Title: I^+ and de Sitter - esque Holography?

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Abstract: We will discuss two topics. First we will revisit the asymptotic structure of classical de Sitter space. In particular we will construct charges at future infinity (I^+) and obtain the asymptotic symmetry group drawing parallels with the BMS group of flat space. Secondly, move away from the region I^+ and study the space living near the cosmological horizon by considering large rotating Nariai black holes whose size tends to that of the cosmological horizon. We will examine the resulting near (cosmological) horizon geometry and find an interesting asymptotic structure containing the Virasoro algebra, suggestive of a holographic interpretation.

$\mathcal{I}^+$ and de Sitter-esque Holography?

Dionysios Anninos

November, 2010

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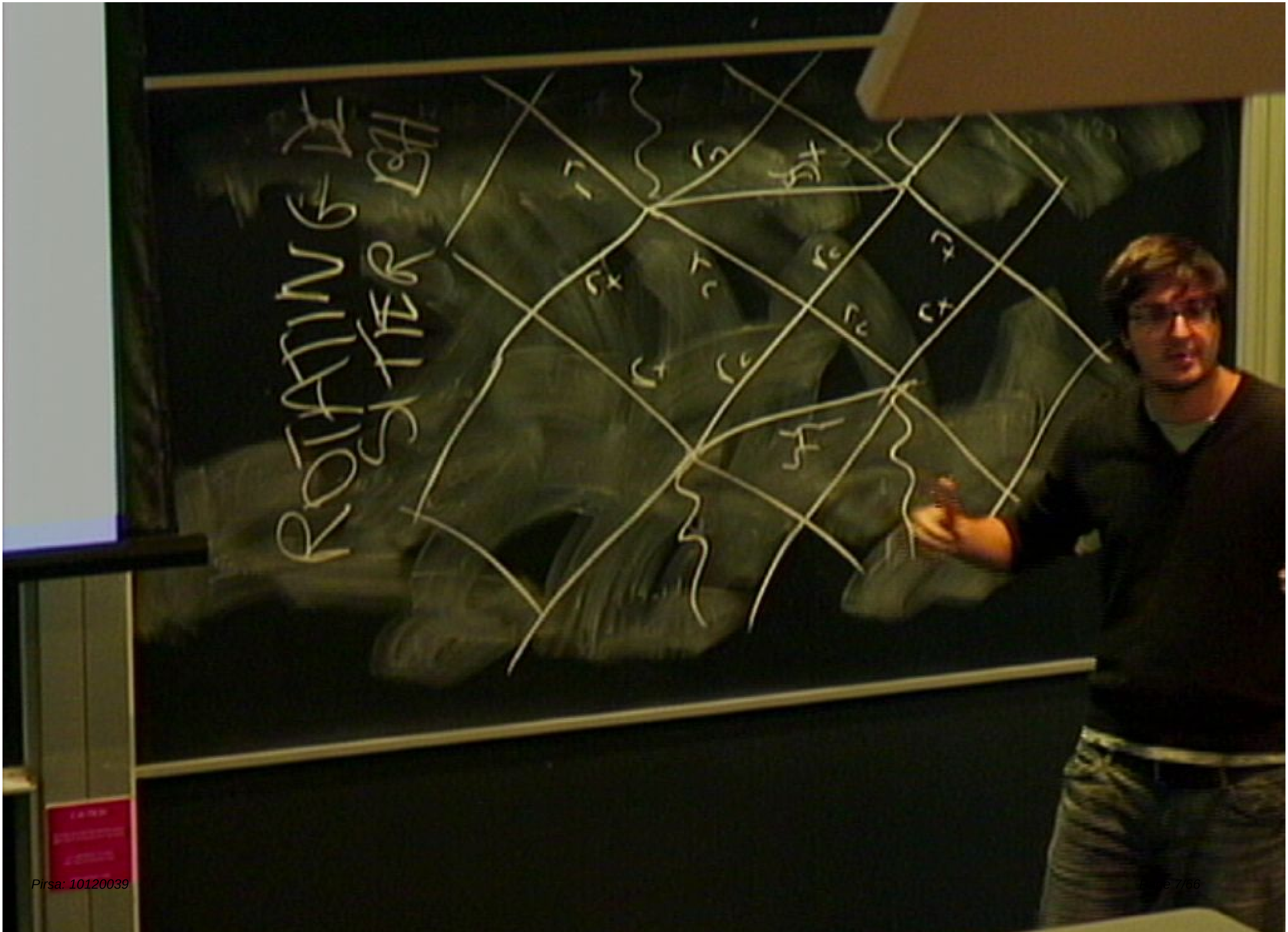
ROTATING DE SITTER BH

Contents

- Brief Review of de Sitter Space
- $\mathcal{I}^+$ in dS_4
- Rotating Black Holes in de Sitter
- Asymptotic Structure of Rotating Nariai
- Scalar Waves in the RN
- Concluding Remarks

de Sitter Space

Maximally symmetric cosmology supported by positive cosmological constant.

Possibly two-de Sitter eras in our Universe: (i) Inflation, (ii) Near future.

Poses an interesting theoretical problem of how holography is defined (if it is at all!) in cosmological spacetimes.

de Sitter Geometry

de Sitter space appears as a solution to Einstein gravity with a positive cosmological constant $\Lambda = +3/\ell^2$. Its metric in the global patch is

$$\frac{ds^2}{\ell^2} = -d\tau^2 + \cosh^2\tau \, d\Omega_d^2$$

No single observer has access to the global patch. The spacetime accessible to a single observer is given by the static patch

$$\frac{ds^2}{\ell^2} = -dt^2(1-r^2) + \frac{dr^2}{(1-r^2)} + r^2 d\Omega_{d-1}^2$$

We notice the appearance of a COSMOLOGICAL HORIZON at $r = 1$.

The conformal boundary is SPACELIKE, and lives at $\mathcal{I}^{\pm}$.


OBSERVER

Hawking Temperature, Entropy

Thus, the observers are immersed in a thermal bath of temperature

$$T_{dS} = \frac{1}{2\pi\ell}$$

and there is a Gibbons-Hawking entropy associated to the static patch given by

$$S_{dS} = \frac{Area}{4G}$$

When the number of spacetime dimensions exceeds THREE, one can have asymptotically de Sitter black holes, whose entropy is always LESS than the entropy of pure de Sitter.

For THREE dimensions, one only has Lorentzian conical defects that are asymptotically de Sitter. Interestingly, the quotients of the 3-sphere, i.e. the spherical three-manifolds, are classified.

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dS/CFT?

The ASYMPTOTIC SYMMETRY GROUP of three-dimensional de Sitter space is two copies of the Virasoro algebra with central charge [Strominger]

$$c_{dS} = \frac{3\ell}{2G}$$

Furthermore, boundary-to-boundary two-point functions of scalar fields were found to behave as

$$\lim_{\tau,\tau'\to\infty} \langle \phi(\tau,x)\phi(\tau',x') \rangle \sim \frac{e^{-h_+(\tau+\tau')}}{|x-x'|^{2h_+}}$$

where $h_+ = 1 + \sqrt{1 - m^2\ell^2}$ is interpreted as the conformal weight of an operator sourced by the bulk scalar.

