Title: Perfect function transfer in two and three dimensions without initialization

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Abstract: We find analytic models that can perfectly transfer, without state initialization or remote collaboration, arbitrary functions in two- and three-dimensional interacting bosonic and fermionic networks. This provides for the possible experimental implementation of state transfer through bosonic or fermionic atoms trapped in optical lattices. A significant finding is that the state of a spin qubit can be perfectly transferred through a fermionic system. Families of Hamiltonians are described that are related to the linear models and that enable the perfect transfer of arbitrary functions. Furthermore, we propose methods for eliminating certain types of errors

Perfect state transfer (PST)

Lian- Ao Wu,

University of the Basque Country
And Basque Science Foundation

Background:

al quantum computer far from reach

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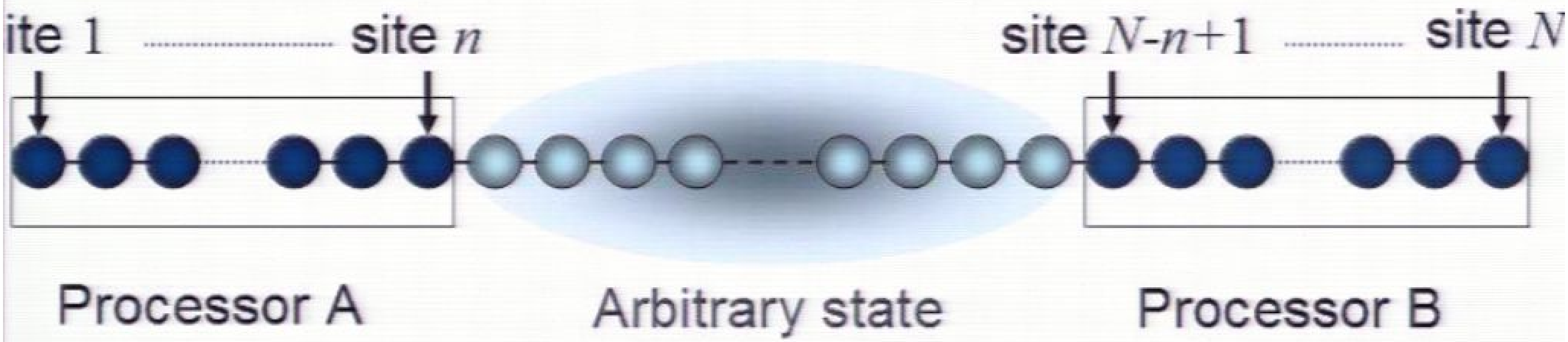
  - avoid interfacing processors with light
  - no switch on/off

Introduction to State transfer:



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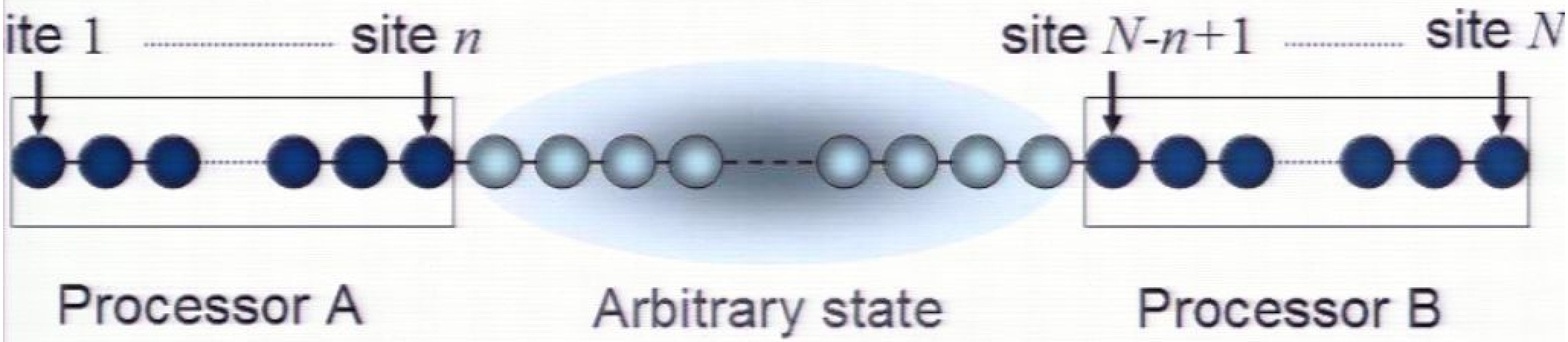

One qubit per processor. An unknown state transferred ideally:

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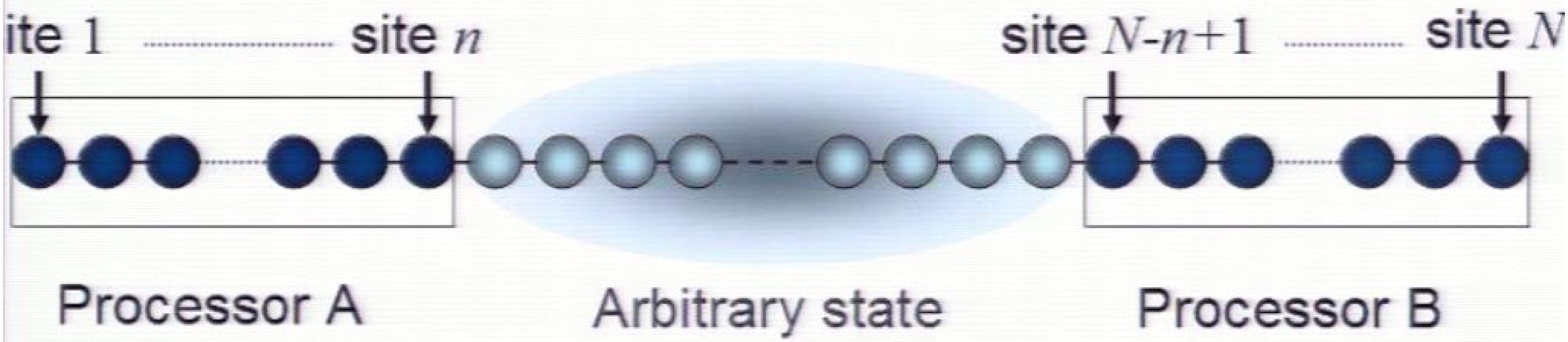

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$$|\phi\rangle_A = a|0\rangle_A + b|1\rangle_A \mapsto |\phi\rangle_B = a|0\rangle_B + b|1\rangle_B$$

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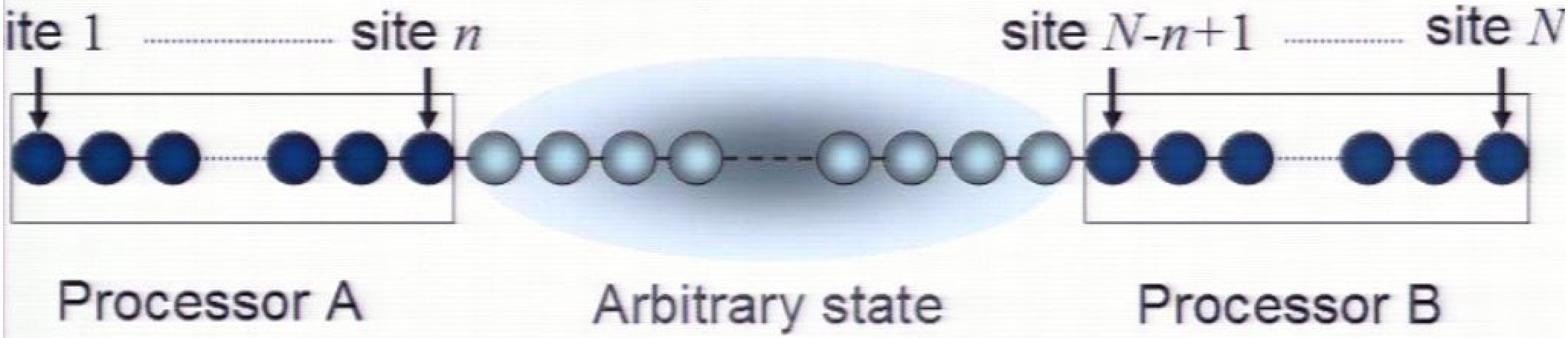

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PST: F=1. XY model 3 qubits, Heisenberger Model, 2 qubits

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Reduced density matrix at A → B after time τ

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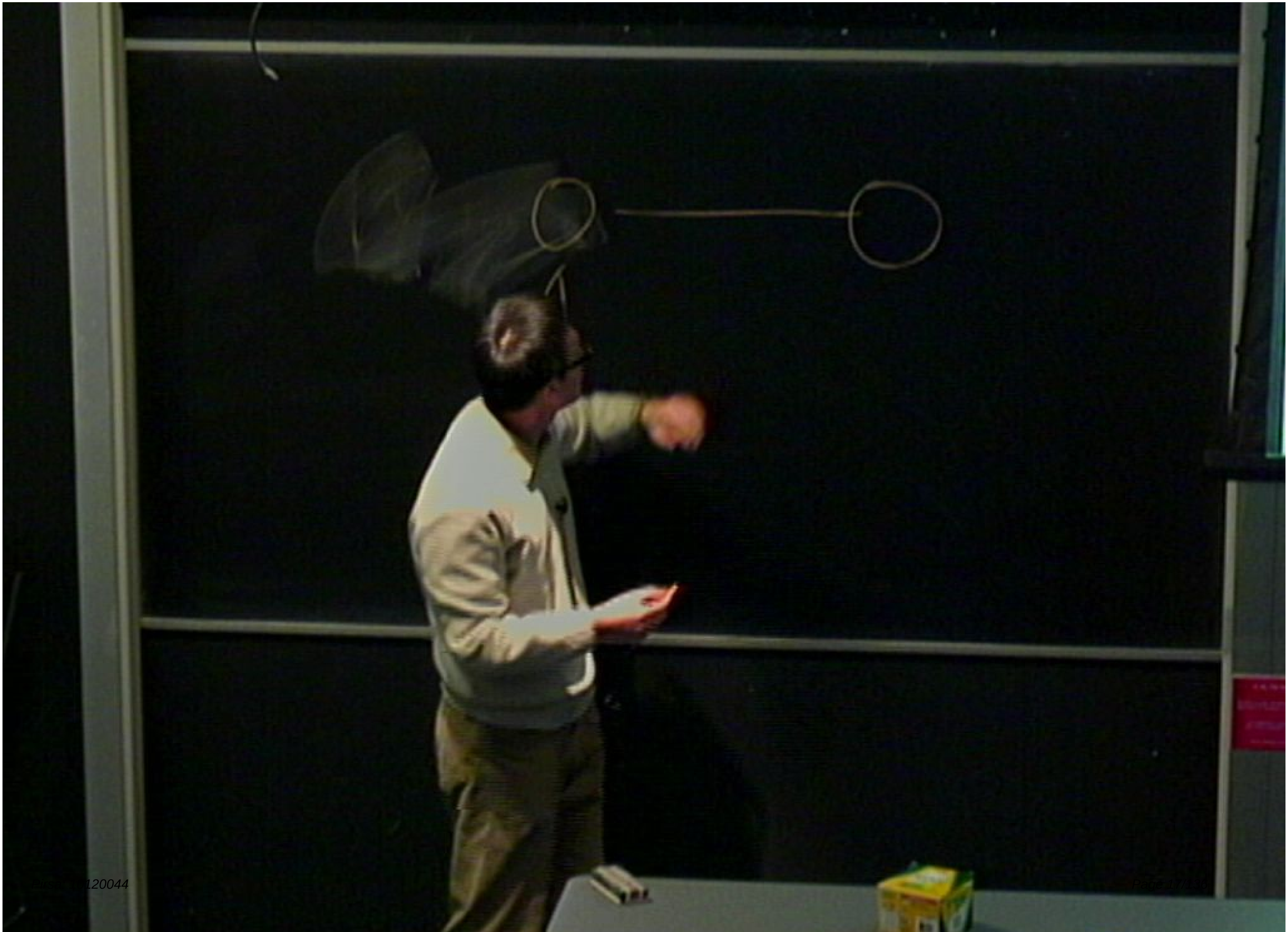
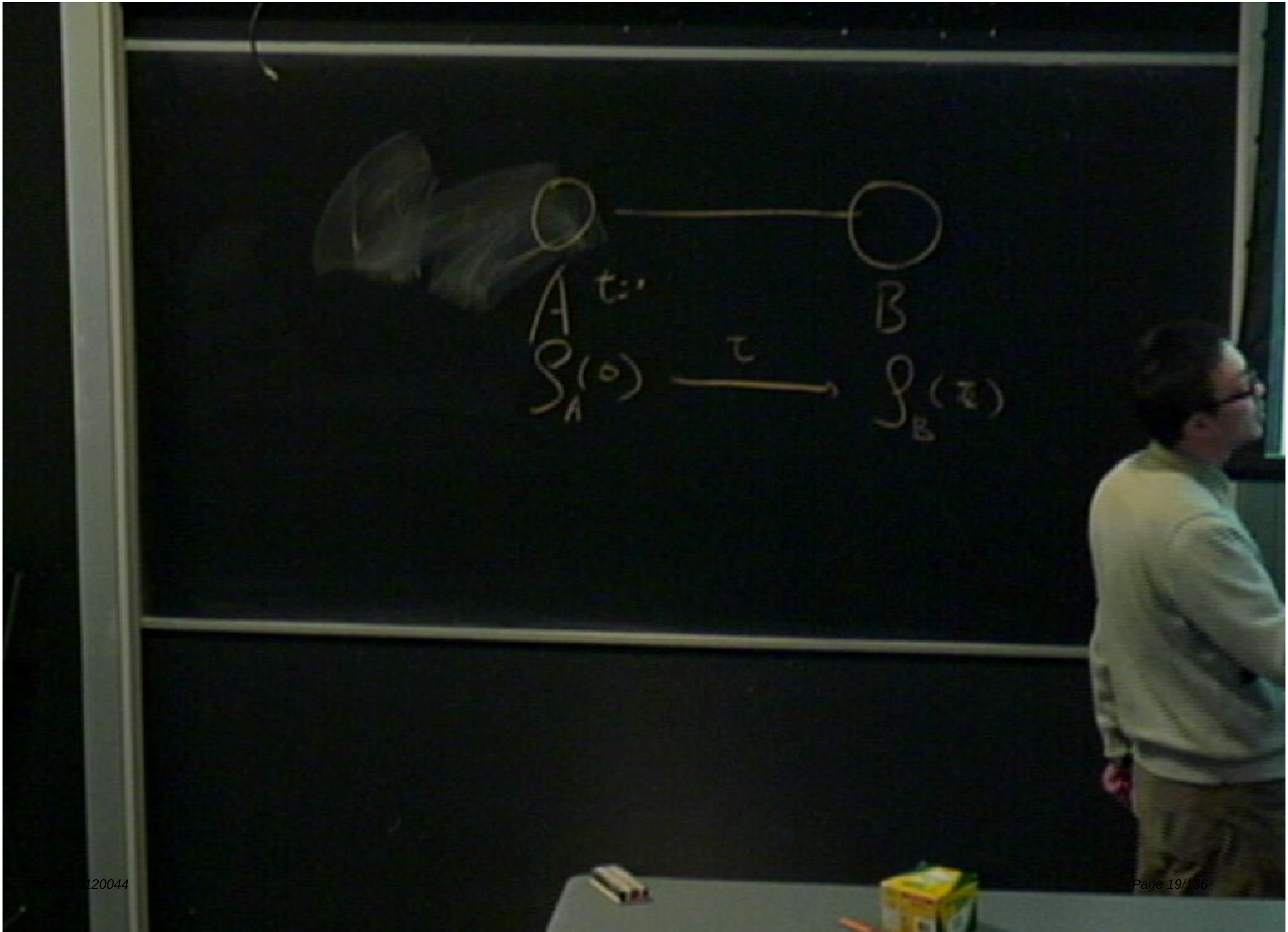



A
B
τ
ρA(0)
ρB(τ)


A
t=0
B
τ
ρ_A(0)
ρ_B(τ)

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$$G(\tau)\left|\Phi_k(0)\right\rangle_\tau = \exp(i\phi_k)\left|\Phi_k(0)\right\rangle_\tau$$

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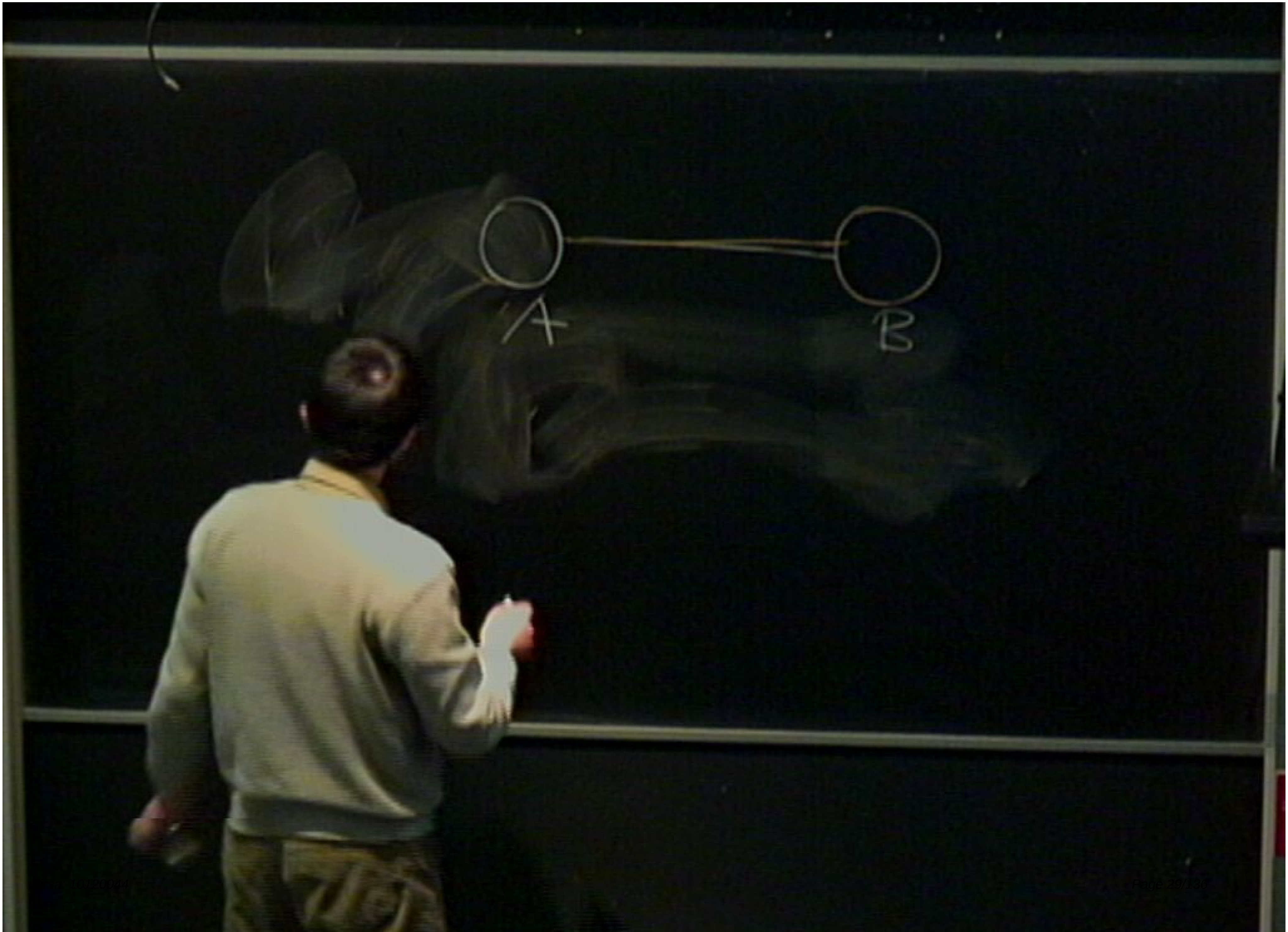
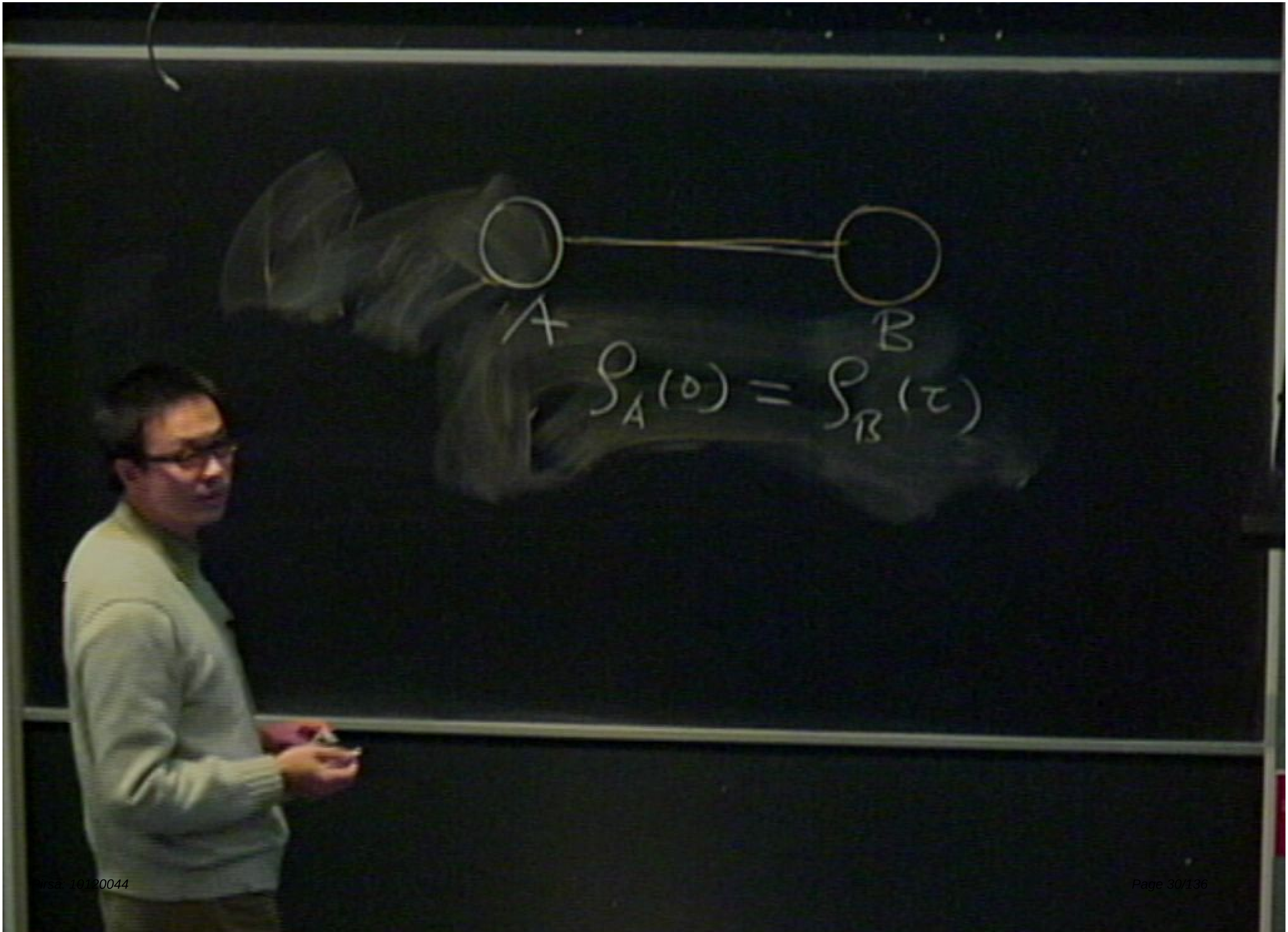
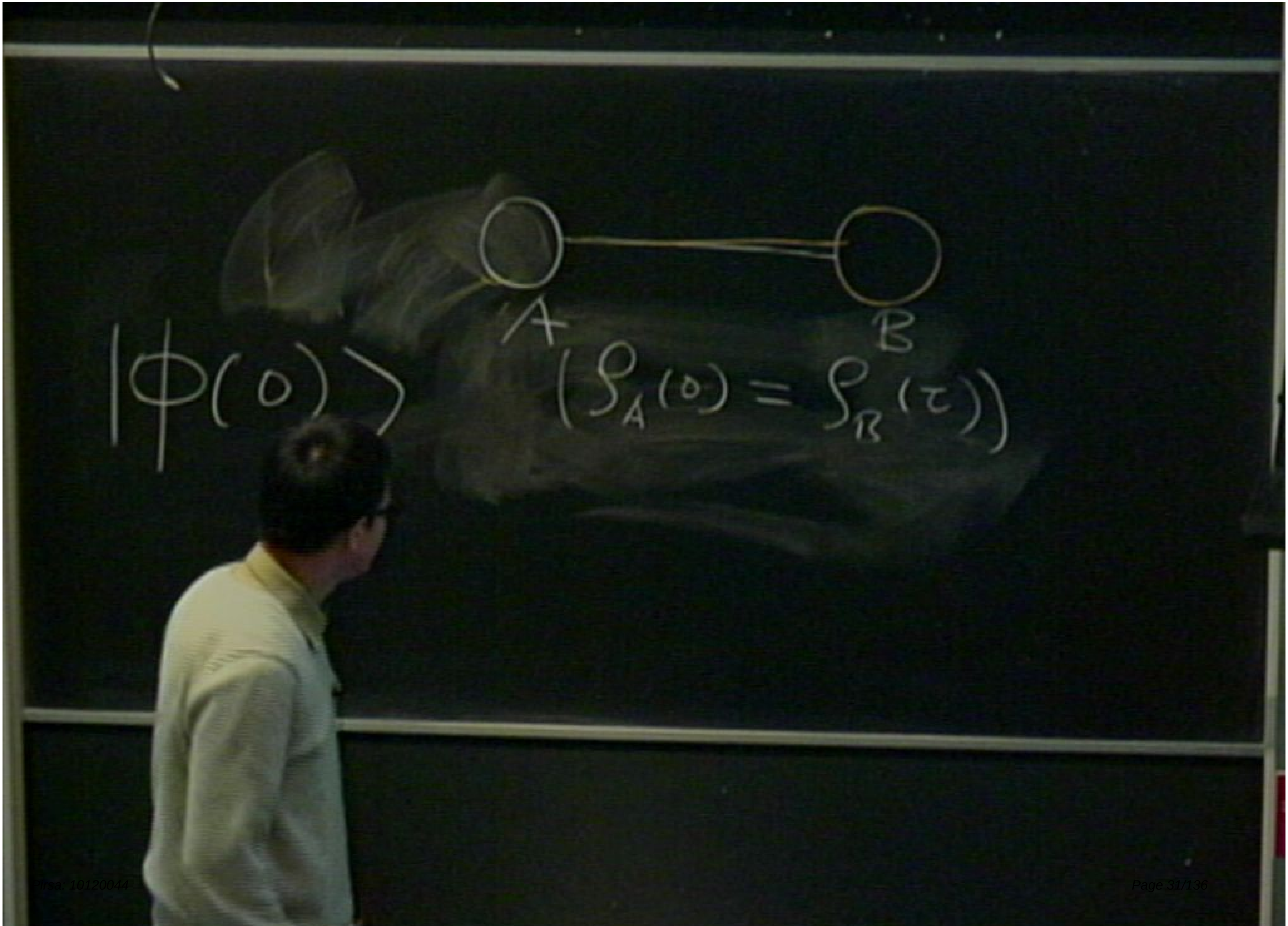
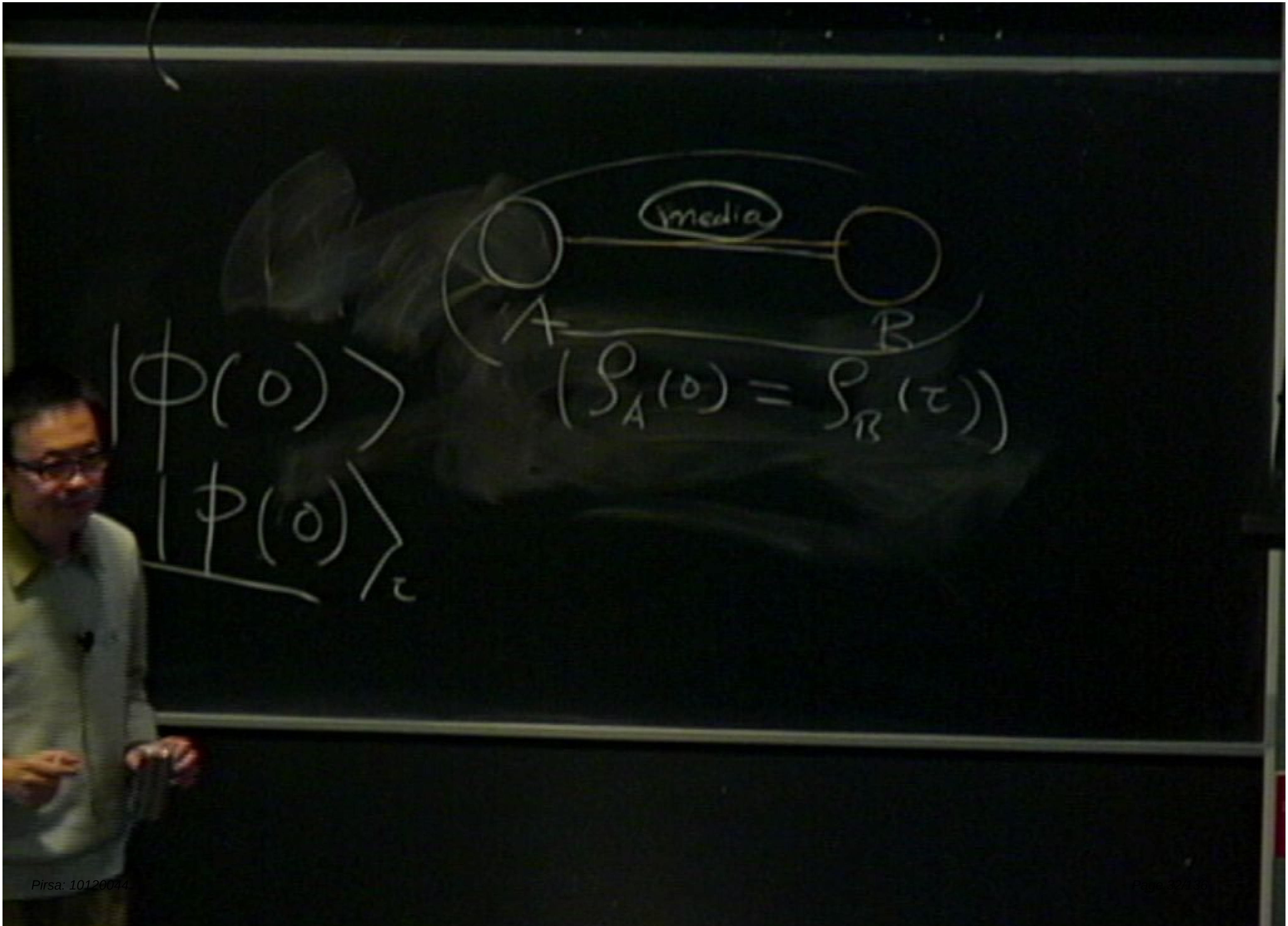
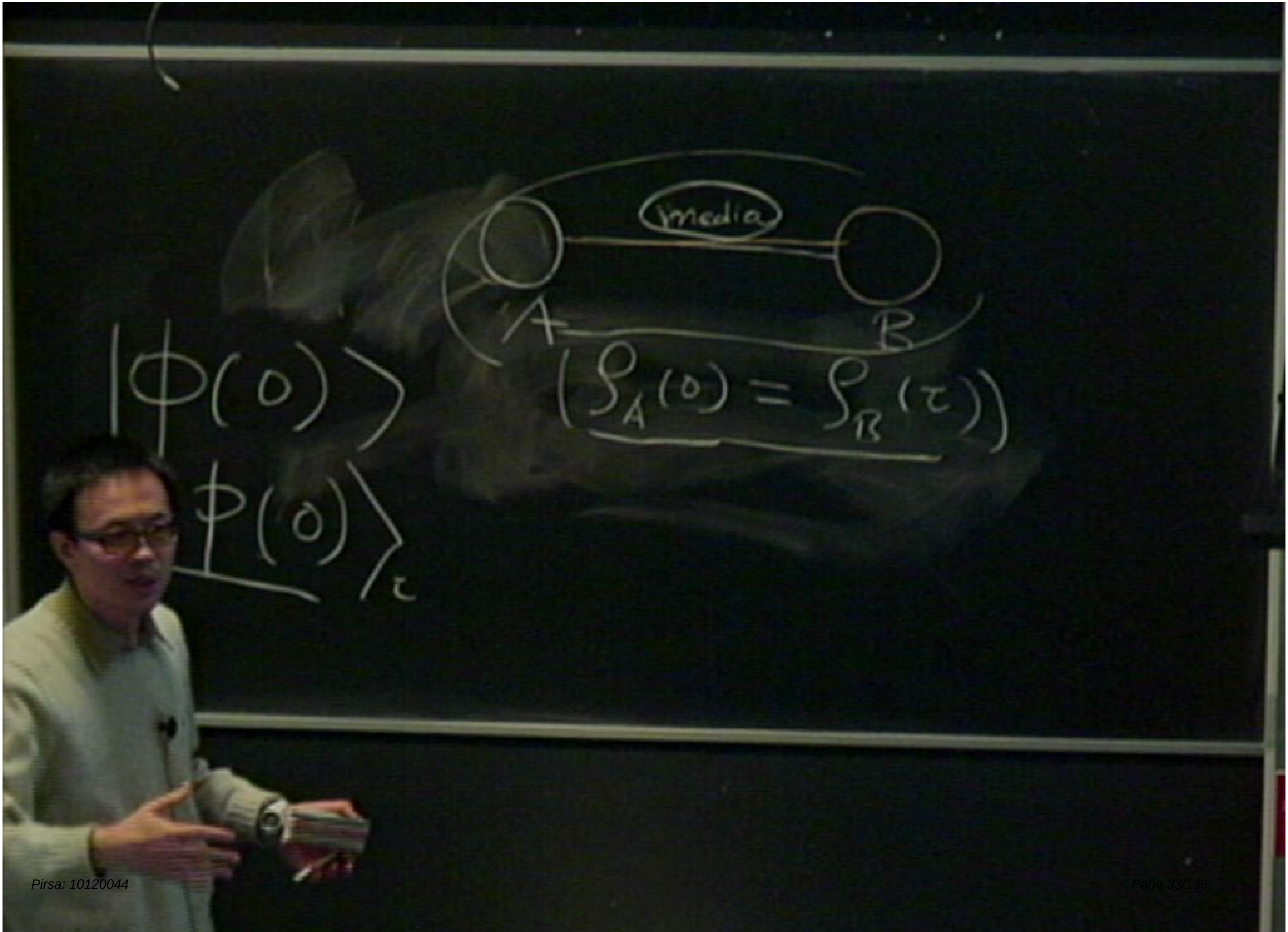

