Title: Quantum Phase Transitions from AdS2: Beyond the Landau Paradigm

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Quantum Phase Transitions from AdS_2: Beyond the Landau Paradigm

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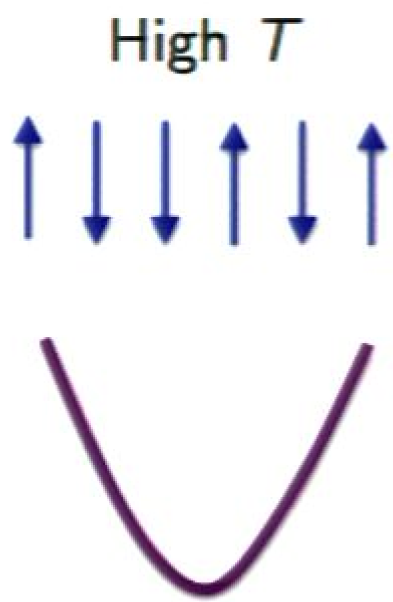
in collaboration with Hong Liu, Mark Mezei, Qimiao Si.

Prelude: Phase Transitions in General



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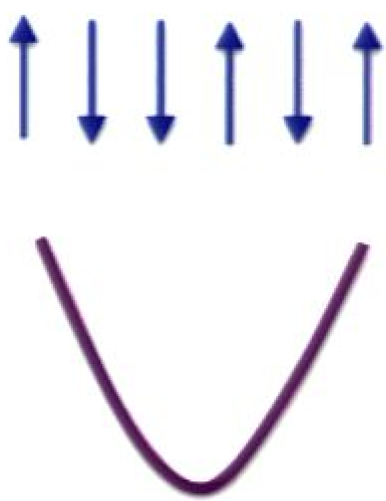
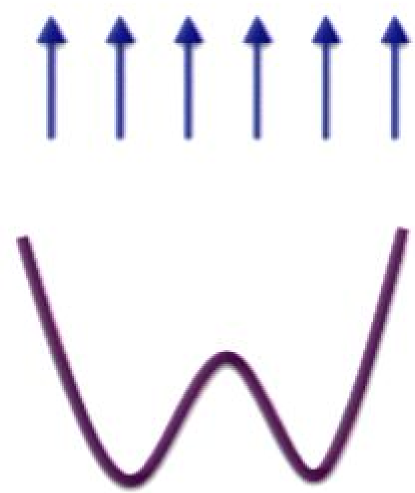


Low T



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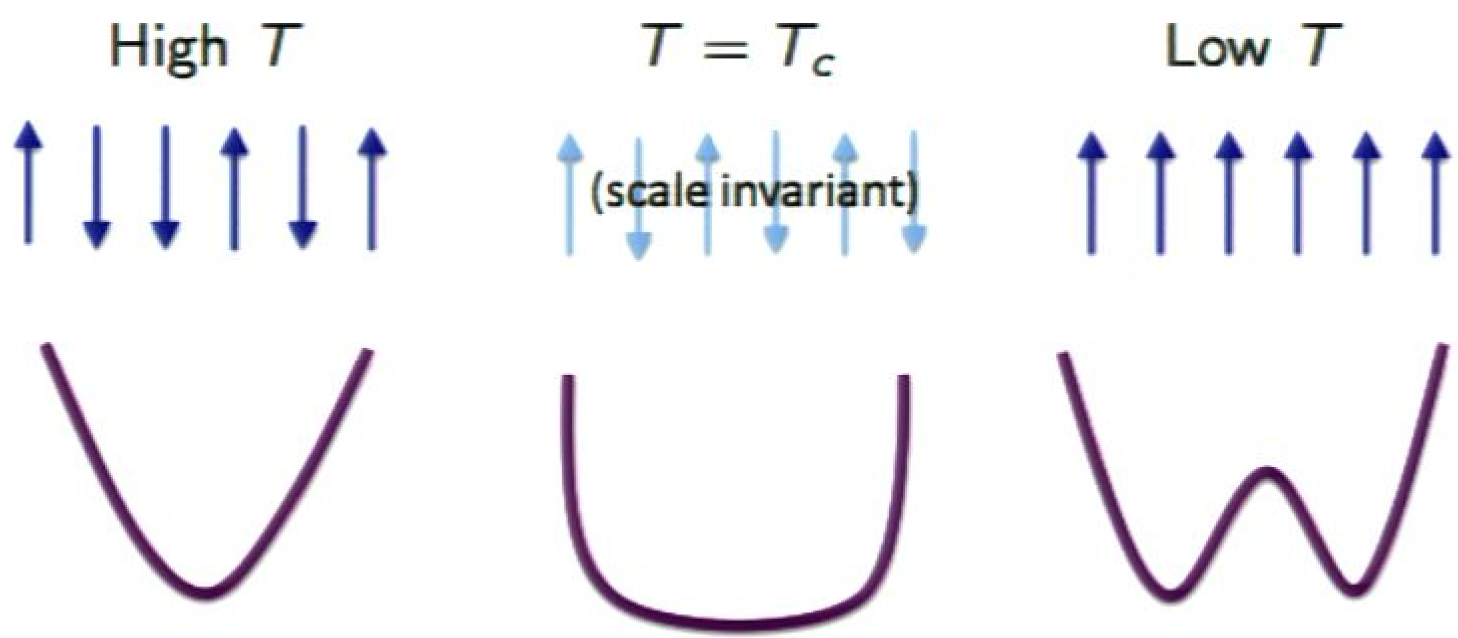

