Title: Aspects of confinement from the functional Renormalization Group

Date: Dec 01, 2010 04:00 PM

URL: http://pirsa.org/10120025

Abstract: The functional Renormalization Group is a continuum method to study quantum field theories in the non-perturbative regime. In Yang-Mills theory, it can be used to relate fully nonperturbative low-order correlation functions in particular gauges to observables such as confinement order parameters. As a special application, we determine the order of the phase transition and the critical temperature for various gauge groups (SU(N), N=3,.,12, Sp(2) and E(7)). This also allows to investigate what determines the order of the deconfinement phase transition. Furthermore we study the non-perturbative effective potential for the field strength, where we observe the formation of a gluon condensate in the vacuum.

Aspects of confinement from functional Renormalization Group methods

Astrid Eichhorn

Friedrich-Schiller-Universität Jena

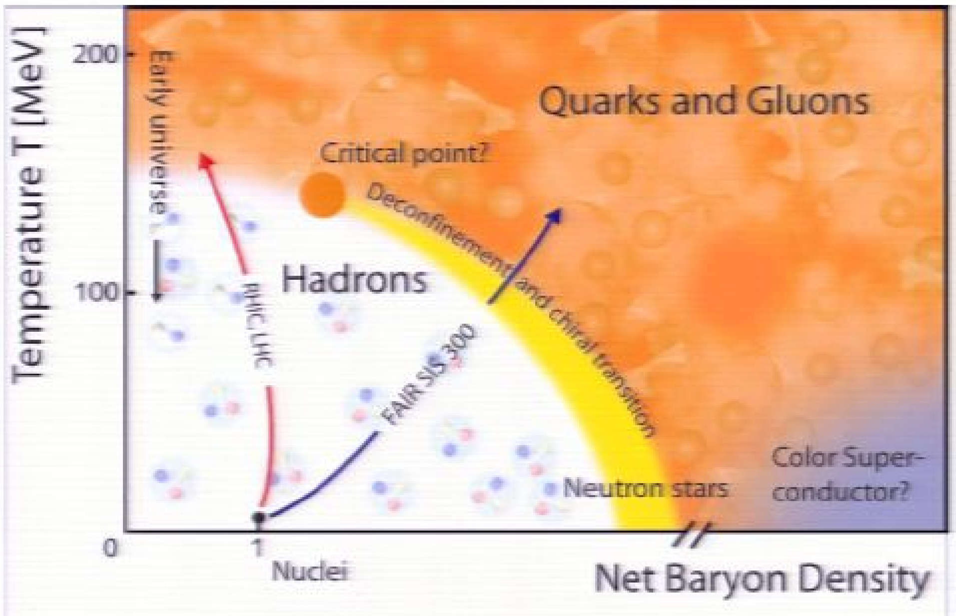
in collaboration with Jens Braun, Holger Gies and Jan M. Pawlowski

Perimeter Institute for Theoretical Physics, 1.12.2010

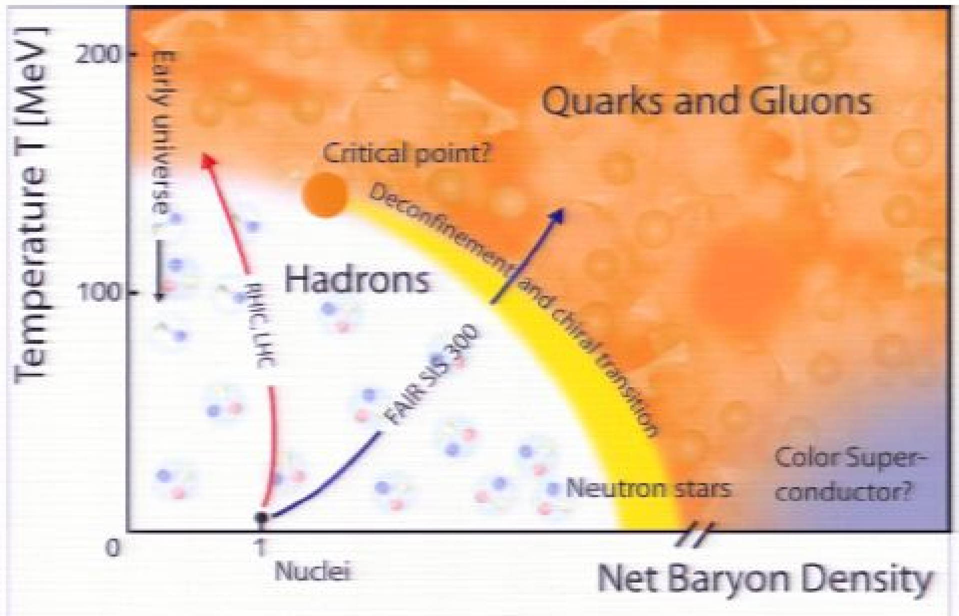
seit 1558

RESEARCH TRAINING GROUP
QUANTUM AND GRAVITATIONAL FIELDS

QCD phase diagram



QCD phase diagram



open questions:

- existence of crit. point?
- relation between chiral symmetry breaking and confinement?
- phys. mechanism of confinement

$\rightarrow$ start by understanding Yang-Mills theory!

table of contents

1. functional Renormalization Group
2. Confinement phase transition at finite temperature
3. Gluon condensate in Yang-Mills theory

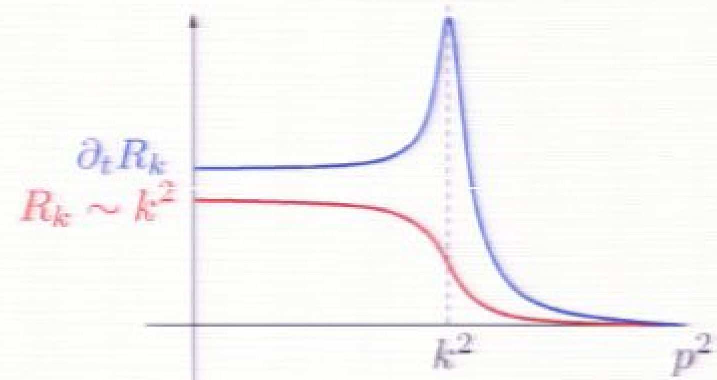
functional Renormalization Group

evaluate full quantum effective action:
generating functional: $Z[J] = \int_\Lambda \mathcal{D}\varphi \, e^{-S[\varphi]+\int J\varphi}$

do path-integral momentum-shell wise:

$$Z_k[J] = \int_\Lambda \mathcal{D}\varphi \, e^{-S[\varphi]+\int J\varphi - \frac{1}{2}\Delta S_k}$$

$$\Delta S_k = \int \frac{d^d p}{(2\pi)^d} \varphi(-p) R_k(p) \varphi(p)$$



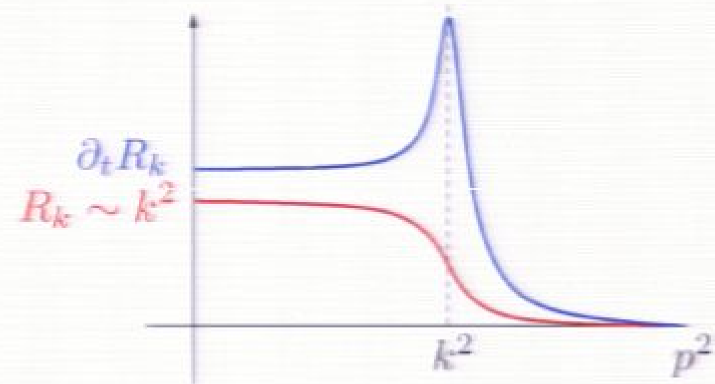
functional Renormalization Group

evaluate full quantum effective action:
generating functional: $Z[J] = \int_\Lambda \mathcal{D}\varphi\, e^{-S[\varphi]+\int J\varphi}$

do path-integral momentum-shell wise:

$$Z_k[J] = \int_\Lambda \mathcal{D}\varphi\, e^{-S[\varphi]+\int J\varphi - \frac{1}{2}\Delta S_k}$$

$$\Delta S_k = \int \frac{d^d p}{(2\pi)^d} \varphi(-p) R_k(p) \varphi(p)$$



generating functional for 1-PI- correlation functions:
$\Gamma_k[\phi] = -\ln Z_k[J] - \int J\phi - \frac{1}{2}\Delta S_k$ with $\phi = \langle\varphi\rangle$
$\Gamma_{k\to 0}[\phi] = \Gamma[\phi]$ quantum effective action

Wetterich-equation

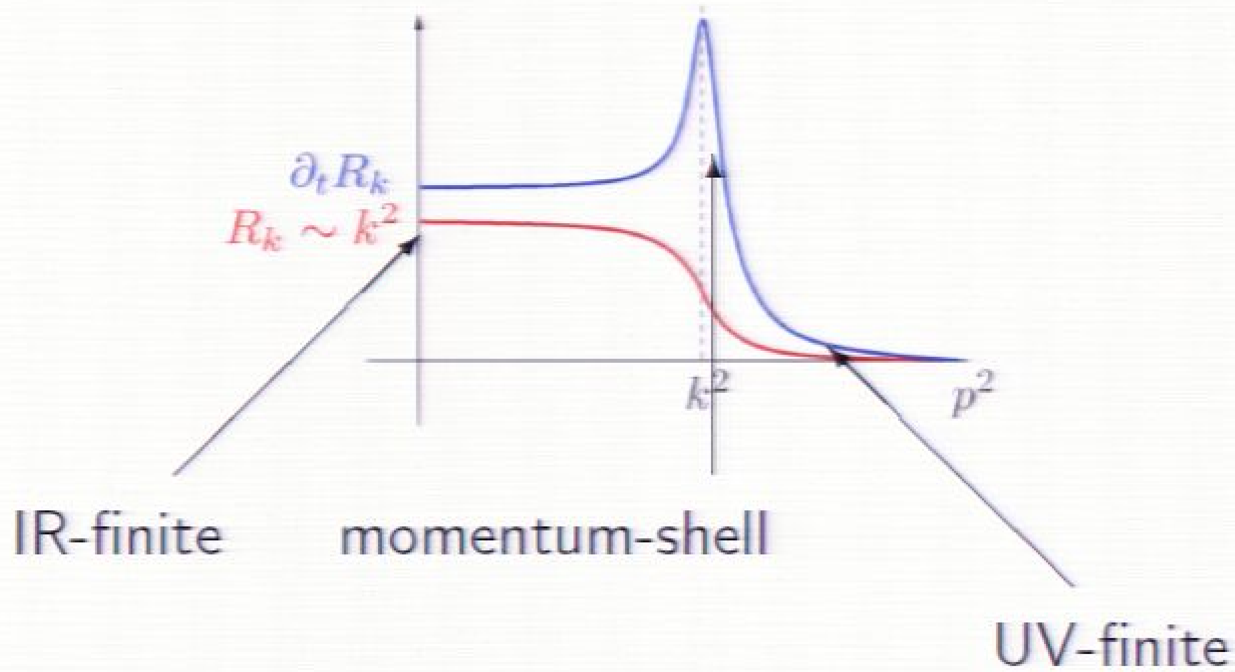
$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

