Title: Topological quantum order and quantum codes

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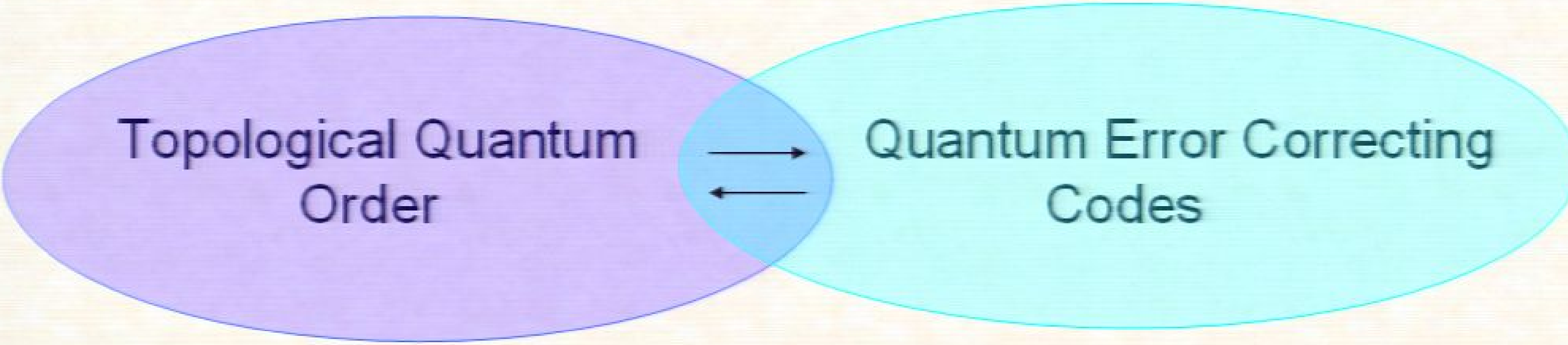
Abstract: Quantum error correcting codes and topological quantum order (TQO) are inter-connected fields that study non-local correlations in highly entangled many-body quantum states. In this talk I will argue that each of these fields offers valuable techniques for solving problems posed in the other one. First, we will discuss the zero-temperature stability of TQO and derive simple conditions that guarantee stability of the spectral gap and the ground state degeneracy under generic local perturbations. These conditions thus can be regarded as a rigorous definition of TQO. Our results apply to any quantum spin Hamiltonian that can be written as a sum of geometrically local commuting projectors on a D-dimensional lattice. This large class of Hamiltonians includes Levin-Wen string-net models and Kitaev's quantum double models. Secondly, we derive upper bounds on the parameters of quantum codes with local check operators and discuss the implications for feasibility of a quantum self-correcting memory.

Topological quantum order and quantum error correcting codes

Sergey Bravyi

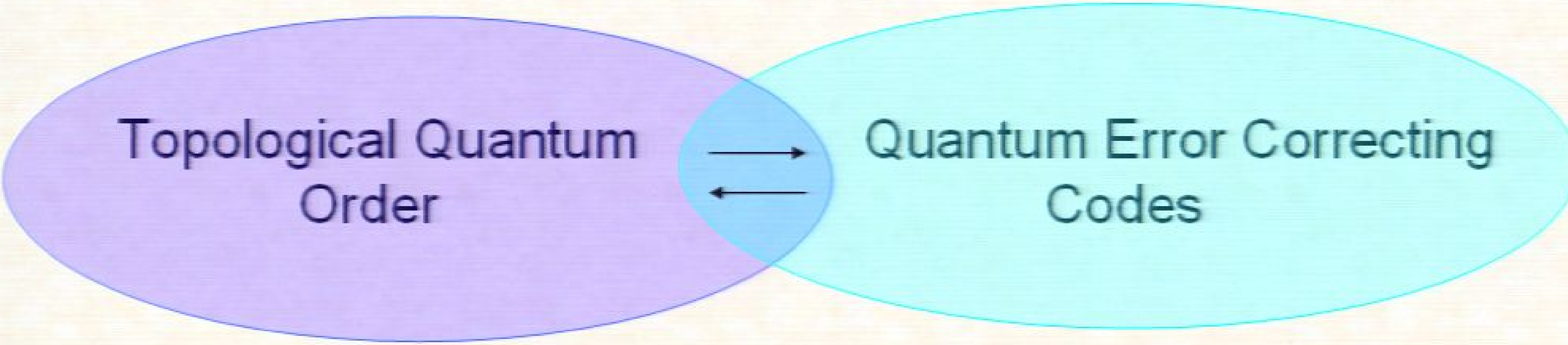
IBM Research

Perimeter Institute
December 1, 2010



Local Hamiltonians	Quantum codes with local check operators
Ground state	Codespace
Braiding and fusion of anyons	Logical operators
Spectral gap, phase transitions	Code distance, error threshold

Let's try to solve problems posed in one field using techniques and ideas developed in the other



Stability of the spectral gap under local perturbations

Upper bounds on the distance of geometrically local codes

Implementation of TQO models using ``perturbation gadgets''

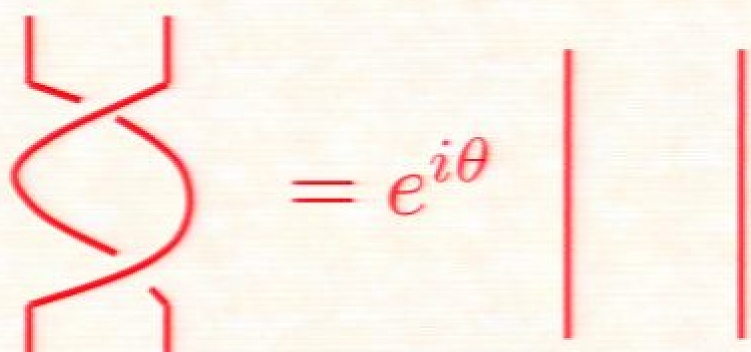
Topological Quantum Order in 2D (Xiao-Gang Wen 90)

Collective property of some strongly interacting 2D spin or electron systems. Main signatures are

1. All excitations are gapped; correlation functions decay exponentially.

2. Ground state is degenerate on a torus; the degeneracy can not be lifted by local perturbations.

3. Excitations are anyons: quasiparticles transforming according to non-trivial representations of the braid group.



Mathematical perspective

TQO

- **Topological Quantum Field Theory** — Chern-Simons gauge theory
- **Knot theory** — Topological invariants of knots and links
- **Conformal Field Theory** — Anyons = primary fields; Wave functions of anyons = conformal blocks
- **Unitary Modular Categories** — Abstract axioms
- **Quantum Error Correcting Codes** — Ground states are degenerate but locally indistinguishable

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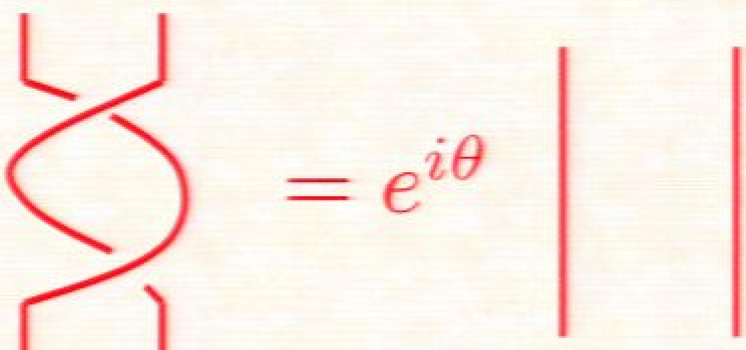

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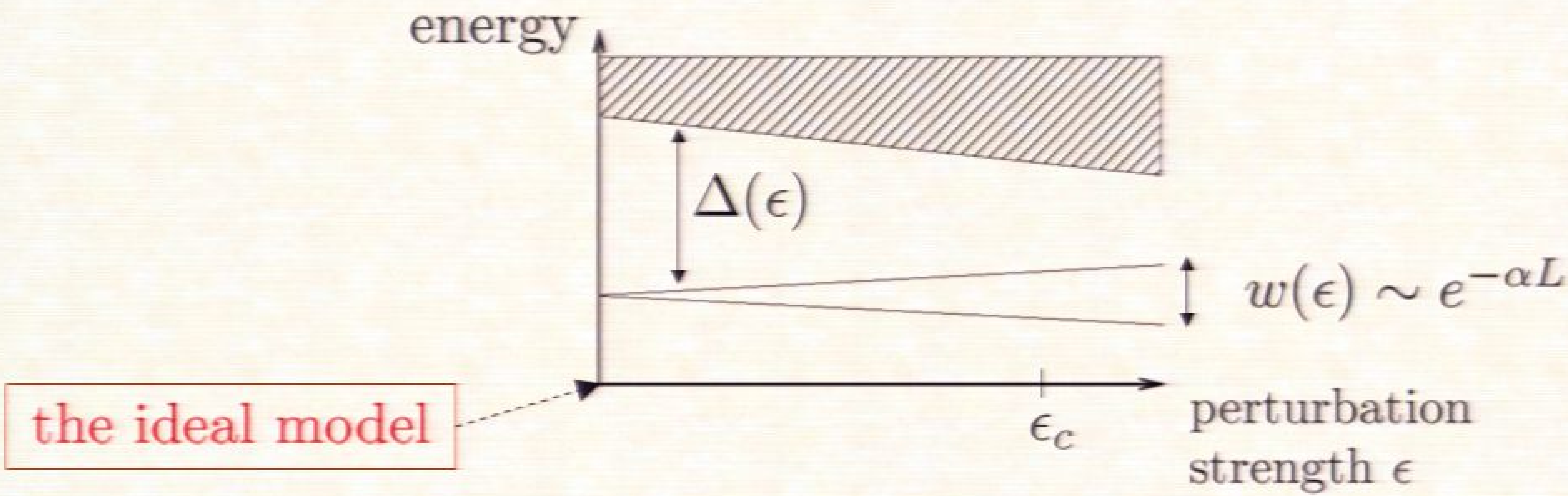
Quantum Error Correcting Codes	Ground states are degenerate but locally indistinguishable

Problem 1: stability of the spectral gap



Goal: find a realistic spin lattice Hamiltonian H_0 that combines these properties:

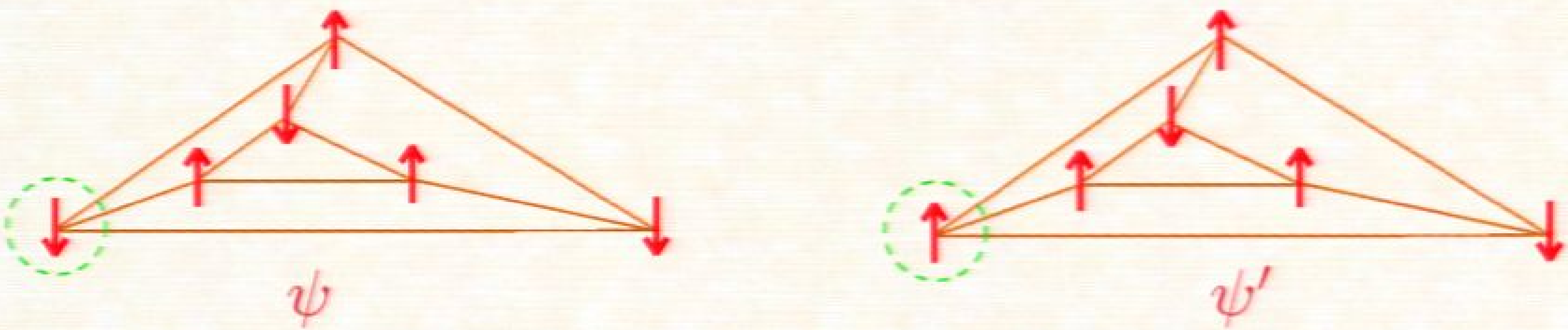
(1) Degenerate ground state.

(2) Constant spectral gap.

(3) These properties are retained in a presence of a generic perturbation ϵV.

(4) Stability radius ϵ_c is independent of the system size L.

The role of the quantum entanglement



Two globally distinct classical states (spin configurations) must be locally distinct.

In the classical world the ground state degeneracy can always be lifted by a local perturbation.

