Title: Evolution of Circumbinary Disks Following Super-massive Black Hole Mergers

Date: Dec 02, 2010 03:00 PM

URL: http://pirsa.org/10120019

Abstract: There has been a growing interest in electromagnetic counterparts to gravitational wave signals. Of particular interest here, are counterparts to gravitational wave signals from super-massive black hole mergers. We consider a circumbinary disk, hollowed out by torques from the binary, and provide an analytic solution to its response following merger. There are two changes to the potential which occur during the merger process: an axisymmetric mass-energy loss and asymmetric recoil kick given to the resulting super-massive black hole. With a brief literature search we argue that, for fiducial disk values and for black hole spins aligned and anti-aligned with the orbital angular momentum, throughout the majority of parameter space the mass loss well dominates the effects of the recoil kicks on the circumbinary disk. This, along with assuming vertical hydrodynamic equilibrium, reduces the problem to one dimension. Using a 1D hydrodynamical code we explore the majority of parameter space and describe the different possible flows. In the 1D case, we give analytic approximations for the locations of the first shocks, their strengths, and the final density after the disk has again reached a steady state. This allows one to determine the temperature jump across the shock front and determine the observability, modulo the yet unknown disk mass.

Evolution of Circumbinary Disks Following SMBH Mergers: Analytics & Simulations

Nate Bode

advisor: Sterl Phinney

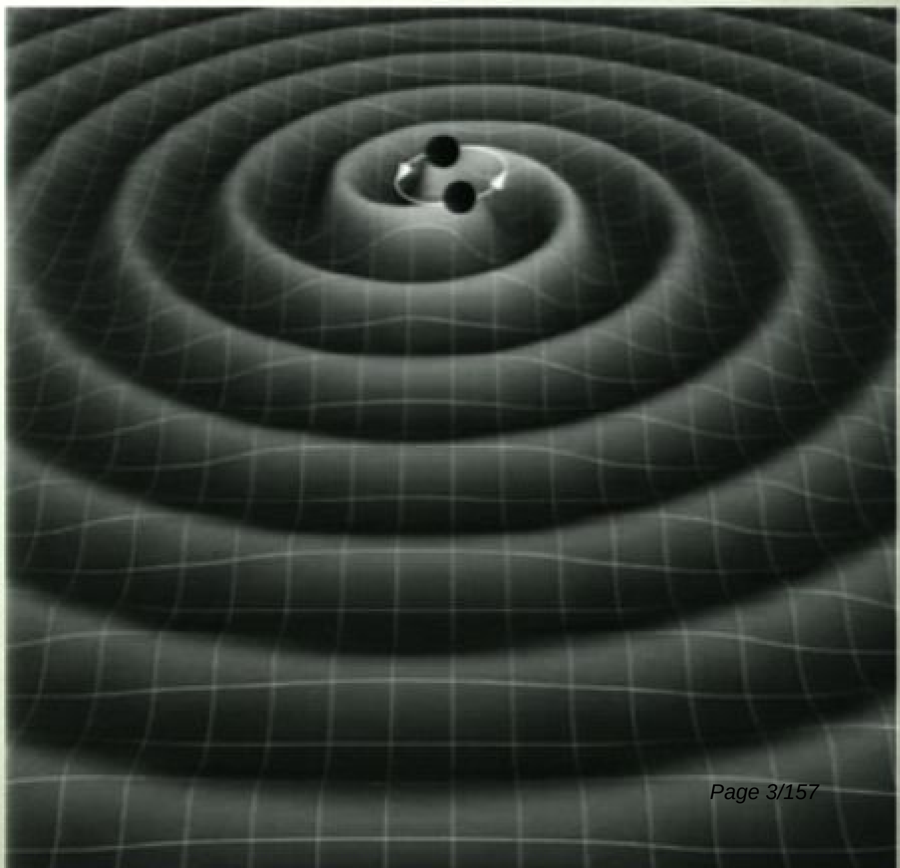
Discussion of Upcoming 2 Paper Series

Introduction: The Setting

A long time ago in a galaxy far, far away....

there was SMBH binary merger...

and it radiated grav. waves.



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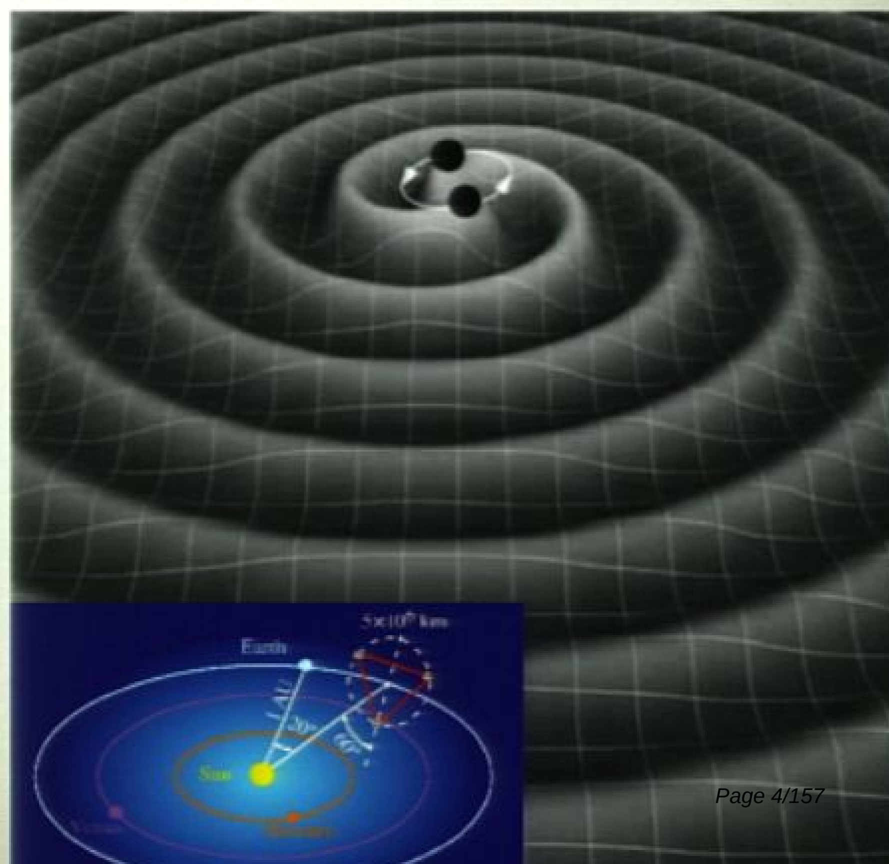

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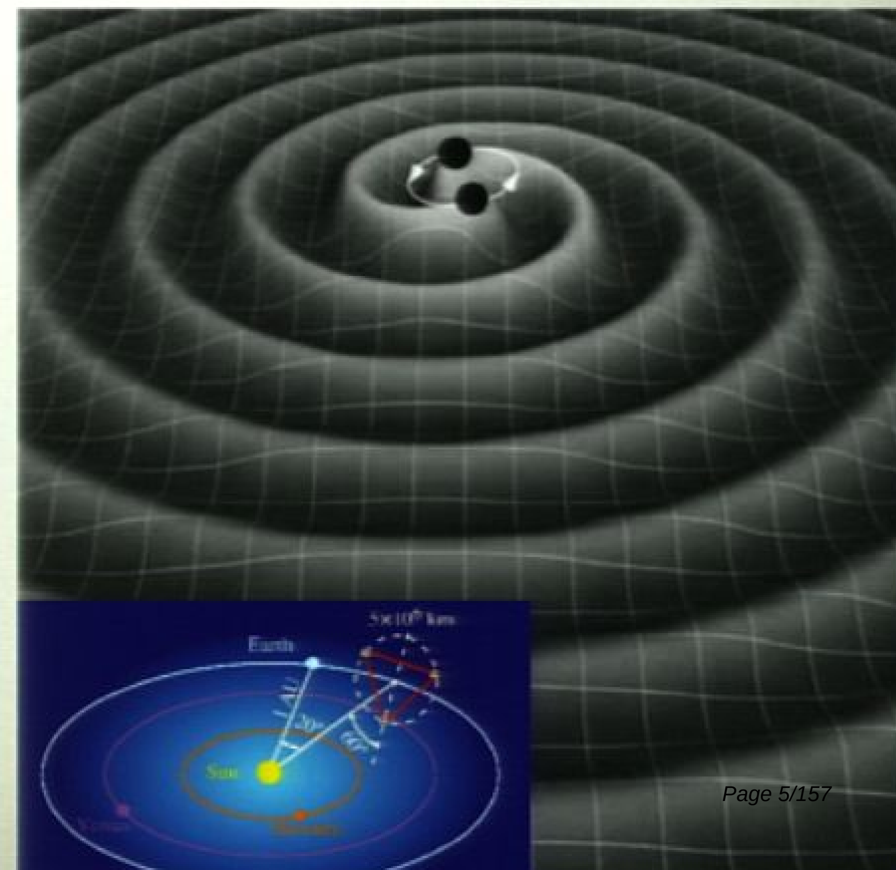
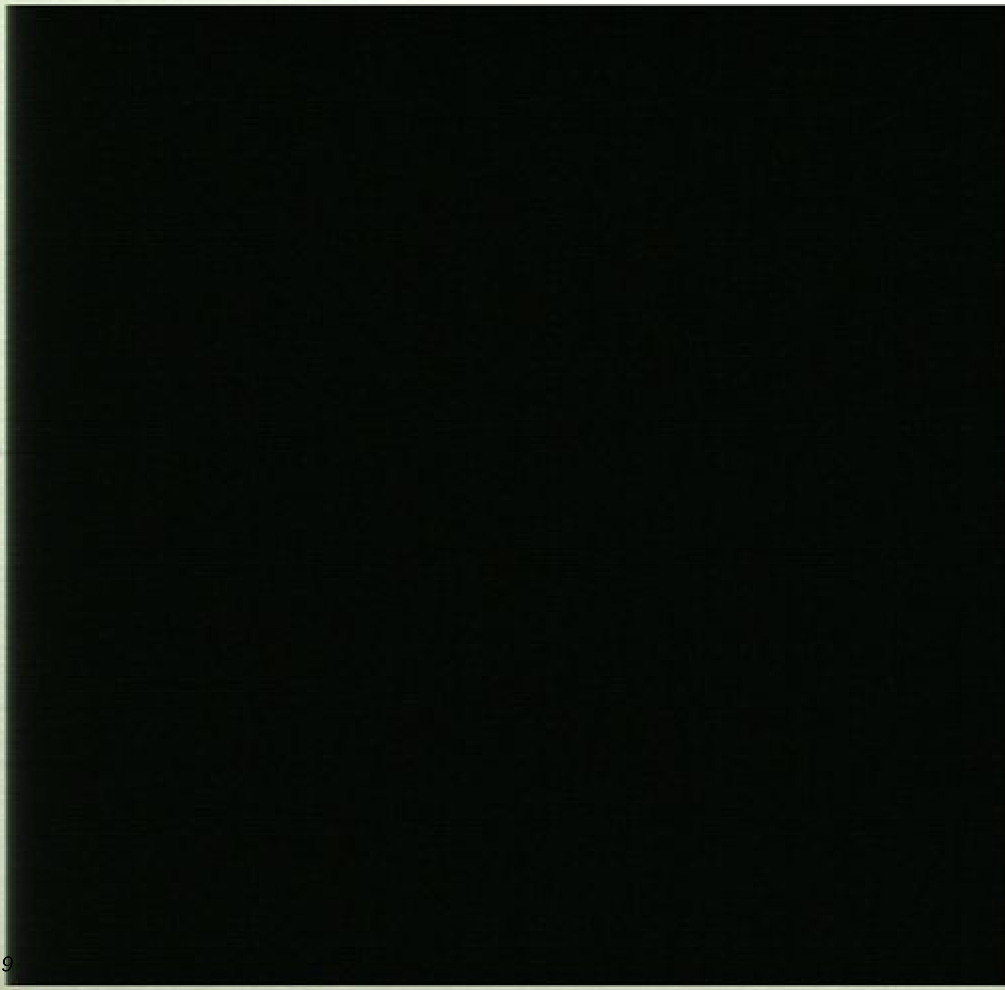


Fortunately, there was LISA to detect it...

And LISA told us where to look in the sky to see it...

INTRODUCTION: THE SETTING

It's somewhere in a 1deg x 1deg field of view:



Hubble Ultra Deep Field x324

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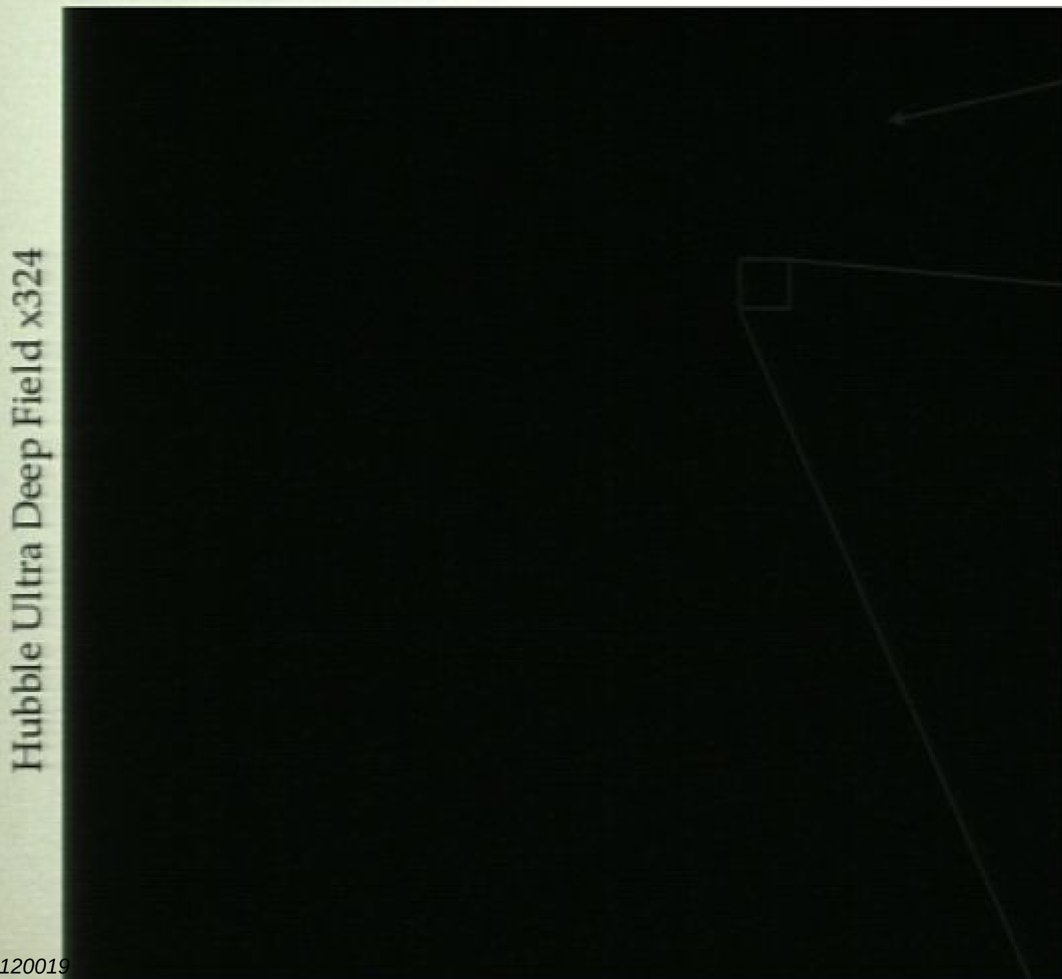

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But, why would we bother?

EM COUNTERPARTS: WHY?

An EM counterpart to a GW signal ...

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- ... finds the host galaxy, helping us understand

  1) the D_L-z relation to extract Ω_M and Ω_Λ. (Holz & Hughes 05)

  2) galaxies having undergone recent mergers (even if we can't tell based on morphology)

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- ... gives an unprecedented opportunity to view all subsequent transient signals from beginning to end.

What I'm Talking About

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- One of the more likely prompt EM sources, is a gaseous disk surrounding the binary.
- I am presenting both an analytic solution to and a numerical study of the evolution of such a circumbinary disk just following merger.

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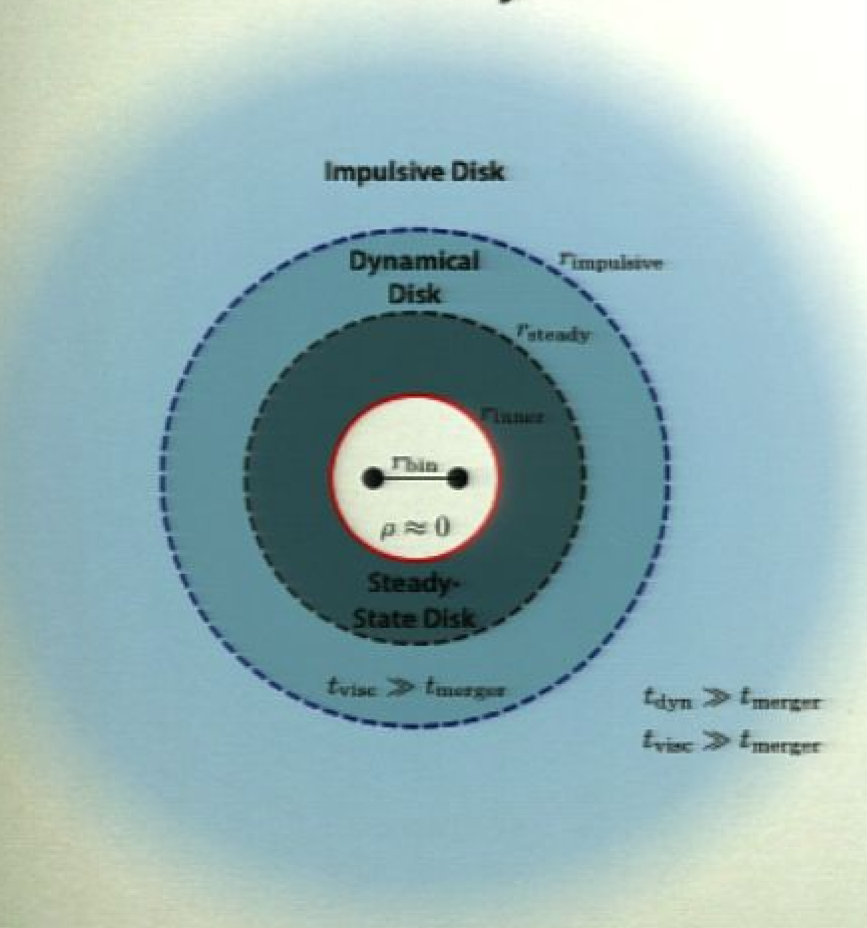

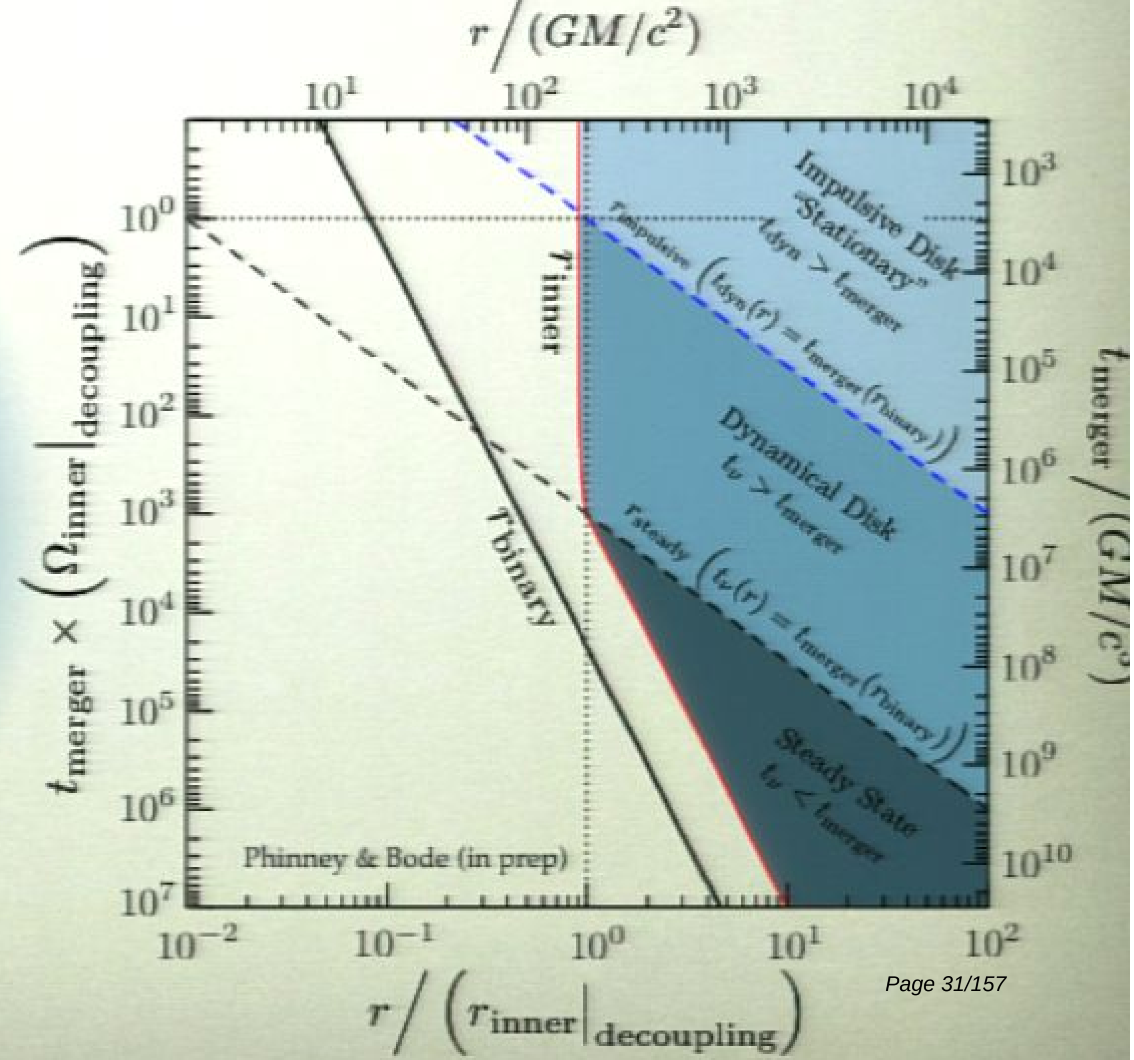

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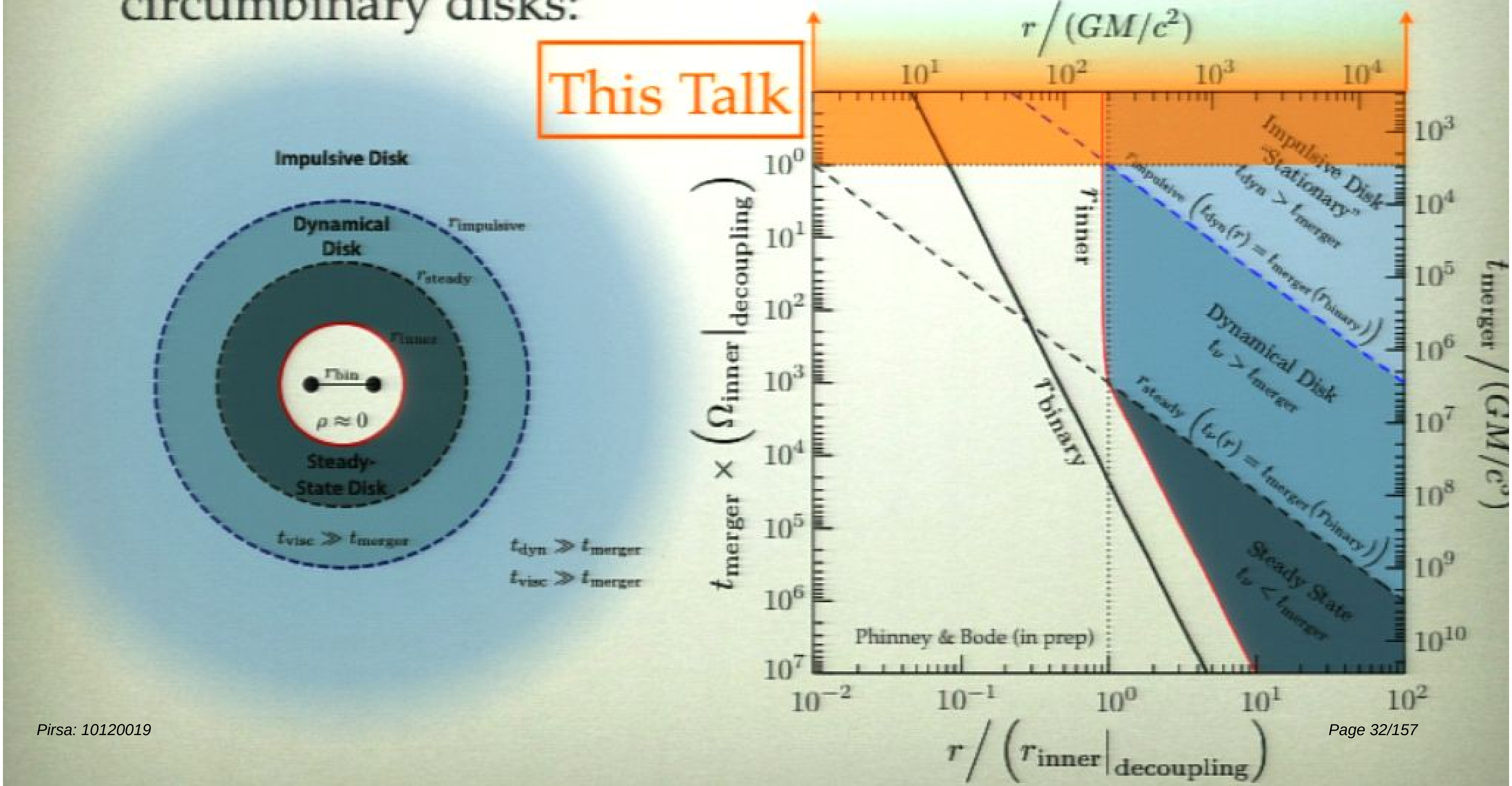

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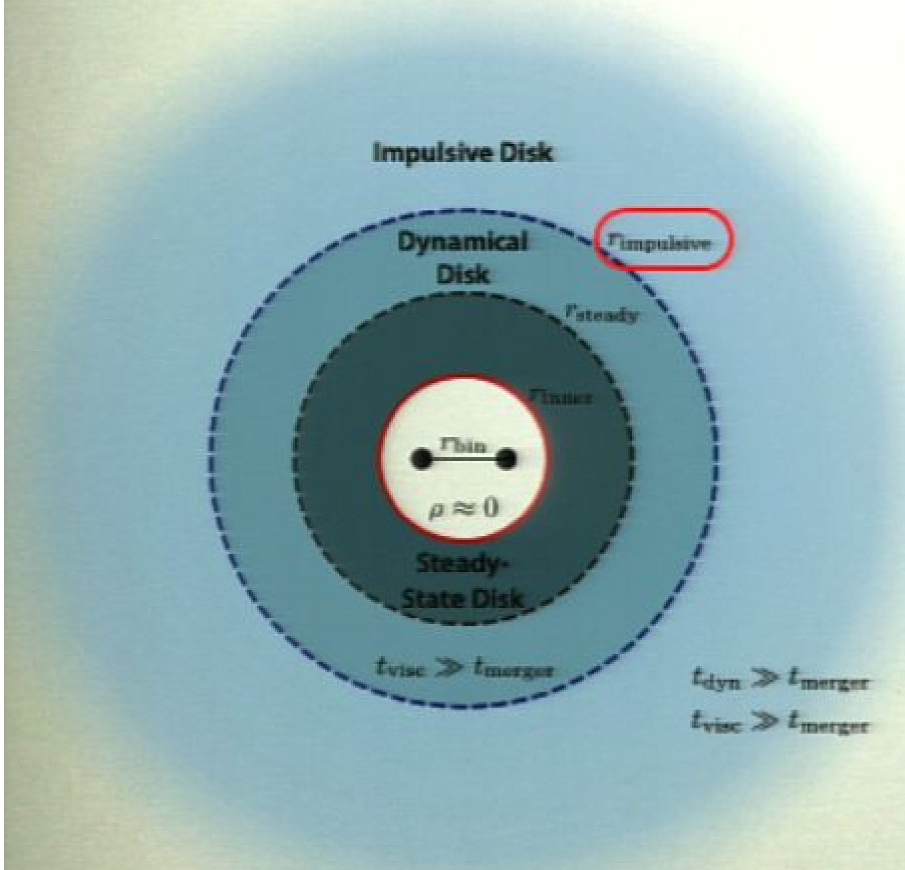

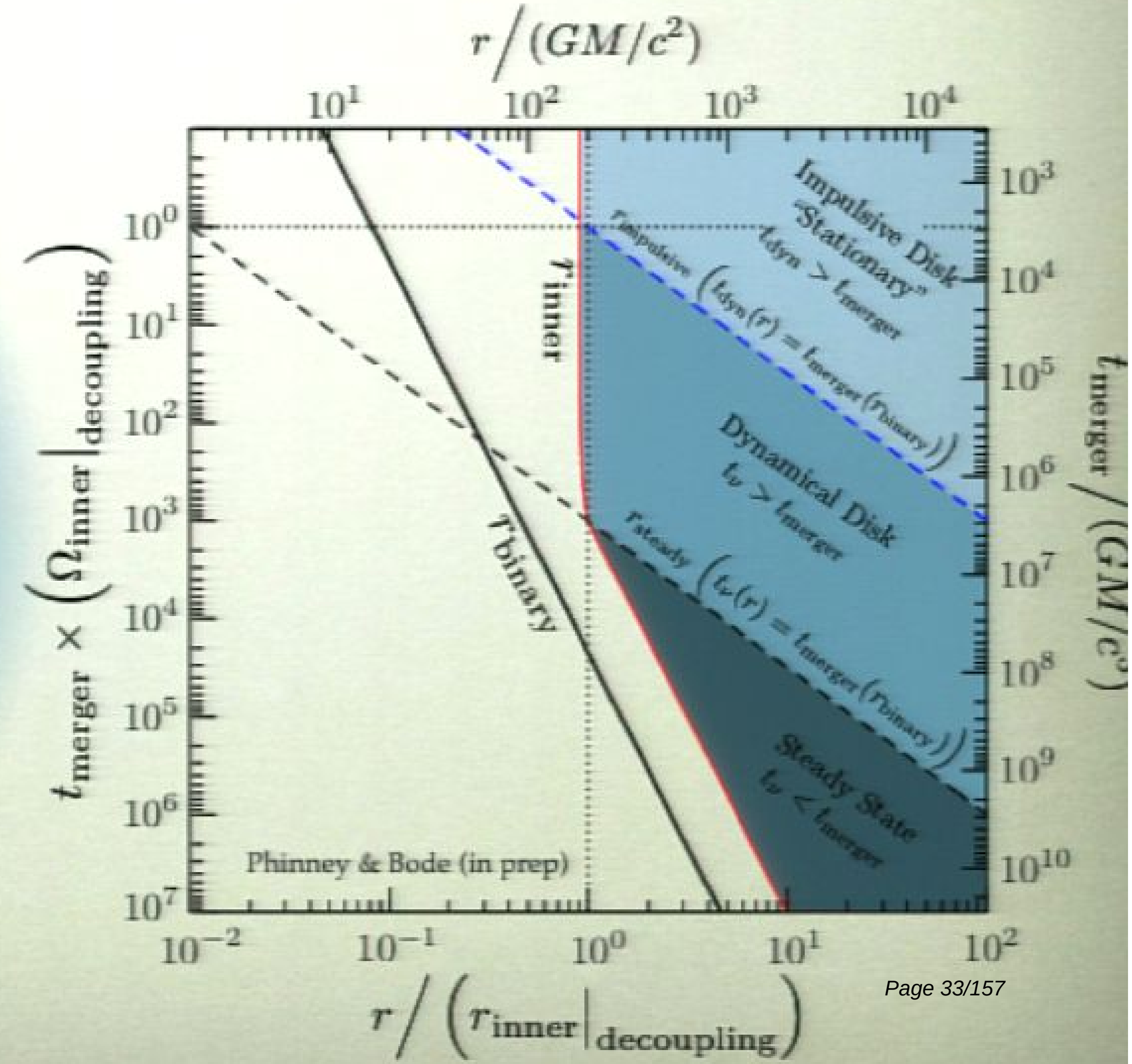

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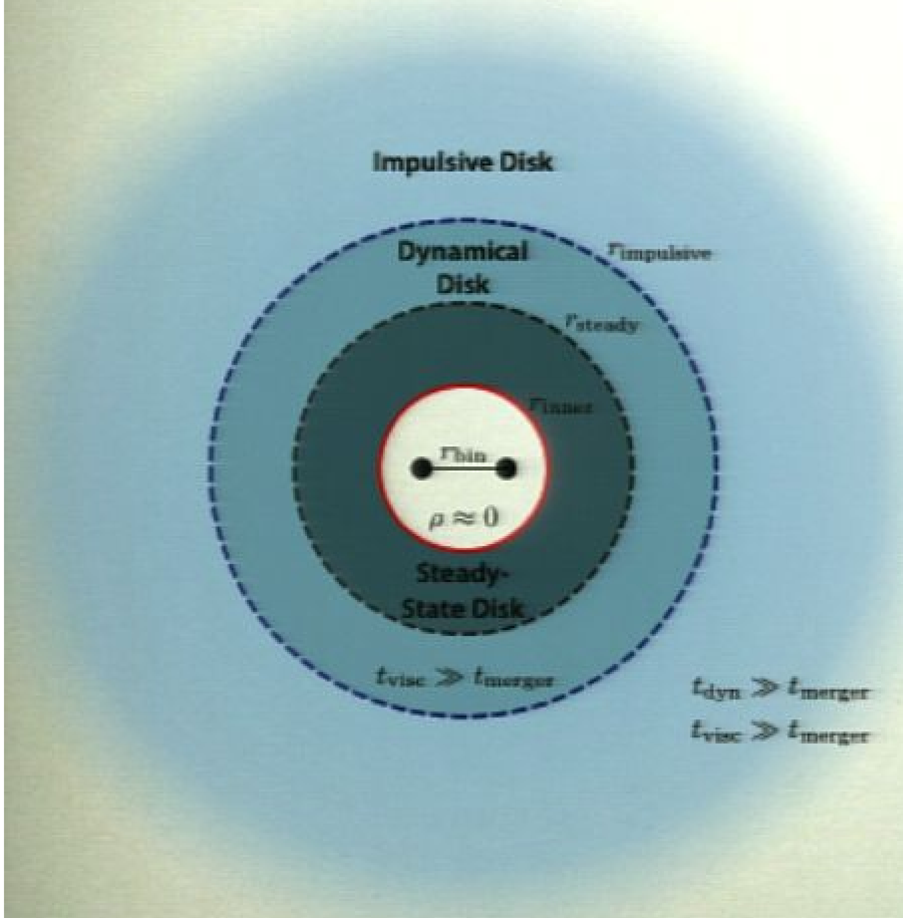