Wetterich, 1992

Wetterich-equation

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

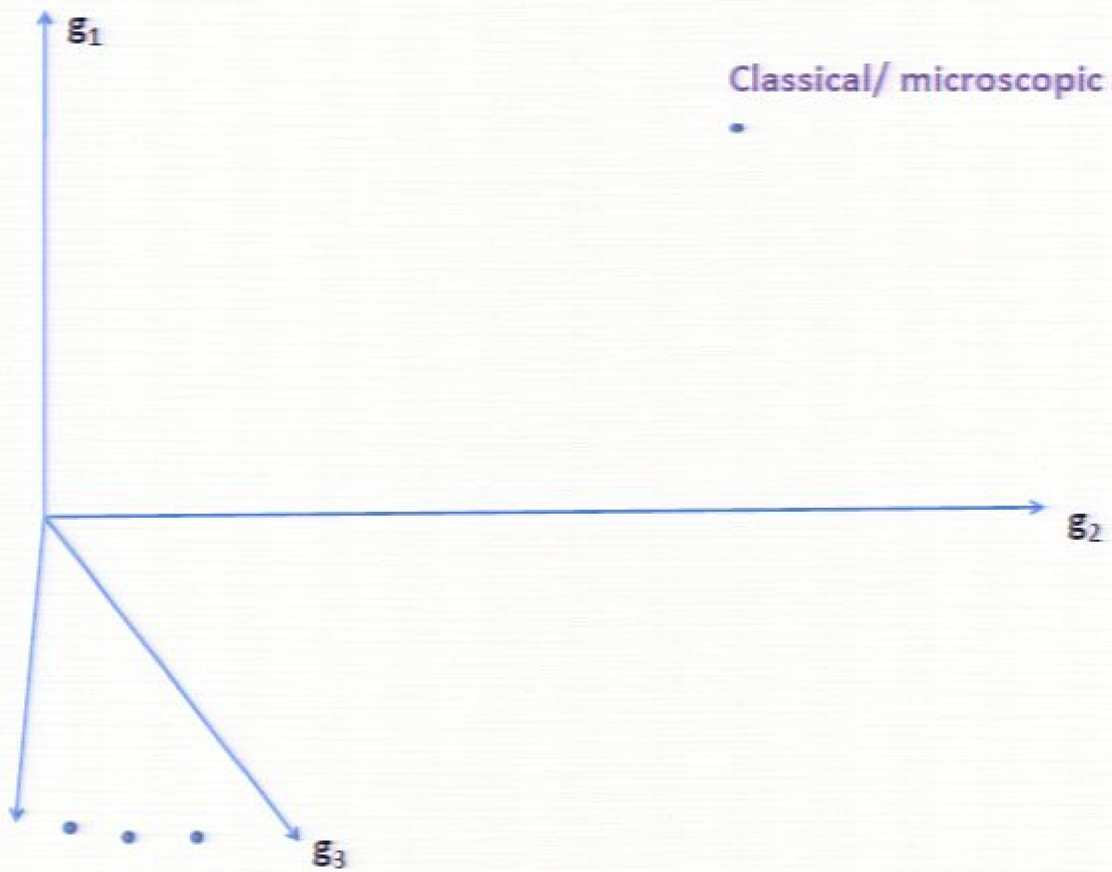
Wetterich, 1992



Wetterich-equation

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

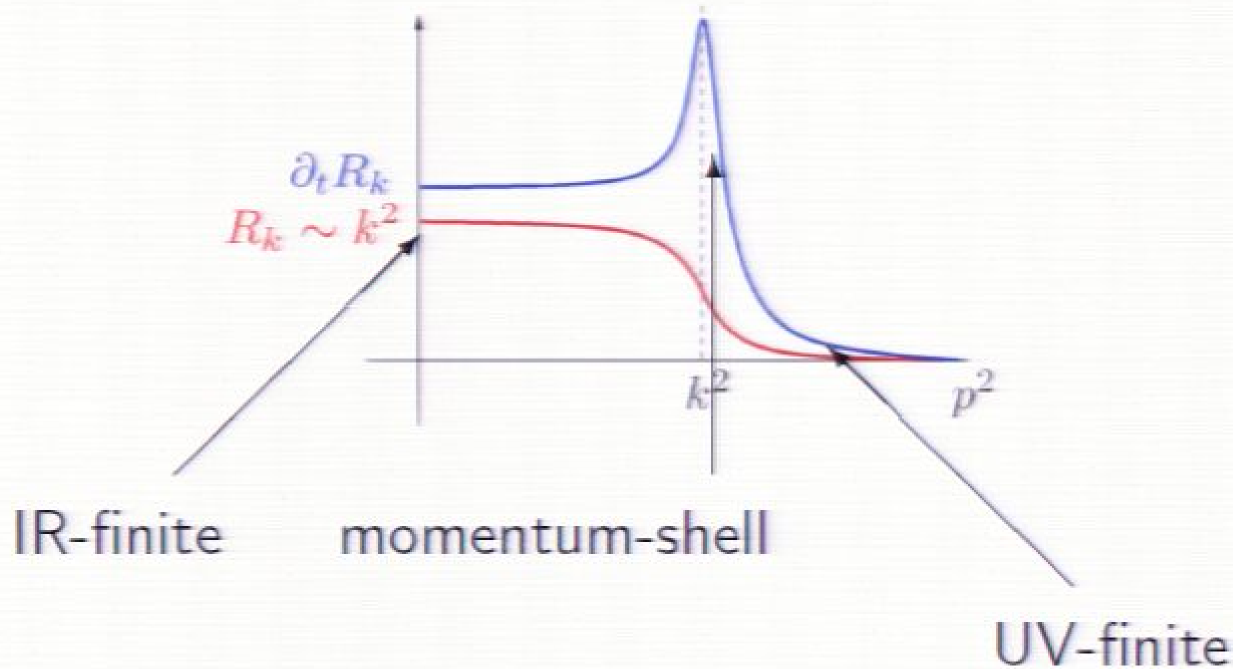
Wetterich, 1992



Wetterich-equation

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

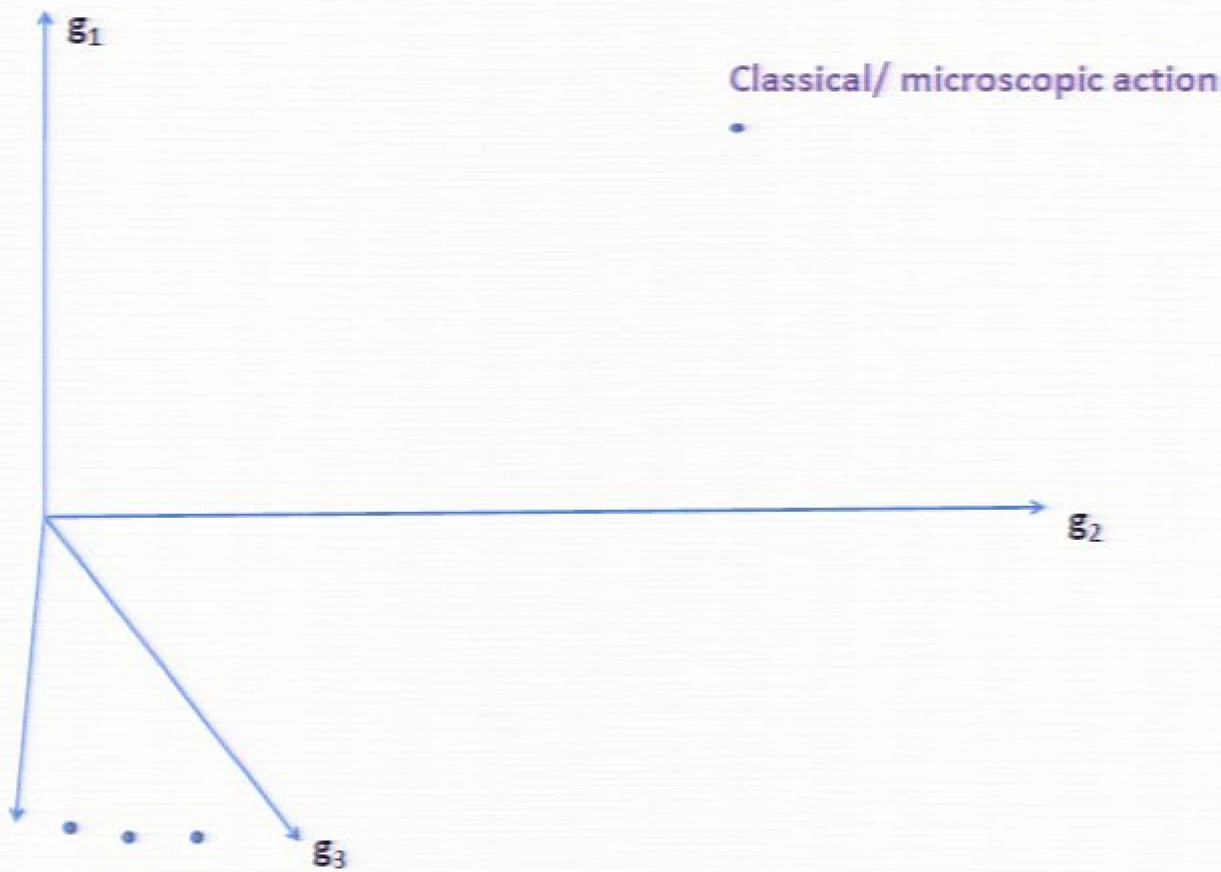
Wetterich, 1992



Wetterich-equation

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

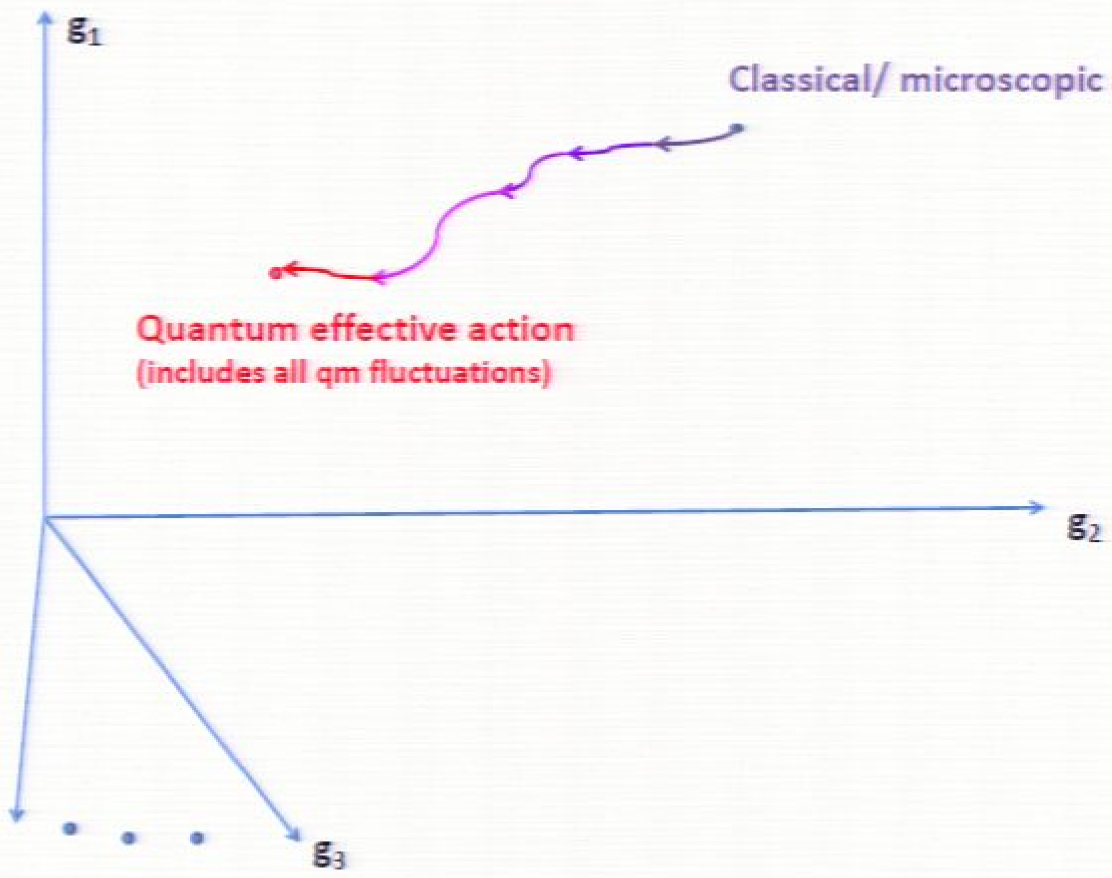
Wetterich, 1992



Wetterich-equation

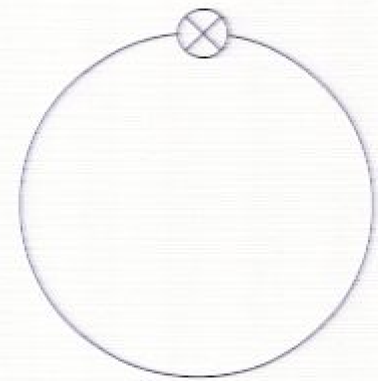
$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

Wetterich, 1992



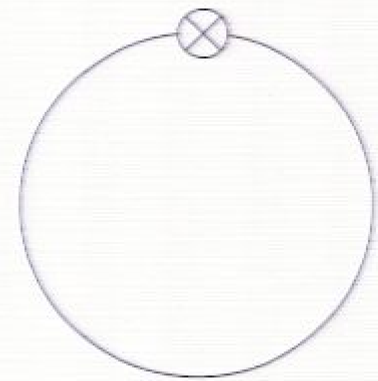
Wetterich-equation

$$k\,\partial_k \Gamma_k = \tfrac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad = \quad \tfrac{1}{2}$$



Wetterich-equation

$$k\,\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad = \quad \frac{1}{2}$$

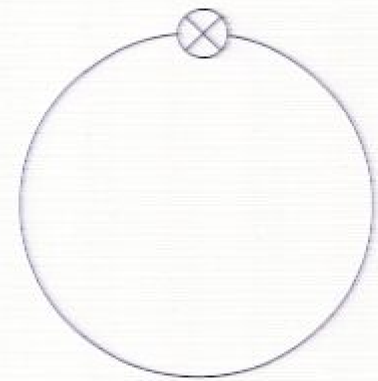


microscopic (classical) action enters as initial condition:

$$\Gamma_{k\to 0} = \Gamma_\Lambda - \frac{1}{2}\int_0^\Lambda \frac{dk}{k}\,\mathrm{STr}\,k\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

Wetterich-equation

$$k\,\partial_k \Gamma_k = \tfrac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad = \quad \tfrac{1}{2}$$



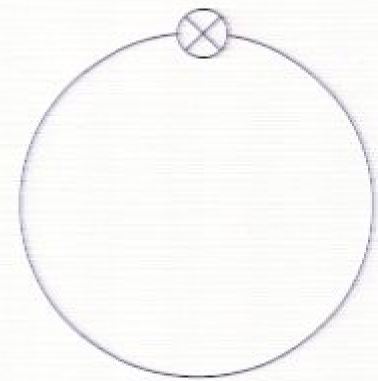
microscopic (classical) action enters as initial condition:

$$\Gamma_{k\to 0} = \Gamma_\Lambda - \tfrac{1}{2}\int_0^\Lambda \frac{dk}{k}\,\mathrm{STr}\, k\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

- exact 1-loop equation

Wetterich-equation

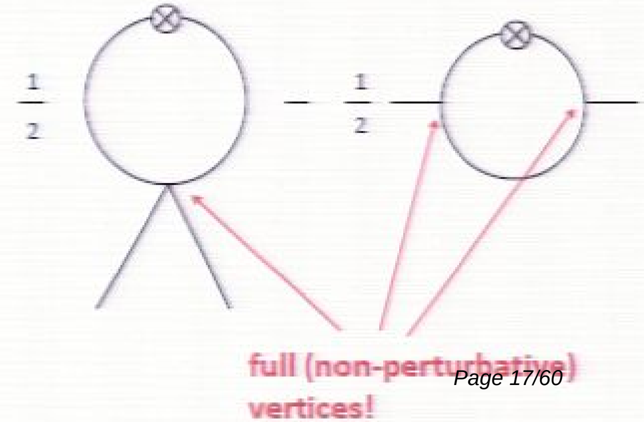
$$k\,\partial_k \Gamma_k = \tfrac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad = \quad \tfrac{1}{2}$$



microscopic (classical) action enters as initial condition:

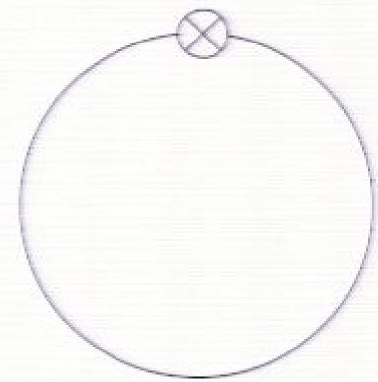
$$\Gamma_{k\to 0} = \Gamma_\Lambda - \tfrac{1}{2}\int_0^\Lambda \tfrac{dk}{k}\,\mathrm{STr}\,k\partial_k\,R_k\left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

- exact 1-loop equation, e.g. $k\,\partial_k \Gamma_k^{(2)} =$
- truncations necessary



Wetterich-equation

$$k\,\partial_k \Gamma_k = \tfrac{1}{2}\mathrm{STr}\, k\,\partial_k\, R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} \quad = \quad \tfrac{1}{2}$$

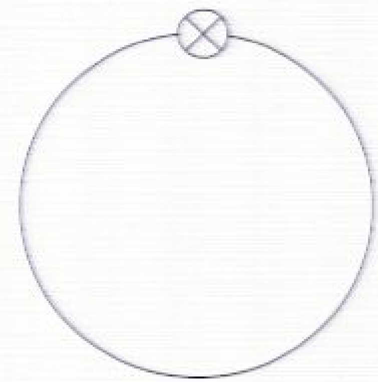


range of applicability:

- perturbative regime
- strongly-interacting regime
  (phase transitions (e.g. BEC-BCS-crossover), quantum gravity
  (asymptotic safety), gauge theories...)

Wetterich-equation

$$k\,\partial_k \Gamma_k = \tfrac{1}{2}\mathrm{STr}\,k\,\partial_k\,R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} = \tfrac{1}{2}$$

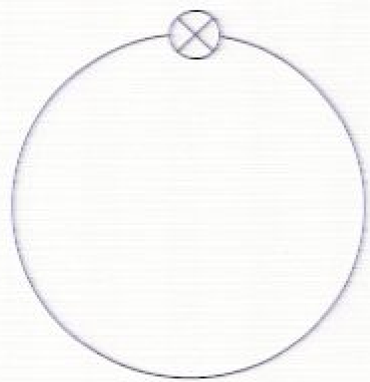
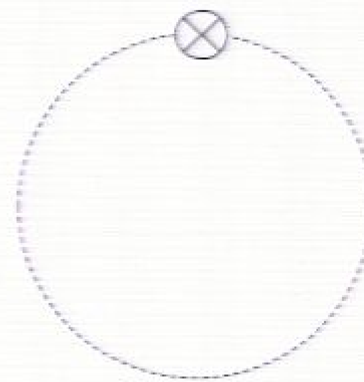


range of applicability:

- perturbative regime
- strongly-interacting regime
  (phase transitions (e.g. BEC-BCS-crossover), quantum gravity (asymptotic safety), gauge theories...)
  $\rightarrow$ test quality of truncation? (reg. dependence, larger truncation)

Wetterich-equation for gauge theories

gauge-fixed formulation
→ Faddeev-Popov-ghosts

$$k\,\partial_k \Gamma_k = \quad \tfrac{1}{2}$$  $$-$$ 

$$\tfrac{1}{2}\mathrm{Tr}\, k\,\partial_k\, R_k \left(\Gamma^{(2)}_{k\,A} + R_k\right)^{-1} \qquad -\mathrm{Tr}\, k\,\partial_k\, R_k \left(\Gamma^{(2)}_{k\,c} + R_k\right)^{-1}$$

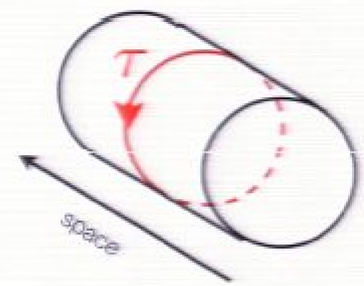
Tools for QCD/ Yang-Mills in the strongly-interacting regime

- lattice gauge theory: need to take continuum limit
  fermions problematic
  finite chemical potential: sign-problem
  no truncation: full QCD
- functional methods: continuum formulation
  fermions "easy"
  but: need to truncate!

