Title: The North American Nanohertz Observatory of Gravitational Waves (NANOGrav)

Date: Dec 16, 2010 03:00 PM

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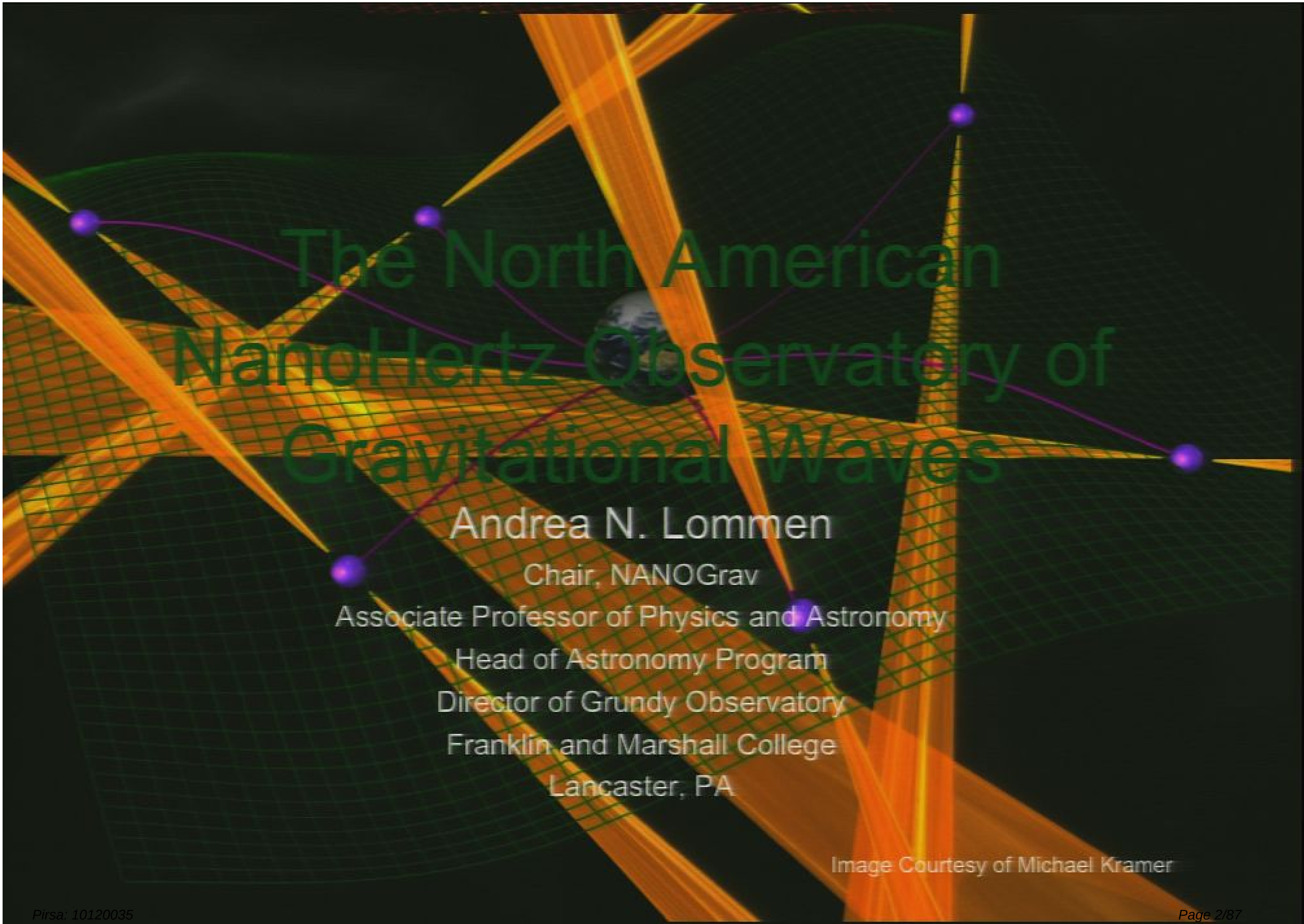
Abstract: NANOGrav is a consortium of radio astronomers and gravitational wave physicists whose goal is to detect gravitational waves using an array of millisecond pulsars as clocks. Whereas interferometric gravitational wave experiments use lasers to create the long arms of the detector, NANOGrav uses earth-pulsar pairs. The limits that pulsar timing places on the energy density of gravitational waves in the universe are on the brink of limiting models of galaxy formation and have already placed limits on the tension of cosmic strings. Pulsar timing has traditionally focused on stochastic sources, but most recently I have been investigating the idea of detecting individual gravitational wave bursts wherein there are some interesting advantages. I will also demonstrate how the array can be used to reconstruct the waveform and obtain its direction.



The North American NanoHertz Observatory of Gravitational Waves

Andrea N. Lommen

Chair, NANOGrav
Associate Professor of Physics and Astronomy
Head of Astronomy Program
Director of Grundy Observatory
Franklin and Marshall College
Lancaster, PA

Image Courtesy of Michael Kramer

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- Anne Archibald, McGill University
- Zaven Arzoumanian, Goddard Space Flight Center
- Don Backer, University of California, Berkeley
- Adam Brazier, Cornell University
- Jason Boyles, West Virginia University
- Brian Burt, Franklin and Marshall College
- Jim Cordes, Cornell University
- Paul Demorest, National Radio Astronomy Observatory
- Justin Ellis, West Virginia University
- Rob Ferdman, CNRS, France
- L. Samuel Finn, Center of Gravitational Physics at Penn State Univers
- Paulo Freire, National Astronomy and Ionospheric Center
- Alex Garcia, University of Texas, Brownsville
- Marjorie Gonzalez, University of British Columbia
- Rick Jenet, University of Texas, Brownsville, CGWA
- Victoria Kaspi, McGill University
- Joseph Lazio, Naval Research Laboratories
- Andrea Lommen, Franklin and Marshall College
- Duncan Lorimer, West Virginia University
- Ryan Lynch, University of Virginia
- Maura McLaughlin, West Virginia University
- Jonathan Nelson, Oberlin College
- David Nice, Bryn Mawr College
- Nipuni Palliyaguru, West Virginia University
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- Larry Price, University of Wisconsin
- Scott Ransom, National Radio Astronomy Observatory
- Ryan Shannon, Cornell University
- Xavi Siemens, University of Wisconsin
- Ingrid Stairs, University of British Columbia
- Dan Stinebring, Oberlin College

The EPTA: partners

NANOGrav

University of Manchester, JBO, UK

ASTRON, NL

Max-Planck Insitut fur Radioastronomie, GER

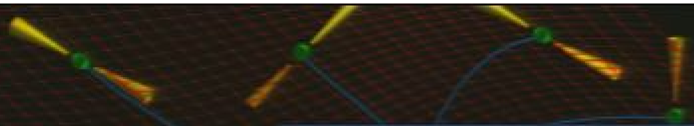
Nancay Observatory, FR

INAF Osservatorio Astonomico di Cagliari, IT

The Parkes Pulsar Timing Array Project

Collaborators:

- Australia Telescope National Facility, CSIRO, Sydney
  - Dick Manchester, George Hobbs, David Champion, John Sarkissian, John Reynolds, Mike Kesteven, Grant Hampson, Andrew Brown, David Smith, Jonathan Khoo, (Russell Edwards)
- Swinburne University of Technology, Melbourne
  - Matthew Bailes, Ramesh Bhat, Willem van Straten, Joris Verbiest, Sarah Burke, Andrew Jameson
- University of Texas, Brownsville
  - Rick Jenet
- Franklin & Marshall College, Lancaster
  - Andrea Lommen
- University of Sydney, Sydney
  - Daniel Yardley
- National Observatories of China, Beijing
  - Johnny Wen
- Peking University, Beijing
  - Kejia Lee
- Southwest University, Chongqing
  - Xiaopeng You
- Curtin University, Perth
  - Aidan Hotan



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The International Pulsar Timing Array

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Talk represents work with:

- NANOGrav
- European Pulsar Timing Array
- Parkes Pulsar Timing Array

Thrilled with the work of:

- A. Sesana, A. Vecchio, M. Volunteri, C. N. Colacino
- Melissa Anholm, Xavier Siemens, Larry Price, U. Milwaukee
- Joe Romano, Graham Woan
- Chris Messenger, AEI

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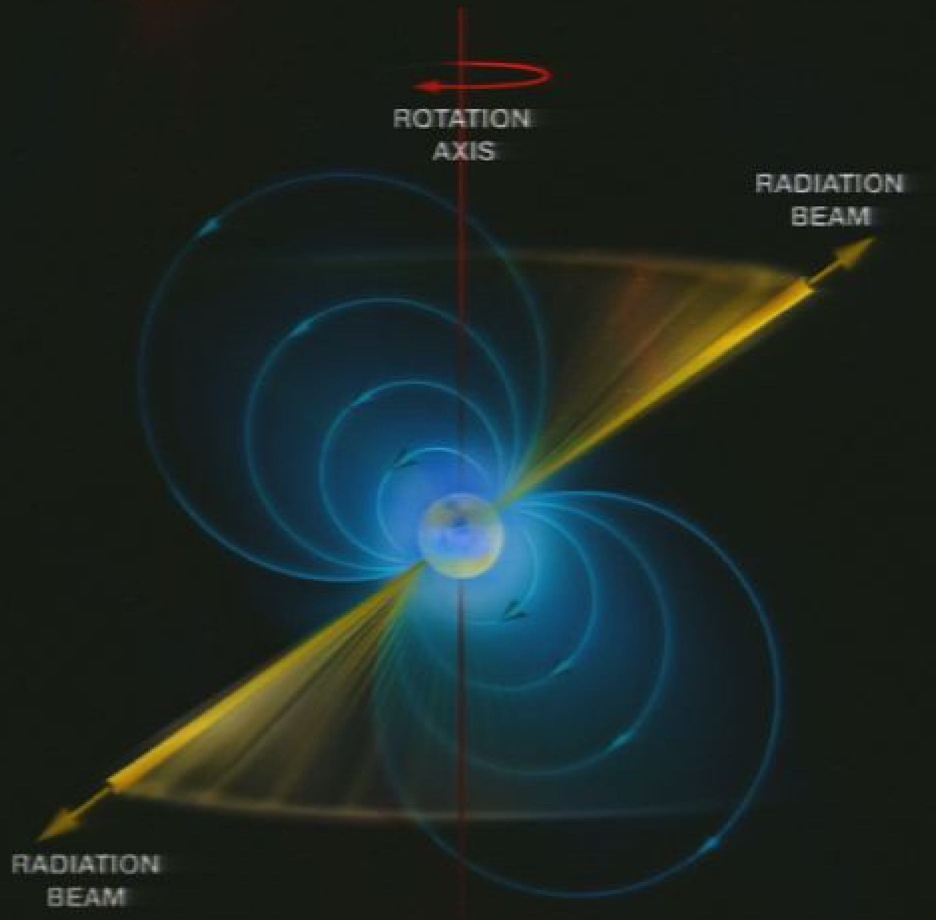
What is a pulsar?



Millisecond Pulsars are *Very* Precise Clocks

PSR B1937+21

At midnight on 5 Dec, 1998:

P = 1.5578064688197945 ms
+/- 0.0000000000000004 ms

The last digit changes by about 1 per second!

This extreme precision is what allows us to ***use pulsars as tools*** to do unique physics!

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Typical Pulsar Model

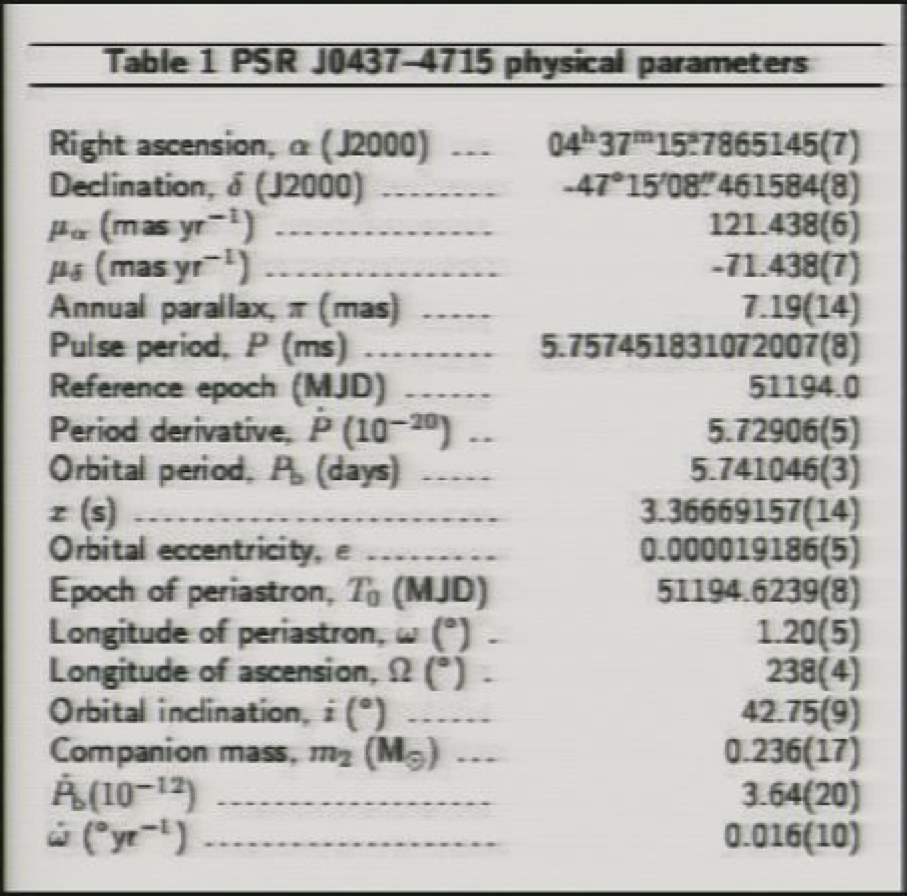
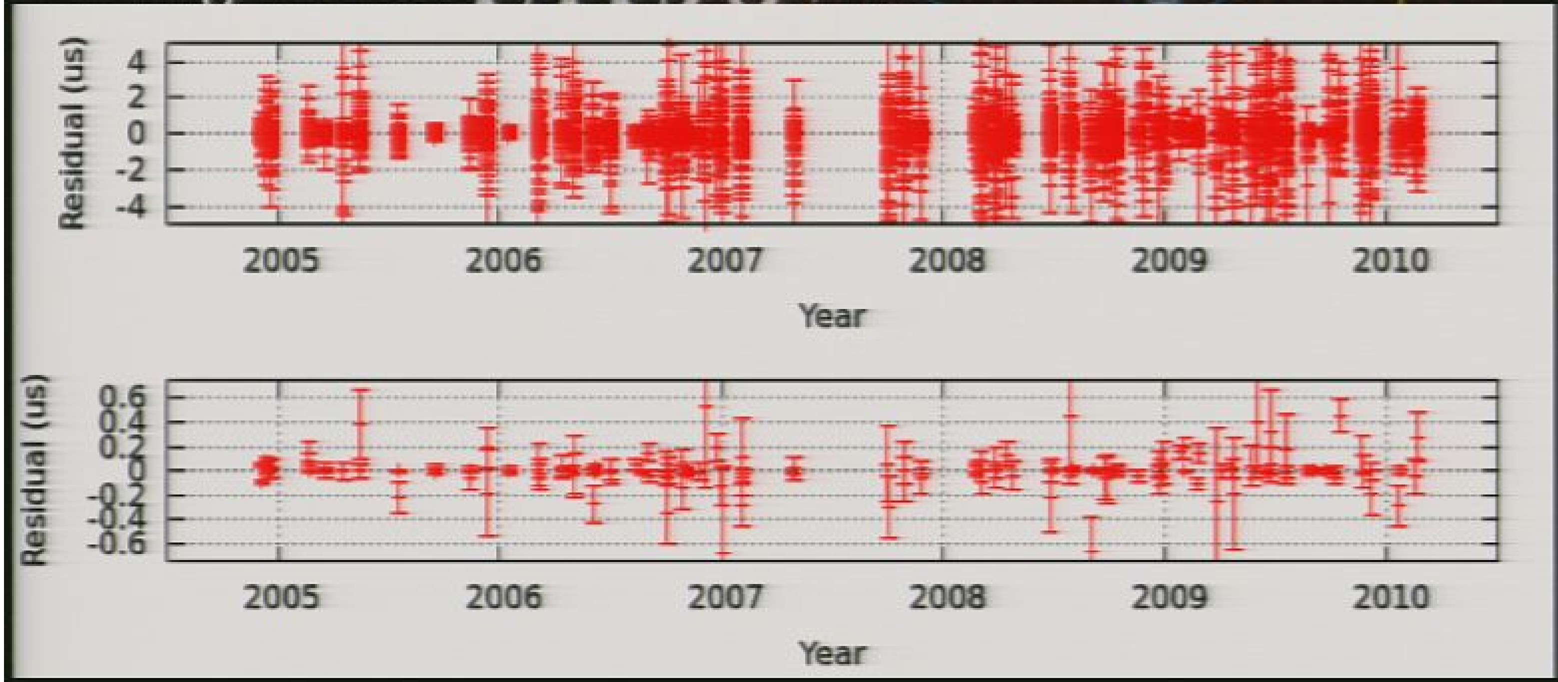


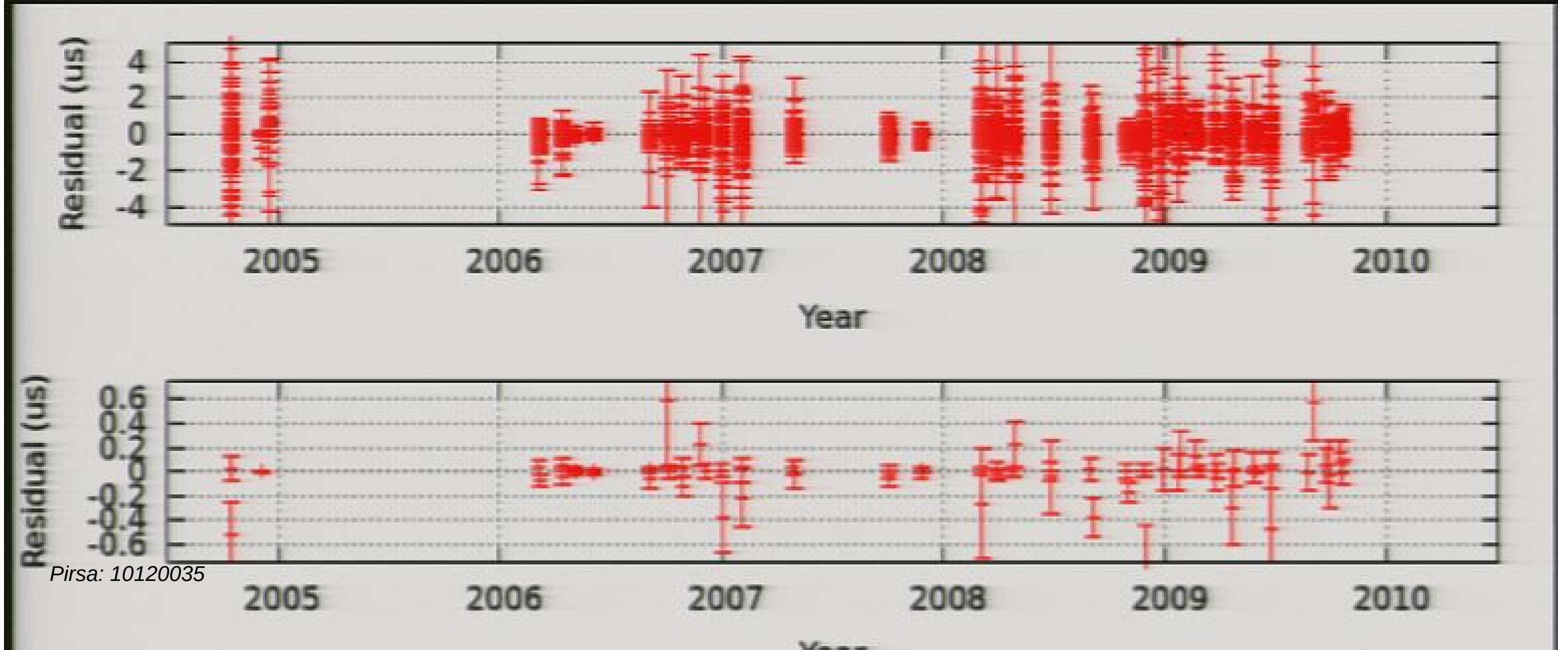
Table 1 PSR J0437–4715 physical parameters

Right ascension, α (J2000) ...	$04^h37^m15^s.7865145(7)$
Declination, δ (J2000)	$-47°15'08''.461584(8)$
μ_α (mas yr^{-1})	121.438(6)
μ_δ (mas yr^{-1})	-71.438(7)
Annual parallax, π (mas)	7.19(14)
Pulse period, P (ms)	5.757451831072007(8)
Reference epoch (MJD)	51194.0
Period derivative, $\dot{P}$ (10^{-20}) ..	5.72906(5)
Orbital period, P_b (days)	5.741046(3)
x (s)	3.36669157(14)
Orbital eccentricity, e	0.000019186(5)
Epoch of periastron, T_0 (MJD)	51194.6239(8)
Longitude of periastron, ω (°) .	1.20(5)
Longitude of ascension, Ω (°) .	238(4)
Orbital inclination, i (°)	42.75(9)
Companion mass, m_2 ($M_\odot$) ...	0.236(17)
$\dot{P}_b$ (10^{-12})	3.64(20)
$\dot{\omega}$ (° yr^{-1})	0.016(10)

Timing residuals versus time:



J1713+0747



J1909-3744

Slide courtesy of Paul

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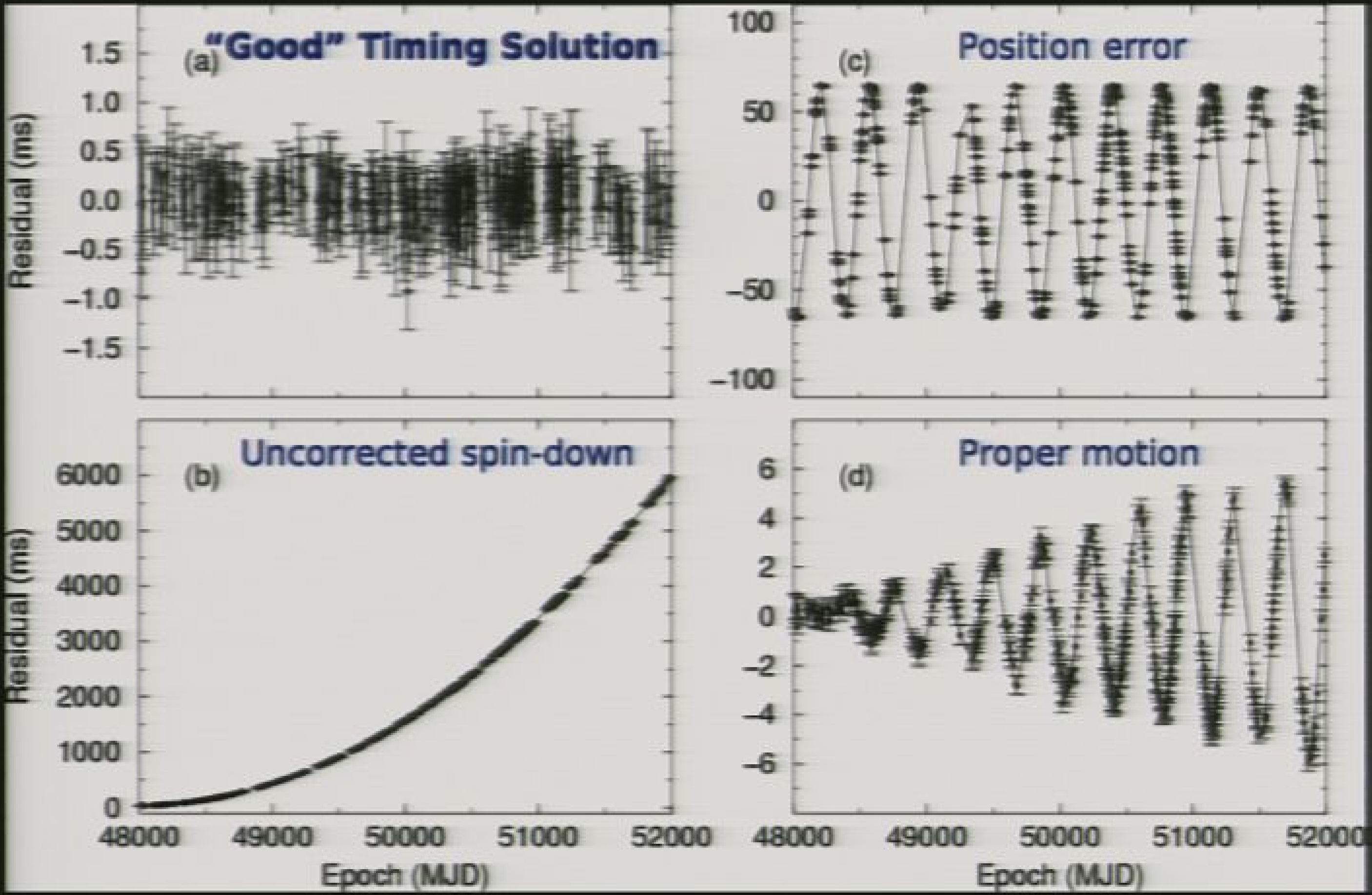
The science is in the residuals!:

RMS precision ~ 10^{-5}-10^{-3} P



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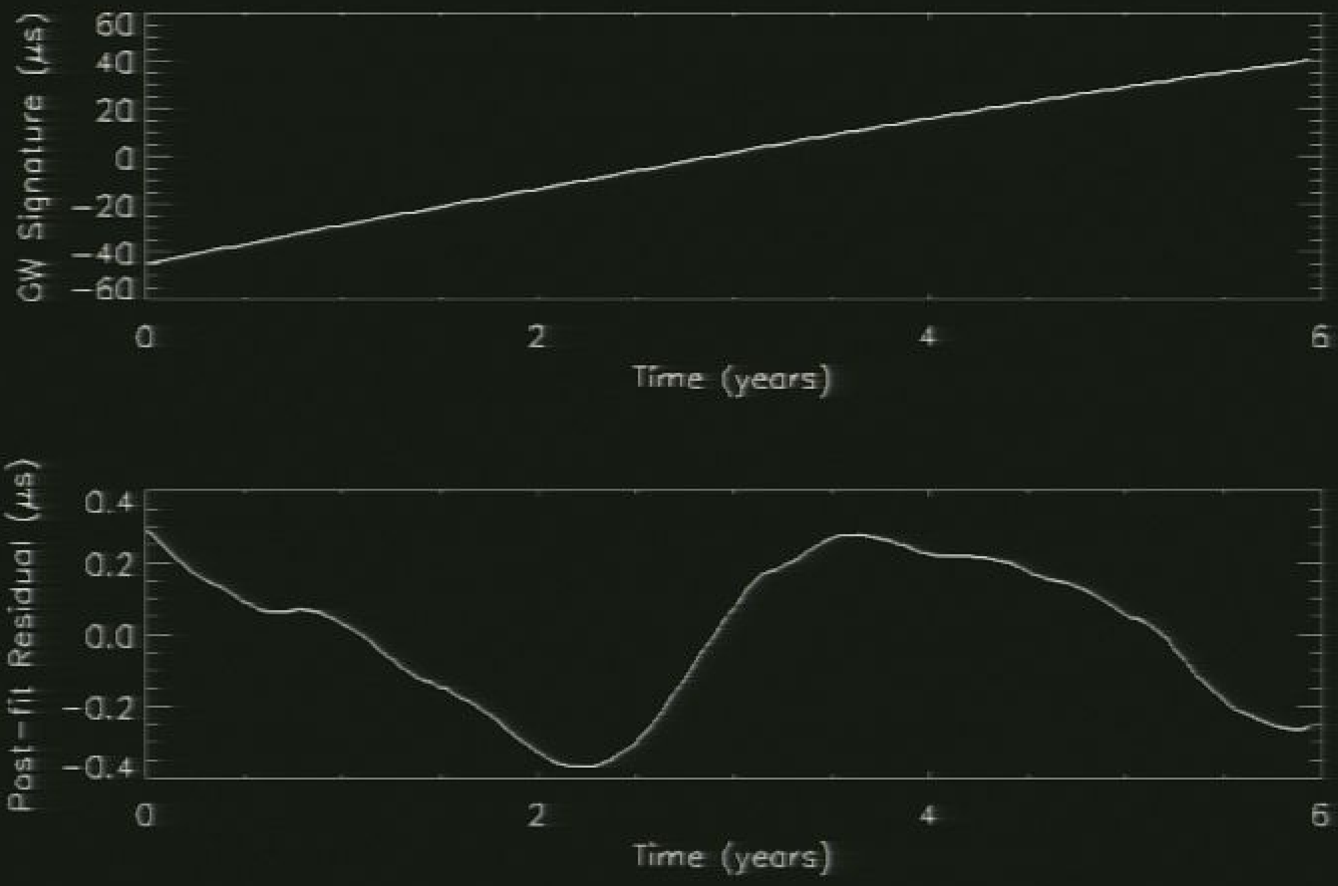
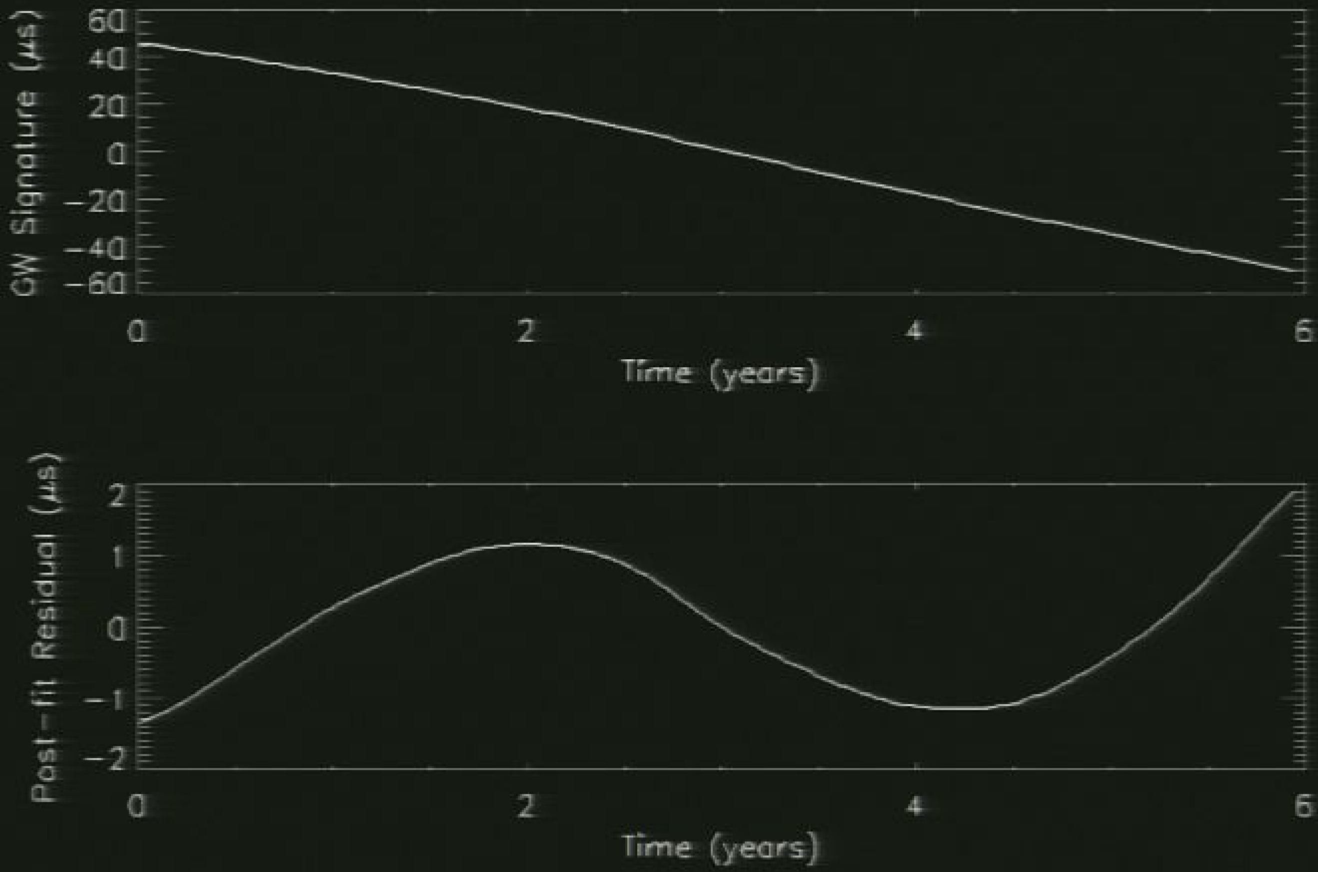


GW Signature (μs)
60
40
20
0
−20
−40
−60
0
2
4
6
Time (years)
Post−fit Residual (μs)
0.4
0.2
0.0
−0.2
−0.4
0
2
4
6
Time (years)