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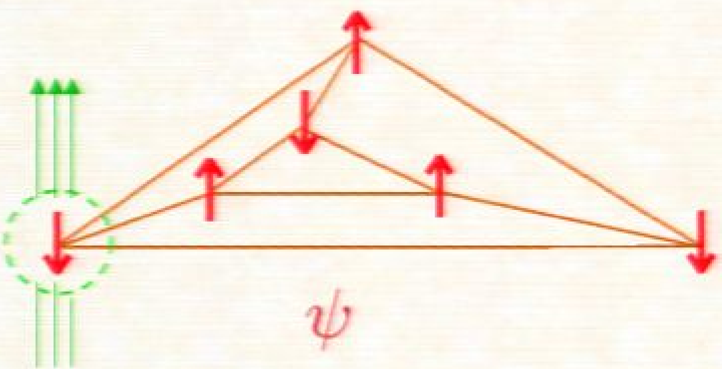

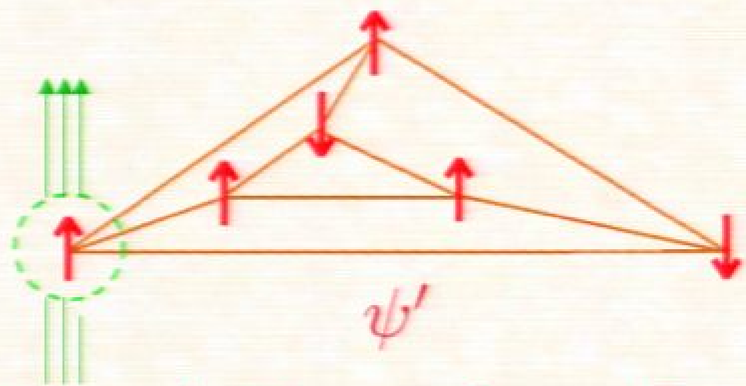

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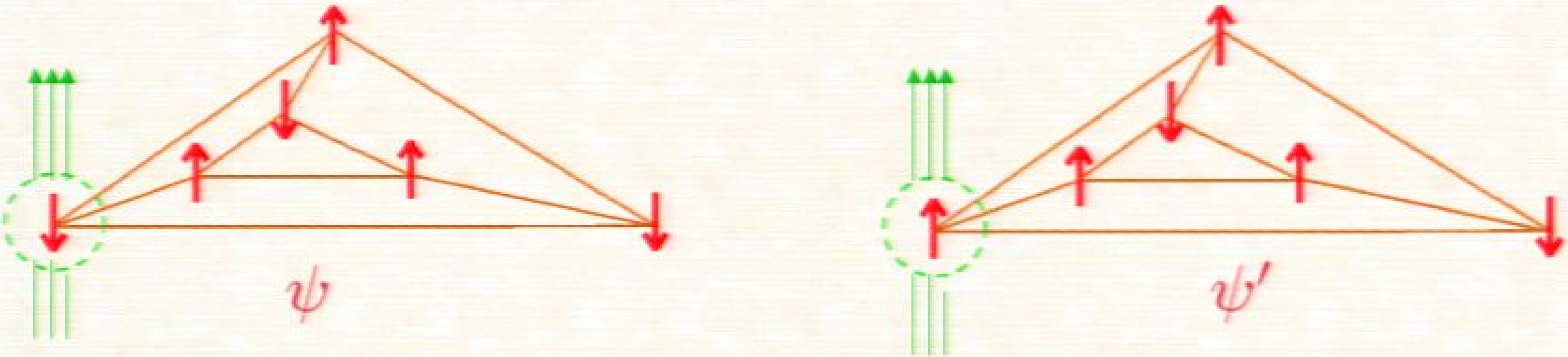

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Quantum entangled states can be globally distinct but locally indistinguishable.

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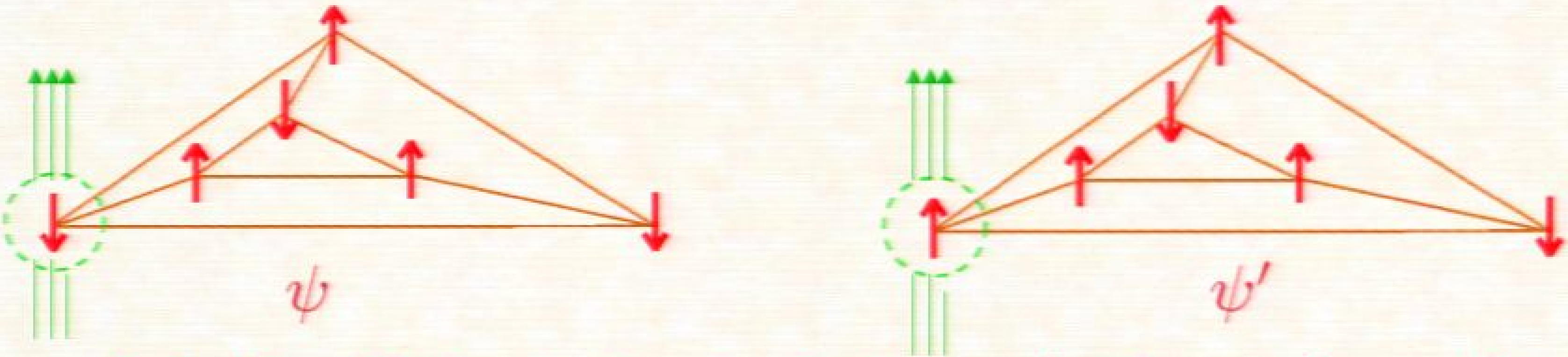

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However $\psi + \psi'$ and $\psi - \psi'$ are locally distinguishable...

Quantum error correcting codes

$$|\psi\rangle \sim |0,0,0,0\rangle + |1,1,1,1\rangle$$

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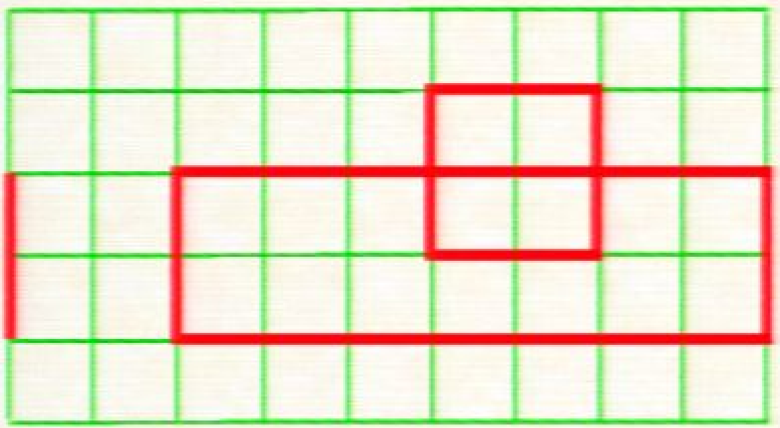
A linear subspace $\mathcal{L}$ is a quantum code with distance d iff no operator O acting on less than d qubits can distinguish different states in $\mathcal{L}$:

$$\langle\psi|O|\psi\rangle = \langle\psi'|O|\psi'\rangle \quad \text{for all } \psi,\psi' \in \mathcal{L}$$

Toric code state (Kitaev 97)

Qubits live on links of a 2D lattice with periodic boundary conditions.



Cycle is a set of links C such that any site has even number of incident links from C.

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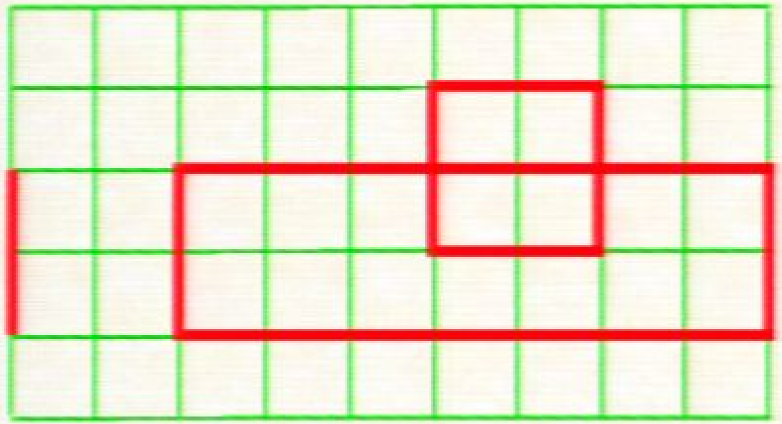

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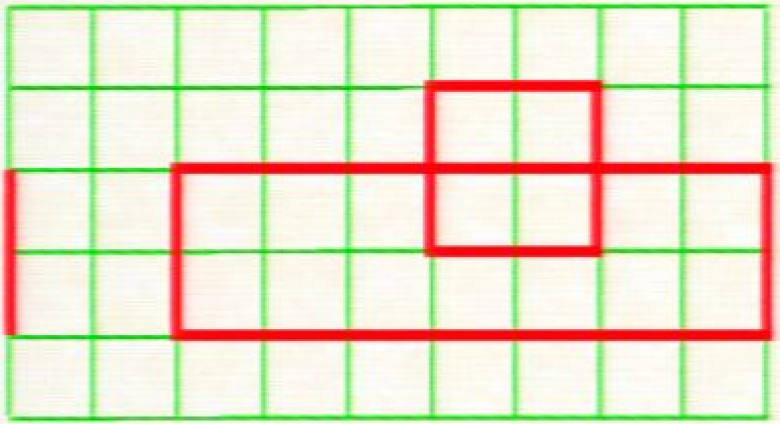

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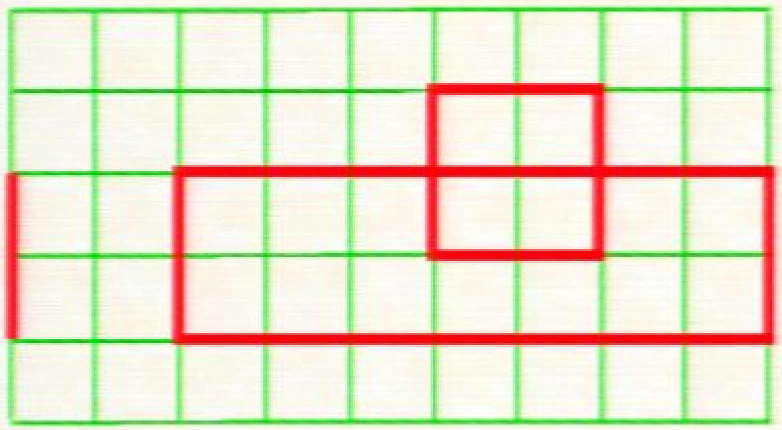

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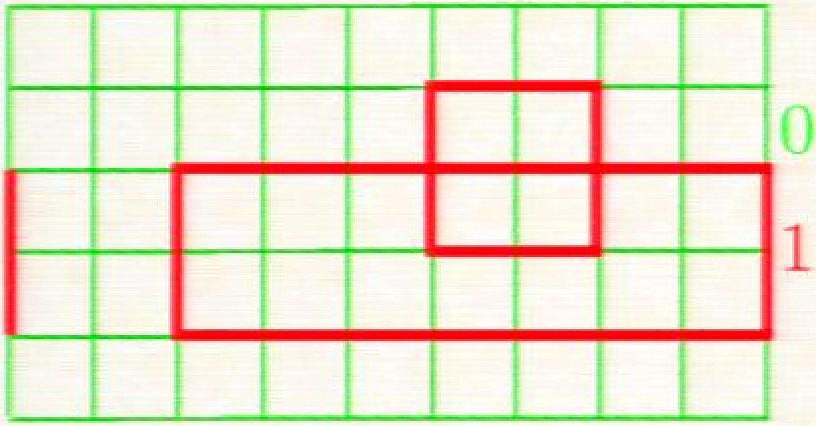

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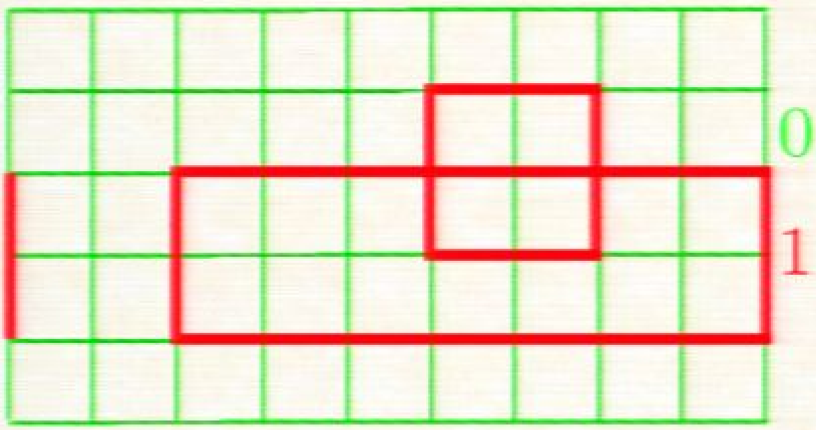

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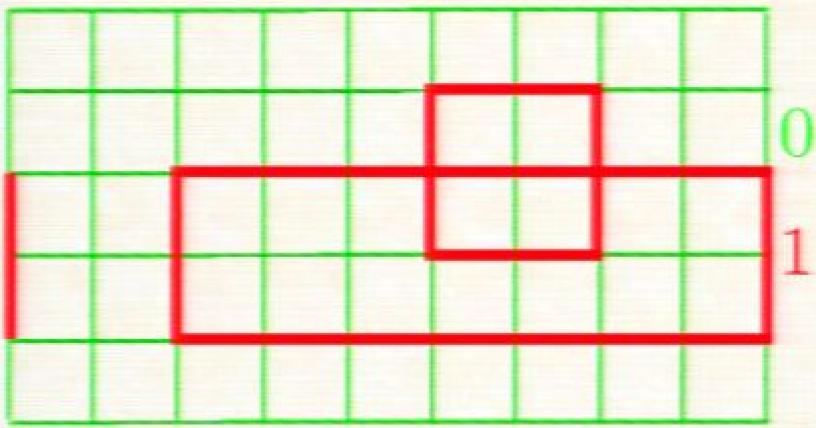