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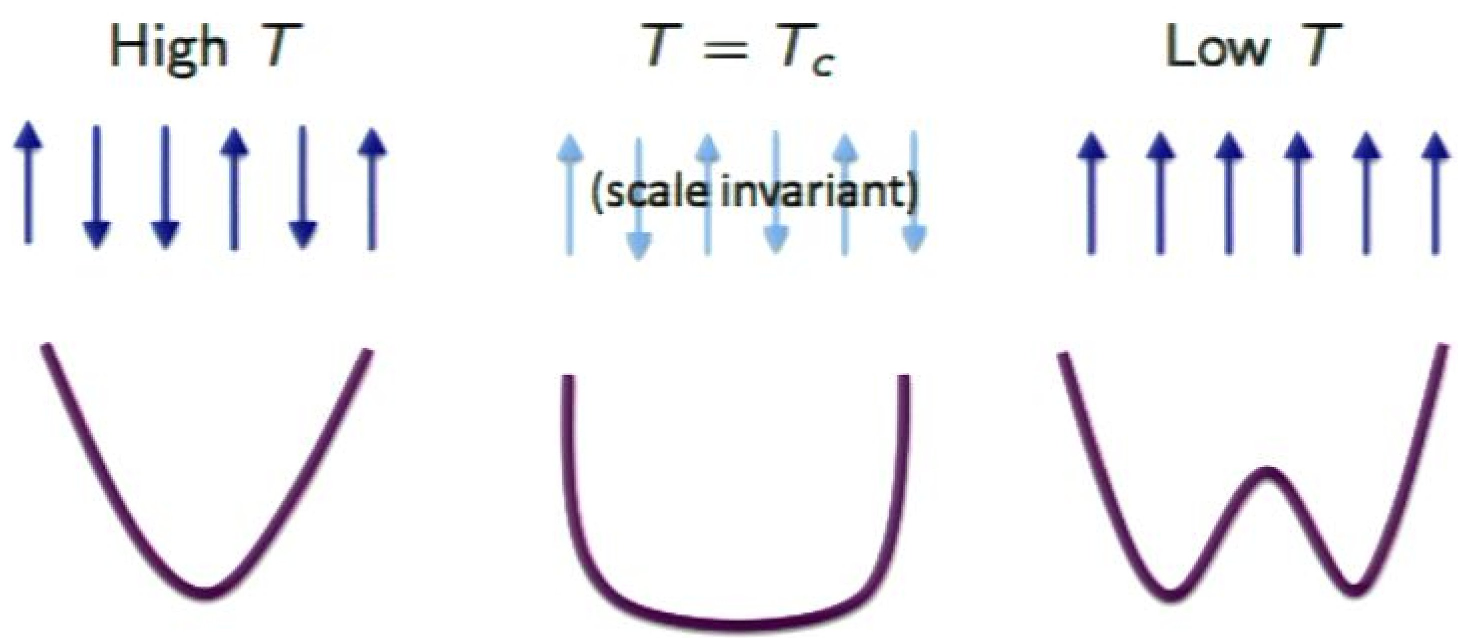

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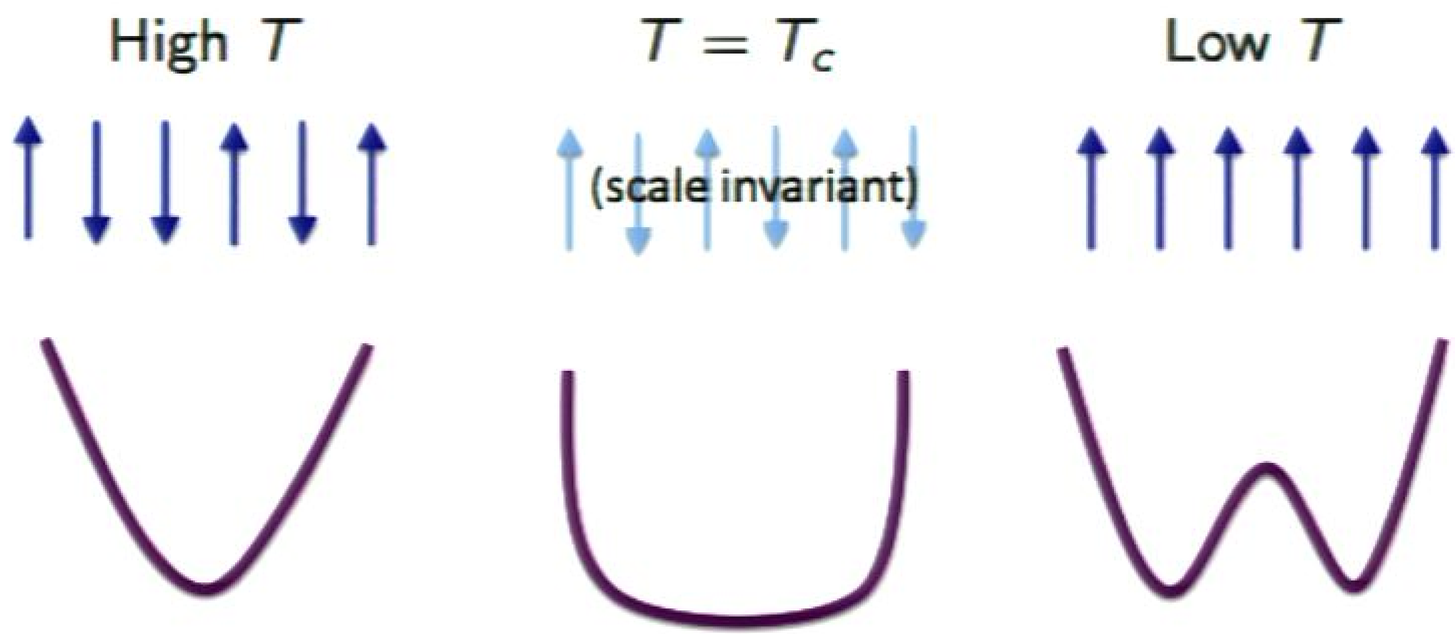

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What other things happen?

Introduction

Holographic phase transitions

New types of critical phenomena

Behavior of the condensed phase

Conclusions

Holographic states of matter

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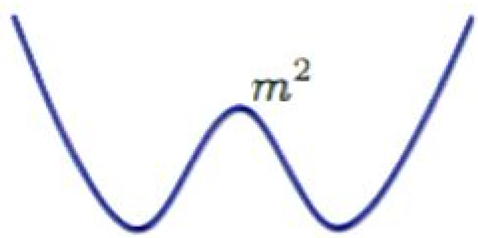
$$\mathcal{L}_\phi = -\frac{1}{\lambda}\left((D\phi)^2 + V(\phi)\right)$$

$V(\phi) =$



(For now, remain agnostic about scalar charge and mass; will specify soon.) What can one do with this model?

Finite density states in holography

Turn on a chemical potential μ for J_μ (and a finite temperature T).

So in the gravity dual, examine Reissner-Nordstrom-AdS$_4$ background.

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}\frac{dr^2}{f(r)} \qquad A_t(r \to \infty) = \mu$$

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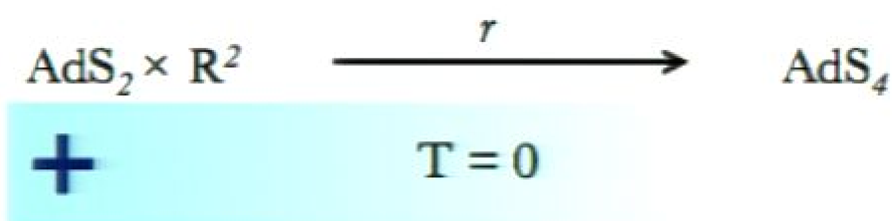
Important fact: at $T = 0$, near-horizon geometry factorizes into $AdS_2 \times \mathbb{R}^2$. (Electric flux does not allow $\mathbb{R}^2$ to shrink.)



This AdS_2 will be very important.

The IR CFT and instabilities

AdS_2: Emergent conformal symmetry in IR.

