Title: Holographic Dual of Free Field Theory

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Abstract: We derive a holographic dual description of free quantum field theory in arbitrary dimensions, by reinterpreting the exact renormalization group, to obtain a higher spin gravity theory of the general type which had been proposed and studied as a dual theory

Holographic dual of free field theory

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Motivation and Goals

- Many examples of AdS/CFT correspondence are well established by now.
- However, there is no direct derivation of the duality in any example.
- Many approaches for such derivation have been (and are being) tried, *e.g*
  - Explicit rewriting of field theory Feynman diagrams as worldsheet correlators. Gopakumar...
  - Taking the zero radius limit of the string sigma model. Berkovits...
  - ...
- In this talk we will outline yet another approach:
  - It is believed that the extra, radial, direction of AdS is related to the RG scale.
  - This observation is in the heart of holographic renormalization.
  - We will try to make the relation between RG and holography more precise.
  - In particular we will claim that the exact RG equations for a free scalar theory are equivalent to higher spin gravity in AdS.

Outline

- General comments
- Exact RG equations
- RG equations as Bulk equations: $RG \quad \leftrightarrow \quad GR$
- Correlators

General comments - Free theory

- The field theory we will discuss is a free scalar theory,

$$S = \int d^D x \partial^\mu \phi^A(x) \partial_\mu \bar{\phi}^A(x), \qquad A = 1 \cdots N.$$

- The analogues of the single trace operators are operators built from one ϕ and one $\bar{\phi}$,

$$\mathcal{O}^{\mu 1, \dots, \mu s; \nu 1, \dots \nu t} = \partial^{\mu 1} \cdots \partial^{\mu s} \phi^A(x) \partial^{\nu 1} \cdots \partial^{\nu t} \bar{\phi}^A(x).$$

- The boundary theory has conserved currents of arbitrarily high spin which should be dual to bulk higher spin gauge fields.
- It is conjectured to be dual to higher spin gravity in *AdS*. Sundborg, Sezgin-Sundell, Klebanov-Polyakov ...
- Such interacting higher spin theory in *AdS* was constructed by Vasiliev and *co.*.
- Recently S. Giombi and X. Yin have verified that the 3pt functions in Vasiliev's theory agree with the free theory.
- The dual theory is not a string theory. For example, the large N expansion is trivial and 't Hooft argument does not apply.

General comments - RG and Holography

- In Holography we have
$$S = S_0 + \int \phi_I(x)\,\mathcal{O}_I(x),$$
with $\mathcal{O}_I(x)$ being operators and $\phi_I(x)$ the sources. In the bulk we have fields $\Phi_I(x,r)$ dual to $\mathcal{O}_I$ which have "boundary values" $\phi_I(x)$.

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- In RG the situation is very similar. We have an action defined at some cut-off scale Λ_0
$$S = S_0 + \int \phi_I(x)\,\mathcal{O}_I(x).$$
Then we define the theory at scale Λ below Λ_0.
$$S = S_0' + \int \Phi_I(x,\Lambda)\,\mathcal{O}_I(x),$$
such that the "effective couplings" $\Phi_I(x,\Lambda)$ go to $\phi_I(x)$ for Λ going to Λ_0.

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- We will try to argue that essentially the above two statements are equivalent, at least in the particular example we will discuss.
- Anything one directly obtains from the field theory is in a "fixed gauge" with respect to bulk gauge symmetry.
- One has to add the redundancy implied by having gauge symmetry, which is not always a simple thing to do.

Exact RG - Polchinski's equation

- We start with a partition function

$$Z = \int [\mathcal{D}\phi]\, e^{-S_{kin} - S_{int}},$$

- and introduce a smooth cut-off

$$S_{kin} = \int d^D p\, p^2 \phi(p)\bar{\phi}(p) \quad \rightarrow \quad S_{kin}(\Lambda) = \int d^D p\, p^2\, K^{-1}(p^2/\Lambda^2)\phi(p)\bar{\phi}(p),$$

such that $K(x)$ interpolates between 1 at $x = 0$ and 0 at $x = \infty$.