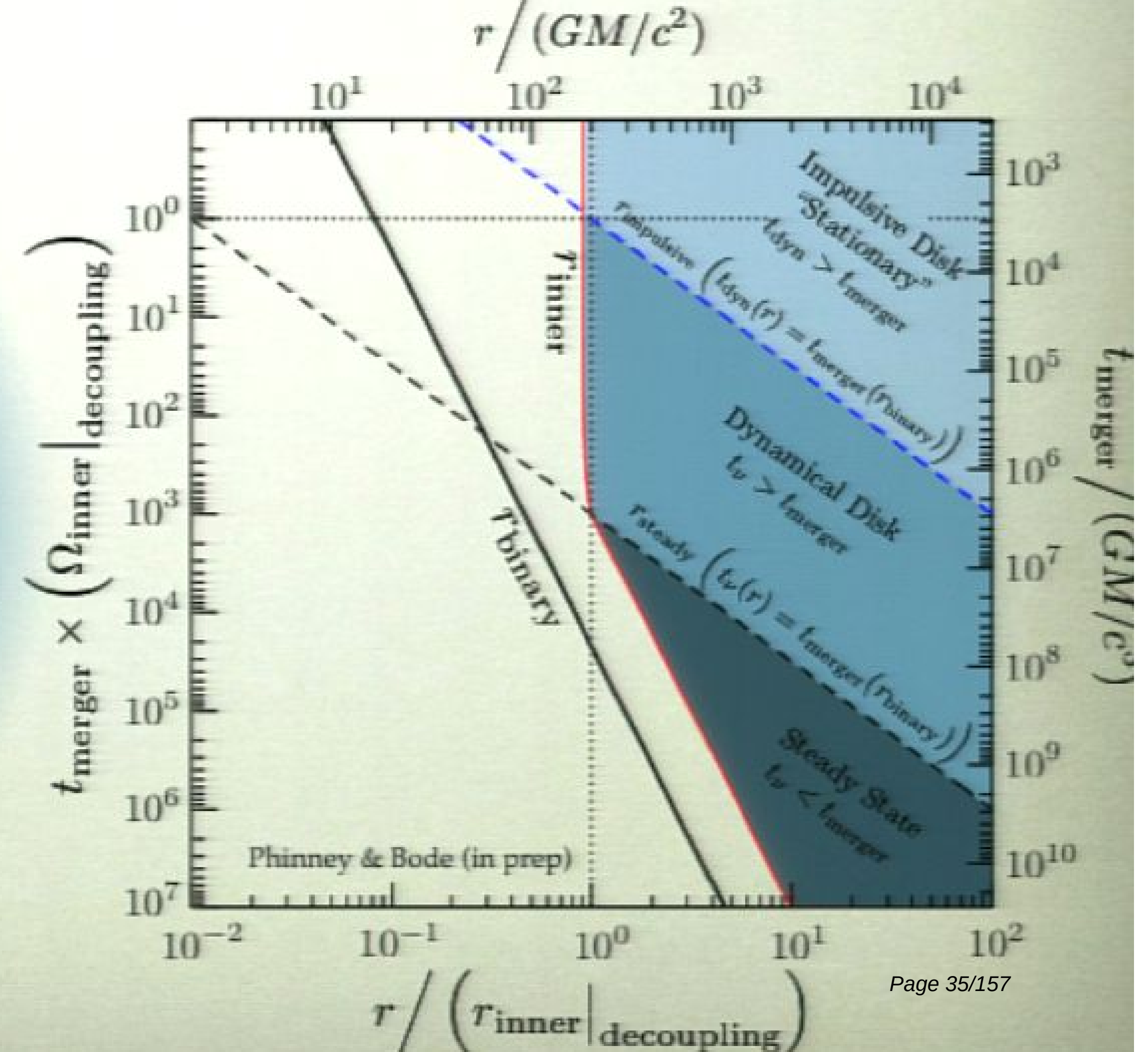

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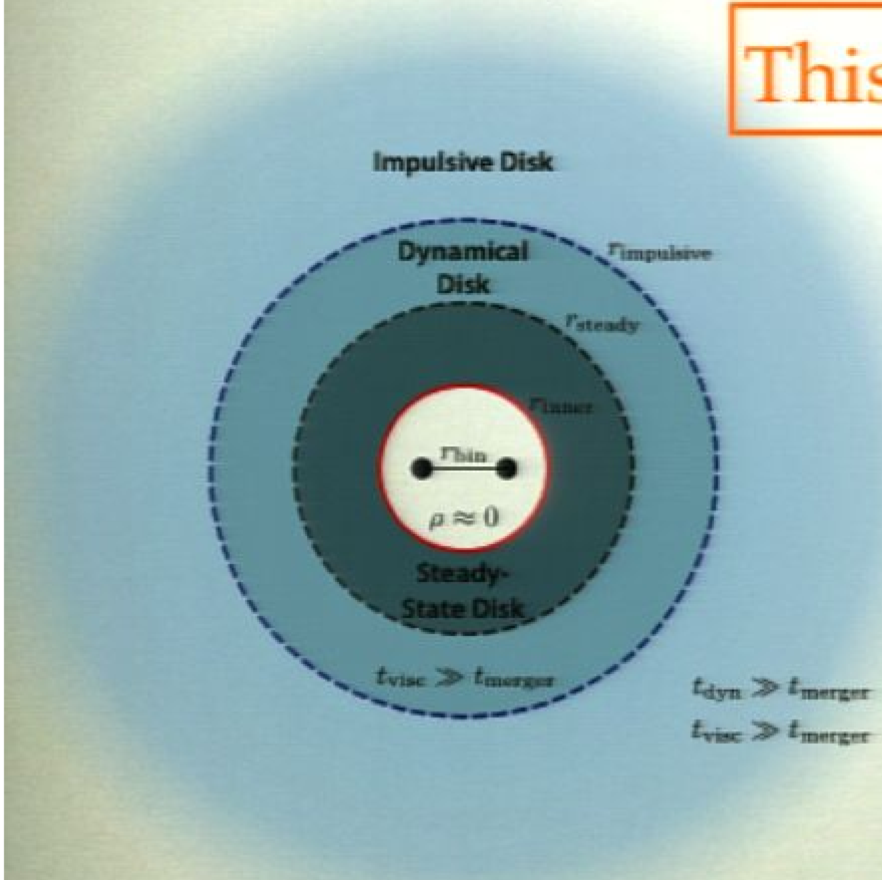
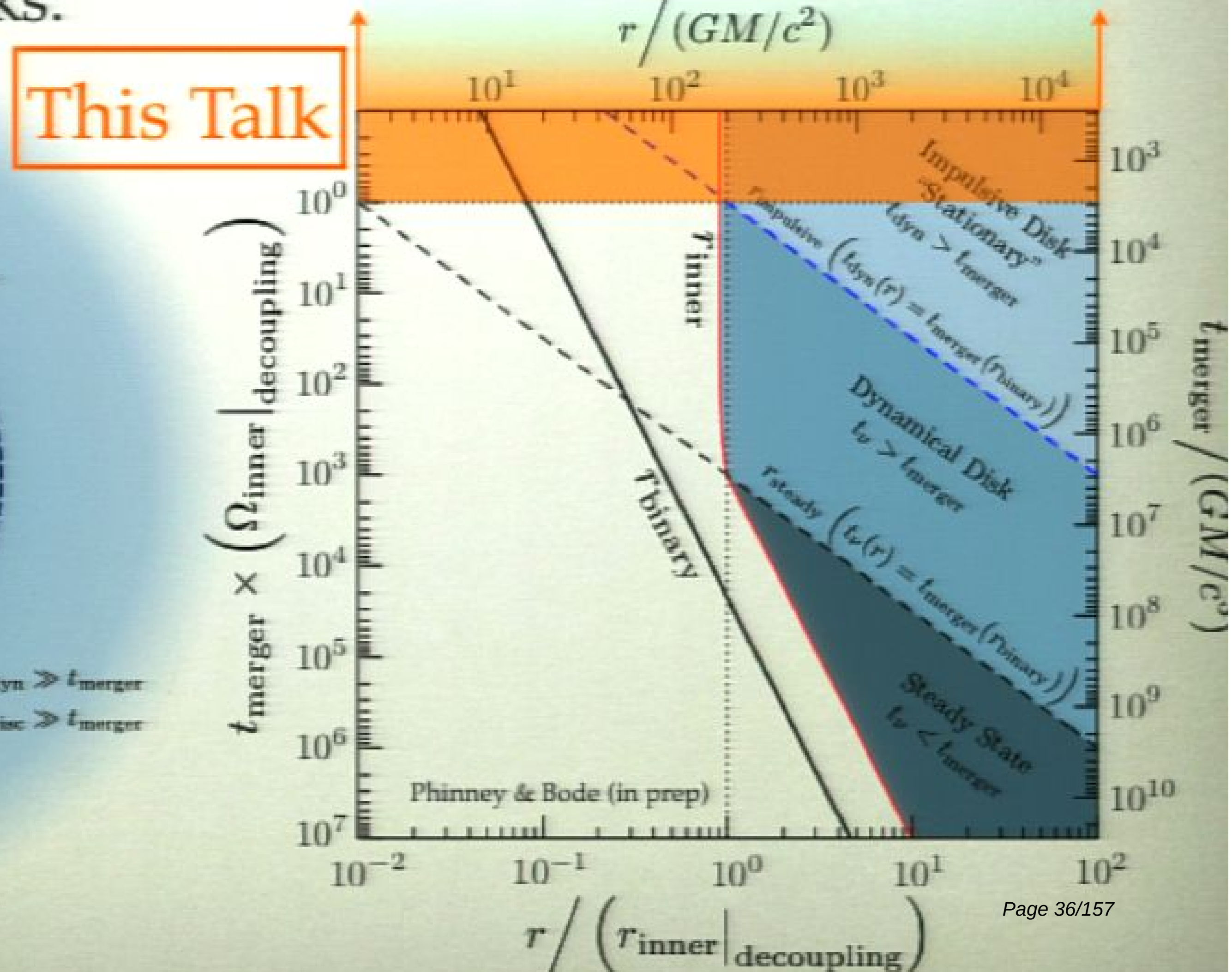
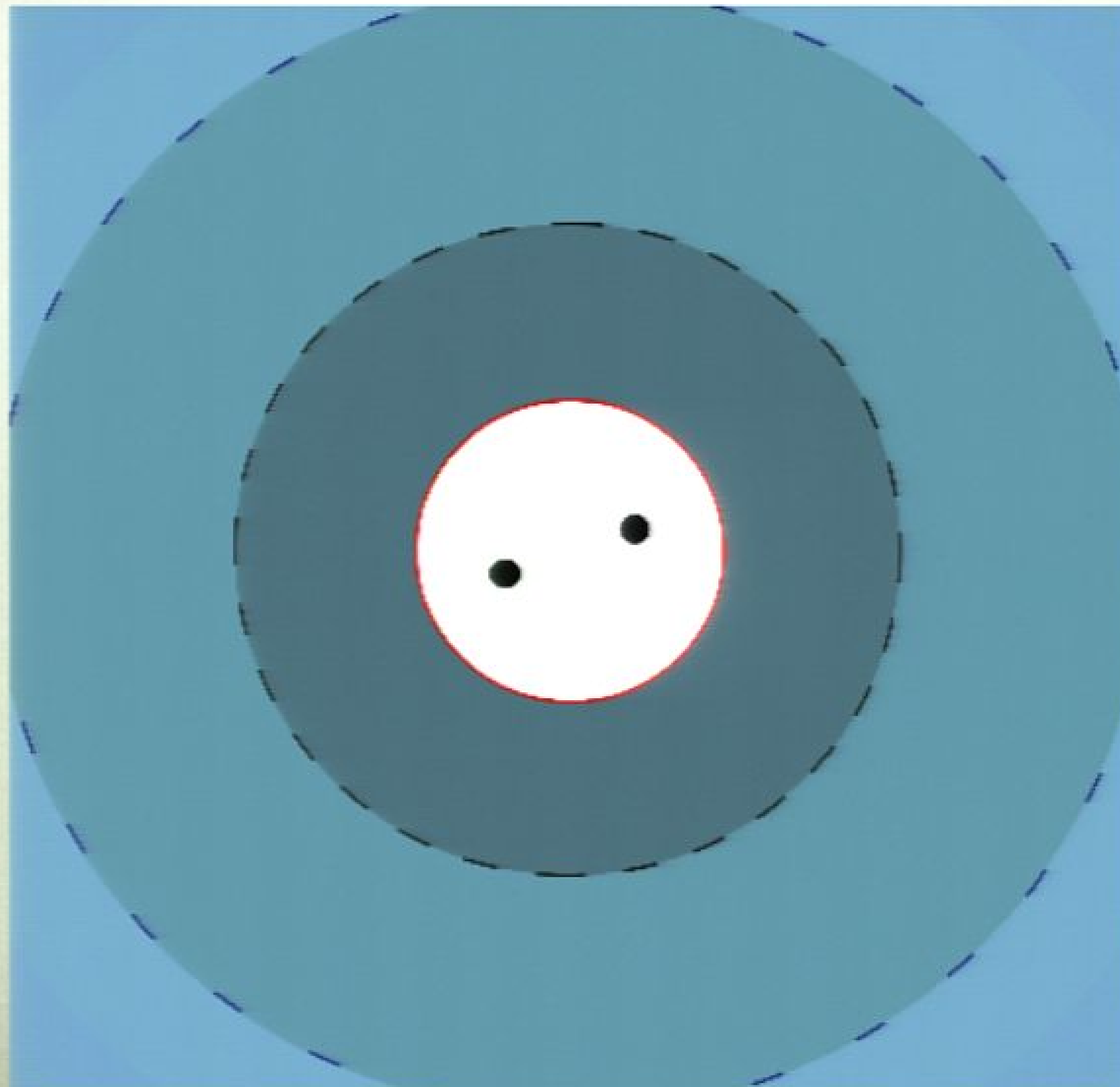




Cartoon of Premerger Evolution



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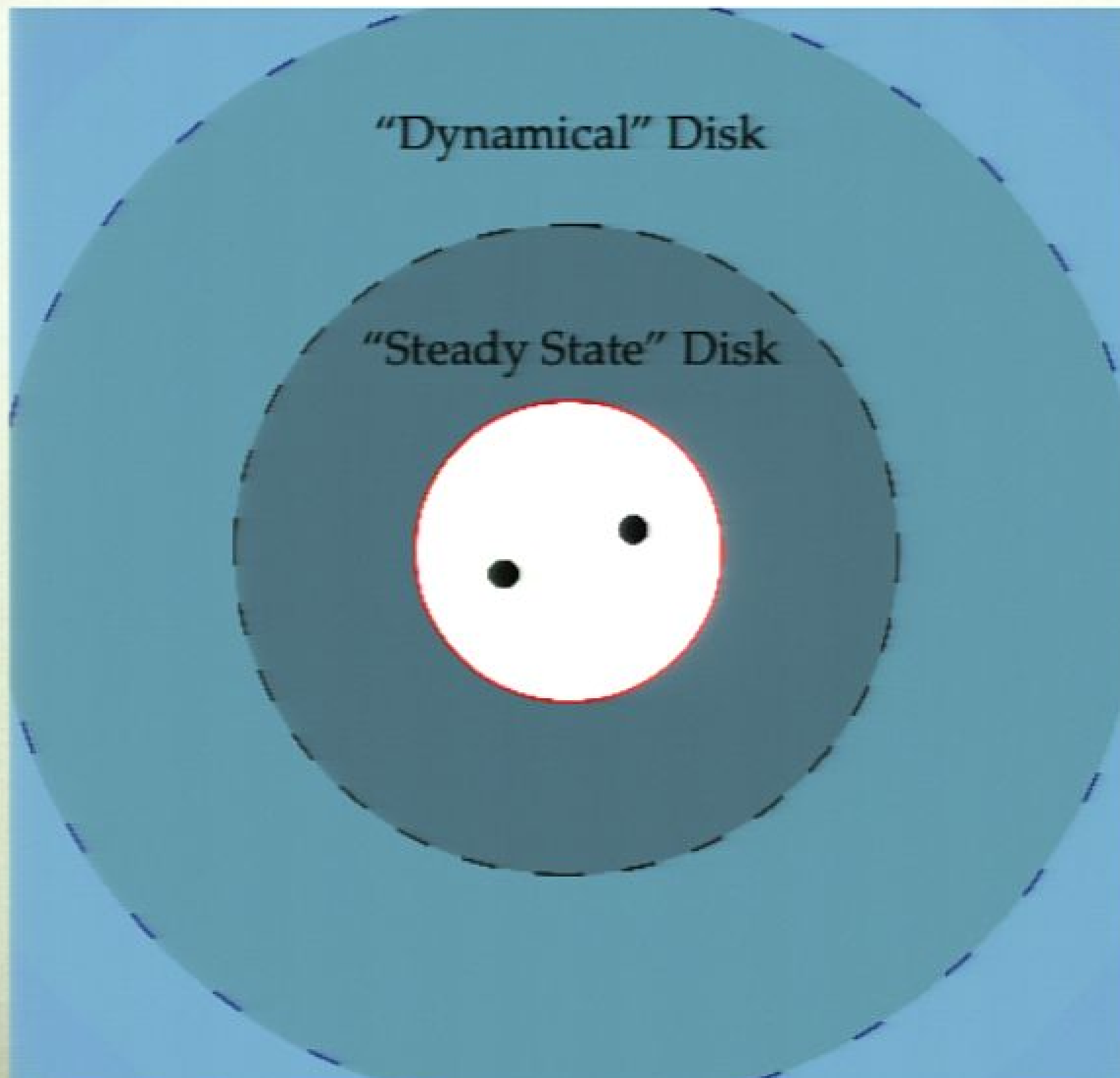

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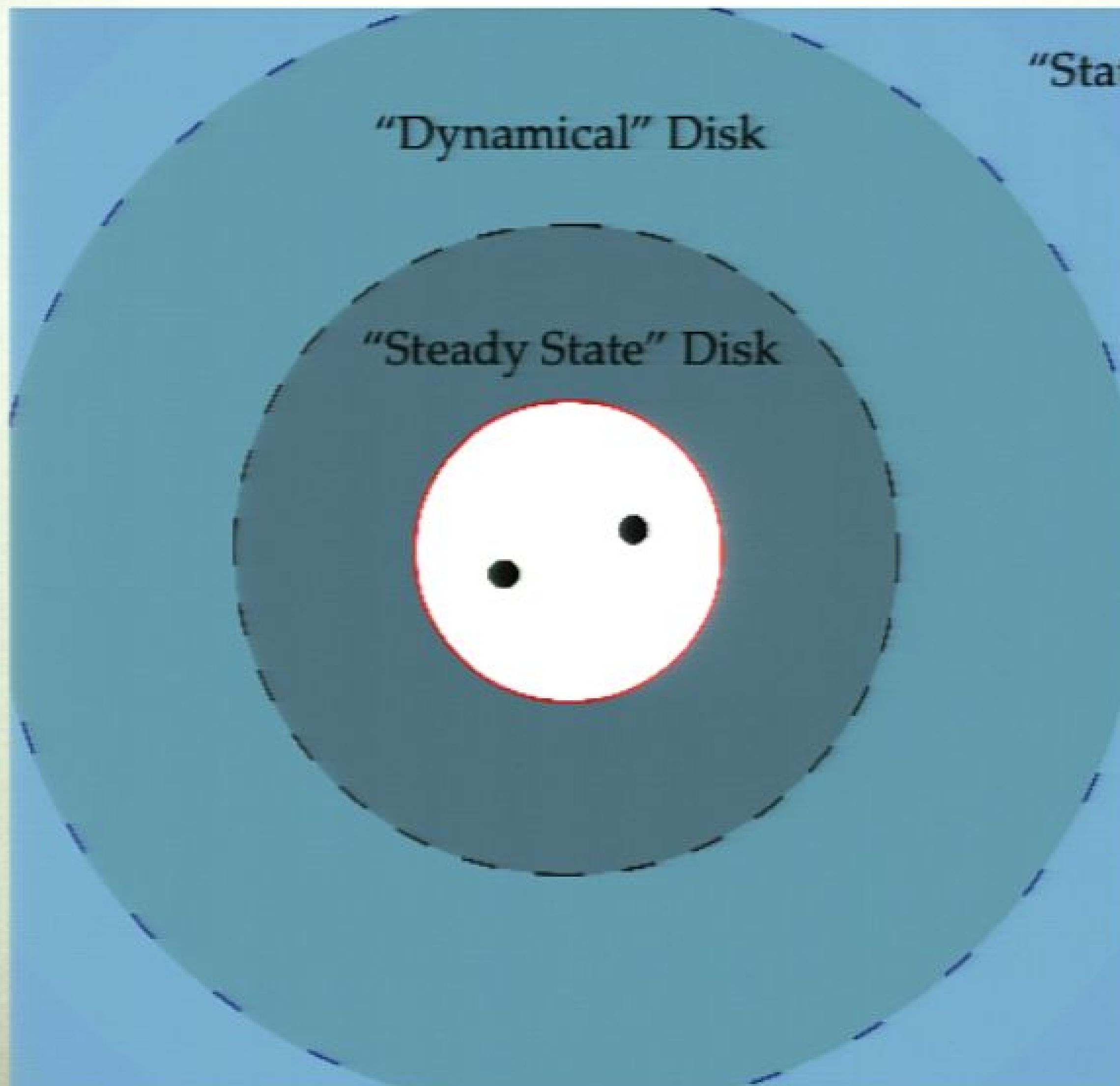

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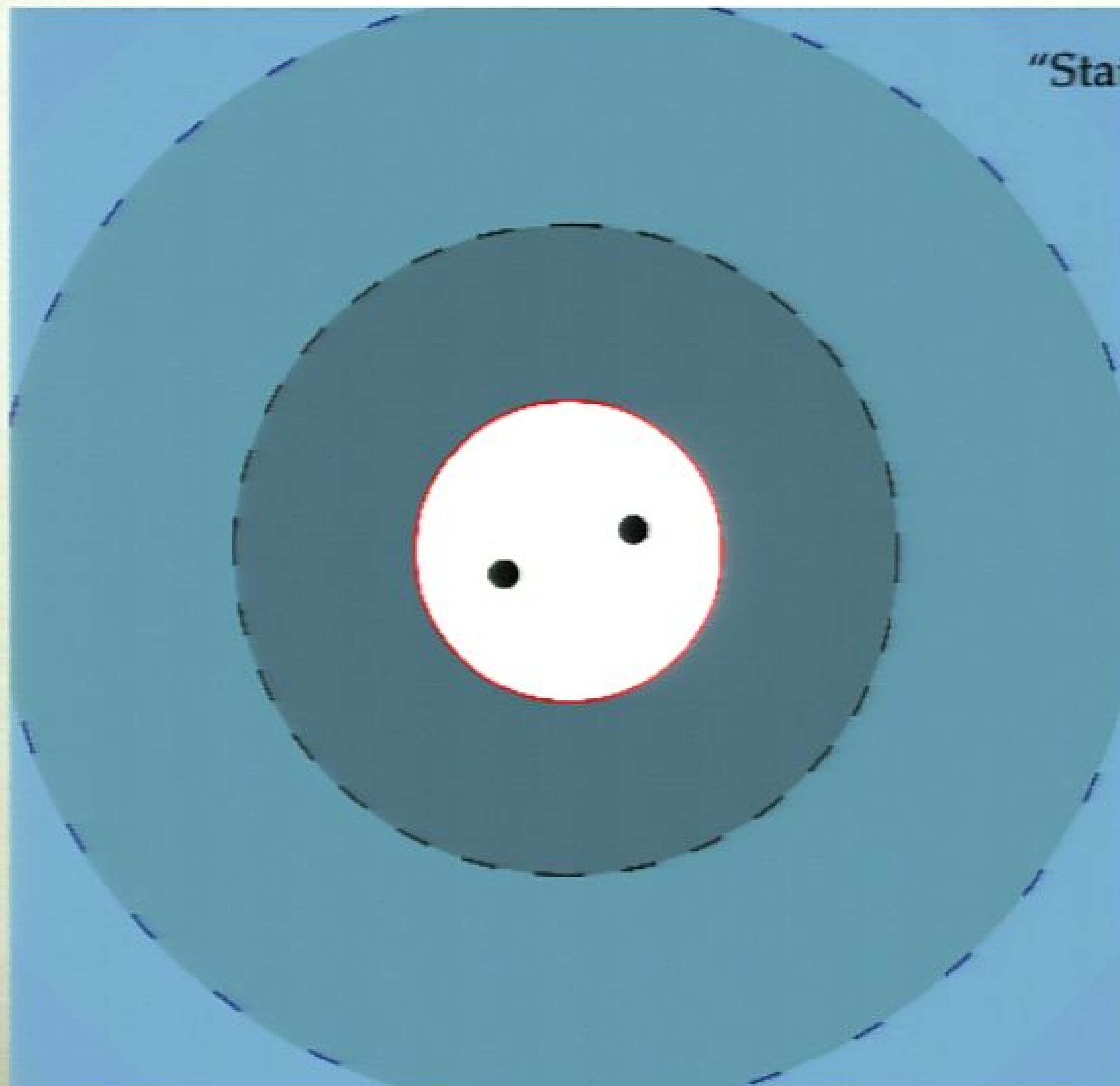

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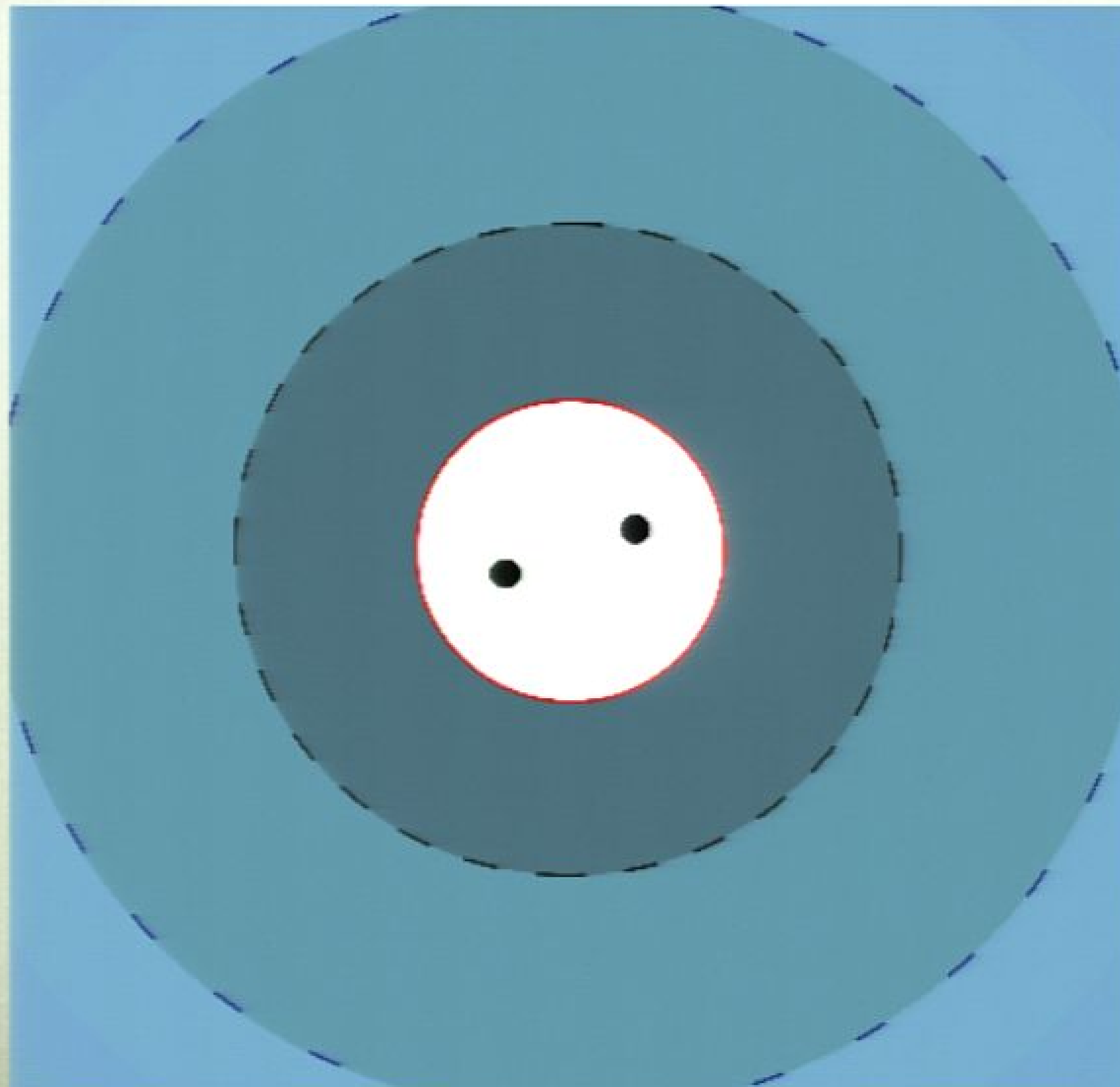

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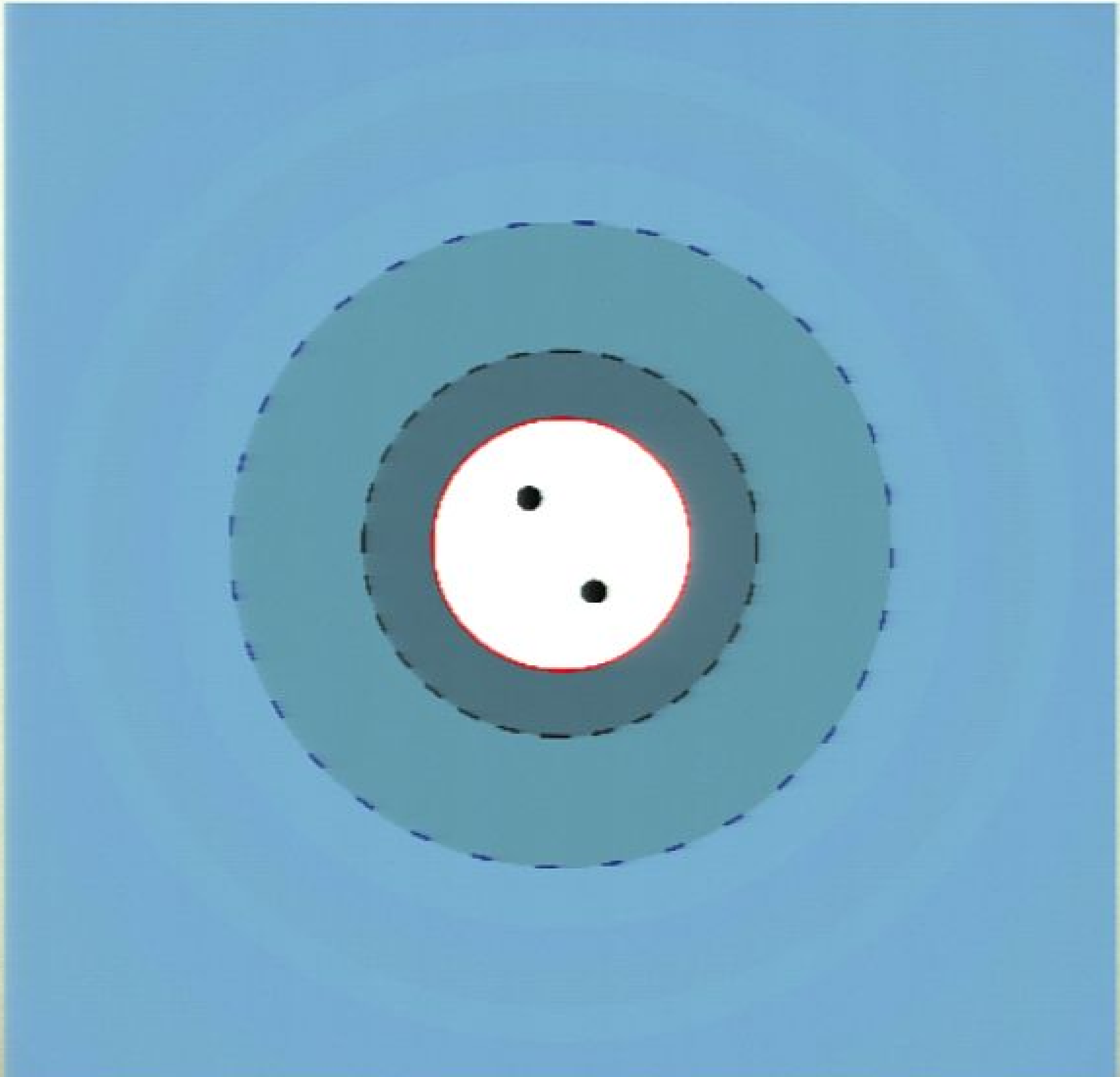

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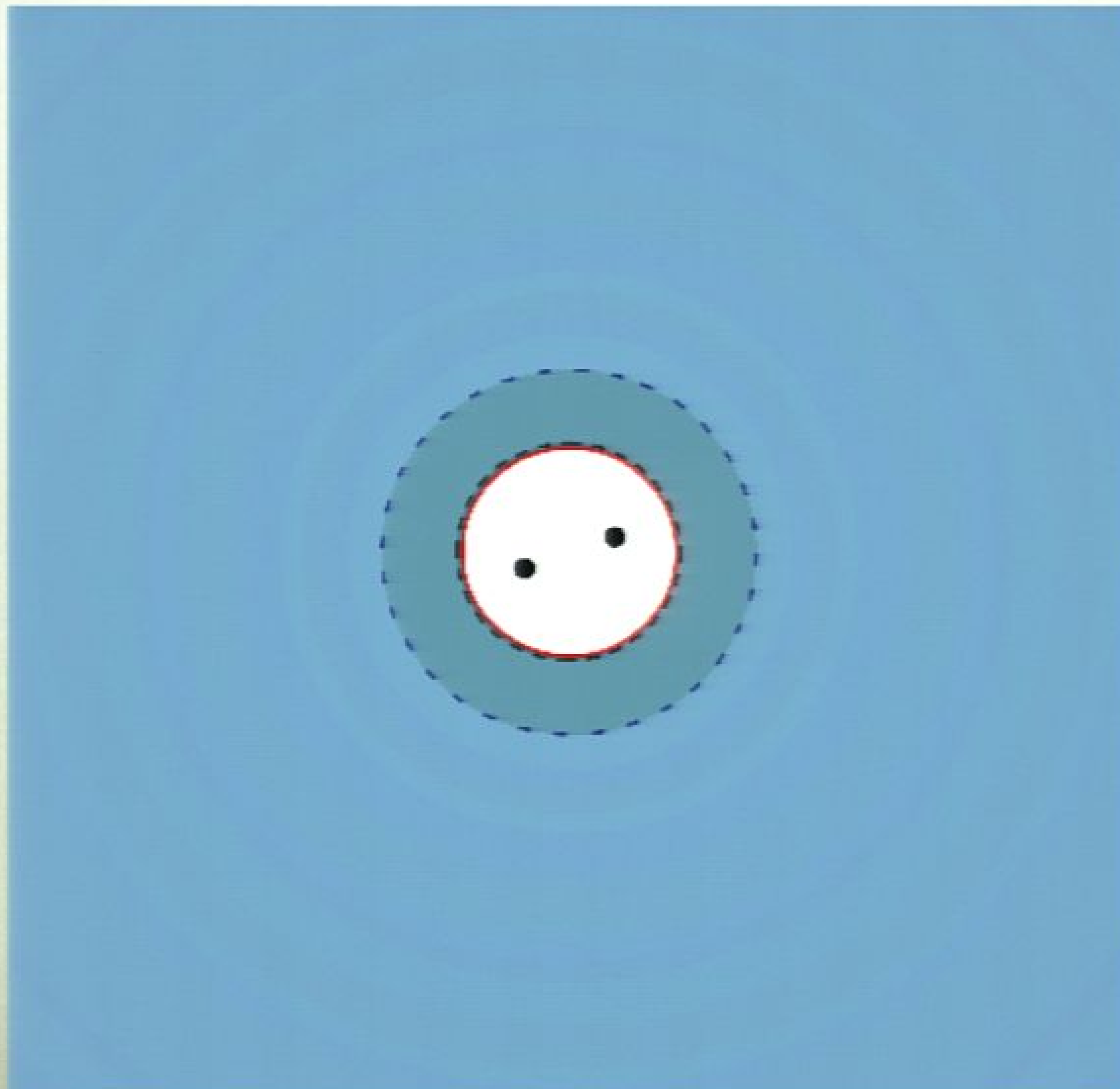

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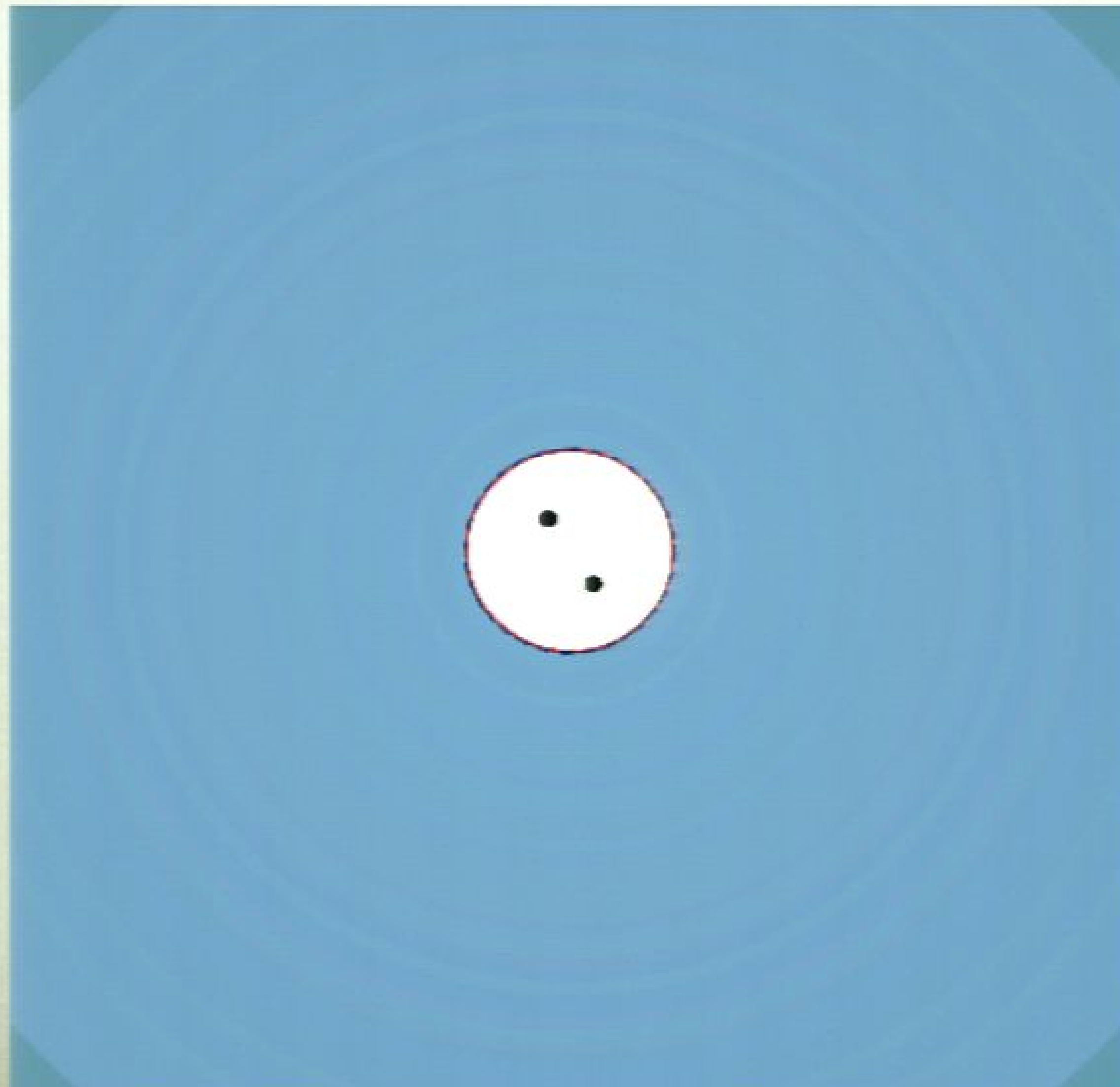

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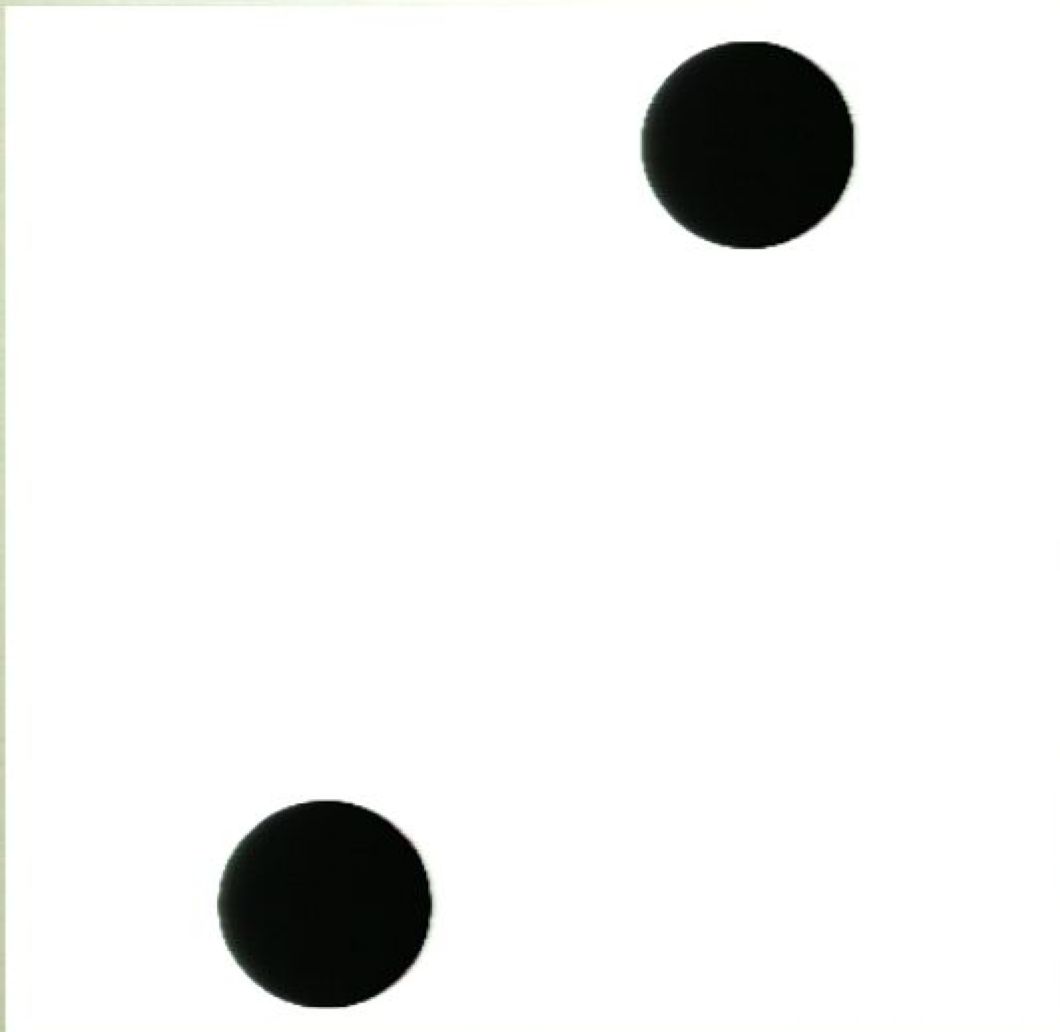

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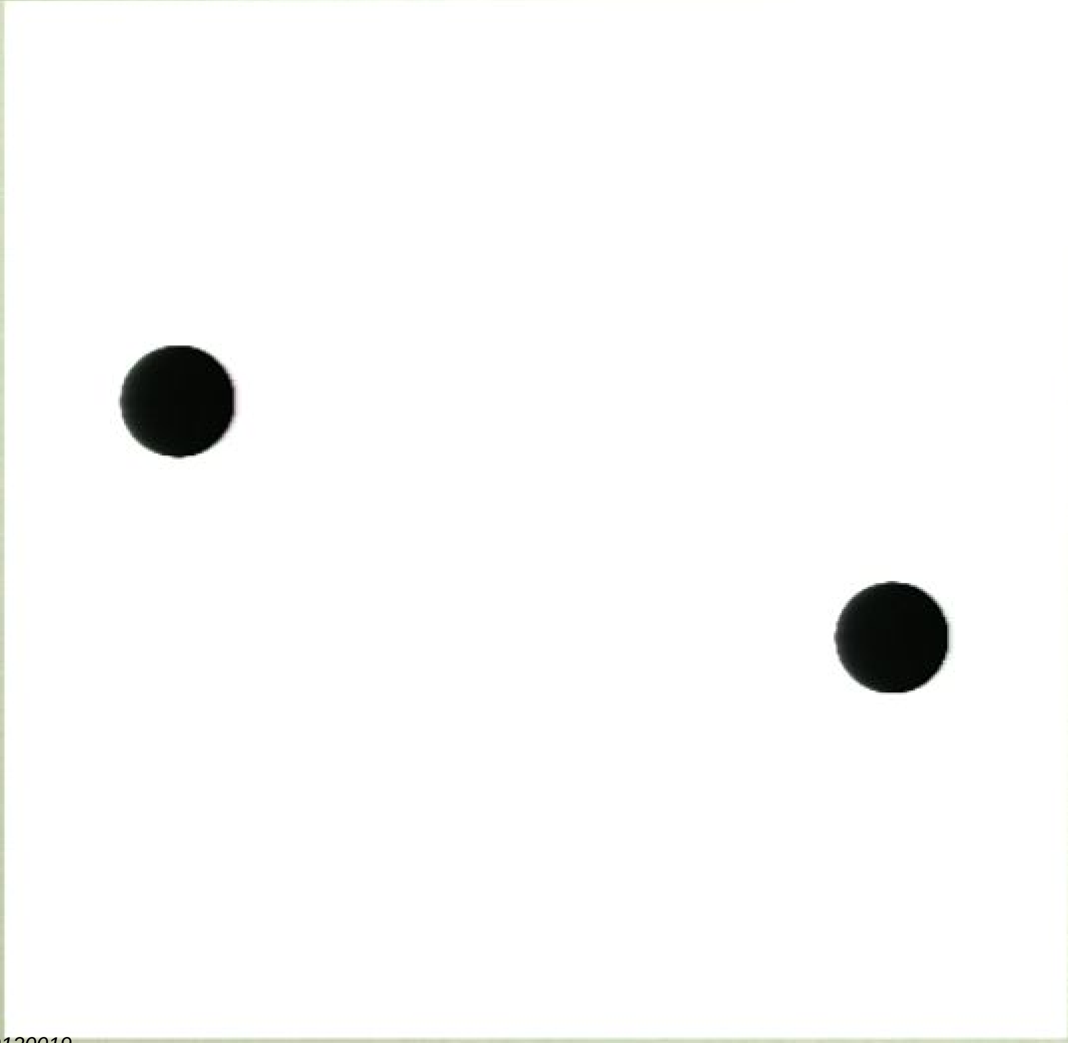

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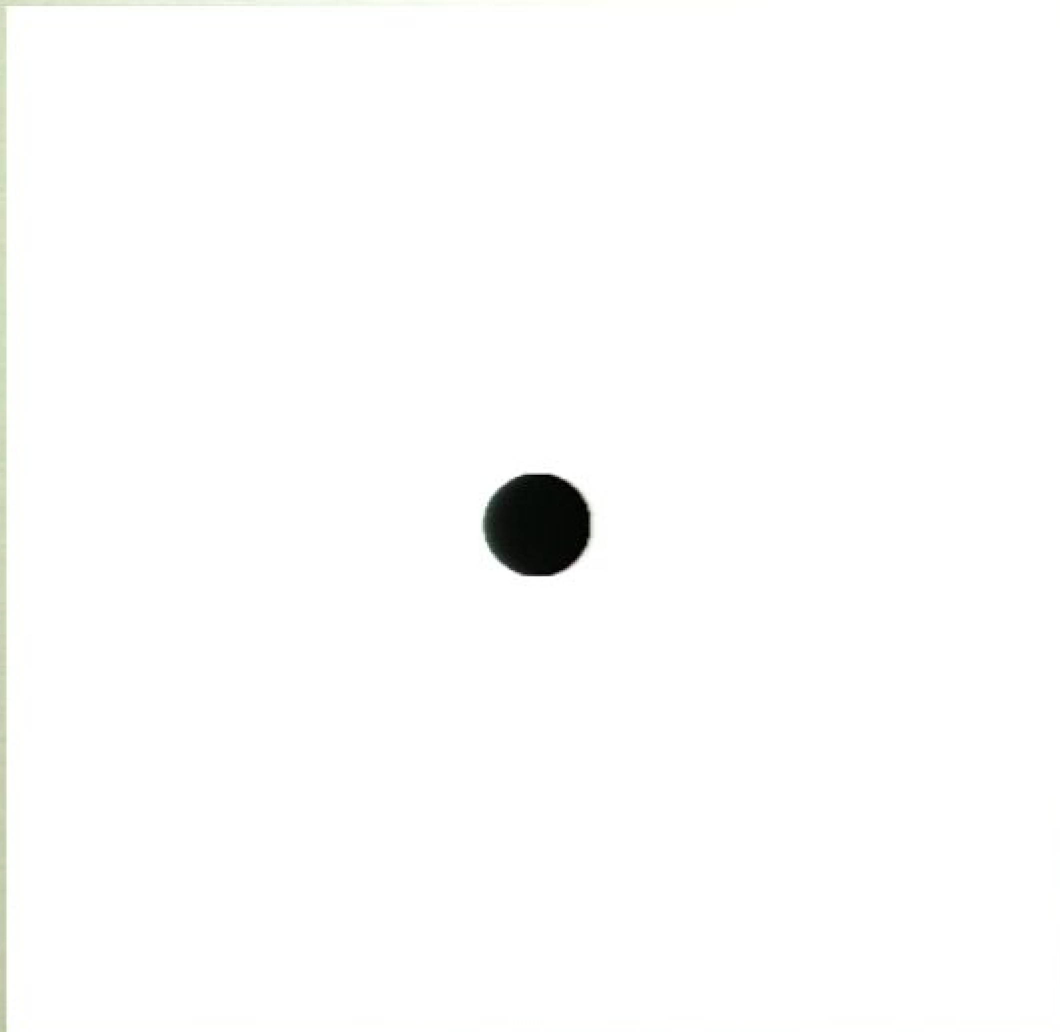