NANOGrav


GW Signature (μs)
60
40
20
0
−20
−40
−60
0
2
4
6
Time (years)
Post−fit Residual (μs)
2
1
0
−1
−2
0
2
4
6
Time (years)

NANOGrav

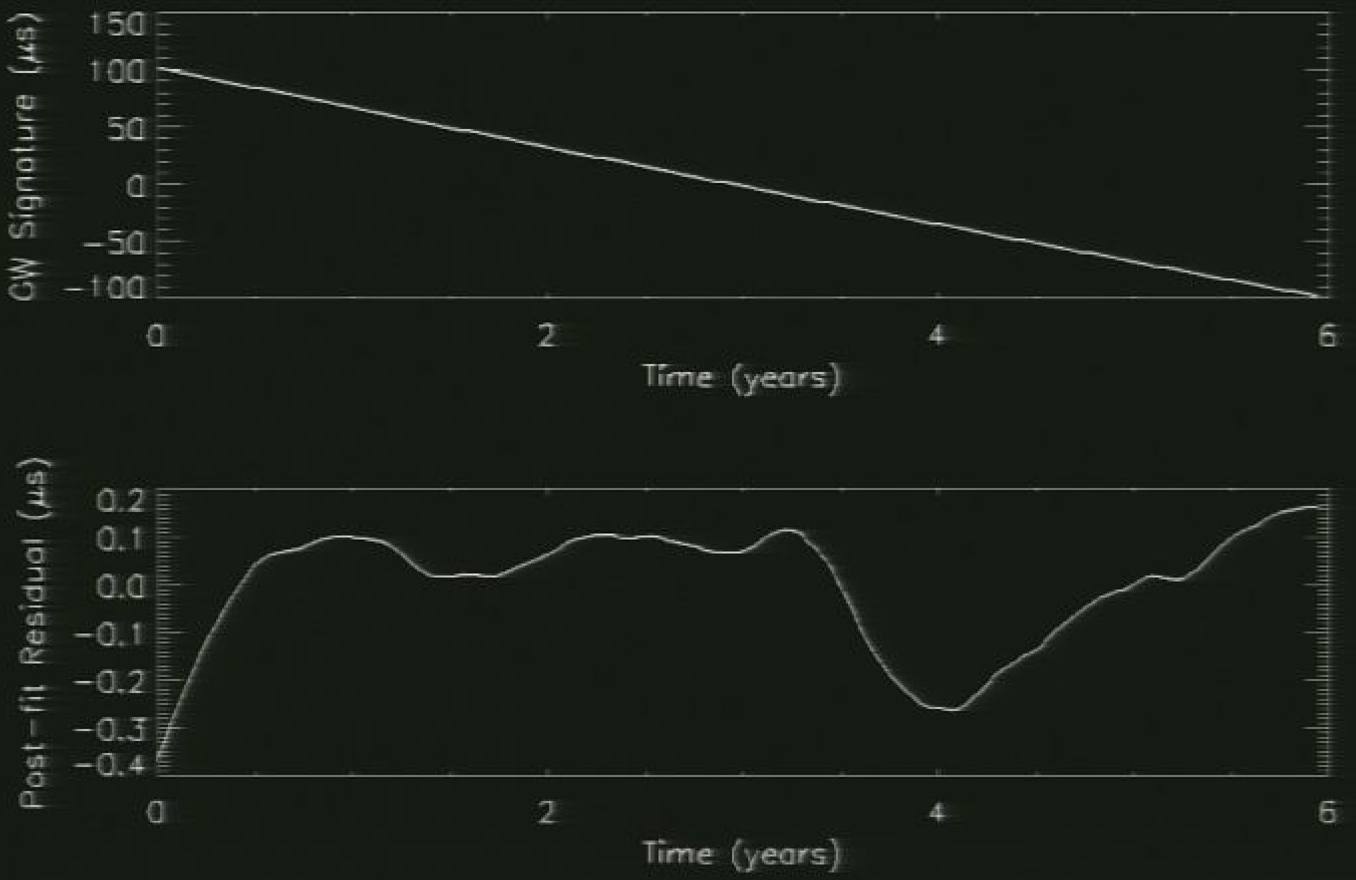

GW Signature (μs)
150
100
50
0
−50
−100
0
2
4
6
Time (years)
Post−fit Residual (μs)
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2
4
6
Time (years)

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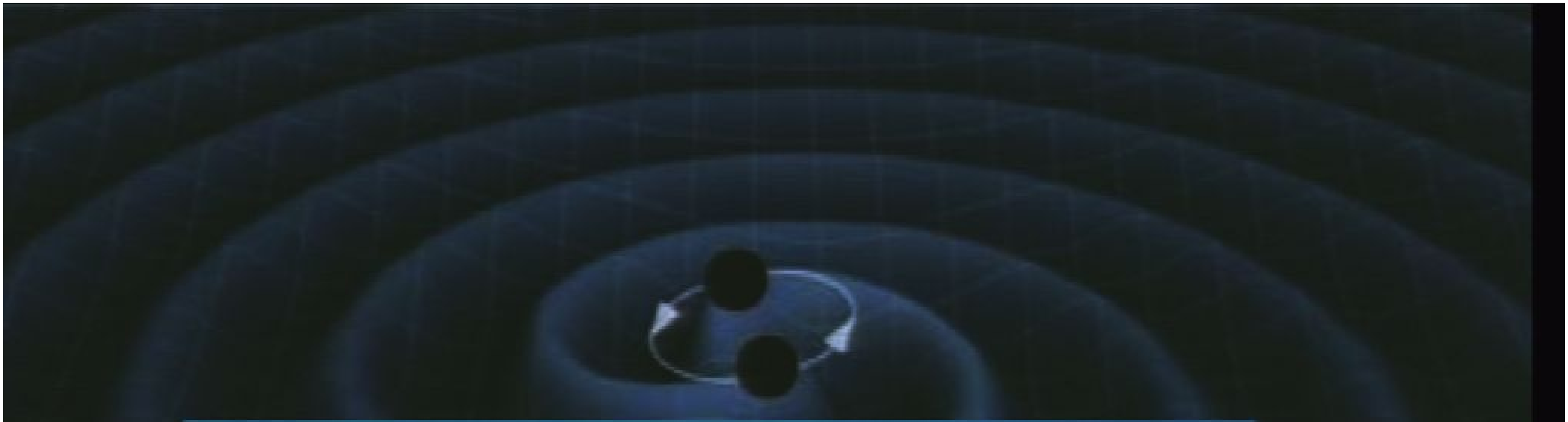
Photo Courtesy of Virgo

Adapted from NASA figure



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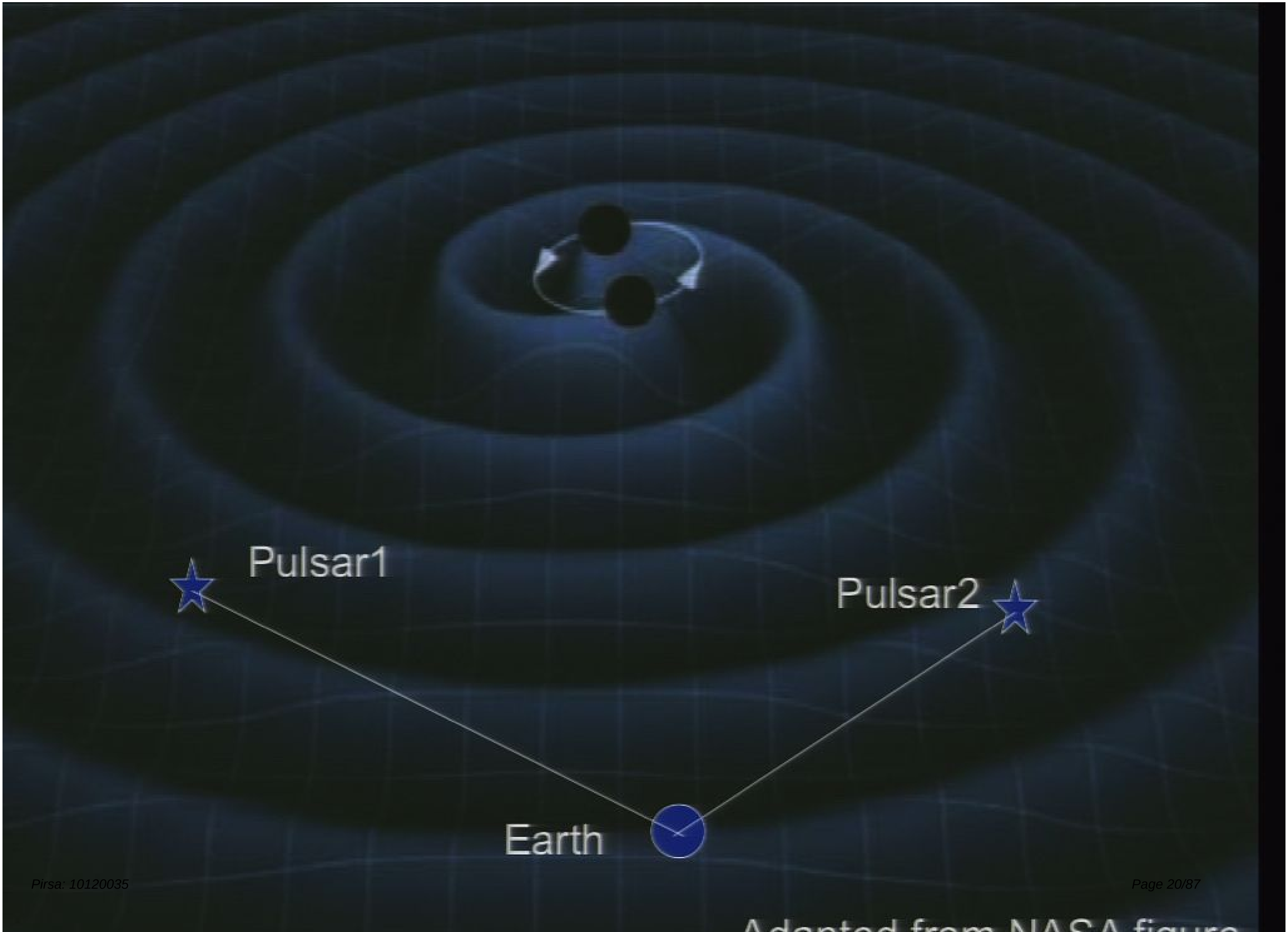

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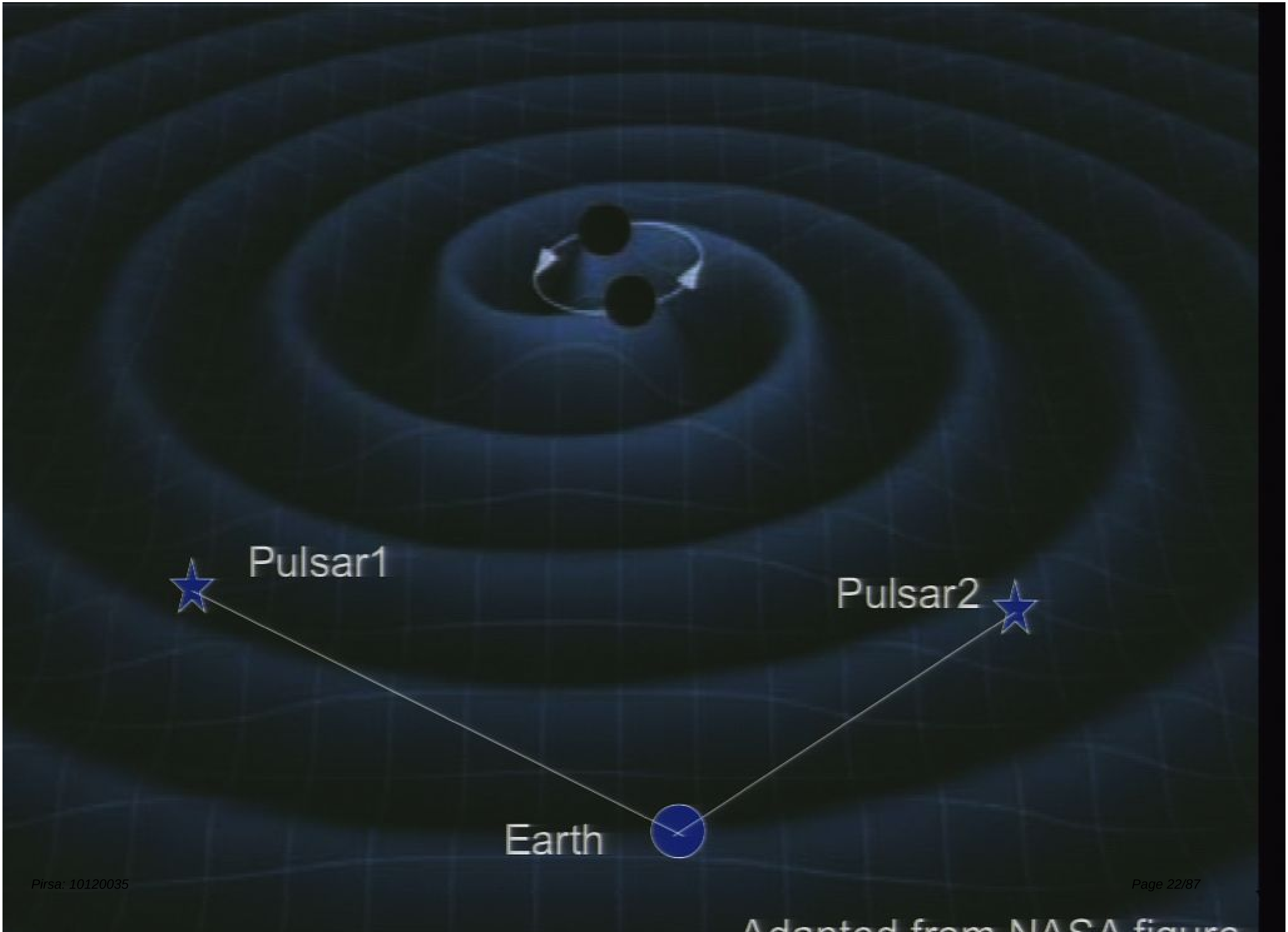


Adapted from NASA figure

Detectability of a Waveform

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

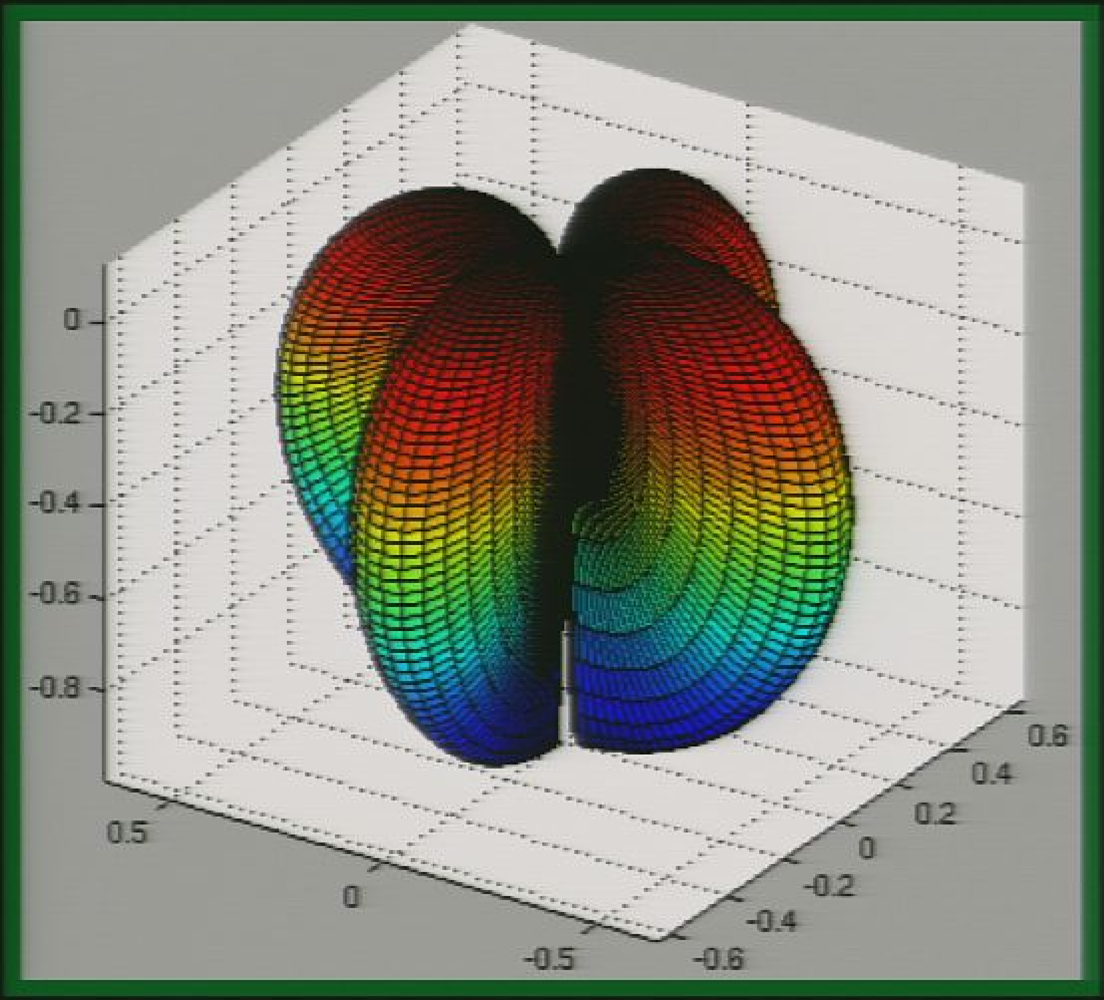
$$\left(\frac{dt}{d\lambda}\right)^2 = \delta_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} + \frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda}h_{jk}(t,\vec{x})$$

$$\int dt = \int d\lambda - \frac{1-k_m n^m}{1+k_m n^m}\int d\lambda n^j n^k h_{jk}\left(t(\lambda),\vec{x}(\lambda)\right)$$

$$\text{Residual} = e_{jk}n^j n^k \frac{h_0}{2}\left(1-k_m n^m\right)\left[f\left(t_0\right) - f\left(t_0 - L\left(1+k_m n^m\right)\right)\right]$$

where $h_{jk}(t,\vec{x}) = h_0 f'\left(t-\hat{k}\cdot\vec{x}\right)$e and L is the distance to the pulsar

The shape of the GW response



Thanks Bill Coles

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A sense of what's detectable

$$h = \frac{M^{5/3}}{P^{2/3} d}$$

$$\tau = hP$$

$$\tau = \frac{M^{5/3} P^{1/3}}{d}$$

$$\tau = 50ns \frac{\left(\frac{M}{2 \times 10^9 M_\Theta}\right)^{5/3} \left(\frac{P}{1 year}\right)^{1/3}}{\left(\frac{d}{100 Mpc}\right)}$$


Radio Galaxy 3C66B
VLA 20cm image
Copyright (c) NRAO/AUI 1999

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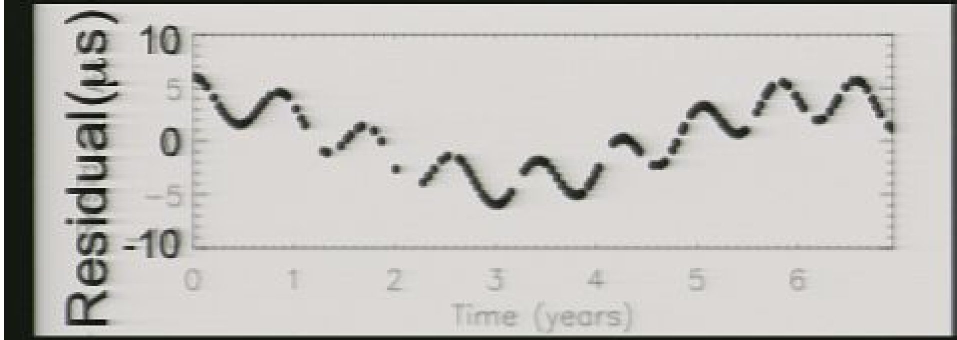
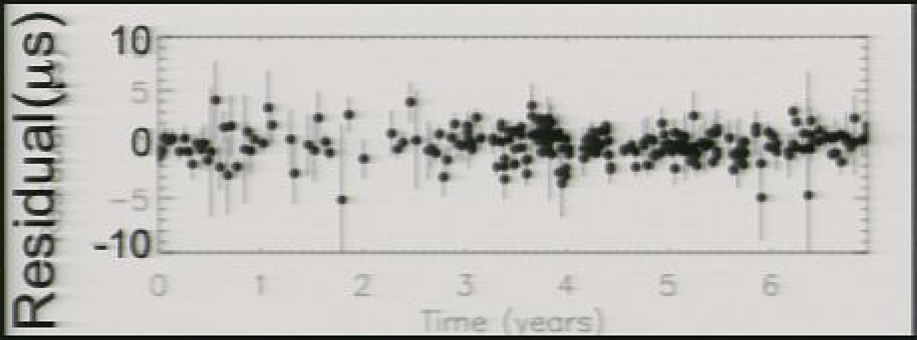
Orbital Motion in the Radio Galaxy 3C 66B: Evidence for a Supermassive Black Hole Binary Sudou, Iguchi, Murata, Taniguchi (2003) Science 300: 1263-1265.

Constraining the Properties of Supermassive Black Hole Systems Using Pulsar Timing: Application to 3C 66b, Jenet, Lommen, Larson and Wen (2004) ApJ 606:799-803.



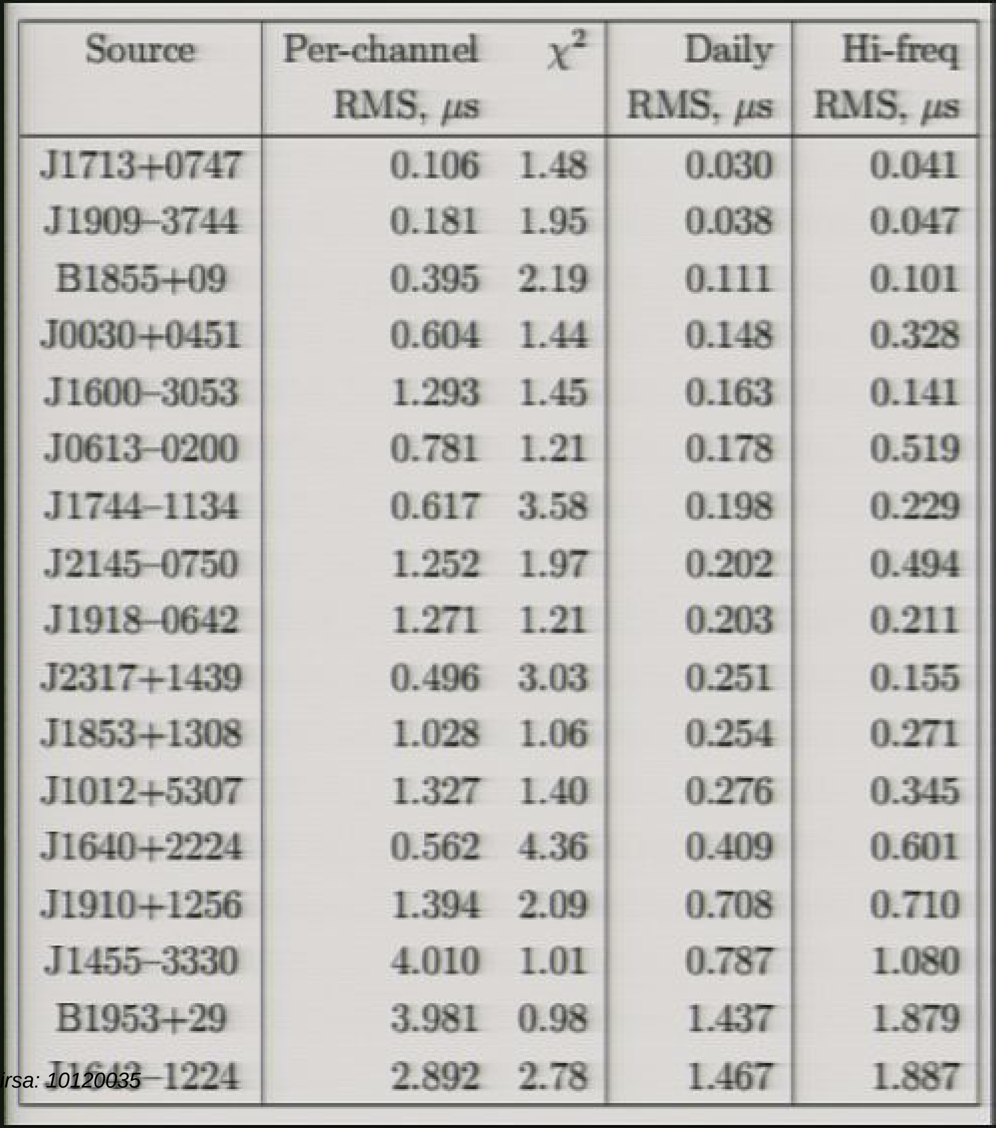
Simulated residuals due to 3c66b



Data from Kaspi, Taylor, Ryba 1994

NANOGrav 5-year timing results summary

(analysis currently ongoing; PD, M. Gonzalez, D. Nice, I. Stairs, S. Ransom, R. Ferdman)



Source	Per-channel RMS, μs	χ^2	Daily RMS, μs	Hi-freq RMS, μs
J1713+0747	0.106	1.48	0.030	0.041
J1909–3744	0.181	1.95	0.038	0.047
B1855+09	0.395	2.19	0.111	0.101
J0030+0451	0.604	1.44	0.148	0.328
J1600–3053	1.293	1.45	0.163	0.141
J0613–0200	0.781	1.21	0.178	0.519
J1744–1134	0.617	3.58	0.198	0.229
J2145–0750	1.252	1.97	0.202	0.494
J1918–0642	1.271	1.21	0.203	0.211
J2317+1439	0.496	3.03	0.251	0.155
J1853+1308	1.028	1.06	0.254	0.271
J1012+5307	1.327	1.40	0.276	0.345
J1640+2224	0.562	4.36	0.409	0.601
J1910+1256	1.394	2.09	0.708	0.710
J1455–3330	4.010	1.01	0.787	1.080
B1953+29	3.981	0.98	1.437	1.879
J1643–1224	2.892	2.78	1.467	1.887

Analysis features:

Two independent calibration/processing pipelines -- psrchive and ASPfitsreader

DM(t) and timing model in single fit.

Fit includes systematic timing vs freq correction (profile shape evolution).

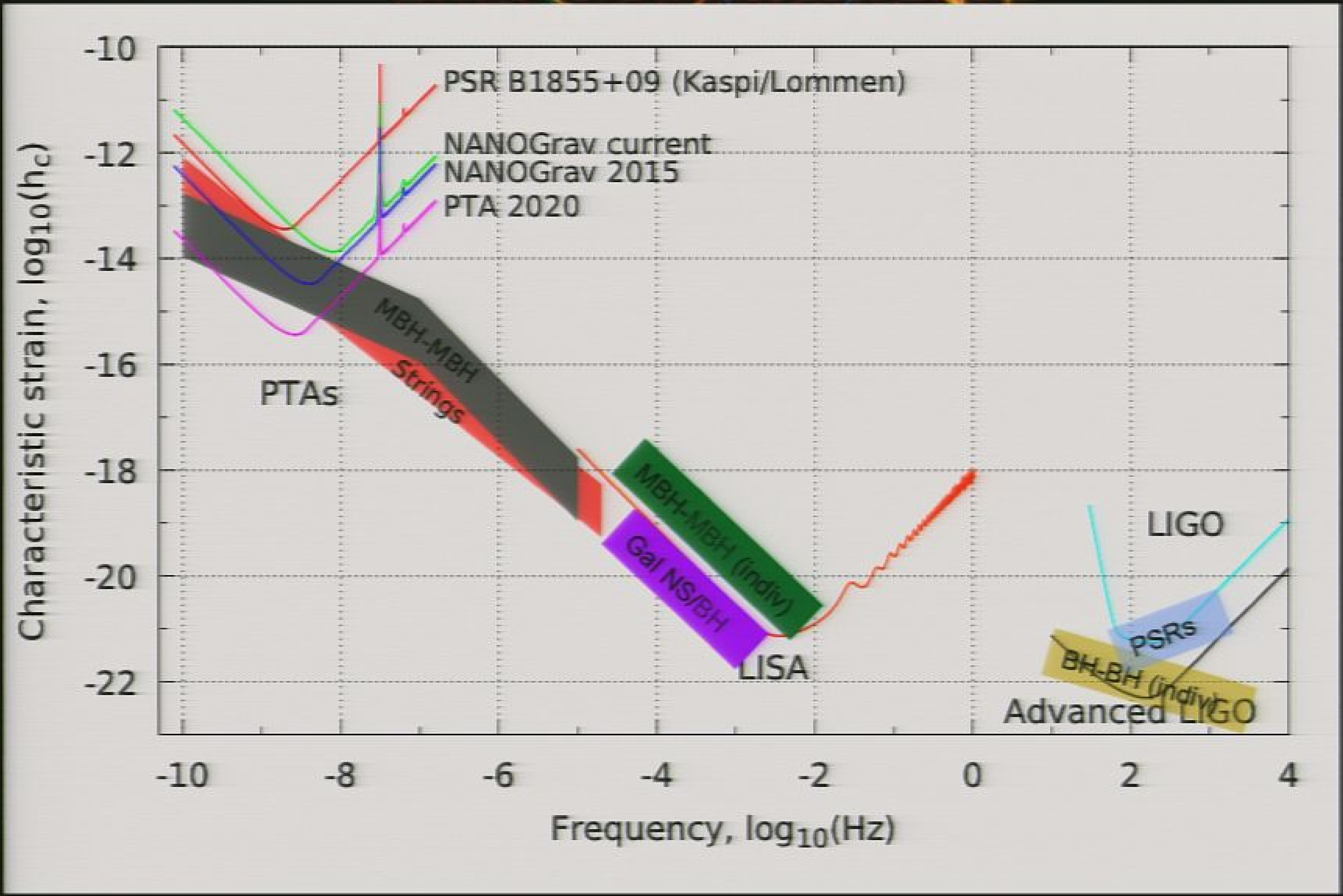
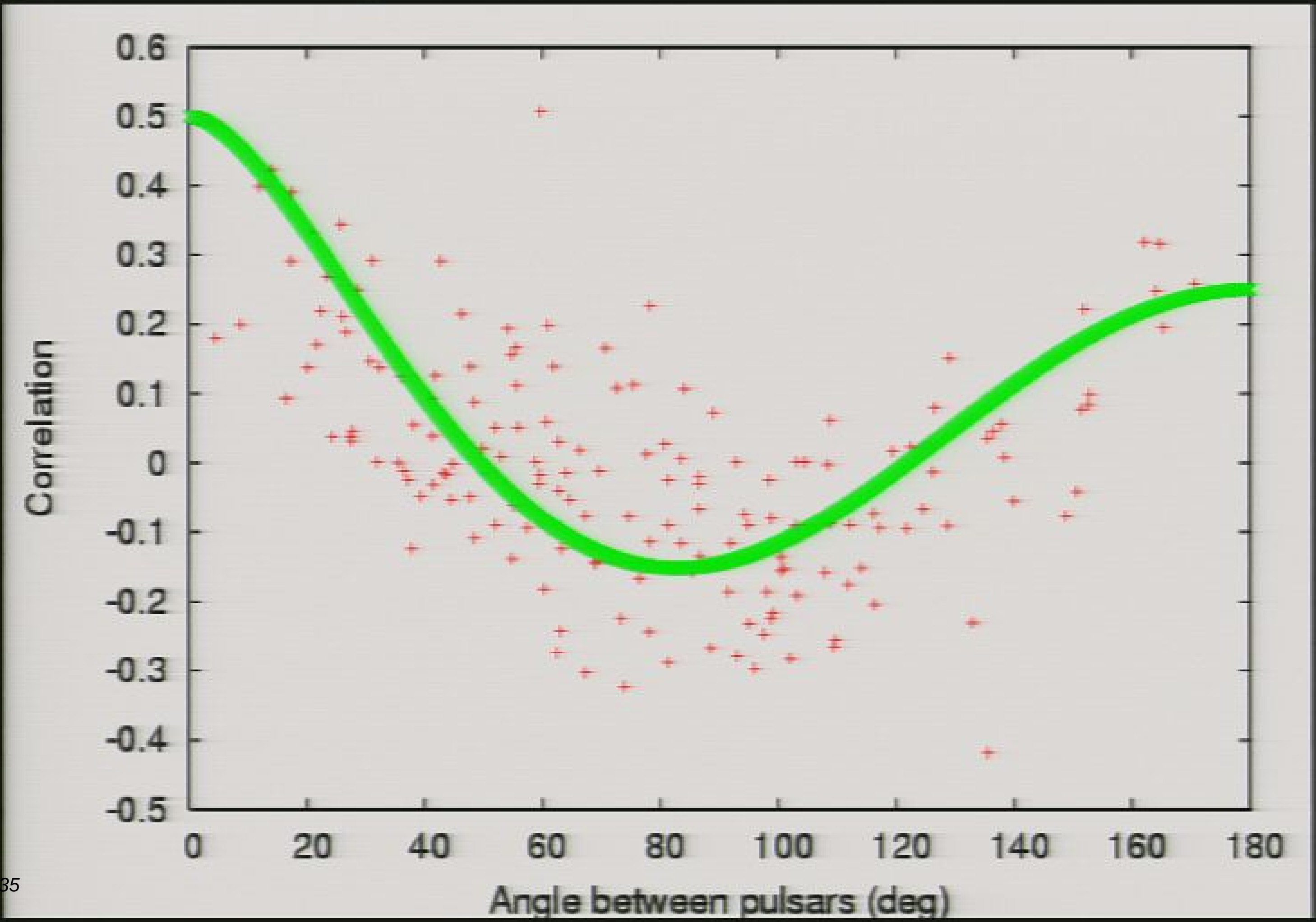