The above was taken as evidence for a conjecture of a dS/CFT correspondence with the Euclidean CFT living at $\mathcal{I}^+$ (and/or $\mathcal{I}^-$) [Bousso,Maloney,Strominger;Spradlin,Volovich;Guijosa,Lowe;Balasubramanian,de Boer,Minic].

Problems

The conjecture came with some unanswered questions.

The infinite asymptotic symmetries argument only holds for three-dimensions so it cannot be used as non-trivial evidence in higher dimensions. In fact, the story is significantly different in higher dimensions.

A two-dimensional CFT has an exponential asymptotic growth of states, whereas there are NO dS_3 BTZ-like black holes. Even so, Cardy's formula remarkably works for the de Sitter entropy on rotating conical singularities [Bousso,Maloney,Strominger]! More recently, three-dimensional black holes have with warped dS_3 asymptotics [D.A.]...

It is not clear how the two boundaries $\mathcal{I}^{\pm}$ come into play, particularly unlike AdS, it is unclear what the boundary interpretation of the static patch thermal density matrix is.

$\mathcal{I}^+$ IN FOUR-DIMENSIONS

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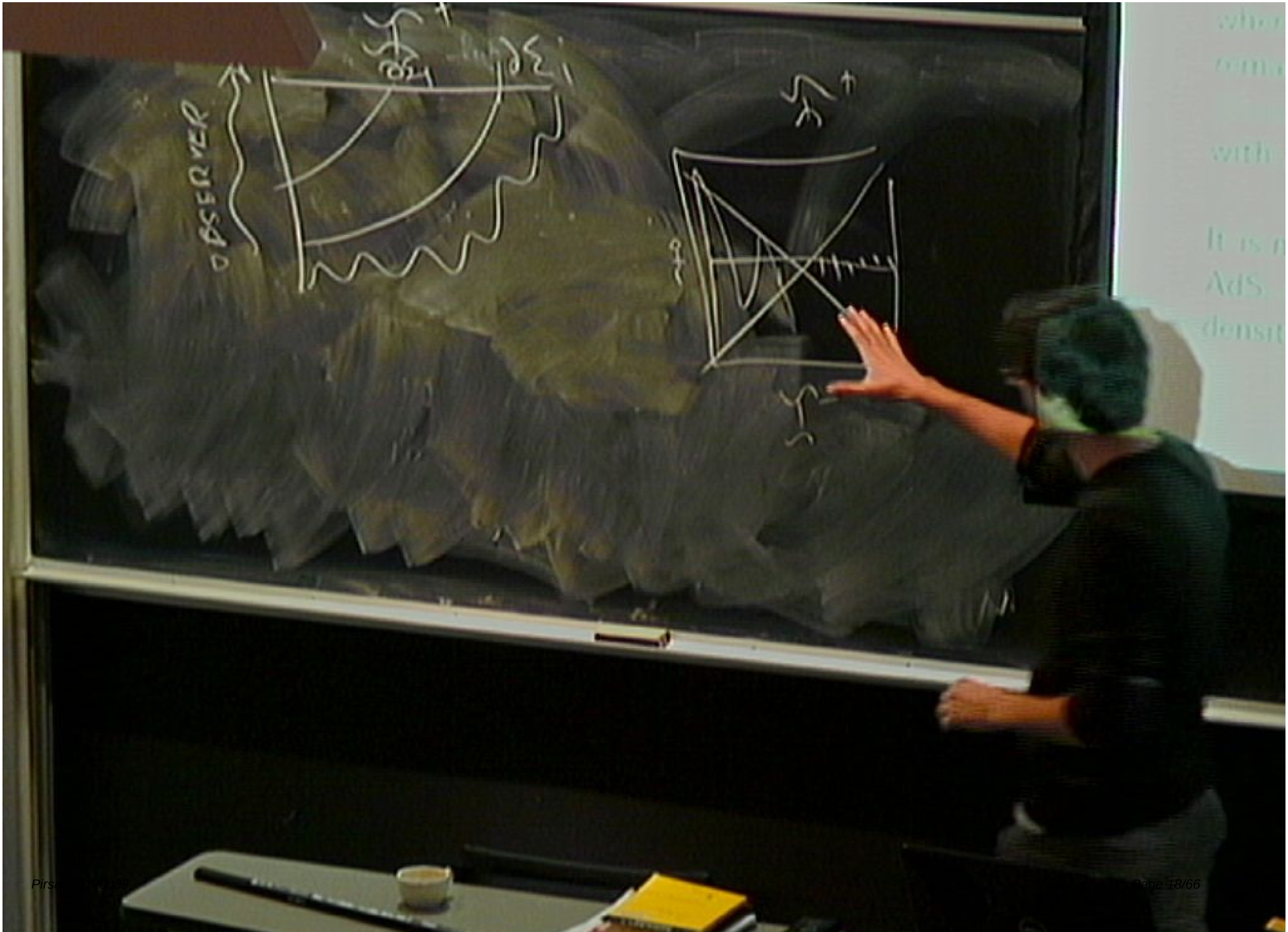
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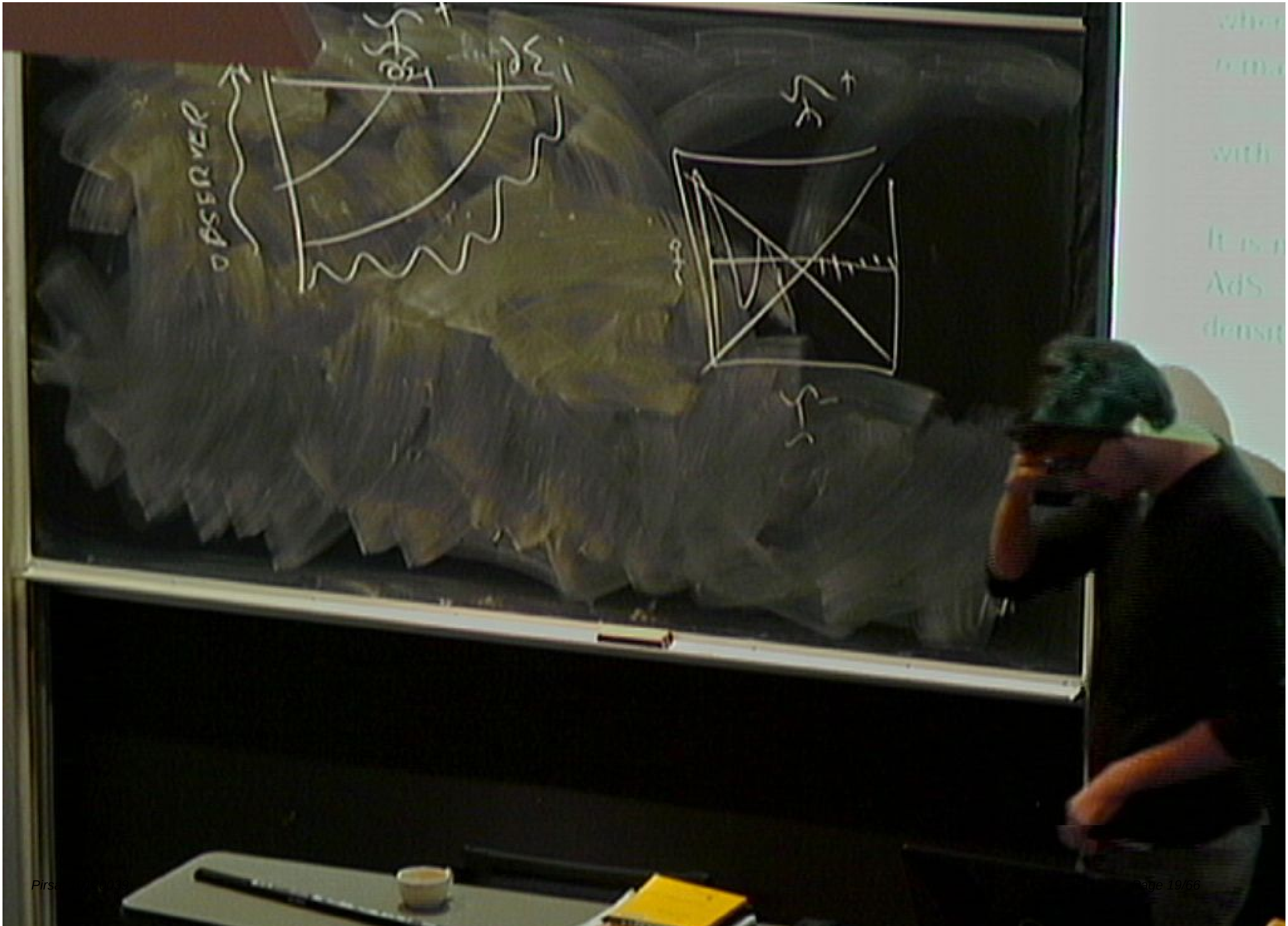
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Asymptotic Expansion