- Then we demand that

$$Z_\Lambda = \int [\mathcal{D}\phi]\, e^{-S_{kin}(\Lambda) - S_{int}(\Lambda)}, \qquad \frac{d}{d\Lambda} Z_\Lambda = 0.$$

- This implies that the effective action has to satisfy

$$\frac{d}{d\Lambda} S_{int}(\Lambda) = -\int d^D p \frac{1}{p^2} \frac{\partial K(p^2/\Lambda^2)}{\partial \Lambda} \left\{ \frac{\delta S_{int}}{\delta \phi(p)} \frac{\delta S_{int}}{\delta \bar{\phi}(p)} - \frac{\delta^2 S_{int}}{\delta \phi(p) \delta \bar{\phi}(p)} \right\}.$$

Exact RG - Free theory

- For generic theories Polchinski's equation gives an infinite set of equations for the infinite number of possible couplings.
- In a free theory the only couplings are quadratic in the fields

$$S_{int} = -\int d^D p d^D q \, B(p,q) \, \phi(p)\bar{\phi}(q) \; + F$$

and we get only two equations for B and F.

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$$d_\Lambda B(p,q) = -\int d^D s \, d^D s' \, B(p,s) \, \alpha(s,s') \, B(s',q)$$

$$d_\Lambda F = \int d^D p d^D q \, \alpha(p,q) \, (P(q,p) + B(q,p)) \, ,$$

where we define

$$P(p,q) = p^2 K^{-1}(p^2/\Lambda^2) \delta^{(D)}(p-q), \qquad \alpha = d_\Lambda P^{-1} \, .$$

- The free energy (and thus the bulk on-shell action in the AdS/CFT correspondance) is given by

$$\tilde{F} = \int d\Lambda \, d_\Lambda F \sim \int d\Lambda \mathrm{Tr} \alpha \cdot B \, . \qquad (= -\mathrm{Tr}\log(P^{-1}B) \,)$$

RG equations as matrix equations

- Let us expand B in powers of p/Λ and q/Λ

$$B = \sum_{s,t=0}^{\infty} \Lambda^{-s-t} B_{a_1 \ldots a_s, b_1 \ldots b_t} p^{a_1} \ldots p^{a_s} q^{b_1} \ldots q^{b_t} \equiv \Lambda^{-s-t} B_{\underline{s}\underline{t}} p^{\underline{s}} q^{\underline{t}},$$

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$$\alpha^{\underline{s}\underline{t}} = \Lambda^{-s-t} \int d^D p \int d^D q \, \alpha(p, q) p^{\underline{s}} q^{\underline{t}}.$$

- Then the RG equation can be written as a matrix equation

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- We will soon interpret this equation as a radial component of a bulk equation of motion. What about other directions?

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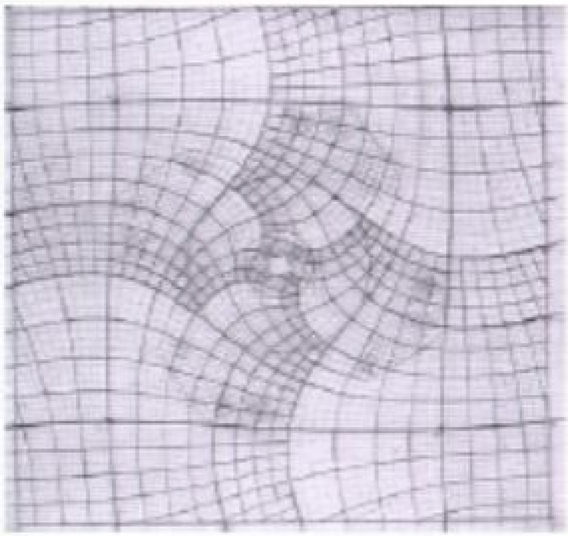
A Generalization

We have discussed till now a translation invariant RG. However, this can be generalized by introducing an explicit dependence on the reference coordinate a into the cut-off function $K(p^2/\Lambda^2) \quad \rightarrow \quad K(p^2/\Lambda^2, a)$, *i.e.* "position dependent momentum cut-off" or equivalently putting the theory on a grid of spatially varying size.