Figure by Paul Demorest (see

Hellings and Downs Curve (Overlap Reduction Function)



Courtesy of Rick Jenet and George Hobbs. Original figure from Hellings and Downs (1983)

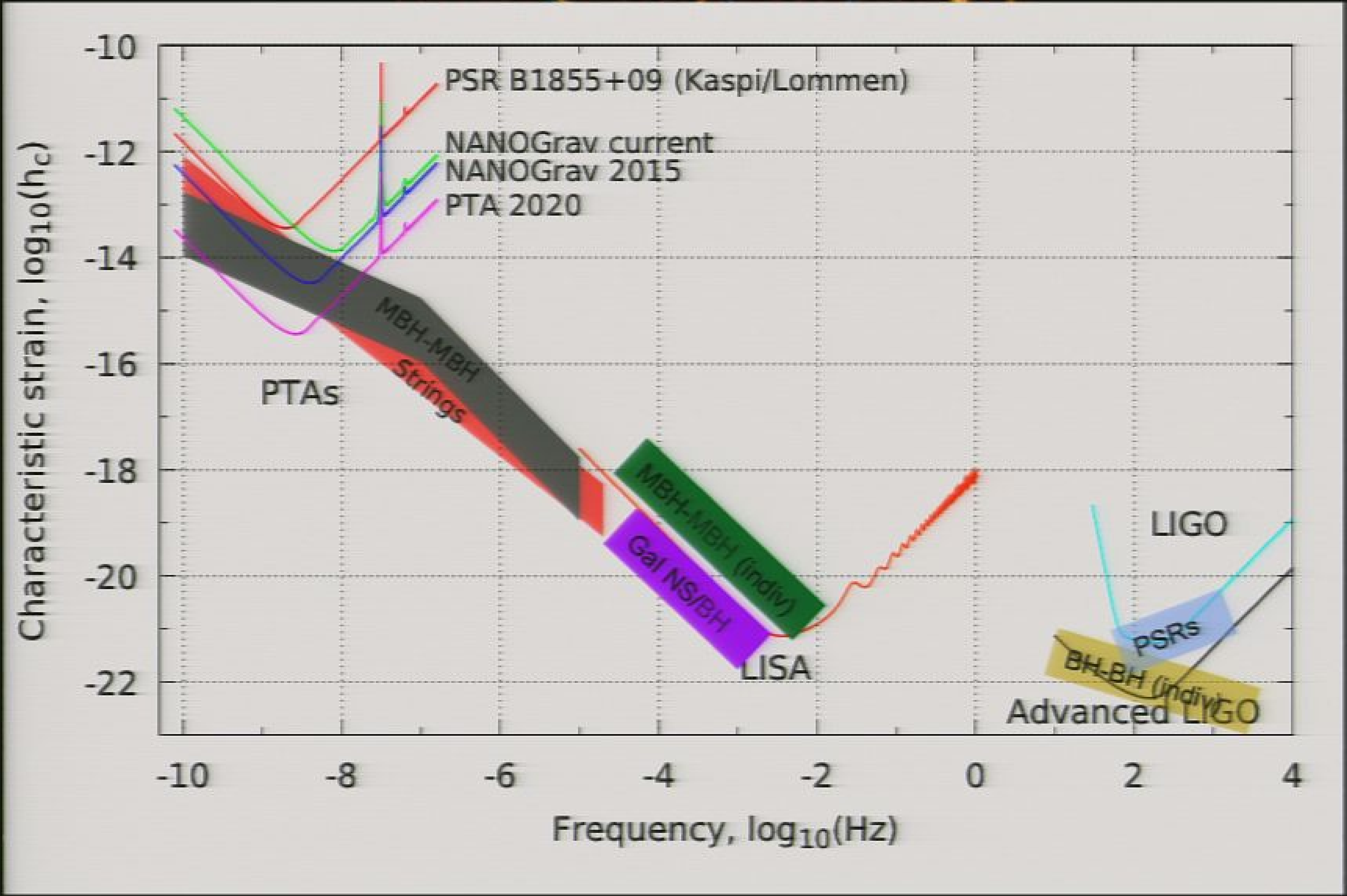


Figure by Paul Demorest (see arXiv:0902.2968)

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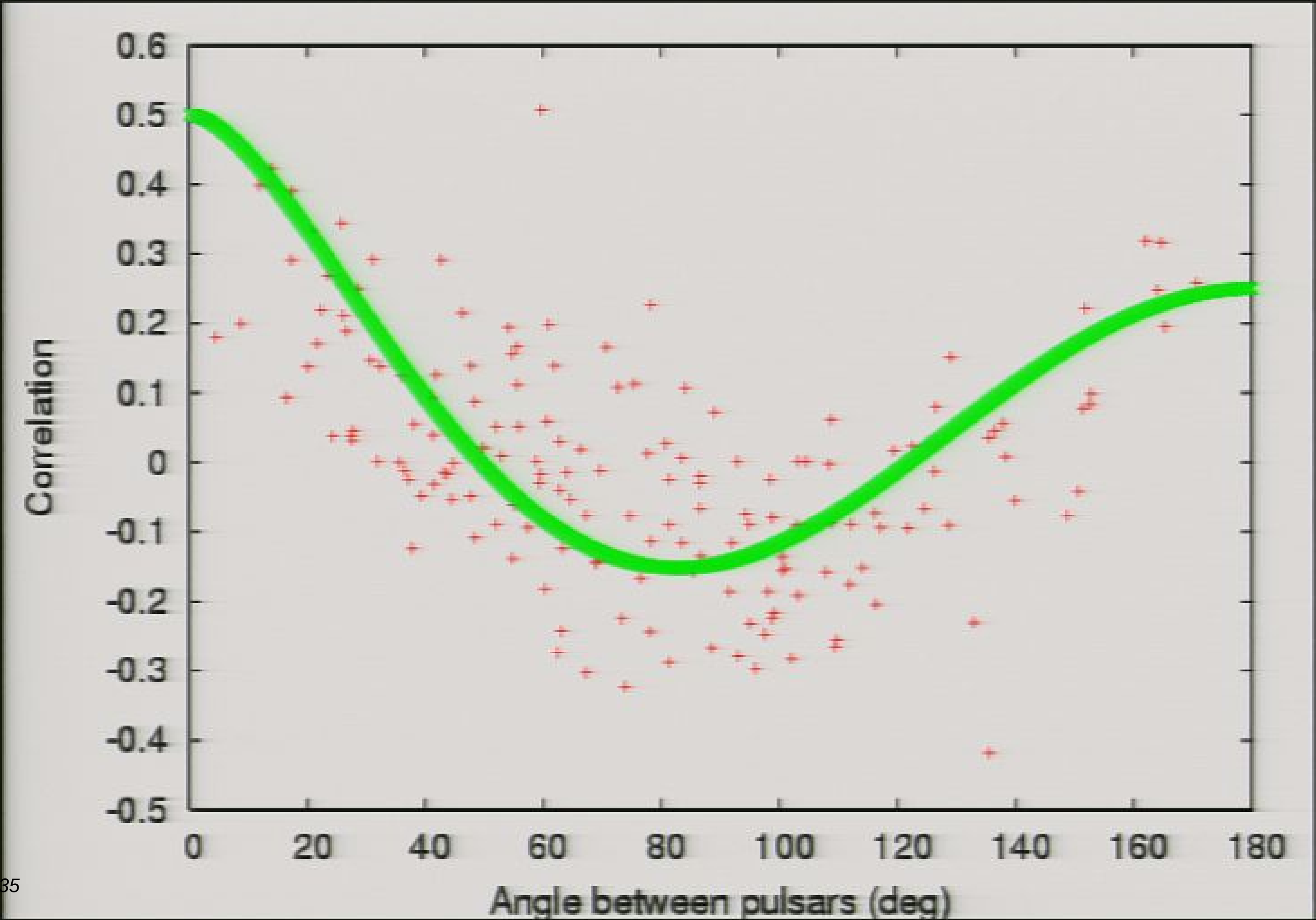

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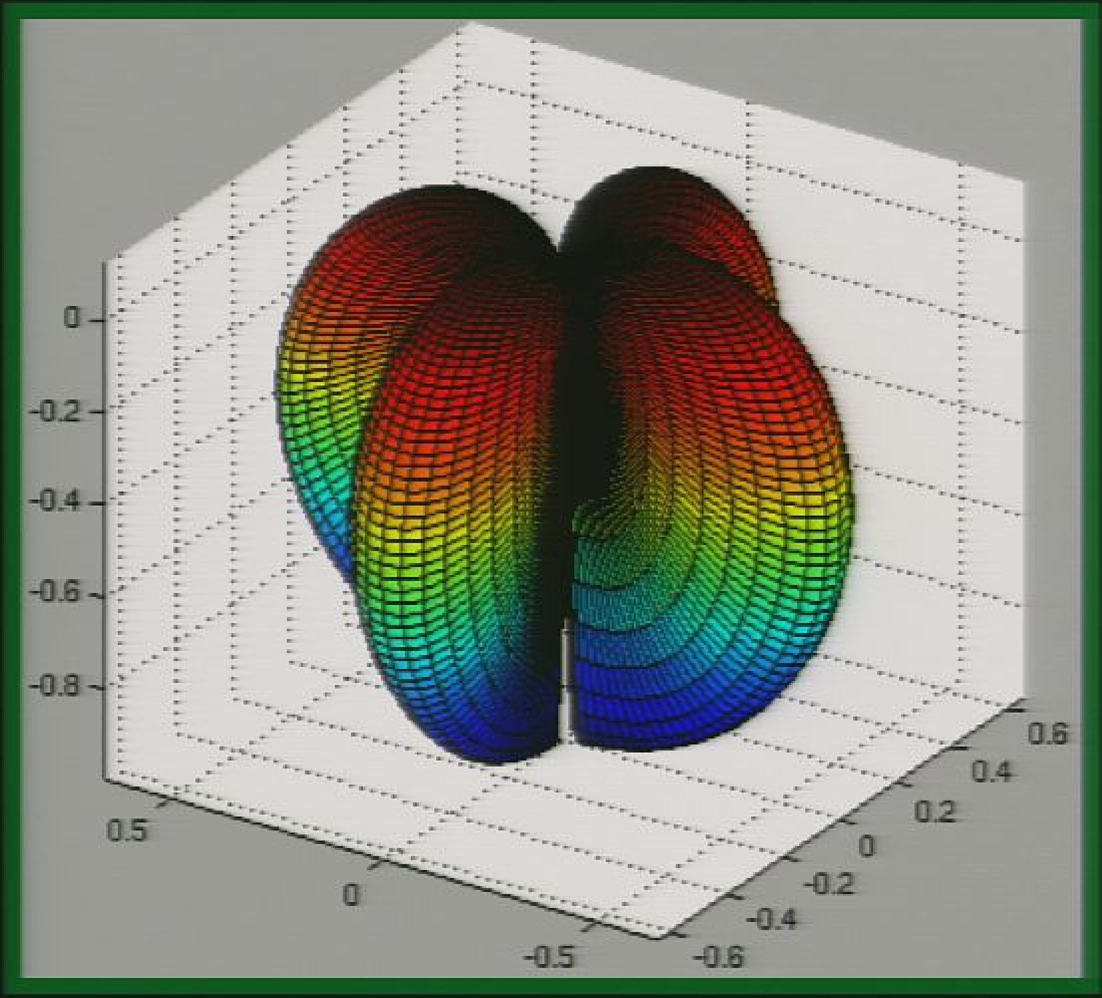
The shape of the GW response



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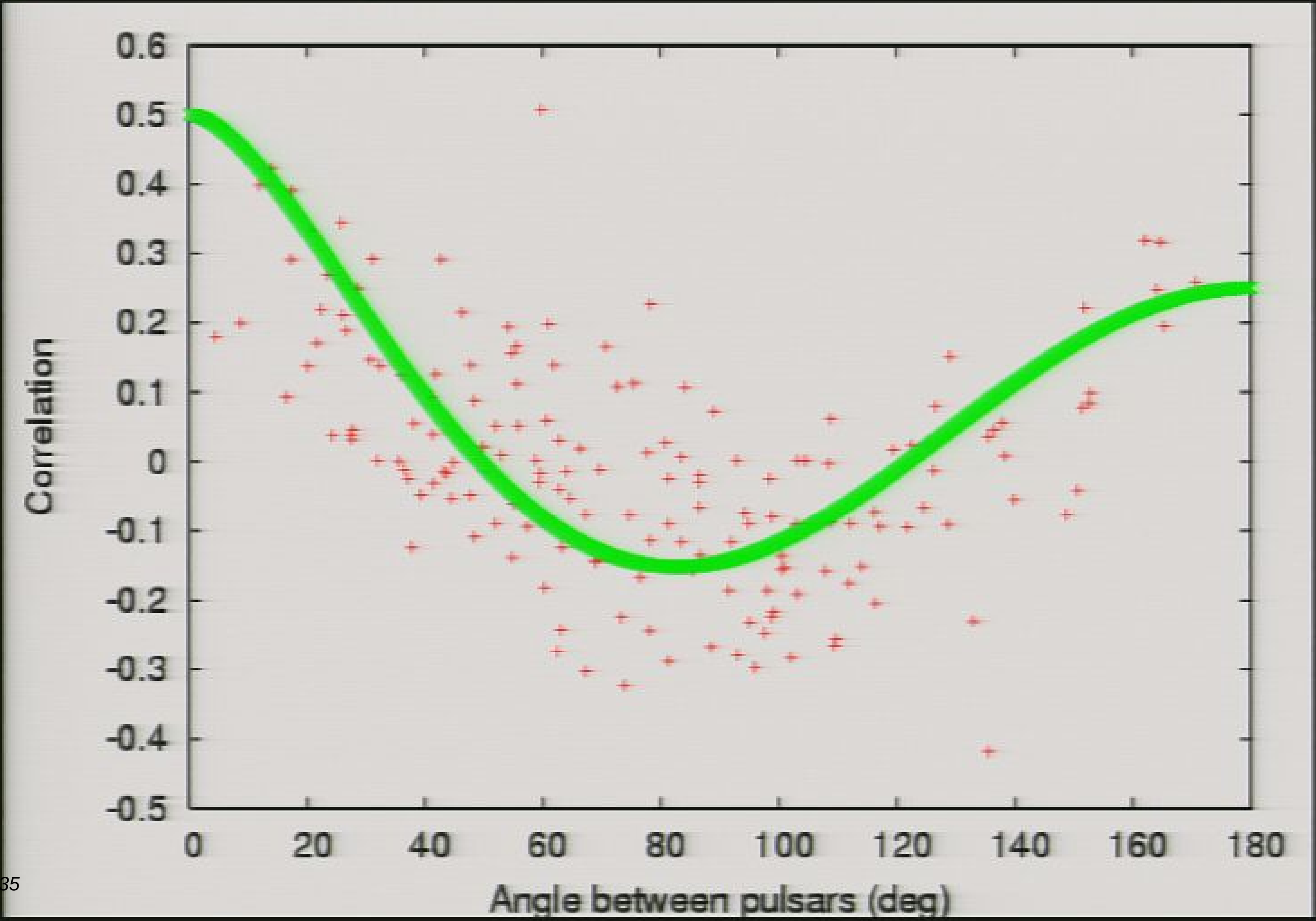

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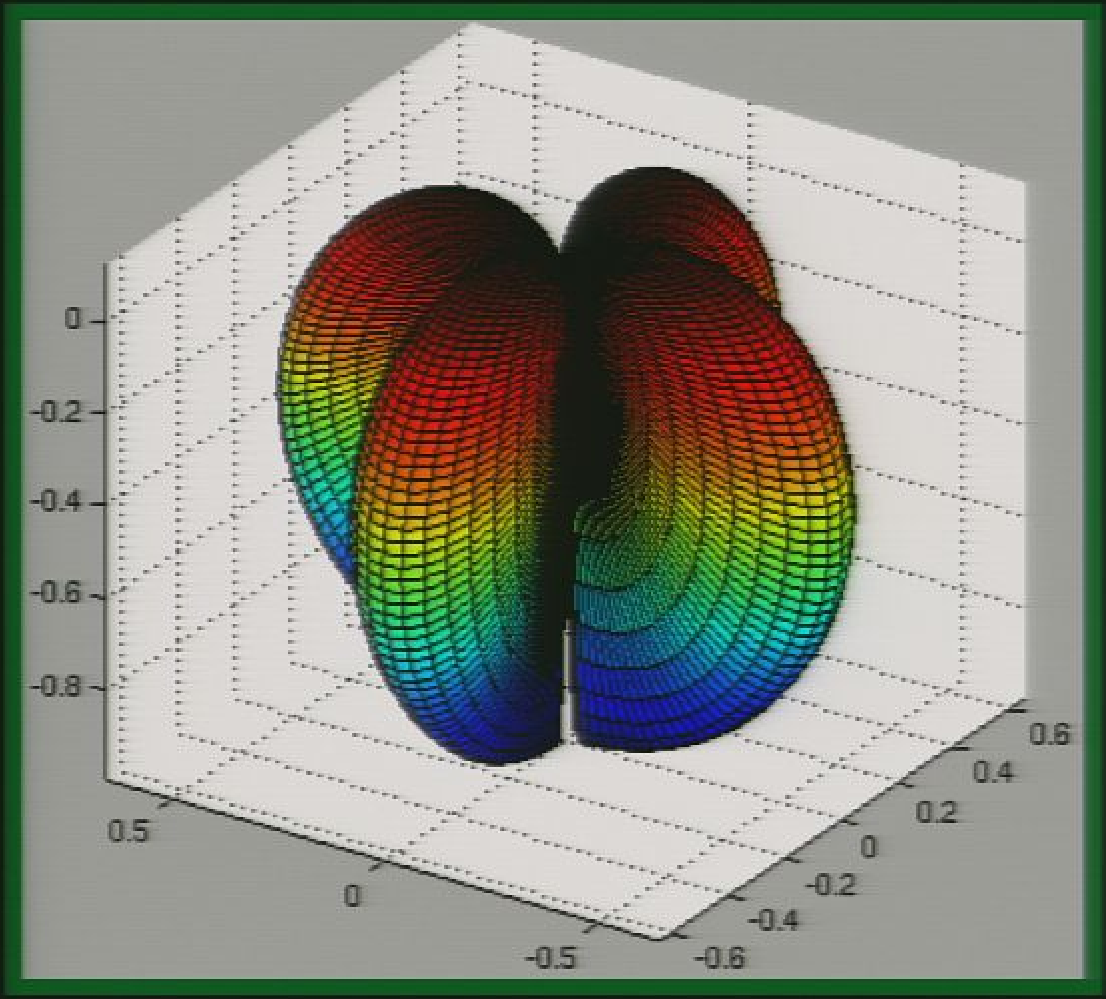

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From Rodin, astroph: 1012.3038 (14 Dec 2010)

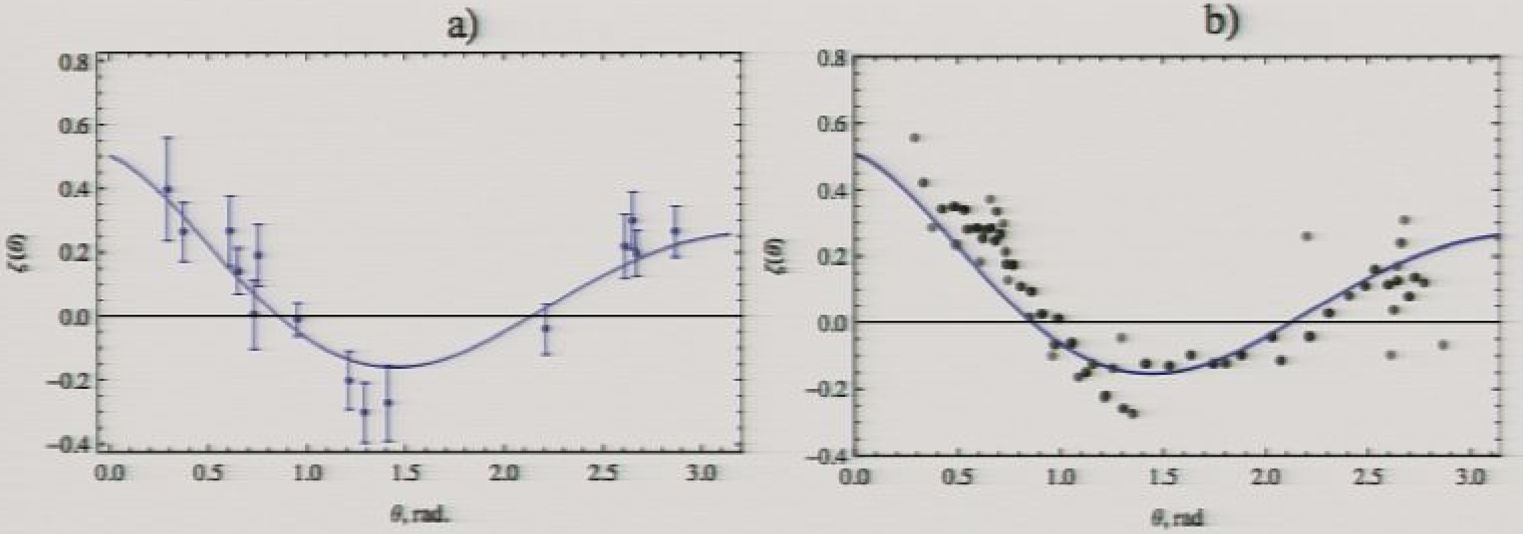
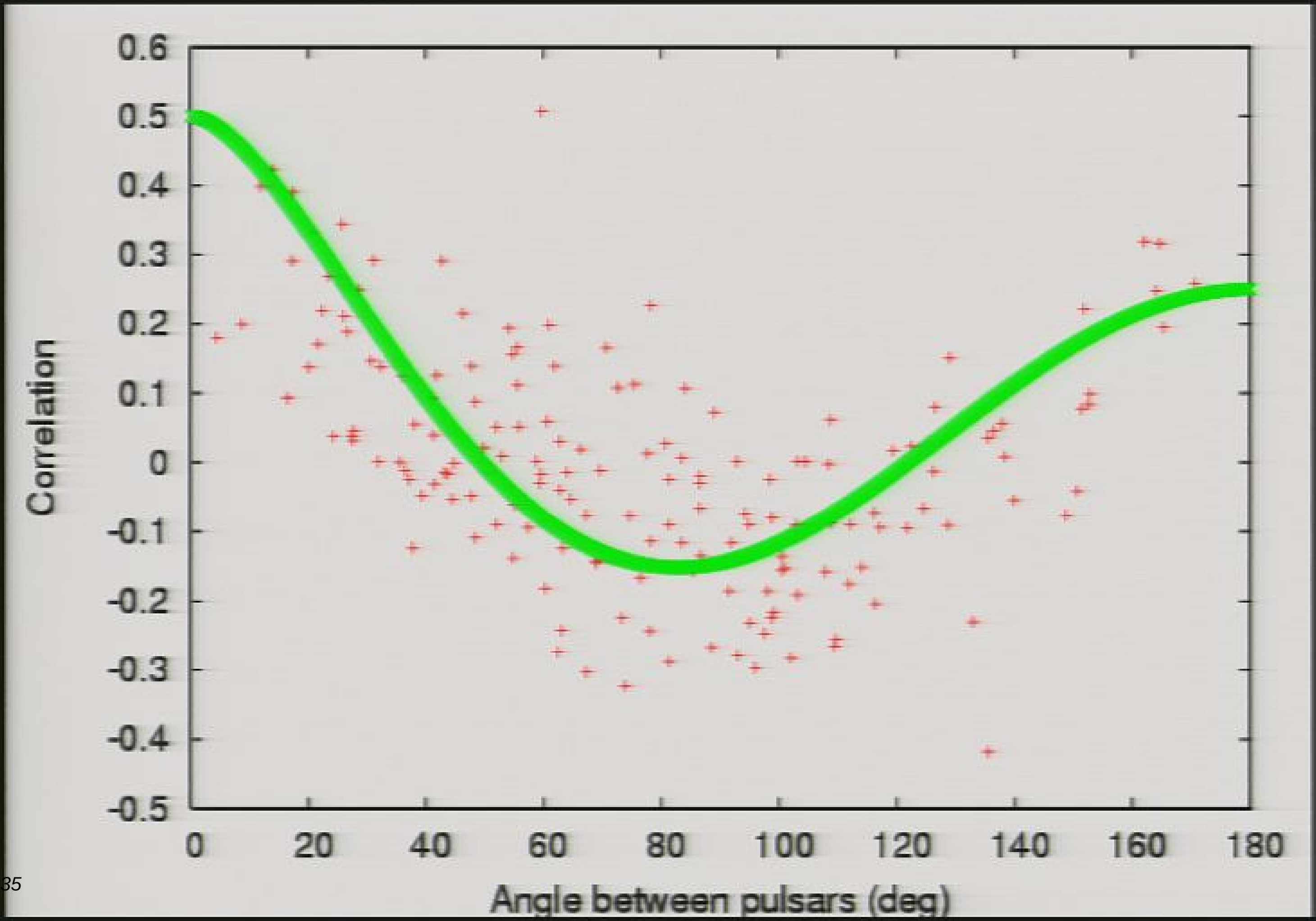


FIGURE 2. a) Results of the numerical simulation of the two-point correlation function for $M = 6$ pulsars and $N = 50$ TOAs. Signal-to-noise ratio $SNR \simeq 10$. b) Experimental plot of the two-point correlation function. Moving average by 2-5 points were applied. Correlation coefficient between theoretical curve (solid line) and experimental points $\rho = 0.82 \pm 0.07$

Hellings and Downs Curve (Overlap Reduction Function)



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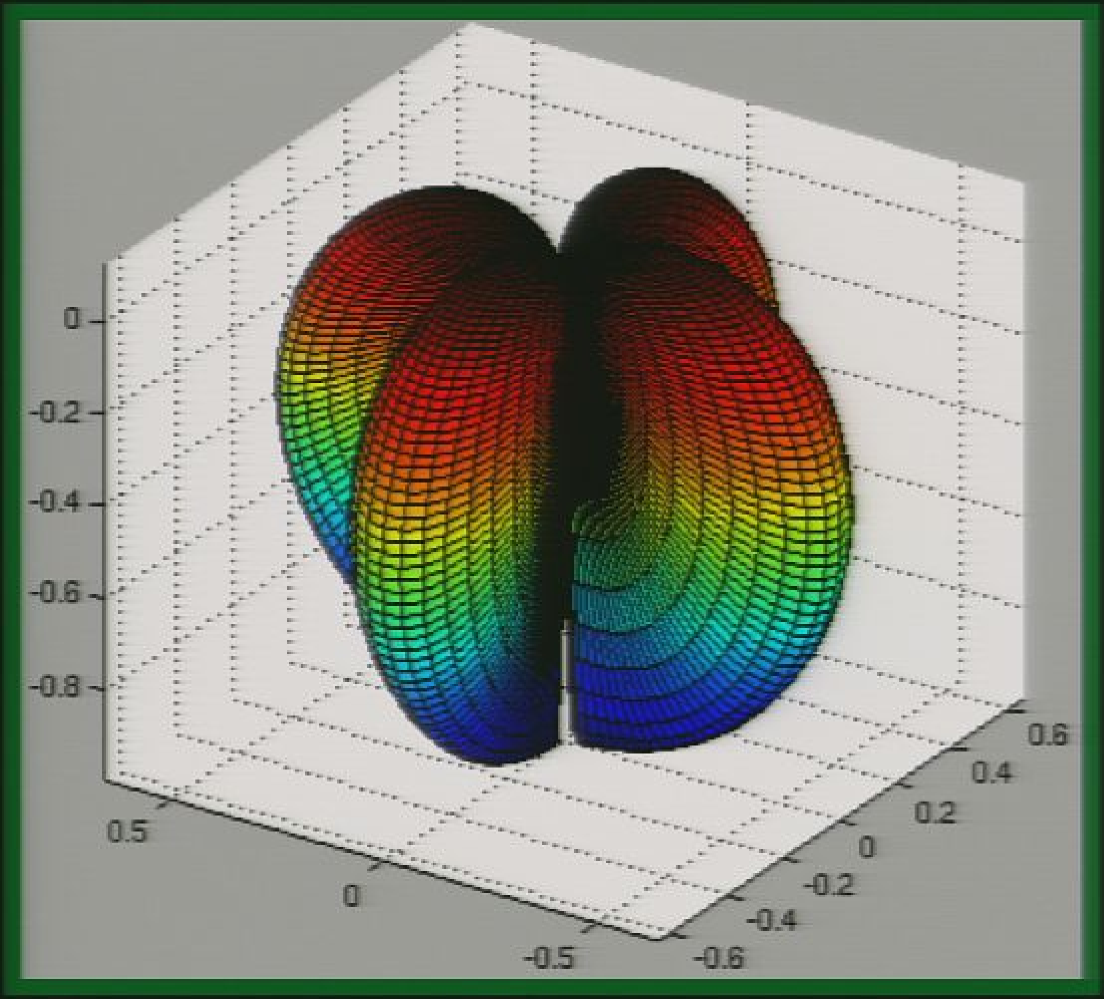

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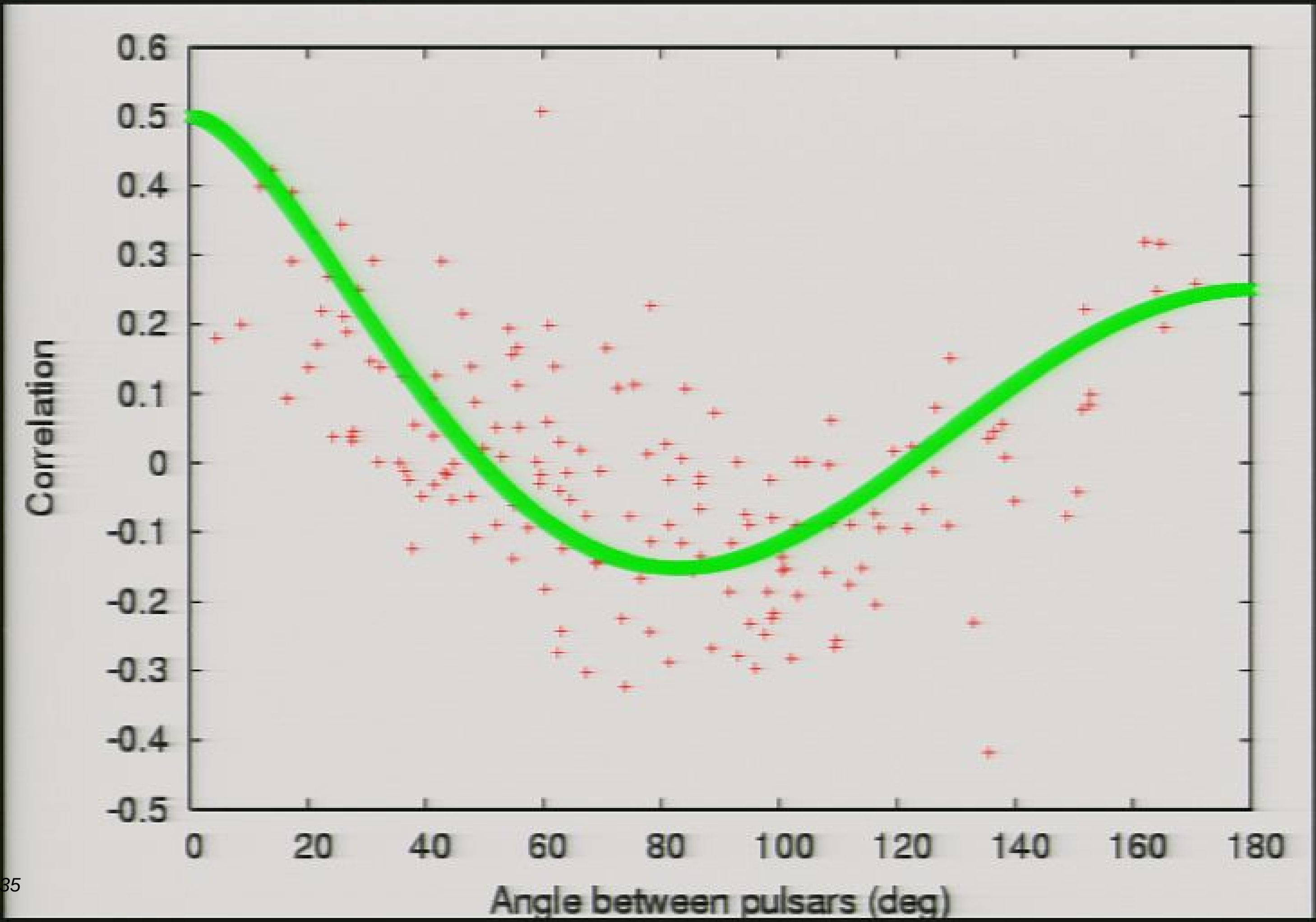

