Title: A Lee-Wick Extension of the Standard Model

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Abstract: Higher derivative extensions of the Standard Model are renormalizable but without a quadratic divergent higgs mass. Electroweak presision data constraint the scale of the higher derivatives to at least a few TeV, but then these models have no flavor problem. We skim through these and other interesting results, most remarkably causality as an emergent characteristic at long distances. But we start by explaining the indefinite metric quantization procedure proposed by Lee and Wick which is necessary for unitary.

A Lee-Wick Extension of the Standard Model

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Waterloo, 10 December, 2010, 10F

Work mostly with Donal O'Connell and Mark Wise

Incomplete list of references

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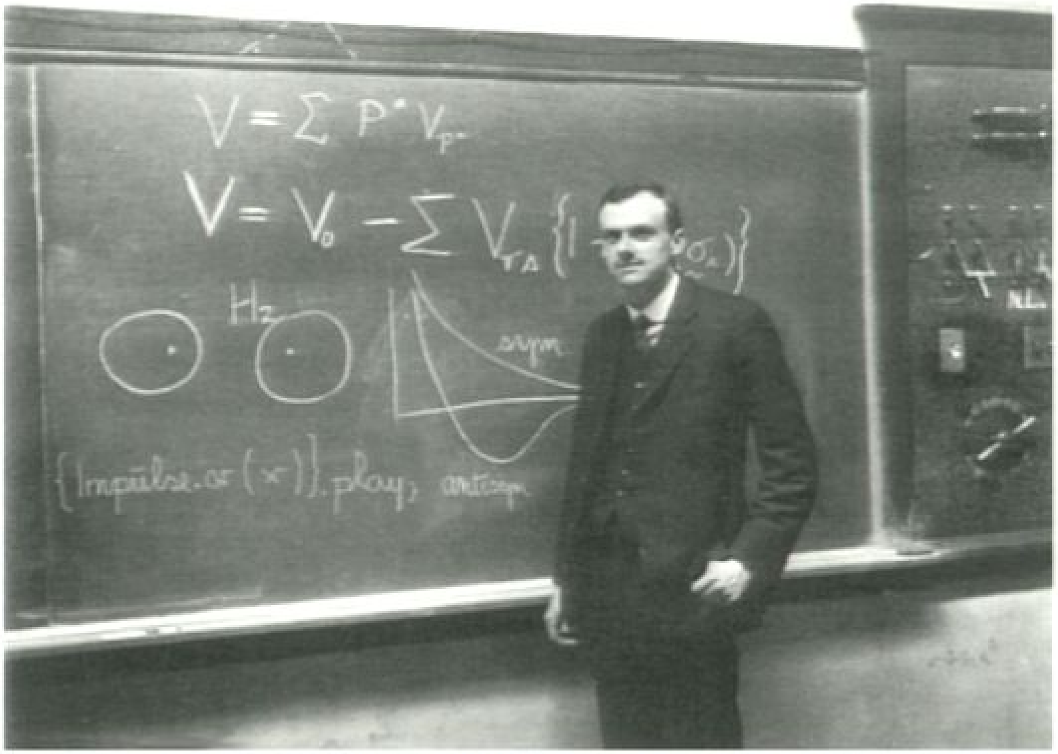
Quantum Mechanics with Indefinite Metric



Paul Dirac at a SuperCollider workshop in the early 1930s.

There is a sign error in the Maxwell equations.

Indefinite Metric Quantization

$$\langle i|j\rangle = \eta_{ij}$$

- Hamiltonian is self-adjoint but not hermitian

$$\bar{H} = H \qquad\qquad \bar{H} = \eta H^\dagger \eta$$

- H eigenvalues are either
  - real with non-zero norm

$$E_r^* = E_r \qquad \langle r|r\rangle \neq 0$$

  - complex, in c.c. pairs, with zero norm

$$E_\pm = E_R \pm iE_I \qquad \langle +|+\rangle = \langle -|-\rangle = 0 \qquad \langle +|-\rangle = 1$$

- H self-adjoint implies S-matrix is pseudo-unitary

$$S^\dagger \eta S = \eta$$

- LW condition: all eigenstates with real eigenvalues have positive norm
  - restriction of S-matrix to states with real eigenvalues gives a unitary S-matrix

$$S^\dagger S = 1 \qquad \langle r|r\rangle > 0$$

Don't be afraid of indefinite metric:

- Lorentz metric is indefinite
- Gauge fields have a negative metric component
  - Combined with the longitudinal mode give pairs of zero norm states
  - S-matrix is unitary because they are not allowed as external asymptotic states (and current conservation)
- Likewise in string theory (X^0 component has negative norm)

TD Lee
and
Giancarlo Wick

Basic idea: unitary S-matrix possible if negative metric states are unstable

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- Strategy (arranging for real eigenvalue states to have positive norm automatically):
  - In absence of interactions have "heavy" (n) negative metric states and "light" (p) positive metric states
  - Turn on interactions; a pp state is degenerate with an n state; n unstable
  - n and pp states mix; complex eigen-energy (c.c. pair), zero norm

$$|\pm\rangle = \frac{|pp\rangle \pm |n\rangle}{\sqrt{2}}$$

  - all negative metric states have disappeared

Consider an example

Three equivalent Lagrangians:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

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$$\phi = \hat{\phi} + \chi$$

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}M^2\chi^2 - V(\phi - \chi)$$

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- Indefinite metric problem explicit

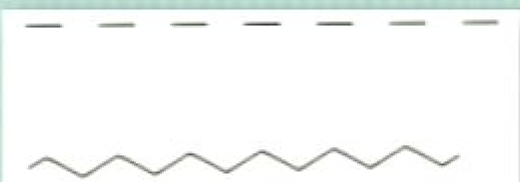
To explain basic ideas consider toy model for simplicity: $g\phi^3$

Recall, equivalent lagrangians

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}M^2\chi^2 - V(\phi - \chi)$$

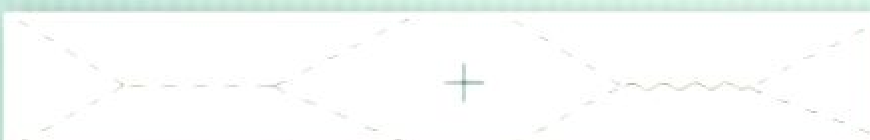
$$g\phi^3 \to g(\phi - \chi)^3$$



$$\frac{i}{p^2 - m^2}$$

$$\frac{-i}{p^2 - M^2}$$

Scattering:



$$= -ig^2\left(\frac{1}{p^2 - m^2} - \frac{1}{p^2 - M^2}\right)$$

$$\Rightarrow \quad \mathrm{Im}\,\mathcal{A}_{\mathrm{fwd}} = \pi g^2\left[\delta(p^2 - m^2) - \delta(p^2 - M^2)\right]$$