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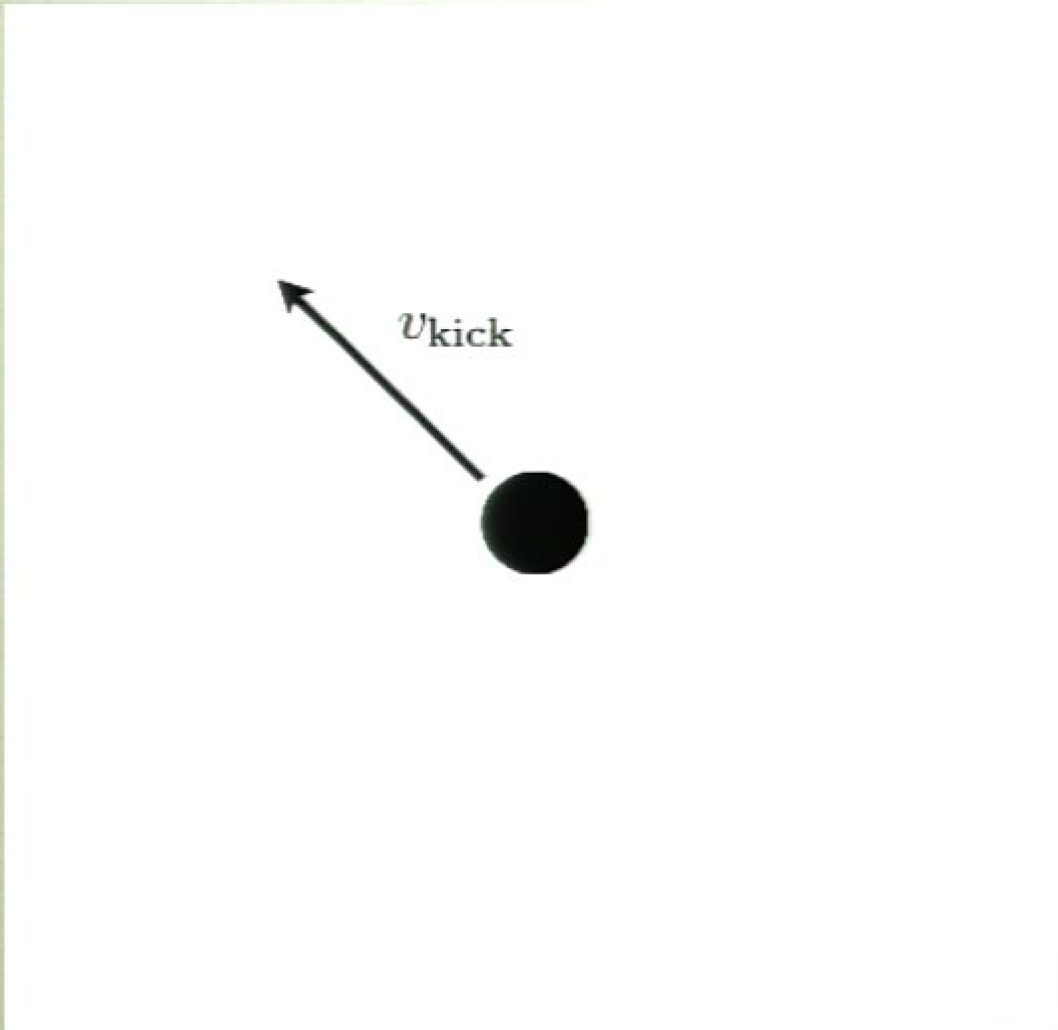

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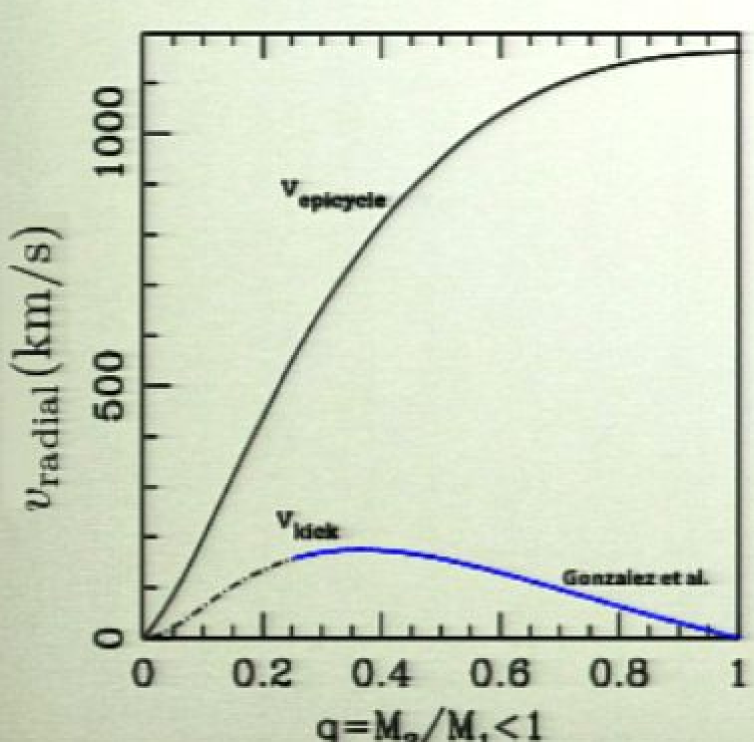
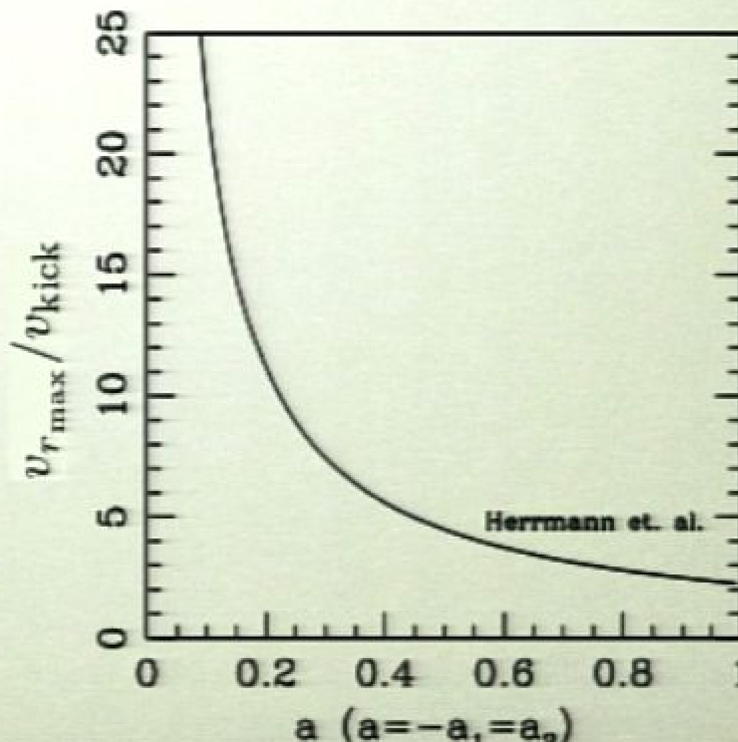
$$v_{\mathrm{kick}}^{(\mathrm{aligned})} \lesssim 200\mathrm{km/s}$$

Analytic Solution: Merger with a Kick

- Initial conditions determine both mass loss and kick magnitude/direction.





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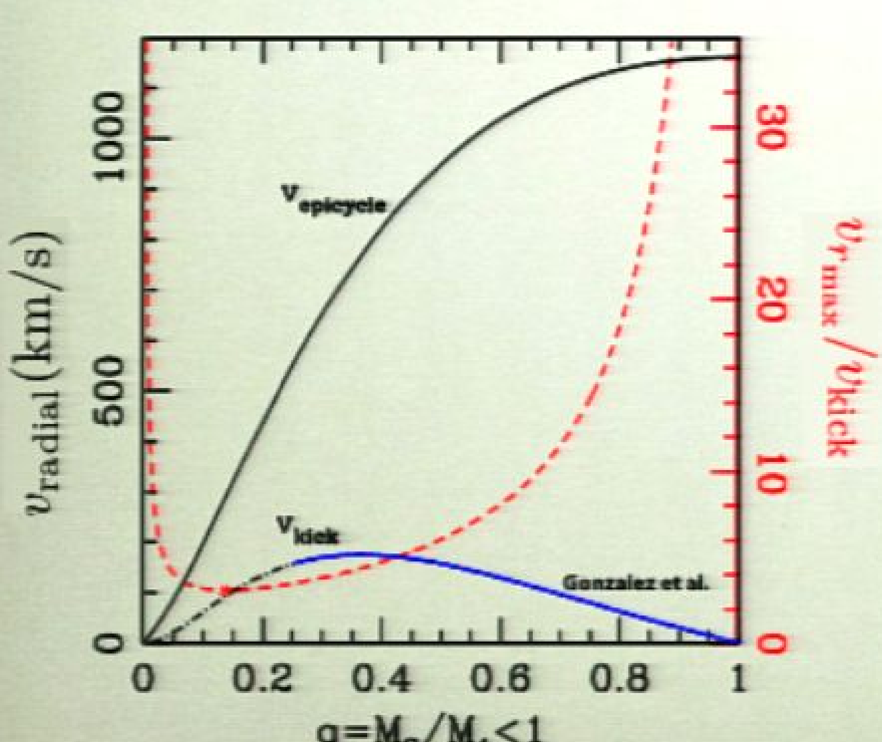

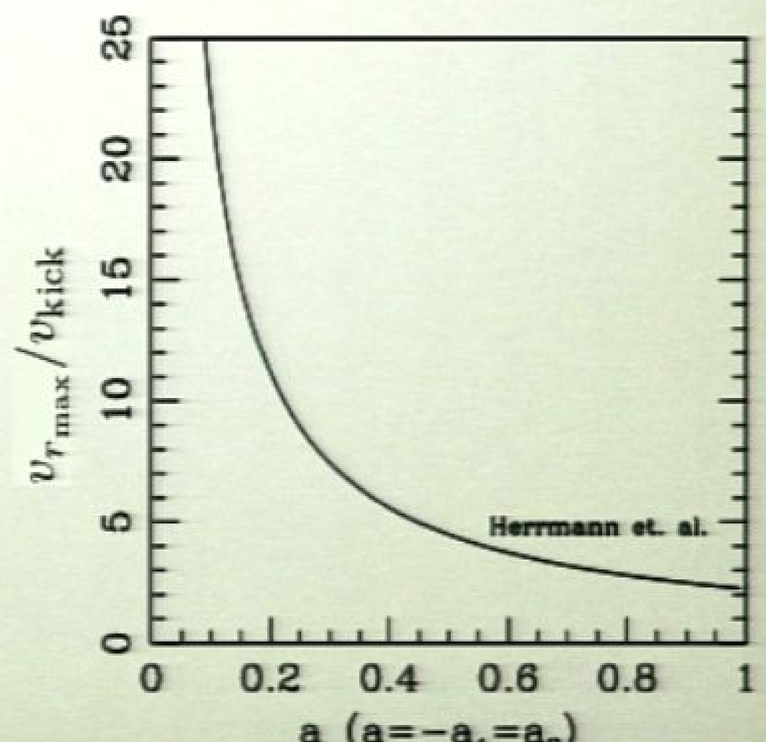

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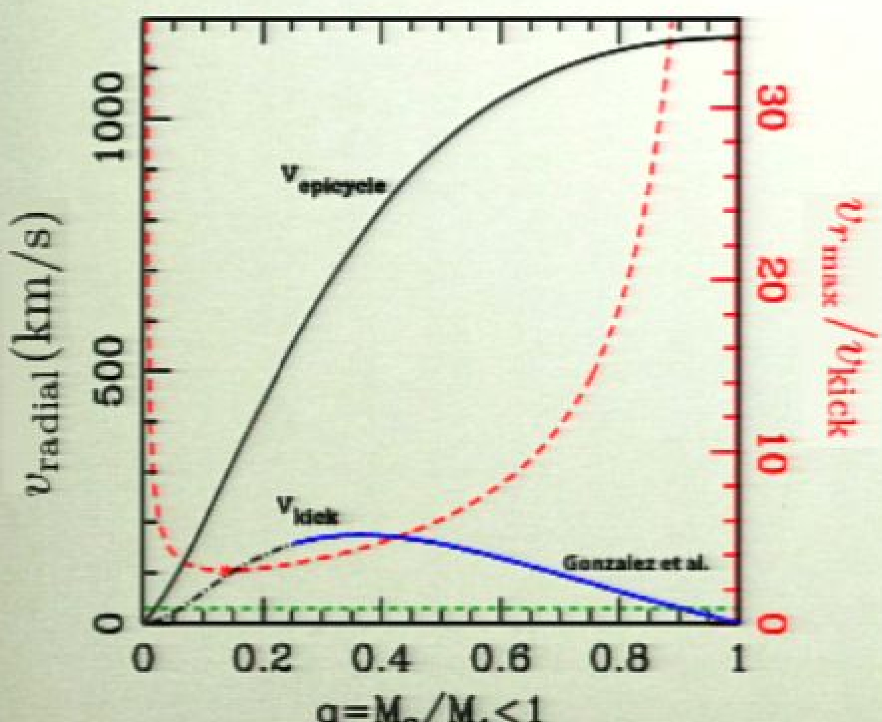
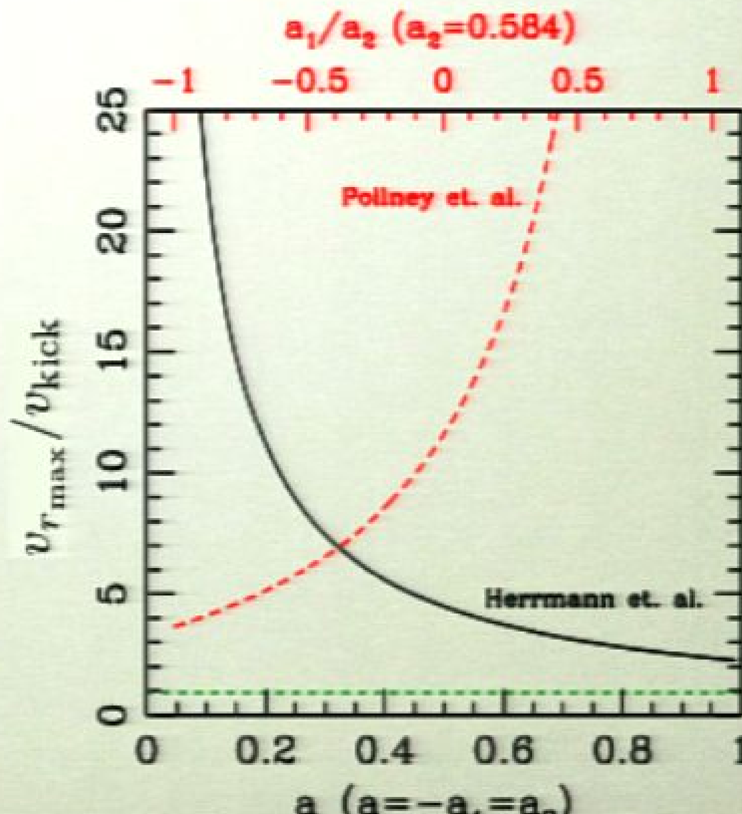
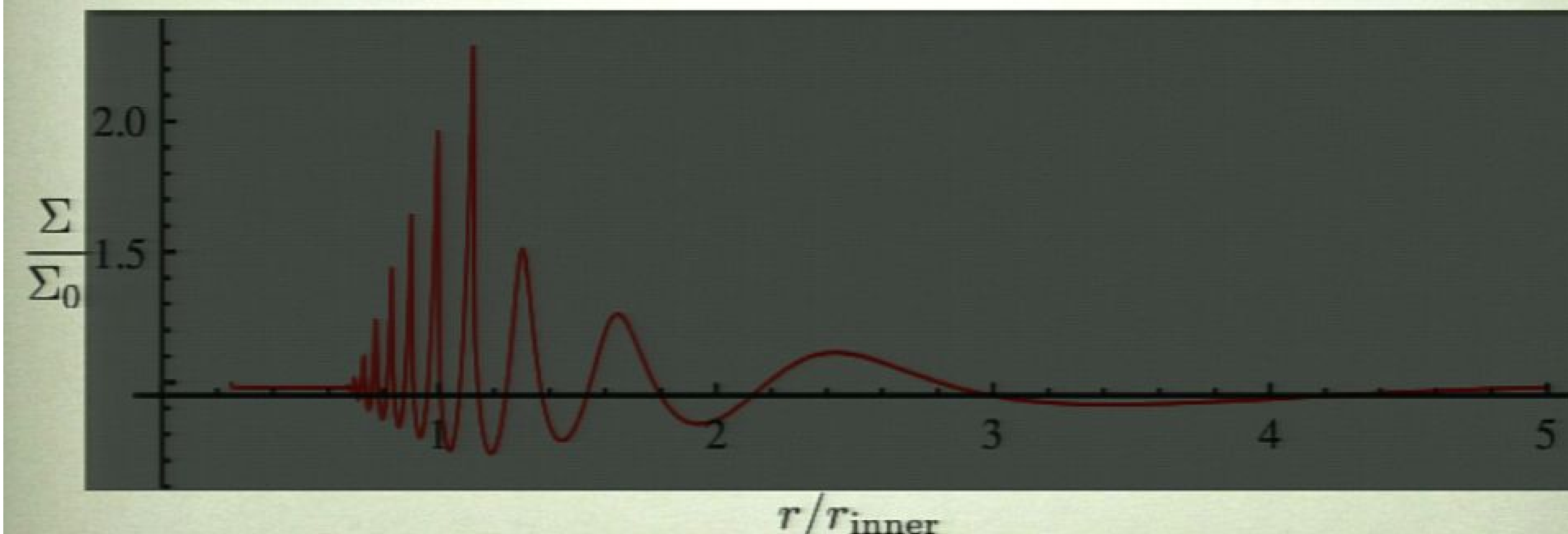




SUMMARY



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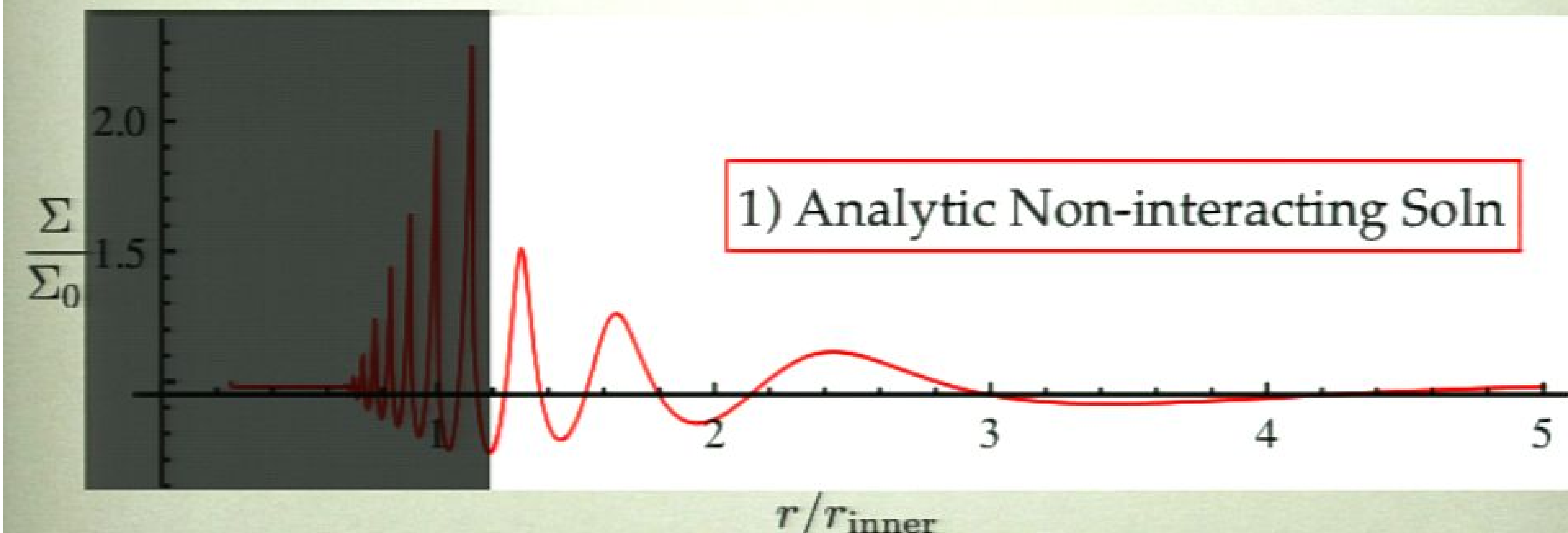
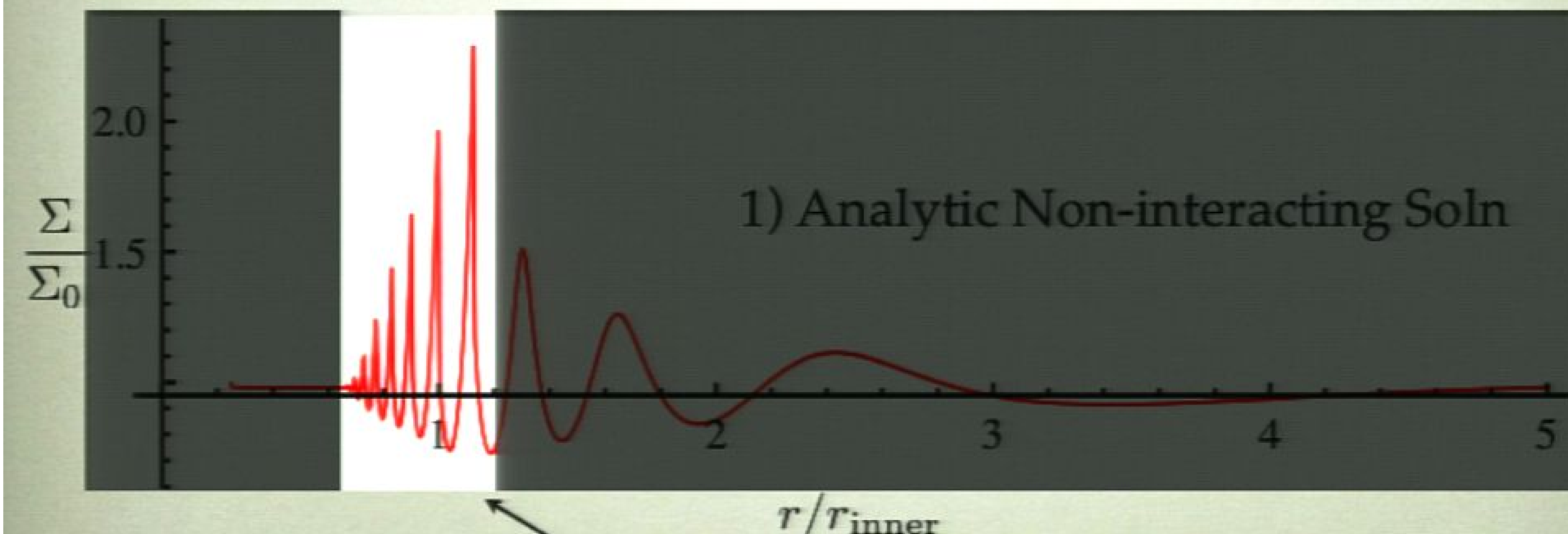


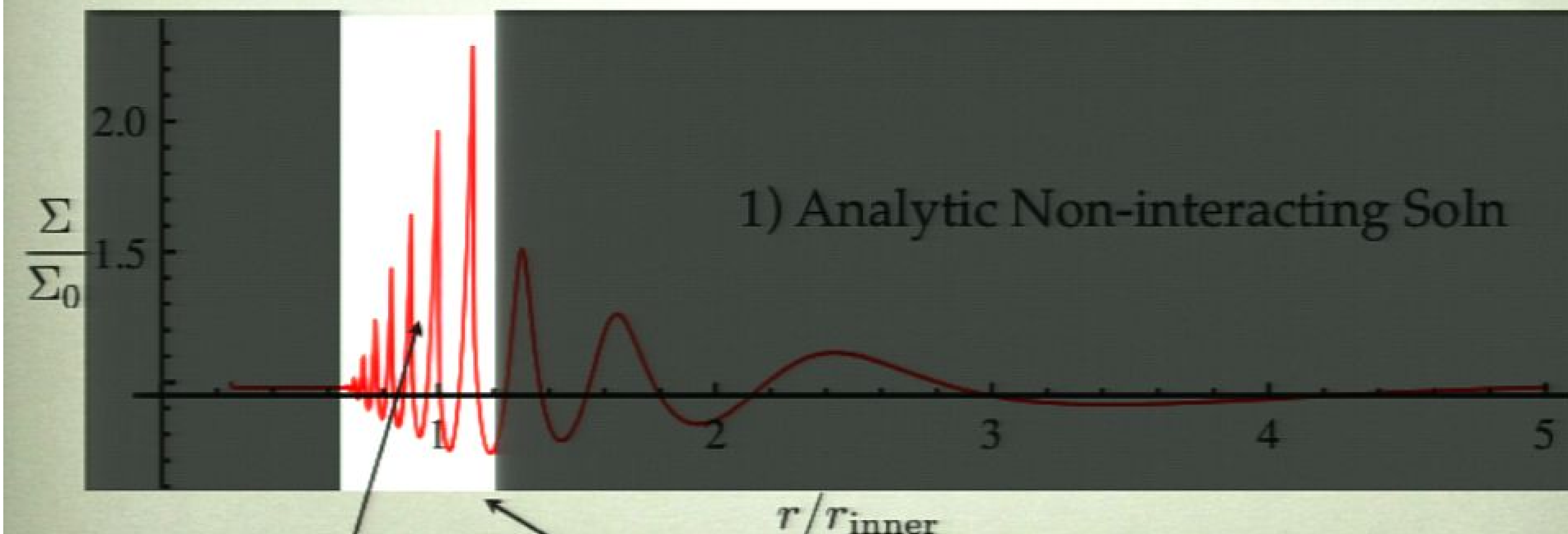
2.0
1.5
$\frac{\Sigma}{\Sigma_0}$
1) Analytic Non-interacting Soln
1
2
3
4
5
$r/r_{\rm inner}$

SUMMARY



2) Describe where the shocks are.

SUMMARY



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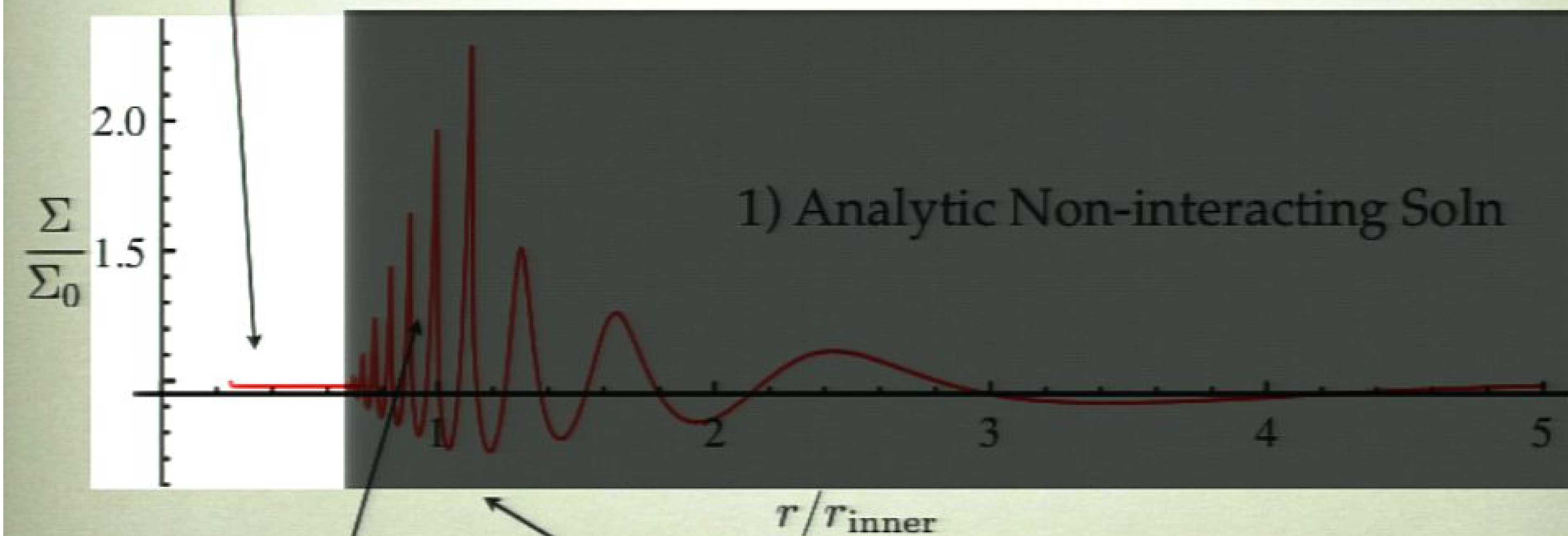
SUMMARY

4) Describe what happens after shocks.



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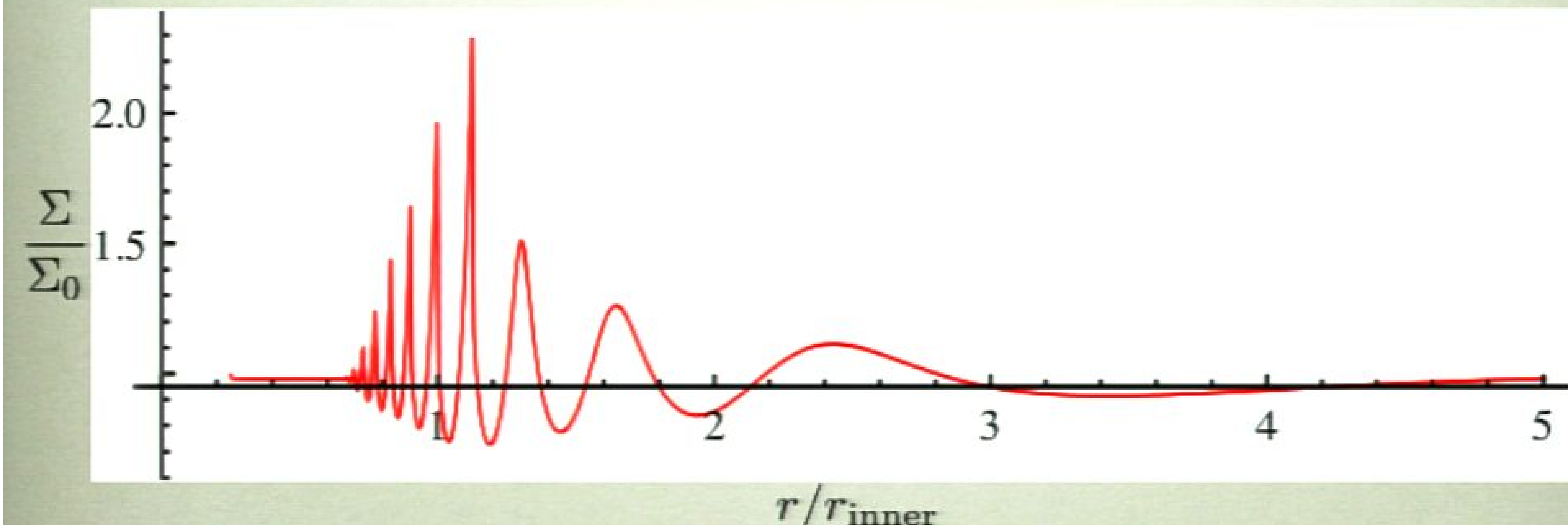
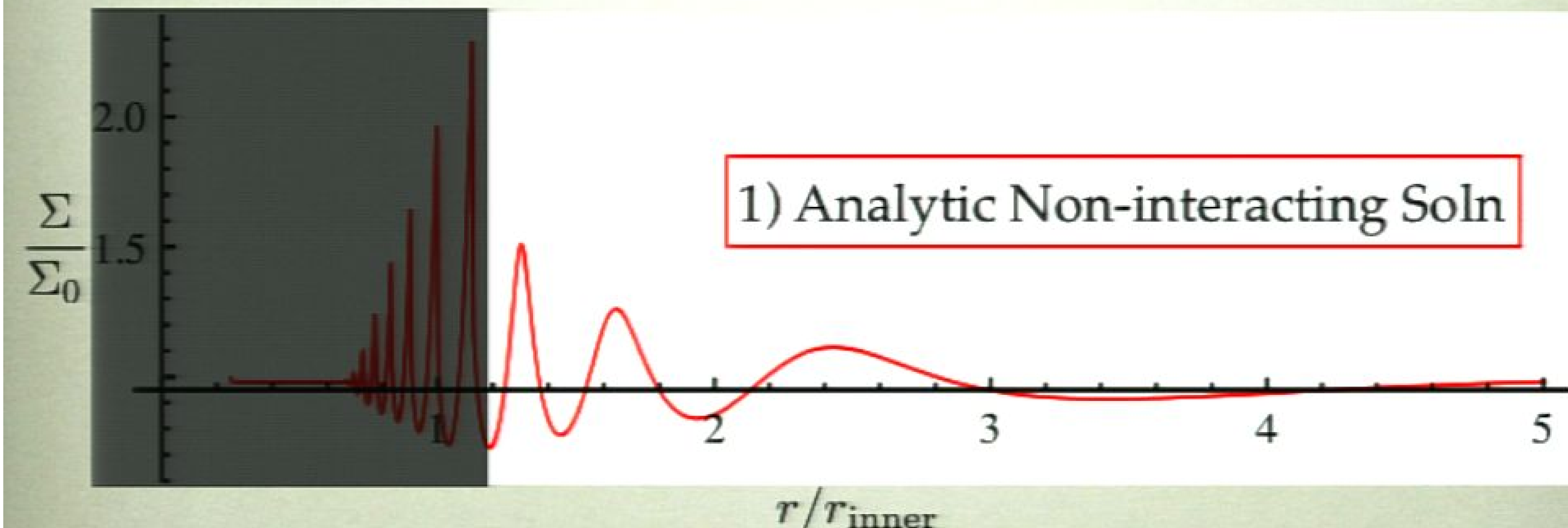
SUMMARY

And we finally show the theory is reflected in a 1D gaseous disk.



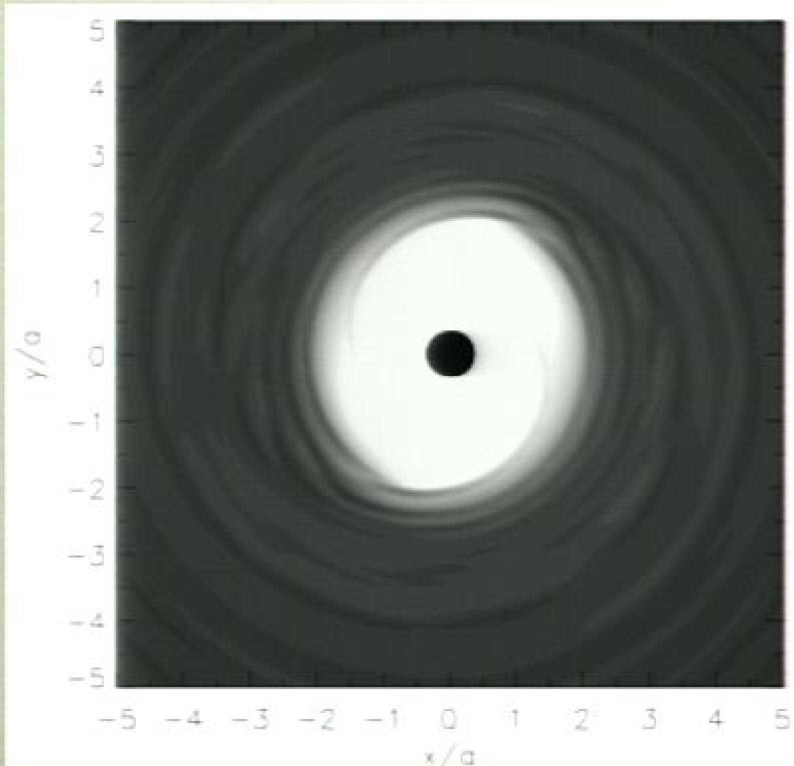
Where We're Going



The Disk's Response

- How do we proceed?

