Title: Limits on non-local correlations from the structure of the local state space

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Abstract: Nonlocality is arguably one of the most remarkable features of quantum mechanics. On the other hand nature seems to forbid other no-signaling correlations that cannot be generated by quantum systems. Usual approaches to explain this limitation is based on information theoretic properties of the correlations without any reference to physical theories they might emerge from. However, as shown in [PRL 104, 140401 (2010)], it is the structure of local quantum systems that determines the bipartite correlations possible in quantum mechanics. We investigate this connection further by introducing toy systems with regular polygons as local state spaces. This allows us to study the transition between bipartite classical, no-signaling and quantum correlations by modifying only the local state space. It turns out that the strength of nonlocality of the maximally entangled state depends crucially on a simple geometric property of the local state space, known as strong self-duality. We prove that the limitation of nonlocal correlations is a general result valid for the maximally entangled state in any model with strongly self-dual local state spaces, since such correlations must satisfy the principle of macroscopic locality. This implies notably that TsirelsonÃ¢ÂÂ€ÂÂ™s bound for correlations of the maximally entangled state in quantum mechanics can be regarded as a consequence of strong self-duality of local quantum systems. Finally, our results also show that there exist models which are locally almost identical to quantum mechanics, but can nevertheless generate maximally nonlocal correlations.

Limits on nonlocal correlations from the structure of the local state space

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Foundations of Quantum mechanics (QM)

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- More restrictive principles: Information Causality? Transitivity? Purification? Teleportation?

- How do these principles relate to each other?
- Might there be alternatives to quantum mechanics?

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  - Convexity
  - States full characterized by measurement statistics
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- Useful mathematical structure

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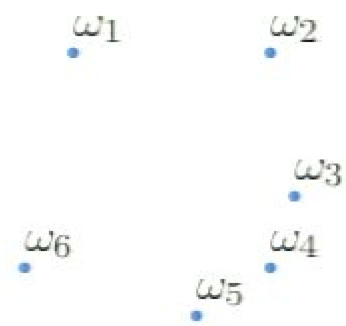
  - Local tomography
- Useful mathematical structure
- QM and classical probability theory included as special cases
- What distinguishes QM from other theories?

States $\leftrightarrow$ Measurement outcomes

- States ω given by $\{p_e\}$



States $\leftrightarrow$ Measurement outcomes

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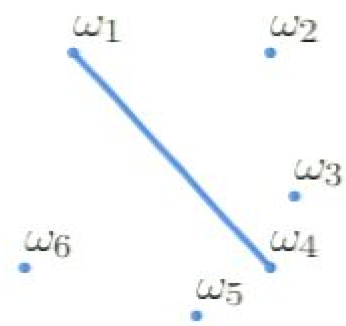
$$\omega = p\,\omega_1 + (1-p)\,\omega_4$$



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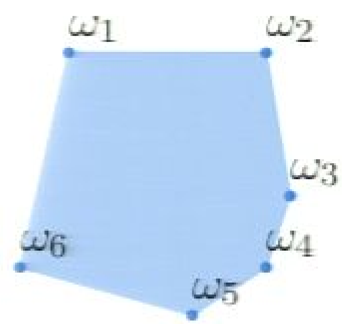

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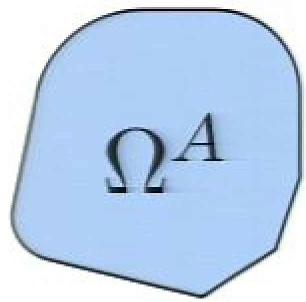
$\Rightarrow$ Convex state space Ω^A



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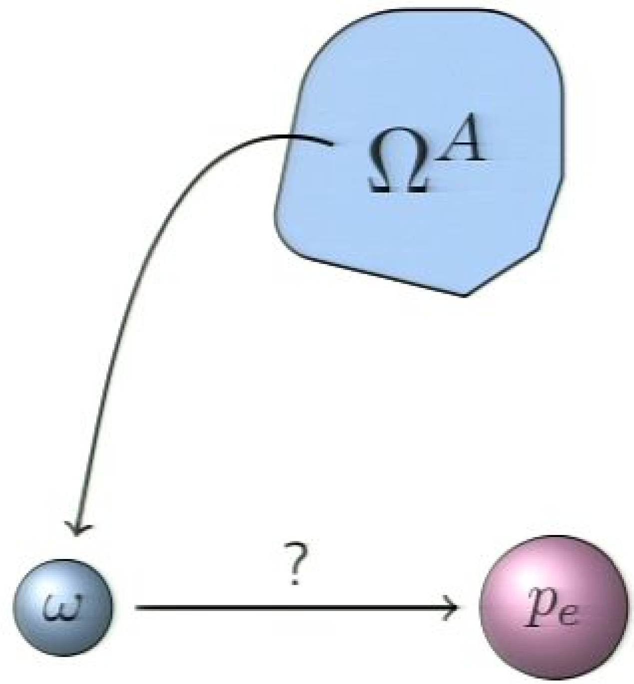

Probability of outcome e:

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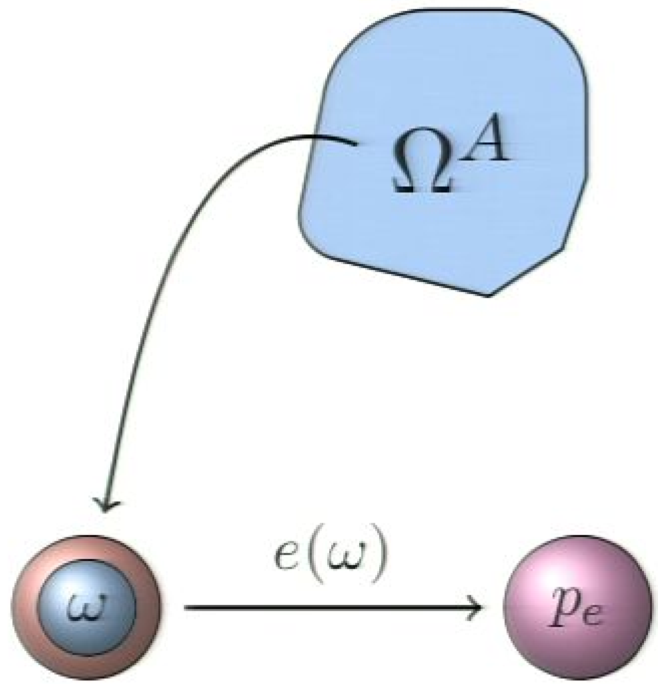
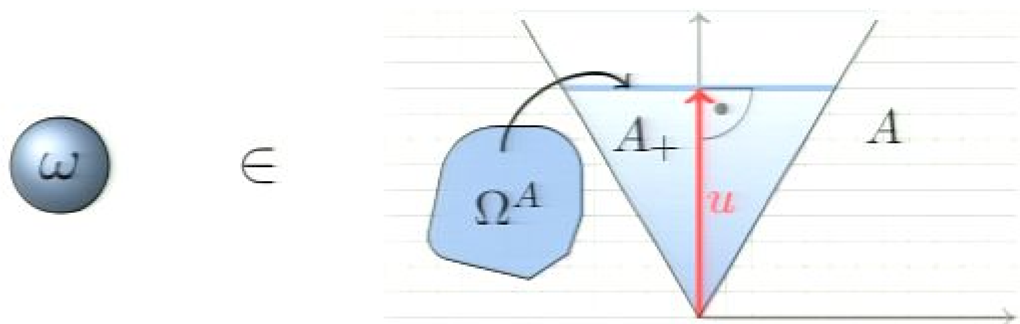


Probability of outcome e:

- linear functional (effect)
  $e : p_e = e(\omega)$
- certain outcome
  $u : p_u = u(\omega) = 1$

Positive Cones

- State space: vectors on a hyperplane $u(\omega) = 1$
- Unnormalized states form cone A_+



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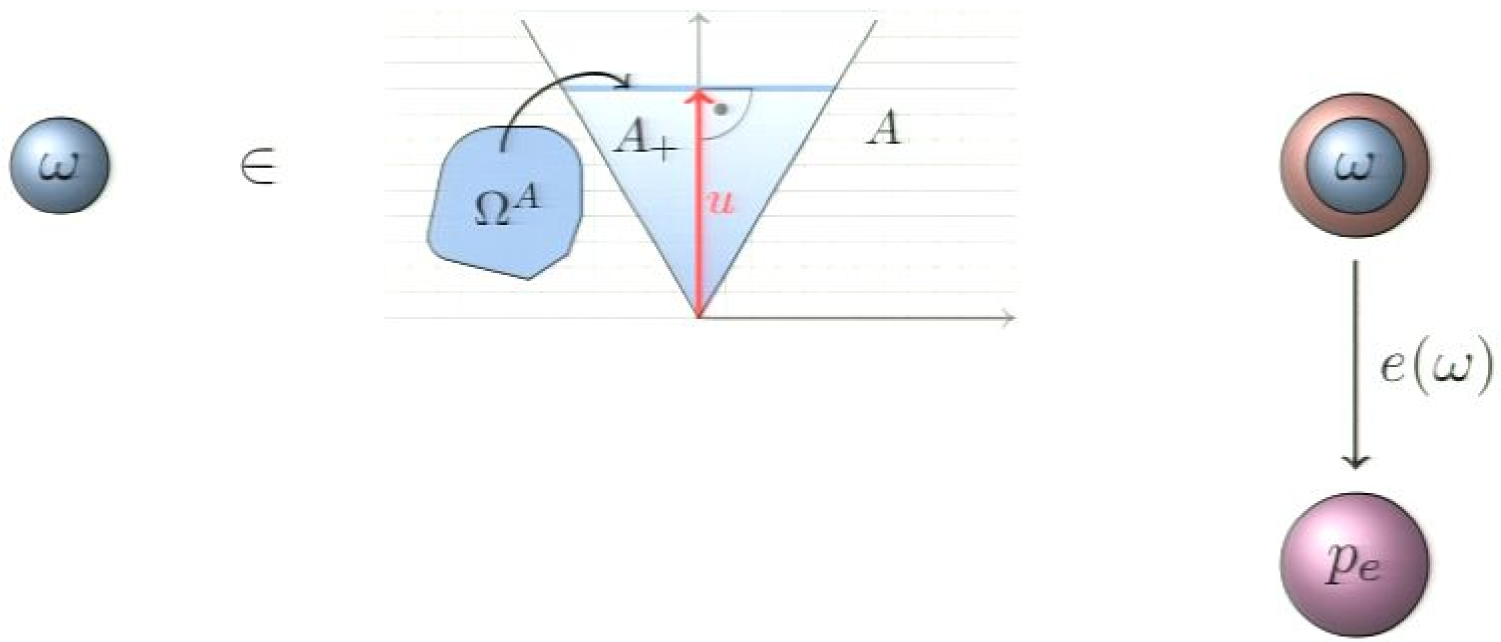

