Title: From N=4 SYM to N=2 Superconformal QCD: Spin chains and Holography

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Abstract: The Z2 orbifold of N=4 SYM can be connected to N=2 superconformal QCD by a marginal deformation. The spin chains in this marginal family of theories have sufficient symmetry that allows for an all-loop determination of dispersion relation of BMN magnons. The exact two body S matrix is also fixed up to an overall phase. The exact dispersion relation of the magnon can be obtained from the matrix model of lowest modes on S^3, as well. I'll also talk briefly about some progress made towards the string dual of N=2 superconformal QCD, the endpoint of the deformation.

# $\mathcal{N} = 4$ SYM $\leftrightarrow$ $IIB$ on $AdS_5 \times S^5$

## How general is the gauge/string correspondence?

This is the most well understood gauge/gravity correspondence.
Other $4d$ instances can be engineered by considering D3 branes probing singularities.
Universality class of $\mathcal{N} = 4$, Common features:

- Adjoint and/or bifundamental matter
  Fundamental matter is added in probe approximation $N_f \ll N_c$
- Anomaly coefficients $a = c$ at large $N_c$
- Dual geometries are $10d$

# Bottom up approach

## Studying operator spectrum of gauge theory

- Operators of gauge theory $\Longrightarrow$ Spin chain

$$\mathrm{Tr}[\ldots XYY\ldots YXX\ldots YYX\ldots] \Longrightarrow \ldots\uparrow\downarrow\downarrow\ldots\downarrow\uparrow\uparrow\ldots\downarrow\downarrow\uparrow\ldots$$

- Anomalous dimension $\delta\Delta \Longrightarrow$ Hamiltonian on the spin chain
- For $\mathcal{N}=4$ SYM, spin chain is *integrable*, the spectrum can be obtained exactly using *Bethe ansatz*
[Minahan,Zarembo][Beisert,Staudacher][Gromov, Kazakov, Vieira]
- Asymptotic Bethe ansatz:

Energy of single spin chain excitation + *scattering matrix* of the excitations = the complete spectrum (for long spin chains)

- A dual description of the excitations $\Longleftrightarrow$ "Giant magnon"
[Hofman,Maldacena]

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Energy of single spin chain excitation + *scattering matrix* of the excitations = the complete spectrum (for long spin chains)

- A dual description of the excitations $\Longleftrightarrow$ "Giant magnon" [Hofman,Maldacena]

# Outline

1. Spin Chains
   - Review of $\mathcal{N}=4$ SYM : $SU(2|2)$ at work
   - $\mathbb{Z}_2$ orbifold & $\kappa$ deformation
   - $SU(2|2)$ at work, again
2. Dual/emergent description
   - Giant Magnon in $\mathcal{N}=4$
   - Emergent Magnon of $\kappa$ deformed theory
3. SCQCD spin chains and Dual
   - SCQCD spin chain
   - SCQCD dual

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# Fields of $\mathcal{N}=4$

- A gauge field $A_\mu$, four Weyl fermions $\lambda^A_\alpha$ and six scalars $X^{AB}$, where $A, B$ are $SU(4)_R$ indices
- Pick $U(1) \subset SU(4)$, $SU(4) \to SU(2_I)_R \times SU(2_{\hat{I}})_L \times U(1)_r$
- fields branch $\lambda^A \to \lambda^I, \lambda^{\hat{I}}$ and $X^{AB} \to \mathcal{X}^{I\hat{I}}, \Phi$

$$X_{AB} = \frac{1}{\sqrt{2}} \left( \begin{array}{cc|cc} 0 & \Phi & & \mathcal{X}^{I\hat{I}} \\ -\Phi & 0 & & \\ \hline & & 0 & \bar{\Phi} \\ -\mathcal{X}^{\hat{I}I} & & -\bar{\Phi} & 0 \end{array} \right)$$

- $SU(2_I)_R \times SU(2_{\hat{I}})_L = SO(4)$ rotations in $6, 7, 8, 9$
  $U(1)_r = SO(2)$ rotations in $4, 5$
- Full symmetry of $\mathcal{N}=4$ $PSU(2_\alpha, 2_{\dot{\alpha}}|4_A)$
- The operator $\mathrm{Tr}[\Phi^l]$ breaks
  $PSU(2_\alpha, 2_{\dot{\alpha}}|4_A) \to PSU(2_{\dot{\alpha}}|2_I) \times PSU(2_\alpha|2_{\hat{I}}) \times \mathbb{R}_{\Delta-r}$

# BMN magnons

## Excitations of BMN vacuum $\mathrm{Tr}[\Phi\ldots\Phi]$

- $\mathrm{Tr}[\Phi\ldots\Phi]$ is thought of as "ferromagnetic vacuum"
- The broken symmetries as *Goldstone* excitations

| | $SU(2_{\dot\alpha})$ | $SU(2_I)$ | $SU(2_\alpha)$ | $SU(2)_{\hat I}$ |
|---|---|---|---|---|
| $SU(2_{\dot\alpha})$ | $\mathcal{L}^{\dot\alpha}_{\dot\beta}$ | $\mathcal{Q}^{\dot\alpha}_{J}$ | $D^{\dagger\dot\alpha}_{\beta}$ | $\lambda^{\dagger\dot\alpha}_{\hat J}$ |
| $SU(2_I)$ | $\mathcal{S}^{I}_{\dot\beta}$ | $\mathcal{R}^{I}_{J}$ | $\lambda^{\dagger I}_{\beta}$ | $\mathcal{X}^{\dagger I}_{\hat J}$ |
| $SU(2_\alpha)$ | $D^{\alpha}_{\dot\beta}$ | $\lambda^{\alpha}_{J}$ | $\mathcal{L}^{\alpha}_{\beta}$ | $\mathcal{Q}^{\alpha}_{\hat J}$ |
| $SU(2_{\hat I})$ | $\lambda^{\hat I}_{\dot\beta}$ | $\mathcal{X}^{\hat I}_{J}$ | $\mathcal{S}^{\hat I}_{\beta}$ | $\mathcal{R}^{\hat I}_{\hat J}$ |

- Anomalous dimension $\rightarrow$ Hamiltonian
- Integrable spin chains: magnon dispersion relation & two body S matrix leads to the spectrum through *Bethe equations*

# BMN magnons

## Excitations of BMN vacuum $\mathrm{Tr}[\Phi\ldots\Phi]$

- Unique S matrix with $SU(2|2)$ symmetry [Beisert]
- The S matrix of magnons factorizes

$$S_{(\alpha\dot{\alpha}I\hat{I})\times(\gamma\dot{\gamma}K\hat{K})}^{(\beta\dot{\beta}J\hat{J})\times(\delta\dot{\delta}L\hat{L})}=S_{(\alpha\hat{I})\times(\gamma\hat{K})}^{(\beta\hat{J})\times(\delta\hat{L})}\otimes S_{(\dot{\alpha}I)\times(\dot{\gamma}K)}^{(\dot{\beta}J)\times(\dot{\delta}L)}.$$

- Dispersion relation is completely determined
- Focus on $SU(2_{\dot{\alpha}}|2_I)$ sector: $\mathcal{X}^I_{\hat{+}}\equiv\mathcal{X}^I$ and $\lambda^{\dot{\alpha}}_{\hat{+}}\equiv\lambda^{\dot{\alpha}}$

# $SU(2|2)$ at work [Beisert]

## Magnon dispersion relation

$$\{\mathcal{Q}^{\dot\alpha}_I, \mathcal{S}^J_{\dot\beta}\} = \delta^J_I \mathcal{L}^{\dot\alpha}_{\dot\beta} + \delta^{\dot\alpha}_{\dot\beta}\mathcal{R}^J_I + \delta^J_I \delta^{\dot\alpha}_{\dot\beta}\mathcal{C}, \qquad \{\mathcal{Q},\mathcal{Q}\} = \{\mathcal{S},\mathcal{S}\} = 0$$

- Particle $\mathcal{X}$ of momentum $p$ propagating on the vacuum.

$$\Psi(p) = \sum_{l=-\infty}^{\infty} e^{ipl}|\mathcal{X}(l)\rangle$$

For $p=0$, short multiplet with energy $\mathcal{C}=0$.

- For $p>0$, still short as there are not other d.o.f
- Central charges

$$\{\mathcal{Q},\mathcal{Q}\} = \mathcal{P}, \quad \{\mathcal{S},\mathcal{S}\} = \mathcal{K}.$$

# $SU(2|2)$ at work

## Magnon disersion relation

- Let $V$ be the representation of $\mathcal{X}^I, \lambda^{\dot{\alpha}}$
- $(\mathcal{Q}, \mathcal{S}) : V \to V$

$$\begin{aligned} \mathcal{Q}|\mathcal{X}\rangle &= a\,|\lambda\rangle \\ \mathcal{Q}|\lambda\rangle &= b\,|\mathcal{X}\Phi^+\rangle \\ \mathcal{S}|\mathcal{X}\rangle &= c\,|\lambda\Phi^-\rangle \\ \mathcal{S}|\lambda\rangle &= d\,|\mathcal{X}\rangle \end{aligned} \qquad \begin{aligned} \mathcal{P}|\mathcal{X}\rangle &= ab\,|\mathcal{X}\Phi^+\rangle \\ \mathcal{K}|\mathcal{X}\rangle &= cd\,|\mathcal{X}\Phi^-\rangle \\ \mathcal{C}|\mathcal{X}\rangle &= \frac{1}{2}(ad+bc)\,|\mathcal{X}\rangle \end{aligned}$$

- $\{\mathcal{Q}, \mathcal{Q}\}$ and $\{\mathcal{S}, \mathcal{S}\}$ from the Lagrangian

$$\begin{aligned} \mathcal{Q}\mathcal{X} &= \lambda \\ \mathcal{Q}\lambda &= \frac{\partial W}{\partial \mathcal{X}} = \frac{\mathbf{g}}{\sqrt{2}}[\mathcal{X}, \Phi] \end{aligned} \qquad \mathbf{g}^2 = \frac{g_{YM}^2 N}{8\pi^2} + O(g^4)$$

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$$\mathcal{P}|\mathcal{X}\rangle = \frac{\mathrm{g}}{\sqrt{2}}(e^{-ip}-1)|\mathcal{X}\Phi^+\rangle$$
$$\mathcal{K}|\mathcal{X}\rangle = \frac{\mathrm{g}}{\sqrt{2}}(e^{+ip}-1)|\mathcal{X}\Phi^-\rangle$$

### All loop dispersion relation

$$\Delta - |r| = 2\mathcal{C} = \sqrt{1+8\mathrm{g}^2\sin^2\frac{p}{2}}$$

- A priori, $\mathrm{g}^2 = \frac{g_{YM}^2 N}{8\pi^2} + O(g_{YM}^4)$, for $\mathcal{N}=4$ it turns out $O(g_{YM}^4)=0$.

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$$K = \frac{\check{g}}{g}$$

$$\{Q, Q\} \sum_{\ell} e^{ip\ell} X_\ell$$

$$\sum_{\ell} e^{ip\ell} [X_\ell, \phi]$$

$$e^{ip} - 1$$

# $SU(2|2)$ at work

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# $SU(2|2)$ at work [Beisert]

## Magnon S matrix

- Central charges with hermiticity determine $a, b, c, d$ hence the complete representation in terms of momentum $p$.
- Two body wave function

$$\psi(p_1, p_2) = \sum e^{ip_1x_1+ip_2x_2}|\ldots\mathcal{X}^I(x_1)\ldots\mathcal{X}^J(x_2)\rangle + S_{sc}{}^{IJ}{}_{KL}(p_2, p_1)e^{ip_1x_2+ip_2x_1}|\ldots\mathcal{X}^K(x_1)\ldots\mathcal{X}^L(x_2)\rangle$$

- $S_{sc}(p_1, p_2)$ is the S matrix, $S_{sc} : V\otimes V \to V \otimes V$
- $\mathcal{Q}(V\otimes V) = \mathcal{Q}V\otimes V + (-1)^F V\otimes \mathcal{Q}V$, similarly for $\mathcal{S}$
- $[S_{sc}, \mathcal{Q}] = 0$, $[S_{sc}, \mathcal{S}] = 0$ determine S matrix completely
- Rapidity variables $x^+, x^-$ , with $\frac{x^+}{x^-} = e^{ip}$

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# Orbifold projection

- Orbifold by $\mathbb{Z}_2 \subset SU(2)_L$, $\mathcal{X}^I_{\hat{I}} \to \pm \mathcal{X}^I_{\hat{I}}$
- For $SU(2N_c)$ $\mathcal{N}=4$, pick $\mathbb{Z}_2 = \{\mathbb{I}_{2N_c\times 2N_c}, \gamma\}$

$$\gamma = \begin{pmatrix} \mathbb{I}_{N_c\times N_c} & 0 \\ & -\mathbb{I}_{N_{\check{c}}\times N_{\check{c}}} \end{pmatrix}$$

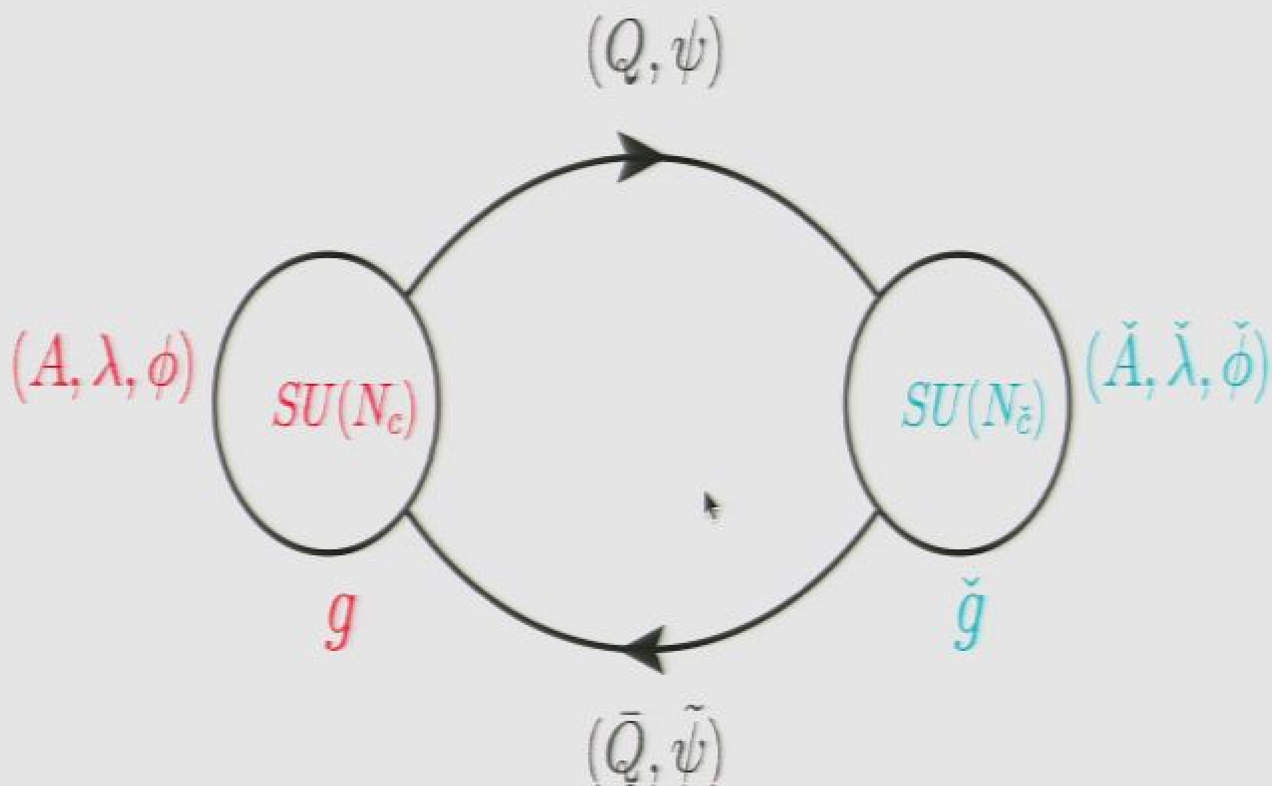
- $\mathcal{N}=4$ fields surviving this projection are

$$A_\mu = \begin{pmatrix} A_\mu & 0 \\ 0 & \check{A}_\mu \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi & 0 \\ 0 & \check{\phi} \end{pmatrix}, \quad \lambda^I = \begin{pmatrix} \lambda^I & 0 \\ 0 & \check{\lambda}^I \end{pmatrix},$$

$$\mathcal{X}^I_{\hat{I}} = \begin{pmatrix} 0 & (Q^I_{\hat{I}})^a_{\check{a}} \\ (\bar{Q}^I_{\hat{I}})^{\check{a}}_a & 0 \end{pmatrix}, \quad \lambda^{\hat{I}} = \begin{pmatrix} 0 & (\psi^{\hat{I}})^a_{\check{a}} \\ (\tilde{\psi}^{\hat{I}})^{\check{a}}_a & 0 \end{pmatrix}.$$