This is a disaster: optical theorem is violated

$$\mathrm{Im}\,\mathcal{A}_{\mathrm{fwd}} = \pi\sqrt{s(s - 4m^2)}\sigma_T > 0$$

Reorganize perturbation theory (old school, resonances, think W/Z):
(i) Replace all propagators by dressed propagators
(ii) Define amplitude by analytic continuation from positive and large $\mathrm{Im}(p^2)$

$iG^{(2)}$ $\quad$ $i\Delta$ $\quad$ $i\Delta \quad i\Pi \quad i\Delta$

$$\Rightarrow iG^{(2)} = \frac{i}{\Delta^{-1} + \Pi}$$

very familiar, but now use $\quad i\Delta = \dfrac{-i}{p^2 - M^2}\quad$ to get the surprising

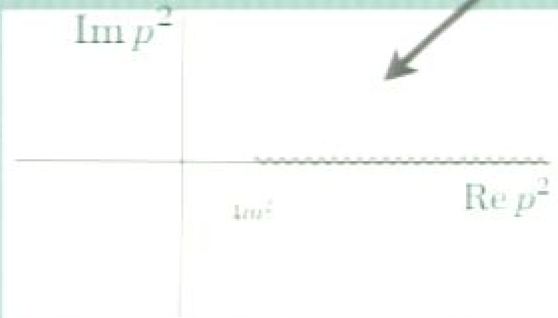
$$iG^{(2)} = \frac{-i}{p^2 - M^2 - \Pi}$$

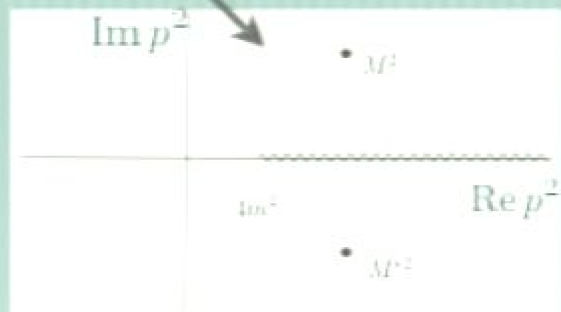
Compare this with normal case: $\quad iG^{(2)} = \dfrac{i}{p^2 - m^2 + \Pi}$

Π itself is very "normal," it is the same for normal and LW fields:

Pole in the scattering amplitude!

$$i\mathcal{A} = -ig^2\left[\frac{1}{p^2-m^2+\Pi} - \frac{1}{p^2-M^2-\Pi}\right]$$





so in fact, the LW propagator is
$$G^{(2)} = -\frac{A}{p^2-\hat{M}^2} - \frac{A^*}{p^2-\hat{M}^{*2}} + \int_{4m^2}^{\infty} d\mu^2\, \frac{\rho(\mu^2)}{p^2-\mu^2}$$

properties: $\rho(\mu^2) \geq 0$ $\quad -A - A^* + \int d\mu^2 \rho(\mu^2) = -1$

positive spectral density, just as in normal theories

Imaginary part of forward amplitude: complex dipole cancels out

$$\mathrm{Im}\,\mathcal{A}_{\mathrm{fwd}} = \pi g^2\left[\rho_{\mathrm{normal}}(\mu^2) + \rho_{\mathrm{LW}}(\mu^2)\right]$$

This is a positive discontinuity.

You can see it is precisely the total cross section (to the order we have carried this out)

Above calculation ok because single LW-resonance in intermediate state can never go "on-shell" when energies of incoming particles are real

Subtleties first encountered in 1-loop amplitude:
with real energy may still produce two LW-resonances with masses M and M^*

$$I = \int \frac{d^4p}{(2\pi)^4} \frac{-i}{(p+q)^2 - M_1^2} \frac{-i}{p^2 - M_2^2},$$

has poles at $p^0 = \pm\sqrt{\mathbf{p}^2 + M_2^2}$ and $p^0 = -q^0 \pm \sqrt{\mathbf{p}^2 + M_1^2}$

Lee & Wick:

Start from $g = 0$, masses real, take usual Feynman contour.
Turn on interaction. As M develops imaginary part deform contour to avoid crossing poles

CLOP:

Issue arises when contour is pinched, which can only happen(for real q^0) when $M_1{}^* = M_2$
Take M_1 and M_2 independent, $M_2 - M_1 = i\delta$
After integration complete take $\delta \to 0$

Clearly works at one loop. How about all orders?

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- We have solved the $\mathrm{O}(N)$ model in large N limit. The width of LW resonance is $O(1/N)$, so positivity of spectral function easily shown. Hence example exists for which

i) used LW-CLOP prescription

ii) S-unitary shown explicitly (directly checked optical theorem)

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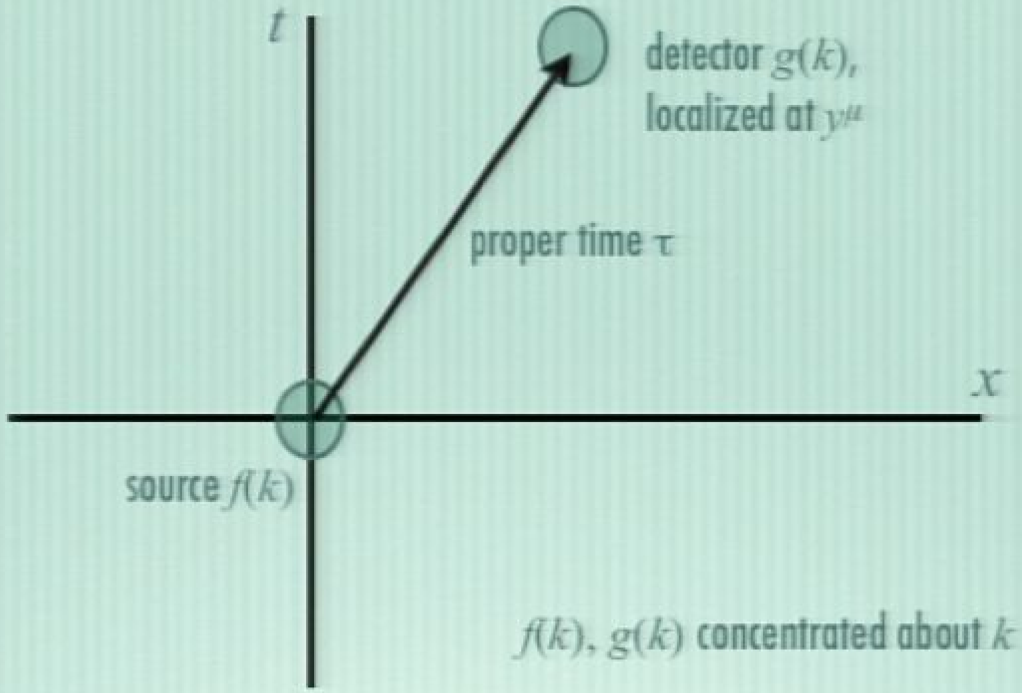
Indefinite metric quantization: Dirac, Pauli, ...

