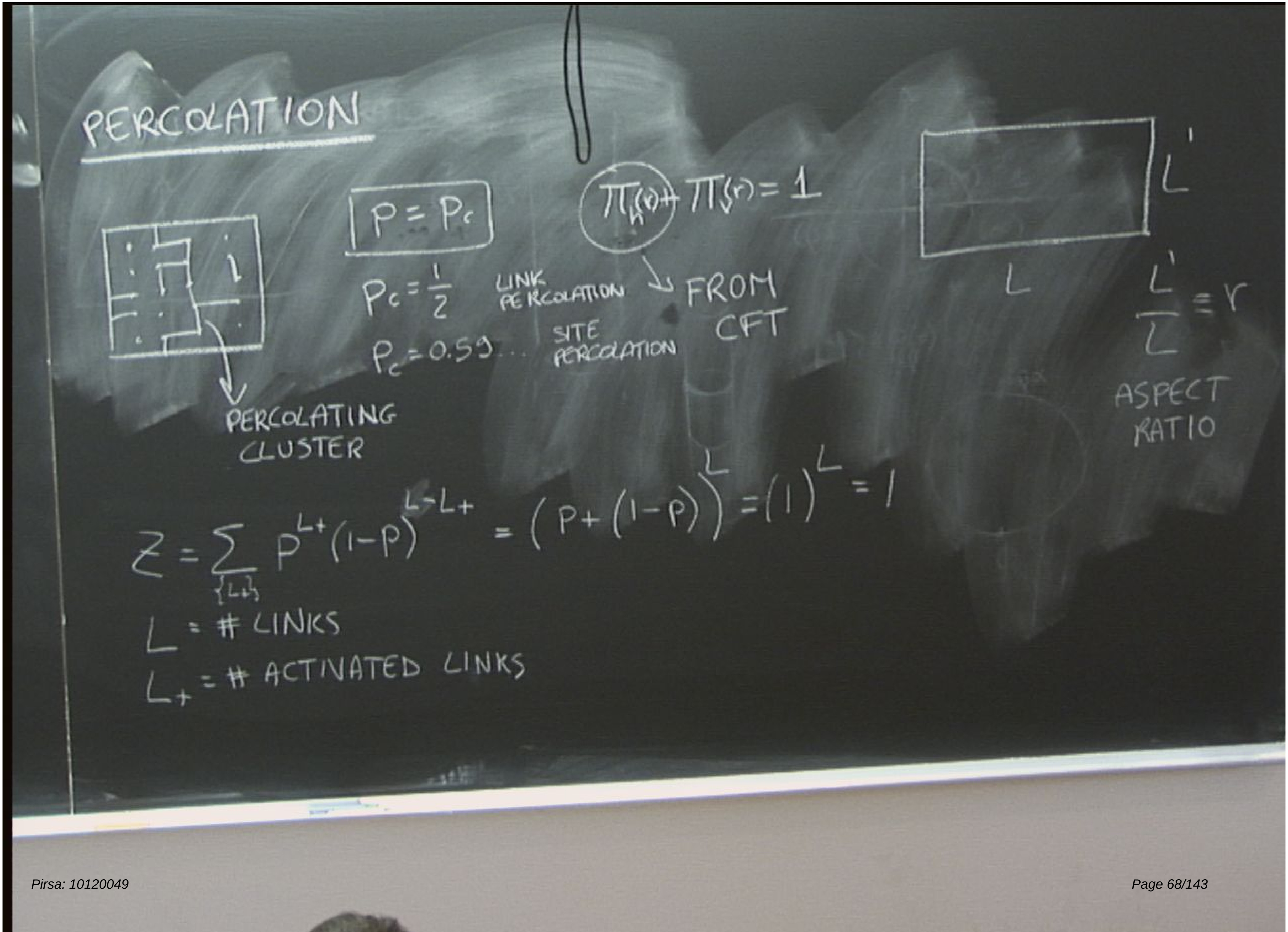

PERCOLATION
P = Pc
πh + πv = 1
L'
L
L'
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1−p)^(L−L+) = (p + (1−p))^L = (1)^L = 1
L = # LINKS
L+ = # ACTIVATED LINKS

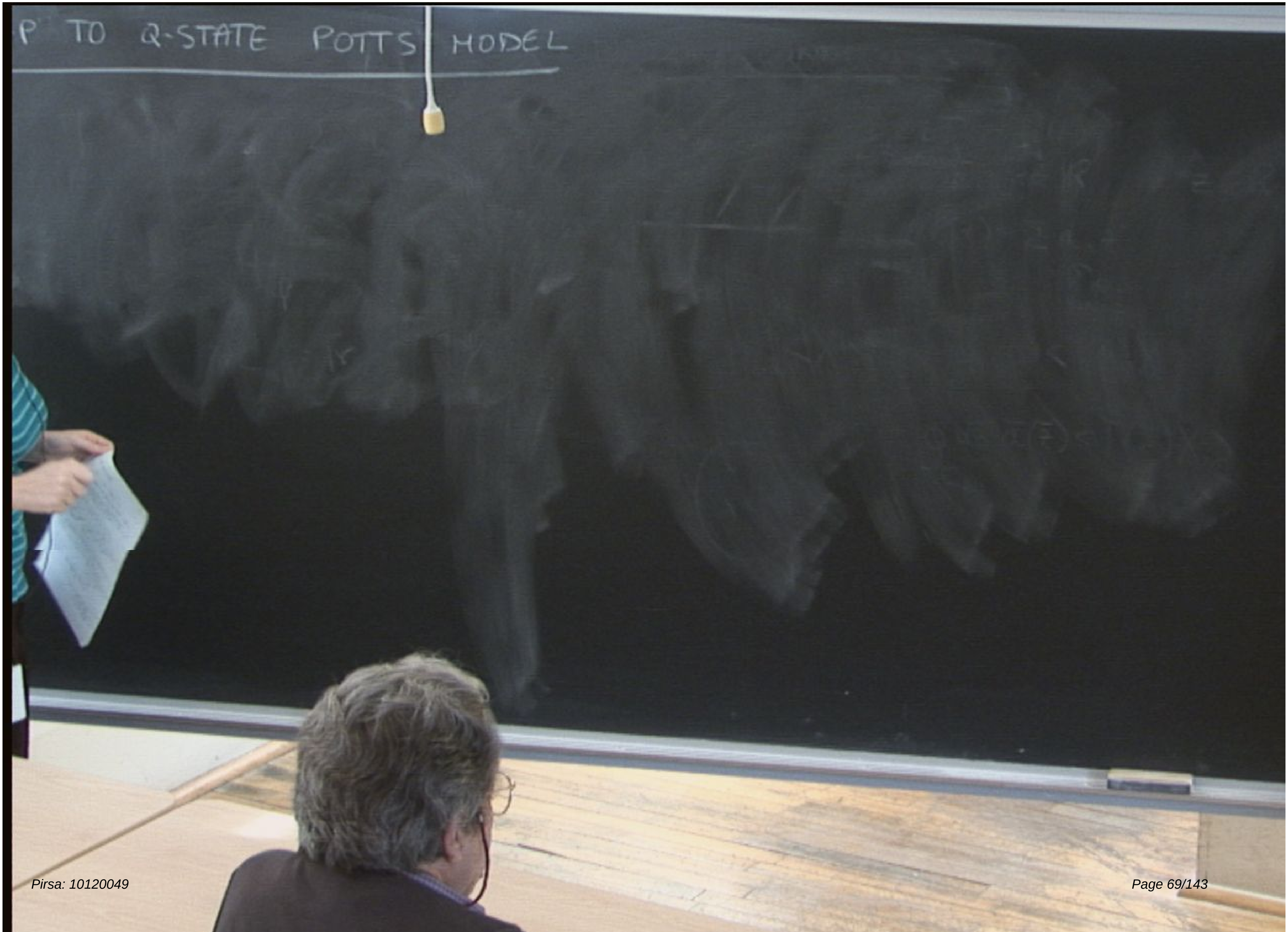

PERCOLATION
$P = P_c$
$\pi_h(r) + \pi_v(r) = 1$
FROM CFT
L'
L
$\frac{L'}{L} \equiv r$
ASPECT RATIO
PERCOLATING CLUSTER
$Z = \sum_{\{L_+\}} p^{L_+}(1-p)^{L-L_+} = (p+(1-p))^L = (1)^L = 1$
L = # LINKS
L_+ = # ACTIVATED LINKS

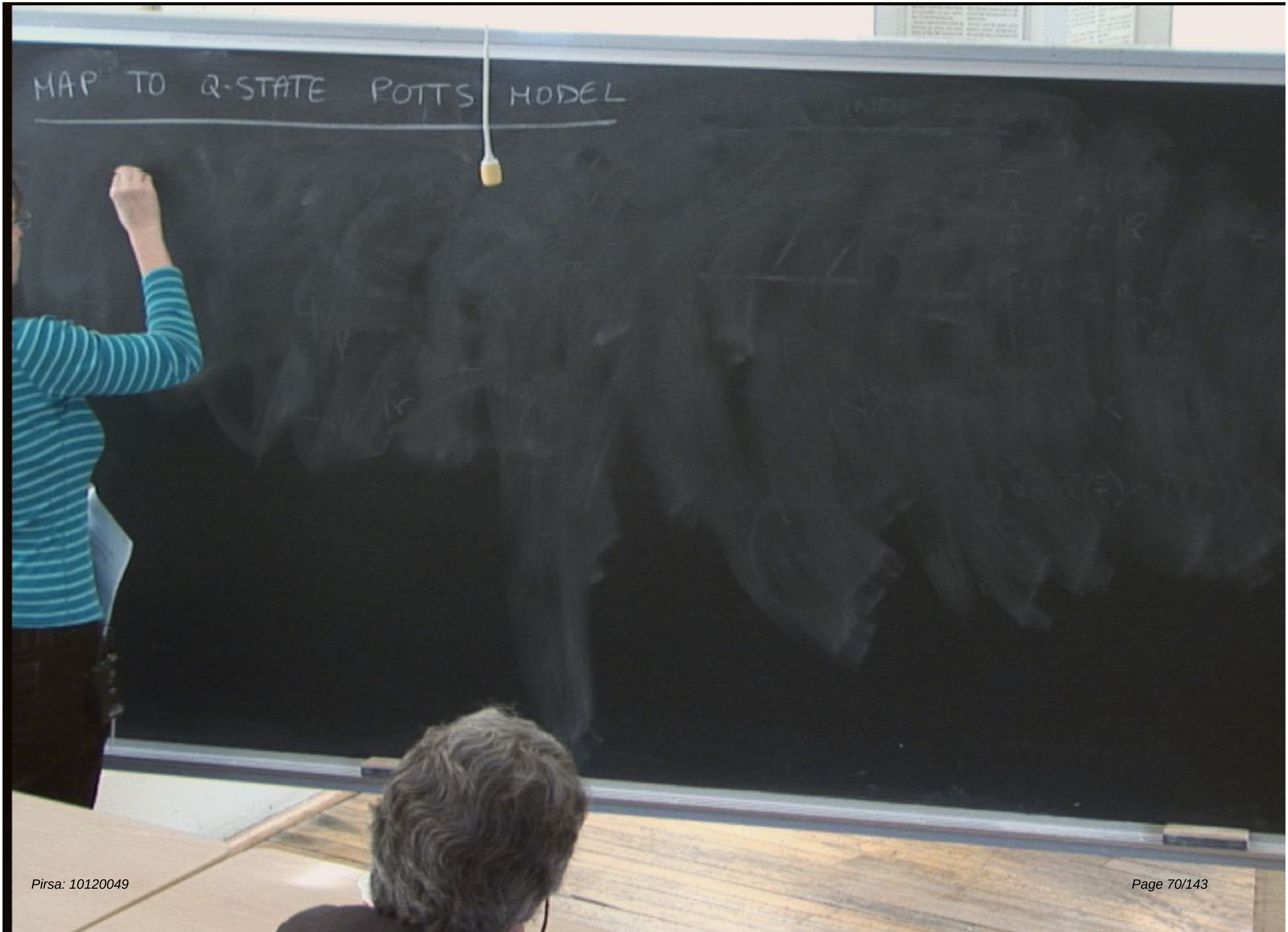

PERCOLATION
P = Pc
Πh(r) + Πv(r) = 1
FROM CFT
L'
L
L'/L = r
ASPECT RATIO
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1−p)^(L−L+) = (p + (1−p))^L = (1)^L = 1
L = # LINKS
L+ = # ACTIVATED LINKS

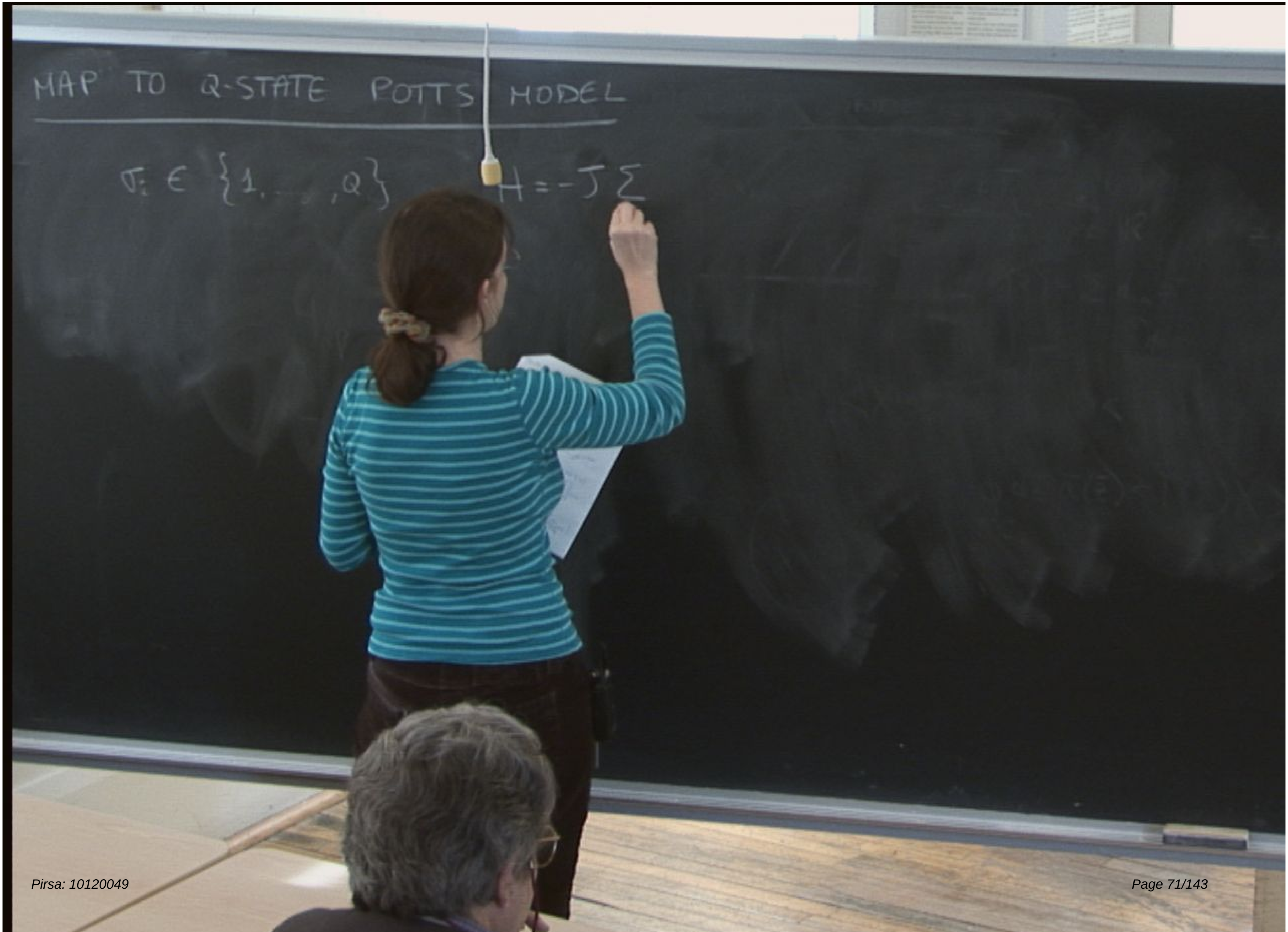

PERCOLATION
P = Pc
PERCOLATING CLUSTER
L
L' / L ≡ r
ASPECT RATIO
Z = Σ p^(L+) (1−p)^(L−L+) = (p + (1−p))^L = (1)
{L+}
L = # LINKS
L+ = # ACTIVATED LINKS

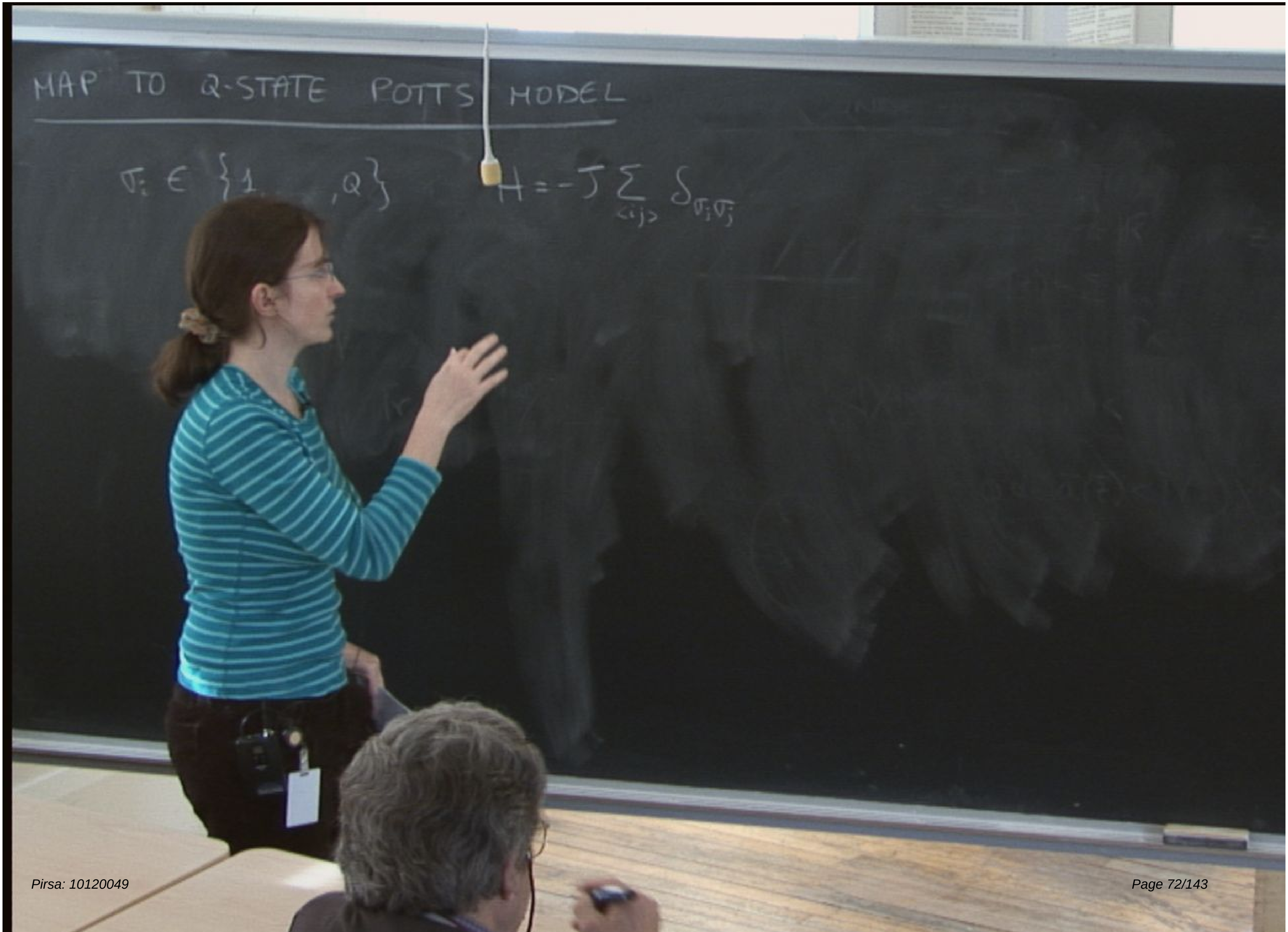

PERCOLATION
$P = P_c$
$\pi_h(r) + \pi_v(r) = 1$
FROM CFT
PERCOLATING CLUSTER
L'
L
$\frac{L'}{L} \equiv r$
ASPECT RATIO
$Z = \sum_{\{L_+\}} p^{L_+}(1-p)^{L-L_+} = (p+(1-p))^L = (1)^L = 1$
L = # LINKS
L_+ = # ACTIVATED LINKS

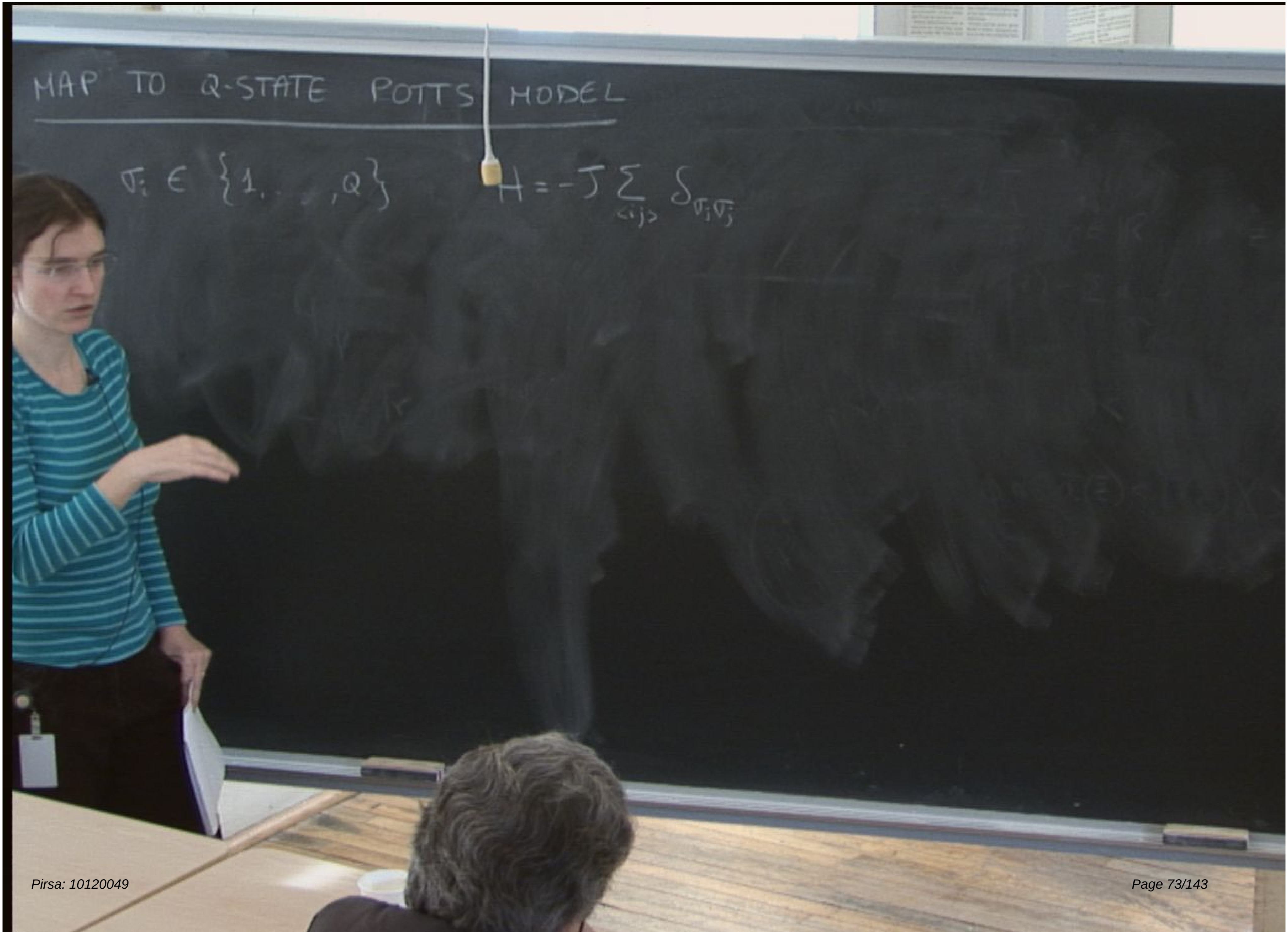

PERCOLATION
P = Pc
Pc = 1/2 LINK PERCOLATION
Pc = 0.59... SITE PERCOLATION
πh(r) + πv(r) = 1
FROM CFT
L'
L
L'/L ≡ r
ASPECT RATIO
PERCOLATING CLUSTER
Z = Σ{L+} p^L+ (1-p)^(L-L+) = (p + (1-p))^L = (1)^L = 1
L = # LINKS
L+ = # ACTIVATED LINKS