# Orbifold projection

- $\mathcal{N}=2$ quiver gauge theory



# Orbifold spin chain

- Orbifolding breaks $SU(2_\alpha|2_{\hat{I}}) \to SU(2_\alpha)\times SU(2_{\hat{I}})$ , only globally for $\check{g}=g$ but locally for $\check{g}\neq g$.
- $SU(2_{\dot{\alpha}}|2_I)$ is intact!
- Two degenerate vacua $\mathrm{Tr}[\phi\ldots\phi]$ and $\mathrm{Tr}[\check{\phi}\ldots\check{\phi}]$
- In infinite chain, four sectors with different boundary conditions

$$\ldots\phi\phi\phi(\lambda^I_\alpha, D^{\dot{\alpha}}_\alpha)\phi\phi\phi\ldots \qquad \ldots\phi\phi\phi(Q^I_{\hat{I}}, \psi^{\dot{\alpha}}_{\hat{I}})\check{\phi}\check{\phi}\check{\phi}\ldots$$

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- Focus on $(Q^I_{\hat{\dagger}}, \psi^{\dot{\alpha}}_{\hat{\dagger}}) \equiv (Q^I, \psi^{\dot{\alpha}})$ and $(\bar{Q}^I_{\hat{\dagger}}, \tilde{\psi}^{\dot{\alpha}}_{\hat{\dagger}}) \equiv (\bar{Q}^I, \tilde{\psi}^{\dot{\alpha}})$ as the remaining mimic $\mathcal{N}=4$

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# $\kappa$ deformed spin chain

- Two representations for two types of "species"

$$
\begin{aligned}
\mathcal{Q}|Q\rangle &= a|\psi\rangle & \mathcal{Q}|\bar{Q}\rangle &= \tilde{a}|\tilde{\psi}\rangle \\
\mathcal{Q}|\psi\rangle &= b|Q\check{\phi}^{+}\rangle & \mathcal{Q}|\tilde{\psi}\rangle &= \tilde{b}|\bar{Q}\phi^{+}\rangle \\
\mathcal{S}|Q\rangle &= c|\psi\check{\phi}^{-}\rangle & \mathcal{S}|\bar{Q}\rangle &= \tilde{c}|\tilde{\psi}\phi^{-}\rangle \\
\mathcal{S}|\psi\rangle &= d|Q\rangle & \mathcal{S}|\tilde{\psi}\rangle &= \tilde{d}|\bar{Q}\rangle.
\end{aligned}
$$

- $\mathcal{P}|(Q,\psi)\rangle = \frac{g}{\sqrt{2}}(e^{-ip}\sqrt{\kappa} - \frac{1}{\sqrt{\kappa}})|(Q,\psi)\check{\phi}^{+}\rangle$
- $\mathcal{P}|(\bar{Q},\tilde{\psi})\rangle = \frac{g}{\sqrt{2}}(e^{-ip}\frac{1}{\sqrt{\kappa}} - \sqrt{\kappa})|(\bar{Q},\tilde{\psi})\phi^{+}\rangle$
- $\mathcal{K} = \mathcal{P}^{\dagger}$

# Dispersion relation [AG, Rastelli, to appear]

## in $\kappa$ deformed spin chain

- As before supersymmetry transformations determine the dispersion relation
- It is same for $(Q,\psi)$ and $(\bar{Q},\tilde{\psi})$ multiplet

**All loop dispersion relation!**

$$\Delta - |r| = \sqrt{1 + 8\mathrm{g}^2\left(\sin^2\frac{p}{2} + \frac{1}{4}(\sqrt{\kappa} - \frac{1}{\sqrt{\kappa}})^2\right)} \qquad \kappa = \frac{\check{g}}{g}$$

$$\mathrm{g}^2 = \frac{g_{YM}\check{g}_{YM}N}{8\pi^2} + O(g_{YM}^4)$$

- Matches with the one loop dispersion relation [AG, Elli Pomoni, Rastelli]

# Magnon S matrix [AG, Rastelli, to appear]

- The two representations are intertwined through scattering

$$
\begin{aligned}
S|Q_1^I \bar{Q}_2^J\rangle &= A|Q_2^{\{I}\bar{Q}_1^{J\}}\rangle + B|Q_2^{[I}\bar{Q}_1^{J]}\rangle + \frac{1}{2}C\epsilon^{IJ}\epsilon_{\dot{\alpha}\dot{\beta}}|\psi_2^{\dot{\alpha}}\tilde{\psi}_1^{\dot{\beta}}\phi^-\rangle \\
S|\psi_1^{\dot{\alpha}}\tilde{\psi}_2^{\dot{\beta}}\rangle &= D|\psi_2^{\{\dot{\alpha}}\tilde{\psi}_1^{\dot{\beta}\}}\rangle + E|\psi_2^{[\dot{\alpha}}\tilde{\psi}_1^{\dot{\beta}]}\rangle + \frac{1}{2}F\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon_{IJ}|Q_2^I\bar{Q}_1^J\phi^+\rangle \\
S|Q_1^I\tilde{\psi}_2^{\dot{\beta}}\rangle &= G|\psi_2^{\dot{\beta}}\bar{Q}_1^I\rangle + H|Q_2^I\tilde{\psi}_1^{\dot{\beta}}\rangle \\
S|\psi_1^{\dot{\alpha}}\bar{Q}_2^J\rangle &= K|\psi_2^{\dot{\alpha}}\bar{Q}_1^J\rangle + L|Q_2^J\tilde{\psi}_1^{\dot{\alpha}}\rangle .
\end{aligned}
$$

- Nontrivial $\kappa$ dependence, unlike $(\lambda, D)$ and $(\check{\lambda}, \check{D})$ which scatter among themselves separately

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# Magnon S matrix

- Complicated all loop expressions, in terms of $e^{ip} = \frac{x^+}{x^-} = \frac{\tilde{x}^+}{\tilde{x}^-}$, $\kappa$ deformed rapidities
- At one loop:

$$
\begin{aligned}
A &= E = -\frac{1+e^{ip_1+ip_2}-2\kappa e^{ip_2}}{1+e^{ip_1+ip_2}-2\kappa e^{ip_1}}, \\
B &= D = -1, \\
C &= F = 0, \\
G &= L = -\kappa\frac{e^{ip_1}-e^{ip_2}}{1+e^{ip_1+ip_2}-2\kappa e^{ip_1}}, \\
H &= K = -\frac{1+e^{ip_1+ip_2}-\kappa(e^{ip_1}+e^{ip_2})}{1+e^{ip_1+ip_2}-2\kappa e^{ip_1}}.
\end{aligned}
$$

- This agrees with S matrix obtained from one loop hamiltonian [Liendo, Pomoni, Rastelli, to appear]

# One Loop Hamiltonian
## in the scalar sector [AG, Pomoni, Rastelli]

$$H_{k,k+1} =$$

$$\begin{array}{c|cccccccc}
 & \phi\phi & Q\bar{Q} & \check{\phi}\check{\phi} & \bar{Q}Q & \phi Q & Q\check{\phi} & \check{\phi}\bar{Q} & \bar{Q}\phi \\
\hline
\phi\phi & (2+\mathbb{K}-2\mathbb{P}) & \mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
Q\bar{Q} & \mathbb{K} & (2-\mathbb{K})\hat{\mathbb{K}}+2\kappa^2\mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
\check{\phi}\check{\phi} & 0 & 0 & \kappa^2(2+\mathbb{K}-2\mathbb{P}) & \kappa^2\mathbb{K} & 0 & 0 & 0 & 0 \\
\bar{Q}Q & 0 & 0 & \kappa^2\mathbb{K} & \kappa^2(2-\mathbb{K})\hat{\mathbb{K}}+2\mathbb{K} & 0 & 0 & 0 & 0 \\
\phi Q & 0 & 0 & 0 & 0 & 2 & -2\kappa & 0 & 0 \\
Q\check{\phi} & 0 & 0 & 0 & 0 & -2\kappa & 2\kappa^2 & 0 & 0 \\
\check{\phi}\bar{Q} & 0 & 0 & 0 & 0 & 0 & 0 & 2\kappa^2 & -2\kappa \\
\bar{Q}\phi & 0 & 0 & 0 & 0 & 0 & 0 & -2\kappa & 2
\end{array}$$

- The dispersion relation on the magnons, S matrix was calculated
- This spin chain was shown not to be integrable

# Outline

1. Spin Chains
   - Review of $\mathcal{N}=4$ SYM : $SU(2|2)$ at work
   - $\mathbb{Z}_2$ orbifold & $\kappa$ deformation
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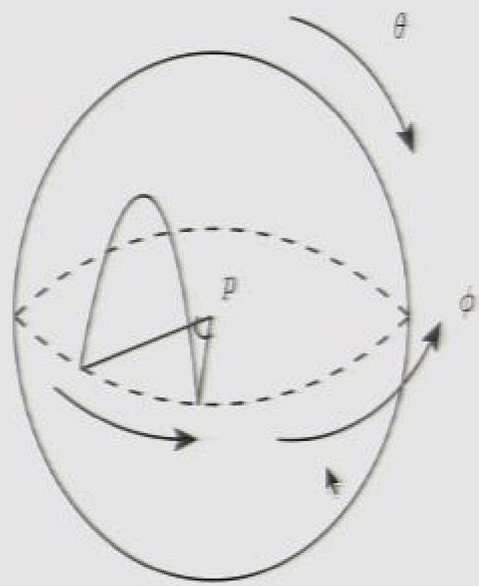
# Giant Magnon

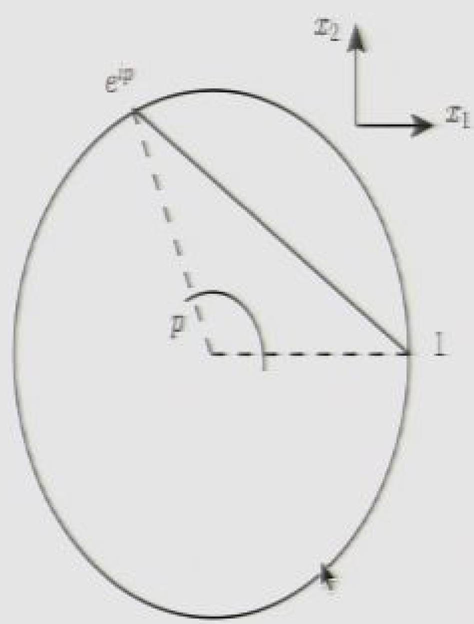
## of Hofman and Maldacena

- $d\Omega_5^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega_3^2 \qquad \phi \in [0, 2\pi), \theta \in [0, \frac{\pi}{2})$




- Length of the "string bit" is the energy of the magnon
- More geometric in LLM coordinates

# Giant Magnon
## in LLM coordinates




- Central charge $\mathcal{P} = x_1 - ix_2 = \frac{g}{\sqrt{2}}(e^{-ip} - 1)$ and $\mathcal{K} = \mathcal{P}^\dagger = \frac{g}{\sqrt{2}}(e^{ip} - 1)$

# Emergent Magnon

Using BMN matrix QM [Berenstein, Correa, Vazquez]

$$S=\int dt\mathrm{Tr}_{a,b}\left(\sum_{i=1}^{6}\frac{1}{2}(D_tX^i)^2-\frac{1}{2}(X^i)^2-\sum_{i,j=1}^{6}\frac{g_{YM}^2}{8\pi^2}[X^i,X^j][X^j,X^i]\right)$$

- Look at the operators with $\Delta - r_{12} = 0$,
- $\frac{1}{2}$ BPS: $\mathrm{Tr}[\Phi^r]$ where $\Phi \equiv X^1 + iX^2$
- Eigenvalues distribute themselves on a circle of radius $\sqrt{\frac{N}{2}}$
- The offdiagonal excitations $X^{6,7,8,9}$ in probe approximation

$$H_{off-diag} = \sum_{a\neq b}\frac{1}{2}(\Pi_{I\hat{I}})^a_b(\Pi^{I\hat{I}})^b_a+\frac{1}{2}\omega_{ab}^2(\mathcal{X}^{I\hat{I}})^a_b(\mathcal{X}_{I\hat{I}})^b_a$$

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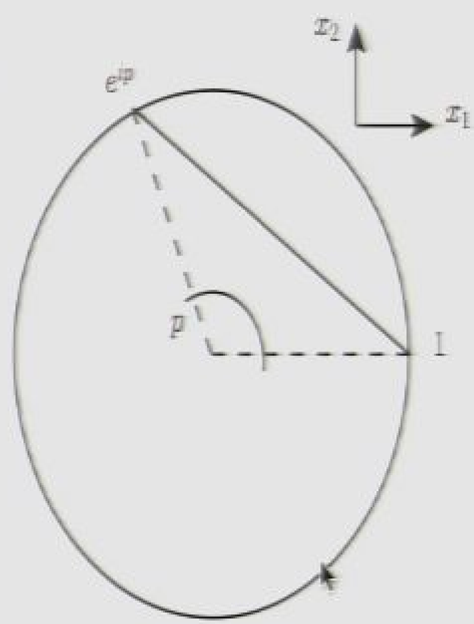
# Giant Magnon