$\Rightarrow$ complementary methods!

Confinement phase transition at finite temperature

Quark confinement and center symmetry breaking at finite temperature

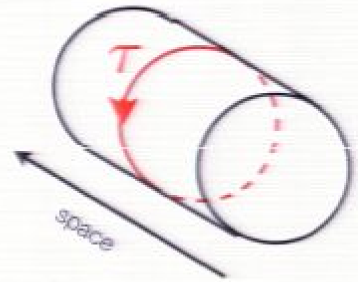


infinitely heavy quark:

Polyakov-loop: $L[A_0] = \frac{1}{N_c} \mathrm{Tr}_{\mathrm{fund}} \mathcal{P} e^{ig \int_0^\beta dx^0 A_0}$

Quark confinement and center symmetry breaking at finite temperature

infinitely heavy quark:



Polyakov-loop: $L[A_0] = \frac{1}{N_c} \mathrm{Tr}_{\mathrm{fund}} \mathcal{P} e^{ig \int_0^\beta dx^0 A_0}$

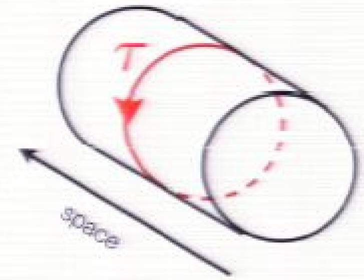
$L[A_0]$ transforms non-trivially under center symmetry

(e.g. $L \to zL$ for $z = \mathbf{1} e^{2\pi i \frac{n}{N}}$ for SU(N))

$\to$ order parameter for center symmetry breaking

Quark confinement and center symmetry breaking at finite T

Polyakov-loop: $L[A_0] = \frac{1}{N_c} \mathrm{Tr}_{\mathrm{fund}} \mathcal{P} e^{ig \int_0^\beta dx^0 A_0}$



$\langle L[A_0] \rangle \sim e^{-\beta F}$, where F is the free energy of a static quark

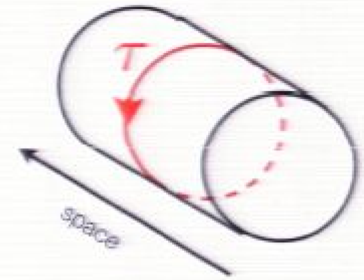
$L[A_0]$ transforms non-trivially under center symmetry

deconfinement: F finite $\Rightarrow \langle L \rangle \neq 0 \Rightarrow$ center symmetry broken

confinement: $F \to \infty \Rightarrow \langle L \rangle = 0 \Rightarrow$ center symmetry restored

Quark confinement and center symmetry breaking at finite T

Polyakov-loop: $L[A_0] = \frac{1}{N_c} \mathrm{Tr}_{\mathrm{fund}} \mathcal{P} e^{ig \int_0^\beta dx^0 A_0}$



$\langle L[A_0] \rangle \sim e^{-\beta F}$, where F is the free energy of a static quark

$L[A_0]$ transforms non-trivially under center symmetry

deconfinement: F finite $\Rightarrow \langle L \rangle \neq 0 \Rightarrow$ center symmetry broken

confinement: $F \to \infty \Rightarrow \langle L \rangle = 0 \Rightarrow$ center symmetry restored

dynamical quarks: break center symmetry explicitly and $\langle L \rangle \neq 0$ in

Quark confinement at finite T

strategy:

- evaluate order-parameter potential $V[\langle A_0 \rangle]$ from flow equation
- determine $\langle A_0(T) \rangle$
- evaluate $L[\langle A_0(T) \rangle] \geq \langle L[A_0] \rangle$

Quark confinement at finite T

strategy:

- evaluate order-parameter potential $V[\langle A_0 \rangle]$ from flow equation
- determine $\langle A_0(T) \rangle$
- evaluate $L[\langle A_0(T) \rangle] \geq \langle L[A_0] \rangle$

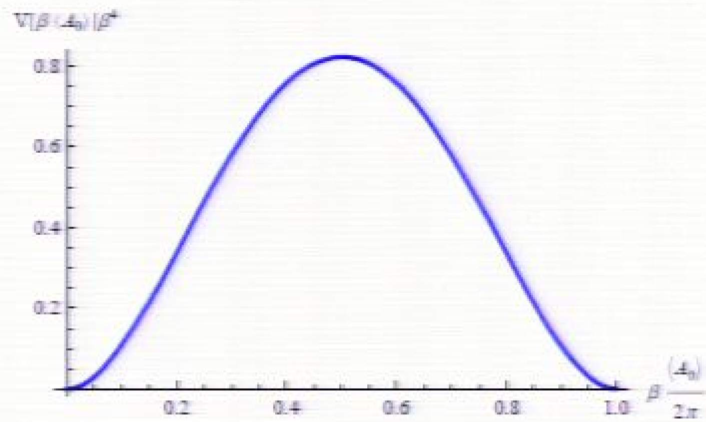
perturbative potential:[Gross, Pisarski, Yaffe, 1981; Weiss 1981, 1982]

$$V[\langle A_0 \rangle] = \frac{\Gamma[\langle A_0 \rangle]}{\Omega} = \frac{1}{2\Omega} \mathrm{Tr} \ln S_A^{(2)}[\langle A_0 \rangle] - \frac{1}{\Omega} \mathrm{Tr} \ln S_c^{(2)}[\langle A_0 \rangle]$$

where
$S^{(2)}[\langle A_0 \rangle] \sim S_c^{(2)}[\langle A_0 \rangle] \sim -D^2[\langle A_0 \rangle] \sim (2\pi n T - \langle A_0 \rangle)^2 + \vec{p}^2$

no confinement from perturbation theory

e.g. SU(2): $L[\langle A_0 \rangle] = cos\frac{\beta \langle A_0 \rangle}{2}$

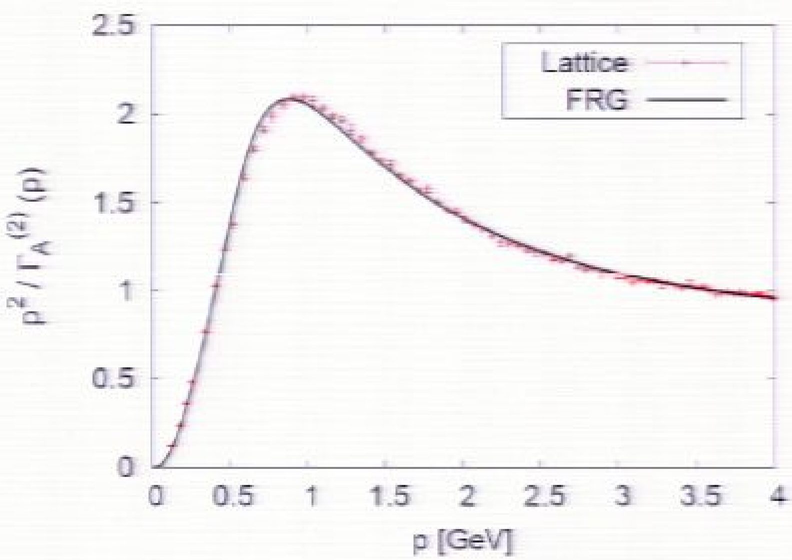


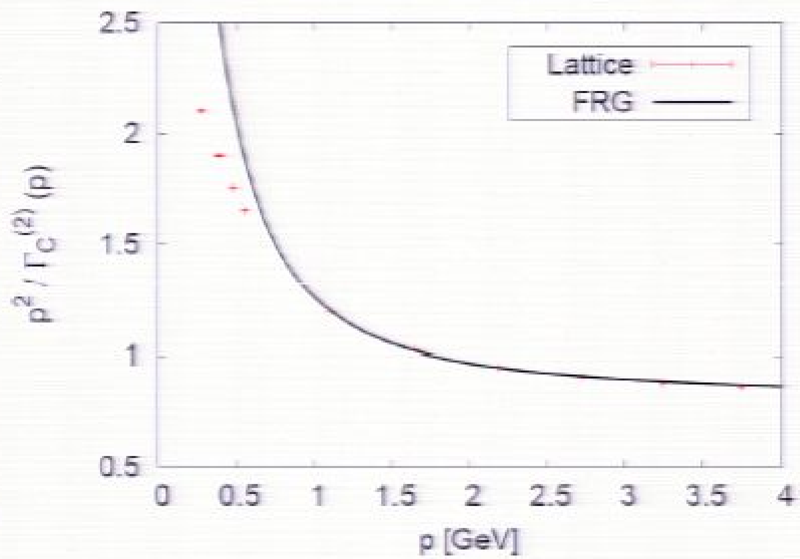
$\langle A_0 \rangle_{min} = 0 \rightarrow L[\langle A_0 \rangle] = 1 \rightarrow$ deconfinement!

need $\frac{\beta}{2\pi}\langle A_0 \rangle = \frac{1}{2}$ for confinement $\rightarrow$ inverted potential!

Confinement in non-perturbative regime

non-perturbative propagators in Landau gauge:





infrared asymptotics: $\Gamma_A^{(2)}(p^2) \sim (p^2)^{1+\kappa_A}$ $\qquad$ $\Gamma_c^{(2)}(p^2) \sim (p^2)^{1+\kappa_c}$

results from functional methods: $\kappa_A = -2\kappa_c$ and $\kappa_c \approx 0.595$

satisfy the Kugo-Ojima color confinement criterion $\kappa_c > \frac{1}{2}$

Quark confinement at finite T

strategy:

- evaluate order-parameter potential $V[\langle A_0 \rangle]$ from flow equation
- determine $\langle A_0(T) \rangle$
- evaluate $L[\langle A_0(T) \rangle] \geq \langle L[A_0] \rangle$

perturbative potential:[Gross, Pisarski, Yaffe, 1981; Weiss 1981, 1982]