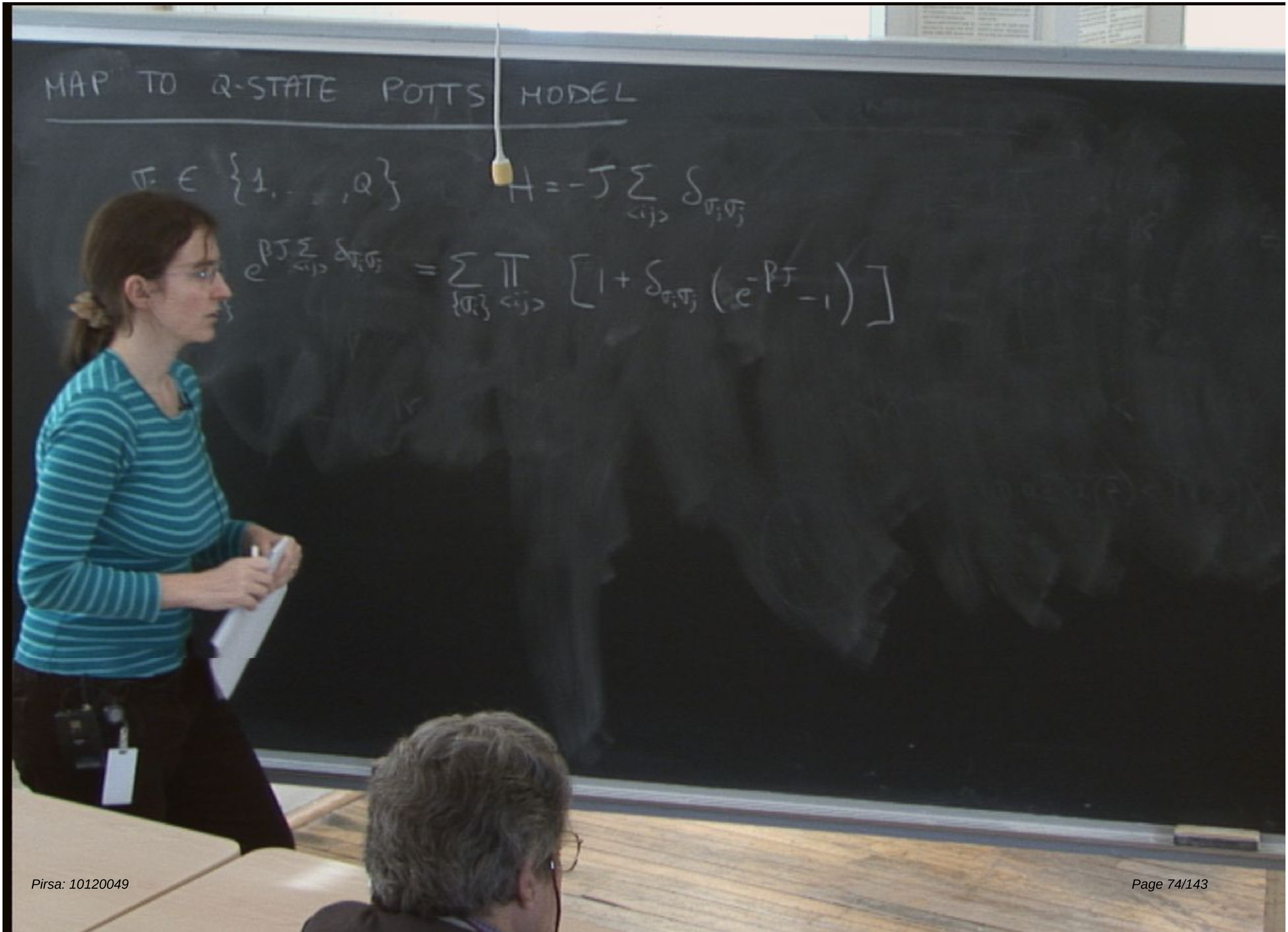

P TO Q-STATE POTTS MODEL

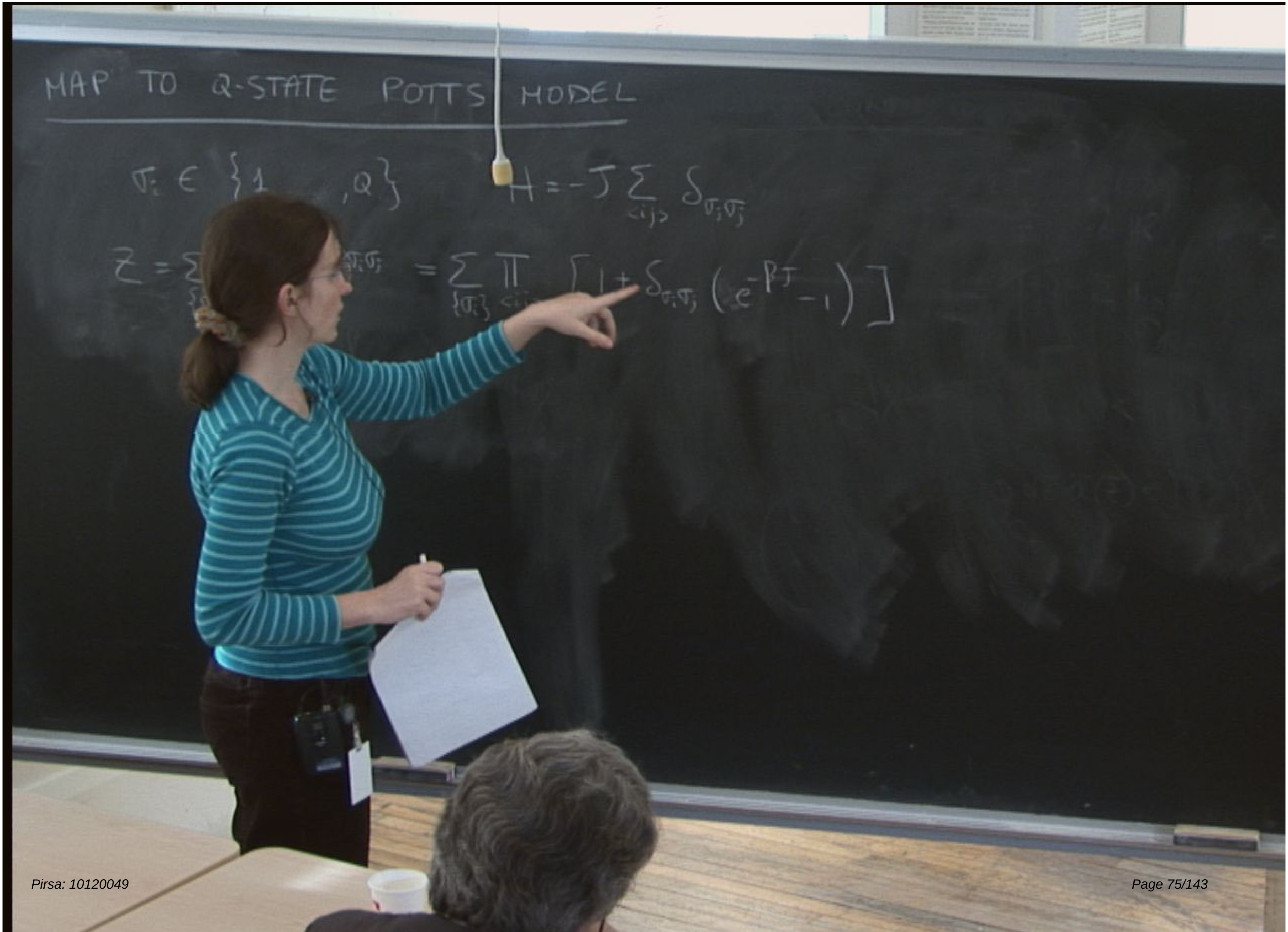

MAP TO Q-STATE POTTS MODEL

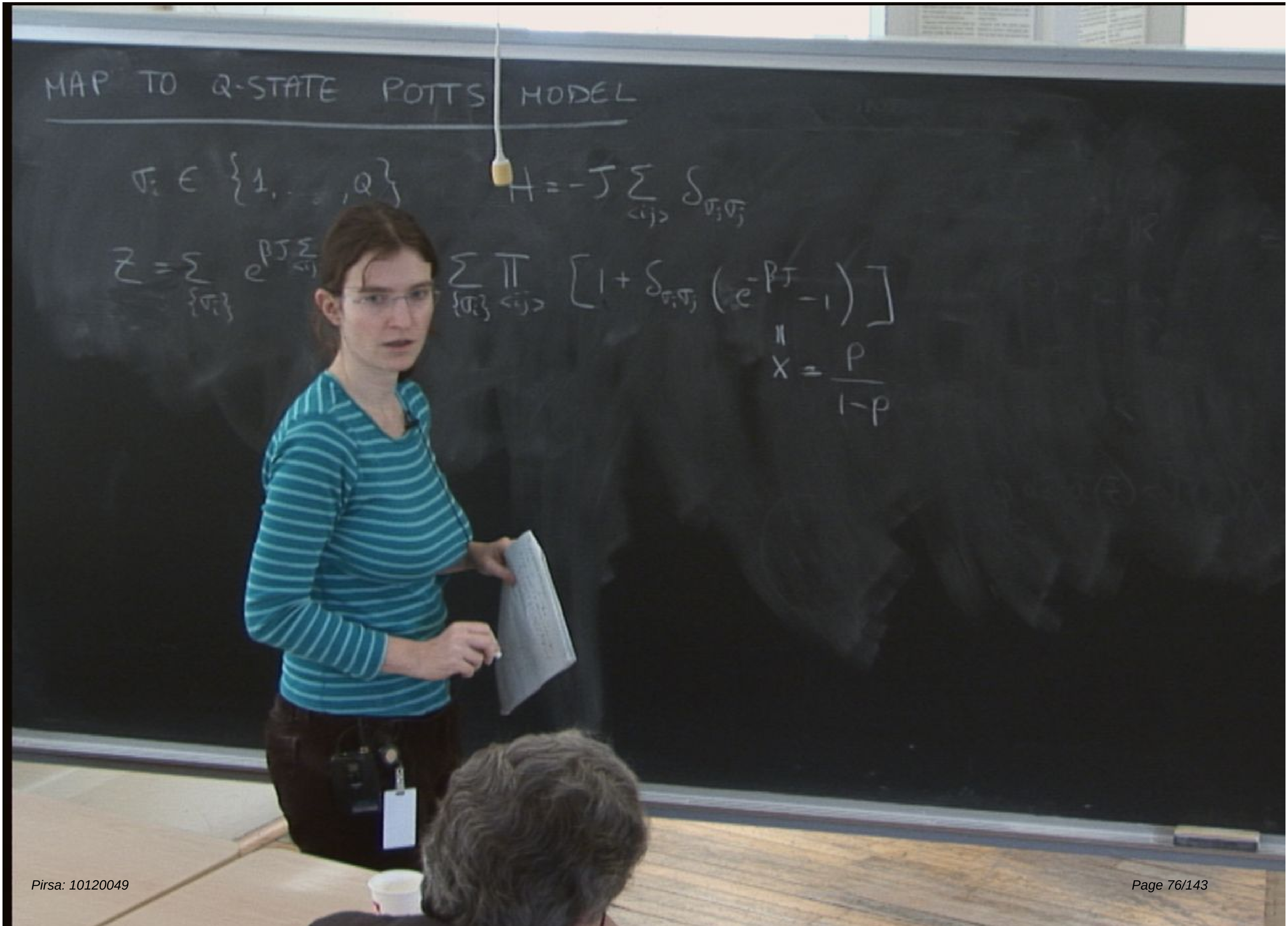

MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum$

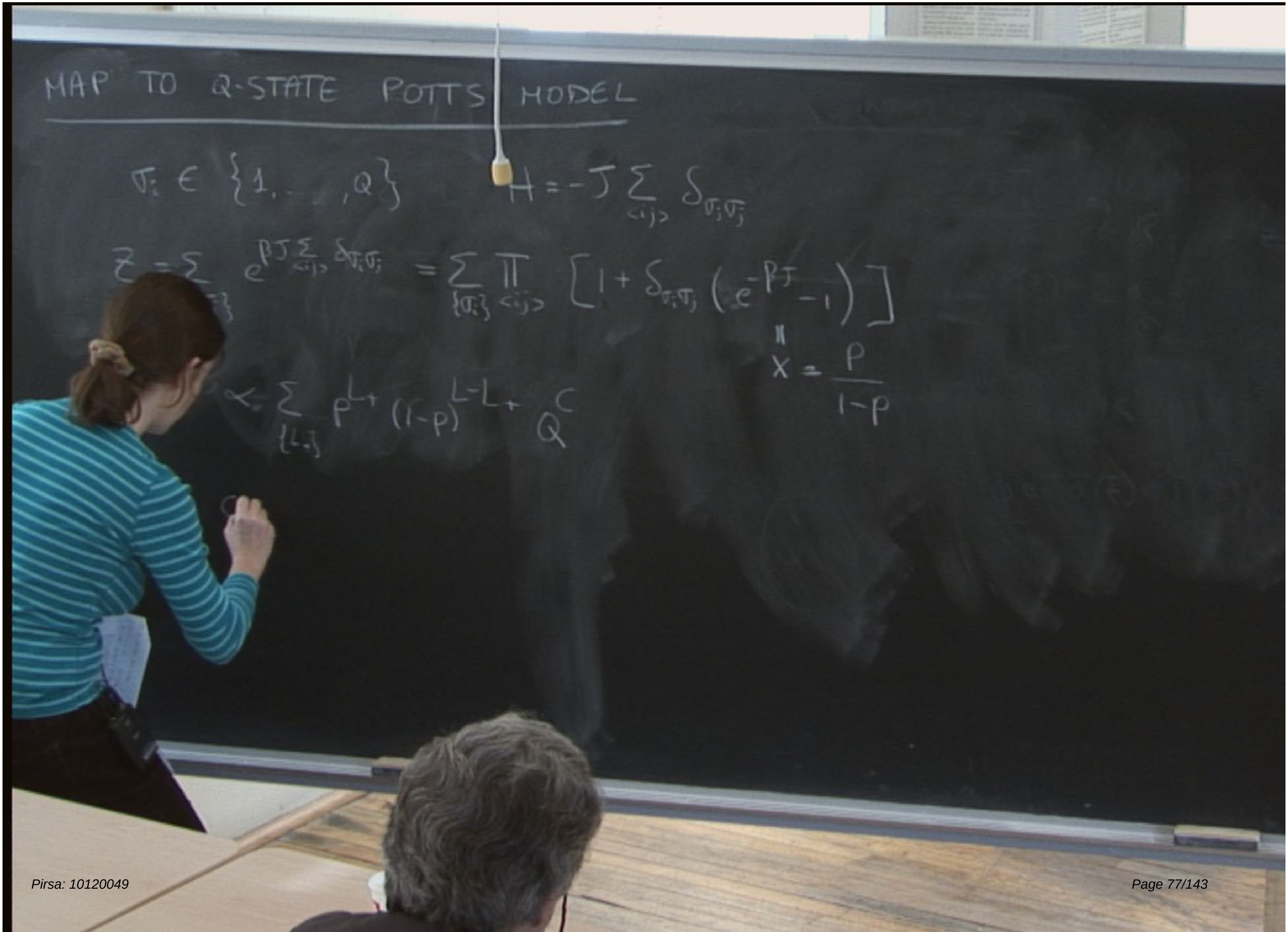

MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$

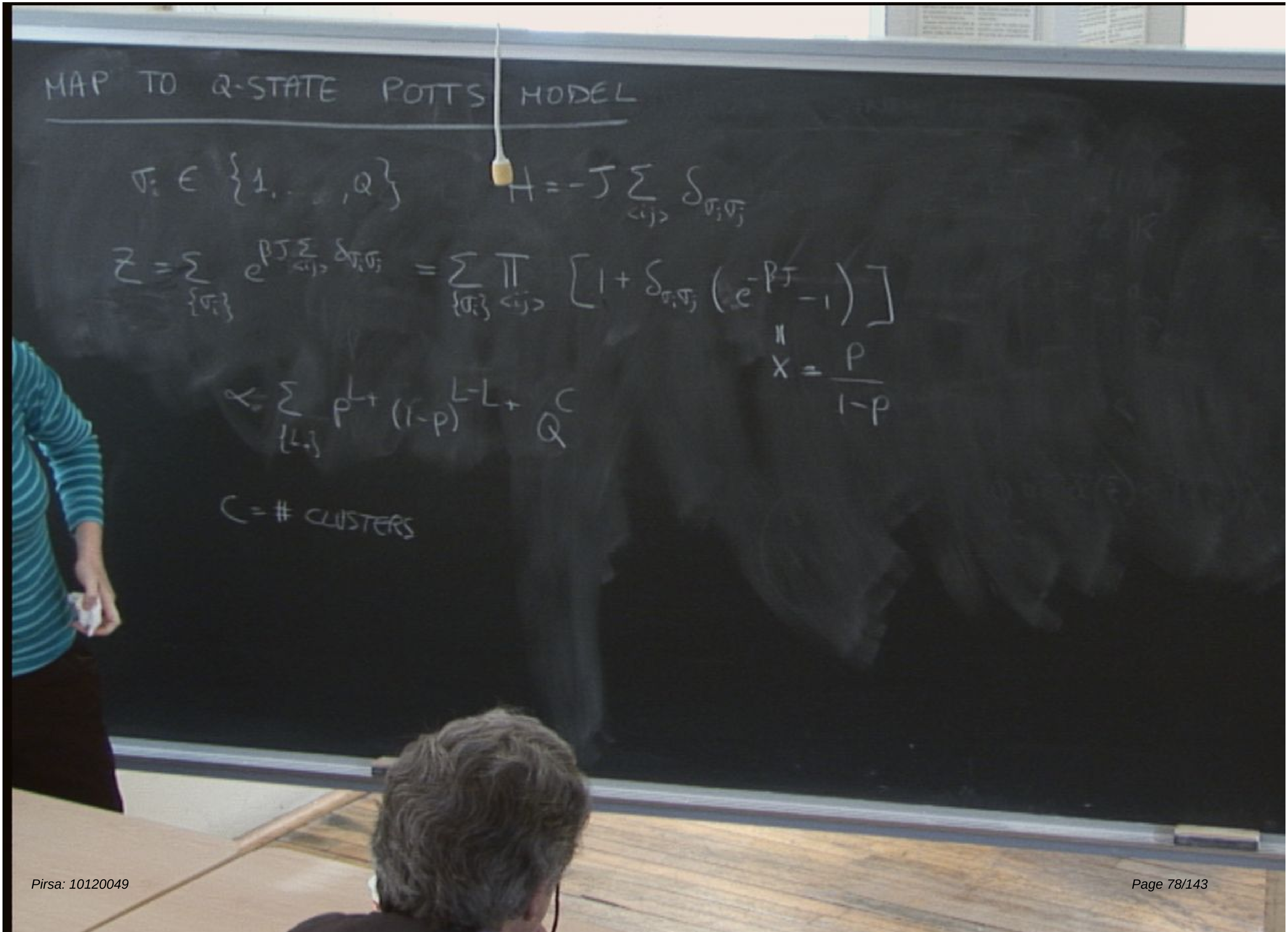

MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$

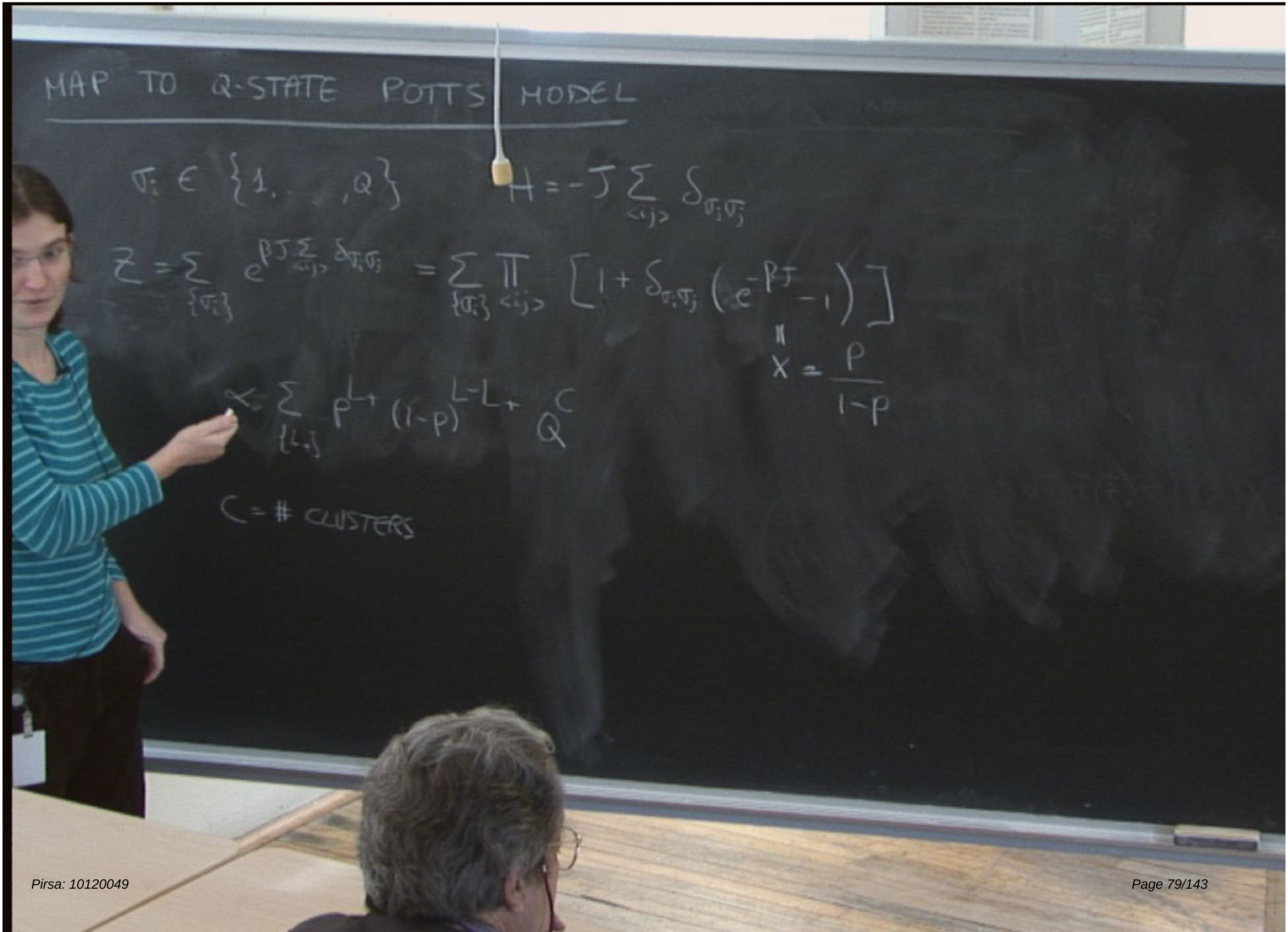

MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$

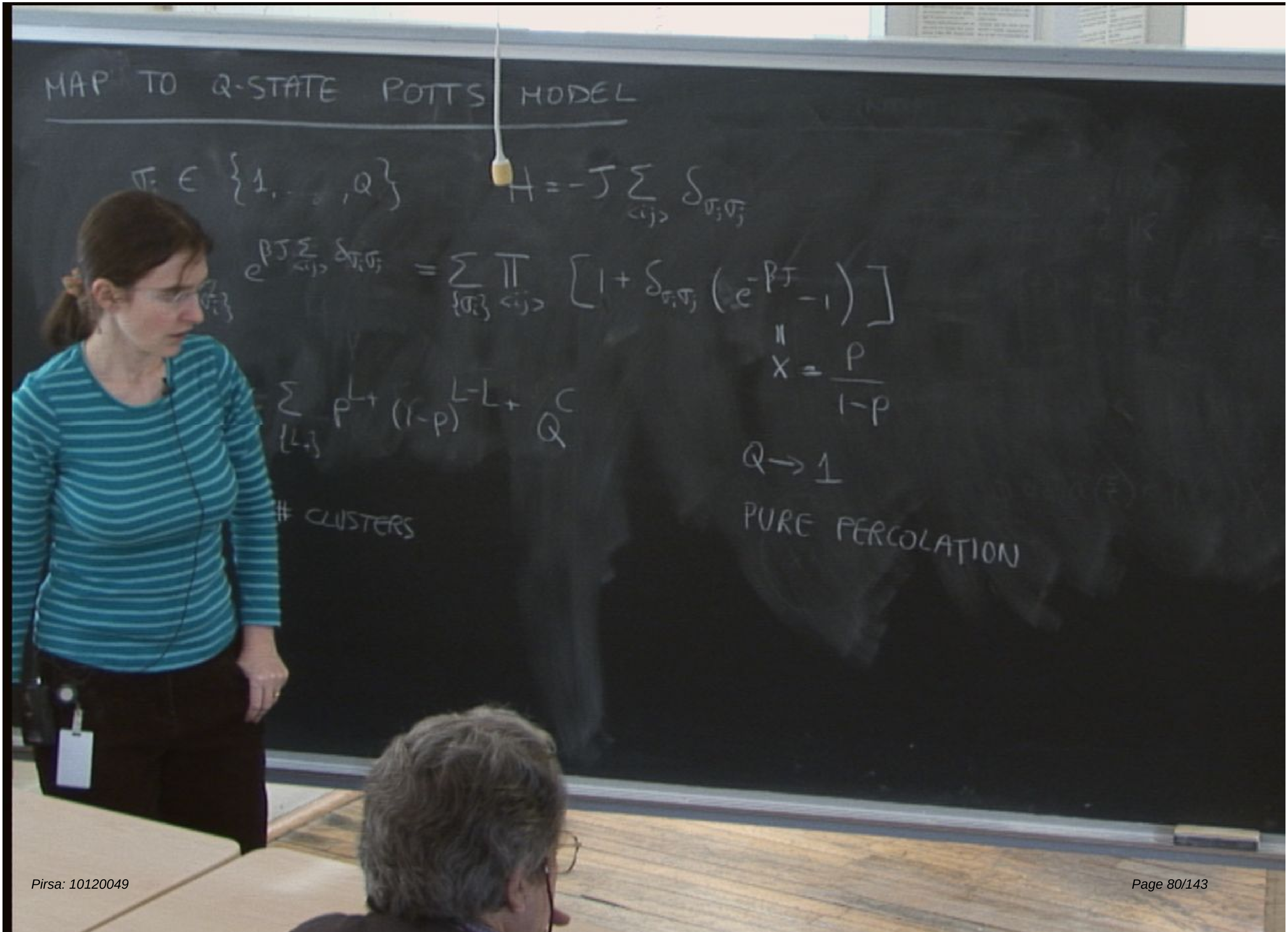

MAP TO Q-STATE POTTS MODEL
σi ∈ {1 ,Q}
H = -J Σ<ij> δσiσj
Z = Σ
= Σ{σi} Π [1 + δσiσj (e^-βJ - 1)]

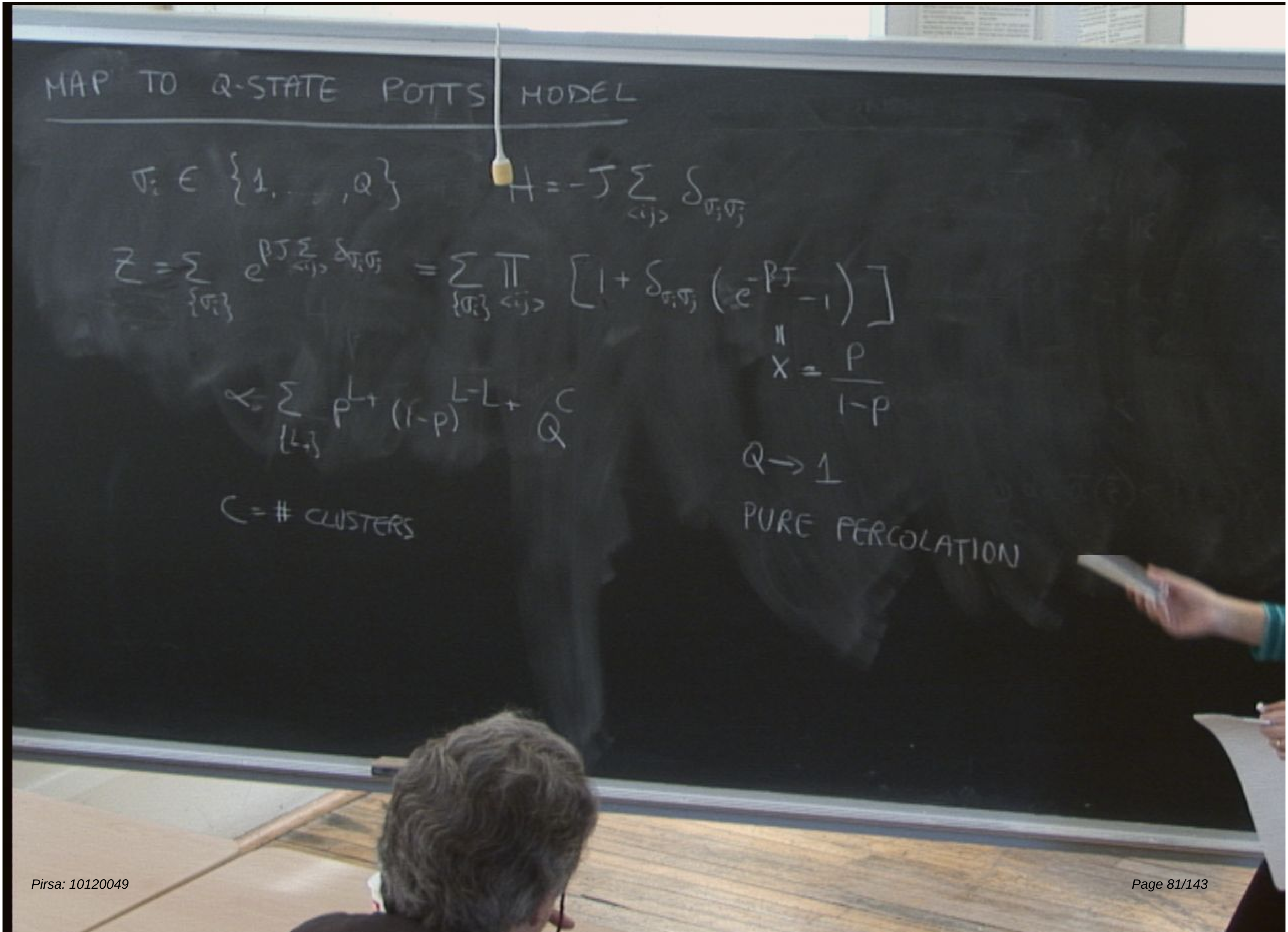

MAP TO Q-STATE POTTS MODEL
σi ∈ {1, . . . , Q}
H = -J Σ<ij> δσiσj
Z = Σ{σi} e^βJΣ<ij> ... Σ{σi} Π<ij> [1 + δσiσj (e^-βJ - 1)]
X = p / 1-p

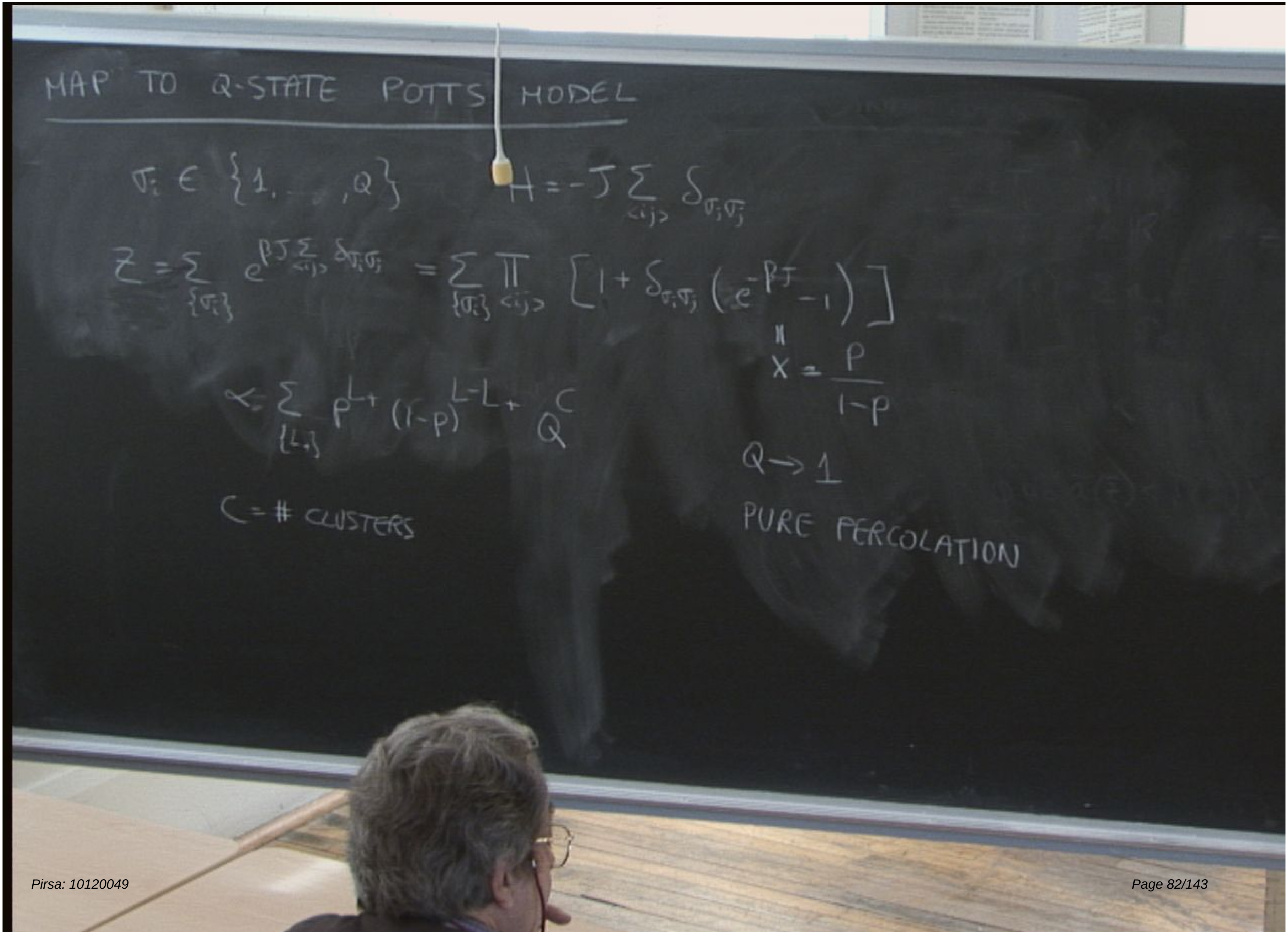

MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$
$x = \frac{p}{1-p}$
$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$


MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$
$X = \frac{p}{1-p}$
$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$
C = # CLUSTERS


MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{\beta J} - 1 \right) \right]$
$x = \frac{p}{1-p}$
$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$
C = # CLUSTERS


MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$
$X = \frac{p}{1-p}$
$\sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$
CLUSTERS
$Q \to 1$
PURE PERCOLATION


MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$
$X = \frac{p}{1-p}$
$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$
$C = \#$ CLUSTERS
$Q \to 1$
PURE PERCOLATION


MAP TO Q-STATE POTTS MODEL
$\sigma_i \in \{1, \ldots, Q\}$
$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$
$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{\beta J} - 1 \right) \right]$
$X = \frac{p}{1-p}$
$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$
$Q \to 1$
C = # CLUSTERS
PURE PERCOLATION

MAP TO Q-STATE POTTS MODEL

$$\sigma_i \in \{1, \ldots, Q\} \qquad H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}$$

$$Z = \sum_{\{\sigma_i\}} e^{\beta J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j}} = \sum_{\{\sigma_i\}} \prod_{\langle ij \rangle} \left[1 + \delta_{\sigma_i \sigma_j} \left(e^{-\beta J} - 1 \right) \right]$$

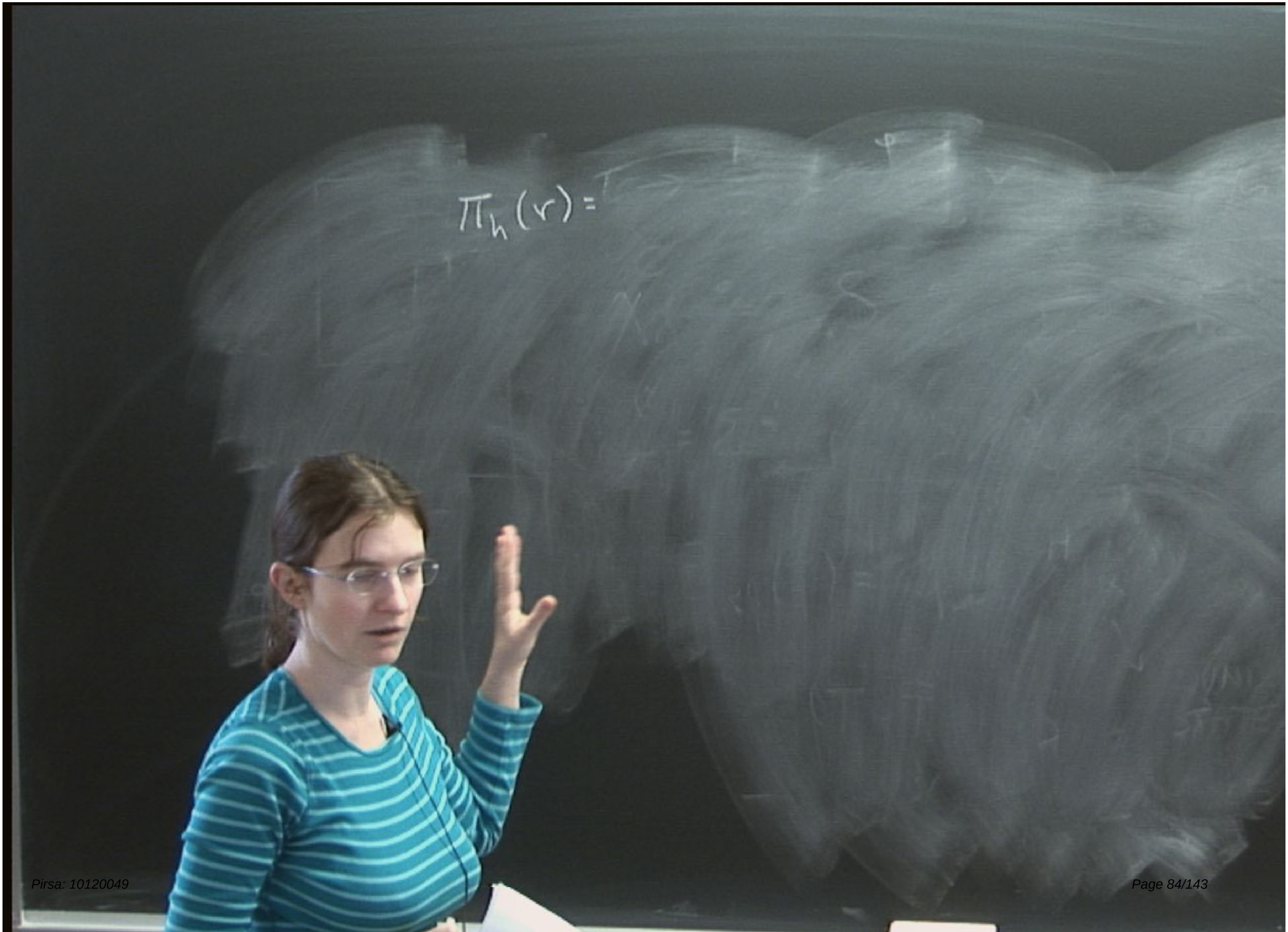
$$\left(e^{-\beta J} - 1 \right) = X = \frac{p}{1-p}$$

