

Title: Holographic Dual of Free Field Theory

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Abstract: We derive a holographic dual description of free quantum field theory in arbitrary dimensions, by reinterpreting the exact renormalization group, to obtain a higher spin gravity theory of the general type which had been proposed and studied as a dual theory

# Holographic dual of free field theory

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## Motivation and Goals

- Many examples of *AdS/CFT* correspondence are well established by now.
- However, there is no direct derivation of the duality in any example.
- Many approaches for such derivation have been (and are being) tried, e.g.
  - ▶ Explicit rewriting of field theory Feynman diagrams as worldsheet correlators. [Gopakumar...](#)
  - ▶ Taking the zero radius limit of the string sigma model. [Berkovits...](#)
  - ▶ ...
- In this talk we will outline yet another approach:
  - ▶ It is believed that the extra, radial, direction of *AdS* is related to the RG scale.
  - ▶ This observation is in the heart of holographic renormalization.
  - ▶ We will try to make the relation between RG and holography more precise.
  - ▶ In particular we will claim that the exact RG equations for a free scalar theory are equivalent to higher spin gravity in *AdS*.



# Outline

- General comments
- Exact RG equations
- RG equations as Bulk equations:  $RG \leftrightarrow GR$
- Correlators



## General comments - Free theory

- The field theory we will discuss is a free scalar theory,

$$S = \int d^D x \partial^\mu \phi^A(x) \partial_\mu \bar{\phi}^A(x), \quad A = 1 \dots N.$$

- The analogues of the single trace operators are operators built from one  $\phi$  and one  $\bar{\phi}$ ,

$$\mathcal{O}^{\mu_1, \dots, \mu_s; \nu_1, \dots, \nu_t} = \partial^{\mu_1} \dots \partial^{\mu_s} \phi^A(x) \partial^{\nu_1} \dots \partial^{\nu_t} \bar{\phi}^A(x).$$

- The boundary theory has conserved currents of arbitrarily high spin which should be dual to bulk higher spin gauge fields.
- It is conjectured to be dual to higher spin gravity in  $AdS$ . Sundborg, Sezgin-Sundell, Klebanov-Polyakov ...
- Such interacting higher spin theory in  $AdS$  was constructed by Vasiliev and co..
- Recently S. Giombi and X. Yin have verified that the 3pt functions in Vasiliev's theory agree with the free theory.
- The dual theory is **not** a string theory. For example, the large  $N$  expansion is trivial and 't Hooft argument does not apply.



## General comments - RG and Holography

- In Holography we have

$$S = S_0 + \int \phi_I(x) \mathcal{O}_I(x),$$

with  $\mathcal{O}_I(x)$  being operators and  $\phi_I(x)$  the sources. In the bulk we have fields  $\Phi_I(x, r)$  dual to  $\mathcal{O}_I$  which have "boundary values"  $\phi_I(x)$ .



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- In RG the situation is very similar. We have an action defined at some cut-off scale  $\Lambda_0$

$$S = S_0 + \int \phi_I(x) \mathcal{O}_I(x).$$

Then we define the theory at scale  $\Lambda$  below  $\Lambda_0$ ,

$$S = S'_0 + \int \Phi_I(x, \Lambda) \mathcal{O}_I(x),$$

such that the “effective couplings”  $\Phi_I(x, \Lambda)$  go to  $\phi_I(x)$  for  $\Lambda$  going to  $\Lambda_0$ .



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- We will try to argue that essentially the above two statements are equivalent, at least in the particular example we will discuss.
- Anything one directly obtains from the field theory is in a “fixed gauge” with respect to bulk gauge symmetry.
- One has to add the redundancy implied by having gauge symmetry, which is not always a simple thing to do.

## Exact RG - Polchinski's equation

- We start with a partition function

$$Z = \int [D\phi] e^{-S_{kin} - S_{int}},$$

- and introduce a smooth cut-off

$$S_{kin} = \int d^D p p^2 \phi(p) \bar{\phi}(p) \rightarrow S_{kin}(\Lambda) = \int d^D p p^2 K^{-1}(p^2/\Lambda^2) \phi(p) \bar{\phi}(p),$$

such that  $K(x)$  interpolates between 1 at  $x = 0$  and 0 at  $x = \infty$ .