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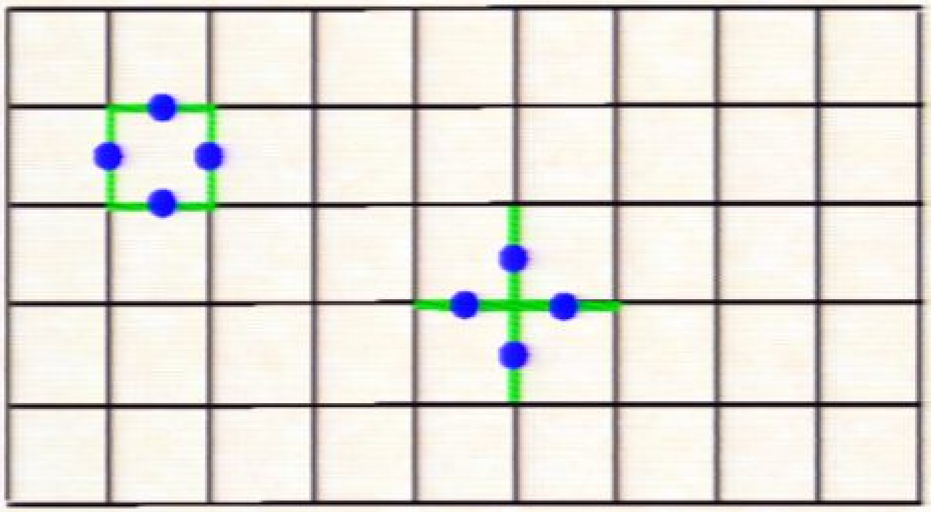
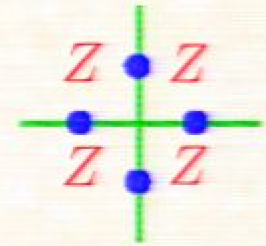
Toric code Hamiltonian



Qubits = links

Star operators:

$A_s =$ 

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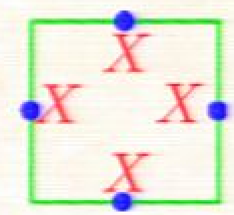
$B_p =$ 

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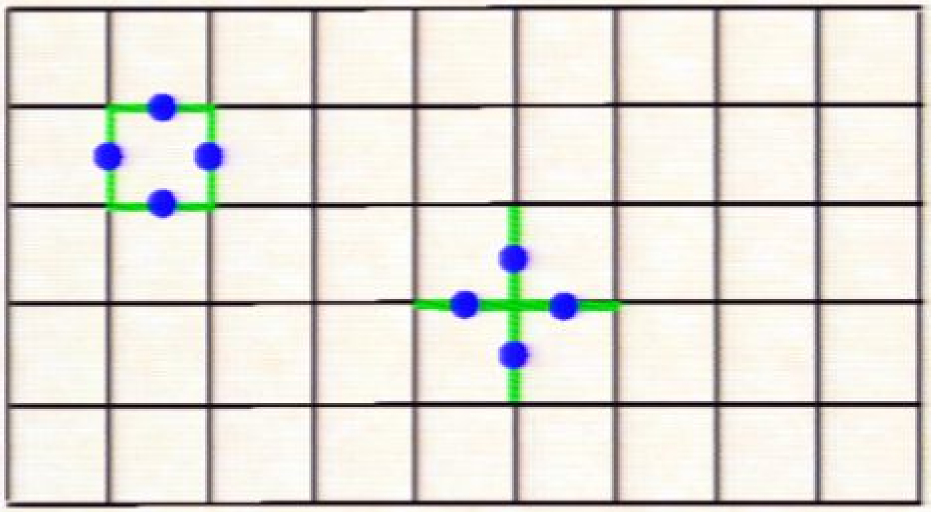

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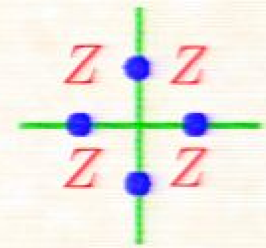
$A_s =$ 

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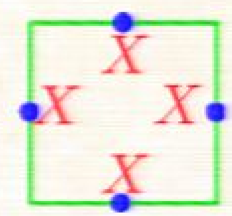
$B_p =$ 

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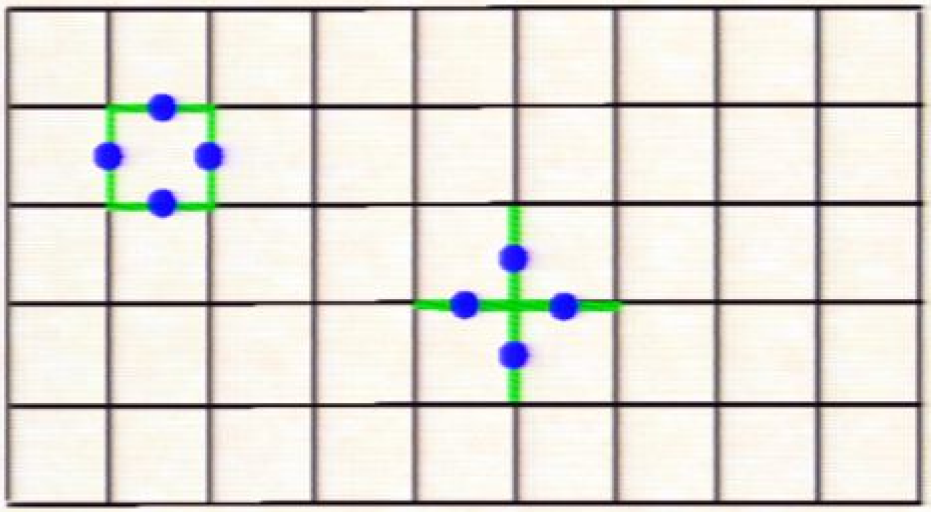

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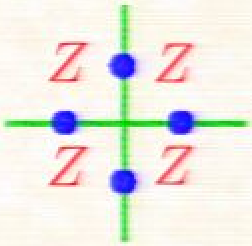
$A_s =$ 

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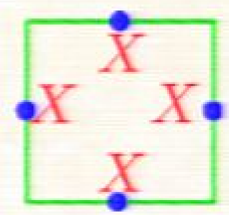
$B_p =$ 

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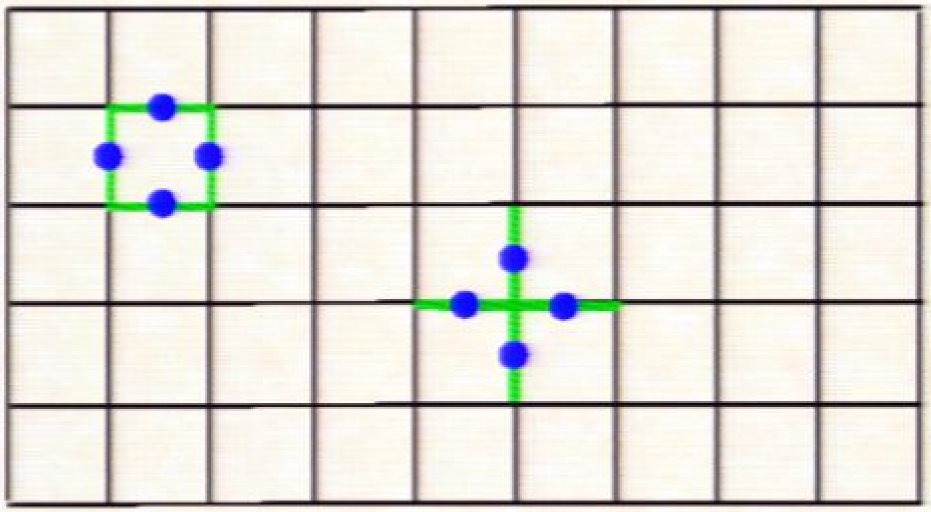

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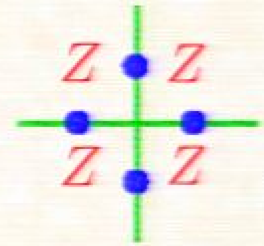
$A_s =$ 

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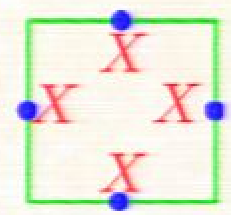
$B_p =$ 

Exact solvability: all A_s, B_p commute with each other

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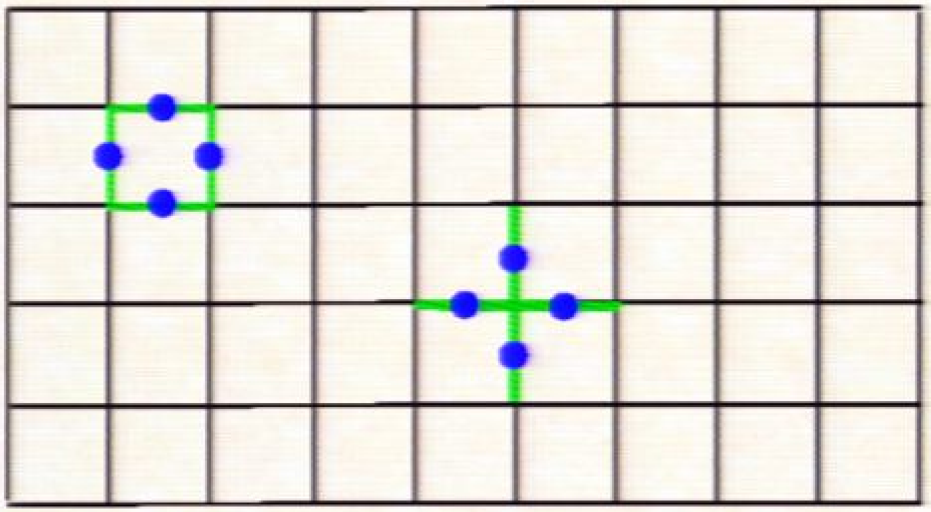

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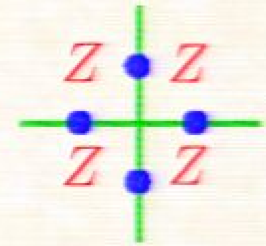

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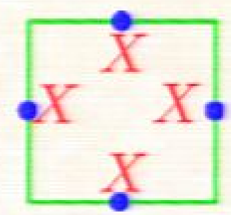

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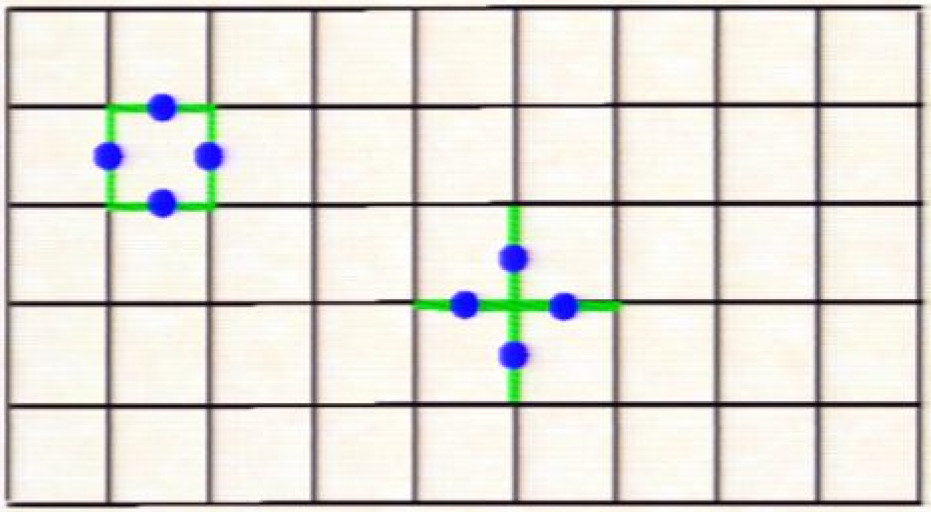

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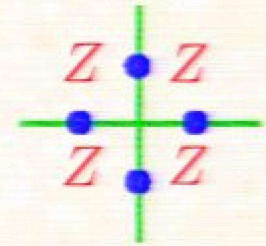
$A_s =$ 

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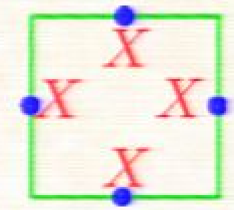
$B_p =$ 