Solutions to Einstein equations with positive cosmological constant can be organized asymptotically in the Fefferman-Graham expansion [Starobinsky]:

$$\frac{ds^2}{\ell^2} = -\frac{d\eta^2}{\eta^2} + \frac{dx^i dx^j}{\eta^2}\left(g_{ij}^{(0)} + \eta^2 g_{ij}^{(2)} + \ldots\right) + \frac{dx^i dx^j}{\eta^2}\left(\eta^3 g_{ij}^{(3)} + \ldots\right)$$

where $\mathcal{I}^+$ lives at $\eta \to 0$.

In four-dimensions, unlike three, there exist propagating gravitons. Generic initial conditions for the graviton lead to variations of $g^{(0)}$ and $g^{(3)}$.

Note that $\mathrm{Tr} g_{ij}^{(3)} = \nabla^i g_{ij}^{(3)} = 0$, $g_{ij}^{(k)}$ are functions of $g^{(0)}$ and $g^{(3)}$ for $k > 3$.

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FG Preserving Diffeomorphisms

The Fefferman-Graham expansion is fully specified with the boundary data $(g^{(0)}, g^{(3)})$ and has a conformal structure in that it is defined up to conformal transformations of $g^{(0)}$. The conformal weights of $g^{(0)}_{ij}$ and $g^{(3)}_{ij}$ are $s = 2$ and $s = -1$ respectively.

Furthermore, the Fefferman-Graham preserving diffeororphisms are given by:

$$\begin{aligned} \xi^\eta &= \eta\delta\sigma(\vec{x}) \\ \xi^i &= \phi^i(\vec{x}) + \frac{\eta^2}{2} g^{(0)ij}\partial_j\delta\sigma(\vec{x}) + \dots \end{aligned}$$

where the purely $\delta\sigma$ diffeomorphisms give rise to scale transformations and the ϕ^i diffeomorphisms are tangent to $\mathcal{I}^+$.

Boundary Conditions

Given that gravitons in asymptotically dS_4 will generically deform both $g^{(0)}$ and $g^{(3)}$, we are led to propose that the appropriate boundary conditions for dS_4 are all spacetimes obeying the Fefferman-Graham expansion, with boundary data within the same conformal class identified [D.A.,Ng,Strominger].

Notice that these boundary conditions differ from those of AdS_4 which freeze $g^{(0)}$, i.e. Dirichlet boundary conditions. In AdS_4 we are allowed to do this because $g^{(0)}$ lives at spacelike infinity where we can freely impose boundary conditions.

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ASG

Our task is to understand what the asymptotic symmetry group of dS_4^+ is. From the geometric structure of $\mathcal{I}^+$ we are already hinted at:

$$\xi_{ASG} = \phi^i(x^i)\partial_i \ .$$

i.e. diffeomorphisms tangent to $\mathcal{I}^+$. We would like to render diffeomorphisms corresponding to conformal transformations of the boundary metric TRIVIAL, given that they are maps between members of the same conformal class.

The infinite dimensional group is closer in spirit to the BMS group of asymptotically flat space in four-dimensions than the finite dimensional group of AdS_4.

Charges I

We must construct a set of 'conserved' quantities that allow boundary conditions containing both $\delta g^{(0)}$ and $\delta g^{(3)}$. Our construction follows from a recent analysis by Compere and Marolf, in an attempt to render fluctuations of the AdS boundary normalizable.

We begin with the regularized Lagrangian constructed in [de Boer,Balasubramanian, Minic]

$$S_{reg} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\,(R-2\Lambda) - \frac{1}{8\pi G}\int_{\mathcal{I}^+} d^3x\sqrt{\gamma}K + \frac{1}{16\pi G}\int_{\mathcal{I}^+} d^3x\sqrt{\gamma}\,[R[\gamma]-4+\ldots]\,.$$

Using covariant phase space techniques, one usually constructs a symplectic form $\omega_{EH}(\delta g_1, \delta g_2)$ out of the Einstein-Hilbert action and then defines charges with respect an asymptotic symmetry ξ. In our case, such charges would diverge and are ill-defined. However, ω_{EH} is ambiguous up to a total derivative.

Charges II

Following [Compere,Marolf,Wald,Zoupas] we can construct a set of finite and integrable charges based on the following symplectic three-form:

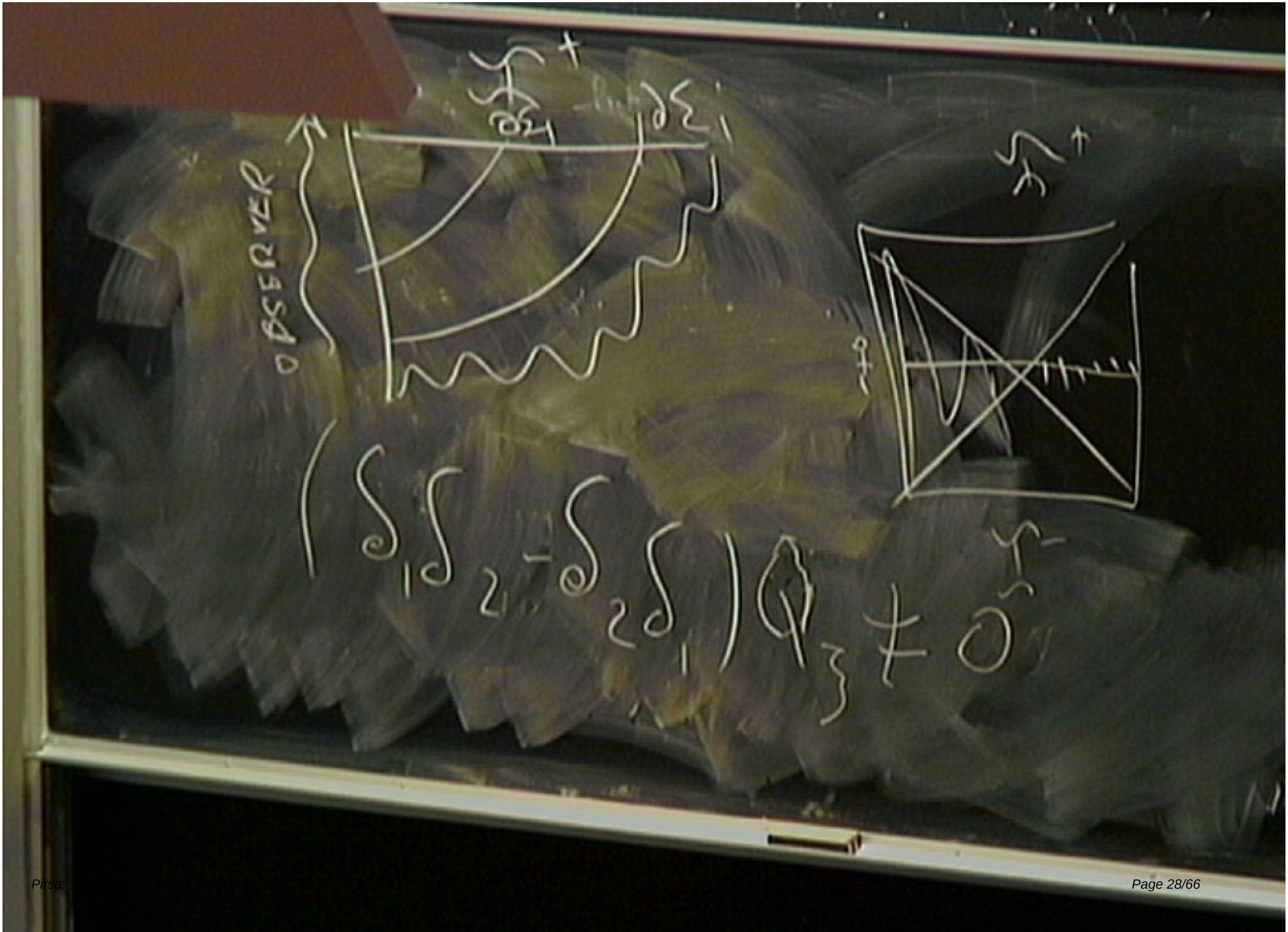
$$\omega_{mod} = \omega_{EH}(\delta g_1, \delta g_2) + d\omega_{ct}(\delta\gamma_1, \delta\gamma_2).$$

The finite charge for an asymptotic symmetry ξ is defined on a two-dimensional cut on $\mathcal{I}^+$ and given by:

$$\delta Q_\xi = \delta\left(\int_{\partial\Sigma} d^2x\sqrt{\sigma}n^i\xi^j T_{ij}^{BY}\right) - \frac{1}{2}\int_{\partial\Sigma} d^2x\sqrt{\sigma}n^k\xi_k T_{BY}^{ij}\delta g_{ij}^{(0)}.$$

Though finite, these charges are non-integrable and non-conserved as is the case for the BMS charges. Wald and Zoupas have constructed a unique boundary counterterm to render the charges integrable (but not conserved since flux leaks through $\mathcal{I}^+$).

One sees that the charges indeed vanish for diffeomorphisms which rescale $g^{(0)}$, i.e. the $\delta\sigma$ diffeomorphisms, as we suspected!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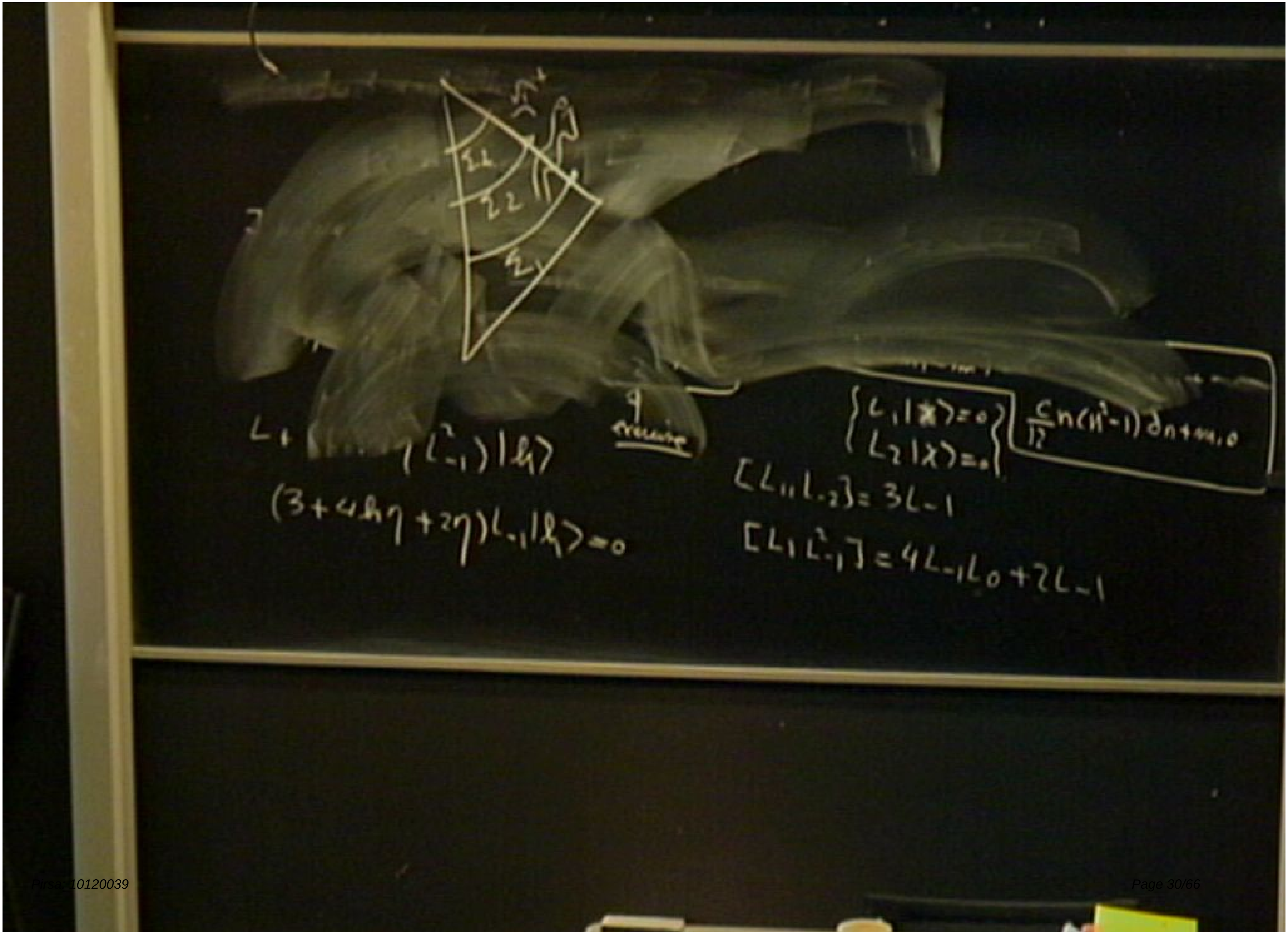