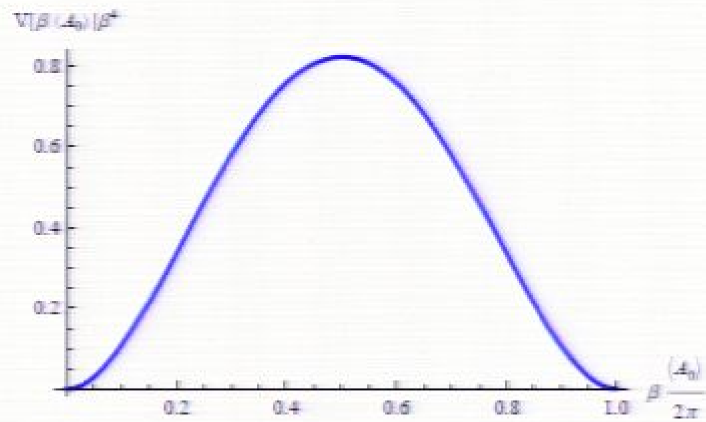
$$V[\langle A_0 \rangle] = \frac{\Gamma[\langle A_0 \rangle]}{\Omega} = \frac{1}{2\Omega} \mathrm{Tr} \ln S_A^{(2)}[\langle A_0 \rangle] - \frac{1}{\Omega} \mathrm{Tr} \ln S_c^{(2)}[\langle A_0 \rangle]$$

where

$$S^{(2)}[\langle A_0 \rangle] \sim S_c^{(2)}[\langle A_0 \rangle] \sim -D^2[\langle A_0 \rangle] \sim (2\pi n T - \langle A_0 \rangle)^2 + \vec{p}^2$$

no confinement from perturbation theory

e.g. SU(2): $L[\langle A_0 \rangle] = cos\frac{\beta\langle A_0\rangle}{2}$

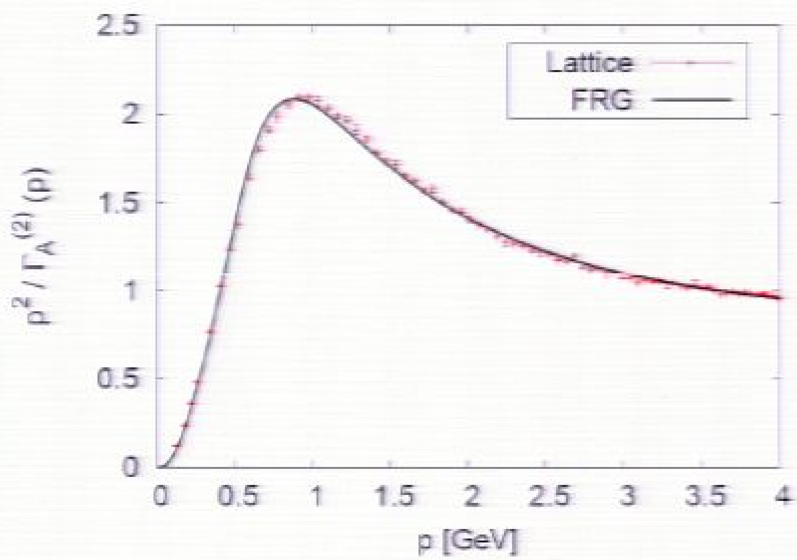


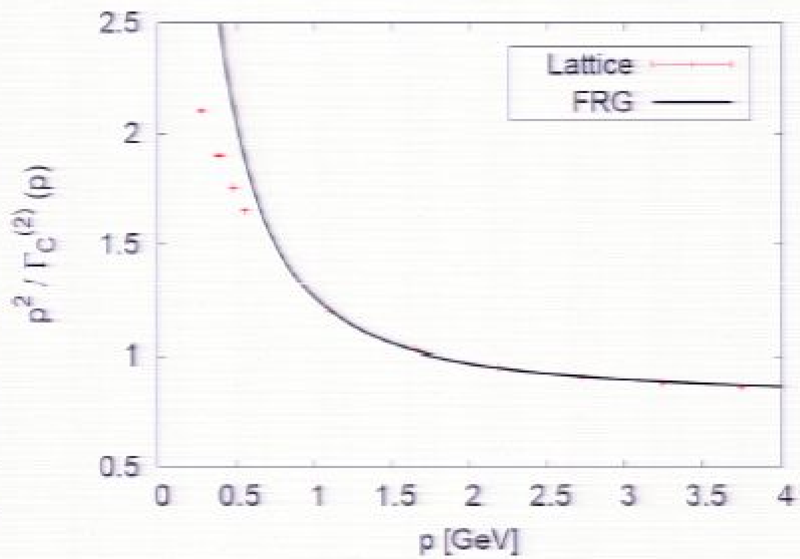
$\langle A_0 \rangle_{min} = 0 \rightarrow L[\langle A_0 \rangle] = 1 \rightarrow$ deconfinement!

need $\frac{\beta}{2\pi}\langle A_0 \rangle = \frac{1}{2}$ for confinement $\rightarrow$ inverted potential!

Confinement in non-perturbative regime

non-perturbative propagators in Landau gauge:





infrared asymptotics: $\Gamma_A^{(2)}(p^2) \sim (p^2)^{1+\kappa_A}$ $\Gamma_c^{(2)}(p^2) \sim (p^2)^{1+\kappa_c}$

results from functional methods: $\kappa_A = -2\kappa_c$ and $\kappa_c \approx 0.595$

satisfy the Kugo-Ojima color confinement criterion $\kappa_c > \frac{1}{2}$

Confinement in non-perturbative regime

$$\Omega V[\langle A_0\rangle] = \frac{1}{2}\mathrm{Tr}\ln\Gamma_A^{(2)}[\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma_c^{(2)}[\langle A_0\rangle] + \mathcal{O}(\partial_k\Gamma_k^{(2)}) + \mathcal{O}(V'')$$

gluon loop | ghost loop | RG improvement | backcoupling of potential

Confinement in non-perturbative regime

$$\Omega V[\langle A_0\rangle] = \tfrac{1}{2}\mathrm{Tr}\ln\Gamma_A^{(2)}[\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma_c^{(2)}[\langle A_0\rangle] + \mathcal{O}(\partial_k\Gamma_k^{(2)}) + \mathcal{O}(V'')$$

gluon loop — ghost loop — RG improvement — backcoupling of potential

potential driven by modes $p \sim T$

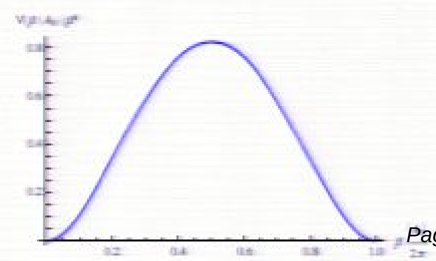
at low $T < T_c$: $\mathrm{Tr}\ \ln\Gamma_{k\,A}^{(2)}[-D^2[A_0]] \to \mathrm{Tr}\ln(-D^2[A_0])^{1+\kappa_A}$

$$= (1+\kappa_A)\,\mathrm{Tr}\ln\left(-D^2[A_0]\right)$$

confinement criterion:[J. Braun, H. Gies, J.M. Pawlowski (2007)]

$$V(\langle A_0\rangle)_{T\to 0} = \frac{1}{\Omega}\left(\frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_c)\right)\mathrm{Tr}\ln\left(-D^2[A_0]\right)$$

transv. gluons — long. gluons — ghosts



Confinement in non-perturbative regime

$$\Omega V[\langle A_0\rangle] = \tfrac{1}{2}\mathrm{Tr}\ln\Gamma_A^{(2)}[\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma_c^{(2)}[\langle A_0\rangle] + \mathcal{O}(\partial_k\Gamma_k^{(2)}) + \mathcal{O}(V'')$$

gluon loop — ghost loop — RG improvement — backcoupling of potential

potential driven by modes $p \sim T$

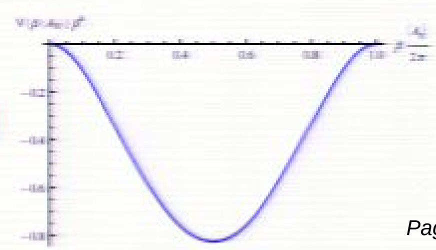
at low $T < T_c$: $\mathrm{Tr}\ \ln\ \Gamma_{k\,A}^{(2)}[-D^2[A_0]] \to \mathrm{Tr}\ln(-D^2[A_0])^{1+\kappa_A}$
$$= (1+\kappa_A)\,\mathrm{Tr}\ln\left(-D^2[A_0]\right)$$

confinement criterion: [J. Braun, H. Gies, J.M. Pawlowski (2007)]

$$V(\langle A_0\rangle)_{T\to 0} = \frac{1}{\Omega}\left(\frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_c)\right)\mathrm{Tr}\ln\left(-D^2[A_0]\right)$$

transv. gluons — long. gluons — ghosts

confinement criterion: $d-2+(d-1)\kappa_A - \kappa_c < 0$



Confinement in non-perturbative regime

confinement criterion: $d - 2 + (d-1)\kappa_A - \kappa_c < 0$

$\rightarrow d = 4 : 3\kappa_A - \kappa_c < -2$

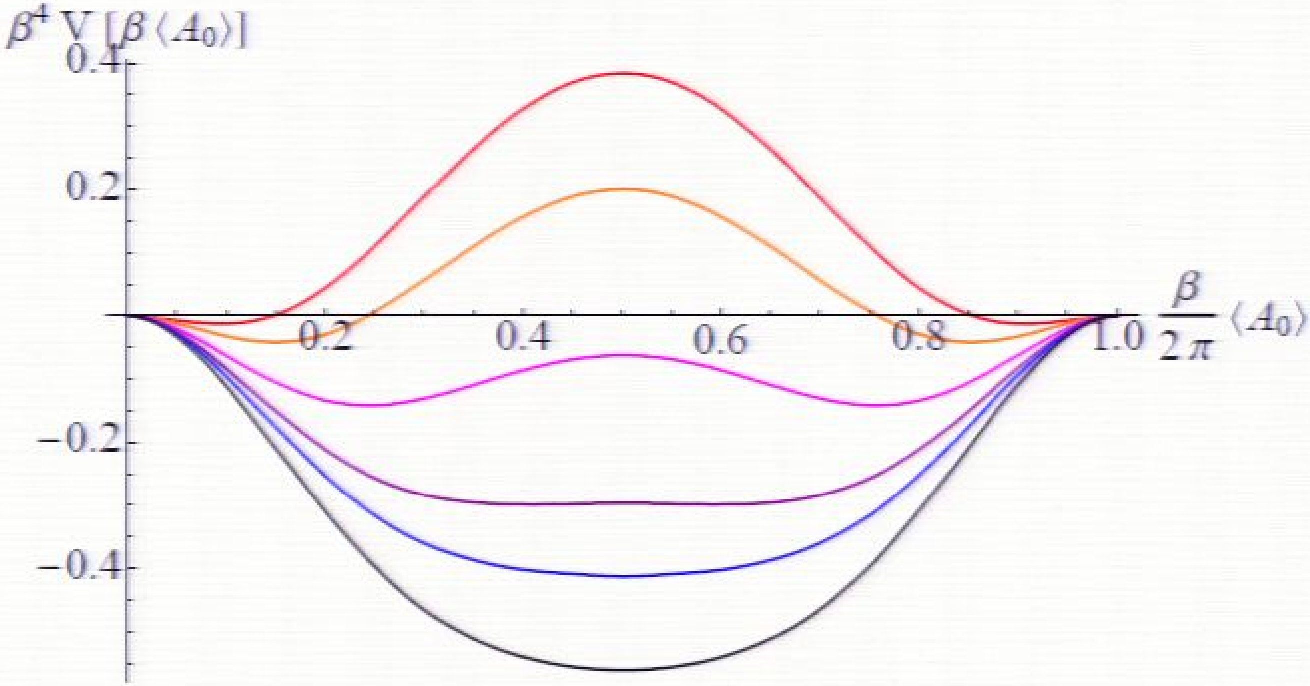
satisfied by Landau-gauge propagators! $(\kappa_c \approx 0.6,\ \kappa_A = -2\kappa_c)$