A
B

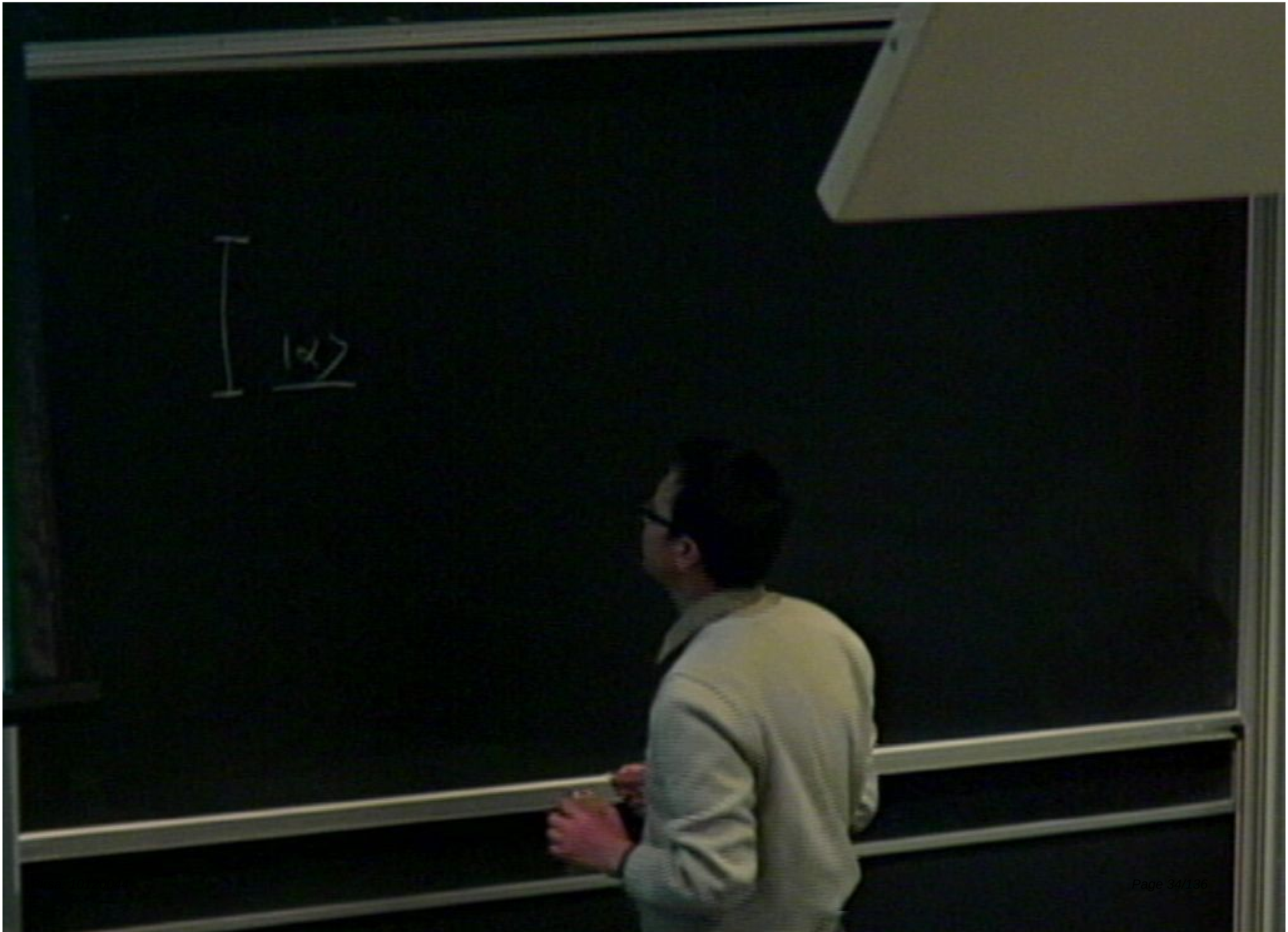

A
B
ρA(0) = ρB(τ)

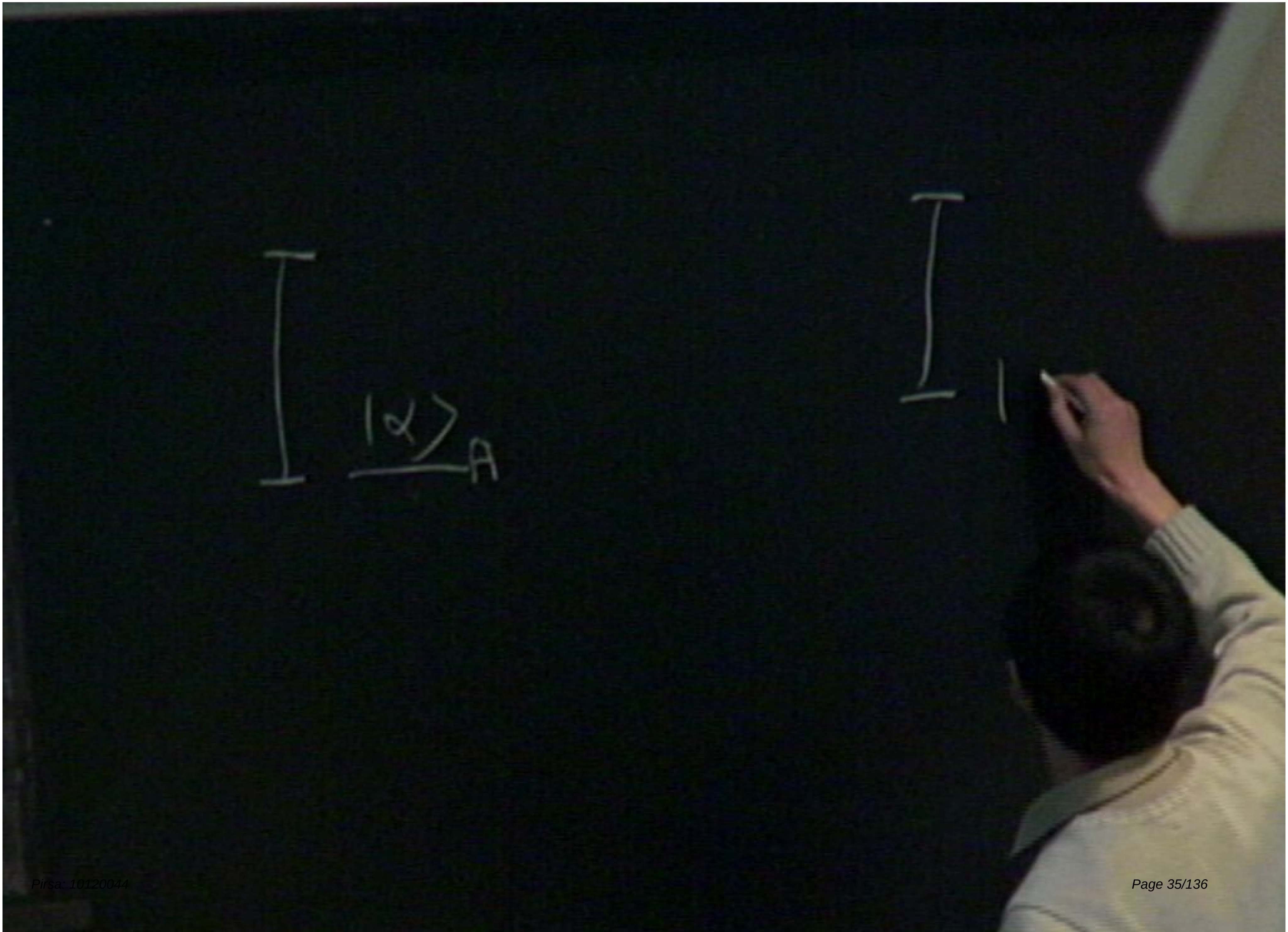

|φ(0)⟩
A
B
(ρ_A(0) = ρ_B(τ))

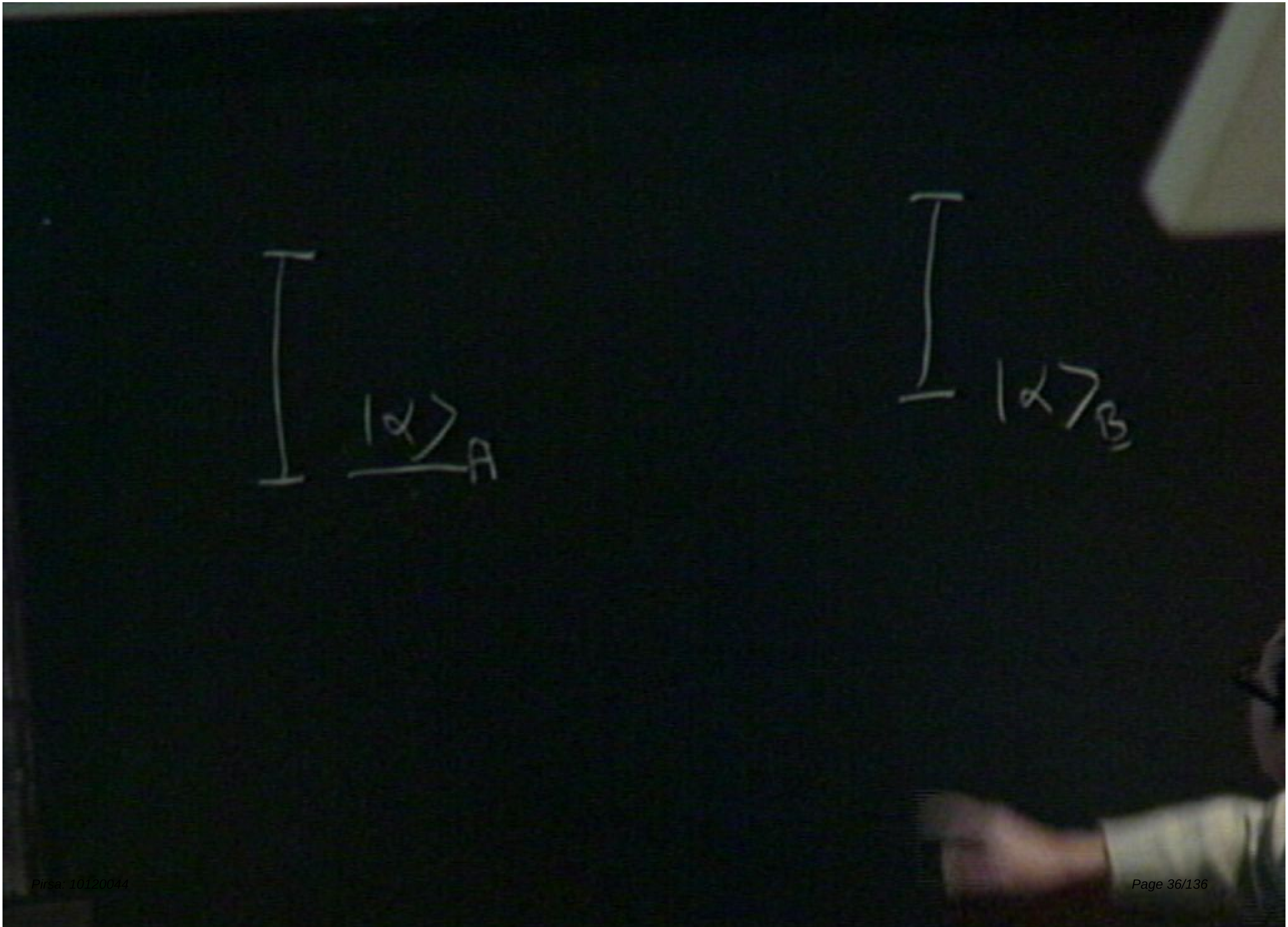

media
A
B
$|\phi(0)\rangle$
$|\phi(0)\rangle_\tau$
$(\rho_A(0) = \rho_B(\tau))$



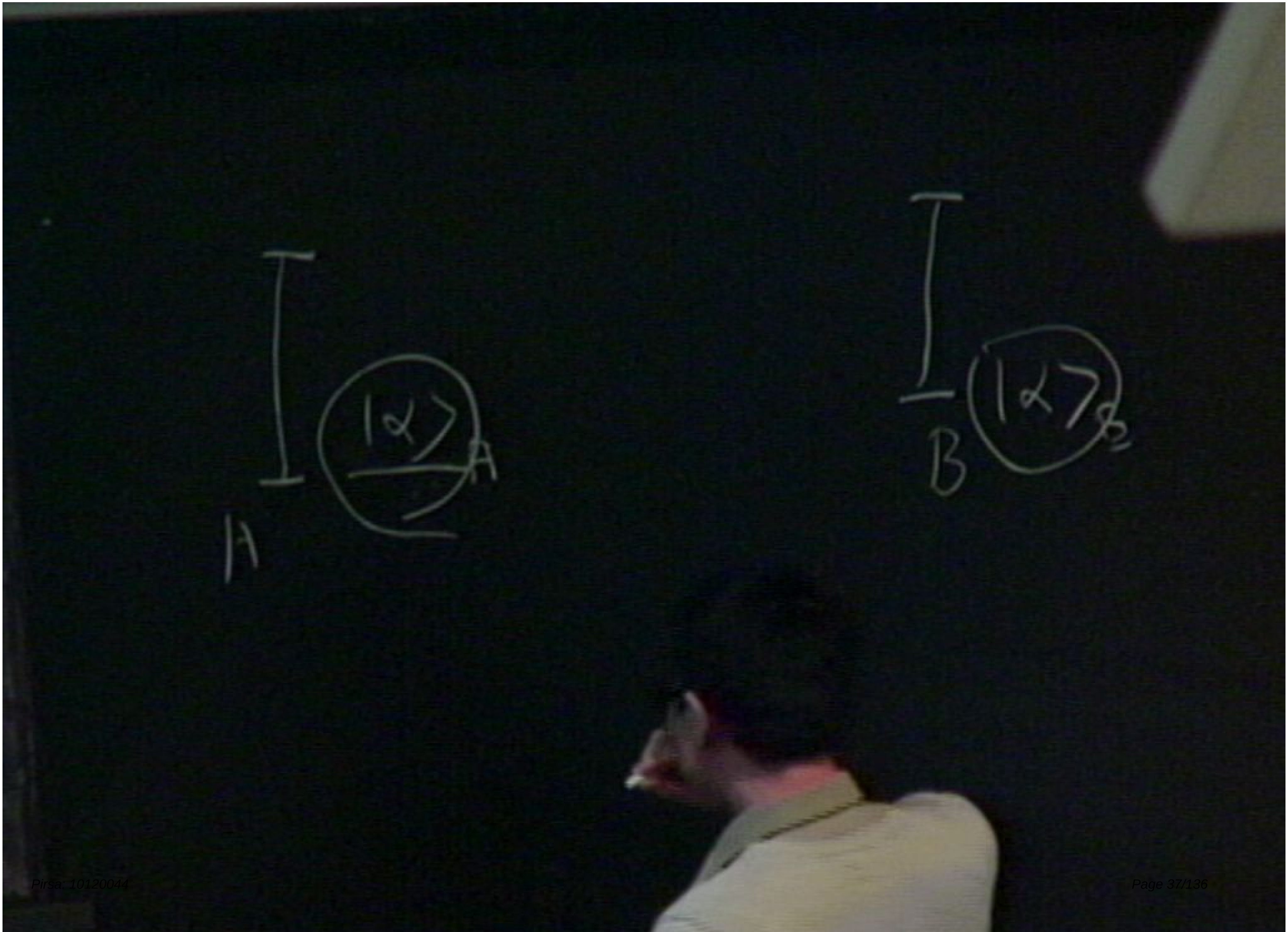
media
A
B
$|\Phi(0)\rangle$
$|\Phi(0)\rangle_\tau$
$(\rho_A(0) = \rho_B(\tau))$

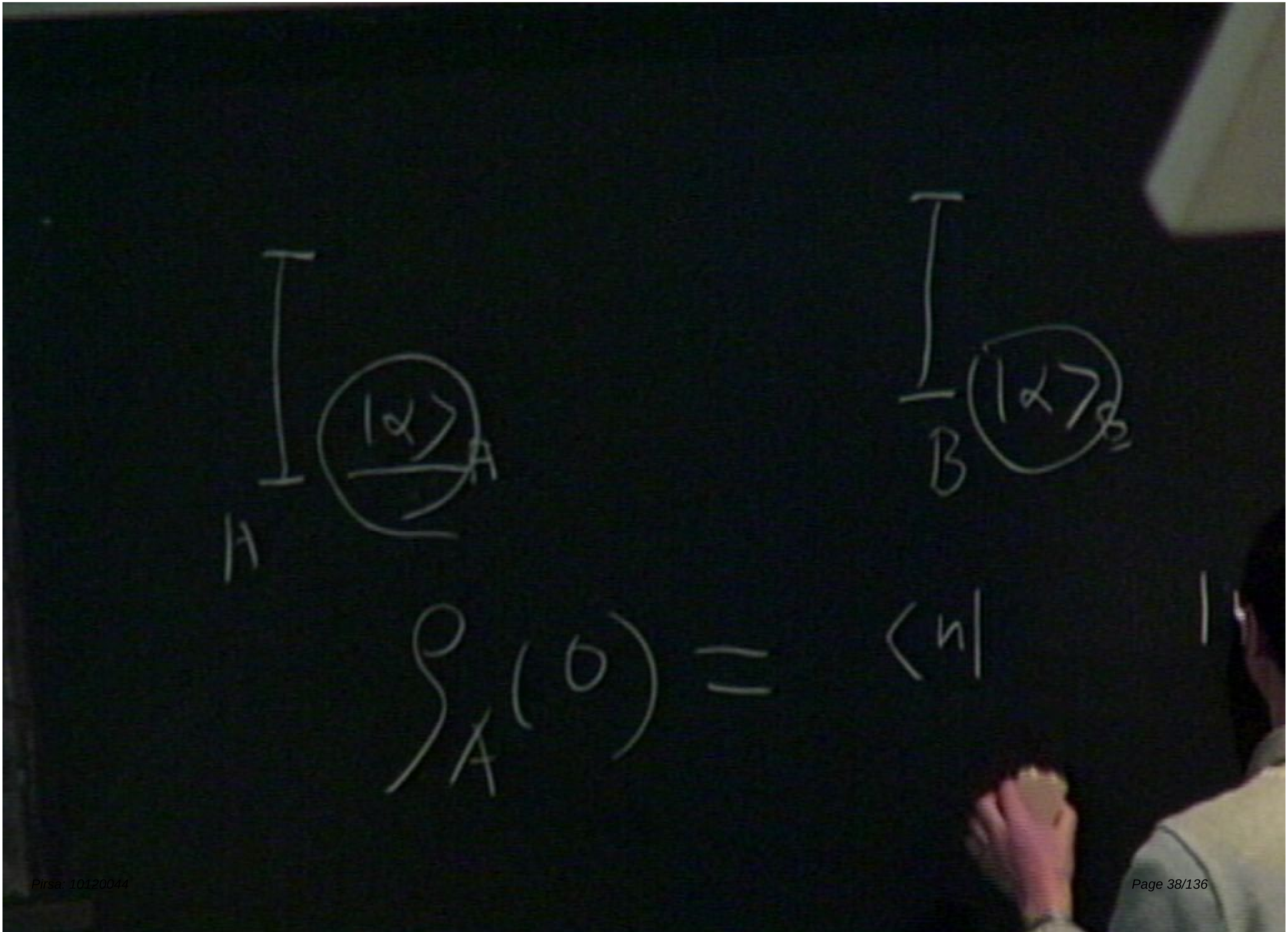



|α⟩ A

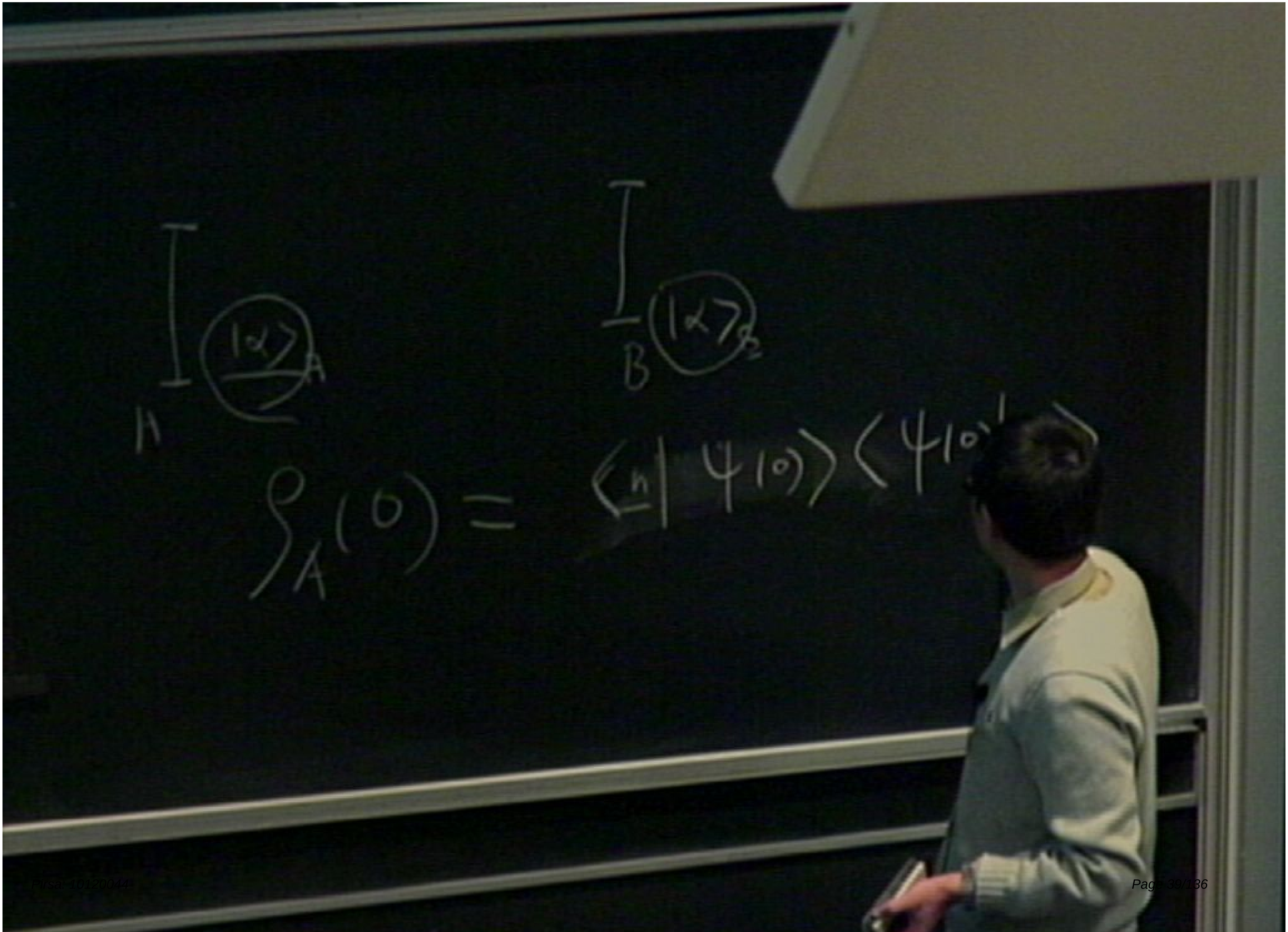


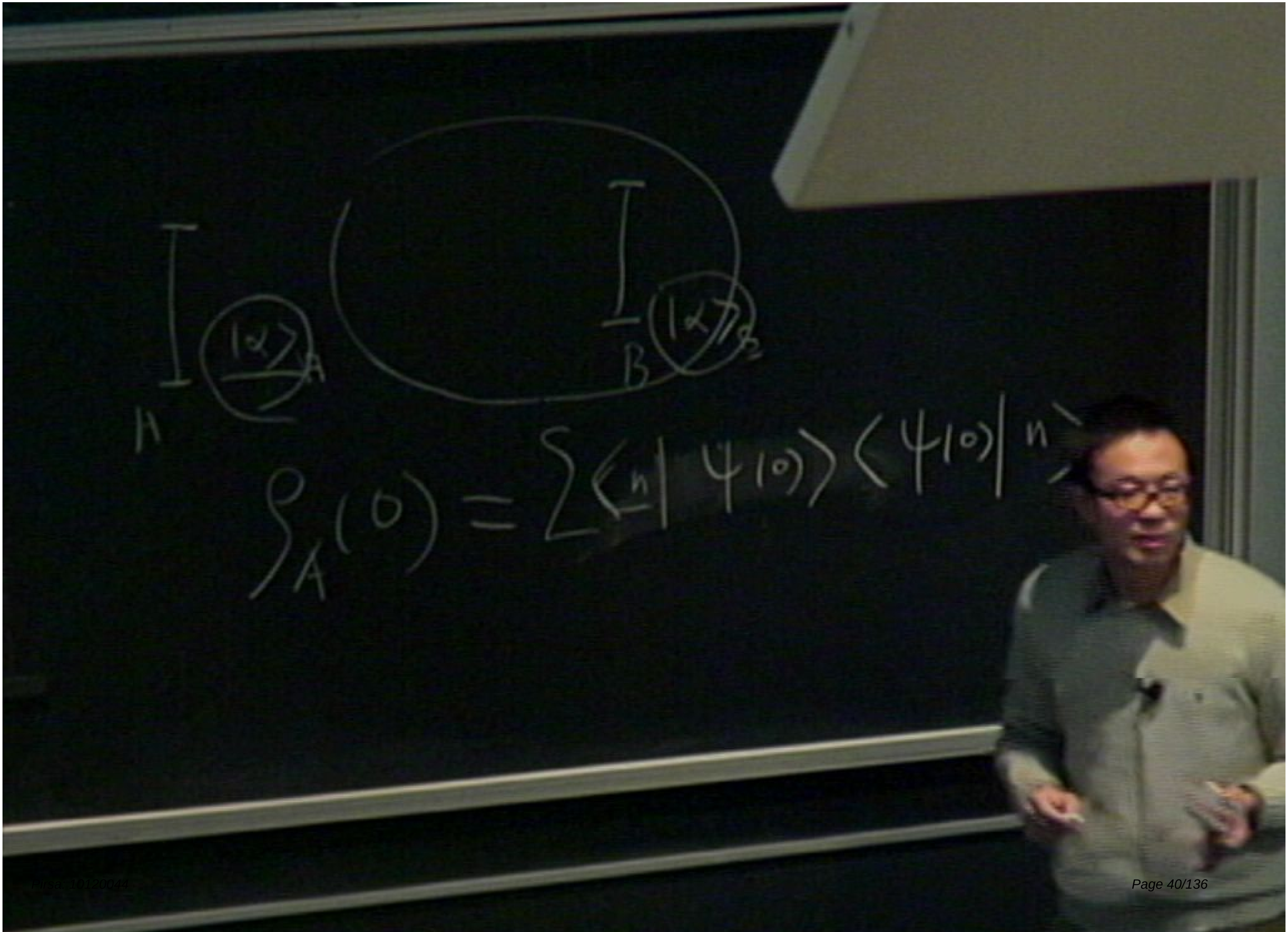
$I \; |\alpha\rangle_A$
$I \; |\alpha\rangle_B$