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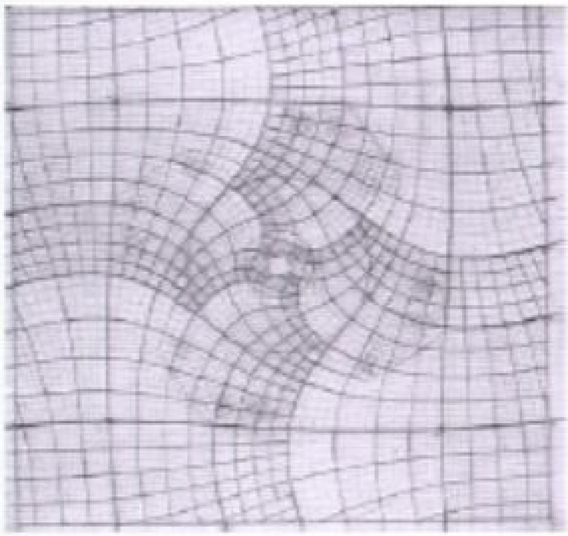

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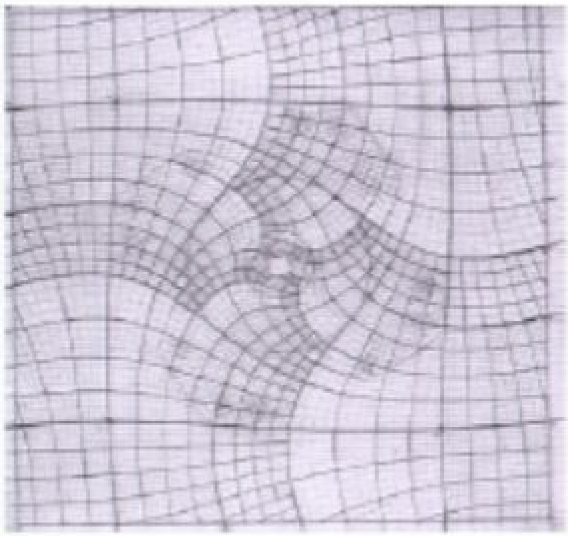

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Short digression: From matrix multiplication to Moyal product

- The matrix multiplication appearing in the equations can be elegantly encoded as a non-commutative Moyal product.
- One introduces auxiliary variables, "oscilators": y^μ, $\bar{y}_\mu$, z^μ, $\bar{z}_\mu$. The index μ takes $D+2$ different values $\{\bullet, r, 0, 1, \ldots, D-1\}$.
- The auxiliary variables are multiplied by a $*$-product which is defined by

$$y^\mu * = y^\mu - \frac{1}{2}\left(\frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu}\right)\hat{\eta}^\mu_\nu, \qquad z^\mu * = z^\mu - \frac{1}{2}\left(\frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu}\right)\hat{\eta}^\mu_\nu,$$
$$\bar{y}_\mu * = \bar{y}_\mu + \frac{1}{2}\left(\frac{\partial}{\partial y^\nu} - \frac{\partial}{\partial z^\nu}\right)\hat{\eta}^\nu_\mu, \qquad \bar{z}_\mu * = \bar{z}_\mu + \frac{1}{2}\left(\frac{\partial}{\partial y^\nu} - \frac{\partial}{\partial z^\nu}\right)\hat{\eta}^\nu_\mu.$$

- In particular $[\bar{y}_\alpha, y^\beta]_* = \hat{\eta}^\beta_\alpha$, $\quad [\bar{z}_\alpha, z^\beta]_* = -\hat{\eta}^\beta_\alpha$. The metric on this auxiliary space is $\hat{\eta}^\beta_\alpha = (-1, 1, \eta)$, where η is the metric on the original flat space-time.