## in LLM coordinates

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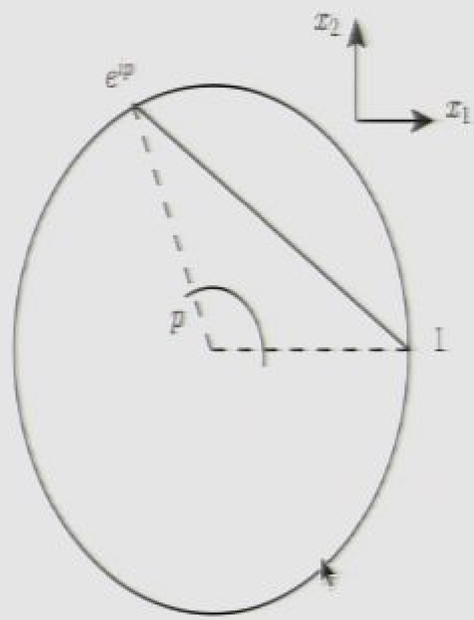


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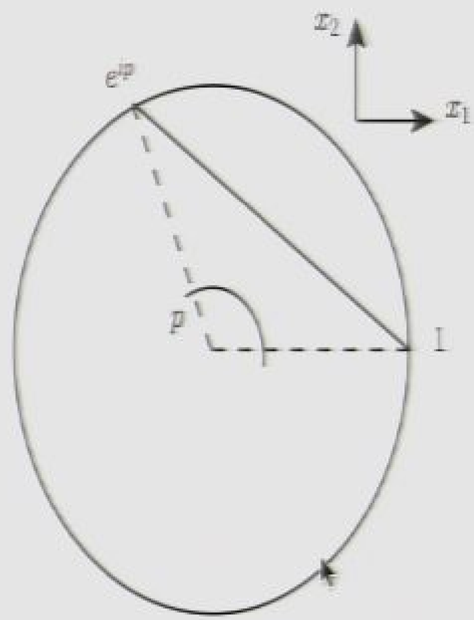
# Giant Magnon
## in LLM coordinates

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# Magnon of $\kappa$ deformed orbifold [AG, Rastelli, to appear]

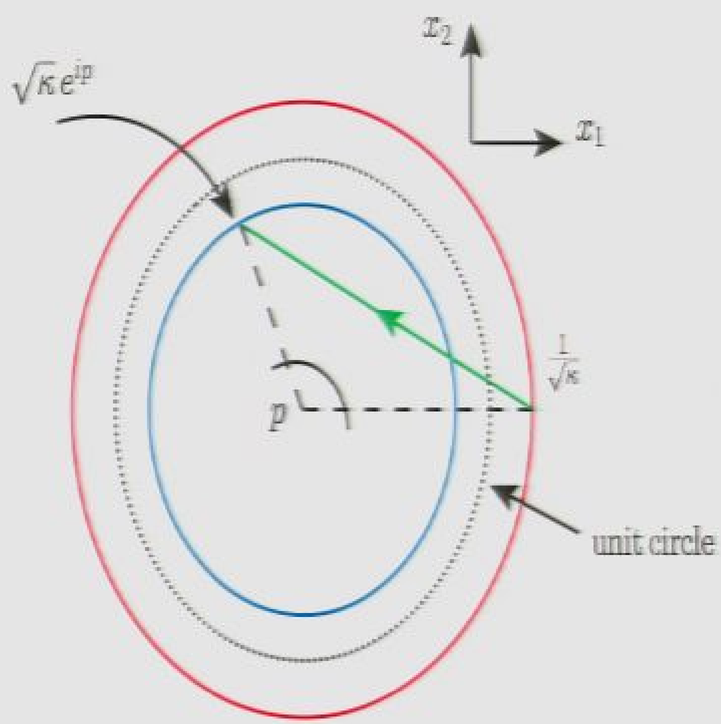
- The matrix model:

$$S = \int dt \mathrm{Tr}(\text{kinetic} + \text{mass})$$
$$- \frac{g_{YM}^2}{8\pi^2}\left([\bar{\phi},\phi]^2 + \sqrt{2}Q^{I\hat{I}}\bar{Q}_{I\hat{I}}(\phi\bar{\phi}+\bar{\phi}\phi) + Q^{I\hat{I}}\bar{Q}_{J\hat{I}}Q^{J\hat{J}}\bar{Q}_{\hat{J}I} - \frac{1}{2}Q^{I\hat{I}}\bar{Q}_{I\hat{I}}Q^{J\hat{J}}\bar{Q}_{J\hat{J}}\right)$$
$$- \frac{\check{g}_{YM}^2}{8\pi^2}\left([\bar{\check{\phi}},\check{\phi}]^2 + \sqrt{2}\bar{Q}_{I\hat{I}}Q^{I\hat{I}}(\check{\phi}\bar{\check{\phi}}+\bar{\check{\phi}}\check{\phi}) + \bar{Q}_{I\hat{I}}Q^{J\hat{I}}\bar{Q}_{J\hat{J}}Q^{\hat{J}I} - \frac{1}{2}\bar{Q}_{I\hat{I}}Q^{I\hat{I}}\bar{Q}_{J\hat{J}}Q^{J\hat{J}}\right)$$
$$+ \frac{g_{YM}\check{g}_{YM}}{8\pi^2}(4\bar{Q}_{I\hat{I}}\check{\phi}Q^{I\hat{I}}\phi + h.c.) + \frac{1}{N_c}(\text{double trace})$$

- Turning on $\phi$ and $\check{\phi}$: eigenvalues form a circle of radius $\sqrt{\frac{N_c}{2}}$
- The hamiltonian for off diagonal $(Q^{I\hat{I}})^a_{\bar{b}}$

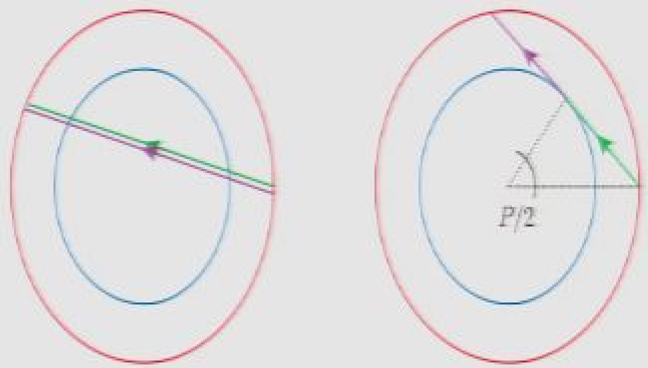
$$H_{off-diag} = \sum \frac{1}{2}(\Pi_{I\hat{I}})^a_{\bar{b}}(\Pi^{I\hat{I}})^{\bar{b}}_a + \frac{1}{2}\omega^2_{a\bar{b}}(Q^{I\hat{I}})^a_{\bar{b}}(\bar{Q}_{I\hat{I}})^{\bar{b}}_a$$
$$\omega^2_{a\bar{b}} = 1 + \frac{1}{2\pi^2}|g_{YM}\phi_a - \check{g}_{YM}\check{\phi}_{\bar{b}}|^2$$

# Emergent Magnon

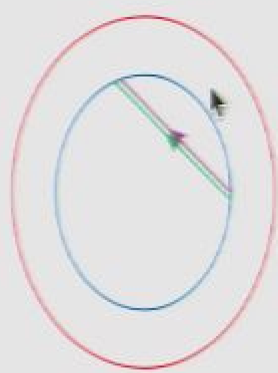



- For $(Q,\psi)$, $\mathcal{P}=x_1-ix_2=\frac{\sqrt{g\check{g}}}{2}(e^{-ip}\frac{1}{\sqrt{\kappa}}-\sqrt{\kappa})$, $\mathcal{K}=\mathcal{P}^\dagger$
- For $(\tilde{Q},\tilde{\psi})$, $\mathcal{P}=\frac{\sqrt{g\check{g}}}{2}(e^{-ip}\sqrt{\kappa}-\frac{1}{\sqrt{\kappa}})$
- $\Delta-r=2\mathcal{C}=\sqrt{1+8g\check{g}\left(\sin^2\frac{p}{2}+\frac{1}{4}(\sqrt{\kappa}-\frac{1}{\sqrt{\kappa}})^2\right)}$

# Bound States




- $Q\bar{Q}$ bound state decays at $P = 2\arccos\kappa$, agrees with one loop [AG, Pomoni, Rastelli]
- $\bar{Q}Q$ bound state is stable



- Interesting to study the wallcrossing phenomenon in the dual

# Dual of $\kappa$ deformation

- $\mathbb{Z}_2$ Orbifold $\Leftrightarrow$ IIB on $AdS_5 \times S^5/\mathbb{Z}_2$
- Collapsed 2-cycle $S^2$

$$\int_{S^2} B_{NSNS} = \frac{1}{2}$$

- Taking $\check{g}_{YM} \neq g_{YM}$ corresponds to changing the $B$ period

$$\frac{1}{g_{YM}^2} + \frac{1}{\check{g}_{YM}^2} = \frac{1}{2\pi g_s}$$

$$\frac{\check{g}_{YM}^2}{g_{YM}^2} \equiv \kappa^2 = \frac{\beta}{1-\beta} \qquad \int_{S^2} B \equiv \beta$$

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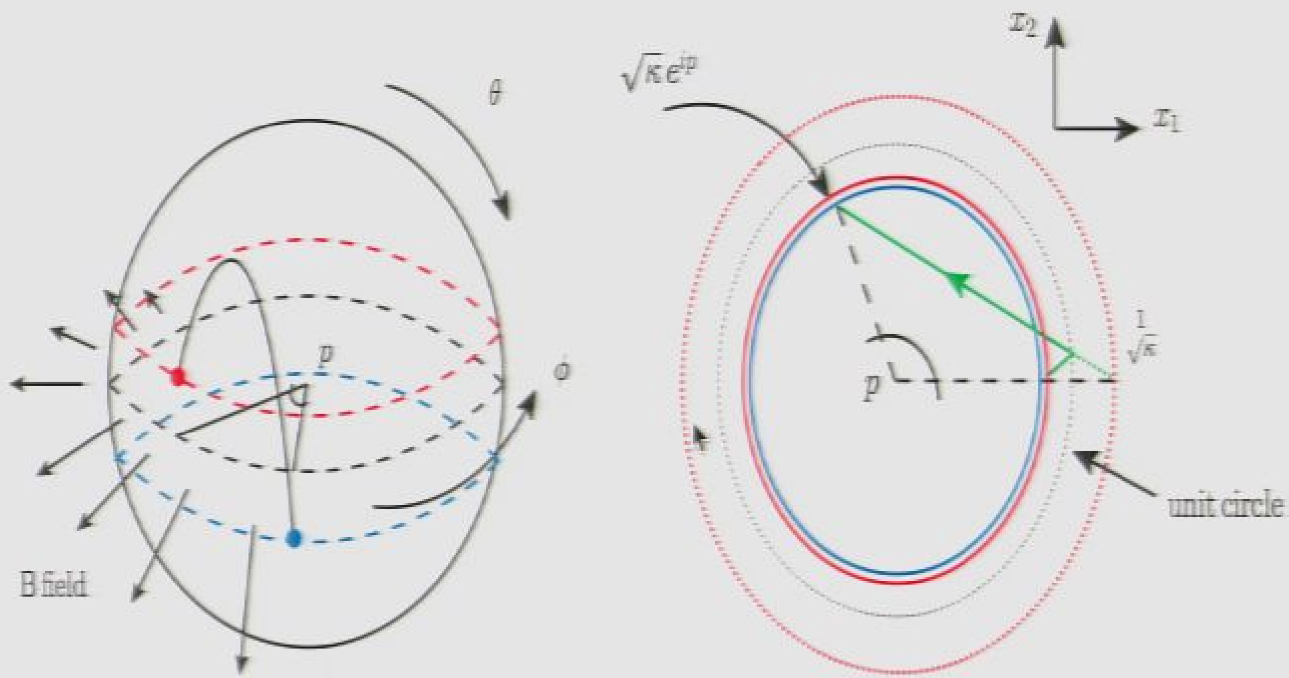
# Magnon in B field

## Effective picture



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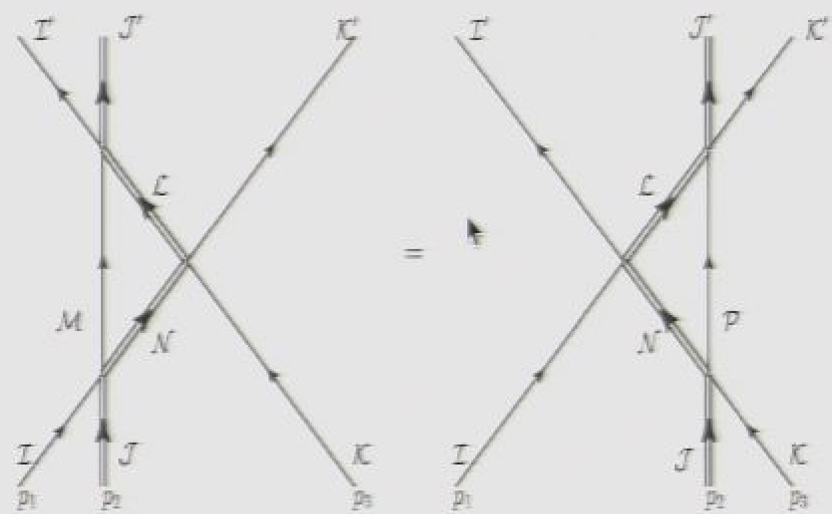
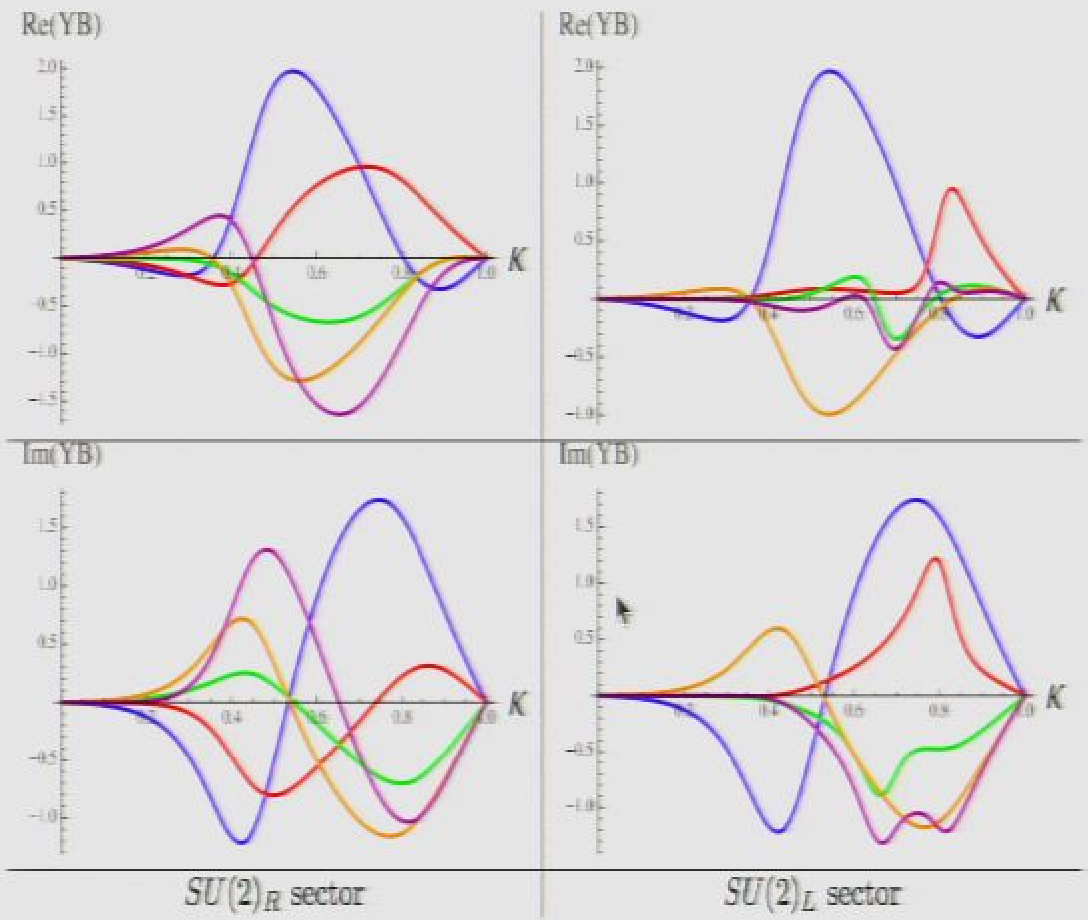
# Yang Baxter equation

- Spin chain for the interpolating theory is not integrable
- It is a necessary condition for integrability
- Implication of factorized scattering



# Yang Baxter equation

## Failure of YBE [AG, Pomoni, Rastelli]




- YBE is satisfied for $\kappa = 1$ and $\kappa = 0$!