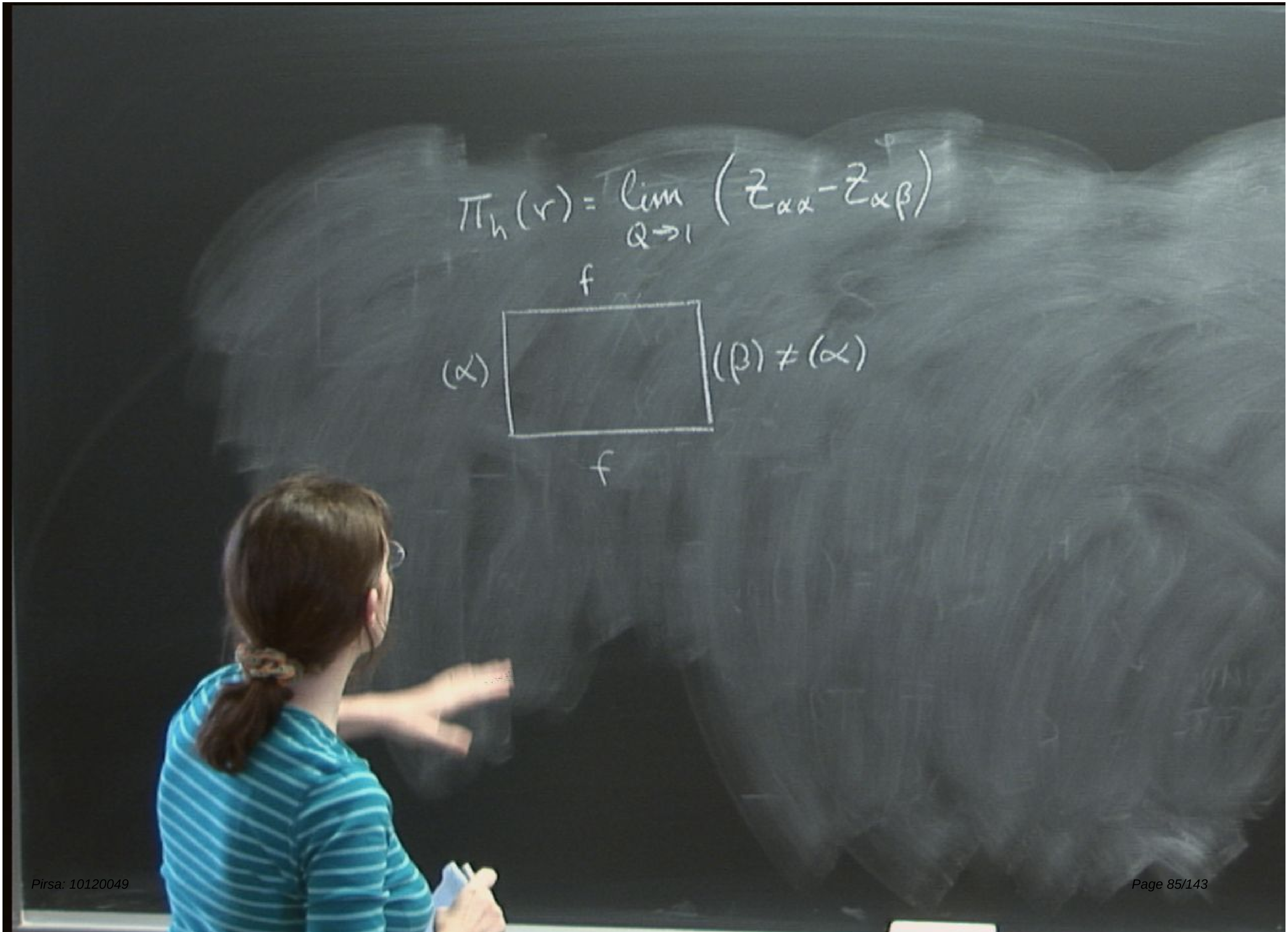
$$\propto \sum_{\{L_+\}} p^{L_+} (1-p)^{L-L_+} Q^C$$

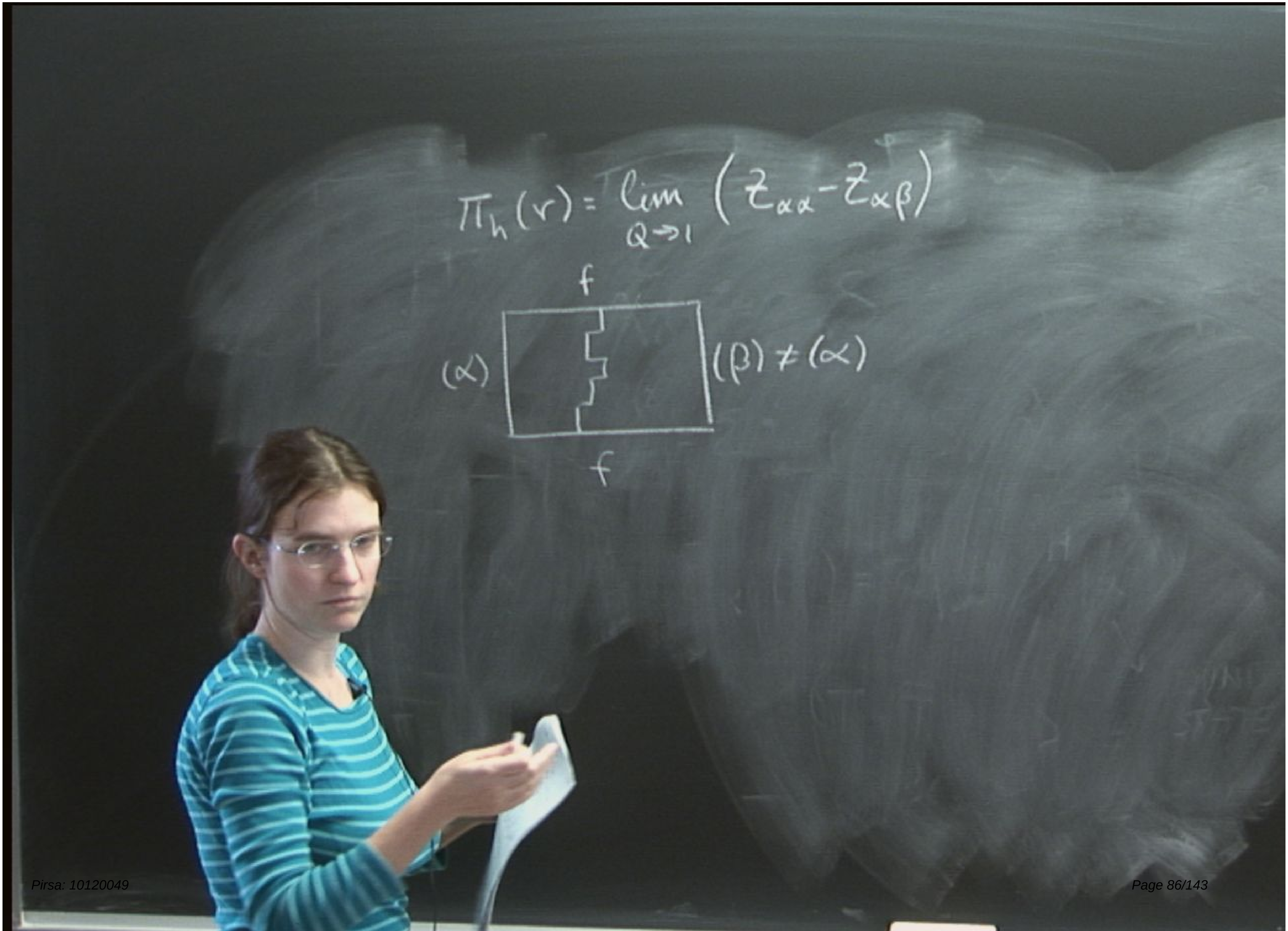
C = # CLUSTERS

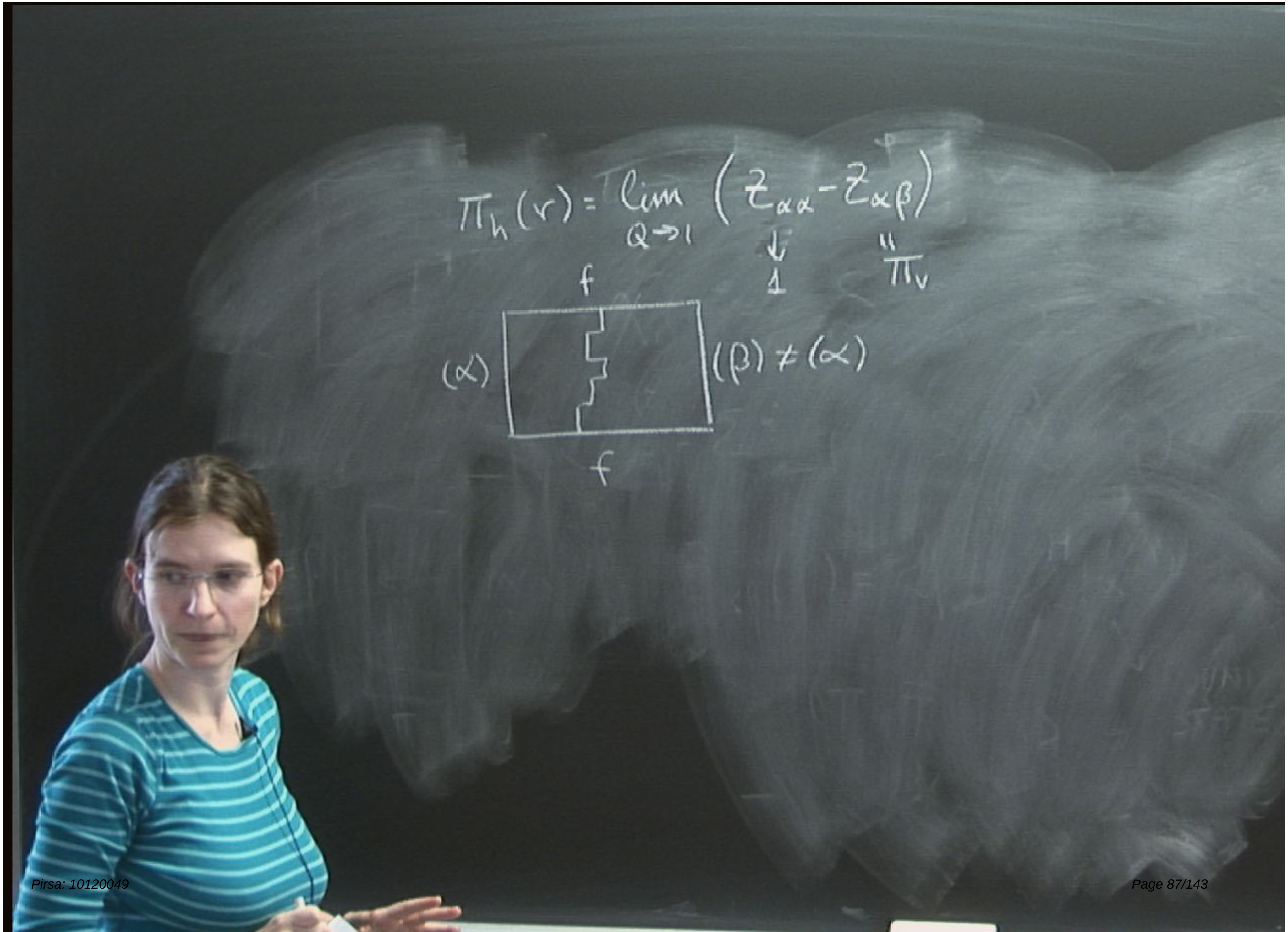
$Q \to 1$

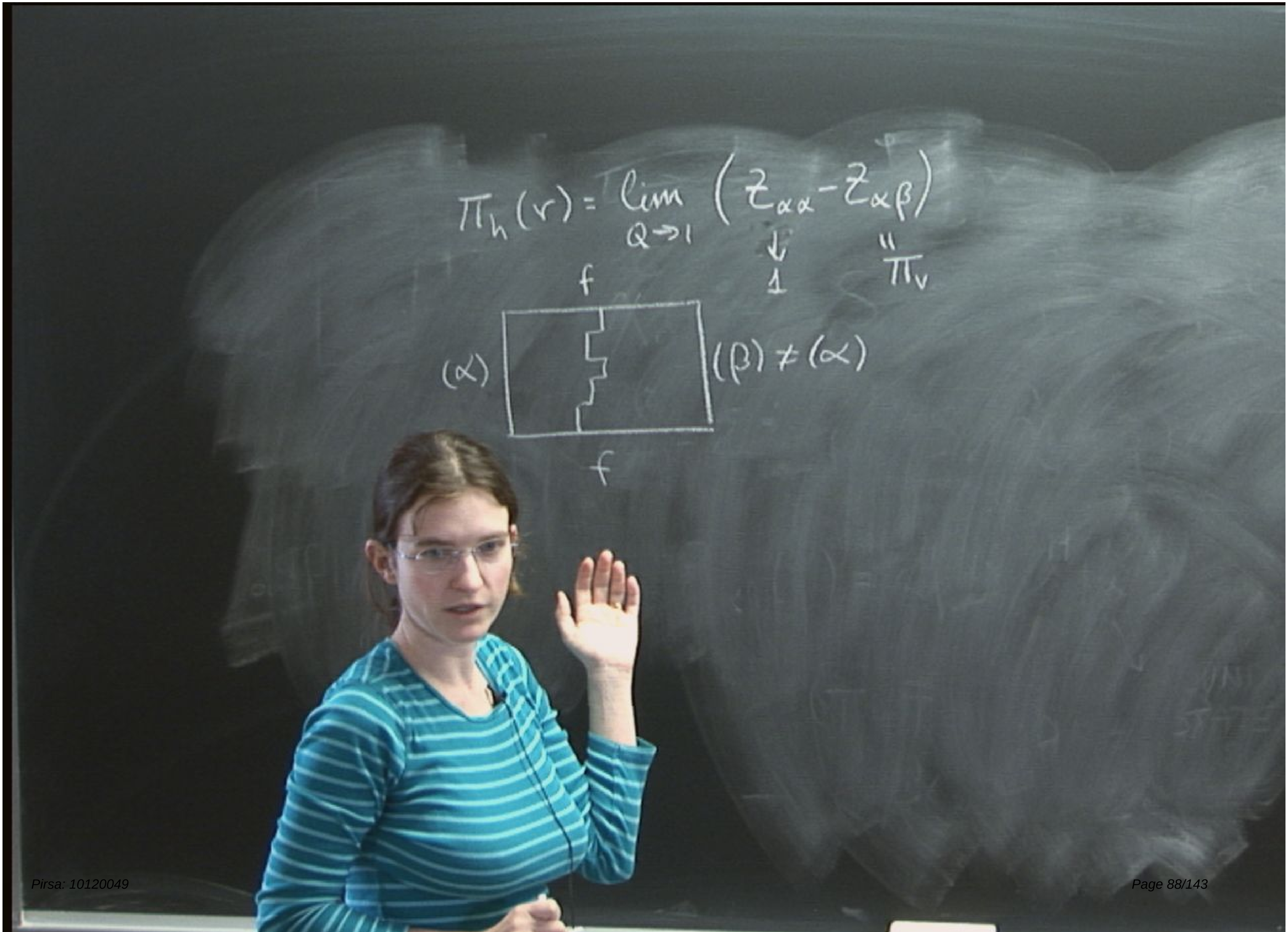
PURE PERCOLATION


$\pi_h(r) =$


$\pi_h(r) = \lim_{Q \to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
f
(α)
$(\beta) \neq (\alpha)$
f


$\pi_h(r) = \lim_{Q \to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
f
(α)
$(\beta) \neq (\alpha)$
f


$\Pi_h(r) = \lim_{Q\to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
$\downarrow$ 1
$= \Pi_v$
f
(α)
(β) $\neq$ (α)
f


$\pi_h(r) = \lim_{Q \to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
$\downarrow$ 1
$= \pi_v$
f
(α)
$(\beta) \neq (\alpha)$
f

PERCOLATION

PERCOLATING CLUSTER

$$\boxed{p = p_c}$$

$p_c = \frac{1}{2}$ LINK PERCOLATION

$p_c = 0.59\ldots$ SITE PERCOLATION

$$\pi_h(r) + \pi_v(r) = 1$$

FROM CFT

L'

L

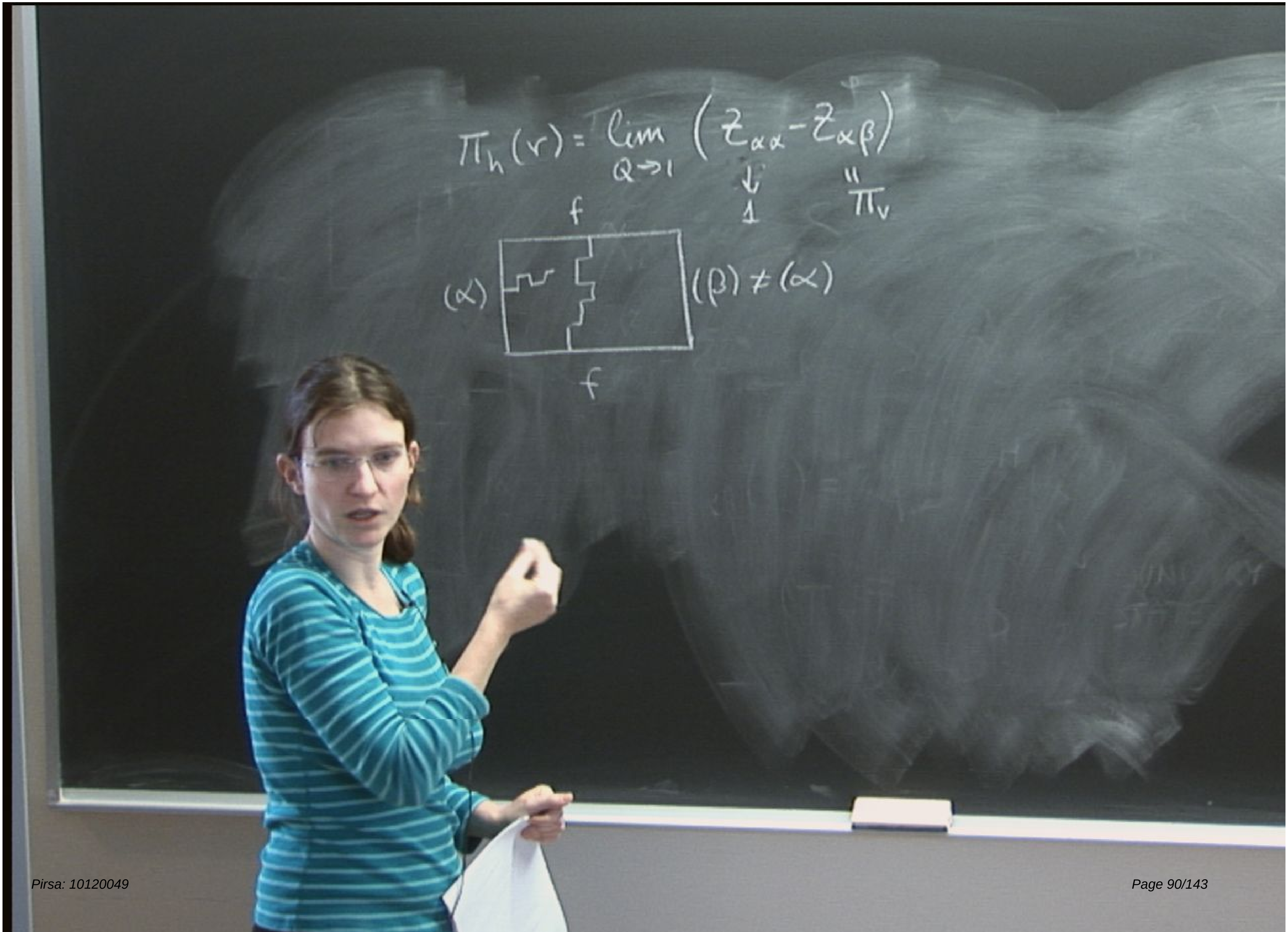
$$\frac{L'}{L} = r$$

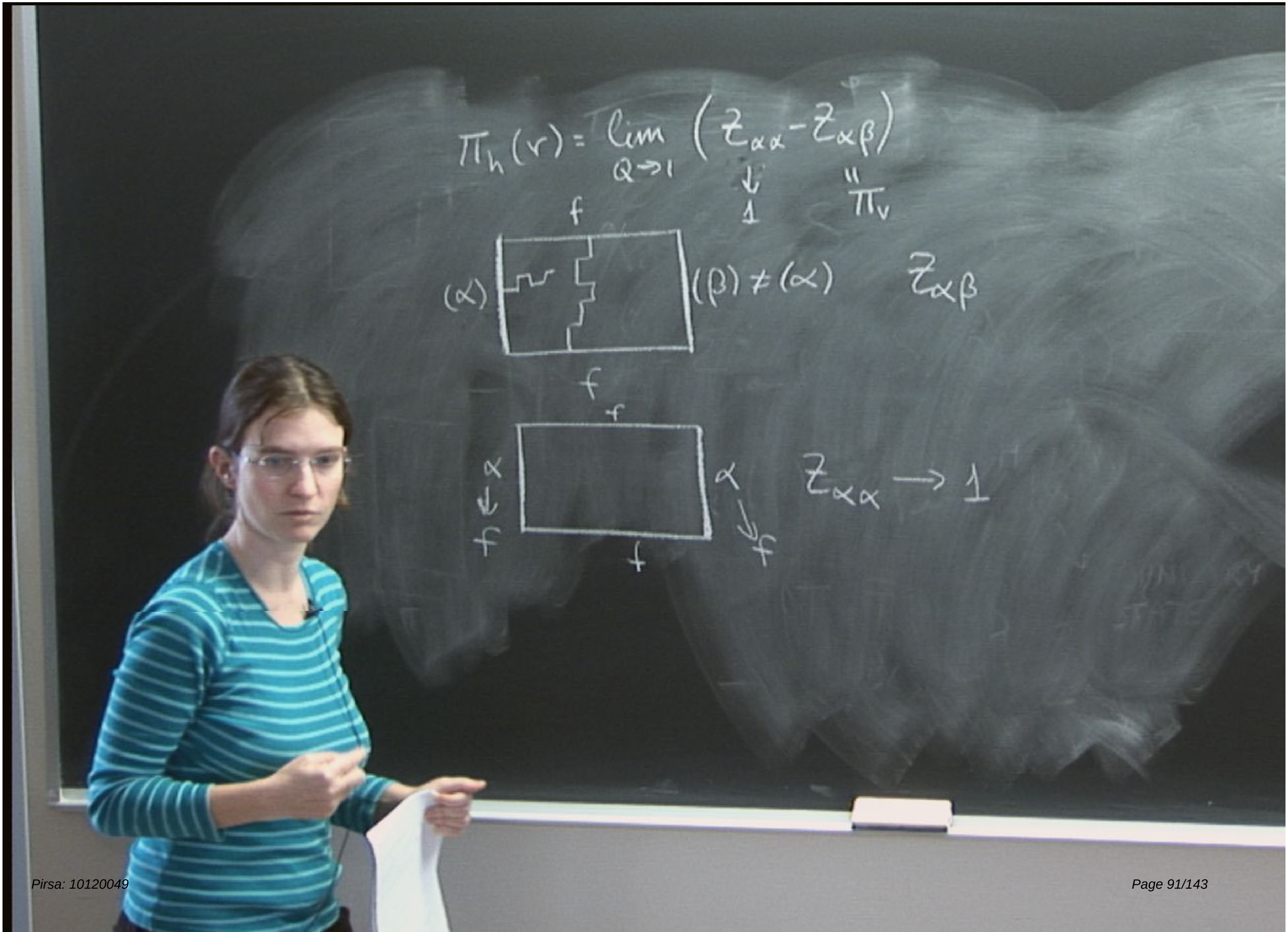
ASPECT RATIO

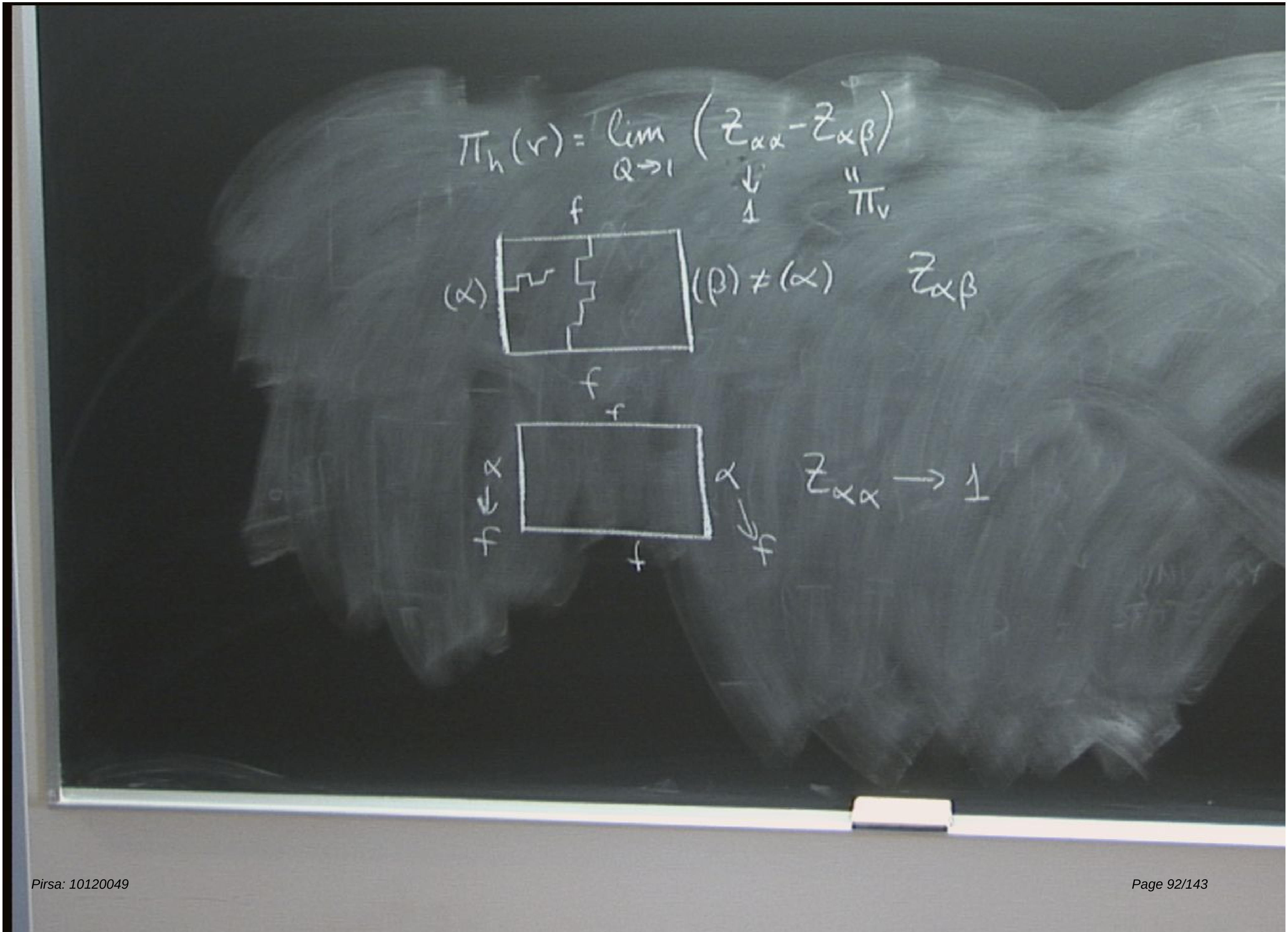
$$Z = \sum_{\{L_+\}} p^{L_+}(1-p)^{L-L_+} = \left(p + (1-p)\right)^L = (1)^L = 1$$

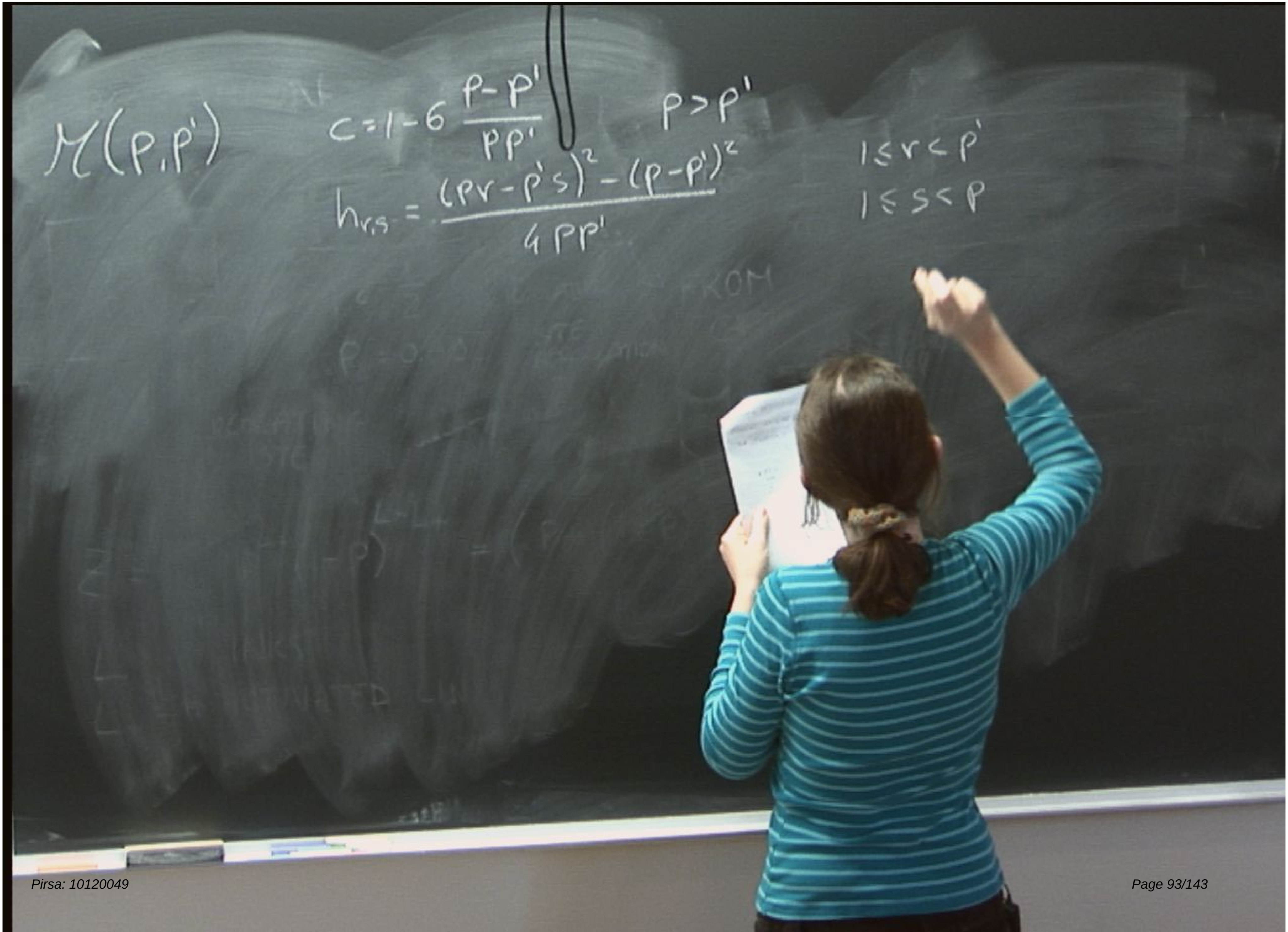
L = # LINKS

L_+ = # ACTIVATED LINKS

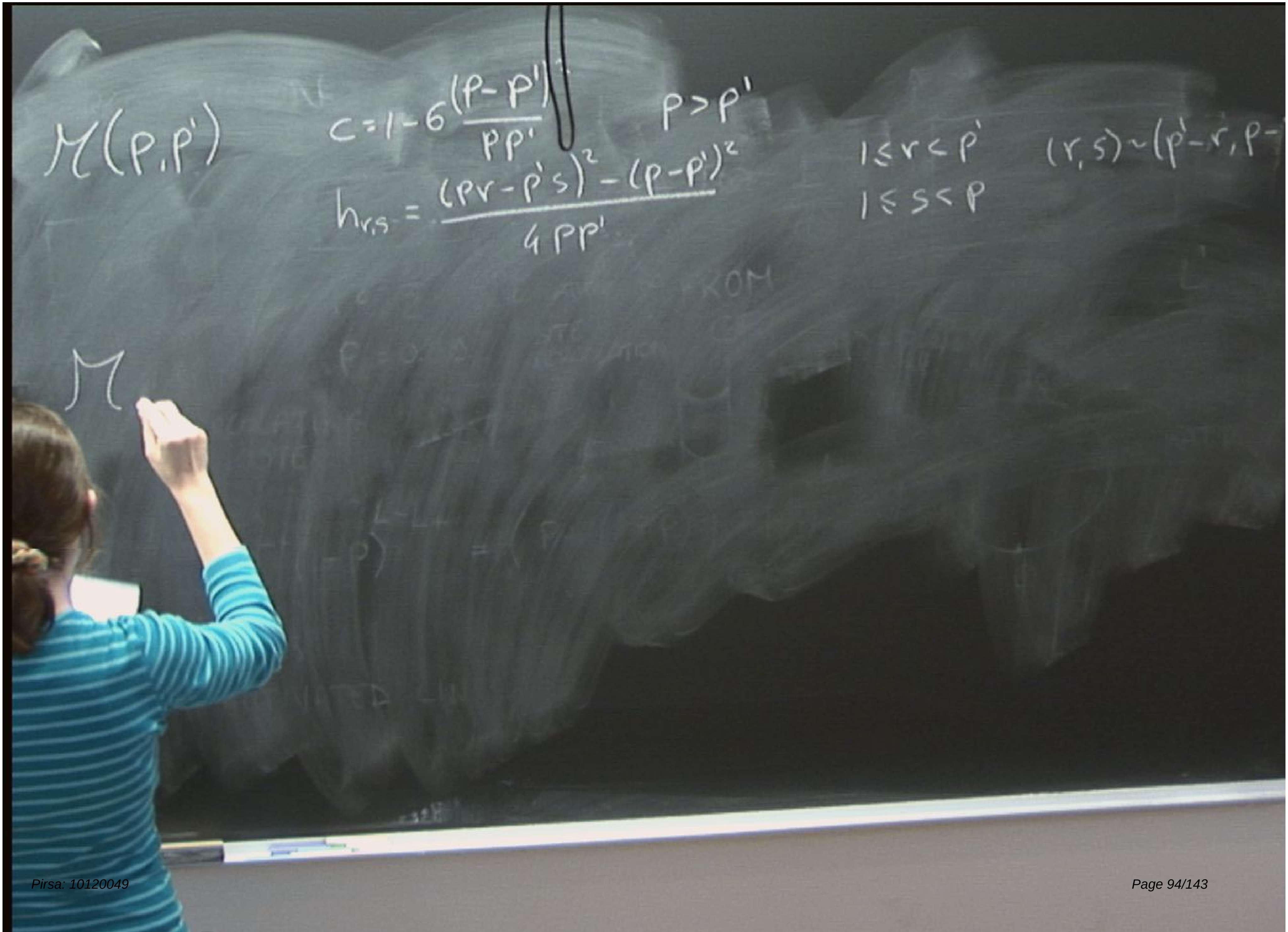

$\pi_h(r) = \lim_{Q\to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
$\downarrow$ 1
= π_v
f
(α)
$(\beta) \neq (\alpha)$
f