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All operators have IR CFT dimensions δ under emergent conformal symmetry. E.g. a charged scalar ϕ_q has:

$$\delta = \frac{1}{2} \pm \nu \qquad \nu = \sqrt{\frac{m^2 R^2}{6} - \frac{q^2}{12} + \frac{1}{4}}$$

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For certain couplings the IR dimension is imaginary; we have violated AdS_2 BF bound (while preserving AdS_4 BF bound). Instability: at $T = 0$, scalar will condense. This is the mechanism driving the holographic superconductor. (Gubser; Hartnoll, Herzog, Horowitz)

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Even for $q = 0$, there is a finite range of masses when this can happen; for simplicity, we focus on this neutral case.

Finite T physics

Warmup: take a neutral scalar with a small mass; search (numerically) for condensed phase solution at various temperatures.

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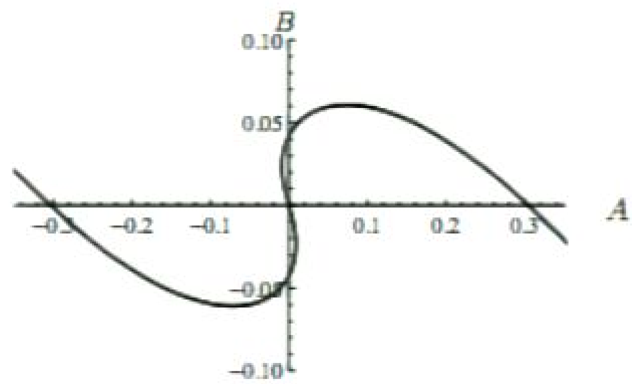

Normalizable solution has $H = 0$.

Increasing the temperature

In linear region, susceptibility χ measures the response to the applied field: $\langle \mathcal{O} \rangle = \chi H$.

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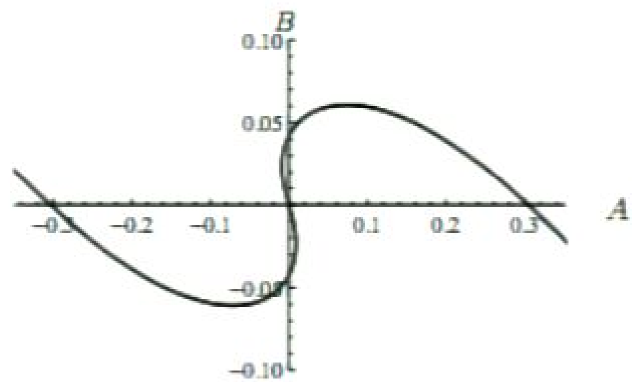

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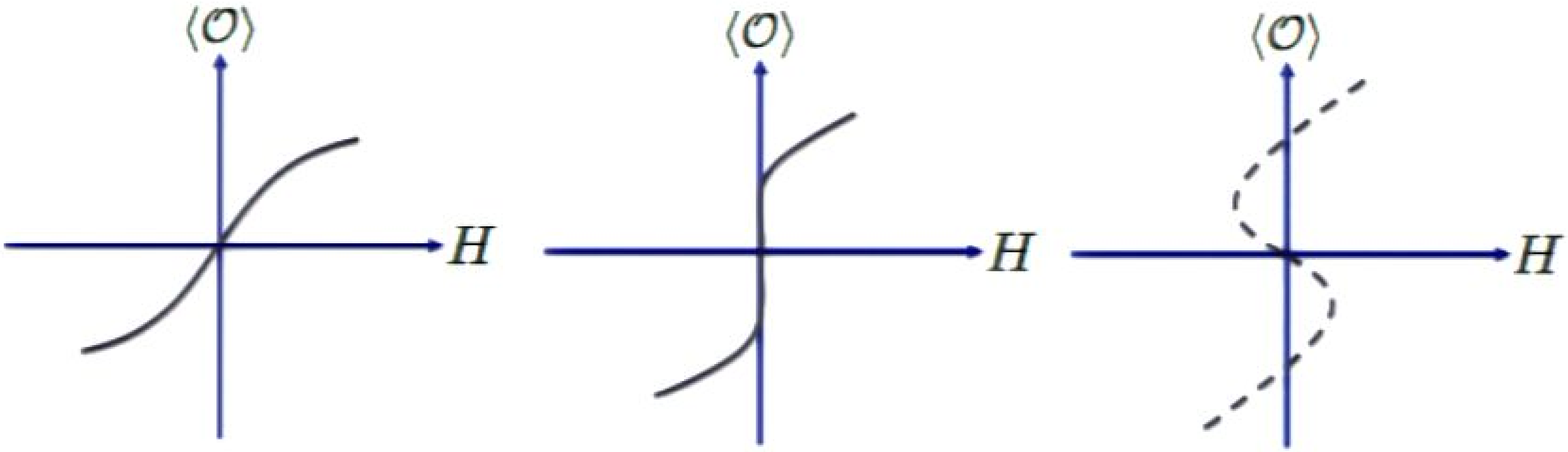

High T. χ finite.

$T = T_c$; Critical point, $\chi \sim \frac{T_c}{T - T_c}$

$T < T_c$; Condensed phase.

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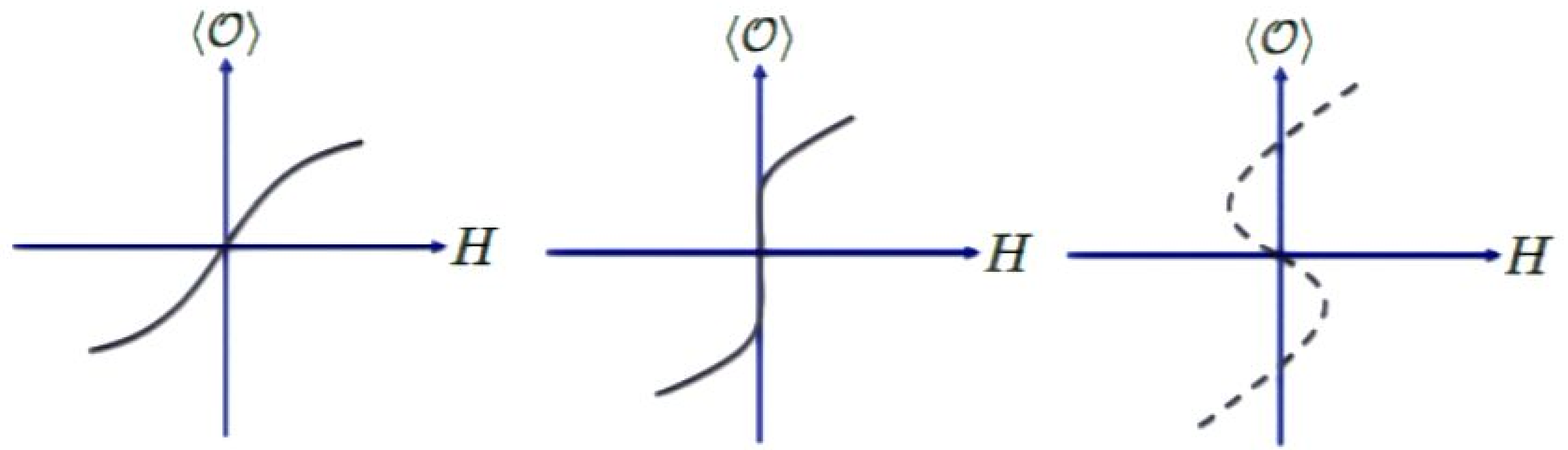

High T. χ finite.	$T = T_c$; Critical point, $\chi \sim \frac{T_c}{T-T_c}$	$T < T_c$; Condensed phase.