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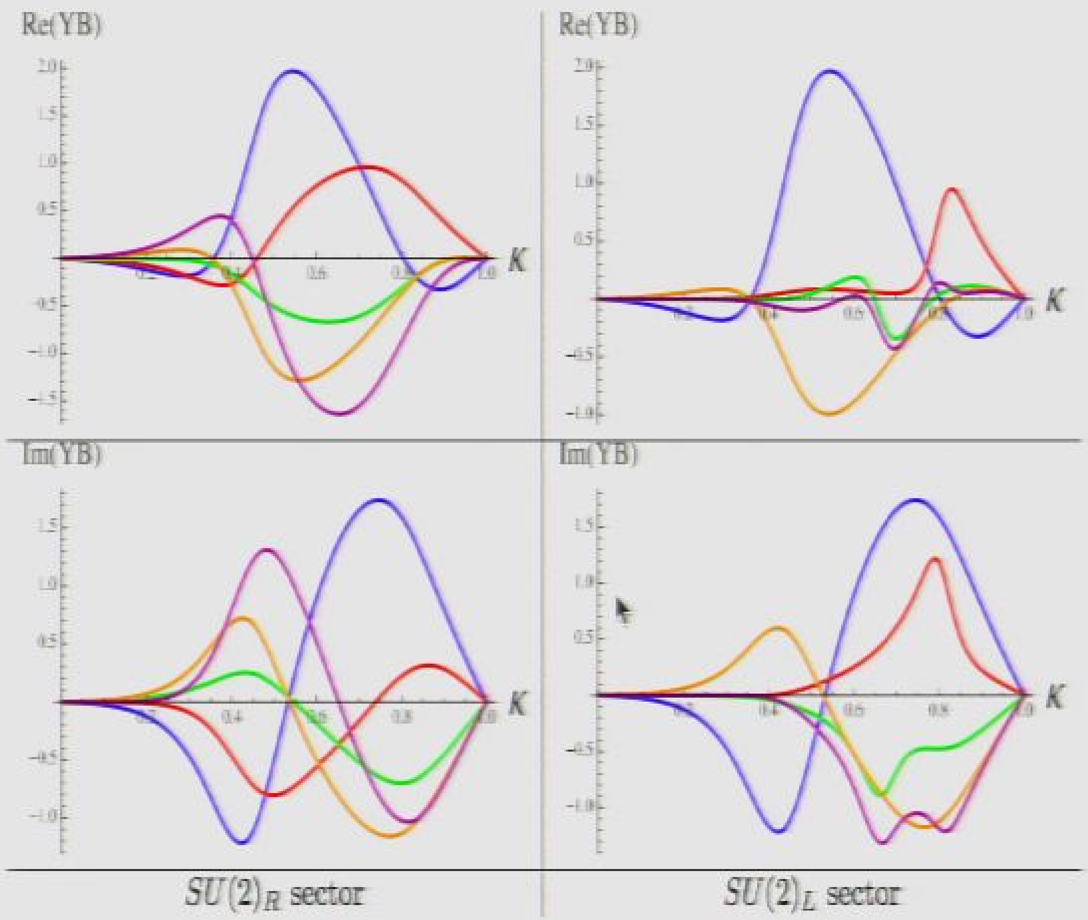