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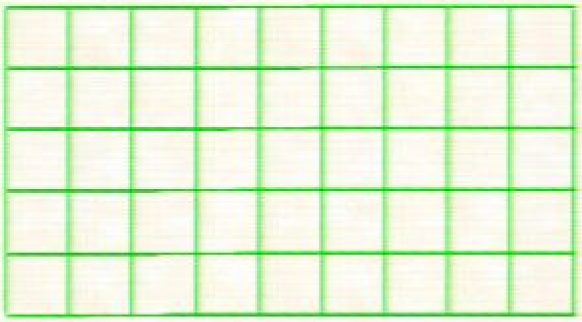
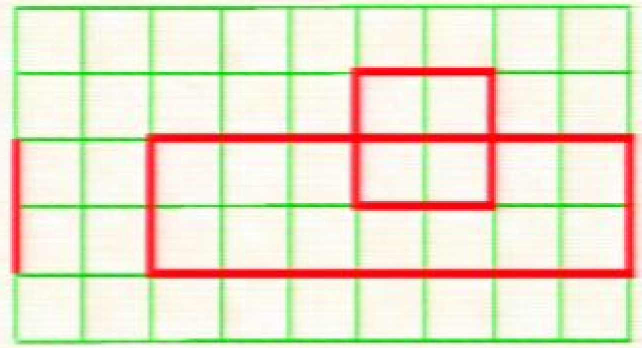
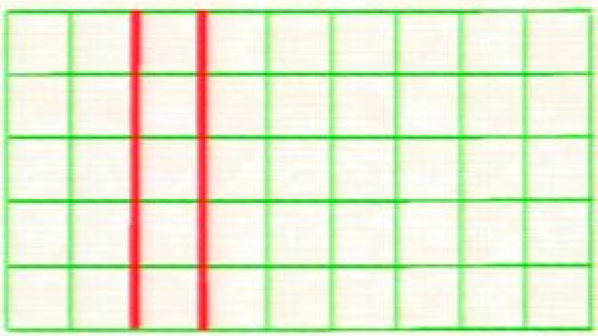
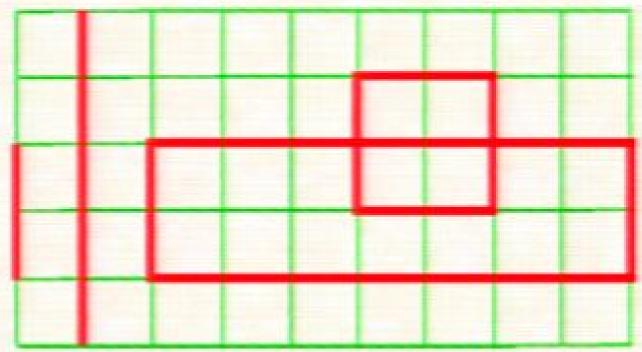
Excited states: $A_s = +1$, $B_p = +1$, Abelian anyons.

Two cycles are homological equivalent iff they differ by a linear combination (mod 2) of contractible loops.

$C_1 =$ 

$C_2 =$ 

$C_3 =$ 

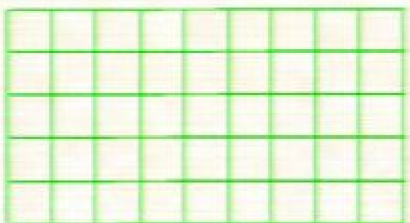
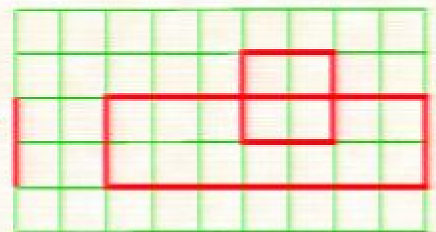
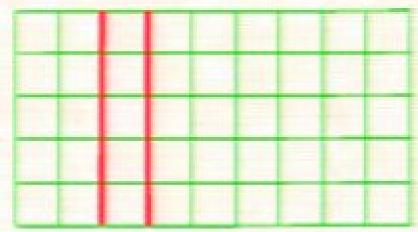
$C_4 =$ 

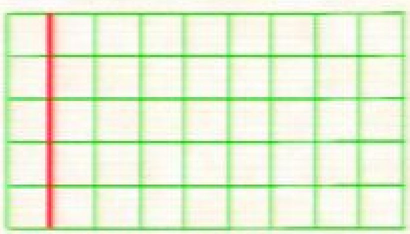
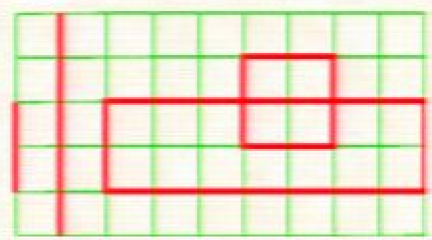
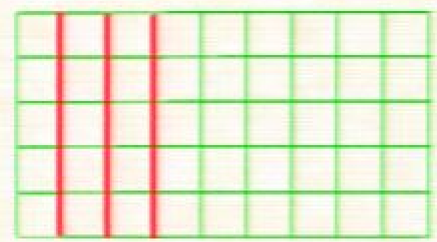
$[C]$ — homological class of a cycle C

$$[C_1] = [C_2] = [C_3] \neq [C_4]$$

Orthogonal ground states are the uniform superpositions of cycles with a fixed homological class:

$$|\psi_\alpha\rangle = \sum_{C:[C]=\alpha} |C\rangle$$

$|\psi_{00}\rangle =$  $+$  $+$  $+ \dots$

$|\psi_{10}\rangle =$  $+$  $+$  $+ \dots$

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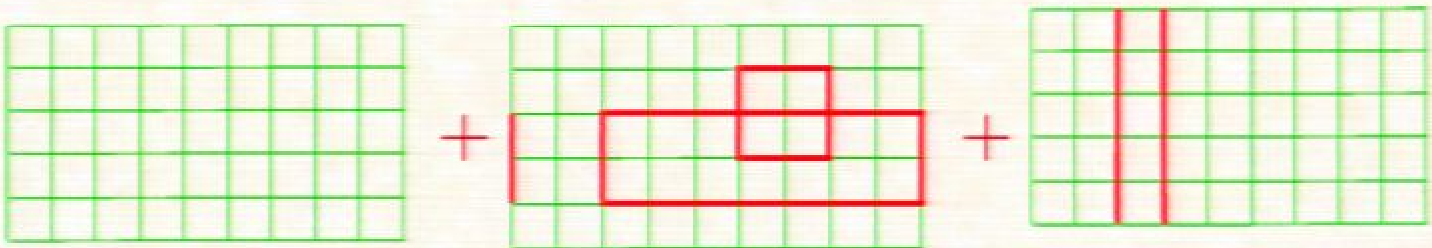
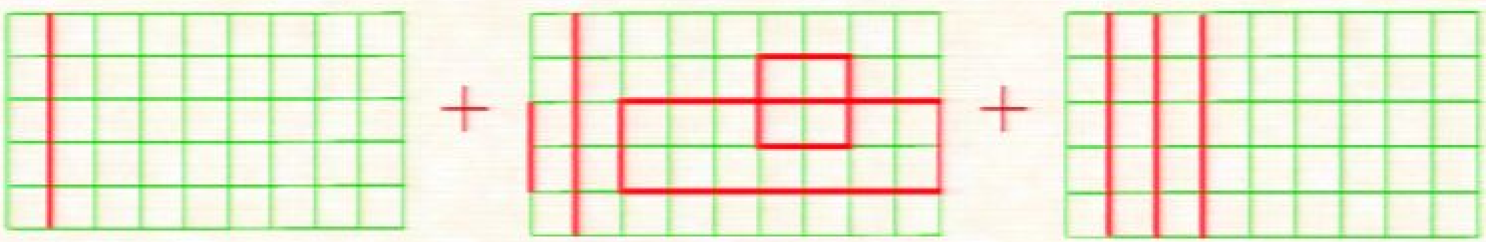
$|\psi_{00}\rangle =$  $+ \ldots$

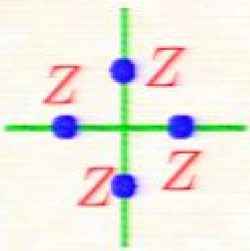
$|\psi_{10}\rangle =$  $+ \ldots$

etc

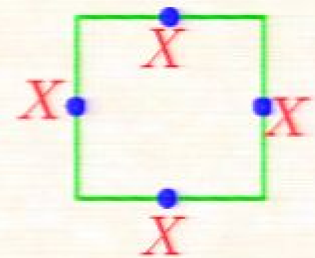
Homological class of a cycle C cannot be determined by looking at any local piece of C. Hence orthogonal ground states are locally indistinguishable. Distance $d \sim L$.

Logical operators in 2D

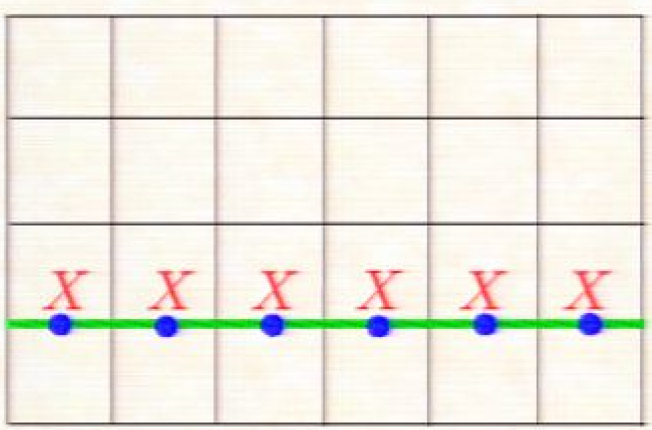
Z-stabilizers



X-stabilizers



Logical X



$$\bar{X} = \prod_{e \in \gamma} X_e$$

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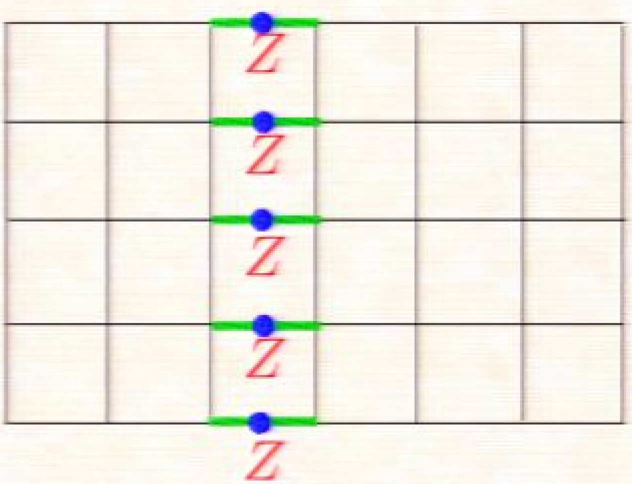
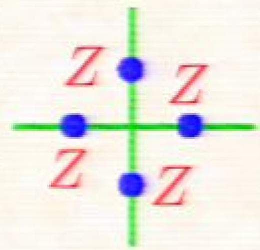
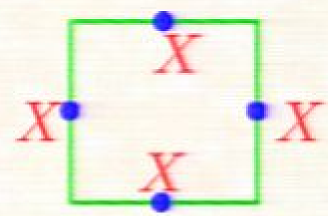


$$\bar{Z} = \prod_{e \in \gamma^*} Z_e$$

Toric codes in higher dimensions

	Qubits	Z-stabilizers	X-stabilizers

Toric codes in higher dimensions

	Qubits	Z-stabilizers	X-stabilizers
2D	links	vertices 	plaquettes 

Toric codes in higher dimensions

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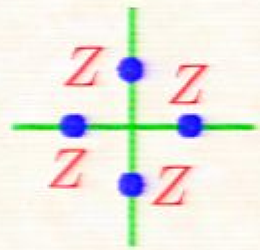
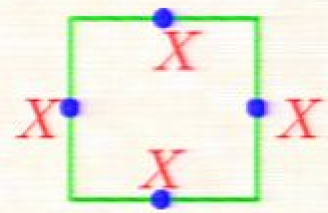
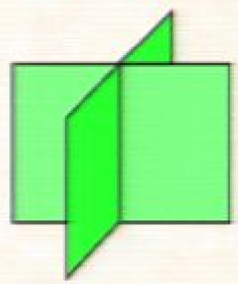
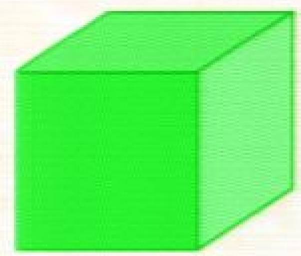
2D	links	vertices 	plaquettes 
3D	plaquettes	links 	cubes 

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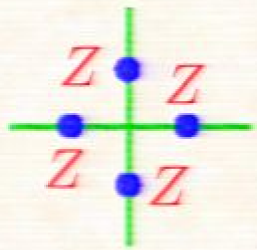
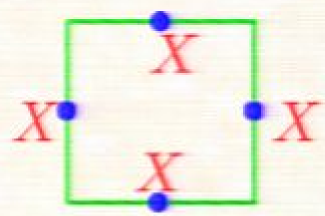
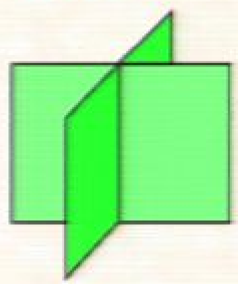
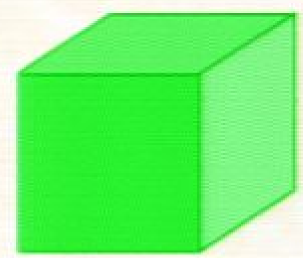
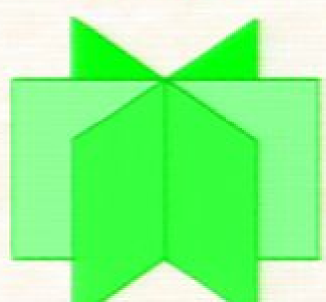
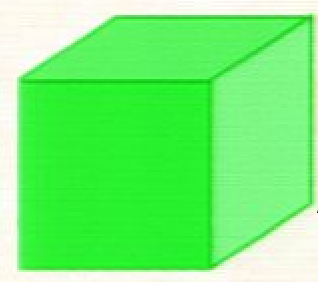
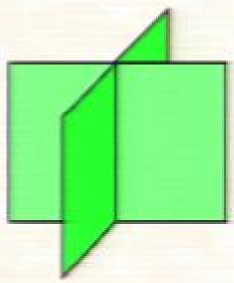
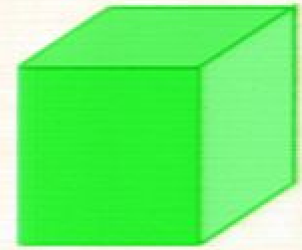
2D	links	vertices 	plaquettes 
3D	plaquettes	links 	cubes 
4D	plaquettes	links 	3-cubes 