π_h(r) = lim_{Q→1} (Z_αα − Z_αβ)
↓ 1
= π_v
f
(α)
(β) ≠ (α)
Z_αβ
f
f
α
↓
f
α
f
f
Z_αα → 1

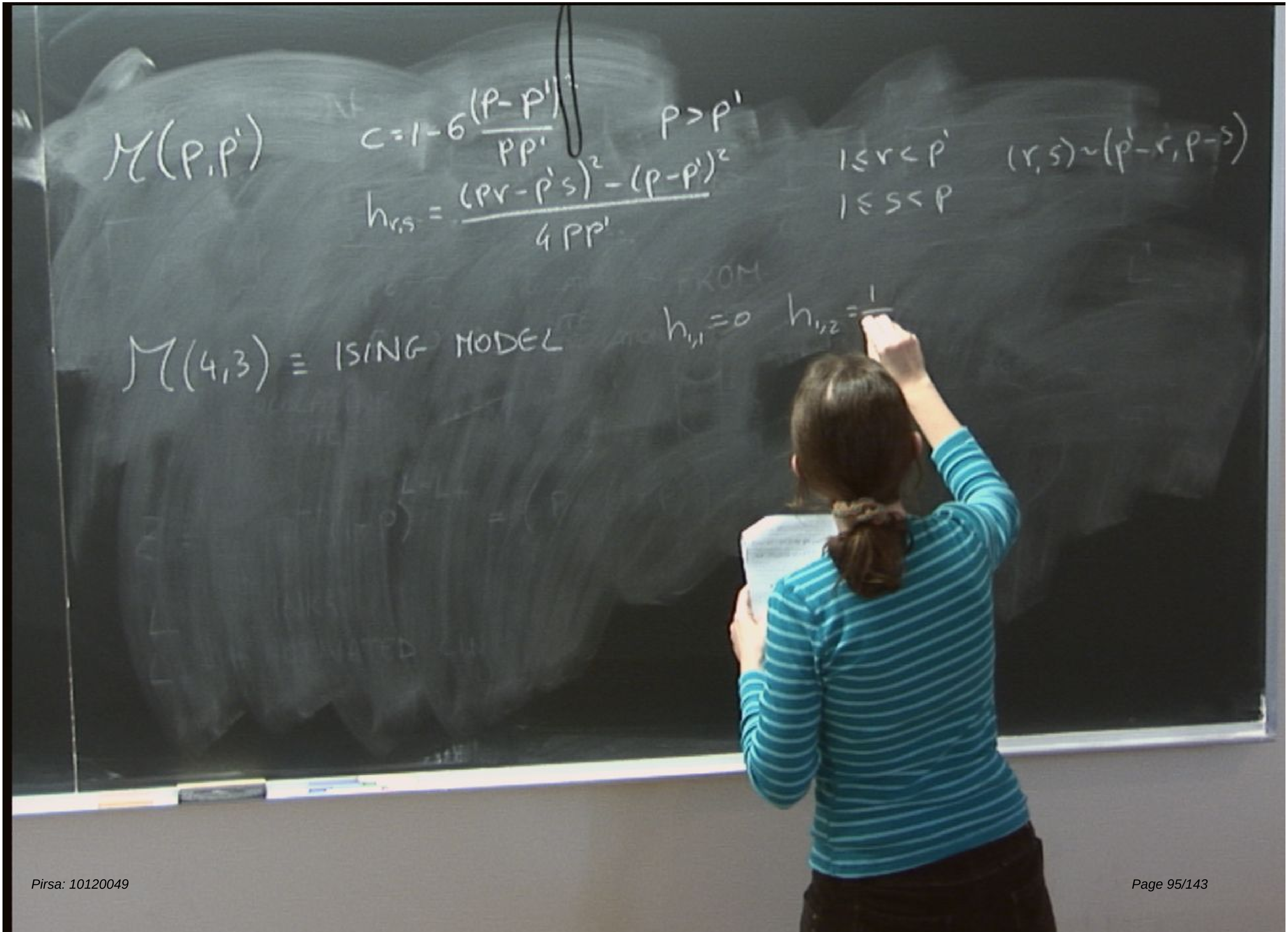

$\pi_h(r) = \lim_{Q\to 1} \left(Z_{\alpha\alpha} - Z_{\alpha\beta} \right)$
$\downarrow$ 1
π_v
f
(α)
$(\beta) \neq (\alpha)$
$Z_{\alpha\beta}$
f
f
α
α
$Z_{\alpha\alpha} \longrightarrow 1$
f
f
f

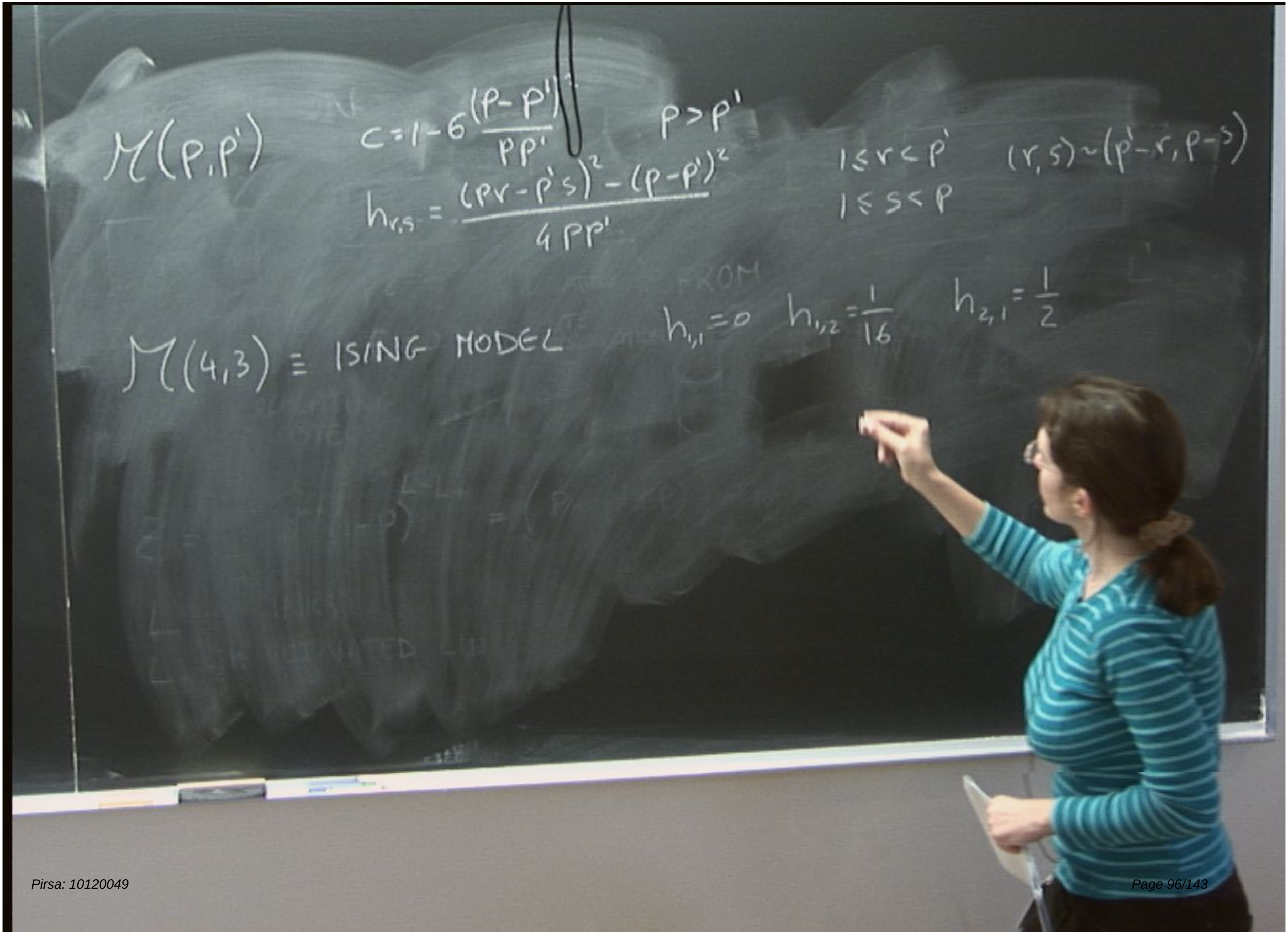


$M(p,p')$
$c = 1 - 6\,\frac{p-p'}{pp'}$
$p > p'$
$h_{r,s} = \frac{(pr - p's)^2 - (p-p')^2}{4pp'}$
$1 \leq r < p'$
$1 \leq s < p$



$\mathcal{M}(p,p')$
$c = 1 - 6\frac{(p-p')^2}{pp'}$
$p > p'$
$h_{r,s} = \frac{(pr-p's)^2-(p-p')^2}{4pp'}$
$1 \le r < p'$
$1 \le s < p$
$(r,s) \sim (p'-r, p-$
$\mathcal{M}$


M(p,p')
c = 1 - 6 (p-p')^2 / pp'
p > p'
1 ≤ r < p'
1 ≤ s < p
(r,s) ~ (p'-r, p-s)
h_{r,s} = ((pr - p's)^2 - (p-p')^2) / 4pp'
M(4,3) ≡ ISING MODEL
h_{1,1} = 0
h_{1,2} =


M(p,p')
c = 1 - 6 (p-p')²/pp'
p > p'
h_{r,s} = ((pr - p's)² - (p-p')²)/4pp'
1 ≤ r < p'
1 ≤ s < p
(r,s) ~ (p'-r, p-s)
M(4,3) ≡ ISING MODEL
h_{1,1} = 0
h_{1,2} = 1/16
h_{2,1} = 1/2

$\mathcal{M}(p,p')$ $\quad c = 1 - 6\frac{(p-p')^2}{pp'}$ $\quad p > p'$

$$h_{r,s} = \frac{(pr - p's)^2 - (p-p')^2}{4pp'}$$

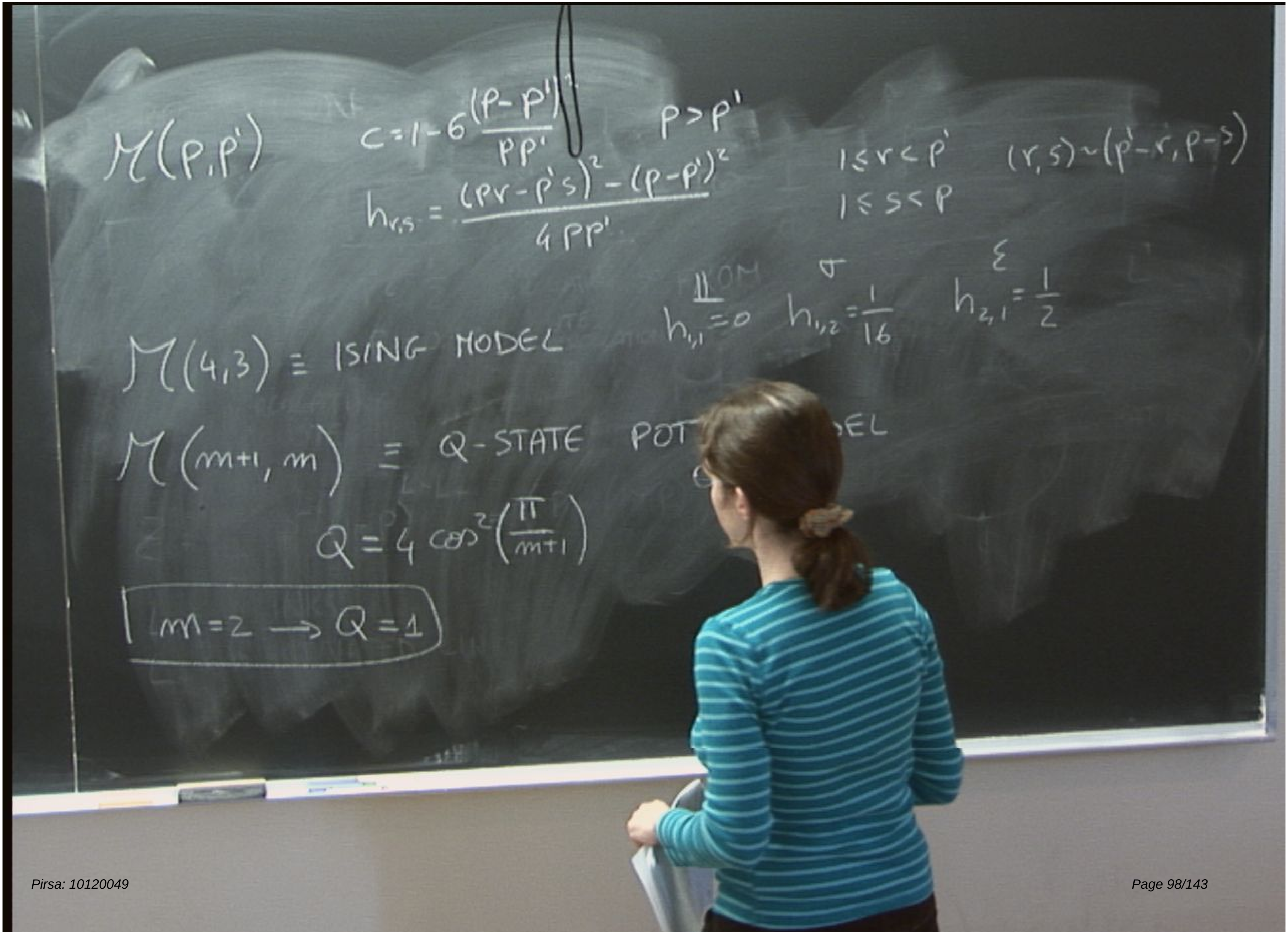
$1 \le r < p'$ $\quad (r,s) \sim (p'-r, p-s)$

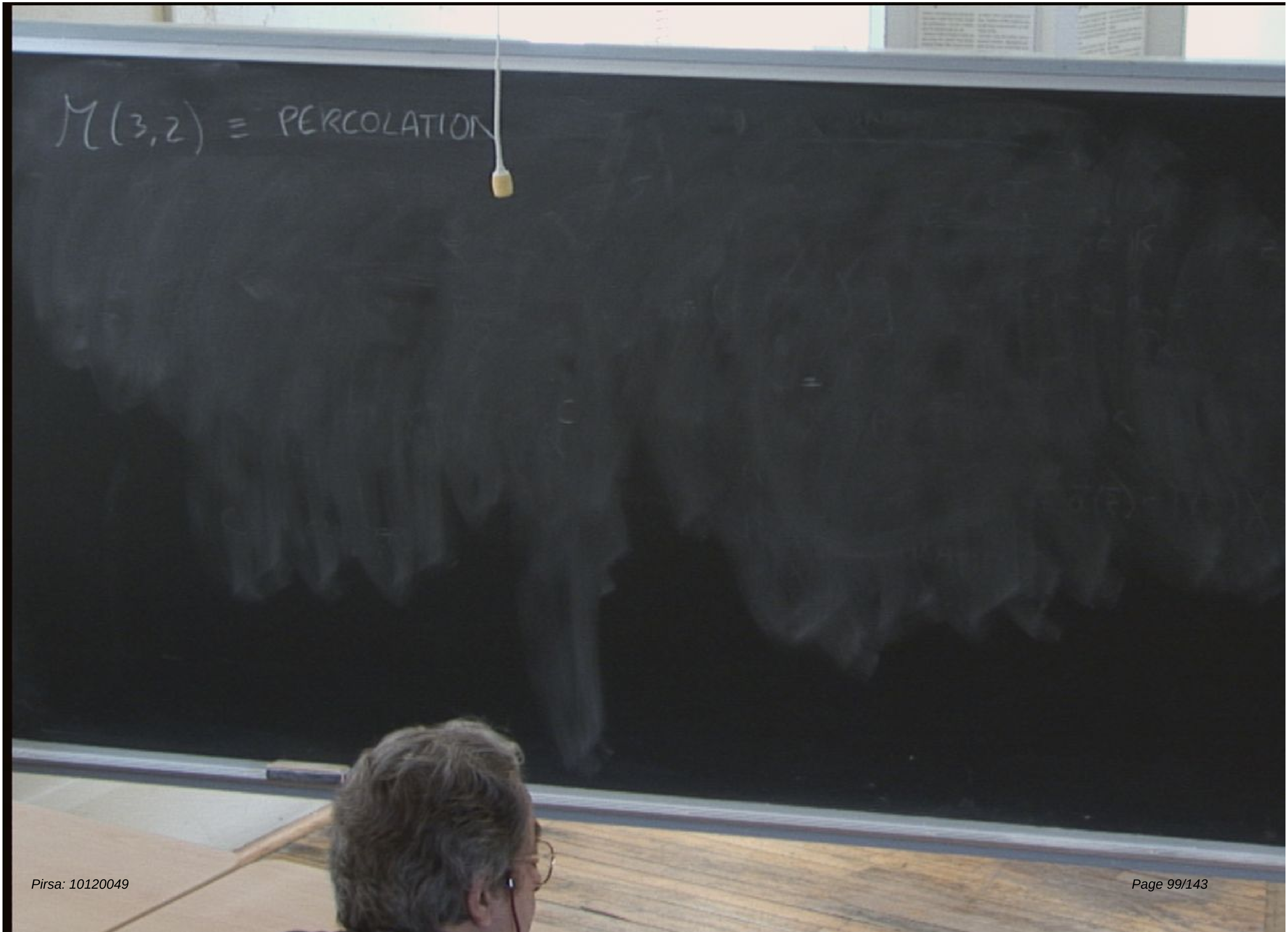
$1 \le s < p$

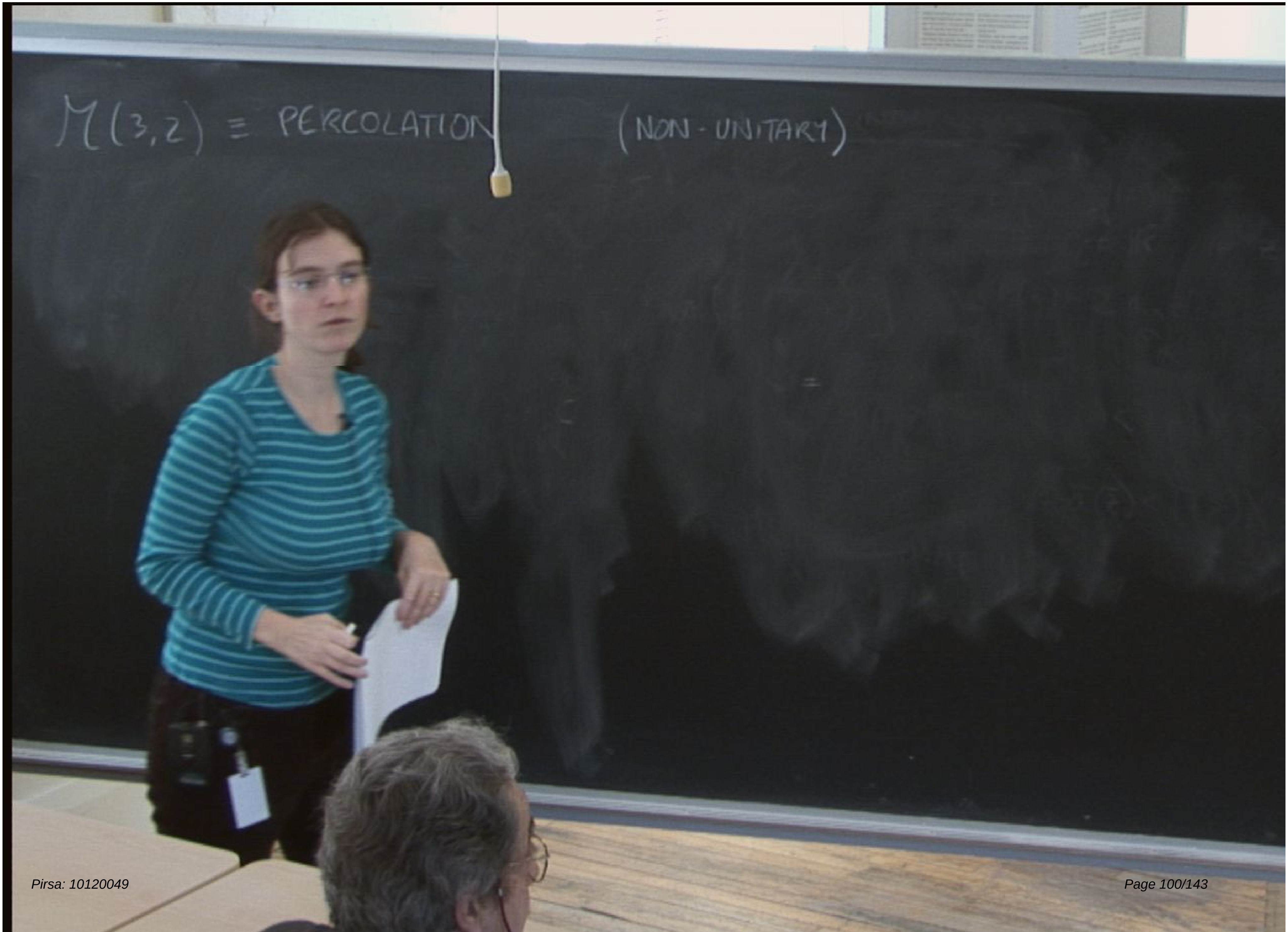
$\mathbb{1}$: $h_{1,1} = 0$ $\quad \sigma$: $h_{1,2} = \frac{1}{16}$ $\quad \varepsilon$: $h_{2,1} = \frac{1}{2}$

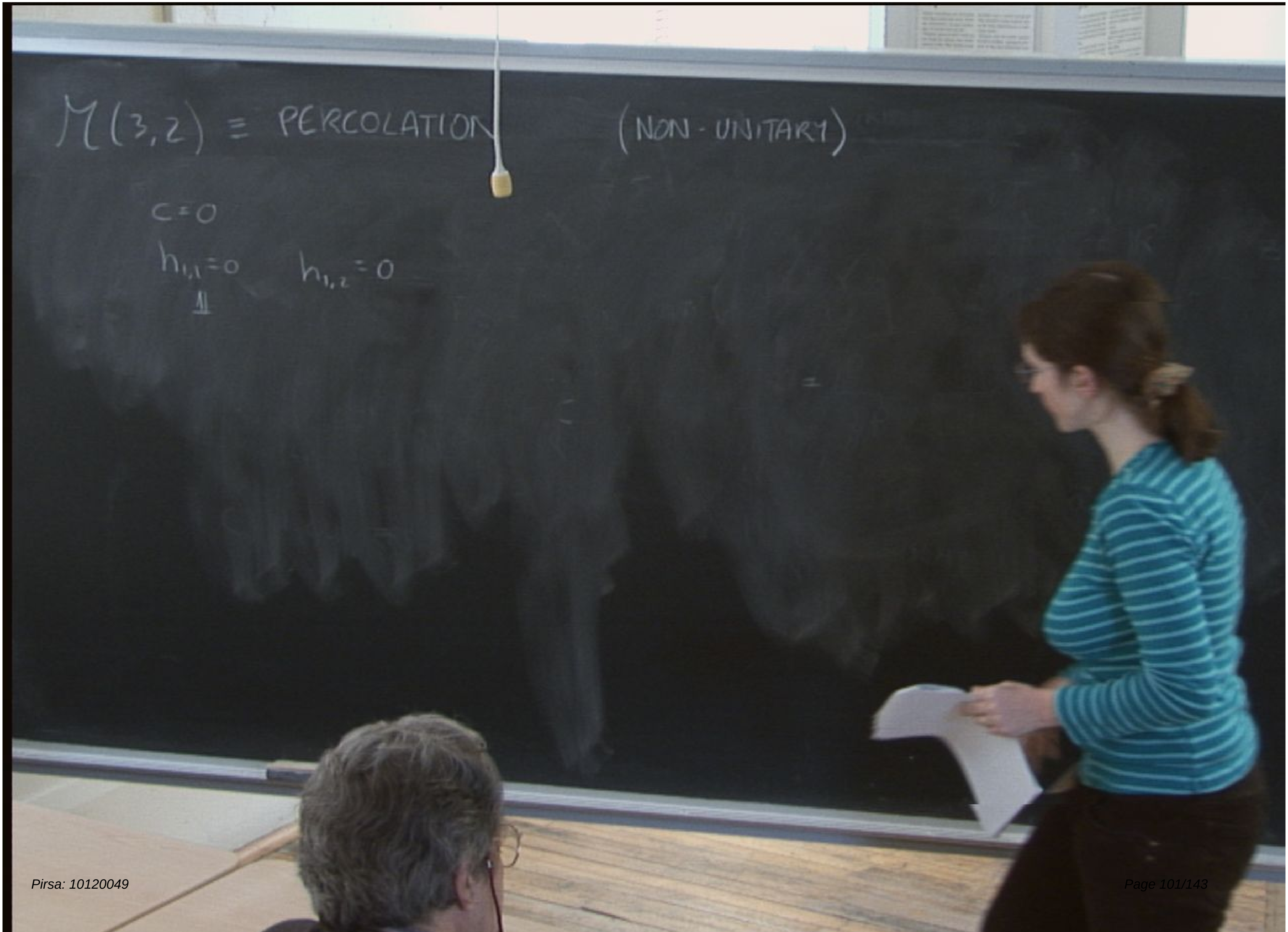
$\mathcal{M}(4,3) \equiv$ ISING MODEL

$\mathcal{M}(m+1, m) \equiv$ Q-STATE POTTS MODEL


M(p,p')
c = 1 - 6 (p - p')² / pp'
p > p'
h_{r,s} = ((pr - p's)² - (p - p')²) / 4pp'
1 ≤ r < p'
1 ≤ s < p
(r,s) ~ (p' - r, p - s)
M(4,3) ≡ ISING MODEL
h_{1,1} = 0
h_{1,2} = 1/16
h_{2,1} = 1/2
M(m+1, m) ≡ Q-STATE
Q = 4 cos²(π/(m+1))
m = 2 → Q = 1