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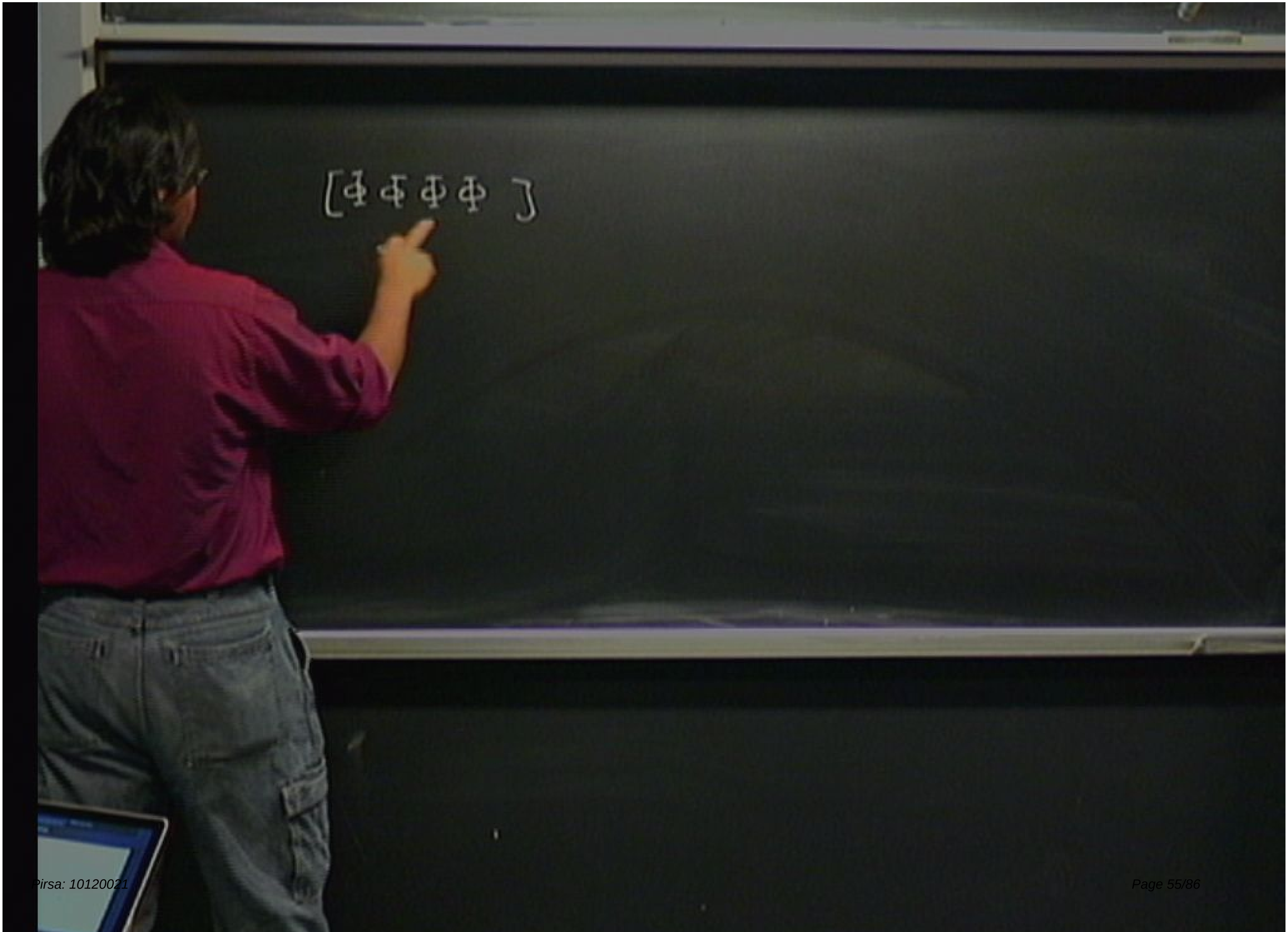
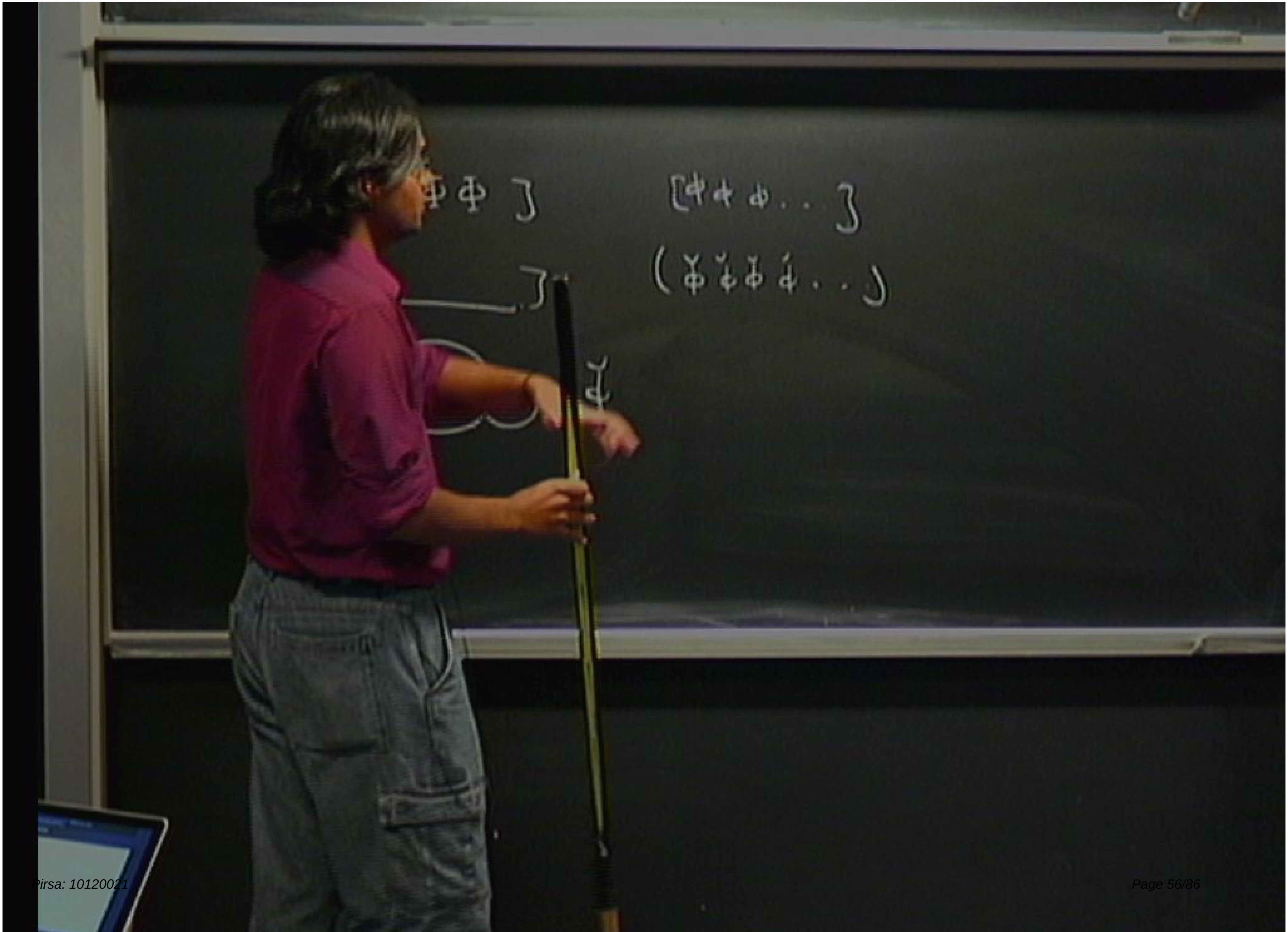
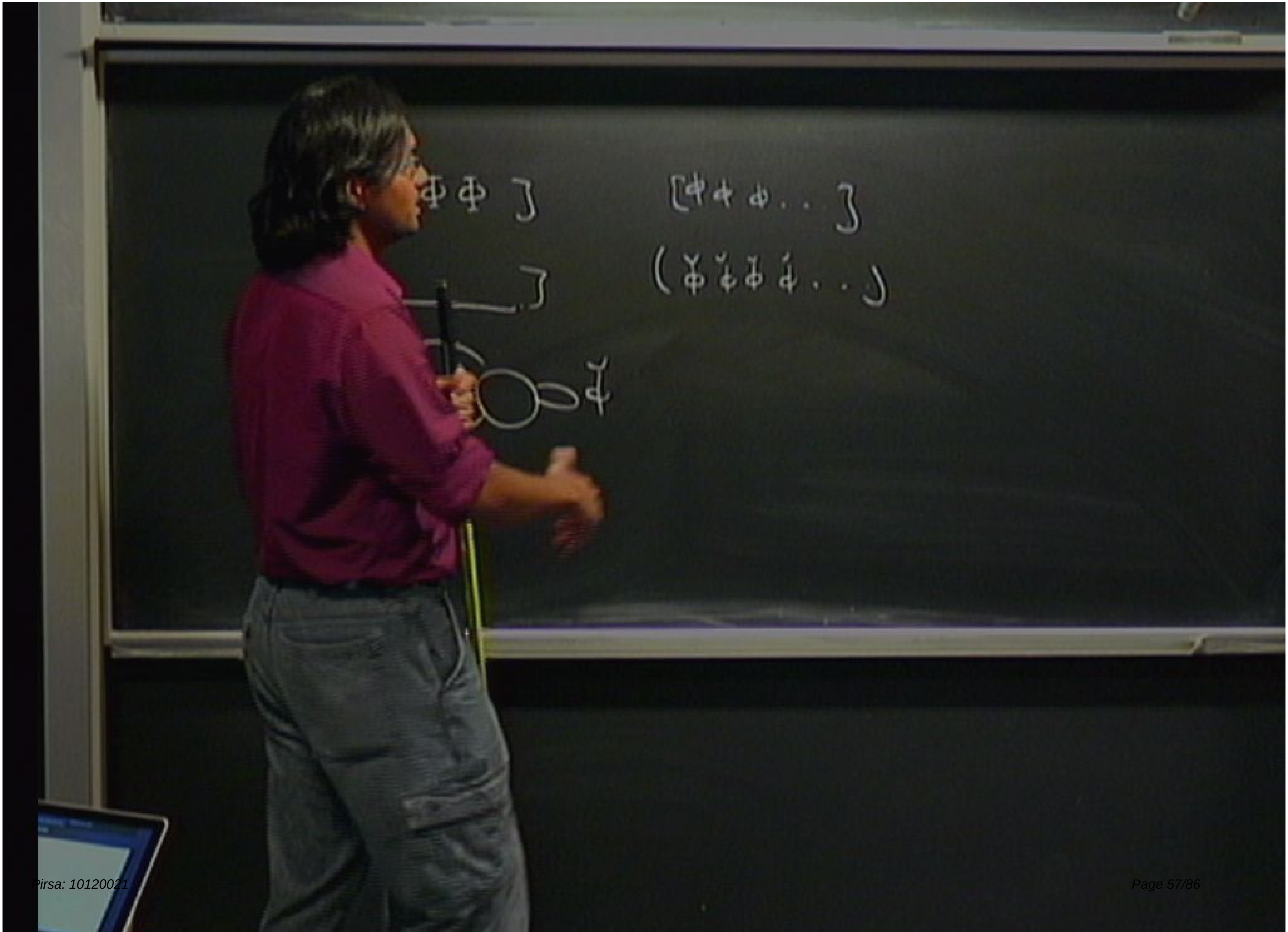
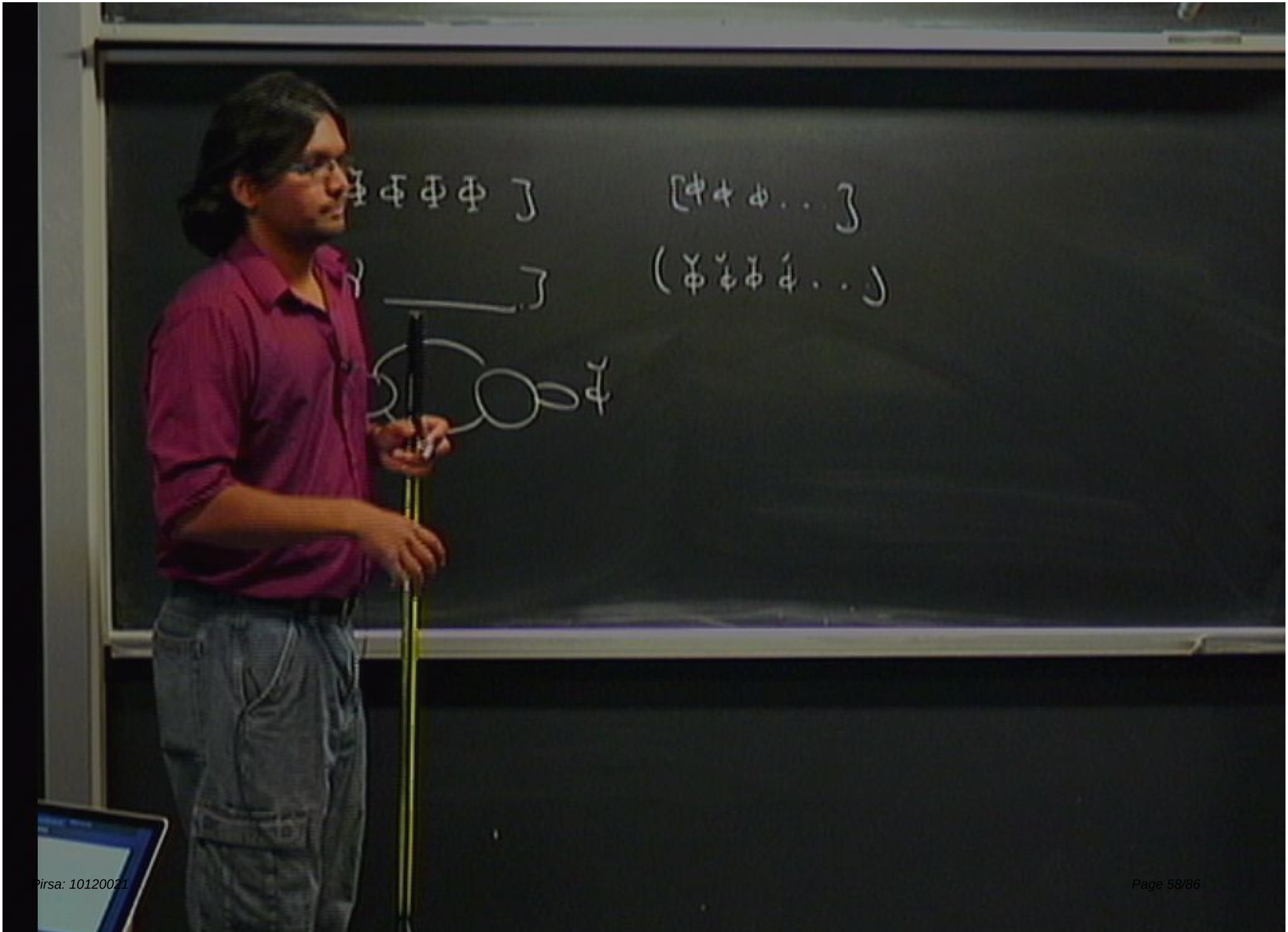
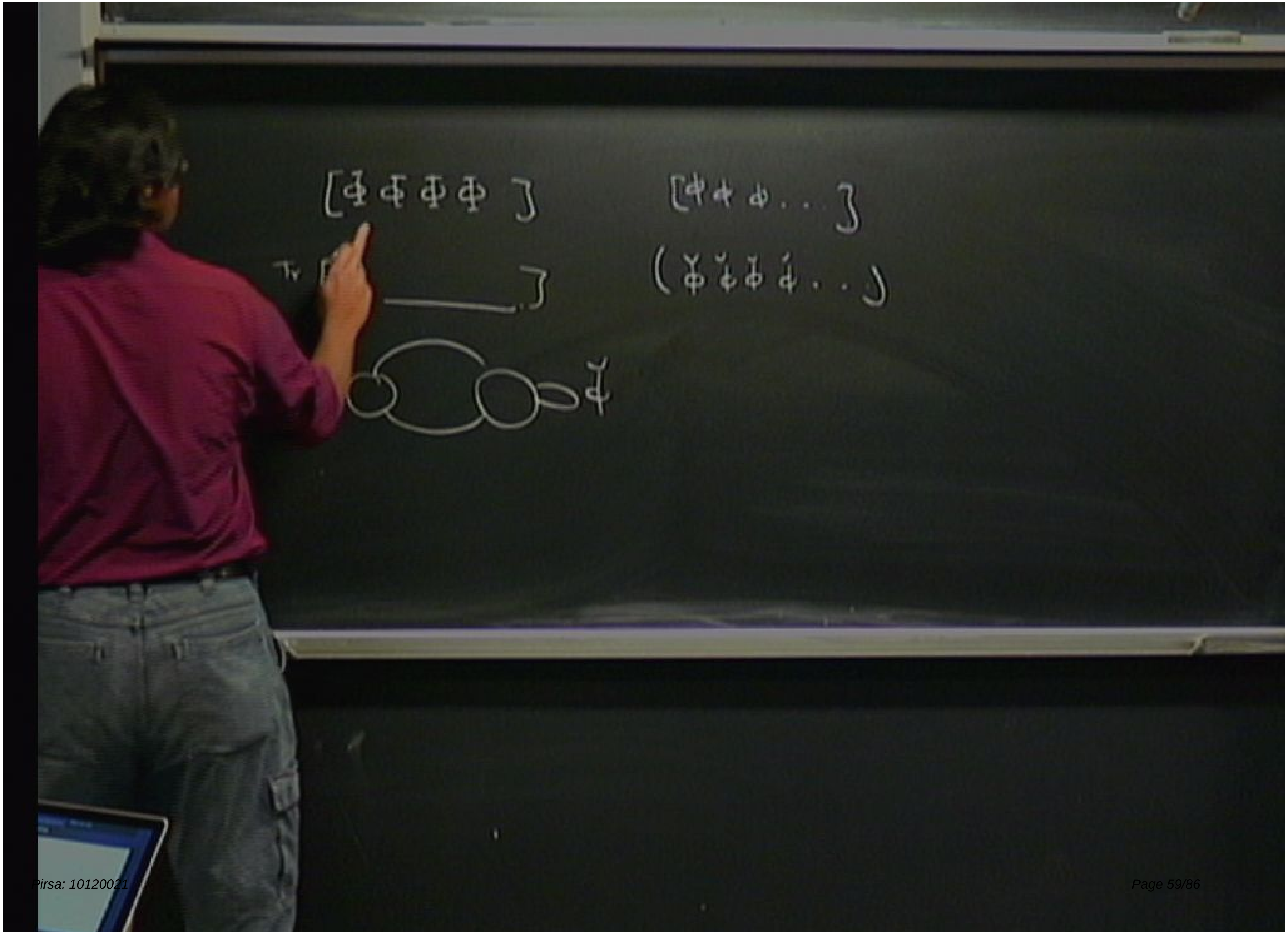
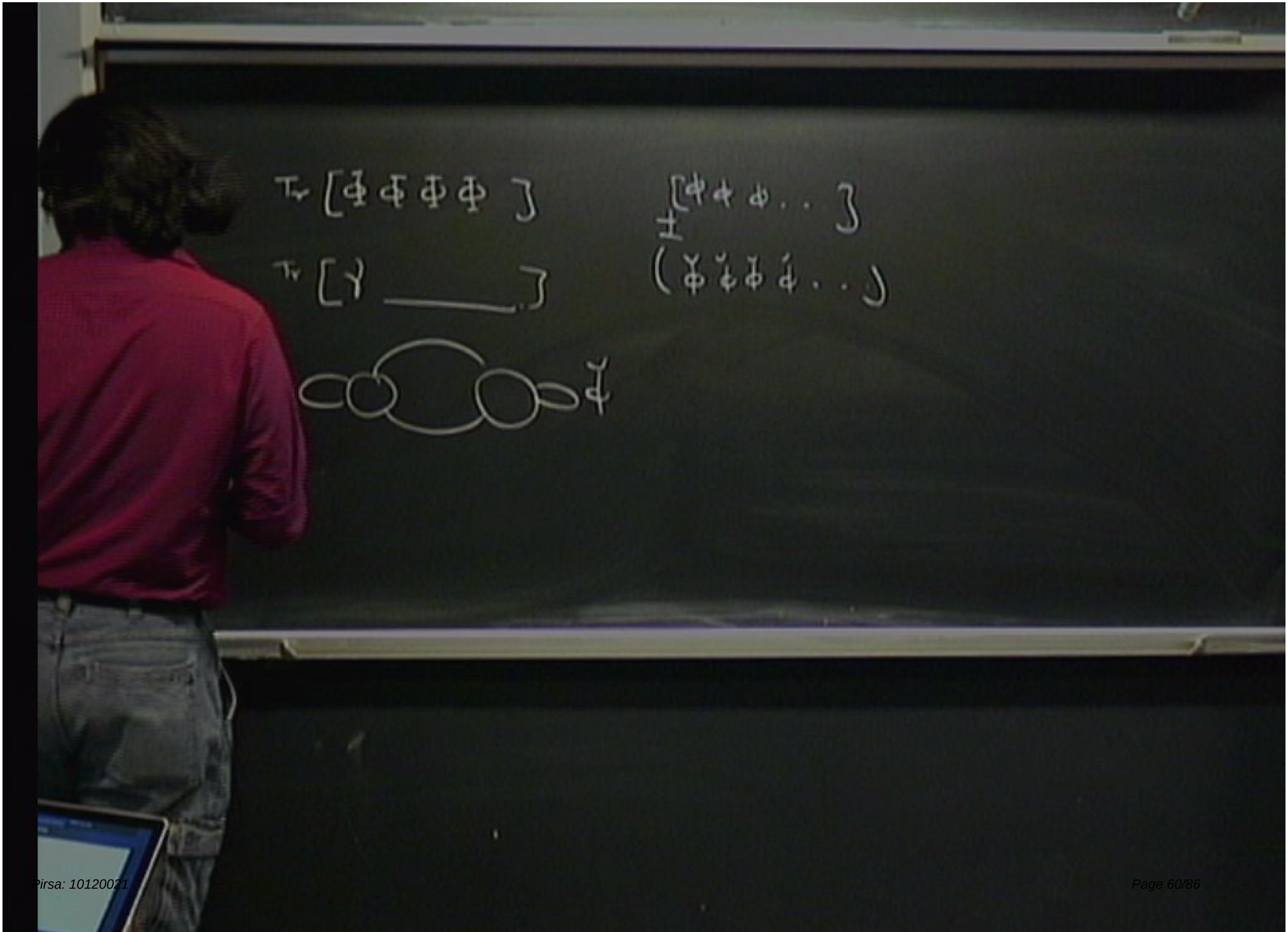