Logical operators in 3D

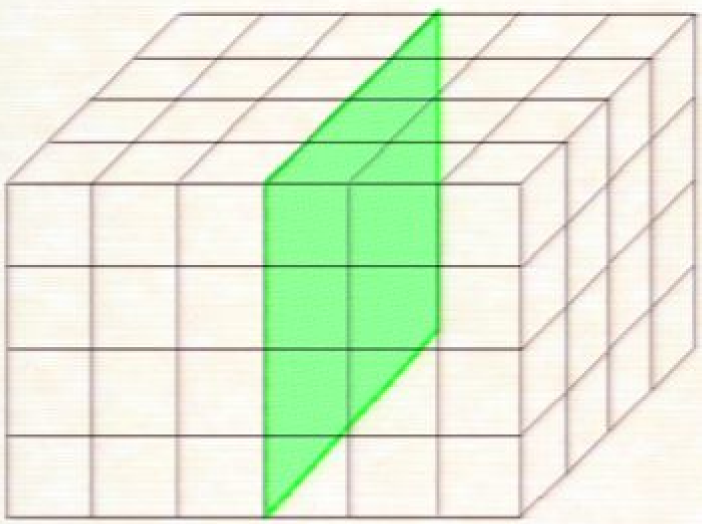
Z-stabilizers



X-stabilizers

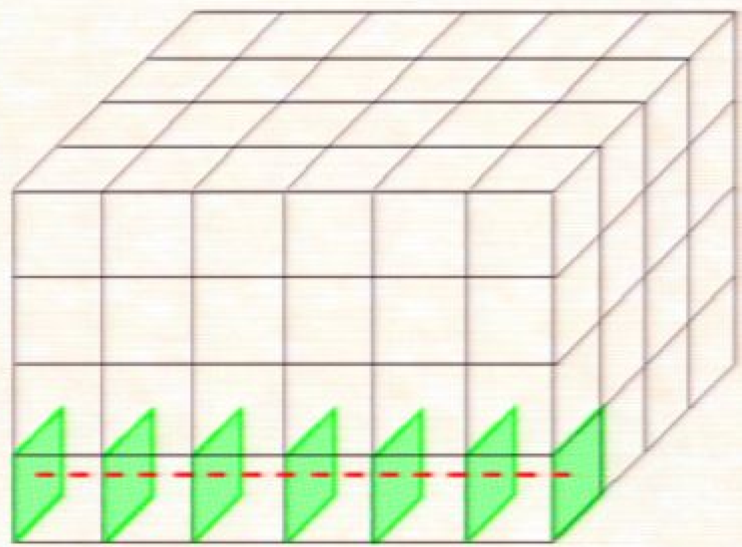


Logical X



$$\bar{X} = \prod_{e \in \gamma} X_e$$

Logical Z



$$\bar{Z} = \prod_{e \in \gamma^*} Z_e$$

The distance of the toric code is determined by the minimum length of homologically non-trivial loop and/or the minimum area of homologically non-trivial surface.

	min. weight of $\bar{Z}$	min. weight of $\bar{X}$
2D	$O(L)$	$O(L)$
3D	$O(L)$	$O(L^2)$
4D	$O(L^2)$	$O(L^2)$

Here L is a linear size of the lattice.

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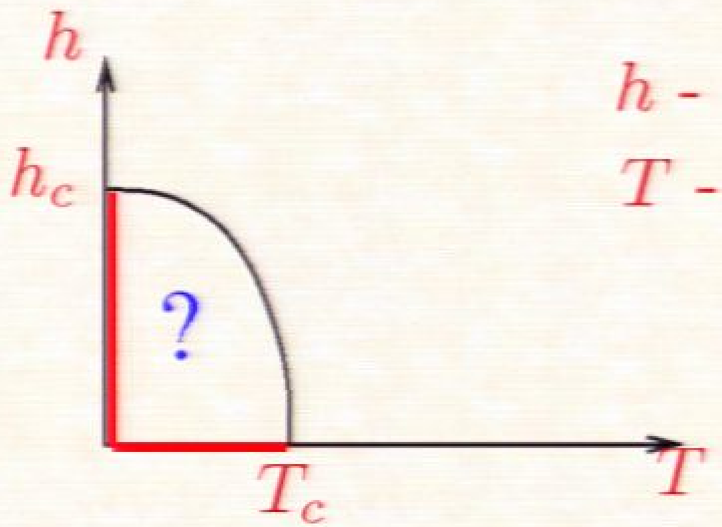
Here L is a linear size of the lattice.

Curious fact: 4D toric codes defined on lattices embedded into 4D Riemannian manifolds may have minimal distance $L^2\sqrt{\log L}$ (Freedman & Meyer 2002)

Stability of topological qubits



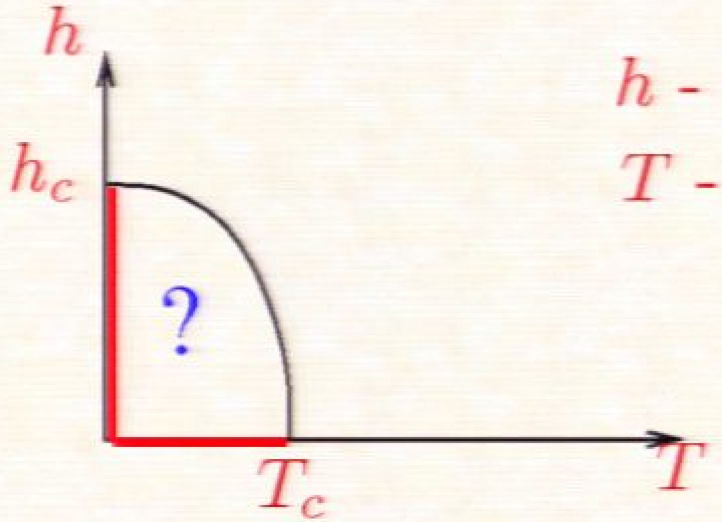
h - perturbation strength

T - temperature

Is TQO stable under thermal fluctuations $(T > 0)$?

Is TQO stable under static perturbations $(T = 0)$?

Stability of topological qubits



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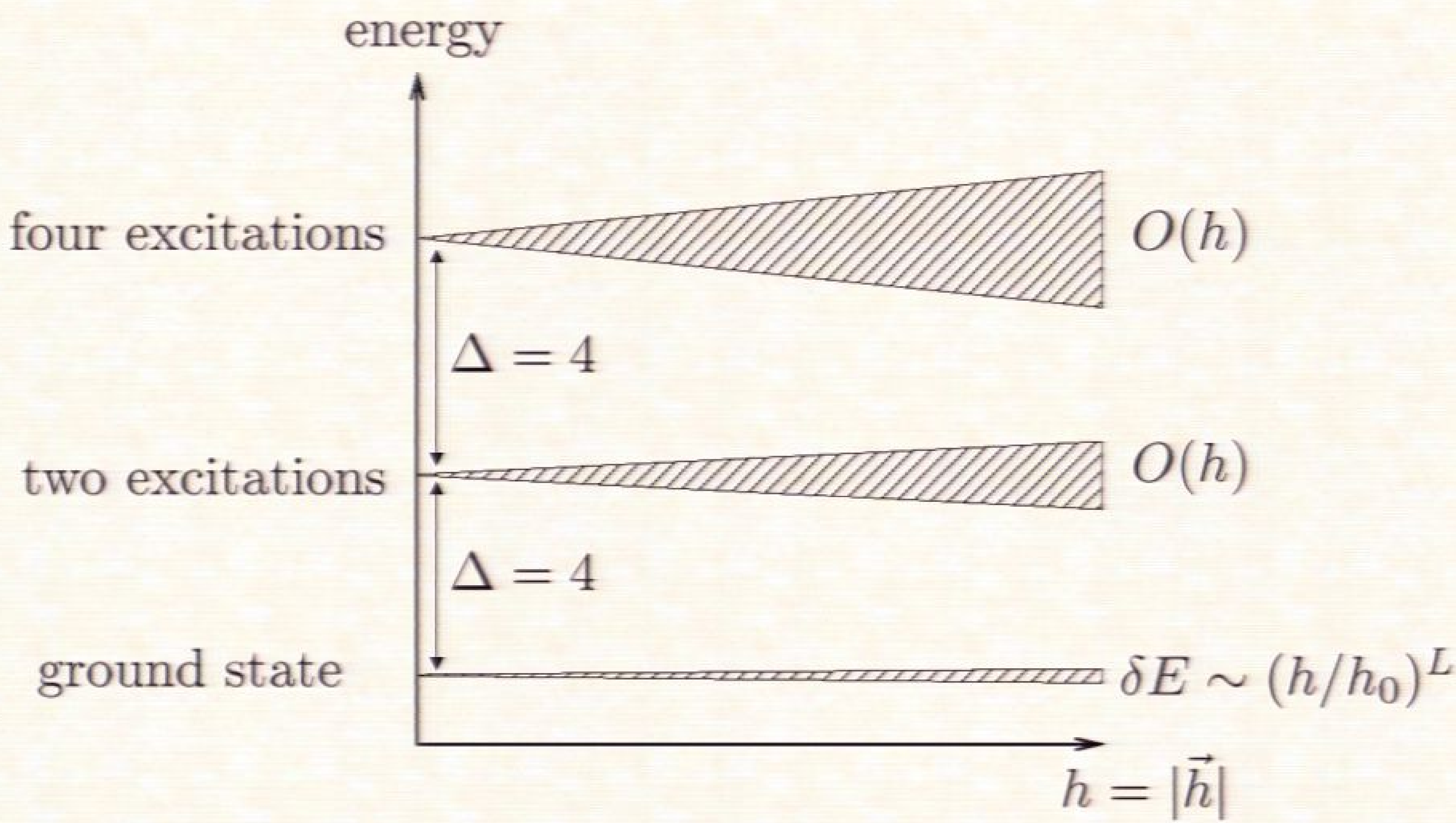
Is TQO stable under static perturbations $(T = 0)$?

Toric codes:

	$T = 0$	$T > 0$
2D	yes	no
3D	yes	no
4D	yes	yes

Dennis, Kitaev, Landahl, Preskill (2001)
Nussinov and Ortiz (2007)
Alicki, Fannes, Horodecki (2008)
Alicki, Horodecki's (2008)
Chesi, Loss, S.B., Terhal (2009)
S.B., Hastings, Michalakis (2009)



Naive perturbation theory:
Anticipated level structure of a perturbed Hamiltonian

$$H = -\sum_{\text{sites}} A_s - \sum_{\text{plaquettes}} B_p - \sum_{\text{links}} h_x X_e + h_y Y_e + h_z Z_e$$

The predictions of the naive PT were confirmed (partially) in several special cases:

Trebst, Werner, Troyer, Shtengel, Nayak (2007)
Magnetic field diagonal in the Z-basis
Reduction to the 2D transverse field Ising model

Vidal, Thomale, Schmidt, Dusuel (2009)
Magnetic field diagonal in the Y-basis
Reduction to the Xu-Moore model; use of self-duality

Klich (2009)
Generic perturbations; non-degenerate ground state
Generic perturbations diagonal in the Z-basis
Cluster expansions for the partition function

Stability requires some extra conditions on H_0

TQO-1: Ground subspace of H_0 is a quantum code with a macroscopic distance.