Peculiar effects: Non-locality?

Recall "response theory"



$f(k)$, $g(k)$ concentrated about $k = k_0$

stable particle

$$\langle \text{detector} | \text{source} \rangle \propto g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} \theta(y^0)$$

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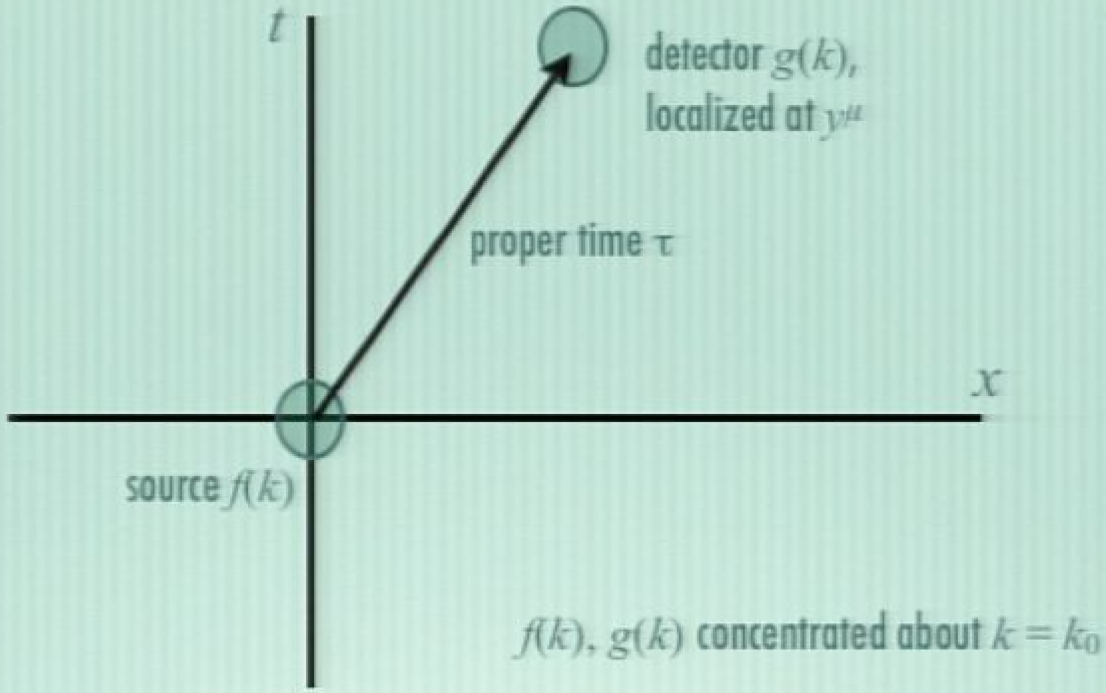

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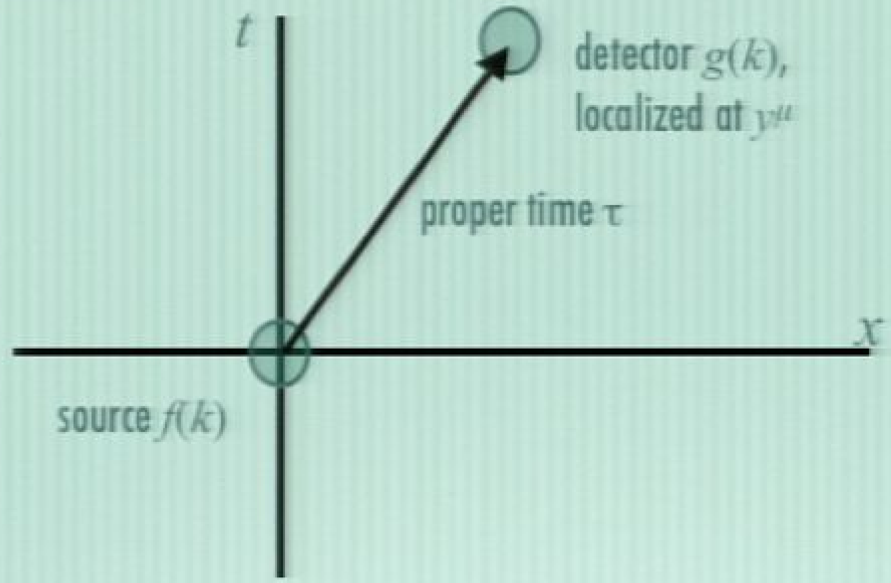
$$\langle \text{detector} | \text{source} \rangle \propto g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} \theta(y^0)$$

and for narrow resonance, production and decay, (pole in second sheet)

$$\langle \text{detector} | \text{source} \rangle \propto g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} e^{-\Gamma\tau/2} \theta(y^0)$$

“source” can be from collision of two normal (non-LW) particles
“detector” from decay into normal particles