These are cartoon pictures; however this is what happens numerically, and they are precisely the predictions of mean-field Landau theory. Not surprising; we are solving classical equations of motion.

Remember: susceptibility diverges as we approach transition.

A quantum phase transition?

That wraps up the finite-T transition.

So: can we tune $T_c \to 0$?

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So: can we tune $T_c \to 0$?

Recall IR dimension is

$$\delta = \frac{1}{2} \pm \nu \qquad \nu = \sqrt{\frac{m^2 R^2}{6} - \frac{q^2}{12} + \frac{1}{4}}$$

Instability only exists if ν imaginary; thus if we can tune $\nu \to 0$, we should have $T_c \to 0$, and a quantum phase transition.

Tuning the IR dimension

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$$\delta_{B,q} = \frac{1}{2} \pm \sqrt{\frac{m^2 R^2}{6} + (6|Bq| - q^2)\frac{\sqrt{1+12B^2}-1}{72B^2} + \frac{1}{4}}$$

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3. Less crude still: study D3D5, find tunable "m^2" as a function of magnetic field (Jensen, Karch, Son, Thompson)

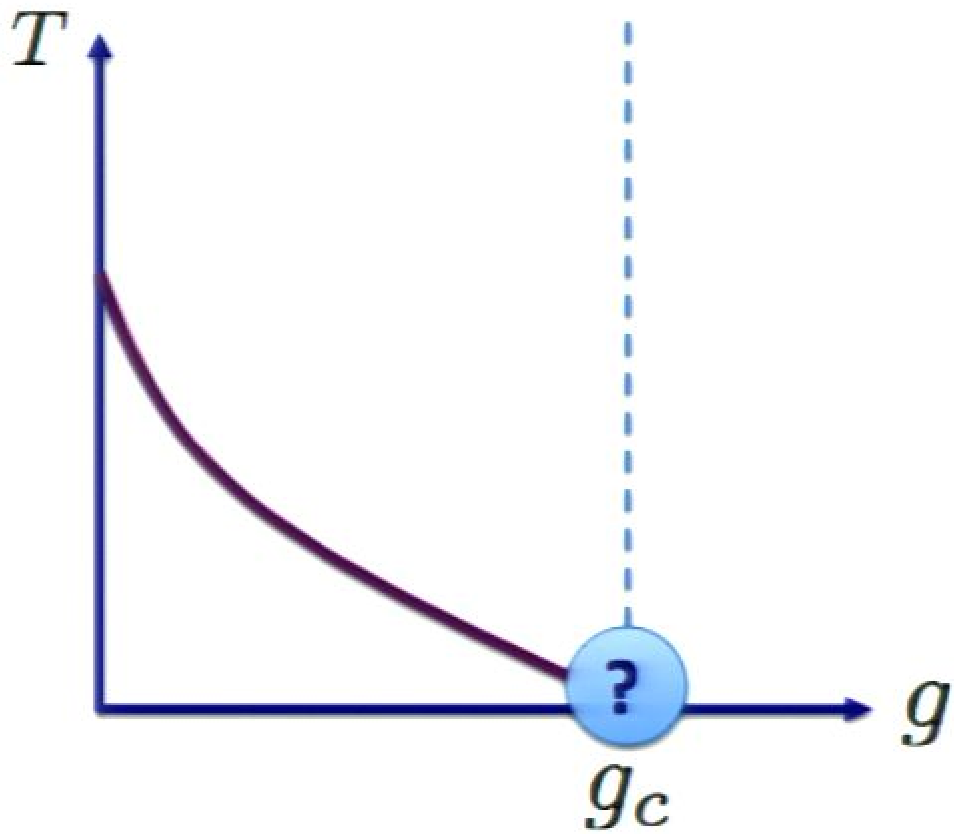
The "universality class" of the transition does not depend on how you do it: so parametrize it with some coupling g:

$$\boxed{\delta = \frac{1}{2} \pm \nu \qquad \nu = \sqrt{g - g_c}}$$

For $g < g_c$ scalar violates AdS_2 BF bound; instability.

The Question

Thus we can drive this critical temperature $T_c \to 0$.



What is the nature of the quantum critical point? What happens if we violate the AdS_2 BF bound?

Quantum phase transition

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Physics slightly below AdS BF bound is an example of annihilation of two conformal fixed points (Kaplan, Lee, Son, Stephanov).
In general, conformality is lost and a new IR scale is generated:

$$\Lambda_{IR} = \Lambda_{UV} \exp\left(-\frac{\pi}{\sqrt{g_c - g}}\right)$$

Peculiar exponential behavior is characteristic of Berezinskii-Kosterlitz-Thouless transition, a classical phase transition involving vortex physics in 2D.

This scale controls physics near the transition: T_c, $\langle\Phi\rangle$, etc. Let us understand how this works in our setup.

Understanding IR scale, part I

Put coordinates on uncondensed AdS_2:

$$\frac{ds^2}{R_2^2} = \frac{-dt^2 + dz^2}{z^2}$$

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Now study scalar wave equation on AdS_2:

$$-\frac{d^2}{dz^2}\phi + \frac{g - g_c - 1/4}{z^2}\phi = \omega^2\phi,$$

Famous $1/z^2$ potential. If $g < g_c$, infinitely many negative "energy" bound states: scalar instability.

Only true if z can go over the whole half-line...

Understanding IR scale, part II

In our problem, there is a UV cutoff on z_{UV}. Imagine putting an IR cutoff z_{IR} as well–then this helps stabilize the spectrum.

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To find z_{IR}: assume $\phi(z_{IR,UV}) = 0$. Study $\omega = 0$ (threshold) solutions:

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Satisfies boundary condition with *no nodes* if

$$\boxed{\log \frac{z_{IR}}{z_{UV}} = \frac{\pi}{\sqrt{g_c - g}}}$$

This is ground state–no nodes! Thus this is the minimum z_{IR} that stabilizes the instability. So a scale is generated! This scale controls the transition.

What provides the scale?