TQO-2: Consistency between the global and the local ground subspaces of H_0 (formal definition will appear later)

We shall prove that TQO-1,2 together are sufficient for stability

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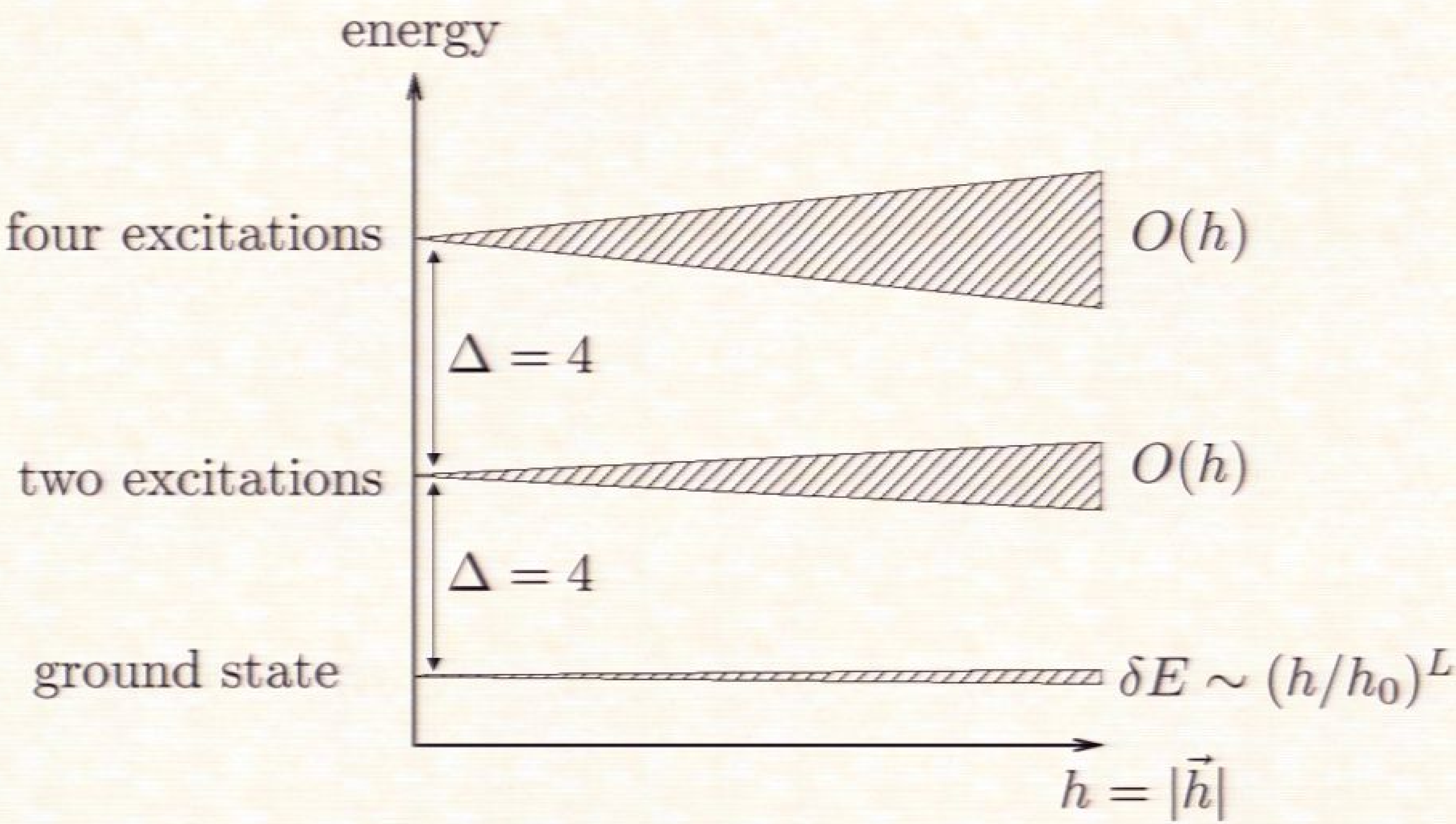

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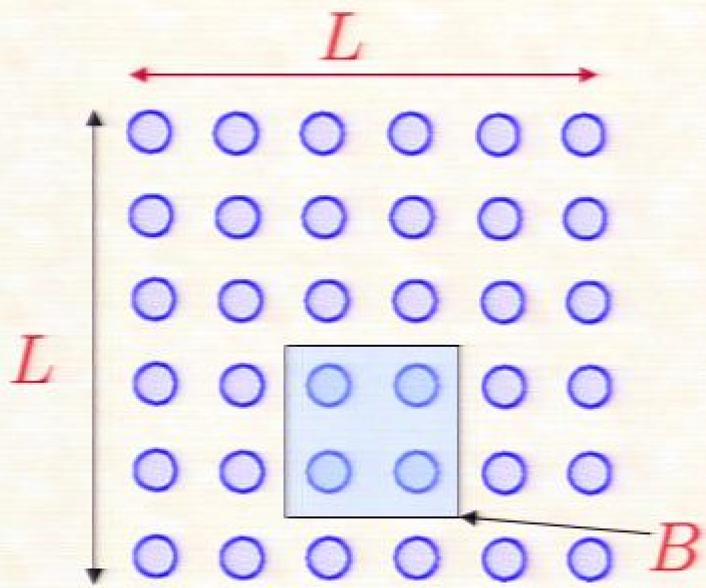


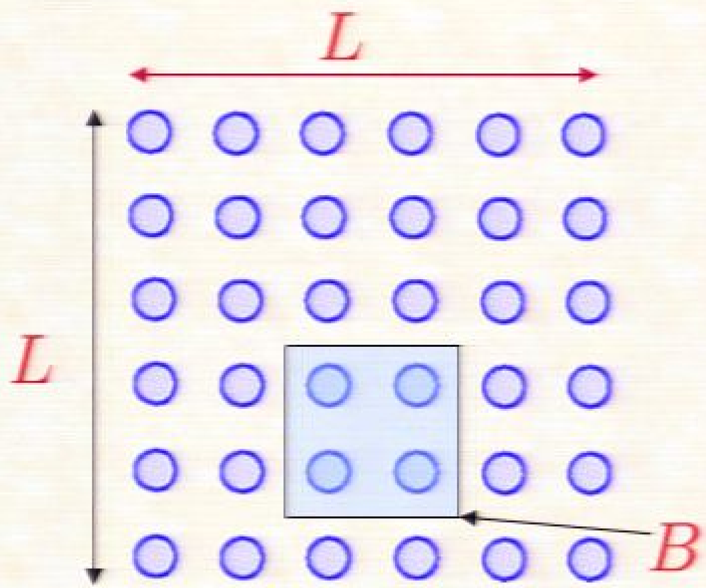
Lattice Λ

Hilbert space: $\mathcal{H} = \bigotimes_{j \in \Lambda} \mathcal{H}_j$

$\dim \mathcal{H}_j = O(1)$

Unperturbed Hamiltonian:

$$H_0 = \sum_{B \subseteq \Lambda} Q_B$$



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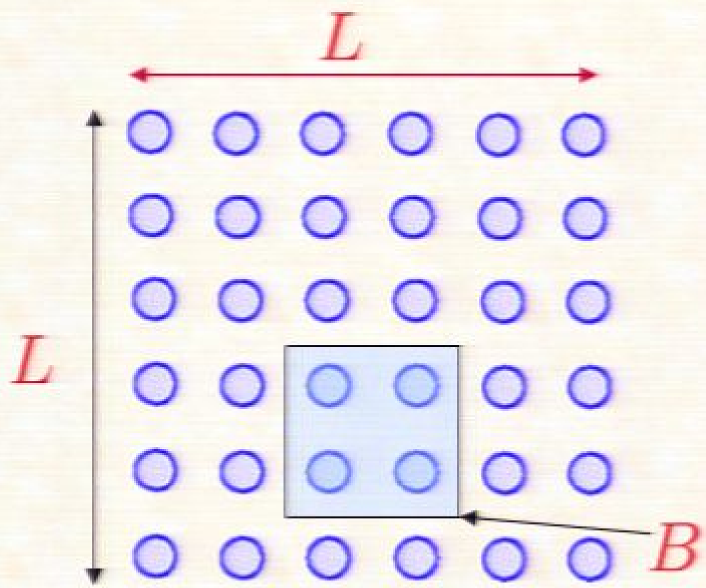
Q_B is a Hermitian operator acting only on a cluster B

Only 2×2 clusters



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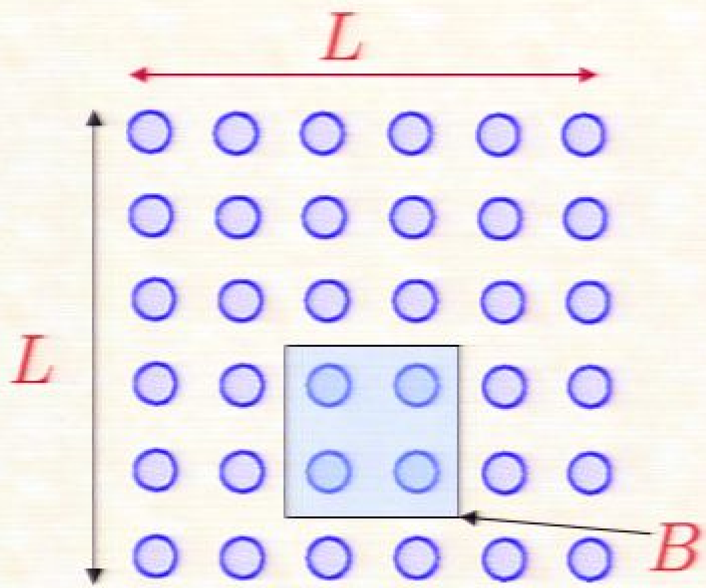
Q_B must be a projector: $Q_B^2 = Q_B$



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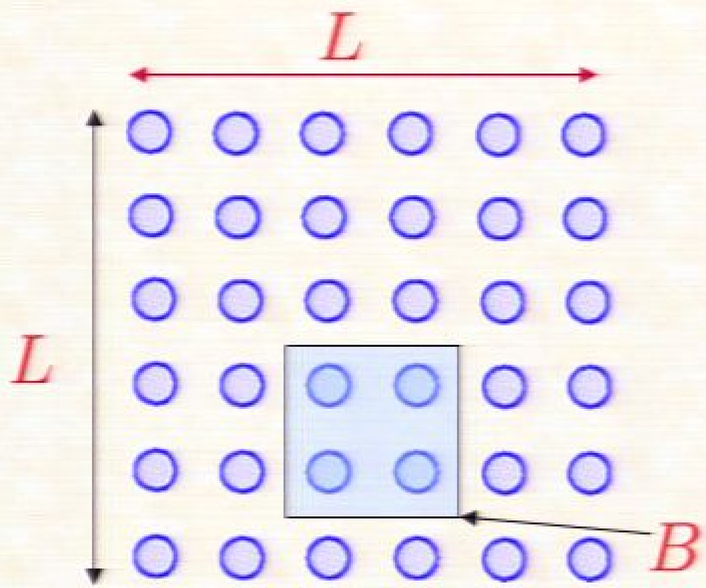
Projectors must pairwise commute: $Q_B Q_C = Q_C Q_B$



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Only 2×2 clusters

Q_B must be a projector: $Q_B^2 = Q_B$

Projectors must pairwise commute: $Q_B Q_C = Q_C Q_B$

Ground states of H_0 are zero-eigenvectors of every projector Q_B

To summarize, we need three properties of the ideal model:

- Spatially local
- Frustration free
- Term-wise commuting

Several extra conditions related to TQO will be introduced later...

Examples:

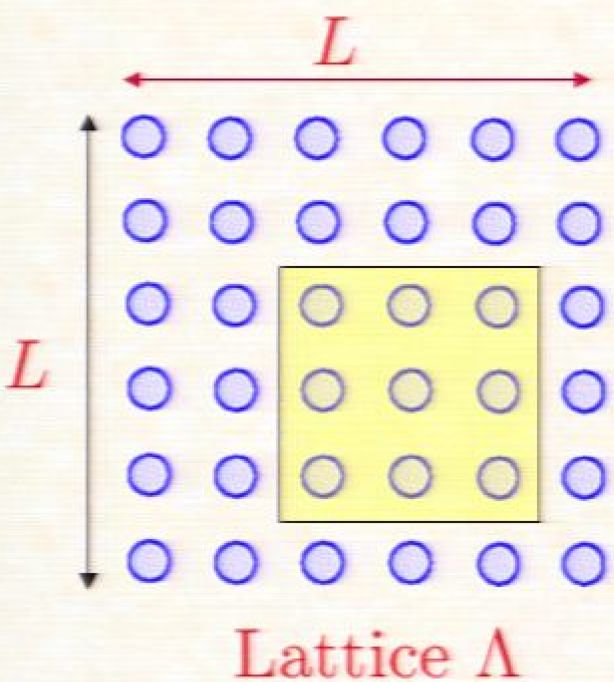
- The toric codes and the surface codes
- Topological color codes
- Quantum double models
- String-net models

$$\left.\right\} \quad H_0 = \sum_B Q_B$$

- Any of the above models with excitations

Kitaev 97
Bombin and Martin-Delgado 06
Levin and Wen 05

Commutativity guarantees that H_0 has constant spectral gap above the ground state !