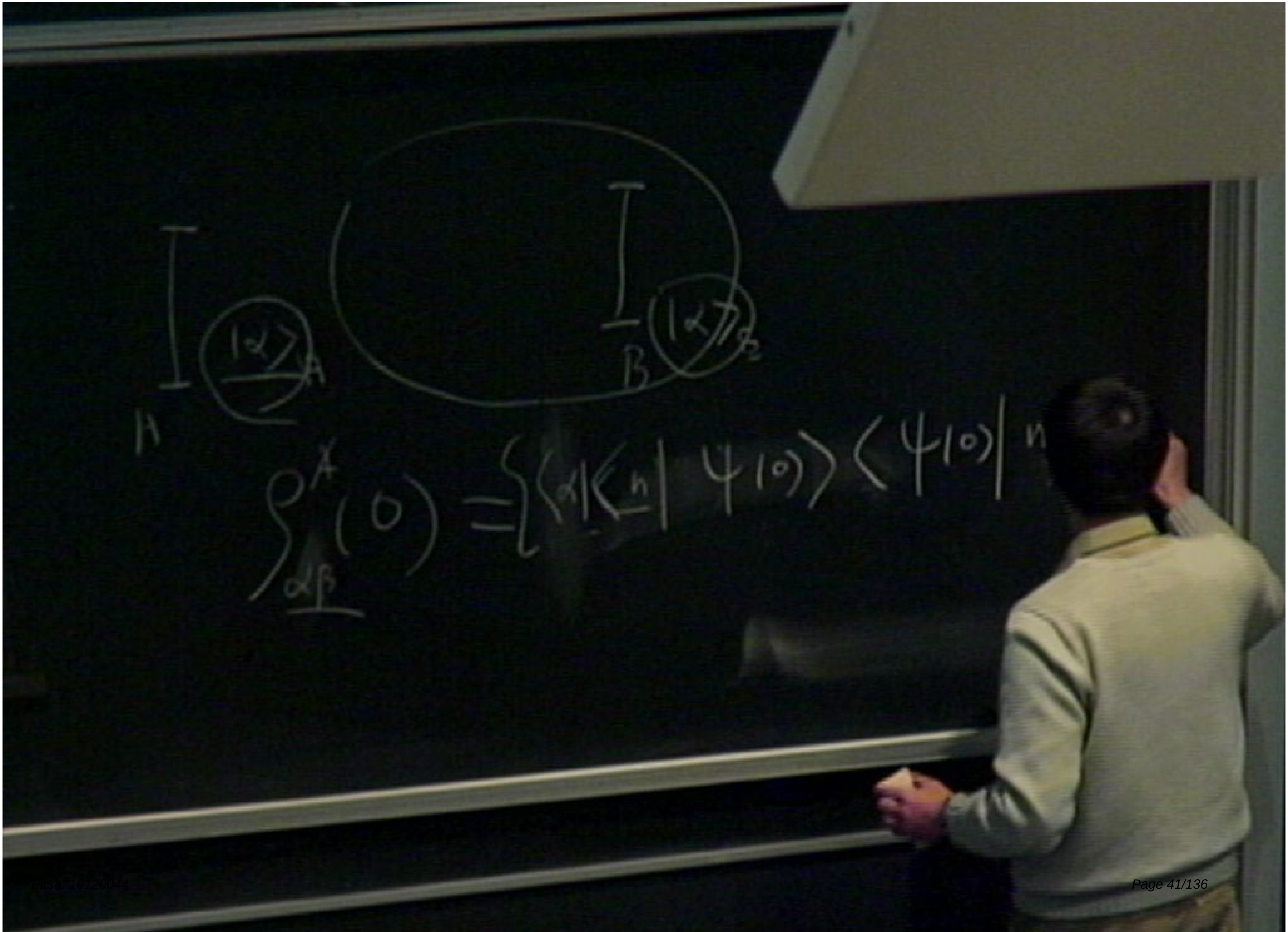


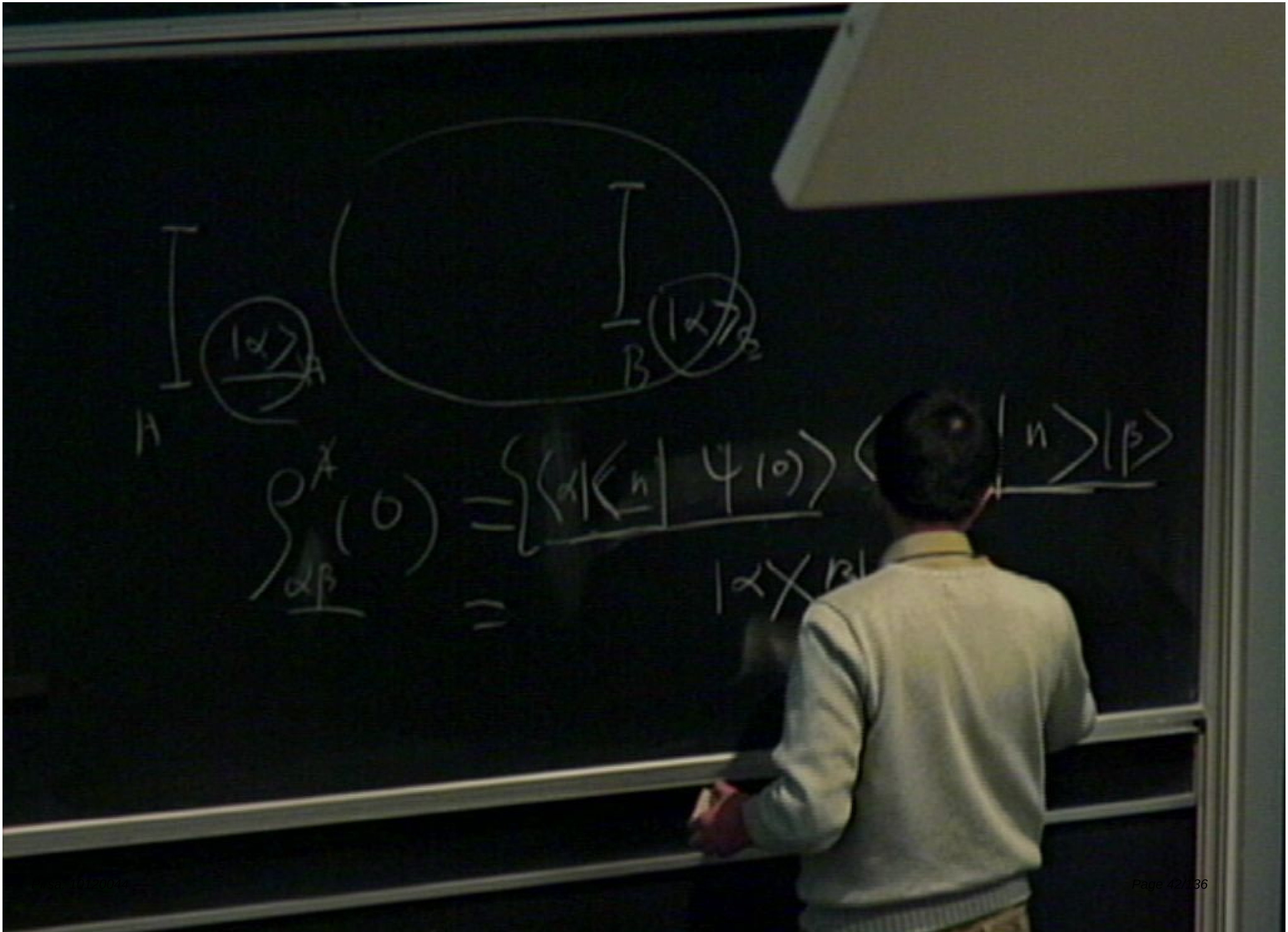
$I_A |\alpha\rangle_A$
$I_B |\alpha\rangle_B$
$\rho_A(0) = \langle n|$

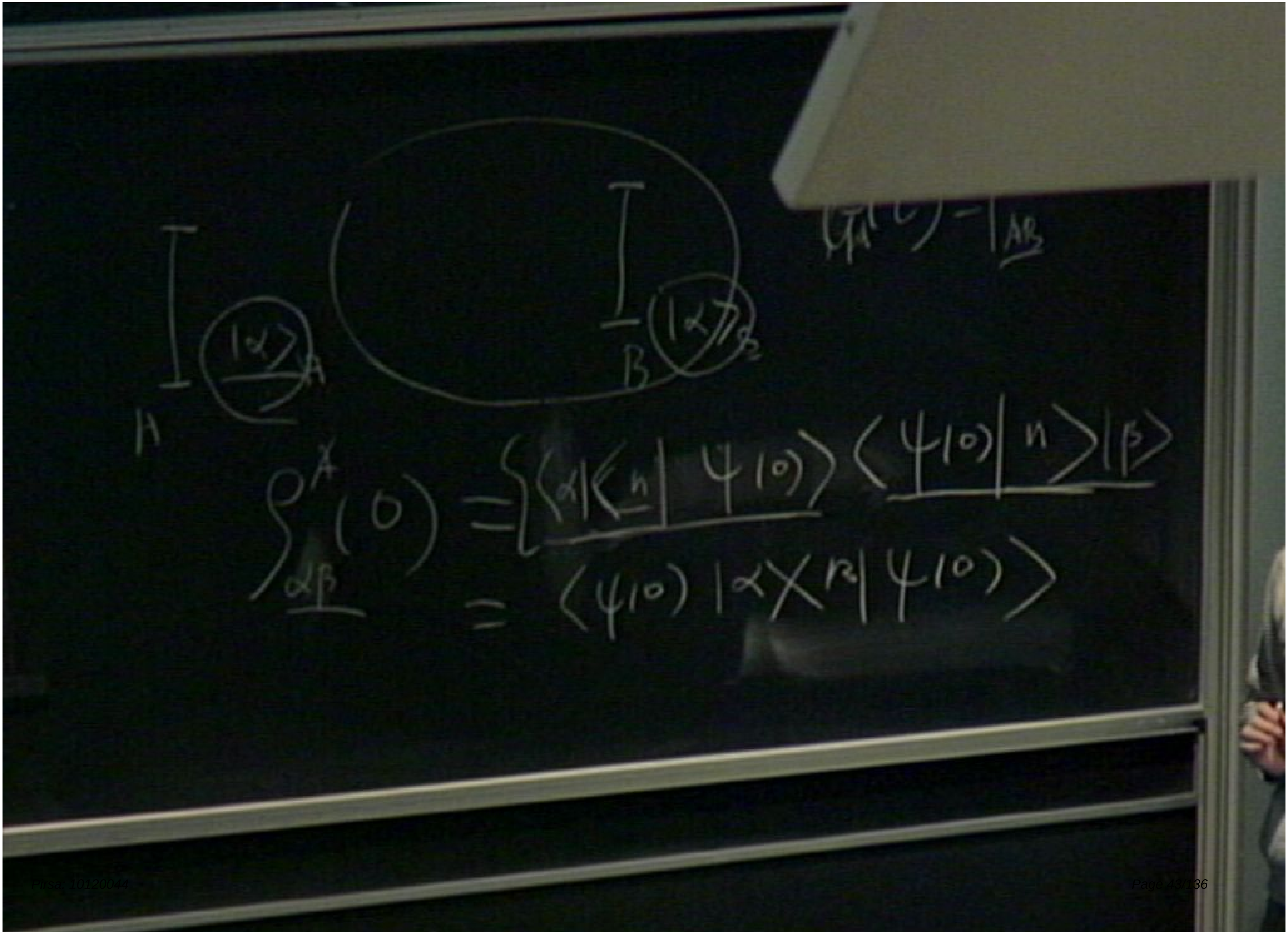

I |α⟩ A
I |α⟩ B
ρ_A(0) = ⟨n| ψ(0)⟩⟨ψ(0)

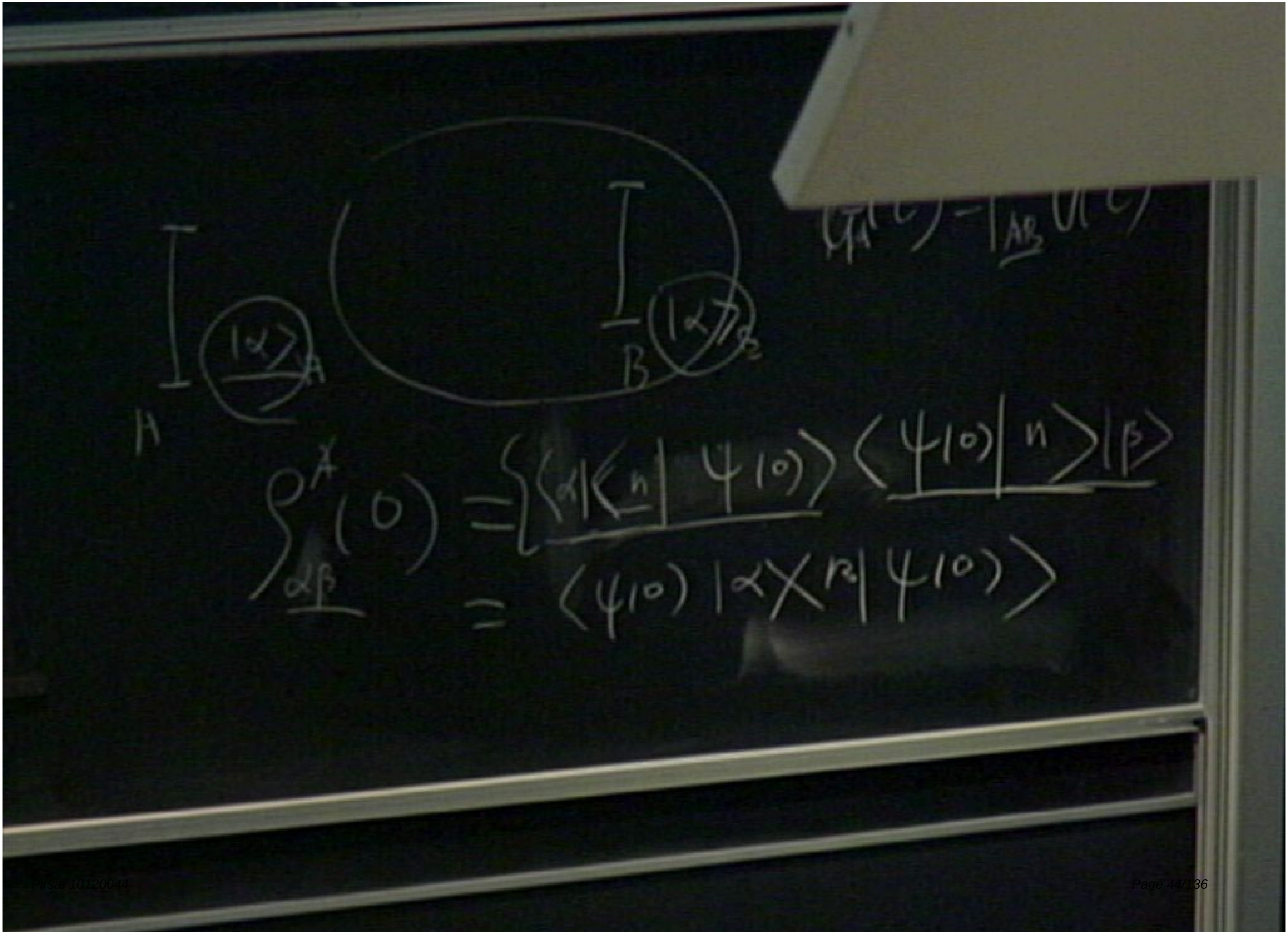


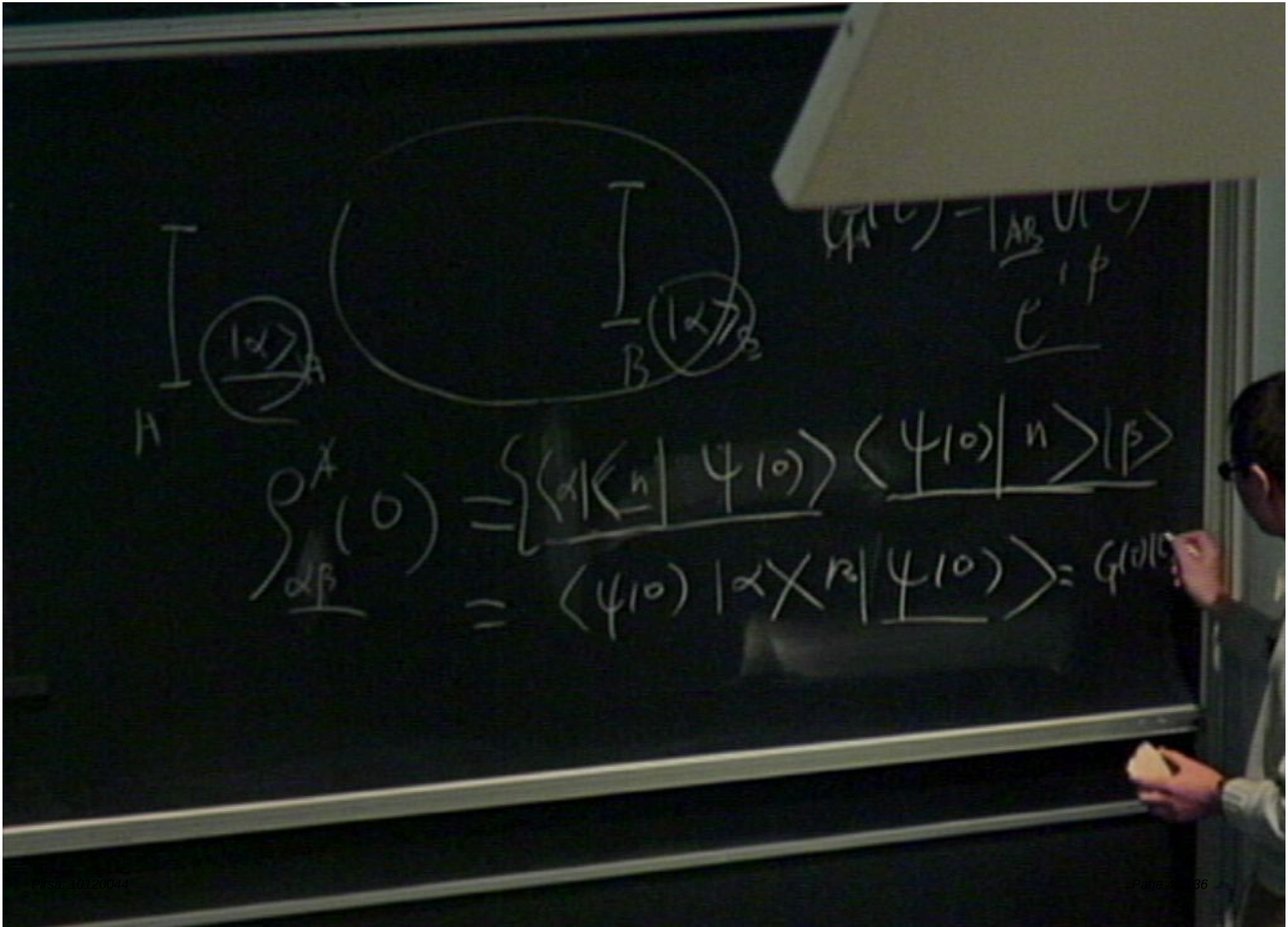
|α⟩_A
|α⟩_B
A
B
ρ_A(0) = Σ ⟨n| ψ(0)⟩⟨ψ(0)| n⟩