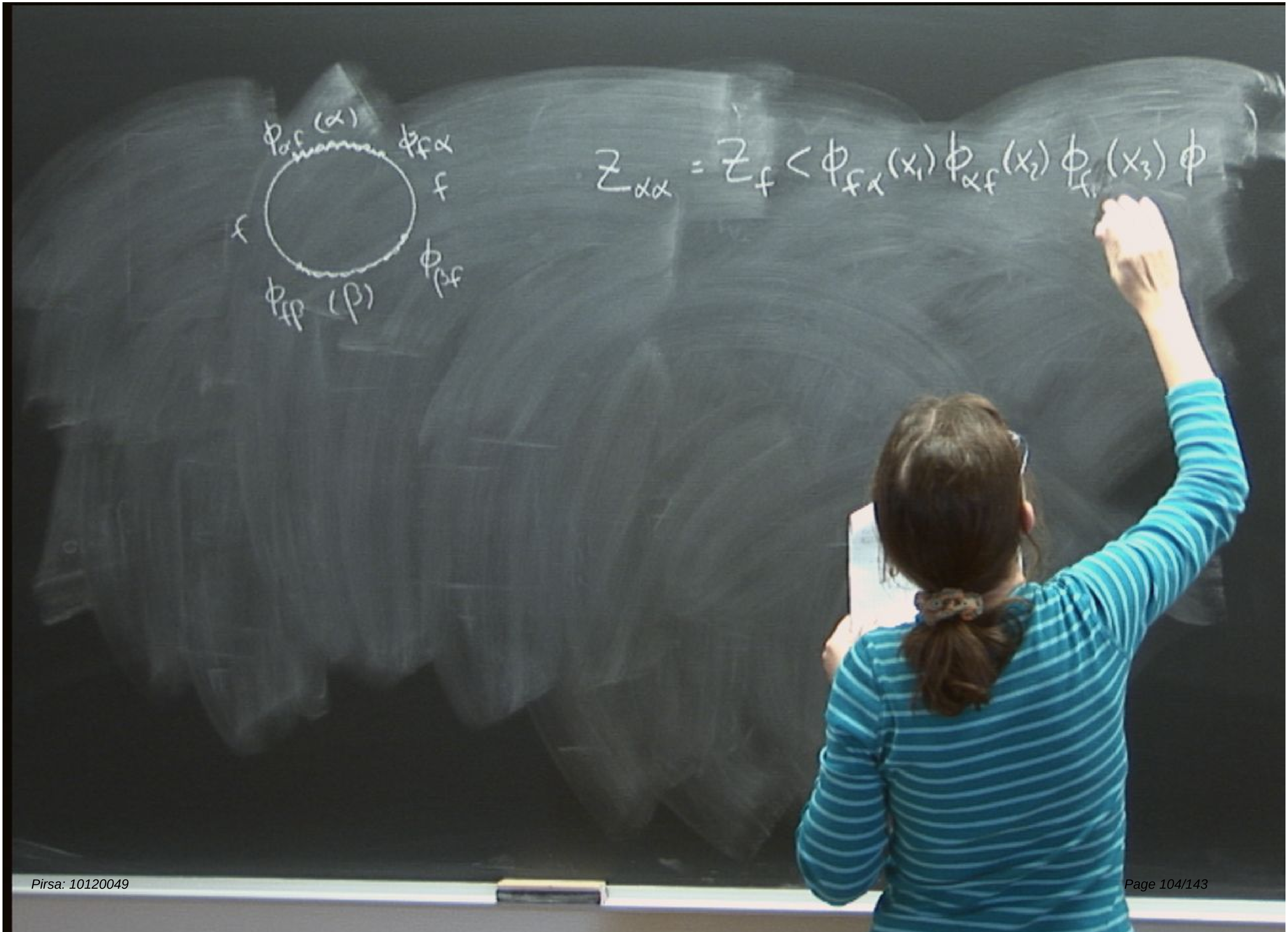

M(3,2) ≡ PERCOLATION

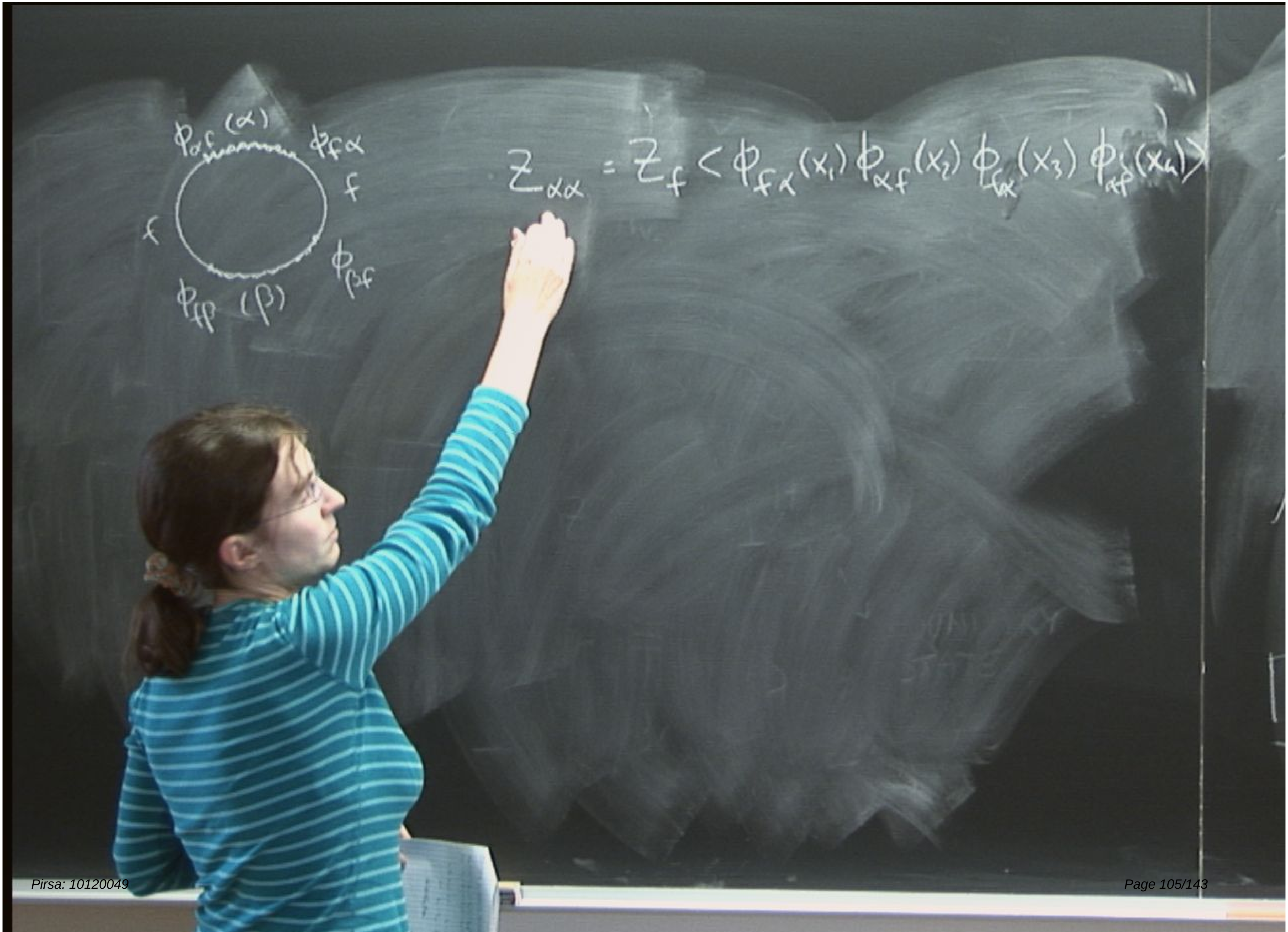

M(3,2) ≡ PERCOLATION
(NON-UNITARY)

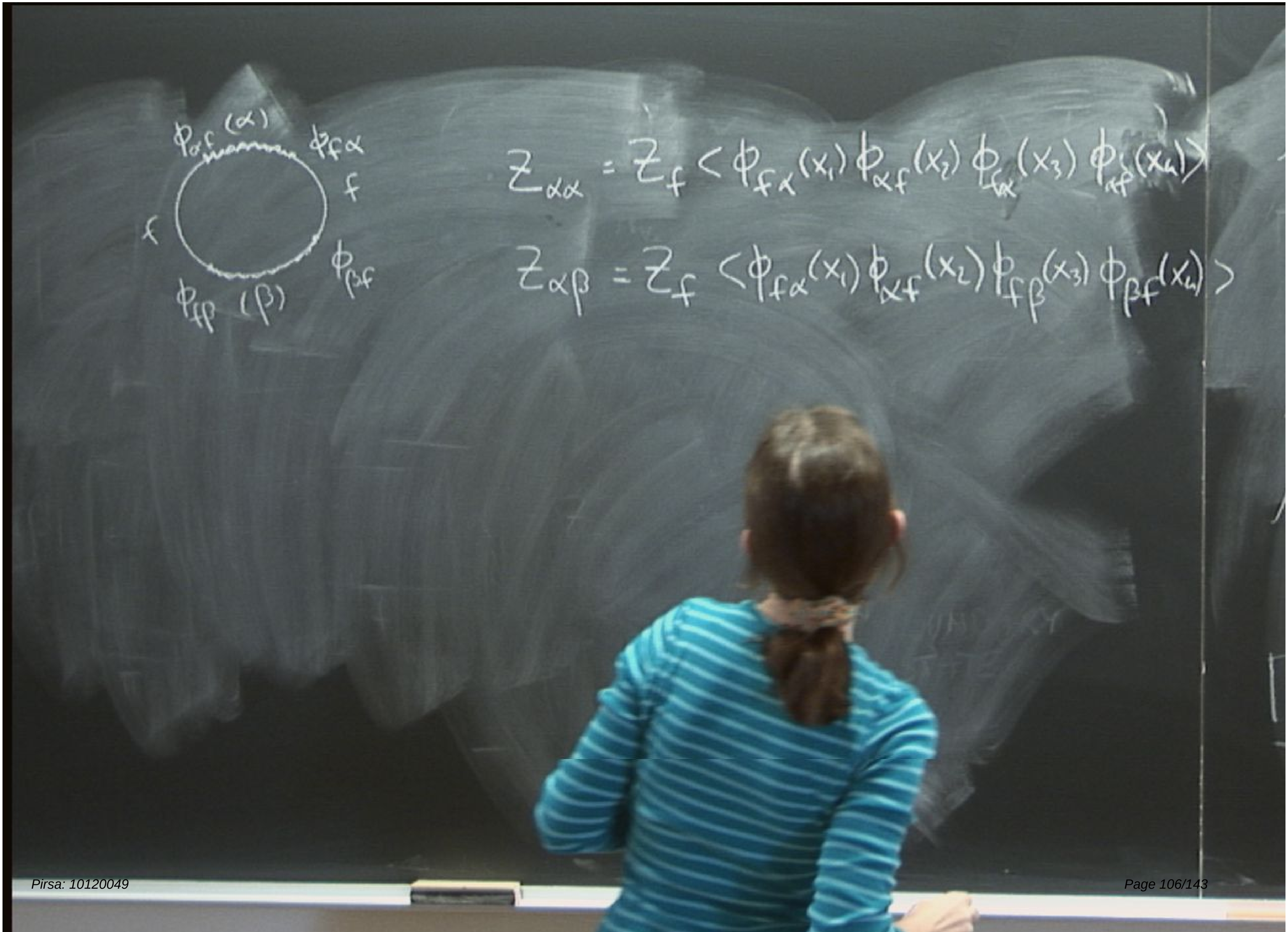

M(3,2) ≡ PERCOLATION
(NON-UNITARY)
c = 0
h1,1 = 0
h1,2 = 0

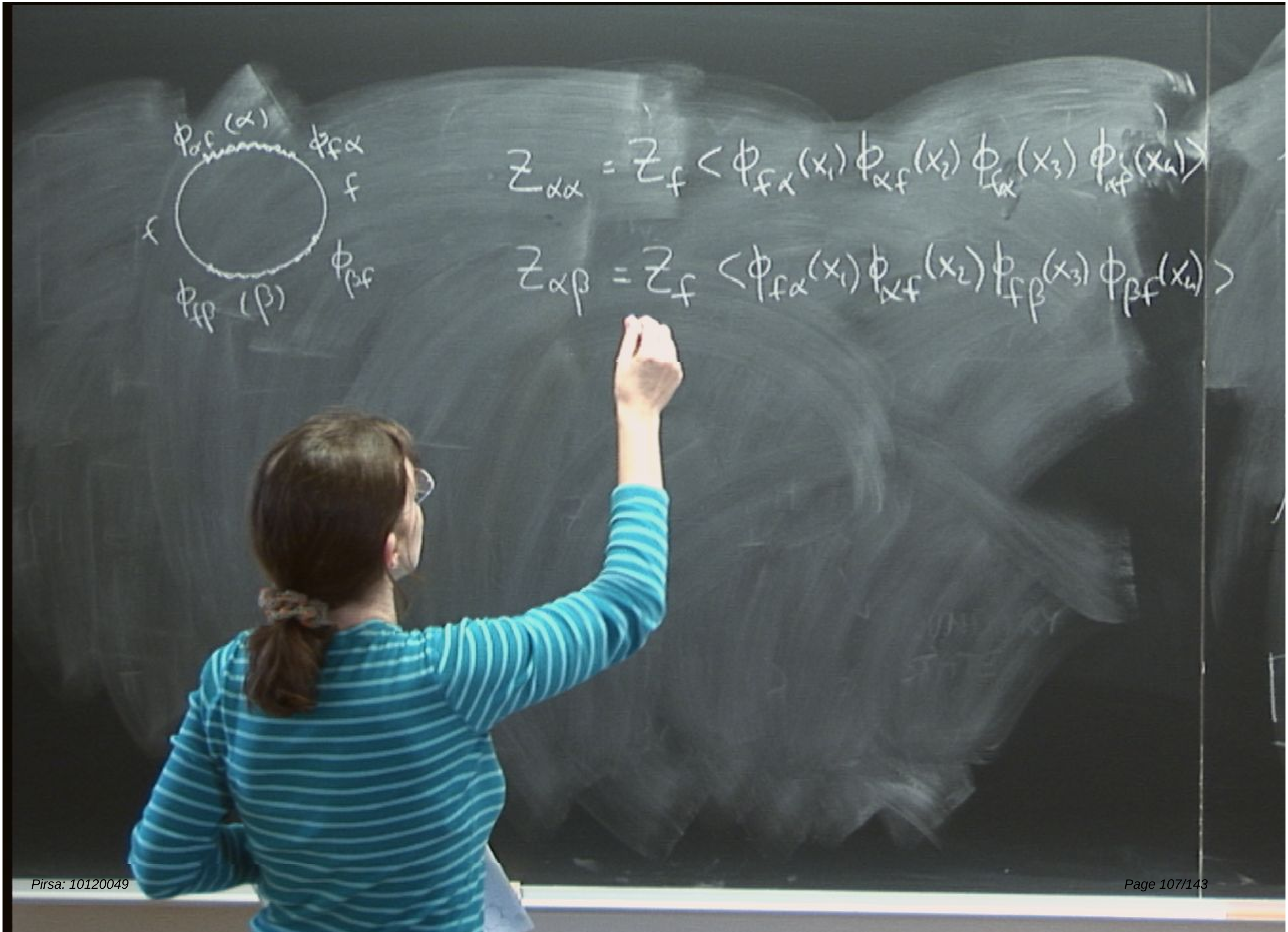

φαf
(α)
φfα
f
f
φβf
φfβ
(β)

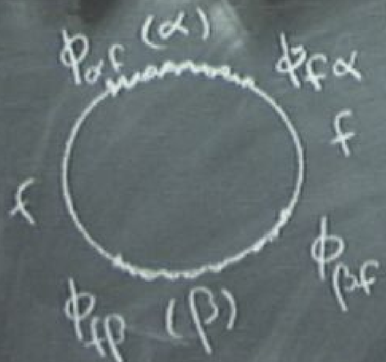

(α)
φαf
φfα
f
f
φβf
(β)
Zαα =


(α)
(β)
f
f


φ_{αf} (α) φ_{fα}
f
f
φ_{βf}
φ_{fβ} (β)
Z_{αα} = Z_f < φ_{fα}(x_1) φ_{αf}(x_2) φ_{fα}(x_3) φ_{αf}(x_4) >


$\phi_{\alpha f}$ (α) $\phi_{f\alpha}$
f
f
$\phi_{\beta f}$
$\phi_{f\beta}$ (β)
$Z_{\alpha\alpha} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\alpha}(x_3)\, \phi_{\alpha f}(x_4) \rangle$
$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\beta}(x_3)\, \phi_{\beta f}(x_4) \rangle$


$\phi_{\alpha f}$ (α) $\phi_{f\alpha}$
f
f
$\phi_{\beta f}$
$\phi_{f\beta}$ (β)
$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\beta}(x_3)\, \phi_{\beta f}(x_4) \rangle$

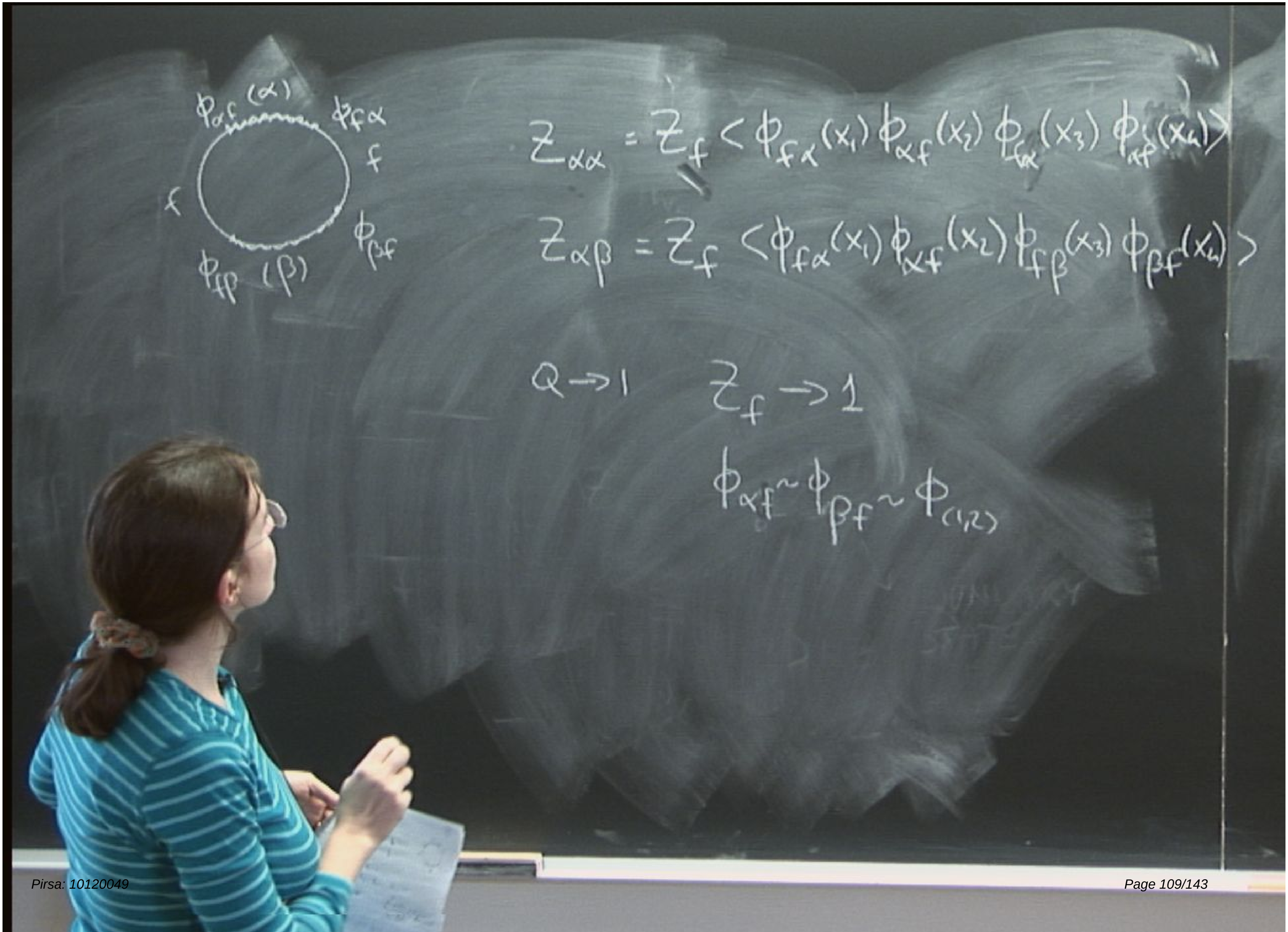


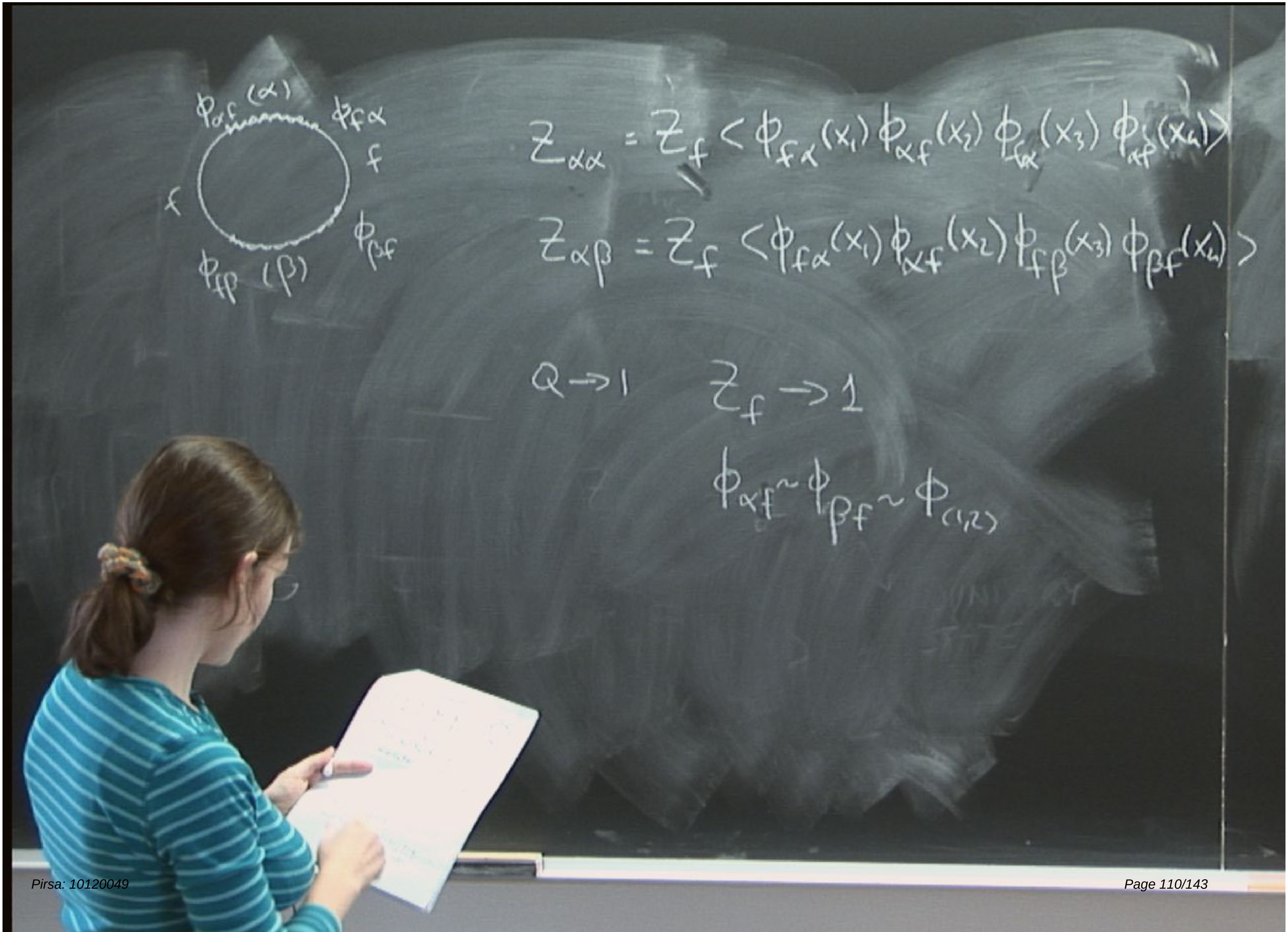
$$Z_{\alpha\alpha} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\alpha}(x_3)\, \phi_{\alpha f}(x_4) \rangle$$

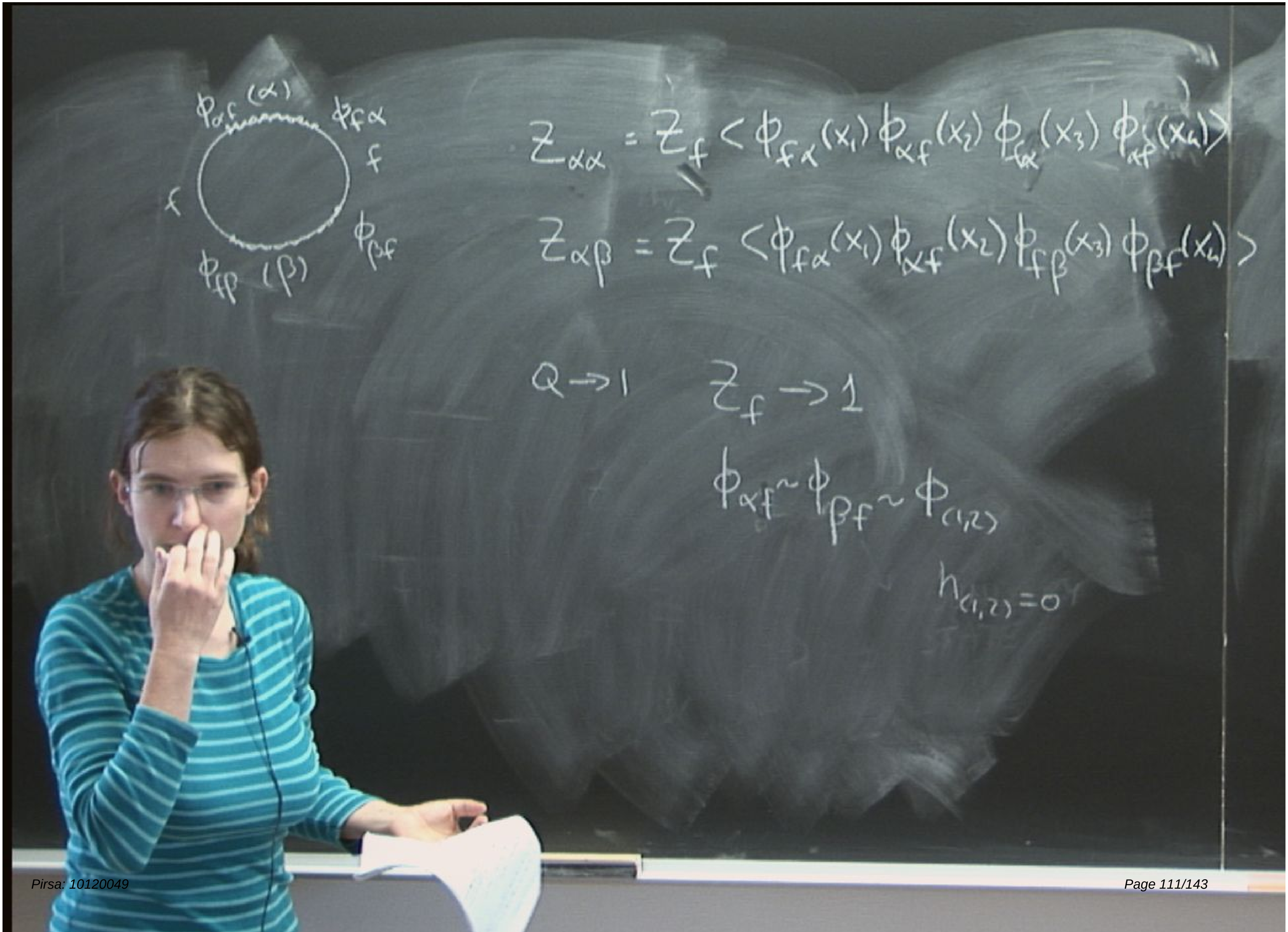
$$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\beta}(x_3)\, \phi_{\beta f}(x_4) \rangle$$

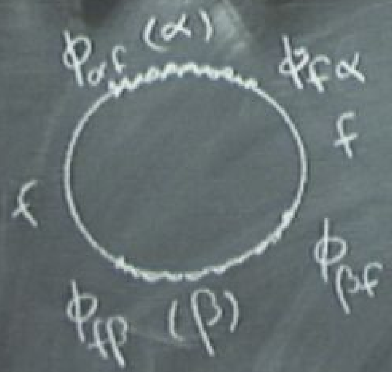
$Q \to 1$ $\quad Z_f \to 1$

$\phi_{\alpha f} \sim \phi_{\beta f} \sim$


$\phi_{\alpha f}$ (α) $\phi_{f\alpha}$
f
f
$\phi_{\beta f}$
$\phi_{f\beta}$ (β)
$Z_{\alpha\alpha} = Z_f \langle \phi_{f\alpha}(x_1)\,\phi_{\alpha f}(x_2)\,\phi_{f\alpha}(x_3)\,\phi_{\alpha f}(x_4)\rangle$
$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\,\phi_{\alpha f}(x_2)\,\phi_{f\beta}(x_3)\,\phi_{\beta f}(x_4)\rangle$
$Q \to 1$
$Z_f \to 1$
$\phi_{\alpha f} \sim \phi_{\beta f} \sim \phi_{(1,2)}$


$\phi_{\alpha f}$ (α) $\phi_{f\alpha}$
f
f
$\phi_{\beta f}$
$\phi_{f\beta}$ (β)
$Z_{\alpha\alpha} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\alpha}(x_3)\, \phi_{\alpha f}(x_4) \rangle$
$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\beta}(x_3)\, \phi_{\beta f}(x_4) \rangle$
$Q \to 1$
$Z_f \to 1$
$\phi_{\alpha f} \sim \phi_{\beta f} \sim \phi_{(1,2)}$


(α)
φαf
φfα
f
f
φβf
φfβ (β)
Zαα = Zf < φfα(x1) φαf(x2) φfα(x3) φαf(x4) >
Zαβ = Zf < φfα(x1) φαf(x2) φfβ(x3) φβf(x4) >
Q → 1
Zf → 1
φαf ~ φβf ~ φ(1,2)
h(1,2) = 0



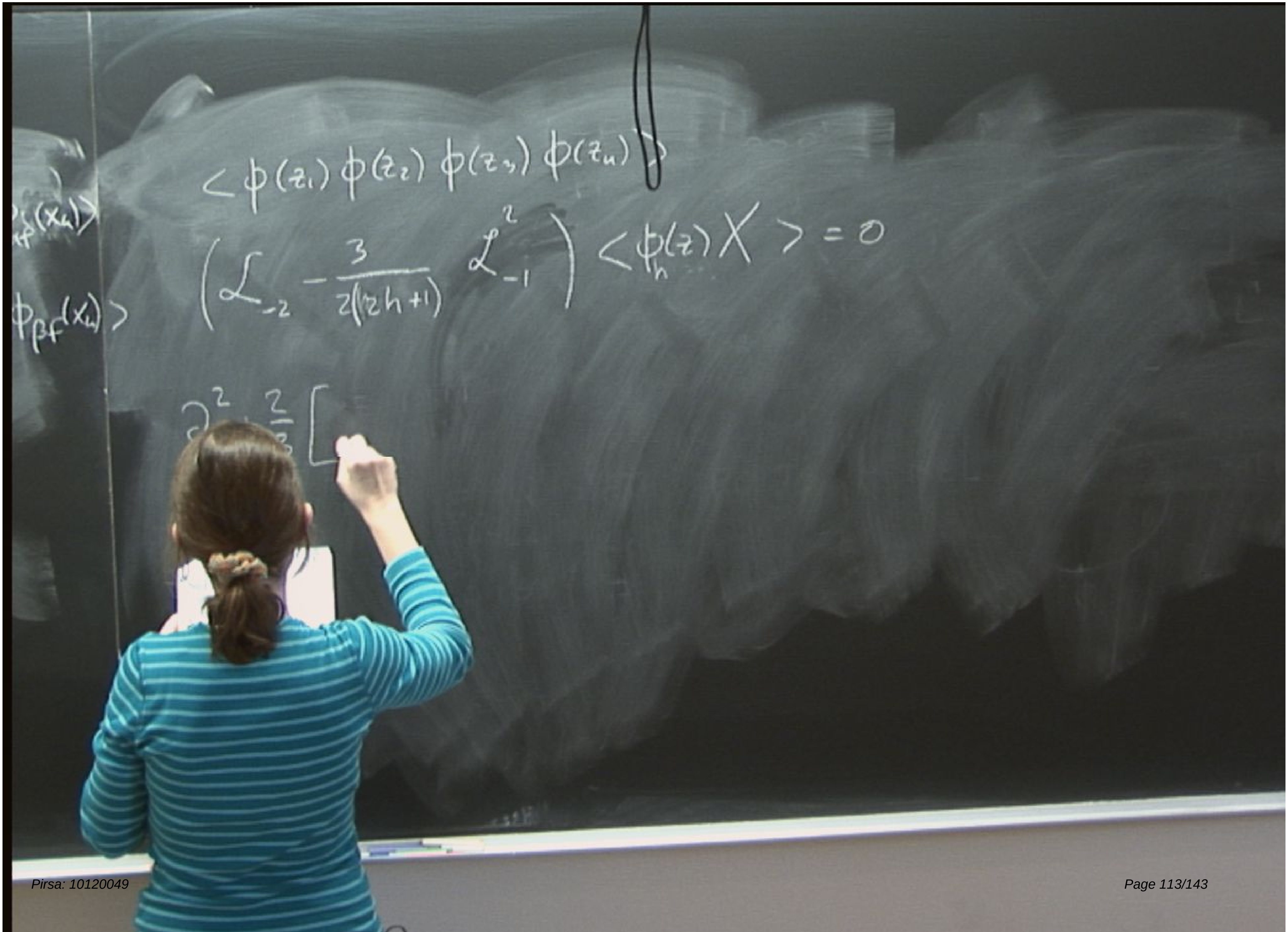
$$Z_{\alpha\alpha} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\alpha}(x_3)\, \phi_{\alpha f}(x_4) \rangle$$

$$Z_{\alpha\beta} = Z_f \langle \phi_{f\alpha}(x_1)\, \phi_{\alpha f}(x_2)\, \phi_{f\beta}(x_3)\, \phi_{\beta f}(x_4) \rangle$$