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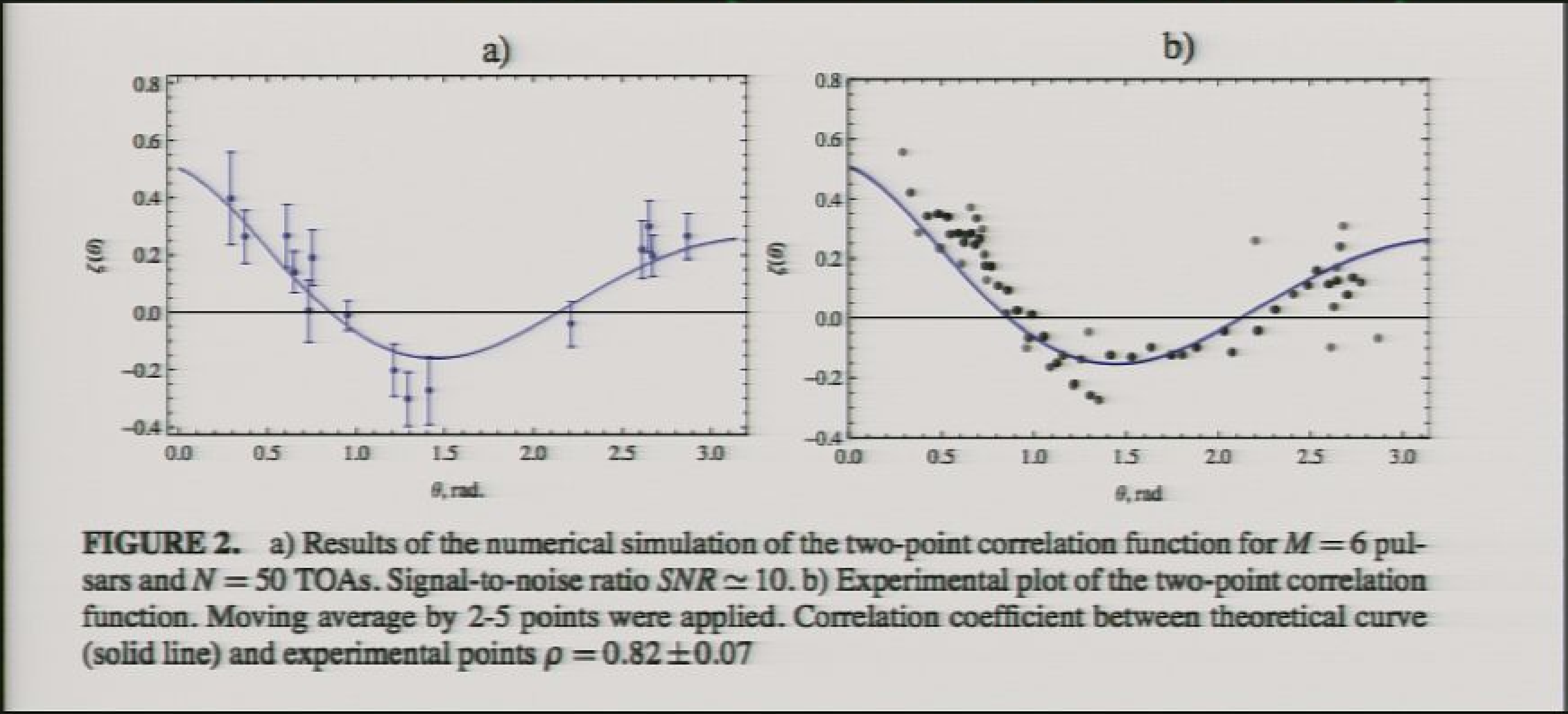

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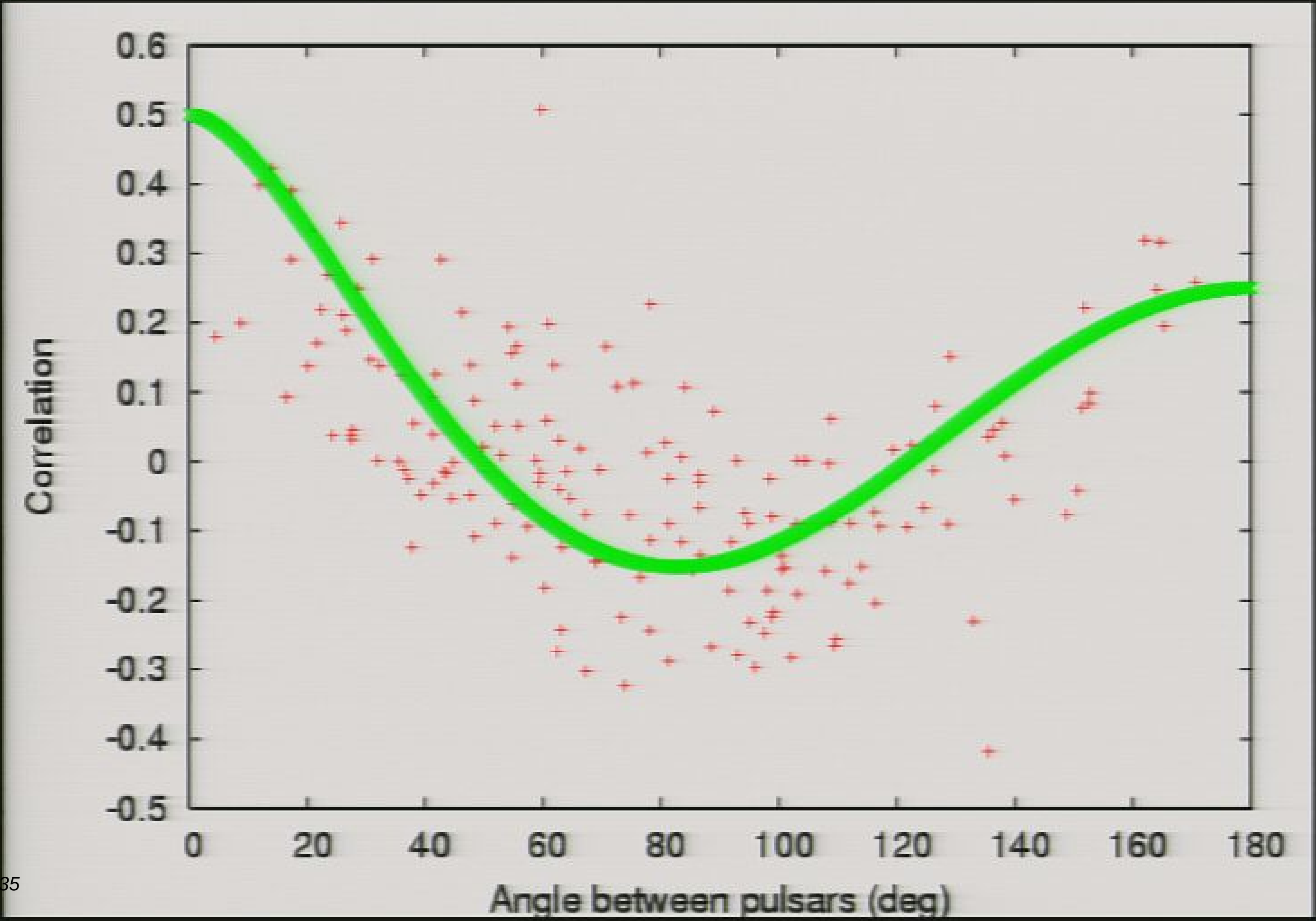

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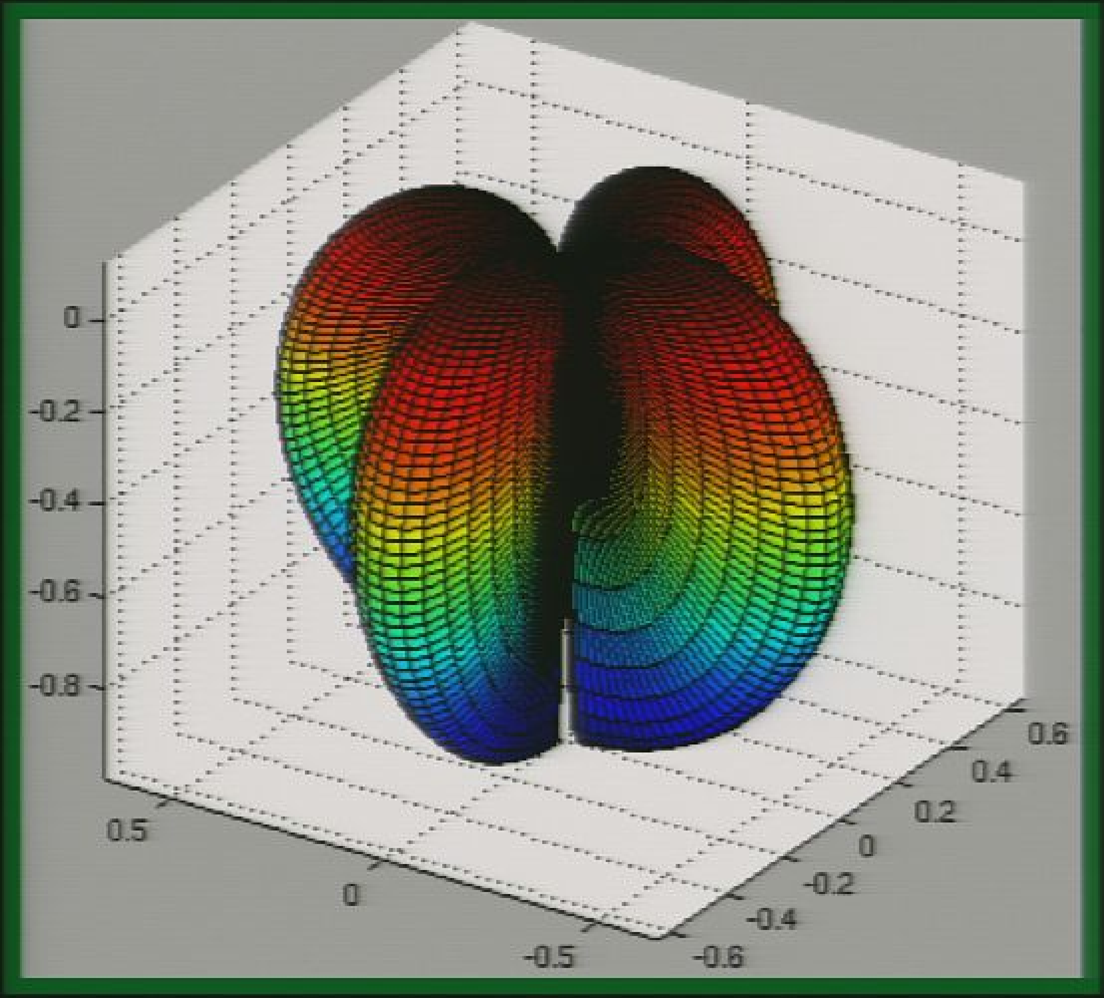

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where $h_{jk}(t,\vec{x}) = h_0 f'\left(t - \hat{k}\cdot\vec{x}\right)$**e** and L is the distance to the pulsar

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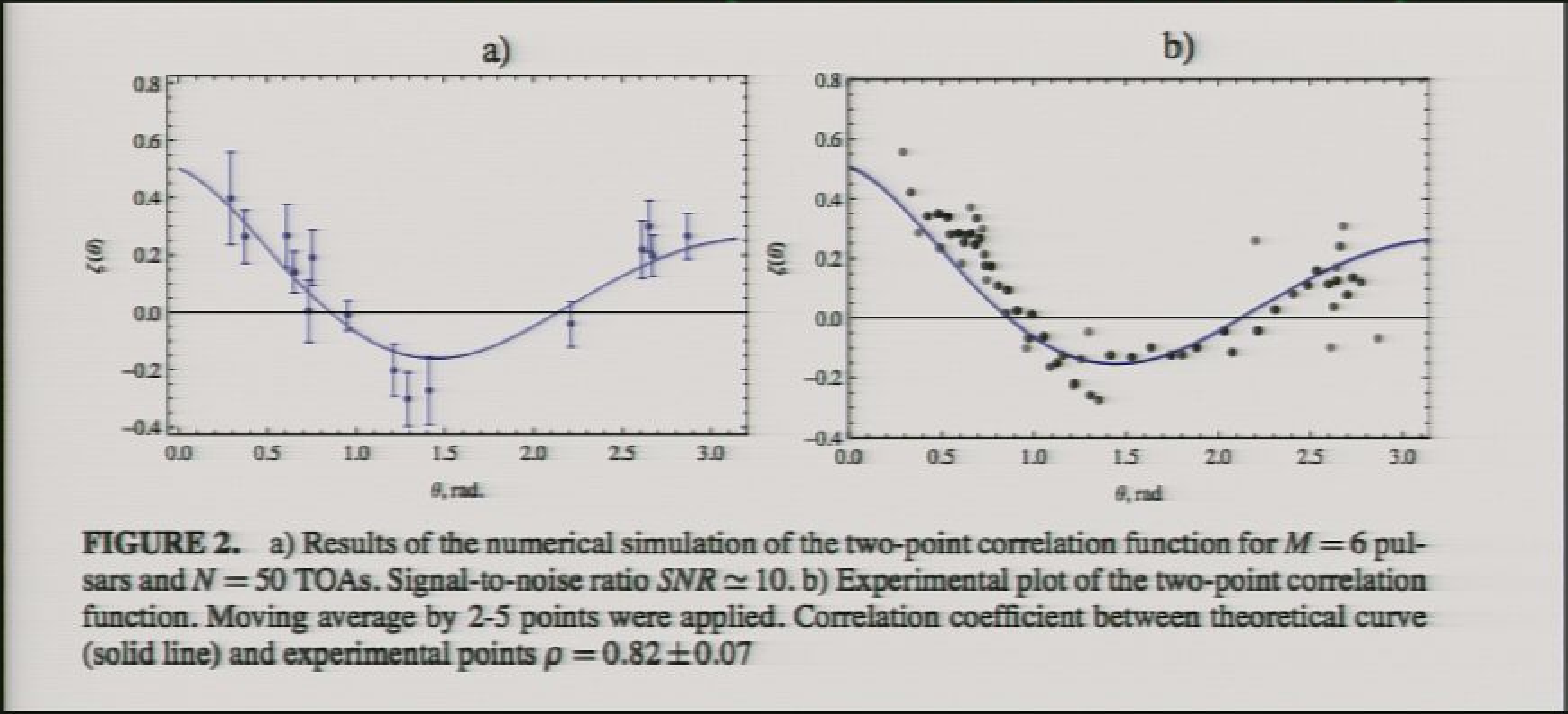

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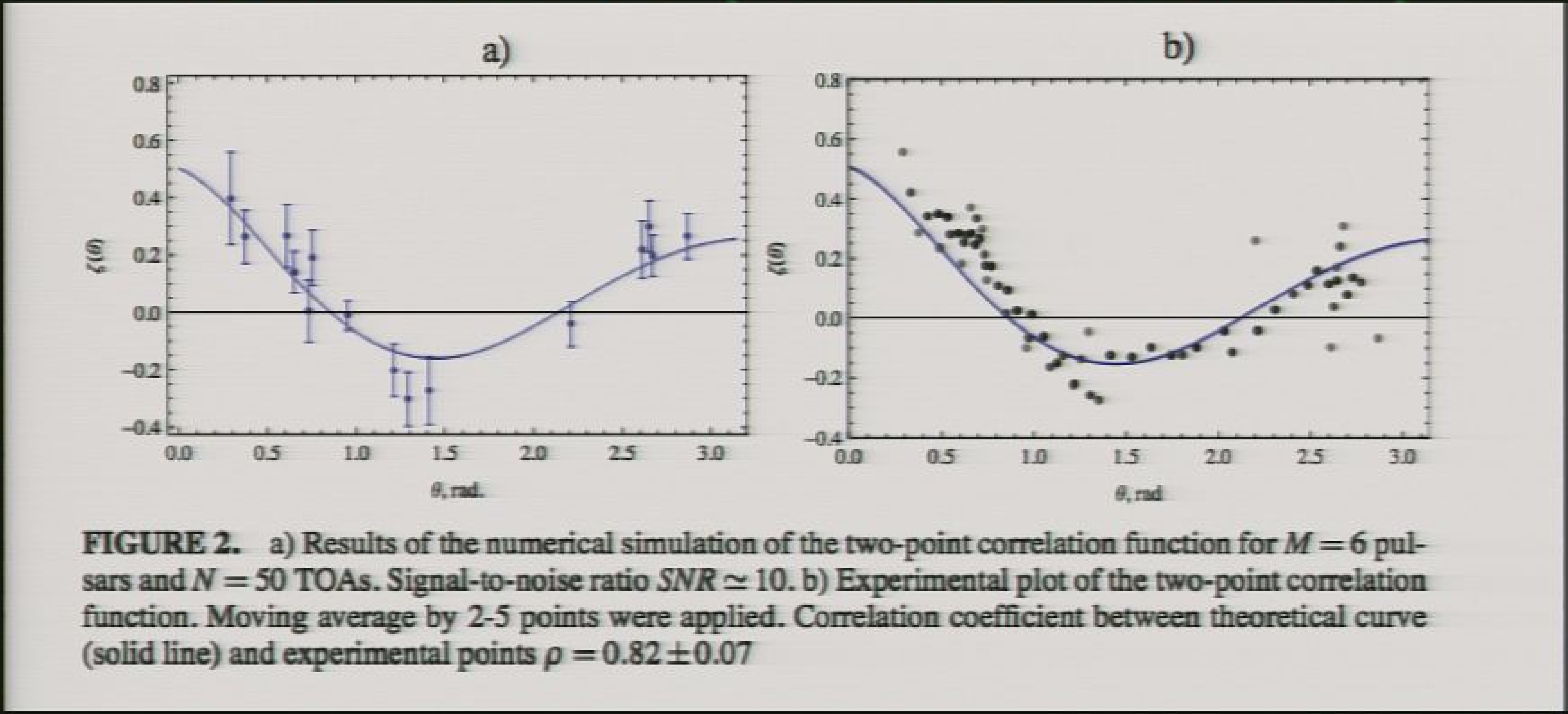

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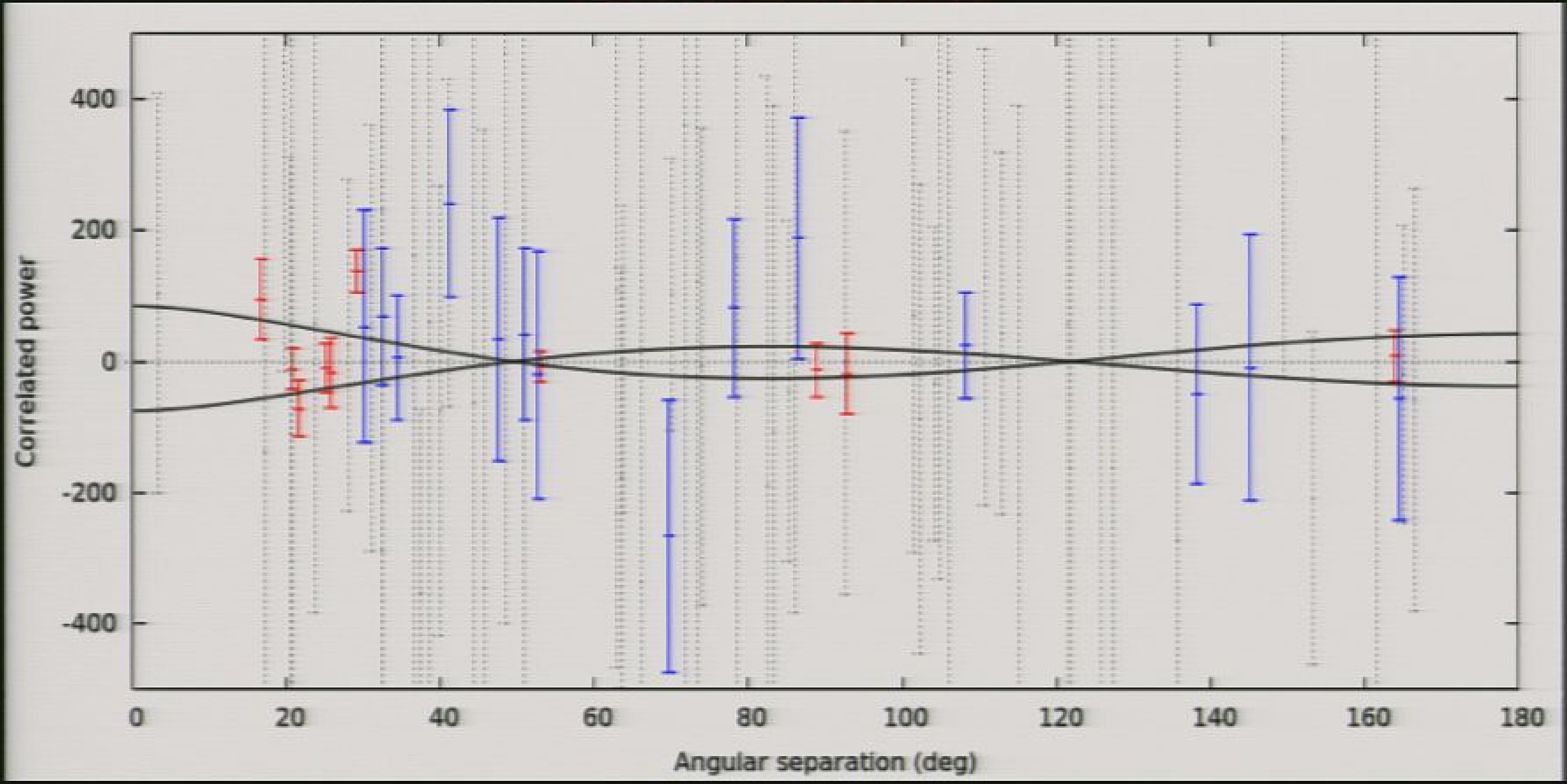
Timing residual GW cross-correlation analysis

NANOGrav



Computed using methods from Demorest PhD (2007):

- Accounts for GW power removed by timing fit.
- Assumes/optimized for -2/3 power law GW spectrum.

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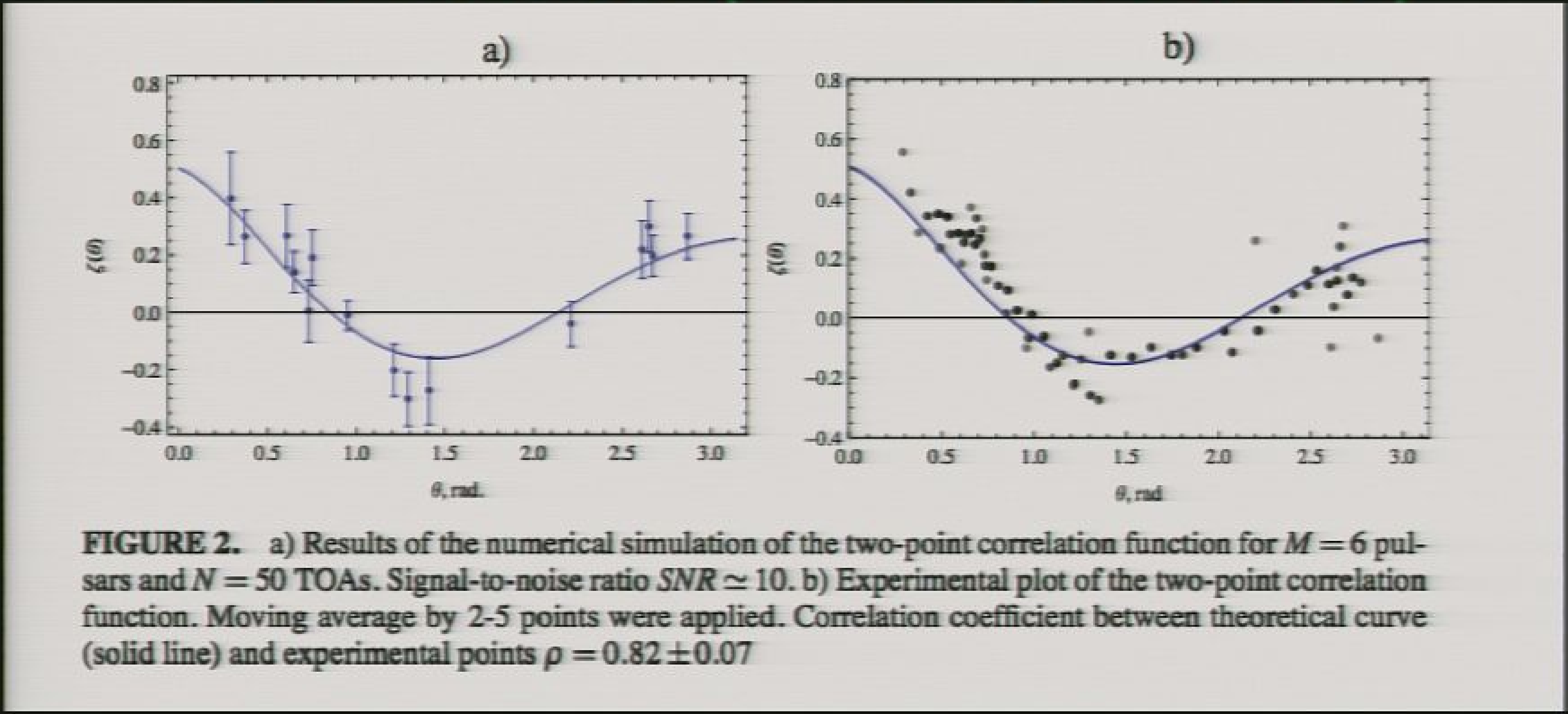
From Rodin, astroph: 1012.3038 (14 Dec 2010)



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Timing residual GW cross-correlation analysis

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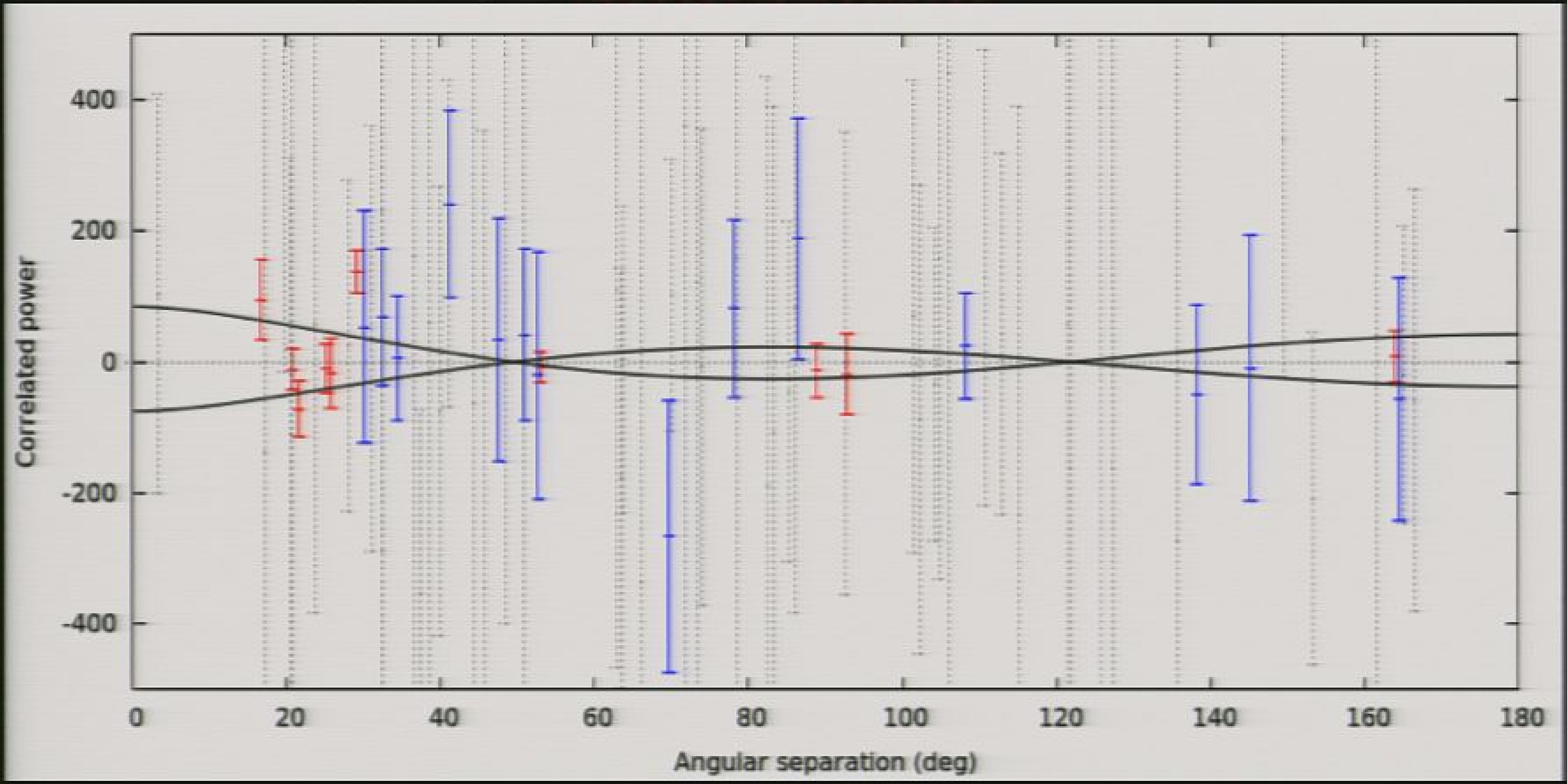

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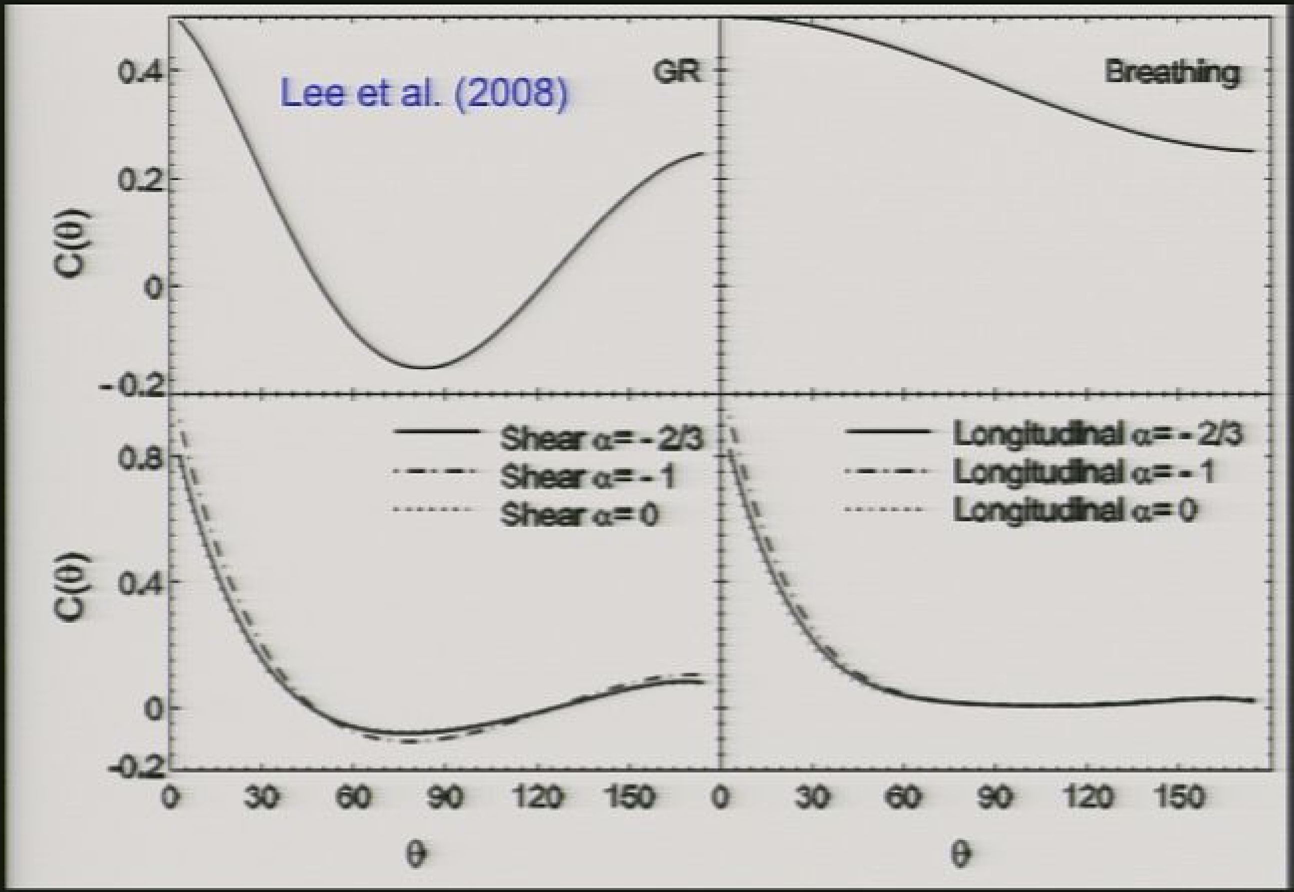
Lee et al. (2008)

- Measure the polarisation properties of gravitational wave
- Test theories of gravity ... !