Kugo-Ojima color confinement criterion: $\kappa_c > \frac{1}{2}$

$\rightarrow$ gluon and ghost propagators that encode color confinement induce a quark confining order-parameter potential!

order-parameter potential at finite T

SU(2):

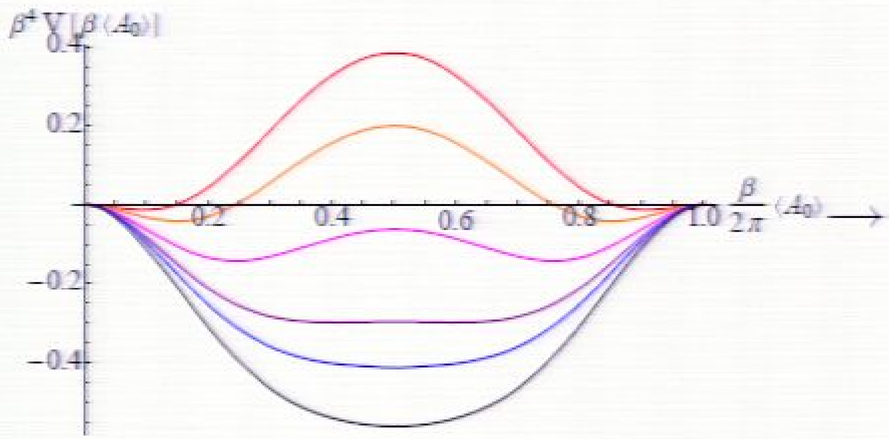


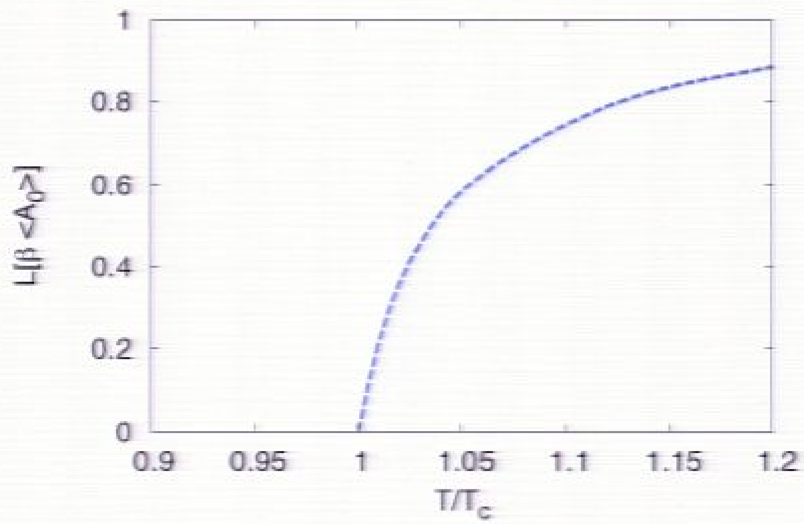
$T_c \approx 265$ MeV

[J. Braun, A.E., H. Gies, J.M. Pawlowski (2010)]

order-parameter potential at finite T

SU(2):

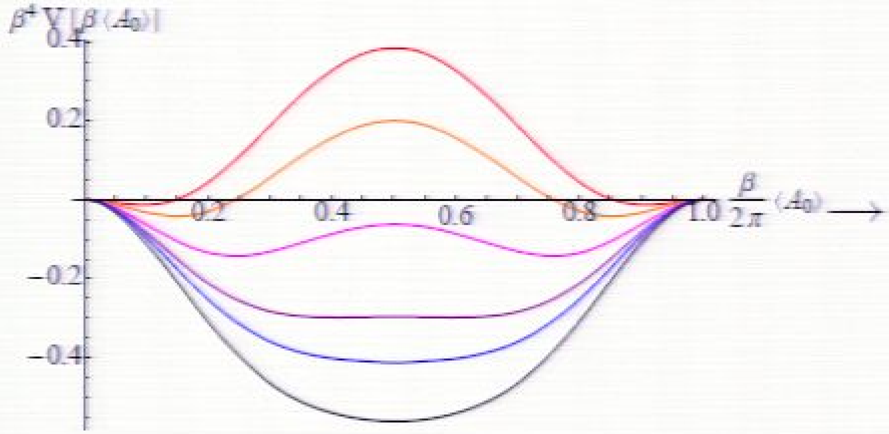


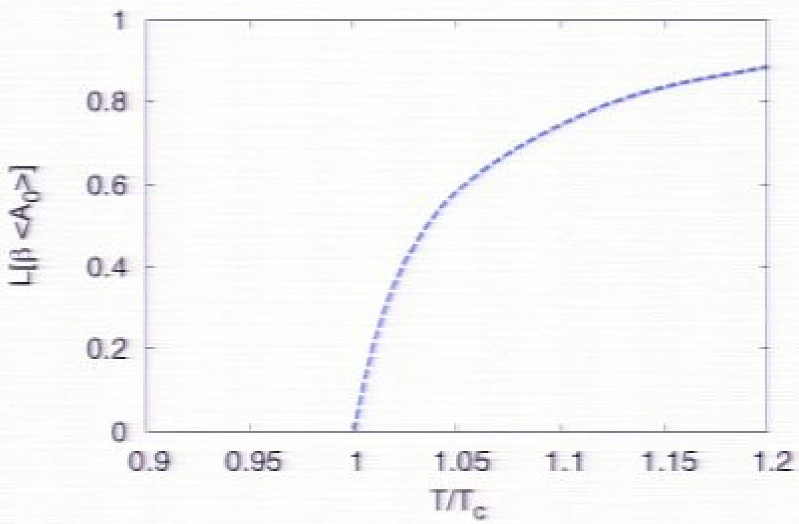


$T_c \approx 265$ MeV $\approx 0.605\sqrt{\sigma}$ (lattice: $T_c \approx 0.709\sqrt{\sigma}$)

order-parameter potential at finite T

SU(2):



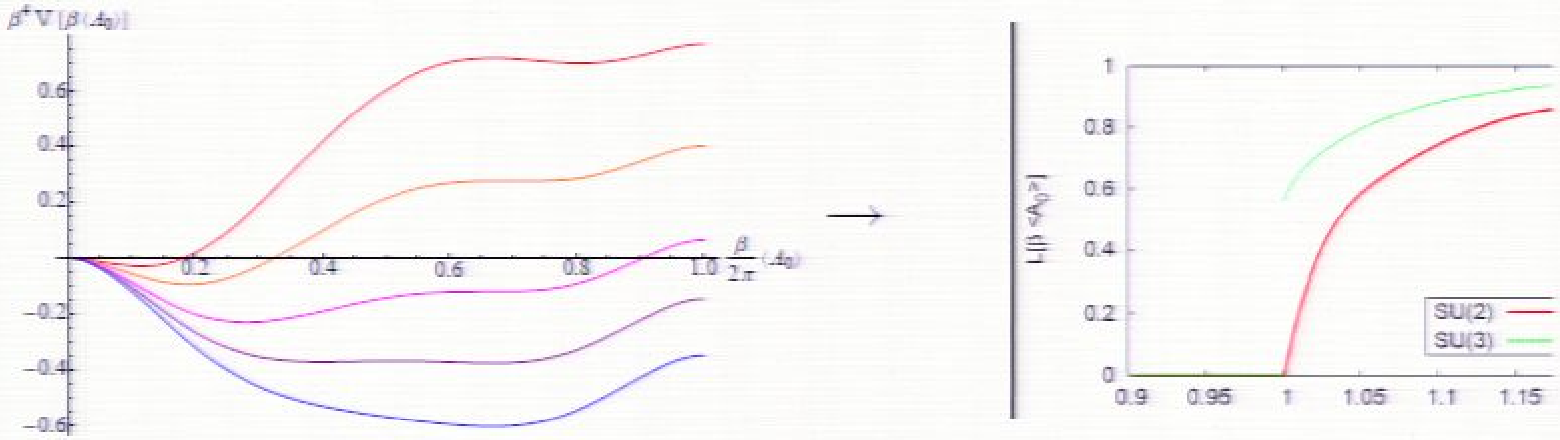


$T_c \approx 265$ MeV $\approx 0.605\sqrt{\sigma}$ (lattice: $T_c \approx 0.709\sqrt{\sigma}$)

$$\Omega V[\langle A_0\rangle] = \frac{1}{2}\mathrm{Tr}\ln\Gamma_A^{(2)}[\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma_c^{(2)}[\langle A_0\rangle] + \mathcal{O}(\partial_k\Gamma_k^{(2)}) + \mathcal{O}(V'')$$

order-parameter potential at finite T

SU(3):

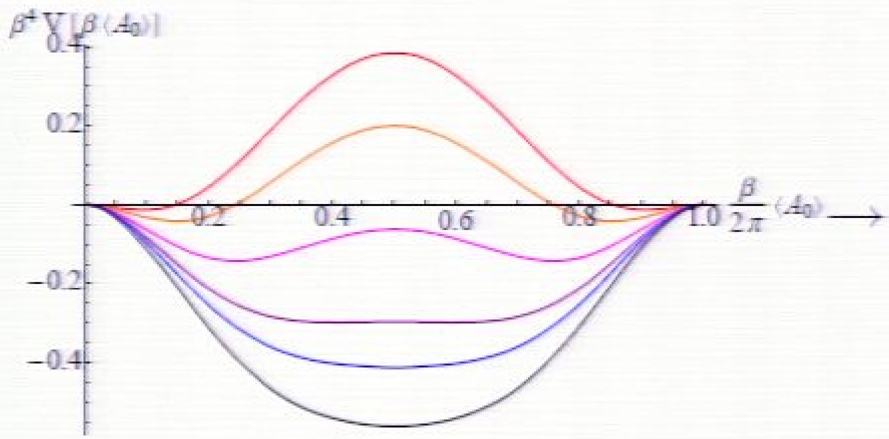


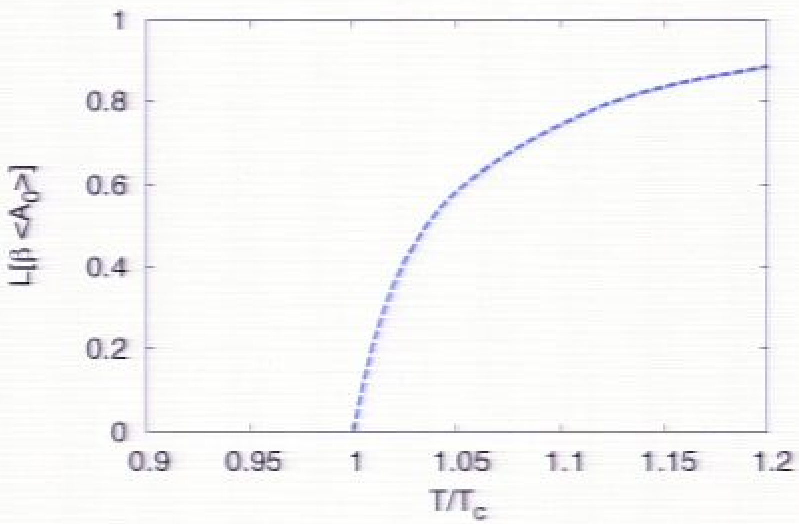
$T_c \approx 291$ MeV $\approx 0.66\sqrt{\sigma}$ (lattice: $T_c \approx 0.65\sqrt{\sigma}$)