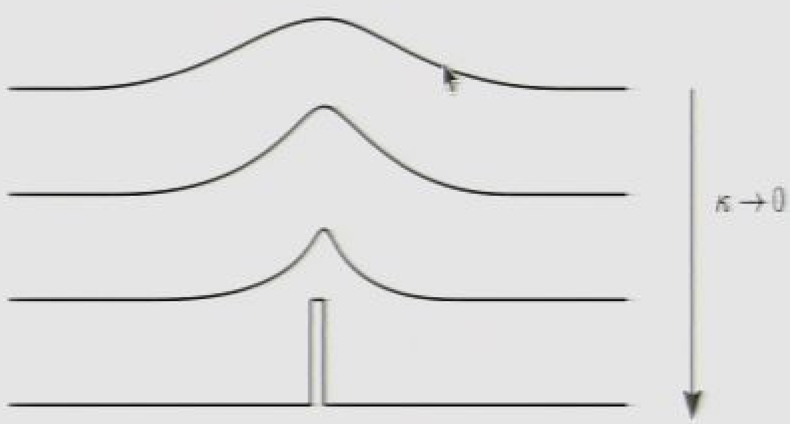
# SCQCD spin chain

## From $\kappa$ deformed spin chain [AG, Pomoni, Rastelli]

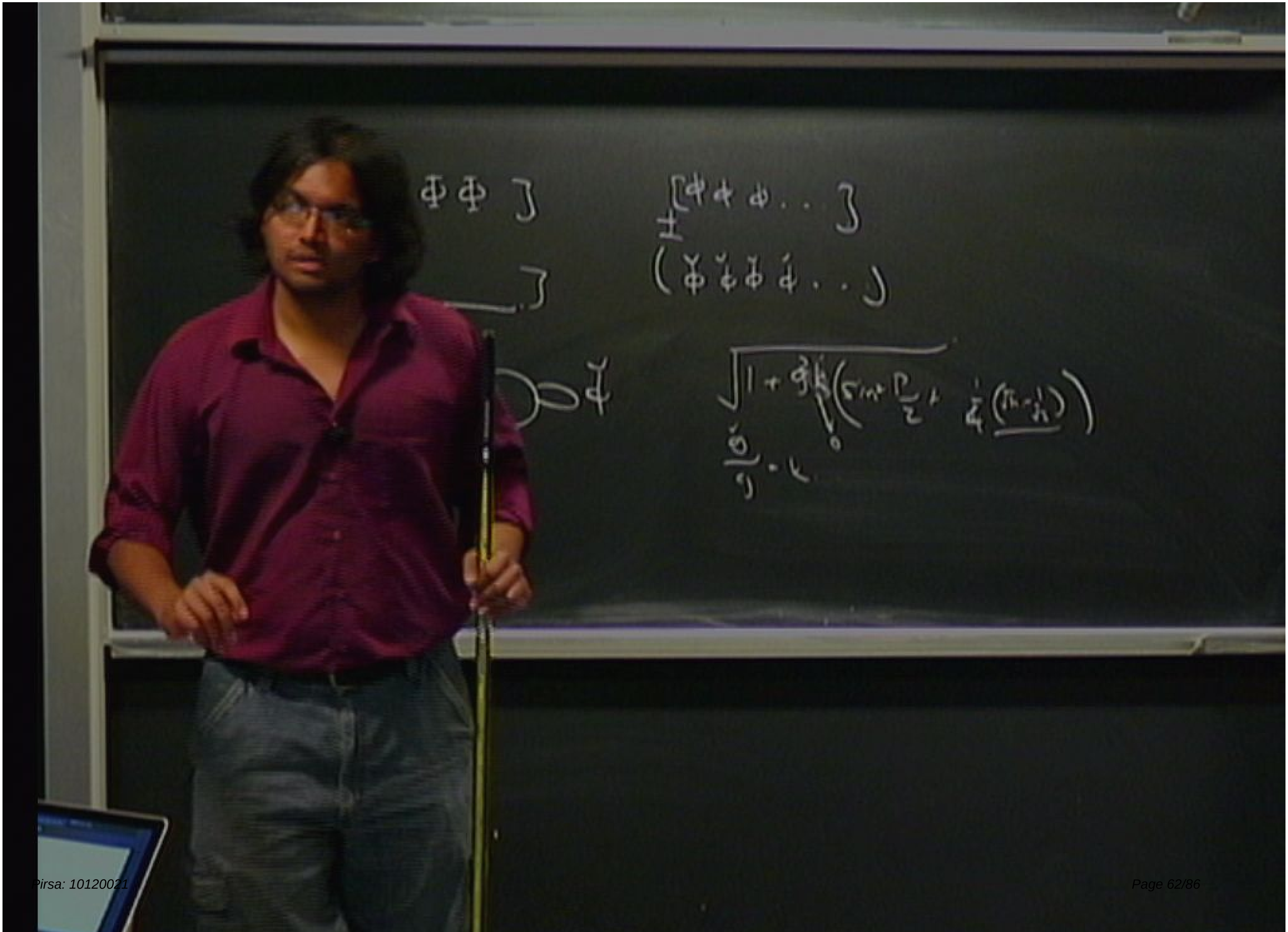
- The dispersion relation of the $Q, \bar{Q}$ magnons at one loop

$$E(p) = g^2((1-\kappa)^2 + 4\kappa \sin^2 \frac{p}{2})$$

- The nontrivial dynamics is carried by bound states
- Wavefunction of bound states of $Q^{I\hat{I}}$ and $\bar{Q}^{J\hat{J}}$ in the $SU(2_{\hat{I}})$ singlet




- Dimeric excitations of SCQCD



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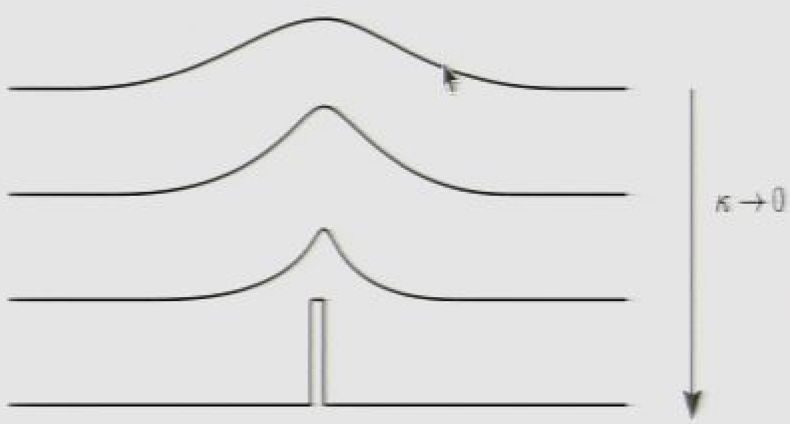


- Diameric excitations of SCQCD

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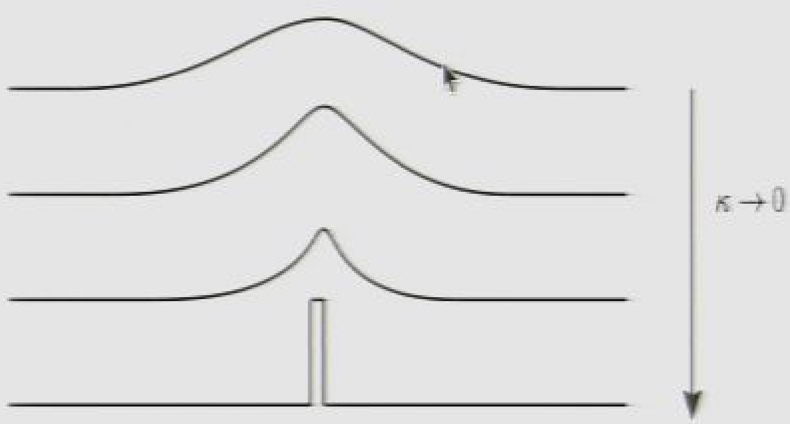


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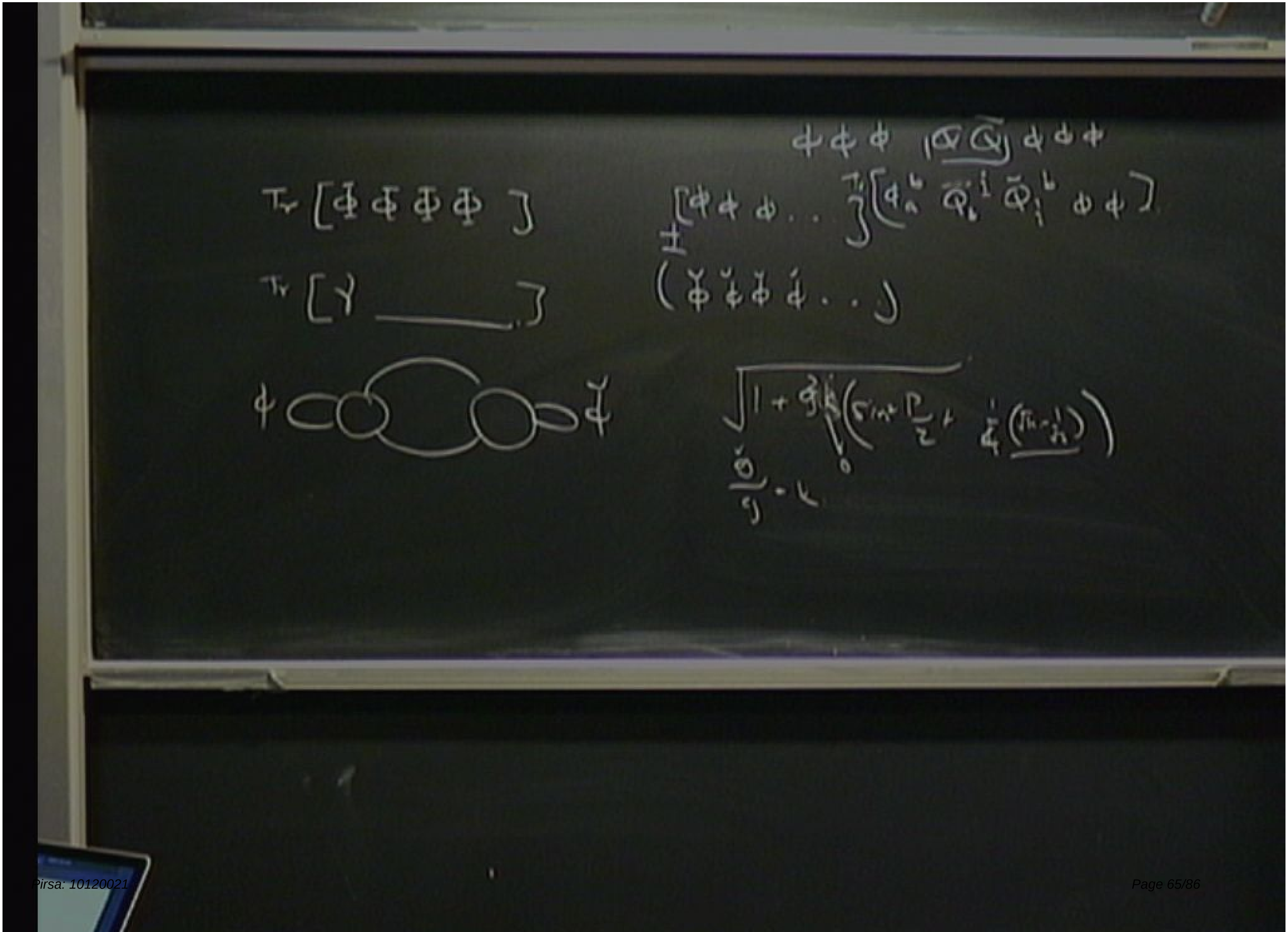

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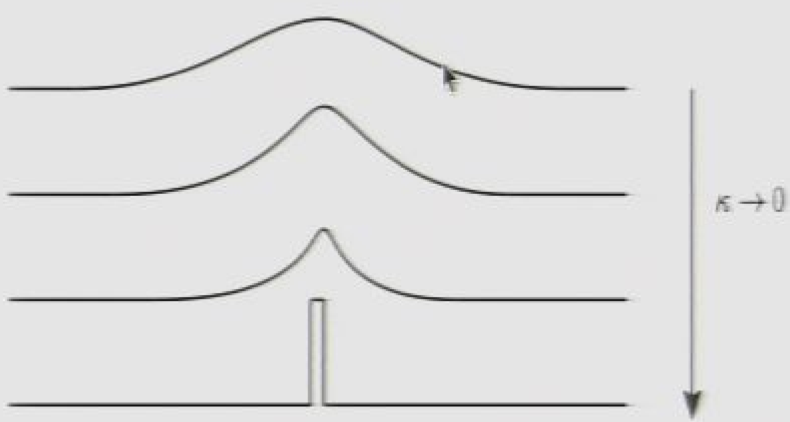




- Diameric excitations of SCQCD

# SCQCD spin chain

- Simplest R matrix for the spin chain doesn't work
- The scattering of diameric excitations in hard to analyze
- One loop limit is somewhat singular: At higher loop, diameric excitations cease to be good asymptotic excitations due to transitions

$$Q\bar{Q}\phi \leftrightarrow \lambda\lambda$$

- At higher loop, the good excitations are $(\lambda_{I\alpha}, D_{\alpha\dot{\alpha}})$ charged under $SU(2_\alpha) \times SU(2_{\dot{\alpha}}|2_I)$
- This symmetry structure is compatible with all loop Bethe ansatz
- The S matrix of $SU(2_{\dot{\alpha}}|2_I)$ is completely fixed (identical to $\mathcal{N} = 4$)
- The S matrix of $SU(2_\alpha)$ is at one loop is also completely fixed (identical to $\mathcal{N} = 4$)
- Investigating integrability in SCQCD: two loop S matrix in $SU(2_\alpha)$