One way: via a finite temperature. Replace AdS_2 with an AdS_2 BH; then horizon cuts off the geometry.

$$T_c \sim \frac{1}{z_{IR}} \sim \Lambda_{UV} \exp\left(\frac{-\pi}{\sqrt{g_c - g}}\right)$$

If $T > T_c$, no instability.

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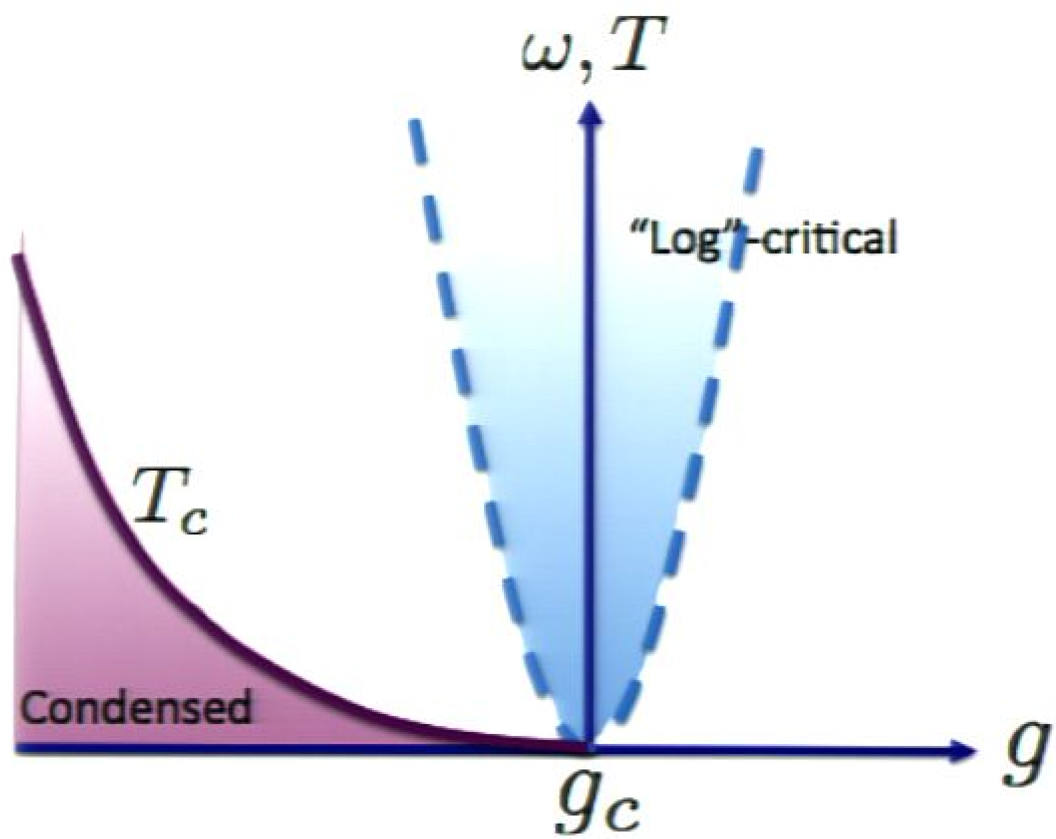
If $T > T_c$, no instability.
Another way: if $T = 0$, then scalar will condense; nonlinearities provide the scale.

$$\langle \mathcal{O} \rangle \sim \exp\left(-\frac{\pi}{2\sqrt{g_c - g}}\right)$$

Will come back to this.

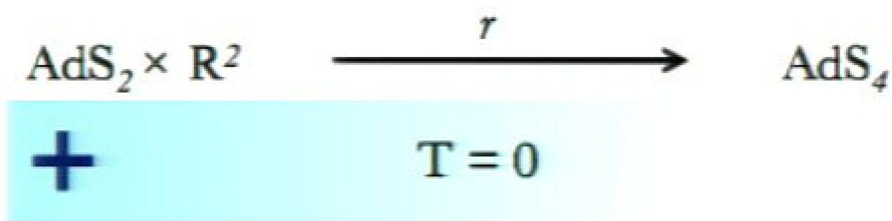
Summary of critical behavior

Thus, we have BKT-generated energy scale in time and a novel quantum critical point.
What is the critical behavior?



Calculational interlude

How do we do finite frequency response at low temperatures? Want to compute retarded correlator $G_R(\omega, k)$ for operator $\mathcal{O}$.



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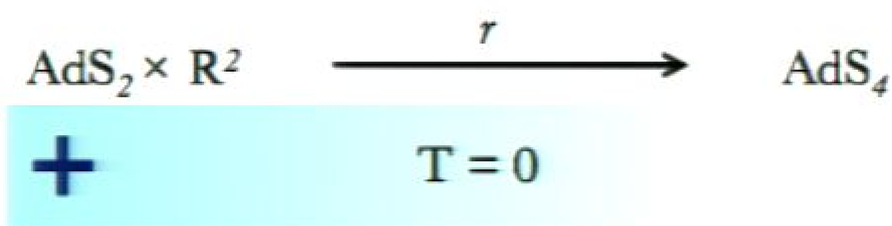

Impose infalling boundary conditions at horizon (Son, Starinets). Find exact solution in AdS_2 region, then match onto UV solution (Faulkner, Liu, McGreevy, Vegh).
Extract answer at boundary: $\phi(r \to \infty) \sim H r^{\mathrm{big}} + \langle \mathcal{O} \rangle r^{\mathrm{small}}$.

$$G_R(\omega, k) \sim \frac{\langle \mathcal{O} \rangle}{H}$$

AdS_2 region contributes interesting non-analyticities in ω, T. UV region does not know about phase transition.

Critical behavior I: a bifurcating critical point

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(α, β are constants from solving UV equation.)

Unlike normal transition, it doesn't diverge at $g = g_c$–instead, bifurcates!