|α⟩_A
|α⟩_B
A
B
ρ^A_{αβ}(0) = Σ⟨α|⟨n| ψ(0)⟩⟨ψ(0)| n

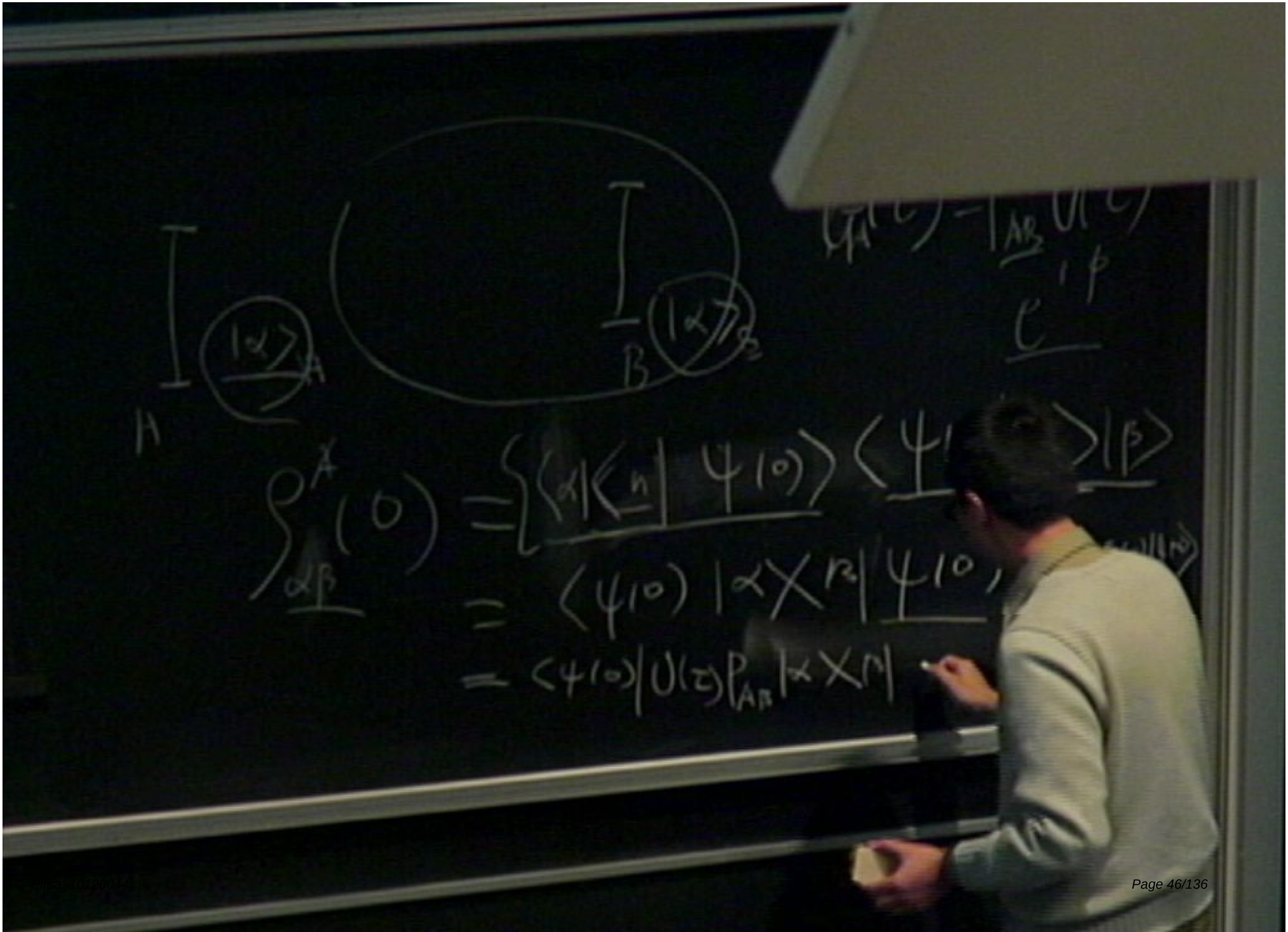


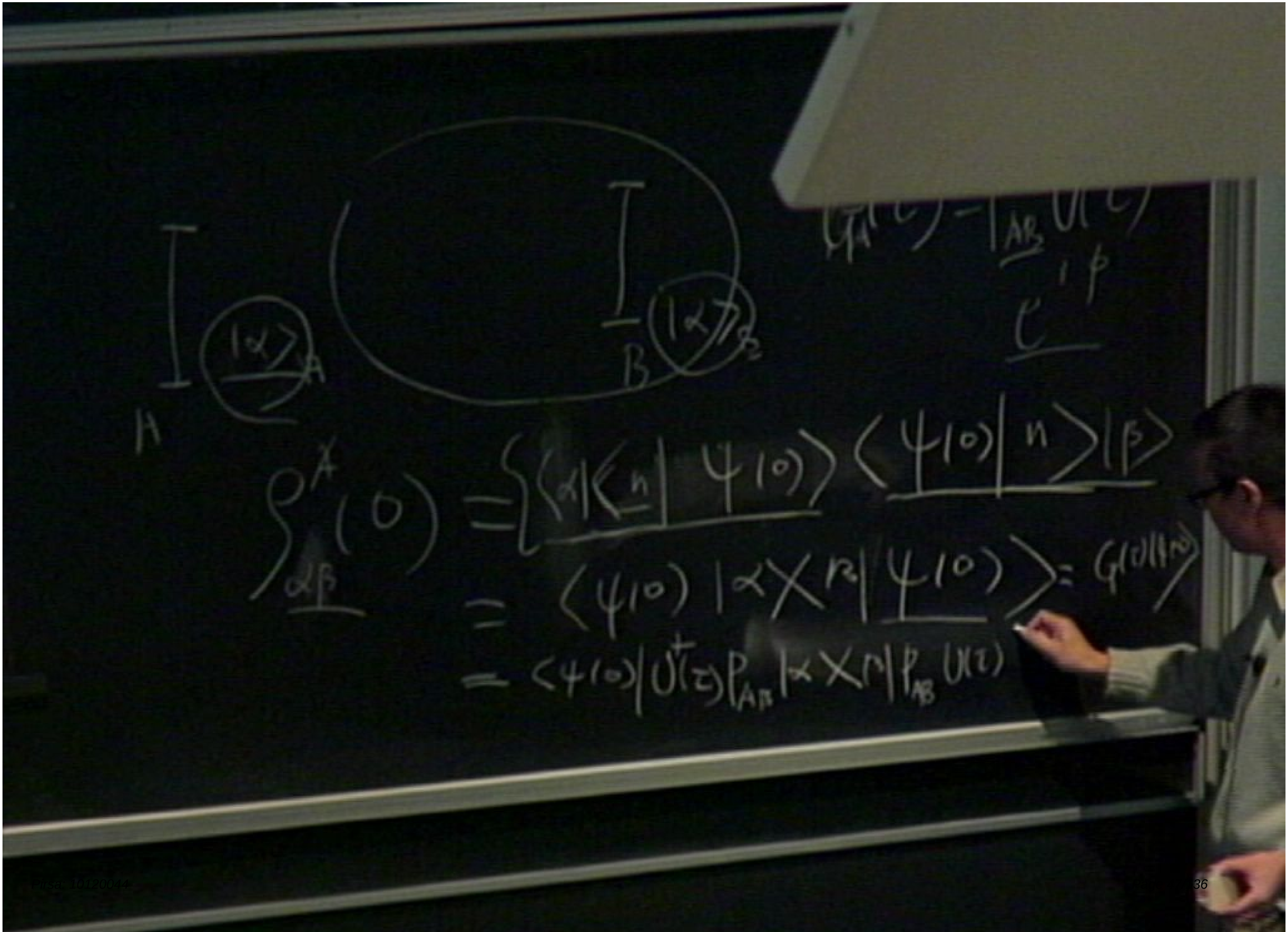

I
|α⟩_A
A
I
|α⟩_B
B
ρ^A_αβ(0) = Σ ⟨α|⟨n| ψ(0)⟩ ⟨ψ(0)| n⟩|β⟩
= ⟨ψ(0)|α⟩⟨β|ψ(0)⟩

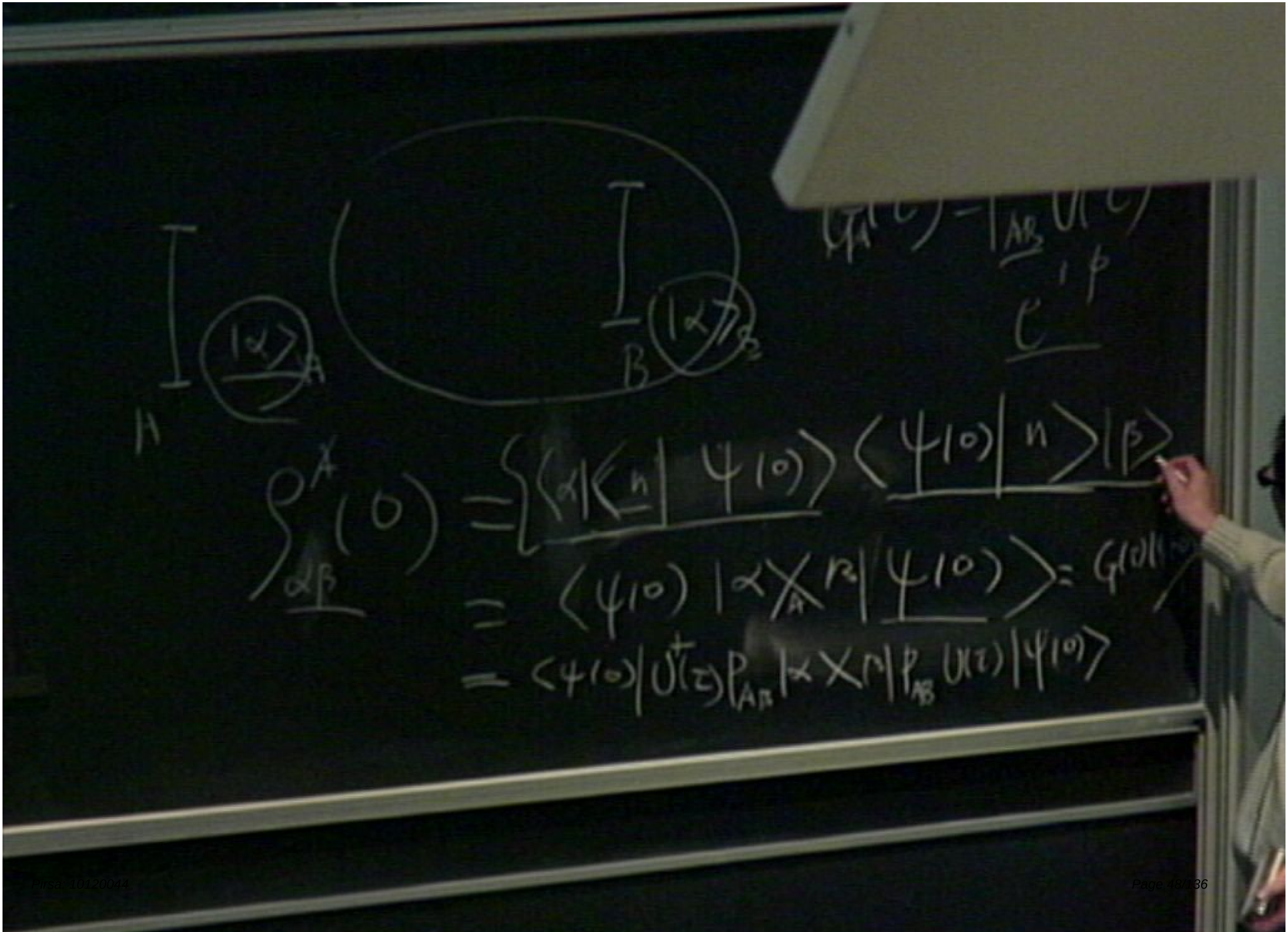


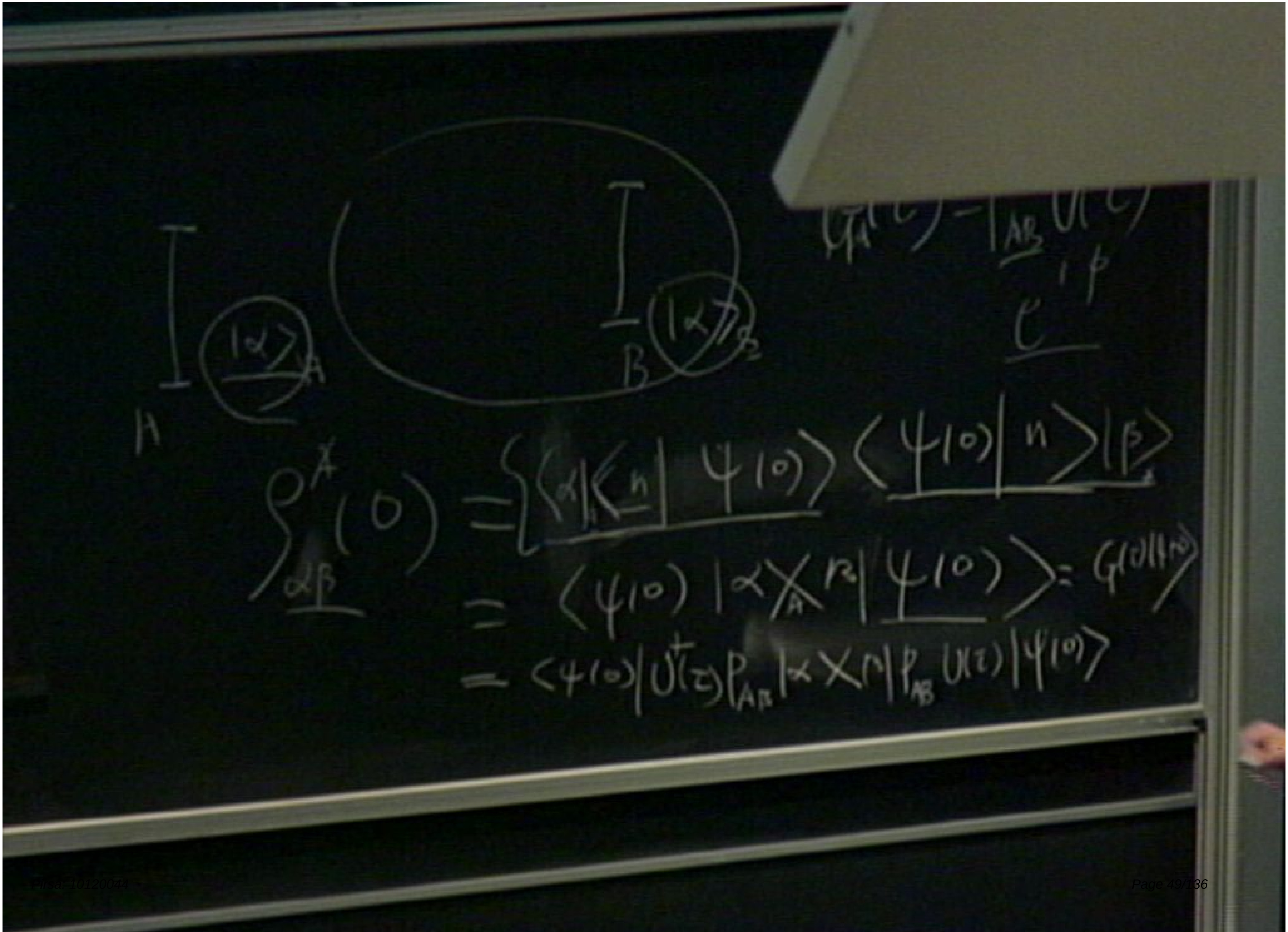


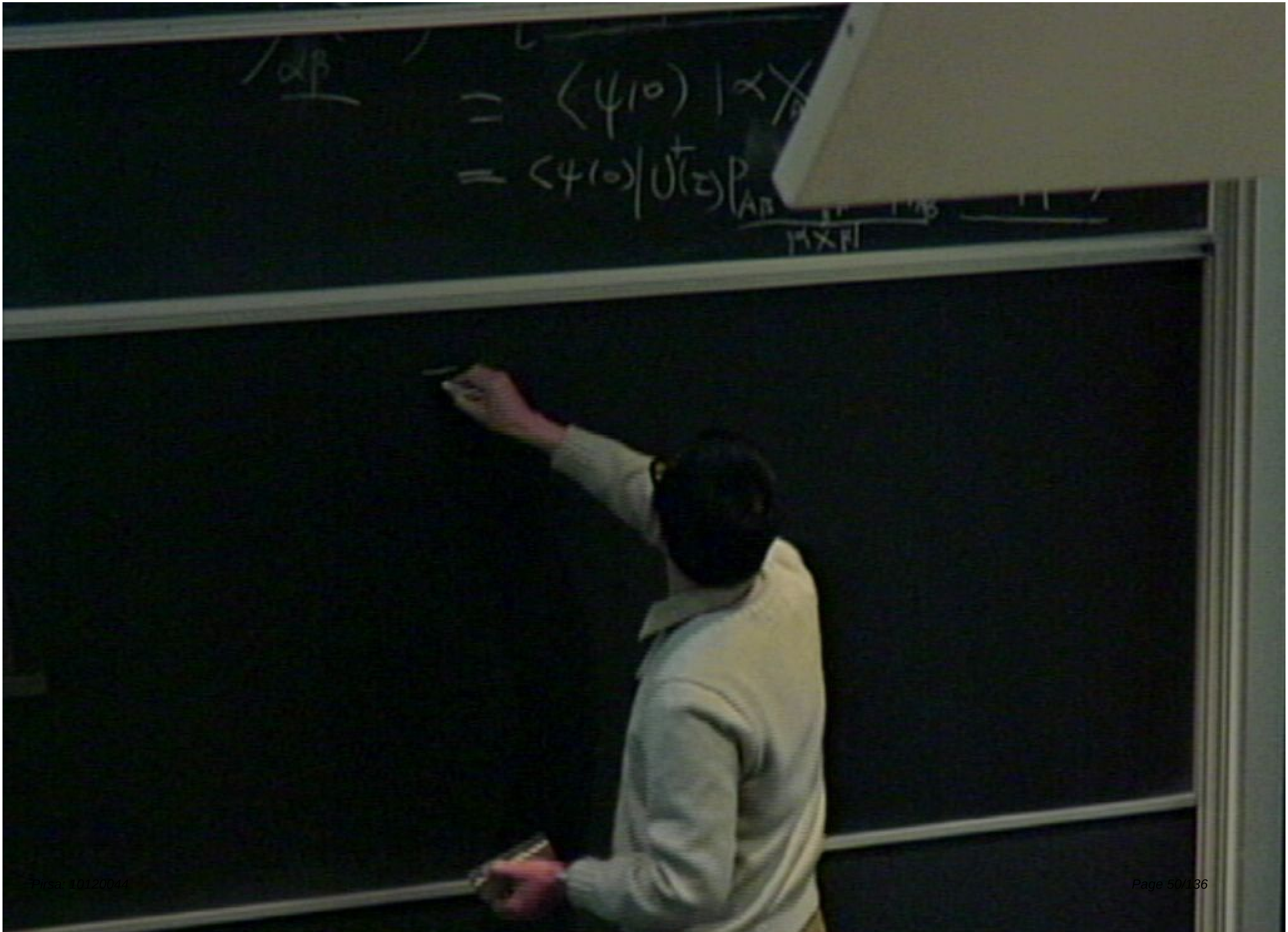
|α⟩_A
|α⟩_B
ρ^A_{αβ}(0) = ⟨α|⟨n|ψ(0)⟩⟨ψ(0)|n⟩|β⟩
= ⟨ψ(0)|α⟩⟨β|ψ(0)⟩ =

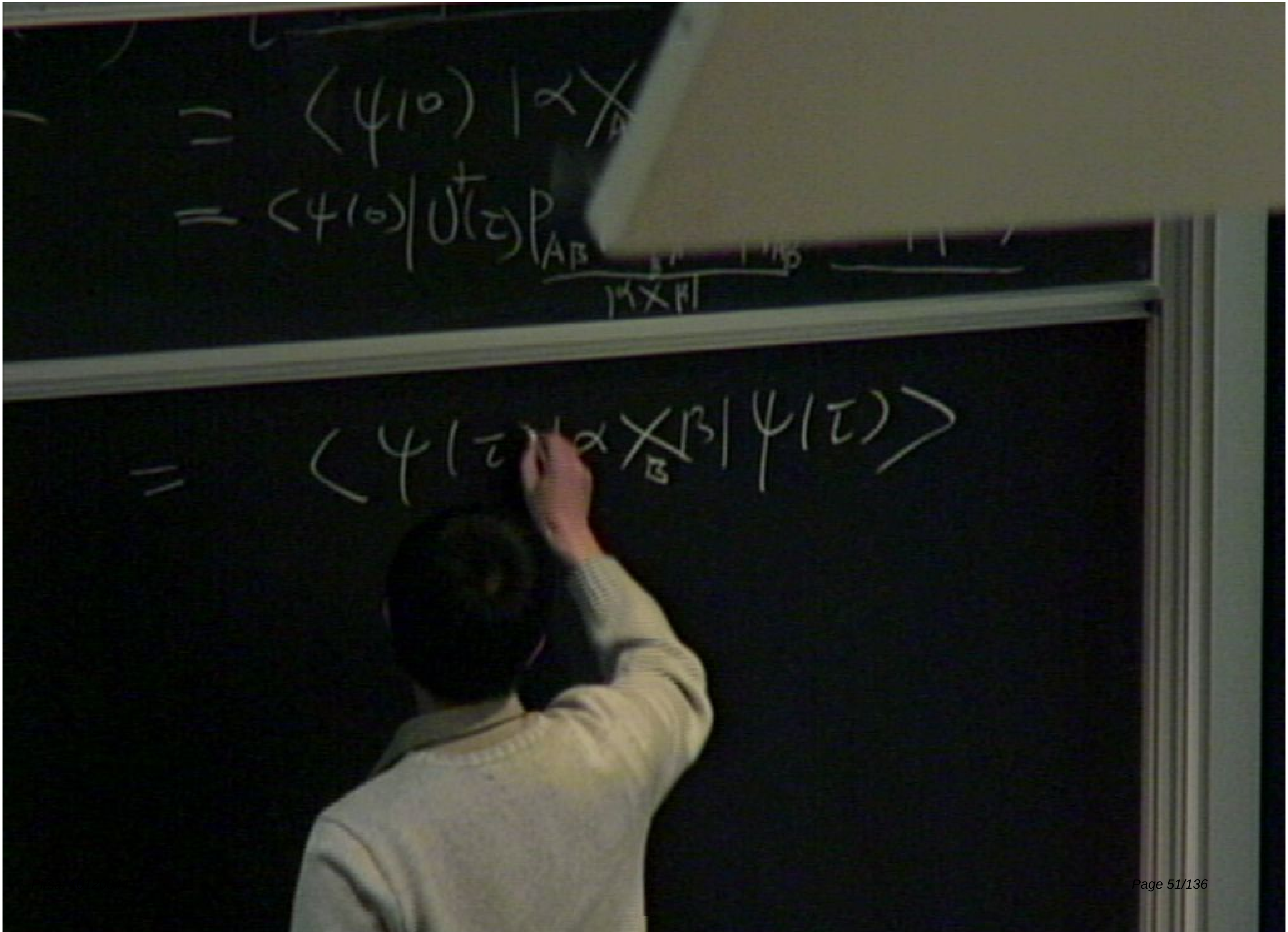


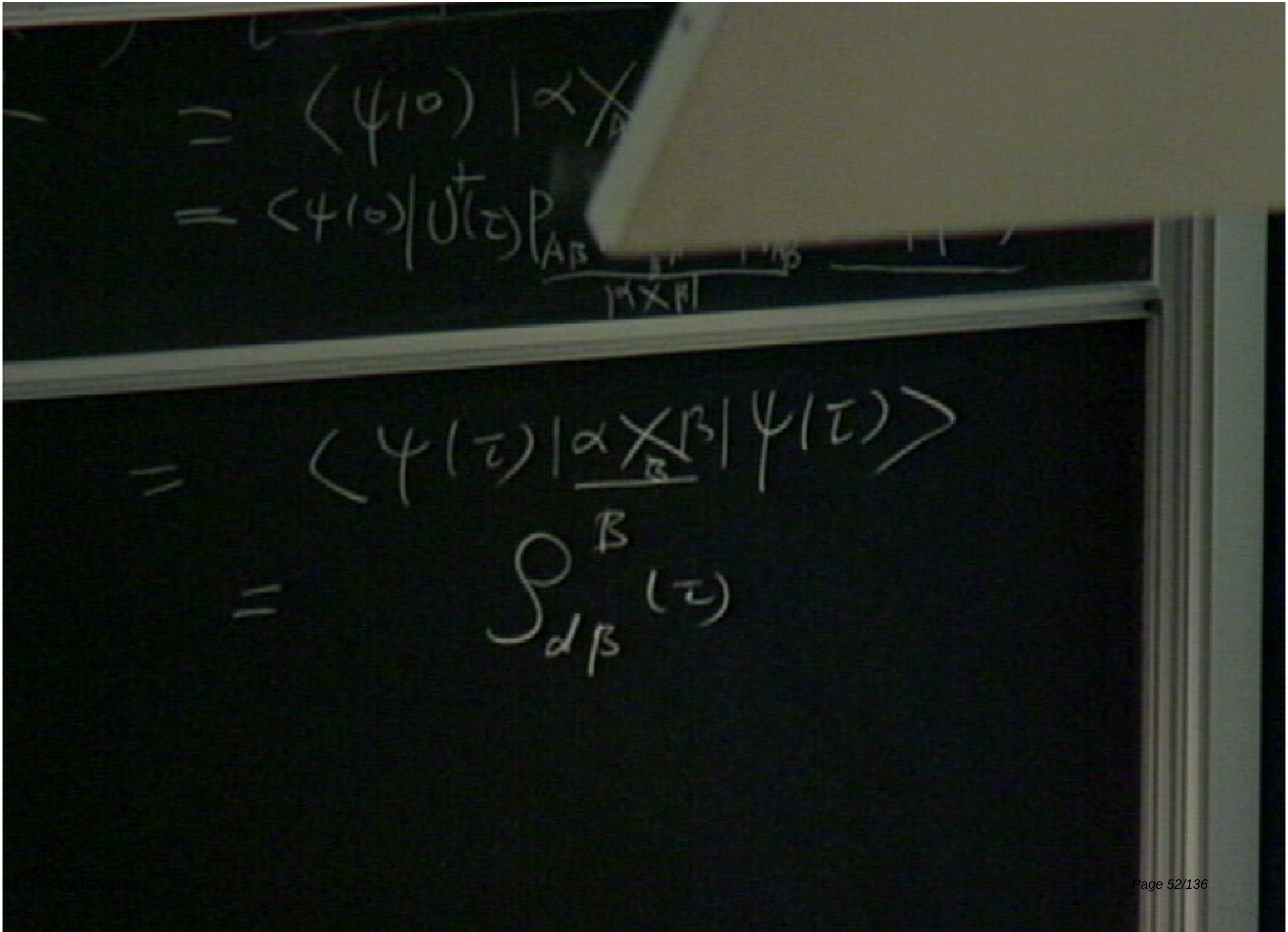

|α⟩_A
|α⟩_B
A
B
= ⟨ψ(0)|α⟩⟨n|ψ(0)⟩
= ⟨ψ(0)|U†(τ)P_AB|α⟩⟨n|P_AB U(τ)




|α⟩_A
|α⟩_B
A
B
$\rho^{A}_{\alpha\beta}(0) = \sum_n \langle\alpha|\langle n|\psi(0)\rangle\langle\psi(0)|n\rangle|\beta\rangle$
$= \langle\psi(0)|\alpha\rangle\langle\beta|\psi(0)\rangle$
$= \langle\psi(0)|U^\dagger(\tau)P_{AB}|\alpha\rangle\langle\beta|P_{AB}U(\tau)|\psi(0)\rangle$




= ⟨ψ(0)|U†(τ)P_AB
|α⟩⟨β|
= ⟨ψ(τ)|α⟩⟨β|ψ(τ)⟩


$= \langle \psi(0) | \alpha \rangle \langle \beta$
$= \langle \psi(0) | U^{\dagger}(\tau) P_{A\beta}$
$|\alpha\rangle\langle\beta|$
$= \langle \psi(\tau) | \alpha \rangle \langle \beta | \psi(\tau) \rangle$
$= \rho^{B}_{\alpha\beta}(\tau)$

The eigenequation is

$$G(\tau)\left|\Phi_k(0)\right\rangle_\tau = \exp(i\phi_k)\left|\Phi_k(0)\right\rangle_\tau$$

Exact state transmissions exist universally, but not all of them are significant for state transfer.

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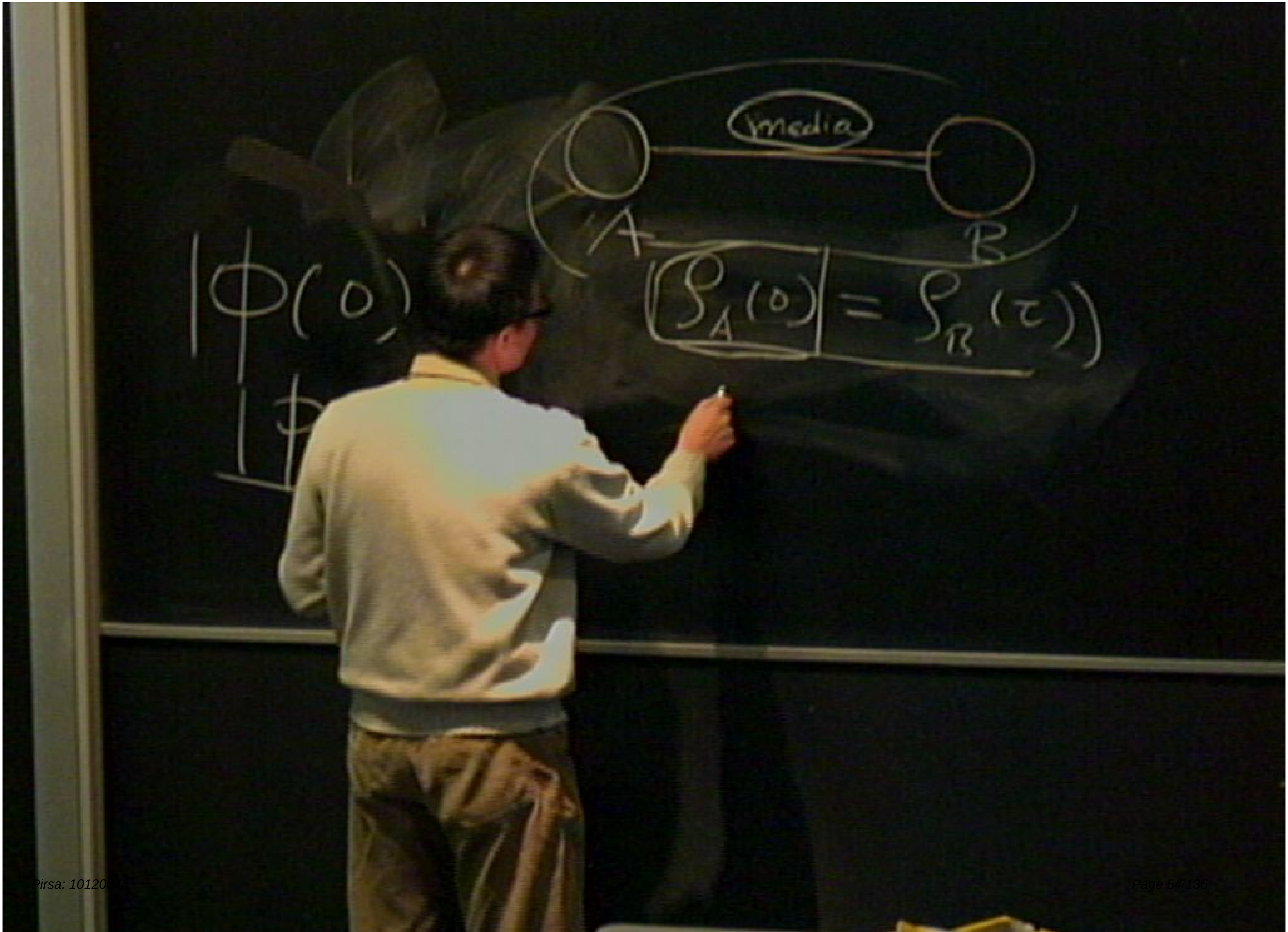
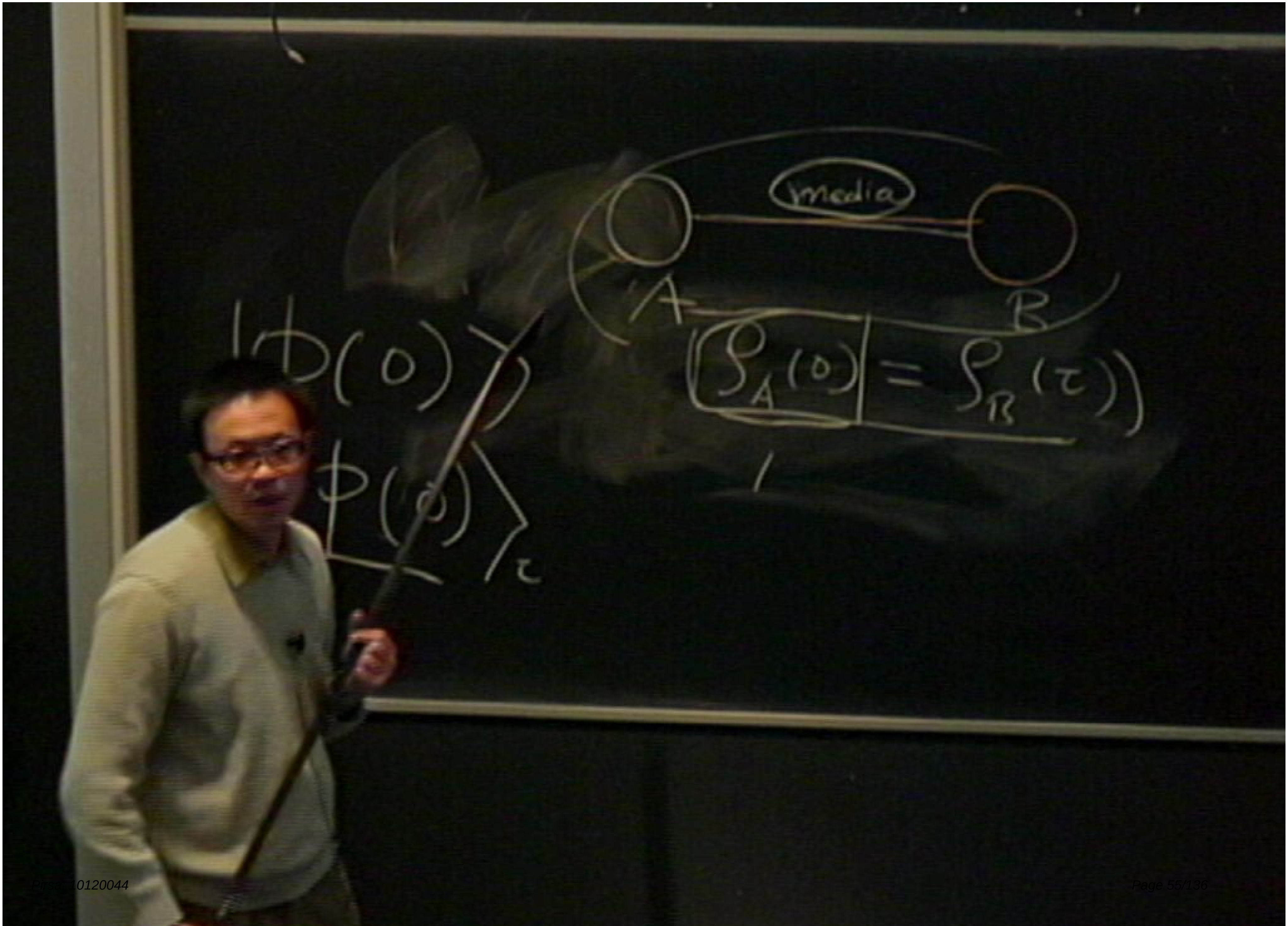

media
A
B
ρA(0) = ρB(τ)
|φ(0)


media
A
B
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Assume that the entire system, processors A, B and the media (connecting A & B), is initially in a state $|\Phi(0)\rangle$

There exists a complete orthogonal set $\{|\Phi_k(0)\rangle\}_\tau$ depending on τ, such that an exact state transmission occurs if the initial state is one of the states in this set.

$\Phi_k(0)\rangle_\tau$ are eigenstates of u-operators $G(\tau) = P_{AB}U(\tau)$ -product of A-B exchange operator and evolution operator.

Existent theorem of exact state transmissions

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$$|\Phi_1(0)\rangle \text{ and } |\Phi_2(0)\rangle$$

Such that $|\Phi(0)\rangle = a|\Phi_1(0)\rangle + b|\Phi_2(0)\rangle$ is also an eigenstate of $G(\tau)$.

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Therefore, PST needs two conditions for the two Φ's

- Localized at processor A
- The two eigenvalues satisfy $\phi_1 = \phi_2 + 2n\pi$

Such that $G(\tau)\ \Phi(0) = \exp(i\phi_1)\ \Phi(0)$. becomes an eigenstate.

Perfect state transfer in 1-D (PRL92,187902(2004))

The naturally-available spin Hamiltonian cannot perform PST, for instance,

$$H = \sum_{i=1} J_i (X_i X_{i+1} + Y_i Y_{i+1})$$

Where X, Y are Pauli's matrices. J=constant-XY model. Using J-W transformation, it becomes

$$H = \sum_{i=1} J_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$$

c's are fermionic operators.

If we pre-engineer J such that $J_i = \sqrt{i(N-i)}$


media
A
B
|φ(0)⟩
(ρA(0) = ρB(τ))
|φ(0)⟩c

The Hamiltonian becomes $H = JL_x$, where

$$L_x = \sum_i \sqrt{i(N-i)}\,(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$$

Mapping: lm ←→ Ni in the way l=(N-1)/2, m=i-(N+2)/2, L_x is the fermi representation of angular momentum.

Example N=4 spin chain :

i 1 2 3 4

m -3/2 -1/2 1/2 3/2

Other components

$$L_x = i\sum_i \sqrt{i(N-i)}\,(c_i^\dagger c_{i+1} - c_{i+1}^\dagger c_i)$$

$$L_z = \sum c_i^\dagger c_i (i - \frac{N+1}{2})$$

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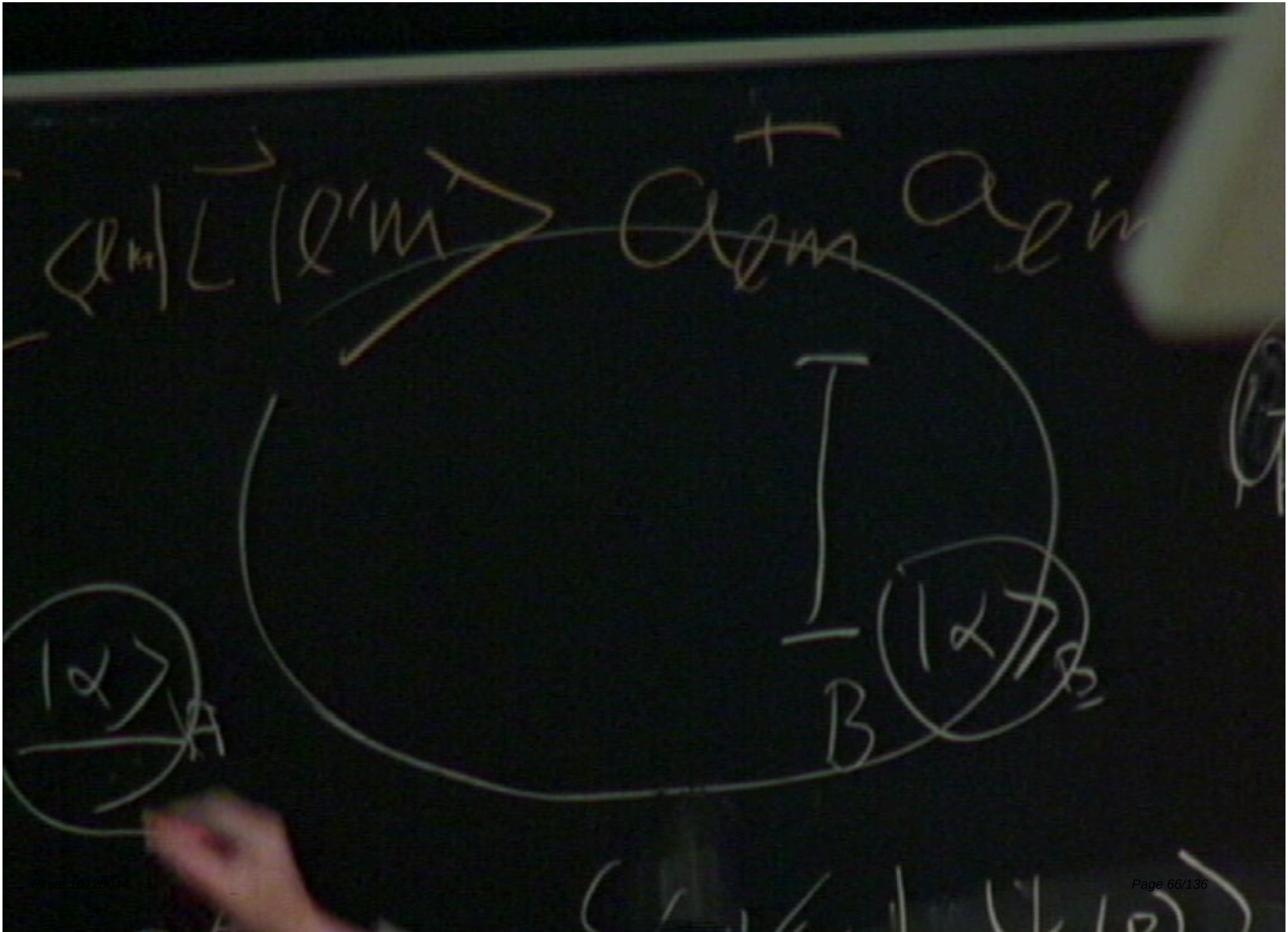
i	1	2	3	4

m	-3/2	-1/2	1/2	3/2

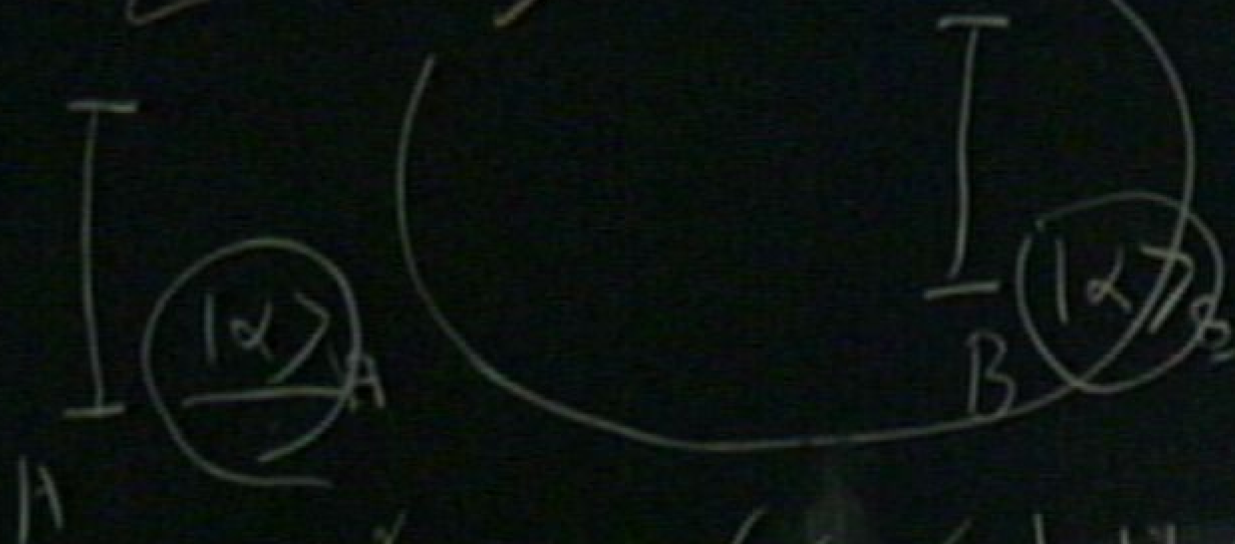
Other components

$$L_x = i\sum_i \sqrt{i(N-i)}(c_i^\dagger c_{i+1} - c_{i+1}^\dagger c_i)$$

$$L_z = \sum c_i^\dagger c_i (i - \frac{N+1}{2})$$



$$\tilde{L} = \sum \langle lm|\vec{L}|l'm'\rangle a^{\dagger}_{lm} a_{l'm'}$$



$$\rho_A(\tau) = P_{AB}$$

$$\rho^A_{\alpha\beta}(0) = \sum \langle\alpha|_A \langle n|\psi(0)\rangle \langle\psi(0)|n\rangle$$

$$= \langle\psi(0)|\alpha\rangle_A \langle\beta|\psi(0)\rangle$$

$$= \langle\psi(0)|U^{\dagger}(\tau) P_{AB} |\alpha\rangle_A \langle\beta| P_{AB} U(\tau)|\psi$$

The Hamiltonian becomes $H = JL_x$, where

$$L_x = \sum_i \sqrt{i(N-i)}(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$$

Mapping: lm←→Ni in the way l=(N-1)/2,m=i-(N+2)/2, L_x is the fermi representation of angular momentum.