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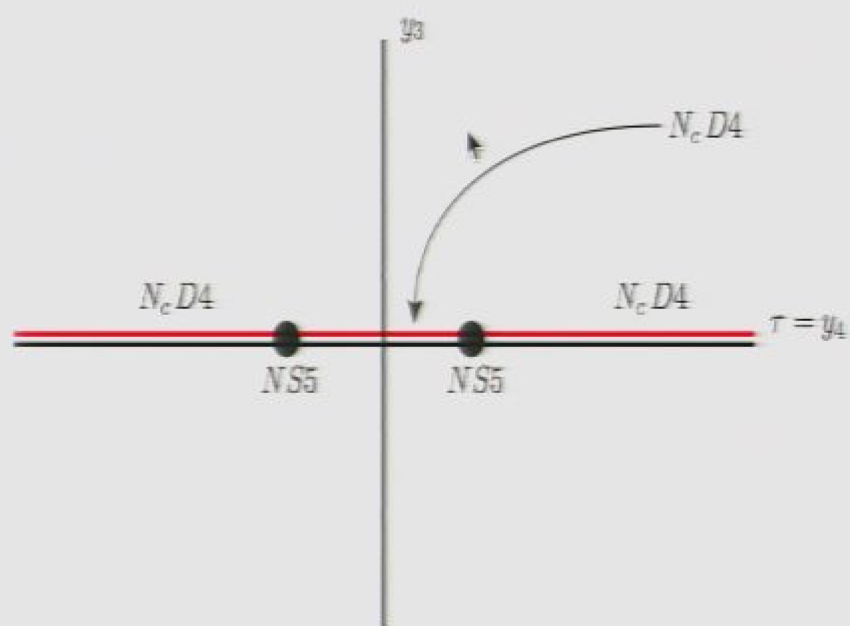
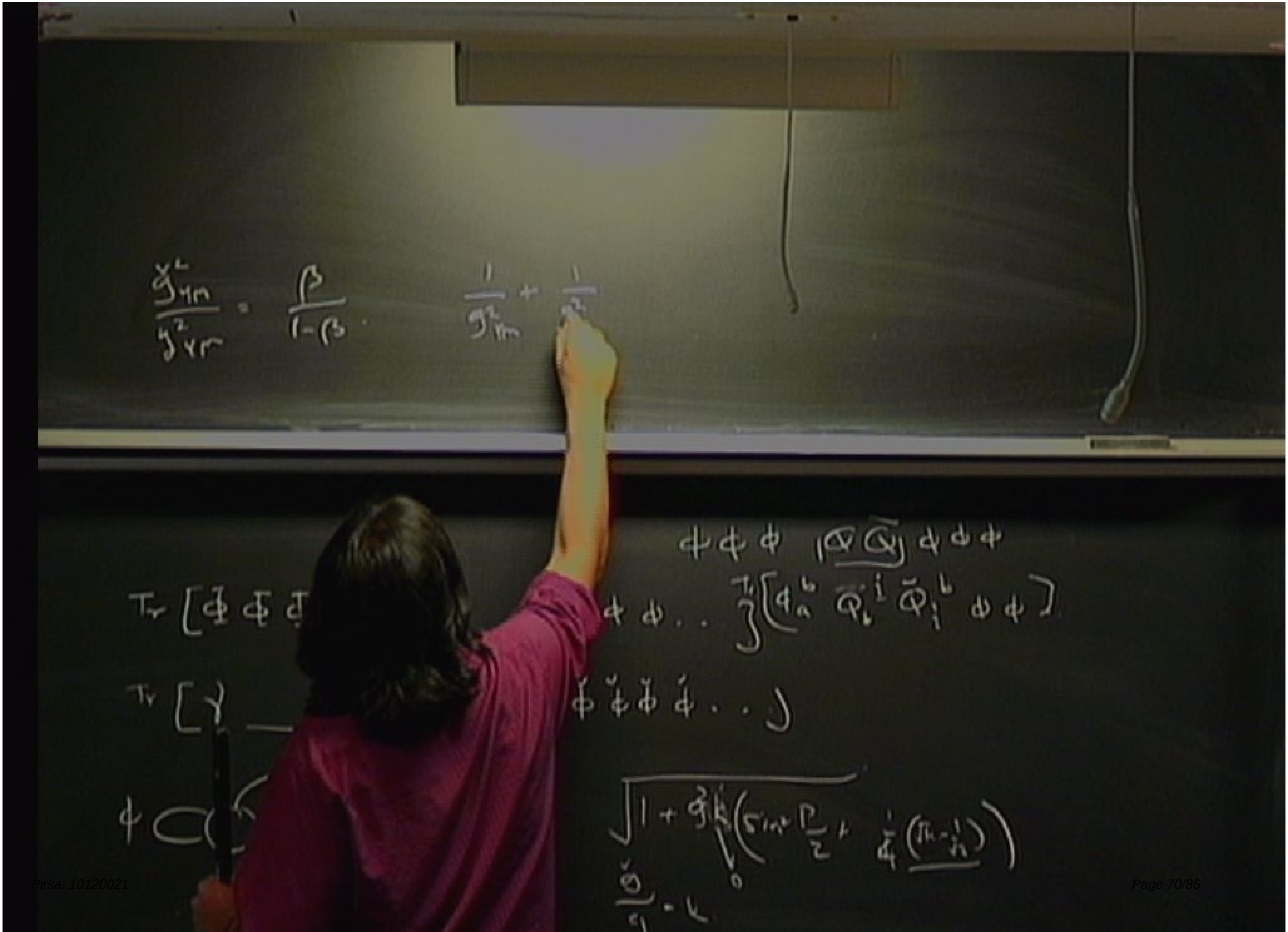
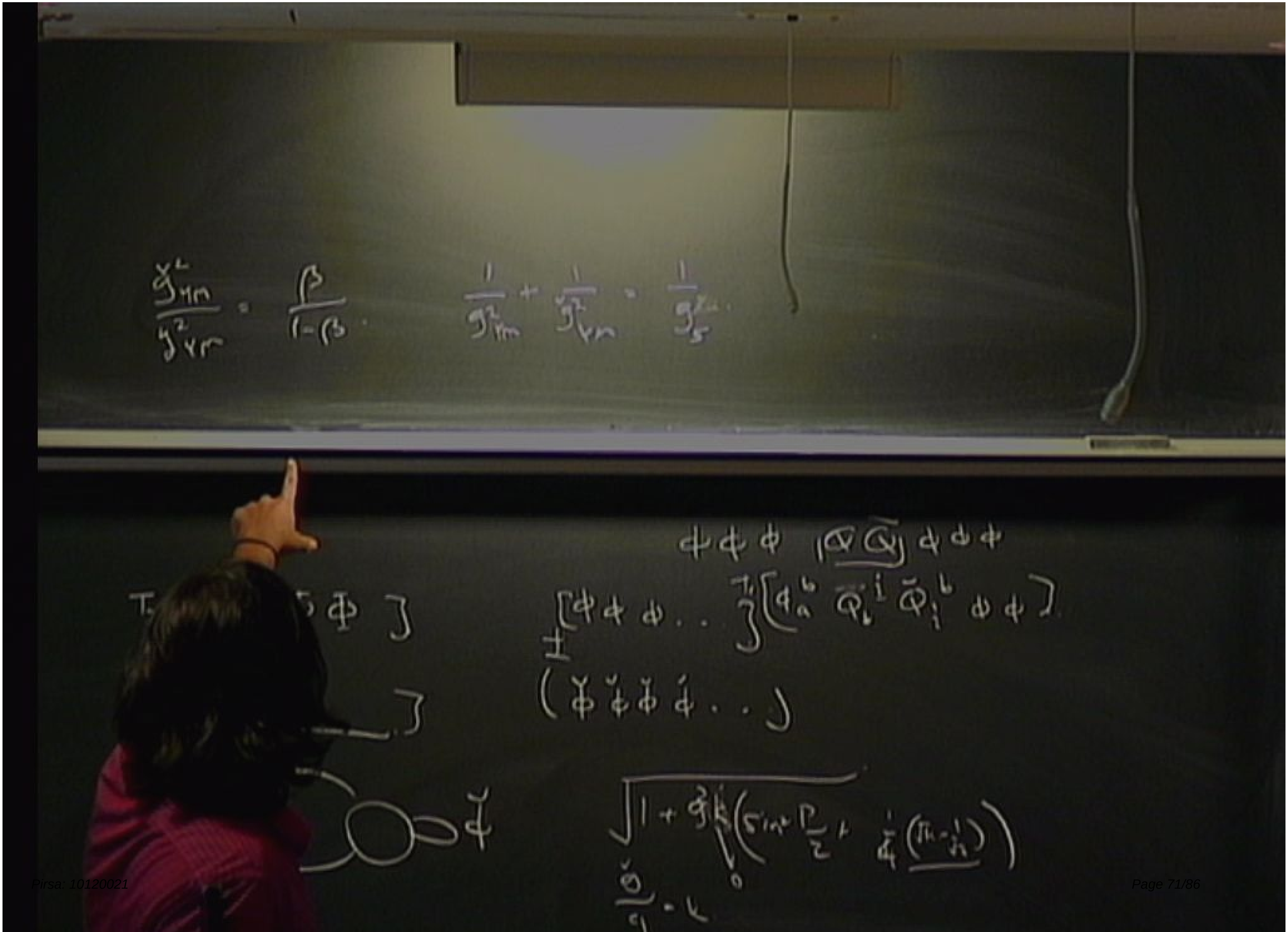
# SCQCD dual

- As $\beta \to 0$, Type IIB description in terms of $AdS^5 \times S_5/\mathbb{Z}_2$ becomes singular
- Convenient to T-dualize and go to Hanany-Witten setup

| IIA | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\tau$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2NS5 | × | × | × | × | × | × | | | | |
| D4 | × | × | × | × | | | × | | | |



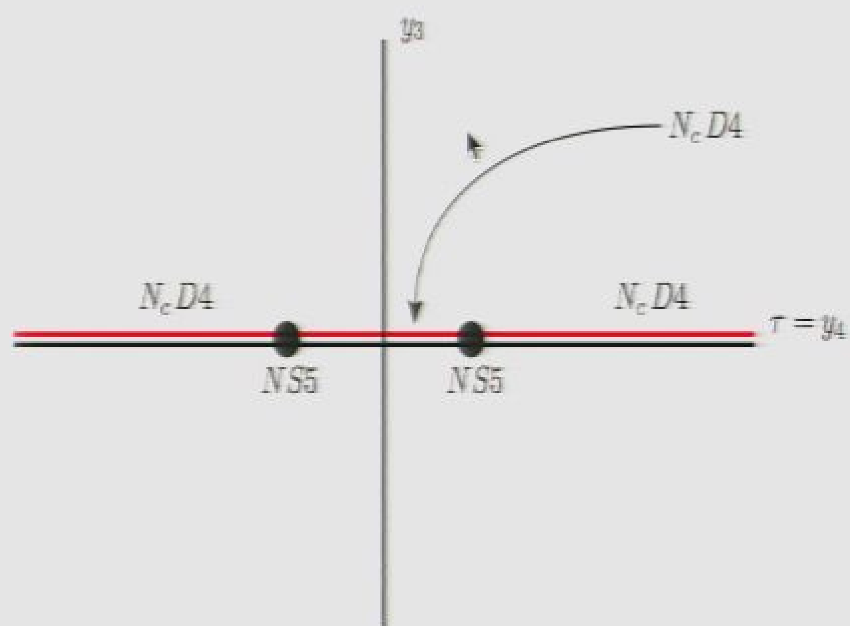




# SCQCD dual

- As $\beta \to 0$, Type IIB description in terms of $AdS^5 \times S_5/\mathbb{Z}_2$ becomes singular
- Convenient to T-dualize and go to Hanany-Witten setup

| IIA | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\tau$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2NS5 | × | × | × | × | × | × | | | | |
| D4 | × | × | × | × | | | × | | | |



# SCQCD dual

- Focus on closed string background, double scaling limit [Giveon, Kutasov]

$$\tau_0 \to 0,\ g_s \to 0 \text{ with } \frac{\tau_0}{g_s l_s} = \frac{1}{g_{eff}} \sim \frac{1}{g_{YM}^2} \text{ fixed} \qquad l_s \text{ fixed}$$

$$l_s \to 0$$

- Closed string $\sigma$ model with target space $\mathbb{R}^{5,1} \times SL(2)_2/U(1)$.

| IIB | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $\rho$ | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| D3 | × | × | × | × | | | | |
| D5 | × | × | × | × | | | × | × |

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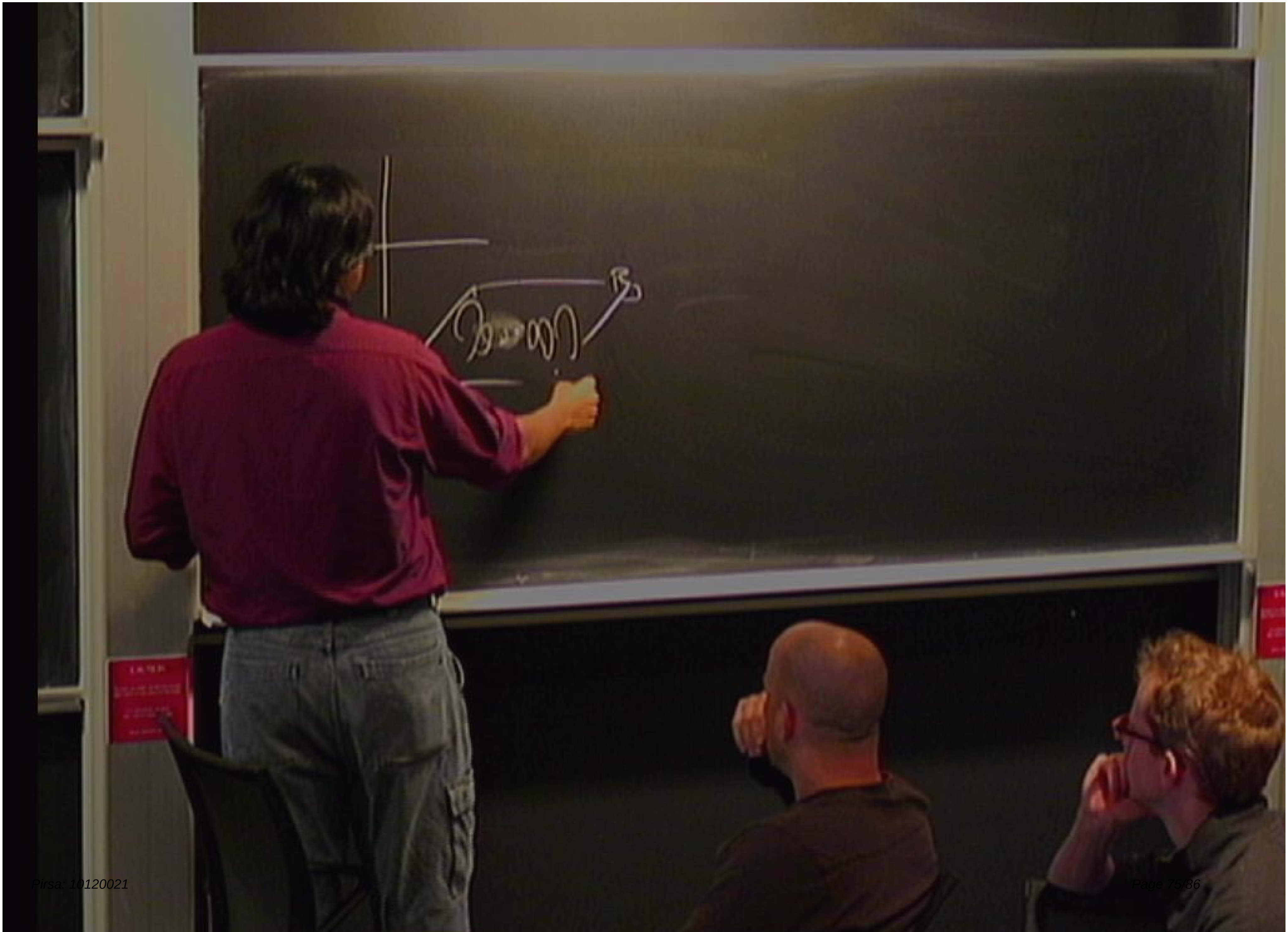
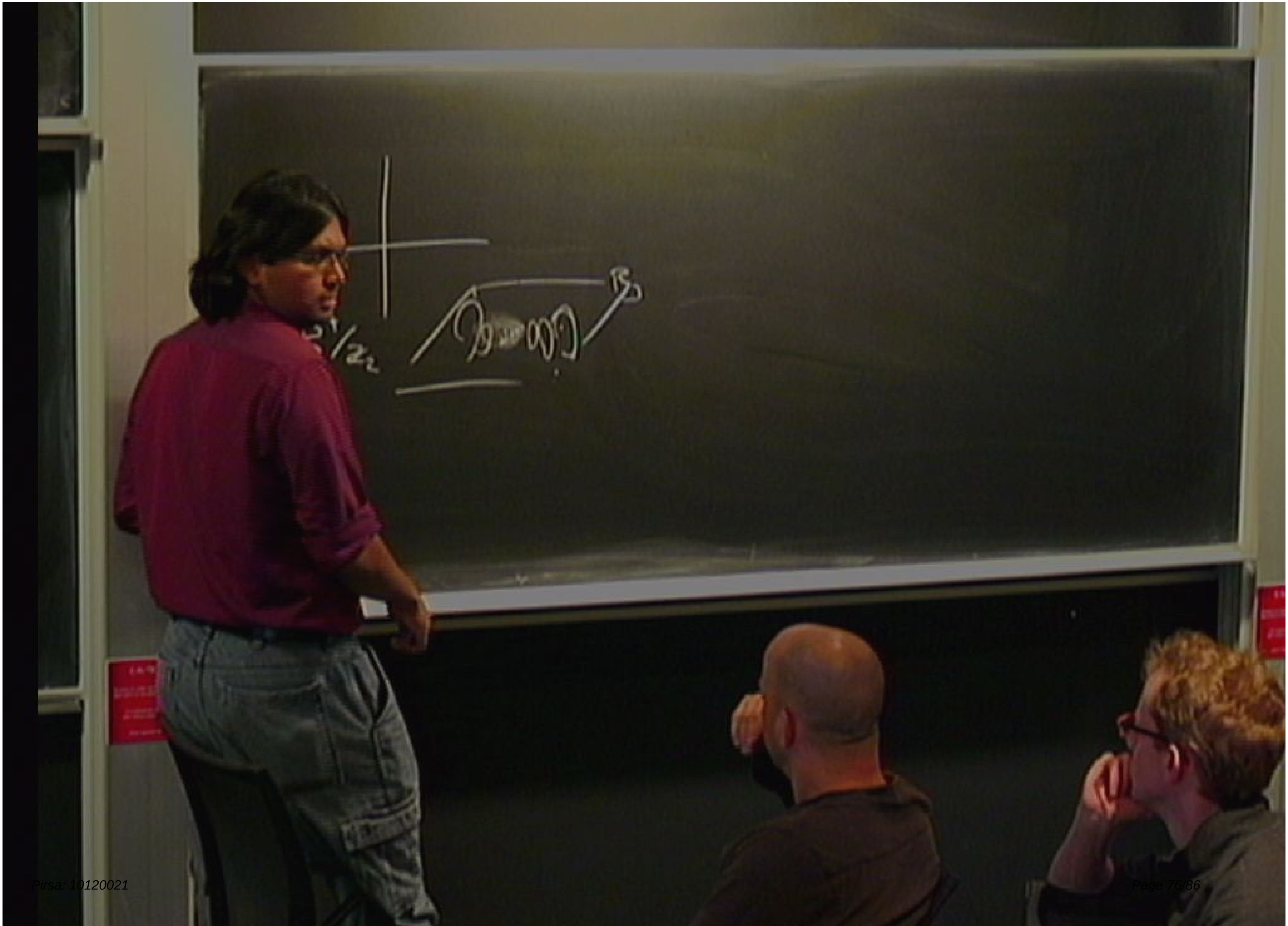
| D5 | × | × | × | × | | | × | × |