Now for LW resonance



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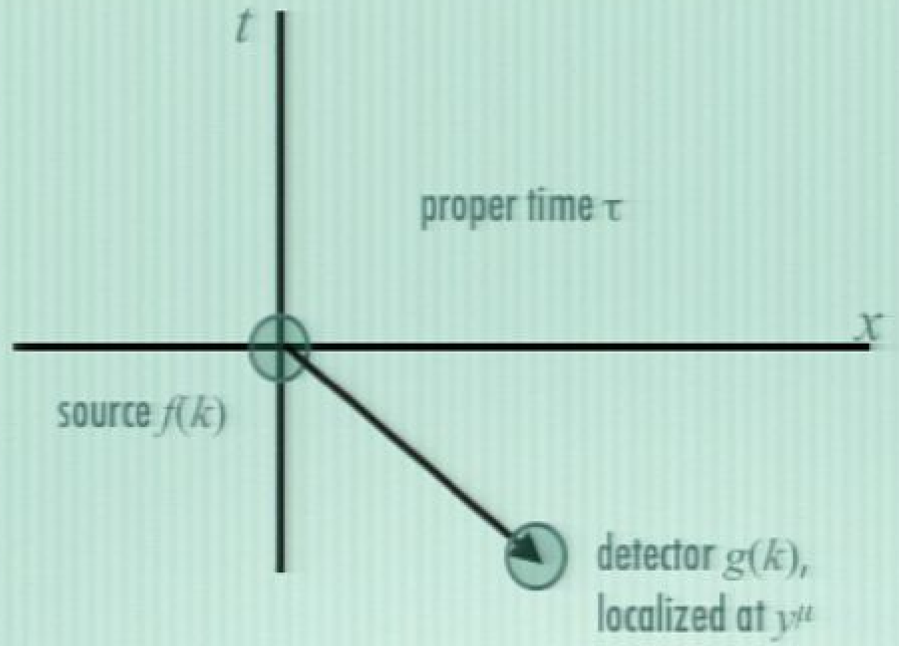
$$\langle \text{detector} | \text{source} \rangle \propto g^*(-my/\tau) f(-my/\tau) \frac{1}{\tau^{3/2}} e^{im\tau} e^{-\Gamma\tau/2} \theta(-y^0)$$

Now for LW resonance



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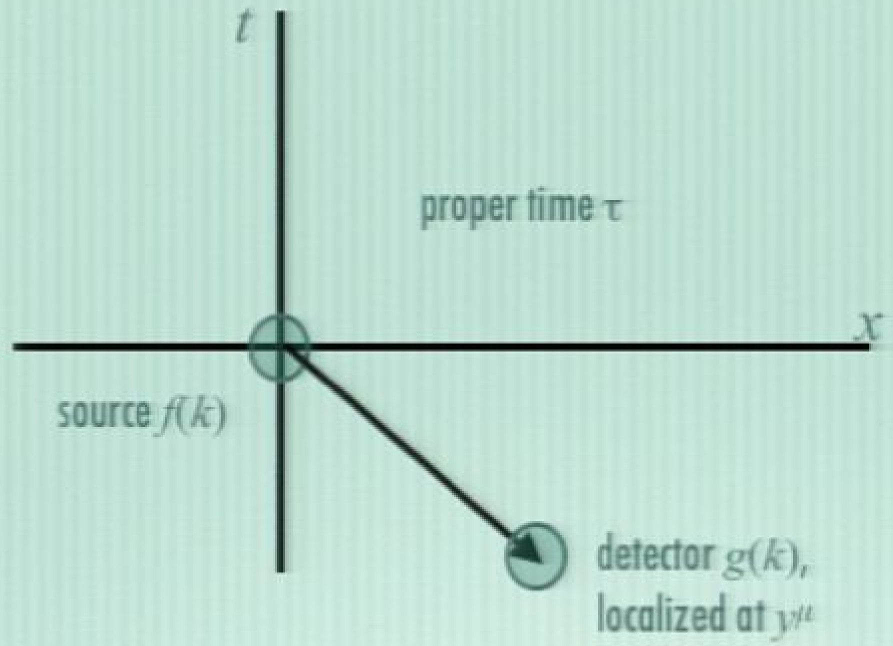

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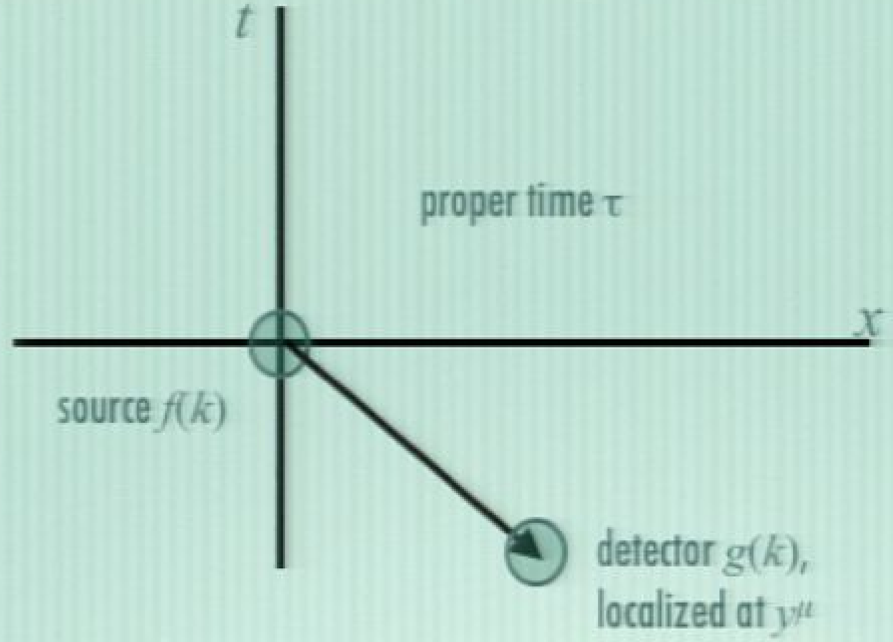

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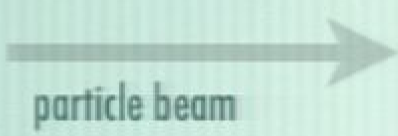

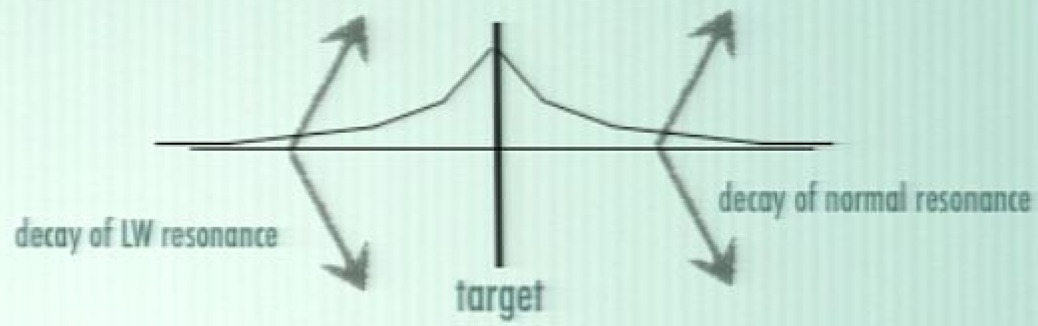

LW-SM: Introduction

- Lore:
  Symmetry+Field Content+Renormalizability+Unitarity = SM
- Higher Derivative (HD) terms:
  - can be made of same fields and preserve symmetries
  - renormalizability preserved
  - unitarity?? Lee-Wick says yes
- Should be explored