Example N=4 spin chain :

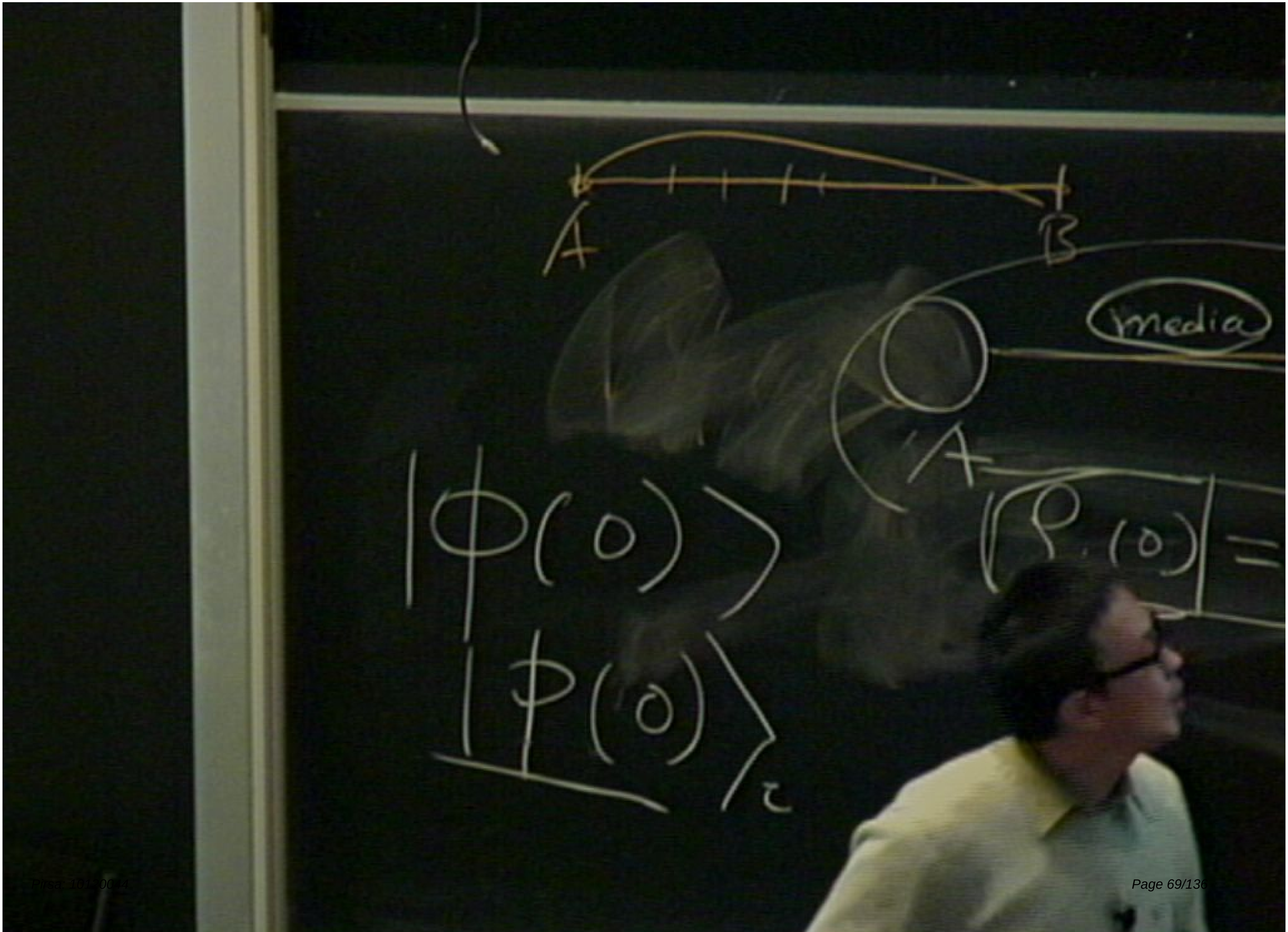
i	1	2	3	4

m	-3/2	-1/2	1/2	3/2

Other components

$$L_x = i\sum_i \sqrt{i(N-i)}(c_i^\dagger c_{i+1} - c_{i+1}^\dagger c_i)$$

$$L_z = \sum c_i^\dagger c_i (i - \frac{N+1}{2})$$


A
B
media
A
$|\Phi(0)\rangle$
$|\Phi(0)\rangle_c$

Evolution operator:

$$U(\tau) = \exp(-iJ\tau L_x)$$

When $J\tau=\pi$, $U(\pi/J)=\exp(-i\pi L_x)$ is a mirror-reflection operator, such that

$$U^\dagger(\pi/J)c_i^\dagger U(\pi/J) = rc_{N-i+1}^\dagger;$$

$$r = \exp(-i\pi\frac{N-1}{2})$$

r is called signature is nuclear physics.

N=1,5,9,...., r=1

N=2,6,10,...., r=-i

N=3,7,11,...., r=-1

N=4,8,12,...., r=i. Z_4 group.

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State transfer can be done $U(\pi/J)$

$$U(\pi/J)f(c_1^\dagger,\ldots,c_n^\dagger)|00......00\rangle$$

$$= f(rc_N^\dagger,\ldots,rc_{N-n+1}^\dagger)|00......00\rangle$$

If processor A has one qubit,

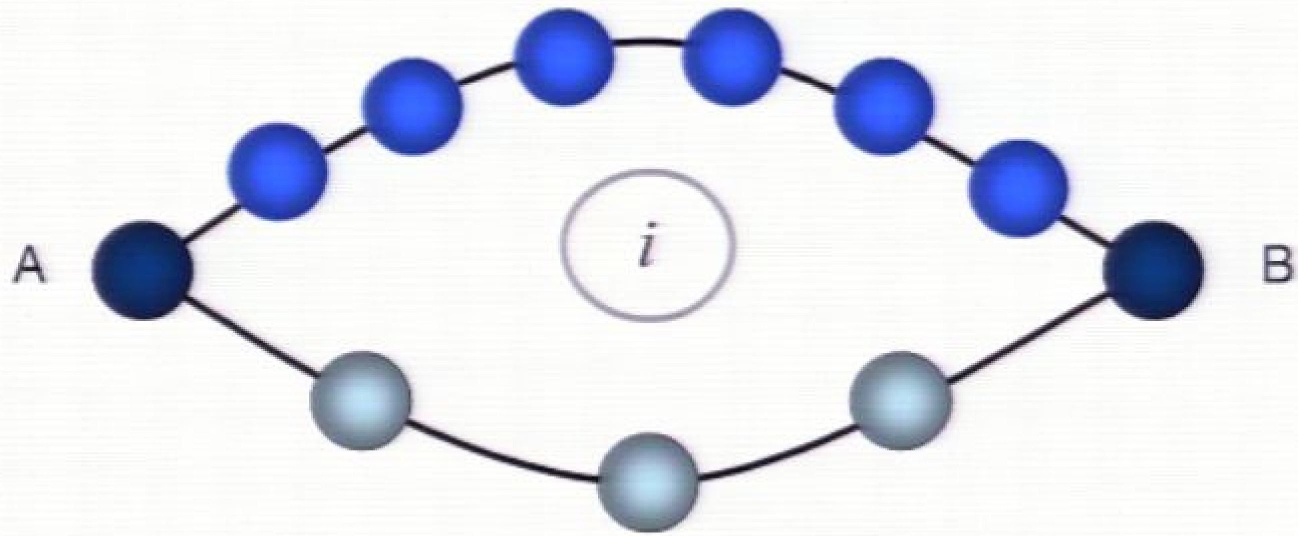
$$U(\pi/J)\left(a|0\rangle_1 + b|1\rangle_1\right)|\Phi(2,3,\ldots,N)\rangle$$

$$= |\Phi'(1,2,\ldots,N-1)\rangle\left(a|0\rangle_N + rb|1\rangle_N\right)$$

where Φ is an arbitary state of the rest of sites.

If r=1, fidelity is one.

Z_4 interference (Wu et al PRA80, 012332 (2009))



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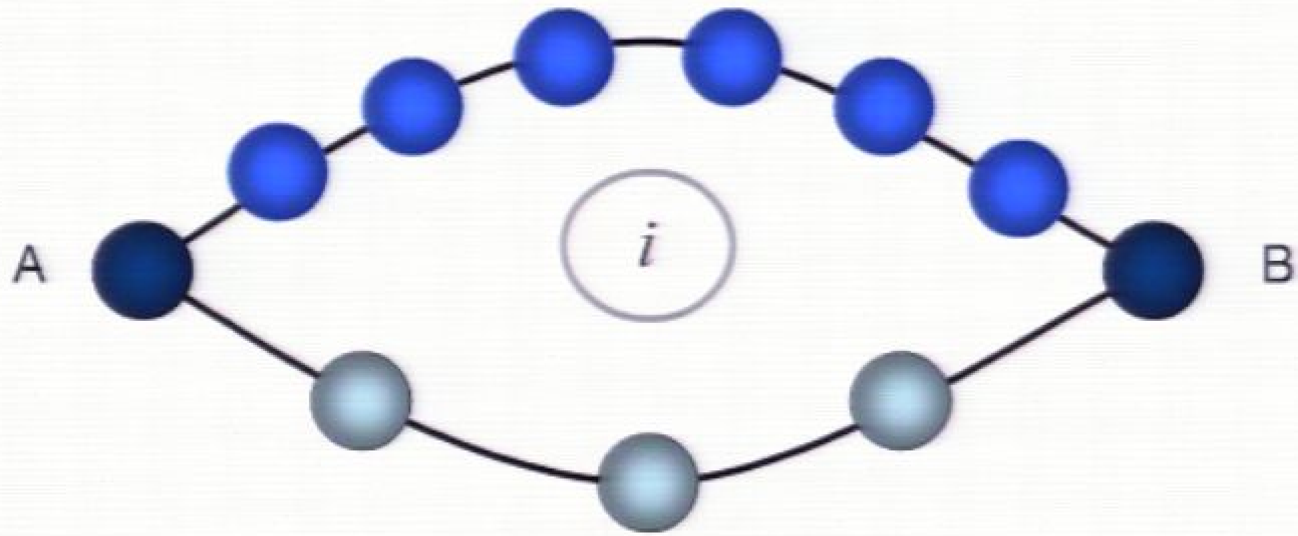

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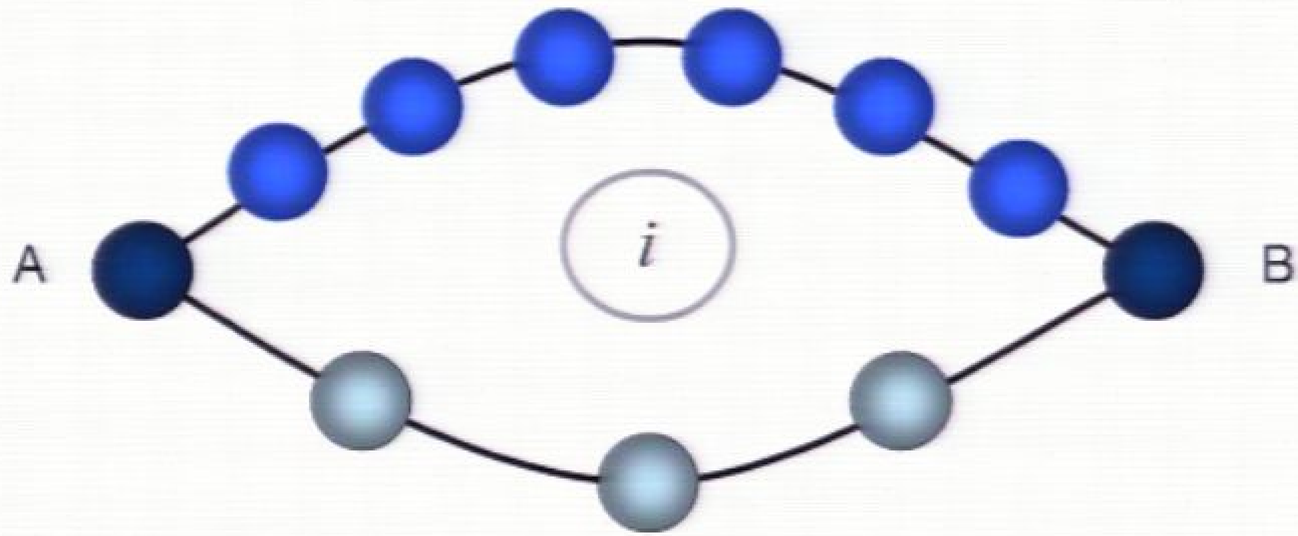

A state can be transferred via two paths with different values of N. (from A to B) .

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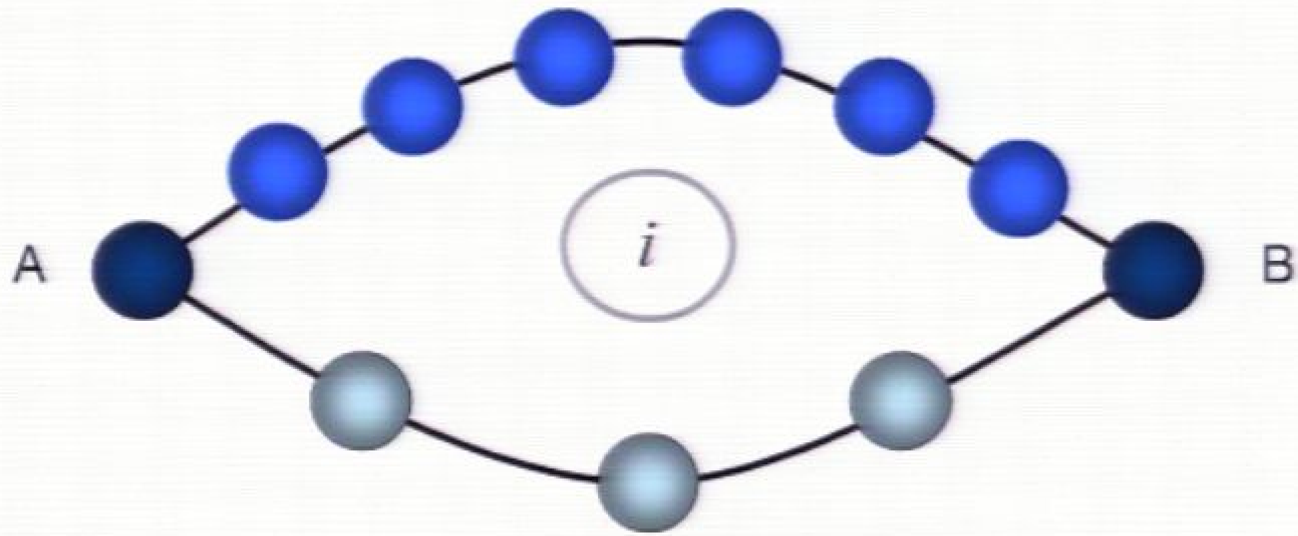

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The intensity at B is $I=2+R+R^*$ ($R=r_A r_B$).

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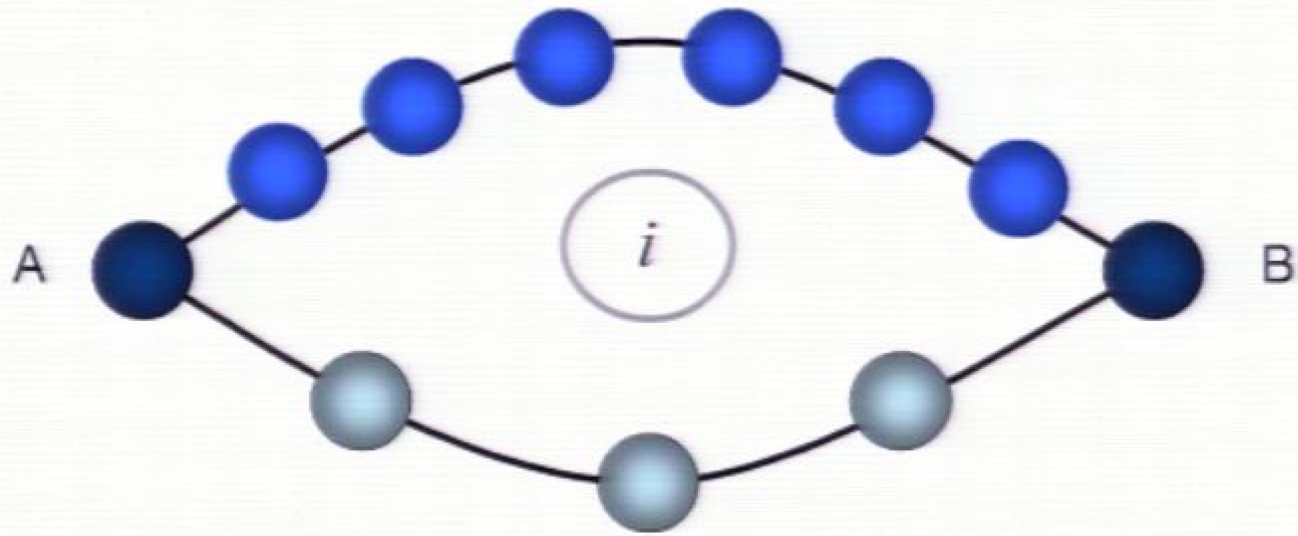

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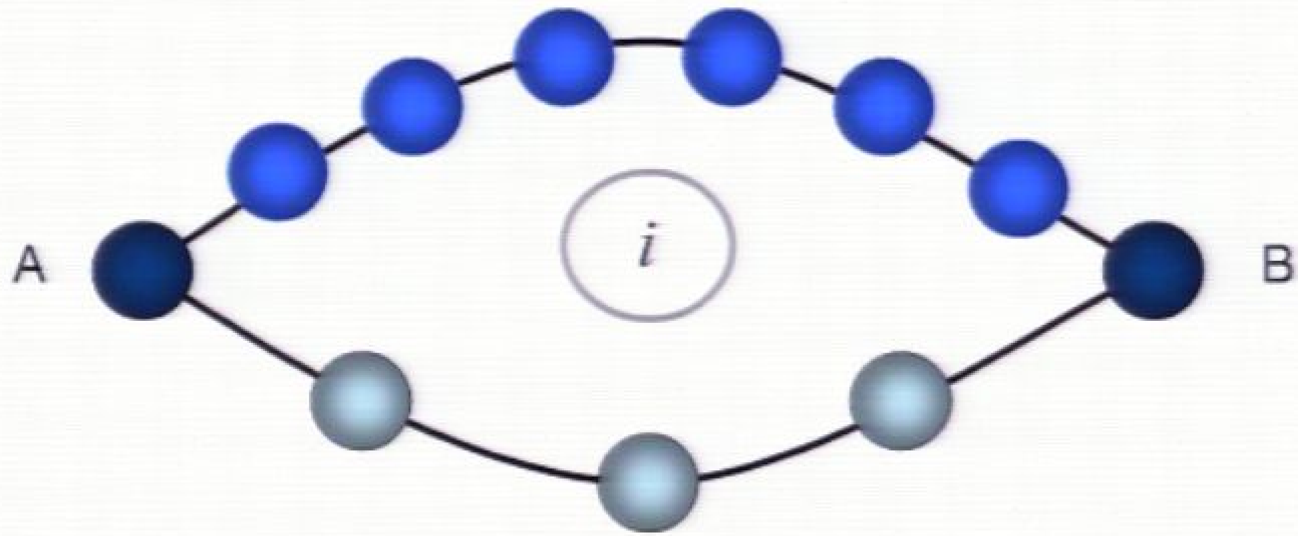

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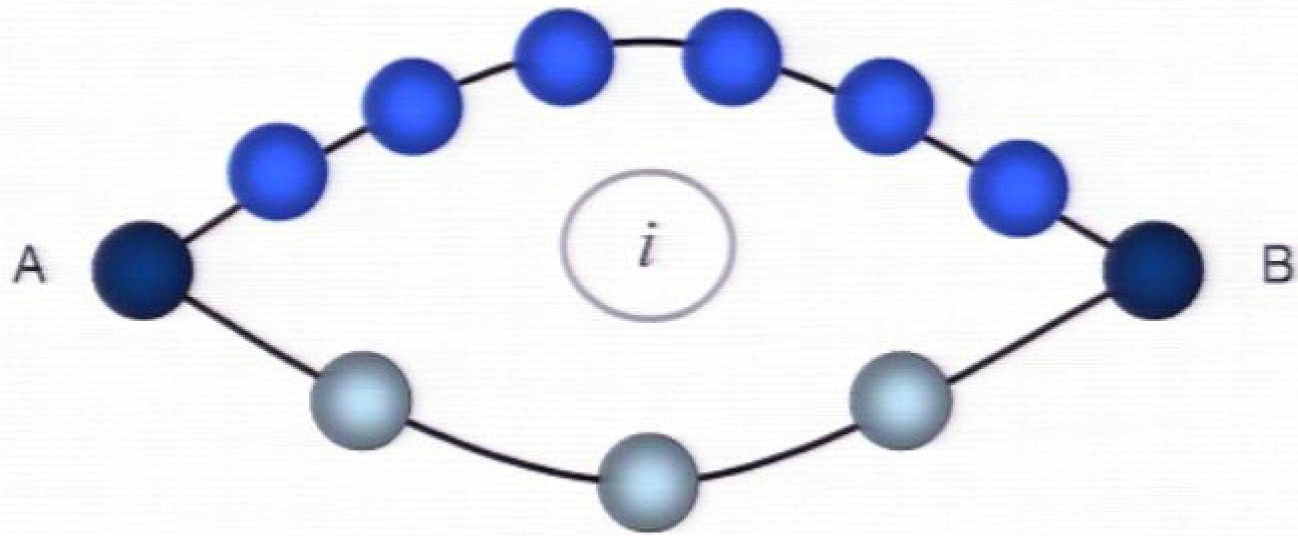

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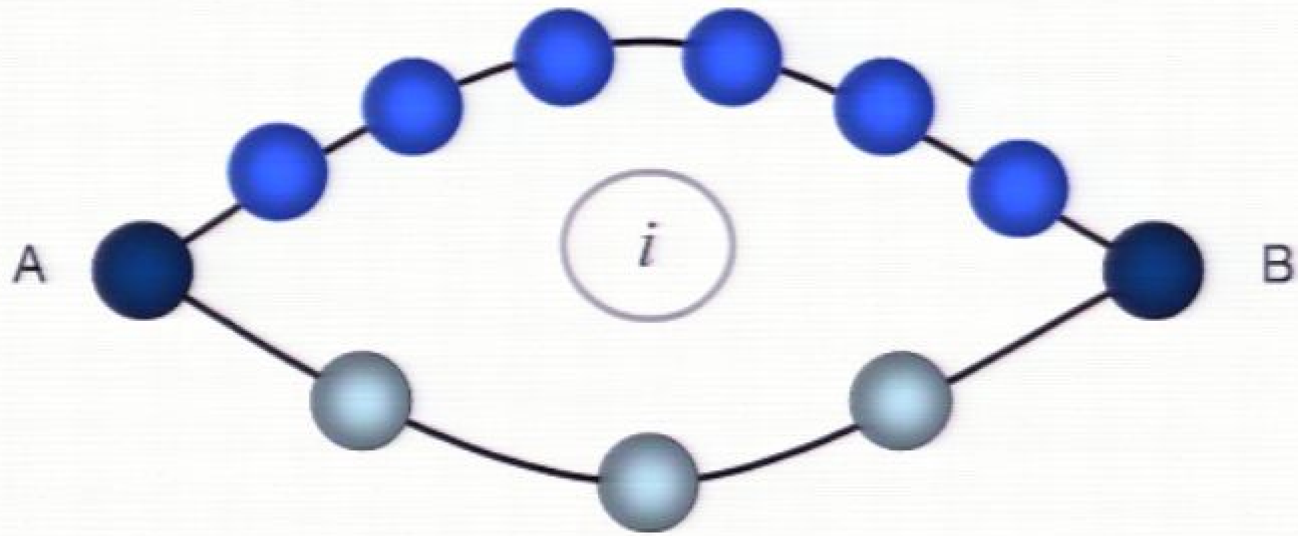

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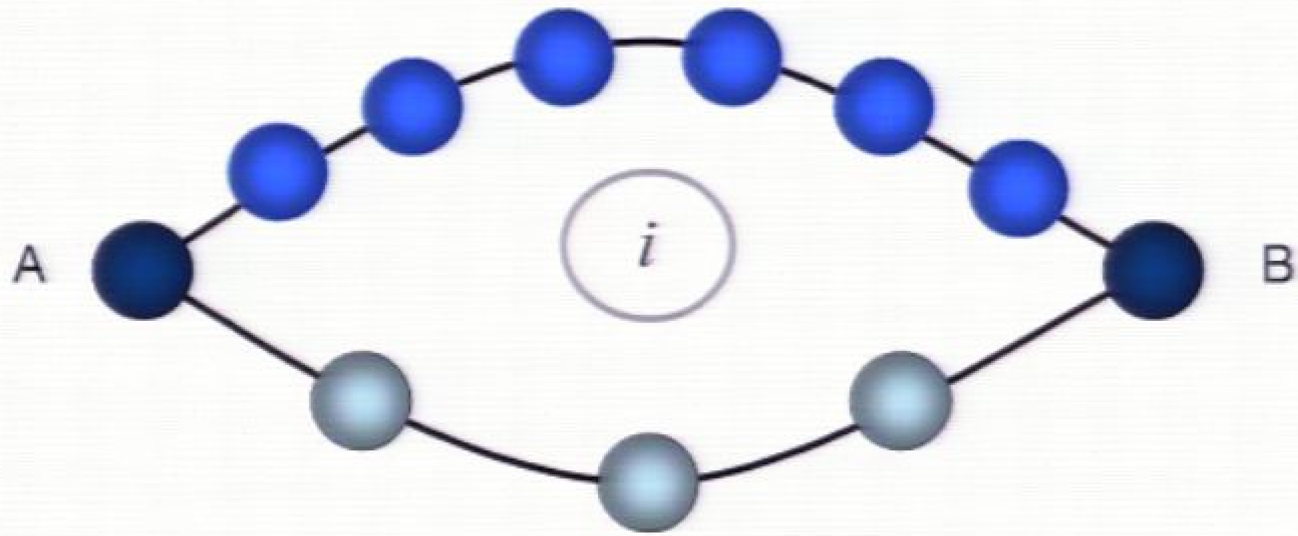

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New interference effect:

Path independent, Size number N Dependent (4 r's).

PST with two-internal-level identical particles (Bosons or fermions) in optical lattices (Wu et al PRA82,052339(2010))

↓ ↑

PST with two-internal-level identical particles (Bosons or fermions) in optical lattices (Wu etal PRA82,052339(2010))

The Hamiltonian is

$$H = \sum_{i,\sigma} J_i (c_{i\sigma}^\dagger c_{i+1\sigma} + c_{i+1\sigma}^\dagger c_{i\sigma}), \;\; J_i = J\sqrt{i(N-i)}$$

$\downarrow \uparrow$

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where we have neglected the on-site interaction and $\sigma=\downarrow\uparrow$ denote the two internal levels.