Σ2
Σ2
Σ1
(L-1^2)|h>
(3+4hη+2η)L-1|h>=0
L1|χ>=0
L2|χ>=0
c/12 n(n^2-1) δn+m,0
[L1, L-2] = 3L-1
[L1, L-1^2] = 4L-1L0 + 2L-1

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Conservation Equation

The on-shell charges (corrected by the WZ term) are found to be:

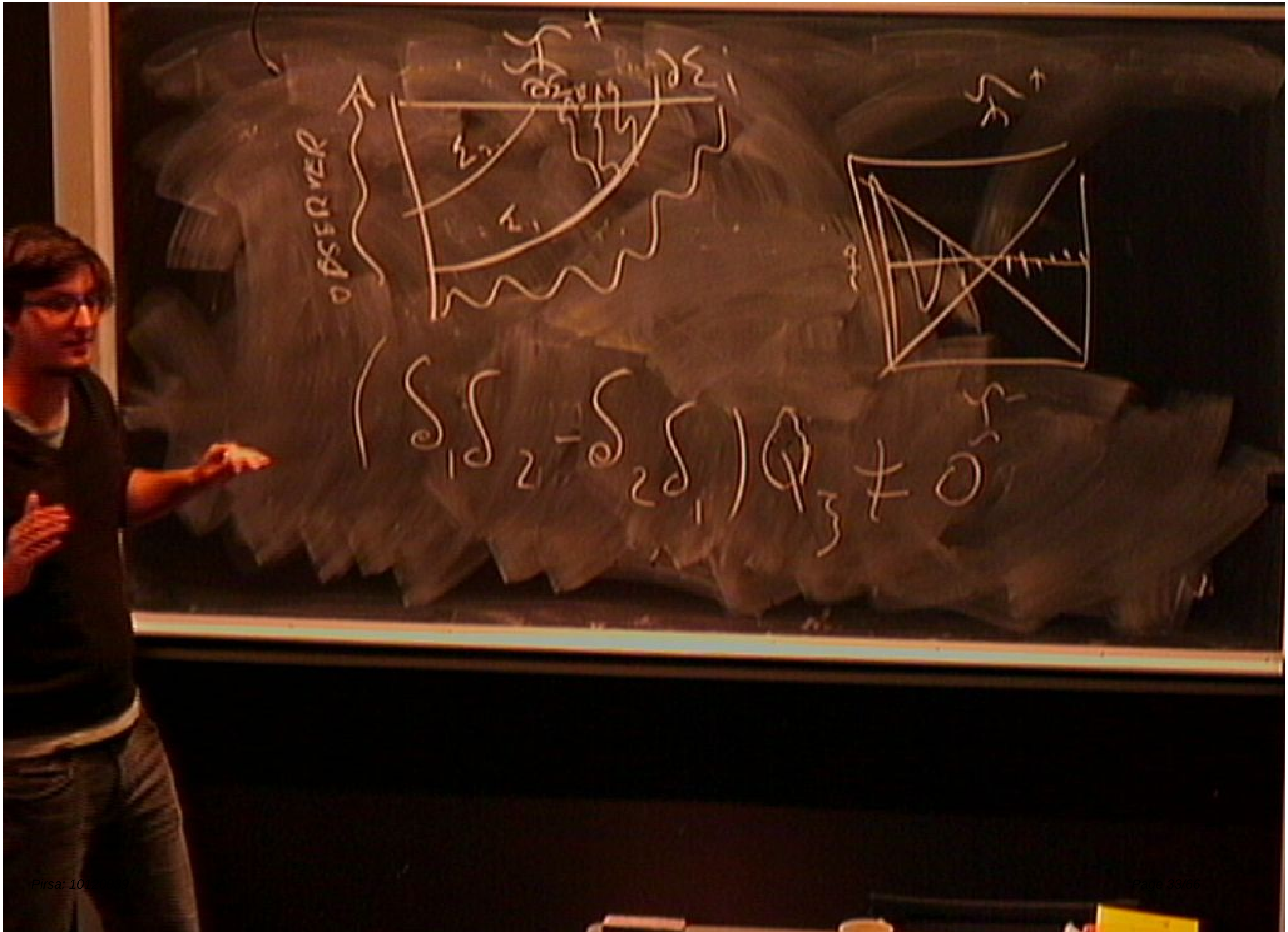
$$Q_\xi = \int_{\partial\Sigma} d^2x \sqrt{\sigma} n^i \xi^j T^{BY}_{ij} = Q_{BY}, \quad \xi^i \in \xi^i_{ASG}.$$

These are the BROWN-YORK charges defined on a 2d submanifold $\partial\Sigma$ of $\mathcal{I}^+$ for the regularized Brown-York stress tensor $T^{BY}_{ij} \sim g^{(3)}_{ij}$ satisfying $\mathrm{Tr}\, T^{BY}_{ij} = \nabla^i T^{BY}_{ij} = 0$.

The charges obey a CONSERVATION EQUATION:

$$Q_\xi[\partial\Sigma_2] - Q_\xi[\partial\Sigma_1] = \frac{1}{2}\int_{\mathcal{B}_{12}} d^3x \sqrt{g^{(0)}}\, T^{ij} \mathcal{L}_\xi g^{(0)}_{ij}.$$