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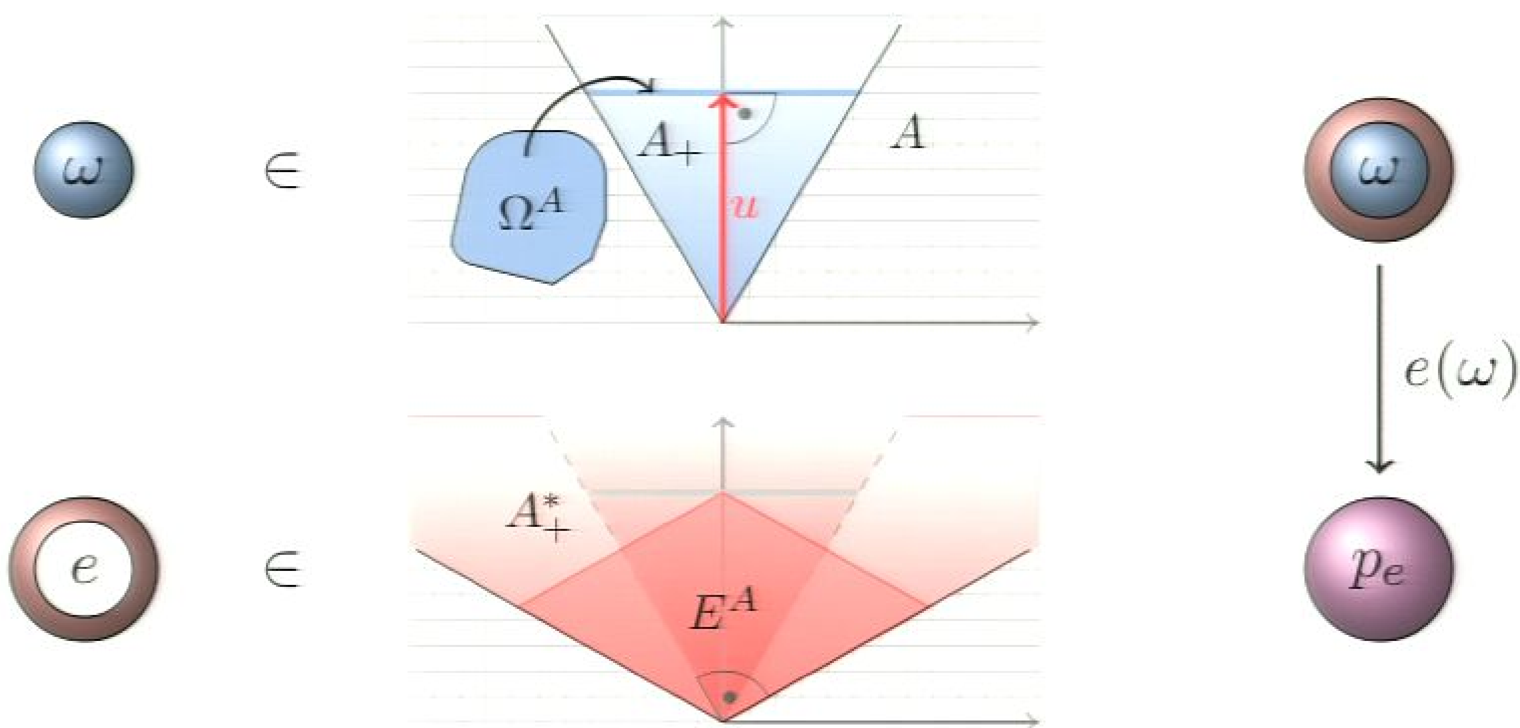
- Effects: Elements of dual cone A_+^*



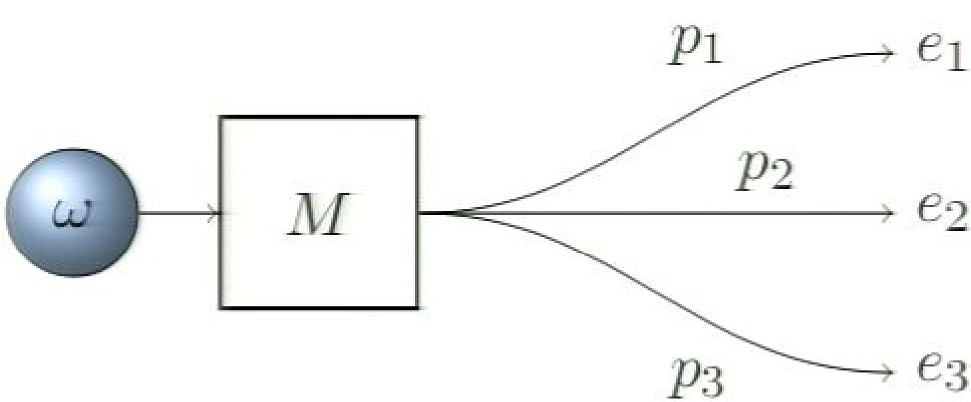
Measurements

Definition

A measurement apparatus M is represented by a set of effects $\{e\}$ each corresponding to one possible outcome e.

$$M = \{e\} \qquad \sum_{e \in M} e = u$$

- A measurement maps a state ω to probability distribution $\{p_e\}$ with $p_e = e(\omega)$



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- The probability p_{any} to get any outcome is:

$$p_{any} = \sum_{e \in M} p_e = \sum_{e \in M} e(\omega) = u(\omega) = 1$$

Cone of joint systems

- Local Tomography + no-signalling $\Rightarrow$ Cone AB_+ of joint system bounded:

$$A_+ \otimes_{\text{min}} B_+ \quad \subseteq \quad AB_+ \quad \subseteq \quad A_+ \otimes_{\text{max}} B_+$$

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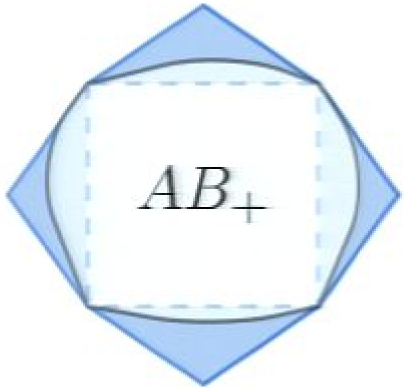

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- Theorie defined by structure of A_+, u^A, B_+, u^B and AB_+ ...

Quantum joint states

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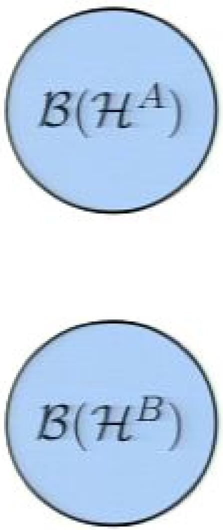
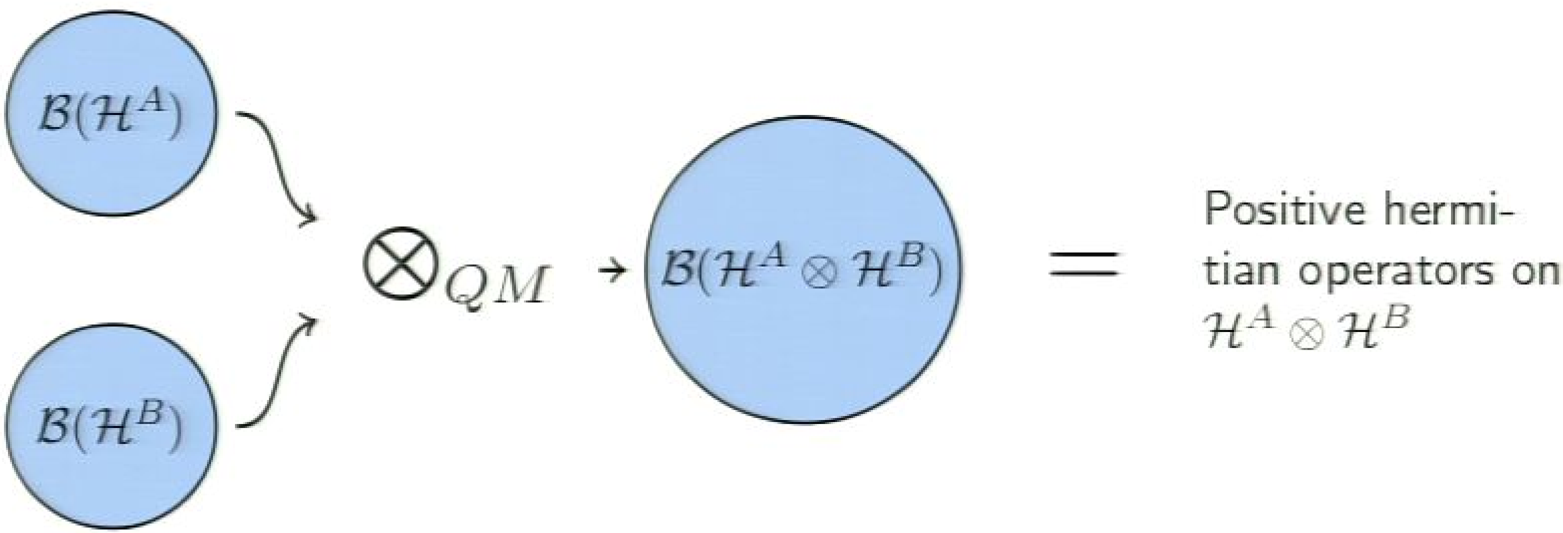


Quantum joint states

- State and effects of local quantum systems are represented by positive hermitian operators $\rho \in \mathcal{B}(\mathcal{H})$ on a Hilbert-Space $\mathcal{H}$ (density matrices)
- In QM the tensor product is chosen to preserve positivity:



- $\mathcal{B}(\mathcal{H}^A) \otimes_{\mathrm{min}} \mathcal{B}(\mathcal{H}^B) \subset \mathcal{B}(\mathcal{H}^A \otimes \mathcal{H}^B) \subset \mathcal{B}(\mathcal{H}^A) \otimes_{\mathrm{max}} \mathcal{B}(\mathcal{H}^B)$

Nonlocal Correlations
in
Quantum Mechanics

Input-Output-Boxes

- Blackbox model for correlations:

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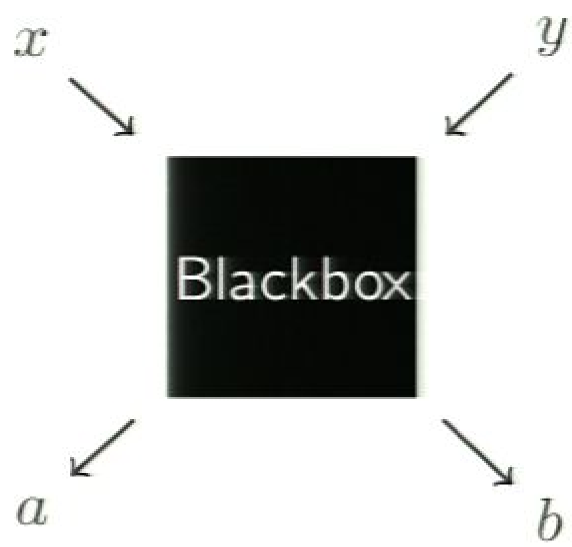