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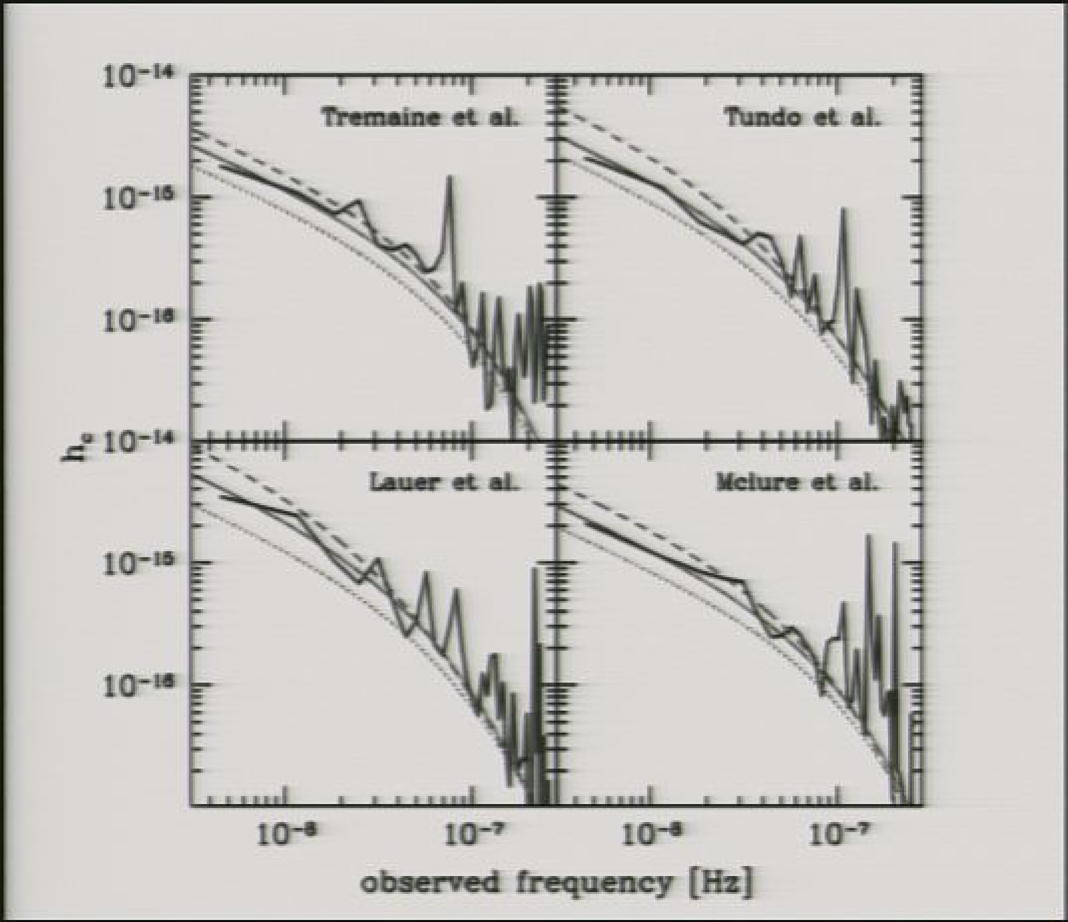
Sesana, Vecchio and Volunteri 2009



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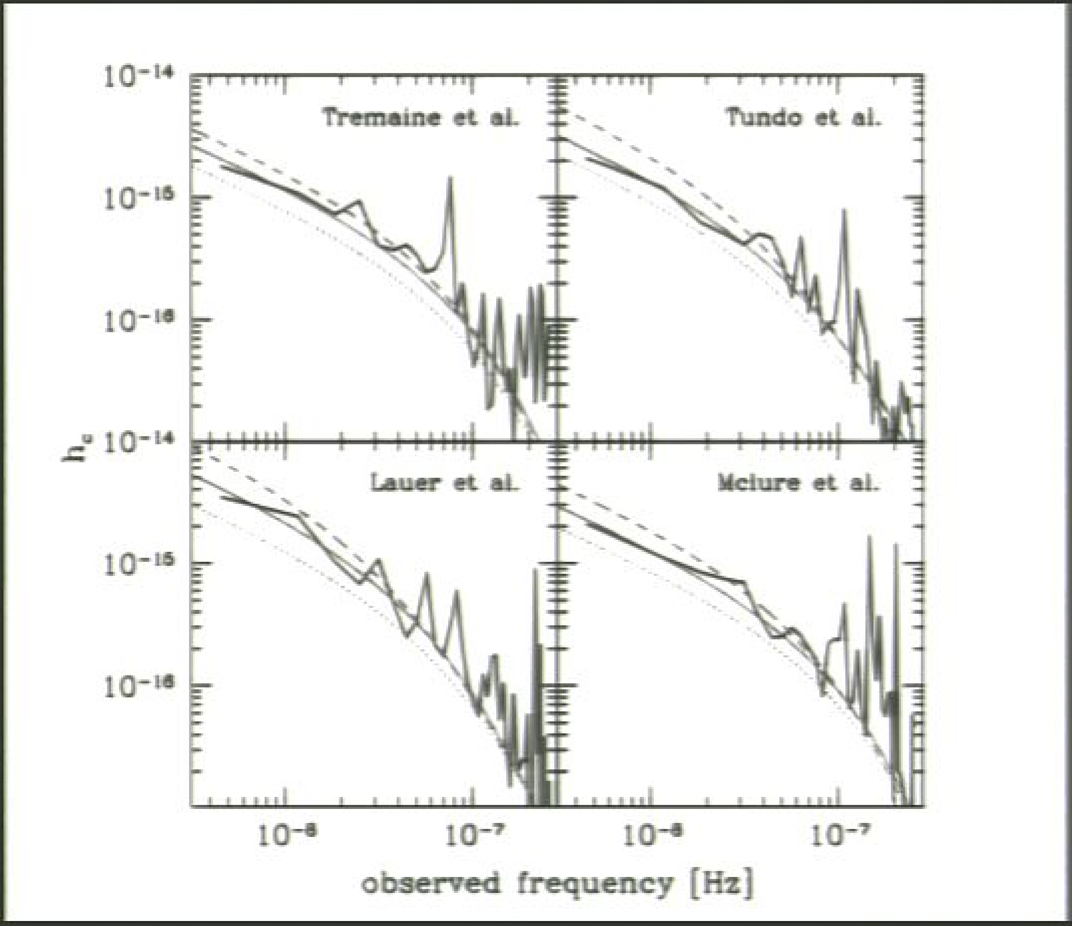
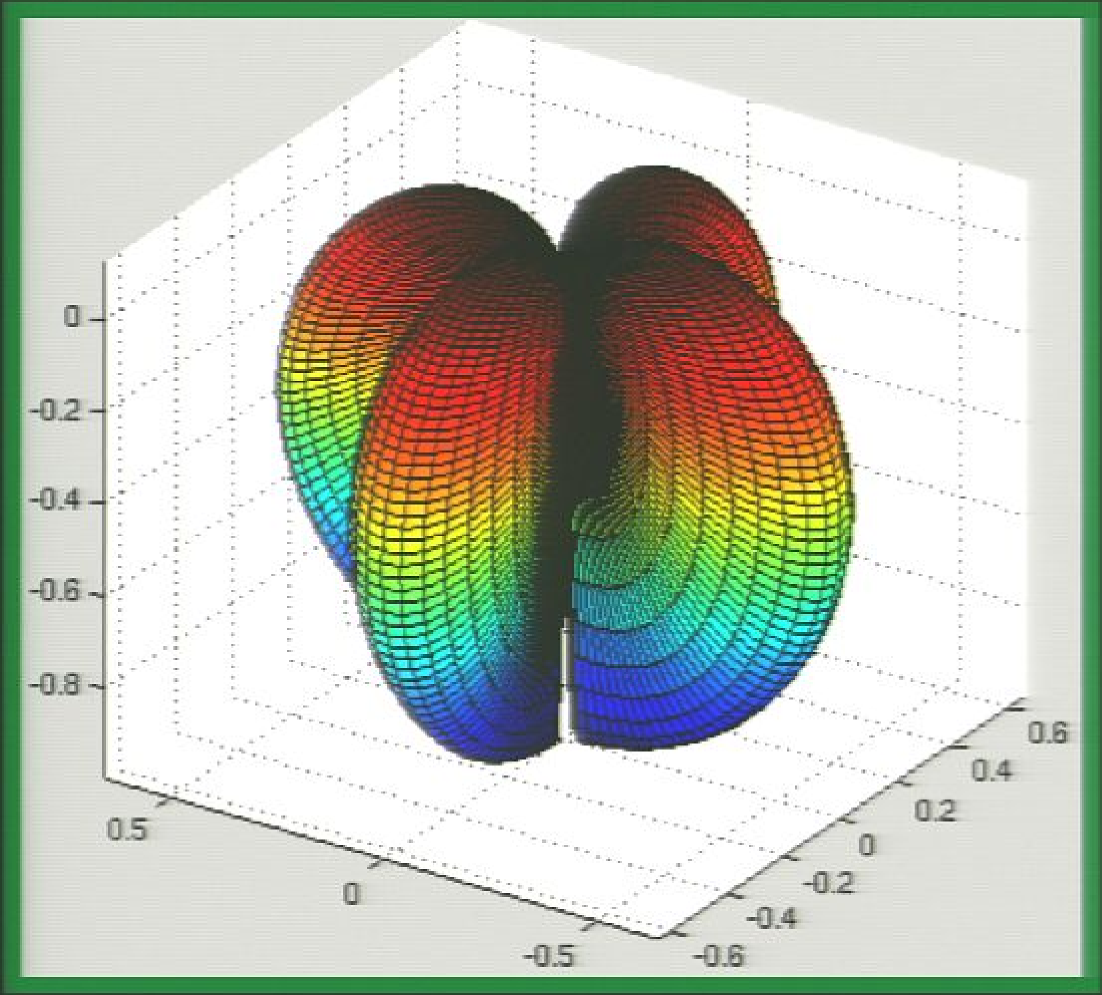
Sesana, Vecchio and Volunteri 2009



The shape of the GW response



Thanks Bill Coles

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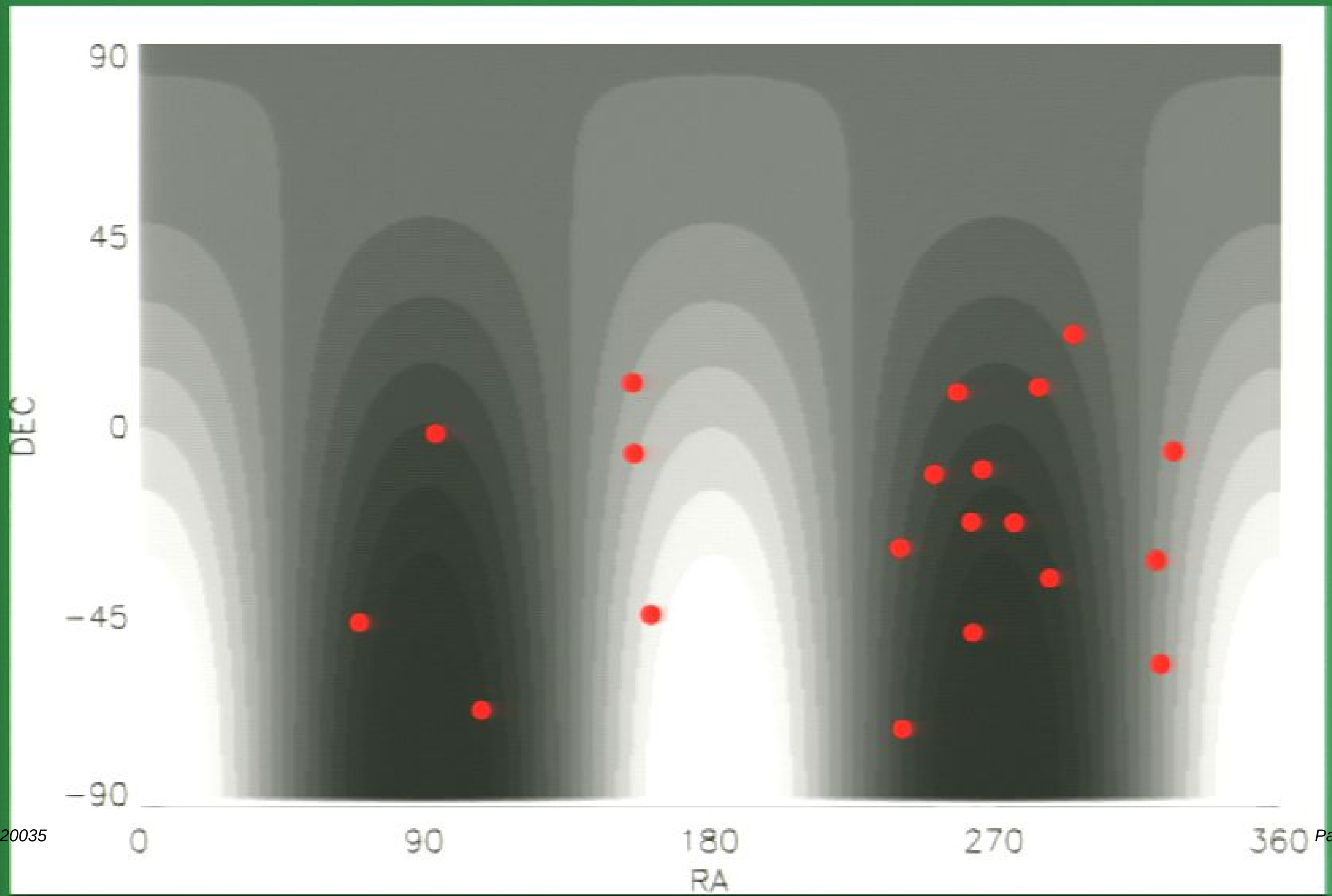


NANOGrav

Maximum Entropy

$$d = n + Rh \Rightarrow n = d - Rh$$

$$p(d \mid h,k,R,N) = \frac{\exp\left[-\frac{1}{2}x^T N^{-1} x\right]}{\sqrt{(2\pi)^{\dim x}\det\|N\|}}$$

where

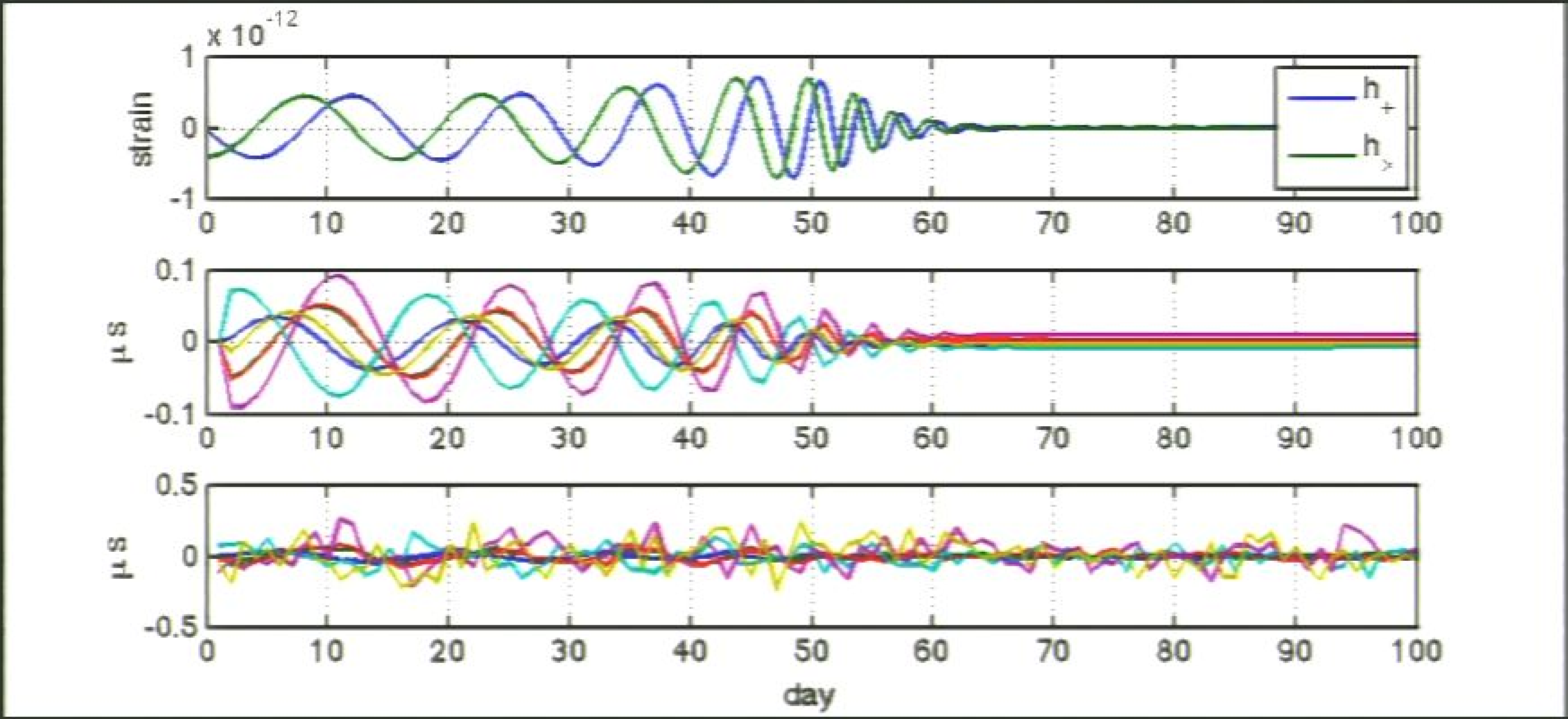
$$x = d - Rh$$

entropy :

$$H(p) = \int dx^n\, p \ln(p)$$

NANOGrav

A 5 x 10^9 solar-mass black hole binary coalescing 100 Mpc away.
30 IPTA pulsars, improved by 10, sampled once a day.



Thank you to Manuela Campanelli, Carlos O. Lousto, Hiroyuki Nakano, and Yosef Zlochower for waveforms. Phys.Rev.D79:084010 (2009). http://ccrg.rit.edu/downloads/waveforms

NANOGrav

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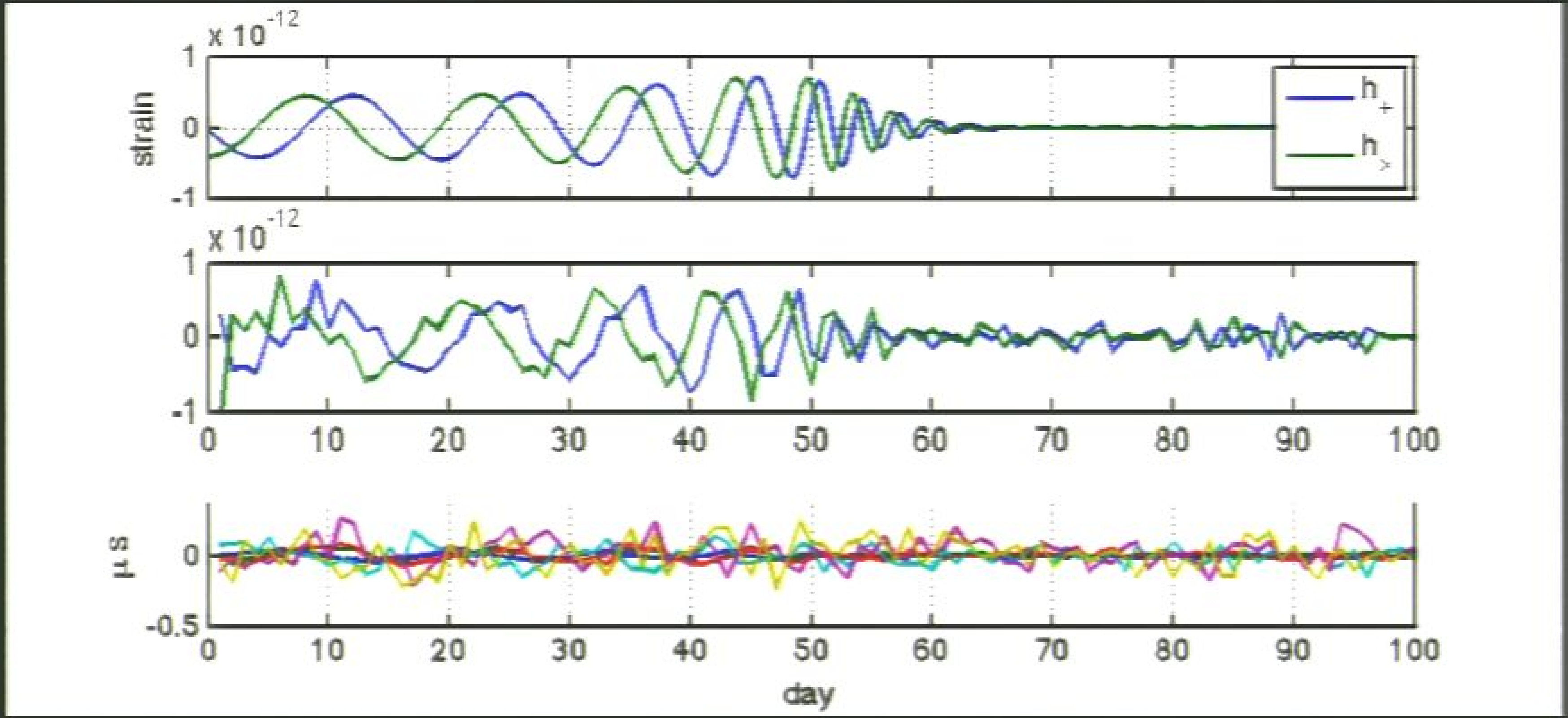
Maximum Entropy based on Summerscales, Burrows, Finn and Ott 2008



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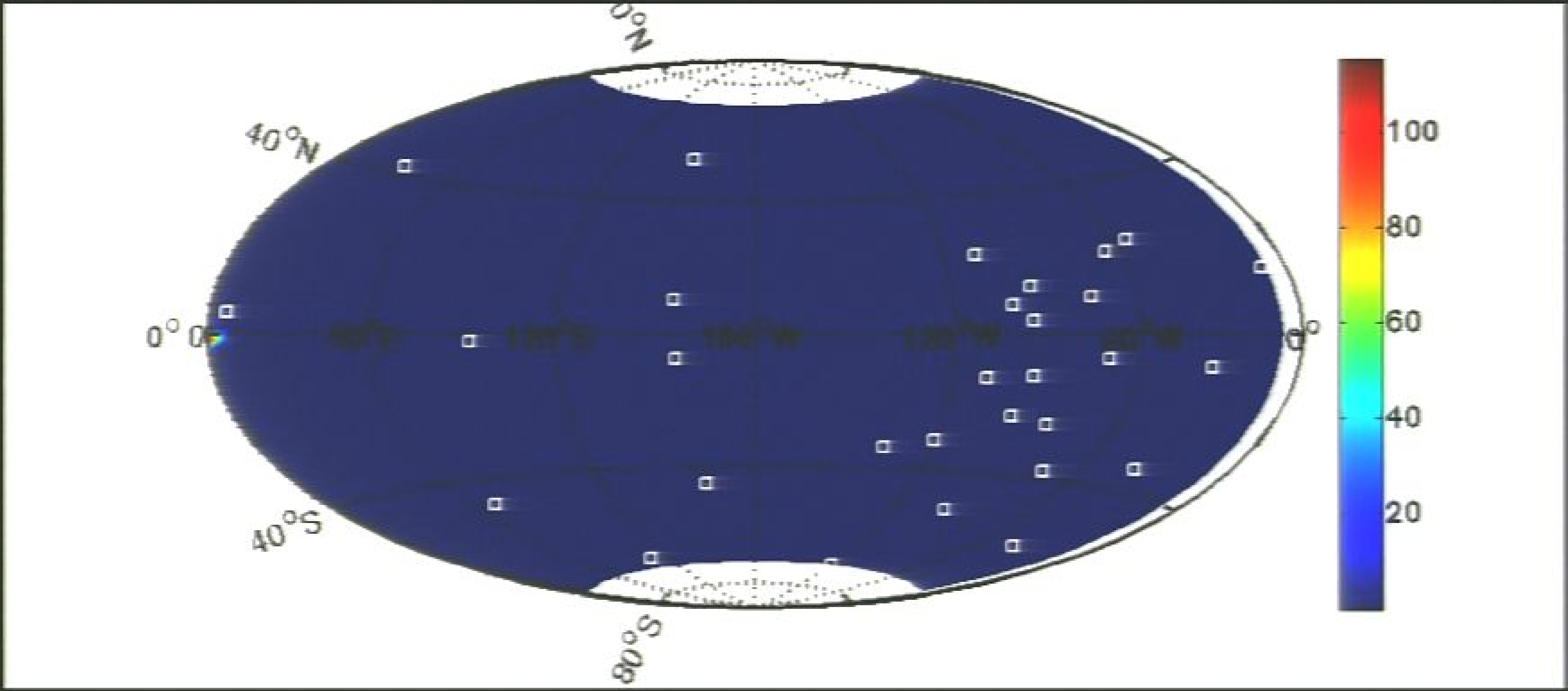
Probability density



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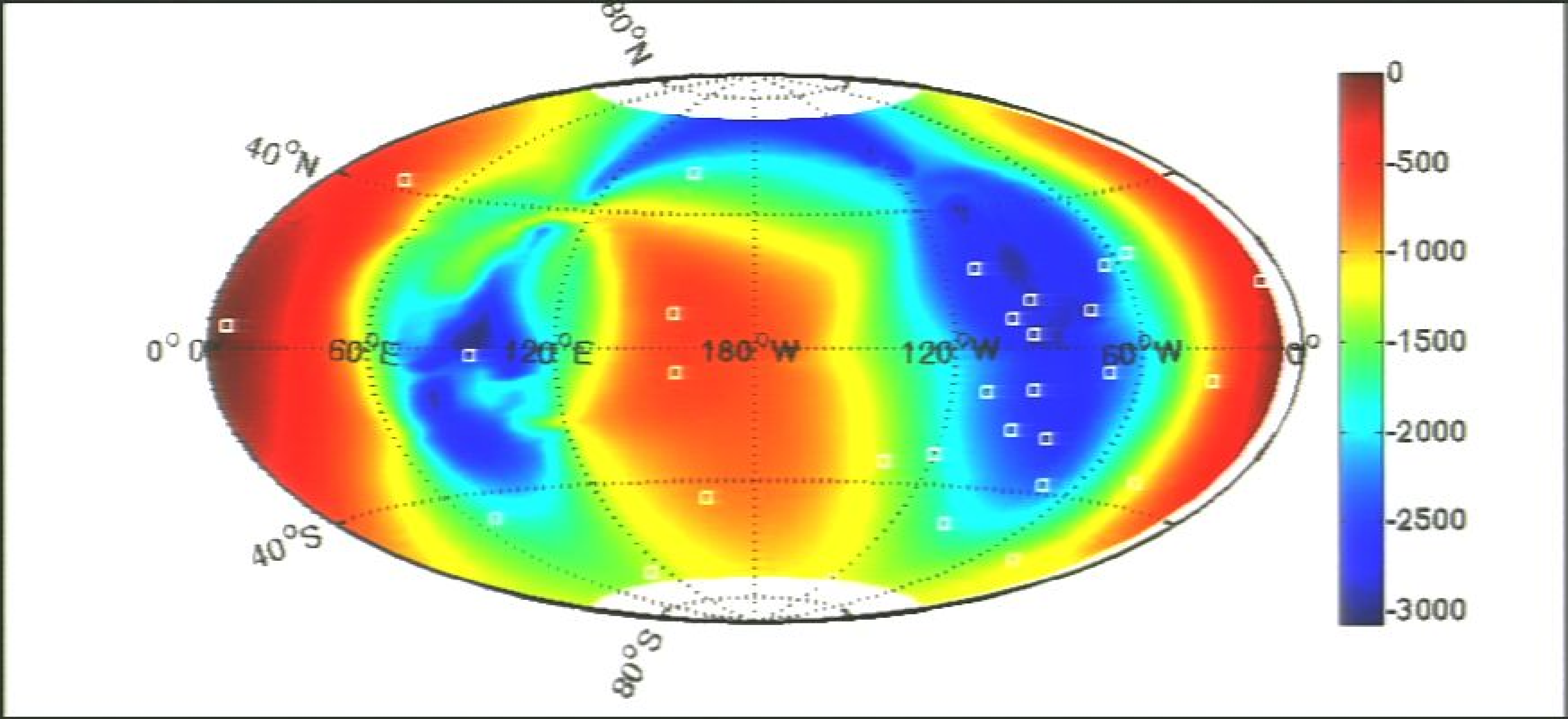
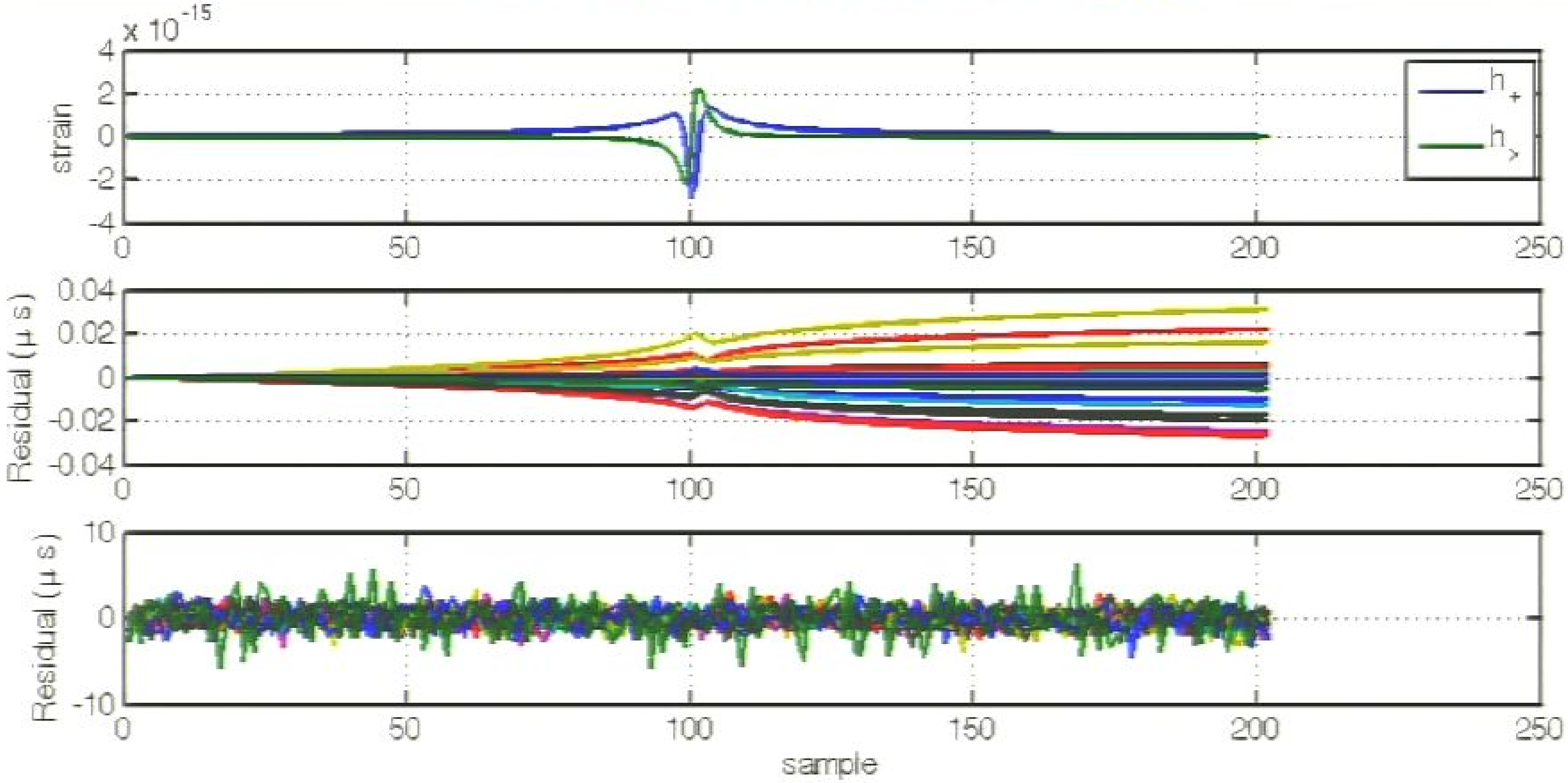
Log of probability density



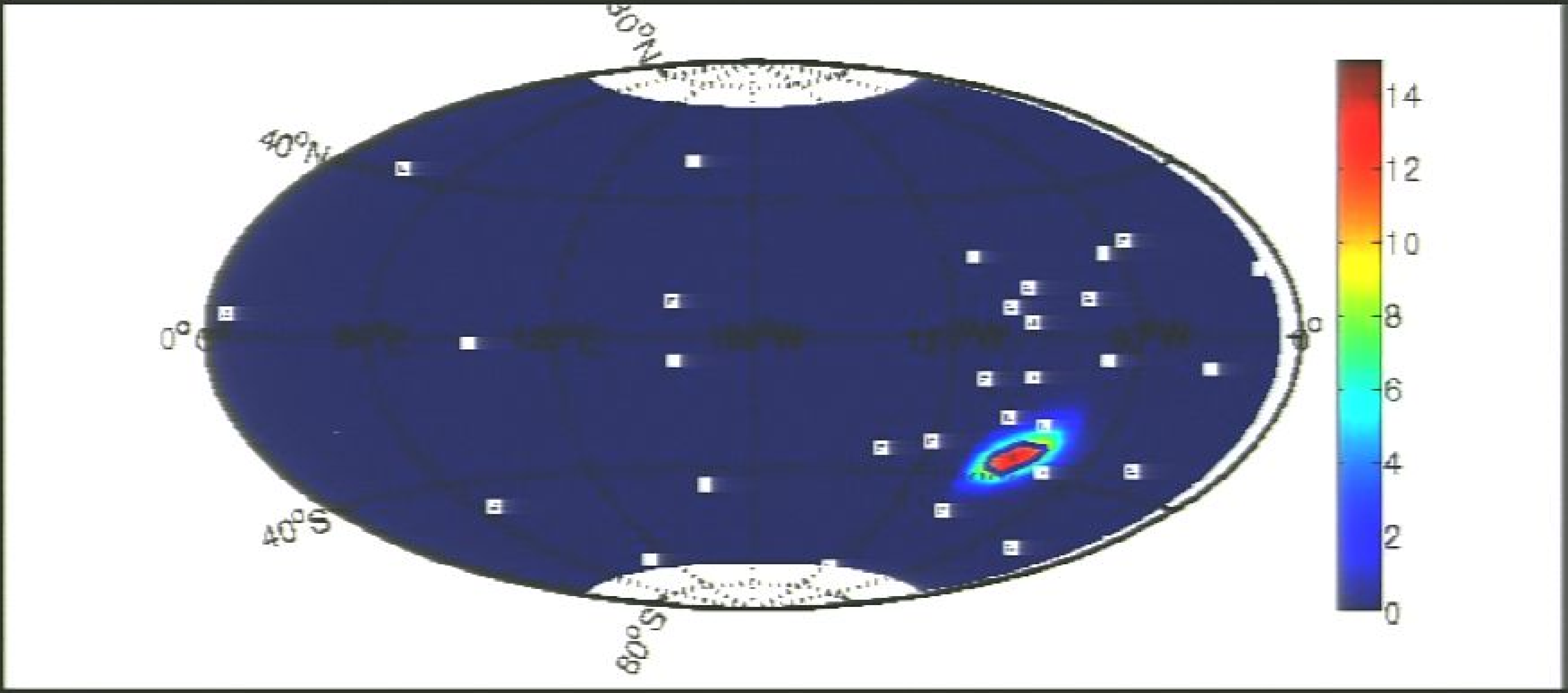


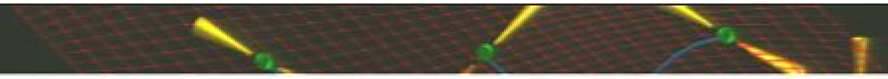
2 10^9 solar mass black holes flying by each other with a separation of 40 Schwarzschild Radii.