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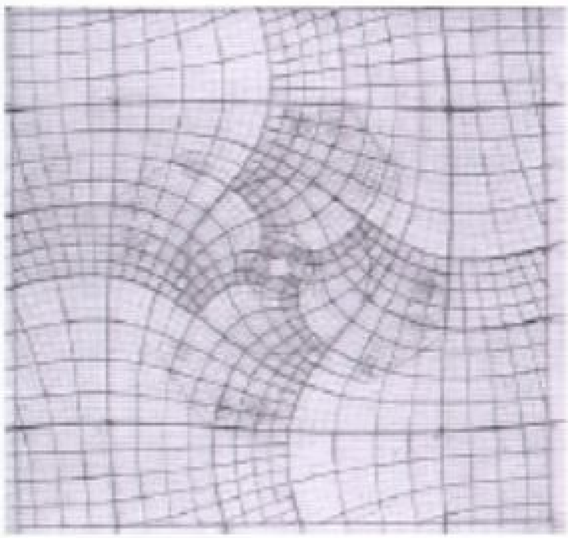

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We identify

$$\boxed{r = \frac{1}{\Lambda}}.$$

Short digression: From matrix multiplication to Moyal product

- The matrix multiplication appearing in the equations can be elegantly encoded as a non-commutative Moyal product.
- One introduces auxiliary variables, "oscilators": y^μ, $\bar{y}_\mu$, z^μ, $\bar{z}_\mu$. The index μ takes $D+2$ different values $\{\bullet, r, 0, 1, \ldots, D-1\}$.
- The auxiliary variables are multiplied by a $*$-product which is defined by

$$y^\mu * = y^\mu - \frac{1}{2}\left(\frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu}\right)\hat{\eta}^\mu_\nu, \qquad z^\mu * = z^\mu - \frac{1}{2}\left(\frac{\partial}{\partial \bar{y}_\nu} - \frac{\partial}{\partial \bar{z}_\nu}\right)\hat{\eta}^\mu_\nu,$$

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- In particular $[\bar{y}_\alpha, y^\beta]_* = \hat{\eta}^\beta_\alpha$, $\quad [\bar{z}_\alpha, z^\beta]_* = -\hat{\eta}^\beta_\alpha$. The metric on this auxiliary space is $\hat{\eta}^\beta_\alpha = (-1, 1, \eta)$, where η is the metric on the original flat space-time.

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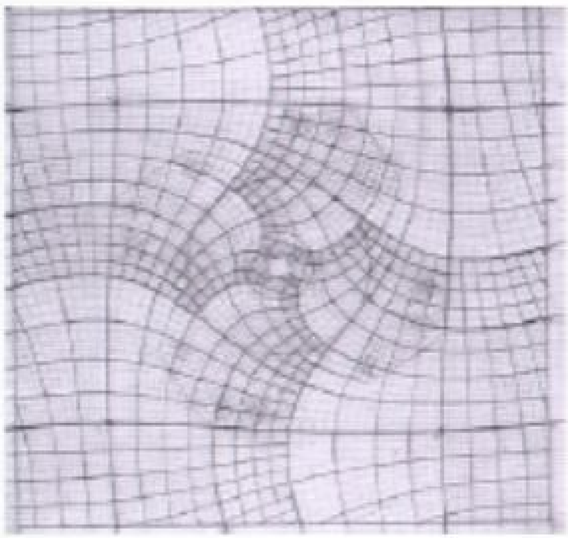

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Short digression: From matrix multiplication to Moyal product II