Generic perturbations:

$$V = \sum_{B \subseteq \Lambda} V_B$$

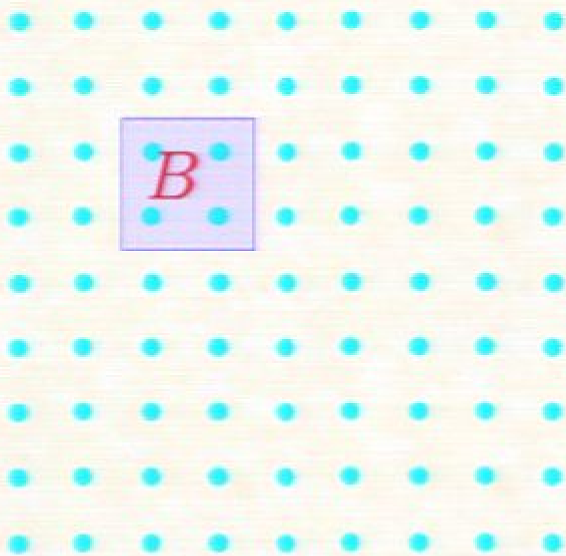
V_B is a Hermitian operator acting only on a cluster B

Exponential decay of interactions

For clusters of size $r \times r$ $\quad \max_B \|V_{r,B}\| \le J e^{-\mu r}$

$J =$ interaction strength

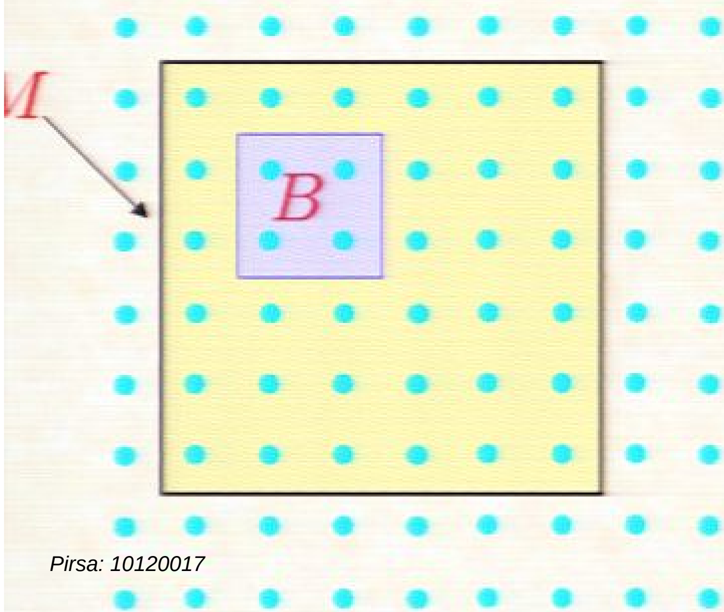
$\mu =$ decay rate (inverse correlation length)



$$H_0 = \sum_B Q_B$$

Global ground subspace:

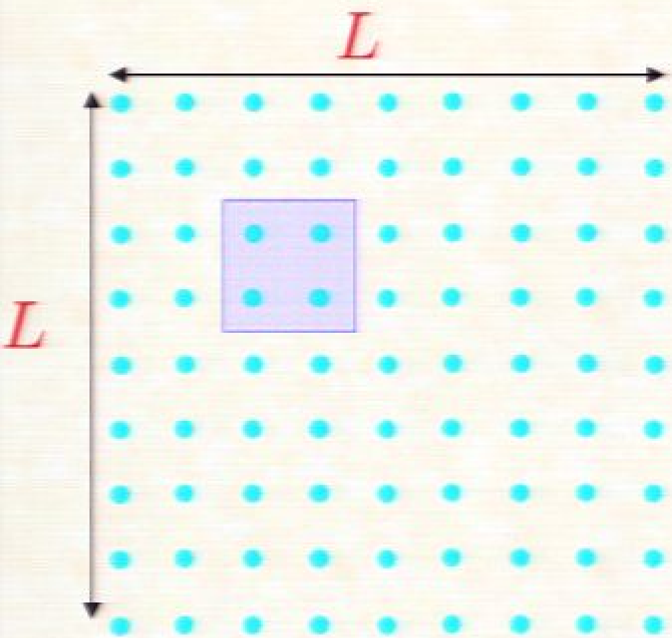
$$P = \{|\psi\rangle \,:\, Q_B\,|\psi\rangle = 0 \quad \forall B\}$$



Local ground subspace:

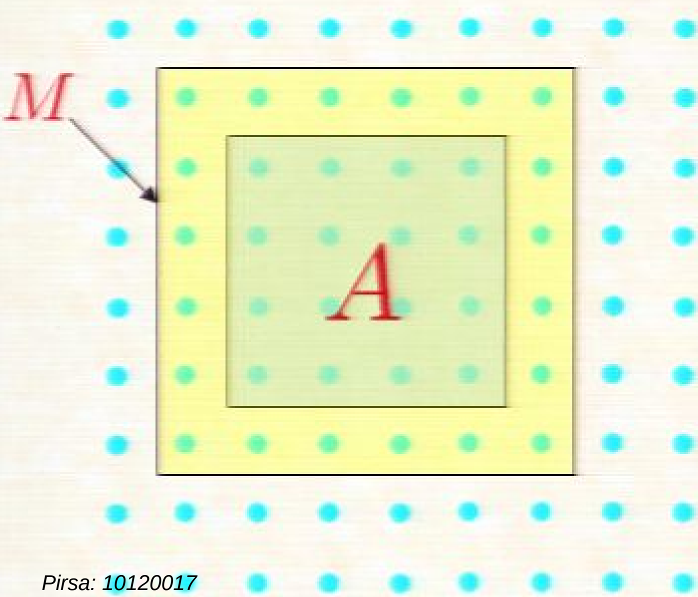
$$P_M = \{|\psi\rangle \,:\, Q_B\,|\psi\rangle = 0 \quad \forall B \subseteq M\}$$

zero eigenvectors only for projectors Q_B whose support is contained in M.



TQO-1 (macroscopic distance):

Global ground states cannot be distinguished by looking at d or less qubits, where $d \sim L^{\alpha}$ for some constant $\alpha > 0$.



TQO-2 (global-local consistency):

Global ground states $\psi \in P$ and local ground states $\phi \in P_M$ cannot be distinguished inside A.

Holds for all M's of size $\leq L^{\alpha}$.

To summarize, we need five properties of the ideal model H_0:

- Spatially local
- Frustration free
- Term-wise commuting
- Macroscopic distance (TQO-1)
- Local-global consistency (TQO-2)

The perturbation V involves exponentially decaying interactions with strength J and decay rate $\mu > 0$.

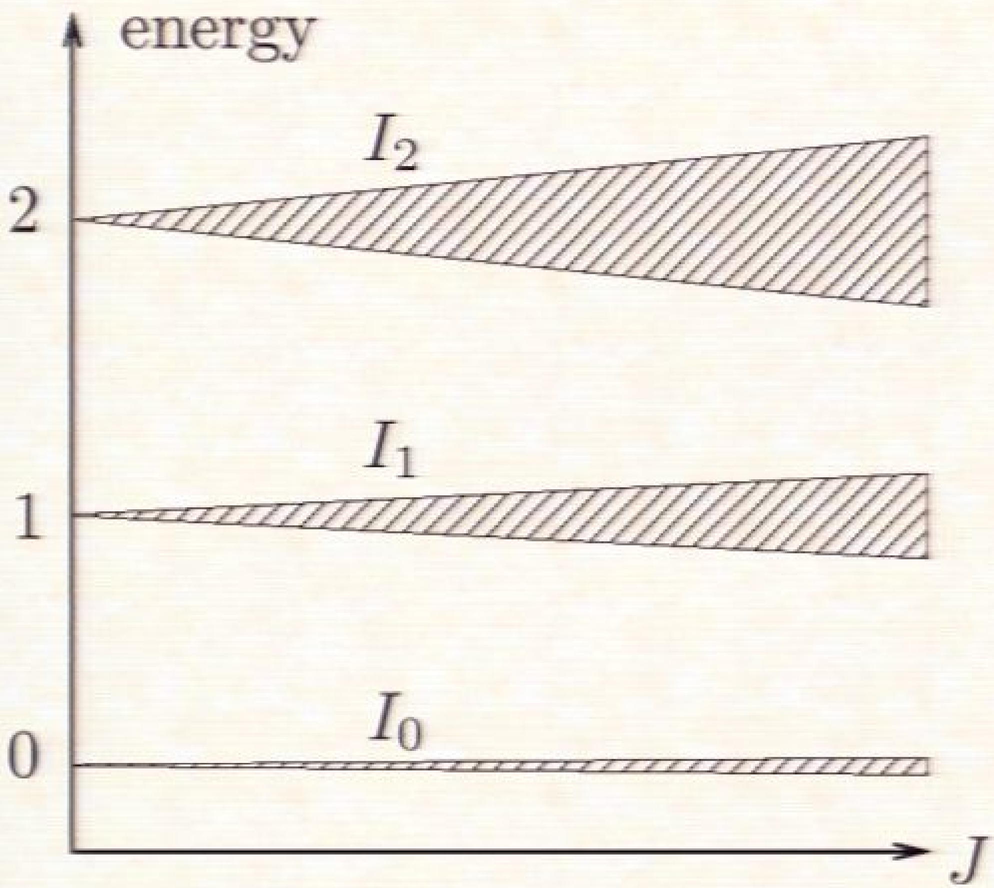
Main result:

If the interaction strength J is below a certain constant threshold $J_0 > 0$ then the spectrum of $H_0 + V$ is contained (up to an overall energy shift) in the union of intervals

$$\bigcup_k I_k$$

- k runs over eigenvalues of H_0
- Interval I_k is centered at k
- $|I_k| = O(Jk)$ for $k > 0$
- $|I_0| = O(J) poly(L) exp(-c\sqrt{L})$

S.B., Hastings, Michalakis, J. Math. Phys. **51**, *093512 (2010)*



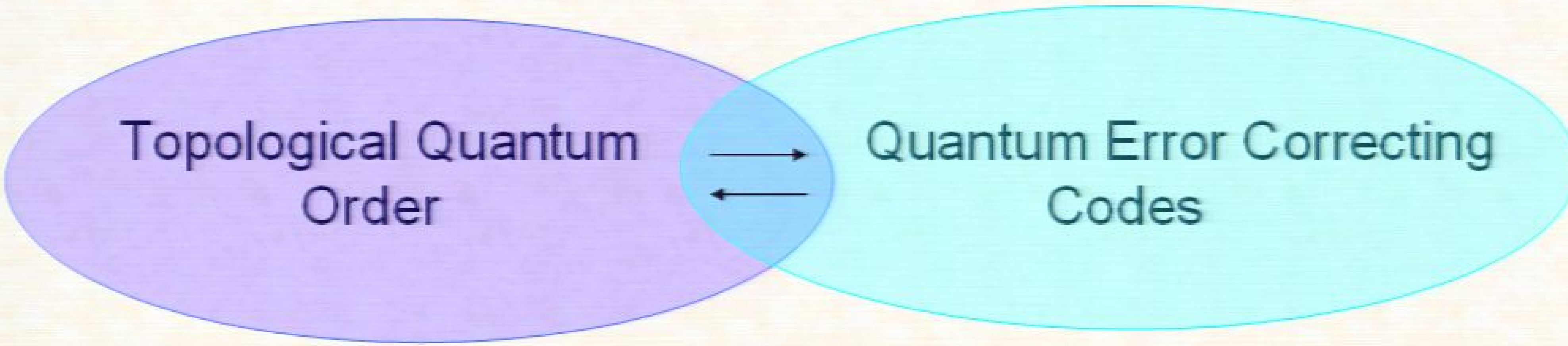
$|I_k| = O(kJ)$ for $k > 0$

$|I_0| = O(1)\exp(-c\sqrt{L})$

Interval I_k is separated from the rest of the spectrum by a gap at least $1/2$ if

$$J < \frac{1}{c(2k+1)}, \quad c = O(1)$$

$\Rightarrow$ number of excitations is well defined for the perturbed model for small enough k.