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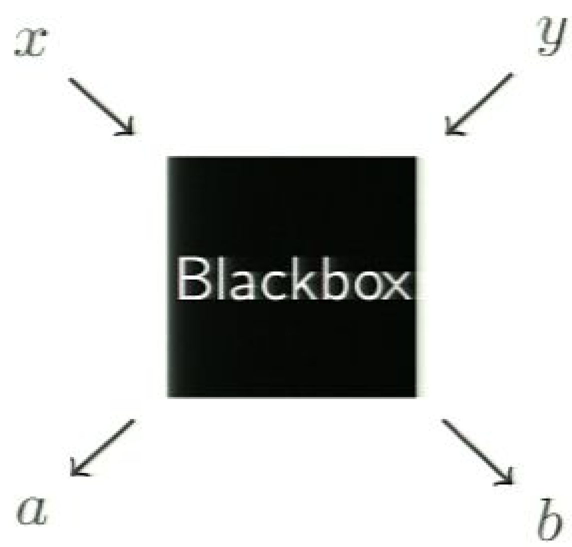
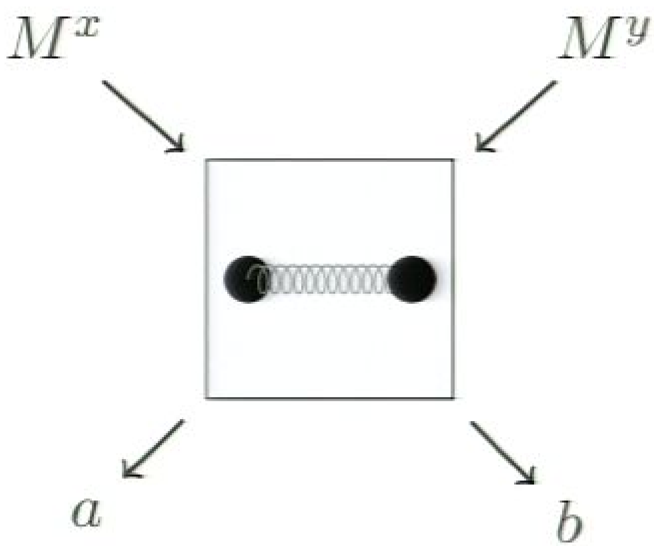


- with probability $p(ab \,|\, xy)$
- Correlation coefficient:

$$E_{xy} = \sum_{a=b} p(ab \,|\, xy) - \sum_{a \neq b} p(ab \,|\, xy)$$

Correlated Measurement Outcomes

- Correlations by local measurements on a joint state:



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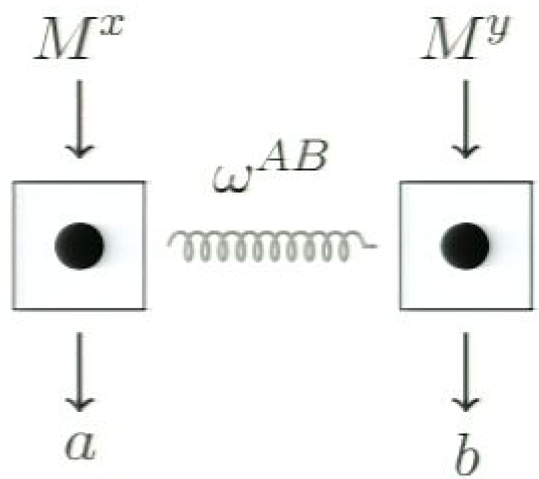

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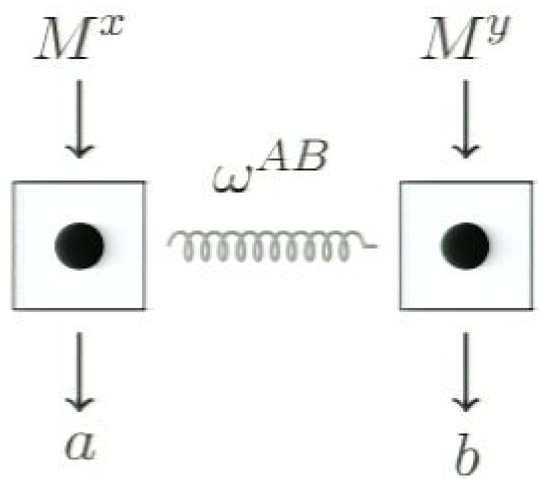

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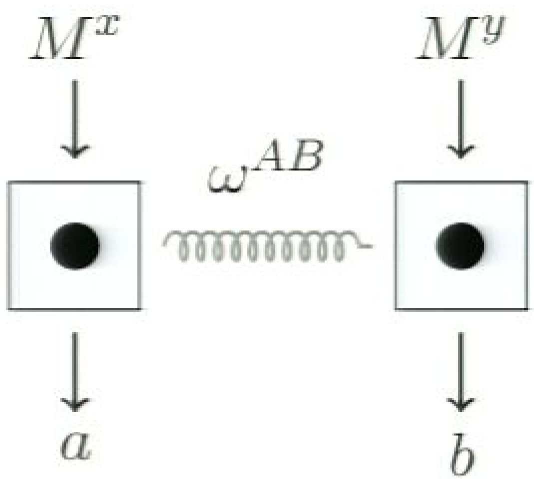

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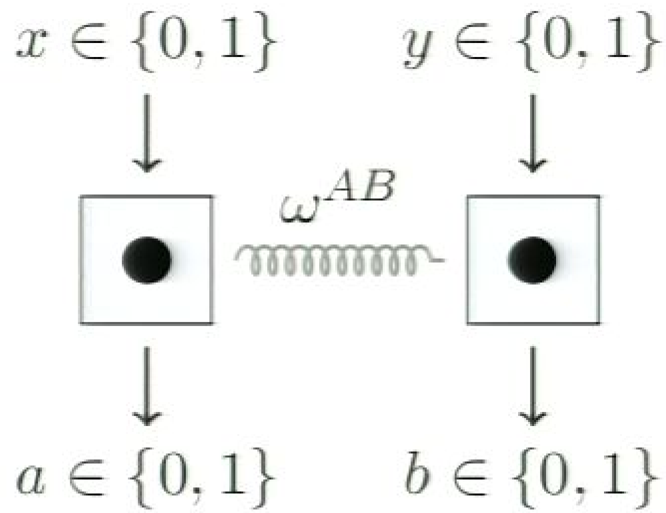
$$p(a\,|\,x) = \sum_b p(ab\,|\,xy) \text{ independent of } B$$

- All $\omega^{AB} \in A_+ \otimes_{\text{max}} B_+$ obey No-Signalling

CHSH-Inequality

- Correlations with 2 binary outcome measurements per site:



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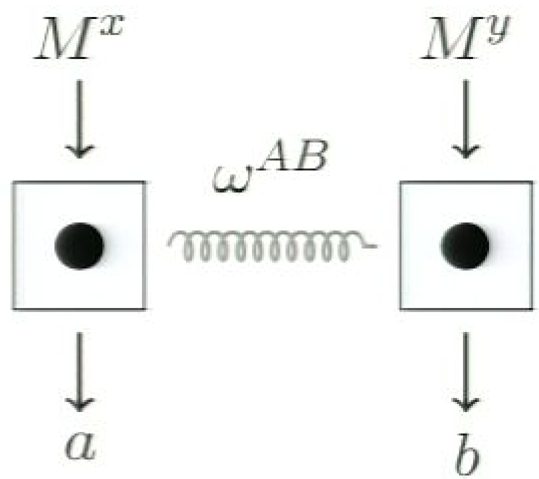

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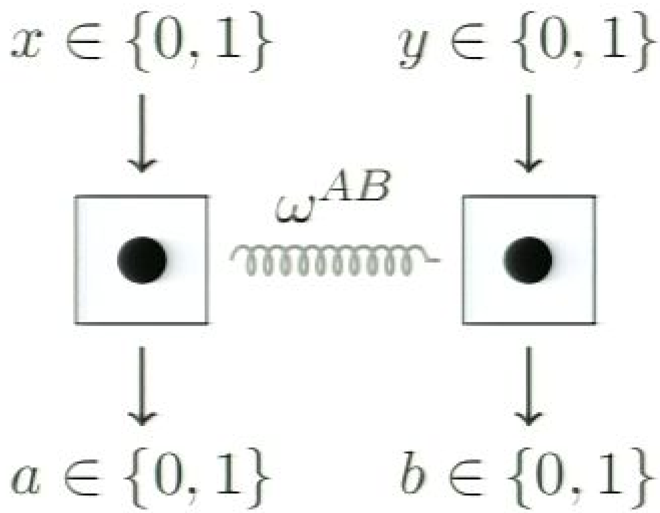

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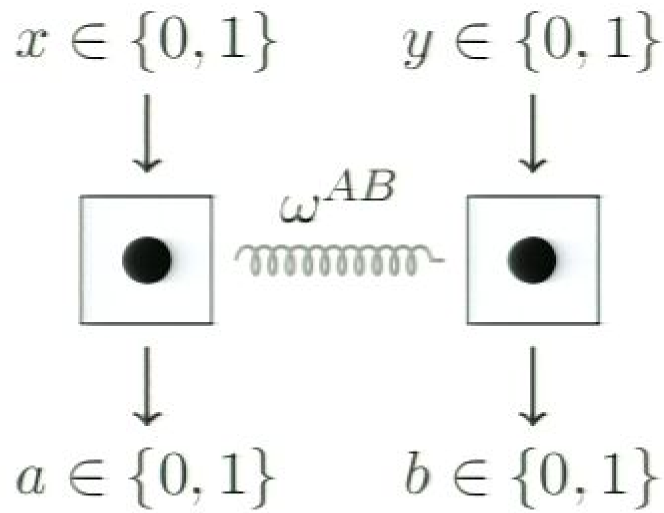

- Local realistic theories obey the CHSH-inequality:

$$S = |E_{00} + E_{01} + E_{10} - E_{11}| \leq 2$$

Maximal CHSH-violation

- Maximal violation in QM (Tsirelson's bound):

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- PR-Boxes:

$$p(ab\,|\,xy) = \begin{cases} \frac{1}{2} & \text{if } x * y = a \oplus b \\ 0 & \text{otherwise} \end{cases}$$

$$p(a\,|\,x) = p(b\,|\,y) = \frac{1}{2}$$

$$\Rightarrow S^{PR} = 4$$

- Why does QM show smaller violations as allowed by No-Signalling?

Maximal Tensor Product of Quantum Subsystems

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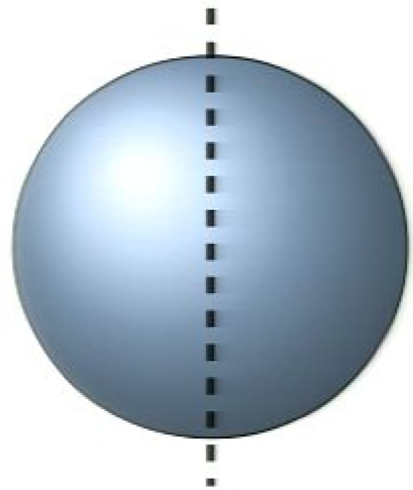

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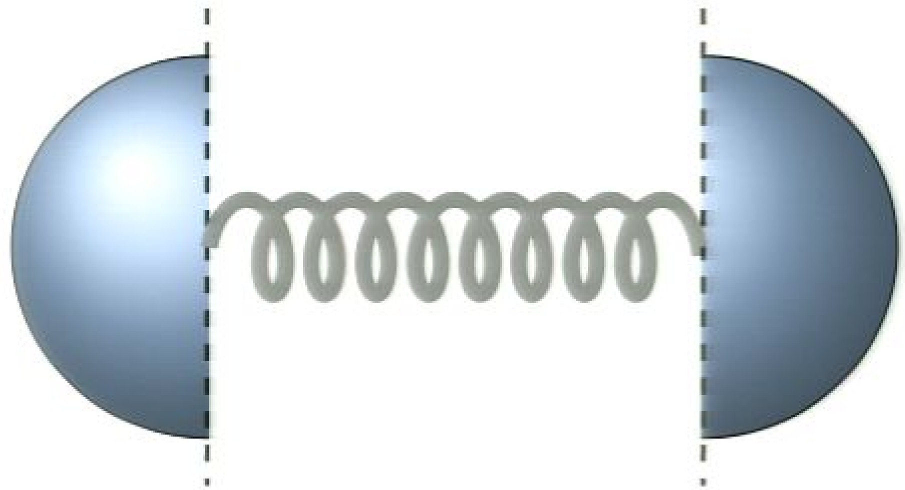