Outline

- Minimalistic presentation of six results:
  - No "big" fine-tuning problem
  - No flavor problem
  - EW precision OK, if mass of new resonances few TeV
  - Renormalization and GUTs
  - High energy vector-vector scattering: the special operators
  - LHC examples

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The LW SM (or HD SM)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm HD}$$

$$\mathcal{L}_{\rm HD} = \frac{1}{2M_1^2}\left(D^\mu F_{\mu\nu}\right)^a \left(D^\lambda F_\lambda{}^\nu\right)^a - \frac{1}{2M_2^2}\left(D_\mu D^\mu H\right)^\dagger \left(D_\nu D^\nu H\right) - \frac{1}{M_3^2}\bar{\psi}_L (i\not{D})^3 \psi_L$$

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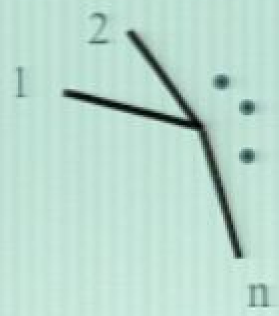
Gauge fixing can be as usual $\quad \mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial\cdot A)^2$

or can include HD's, eg,
(convenient for power counting) $\quad \mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial\cdot A)(1+\frac{\partial^2}{M_3^2})(\partial\cdot A)$

Naive degree of divergence, naively done (but correct!)

propagators $\qquad \sim \dfrac{i}{p^2 - p^4/M^2}$



vertices $\qquad \sim p^{6-n} \quad \text{(leading)}$

L = # of loops

V_n = # of vertices with n lines

I = # of internal propagators

E = # of external lines

naive degree of divergence:

$$D = 4L + \sum_n (6-n) V_n - 4I$$

topological identities

$$L = I - \sum_n V_n + 1 \qquad \sum_n n V_n = 2I + E$$

$$\Rightarrow \boxed{D = 6 - 2L - E}$$

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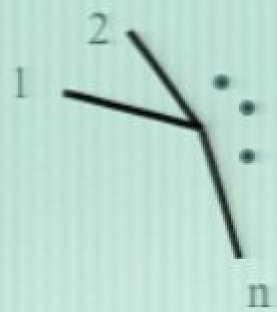

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possible divergences:

$D = \{ 4 - E \quad L = 1$ quadratic only for L=1, E=2

1. Quadratic divergences?

(i) Gauge fields: gauge invariance decreases divergence to $D = 0$

$$= i(p_\mu p_\nu - g_{\mu\nu} p^2)\Pi(p^2)$$

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(ii) Higgs field: quadratic divergence from vertex with 2/3 derivatives

$$(D^2 H)^\dagger (D^2 H) \qquad D^2 H = [\partial^2 + 2igA\cdot\partial + ig(\partial\cdot A)]H$$

Choose gauge $\partial \cdot A = 0$ and integrate by parts: there are at least two derivatives on external field

$$\Rightarrow \boxed{\delta m_H^2 \sim M^2 \ln \Lambda^2}$$

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Notes:

1. Physical mass is gauge independent. Quadratic divergences found in unphysical quantities
2. Result checked by explicit calculation (arbitrary ξ-gauge)

2. FCNC's

There is no need for artificially imposed restrictions
(ie, no need to impose MFV couplings for the HDs)
nor an additional huge superstructure to deal with this
(like in SUSY with gauge mediation).

This merits more study, only studied superficially so far.

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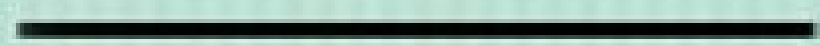
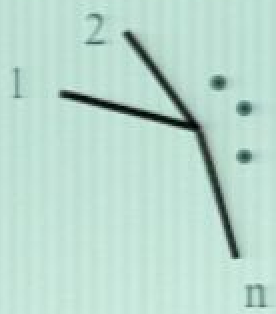
propagators  $\sim \dfrac{i}{p^2 - p^4/M^2}$

vertices 

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(ie, no need to impose MFV couplings for the HDs)
nor an additional huge superstructure to deal with this
(like in SUSY with gauge mediation).

This merits more study, only studied superficially so far.

Notation: SM Yukawas:

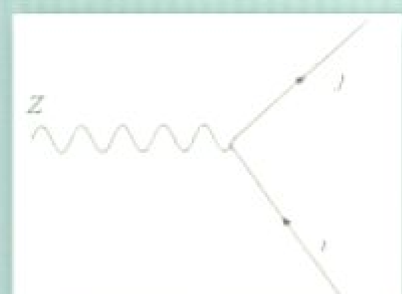
$$\mathcal{L}_{\rm SM} \supset \lambda_U H \bar{q}_L u_R + \lambda_D H^* \bar{q}_L d_R + \lambda_E H^* \bar{\ell}_L e_R$$

For low energy FCNCs treat HDs as small.
Use EOM on HD terms:

$$\frac{1}{M^2} r_{ij} \bar{q}_L^i (i\not{D})^3 q_L^j = \frac{1}{M^2} (\lambda_U^\dagger r \lambda_U)_{ij}\, \bar{u}_R^i H^* i\not{D} (H u_R^j)$$

completely arbitrary matrix (order(1))

:: There are off-diagonal tree level Z couplings, but suppressed



$$\sim \delta_{ij} + \Delta_{ij} \qquad \Delta_{ij} \sim \frac{m_i m_j r_{ij}}{M^2}$$

So, for example, with M = 1 TeV

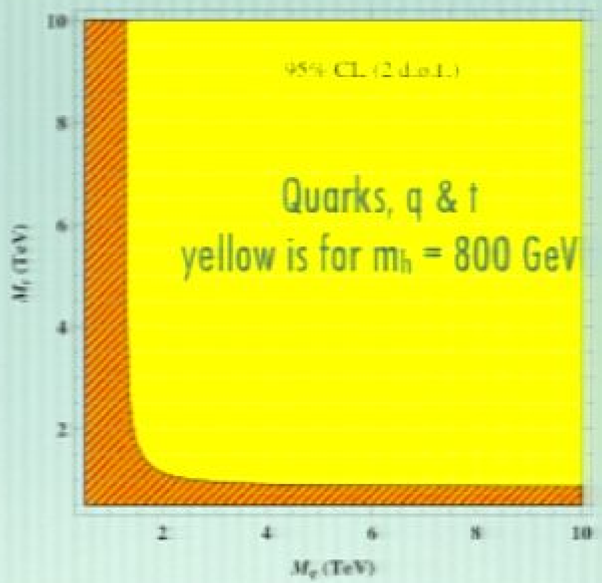
$$\Delta_{bs} \sim \frac{m_b m_s r_{bs}}{M^2} \sim 10^{-6}$$