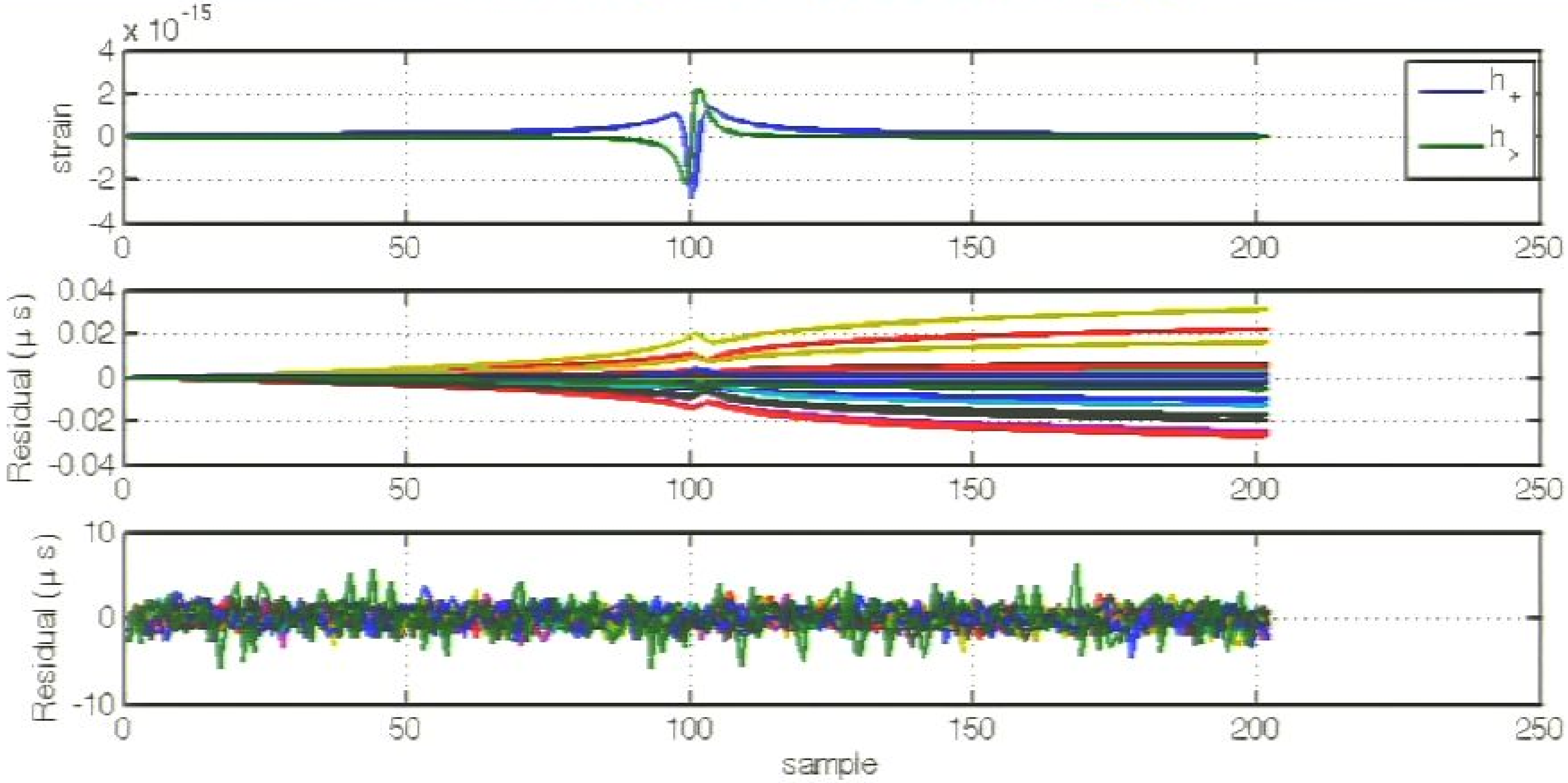
Distance: 100 Mpc

30 IPTA pulsars

Probability density as a function of sky position







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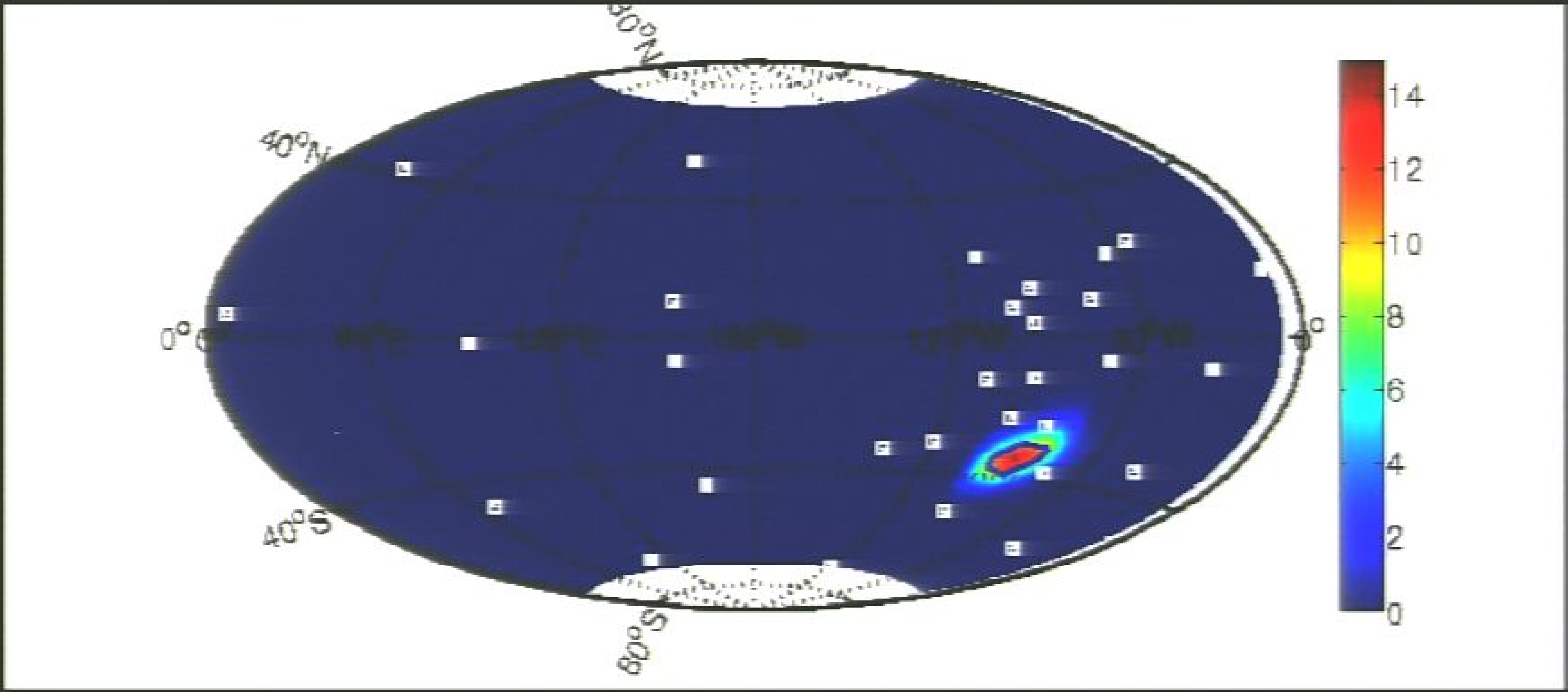
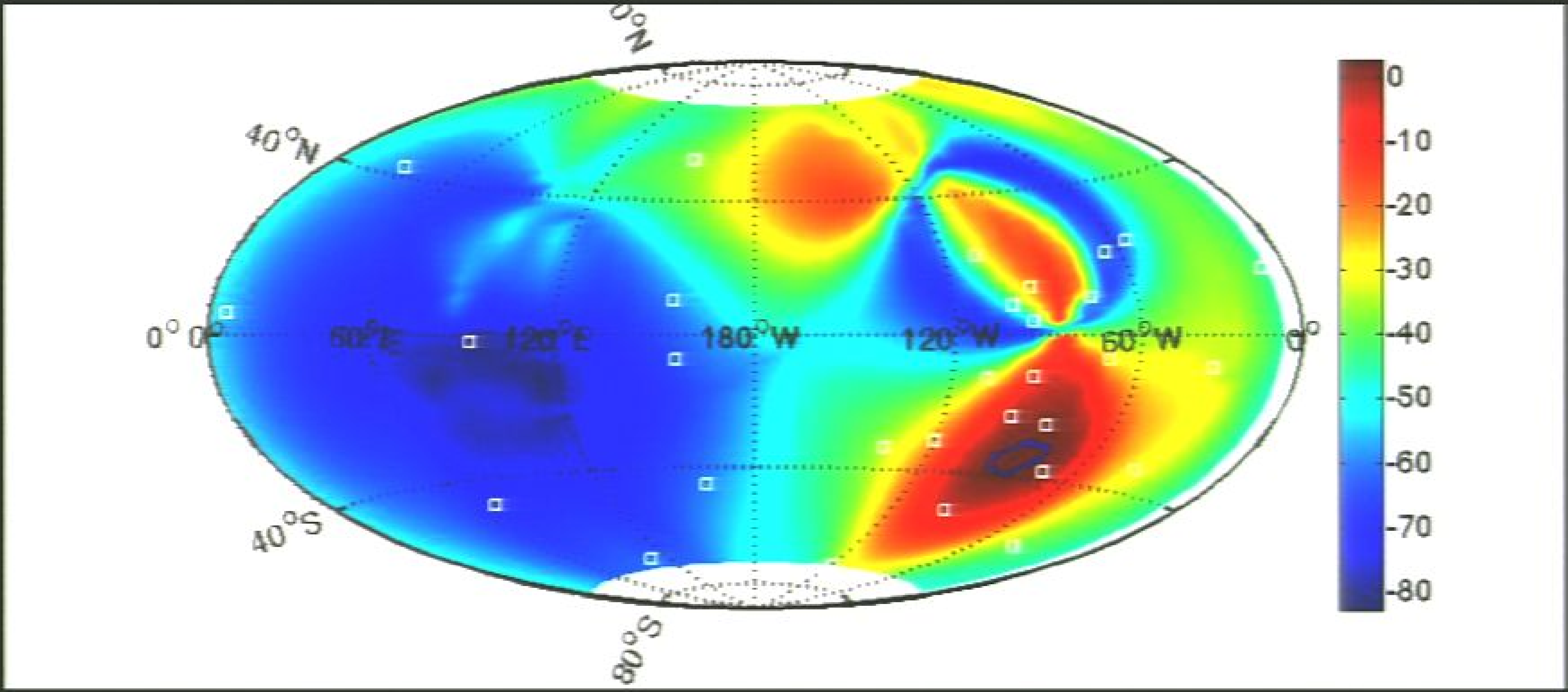
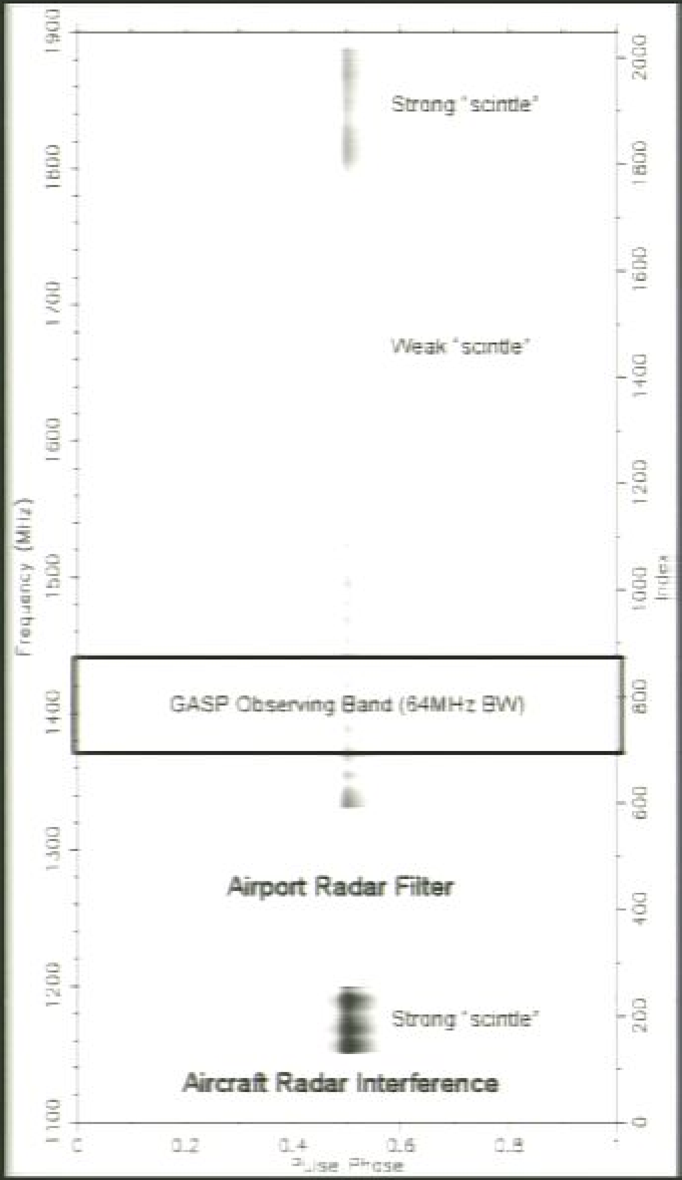


Log(probability density) as a function of sky position

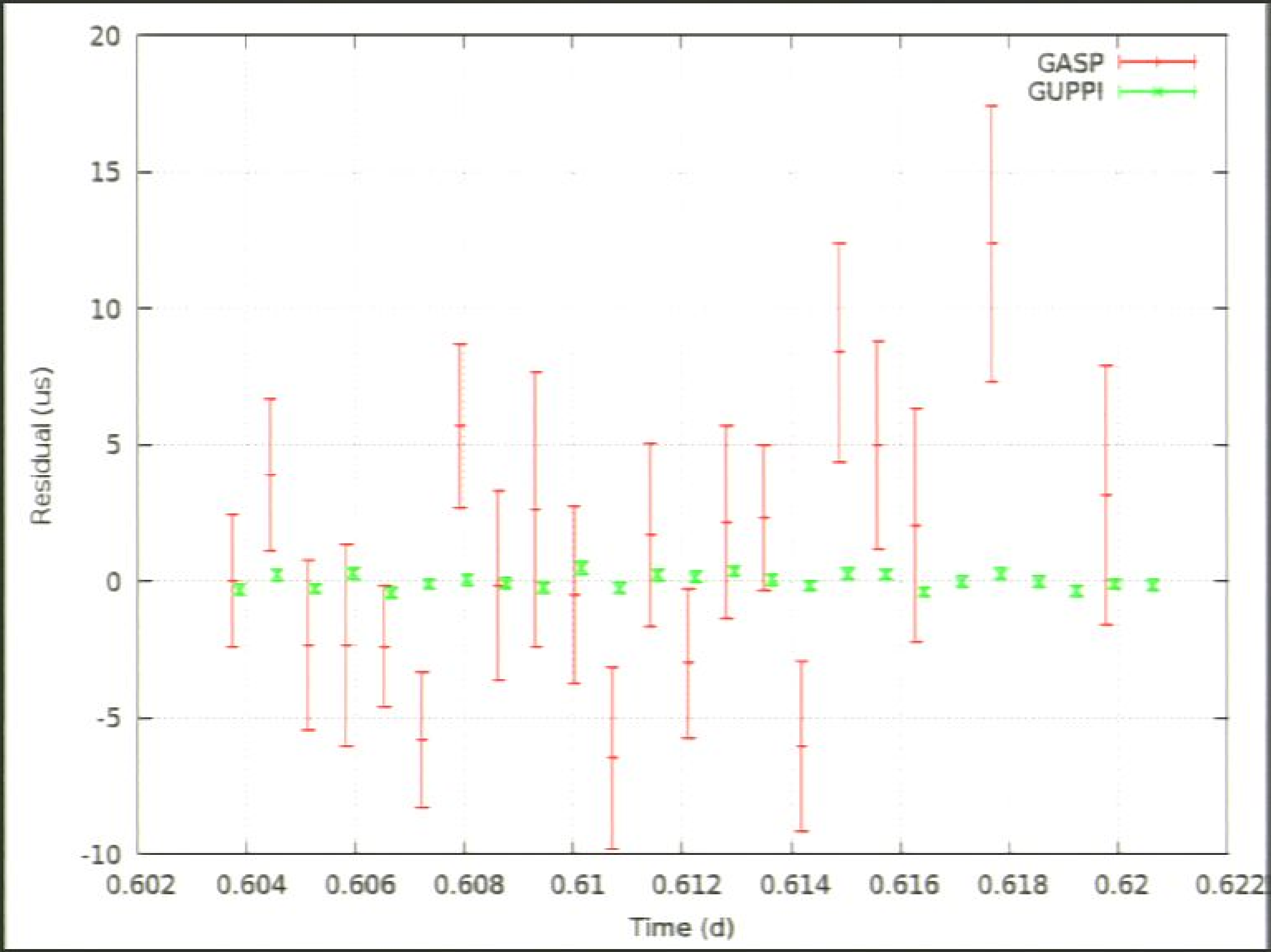


NANOGrav

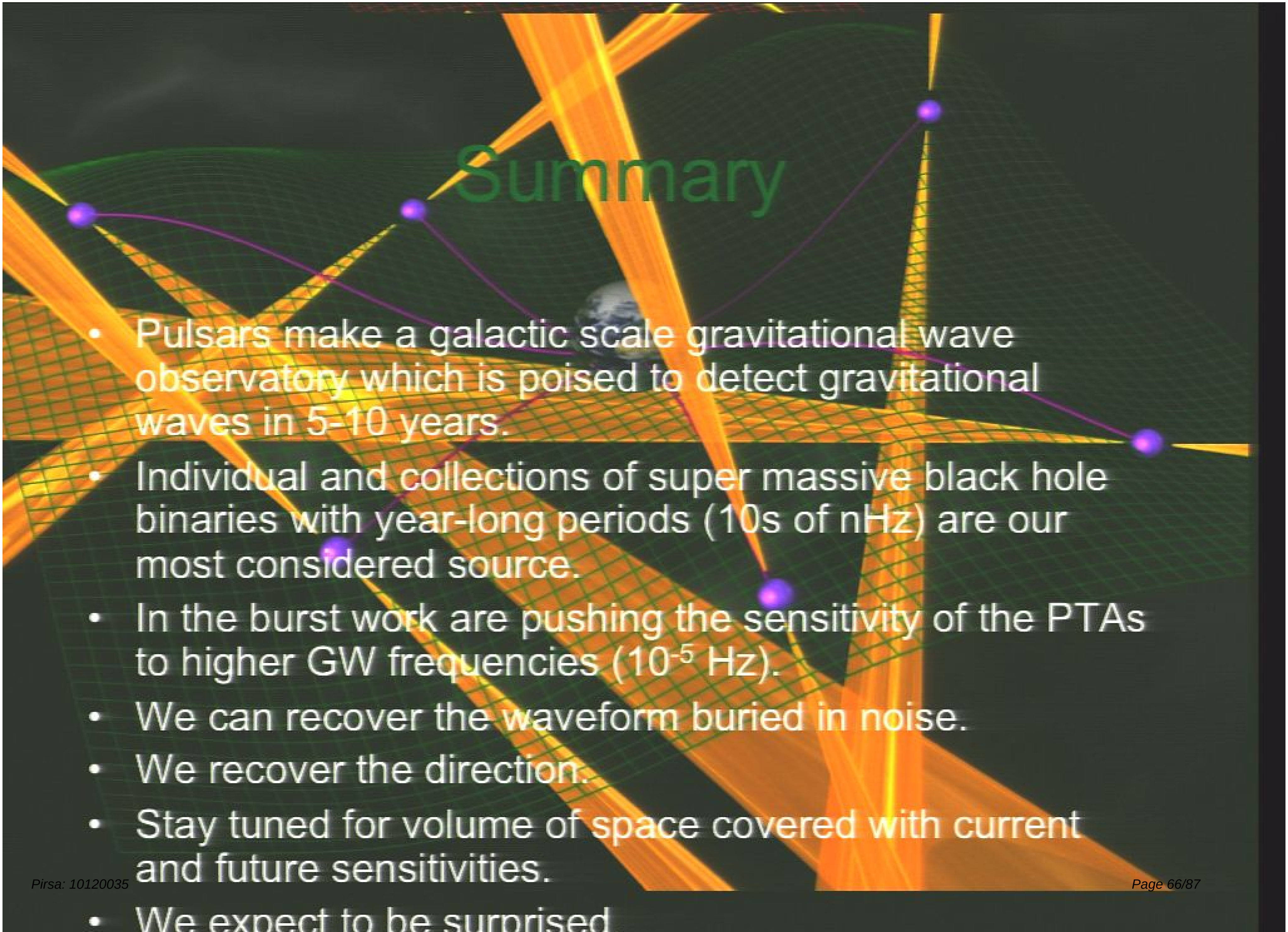
The Advantage of New Wide-band Backend System at Green Bank "GUPPI"



NANOGrav



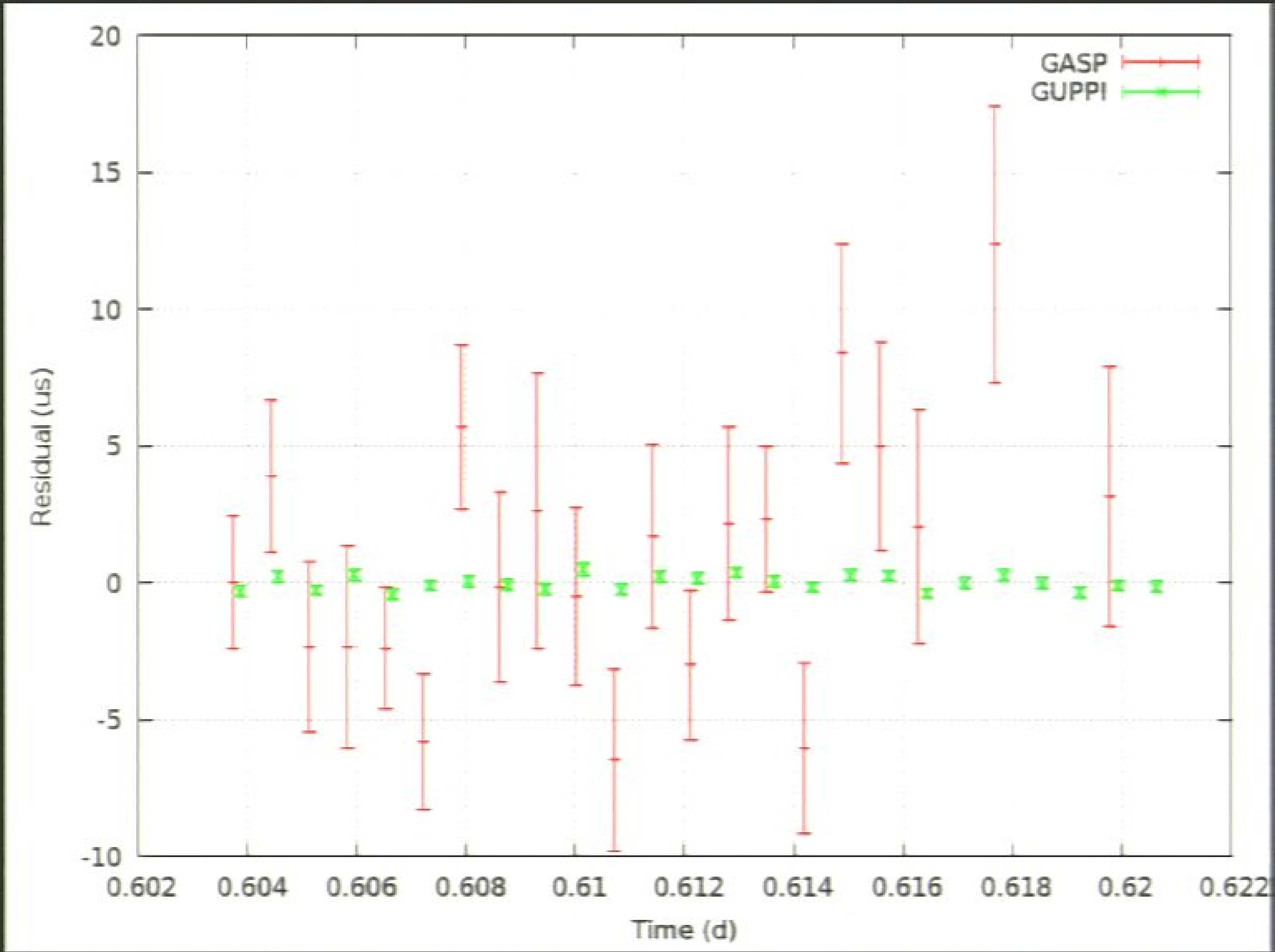
GASP
GUPPI
Residual (us)
Time (d)
20
15
10
5
0
-5
-10
0.602
0.604
0.606
0.608
0.61
0.612
0.614
0.616
0.618
0.62
0.622



Summary

- Pulsars make a galactic scale gravitational wave observatory which is poised to detect gravitational waves in 5-10 years.
- Individual and collections of super massive black hole binaries with year-long periods (10s of nHz) are our most considered source.
- In the burst work are pushing the sensitivity of the PTAs to higher GW frequencies (10^{-5} Hz).
- We can recover the waveform buried in noise.
- We recover the direction.
- Stay tuned for volume of space covered with current and future sensitivities.
- We expect to be surprised

NANOGrav


GASP
GUPPI
Residual (us)
Time (d)
20
15
10
5
0
-5
-10
0.602
0.604
0.606
0.608
0.61
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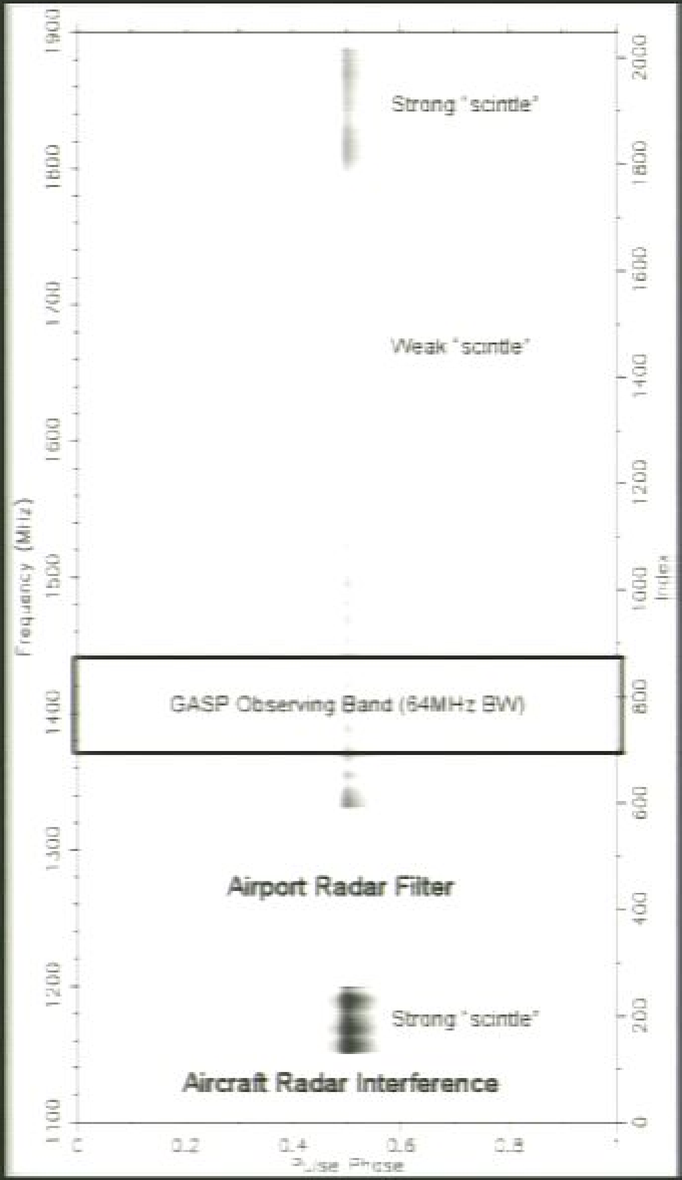
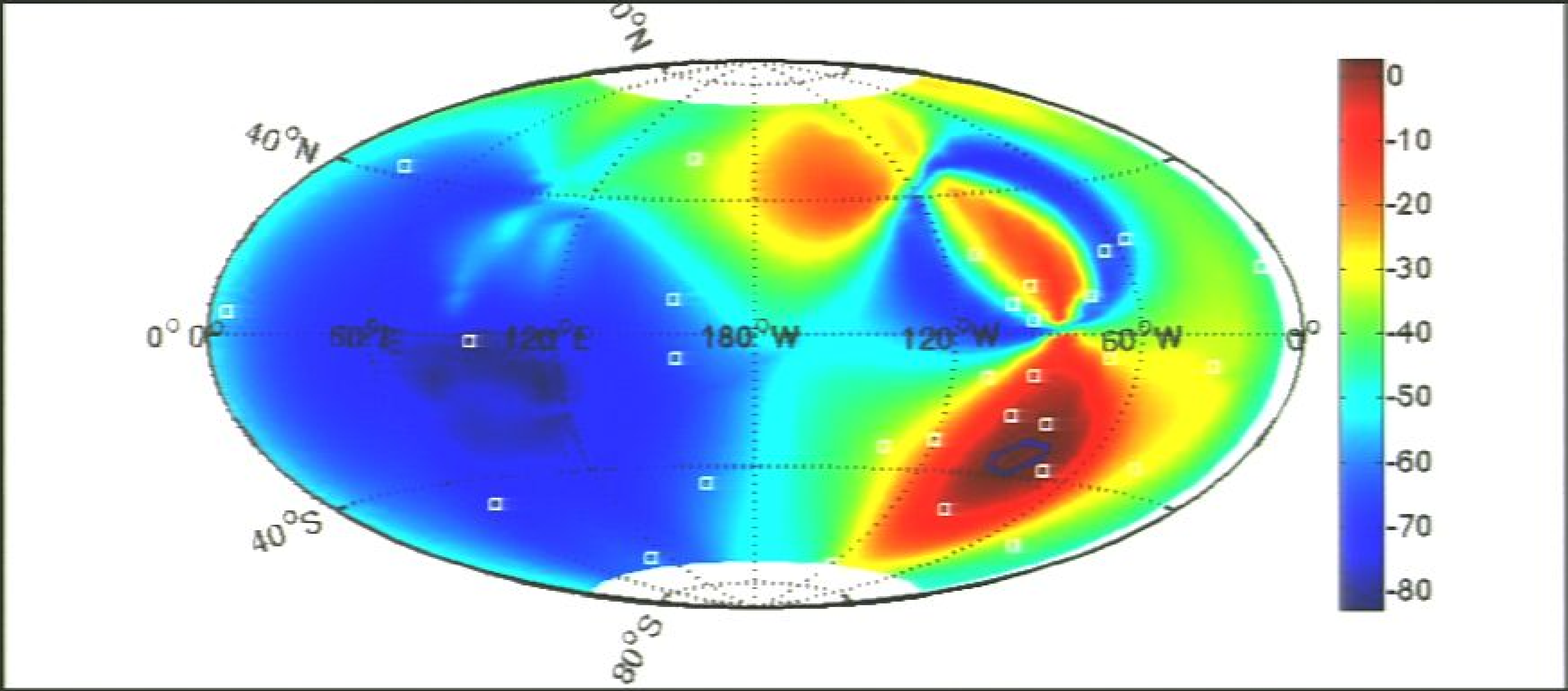
0.622