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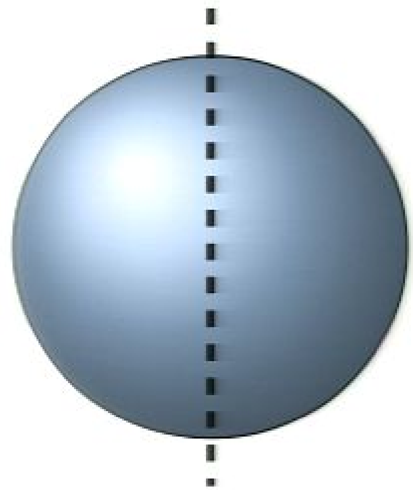

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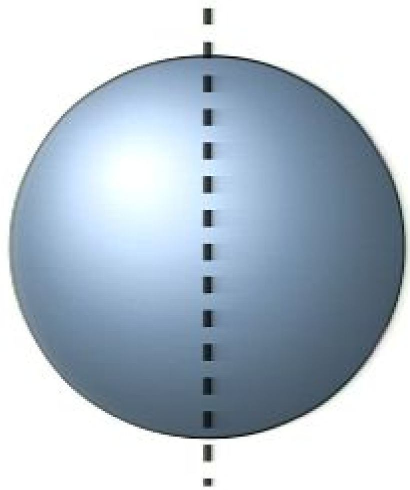

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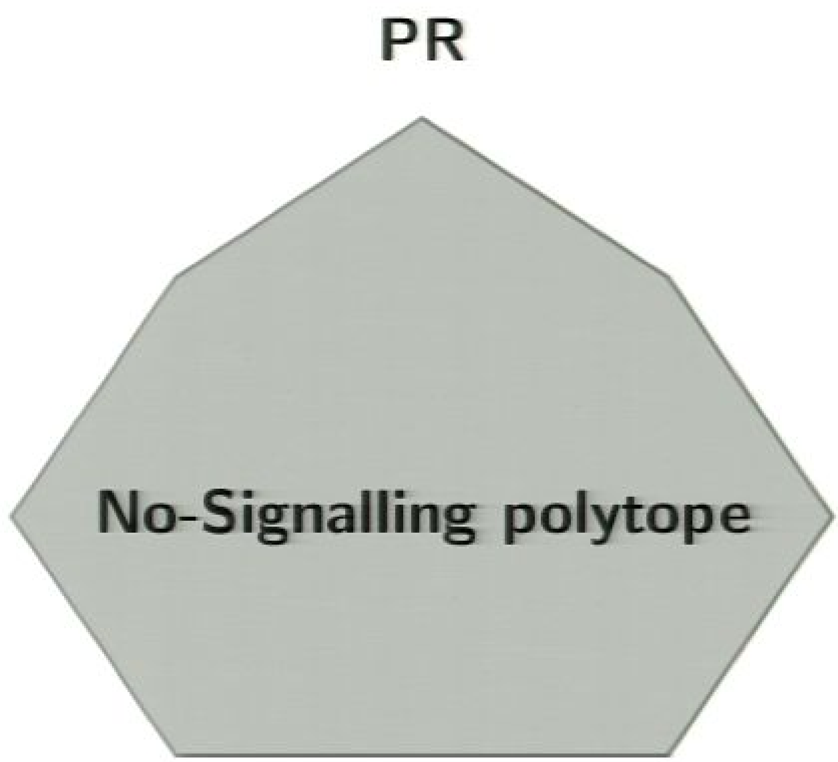
Nonlocality beyond QM

The No-Signalling Polytope

- PR-Boxes are extremals of a 8-dim polytope



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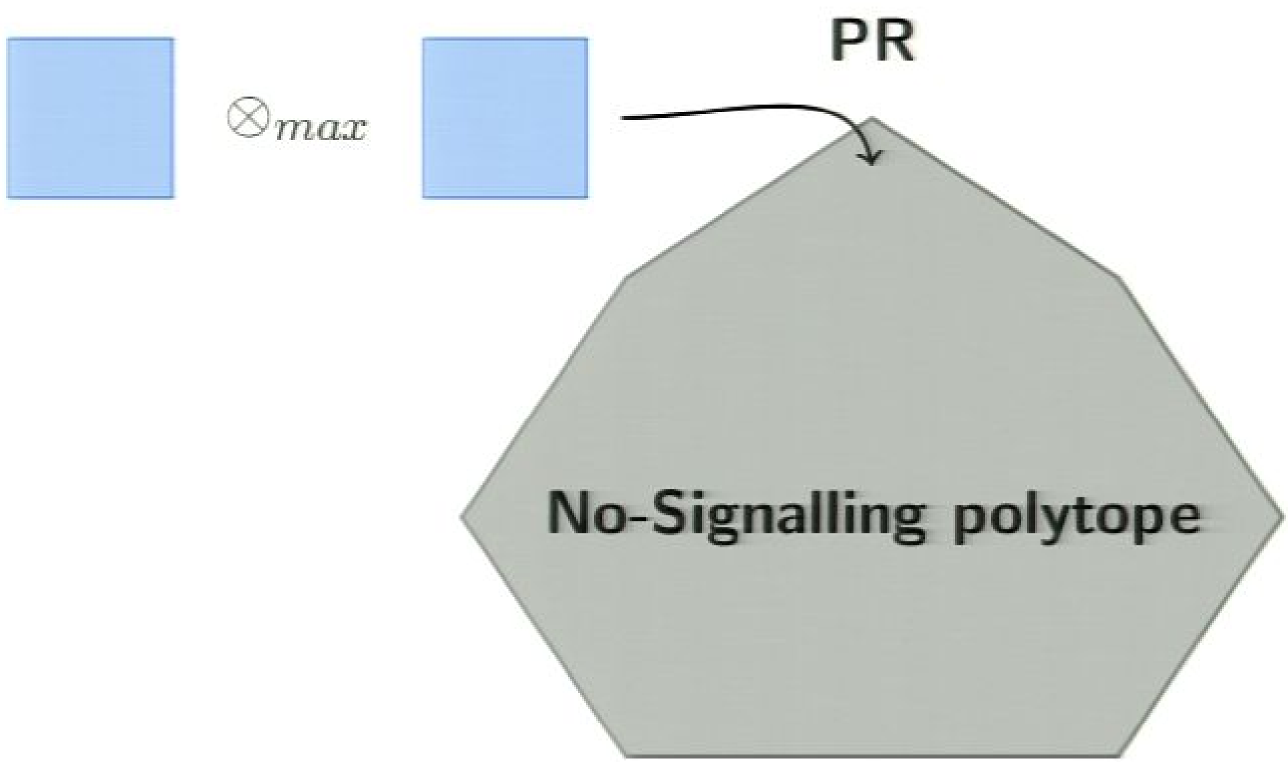

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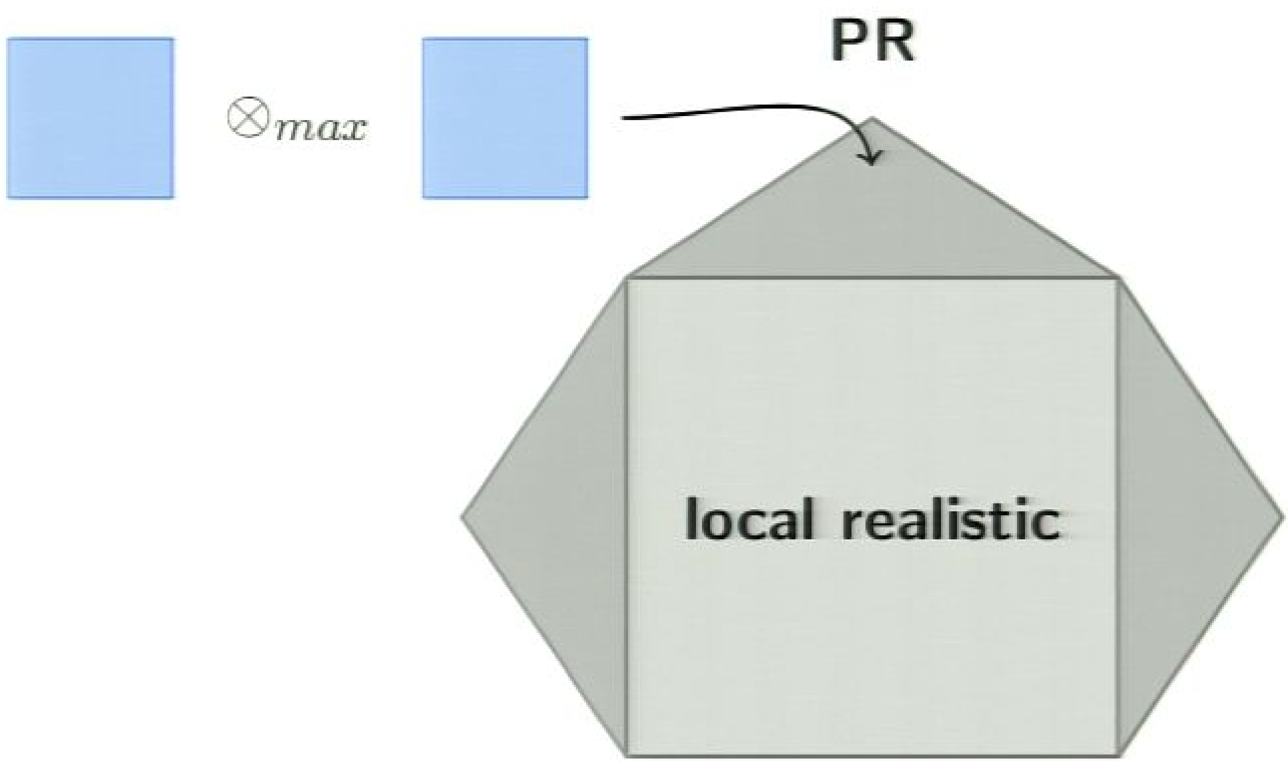