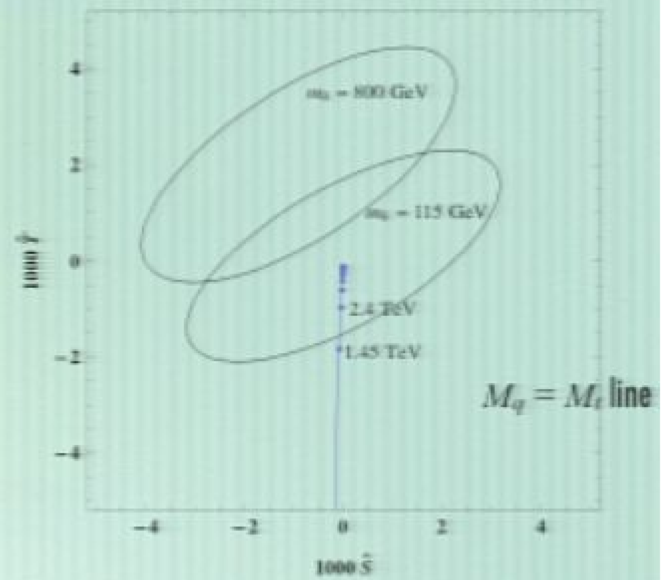
Even for LFV, this mass suppression is sufficient

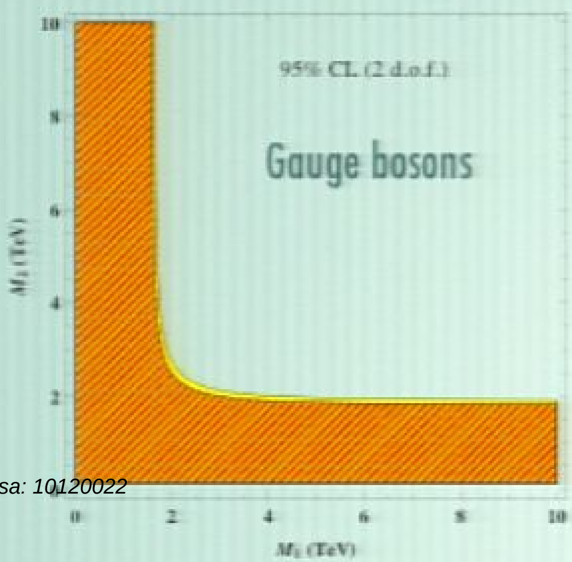
Note: In the absence of Yukawa couplings, $SU(3)_Q \times SU(3)_U \times SU(3)_D$

3. EW precision

Alvarez, Da Rold, Schat & Szynkman, JHEP 0804:026,2008
Underwood & Zwicky, Phys. Rev. D79:035016,2009
Carone & Lebed, Phys. Lett.B668: 221-225,2008
S. Chivukula et al, arXiv:1002.0343 (this reported below)







Bounds, quark or gauge bosons, largely decouple: enter into (S,T), or (W, Y)

Light higgs favored

4. YM-beta function

Background-Field Gauge

1-loop, normal
$$\beta = -\frac{g^3}{16\pi^2} C_2 \left(\frac{10}{3} + \frac{1}{3} \right)$$

1-loop, HD^2 theory
$$\beta = -\frac{g^3}{16\pi^2} C_2 \left(2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6} \right)$$

1/6 is easy to understand: doubling obvious only when longitudinal and transverse modes all have same power counting. Need HD GF. But then get determinant from exponentiation trick:

$$\sqrt{\det(1 + D^2/M^2)} \int [d\alpha] e^{\frac{i}{2\xi} \int d^4x\, \alpha \left(1 + \frac{D^2}{M^2}\right) \alpha} \delta(\partial \cdot A - \alpha)$$

This det is, for UV, same as usual ghosts in BFG. The sqrt gives an additional 1/2

1-loop, HD^3 theory
$$\beta = -\frac{g^3}{16\pi^2} C_2 \left(2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6} + 1 \right)$$

C. Carone, arXiv:0904.2359

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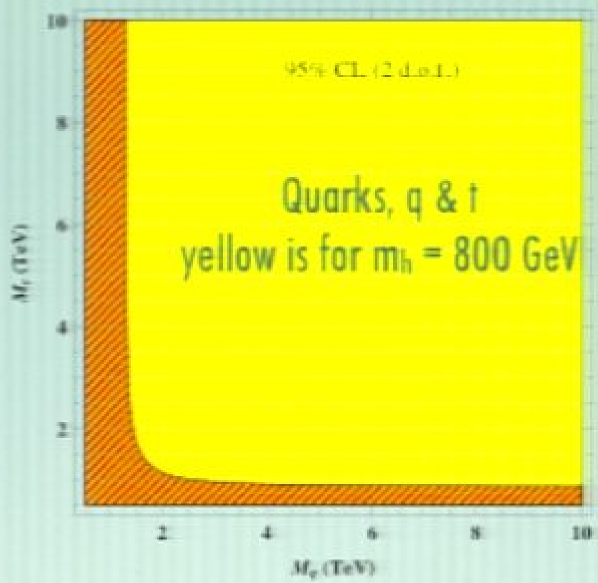