$Q \to 1$ $\quad Z_f \to 1$

$$\phi_{\alpha f} \sim \phi_{\beta f} \sim \phi_{(1,2)}$$

$$h_{(1,2)} = 0$$


< φ(z₁) φ(z₂) φ(z₃) φ(z₄) >
(L₋₂ − 3/(2(2h+1)) L²₋₁) < φₕ(z) X > = 0

$\ldots(x_4)\rangle$

$\phi_{\beta f}(x_4)\rangle$

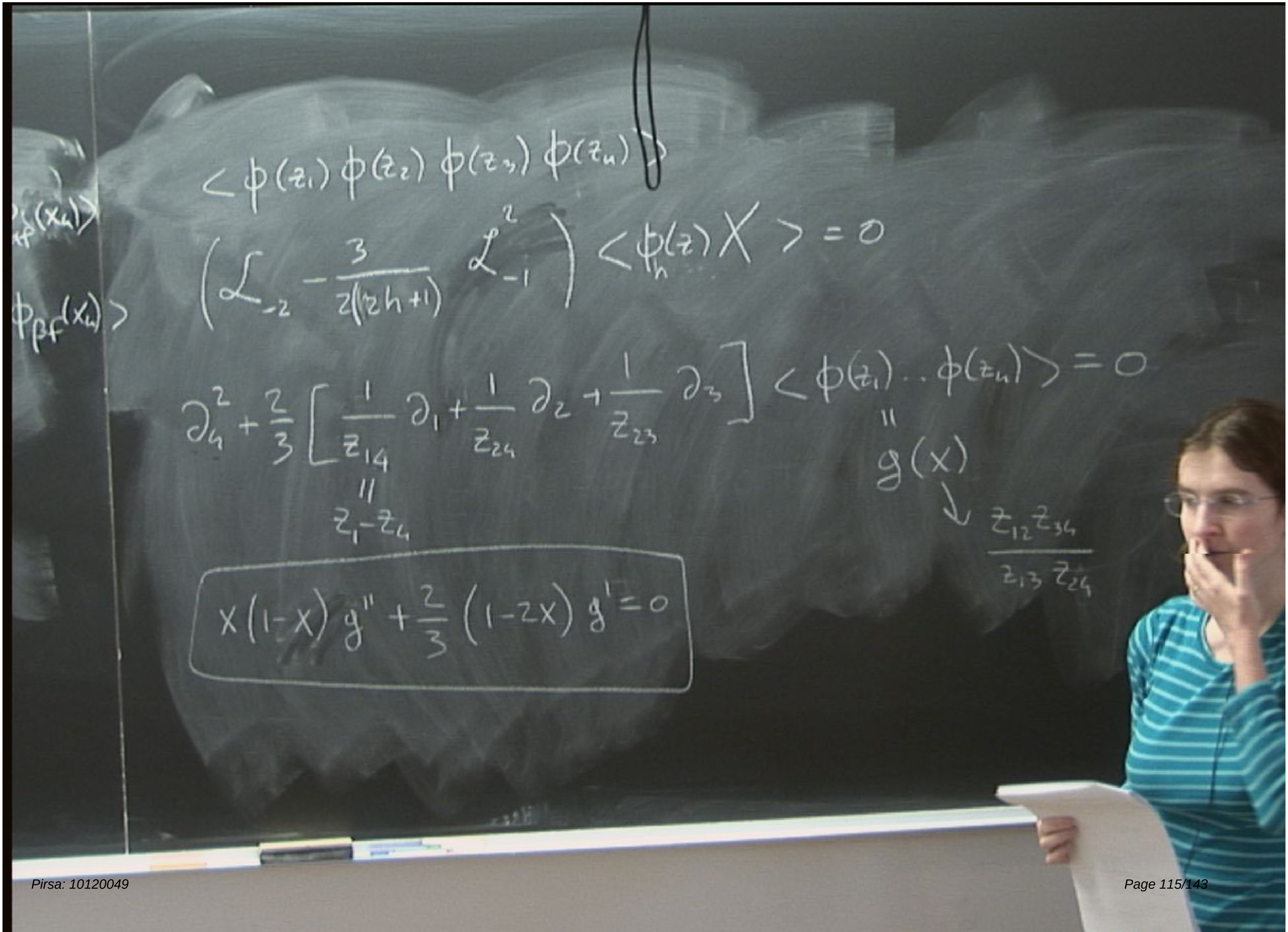
$$\langle \phi(z_1)\,\phi(z_2)\,\phi(z_3)\,\phi(z_4)\rangle$$

$$\left(\mathcal{L}_{-2} - \frac{3}{2(2h+1)}\,\mathcal{L}_{-1}^{2}\right)\langle \phi_h(z)\,X\rangle = 0$$

$$\partial_4^2 + \frac{2}{3}\left[\frac{1}{z_{14}}\,\partial_1 + \frac{1}{z_{24}}\,\partial_2 + \frac{1}{z_{23}}\,\partial_3\right]\langle \phi(z_1)\ldots\phi(z_4)\rangle = 0$$

$$z_{14} = z_1 - z_4$$

$$\langle \phi(z_1)\ldots\phi(z_4)\rangle = g(x), \qquad x \to \frac{z_{12}\,z_{34}}{z_{13}\,z_{24}}$$


$\langle \phi(z_1)\phi(z_2)\phi(z_3)\phi(z_4)\rangle$
$\left(\mathcal{L}_{-2} - \frac{3}{2(2h+1)}\mathcal{L}_{-1}^2\right)\langle \phi_h(z) X \rangle = 0$
$\partial_4^2 + \frac{2}{3}\left[\frac{1}{z_{14}}\partial_1 + \frac{1}{z_{24}}\partial_2 + \frac{1}{z_{23}}\partial_3\right]\langle \phi(z_1)\dots\phi(z_4)\rangle = 0$
$= g(x)$
$z_{14} = z_1 - z_4$
$\frac{z_{12} z_{34}}{z_{13} z_{24}}$
$x(1-x)g'' + \frac{2}{3}(1-2x)g' = 0$

$\phi_{\beta F}(x_4)\rangle$

$$\langle \phi(z_1)\,\phi(z_2)\,\phi(z_3)\,\phi(z_4)\rangle$$

$$\left(\mathcal{L}_{-2} - \frac{3}{2(2h+1)}\,\mathcal{L}_{-1}^2\right)\langle \phi_h(z)\, X\rangle = 0$$

$$\partial_4^2 + \frac{2}{3}\left[\frac{1}{z_{14}}\,\partial_1 + \frac{1}{z_{24}}\,\partial_2 + \frac{1}{z_{23}}\,\partial_3\right]\langle \phi(z_1)\ldots\phi(z_4)\rangle = 0$$

$$z_{14} = z_1 - z_4$$

$$\langle \phi(z_1)\ldots\phi(z_4)\rangle = g(x), \qquad x = \frac{z_{12}\,z_{34}}{z_{13}\,z_{24}}$$

$$\boxed{x(1-x)\,g'' + \frac{2}{3}(1-2x)\,g' = 0}$$

HYPERGEOMETRIC

$$g_1(x) = 1$$

$$g_2(x) = x^{1/3}\, F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x\right)$$

$$\langle \phi(z_1)\,\phi(z_2)\,\phi(z_3)\,\phi(z_4)\rangle$$

$$\left(\mathcal{L}_{-2} - \frac{3}{2(2h+1)}\mathcal{L}_{-1}^{2}\right)\langle \phi_h(z)\, X \rangle = 0$$

$$\partial_4^2 + \frac{2}{3}\left[\frac{1}{z_{14}}\partial_1 + \frac{1}{z_{24}}\partial_2 + \frac{1}{z_{23}}\partial_3\right]\langle \phi(z_1)\ldots\phi(z_4)\rangle = 0$$

$$z_{14} = z_1 - z_4$$

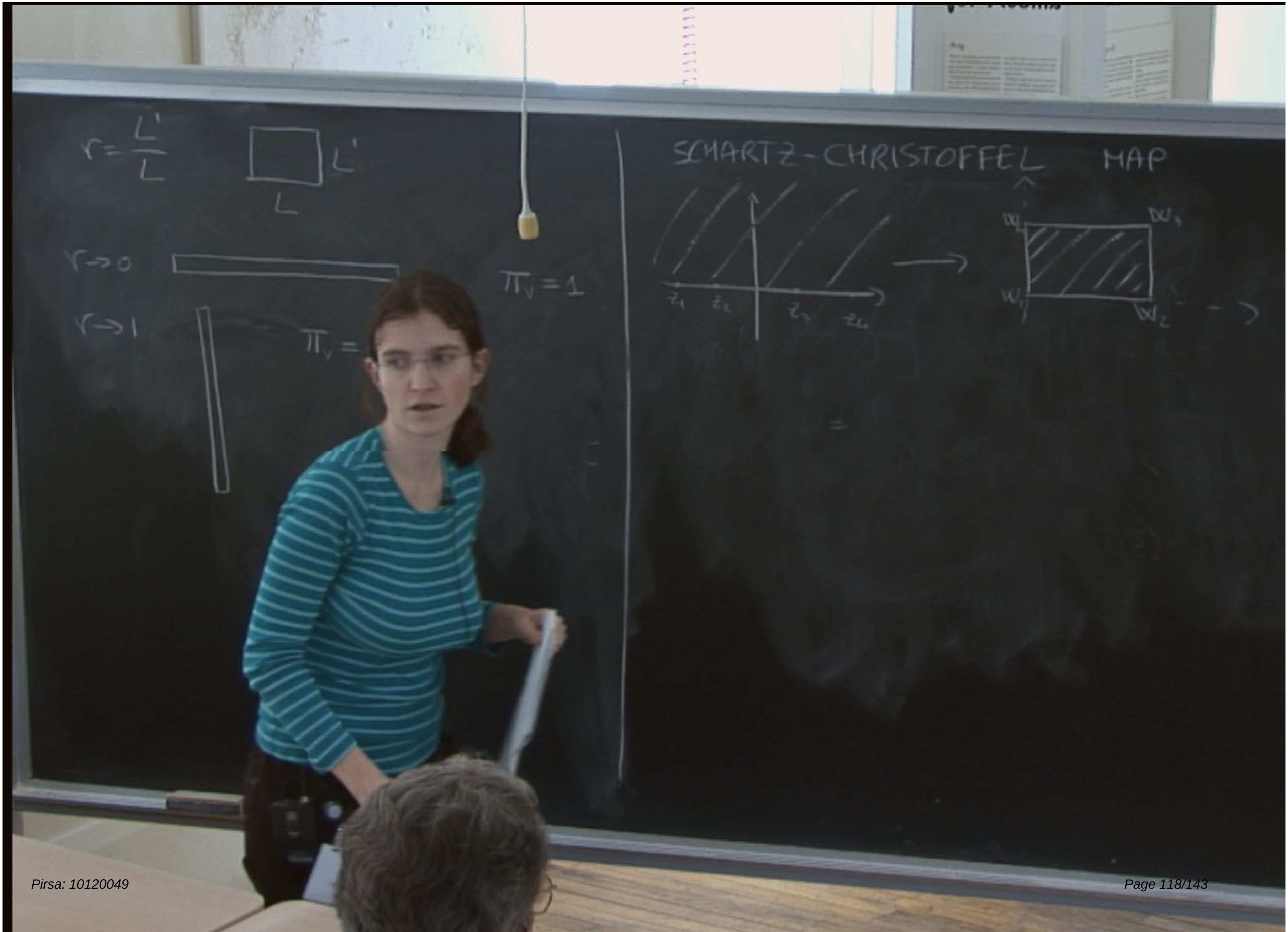
$$\langle \phi(z_1)\ldots\phi(z_4)\rangle = g(x), \quad x \to \frac{z_{12}z_{34}}{z_{13}z_{24}}$$

$$\boxed{x(1-x)\,g'' + \frac{2}{3}(1-2x)\,g' = 0}$$

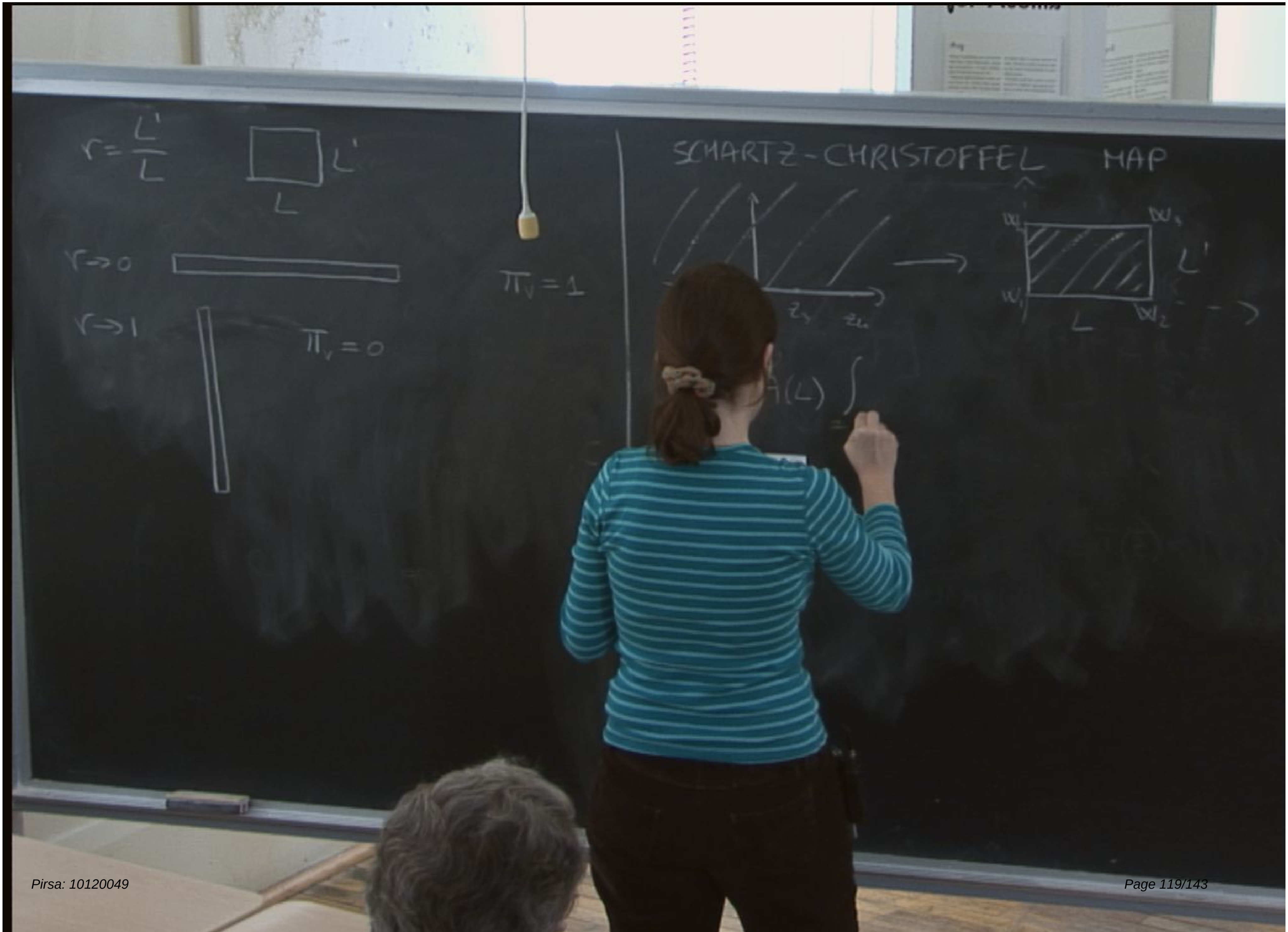
HYPERGEOMETRIC

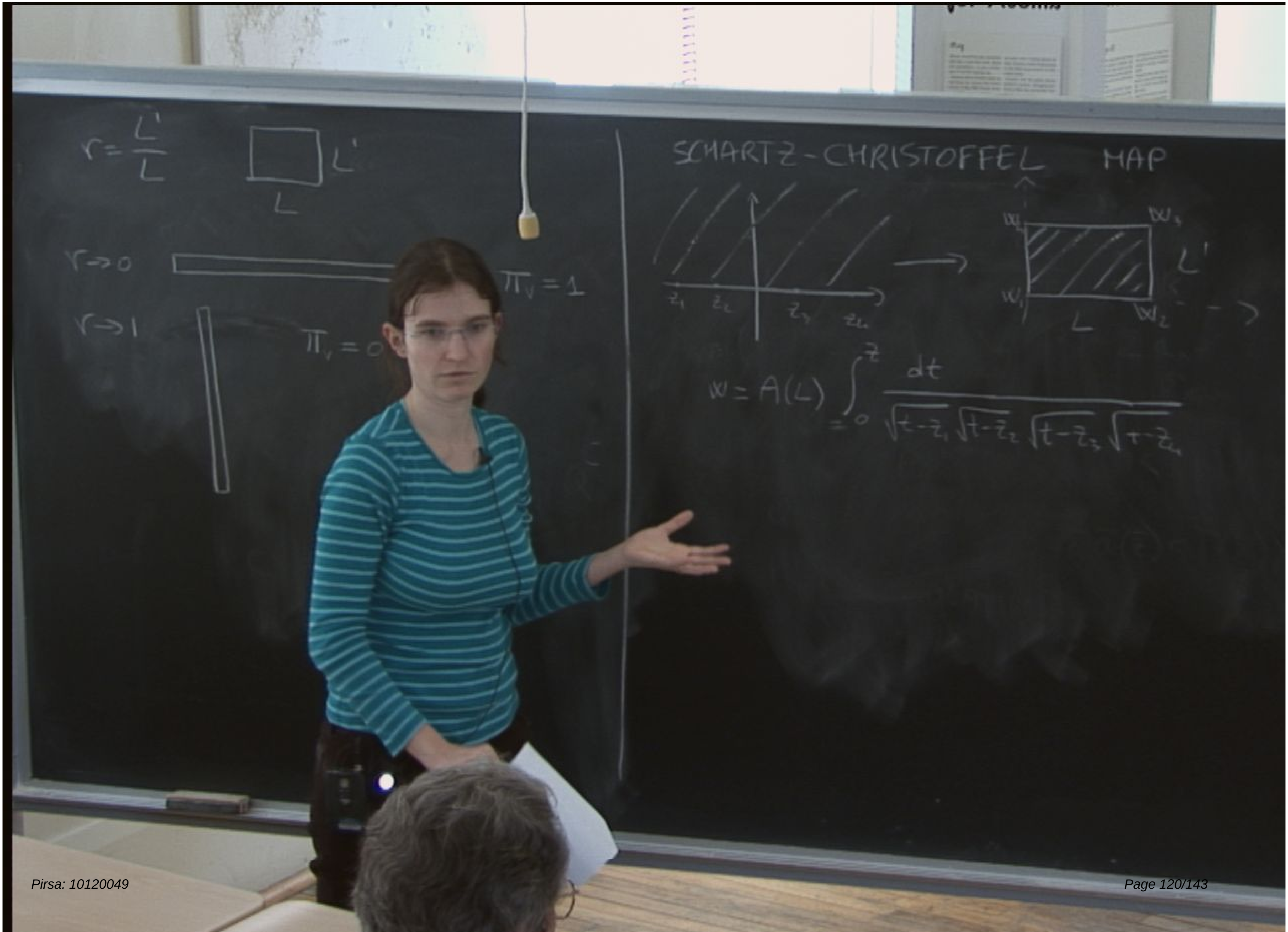
$$g_1(x) = 1$$

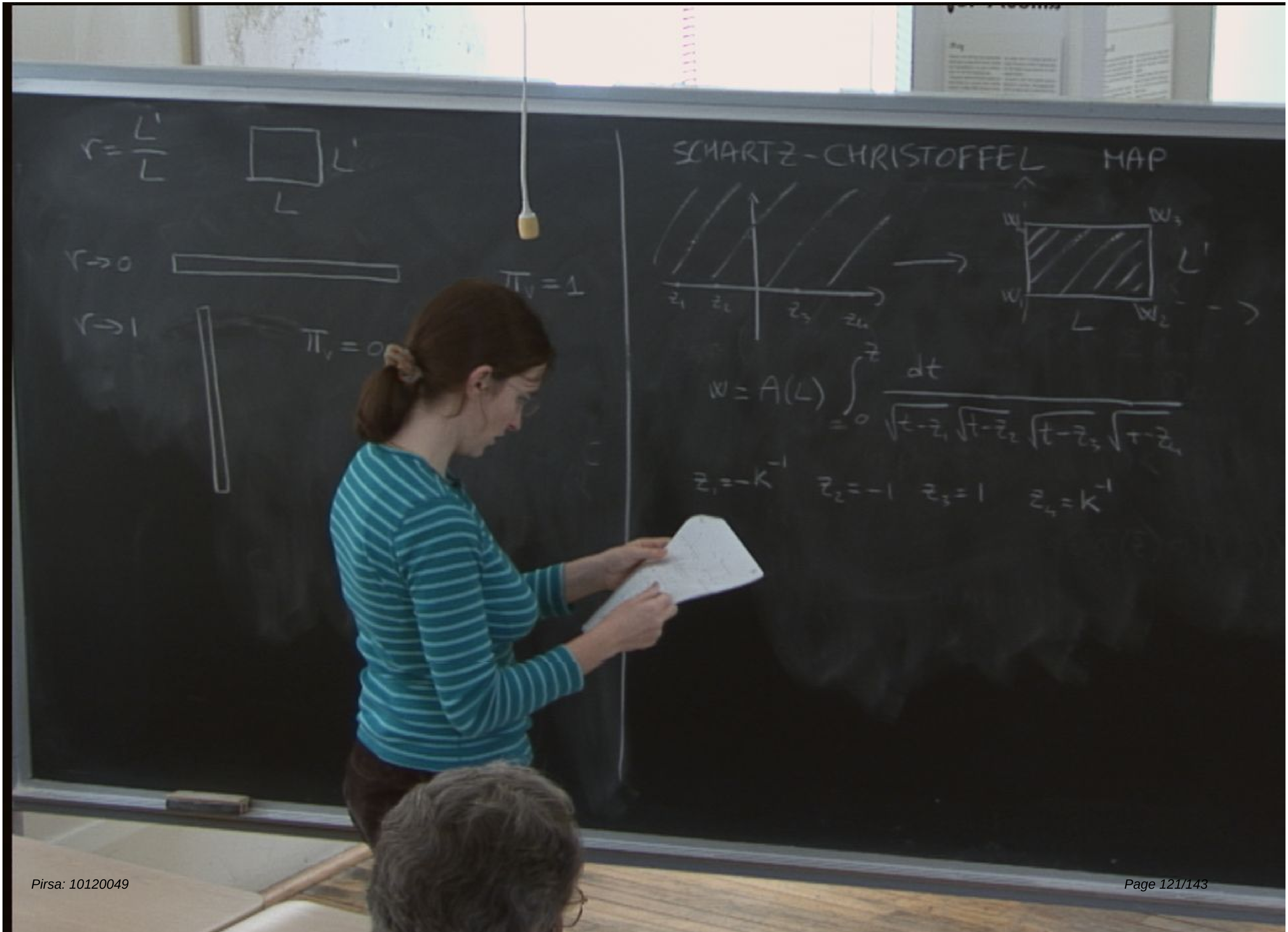
$$g_2(x) = x^{1/3}\,F\left(\tfrac{1}{3}, \tfrac{2}{3}; \tfrac{4}{3}; x\right)$$

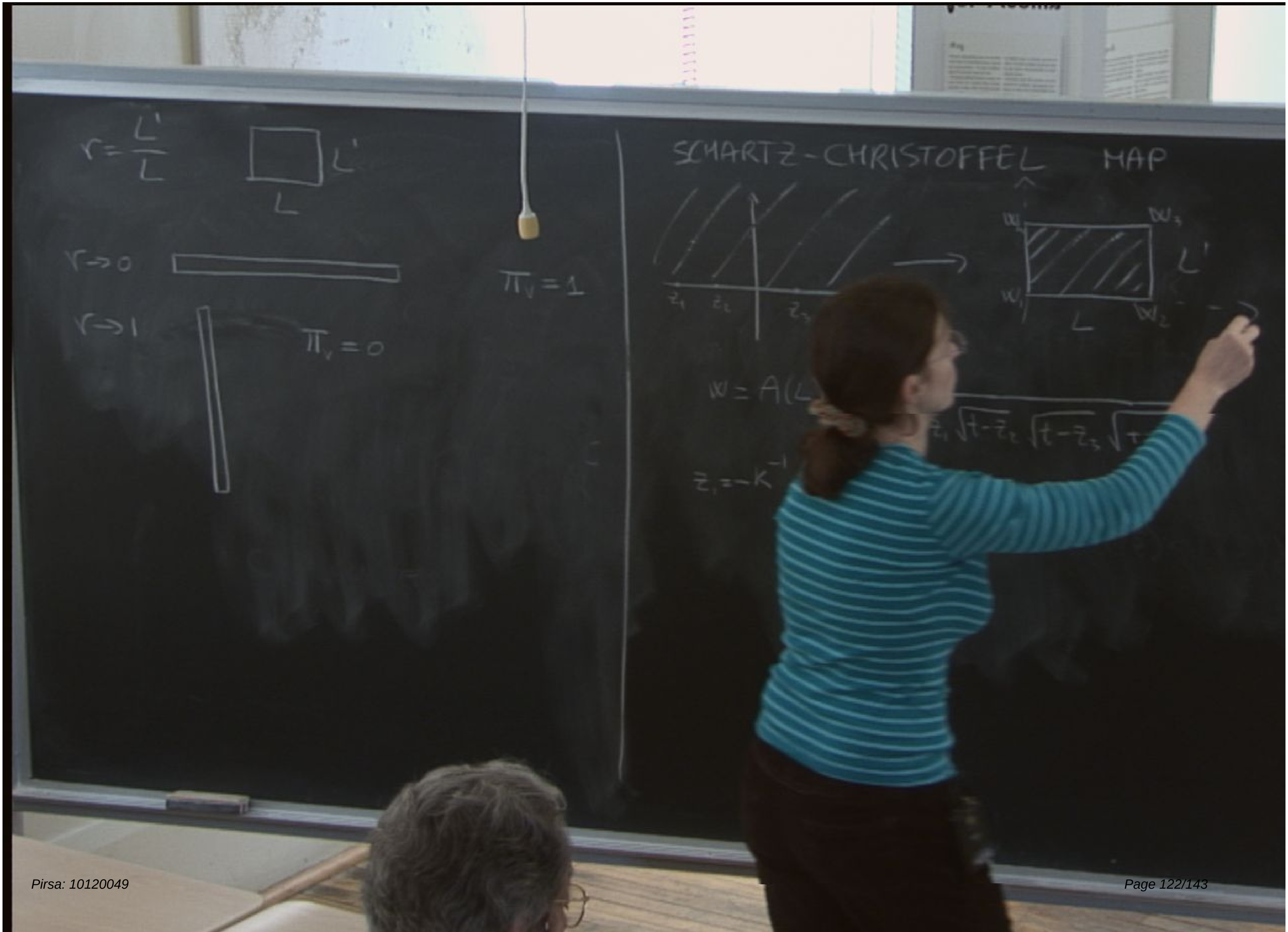


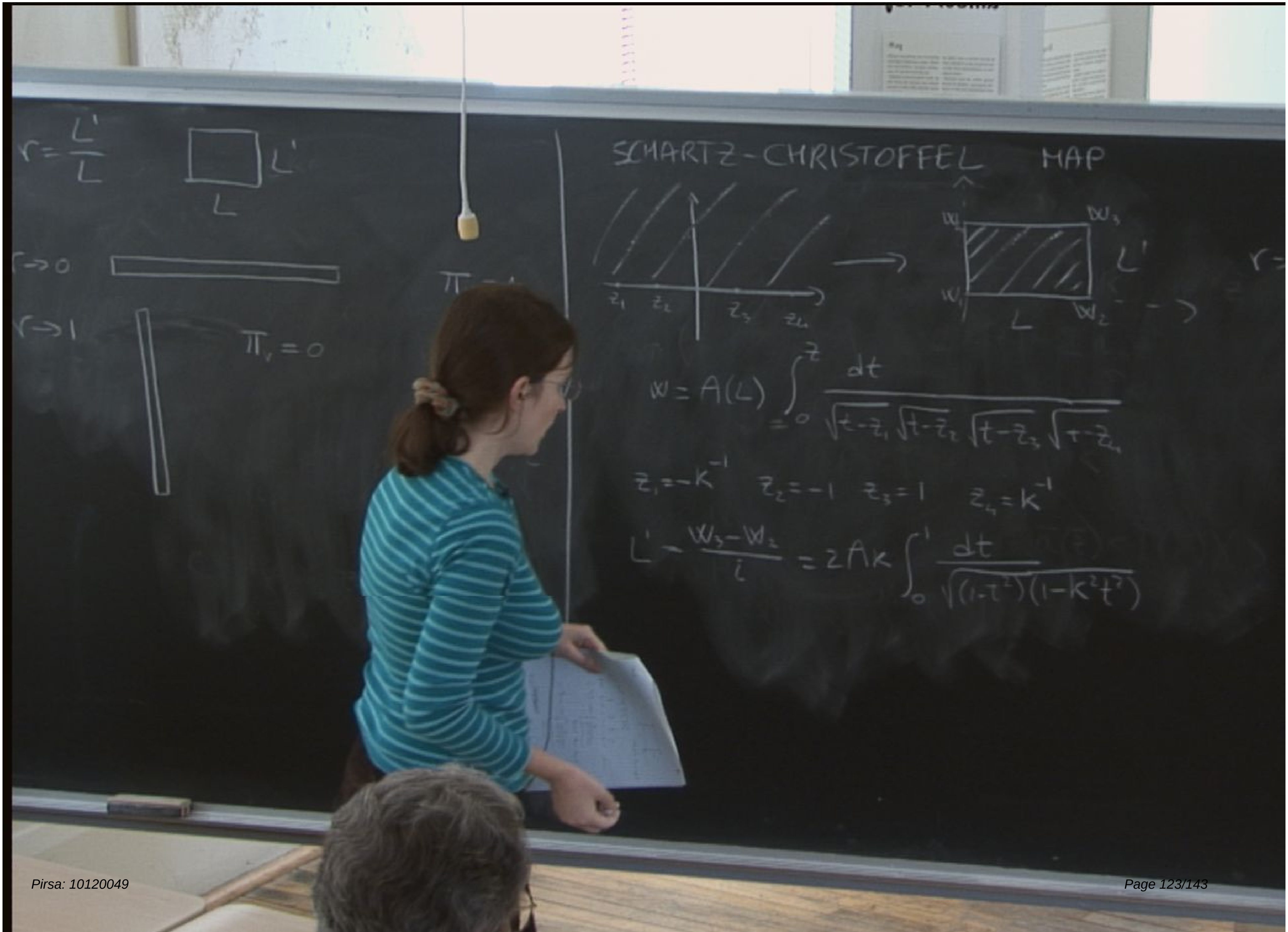
$r = \frac{L'}{L}$
L'
L
$r \to 0$
$\pi_v = 1$
$r \to 1$
$\pi_v =$
SCHARTZ-CHRISTOFFEL MAP
z_1 z_2 z_3 z_4
w_1 w_2 w_3 w_4