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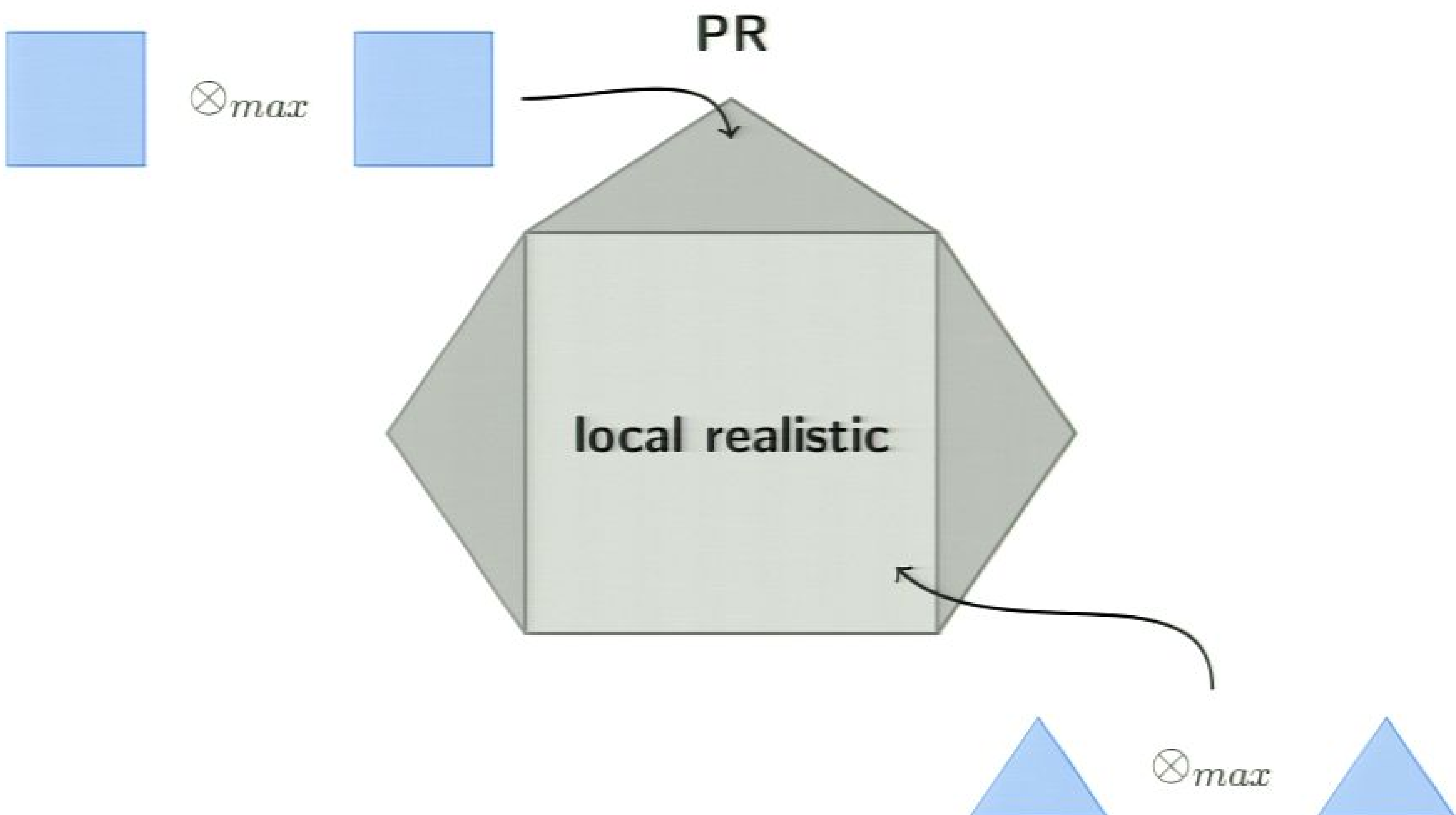

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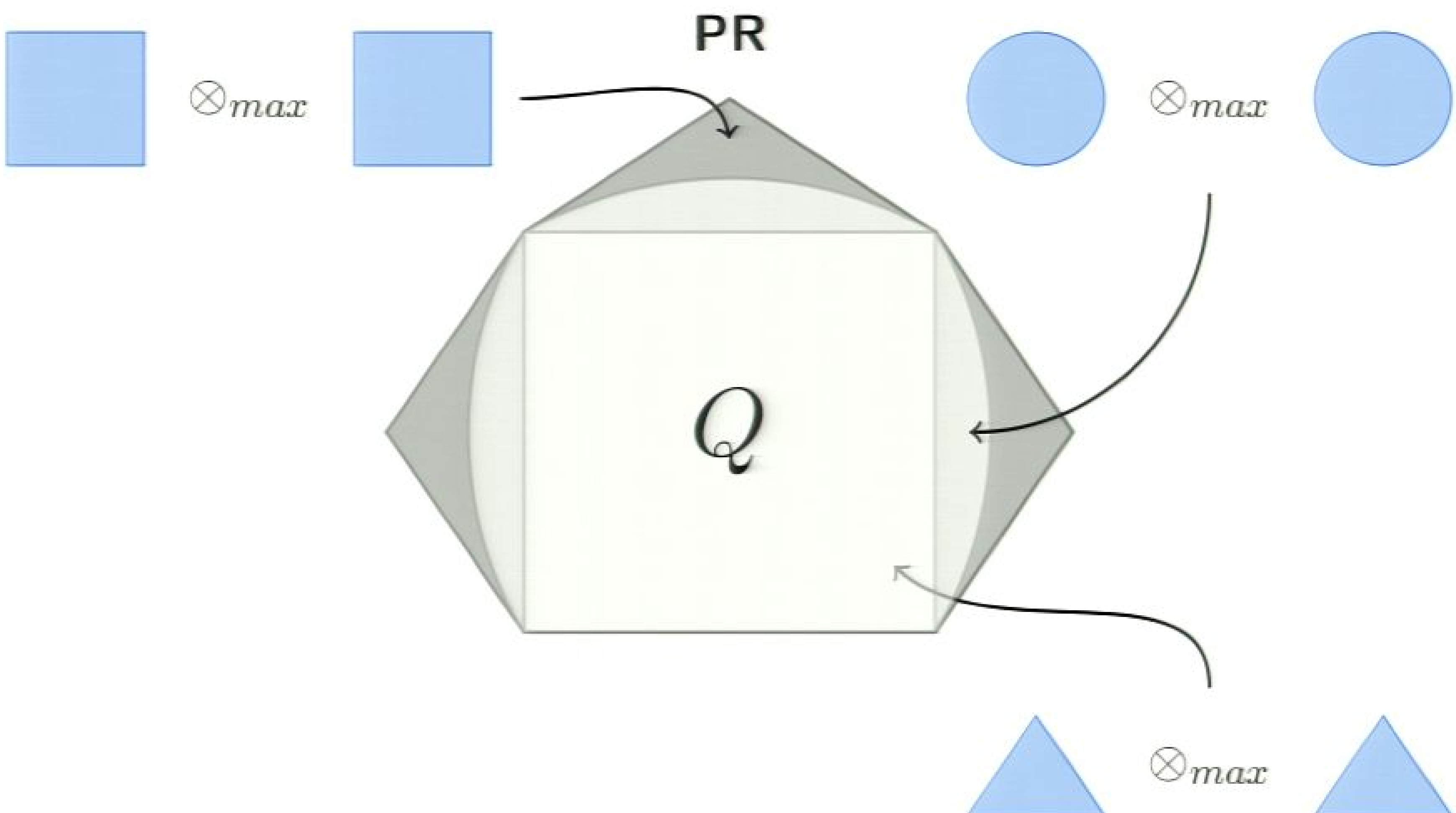
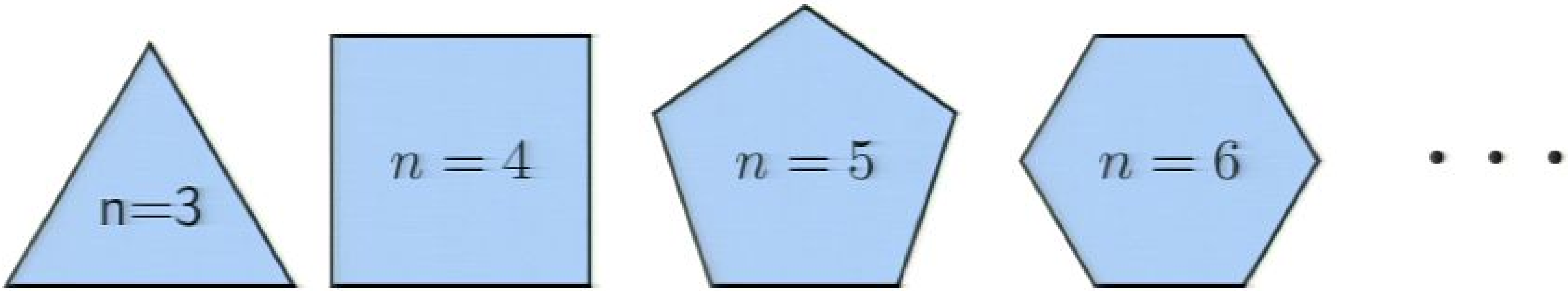


Transition Classical $\rightarrow$ QM Correlations



- $n = 3$: Classical state space
  $n = 4$: GBit leading to all possible no-signalling correlations
  $n \rightarrow \infty$: 2D-intersection of a quantum state space

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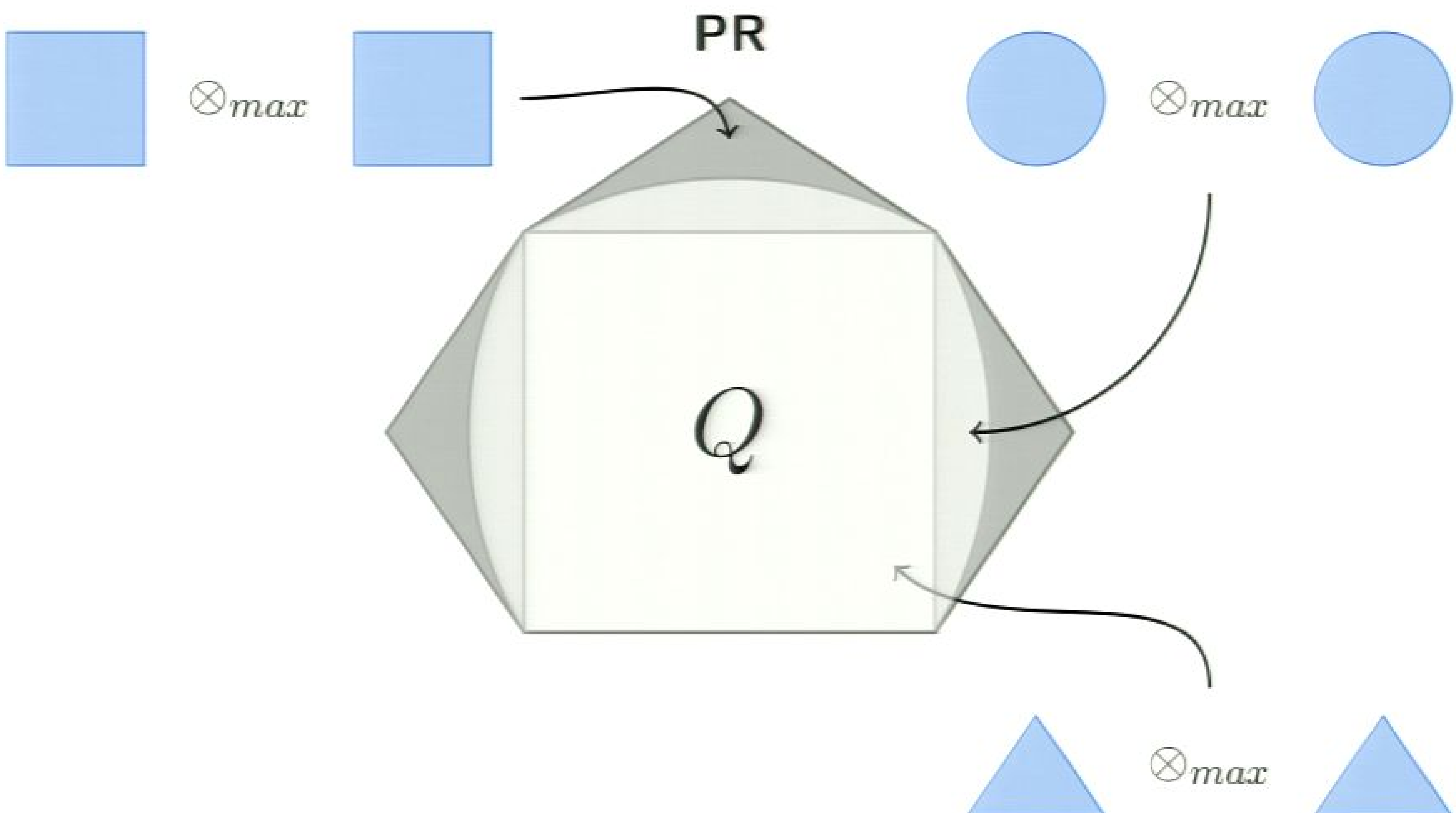