- Cigar circle is at free fermion radius, $U(1)$ gets enhanced to $SU(2)_R$





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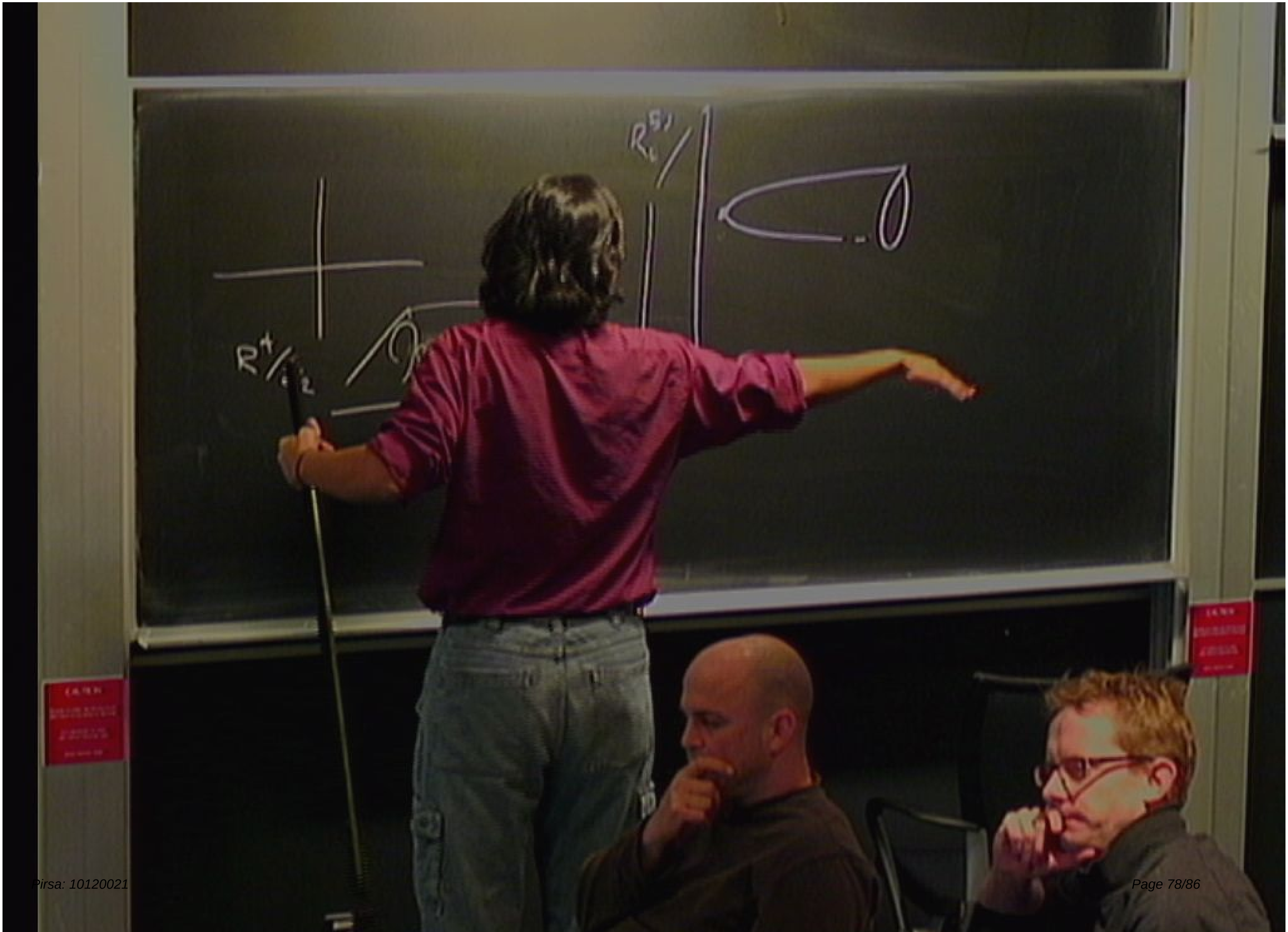
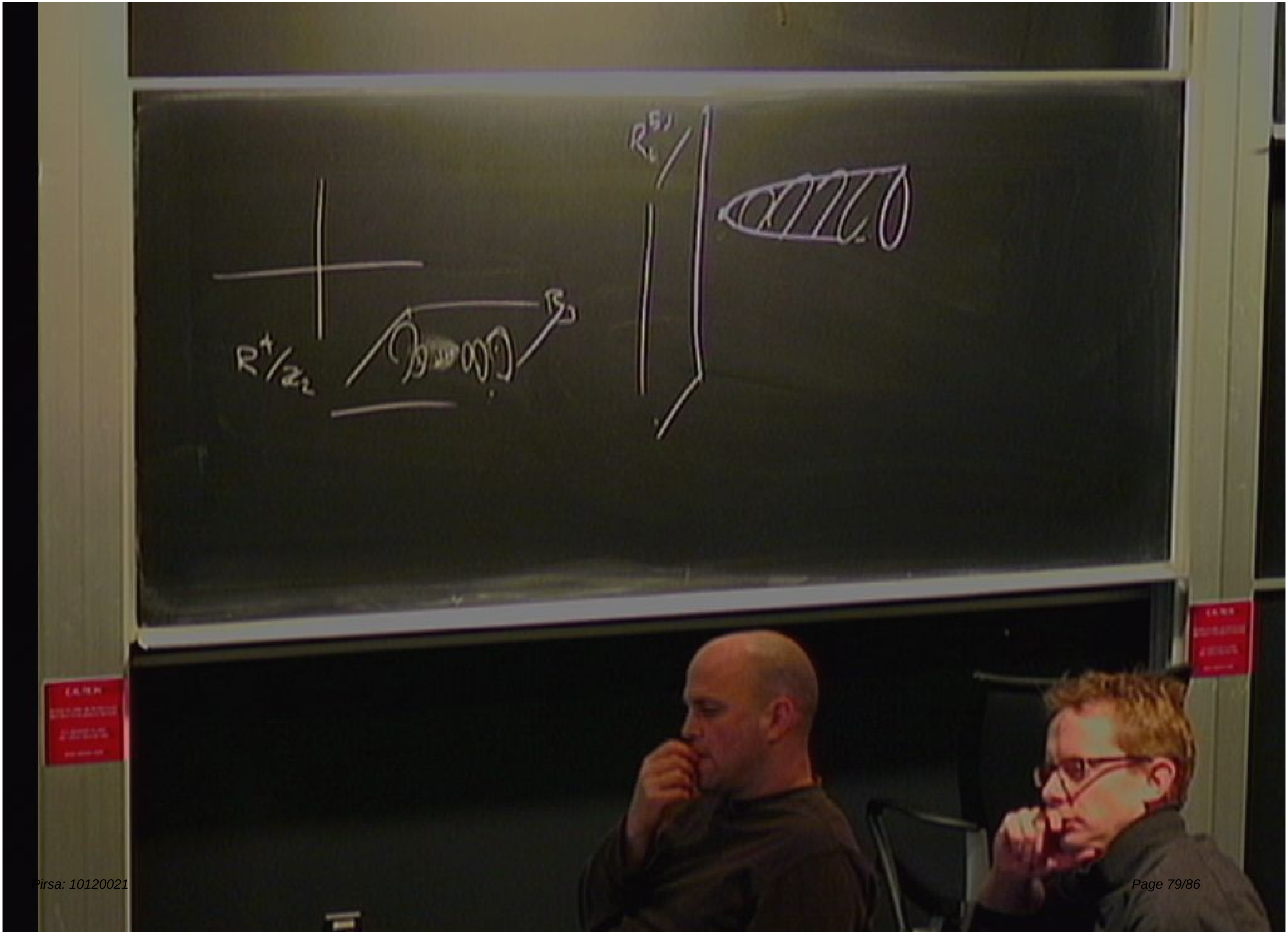
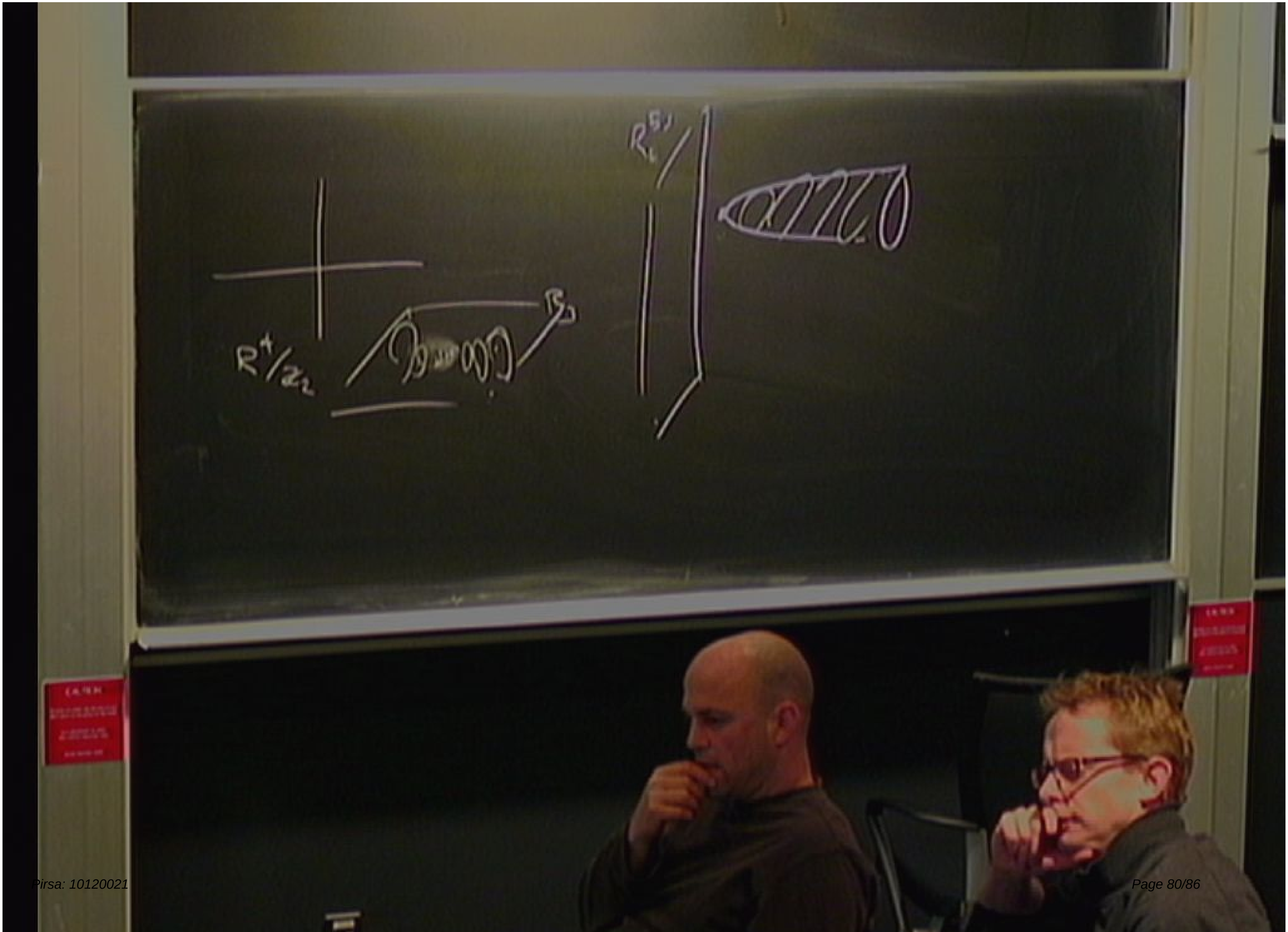



$R^4/Z_2$
$R^{5,1}$


R⁴/Z₂

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# SCQCD dual [AG, Pomoni, Rastelli]

- No real separation of scales
- "Reducing" on the cigar circle gives "effective theory": 7*d* maximal supergravity with $SO(4)$ gauging
- $SU(2)_R$ is symmetry of sugra while $U(1)_r$ is geometric $\Rightarrow$ KK modes
- Qualitative agreement with gauge theory, $U(1)_r$ tower of protected states $\mathrm{Tr}[\phi^r]$ but not $SU(2)_R$
- $AdS_5 \times S_1$ ansatz that is consistent with the symmetries

$$ds^2 = f(y)ds^2_{AdS} + g(y)d\phi^2 + C(y)dy^2$$

more generally, $AdS_5 \times S^1$ fibered over an interval $y \in [0,1]$

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Thank you!