This is reminiscent of the flux going through $\mathcal{I}^+$ in flat space where T^{ij} would be the Bondi news tensor, indicating the presence of gravitational radiation.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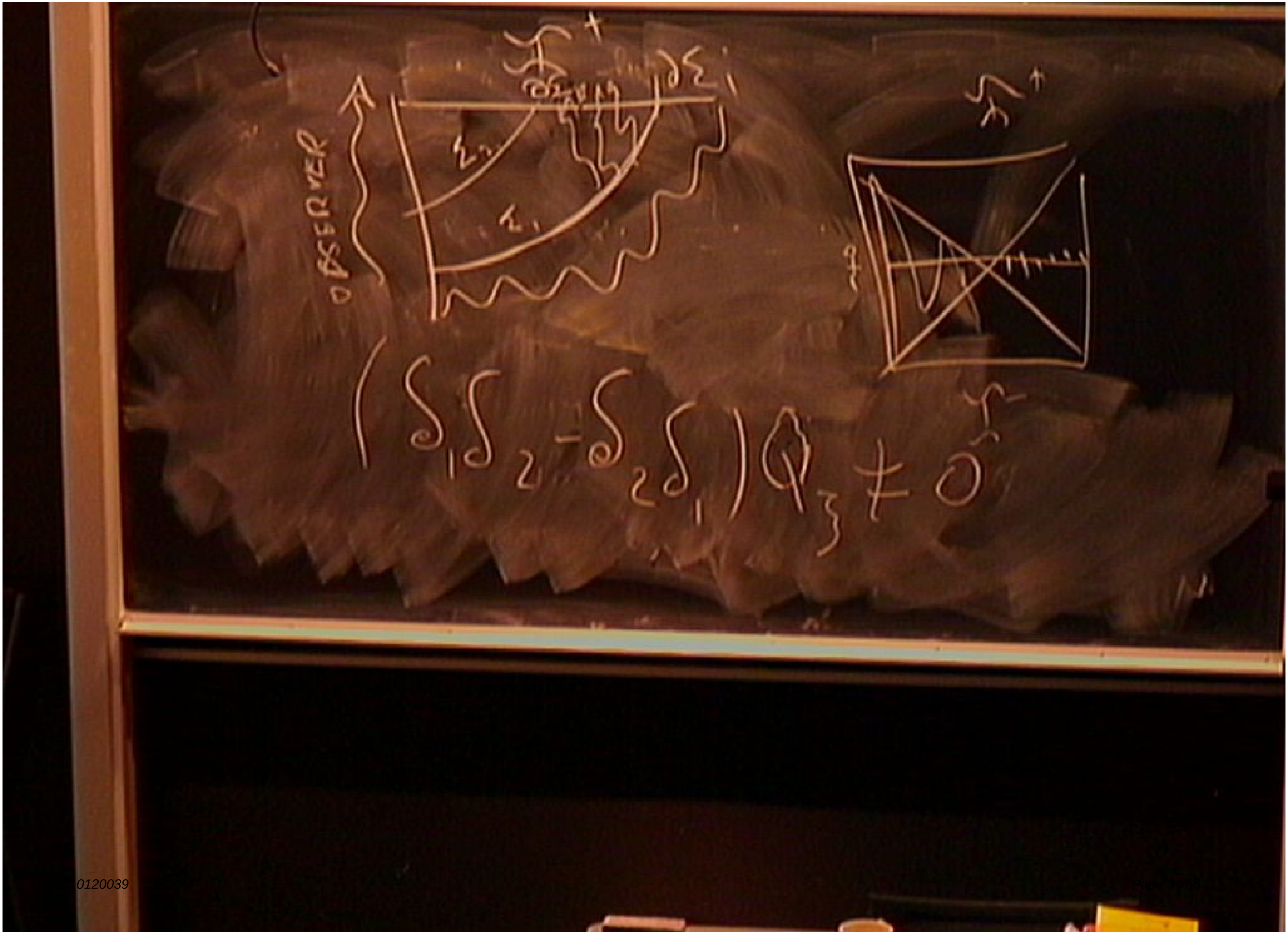

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$\mathcal{I}^+$ - Summary

We have found finite integrable charges at $\mathcal{I}^+$ for all variations of the metric preserving the Fefferman-Graham form.

The asymptotic symmetries associated to these charges are given by the diffeomorphisms on $\mathbb{R}^3$, i.e. those tangent to $\mathcal{I}^+$. Indeed charges with respect to $\delta\sigma \neq 0$ diffeomorphisms (which move $\mathcal{I}^+$) vanish.

This is closer in analogy to the infinite dimensional BMS group which is the ASG of flat space, than the finite dimensional ASG of AdS_4.

We will now move on to an analysis of asymptotic symmetries near the cosmological horizon...

ROTATING BLACK HOLES IN DE SITTER SPACE

Kerr de Sitter Space

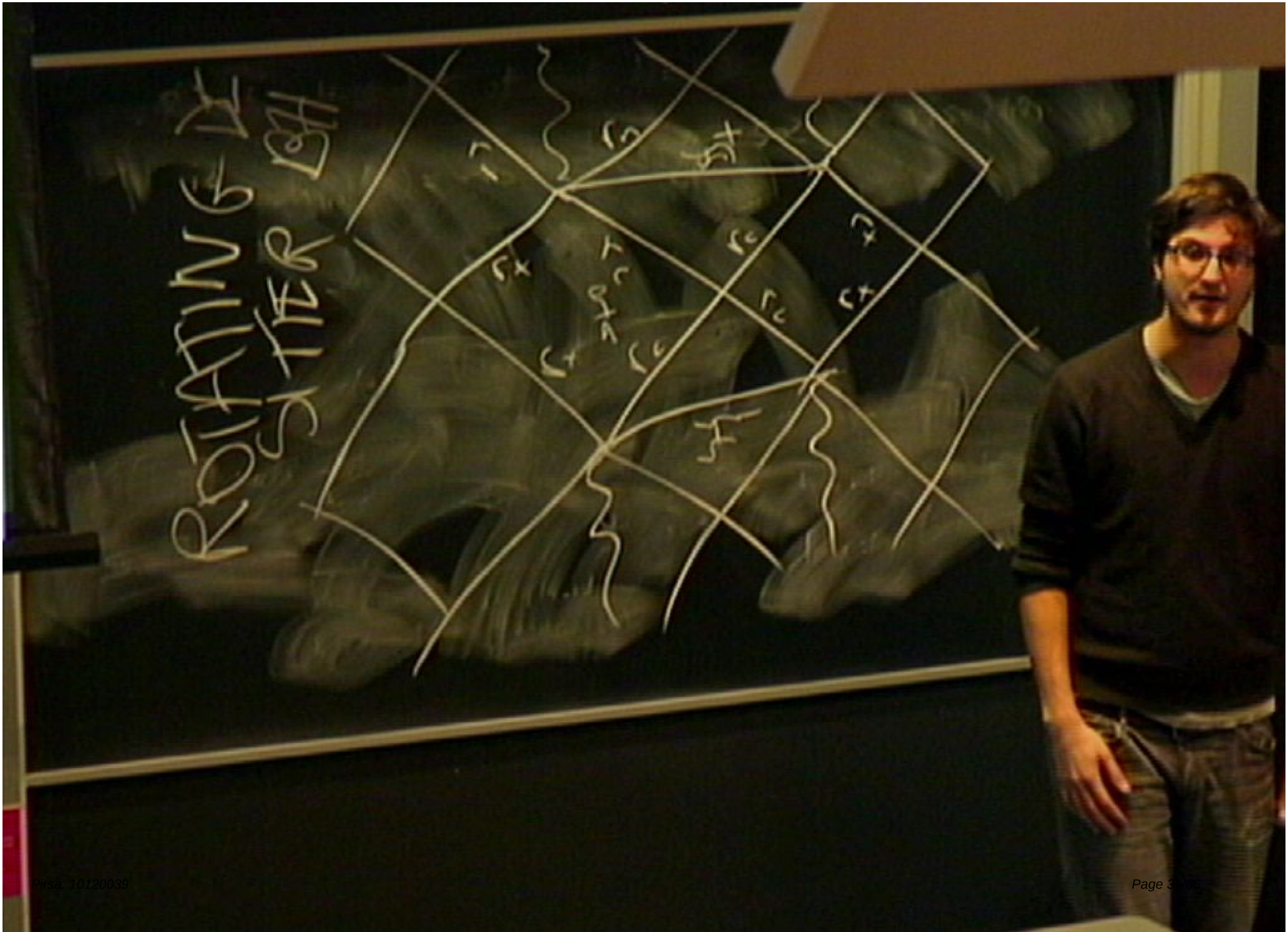
The general rotating black hole solution with positive cosmological constant is given by

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a}{\Xi}\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2$$

where

$$\Delta_r = (r^2+a^2)\left(1-\frac{r^2}{\ell^2}\right) - 2Mr, \quad \Xi = 1 + \frac{a^2}{\ell^2},$$
$$\Delta_\theta = 1 + \frac{a^2}{\ell^2}\cos^2\theta, \quad \rho^2 = r^2 + a^2\cos^2\theta$$

Δ_r has four real roots, the two largest, r_+ and r_c, being the black hole and cosmological horizons.