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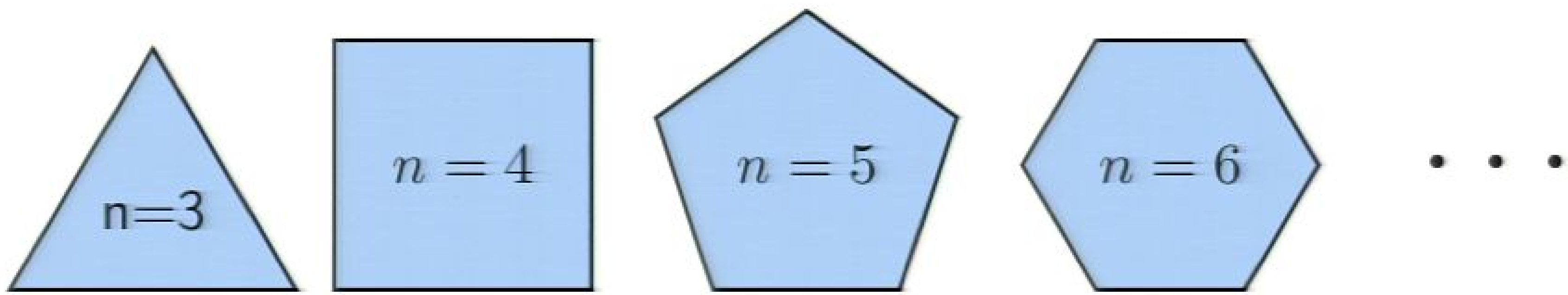

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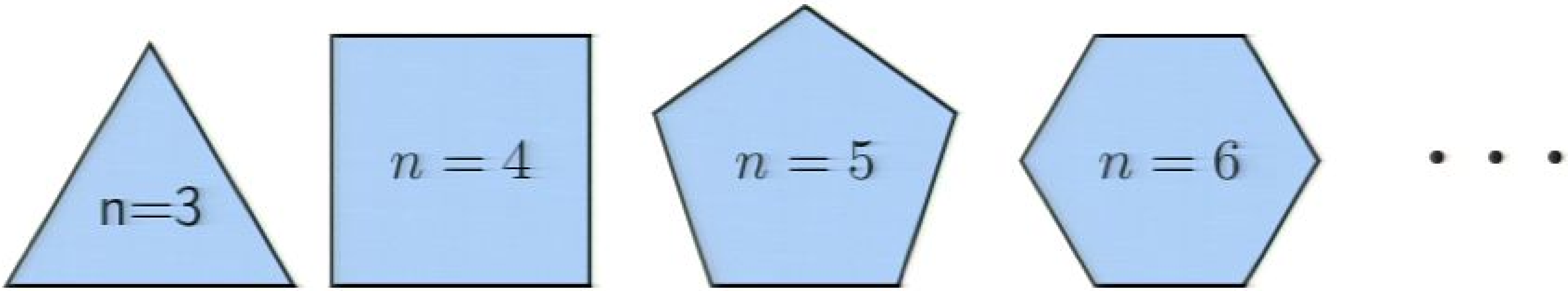
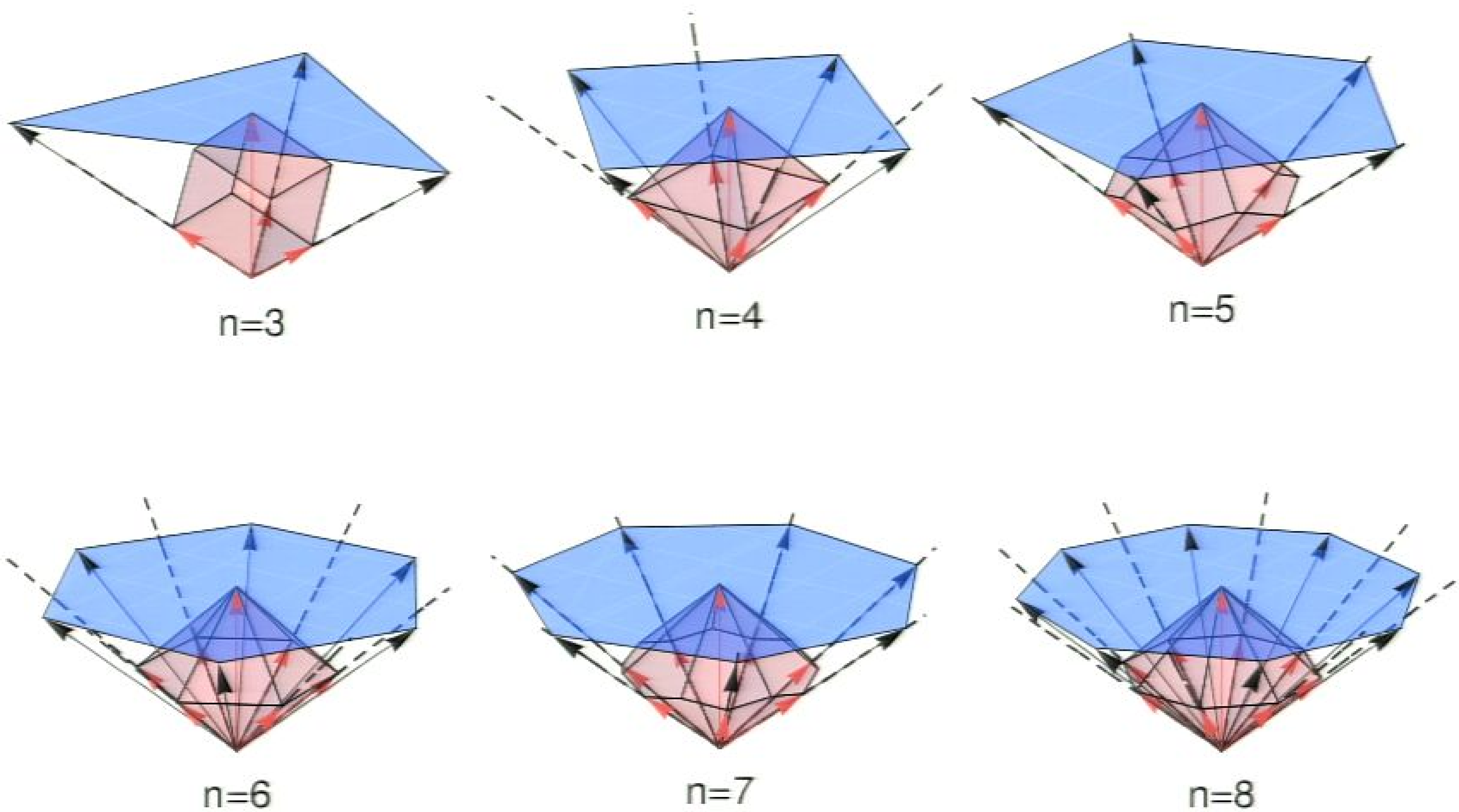
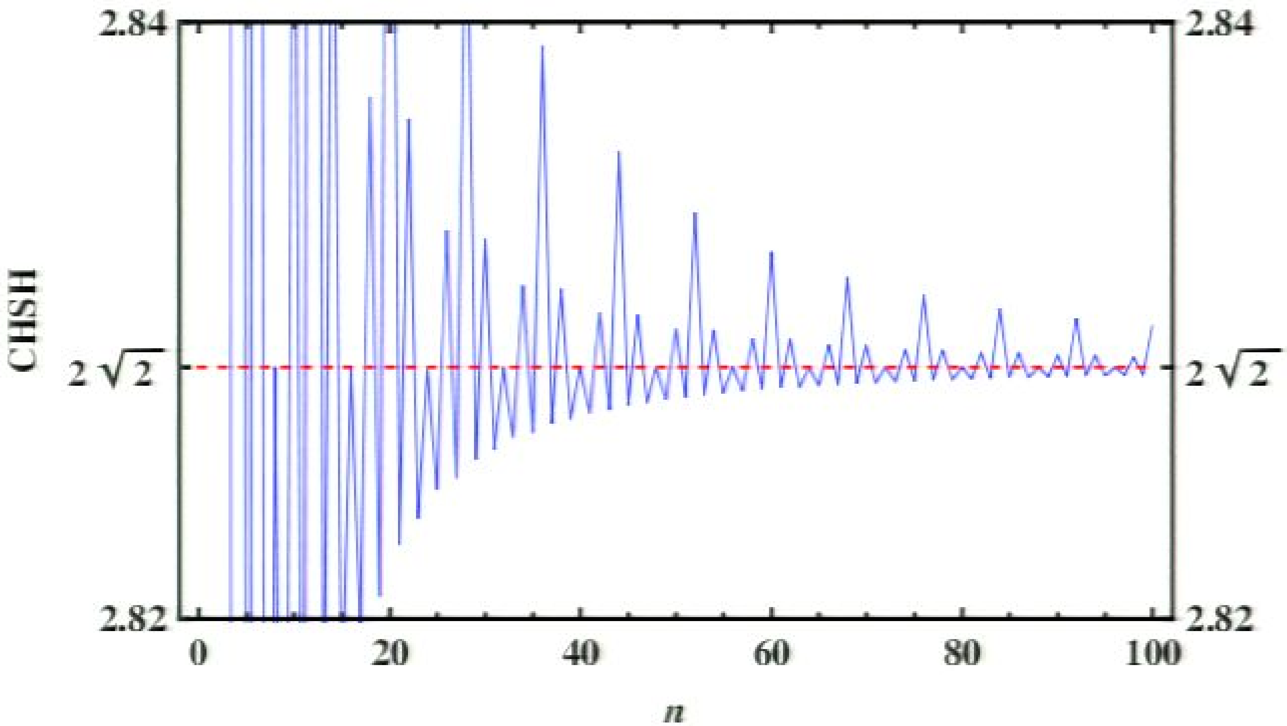


- $n = 3$: Classical state space
  $n = 4$: GBit leading to all possible no-signalling correlations
  $n \rightarrow \infty$: 2D-intersection of a quantum state space
- Full characterization of the maximal tensor product unknown
- Maximally entangled state $\Phi : e_i \rightarrow p\,\omega_i$ is extremal in $\Omega^A \otimes_{\text{max}} \Omega^B$ (for $n > 3$)
- Polygon box or P_n box := Measurement statistics on maximally entangled state Φ of two polygon systems

Weak and strong self-duality



Transition Classical → QM Correlations (Plot)



- Maximal CHSH-coefficients of P_n boxes form 8 classes
- All classes converge to Tsirelson's bound in the limit $n \to \infty$
- Tsirelson's bound appears as natural separation between boxes with odd n and even n

Braunstein-Caves Inequalities

$$S_N = \left| \sum_{j=1}^{N-1} (E_{j,j} + E_{j,j+1}) + E_{N,N} - E_{N,1} \right| \leq 2N - 2$$