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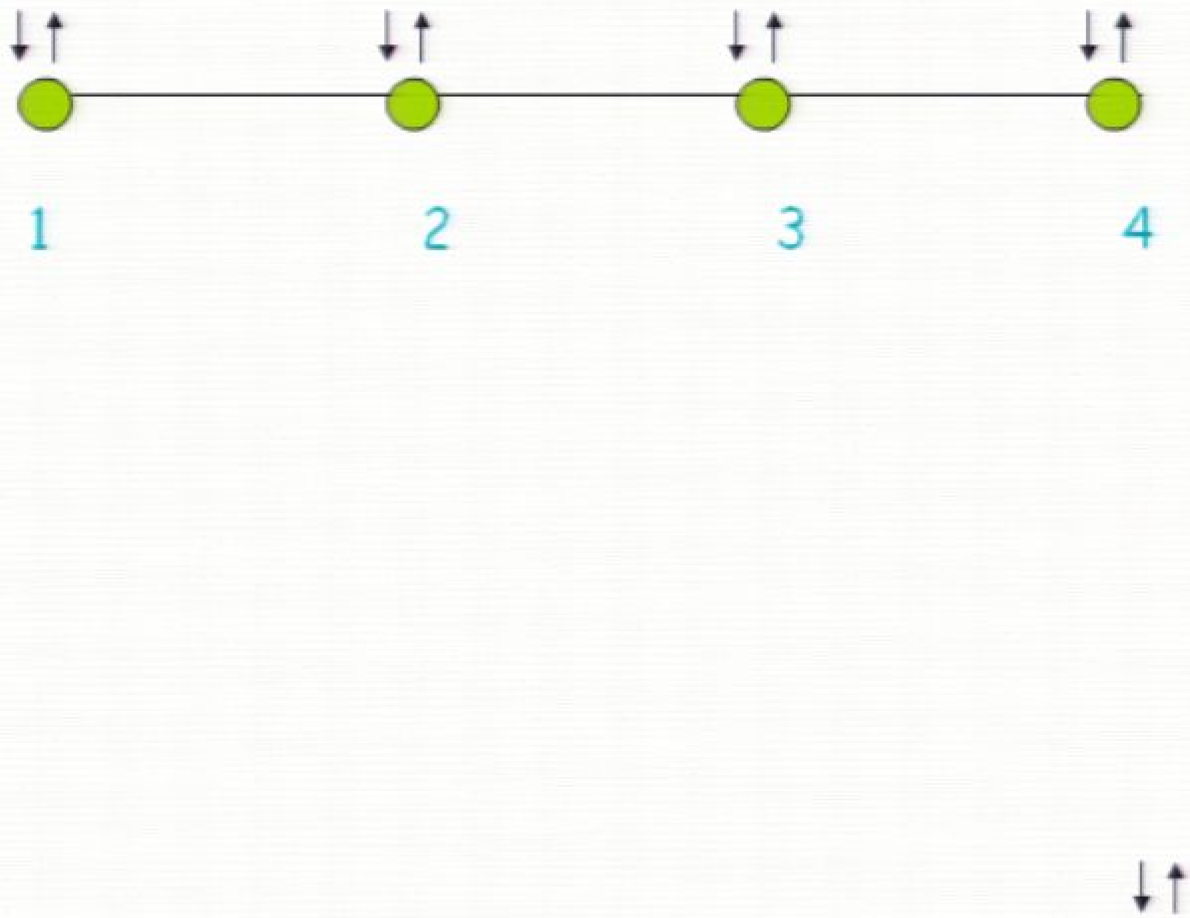
where we have neglected the on-site interaction and $\sigma = \downarrow \uparrow$ denote the two internal levels.

We show that

$$L_x = \sum_{\sigma} \sqrt{i(N-i)} (c_{i\sigma}^\dagger c_{i+1\sigma} + c_{i+1\sigma}^\dagger c_{i\sigma})$$

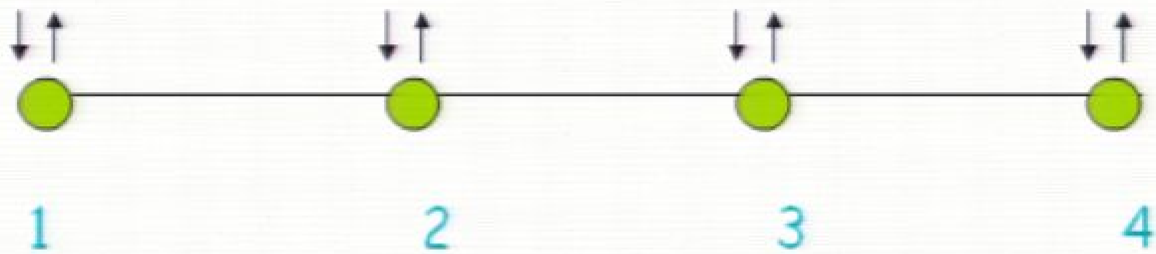
is also the x- component of angular momentum, such that it can be used to do PST as the above.


1
2
3
4

The tunneling matrix elements J's are usually independent of spin directions.

There is a qubit supported by spin-up and down, at each site.

↓↑



The tunneling matrix elements J's are usually independent of spin directions.

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The system is one of proposals for universal quantum computation, with qubit . External field is used to manipulate individual spin (signal-qubit gate, commute with H) and to make them interacted (two-qubit gate).

Quantum device is made of the same qubits, ***no interface***.

PST can be made by H

$$|\phi\rangle_1 = a|\uparrow\rangle_1 + b|\downarrow\rangle_1 \xrightarrow{U(\pi/J)} |\phi\rangle_N = a|\uparrow\rangle_N + b|\downarrow\rangle_N$$

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- The present case cannot be translated into spin chain directly since JW transformation does not work.

Spin Chain: only two states

$|0\rangle_i$ an

present

$|00\rangle_i$, c_i

Two sta $c_{i\downarrow}^{\dagger}|00\rangle_i$

serve as a qubit

Spin Chain: only two states

$|0\rangle_i$ and $c_i^\dagger |0\rangle_i$, serve as qubit

present case: four states

$|00\rangle_i$, $c_{i\uparrow}^\dagger |00\rangle_i$, $c_{i\downarrow}^\dagger |00\rangle_i$ and $c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger |00\rangle_i$

Two states: $|\uparrow\rangle_i = c_{i\uparrow}^\dagger |00\rangle_i$ and $|\downarrow\rangle_i = c_{i\downarrow}^\dagger |00\rangle_i$

serve as a qubit

PST can be made by H

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- The present case cannot be translated into spin chain directly since JW transformation does not work.
- Different from spin chain

The present case makes transfers via particles, while spin chain makes transfers via spin flipping

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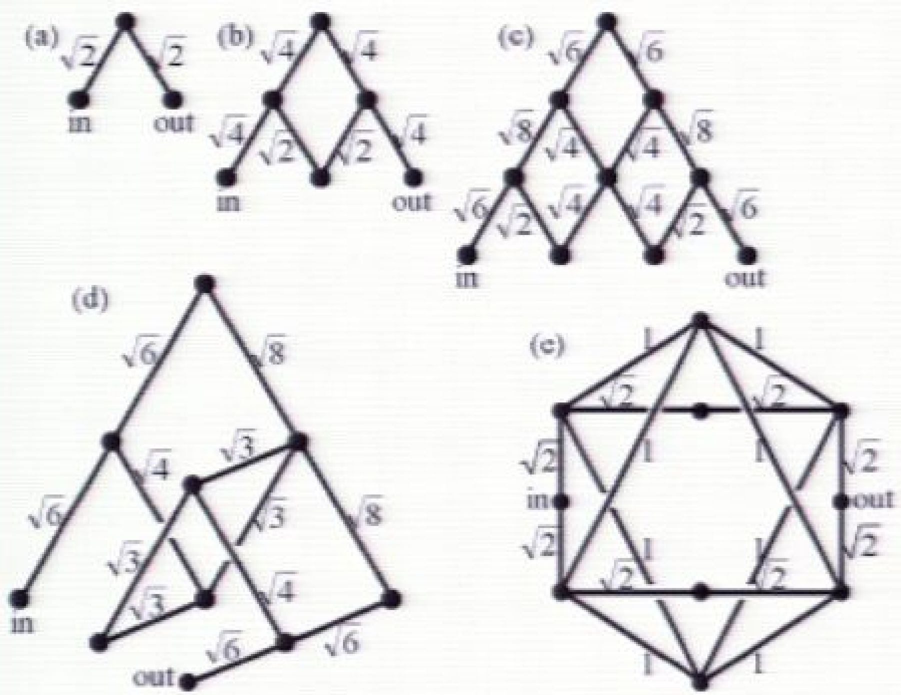
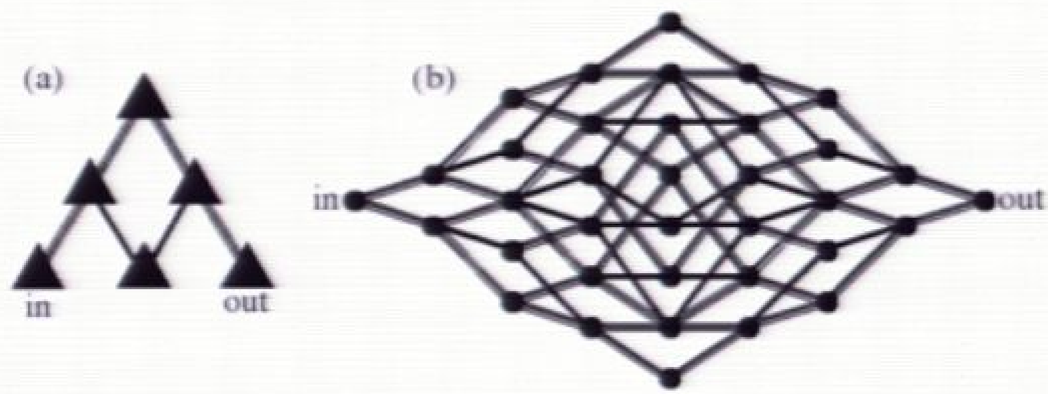
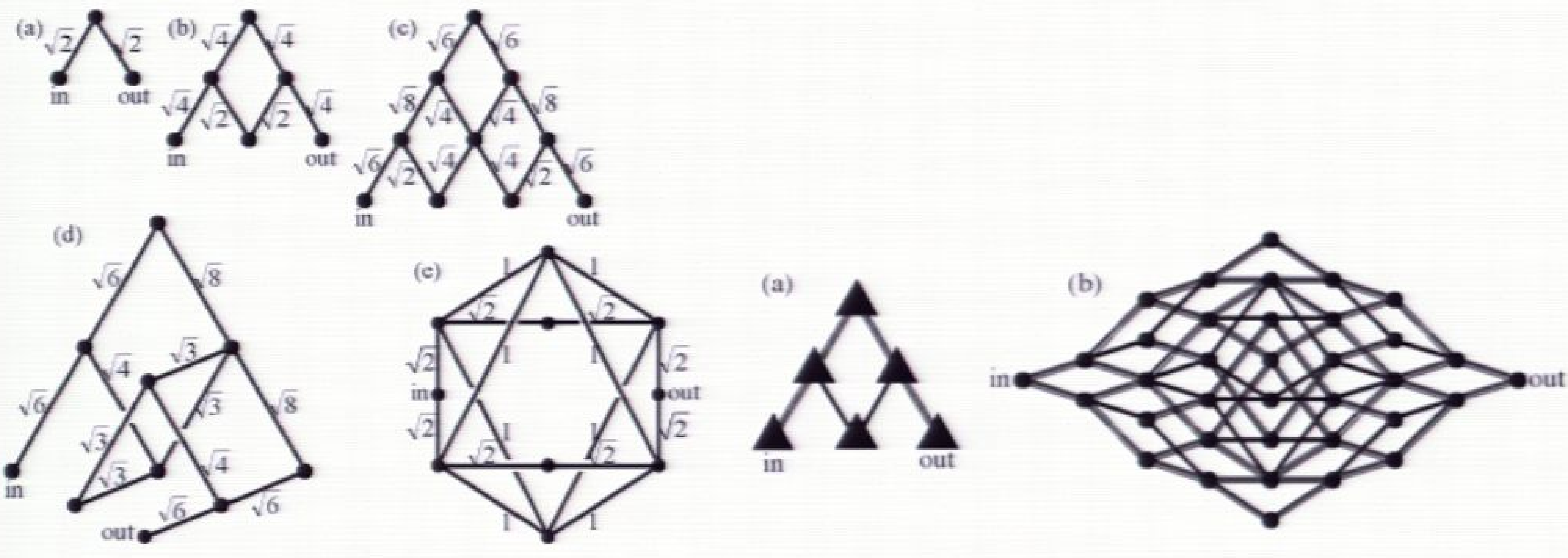
Spin Chain: only two states

$|0\rangle_i$ and $c_i^\dagger |0\rangle_i$, serve as qubit





The present case can be extended to higher dimensions, while higher-D spin PST remains unclear (though there are many interesting explorations, for instance topologic stuctures of the networks, Feder, PRL 97,180502 (2006)))



For 2D identical particles at site (i,k)

(Wu etal PRA 82,052339 (2010).)

First analytical model for higher D

$$H = J^{(1)} L_x^{(1)} + J^{(2)} L_x^{(2)}$$

$$L_x^{(1)} = \sum_{\sigma,k} \sqrt{i(N-i)} (c_{ik\sigma}^\dagger c_{i+1k\sigma} + c_{i+1k\sigma}^\dagger c_{ik\sigma})$$

$$L_x^{(2)} = \sum_{\sigma,i} \sqrt{k(M-k)} (c_{ik\sigma}^\dagger c_{ik+1\sigma} + c_{ik+1\sigma}^\dagger c_{ik\sigma})$$

$$[L_x^{(1)}, L_x^{(2)}] = 0$$

For 2D spin system, one can define similar L_x's , but they don't commute

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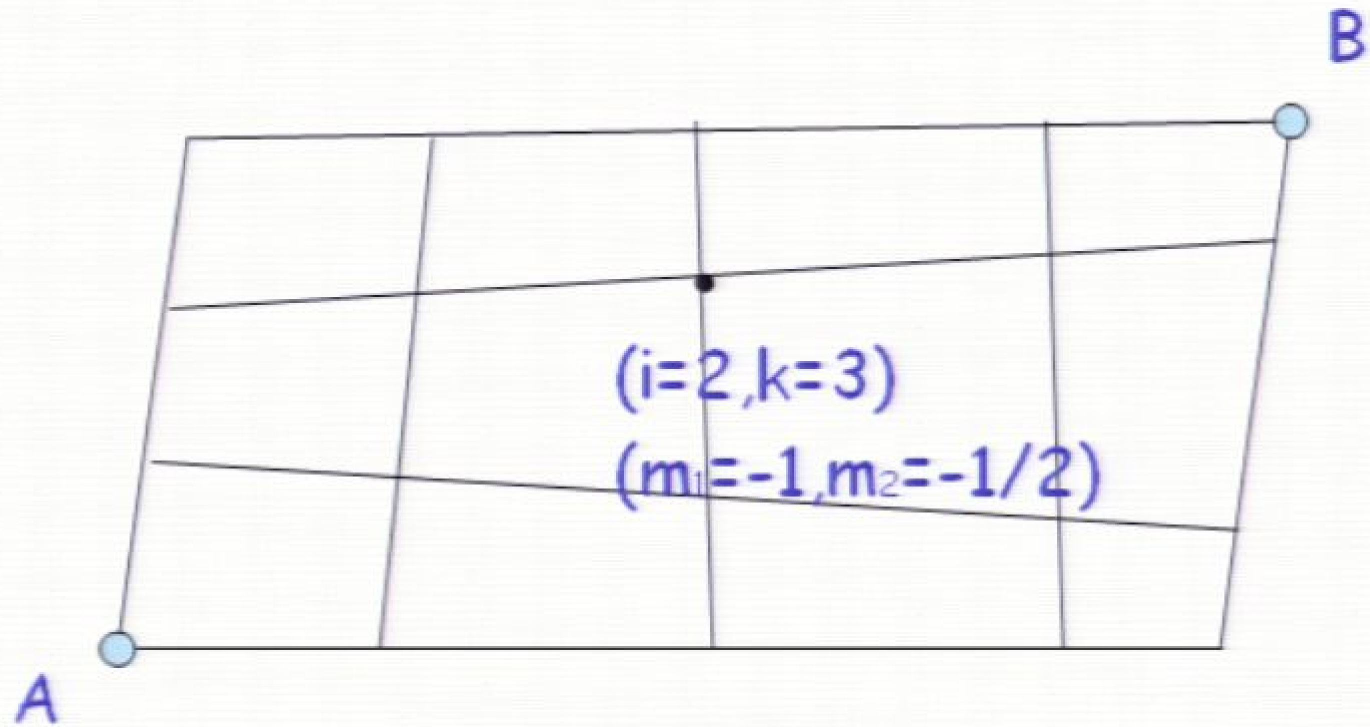
Two-D networks N=4, M=5



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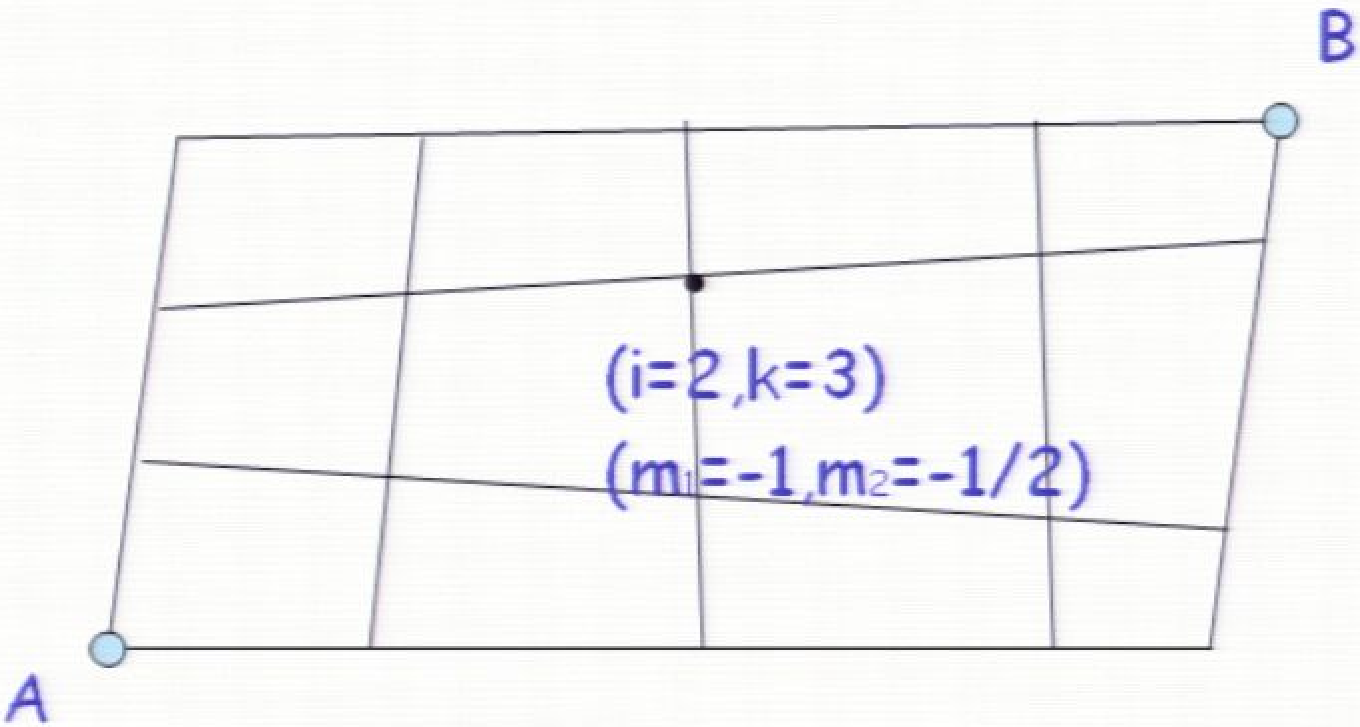
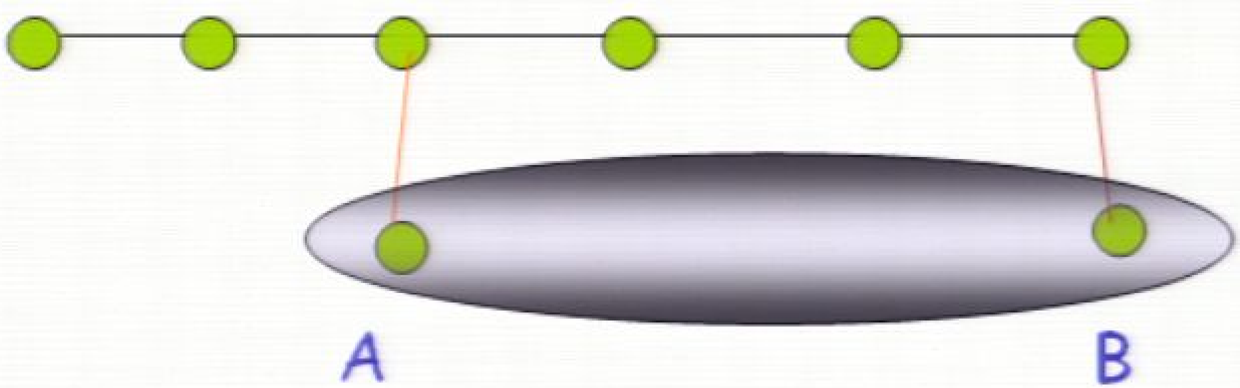
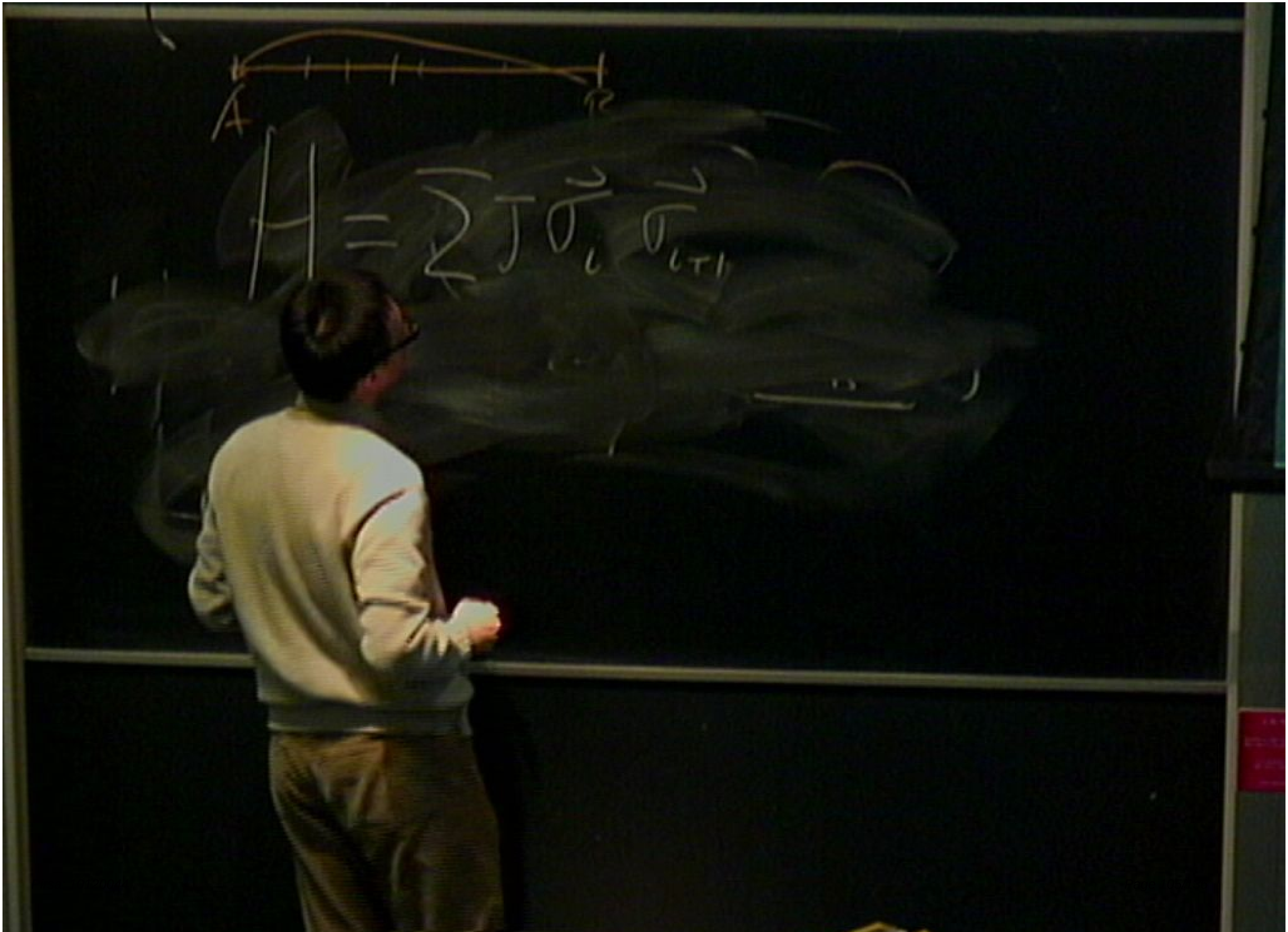
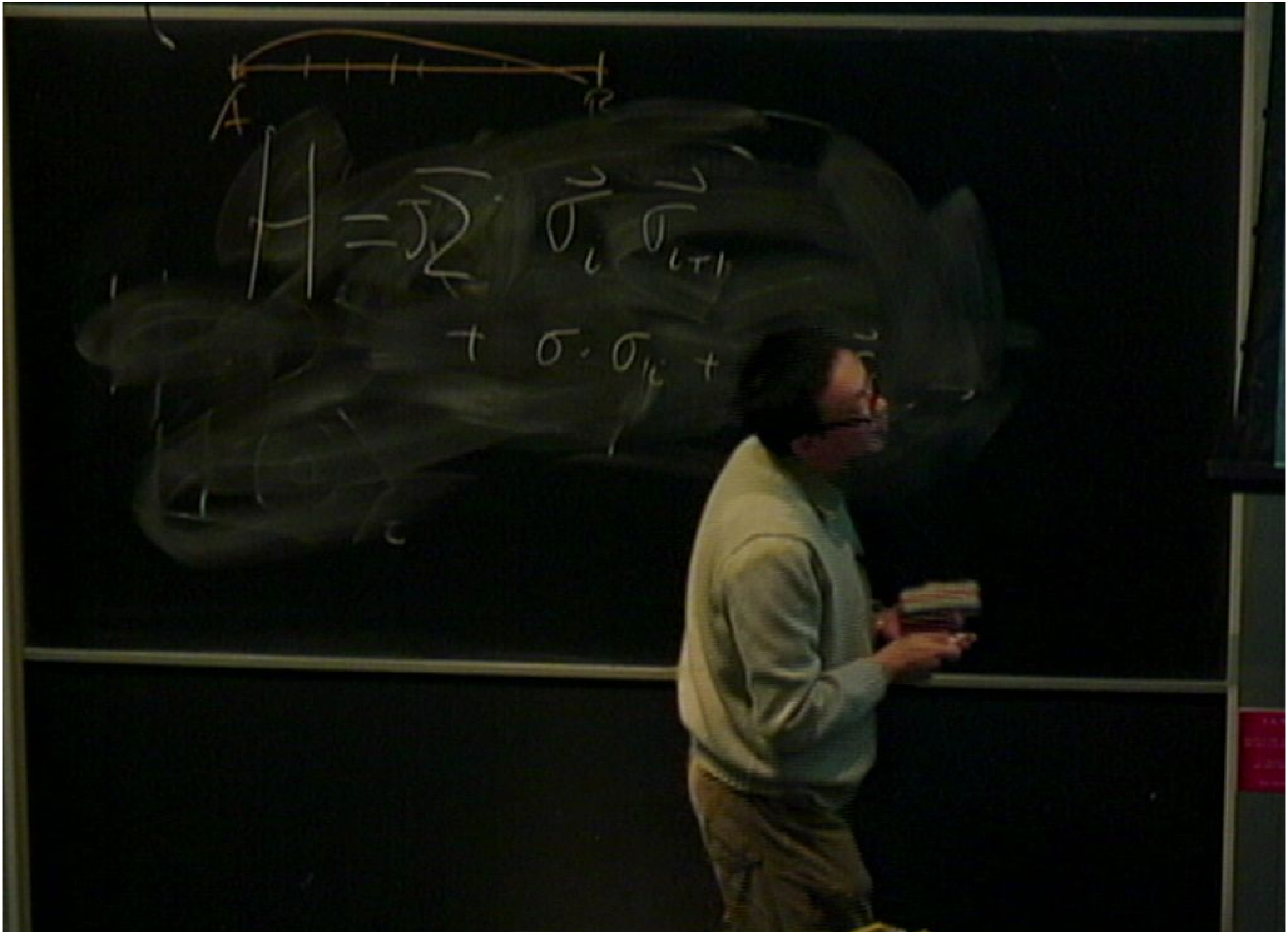
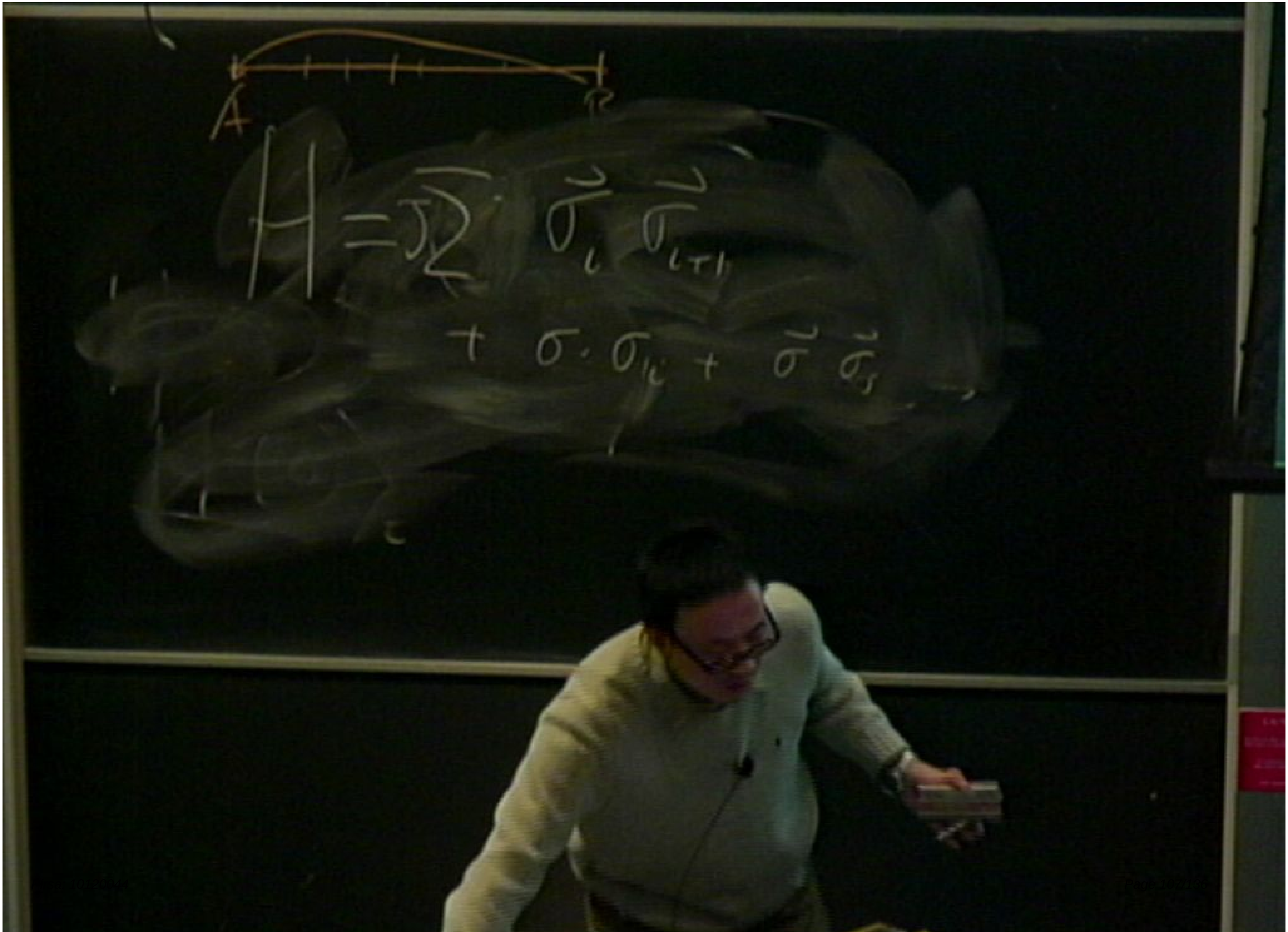