Critical behavior II: diverging correlation length

Now turn on finite k: this contributes to the AdS_2 mass, and so pushes us away from critical point:

$$G_R(\omega = 0, k) = \frac{\beta + \tilde{\beta}\sqrt{(g - g_c) + \frac{k^2}{\Lambda^2}}}{\alpha + \tilde{\alpha}\sqrt{(g - g_c) + \frac{k^2}{\Lambda^2}}}$$

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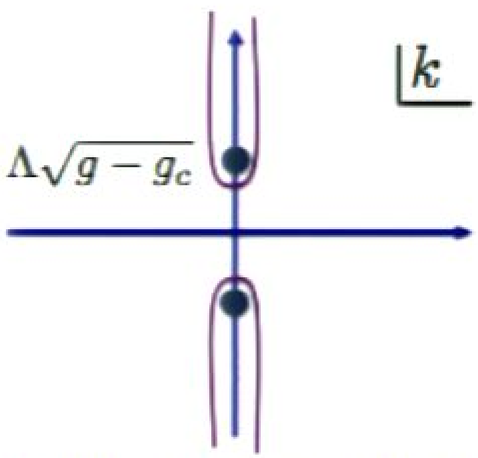
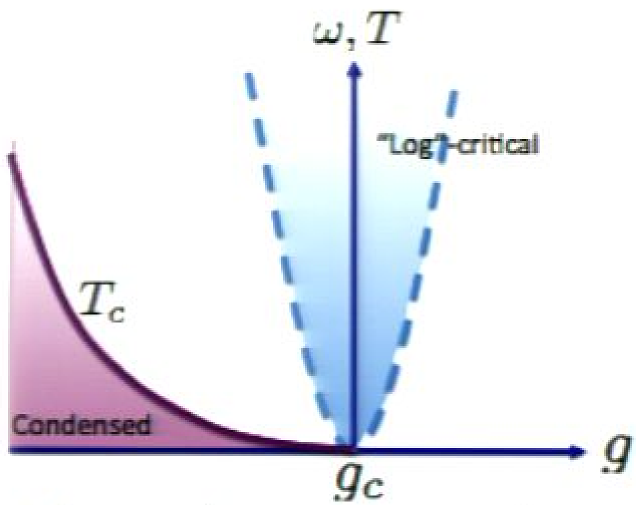


Cuts in complex k-plane; can use these to do Fourier transforms. Find diverging correlation length: $\xi \sim (g - g_c)^{-\frac{1}{2}}$.

Mean-field scaling in k; this is because spatial directions do not take part in non-trivial IR CFT.

Critical behavior III: finite frequency

Turn on a finite ω; we find then in blue region:



$$G_R(\omega \neq 0) = \frac{\beta}{\alpha} \frac{\log\left(\frac{\omega}{\omega_b}\right) - \frac{i\pi}{2}}{\log\left(\frac{\omega}{\omega_a}\right) - \frac{i\pi}{2}}$$

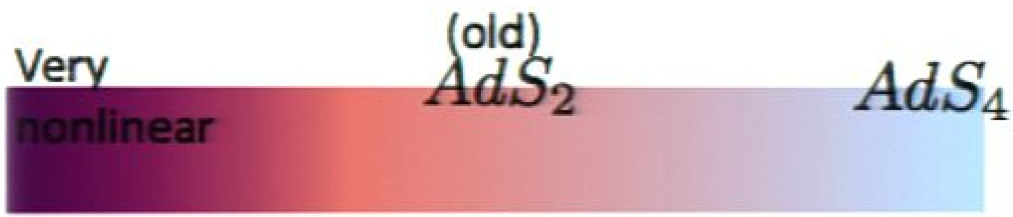
In this region system does not know about $g - g_c$, or about k.

This behavior holds over an exponentially large range of ω: $\log(\omega) \ll \frac{1}{\sqrt{g_c - g}}$.

Across the transition

What if $g < g_c$? Now if we are at $T = 0$ we know we must be in a condensed phase. Deep IR depends on details of nonlinearities.

Split spacetime into three regions:



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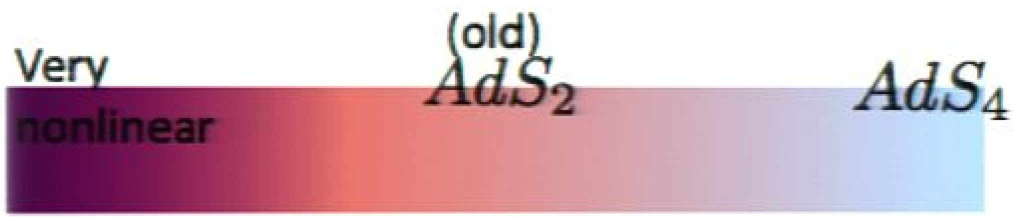
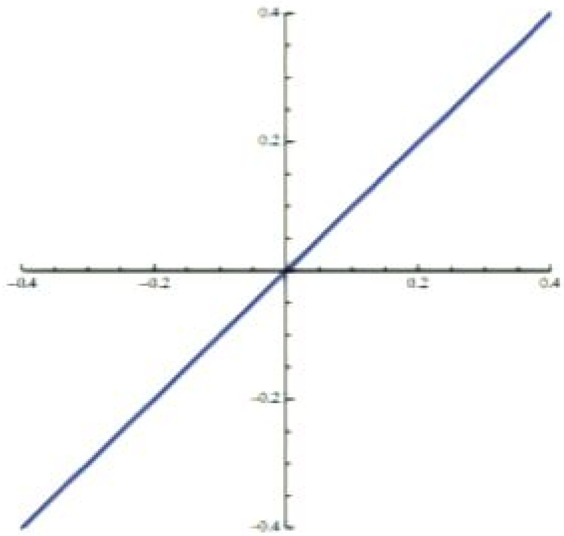


$$\phi(r) =$$

Close to transition we can derive analytic formula for nonlinear response curve (up to a single parameter that is determined by nonlinearities.) We expect it to have some sort of oscillatory character...

The Efimov Spiral



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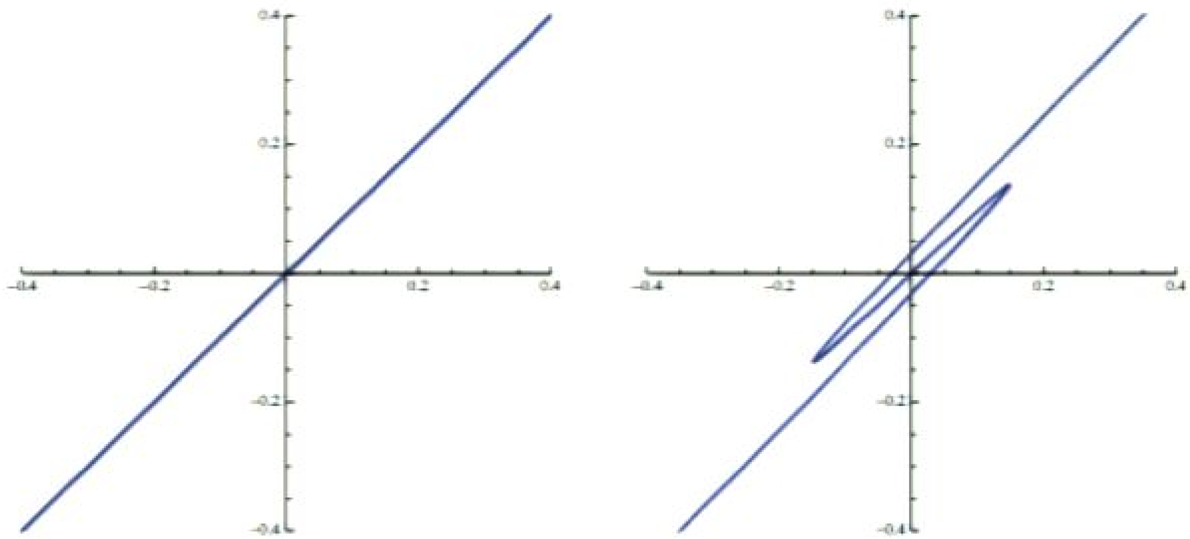