[J. Braun, A.E., H. Gies, J.M. Pawlowski (2010)]

order-parameter potential at finite T

SU(2):



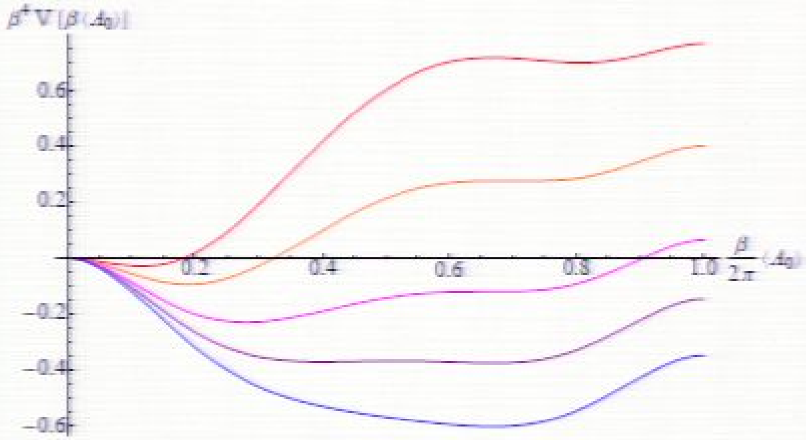


$T_c \approx 265$ MeV $\approx 0.605\sqrt{\sigma}$ (lattice: $T_c \approx 0.709\sqrt{\sigma}$)

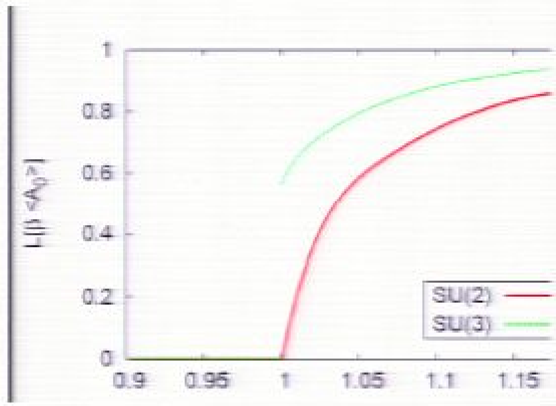
$$\Omega V[\langle A_0\rangle] = \frac{1}{2}\mathrm{Tr}\ln\Gamma_A^{(2)}[\langle A_0\rangle] - \mathrm{Tr}\ln\Gamma_c^{(2)}[\langle A_0\rangle] + \mathcal{O}(\partial_k\Gamma_k^{(2)}) + \mathcal{O}(V'')$$

order-parameter potential at finite T

SU(3):



$\longrightarrow$



$T_c \approx 291$ MeV $\approx 0.66\sqrt{\sigma}$ (lattice: $T_c \approx 0.65\sqrt{\sigma}$)

[J. Braun, A.E., H. Gies, J.M. Pawlowski (2010)]

What determines the order of the phase transition?

- Svetitsky-Yaffe ('82) conjecture: If the phase transition is of second order, the center determines the universality class: scalar field theory in one dimension lower (e.g. SU(2) in 4 d: Ising universality class)

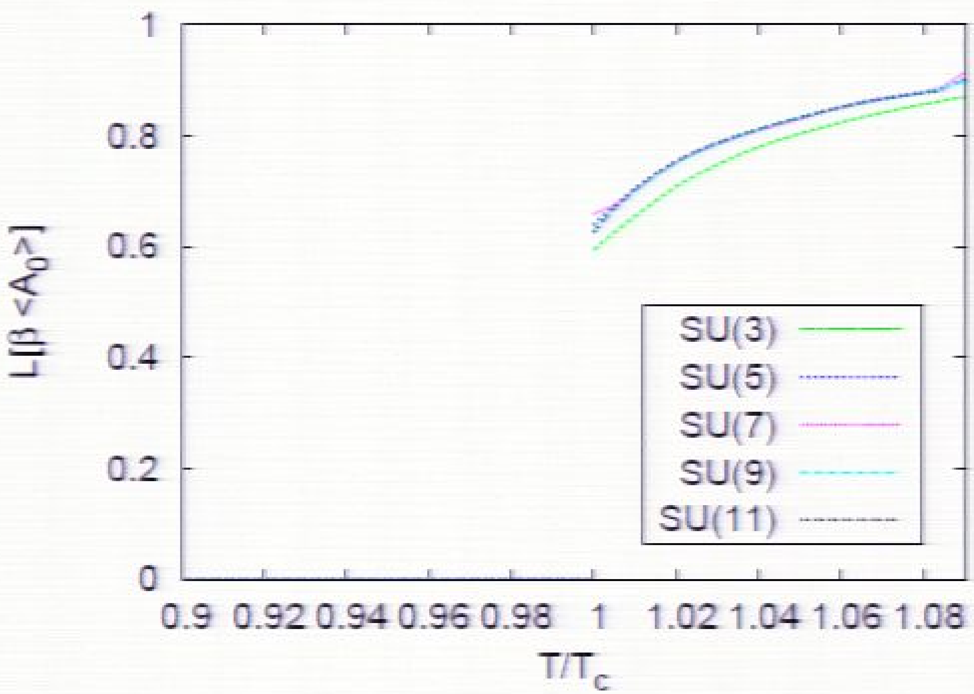
What determines the order of the phase transition?

- Svetitsky-Yaffe ('82) conjecture: If the phase transition is of second order, the center determines the universality class: scalar field theory in one dimension lower (e.g. SU(2) in 4 d: Ising universality class)
  findings from lattice gauge theory: SU(N) for $3 \leq N \leq 8$ is first order $\rightarrow$ gauge theories "do not make use" of available universality class!

What determines the order of the phase transition?

- Svetitsky-Yaffe ('82) conjecture: If the phase transition is of second order, the center determines the universality class: scalar field theory in one dimension lower (e.g. SU(2) in 4 d: Ising universality class)
  findings from lattice gauge theory: SU(N) for $3 \leq N \leq 8$ is first order $\rightarrow$ gauge theories "do not make use" of available universality class!
- mismatch in number of dynamical degrees of freedom: number of glueballs vs. number of gluons $\rightarrow$ expect first order for large gauge groups [Holland, Pepe, Wiese (2003)]

Results for large gauge groups: SU(N)

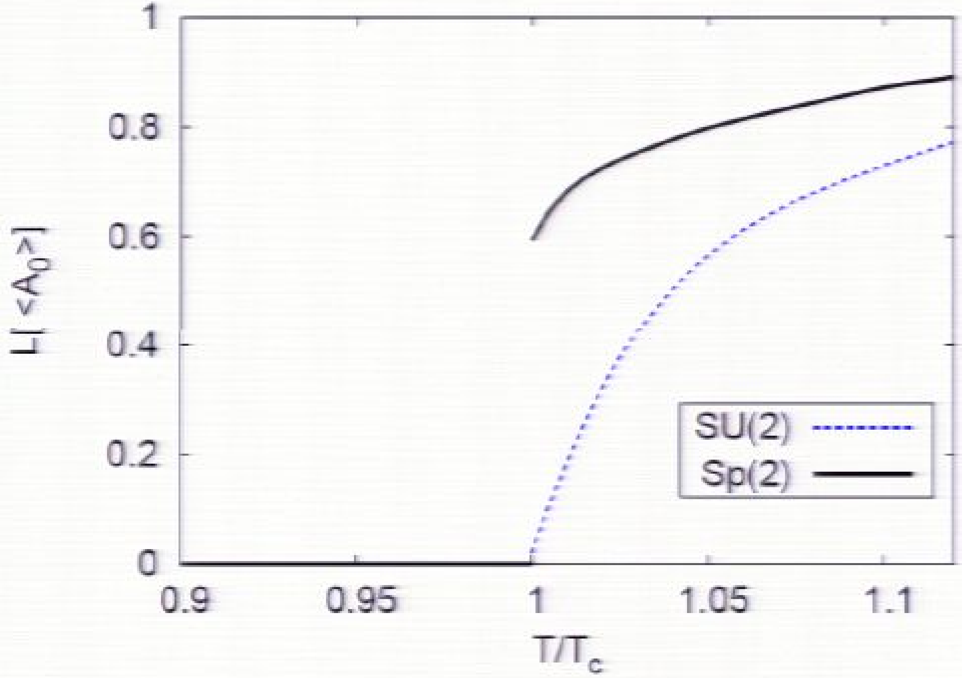


J. Braun, A.E. H.Gies,

J.M.Pawlowski, 2010

- first order transition for $N > 2$
- T_c and height of the jump independent of N for $N > 8$
- confirmation of lattice results (available up to $N = 8$)