- One can define also

$$y^{a} = Y^{a} + Z^{a}, \qquad \bar{y}_{a} = \frac{1}{2}(\bar{Y}_{a} - \bar{Z}_{a}), \qquad z^{a} = Z^{a} - Y^{a}, \qquad \bar{z}_{a} = \frac{1}{2}(\bar{Y}_{a} + \bar{Z}_{a}),$$

and

$$H = e^{-Y\bar{Y} - Z\bar{Z}} = e^{z\bar{y} - y\bar{z}}.$$

- The following property holds

$$(Y^{k} * \bar{Z}^{n} * H * \bar{Y}^{m} * Z^{l}) * (Y^{k'} * \bar{Z}^{n'} * H * \bar{Y}^{m'} * Z^{l'}) =$$
$$= m!l!\, \delta_{m}{}^{k'}\, \delta^{l}{}_{n'}\, (Y^{k} * \bar{Z}^{n} * H * \bar{Y}^{k'} * Z^{l'}).$$

- We define

$$B(y, z, \bar{y}, \bar{z}) \sim i^{s-t}\, B_{\underline{st}}\, Y^{\underline{s}} * H * Z^{\underline{t}} (\bar{z}_r - \bar{z}_{\bullet})^{s+t},$$
$$\alpha_{\mu}(y, z, \bar{y}, \bar{z}) \sim -\frac{(-i)^{t-s}}{s!t!}\, \alpha_{\mu}^{\underline{st}}\, \bar{Z}_{\underline{s}} * H * \bar{Y}_{\underline{t}} (\bar{z}_r - \bar{z}_{\bullet})^{-s-t},$$

- Then

$$B_{\underline{sl}}\, \alpha_{\mu}^{\underline{lj}}\, B_{\underline{jt}} \qquad \leftrightarrow \qquad B * \alpha_{\mu} * B.$$

AdS in terms of the auxiliary variables

- The group of linear transformations on each set of auxiliary variables preserving the metric η_α^β is $SO(D-1,2)$, the group of isometries of AdS_{D+1}. As is well known, we can represent the corresponding Lie algebra as star commutators with generators which are quadratic functions of the oscillators.
- Dilatations and translations are given by

$$P_r = \bar{z}_r z^\bullet - \bar{z}_\bullet z^r, \qquad P_a = \bar{z}_a z^\bullet - \bar{z}_a z^r - \bar{z}_\bullet z^a + \bar{z}_r z^a .$$

- The commutation relation

$$[P_r, P_a]_* = P_a$$

implies that if we define a connection

$$W_\mu^{(0)} = \frac{1}{r} P_\mu ,$$

then it is flat

$$dW^{(0)} + W^{(0)} \wedge * W^{(0)} = 0 ,$$

and it describes an AdS_{D+1} background

$$e_\mu^a = -\frac{1}{r}\delta^a_\mu, \qquad w_\mu{}^{ra} = -w_\mu{}^{ar} = \frac{1}{r}\delta^{a\mu} ,$$

and the metric is

$$ds^2 = \frac{dr^2 + dx^a dx^a}{r^2}$$

Linearized equations describe *AdS* vacuum

- The linearized equations are

$$\frac{d}{d\Lambda} B_{\underline{st}} = \Lambda^{-1}(s+t)\, B_{\underline{st}}\,, \qquad \frac{d}{da_I}\, B_{\underline{st}} p^{\underline{s}} q^{\underline{t}} = i(p^I - q^I)\, B_{\underline{st}} p^{\underline{s}} q^{\underline{t}}.$$

- Using our definitions of the $*$-product and the connection $W^{(0)}$ these are just given by

$$\frac{d}{dx^\mu} B + W^{(0)}_\mu * B - B * W^{(0)}_\mu = 0\,.$$

- The commutator $[W^{(0)}_3, B]_*$ gives an additional term proportional to $z^\bullet - z^r$ which does not appear in the field theory equations of motion since there is no meaning to these components in field theory. Thus, we have to add these components to the discussion remembering that the field theory equations are recovered by setting these to zero.