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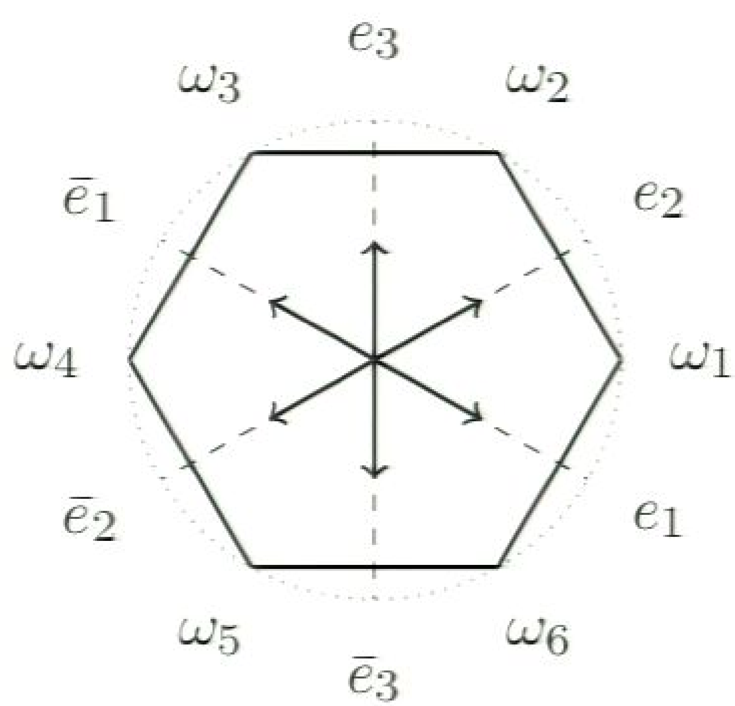

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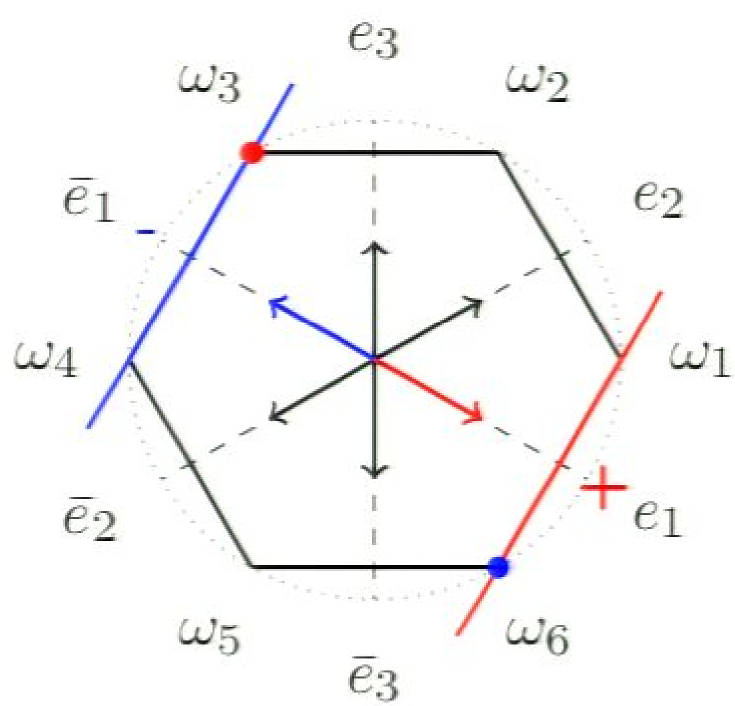

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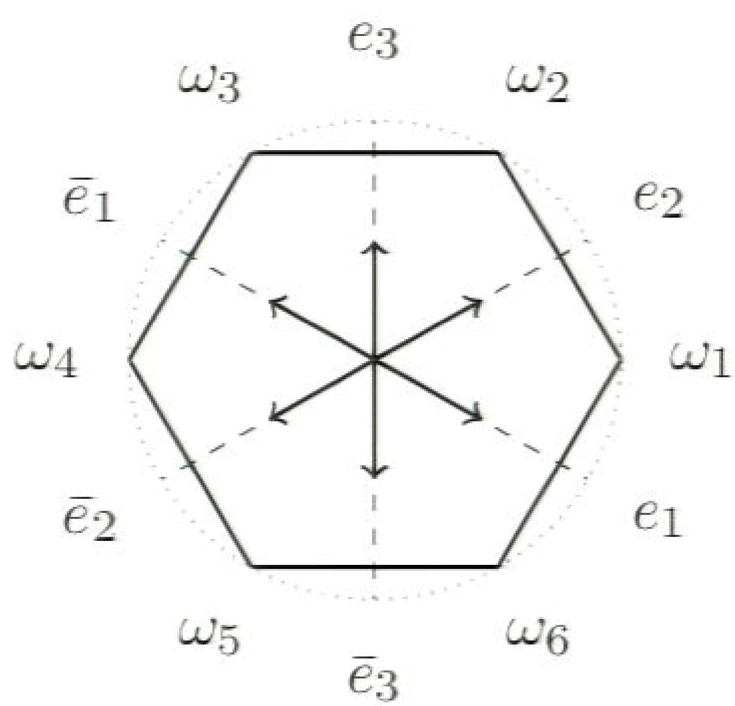

$\Rightarrow$ Secure key distribution

Nonlocality Distillation

- Measurement settings by the first two effects:

$$M_{x=0} = \{e_1, u - e_1\} \quad M_{x=1} = \{e_2, u - e_2\}$$
$$M_{y=0} = \{e_1, u - e_1\} \quad M_{y=1} = \{e_2, u - e_2\}$$

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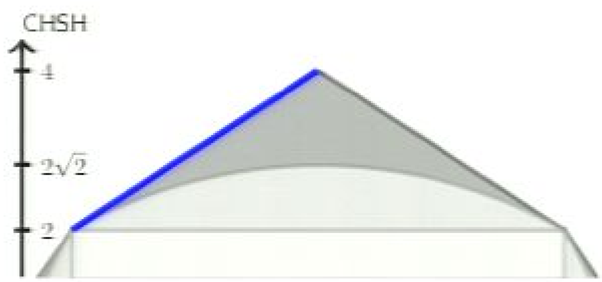
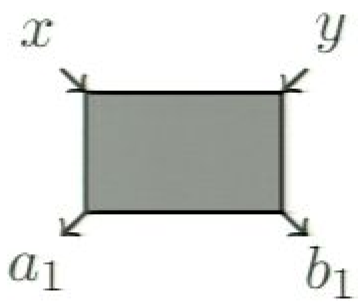
- Measurement statistics $p(ab|xy) = \Phi(e_a^x, e_b^y)$:

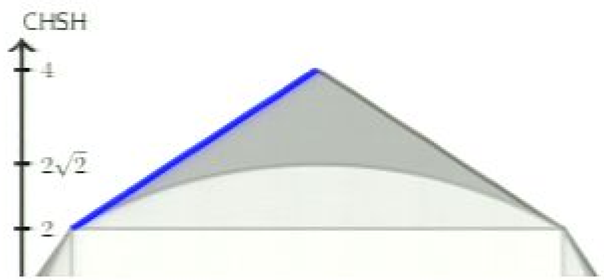
$$p(ab|xy) = \epsilon P^{PR} + (1-\epsilon) P^c$$
$$\epsilon = 1 - \cos\left(\frac{2\pi}{n}\right)$$



Nonlocality Distillation

- Distillation scheme by Brunner et al. 2009





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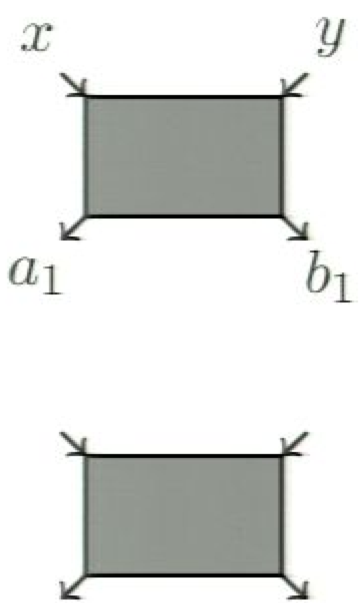

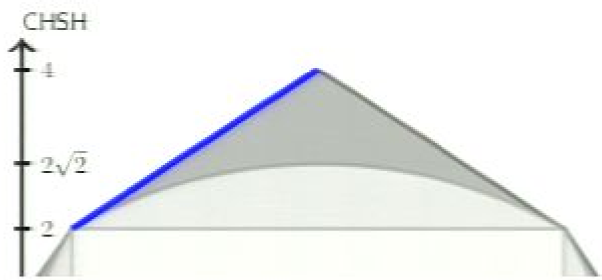

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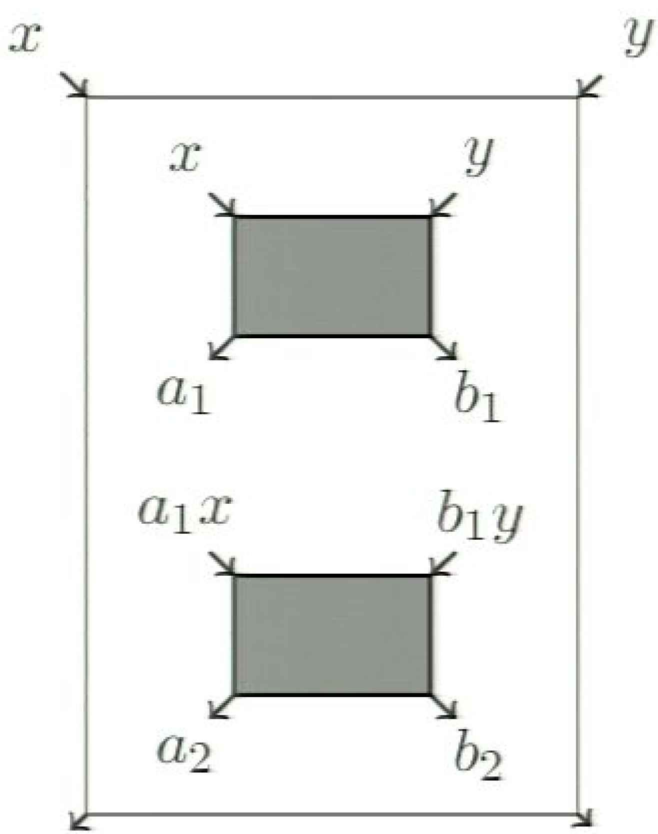
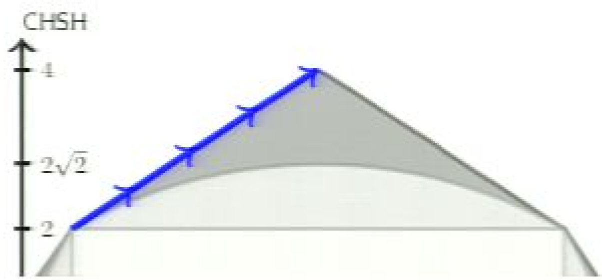