Results for large gauge groups

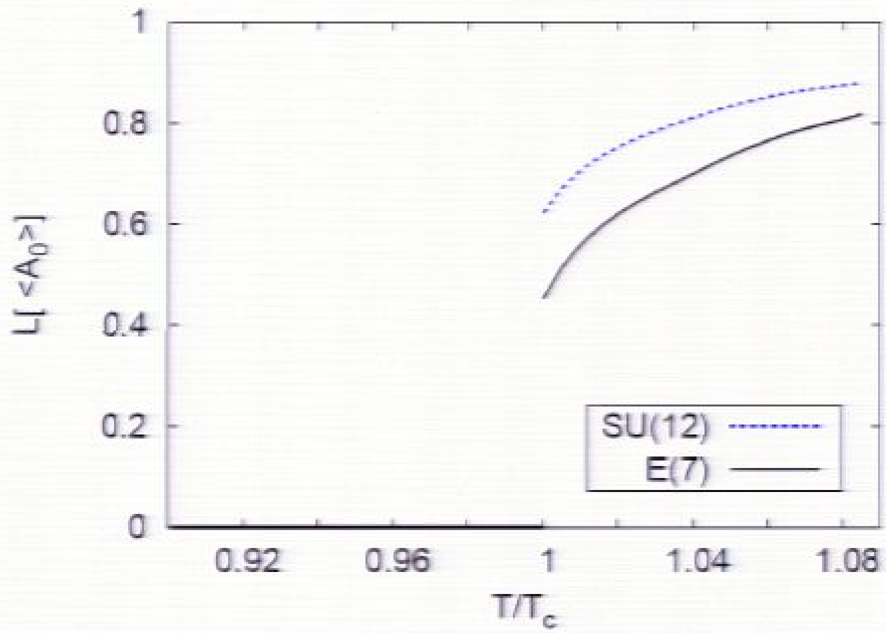


J. Braun, A.E., H.Gies,

J.M.Pawlowski, 2010

- symplectic group Sp(2)
- Sp(2) has same universality class available as SU(2) (Z_2)
- number of generators: 10 $\longrightarrow$ large mismatch
- in agreement with lattice results

Results for large gauge groups



J. Braun, A.E., H.Gies,

J.M.Pawlowski, 2010

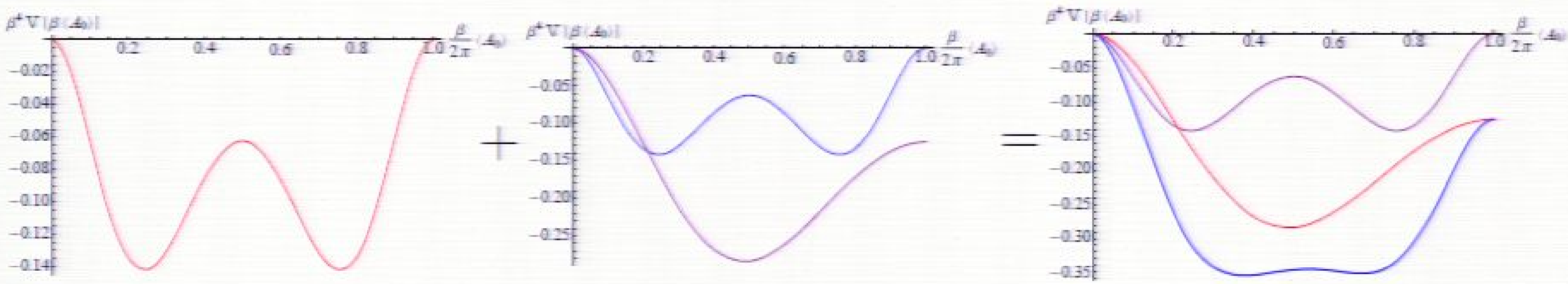
- E(7) has center Z_2, 133 generators
- E(7) has first order phase transition
- surprising: transition is weaker first order than expected

How can we understand this?

"Constructive/destructive interference":

$$V[\langle A_0\rangle] = \frac{1}{2}\sum_{l=1}^{d_{adj}} V_{SU(2)}[\nu_l\langle A_0\rangle]$$

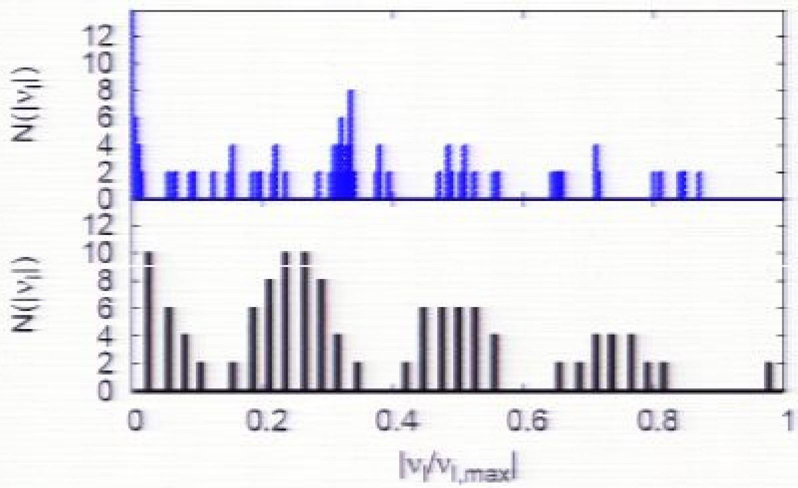
$$\nu_l = spec\{\frac{\left(T^a\langle A_0^a\rangle\right)^{bc}}{|\langle A_0\rangle|}\}$$



e.g. SU(3): $\nu_l : \{-1, 1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 0, 0\}$

Constructive/destructive interference

$$V[\langle A_0\rangle] = \frac{1}{2}\sum_{l=1}^{d_{adj}} V_{SU(2)}[\nu_l\langle A_0\rangle]$$



J. Braun, A.E., H.Gies, J.M.Pawlowski, 2010

constructive interference of SU(2) potentials
$\longrightarrow$ weak first order phase transition

destructive interference of SU(2) potentials:
$\longrightarrow$ strong first order phase transition

Confinement phase transition at finite T

- Landau gauge ghost and gluon propagators encode quark confinement
- order of phase transition determined by mismatch in the number of dynamical degrees of freedom ($\rightarrow$ first order in SU(N),N>3, Sp(2) and E(7) in $d = 4$)
- "constructive interference" of SU(2) potentials weakens the first order phase transition

Gluon condensate in Yang-Mills theory

Gluon condensate

perturbative vacuum unstable in Yang-Mills theory

expect non-trivial vacuum structure in Yang-Mills theory: $\langle F^2 \rangle \neq 0$

1-loop effective action: non-trivial minimum [Savvidy (1977)]

problems:

- covariantly constant colormagnetic background field configuration unstable
- minimum lies beyond the perturbative domain

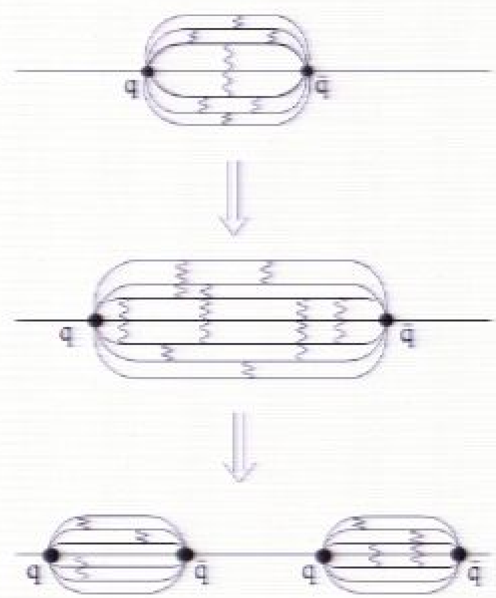
Gluon condensate and confinement

physically interesting:

leading-log model $W_{eff}(F^2) \sim F^2 \ln F^2$

$\rightarrow$ linearly growing quark potential

string-tension:
$\sqrt{\sigma} \sim \left(\frac{1}{3}F^2|_{\min}\right)$ [Adler, Piran (1981)]



Gluon condensate

evaluate non-perturbative potential from flow-equation

generalize Landau-gauge propagators to background:

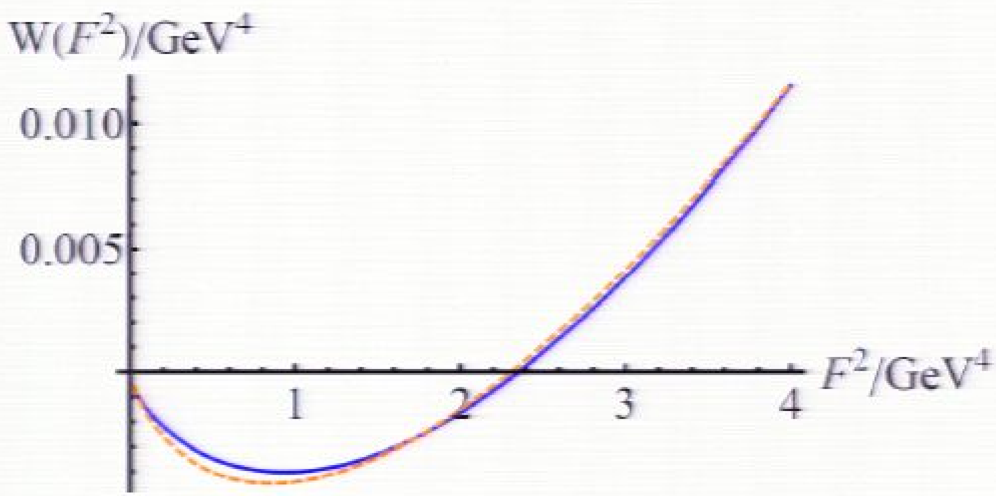
$\Gamma_k^{(2)}[A] = \Gamma_{k\,Landau}^{(2)}[\mathcal{D}] + F_{\mu\nu} f_{\mu\nu}(\mathcal{D})$

here: "minimal reconstruction": $\mathcal{D}_{T\,\mu\nu} = -D^2\delta_{\mu\nu} + 2igF_{\mu\nu}$

spectra of covariant Laplace-type operators on self-dual background[Leutwyler, 1980]

Gluon condensate from functional RG

initial condition at $k_{\mathrm{UV}} = 10$ GeV: $W_{\mathrm{UV}}[F^2] = \frac{1}{4g^2} F^2$



A.E. H.Gies, J.M.Pawlowski, 2010

- functional form qualitatively given by $F^2 \ln F^2$
- $\langle F^2 \rangle \approx 0.93\,\mathrm{GeV}^4$
- string-tension $\sigma^{\frac{1}{2}} \approx 747\mathrm{MeV}$

Conclusions

- functional Renormalization Group methods provide a useful tool to access non-perturbative regime of QCD/ Yang-Mills theories
- color confining gluon and ghost propagators give rise to confining Polyakov loop potential
- mismatch in the number of degrees of freedom determines order of the confinement phase transition
- gluon condensate in Yang-Mills vacuum gives linearly rising quark potential

Outlook

- calculate thermodynamic properties such as pressure and entropy density
- $d \neq 4$: find $N_{\mathrm{crit}}(d)$
- study center-free gauge groups (e.g. G(2))
- study temperature-dependent formation of gluon condensate

Thank you for your attention!