Density plot of surface density of circumbinary disk



MacFadyen & Milosavljevic 2006

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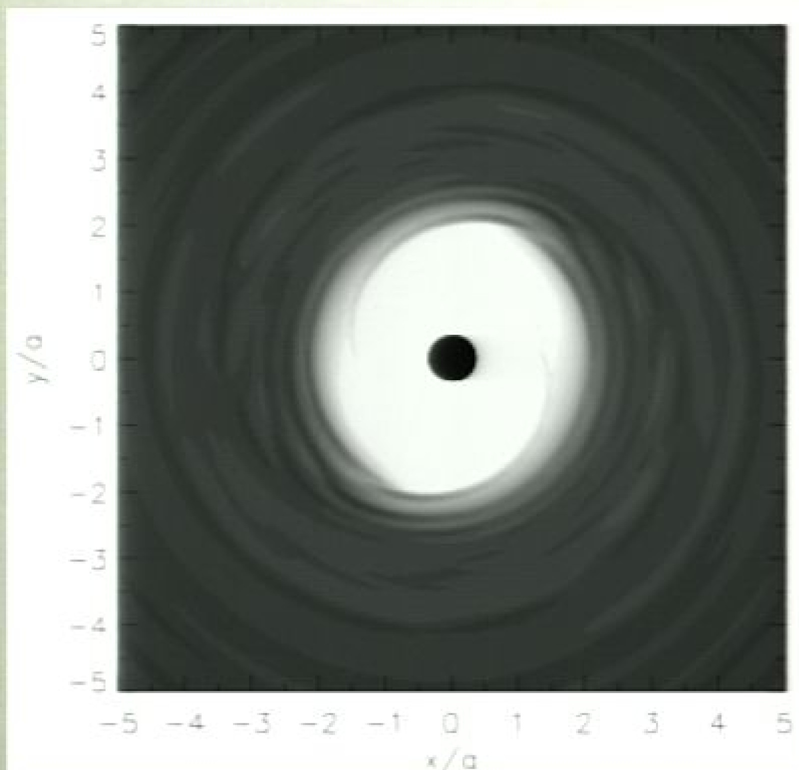

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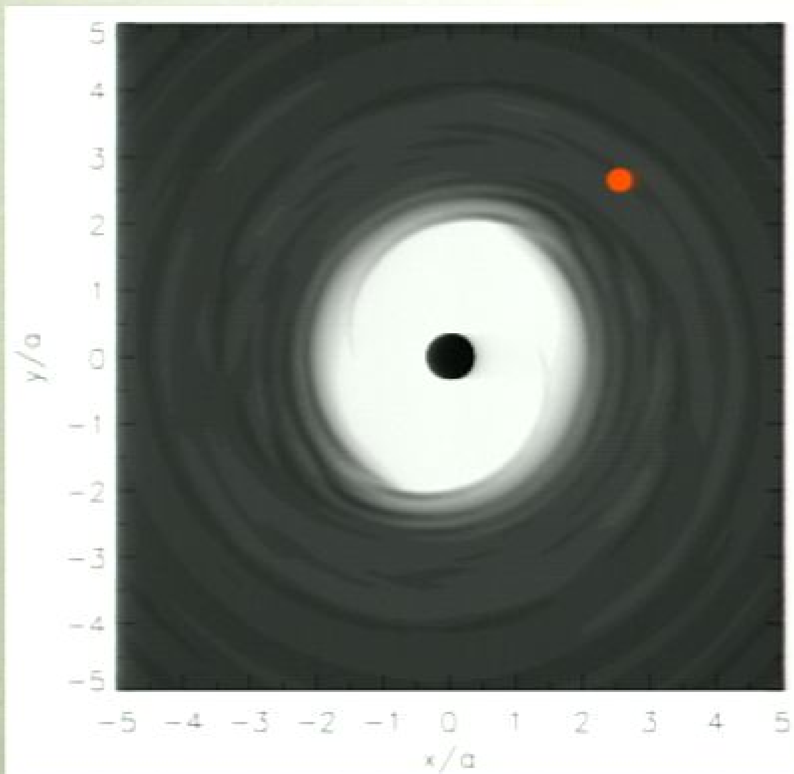

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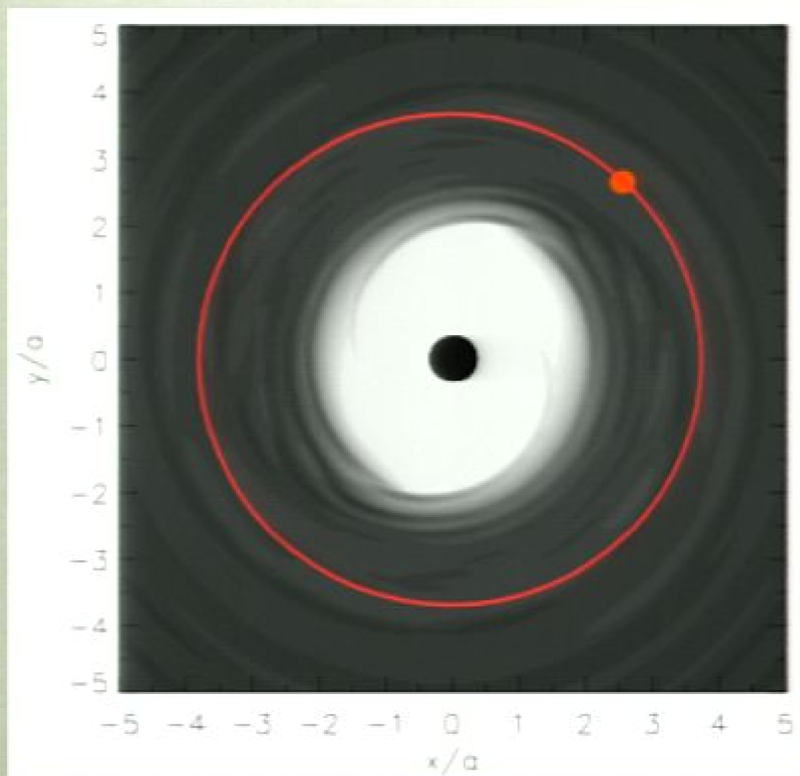

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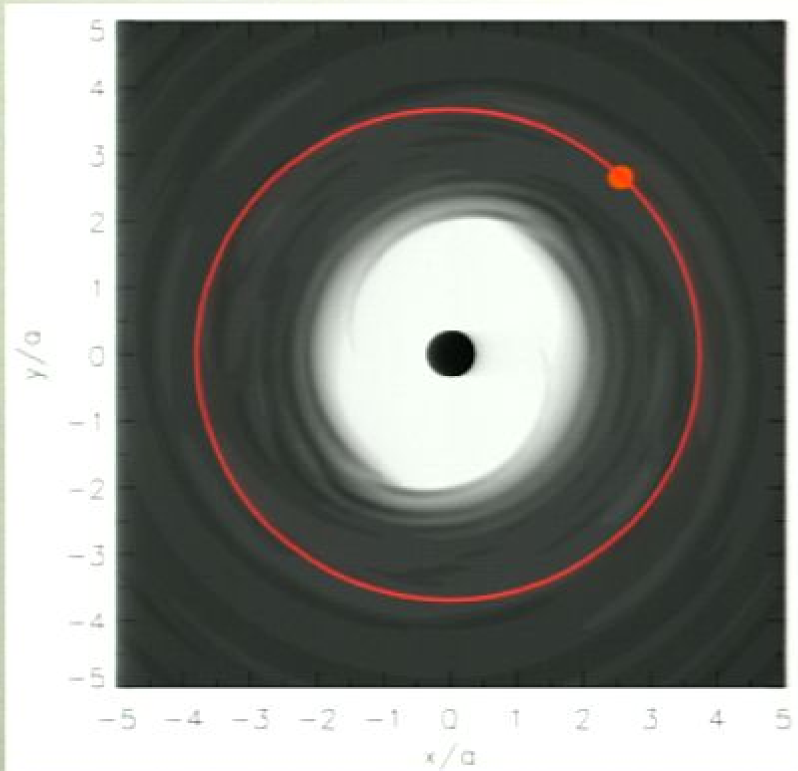

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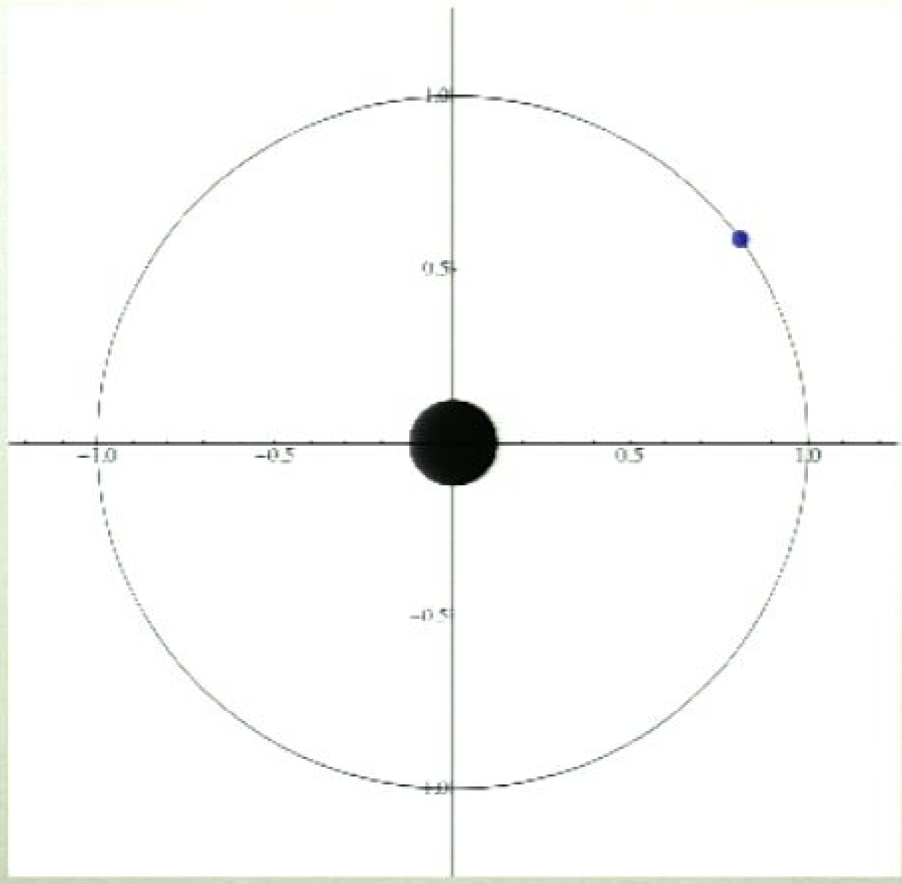

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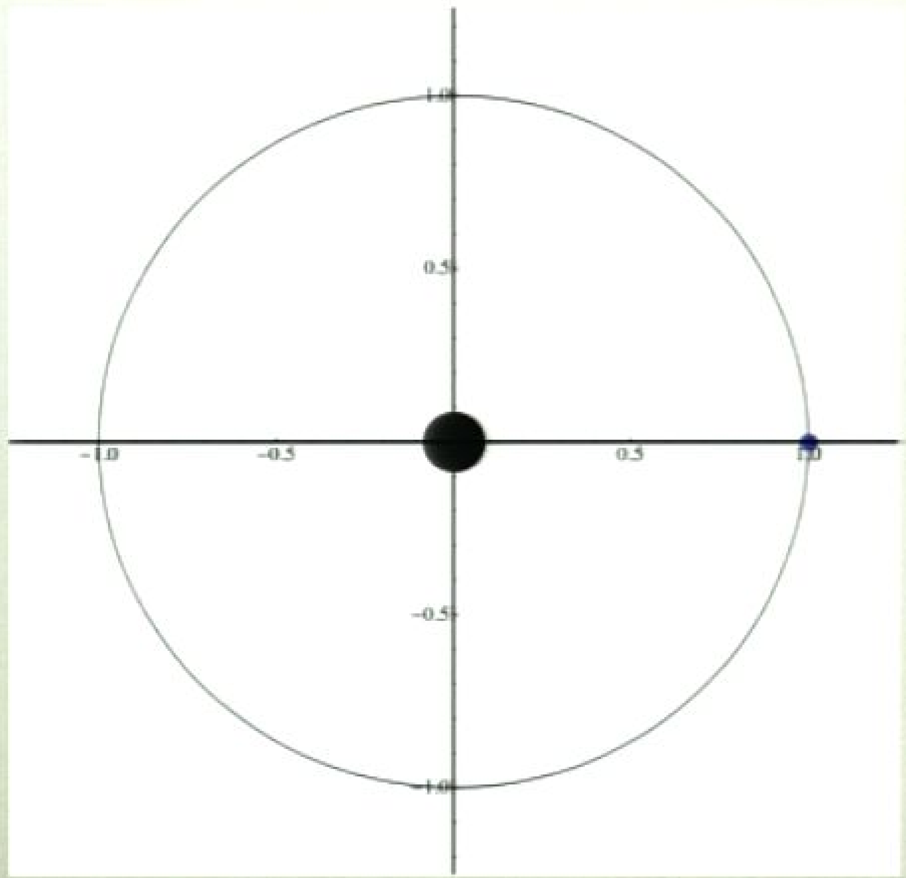

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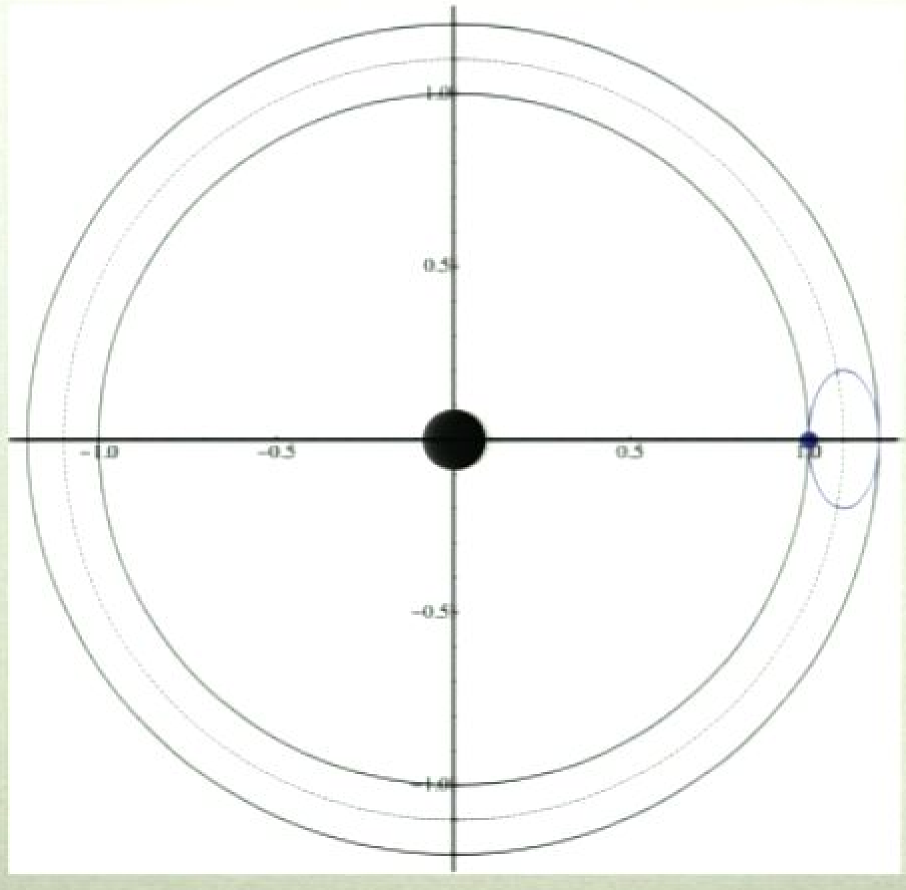

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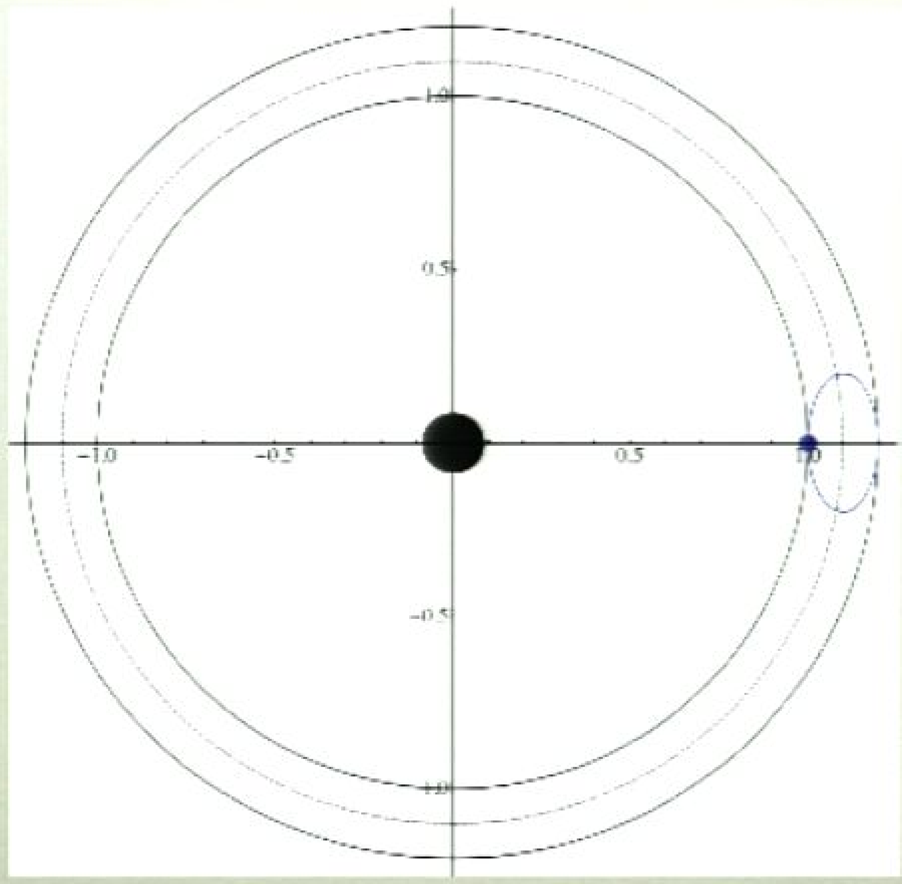

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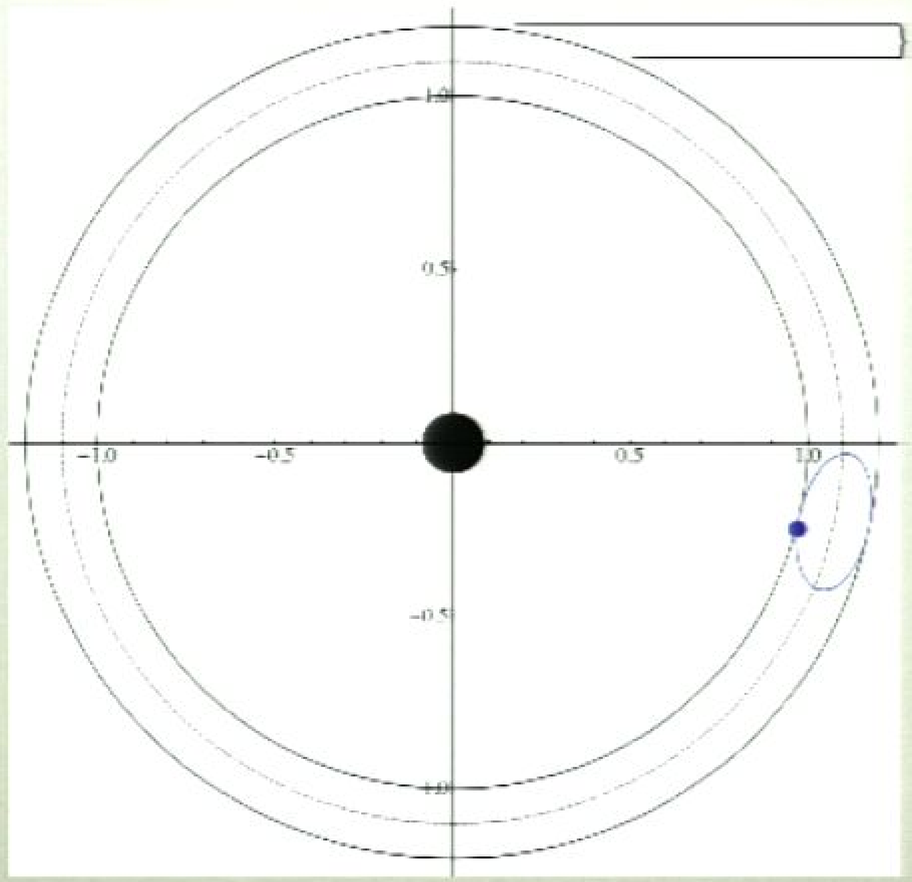

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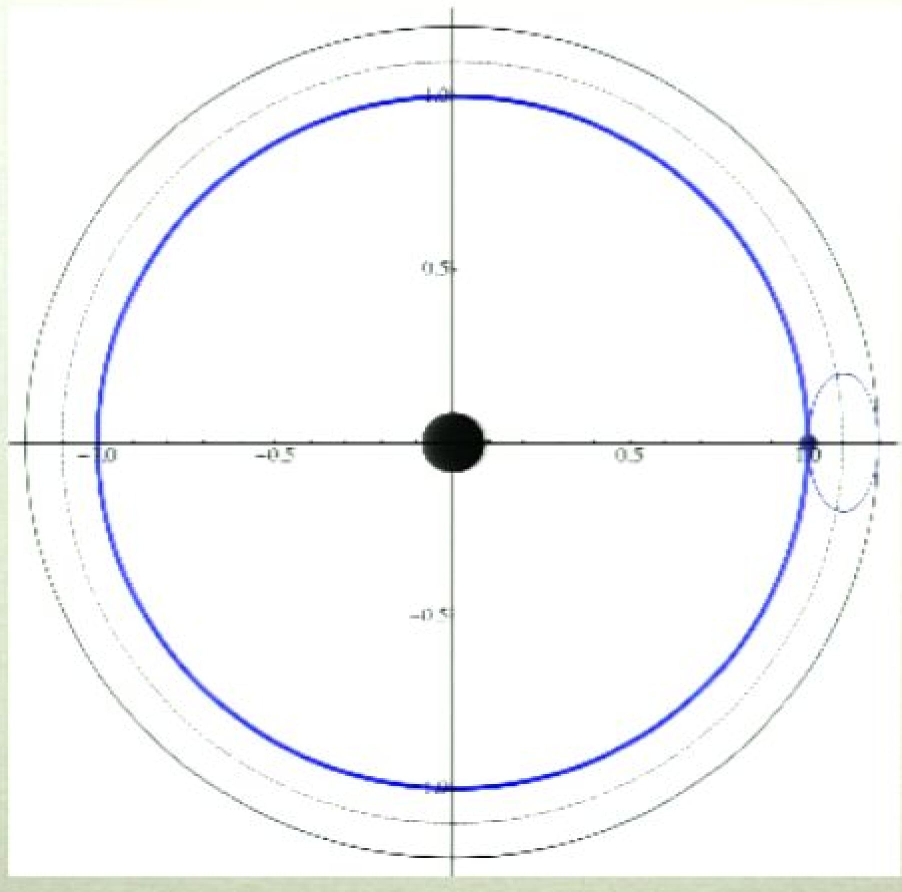

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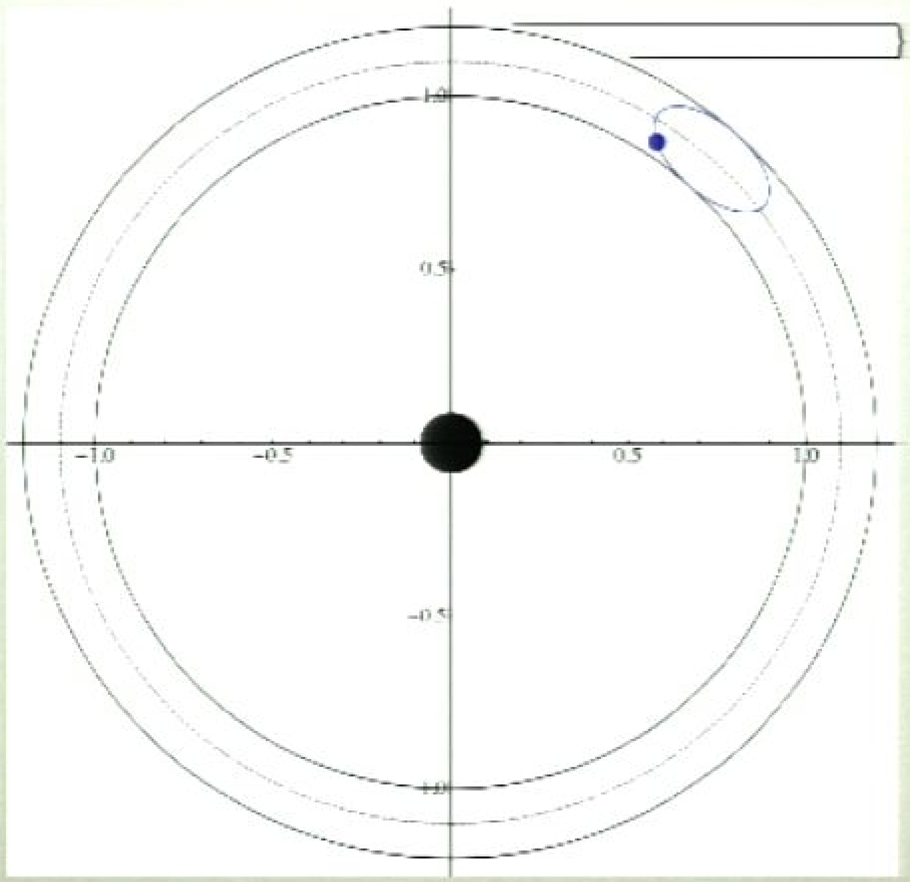

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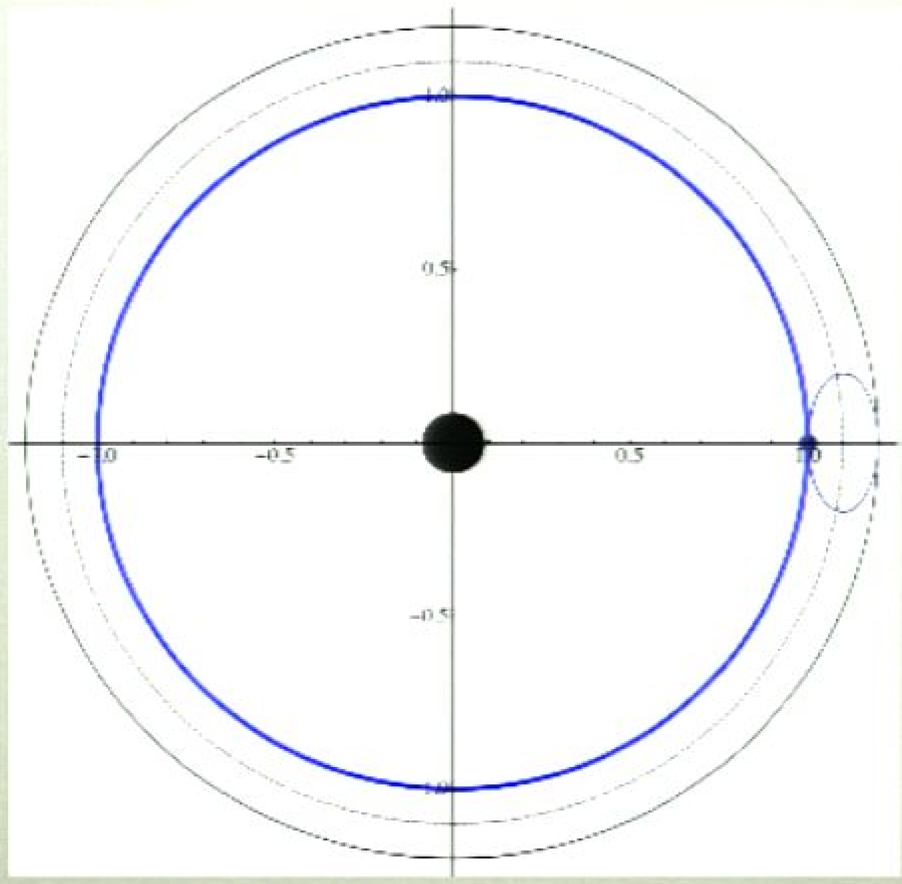

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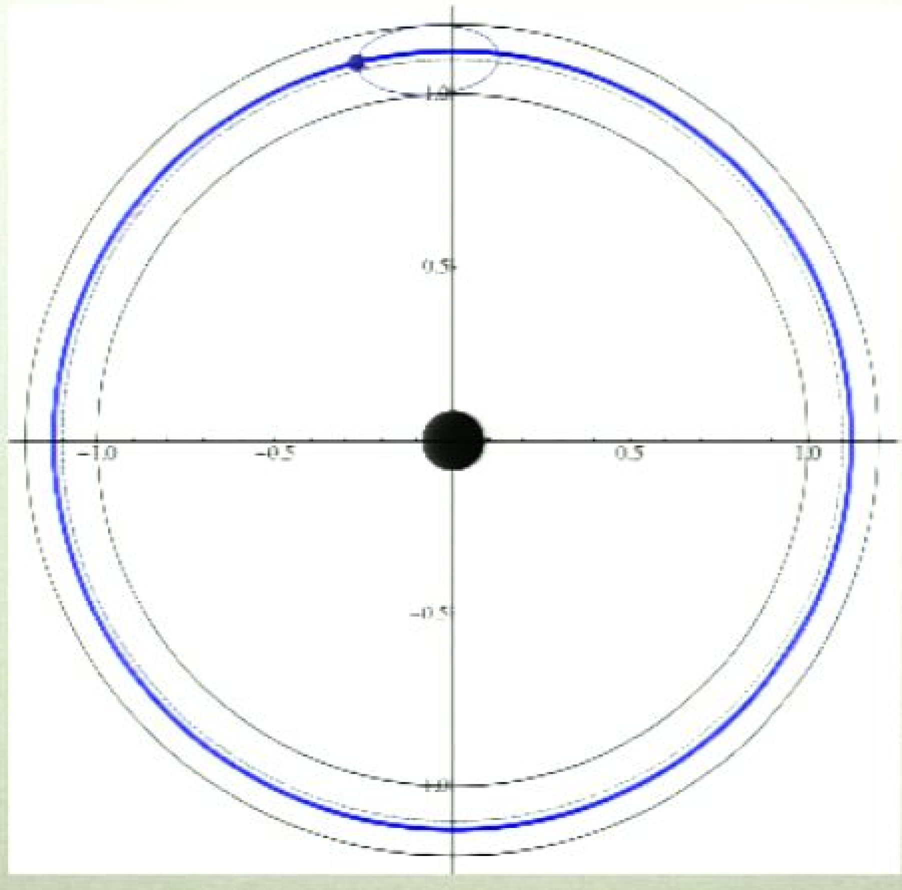

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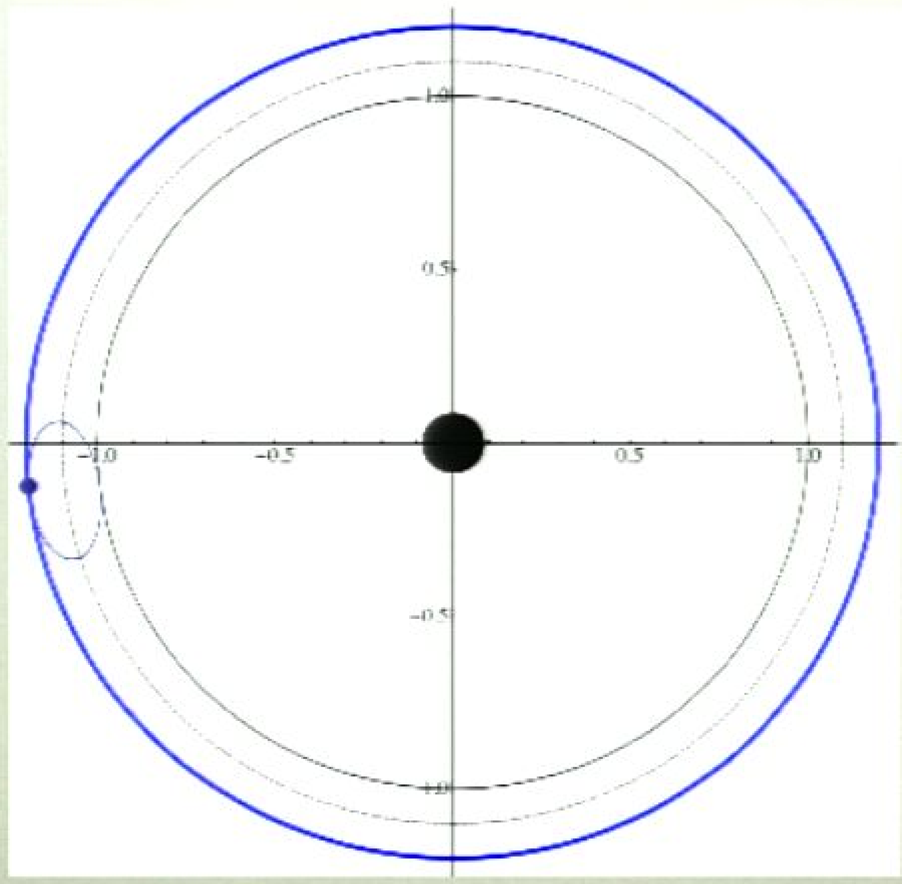

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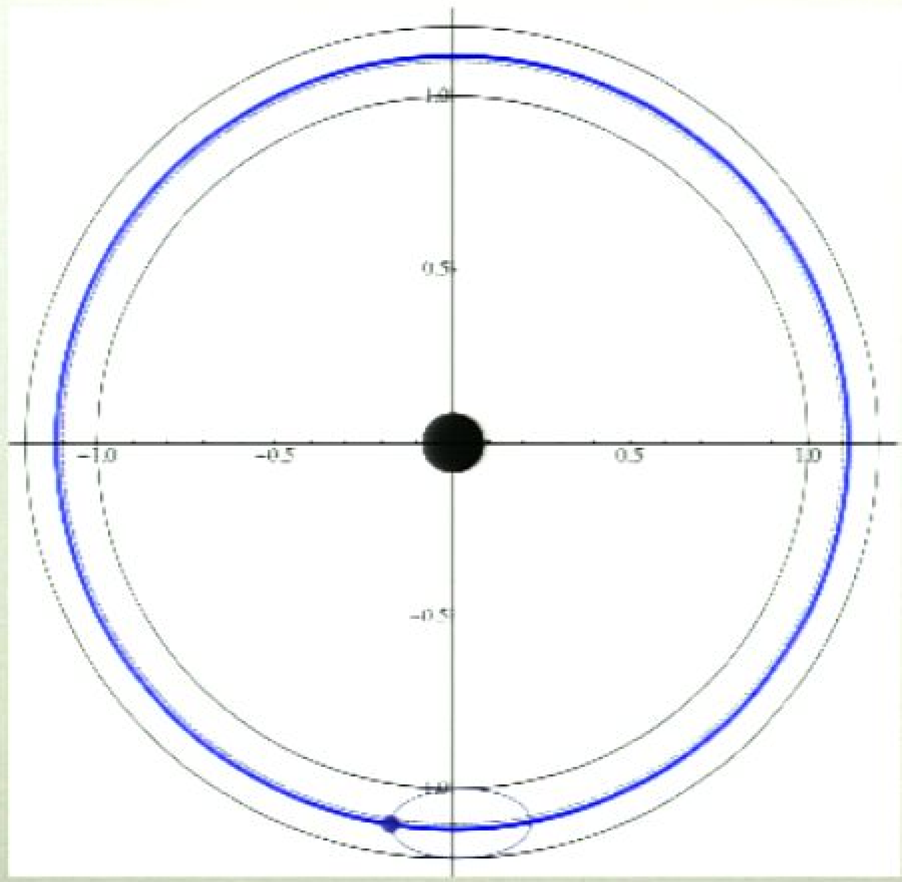

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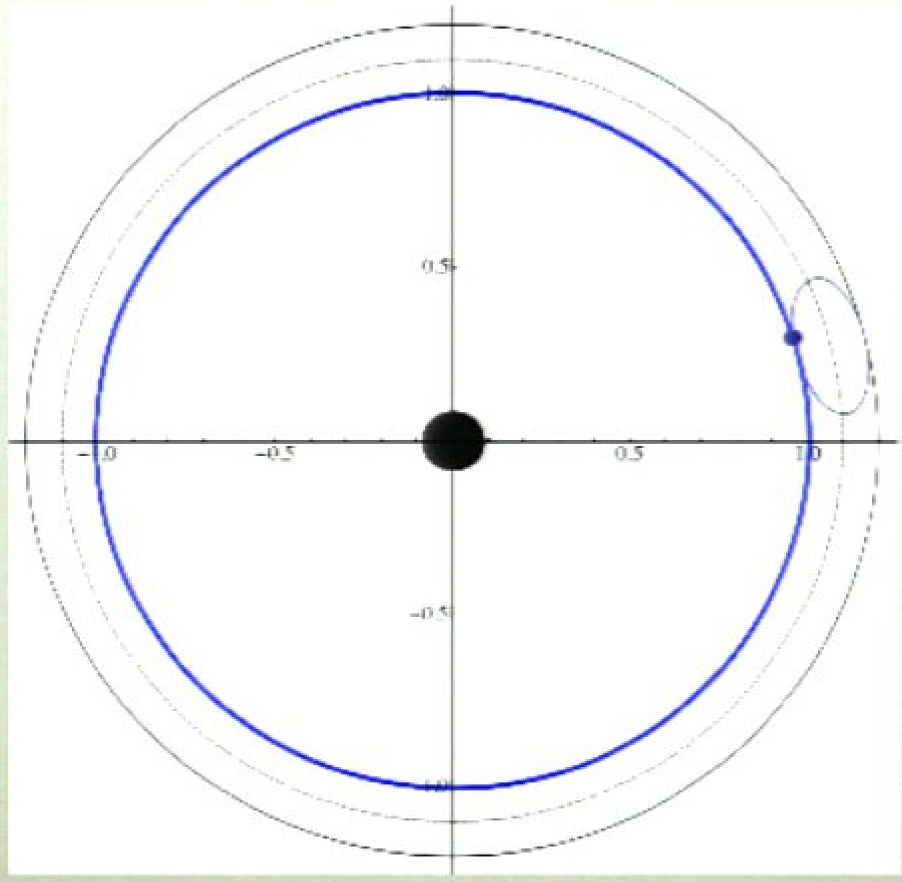

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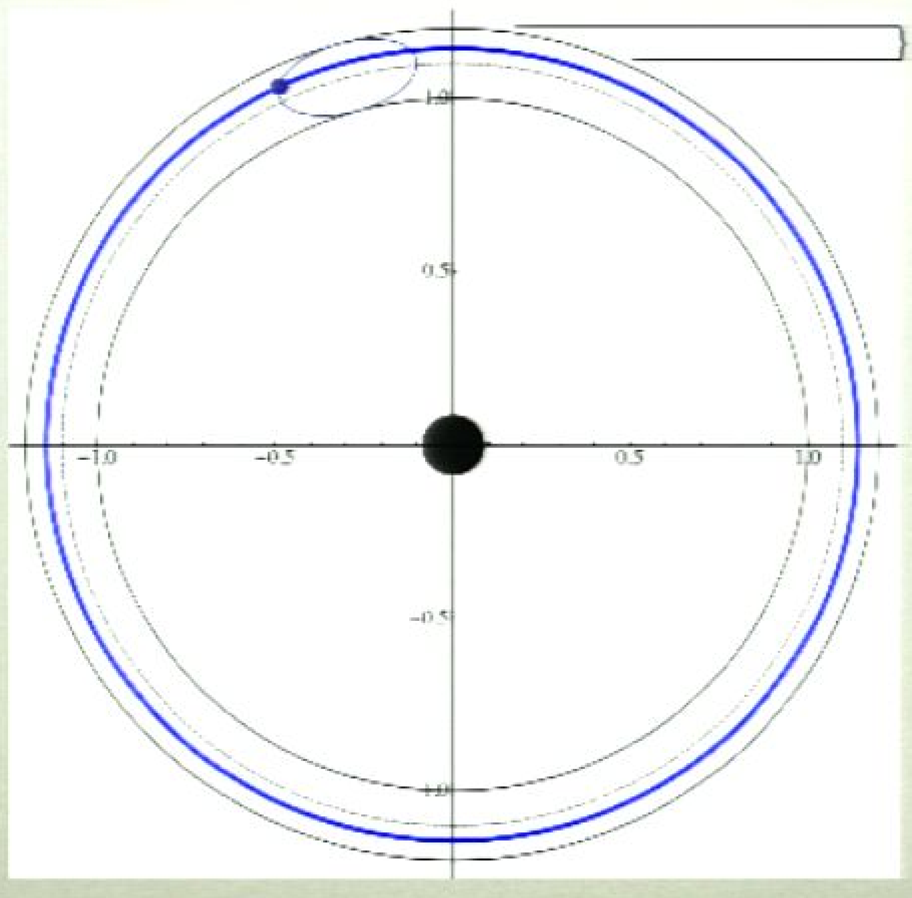

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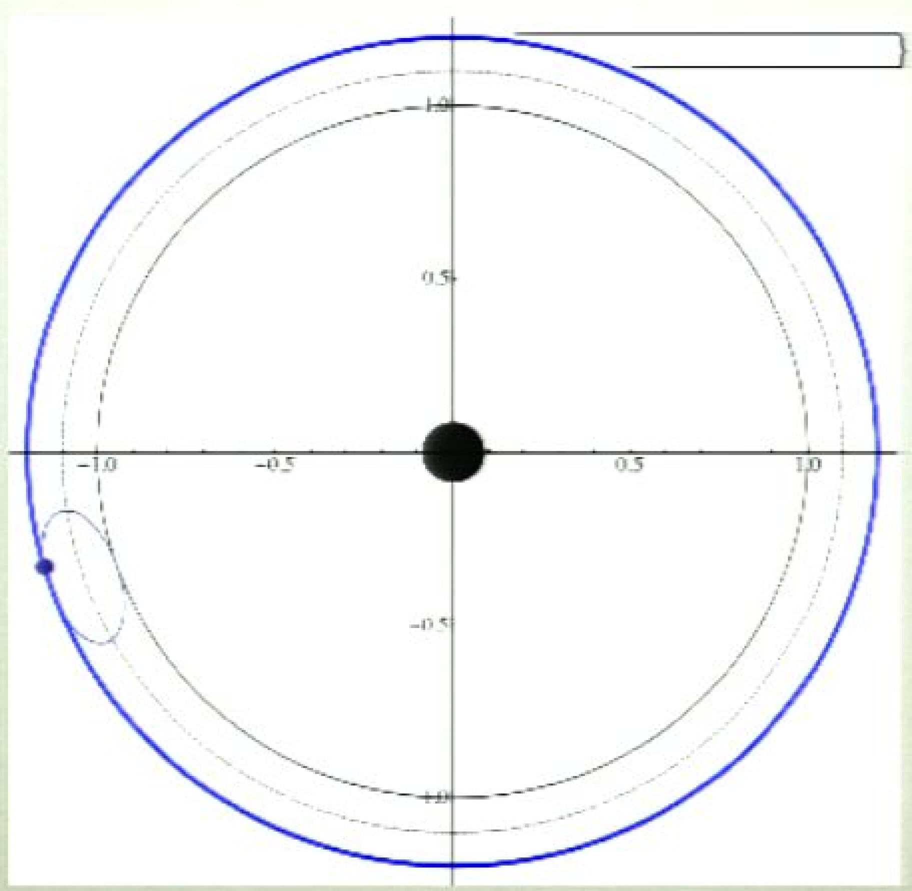

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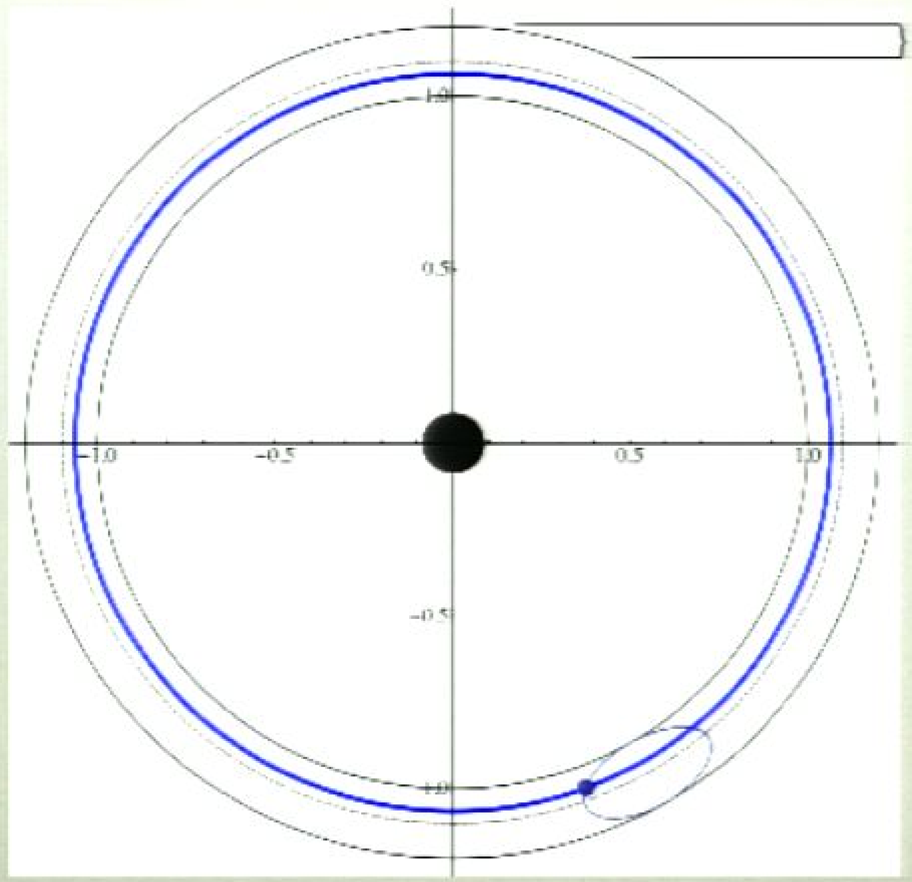