$a = a_1 \oplus a_2$ $b = b_1 \oplus b_2$



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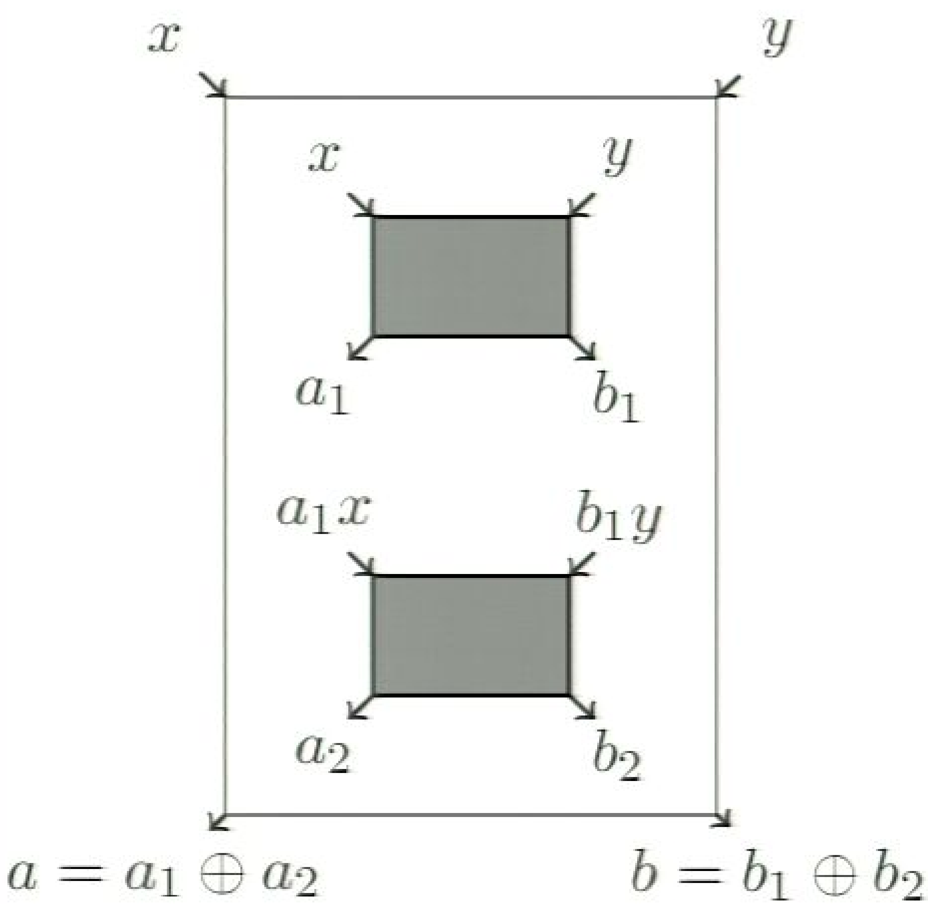

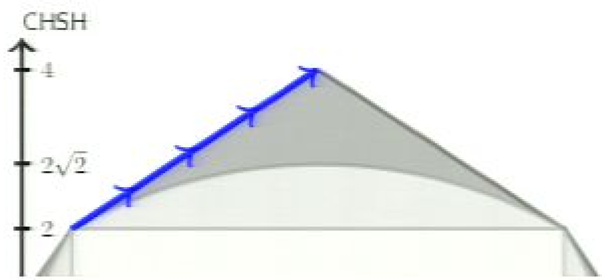

- Polygon boxes with even n inherit powerful features of PR boxes

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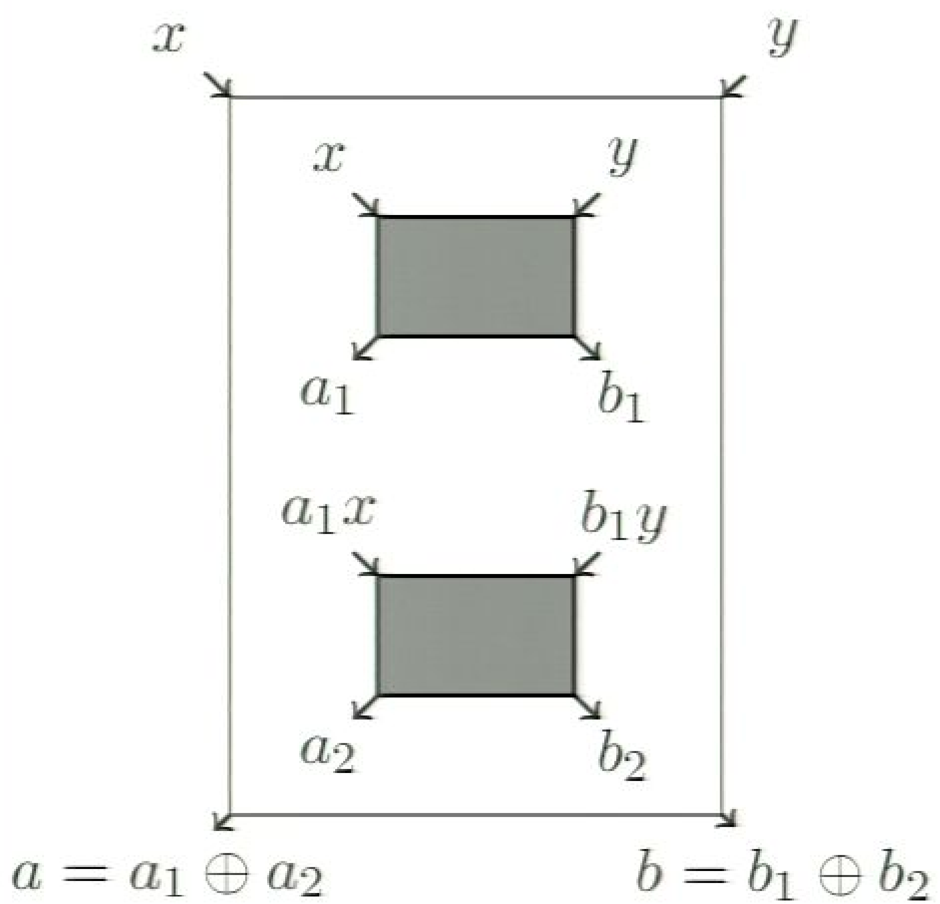

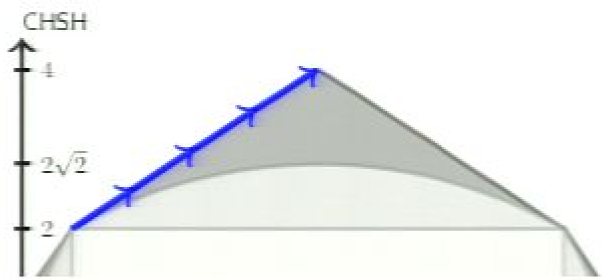

- Polygon boxes with even n inherit powerful features of PR boxes
- Drastically different behavior as QM although almost no difference locally for high n

The set Q_1

$$\gamma = \begin{pmatrix} 1 & \vec{P}_A^T & \vec{P}_B^T \\ \vec{P}_A & \tilde{Q} & \tilde{P}^T \\ \vec{P}_B & \tilde{P} & \tilde{R} \end{pmatrix} \qquad \gamma \geq 0$$

- $\vec{P}_A$ is vector of marginal distributions $P_A(i) = \sum_j \omega(e_i \otimes e_j)$
- $\tilde{P}_{ij} = P(i,j) = \omega(e_i \otimes e_j)$
- Partially determined: $\tilde{Q}_{ii} = P_A(i)$ and $\tilde{R}_{jj} = P_B(j)$
- Other entries can be chosen freely

- $Q \subset Q_1$, bounded by Tsirelson's bound, closed under wirings, macroscopic locality

Strong self-duality $\Rightarrow Q_1$ for Φ

- Sets of local effects under consideration:

$$e := \{e_i\}_{i=1}^{n^A} \text{ and } f := \{f_j\}_{j=1}^{n^B}$$

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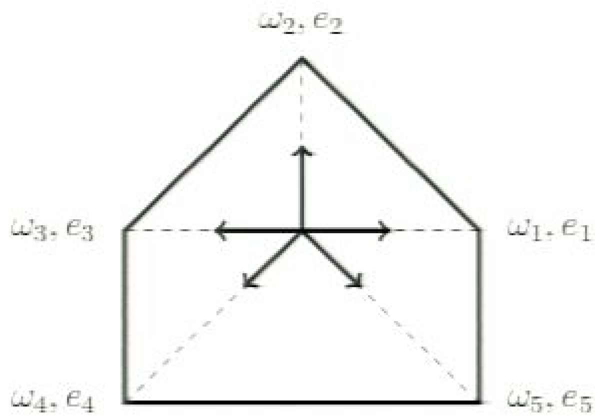
Correlations on general states in strongly self-dual systems

- House-shaped state space is strongly self-dual:



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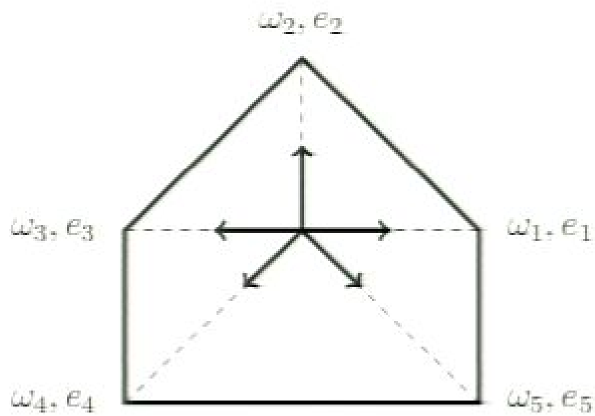

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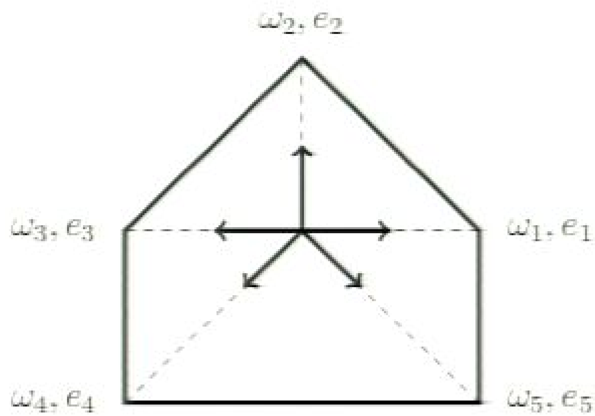

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- Open question: Can strong self-duality strengthened such that all correlations lie in Q_1?
- Does Tsirelson's bound hold for all strongly self-dual systems?

Thank you for your attention!

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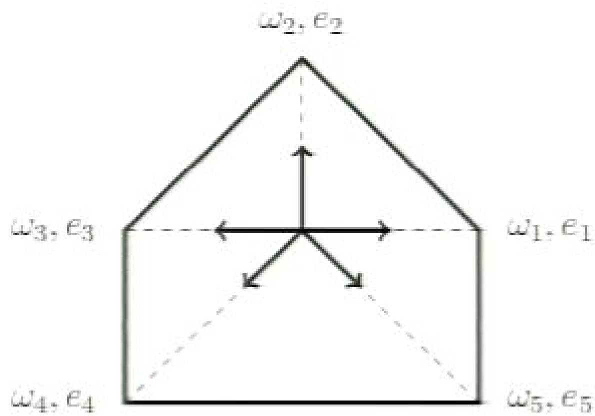

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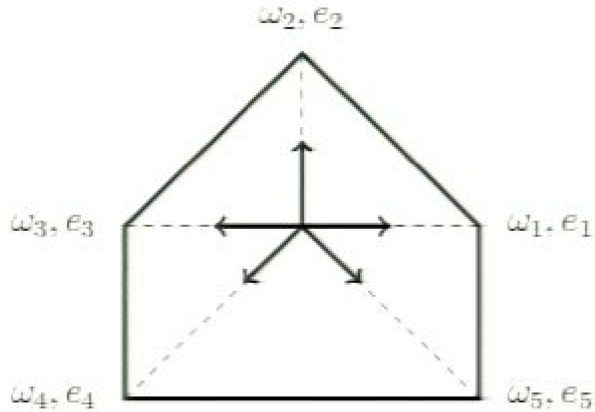

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