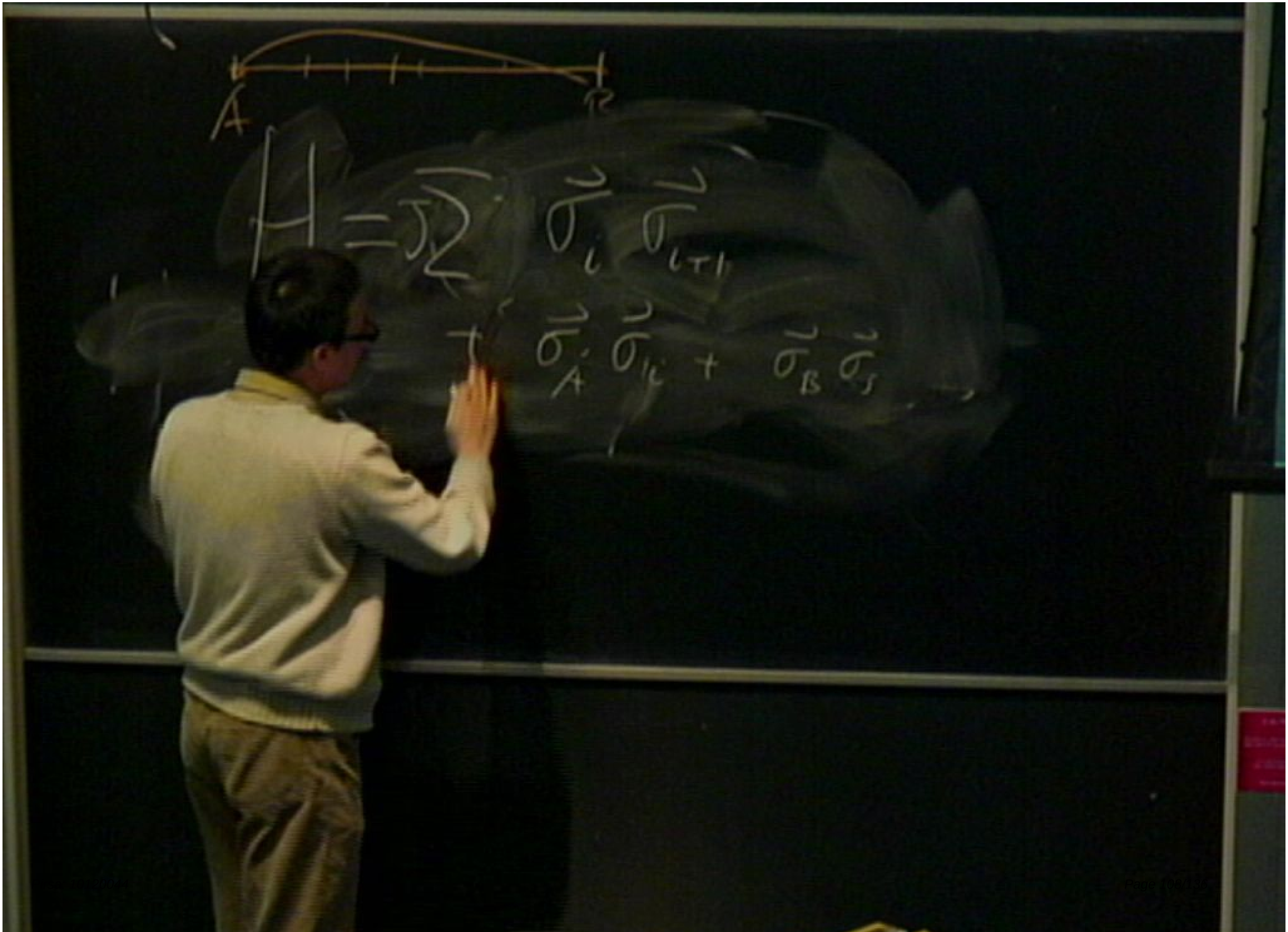
PST with the natural Heisenberg interaction with weakly attached processor A and B via RKKY.

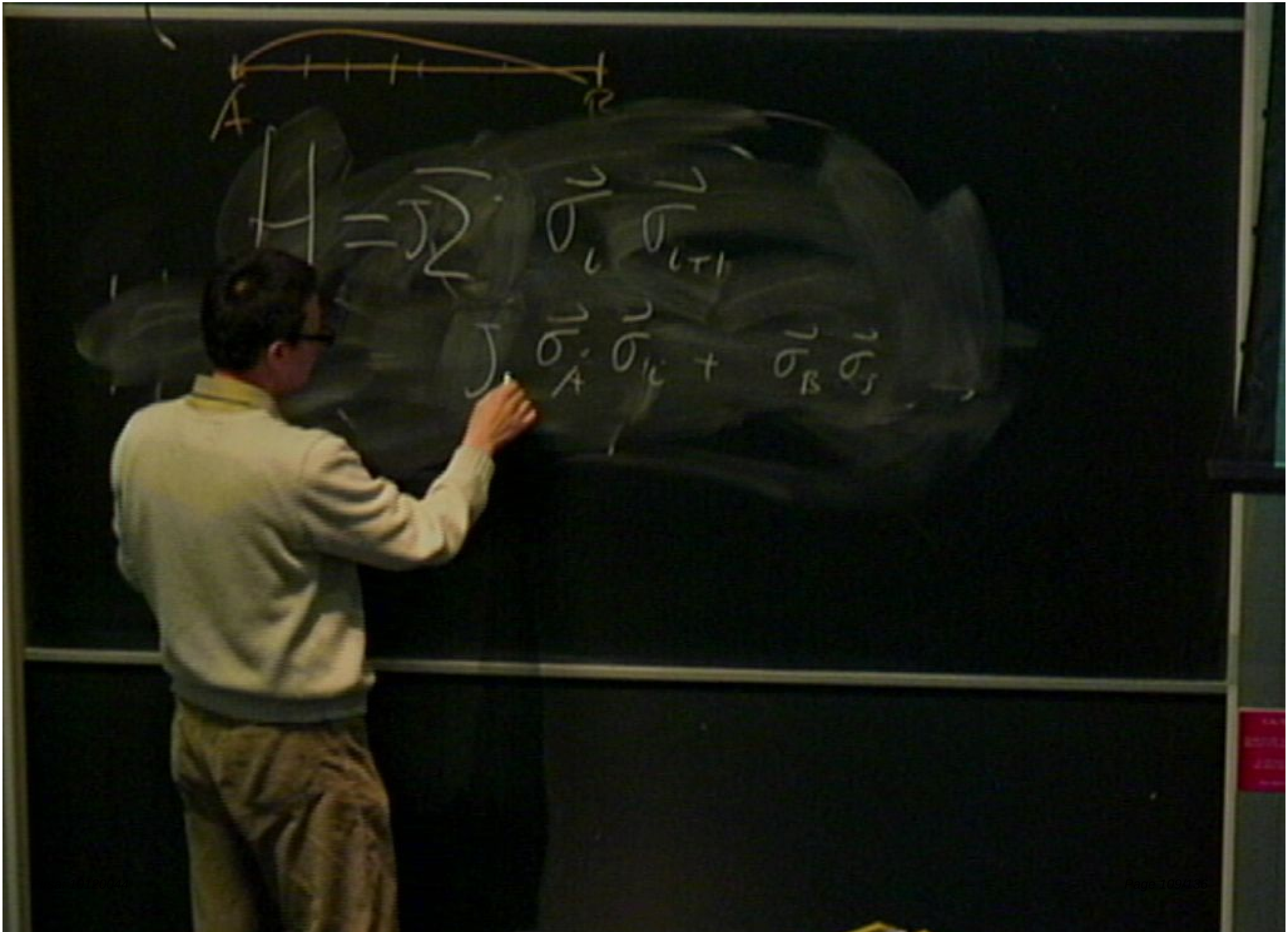


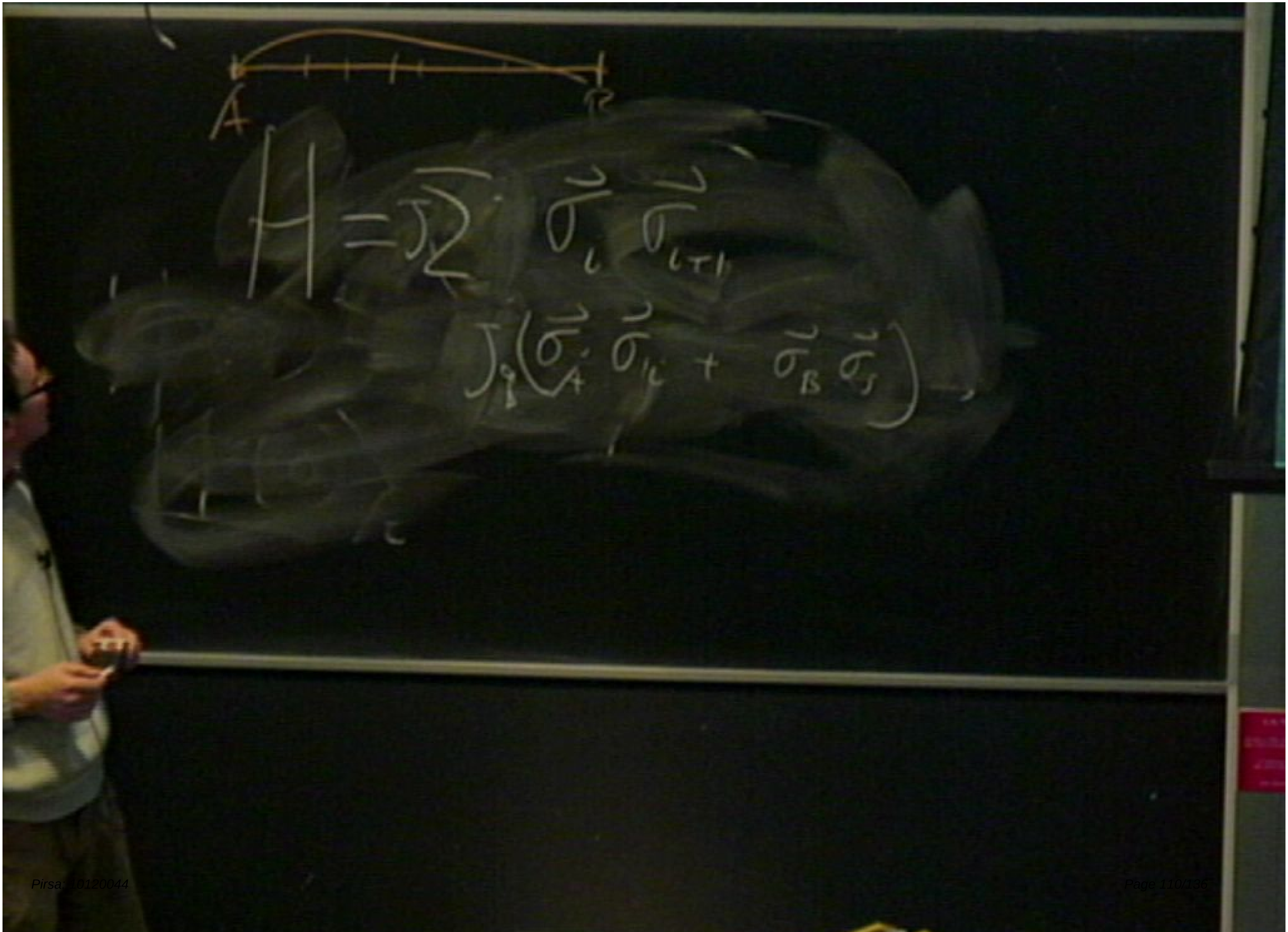

$H = \sum J \vec{\sigma}_i \vec{\sigma}_{i+1}$

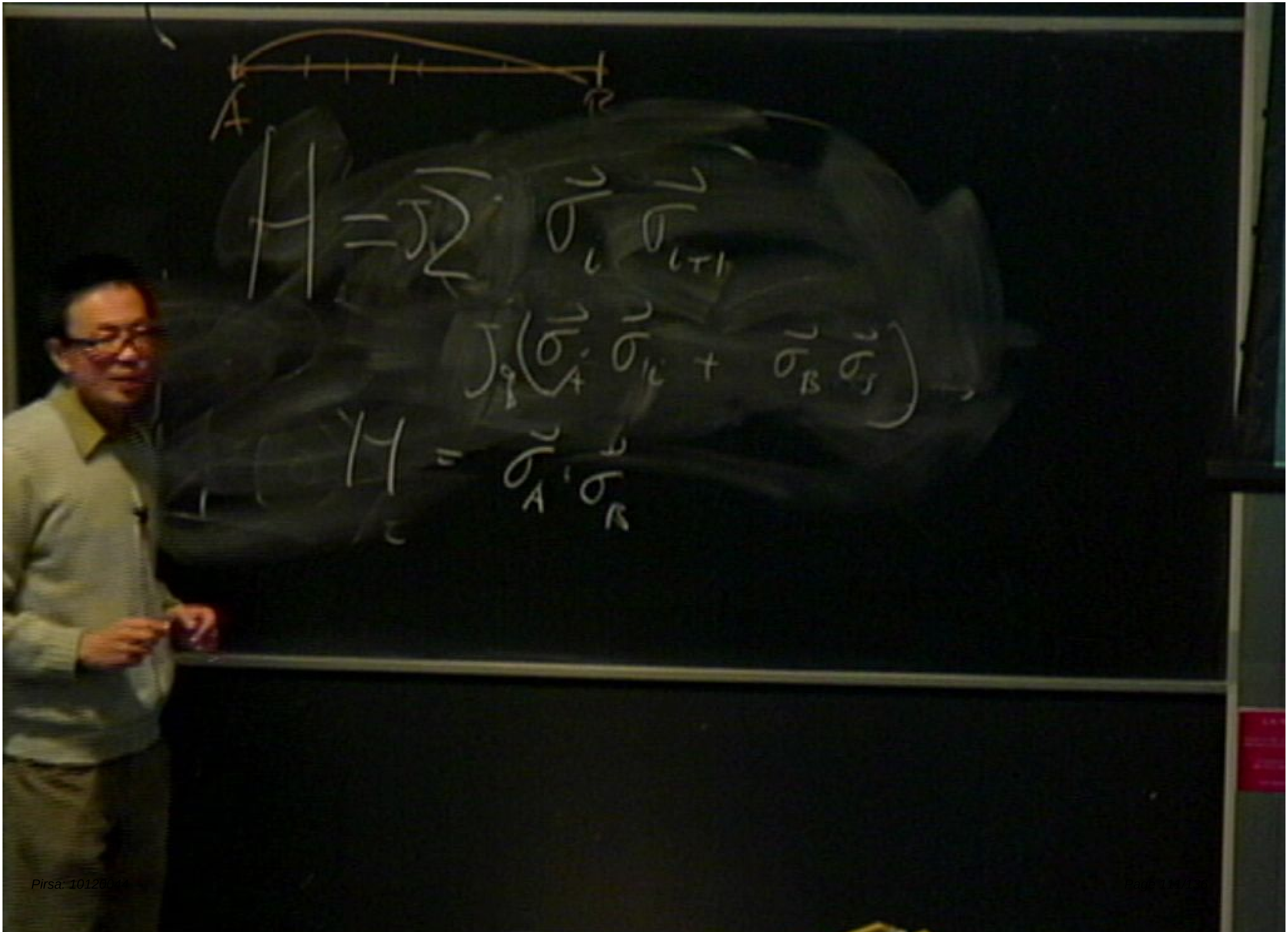

H = JΣ σ_i σ_{i+1}

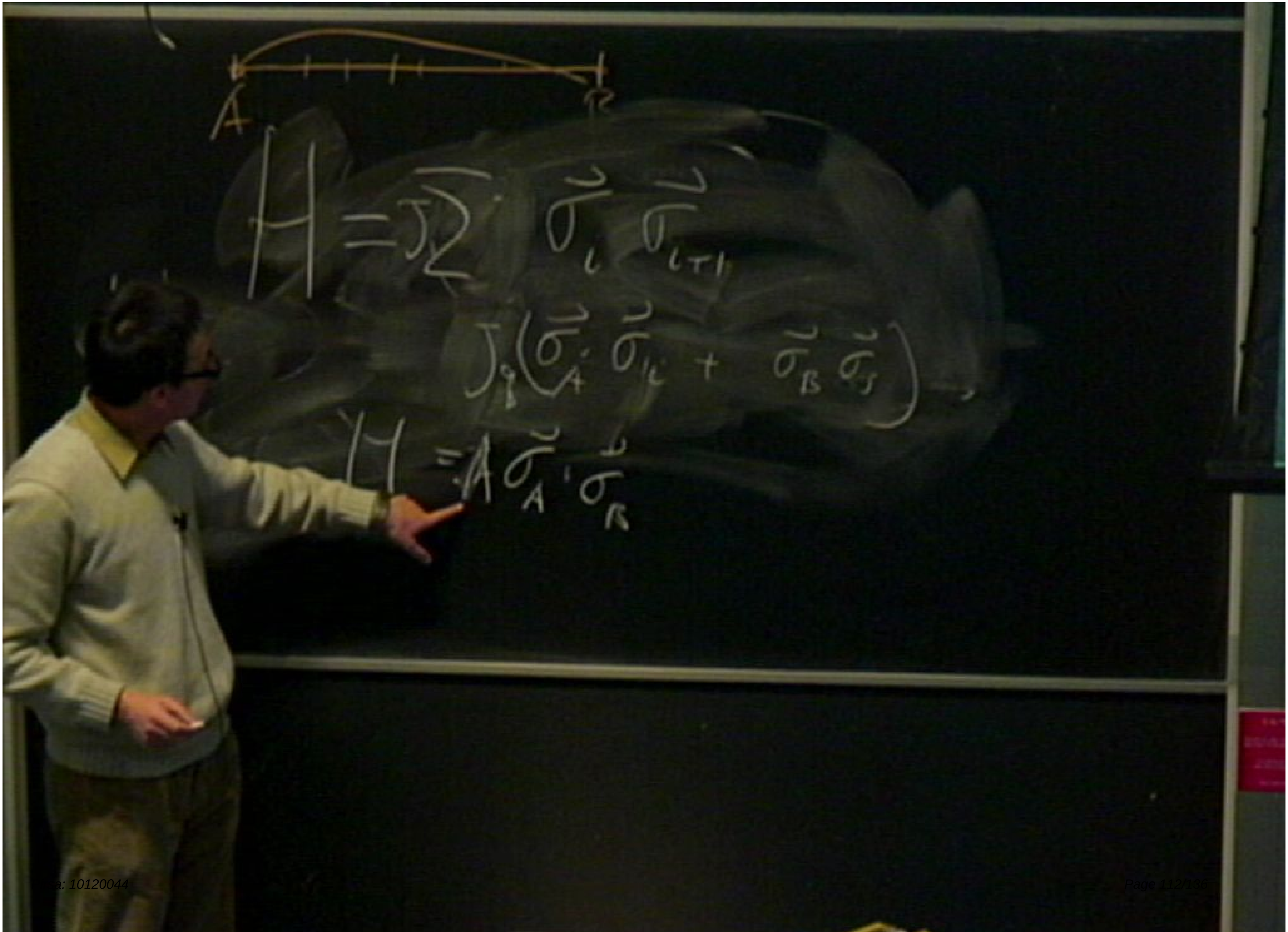


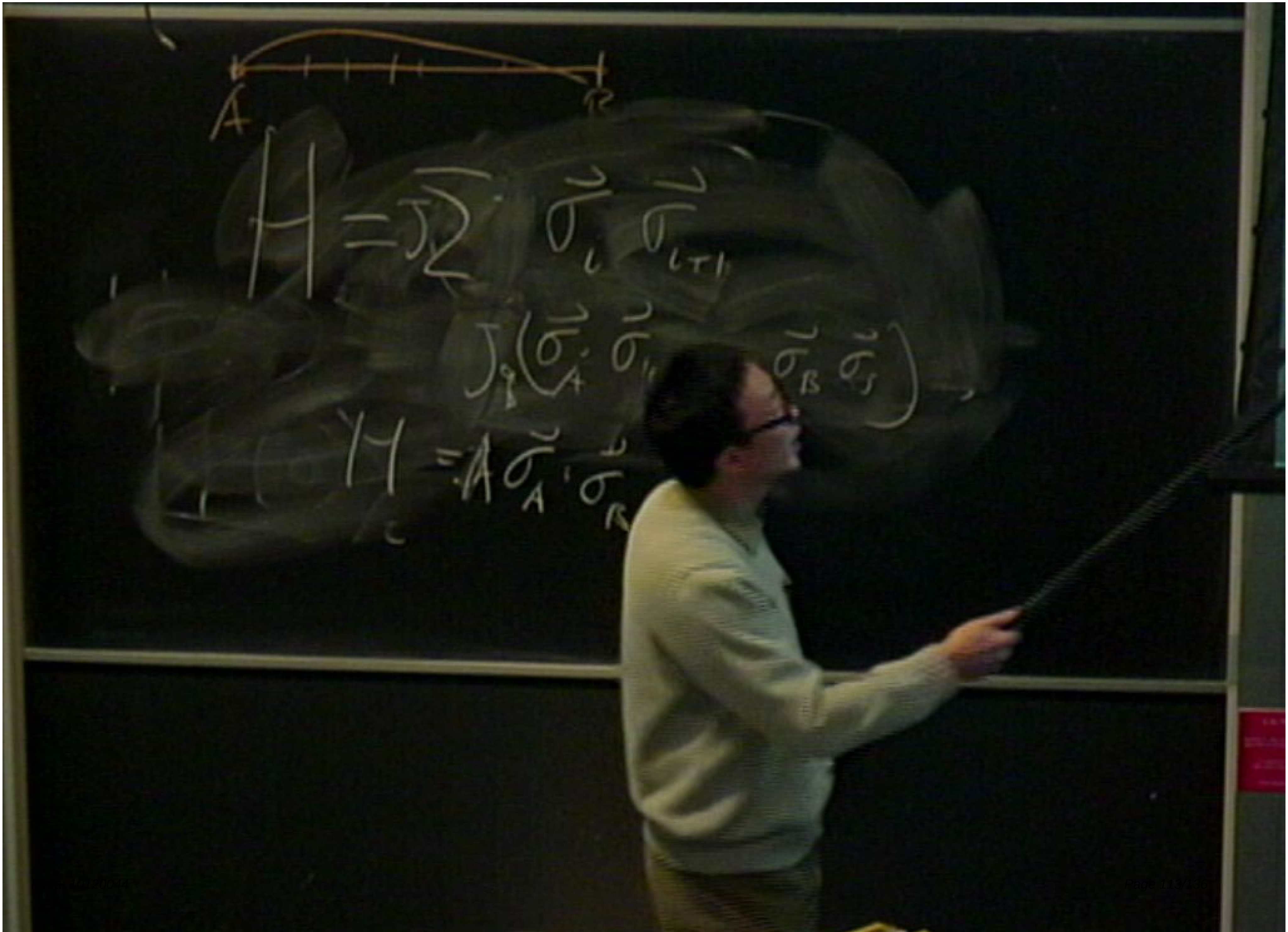

A
B
$H = J\sum \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$
$+ \vec{\sigma}_A \cdot \vec{\sigma}_k + \vec{\sigma}_B \vec{\sigma}_s$


A
B
H = J Σ σ_i · σ_{i+1}
J σ_A · σ_i + σ_B σ_j


A
B


A
B
H = J Σ σ_i · σ_{i+1}
J_B (σ_A · σ_1 + σ_B · σ_5)
H = σ_A · σ_B


A
B
$H = J \sum \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$
$J_B (\vec{\sigma}_A \cdot \vec{\sigma}_1 + \vec{\sigma}_B \vec{\sigma}_J)$
$H = A \vec{\sigma}_A \cdot \vec{\sigma}_B$