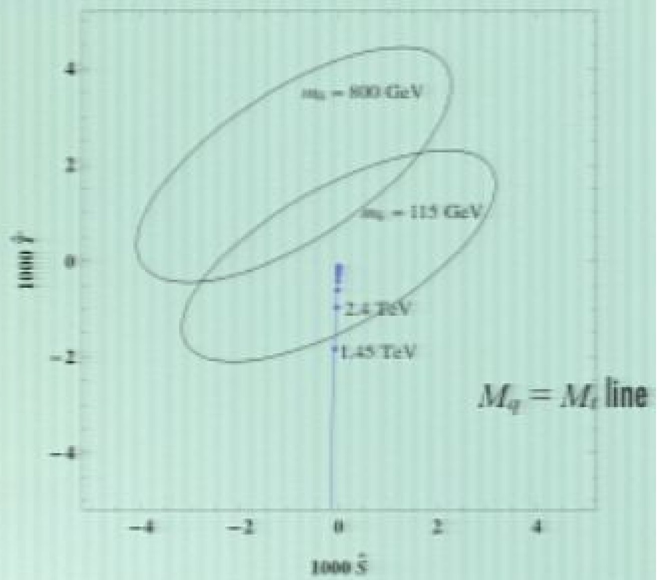

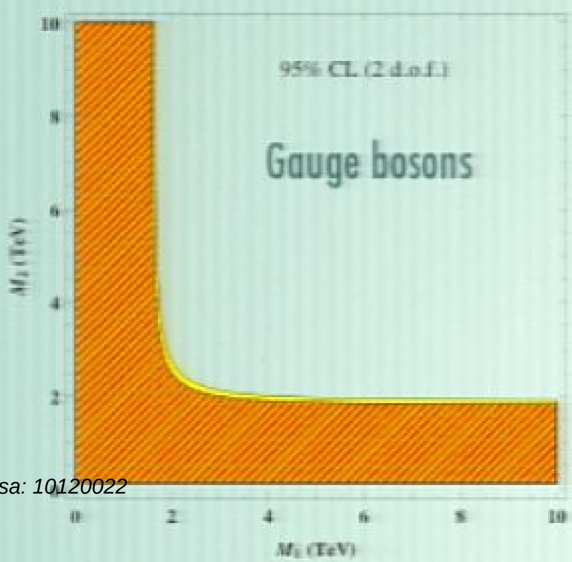

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More generally, in HD² $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\psi + \mathcal{L}_\phi,$

$$\mathcal{L}_A = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{m^2}\mathrm{Tr}(D^\mu F_{\mu\nu})^2 - \frac{i\gamma g}{m^2}\mathrm{Tr}(F^{\mu\nu}[F_{\mu\lambda}, F_\nu{}^\lambda])$$

$$\mathcal{L}_\psi = \bar{\psi}_L i\not{D}\psi_L + \frac{i}{m^2}\bar{\psi}_L\left[\sigma_1 \not{D}\not{D}\not{D} + \sigma_2 \not{D}D^2 + ig\sigma_3 F^{\mu\nu}\gamma_\nu D_\mu + ig\sigma_4(D_\mu F^{\mu\nu})\gamma_\nu\right]\psi_L$$

$$\mathcal{L}_\phi = -\phi^* D^2\phi - \frac{1}{m^2}\phi^*\left[\delta_1(D^2)^2 + ig\delta_2(D_\mu F^{\mu\nu})D_\nu + g^2\delta_3 F^{\mu\nu}F_{\mu\nu}\right]\phi$$

$$\beta(g) = -\frac{g^3}{16\pi^2}\left[\left(\frac{43}{6} - 18\gamma + \frac{9}{2}\gamma^2\right)C_2 - n_\psi\left(\frac{\sigma_1^2 - \sigma_2\sigma_3 + \frac{1}{2}\sigma_3^2}{(\sigma_1+\sigma_2)^2}\right) - n_\phi\left(\frac{\delta_1 + 6\delta_3}{3\delta_1}\right)\right]$$

$$\gamma_\psi(g) = -\frac{g^2}{16\pi^2}\frac{3}{4}C_1\left(\frac{2\sigma_1(2\sigma_2 + \sigma_3 - 2\sigma_4) + \sigma_2(2\sigma_2 + 2\sigma_3 - \sigma_4) - \sigma_3^2 - \sigma_4^2 + \sigma_3\sigma_4}{\sigma_1 + \sigma_2}\right)$$

$$\gamma_\phi(g) = -\frac{g^2}{16\pi^2}\frac{3}{8}C_1\left(\frac{8\delta_1^2 - \delta_2^2 - 4\delta_1\delta_2}{\delta_1}\right)$$

$$\mu\frac{\partial\gamma}{\partial\mu} = 0 \qquad \mu\frac{\partial(g^2\sigma_i)}{\partial\mu} = 2(g^2\sigma_i)\gamma_\psi(g) \qquad \text{and} \qquad \mu\frac{\partial(g^2\delta_i)}{\partial\mu} = 2(g^2\delta_i)\gamma_\phi(g).$$

This is for general HD terms, but not all have good high energy behavior (next section)

5. Massive V V-scattering: Special HD terms

Consider VV-scattering, first in non-HD case:

- if described by massive vector boson lagrangian, $\mathcal{A} \sim E^2 \quad E >> m$

unitarity violated (perturbatively)

- growth could be E^4, $\epsilon_L^\mu(p) = 1/M(p, 0, 0, E)$

but approximate GI at large E reduces growth by E^2, since $\epsilon_L^\mu(p) = p^\mu/M + (M/2E)n^\mu$

$(n^2 = 0)$

- HD:

+ Gauge Invariance (GI) is maintained, exact ward identities

+ Use LW-form (2-fields): amplitude has no inverse powers of M

$$\Rightarrow \quad \mathcal{A} \sim E^0$$

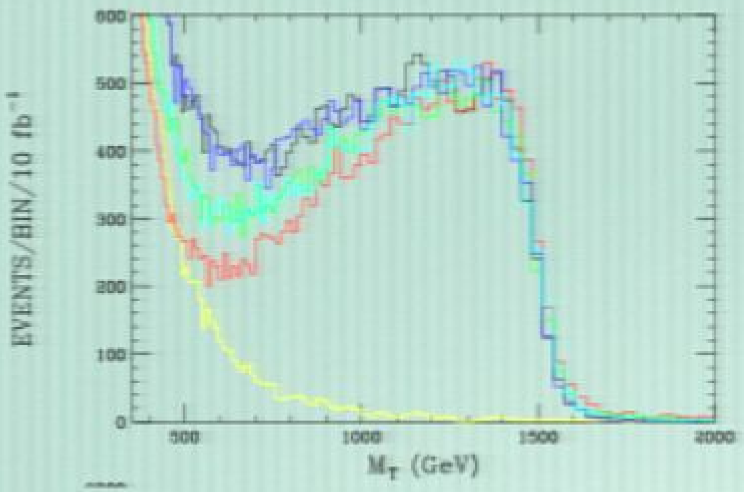
Unacceptable growth is controlled by GI and absence of $1/M$ terms in lagrangian.