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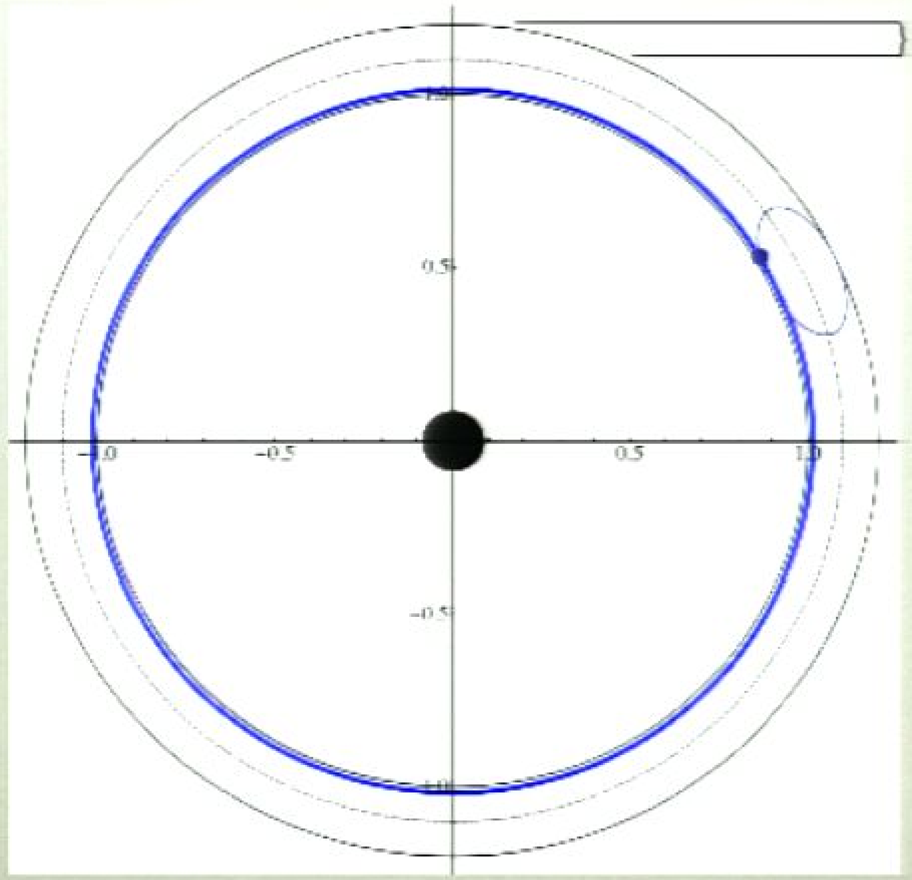

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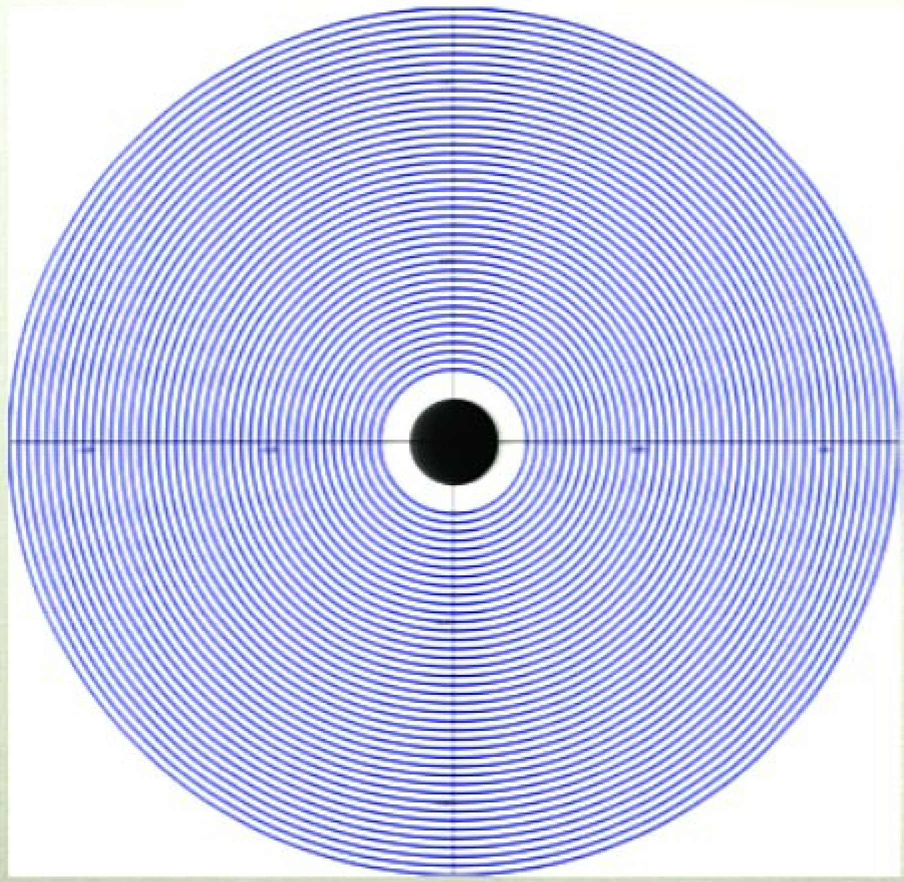

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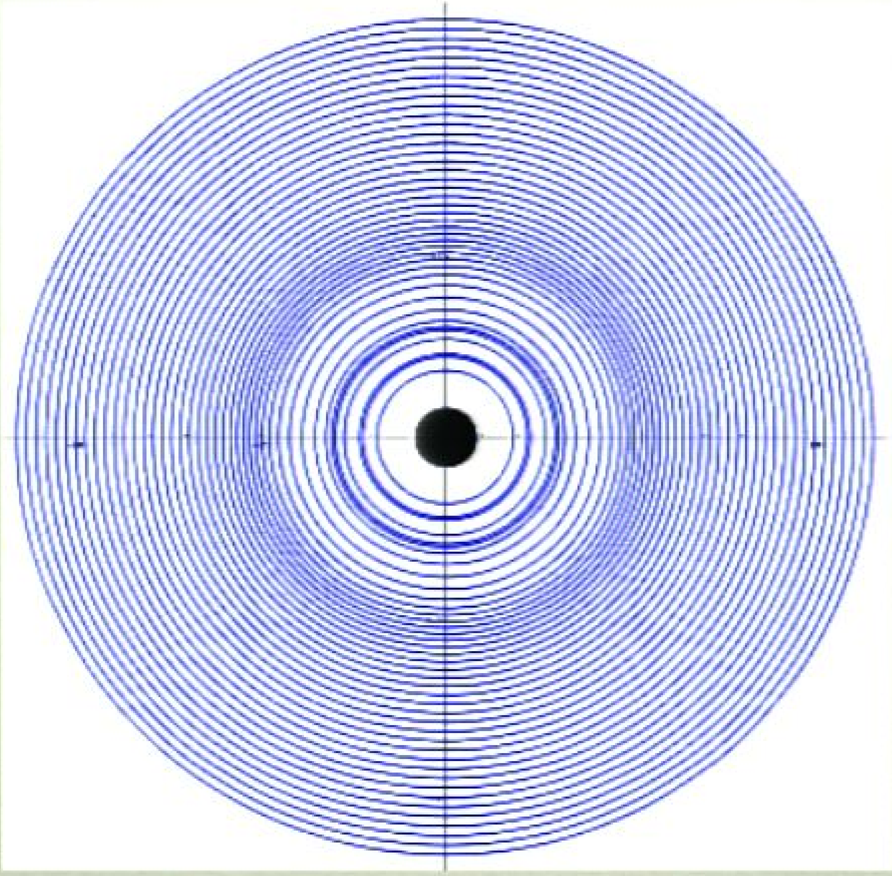

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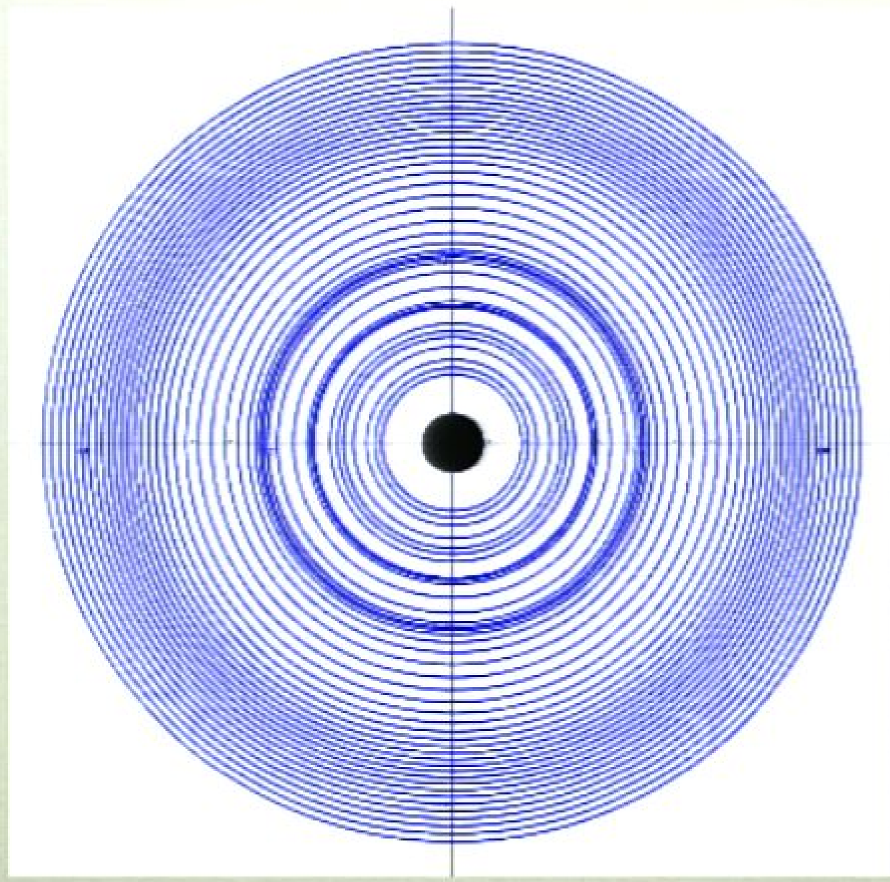

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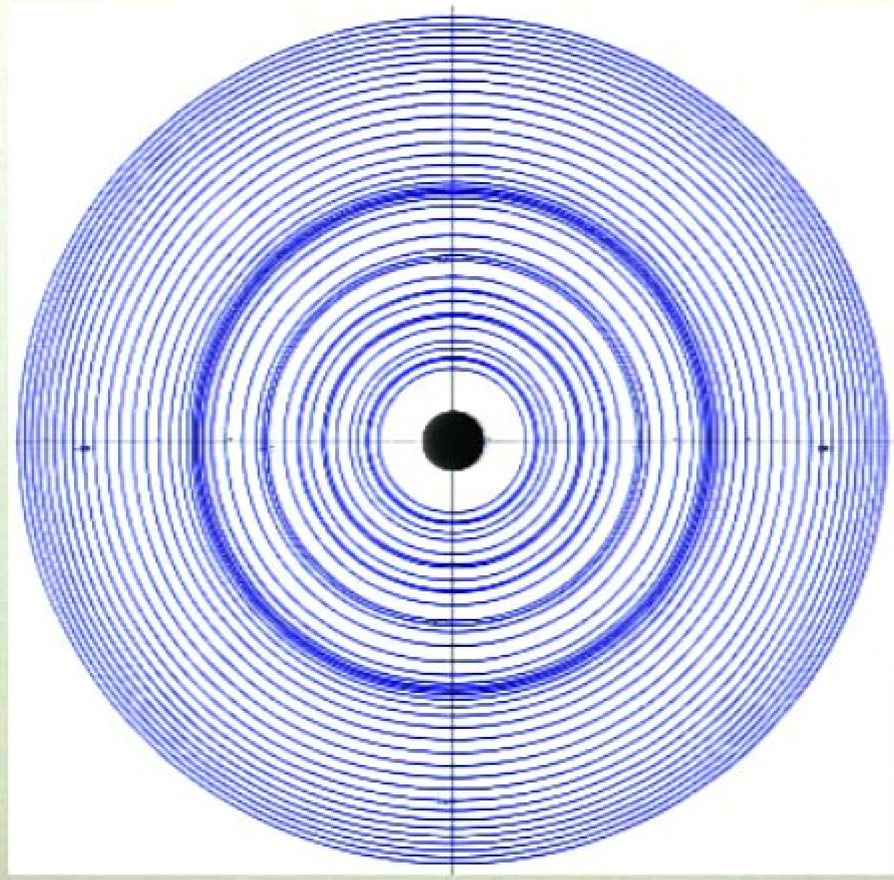

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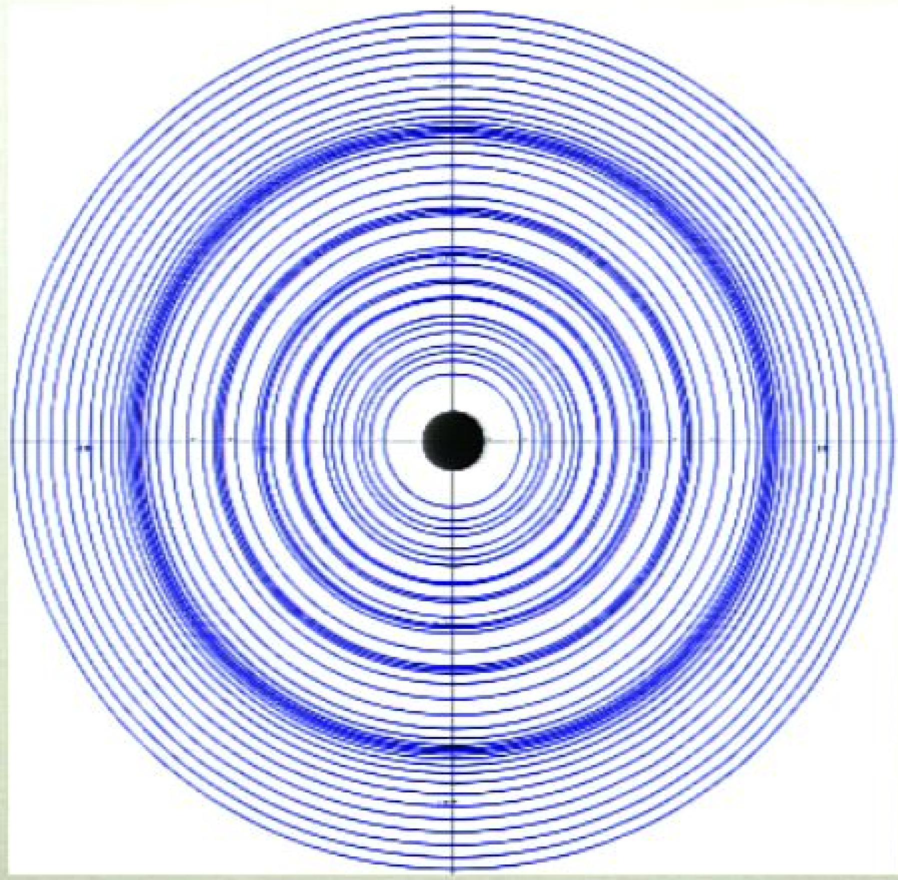

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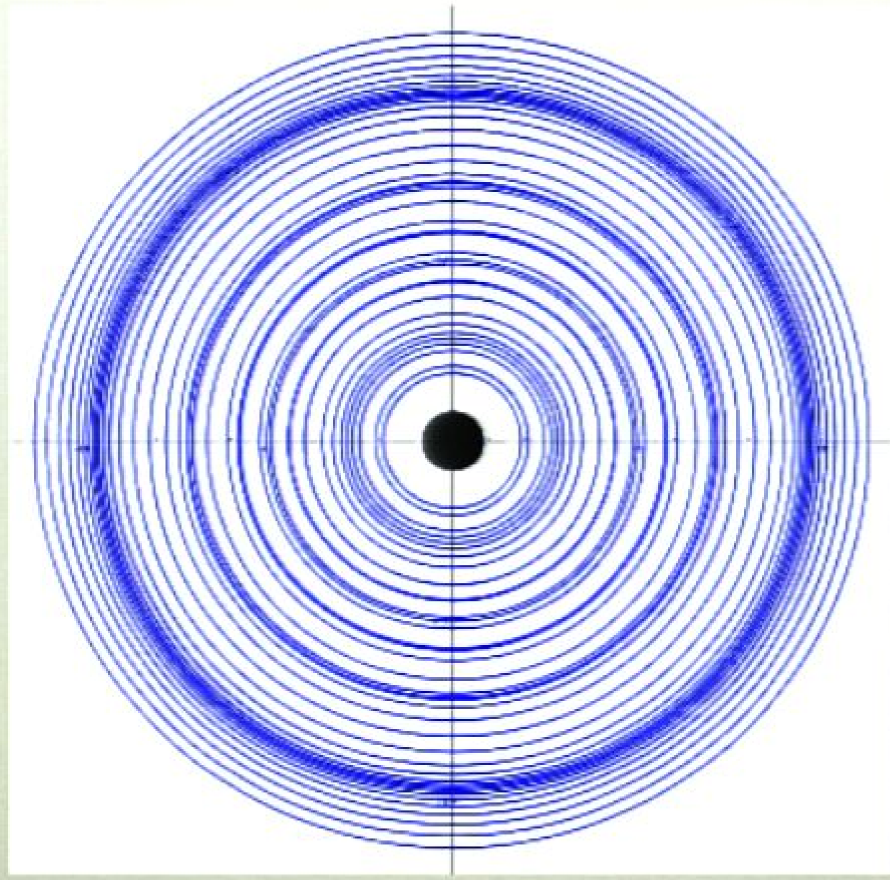

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$$S(r_0, t) = \left(1 + 2\frac{\delta M}{M}\right) - \frac{\delta M}{M}\left\{2\cos[\Omega(a)t] + \frac{3}{2}\Omega_0 t \sin[\Omega(a)t]\right\}$$

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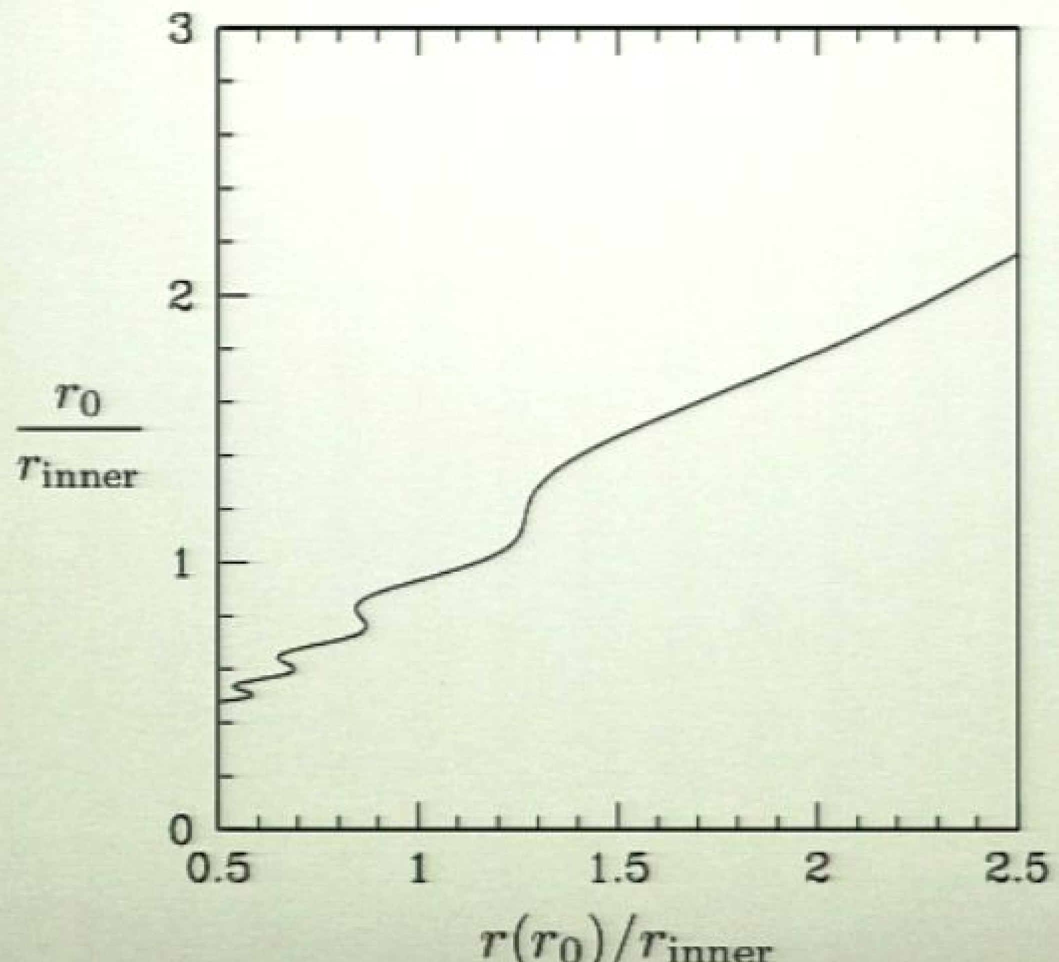
- A simple illustrative example: a circumbinary disk with an 8% mass loss plotted at $\Omega_{\rm inner} t = 10$ (noninteracting).



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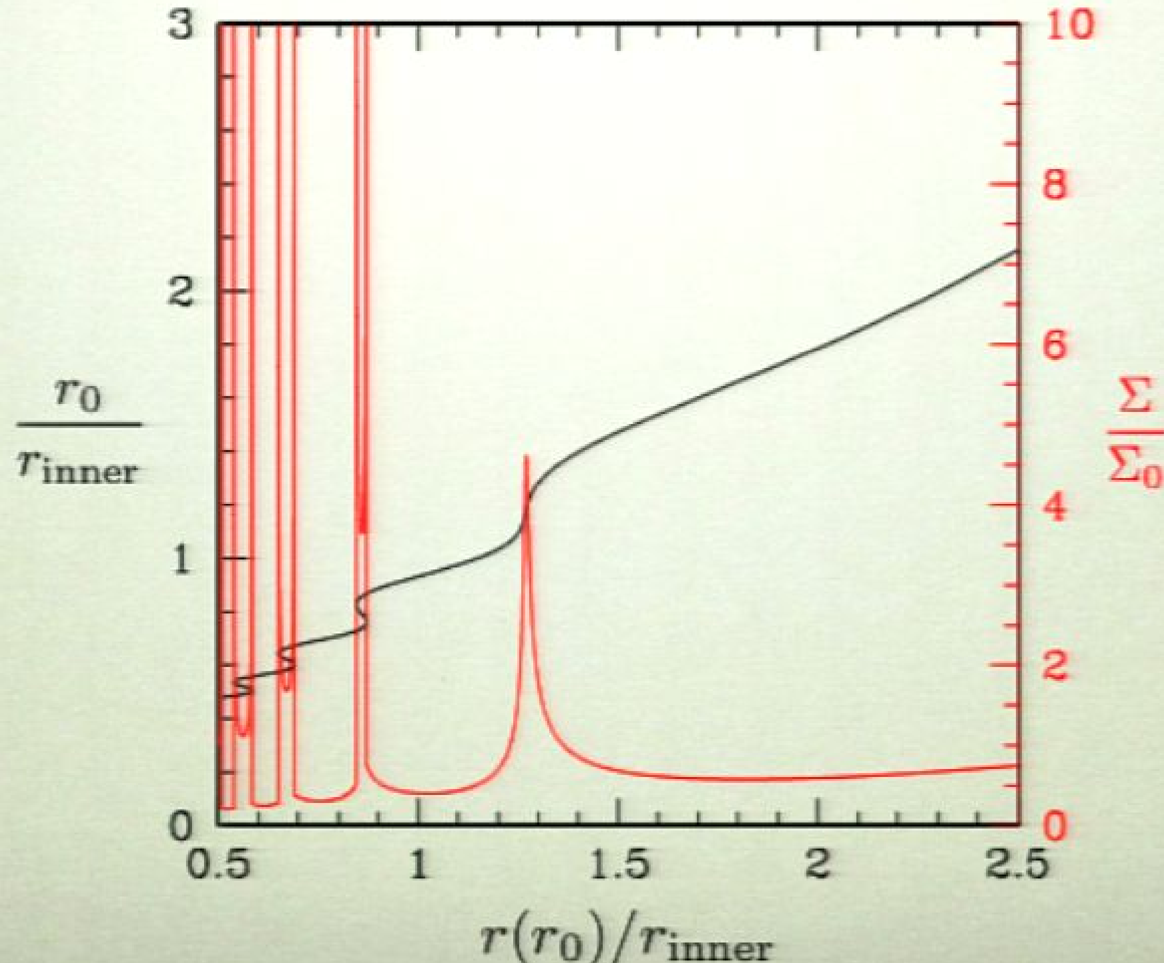

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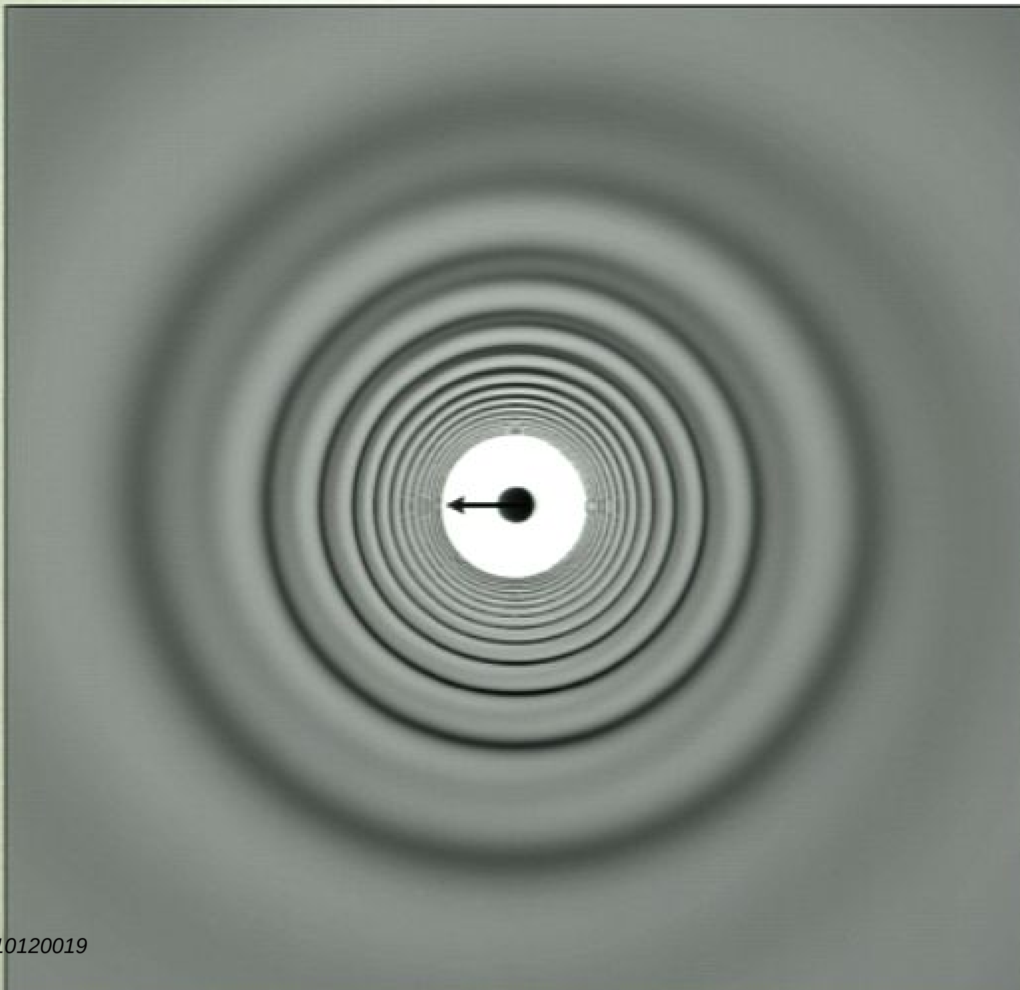
  3) density peaks travel at velocity:

$$\frac{v^{(\rm peak)}_{N_{\rm peak}}}{v_{\rm K_{inner}}} = \frac{1}{3}\frac{1}{N_{\rm peak}-3/4}\frac{1}{\sqrt{r/r_{\rm inner}}}$$

Analytic Solution: Merger with a Kick

Surface density of circumbinary disk with kick to the left.



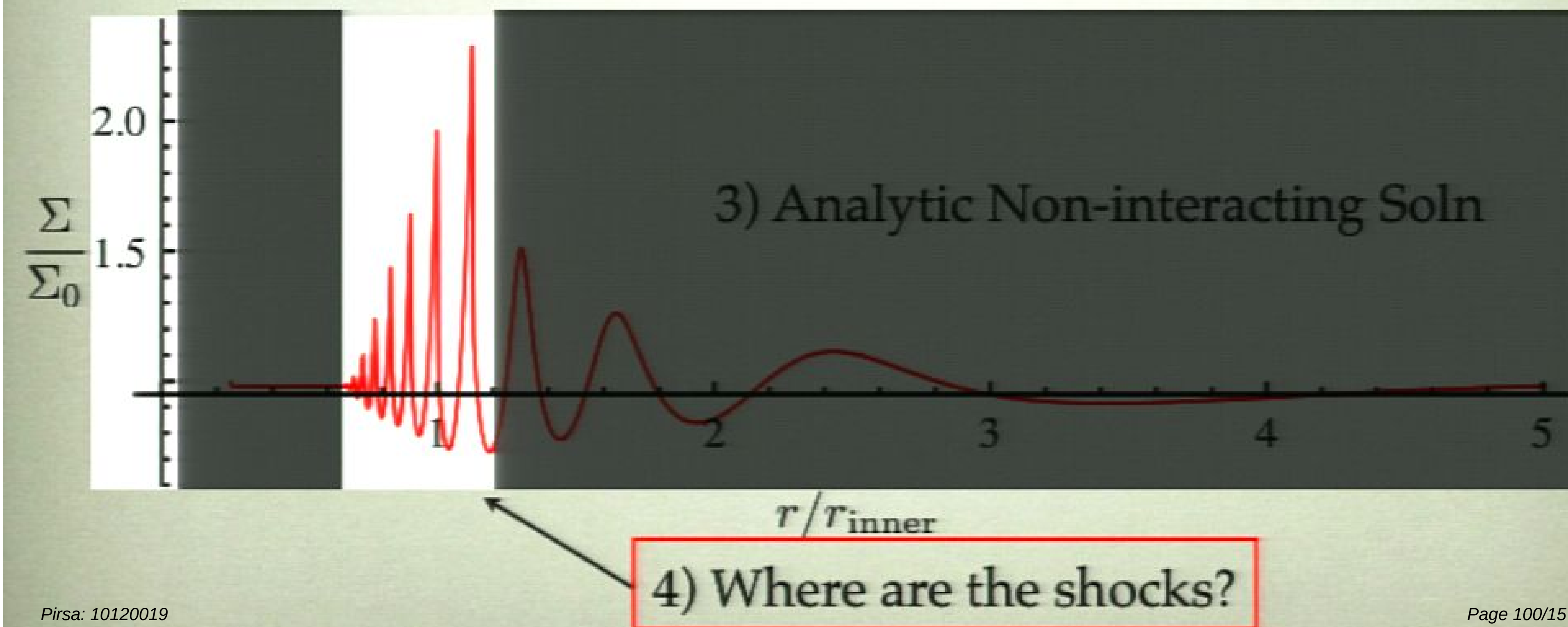
$$\frac{\delta M}{M_0} = 0.0133$$

$$\frac{v_{\rm kick}}{v_0(r_{\rm inner})} = 0.0043$$

$$\Omega_0(r_{\rm inner})t = 90$$

$$r_{\rm inner} = 200M_0$$

Where We're Going



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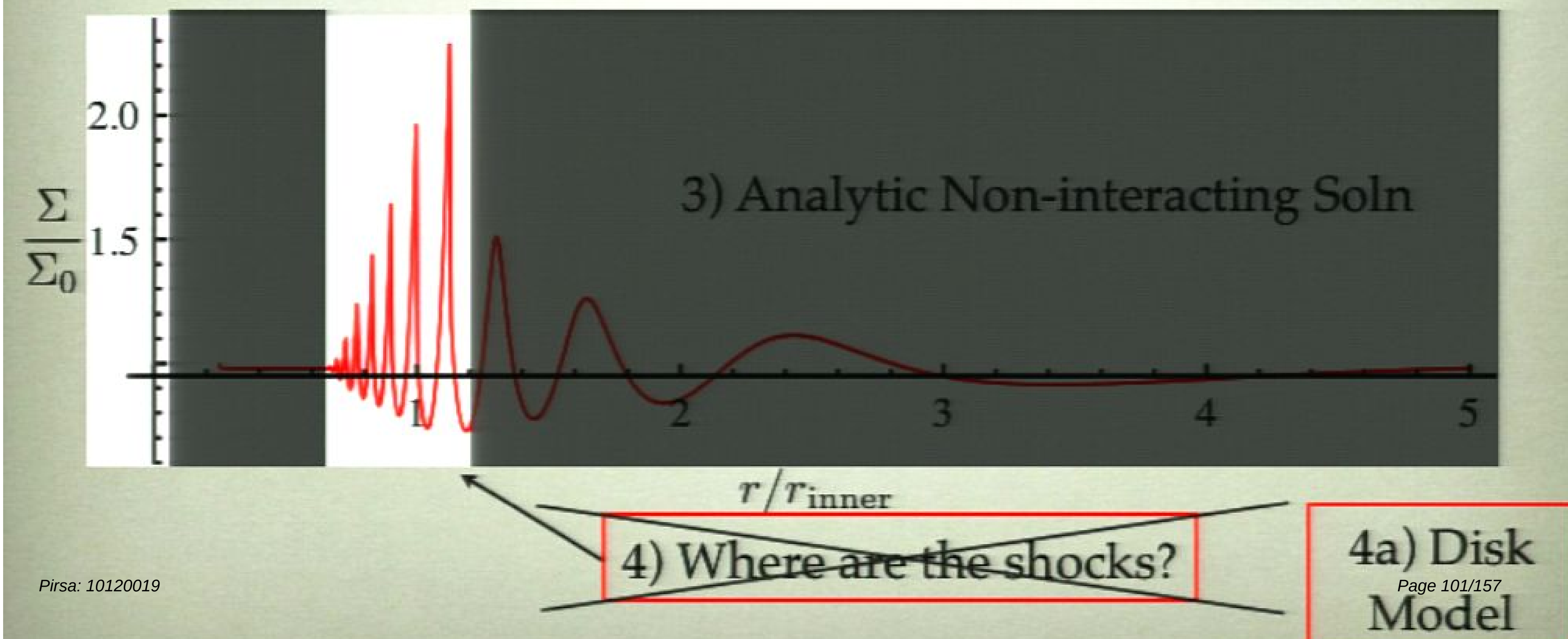

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$$P_{\text{int}} = 2\int_0^\infty \frac{1}{2}\Omega^2 h^2 \rho \, dz = \Omega^2 h^2 \frac{\Sigma}{2} \quad \text{so that}$$

$$\frac{Du_r}{Dt} = -\frac{GM}{r^2} - \frac{\nabla P}{\rho} + \frac{v_\phi^2}{r} \longrightarrow \frac{Du_r}{Dt} = -\frac{GM}{r^2} - \frac{\nabla P_{\text{int}}}{\Sigma} + \frac{v_\phi^2}{r}$$

Analytic Application: Epicyclic Mach Number

- Better understand flow by considering radial Euler equation:

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{GM_{\rm final}}{r^2} - \frac{\nabla P_{\rm int}}{\Sigma} + \frac{u_\phi^2}{r}$$

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- Take ratio of kinetic term to pressure term:

$$M_e^2 \equiv \frac{\text{kinetic term}}{\text{pressure term}} \sim \left(\frac{\delta M/M_0}{h(r)/r}\right)^2$$

Analytic Application: Epicyclic Mach Number

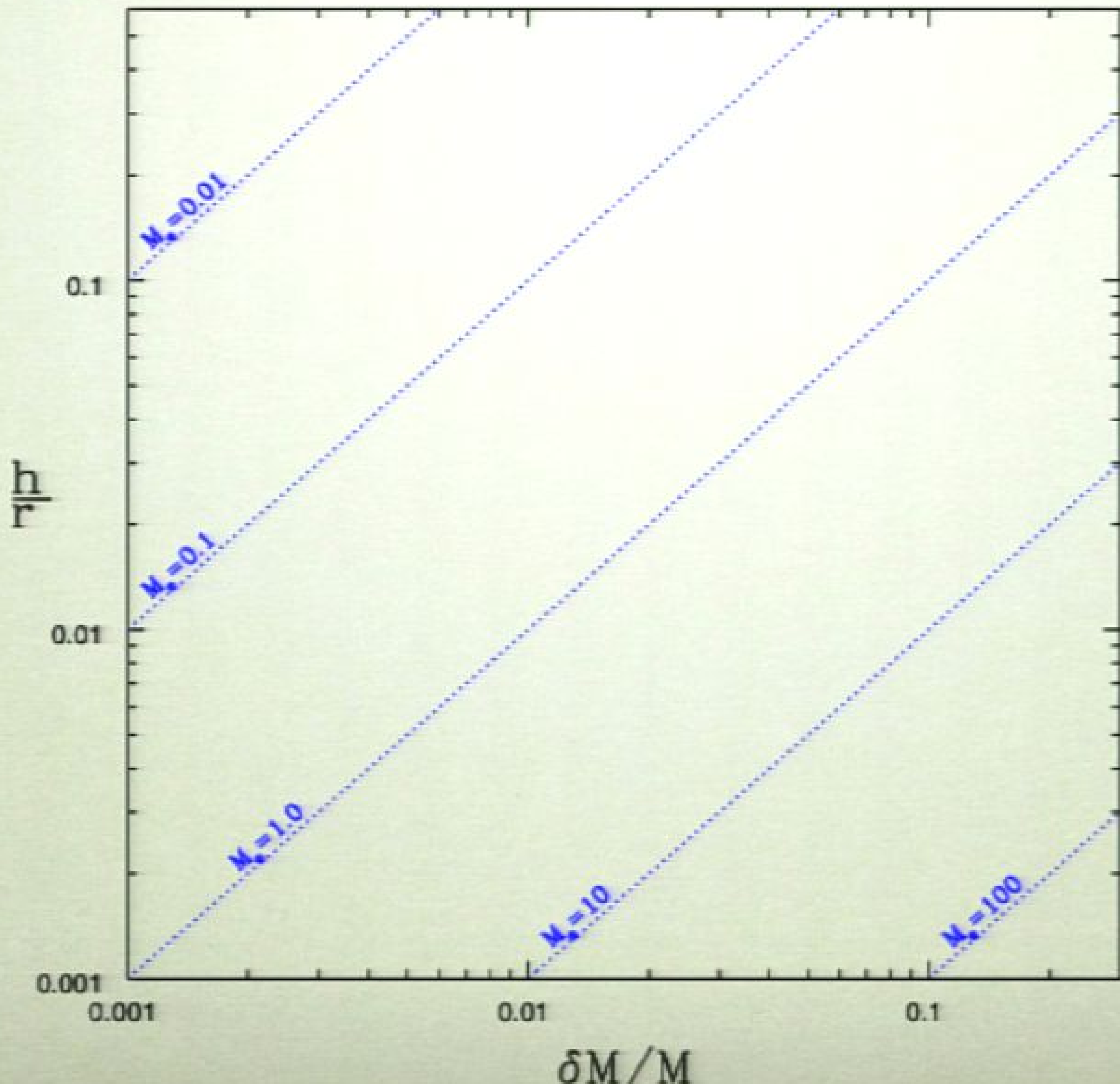
- M_e tells us about how a given peak evolves in time. Keeping in mind that $M_e^2 = \frac{\text{kinetic term}}{\text{pressure term}}$

$M_e \ll 1$: acoustic limit

$M_e \sim 1$: perturbations become shocks rapidly

$M_e \gg 1$: pressureless limit

Analytic Application: Parameter Space



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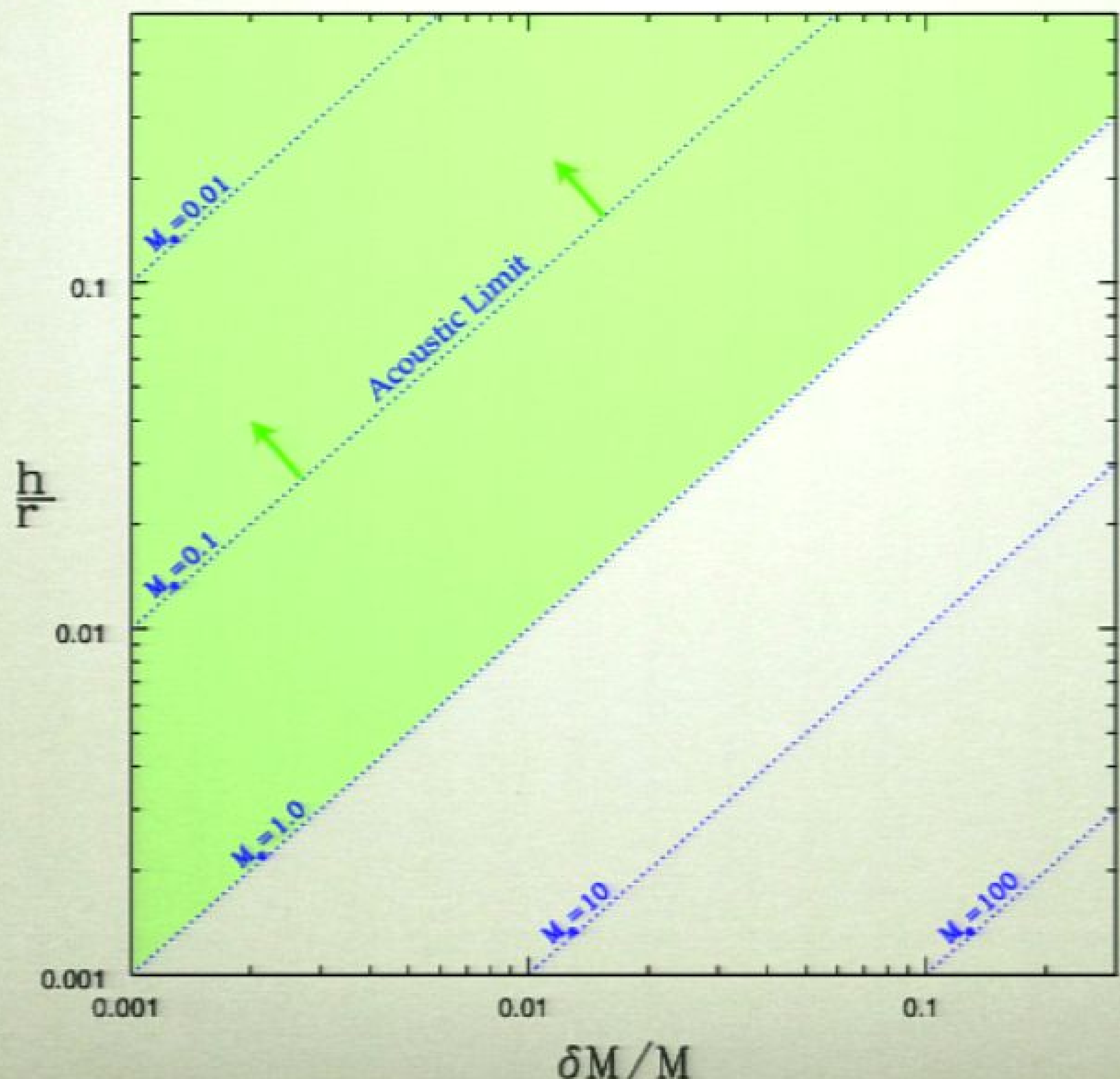