- Then we demand that

$$Z_\Lambda = \int [D\phi] e^{-S_{kin}(\Lambda) - S_{int}(\Lambda)}, \quad \frac{d}{d\Lambda} Z_\Lambda = 0.$$

- This implies that the effective action has to satisfy

$$\frac{d}{d\Lambda} S_{int}(\Lambda) = - \int d^D p \frac{1}{p^2} \frac{\partial K(p^2/\Lambda^2)}{\partial \Lambda} \left\{ \frac{\delta S_{int}}{\delta \phi(p)} \frac{\delta S_{int}}{\delta \bar{\phi}(p)} - \frac{\delta^2 S_{int}}{\delta \phi(p) \delta \bar{\phi}(p)} \right\}.$$

## Exact RG - Free theory

- For generic theories Polchinski's equation gives an infinite set of equations for the infinite number of possible couplings.
- In a free theory the only couplings are quadratic in the fields

$$S_{int} = - \int d^D p d^D q B(p, q) \phi(p) \bar{\phi}(q) + F$$

and we get only two equations for  $B$  and  $F$ .

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$$d_\Lambda B(p, q) = - \int d^D s d^D s' B(p, s) \alpha(s, s') B(s', q)$$

$$d_\Lambda F = \int d^D p d^D q \alpha(p, q) (P(q, p) + B(q, p)),$$

where we define

$$P(p, q) = p^2 K^{-1}(p^2/\Lambda^2) \delta^{(D)}(p - q), \quad \alpha = d_\Lambda P^{-1}.$$

- The free energy (and thus the **bulk on-shell action** in the AdS/CFT correspondence) is given by

$$\tilde{F} = \int d\Lambda d_\Lambda F \sim \int d\Lambda \text{Tr} \alpha \cdot B. \quad (= -\text{Tr} \log(P \overset{\text{im}}{\Lambda} B))$$

## RG equations as matrix equations

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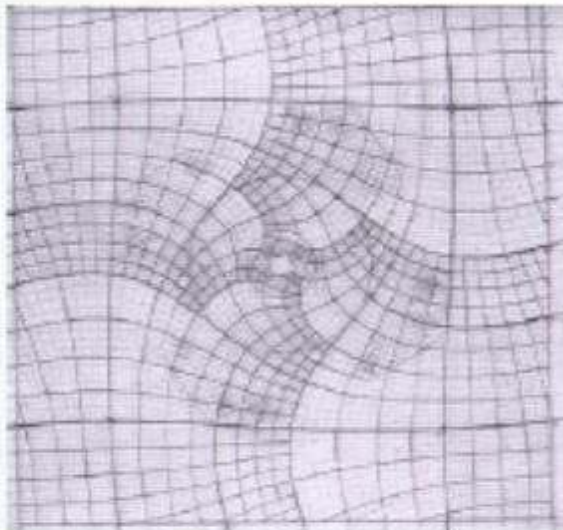
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thus Fourier transforming to momentum space we get

$$\frac{d}{da_l} B_{\underline{s}\underline{t}} p^{\underline{s}} q^{\underline{t}} = i(p^l - q^l) B_{\underline{s}\underline{t}} p^{\underline{s}} q^{\underline{t}}.$$

## A Generalization

We have discussed till now a translation invariant RG. However, this can be generalized by introducing an explicit dependence on the reference coordinate  $a$  into the cut-off function  $K(p^2/\Lambda^2) \rightarrow K(p^2/\Lambda^2, a)$ , i.e. "position dependent momentum cut-off" or equivalently putting the theory on a grid of spatially varying size.



$$\frac{d}{d\Lambda} B_{\underline{st}} = -B_{\underline{st}} \alpha_{\Lambda}^{\underline{t}} B_{\underline{jt}} + \Lambda^{-1}(s+t) B_{\underline{st}},$$

$$\frac{d}{da_l} B_{\underline{st}} p^s q^t = -B_{\underline{st}} \alpha_l^{\underline{t}} B_{\underline{jt}} p^s q^t + i(p^l - \frac{gm^l}{\Lambda}) B_{\underline{st}} p^s q^t.$$