Interesting Regions of Parameter Space

There are several regions in the parameter space of Kerr-de Sitter space worth noting. Particularly we have:

EXTREMAL BLACK HOLES with vanishing temperature.

LUKEWARM BLACK HOLES with equal temperature to the cosmological horizon.

ROTATING NARIAI BLACK HOLES with size equal to the cosmological horizon.

There exist smooth Euclidean instantons corresponding to these geometries [Booth,Mann], unlike most Euclideanizations of black holes in de Sitter which have conical singularities. We will now focus on the rotating Nariai black holes.

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Thermodynamics

The mass, angular momentum and cosmological entropy of the Kerr-de Sitter geometry are given by:

$$\mathcal{Q}_{\partial_t} = -\frac{M}{\Xi^2}, \quad \mathcal{Q}_{\partial_\phi} = \frac{aM}{\Xi^2}, \quad S_c = \frac{\pi\left(r_c^2 + a^2\right)}{\Xi}$$

and follow the usual first law of thermodynamics. These charges are defined at $\mathcal{I}^+$ via the regularized Brown-York stress tensor.

The first law of thermodynamics for Kerr-de Sitter black holes in the rotating Nariai limit becomes:

$$dS_c = \beta_L d\mathcal{Q}_{\partial_\phi}, \quad \beta_L = T_L^{-1} = 2\pi k.$$

Scaling Limit of Rotating Nariai Black Hole

When we take the rotating Nariai limit $r_+ \to r_c$, it is in fact possible to also take a scaling limit with an infinite scaling geometry which we call the rotating Nariai geometry.

$$t' = b\lambda t, \quad r' = \frac{(r - r_+)}{\lambda r_+}, \quad \phi' = \phi - \Omega_{BH} t, \quad \tau \equiv \frac{r_c - r_+}{r_+}.$$

This leads to the following metric upon $\lambda \to 0$ and τ/λ fixed:

$$ds^2 = \Gamma(\theta)\left(-r(\tau - r)dt^2 + \frac{dr^2}{r(\tau - r)}\right) + \gamma(\theta)\left(d\phi + krdt\right)^2 + \alpha(\theta)d\theta^2$$

In global coordinates we find:

$$ds^2 = \Gamma(\theta)\left(-dt^2 + \cosh^2 t d\psi^2\right) + \gamma(\theta)\left(d\phi + k \sinh t d\psi\right)^2 + \alpha(\theta)d\theta^2$$

with $\psi \sim \psi + 2\pi$.

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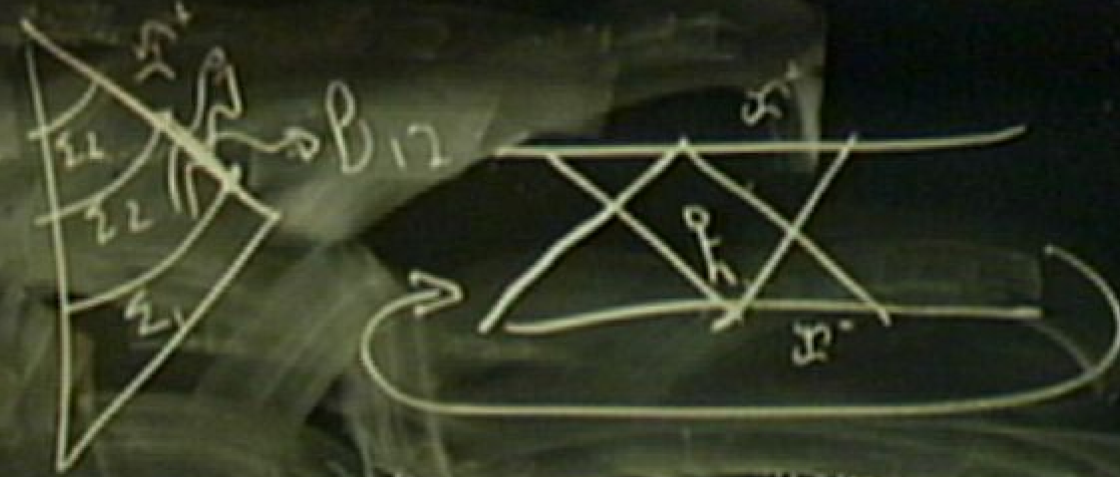

$L_1 \; \ldots \; (L_{-1}^2)|h\rangle$

$(3+4h\eta+2\eta)L_{-1}|h\rangle = 0$

η freezing

$\begin{cases} L_1|\chi\rangle = 0 \\ L_2|\chi\rangle = 0 \end{cases}$ $\quad \frac{c}{12}n(n^2-1)\delta_{n+m,0}$

$[L_1, L_{-2}] = 3L_{-1}$

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Geometry of Rotating Nariai

It is a de Sitter version of the NHEK geometry, i.e. an S^2 fibration over dS_2.

Constant time slices of the global geometry have an $S^1 \times S^2$ topology.

There is an $SL(2,\mathbb{R}) \times U(1)$ four-dimensional isometry group.

The parameter τ is a near-extremality parameter which when non-zero allows for the black hole and cosmological horizons to be preserved.

At constant polar angle θ the geometry becomes that of warped de Sitter space, which is a solution to topologically massive gravity with positive cosmological constant. This maybe a simpler context to study rotating Nariai...

Asymptotic Symmetries I

We would now like to explore the asymptotic symmetry group of the rotating Nariai geometry [D. A., Hartman].

Recall that these are given by the set of diffeomorphisms obeying certain boundary conditions quotiented by the trivial ones. By trivial we mean diffeomorphisms with vanishing Barnich-Brandt charges at $\mathcal{I}^+_{RN}$.

We propose the following boundary conditions at $\mathcal{I}^+_{RN}$ (recall large r is a time coordinate):

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The diffeomorphisms preserving the above boundary conditions are given by:

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such that $[\zeta_m, \zeta_n] = -i(m-n)\zeta_{m+n}$.

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One can find the asymptotic symmetry group of the rotating Nariai geometry, to be a single copy of the Virasoro algebra with REAL, POSITIVE central charge:

$$c_L = \frac{12 r_c^2 \sqrt{(1-3r_c^2/\ell^2)(1+r_c^2/\ell^2)}}{-1+6r_c^2/\ell^2+3r_c^4\ell^4}.$$

As in Kerr/CFT the ASG comes from the $U(1)$ isometries. Notice further that the central charge vanishes when $r_c^2 = \ell^2/3$, this is the non-rotating Nariai geometry.