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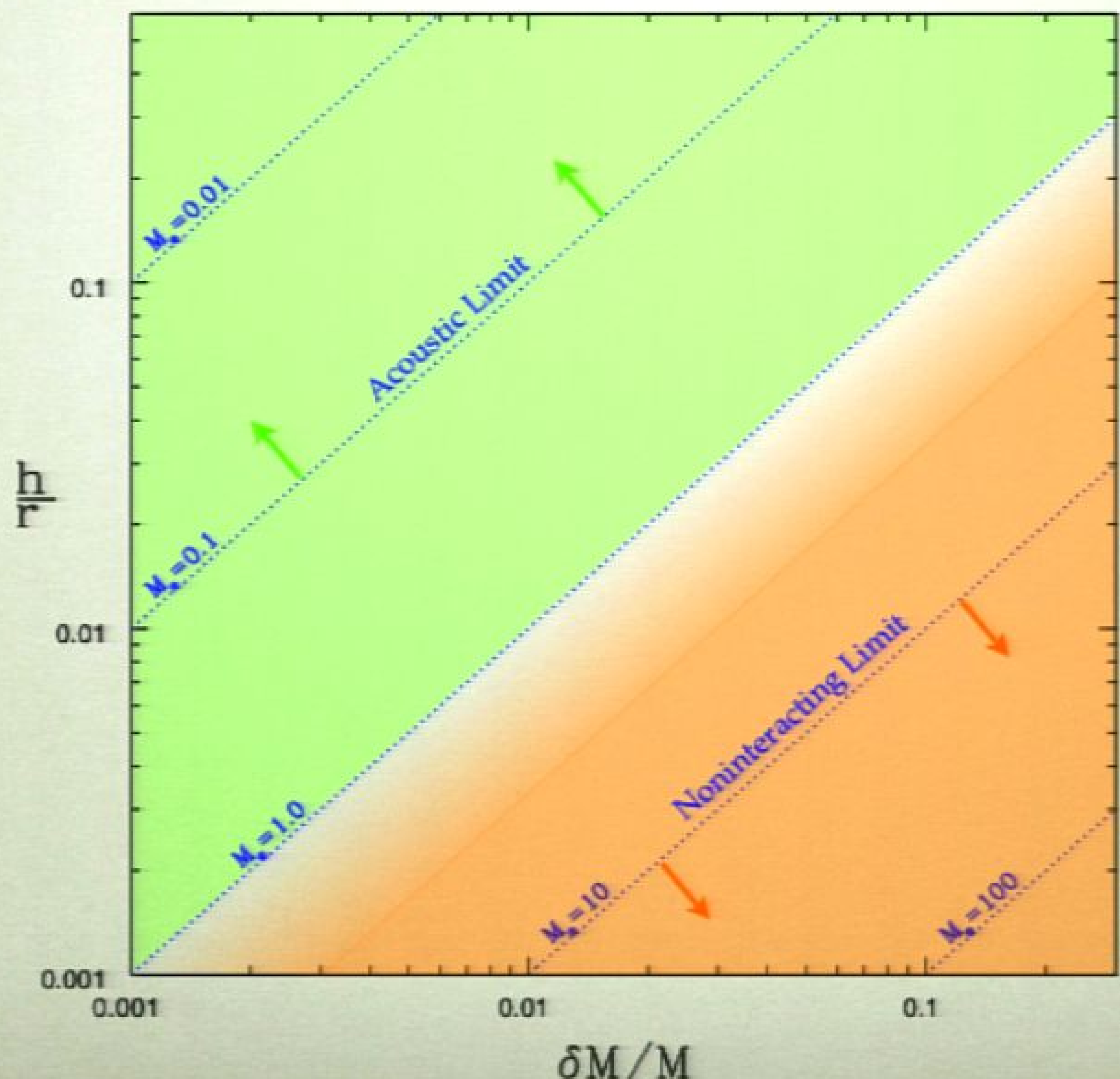

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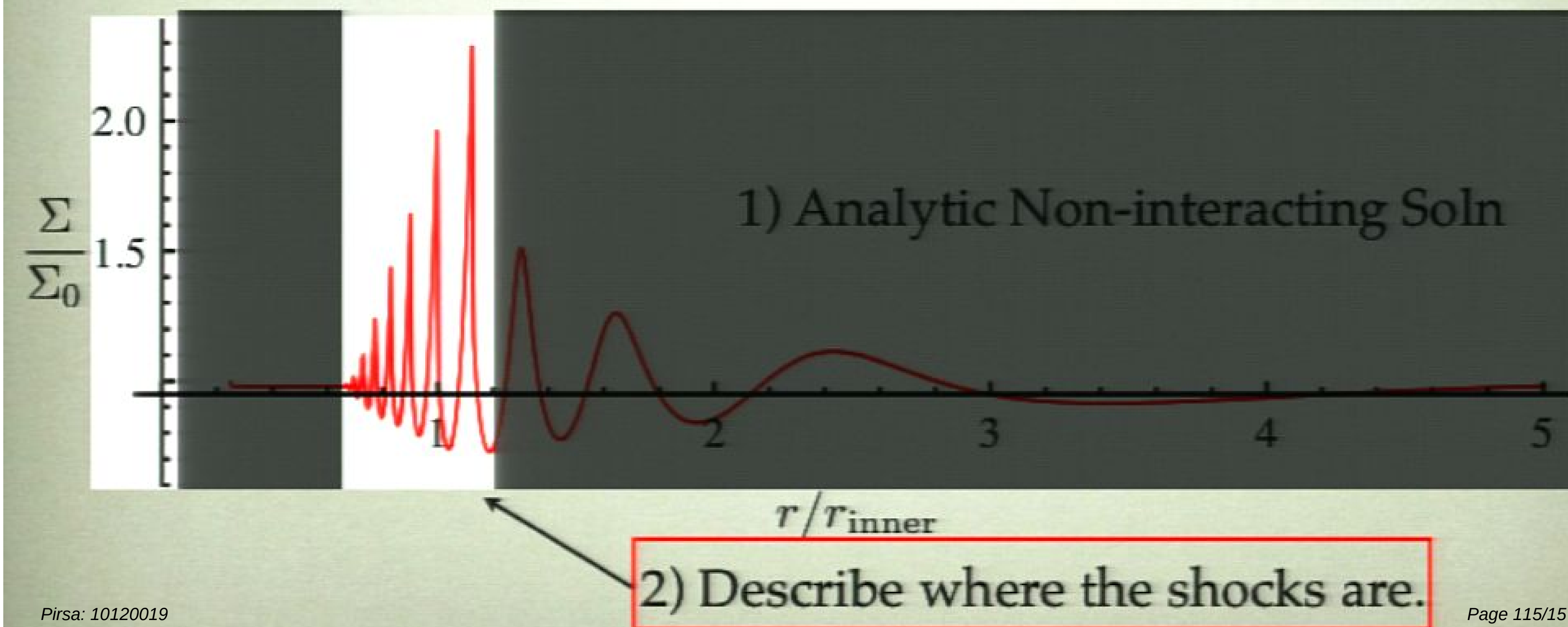

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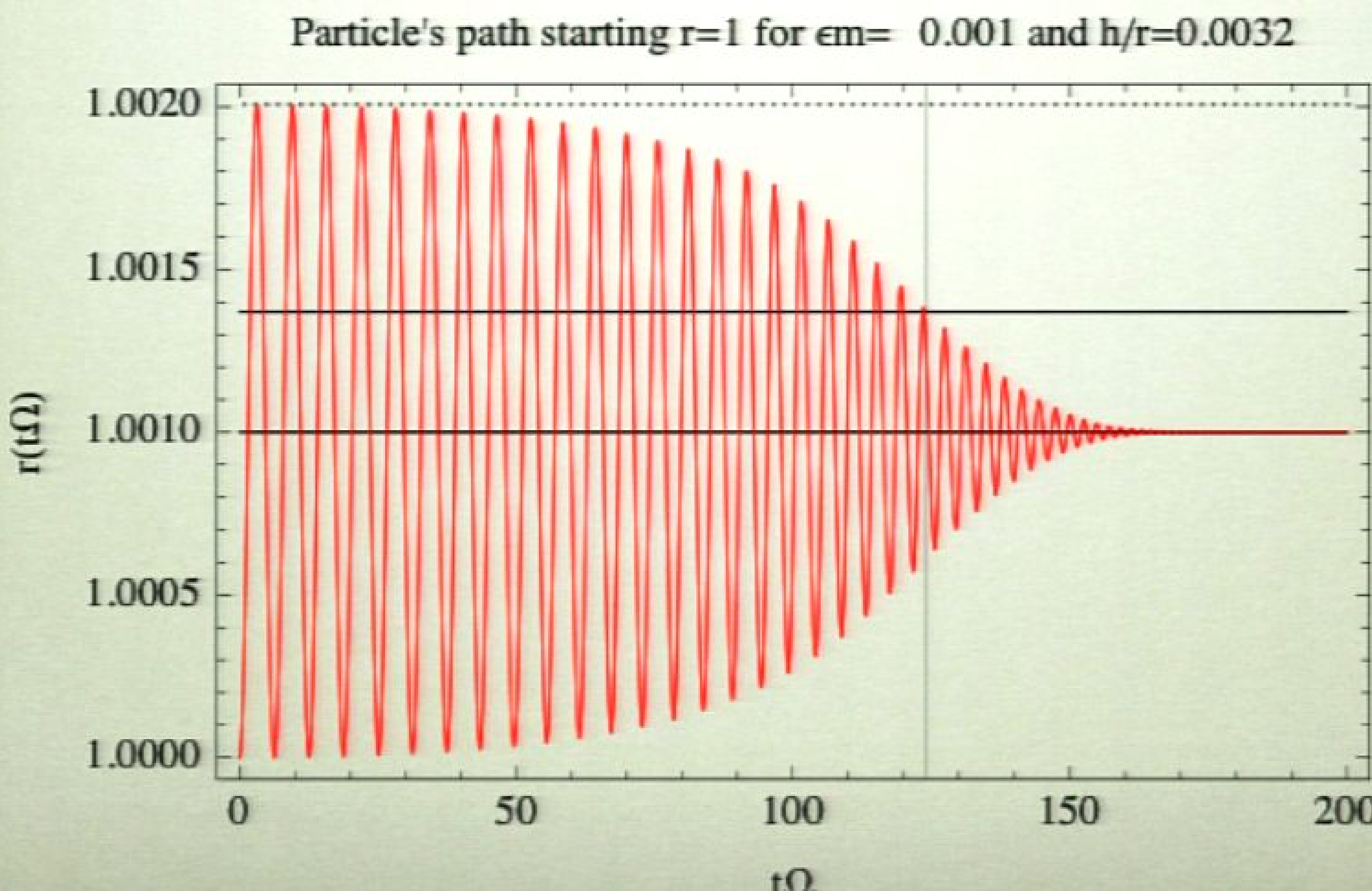
- When will a real gaseous flow deviate from our non-interacting solution?



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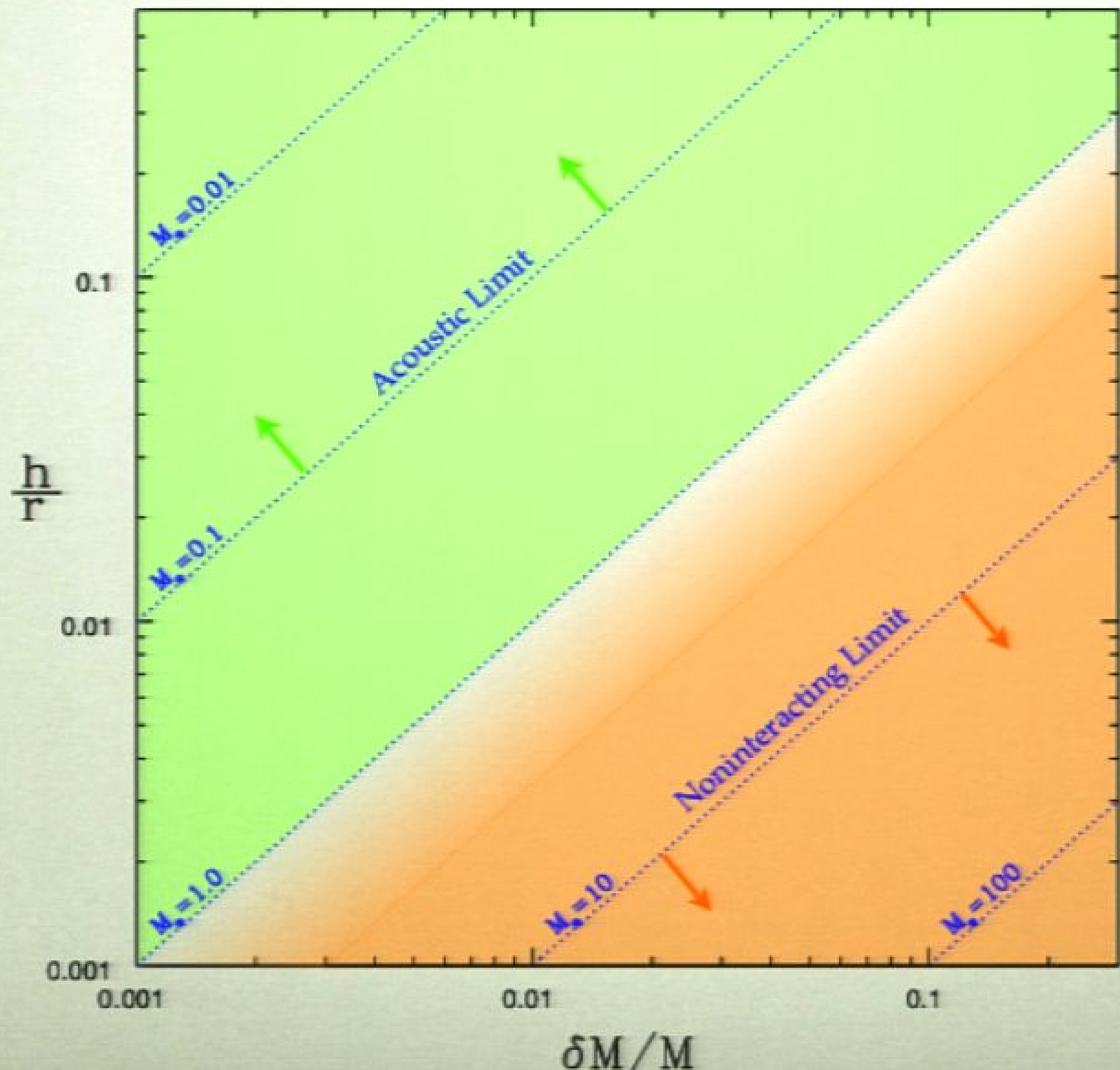
2) When $M_e \gtrsim 3$, then

$$t_{\rm dev} \lesssim t_{\rm caustic} = \left(\frac{2}{3}\frac{1}{\delta M/M}\right) t_{\rm dyn}$$

Analytic Application: Parameter Space



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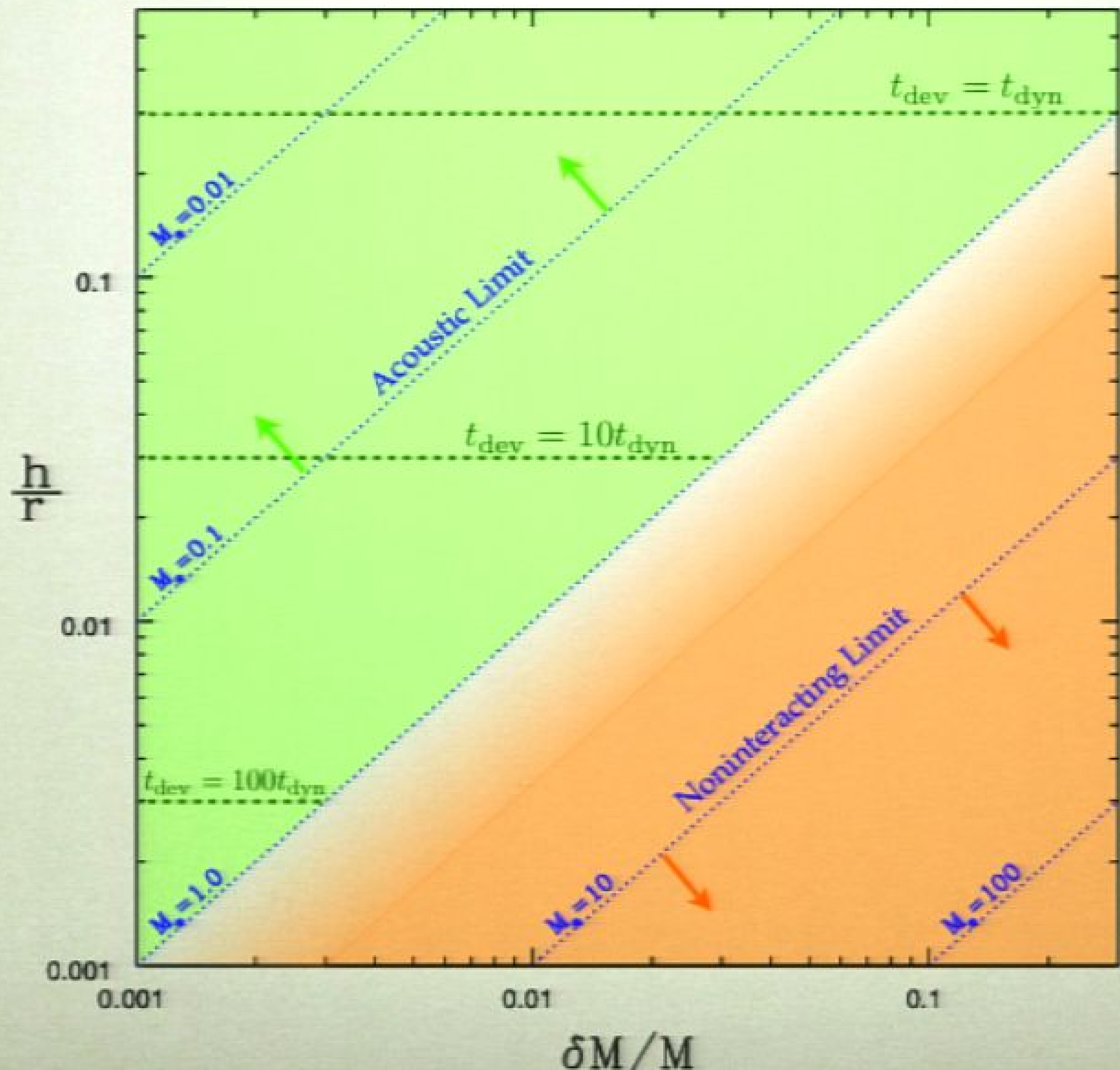

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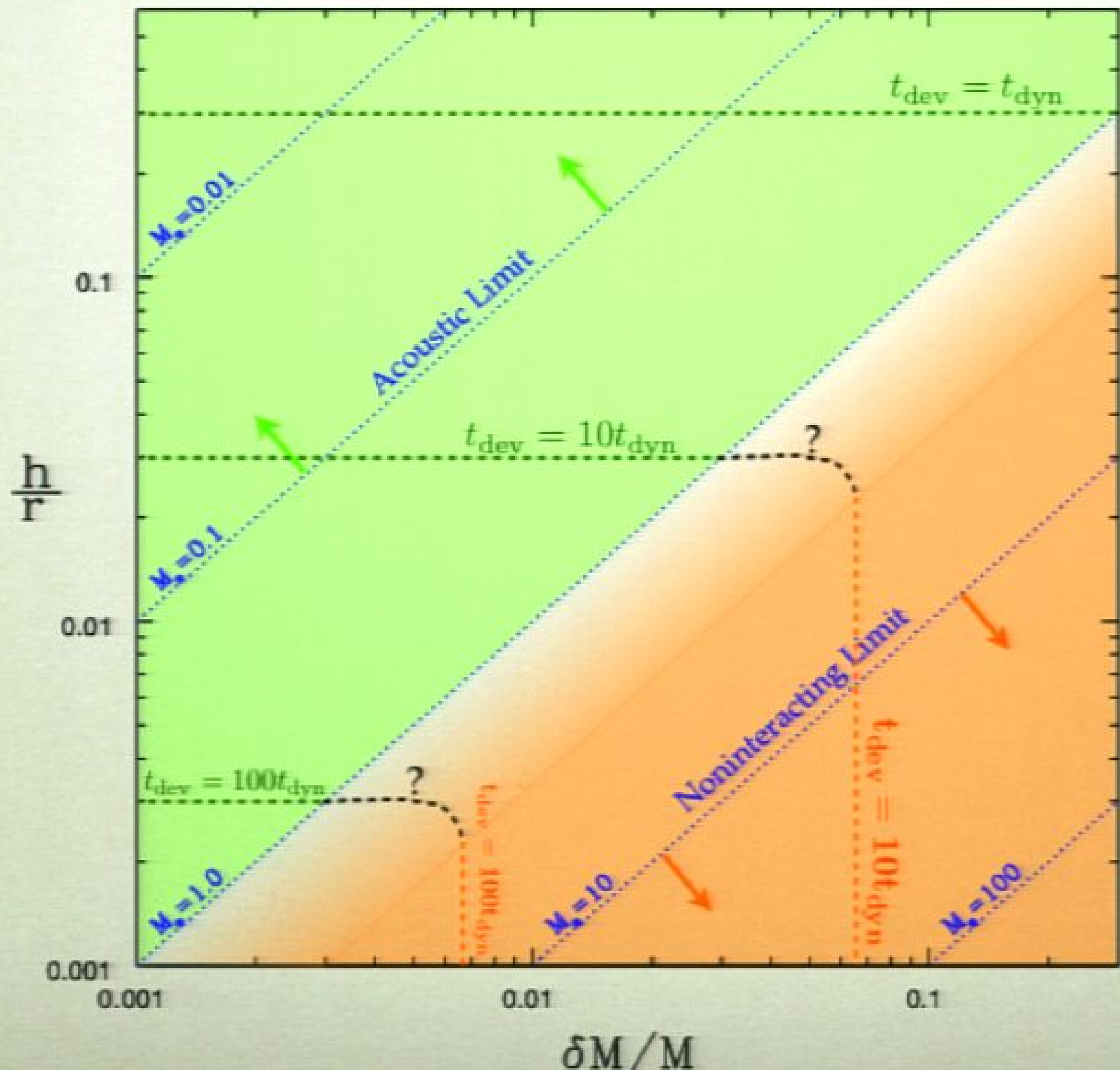

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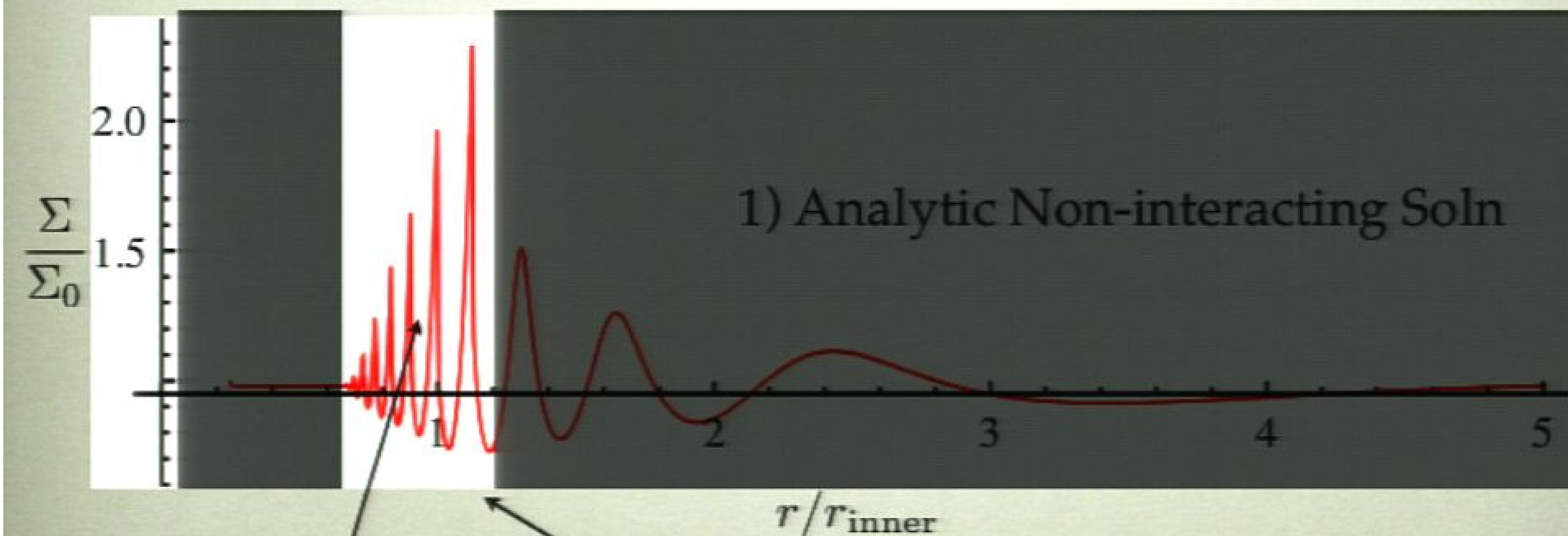

2) Describe where the shocks are.

3) Determine how strong the shocks are

Analytic Application: Shocks and Final Density

- Shocks can go no slower than the density peaks on which they form

  1) So shocks have Mach number

$$M_{\text{shock}}(N_{\text{peak}}) \gtrsim \frac{1}{3\pi(N_{\text{peak}} - 3/4)} \frac{1}{h/r}$$

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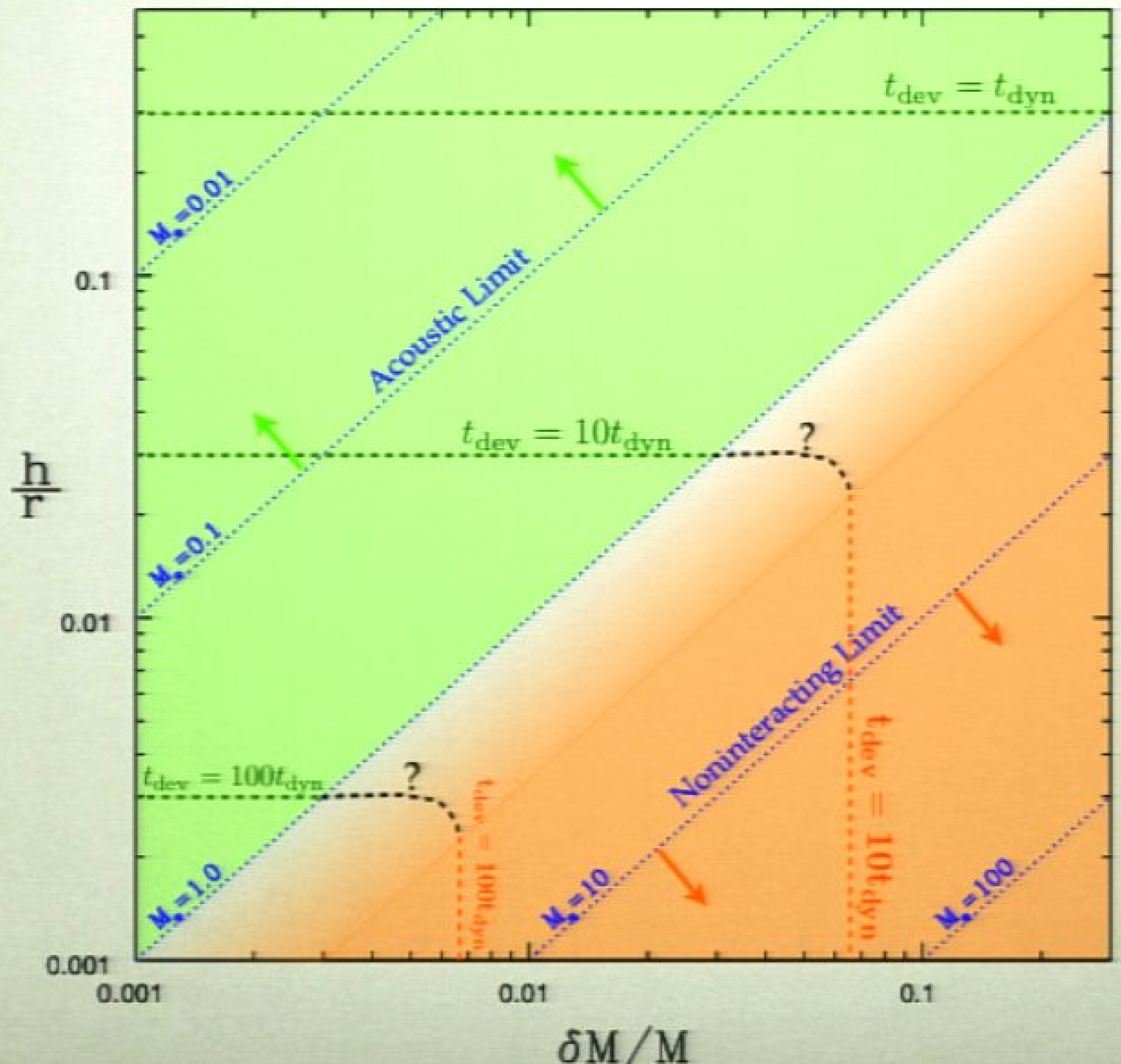
2) Using relations from previous slides we can write:

$$M_{\rm shock} \sim \begin{cases} 2 & \text{when } M_e \leq 1 \\ M_e & \text{when } M_e \gtrsim 3 \end{cases}$$

ANALYTIC APPLICATION: PARAMETER SPACE



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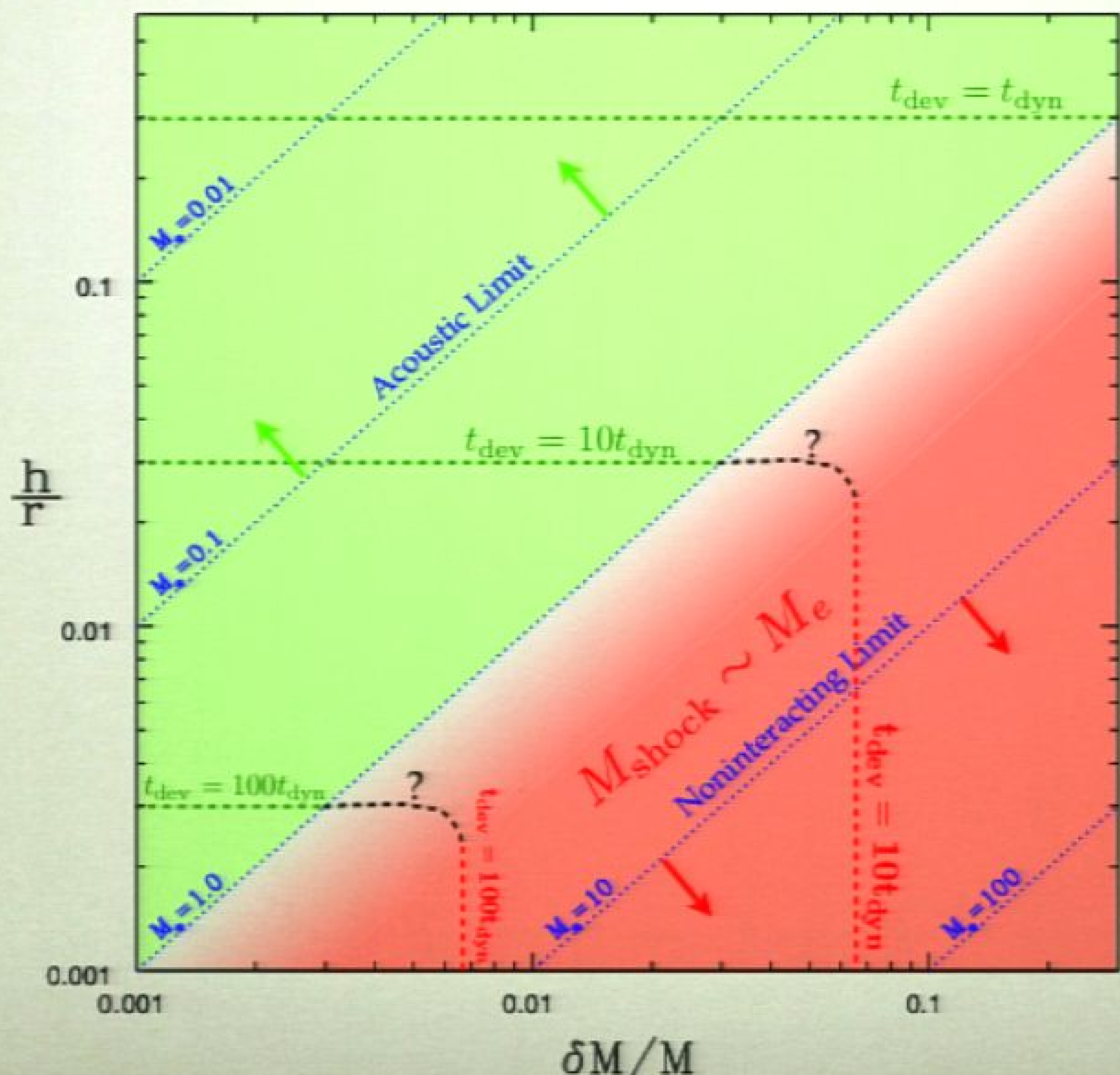
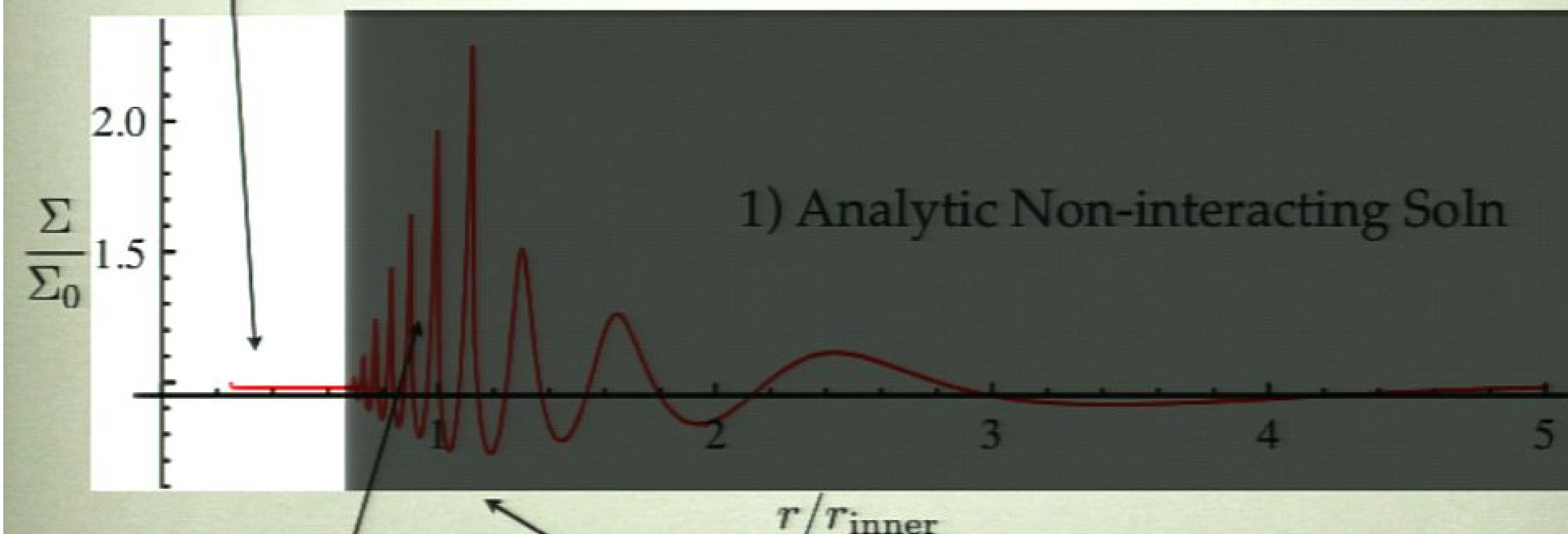


Where We're Going

4) Describe what happens after shocks.



2) Describe where the shocks are.

Analytic Application: Shocks and Final Density

- The viscous time is always much longer than the deviation time, so angular momentum should be conserved.

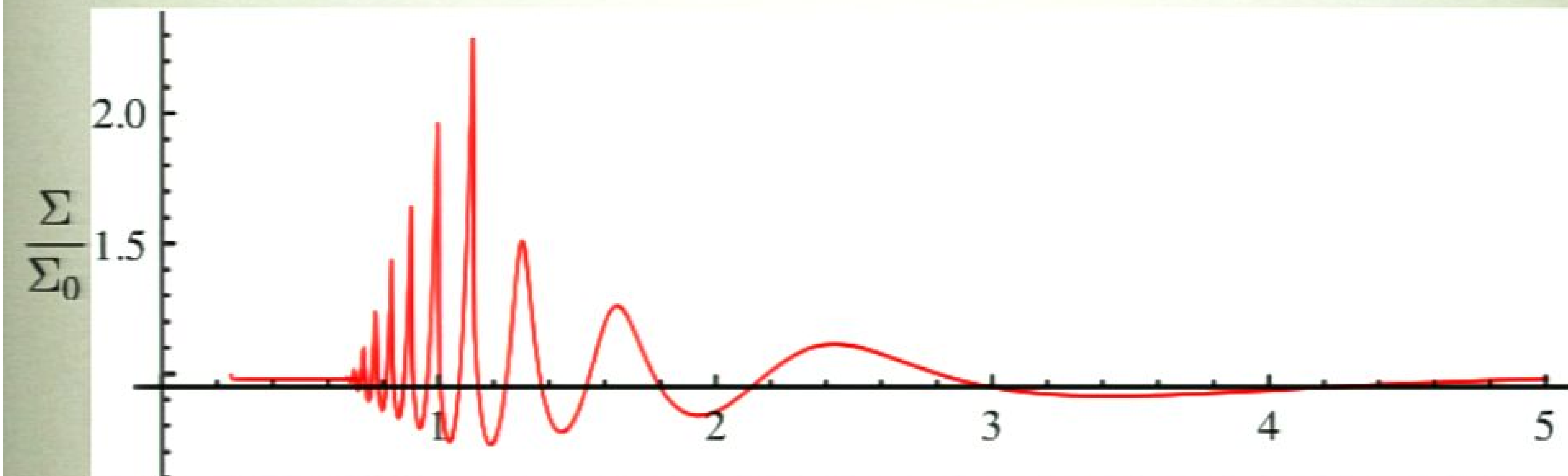
- Surface density after shocks have passed and particles of circularized

  1) Conservation of angular momentum then gives:

$$\Sigma_{\rm final} = \Sigma_0 \left(1 - \frac{\delta M}{M}\right)^2$$

SUMMARY

7) How does this compare to real gaseous disks?