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Non-linear equations

- We wrote the linearized equations as covariant constancy condition with respect to AdS_{D+1} connection.
- The full non-linear equation can be also written as covariant constancy condition with respect to a dynamical connection. We define

$$(\delta \widetilde{W}_\mu)^{\underline{p}}{}_{\underline{q}} = 0, \qquad (\delta W_\mu)_{\underline{q}}{}^{\underline{p}} = B_{\underline{qr}}\, \alpha^{\underline{rp}}{}_\mu \,,$$

and write

$$\frac{d}{dx_\mu} B + W_\mu * B - B * \widetilde{W}_\mu = 0 \,.$$

- The covariant constancy equation is consistent only if the connection is flat

$$dW + W \wedge * W = 0 \,.$$

This equation implies a condition on the cut-off function α_μ

$$d\alpha + W^{(0)} \wedge * \alpha + \alpha \wedge * W^{(0)} = 0 \,.$$

- This condition is satisfied for any position independent cutoff, and basically states that the cutoff function is consistent with AdS isometries.

Some coomments

- The bulk equations of motion we obtain admit gauge transformations

$$\Delta W = d\epsilon + [W, \epsilon]_* , \qquad \Delta B = B * \tilde{\epsilon} - \epsilon * B \quad .$$

- However, the identification

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Vasiliev's theory *vs* RG

- Vasiliev theory of higher spin gauge theory in AdS is formulated using the same kinematic language we used for the RG equations.
- The dynamics is given by

$$dW + W * \wedge W = 0, \qquad dB + W * B - B * \widetilde{W} = 0\,,$$

- supplemented by constraints relating B and W, (schematically)

$$S * B = B * \widetilde{S}, \qquad S * S \sim 1 + B, \qquad dS + W * S + S * W = 0\,.$$

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- By rewriting the RG equations we obtained very similar equations.
- However, the constraint is $\delta W = B * \alpha$ is not gauge invariant.
- One can hope that either,
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On-shell action and correlators

- Remember that the free energy is given by

$$F = -Tr \int d\Lambda B \cdot \alpha \qquad \to \qquad Tr \int dx^{\mu} \delta W_{\mu}.$$

- Thus it is given by a holonomy integral of the combination we identified as the gauge field in the bulk.
- The free energy is identified in AdS/CFT with the *on-shell* bulk action.
- The correlators in field theory are given by variations of the free energy with respect to the sources, B in our case.
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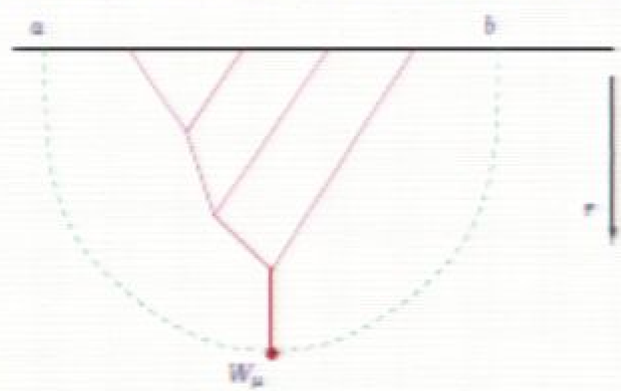

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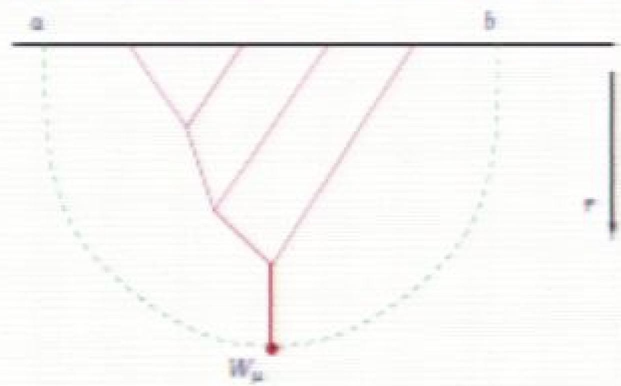

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- We have shown that these equations can be interpreted as equations of motion of higher spin fields propagating in AdS_{D+1}.
- The full on-shell action is given by a holonomy integral.
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- W_μ and B are related by a constraint which breaks the gauge symmetry. Physically it happens because we choose a cut-off prescription.
- Some research directions:
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