SCHARTZ-CHRISTOFFEL MAP
r = L'/L
r→0
r→1
Π_v = 1
Π_v = 0
L
L'
w_1
w_2
w_3
w_4

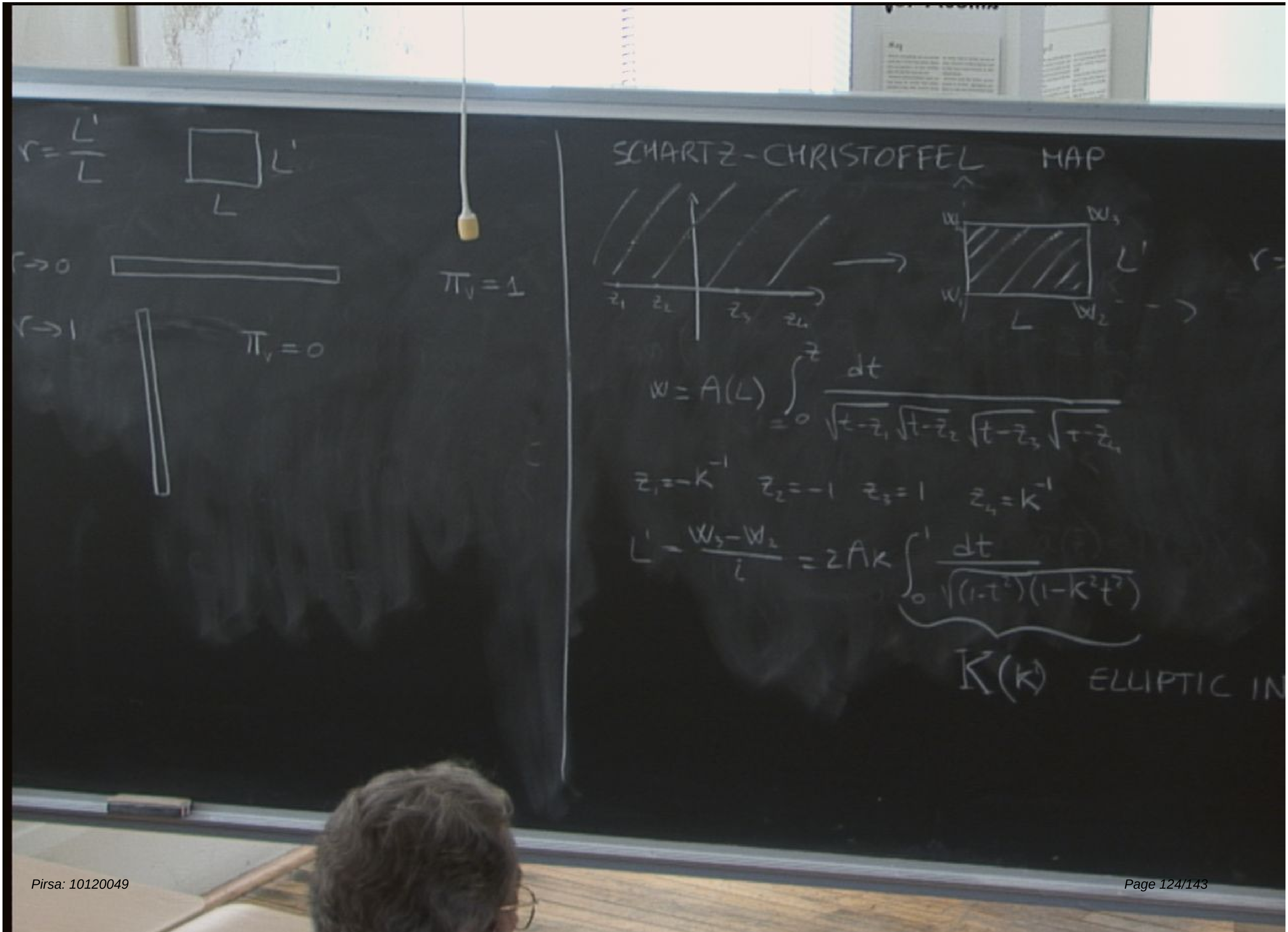

r = L'/L
L'
L
r→0
π_v = 1
r→1
π_v = 0
SCHARTZ-CHRISTOFFEL MAP
z_1 z_2 z_3 z_4
w_1 w_2 w_3 w_4
L'
L
w = A(L) ∫_{z=0}^{z} dt / (√(t−z_1) √(t−z_2) √(t−z_3) √(t−z_4))


SCHARTZ-CHRISTOFFEL MAP
r = L'/L
L'
L
r→0
r→1
π_v = 1
π_v = 0
z1 z2 z3 z4
w1 w2 w3 w4
L'
L
w = A(L) ∫ dt / √(t−z1) √(t−z2) √(t−z3) √(t−z4)
z1 = −k⁻¹ z2 = −1 z3 = 1 z4 = k⁻¹

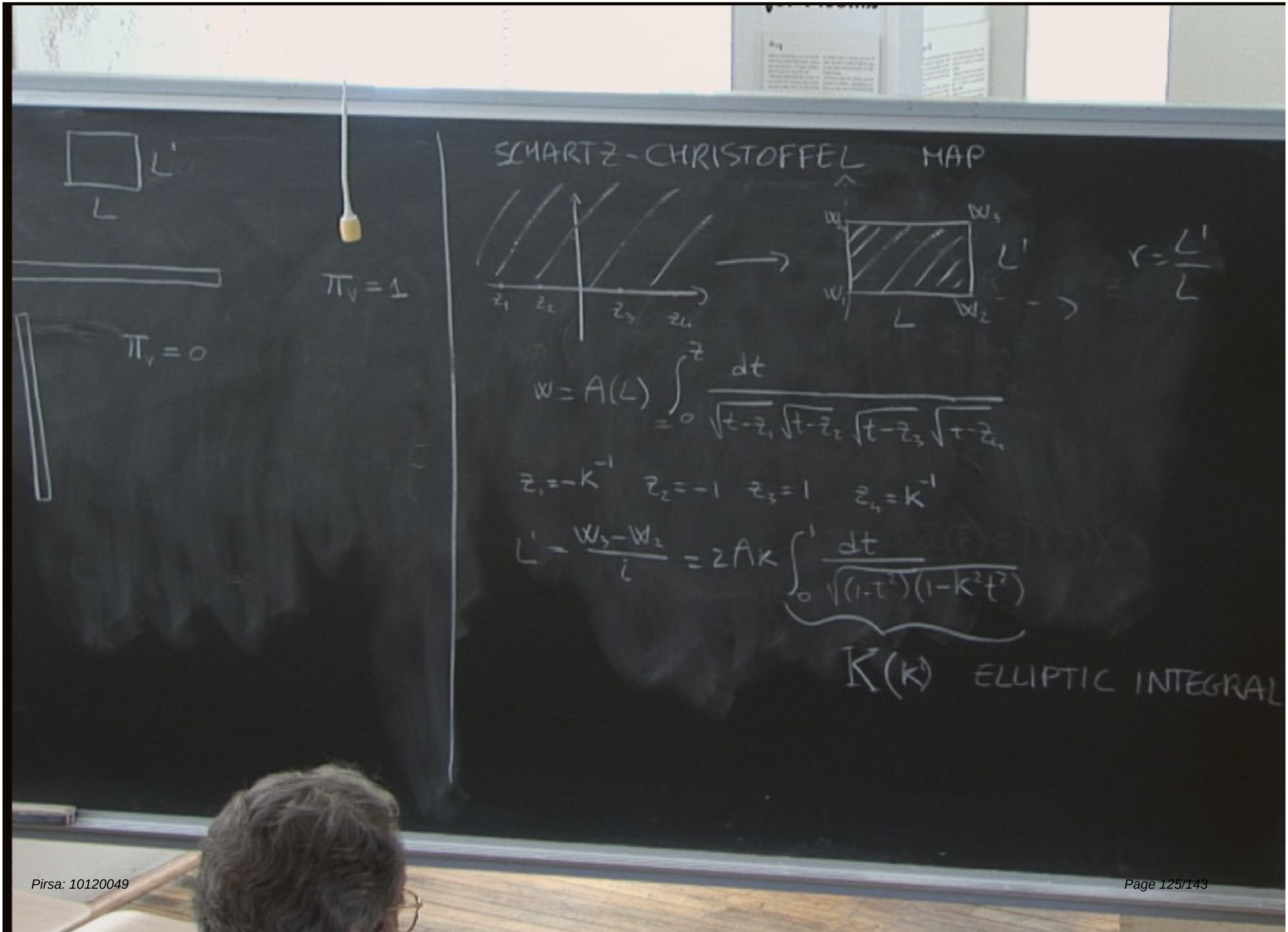

r = L'/L
L'
L
r → 0
r → 1
π_v = 1
π_v = 0
SCHARTZ-CHRISTOFFEL MAP
w₁
w₂
w₃
w₄
z₁
z₂
z₃
w = A(

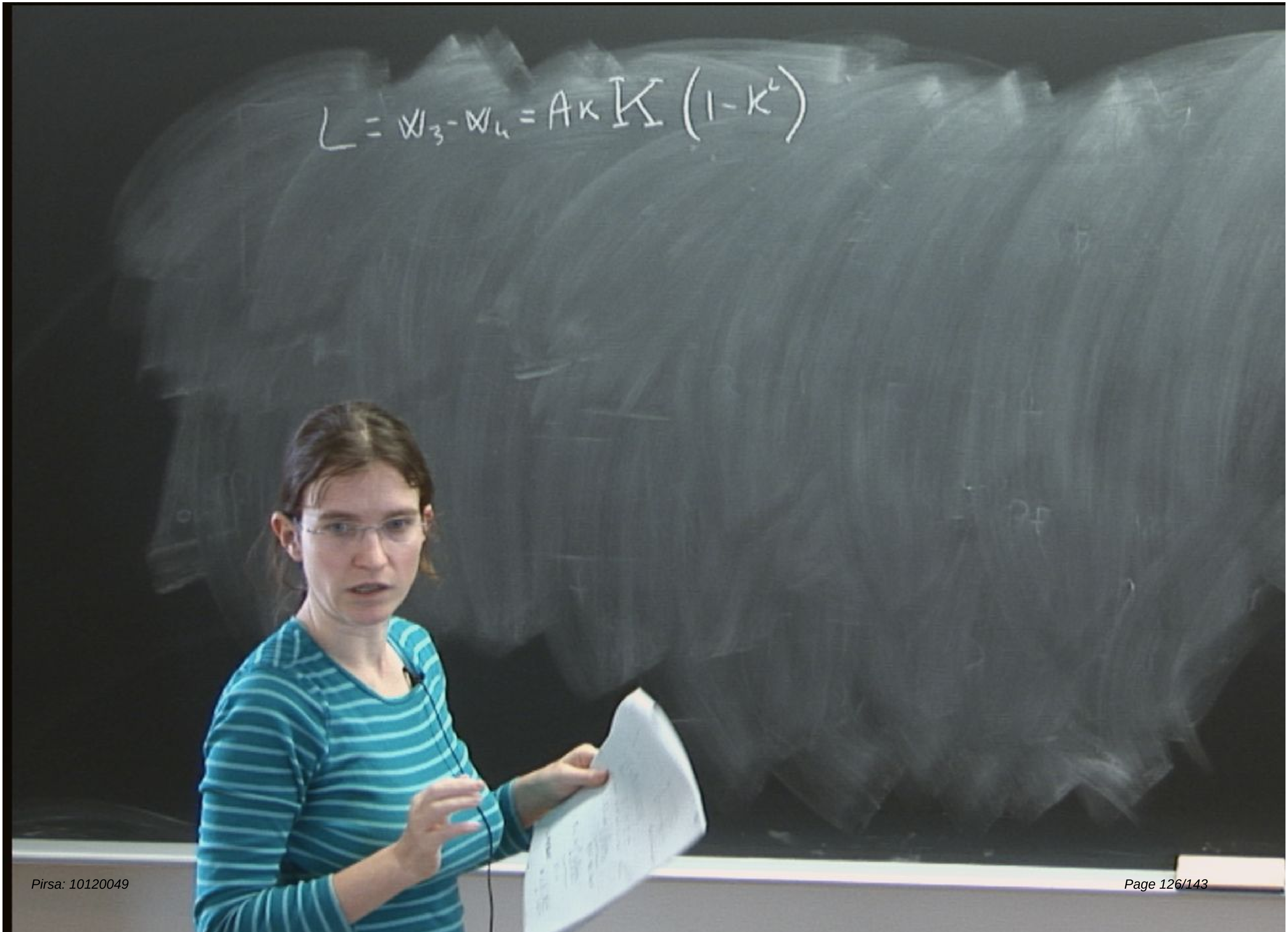


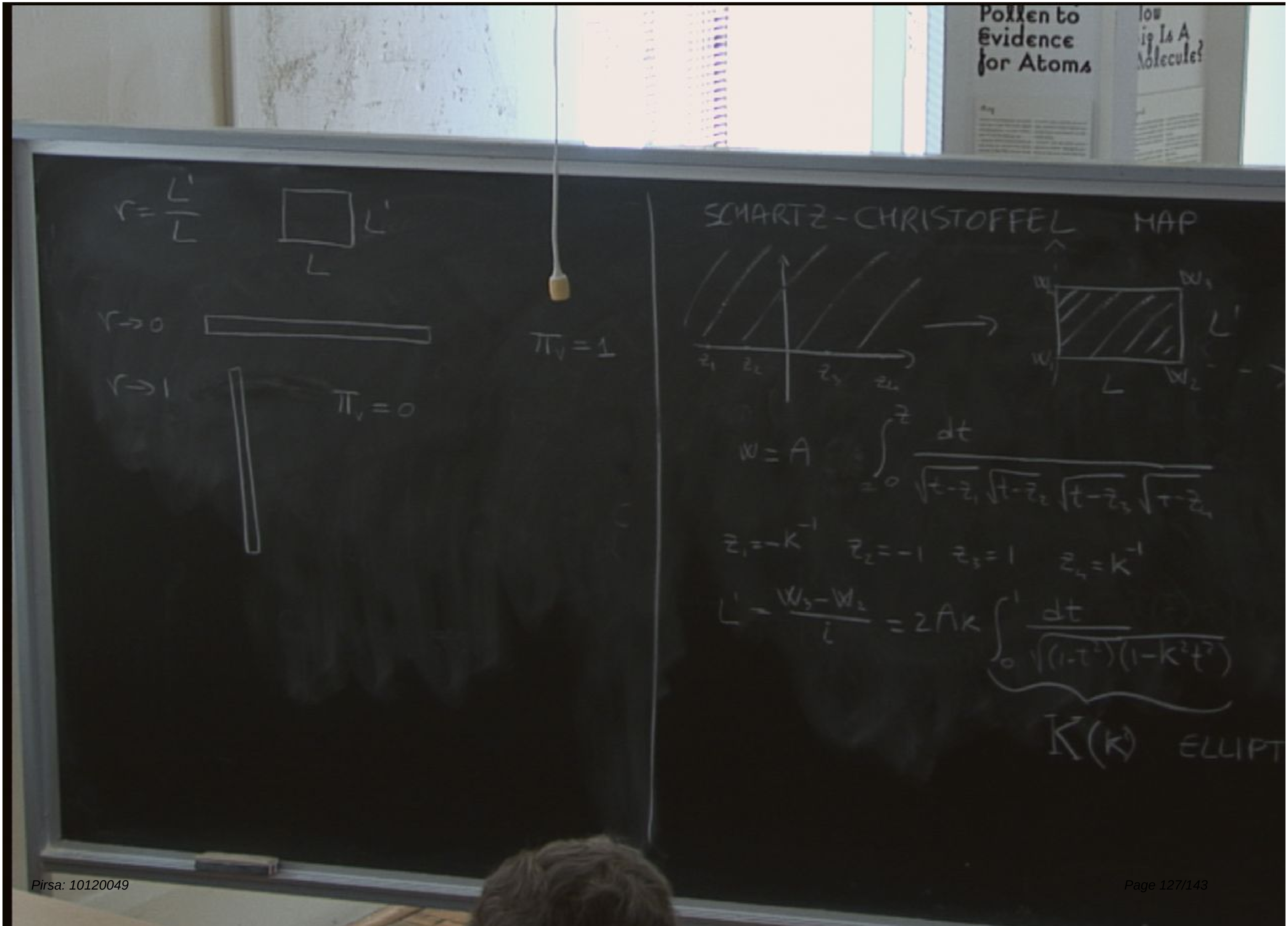
$r = \frac{L'}{L}$
L'
L
$r \to 0$
$r \to 1$
$\pi_v = 0$
SCHARTZ-CHRISTOFFEL MAP
z_1 z_2 z_3 z_4
w_1 w_2 w_3 w_4
L'
L
$w = A(L) \int_{z=0}^{z} \frac{dt}{\sqrt{t-z_1}\sqrt{t-z_2}\sqrt{t-z_3}\sqrt{t-z_4}}$
$z_1 = -k^{-1}$ $z_2 = -1$ $z_3 = 1$ $z_4 = k^{-1}$
$L' = \frac{w_3 - w_2}{i} = 2Ak \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$

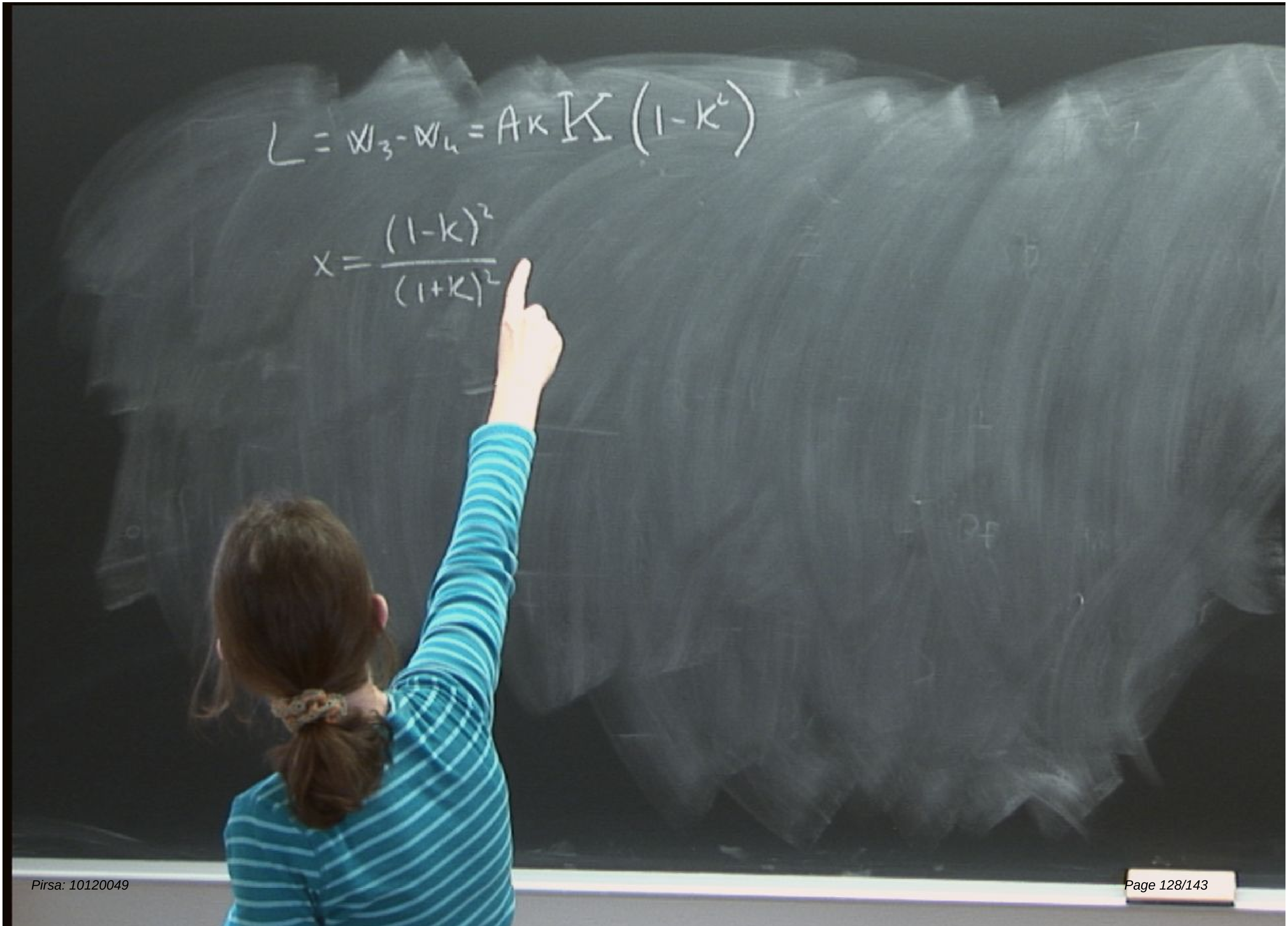


r = L'/L
L'
L
r -> 0
r -> 1
π_v = 1
π_v = 0
SCHARTZ-CHRISTOFFEL MAP
z1 z2 z3 z4
w1 w2 w3 w4
w = A(L) ∫_{z=0}^{z} dt / (√(t−z1) √(t−z2) √(t−z3) √(t−z4))
z1 = −k^{-1} z2 = −1 z3 = 1 z4 = k^{-1}
L' = (w3 − w2)/i = 2Ak ∫_0^1 dt / √((1−t²)(1−k²t²))
K(k) ELLIPTIC IN


SCHARTZ-CHRISTOFFEL MAP
L'
L
π_v = 1
π_v = 0
z1 z2 z3 z4
w4 w3 w1 w2
L'
L
r = L'/L
w = A(L) ∫_0^z dt / (√(t−z1) √(t−z2) √(t−z3) √(t−z4))
z1 = −k^-1   z2 = −1   z3 = 1   z4 = k^-1
L' = (w3 − w2)/i = 2Ak ∫_0^1 dt / √((1−t²)(1−k²t²))
K(k) ELLIPTIC INTEGRAL

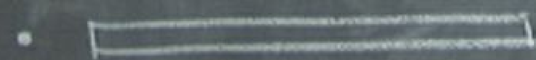

$L = W_3 - W_4 = A\kappa K (1-\kappa^2)$


Pollen to Evidence for Atoms
SCHARTZ-CHRISTOFFEL MAP
r = L'/L
r → 0
r → 1
Π_v = 1
Π_v = 0
w = A ∫_0^z dt / √(t−z_1) √(t−z_2) √(t−z_3) √(t−z_4)
z_1 = −k^{-1}   z_2 = −1   z_3 = 1   z_4 = k^{-1}
L' = (w_3 − w_2)/i = 2Ak ∫_0^1 dt / √((1−t^2)(1−k^2t^2))
K(k) ELLIPT


$L = \omega_3 - \omega_4 = A\kappa \mathbb{K}(1-k^2)$
$x = \frac{(1-k)^2}{(1+k)^2}$

$$L = W_3 - W_4 = A\kappa \, K\,(1-k^2)$$

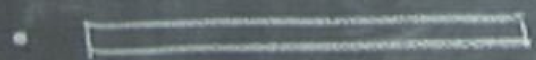
$$x = \frac{(1-k)^2}{(1+k)^2}$$



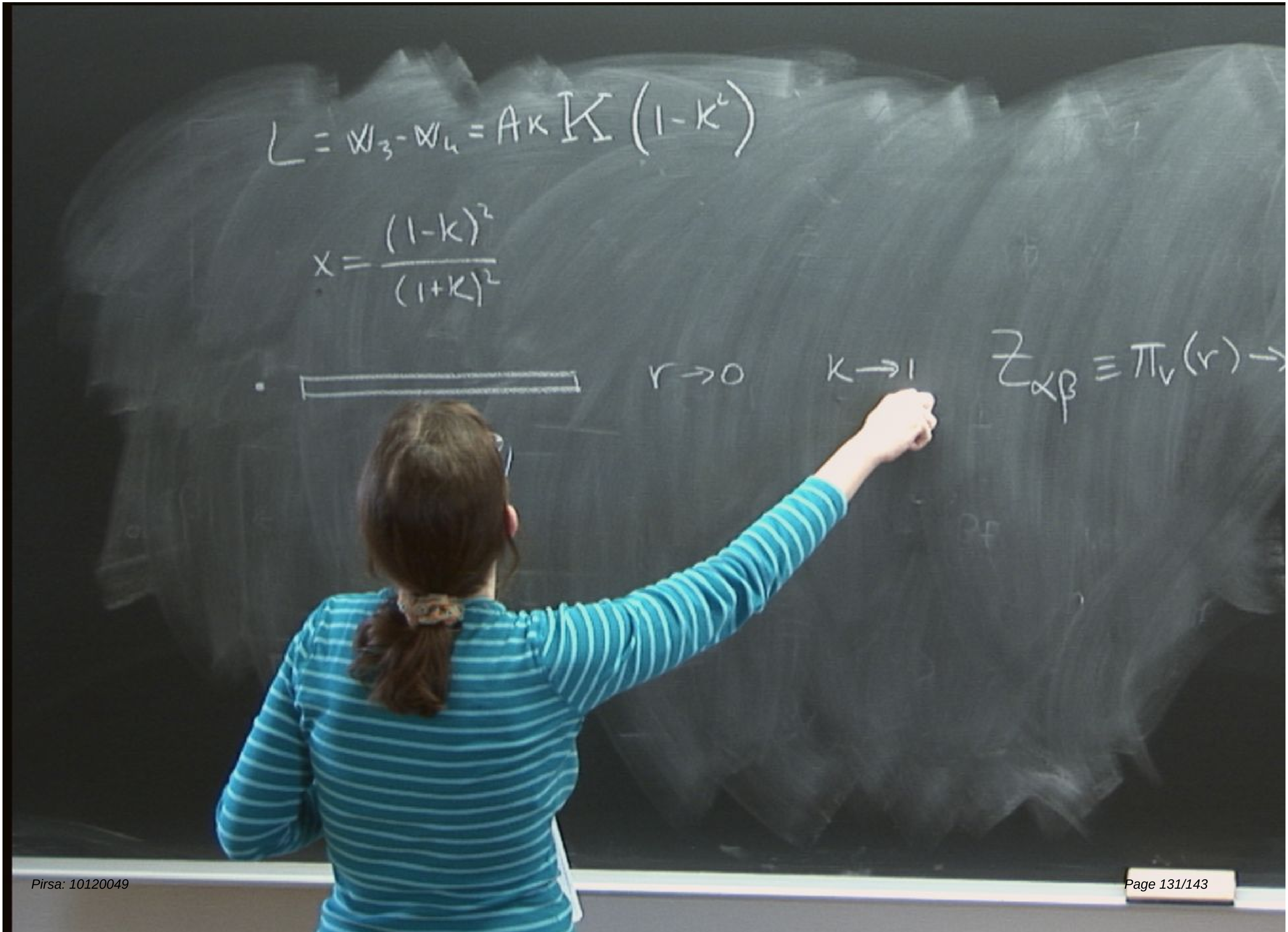
$r \to 0$

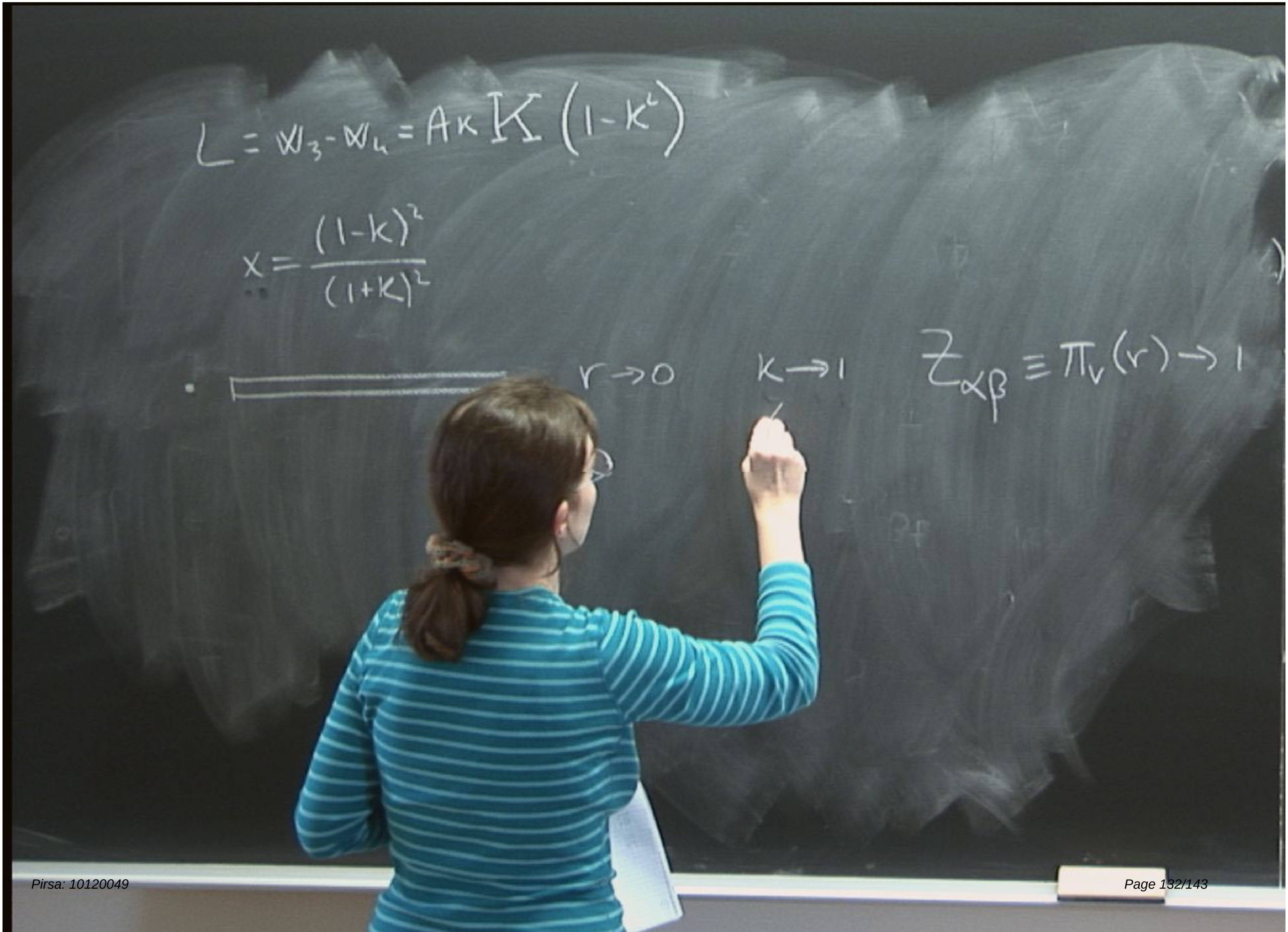
$$L = \omega_3 - \omega_4 = A\kappa\, \mathbb{K}\left(1-k^2\right)$$