SIMULATION

- Written a 1-D Lagrangian finite-differencing leapfrog scheme evolving the four coupled fluid equations:

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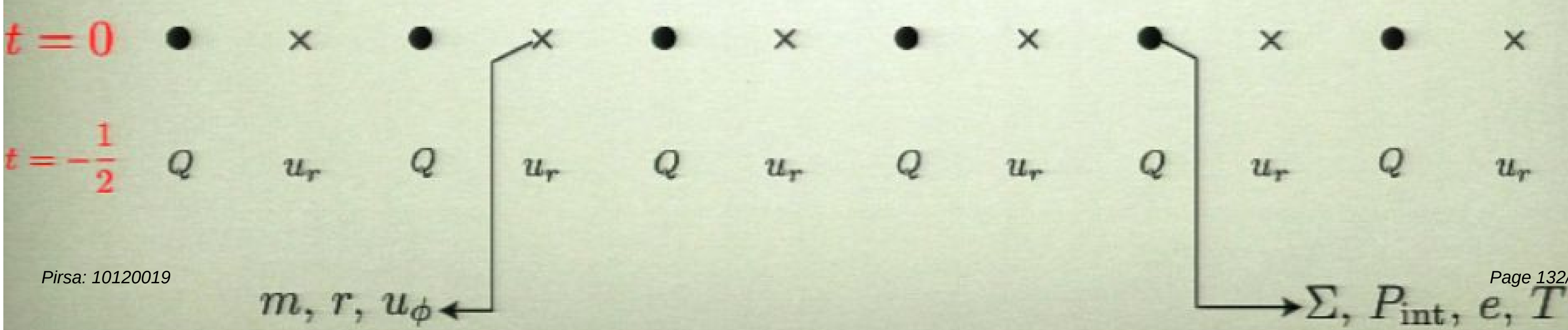

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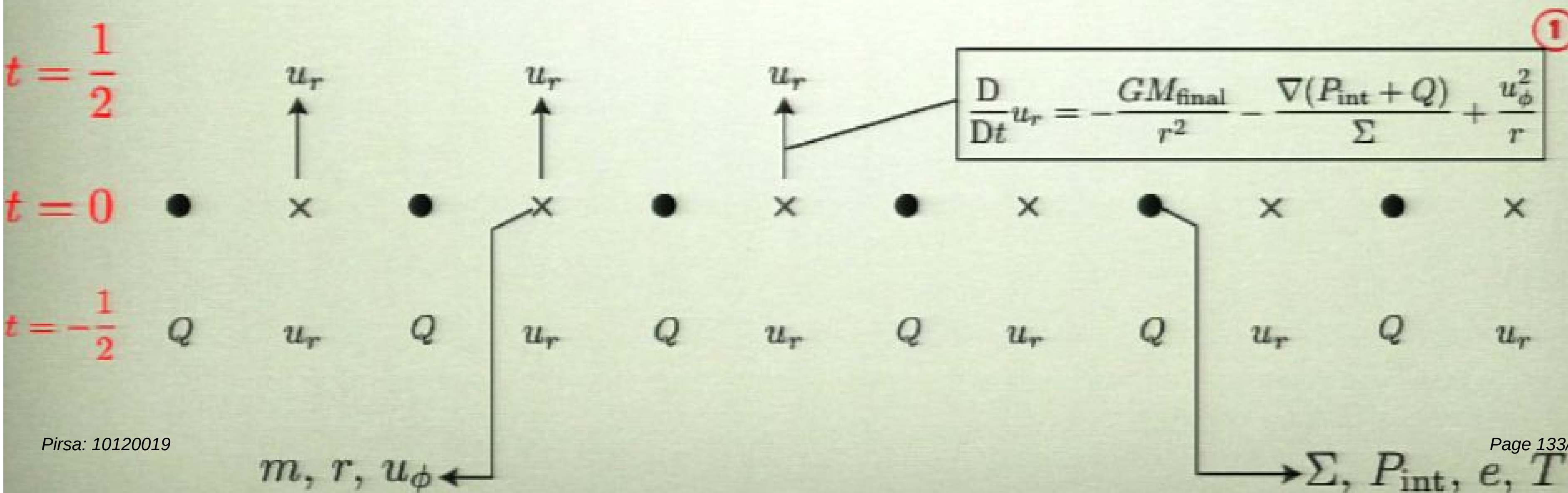

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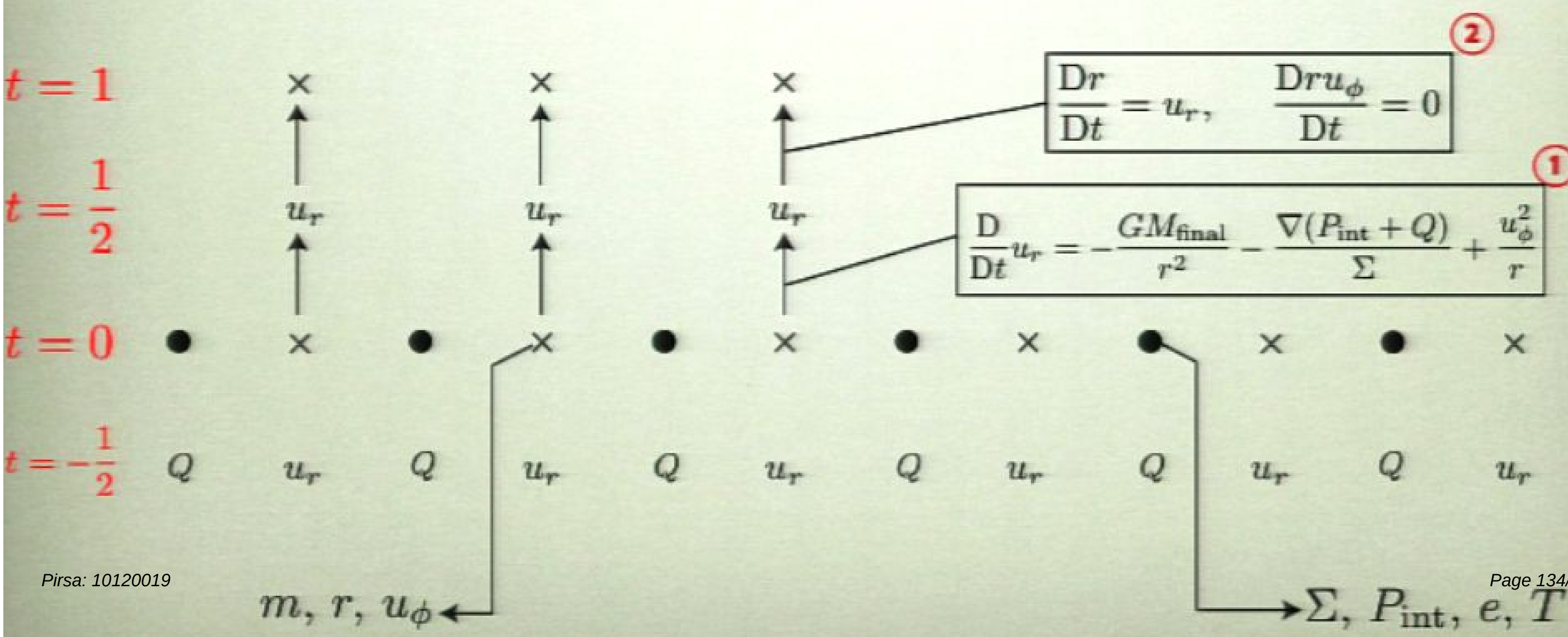

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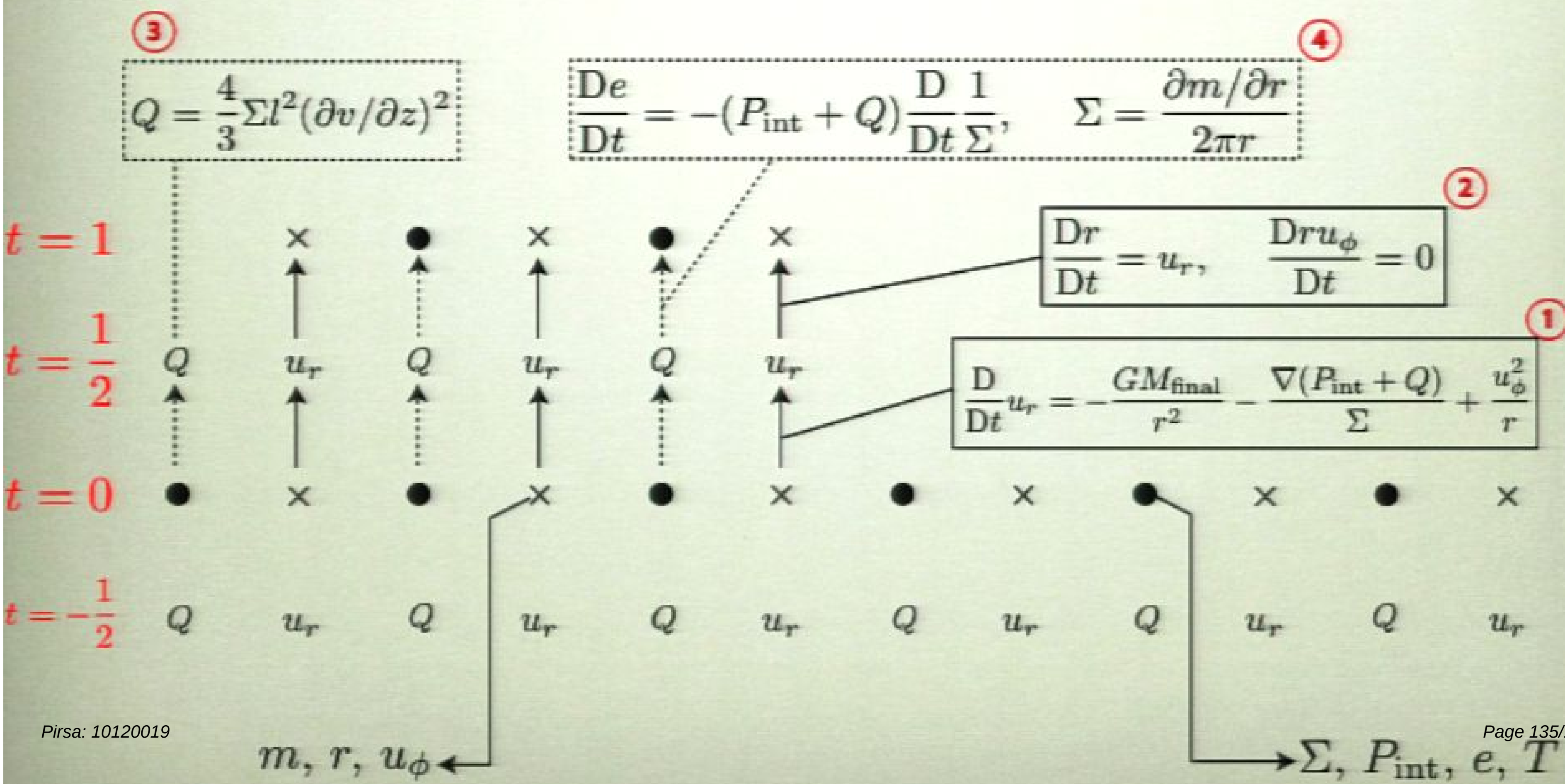
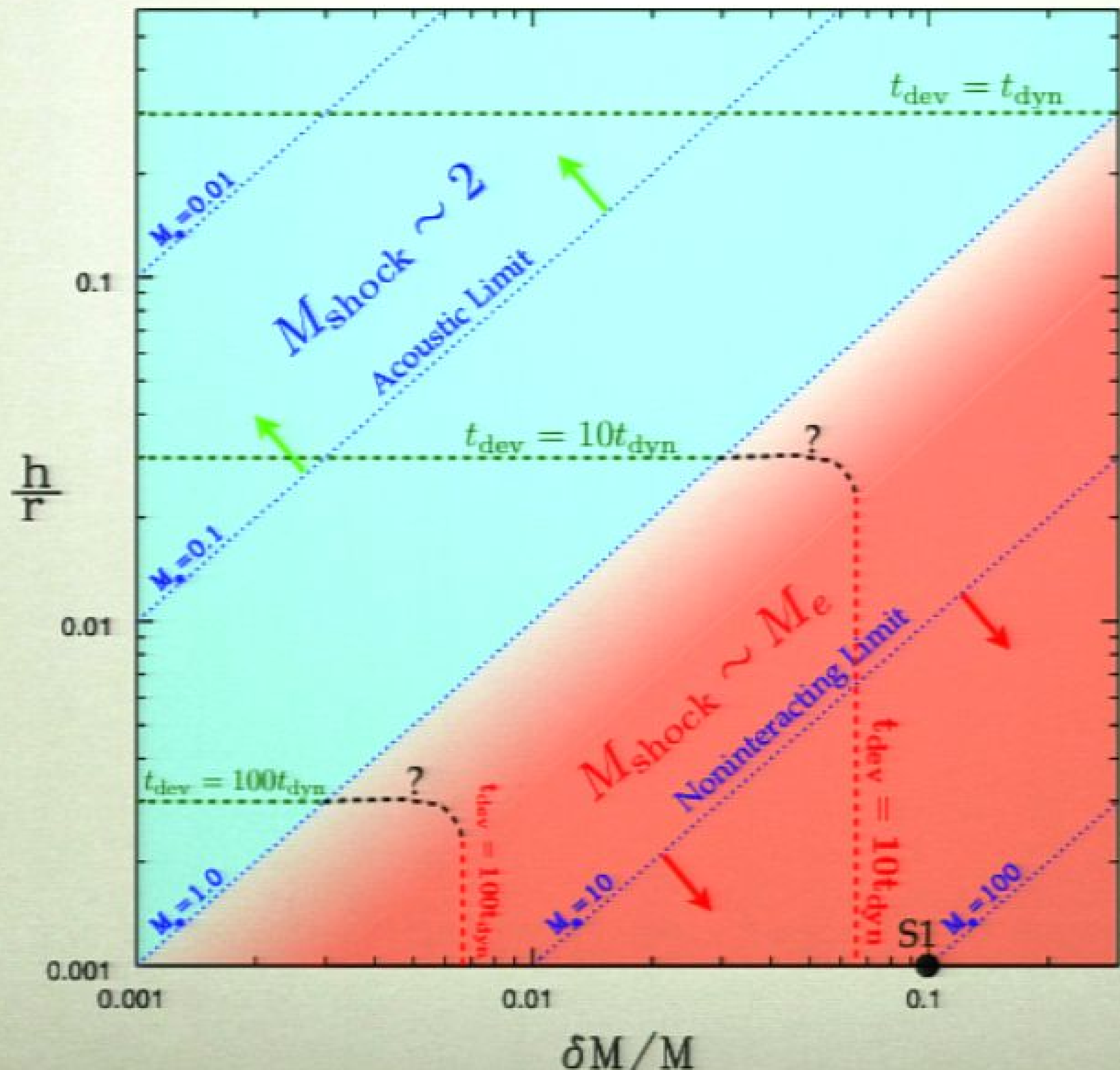
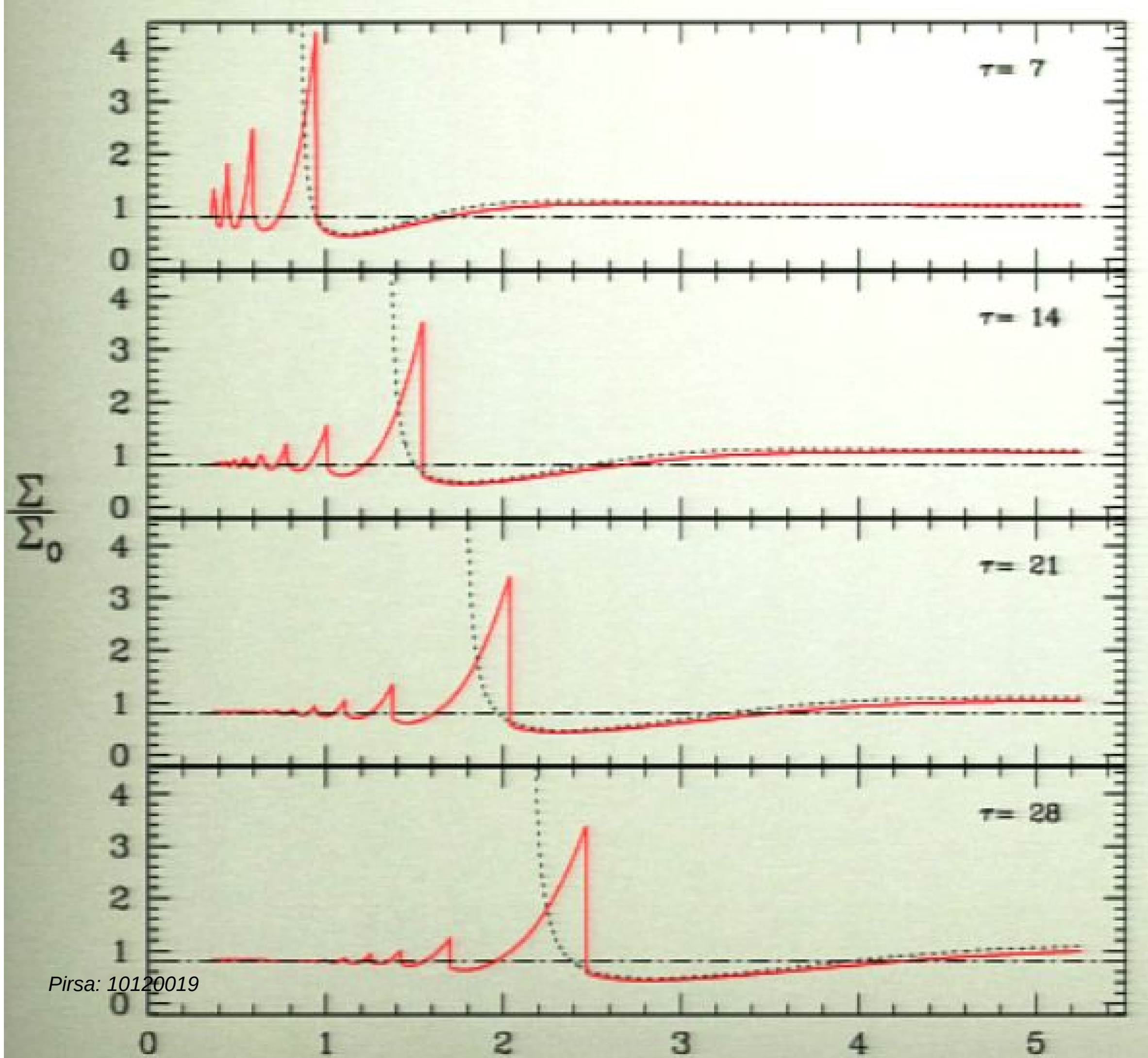


Analytic Application: Parameter Space



Simulation: Results

Ratio of densities.



$$\frac{\delta M}{M_0} = 0.1$$
$$h(r)/r = 0.001$$

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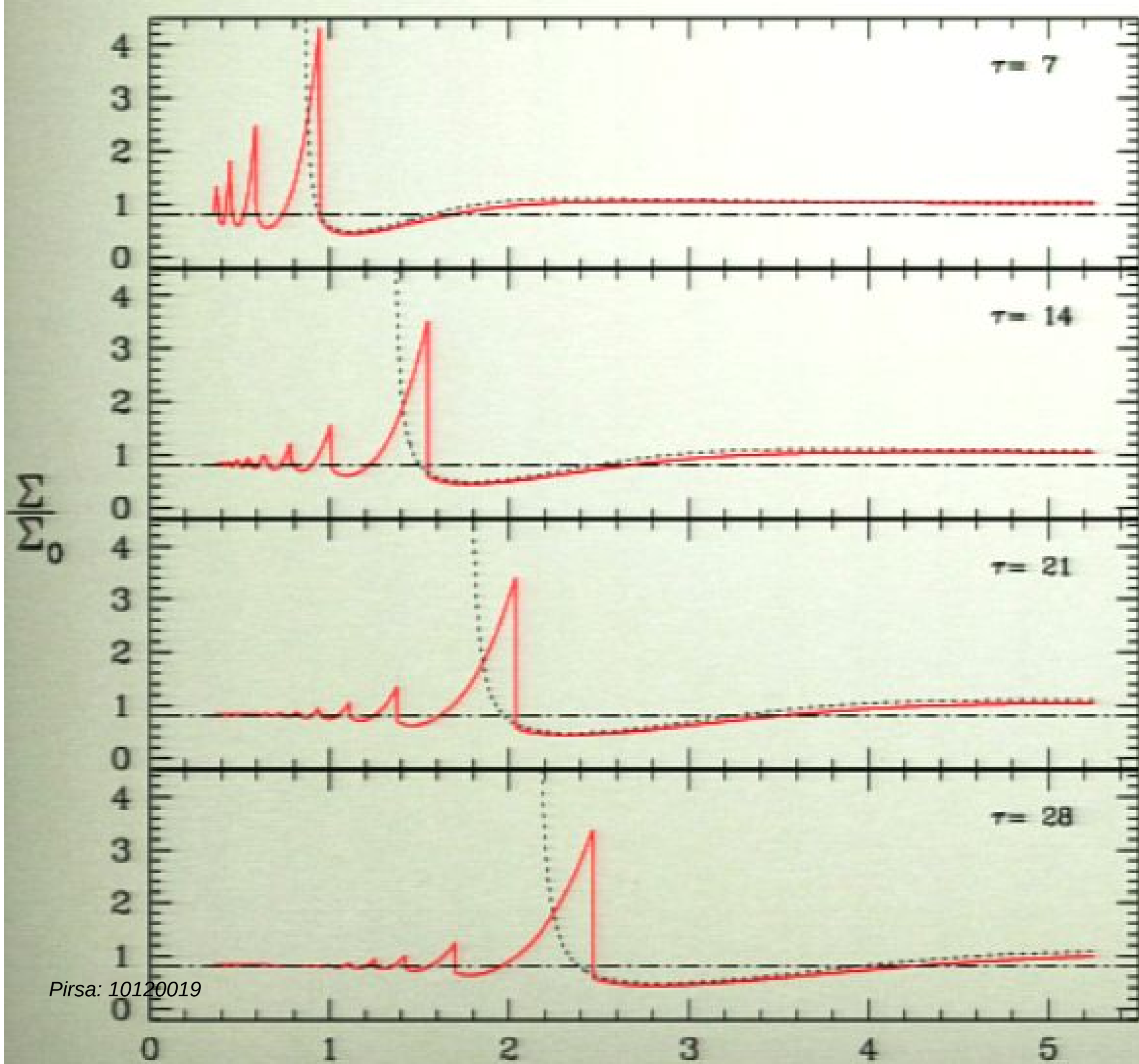

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$$\downarrow$$

$$M_e = 100 \gg 1$$

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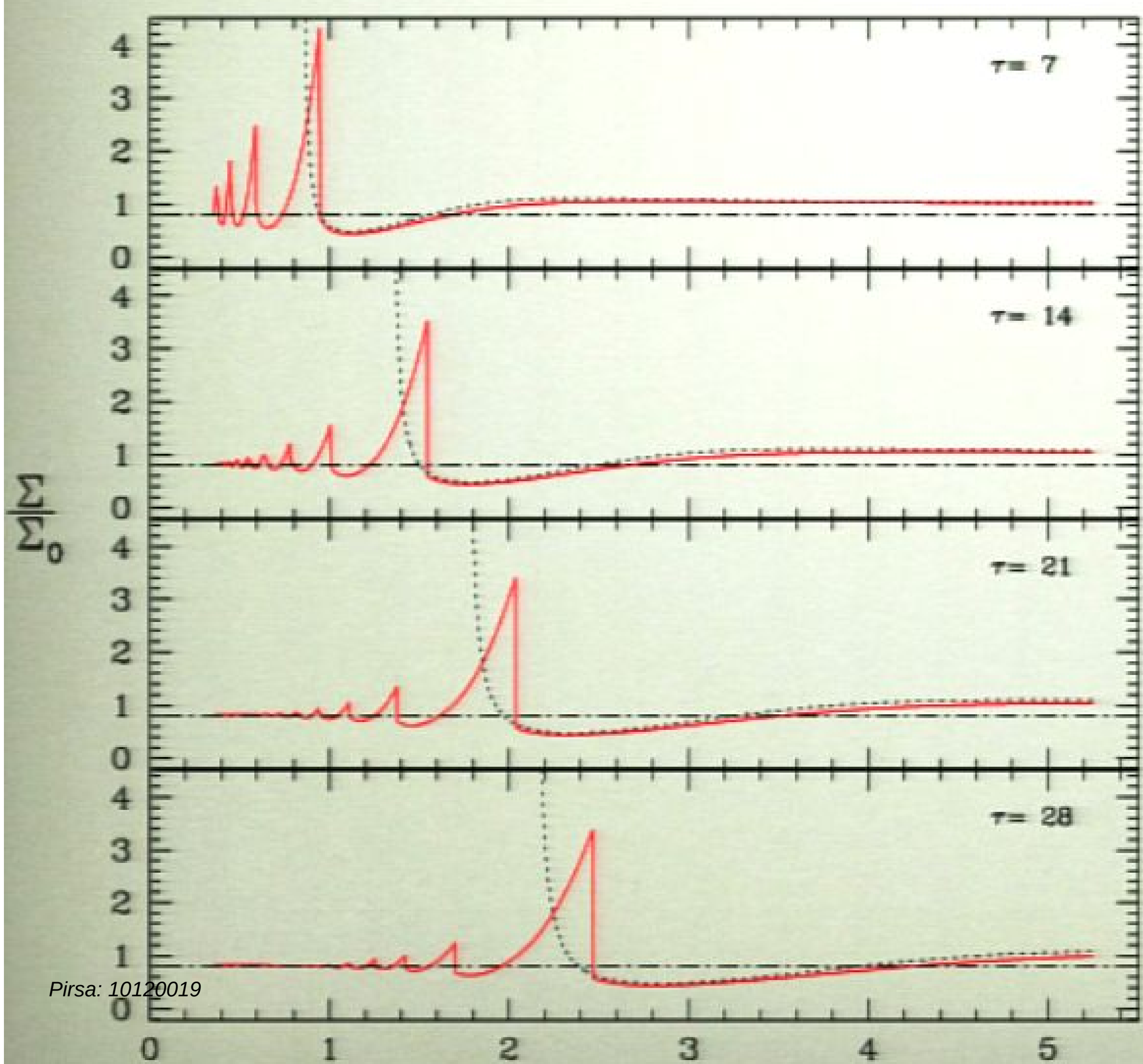

$$\frac{\delta M}{M_0} = 0.1$$
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1. Deviation time is approx. the caustic formation time, but 1st caustic is second peak.

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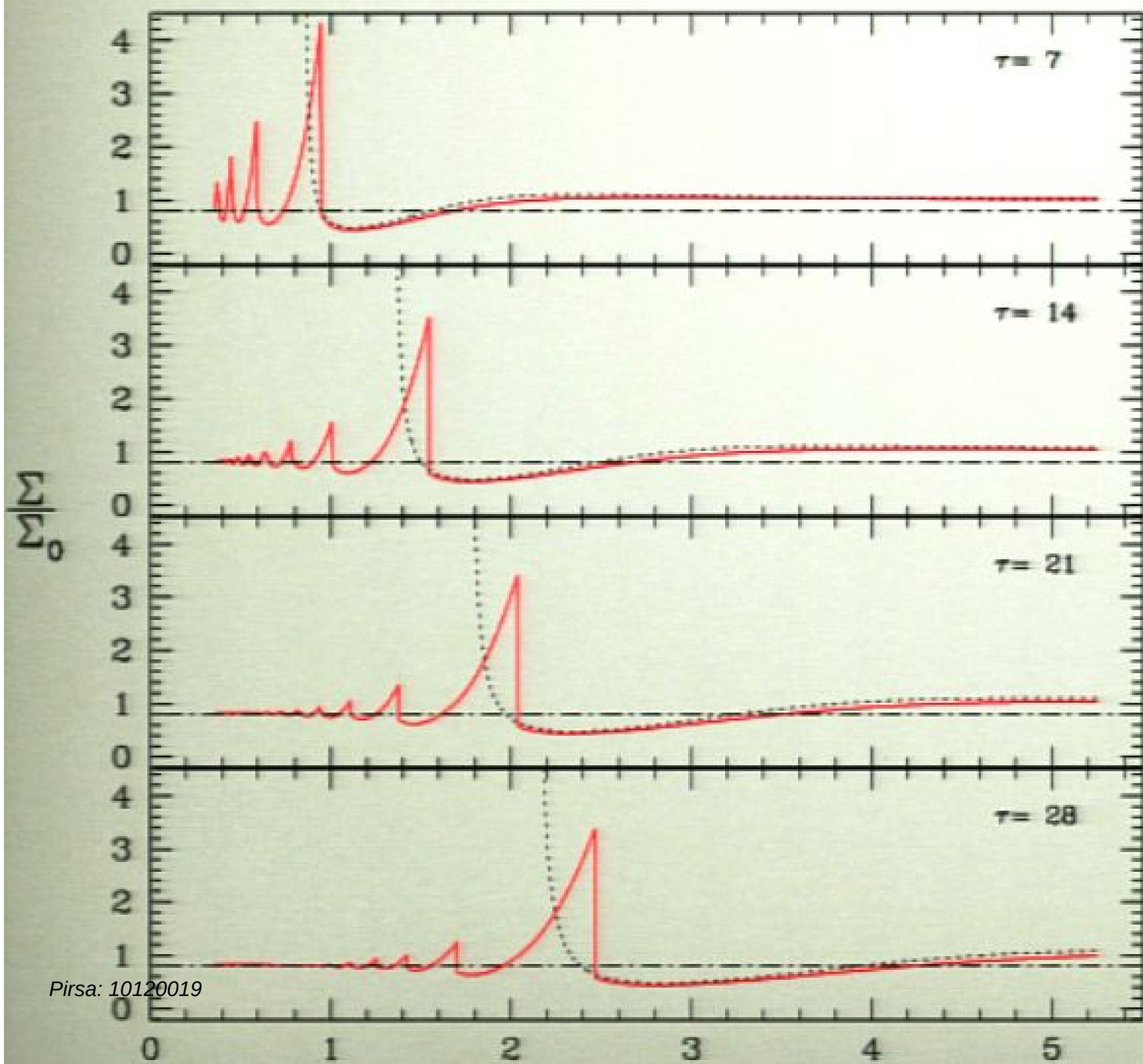

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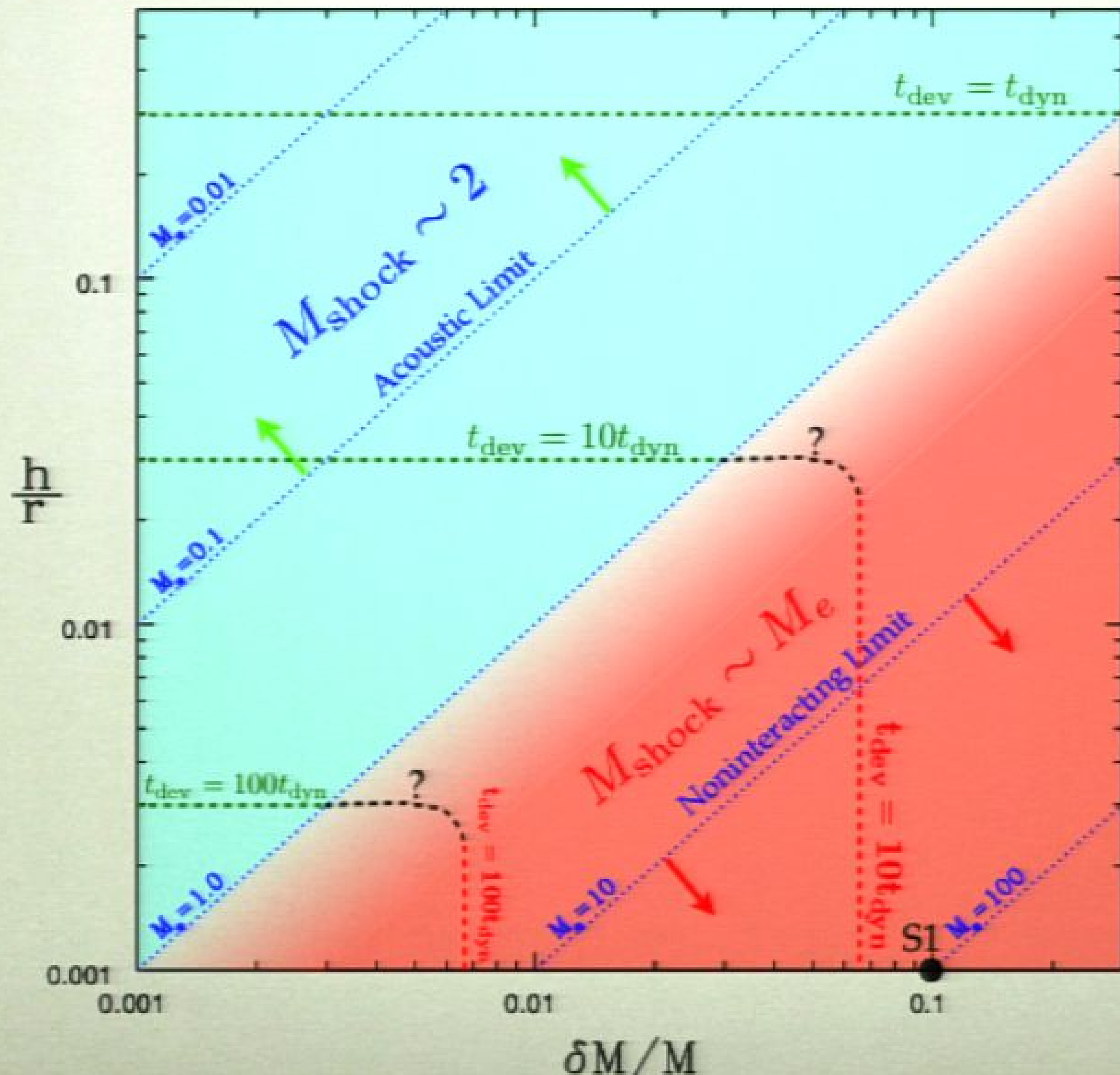
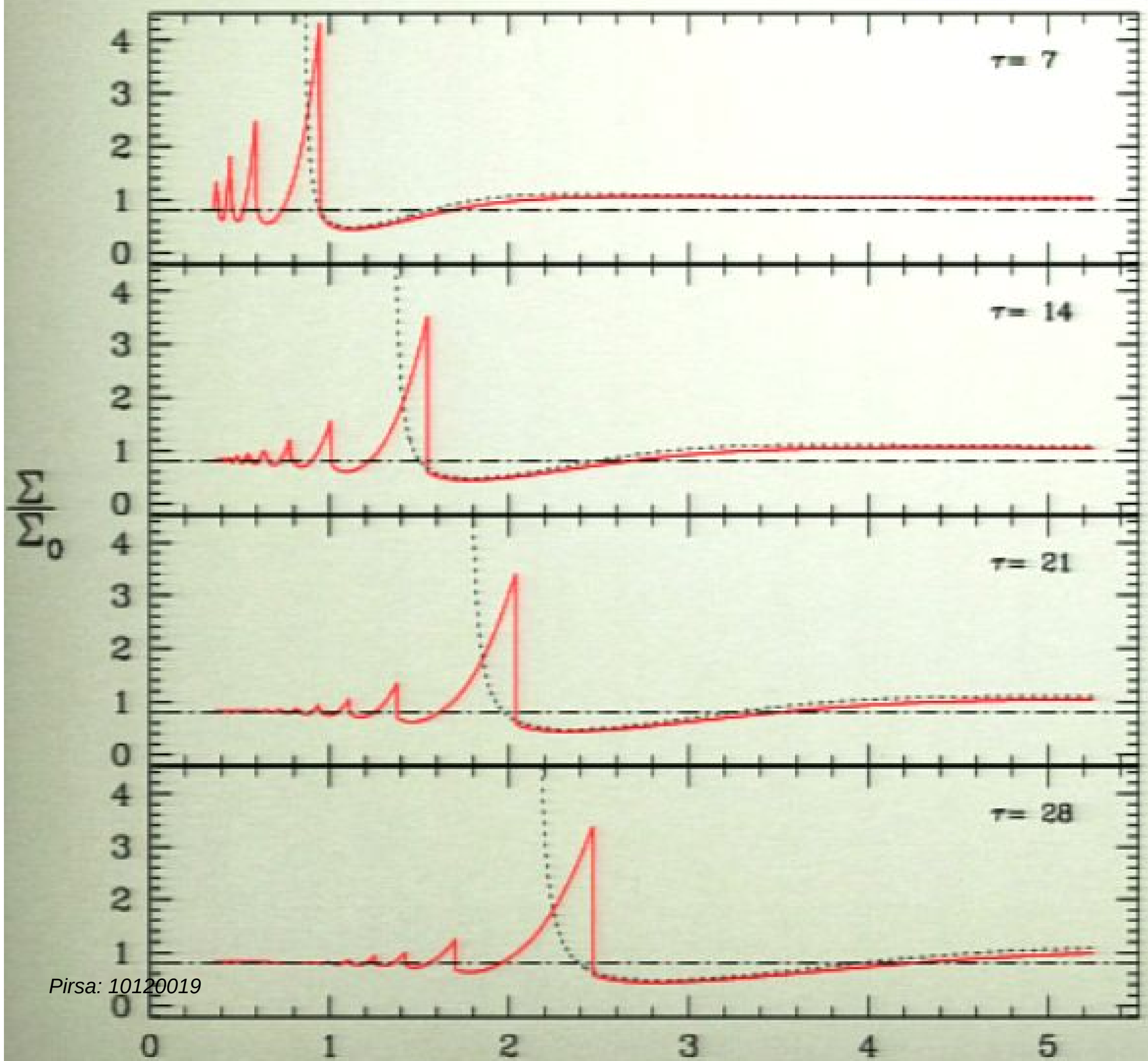
$$M_{\mathrm{shock}} \approx 150$$

Analytic Application: Parameter Space



Simulation: Results

Ratio of densities.



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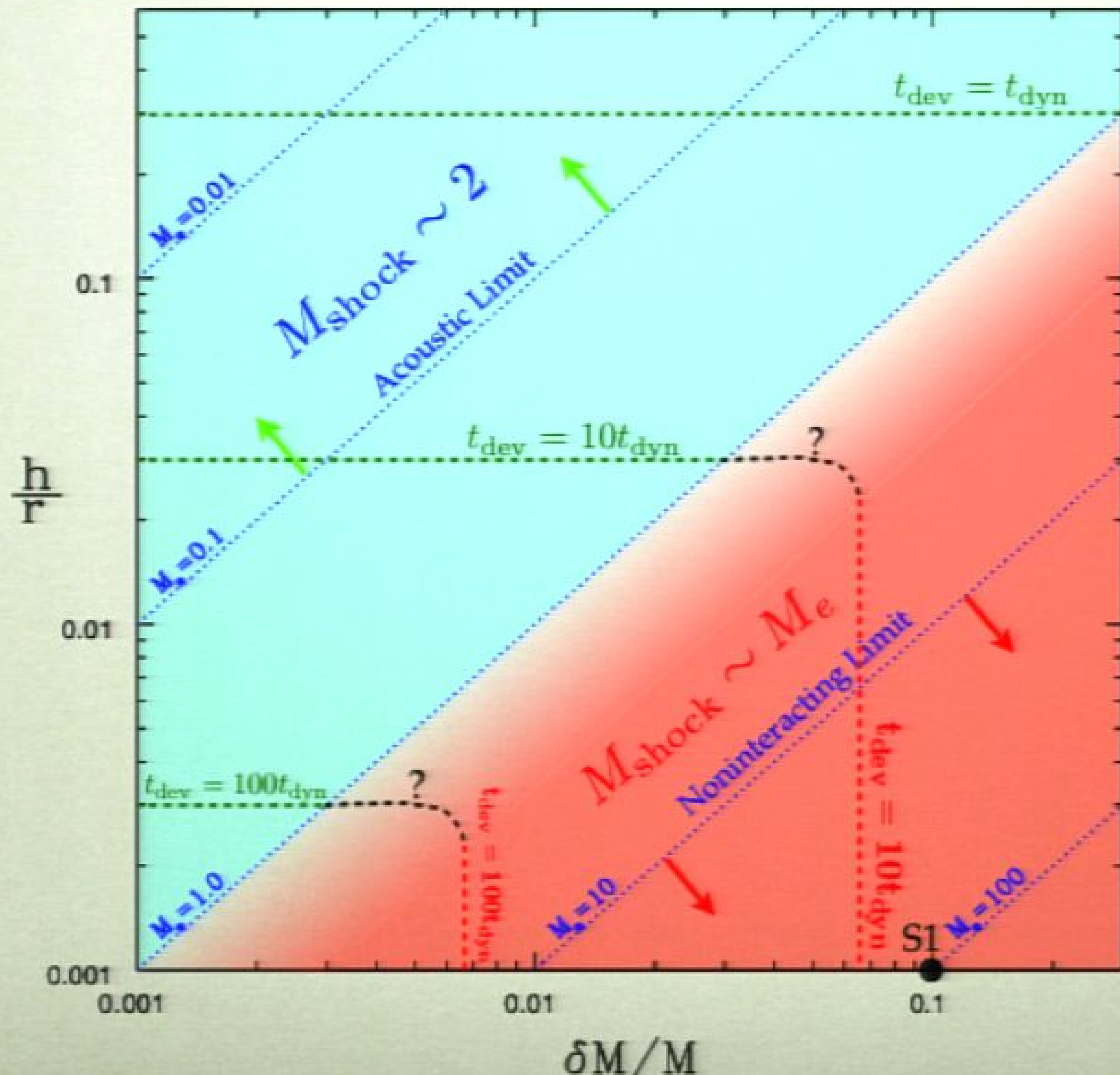
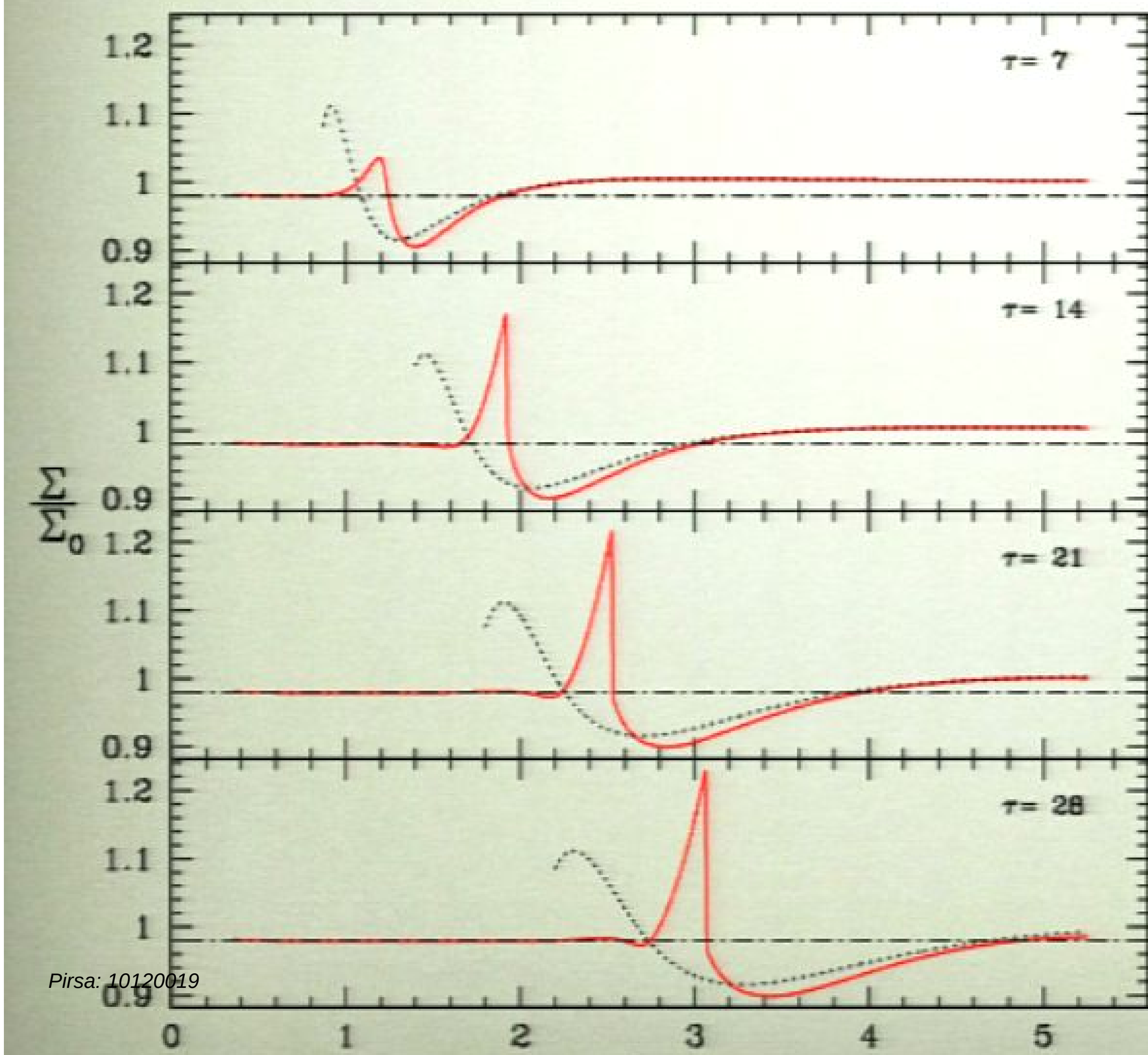


Simulation: Results

Ratio of densities.