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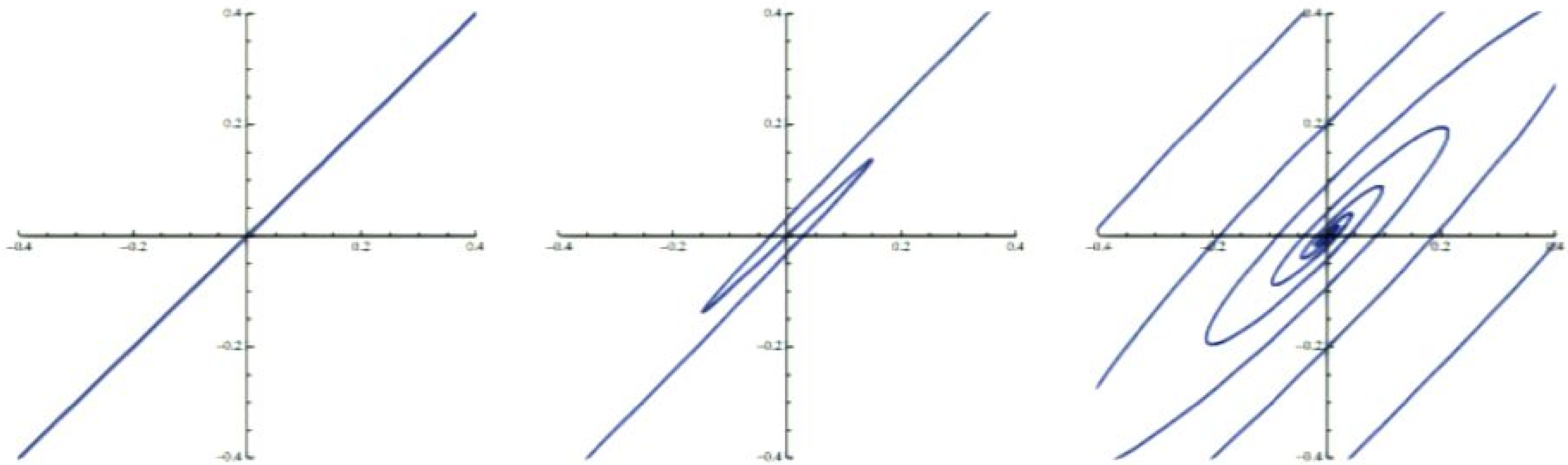
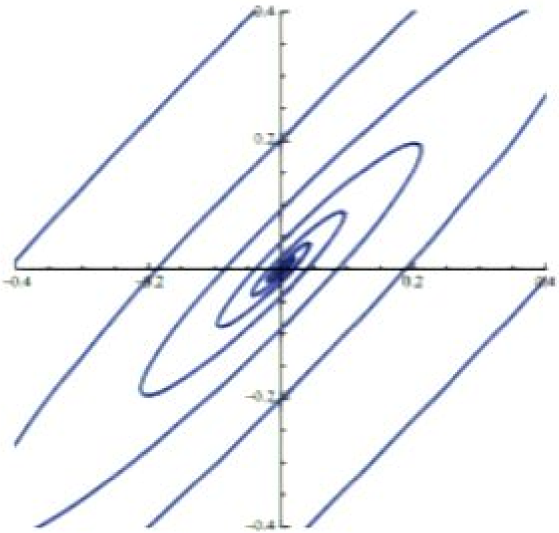


Straight line from linear response explodes into a spiral that goes on forever.

Intersections with $H = 0$ line define an infinite number of normalizable Efimov states.

Thoughts on the spiral



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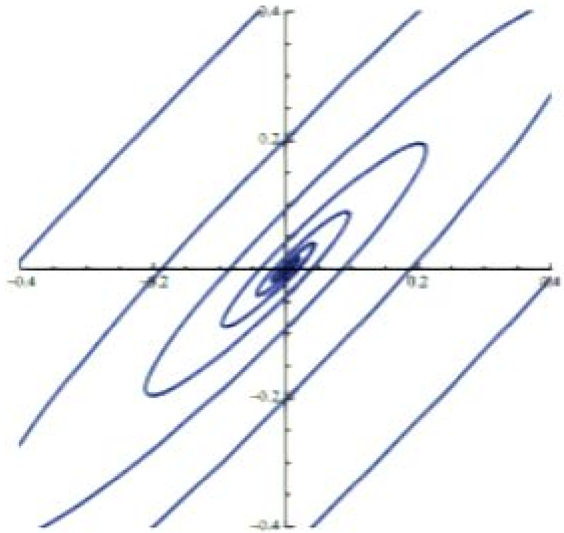
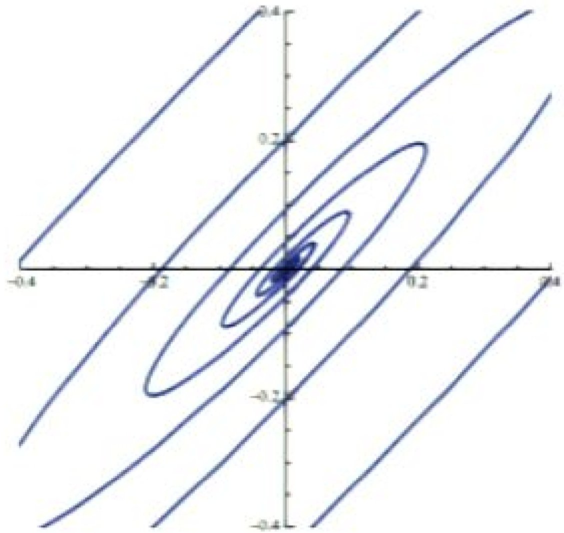


States are exponentially spaced. Discrete scale invariance, broken eventually by UV. Outermost state is thermodynamically stable, $\langle \mathcal{O} \rangle \sim \exp\left(-\frac{\pi}{2\sqrt{g_c - g}}\right)$