We identify

$$r = \frac{1}{\Lambda}.$$

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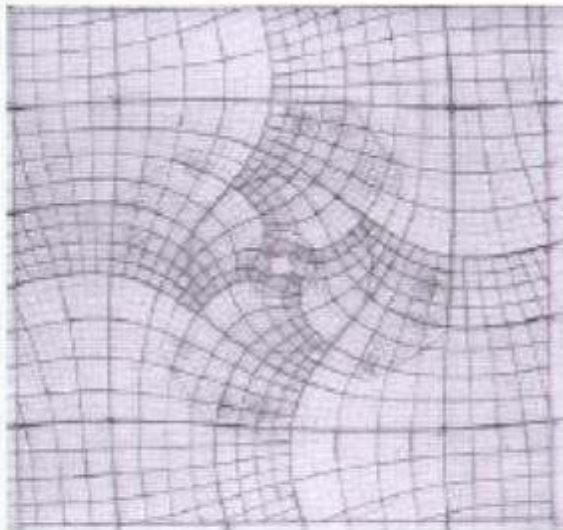
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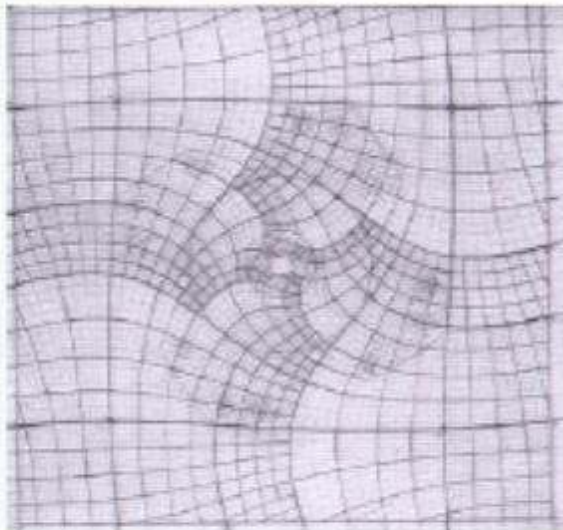
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We identify

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## Short digression: From matrix multiplication to Moyal product

- The matrix multiplication appearing in the equations can be elegantly encoded as a non-commutative Moyal product.
- One introduces auxiliary variables, "oscillators":  $y^\mu, \bar{y}_\mu, z^\mu, \bar{z}_\mu$ . The index  $\mu$  takes  $D + 2$  different values  $\{\bullet, r, 0, 1, \dots, D - 1\}$ .
- The auxiliary variables are multiplied by a  $*$ -product which is defined by

$$y^\mu * = y^\mu - \frac{1}{2} \left( \frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu} \right) \hat{\eta}_\nu^\mu, \quad z^\mu * = z^\mu - \frac{1}{2} \left( \frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu} \right) \hat{\eta}_\nu^\mu,$$

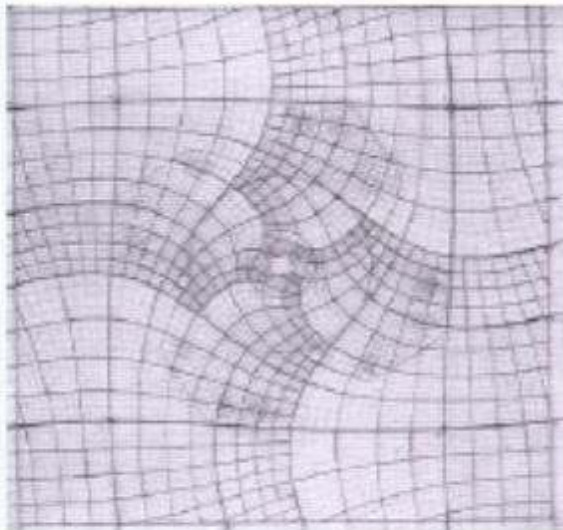
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## A Generalization

We have discussed till now a translation invariant RG. However, this can be generalized by introducing an explicit dependence on the reference coordinate  $a$  into the cut-off function  $K(p^2/\Lambda^2) \rightarrow K(p^2/\Lambda^2, a)$ , i.e. "position dependent momentum cut-off" or equivalently putting the theory on a grid of spatially varying size.



$$\frac{d}{d\Lambda} B_{\underline{st}} = -B_{\underline{st}} \alpha \frac{t}{\Lambda} B_{\underline{jt}} + \Lambda^{-1} (s+t) B_{\underline{st}},$$