$$\frac{\delta M}{M} = 0.01$$

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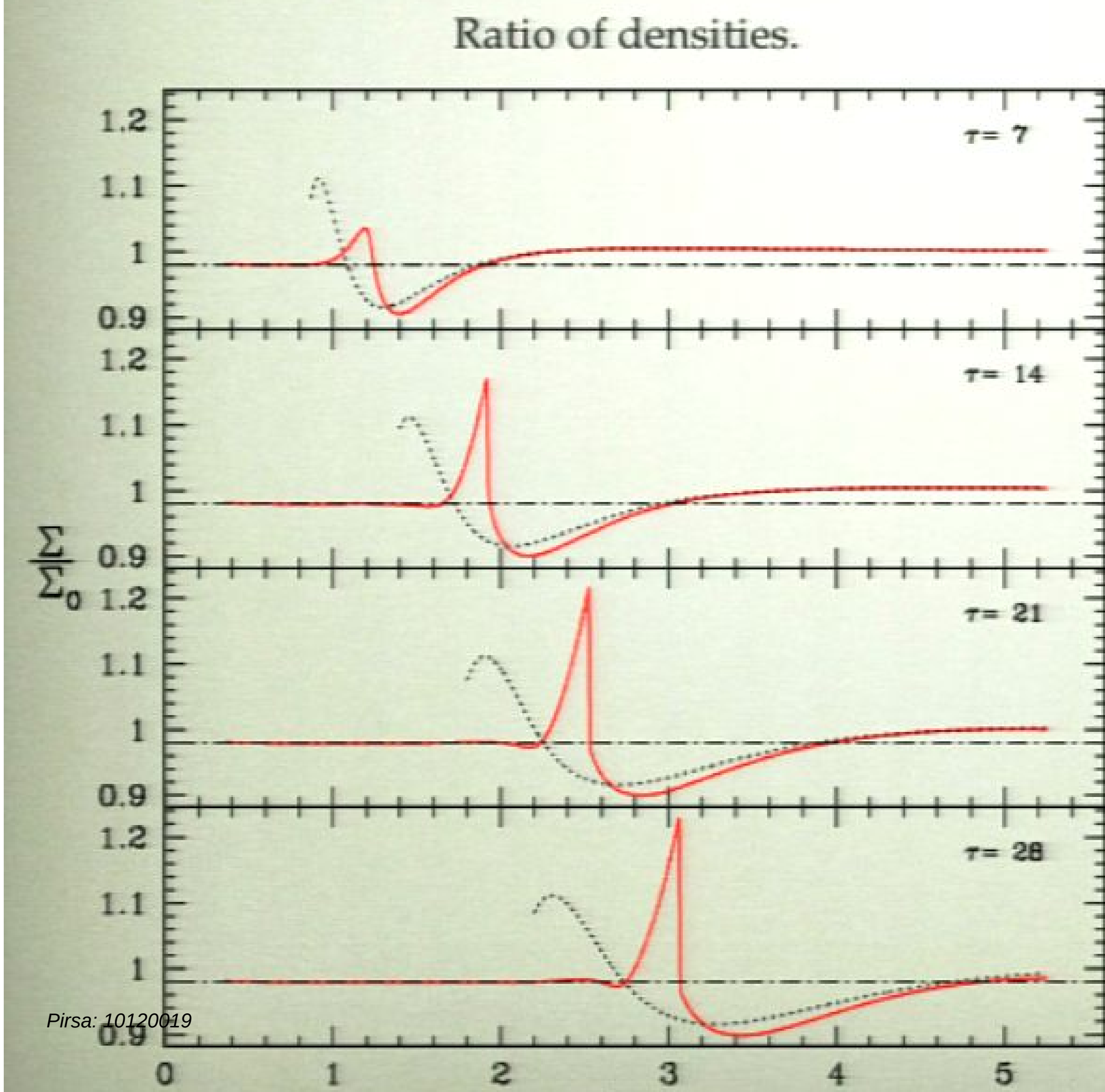
SIMULATION: RESULTS



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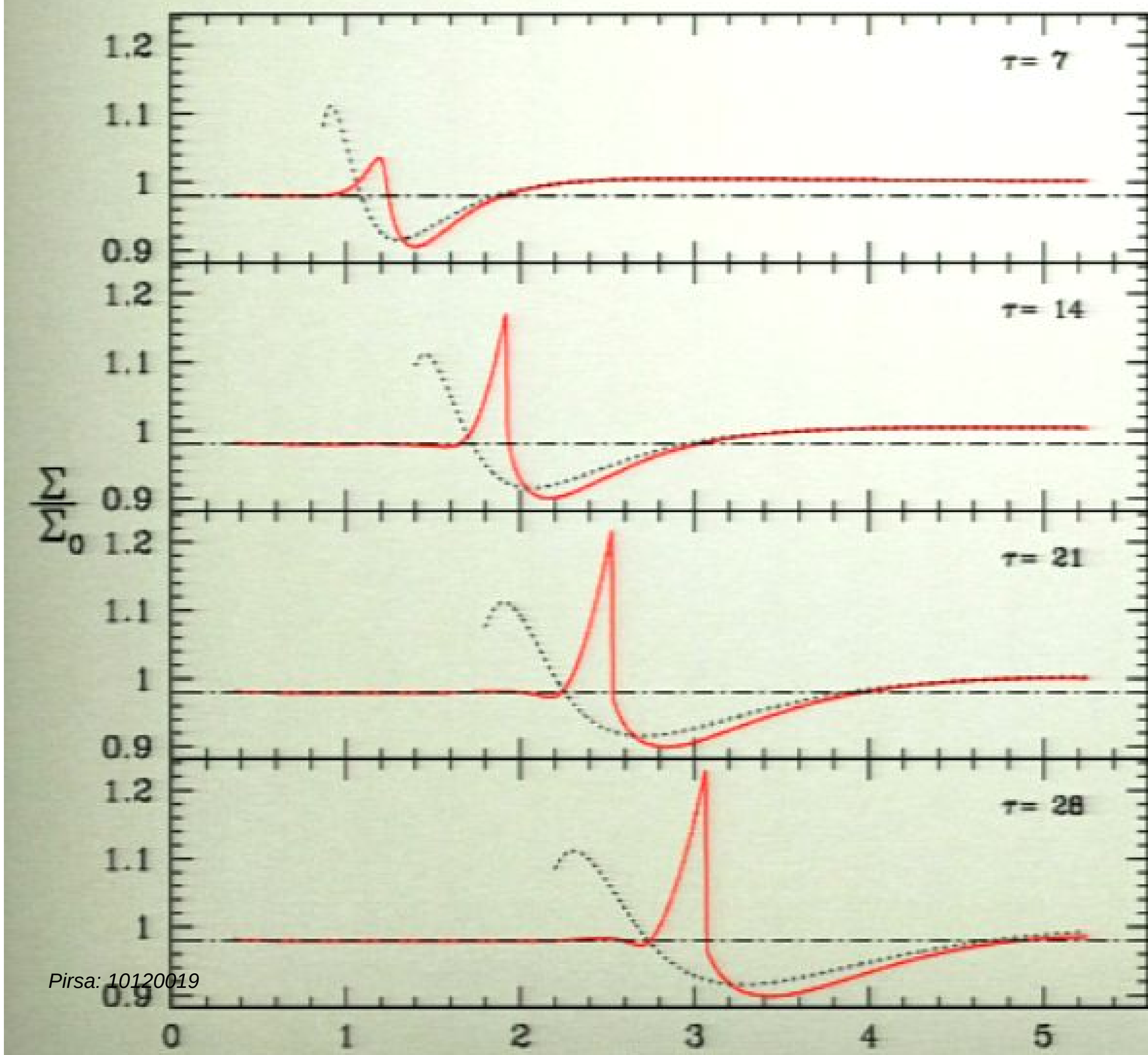
$$\downarrow$$

$$M_e = 0.1$$

Simulation: Results

Ratio of densities.



$$\frac{\delta M}{M} = 0.01$$
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↓

$$M_e = 0.1$$

↓

1. At r=1.4 should have dev. time:

$$t_{\rm dev}\Omega_{\rm inner} = \frac{1}{3}\frac{1.4^{3/2}}{h/r} \approx 6$$

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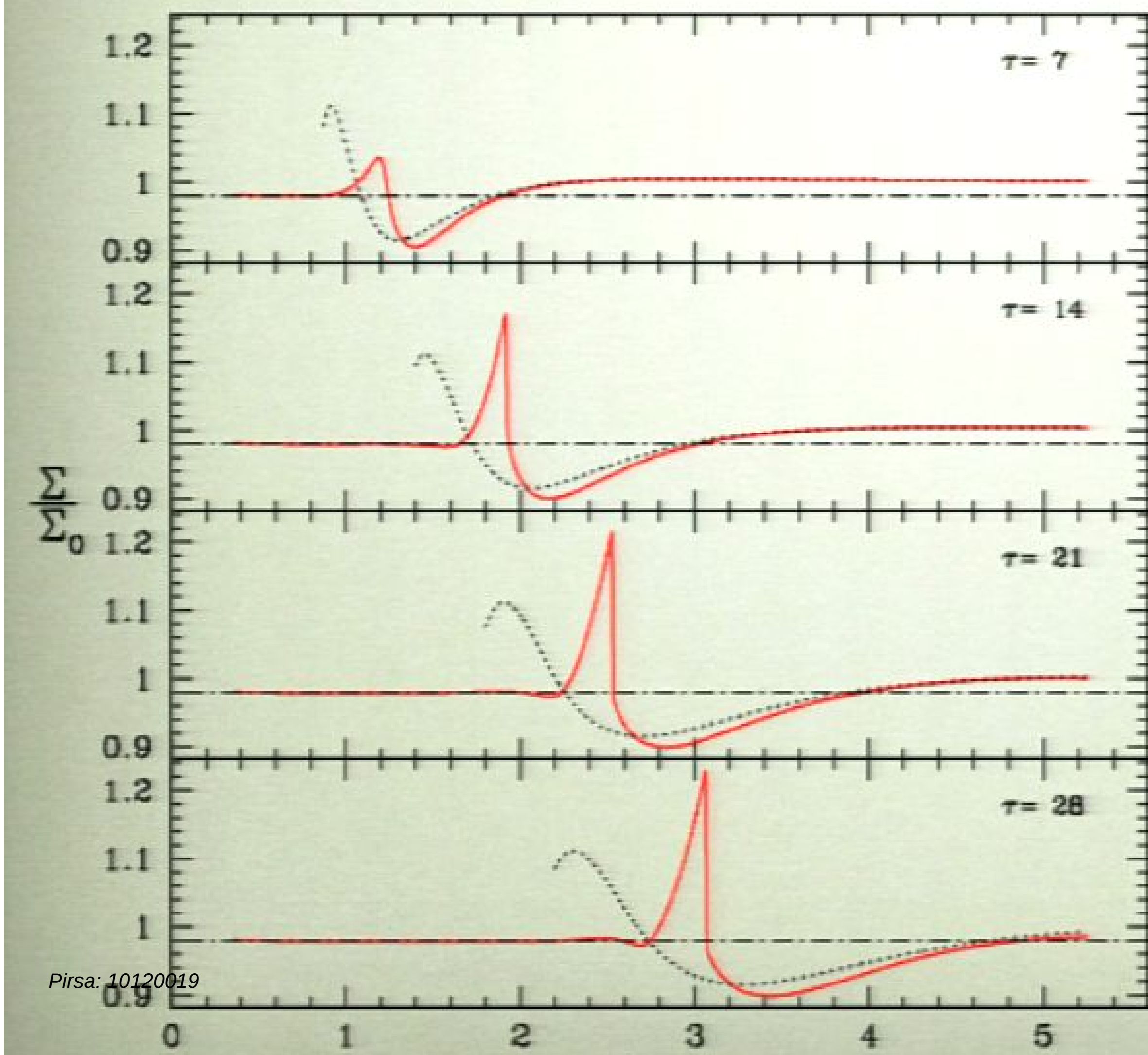

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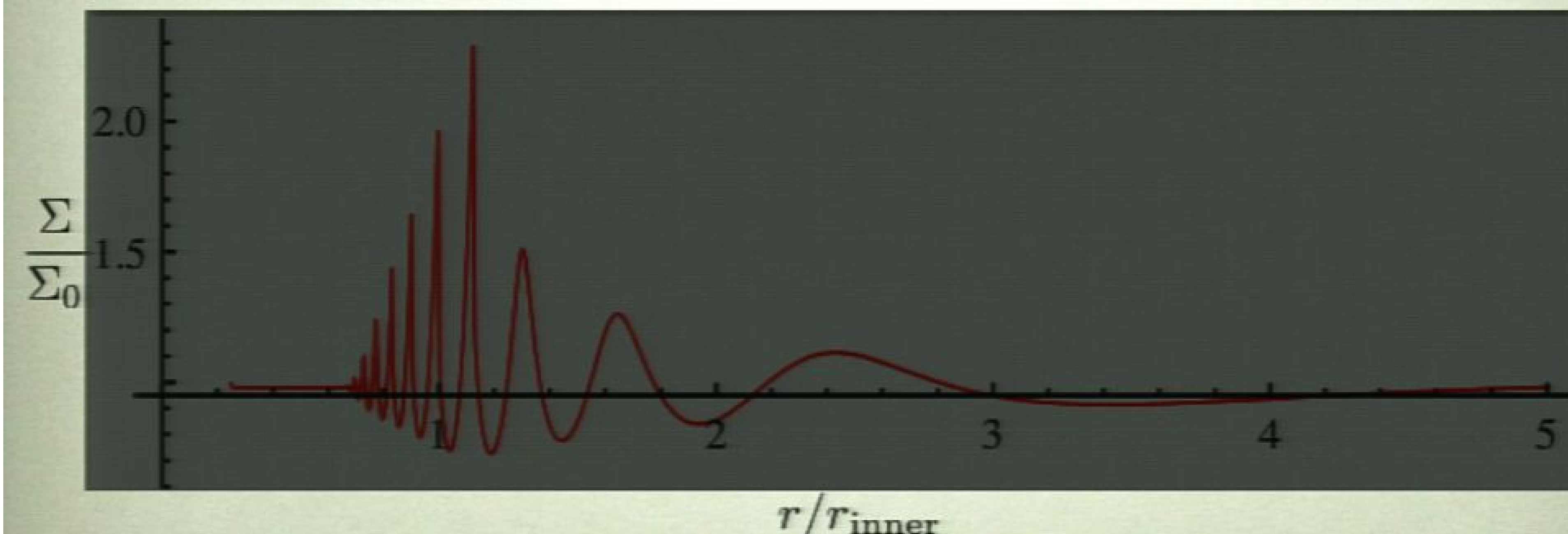
$$t_{\rm dev}\Omega_{\rm inner} = \frac{1}{3}\frac{1.4^{3/2}}{h/r} \approx 6$$

$$M_{\rm shock} \approx 1.5$$

Conclusion



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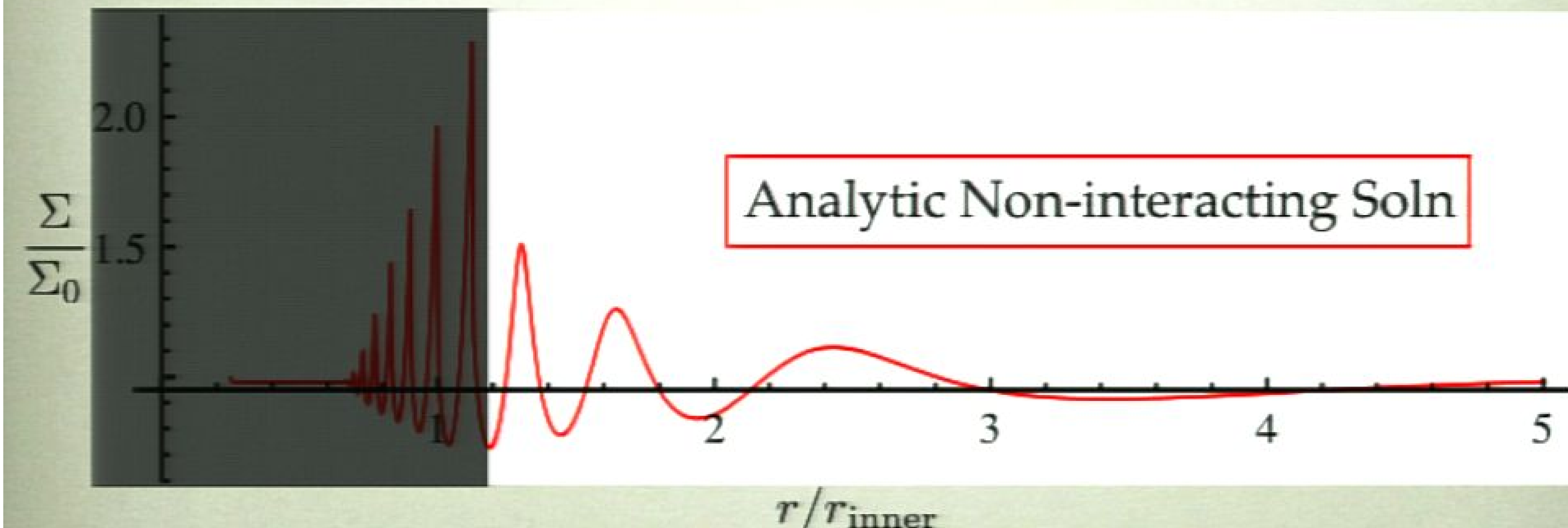
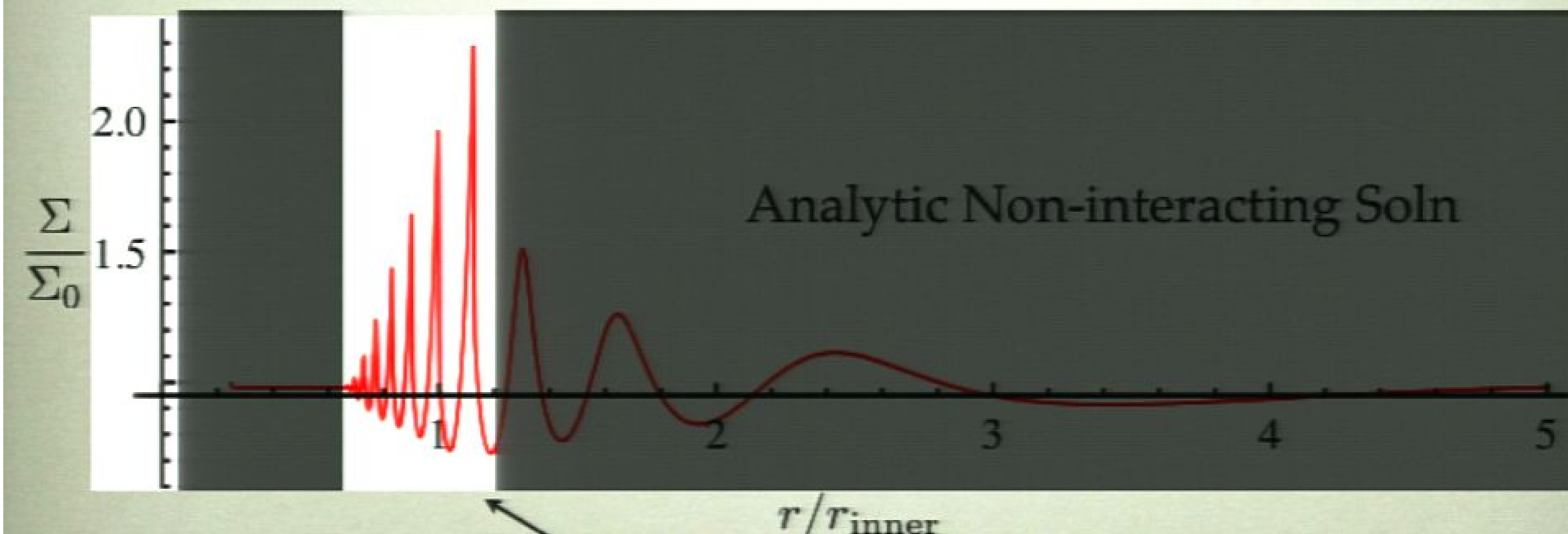


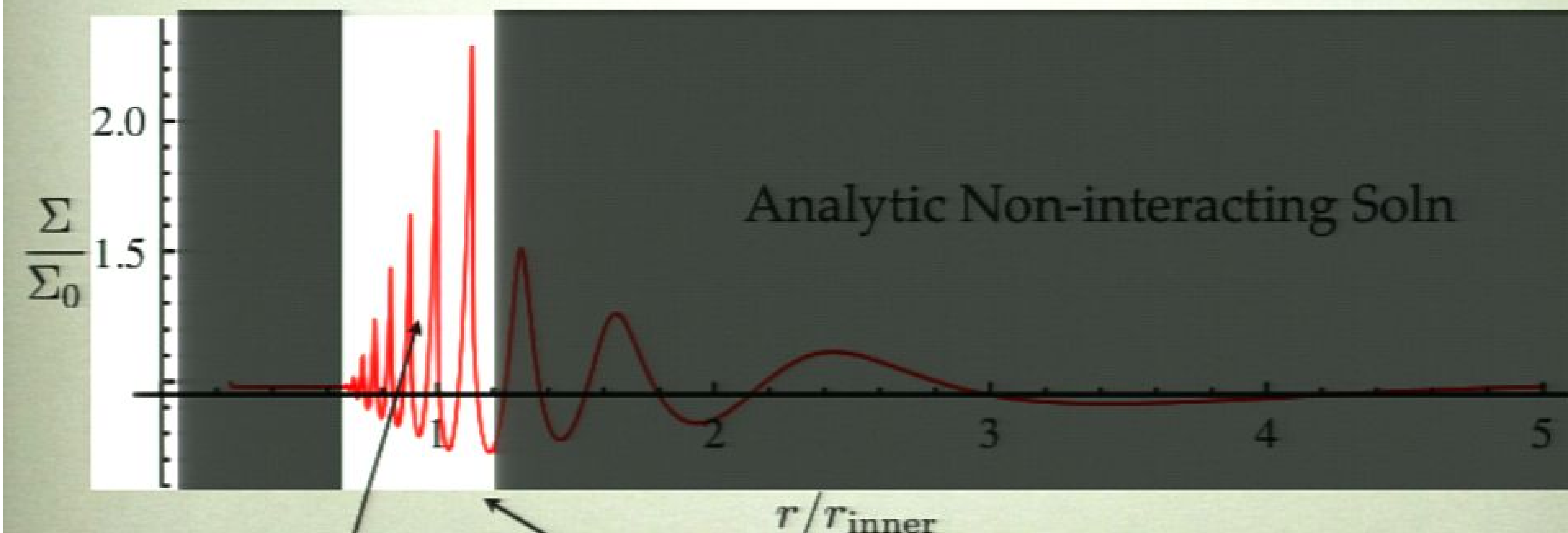
Analytic Non-interacting Soln
$\frac{\Sigma}{\Sigma_0}$
2.0
1.5
1
2
3
4
5
$r/r_{\rm inner}$

Conclusion



Described where the shocks were.

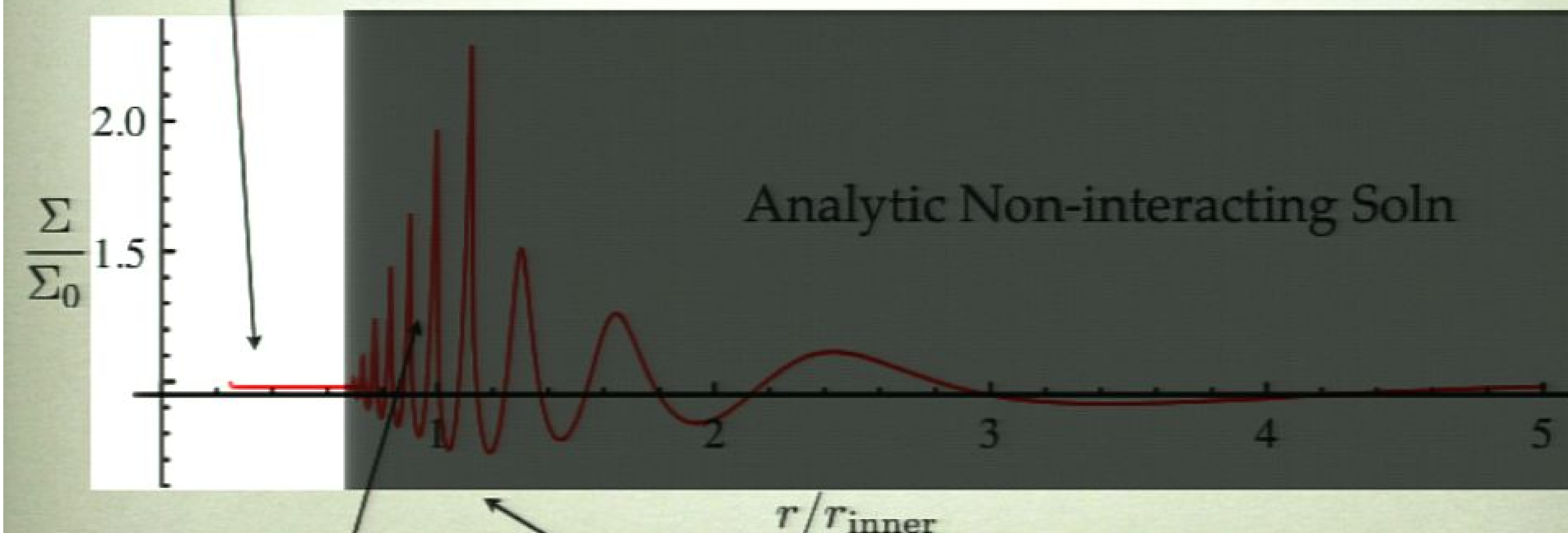
Conclusion



Described where the shocks were.

Conclusion

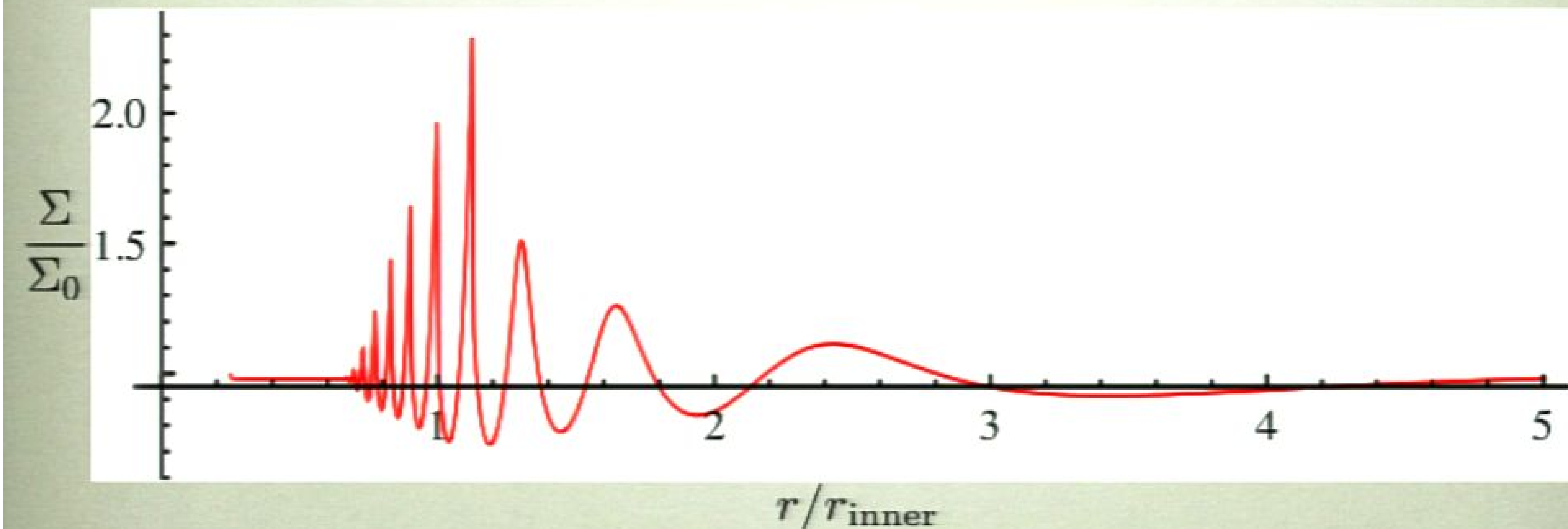
Described what happens after shocks.



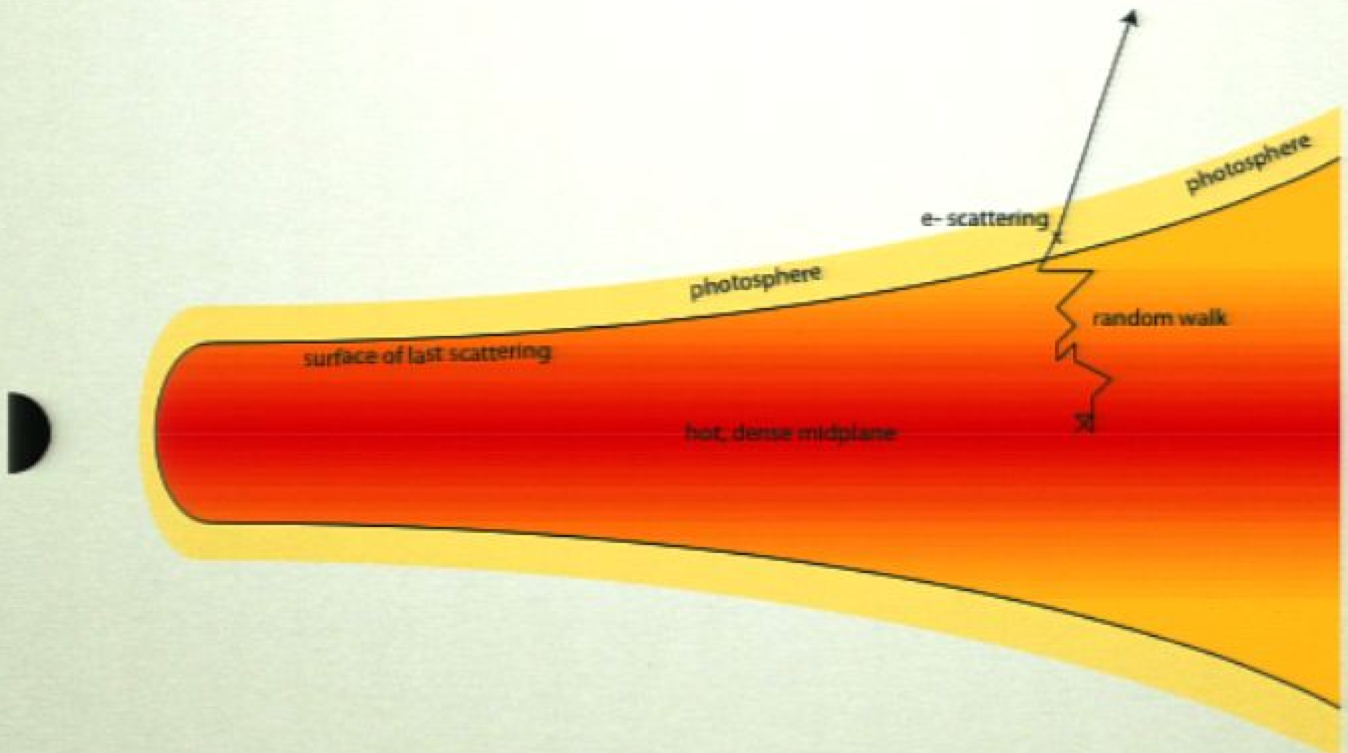
Described where the shocks were.

CONCLUSION

And we showed the theory is reflected in a 1D gaseous disk.



OBSERVABILITY?



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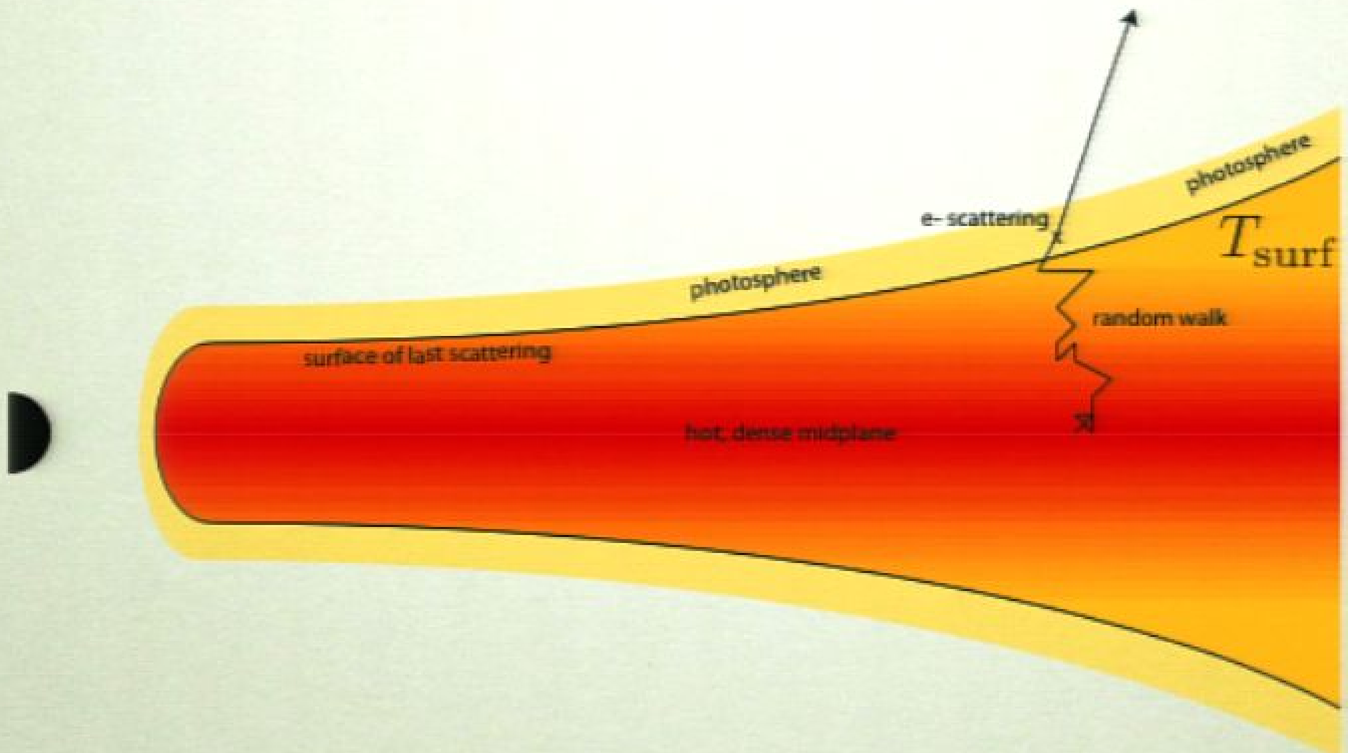
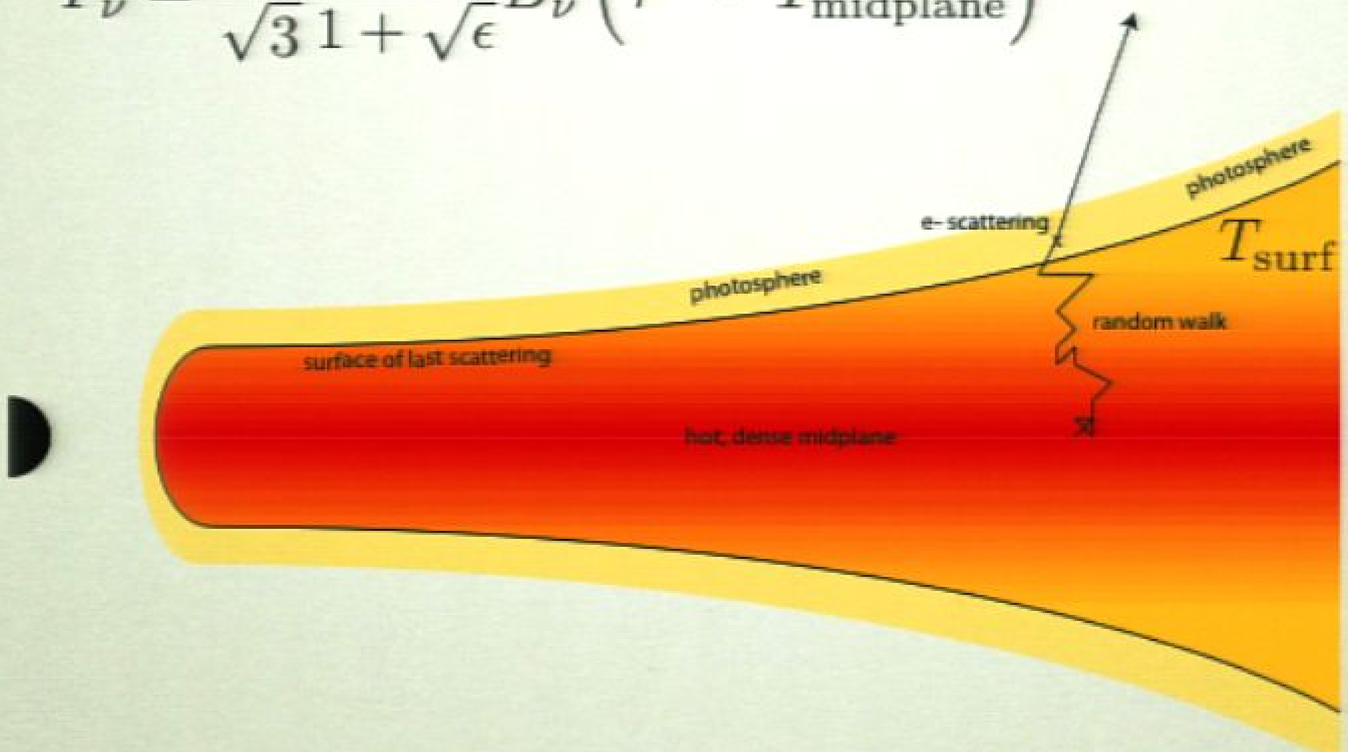


$$T_{\mathrm{surf}} = \tau^{-1/4} T_{\mathrm{midplane}}$$

Observability?

$$F_\nu = \frac{4\pi}{\sqrt{3}}\frac{\sqrt{\epsilon}}{1+\sqrt{\epsilon}}B_\nu\left(\tau^{-1/4}T_{\text{midplane}}\right)$$



$$T_{\text{surf}} = \tau^{-1/4}T_{\text{midplane}}$$

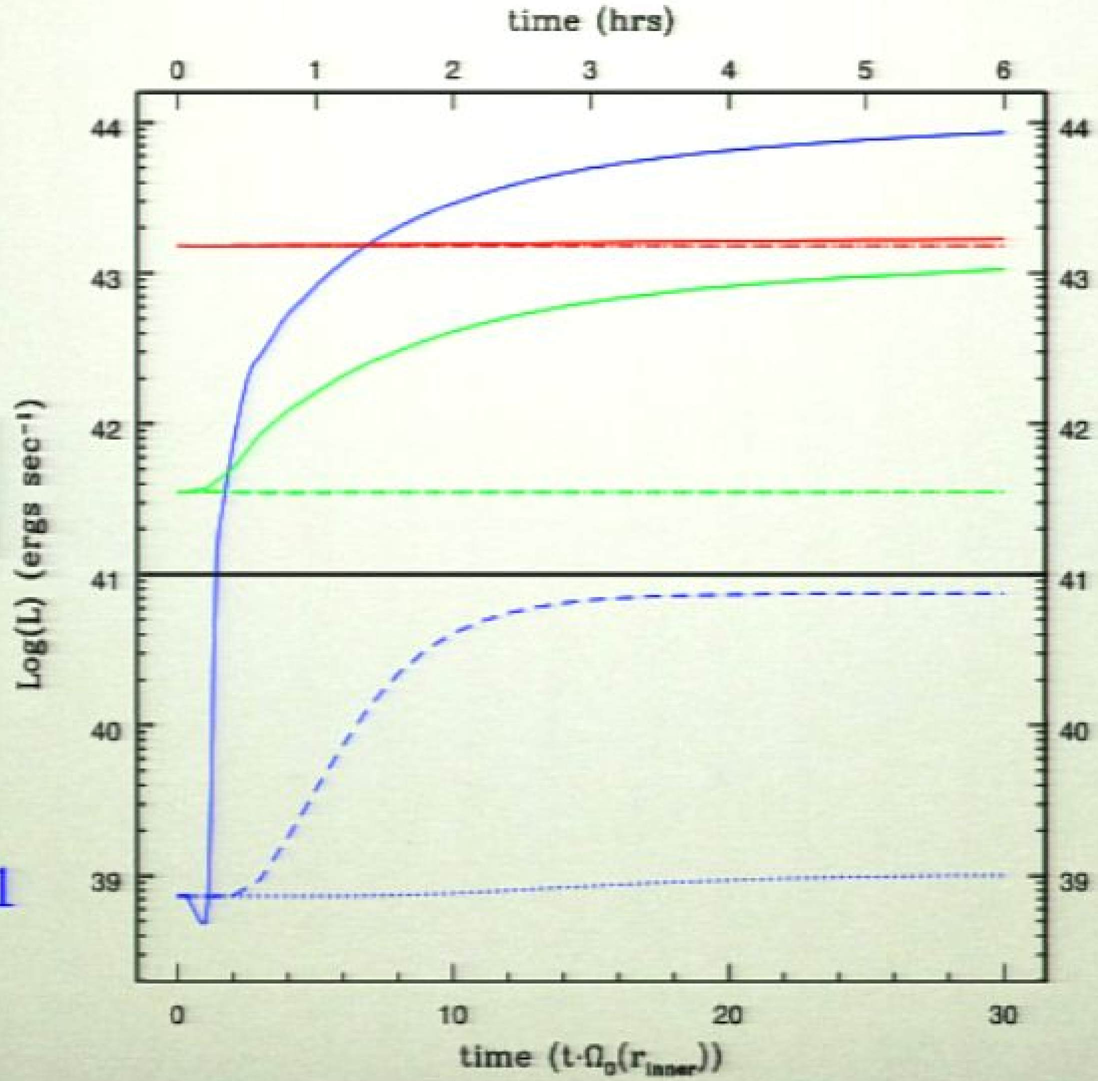
Here $\epsilon \equiv \dfrac{\alpha_\nu}{\sigma_T + \alpha_\nu}$ is the probability per interaction that the photon will be absorbed

Observability?

Red: $\frac{h}{r} = 0.1$

Green: $\frac{h}{r} = 0.01$

Blue: $\frac{h}{r} = 0.001$



$\text{—} \quad \frac{\delta M}{M} = 0.1$

$\text{- - -} \quad \frac{\delta M}{M} = 0.01$

$\cdots \quad \frac{\delta M}{M} = 0.001$