NANOGrav

The Advantage of New Wide-band Backend System at Green Bank “GUPPI”



Log(probability density) as a function of sky position



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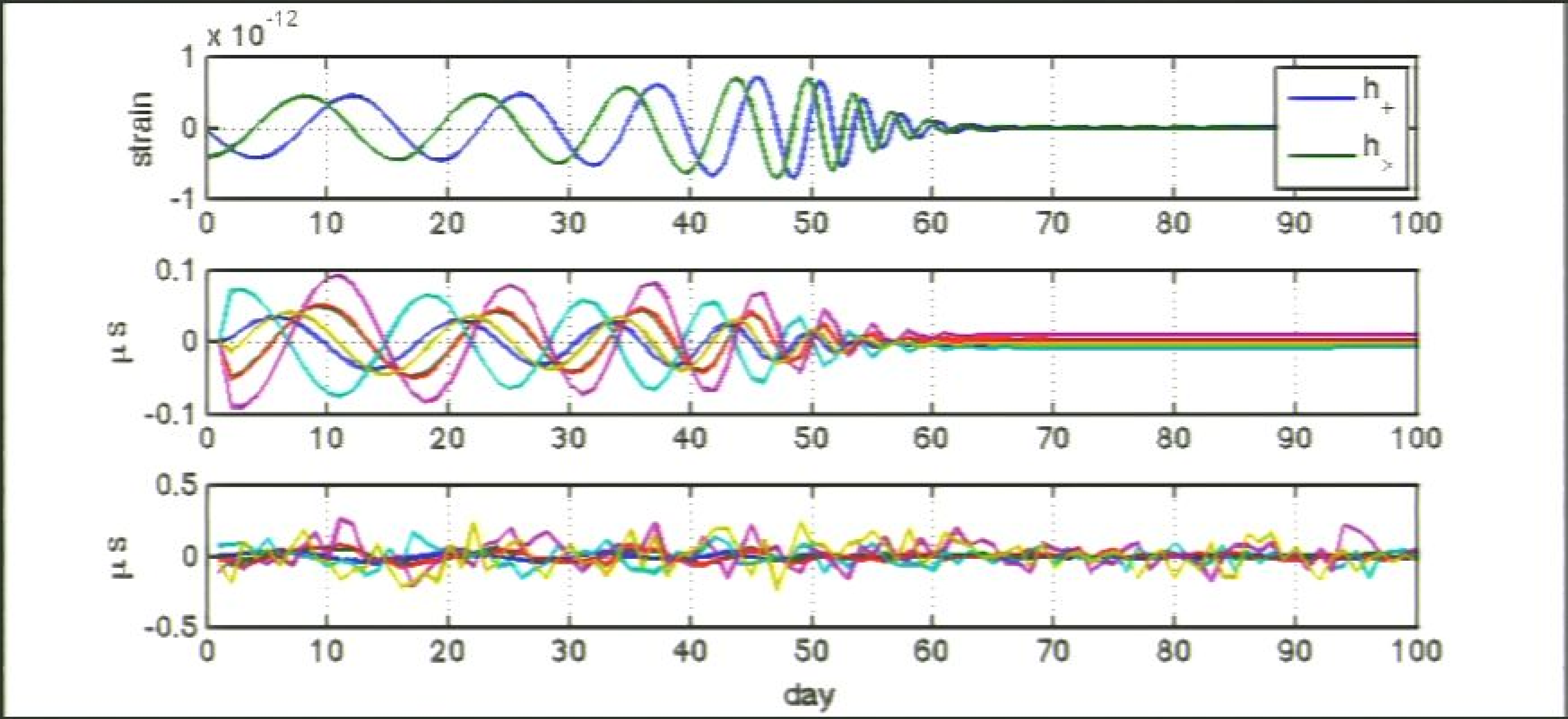

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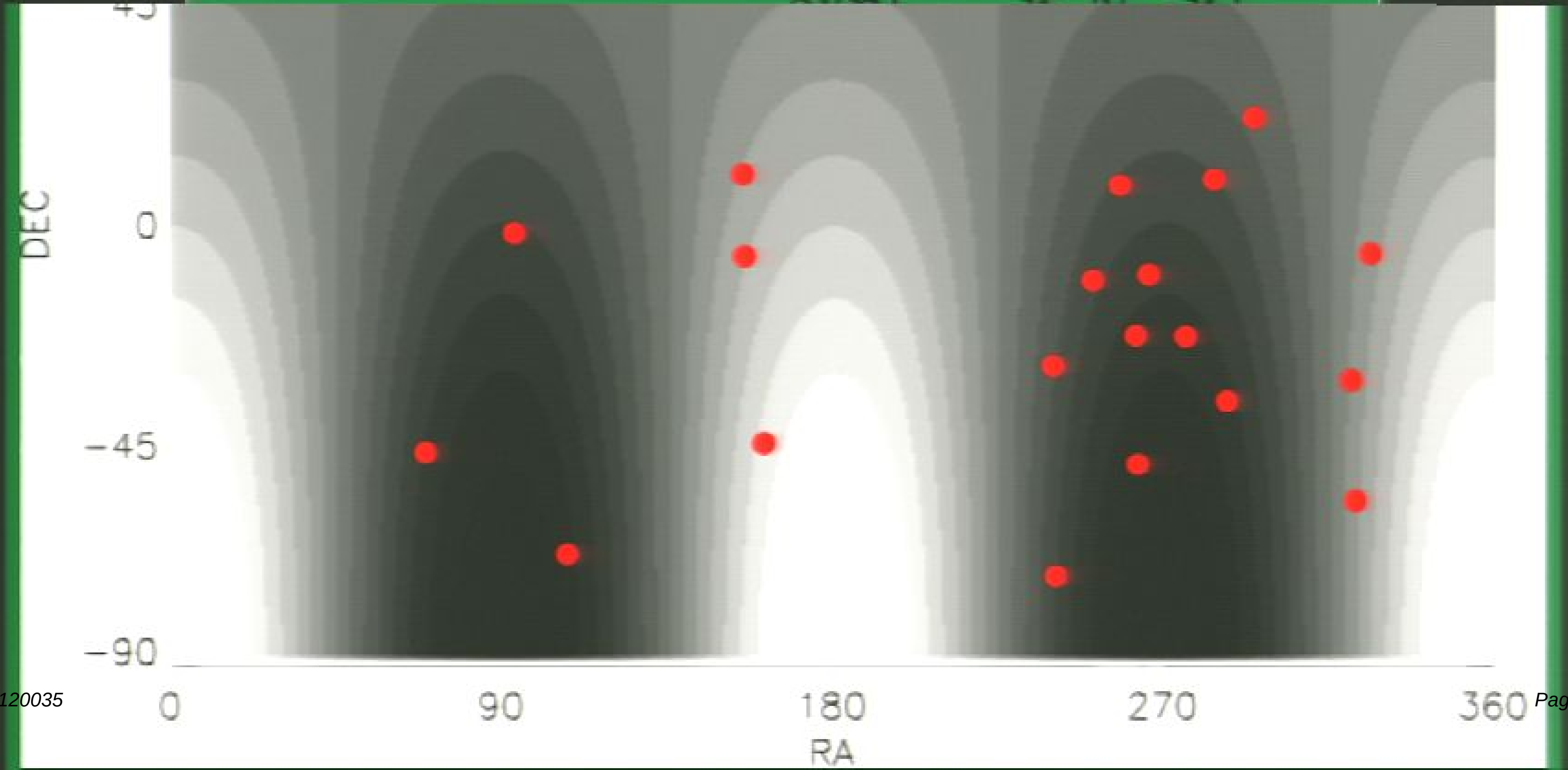
$$\exp\left[\ 1\ n^T N^{-1} n\ \right]$$



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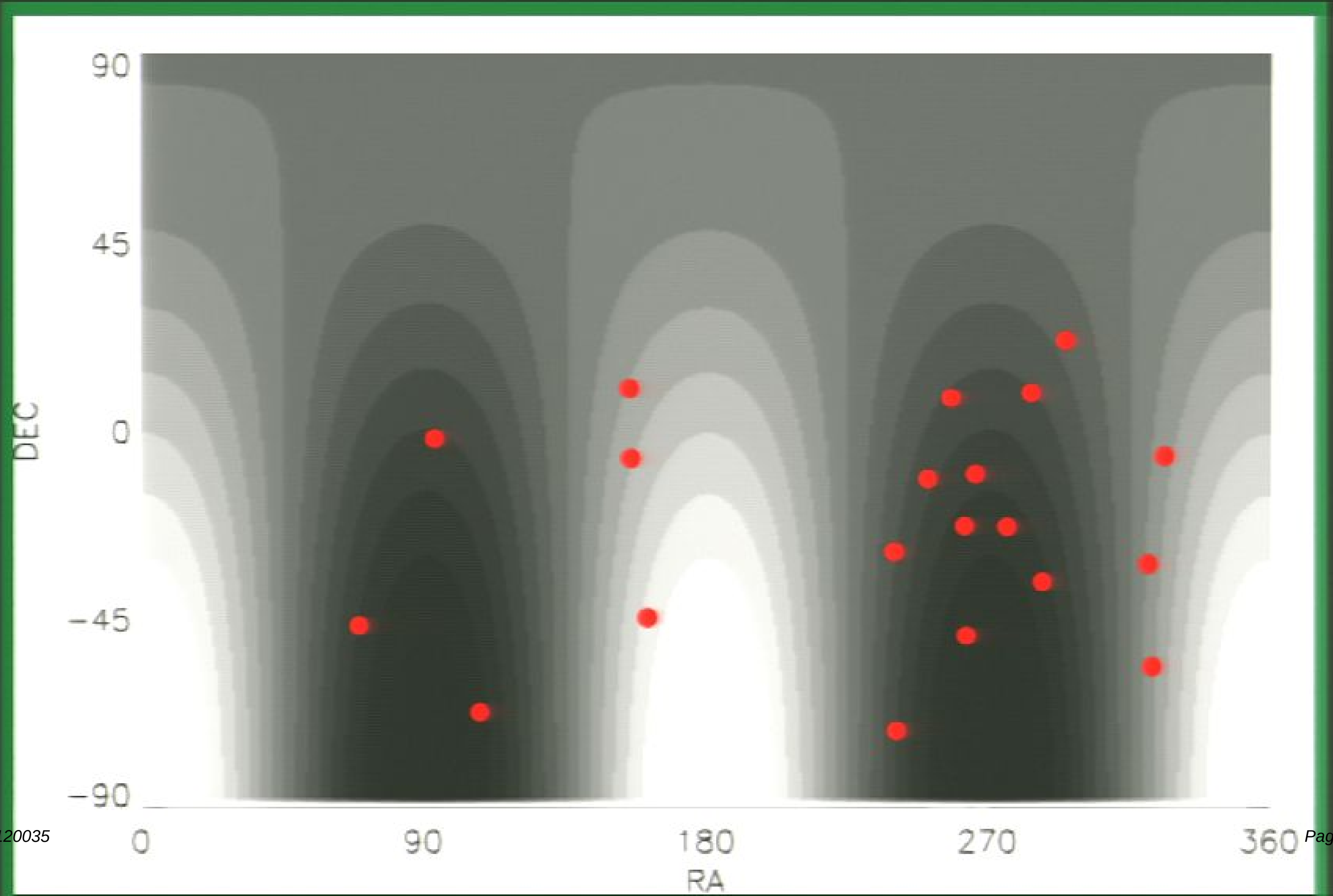

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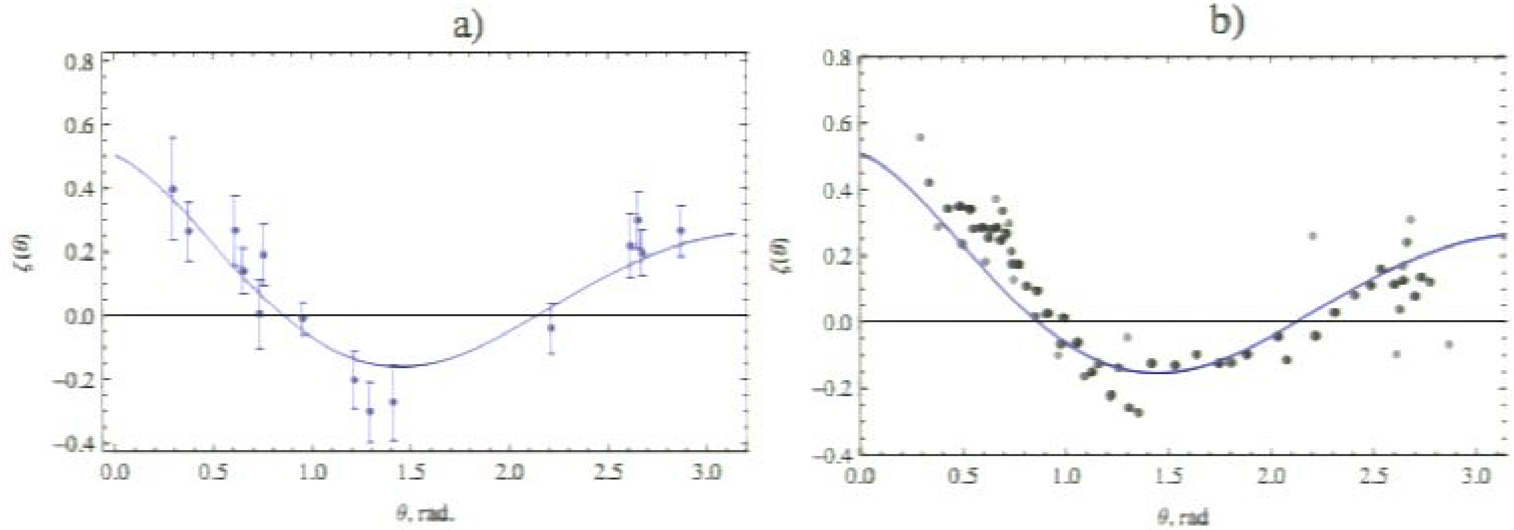

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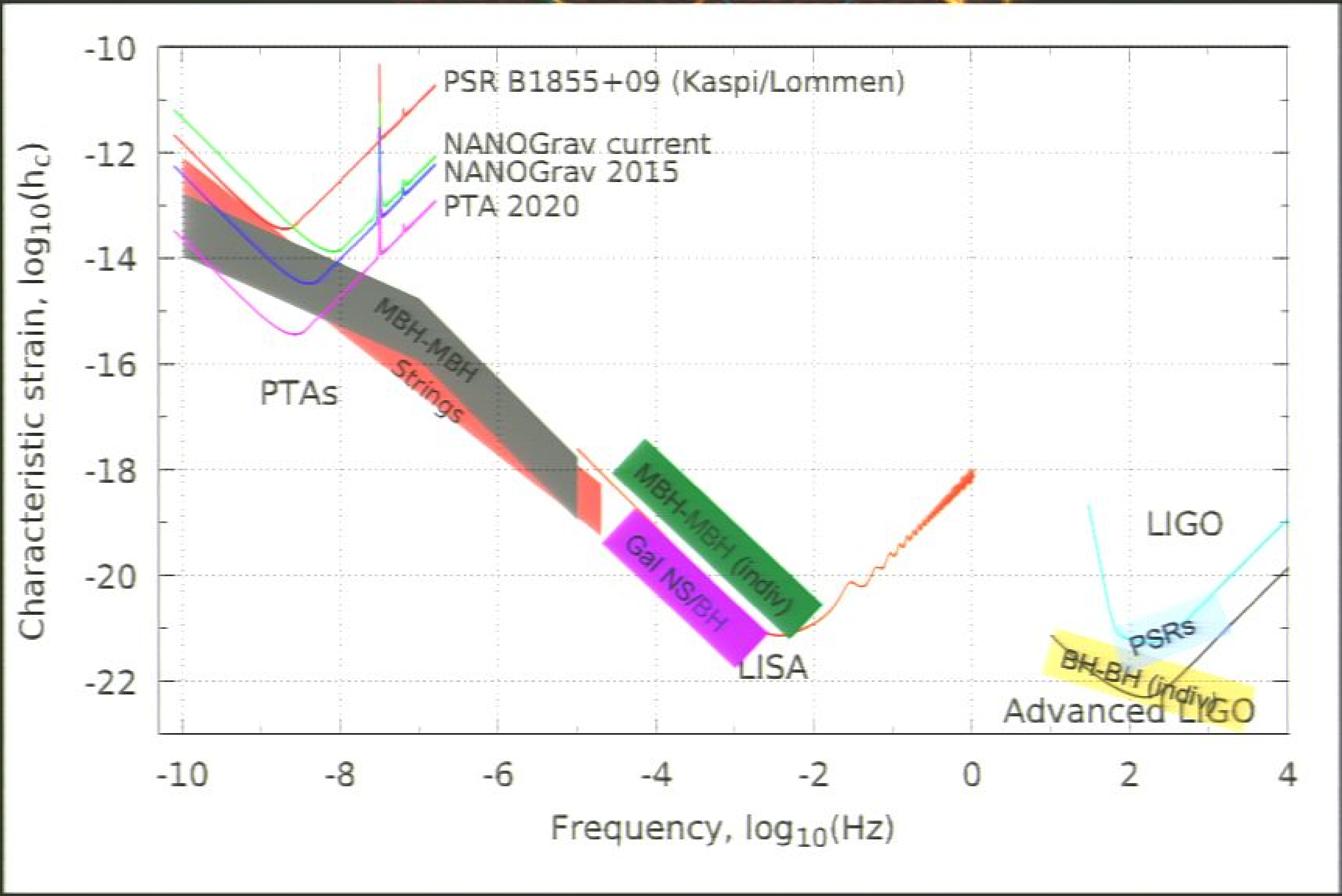

Figure by Paul Demorest (see arXiv:0902.2968)

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Detectability of a Waveform

$$g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon}$$

$$\left(\frac{dt}{d\lambda}\right)^2 = \delta_{jk}\frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda} + \frac{dx^j}{d\lambda}\frac{dx^k}{d\lambda}h_{jk}\left(t,\vec{x}\right)$$

$$\int dt = \int d\lambda - \frac{1-k_m n^m}{1+k_m n^m}\int d\lambda n^j n^k h_{jk}\left(t\left(\lambda\right),\vec{x}\left(\lambda\right)\right)$$

$$\text{Residual} = e_{jk}n^j n^k \frac{h_0}{2}\left(1-k_m n^m\right)\left[f\left(t_0\right) - f\left(t_0 - L\left(1+k_m n^m\right)\right)\right]$$

where $h_{jk}\left(t,\vec{x}\right) = h_0 f'\left(t-\hat{k}\cdot\vec{x}\right)\mathbf{e}$ and L is the distance to the pulsar

Photo Courtesy of Virgo

Adapted from NASA figure

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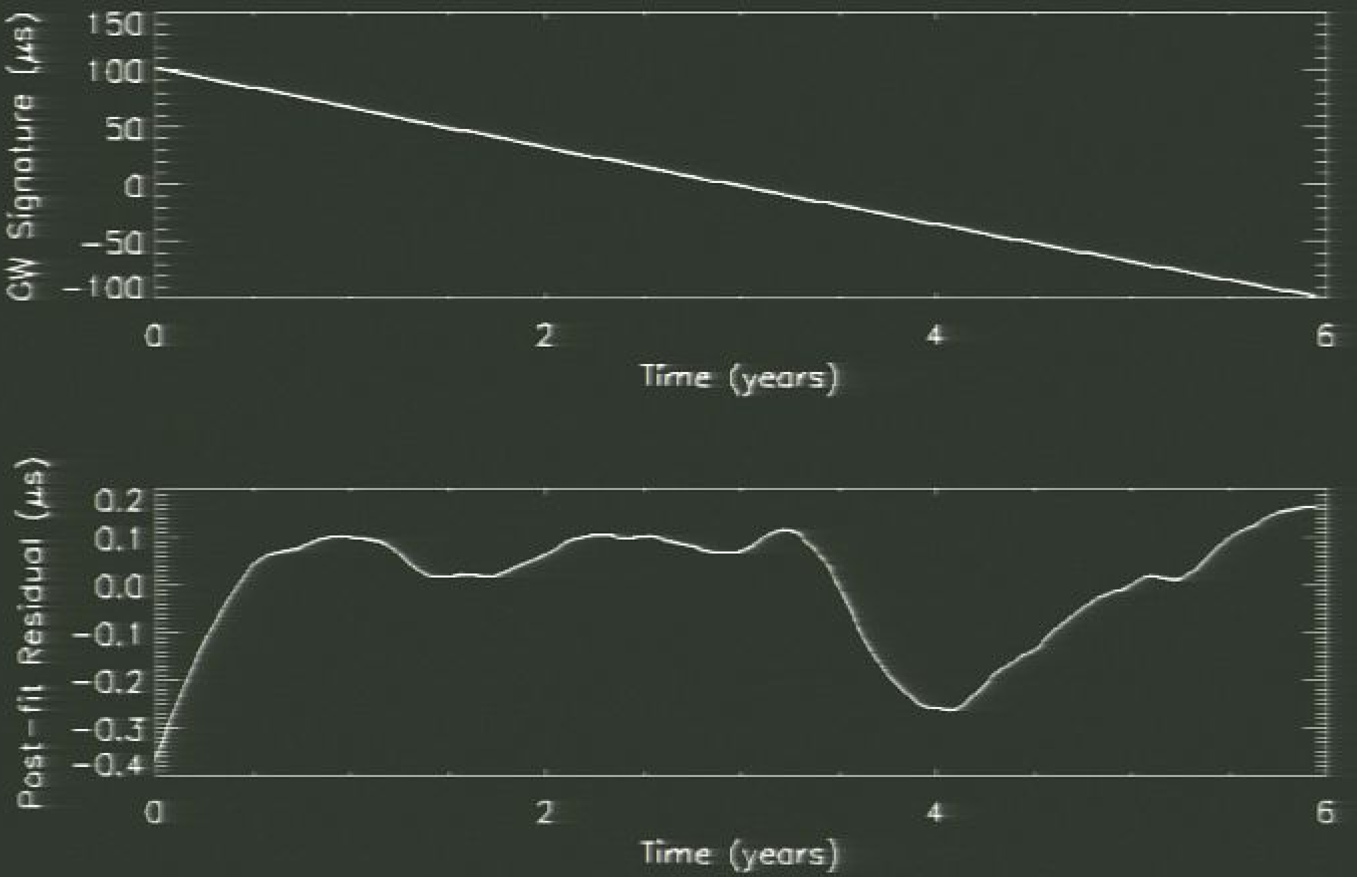
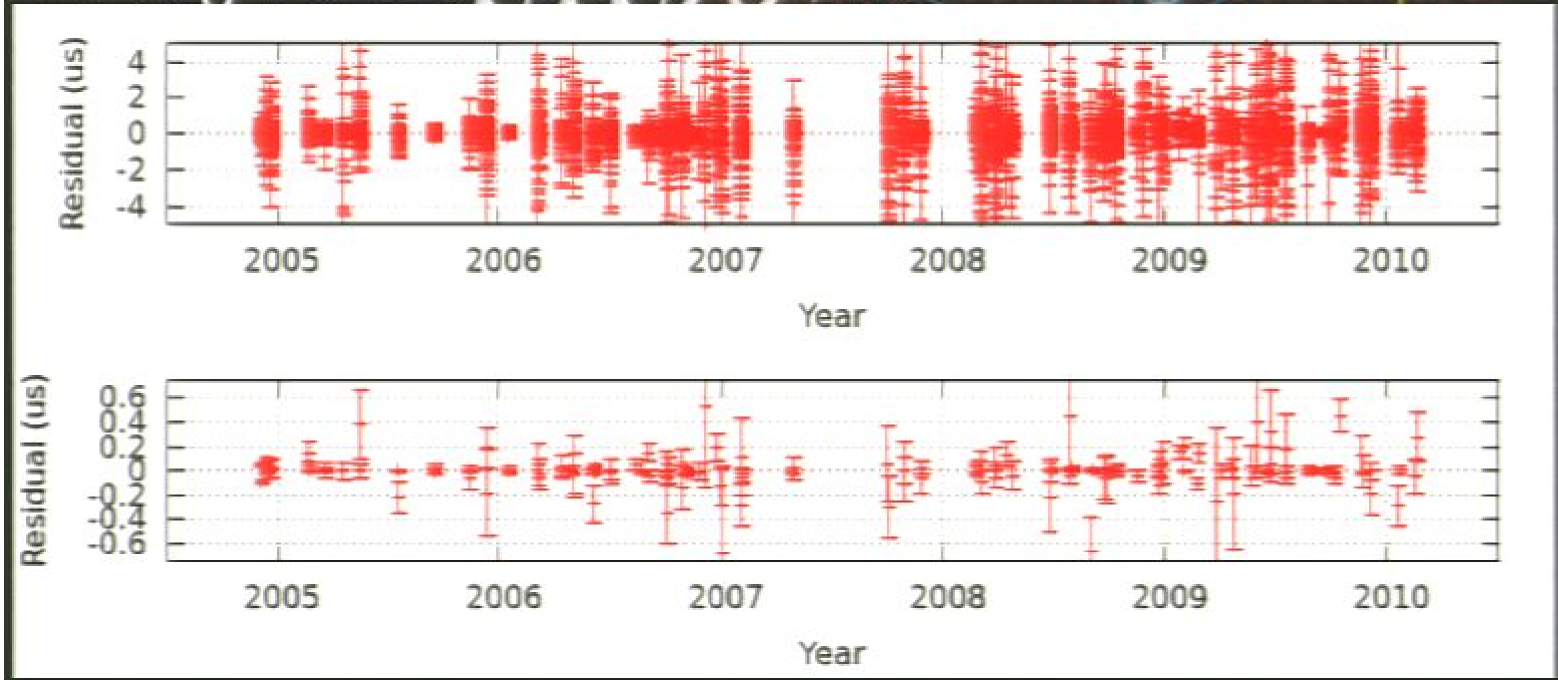
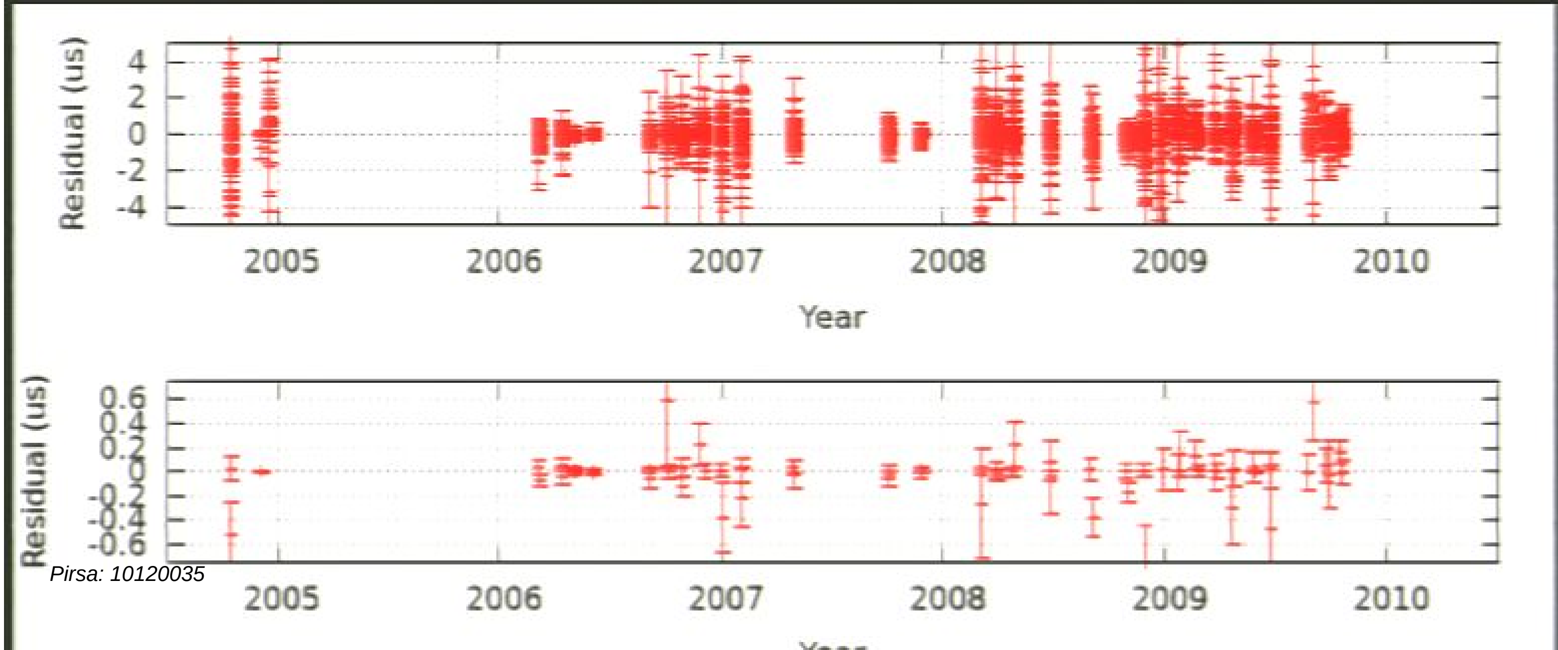


GW Signature (μs)
150
100
50
0
−50
−100
0
2
4
6
Time (years)
Post−fit Residual (μs)
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
2
4
6
Time (years)

Timing residuals versus time:

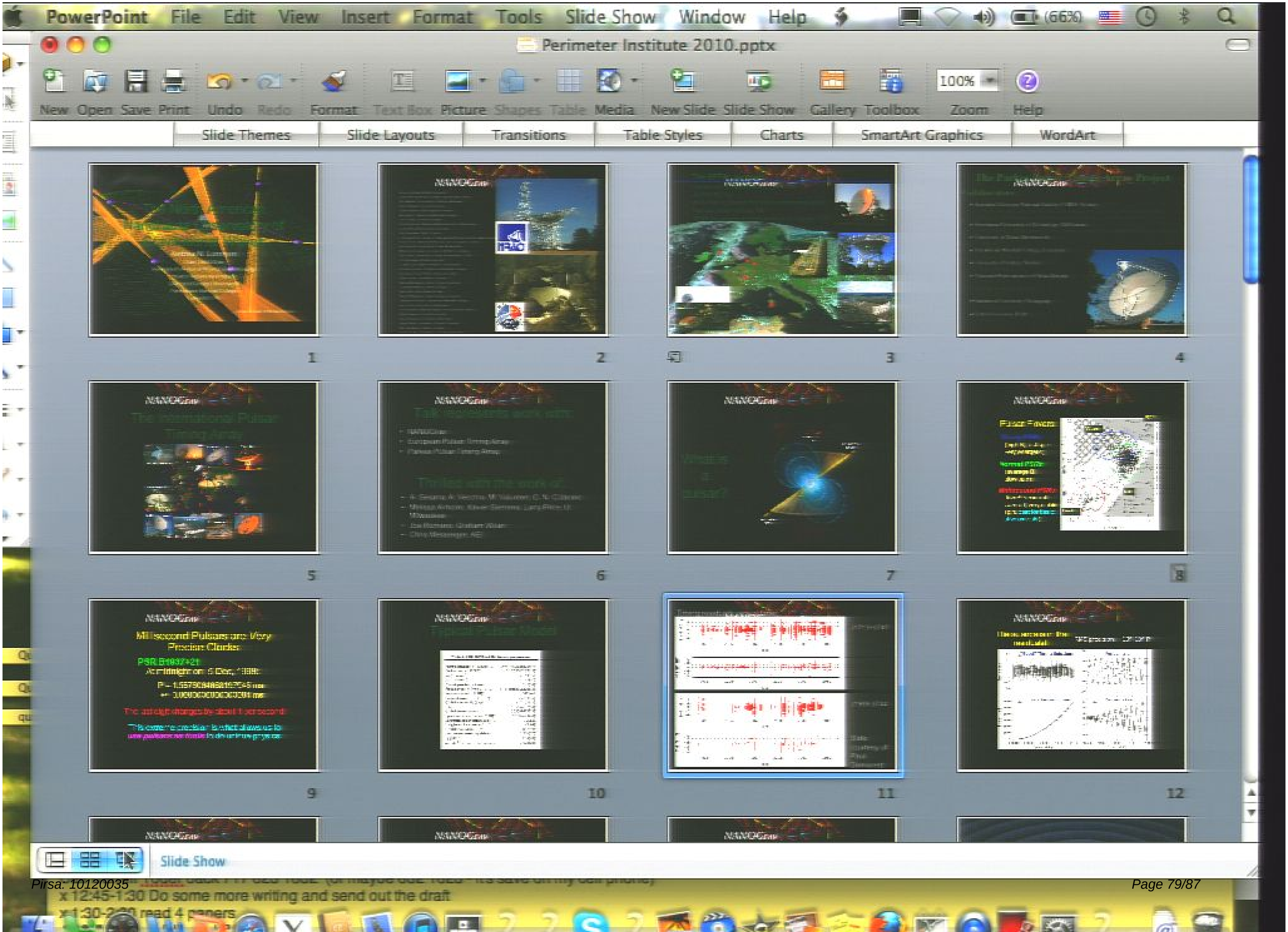


J1713+0747



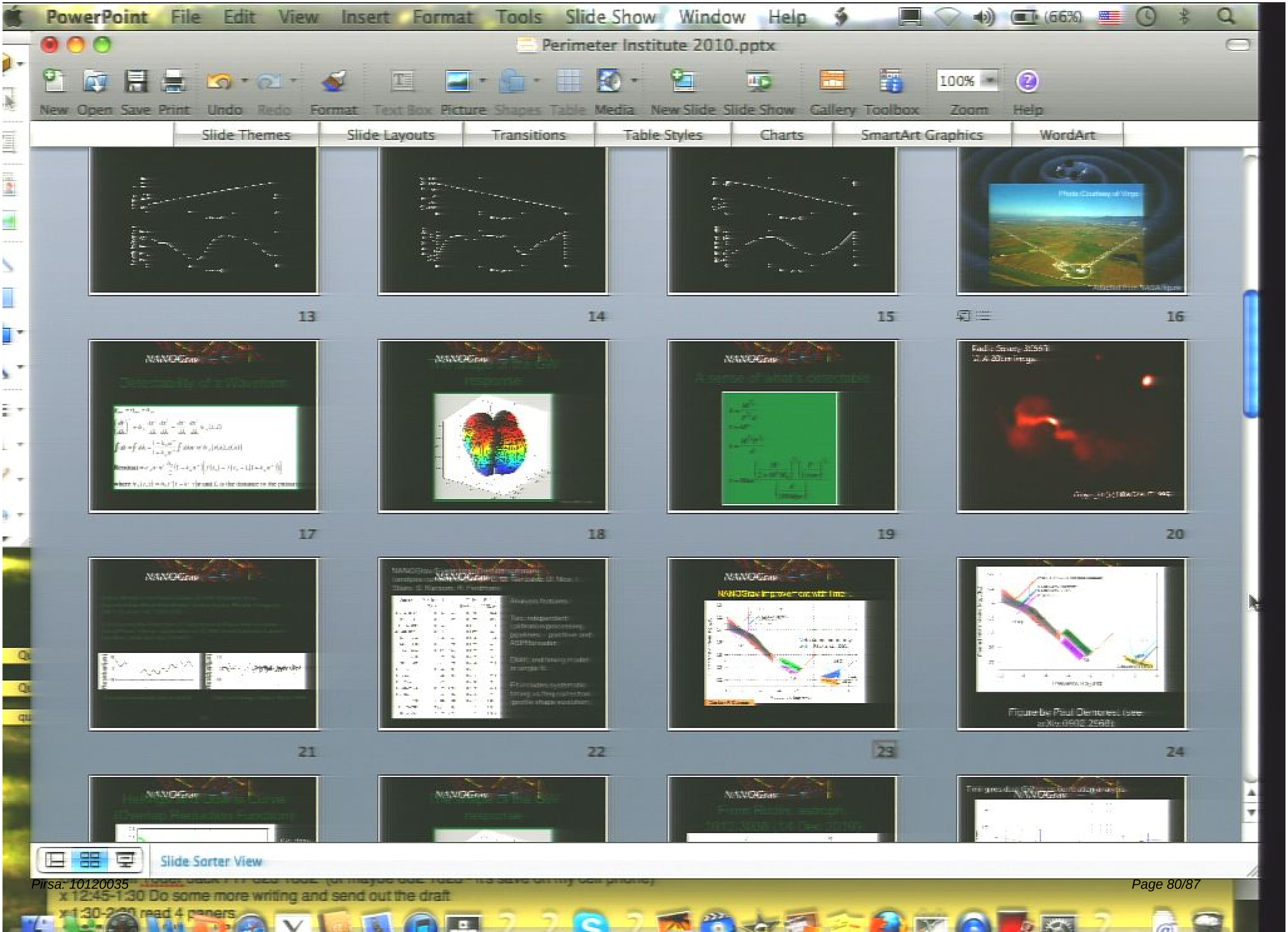
J1909-3744

Slide courtesy of Paul



x 12:45-1:30 Do some more writing and send out the draft

x 1:30-2:30 read 4 papers



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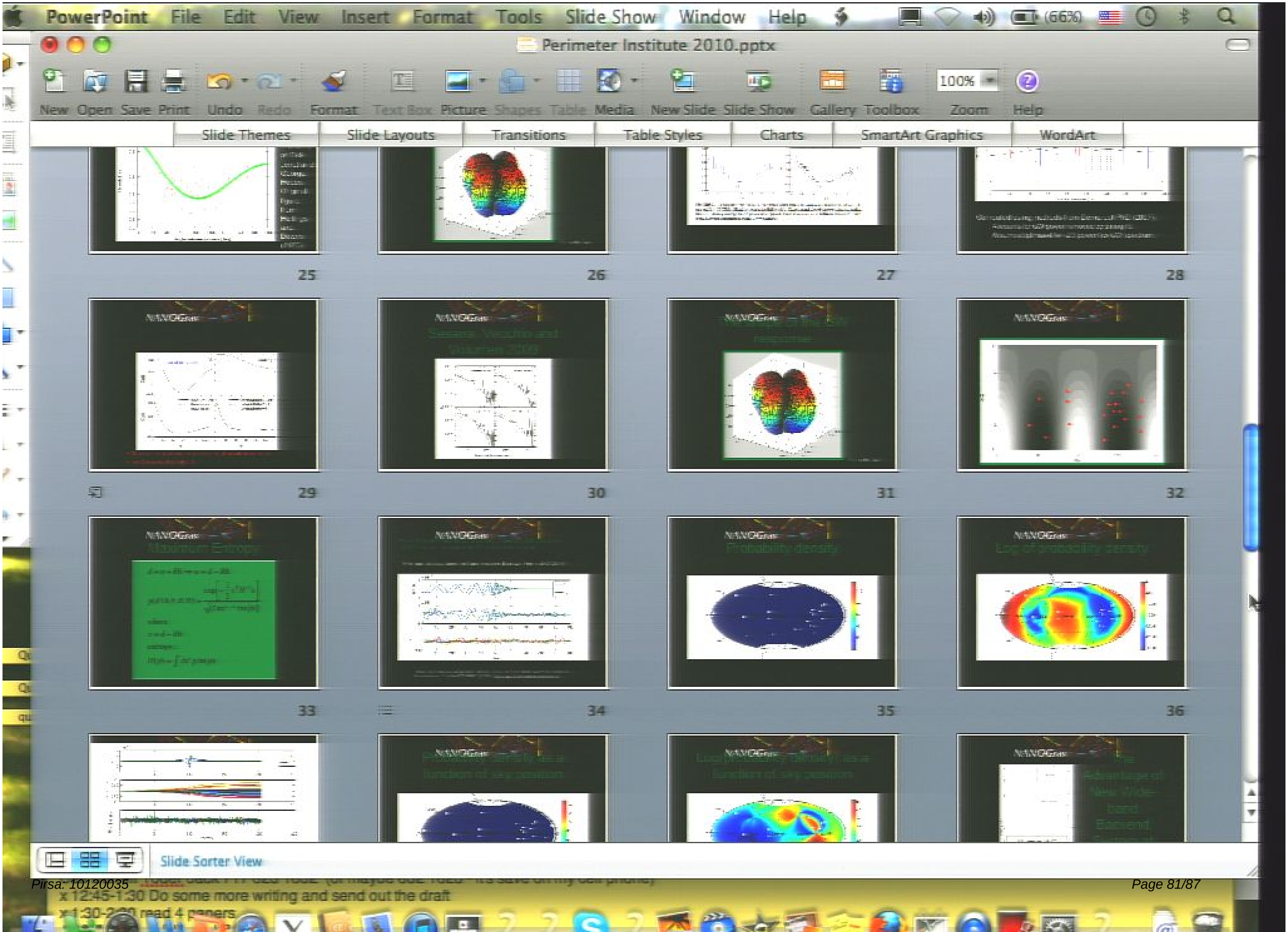

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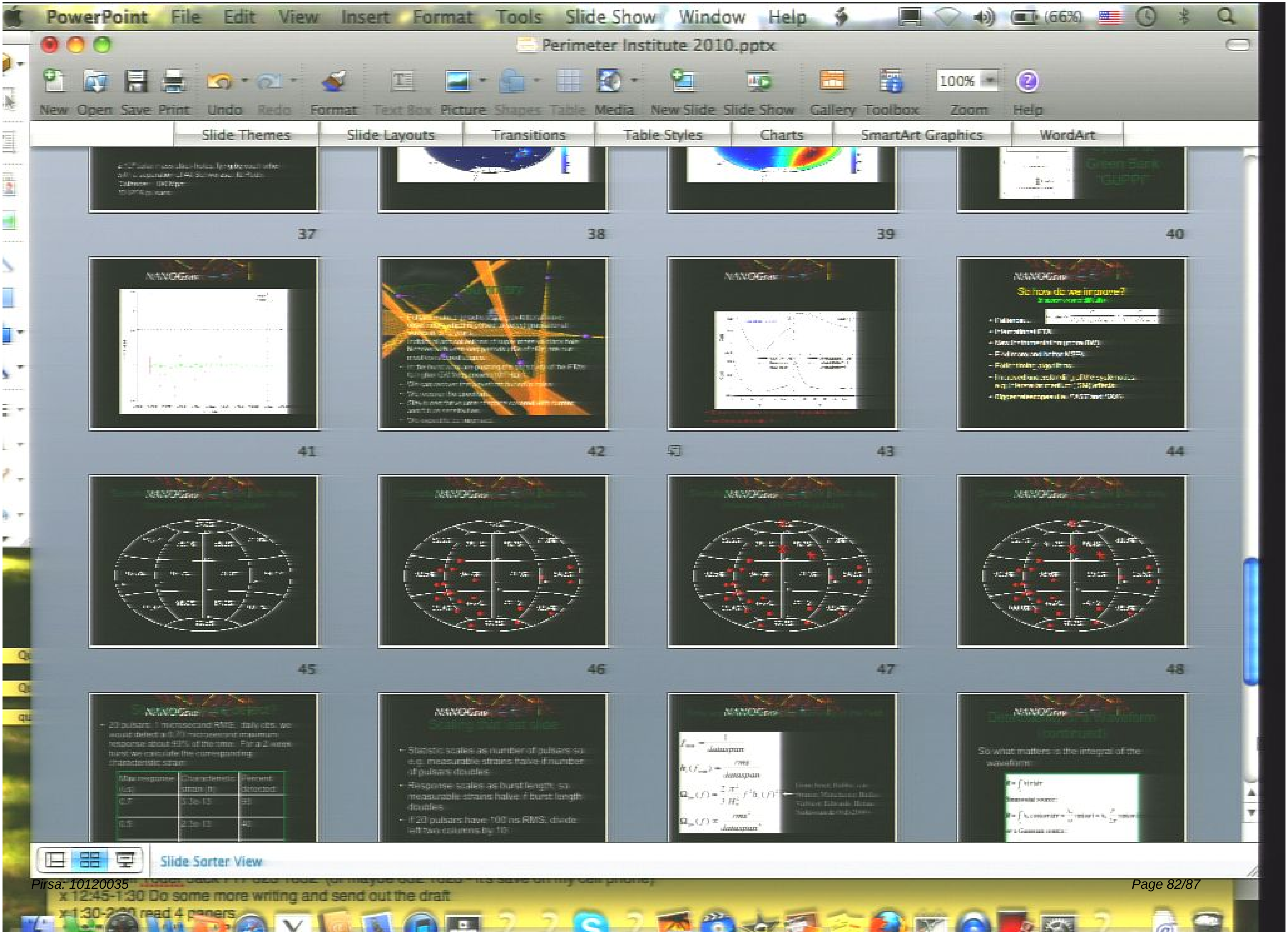

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- Statistic scales as number of pulsars so e.g. measurable strains halve if number of pulsars doubles
- Response scales as burst length, so measurable strains halve if burst length doubles
- If 20 pulsars have 100 ns RMS, divide left two columns by 10

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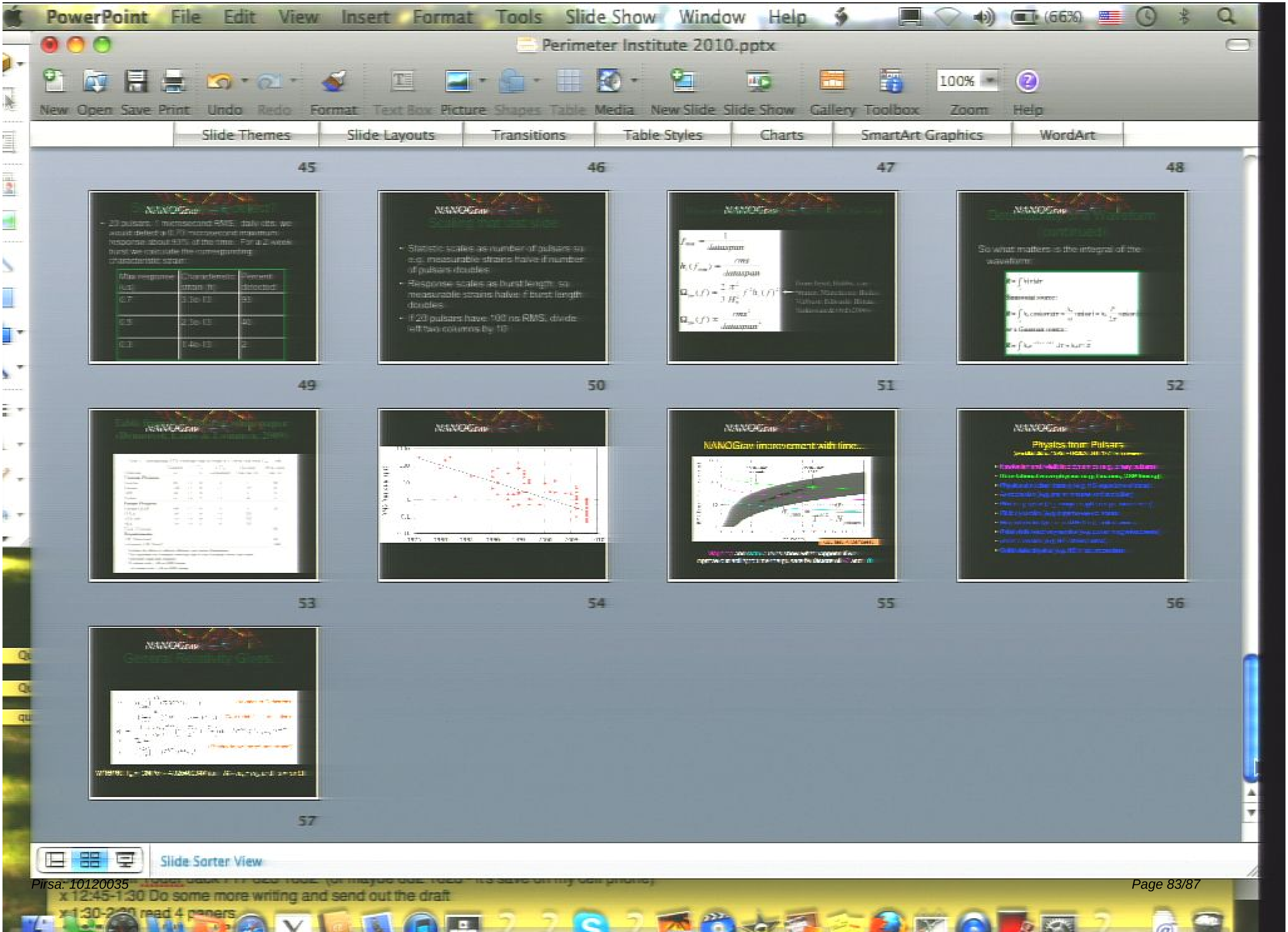
So what matters is the integral of the waveform


PowerPoint
File
Edit
View
Insert
Format
Tools
Slide Show
Window
Help
(66%)
Perimeter Institute 2010.pptx
New
Open
Save
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Undo
Redo
Format
Text Box
Picture
Shapes
Table
Media
New Slide
Slide Show
Gallery
Toolbox
100%
Zoom
Help
Slide Themes
Slide Layouts
Transitions
Table Styles
Charts
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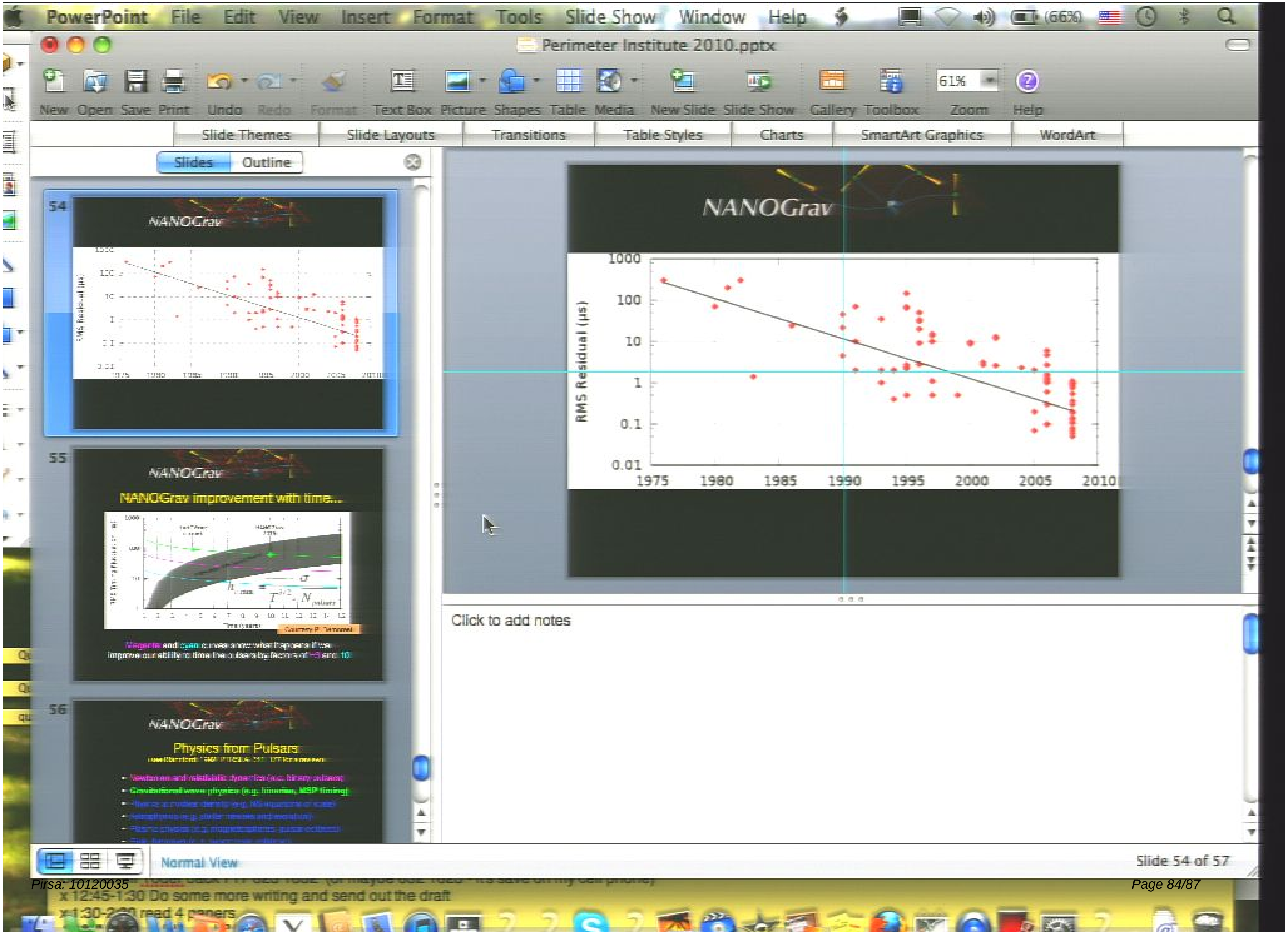
Slide Sorter View

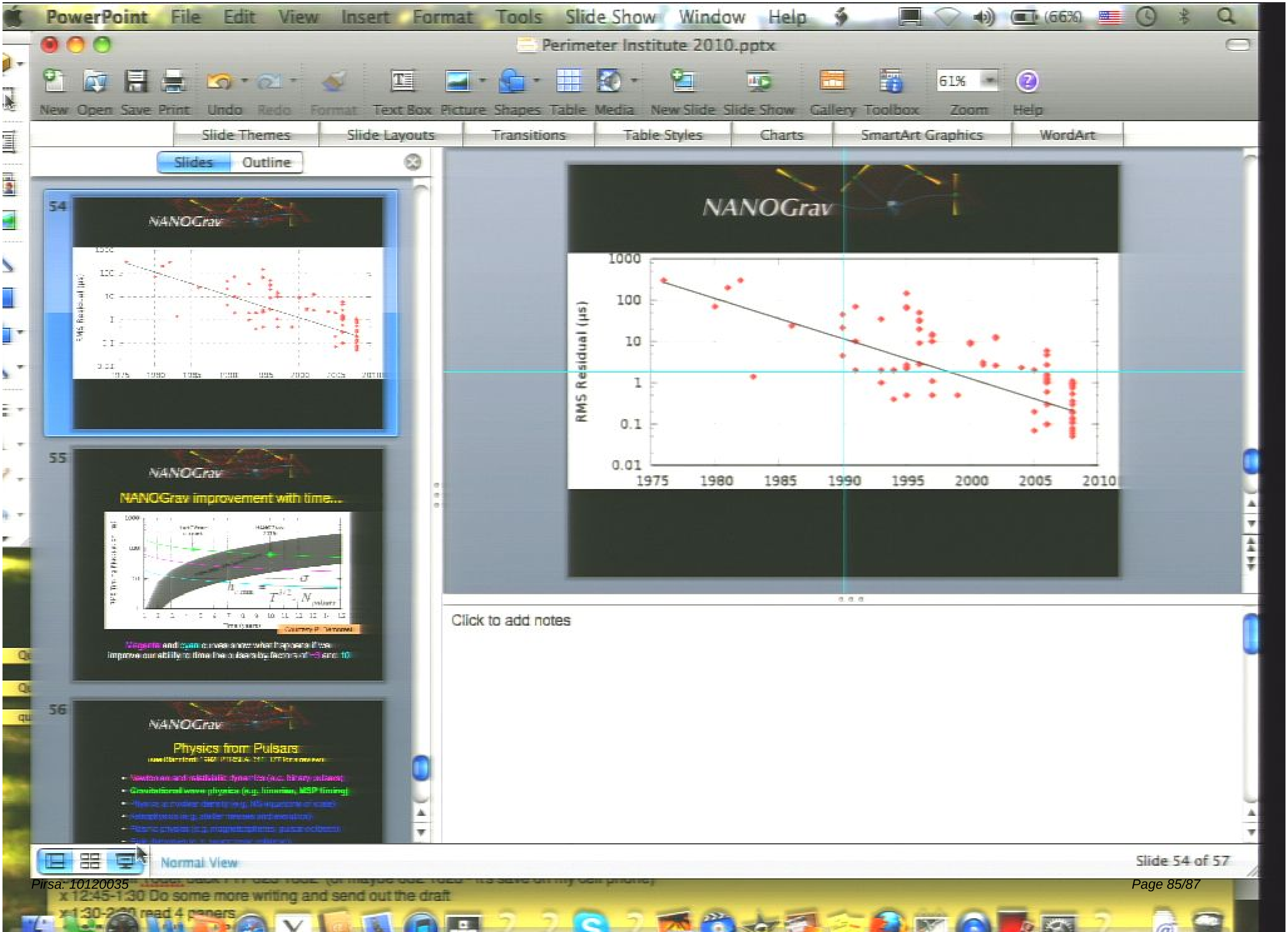


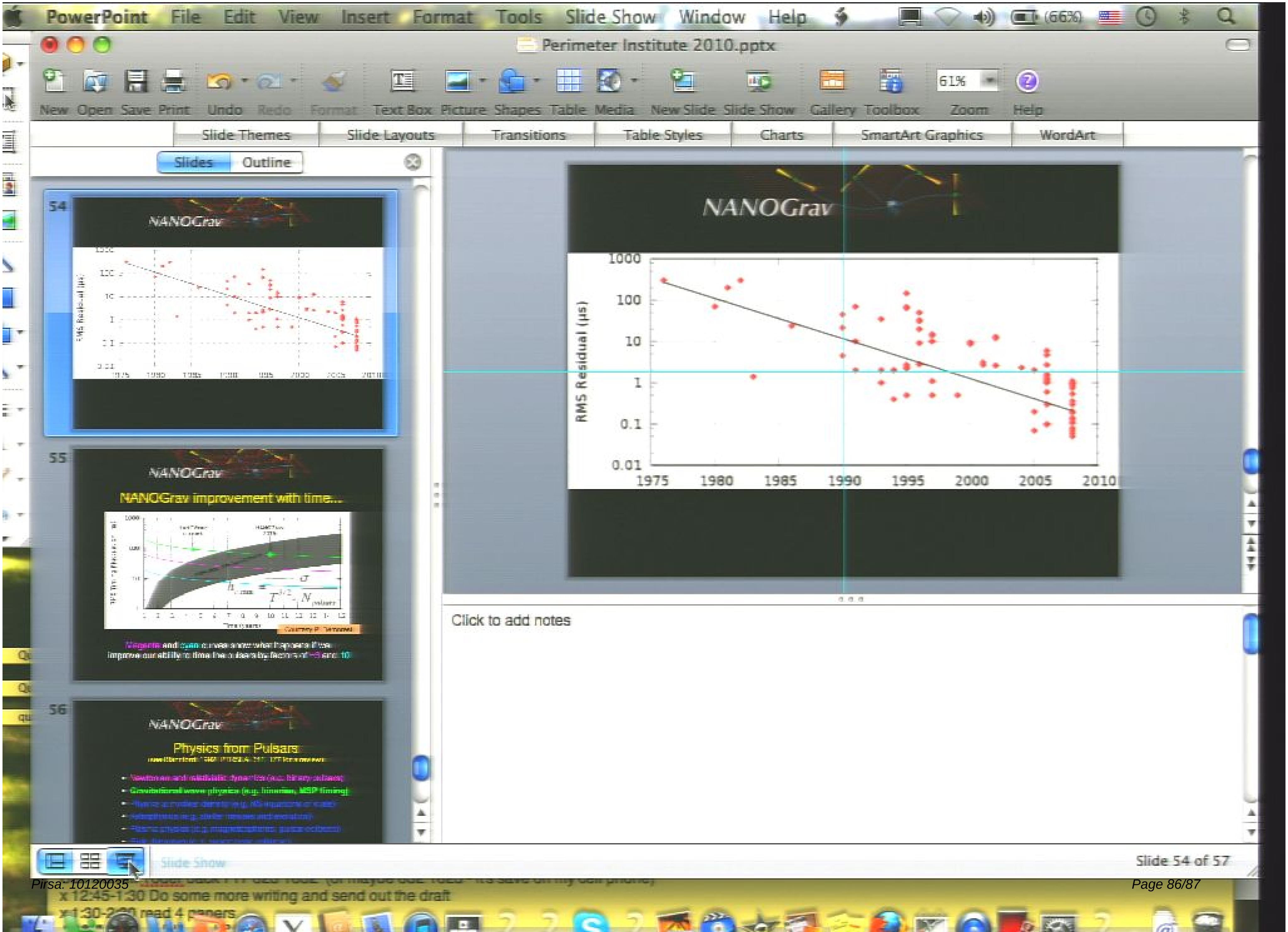
Click to add notes

Slide 54 of 57

x 12:45-1:30 Do some more writing and send out the draft

x 1:30-2:00 read 4 papers


PowerPoint File Edit View Insert Format Tools Slide Show Window Help
(66%)
Perimeter Institute 2010.pptx
New Open Save Print Undo Redo Format Text Box Picture Shapes Table Media New Slide Slide Show Gallery Toolbox Zoom Help
61%
Slide Themes Slide Layouts Transitions Table Styles Charts SmartArt Graphics WordArt
Slides Outline
54
55
56
NANOGrav
NANOGrav improvement with time...
Courtesy P. Demorest
Physics from Pulsars
RMS Residual (μs)
1000
100
10
1
0.1
0.01
1975
1980
1985
1990
1995
2000
2005
2010
Click to add notes
Normal View
Slide 54 of 57
x 12:45-1:30 Do some more writing and send out the draft
x 1:30-2:30 read 4 papers



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Slide 54 of 57

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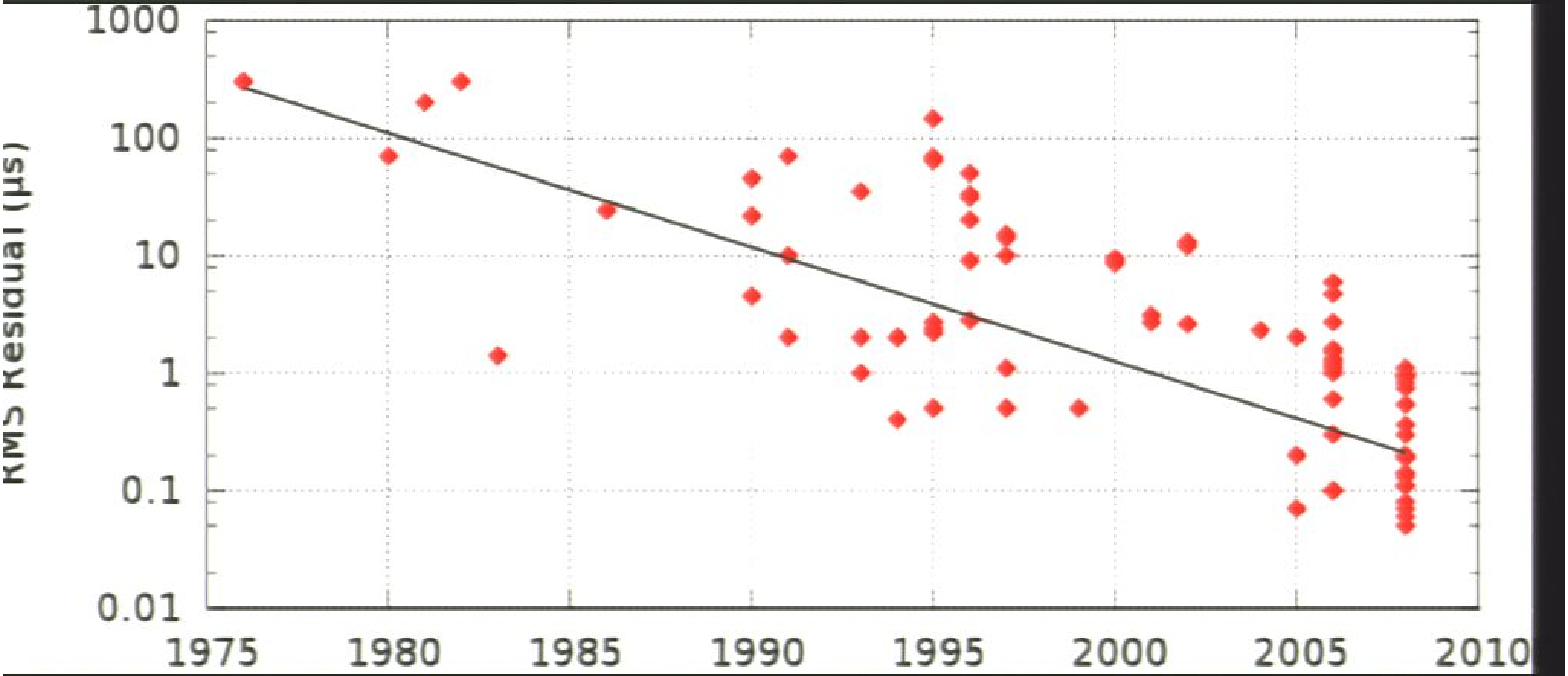
x 1:30-2:?? read 4 papers

NANOGrav


RMS Residual (μs)
1000
100
10
1
0.1
0.01
1975
1980
1985
1990
1995
2000
2005
2010