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We identify

$$r = \frac{1}{\Lambda}.$$

## Short digression: From matrix multiplication to Moyal product

- The matrix multiplication appearing in the equations can be elegantly encoded as a non-commutative Moyal product.
- One introduces auxiliary variables, "oscillators":  $y^\mu, \bar{y}_\mu, z^\mu, \bar{z}_\mu$ . The index  $\mu$  takes  $D + 2$  different values  $\{\bullet, r, 0, 1, \dots, D - 1\}$ .
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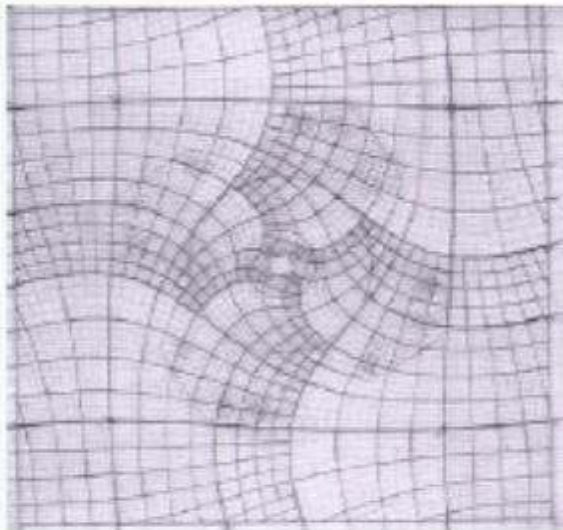
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## Short digression: From matrix multiplication to Moyal product II

- One can define also

$$y^2 = Y^2 + Z^2, \quad \bar{y}_2 = \frac{1}{2}(\bar{Y}_2 - \bar{Z}_2), \quad z^2 = Z^2 - Y^2, \quad \bar{z}_2 = \frac{1}{2}(\bar{Y}_2 + \bar{Z}_2),$$

and

$$H = e^{-Y\bar{Y} - ZZ} = e^{z\bar{y} - yz}.$$

- The following property holds

$$\begin{aligned} (Y^k * \bar{Z}^n * H * \bar{Y}^m * Z^l) * (Y^{k'} * \bar{Z}^{n'} * H * \bar{Y}^{m'} * Z^{l'}) &= \\ &= m!l! \delta_m^{k'} \delta_l^{n'} (Y^k * \bar{Z}^n * H * \bar{Y}^{k'} * Z^{l'}). \end{aligned}$$

- We define

$$B(y, z, \bar{y}, \bar{z}) \sim i^{s-t} B_{s\bar{s}} Y^{\bar{s}} * H * Z^{\bar{s}} (\bar{z}_r - \bar{z}_\bullet)^{s+t},$$

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- Then

$$B_{s\bar{s}} \alpha_{\mu}^{\bar{j}} B_{\bar{j}t} \leftrightarrow B * \alpha_{\mu} * B.$$

## AdS in terms of the auxiliary variables

- The group of linear transformations on each set of auxiliary variables preserving the metric  $\eta_{\alpha\beta}^{\beta}$  is  $SO(D-1, 2)$ , the group of isometries of  $AdS_{D+1}$ . As is well known, we can represent the corresponding Lie algebra as star commutators with generators which are quadratic functions of the oscillators.
- Dilatations and translations are given by

$$P_r = \bar{z}_r z^{\bullet} - \bar{z}_{\bullet} z^r, \quad P_a = \bar{z}_a z^{\bullet} - \bar{z}_{\bullet} z^a - \bar{z}_{\bullet} z^a + \bar{z}_r z^a.$$

- The commutation relation

$$[P_r, P_a]_{\star} = P_a$$

implies that if we define a connection

$$W_{\mu}^{(0)} = \frac{1}{r} P_{\mu},$$

then it is flat

$$dW^{(0)} + W^{(0)} \wedge \star W^{(0)} = 0,$$

and it describes an  $AdS_{D+1}$  background

$$e^a_{\mu} = -\frac{1}{r} \delta^a_{\mu}, \quad W_{\mu}^{ra} = -W_{\mu}^{ar} = \frac{1}{r} \delta^{ra},$$

and the metric is

$$ds^2 = \frac{dr^2 + dx^a dx^a}{r^2}$$

## Linearized equations describe *AdS* vacuum

- The linearized equations are

$$\frac{d}{d\Lambda} B_{\underline{st}} = \Lambda^{-1}(s + t) B_{\underline{st}}, \quad \frac{d}{d\alpha_l} B_{\underline{st}} p^{\underline{s}} q^{\underline{t}} = i(p^l - q^l) B_{\underline{st}} p^{\underline{s}} q^{\underline{t}}.$$