Thoughts on the spiral



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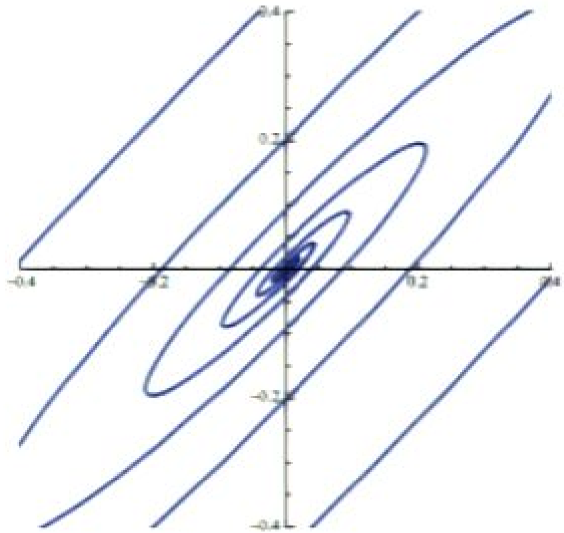

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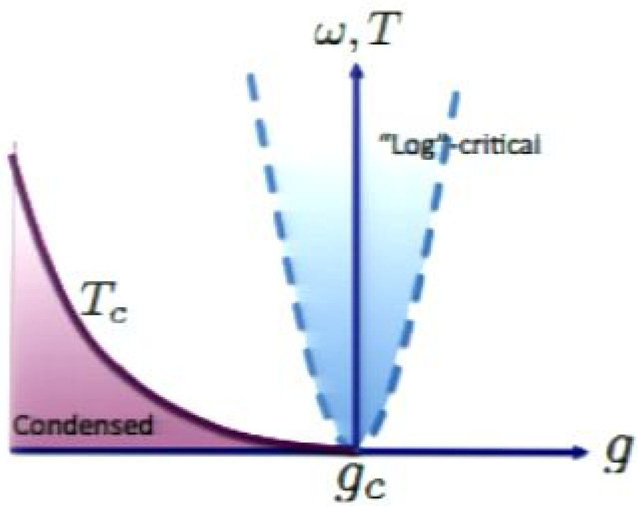
As we approach critical point $g \to g_c^-$, can show analytically that spiral is squeezed into linear response line: recall, finite susceptibility as $g \to g_c^+$.

Summary



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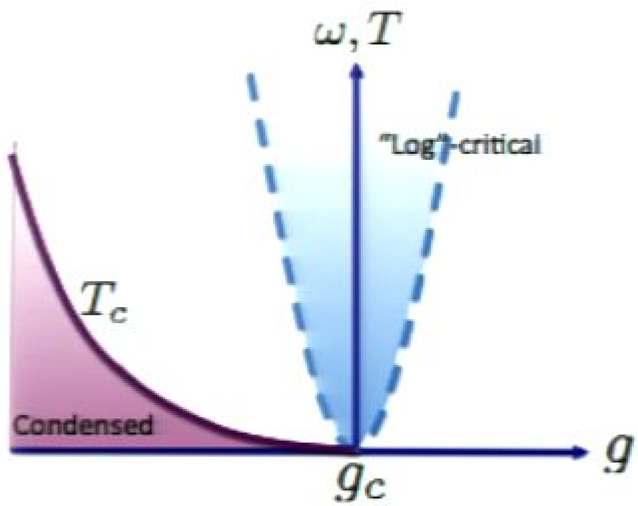

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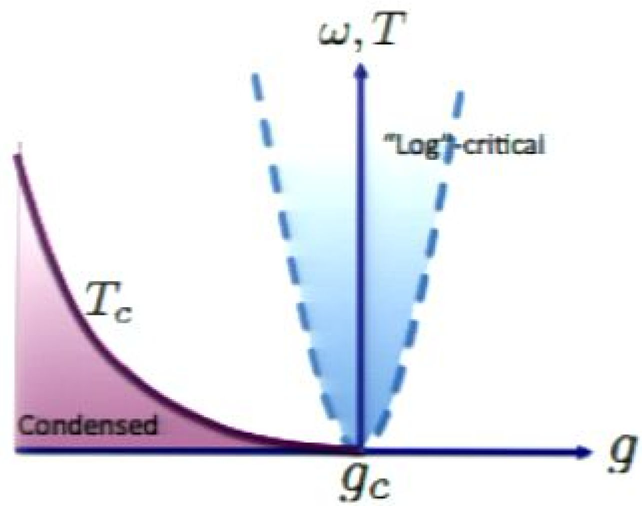

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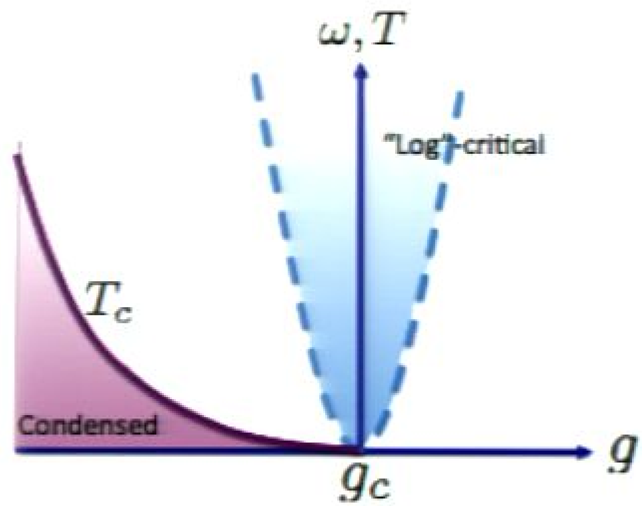

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4. BKT scaling in time; mean-field scaling in space.

Future directions

How do these critical degrees of freedom affect other fields they are coupled to? (e.g. Fermions near quantum criticality.)

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Can we write down an effective field theory for this sort of phase transition? (What replaces Landau?)

Perhaps a semi-holographic construction (Faulkner, Liu, McGreevy, Vegh; Faulkner, Polchinski; Son, Nickel):

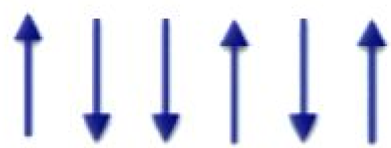
$$S = S_{UV}(\phi) + S_{IR}(\Phi) + \eta \int \Phi\phi$$

This works well for non-Fermi liquids but is rather nontrivial in this case... (in progress with Liu, Mezei).

More speculatively...

Field theoretical physics behind this remains obscure.

Insensitivity to k suggests that somehow different localized patches are going critical individually; like a lattice of uncorrelated degrees of freedom.



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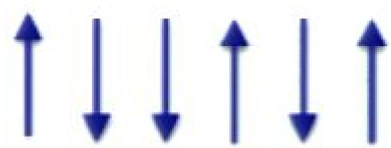

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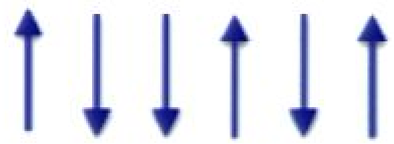

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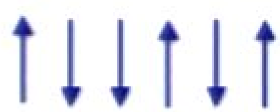

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Nabil Iqbal Quantum Phase Transitions from AdS_2: Beyond the Landau P...