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Geometry of Rotating Nariai

It is a de Sitter version of the NHEK geometry, i.e. an S^2 fibration over dS_2.

Constant time slices of the global geometry have an $S^1 \times S^2$ topology.

There is an $SL(2,\mathbb{R}) \times U(1)$ four-dimensional isometry group.

The parameter τ is a near-extremality parameter which when non-zero allows for the black hole and cosmological horizons to be preserved.

At constant polar angle θ the geometry becomes that of warped de Sitter space, which is a solution to topologically massive gravity with positive cosmological constant. This maybe a simpler context to study rotating Nariai...

Scaling Limit of Rotating Nariai Black Hole

When we take the rotating Nariai limit $r_+ \to r_c$, it is in fact possible to also take a scaling limit with an infinite scaling geometry which we call the rotating Nariai geometry.

$$t' = b\lambda t, \quad r' = \frac{(r - r_+)}{\lambda r_+}, \quad \phi' = \phi - \Omega_{BH} t, \quad \tau \equiv \frac{r_c - r_+}{r_+}.$$

This leads to the following metric upon $\lambda \to 0$ and τ/λ fixed:

$$ds^2 = \Gamma(\theta)\left(-r(\tau - r)dt^2 + \frac{dr^2}{r(\tau - r)}\right) + \gamma(\theta)\left(d\phi + krdt\right)^2 + \alpha(\theta)d\theta^2$$

In global coordinates we find:

$$ds^2 = \Gamma(\theta)\left(-dt^2 + \cosh^2 t d\psi^2\right) + \gamma(\theta)\left(d\phi + k \sinh t d\psi\right)^2 + \alpha(\theta)d\theta^2$$

with $\psi \sim \psi + 2\pi$.

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Rotating Nariai/CFT Proposal

One is thus led to propose that the rotating Nariai geometry is HOLOGRAPHICALLY DUAL to a two-dimensional Euclidean conformal field theory.

Indeed, somewhat mysteriously, the cosmological entropy is given by the Cardy formula

$$S_c = \frac{\pi^2}{3} T_L c_L.$$

Notice further that this is a different class of dualities which is not continuously connected to dS/CFT or Kerr/CFT. The natural test lies in a thorough analysis of fields defined in this geometry.

Beyond $T_L = 0$?

One can compute the change in cosmological entropy when we go away from the Nariai limit. It is in fact POSITIVE.

From the point of view of the rotating Nariai/CFT proposal, leaving the Nariai limit corresponds to adding some right moving energy which in turn also increases the entropy as follows from Cardy:

$$S_c = \frac{\pi^2}{3} T_L c_L + \frac{\pi^2}{3} T_R c_R,$$

so long as $c_L = c_R > 0$.

One can attempt to compute c_R by studying the asymptotic symmetries of an effective 2d theory of gravity obtained from a Kaluza-Klein reduction of a class of 4d metrics containing the rotating Nariai metric.

This is work in progress, but looks promising.

In particular, the picture is EXACTLY realized for a closely related family of solutions in three dimensional gravity known as warped de Sitter spaces.

SCALAR WAVES IN ROTATING NARIAI

Scalar Waves I - ANSATZ

One finds further evidence for the correspondence by studying free scalar perturbations in the rotating Nariai background [D. A.,T.Anous]:

$$\nabla^2\Phi = 0.$$

We choose the following ansatz: $\Phi(t, r, \phi, \theta) = e^{-i\omega t + im\phi} R(r) Y_{lm}(\theta)$.

Then the equations of motion SEPARATE and can be solved exactly in terms of HYPERGEOMETRIC FUNCTIONS.

The large r, i.e. late time, behavior goes as:

$$\Phi \sim r^{-h_\pm}, \quad h_\pm = \frac{1}{2} \pm \frac{i}{2}\sqrt{4m^2k^2 + 4j_{lm} - 1}.$$

Thus, conformal weights are COMPLEX (perhaps this implies principal series irreps are the correct ones [Guijosa,Lowe])

Scalar Waves II - QUASINORMAL MODES

Demanding that the waves have ingoing flux and the black hole horizon and outgoing flux at the cosmological horizon, our spectrum becomes quantized and takes the form:

$$\begin{aligned} m &= -2\pi i T_L(n + h_L), \quad n = 0, 1, 2, 3, \ldots \\ \omega &= -2\pi i T_R\left(n + h_R\right), \quad n = 0, 1, 2, 3, \ldots \end{aligned}$$

where $T_R = \tau/4\pi$ is the temperature of the cosmological horizon in the rotating Nariai geometry and $h_L = h_R = h_\pm$.

These have the form of the poles of a thermal retarded Green's function in a CFT_2 (upon identifying the various CFT quantities).

Scalar Waves III - $\mathcal{I}^+_{RN}$ CORRELATORS

Boundary-to-boundary correlators at $\mathcal{I}^+_{RN}$ take the form of the thermal correlators in a two-dimensional CFT, upon imposing that the scalar modes are purely ingoing at the black hole horizon.

We compute these using the AdS/CFT inspired prescription:

$$\langle \mathcal{O}_{\phi_0}(x)\mathcal{O}_{\phi_0}(y)\rangle = \frac{\delta^2}{\delta\phi_0(x)\delta\phi_0(y)} S_{matter}\left[\phi_0\right]$$

Explicitly, the late time behavior of the scalar is given by:

$$\Phi = \phi_0\left(r^{-h_+} + \gamma r^{-1+h_+}\right)$$

then the boundary-to-boundary correlator at $\mathcal{I}^+$ is found to be γ which has precisely the structure of a thermal correlator in a CFT_2. Particularly, the poles of the boundary-to-boundary correlators precisely agree with the quasinormal modes.

Scalar Waves IV - α-Vacua

As in pure de Sitter space, we can define a complex parameter worth of quantum vacua for the free scalar field known as the α-vacua.

Three interesting vacua are:

- the $|in\rangle$ vacuum (defined by positive frequency modes at $\mathcal{I}^-_{RN}$),
- the $|out\rangle$ vacuum (defined by positive frequency modes at $\mathcal{I}^+_{RN}$) and
- the $|E\rangle$ vacuum (defined by modes which are analytic in the lower hemisphere of the Eucledeanized dS_2).

Interestingly, the rotating Nariai geometry has cosmological particle creation for scalars in ALL dimensions. We find

$$\langle in|a^\dagger_{out} a_{out}|in\rangle = \cosh^2(\pi m k)\,\mathrm{csch}^2(\pi\mu/2)\,.$$

This vacuum structure is NOT given by an analytic continuation of the vacua of free scalars in NHEK.

Conclusions and Challenges

- There is a rich structure residing at $\mathcal{I}^+$ of dS_4 which begs for an interpretation
- Particularly, what is the quantum extension of the classical story, i.e. IR divergences, bubble nucleation...
- Rotating Black holes in de Sitter Space have a rich symmetry group in the Nariai limit.
- What is the non-zero T_R extension of our result? Can we apply near-extremal Kerr/CFT techniques?
- Is there a hidden (broken) conformal symmetry for more general rotating de Sitter black holes?
- Why does Cardy's formula work?