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## Non-linear equations

- We wrote the linearized equations as covariant constancy condition with respect to  $AdS_{D+1}$  connection.
- The full non-linear equation can be also written as covariant constancy condition with respect to a dynamical connection. We define

$$(\delta \tilde{W}_\mu)_{\underline{q}}^{\underline{p}} = 0, \quad (\delta W_\mu)_{\underline{q}}^{\underline{p}} = B_{\underline{q}\underline{r}} \alpha_{\underline{r}\underline{\mu}}^{\underline{p}},$$

and write

$$\frac{d}{dx_\mu} B + W_\mu * B - B * \tilde{W}_\mu = 0.$$

- The covariant constancy equation is consistent only if the connection is flat

$$dW + W \wedge * W = 0.$$

This equation implies a condition on the cut-off function  $\alpha_\mu$

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- This condition is satisfied for any position independent cutoff, and basically states that the cutoff function is consistent with  $AdS$  isometries.

## Some comments

- The bulk equations of motion we obtain admit gauge transformations

$$\Delta W = d\epsilon + [W, \epsilon]_* , \quad \Delta B = B * \tilde{\epsilon} - \epsilon * B .$$

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## Vasiliev's theory vs RG

- Vasiliev theory of higher spin gauge theory in  $AdS$  is formulated using the same kinematic language we used for the RG equations.
- The dynamics is given by

$$dW + W * \wedge W = 0, \quad dB + W * B - B * \bar{W} = 0,$$

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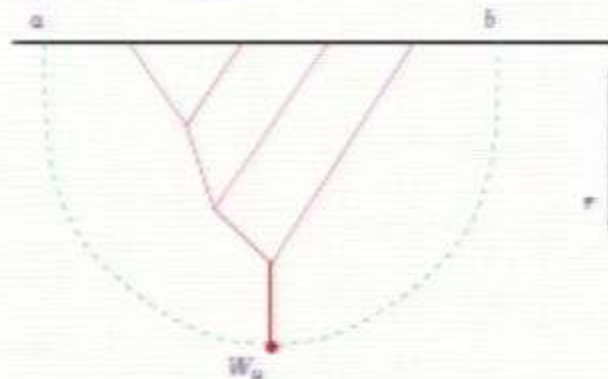
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## On-shell action and correlators

- Remember that the free energy is given by

$$F = -\text{Tr} \int d\Lambda B \cdot \alpha \quad \rightarrow \quad \text{Tr} \int dx^\mu \delta W_\mu.$$

- Thus it is given by a holonomy integral of the combination we identified as the gauge field in the bulk.
- The free energy is identified in *AdS/CFT* with the *on-shell* bulk action.
- The correlators in field theory are given by variations of the free energy with respect to the sources,  $B$  in our case.
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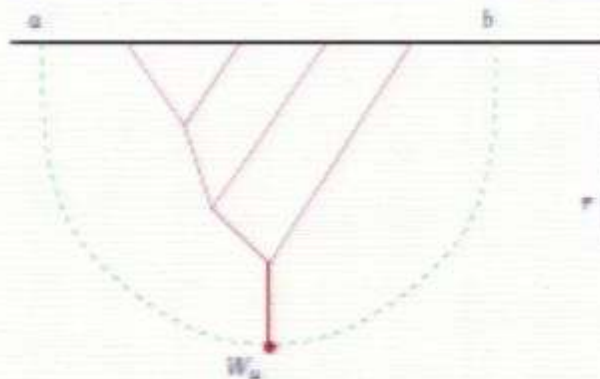
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## Summary and outlook

- We have considered the ERGE for free scalar field theory in  $D$  dimensions with arbitrary sources for “single trace” operators.
- We have shown that these equations can be interpreted as equations of motion of higher spin fields propagating in  $AdS_{D+1}$ .
- The full on-shell action is given by a holonomy integral.
- The equations one obtains from RG are for a gauge field  $W_\mu$  and a scalar  $B$ . These equations enjoy the higher spin gauge symmetry and take the form of covariant constancy and flatness conditions.
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- Some research directions:
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