A
B
H = J Σ σ_i · σ_{i+1}
H_L = A σ_A · σ_B

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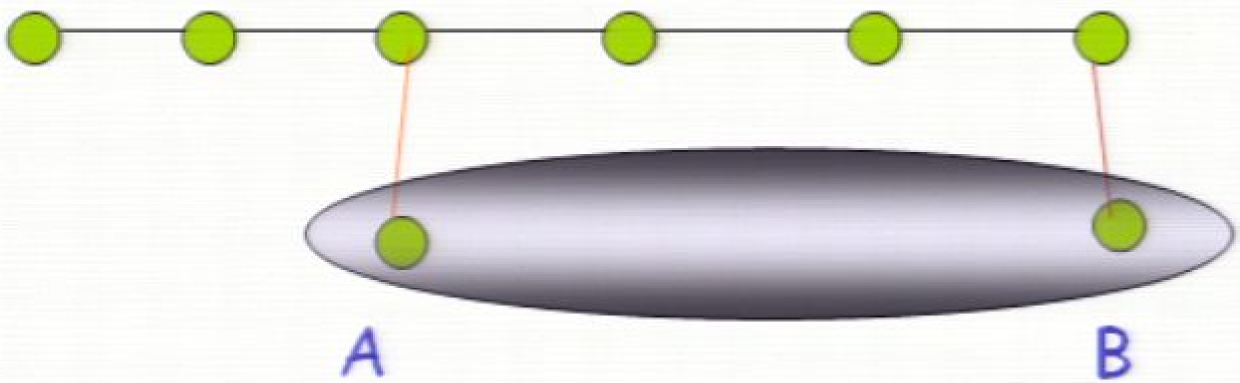
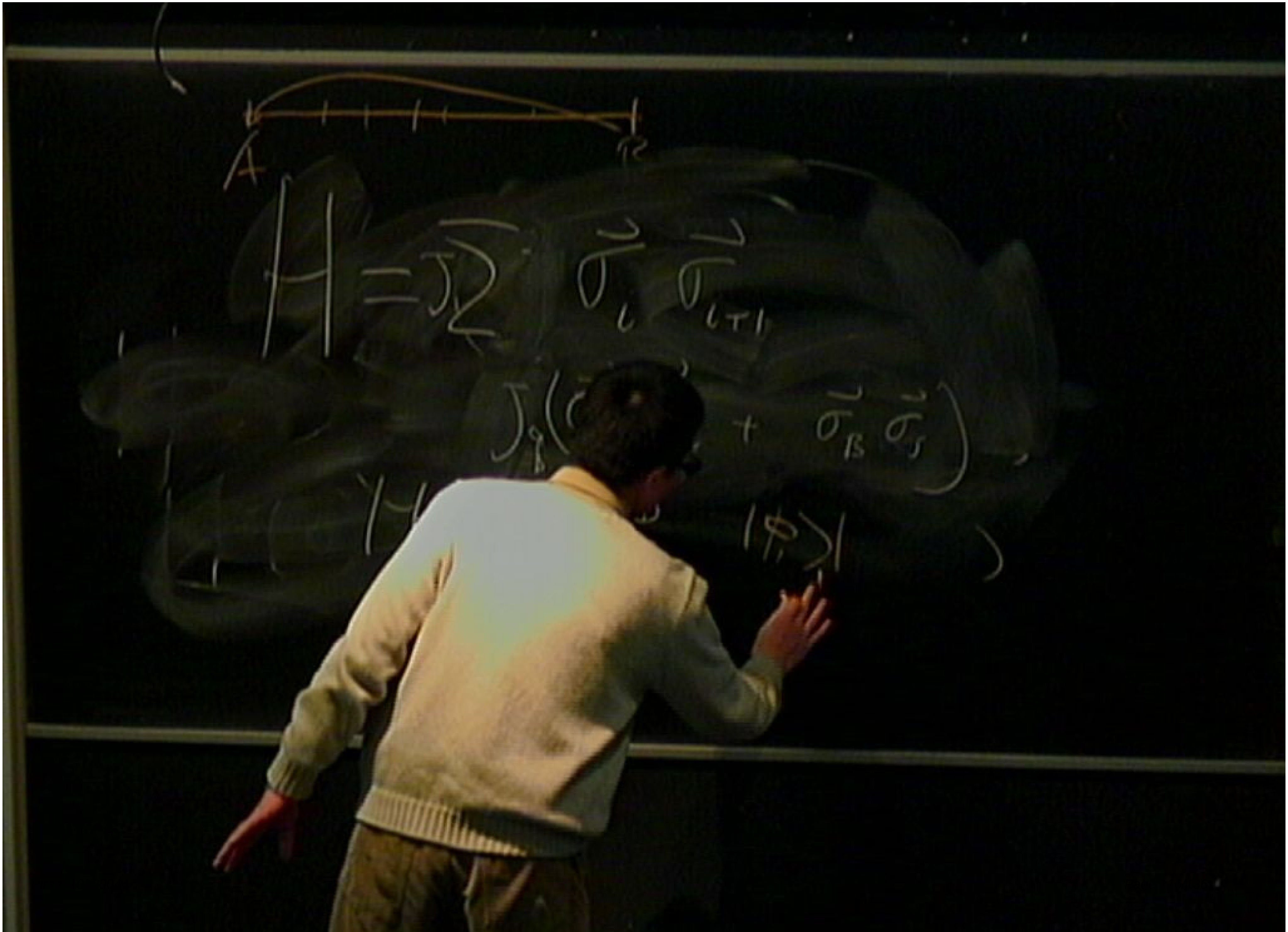
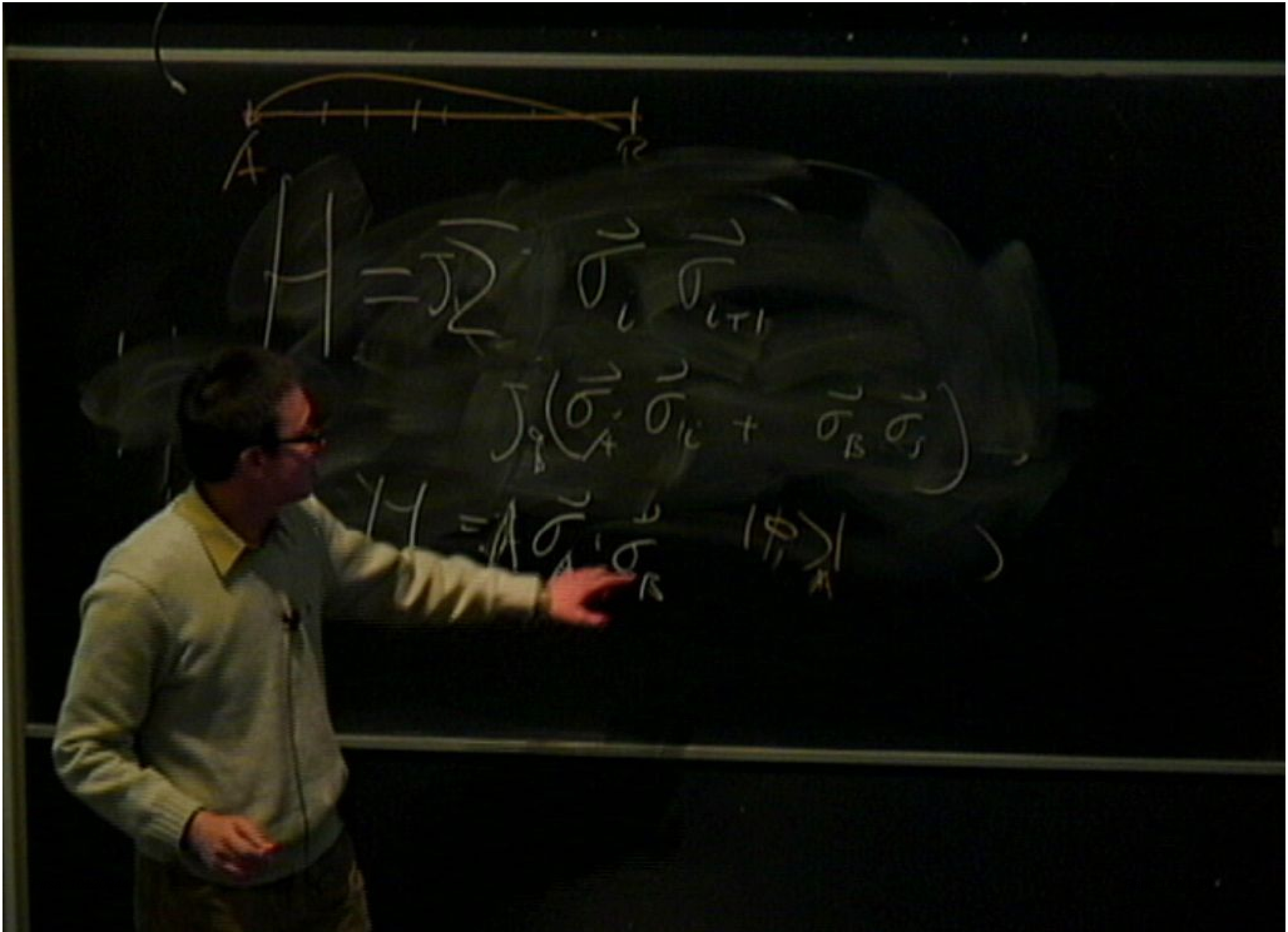
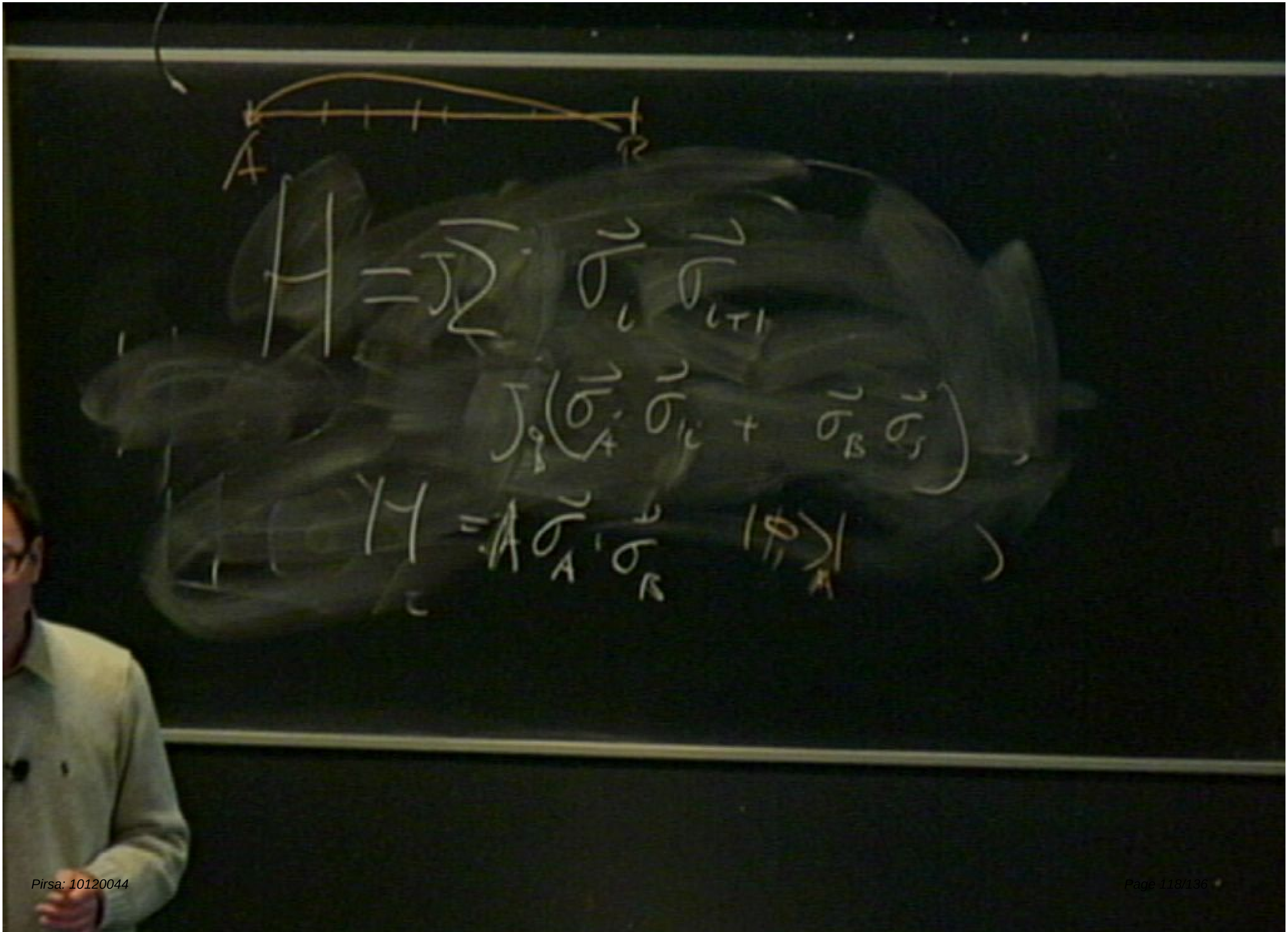


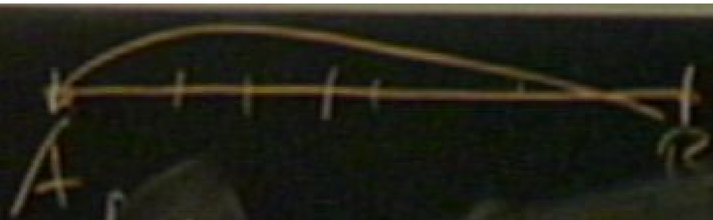
Summary

- Introduction, motivation
- State transfer with Naturally available interaction
- Perfect state transfer in 1-D with spin network
- PST in 1-D with identical particles with 2 levels
- PST in higher dimension with identical particles
- PST through natural Heisenberg chain with attached processors.






A
B
H = J Σ σ_i · σ_{i+1}
J_B (σ_A · σ_1 + σ_B · σ_J)
H = A σ_A · σ_B
|φ_1⟩



$$H = J \sum \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$$

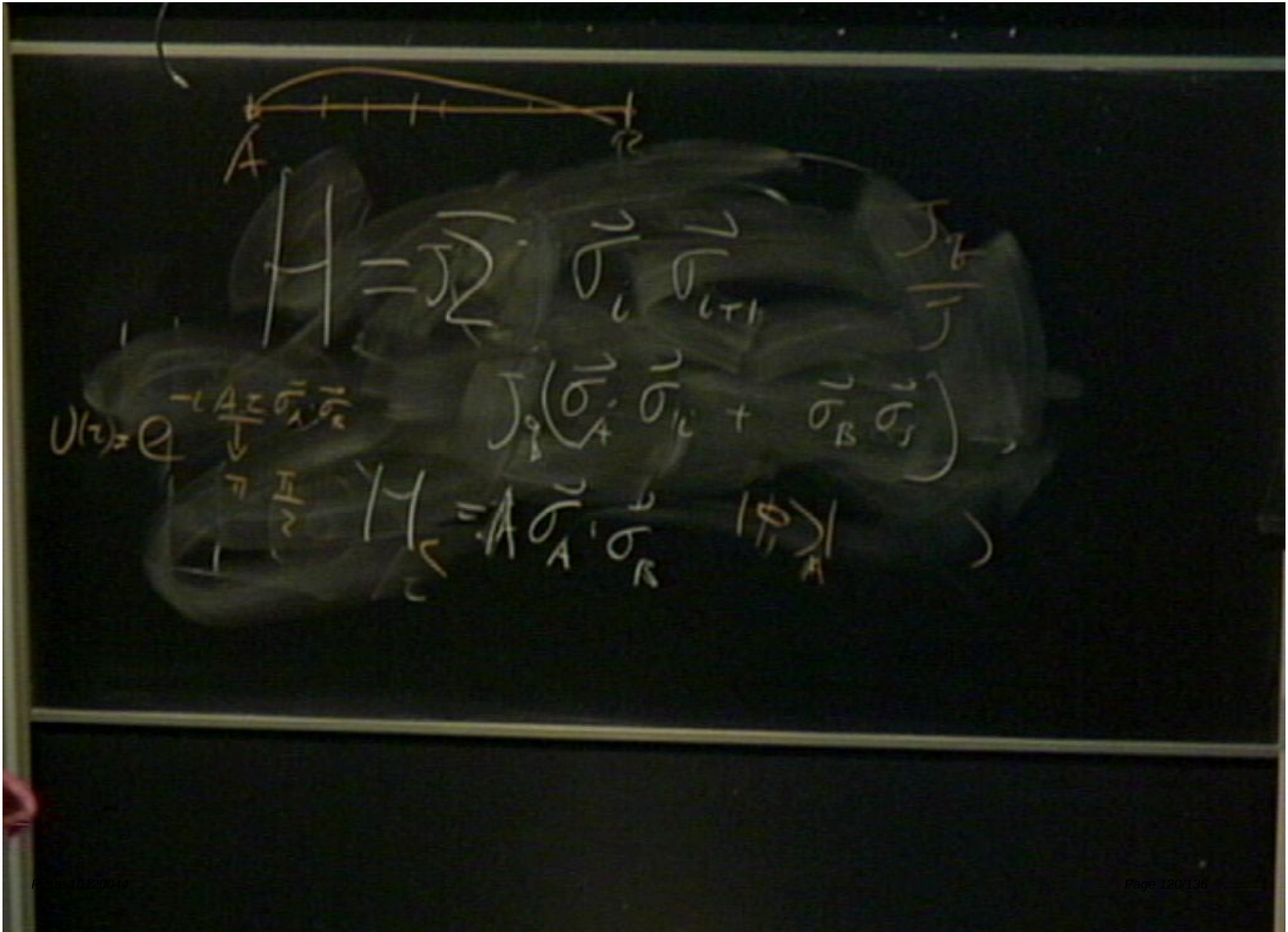
$$J_q\left(\vec{\sigma}_A \cdot \vec{\sigma}_i + \vec{\sigma}_B \cdot \vec{\sigma}_j\right)$$

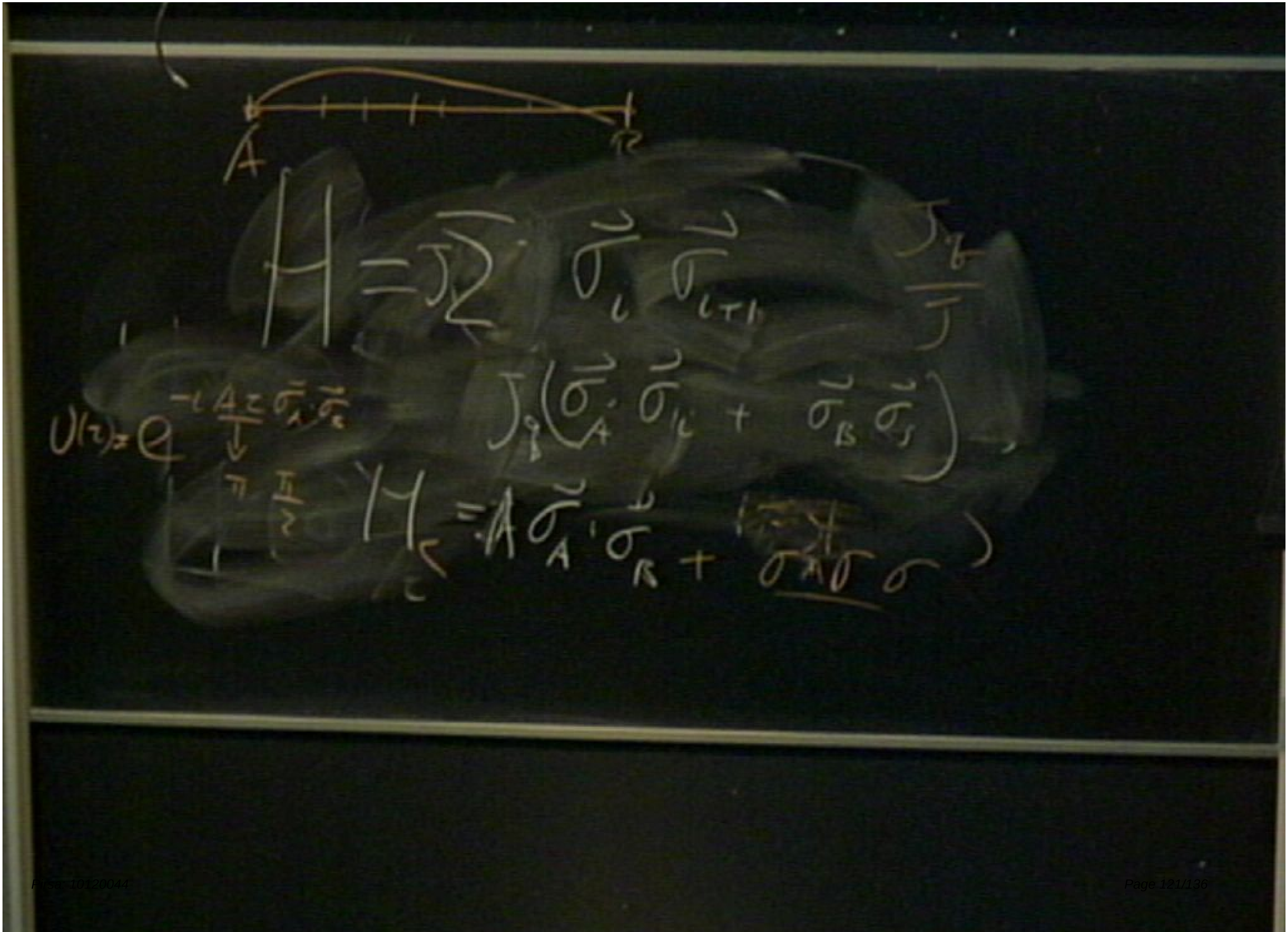
$$U(\tau) = e^{-i A \tau \vec{\sigma}_A \cdot \vec{\sigma}_B}$$

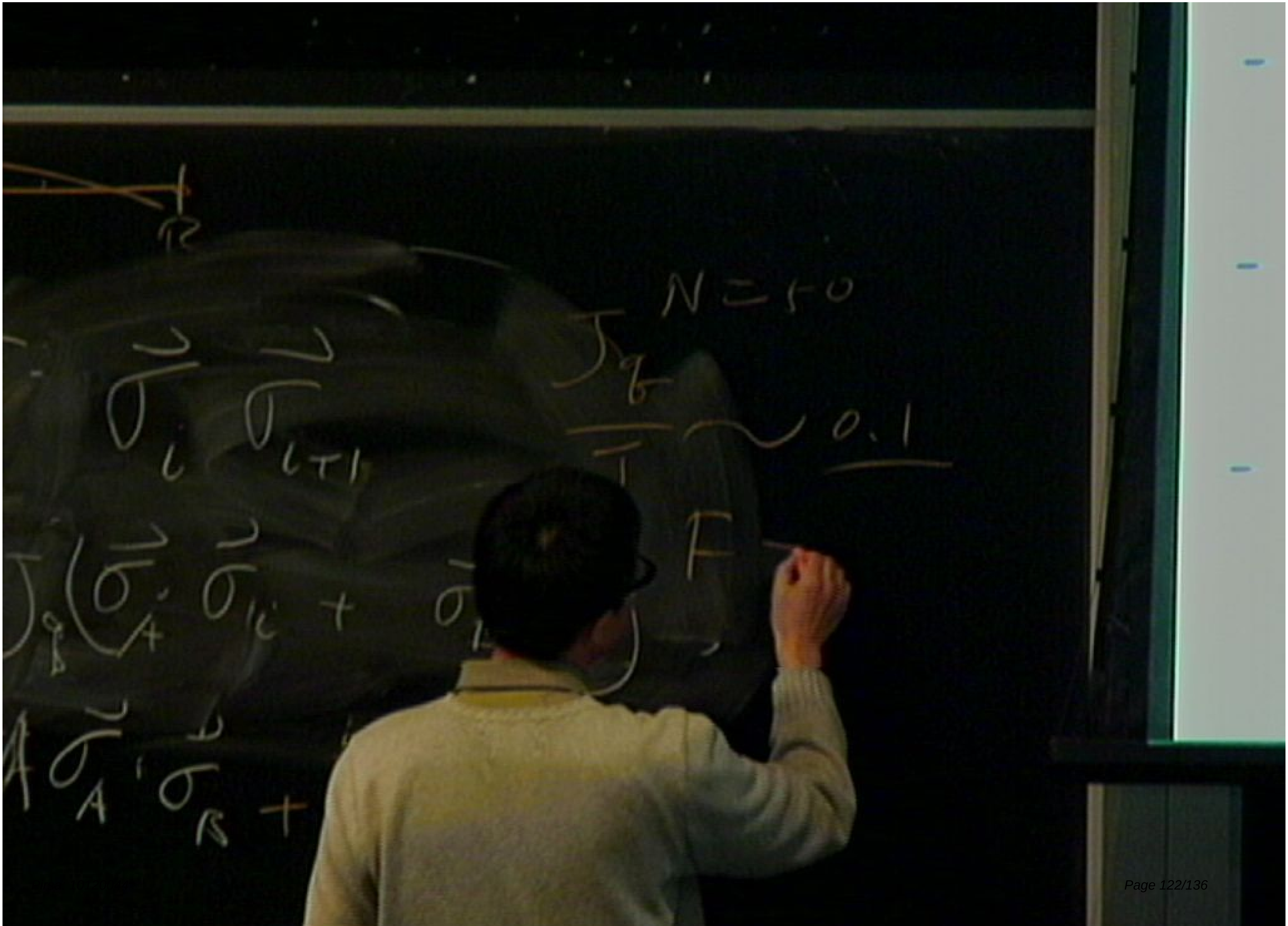
π $\frac{\pi}{2}$

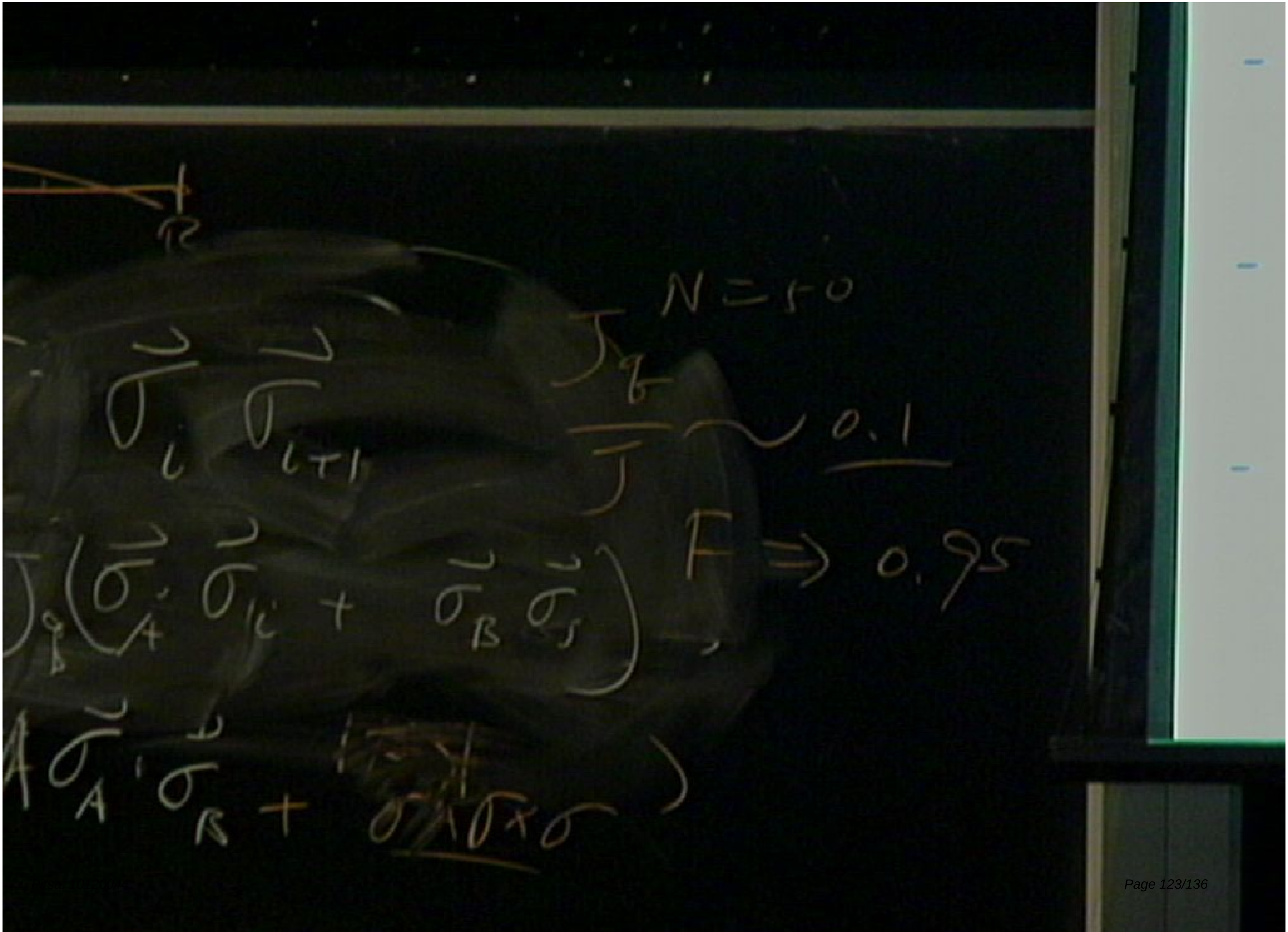
$$H_c = A \vec{\sigma}_A \cdot \vec{\sigma}_B$$

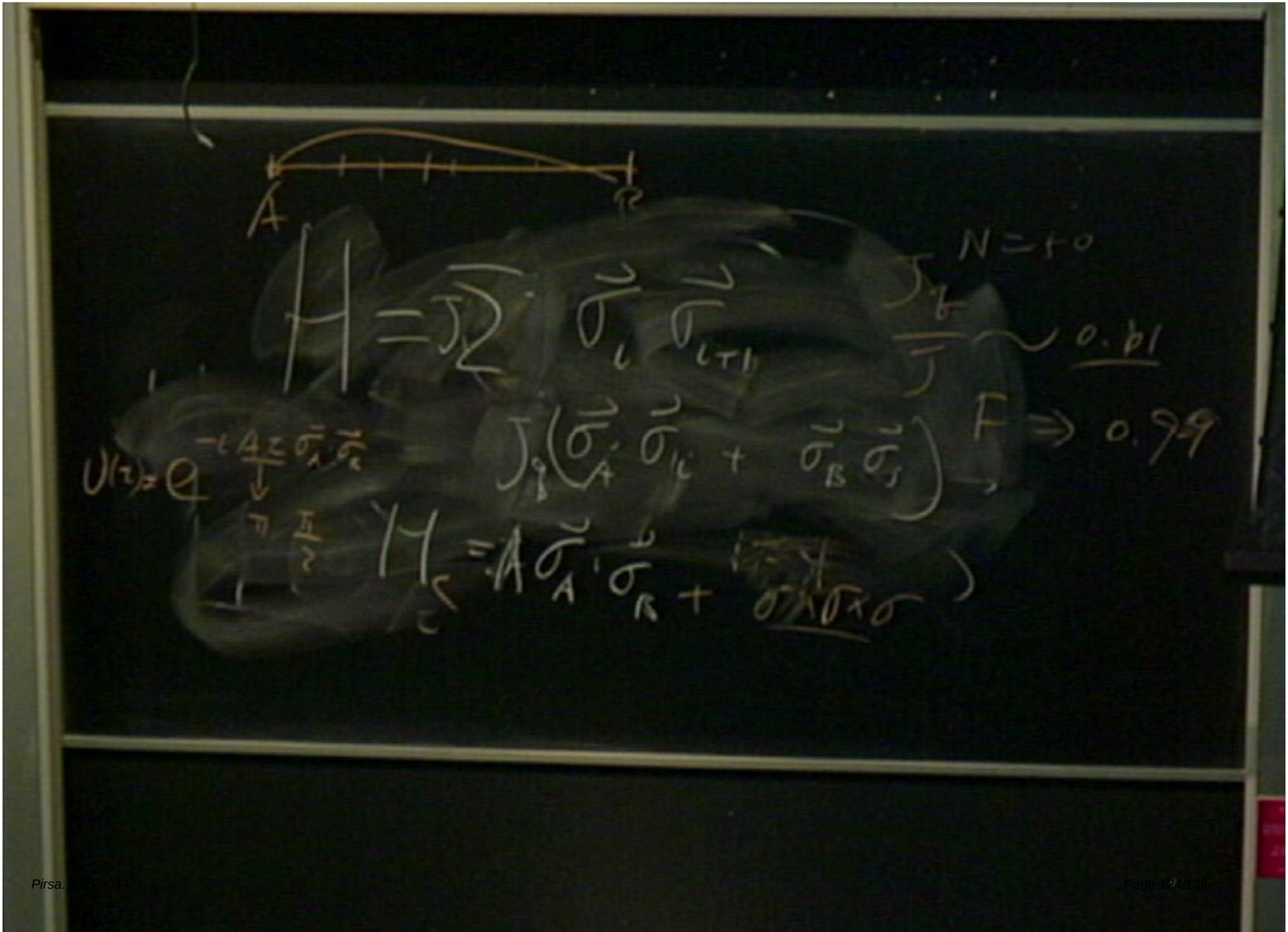
$|\phi_1\rangle_A$

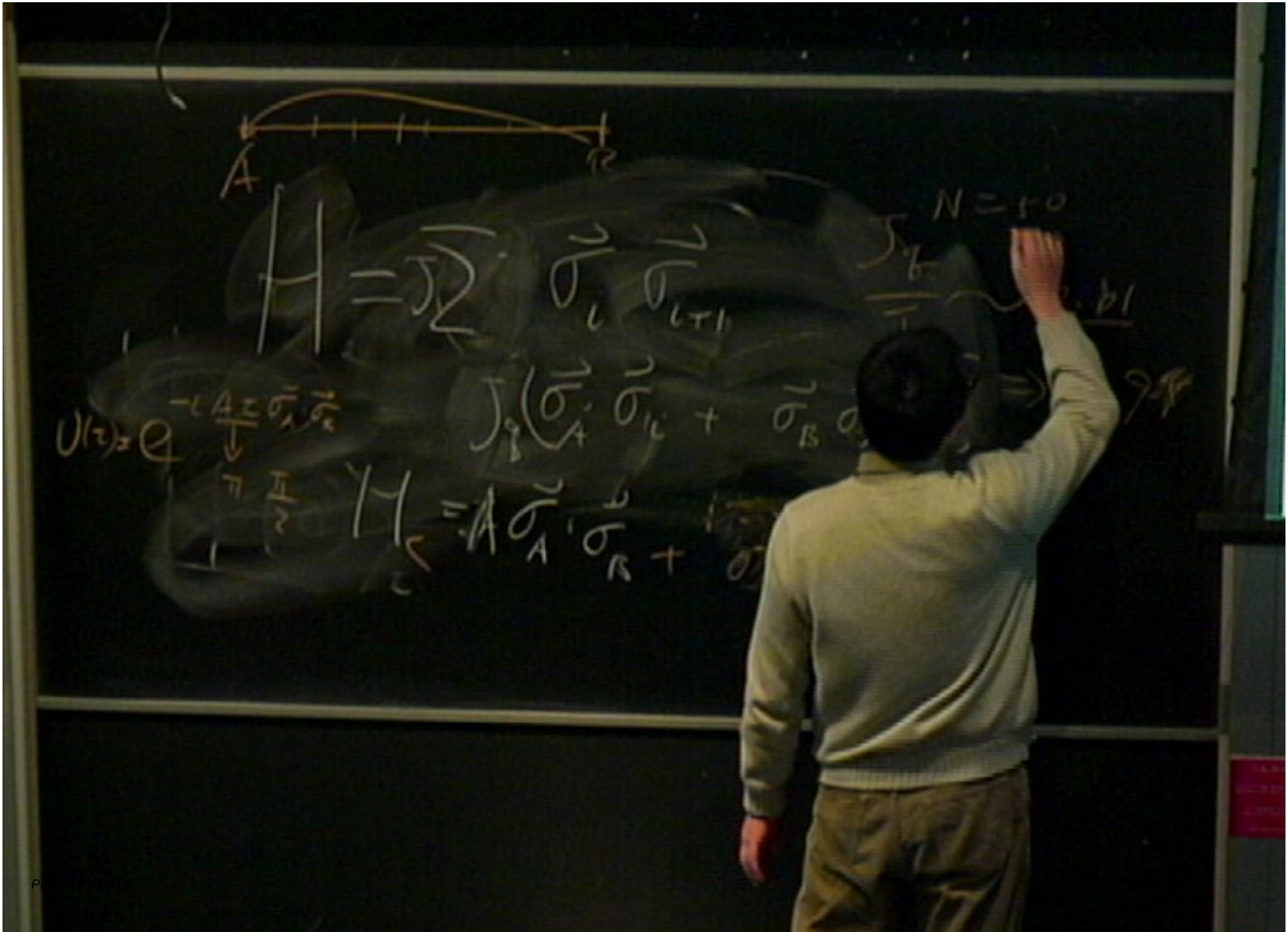



A
B
H = J Σ σ_i σ_{i+1}


N=50
0.1


N = 50
~ 0.1
F ⇒ 0.95


A
N=10
0.61
F ⇒ 0.99


A
N = 10