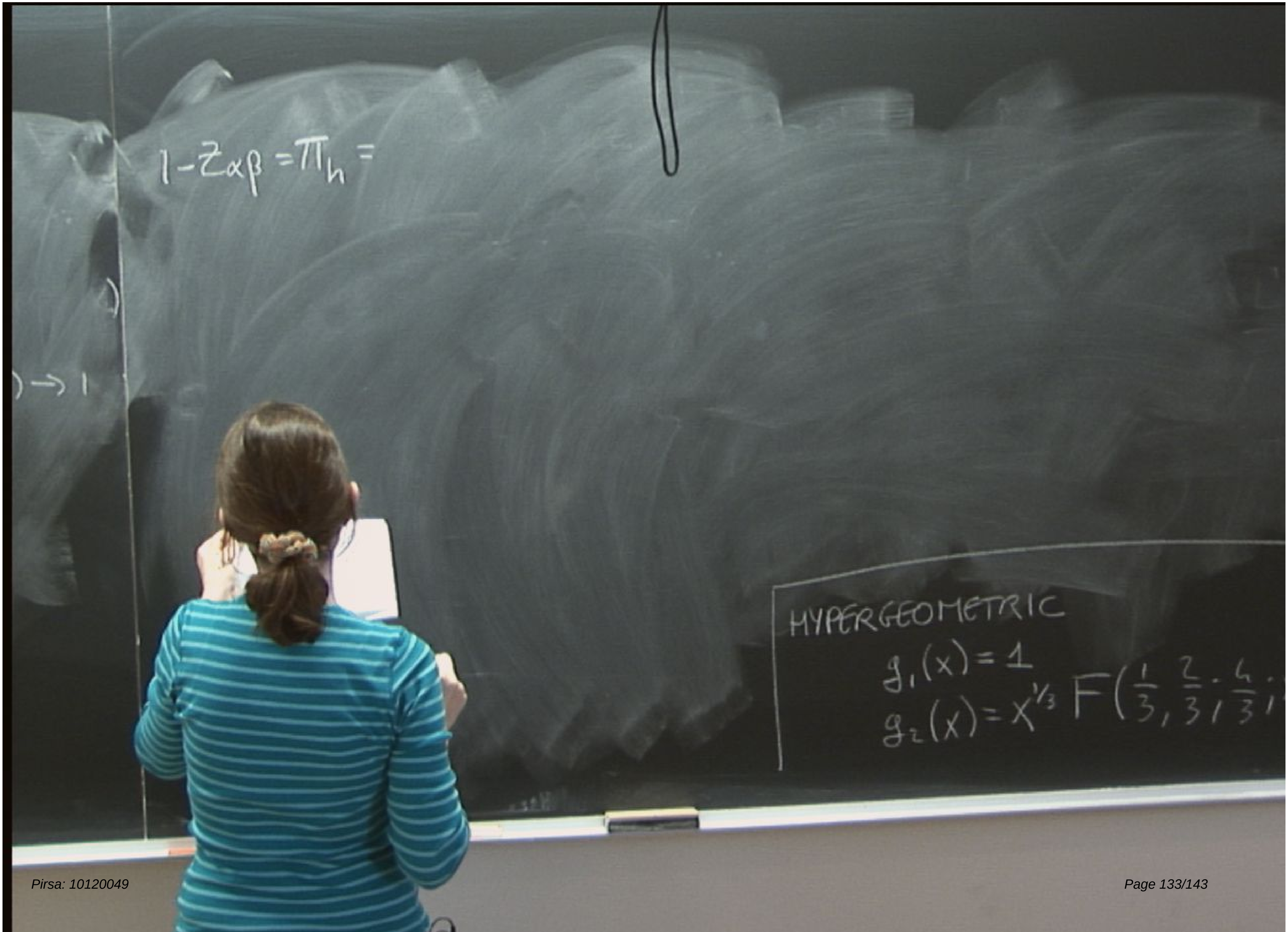
$$x = \frac{(1-k)^2}{(1+k)^2}$$

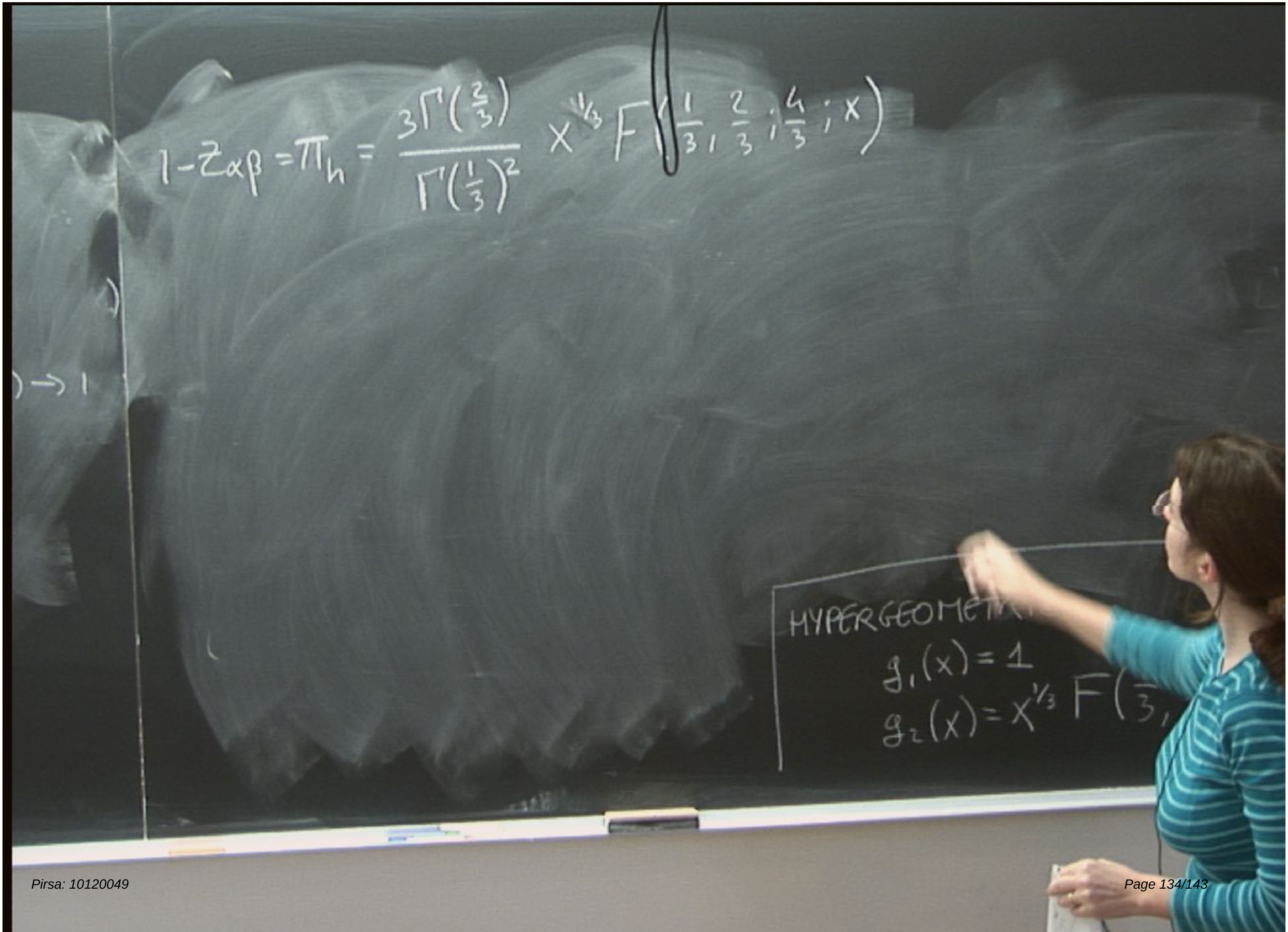


$r \to 0 \qquad k \to 1$


L = W3 - W4 = Aκ K (1-k²)
x = (1-k)² / (1+k)²
r→0
k→1
Zαβ ≡ πv(r) →


$L = \wp_3 - \wp_4 = A\kappa K (1-k^2)$
$x = \frac{(1-k)^2}{(1+k)^2}$
$r \to 0$
$k \to 1$
$Z_{\alpha\beta} \equiv \pi_v(r) \to 1$


$1 - Z\alpha\beta = \pi_h =$
$) \rightarrow 1$
HYPERGEOMETRIC
$g_1(x) = 1$
$g_2(x) = x^{1/3} F(\frac{1}{3}, \frac{2}{3}; \frac{4}{3};$


1 − Zαβ = πh = 3Γ(2/3) / Γ(1/3)² x^(1/3) F(1/3, 2/3; 4/3; x)
) → 1
HYPERGEOMET
g1(x) = 1
g2(x) = x^(1/3) F(

$$1 - Z\alpha\beta = \pi_h = \frac{3\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)^2} \underbrace{x^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x\right)}_{g_2}$$

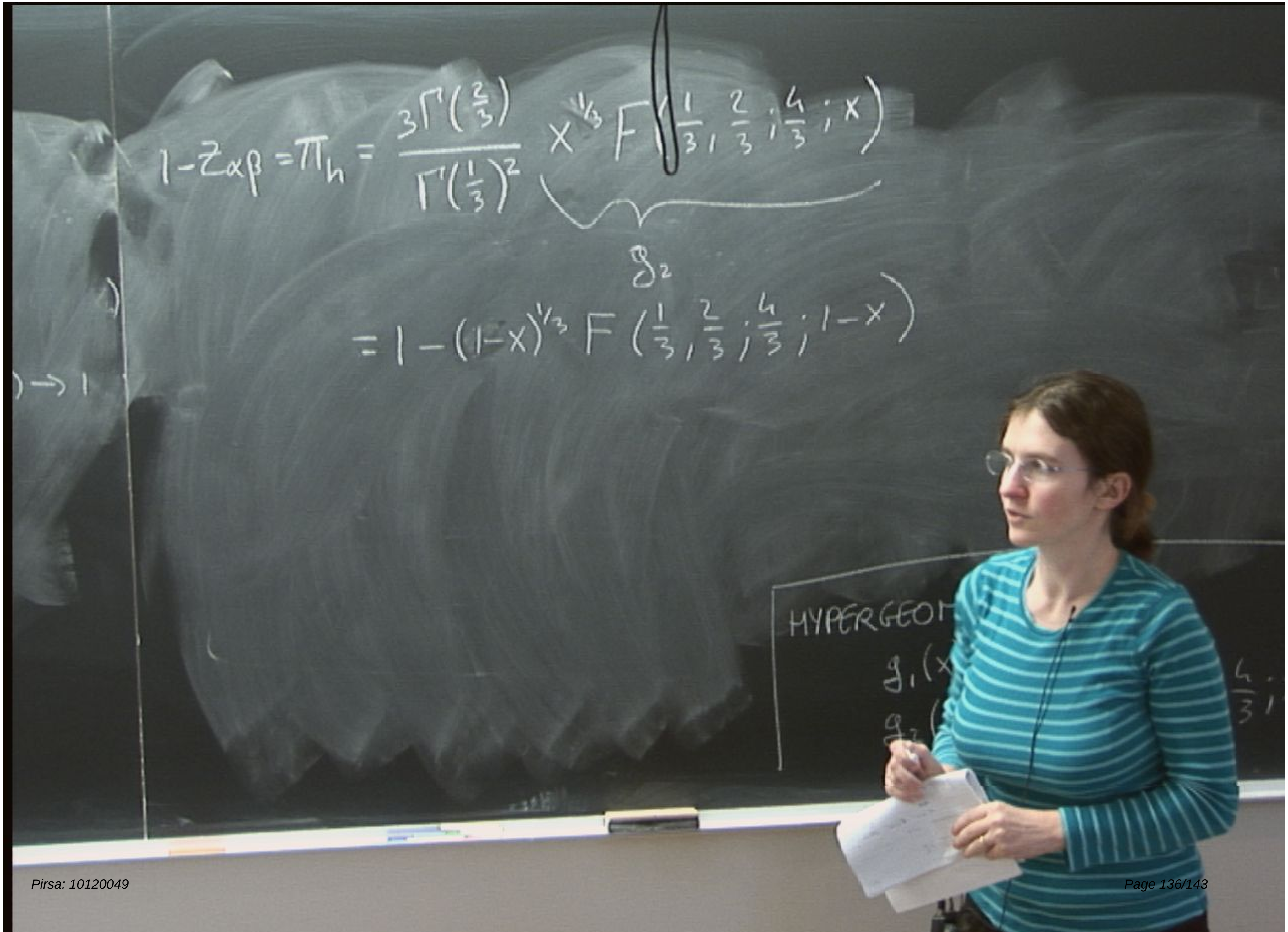
$$= 1 - (1-x)^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; 1-x\right)$$

$) \to 1$

HYPERGEOMETRIC

$g_1(x) = 1$

$g_2(x) = x^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; \ldots\right.$


1−Zαβ = π_h = 3Γ(2/3) / Γ(1/3)² x^(1/3) F(1/3, 2/3; 4/3; x)
g₂
= 1 − (1−x)^(1/3) F(1/3, 2/3; 4/3; 1−x)
HYPERGEOM

$) \to 1$

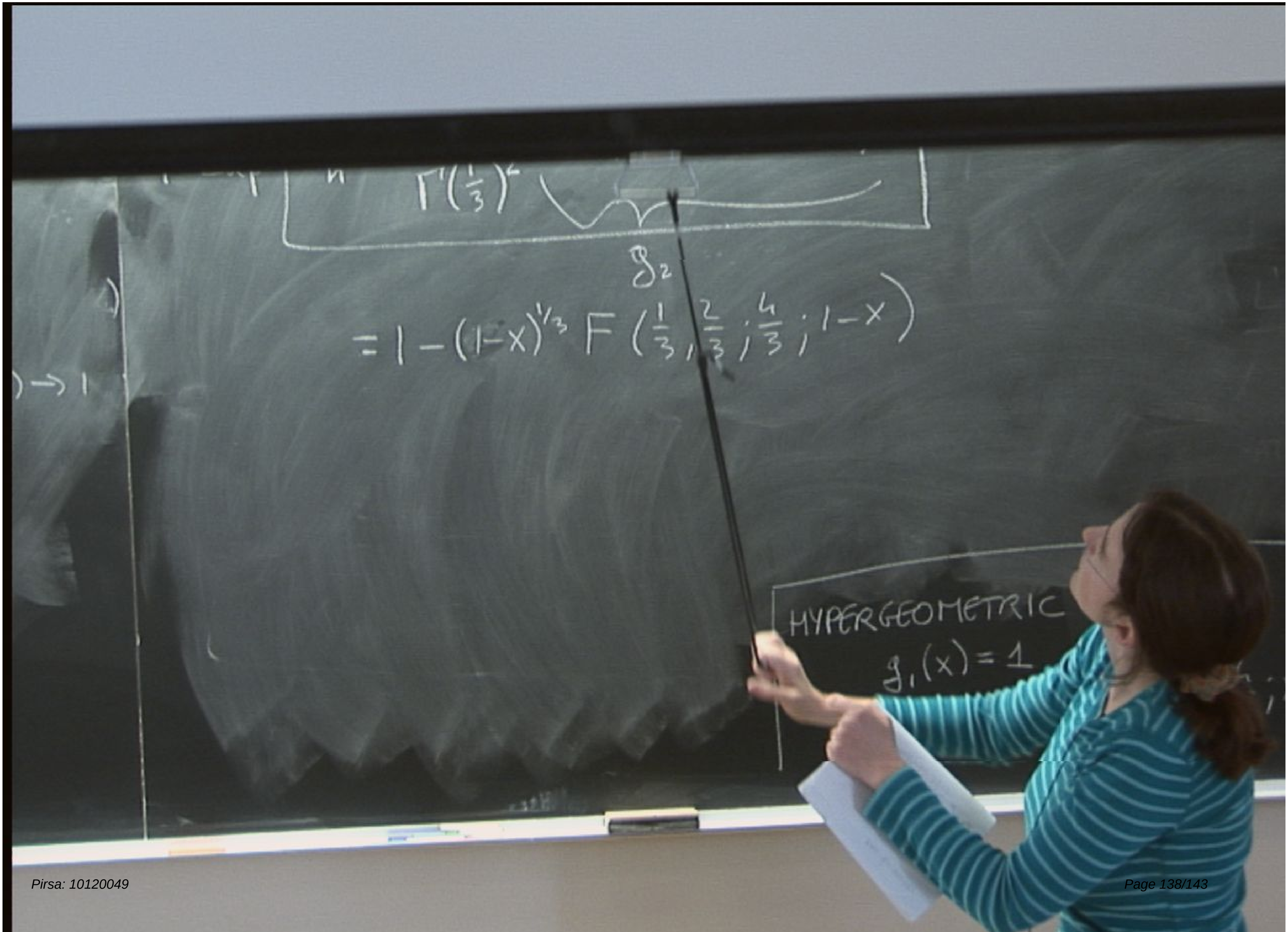
$$1 - Z_{\alpha\beta} = \pi_h = \frac{3\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{3}\right)^2} \underbrace{x^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x\right)}_{g_2}$$

$$= 1 - (1-x)^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; 1-x\right)$$

HYPERGEOMETRIC

$$g_1(x) = 1$$

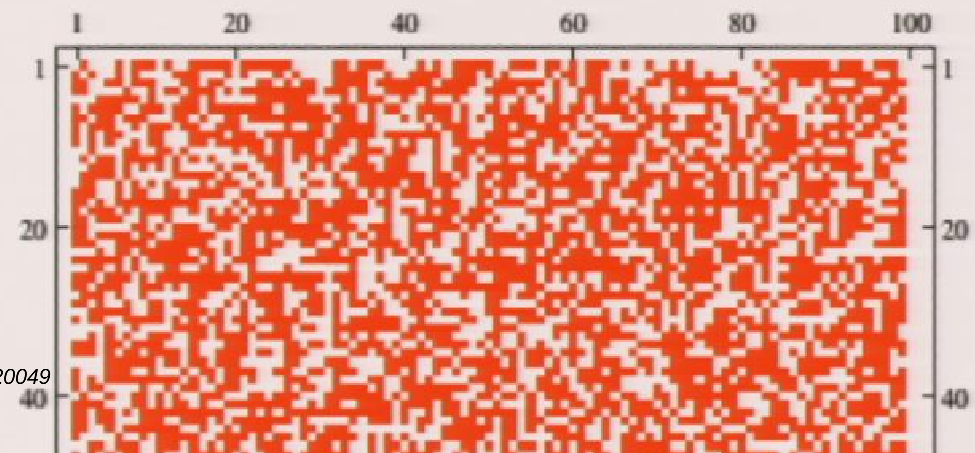
$$g_2(x) = x^{1/3} F\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{\cdot}\right.$$

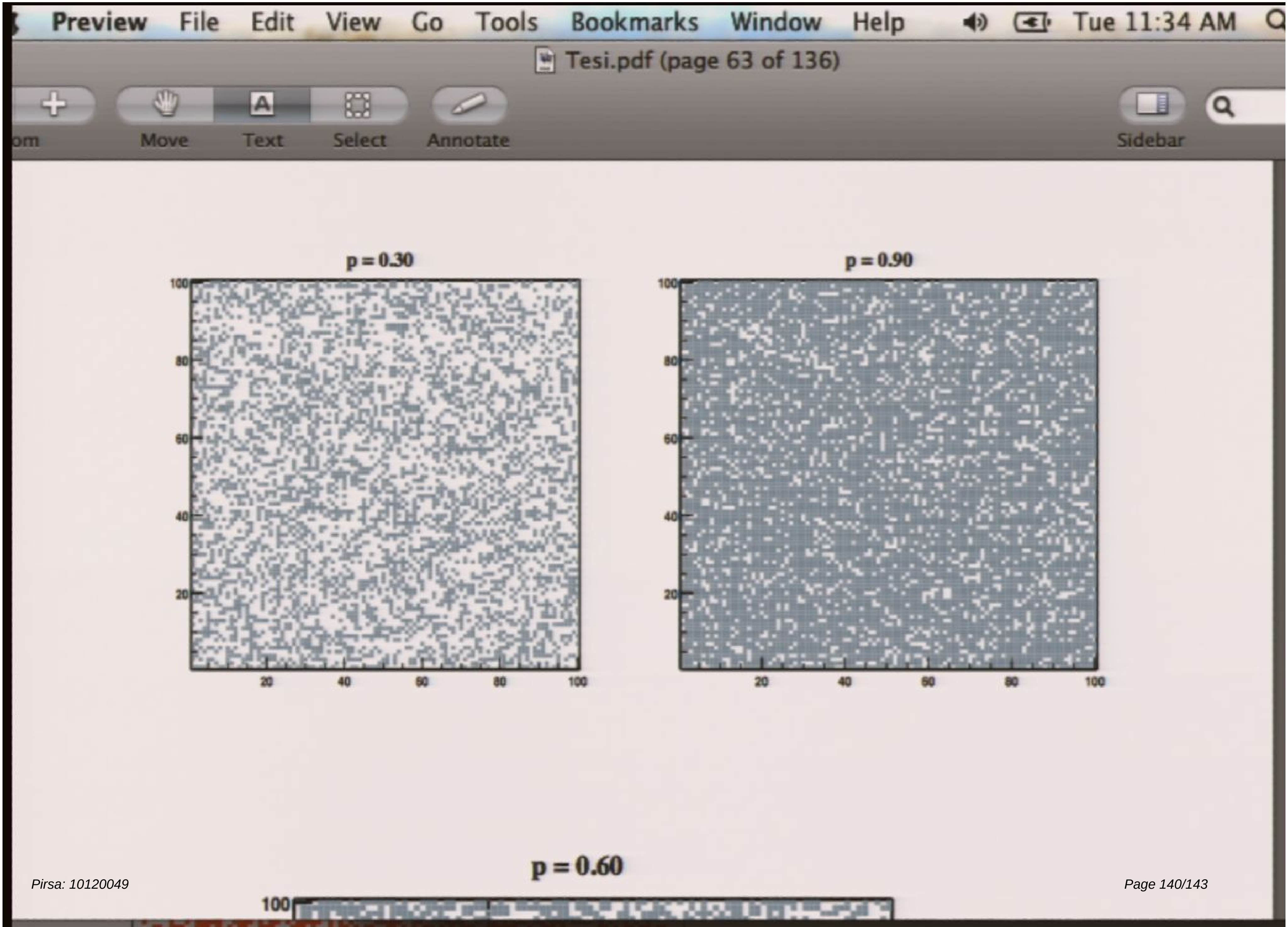

g2
= 1 − (1−x)^1/3 F(1/3, 2/3; 4/3; 1−x)
HYPERGEOMETRIC
g1(x) = 1

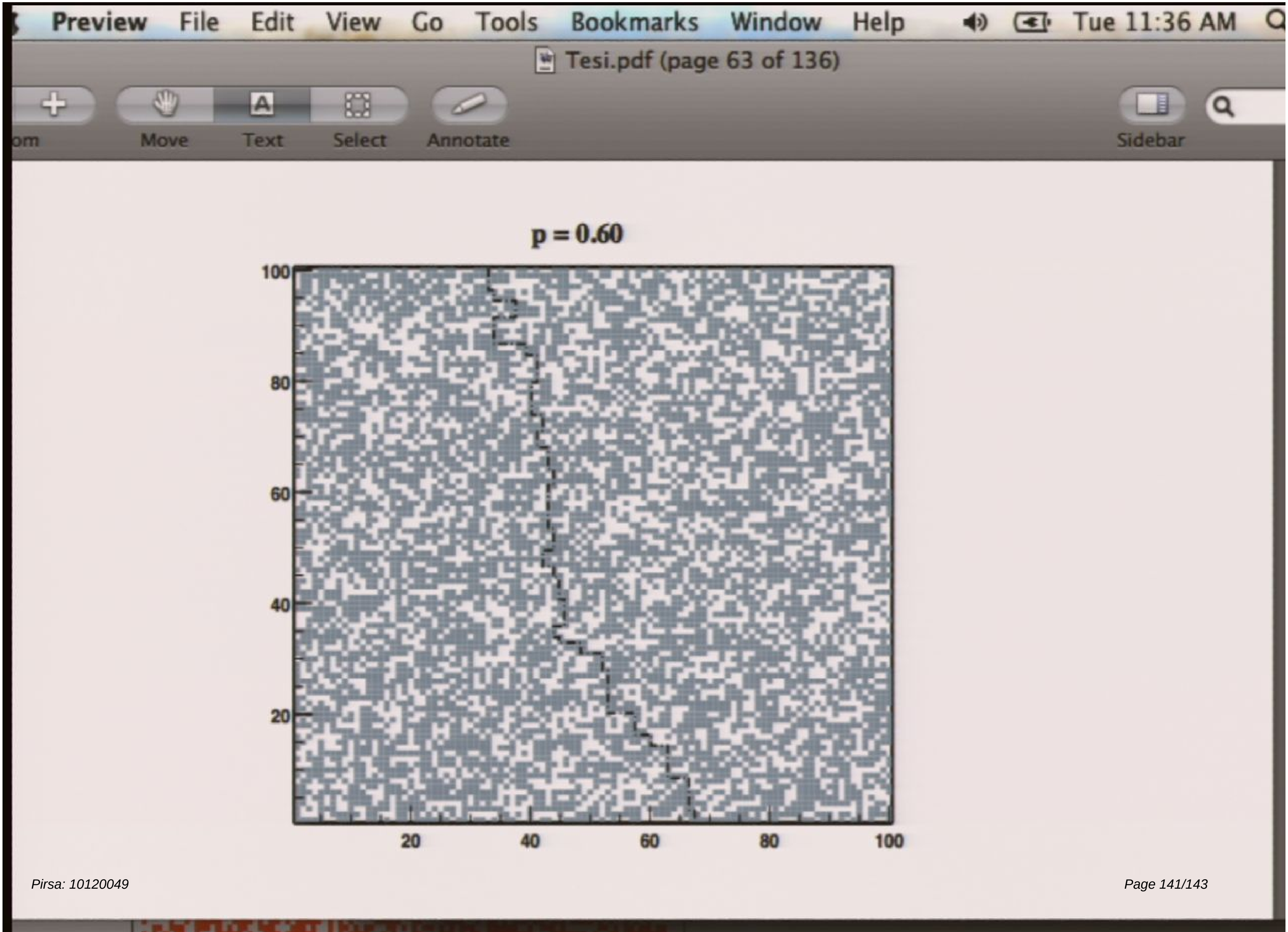
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

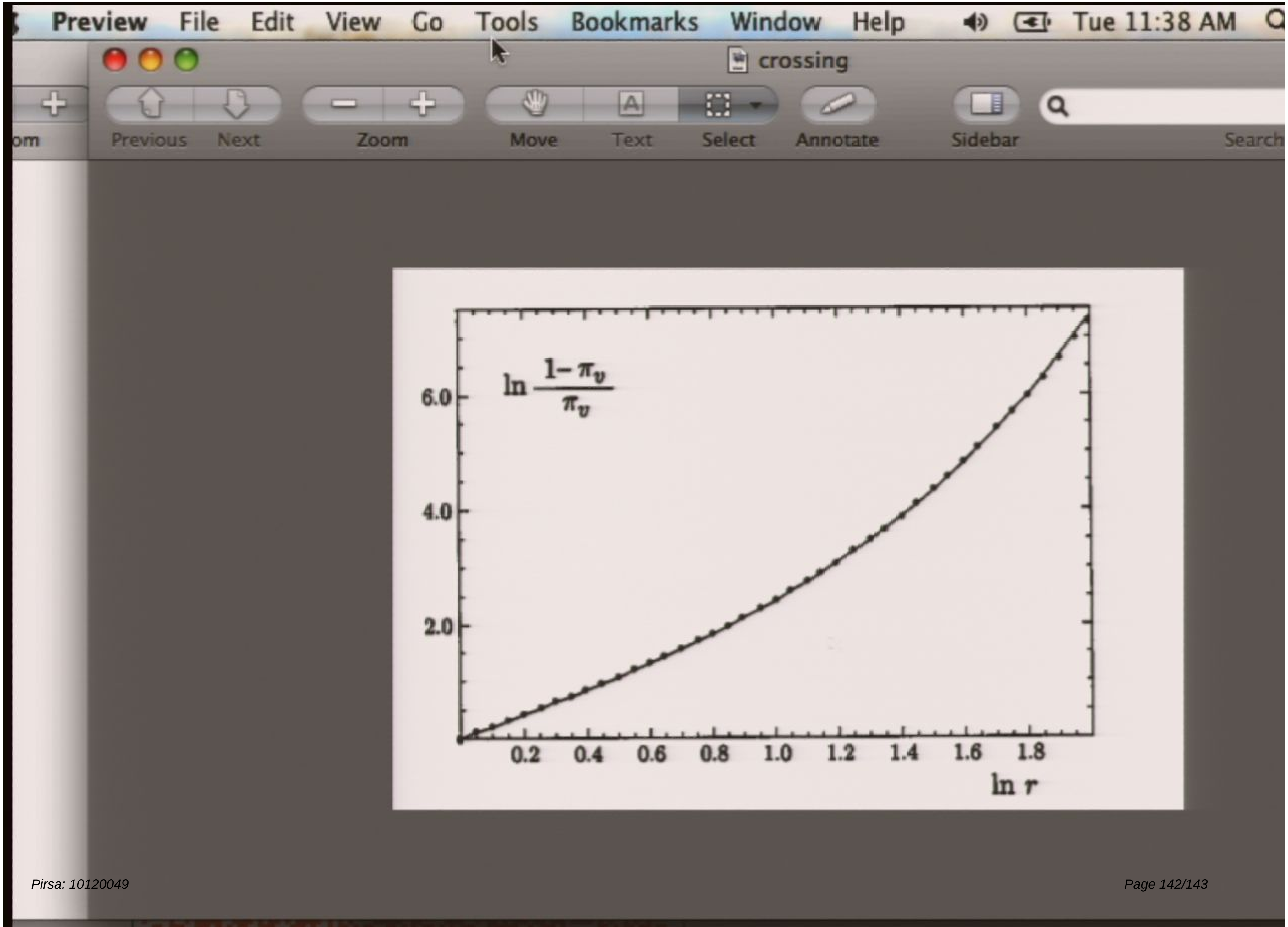
percolation

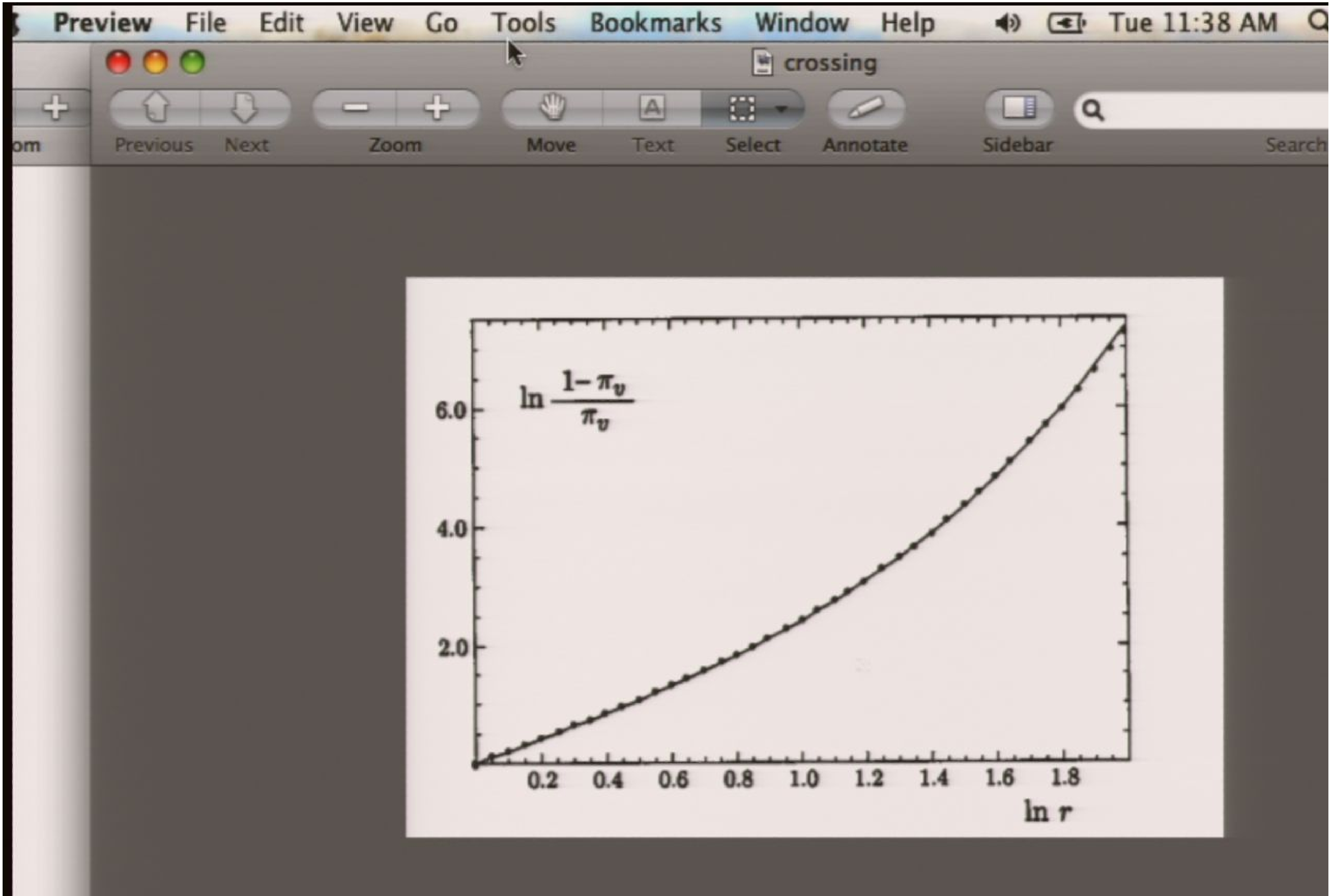
```
n = 100
100

m = 100
100

p = 0.59
0.59

L = SparseArray[{n, m} → 0]
SparseArray[<0>, {100, 100}]

For[i = 1, i < n, i++,
 For[j = 1, j < m, j++,
  r = RandomReal[];
  If[r < p, L[[i, j]] = 1;];
 ]]

MatrixPlot[L]
```




Preview File Edit View Go Tools Bookmarks Window Help
Tue 11:34 AM
Tesi.pdf (page 63 of 136)
Move Text Select Annotate
Sidebar
p = 0.30
p = 0.90
p = 0.60


Preview
File
Edit
View
Go
Tools
Bookmarks
Window
Help
Tue 11:36 AM
Tesi.pdf (page 63 of 136)
Move
Text
Select
Annotate
Sidebar
p = 0.60
100
80
60
40
20
20
40
60
80
100


Preview File Edit View Go Tools Bookmarks Window Help
Tue 11:38 AM
crossing
Previous Next Zoom Move Text Select Annotate Sidebar Search
$\ln \frac{1-\pi_v}{\pi_v}$
6.0
4.0
2.0
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
$\ln r$


Preview File Edit View Go Tools Bookmarks Window Help
Tue 11:38 AM
crossing
Previous Next Zoom Move Text Select Annotate Sidebar
$\ln \frac{1-\pi_v}{\pi_v}$
6.0
4.0
2.0
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
$\ln r$