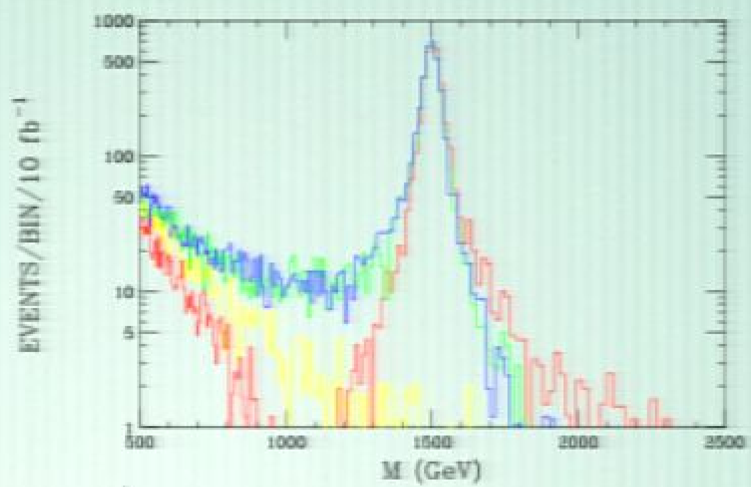
- HD with no LW-form, like F^3, does have E^2 growth at tree level (verified by explicit calculation)

6. LHC examples

T. Rizzo, JHEP 06:070(2007)

LW-Wboson
M=1.5TeV
ATLAS-like cuts
10 fb⁻¹ (14TeV)
(LW=black)





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7. Thermal HD theory

Fornal et al Phys.Lett.B674:330-335,200

Cannot do ideal gas: absent interactions yields theory with ghosts

Solution (maybe!!??): Ideal gas of normal particles, then resonances through 1st virial coeff.

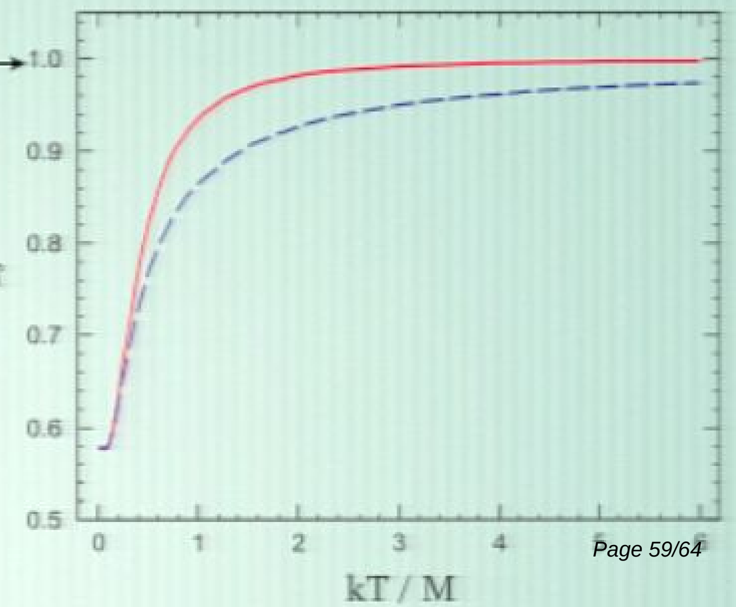
This can be obtained from S-matrix (Dashen, Ma & Bernstein, Phys. Rev. 187: 345, 1969)

At weak coupling massless scalar HD theory gives extra contribution to thermodynamic potential:

$$\Omega_{\rm LW} = -\frac{V}{\beta}\int \frac{\mathrm{d}^3P}{(2\pi)^3}\ln\left(1 - e^{-\beta\sqrt{M^2+\mathbf{P}^2}}\right)$$

Hence negative density and negative pressure

causal limit



High T expansions:

$$\rho_{\rm LW} = -\left[\frac{\pi^2(kT)^4}{30} - \frac{M^2(kT)^2}{24}\right] + \ldots$$

Cancel normal

Equal and positive e.o.s.: w = 1

$$p_{\rm LW} = -\left[\frac{\pi^2(kT)^4}{90} - \frac{M^2(kT)^2}{24} + \frac{M^3(kT)}{12\pi}\right] + \ldots$$

Fornal et al Phys.Lett.B674:330-335,200

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The End

- There exist unitary HD theories (at least large N to all orders g)
- HDSM Solves big fine tuning, flavor OK, EWP fine ($M > 3$ TeV)
- No straightforward GUT. Open questions on completion and gravity
- Acausal (non-local?) at short distances, but does not build macroscopic acausality (at least not in hot Bose gas)
- Other applications? Cosmology?

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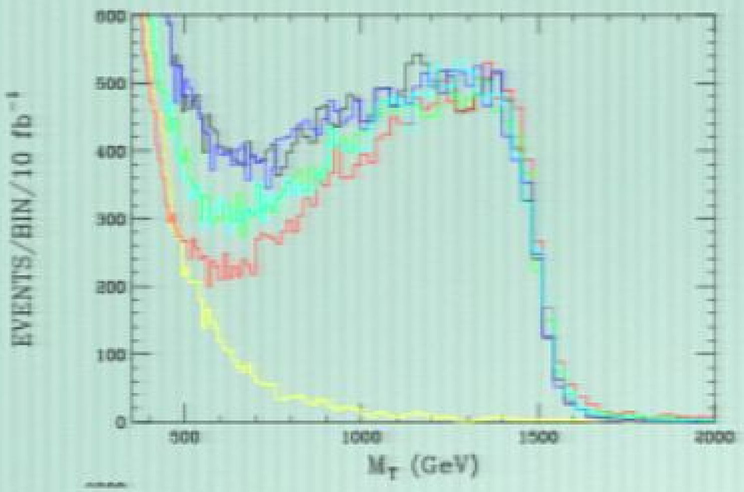

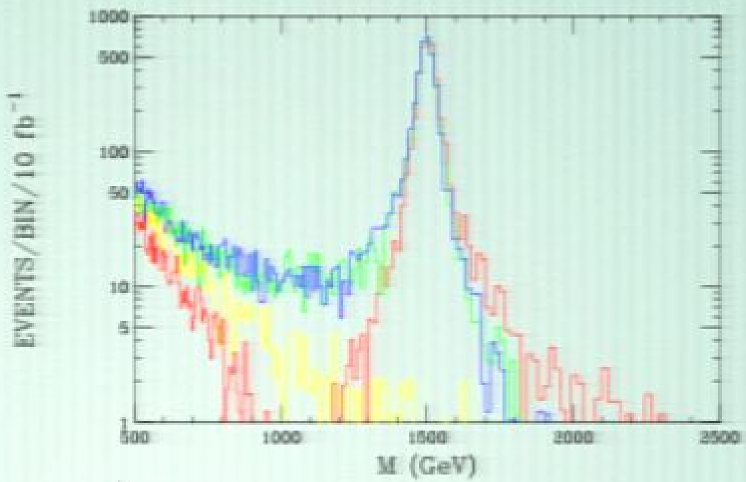

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