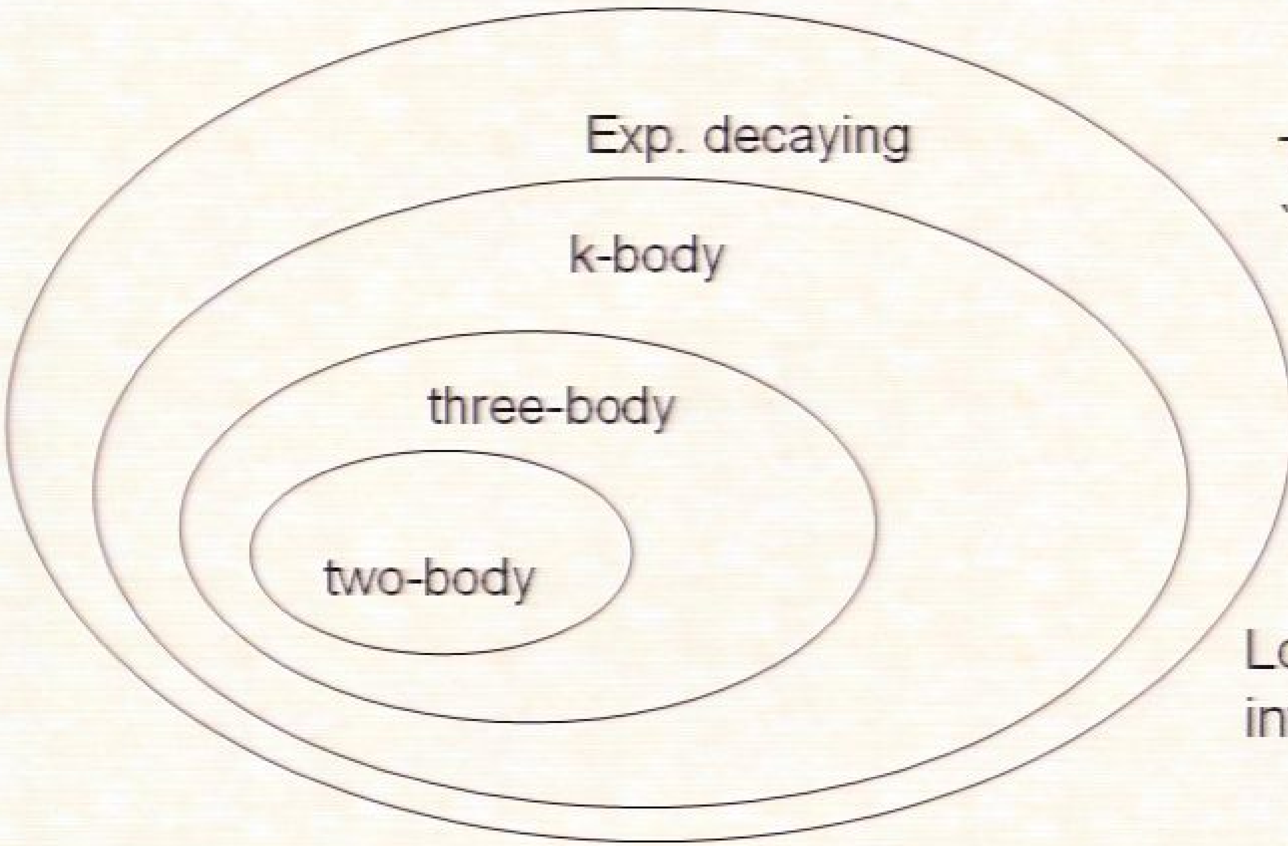


Stability of the spectral gap under local perturbations

Upper bounds on the distance of geometrically local codes

Implementation of TQO models using ``perturbation gadgets''

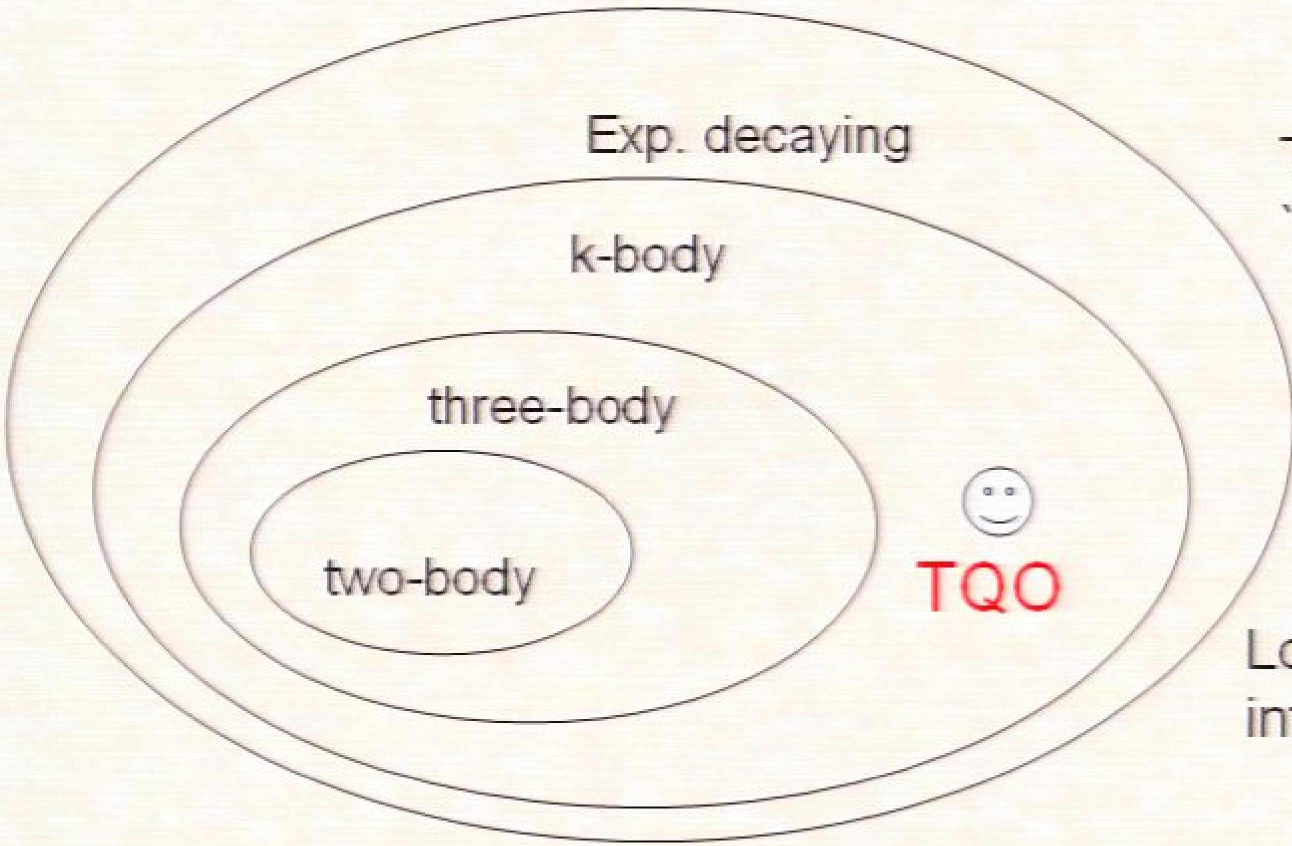
Can TQO emerge in systems with two-body interactions?



The space of all `physical' Hamiltonians

Local bounded-strength interactions

Can TQO emerge in systems with two-body interactions?



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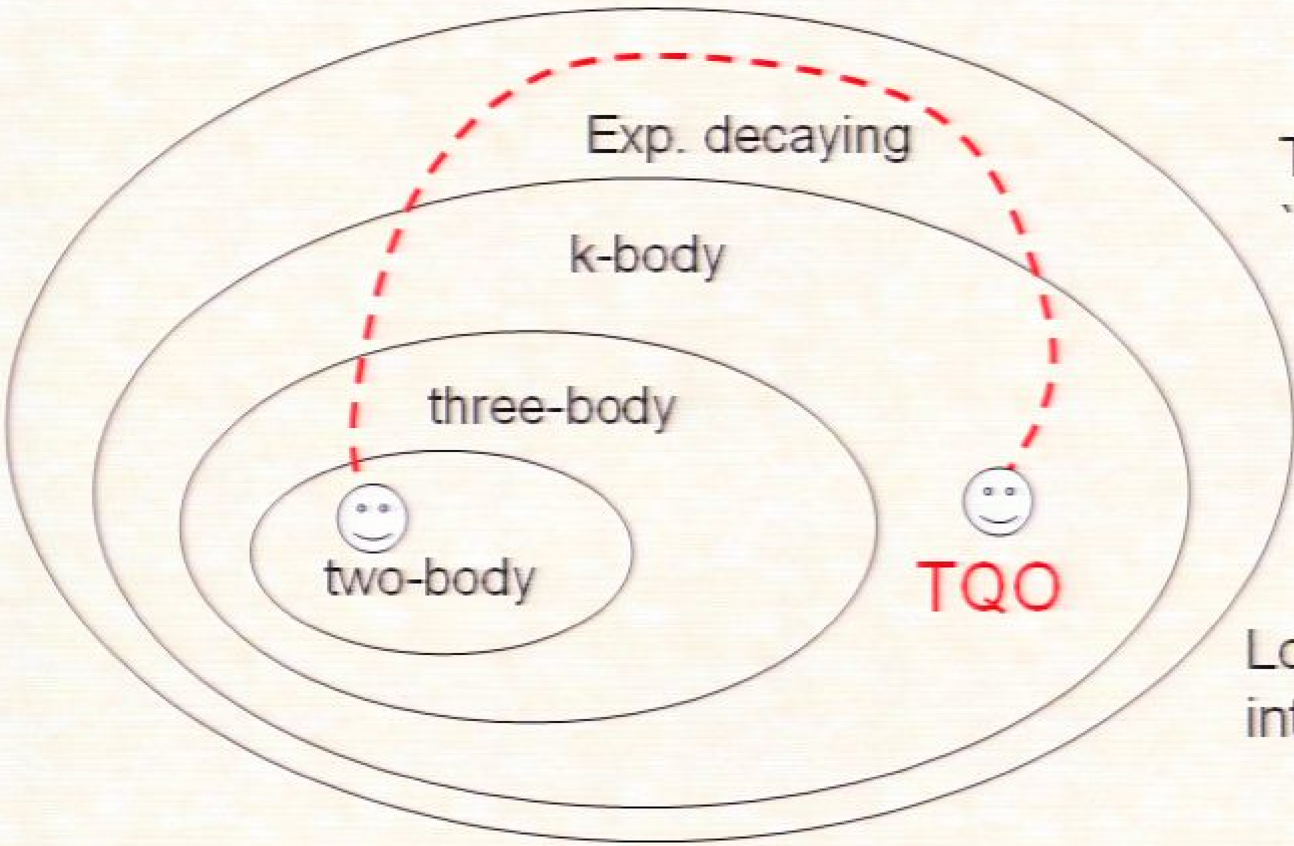


The space of all `physical' Hamiltonians

Local bounded-strength interactions

Corollary: a TQO model with many-body interactions can be connected to a two-body Hamiltonian without closing the gap

Perturbative expansions in physics and QC

Physics

High-energy fundamental theory, full Hamiltonian H

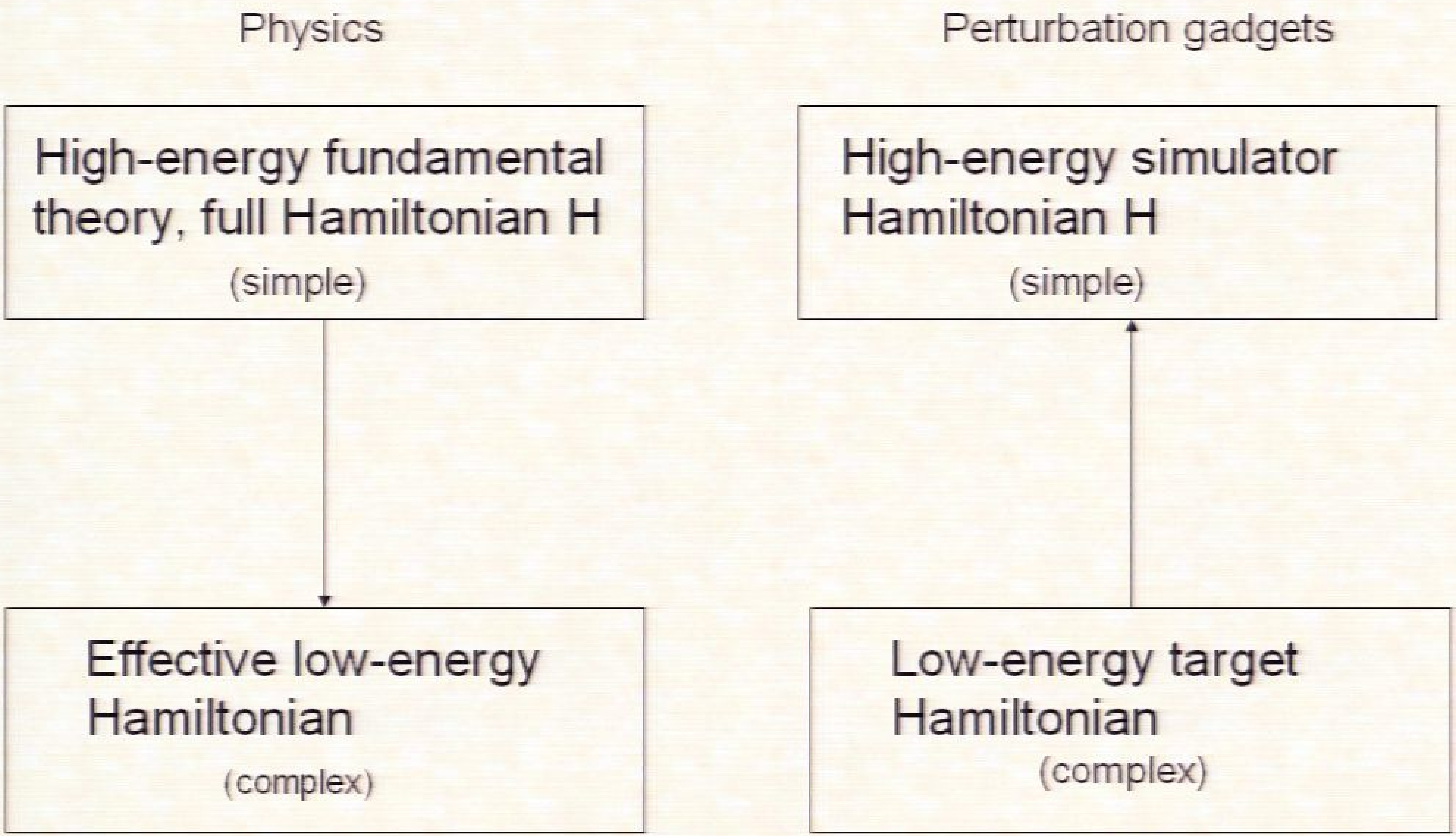
(simple)

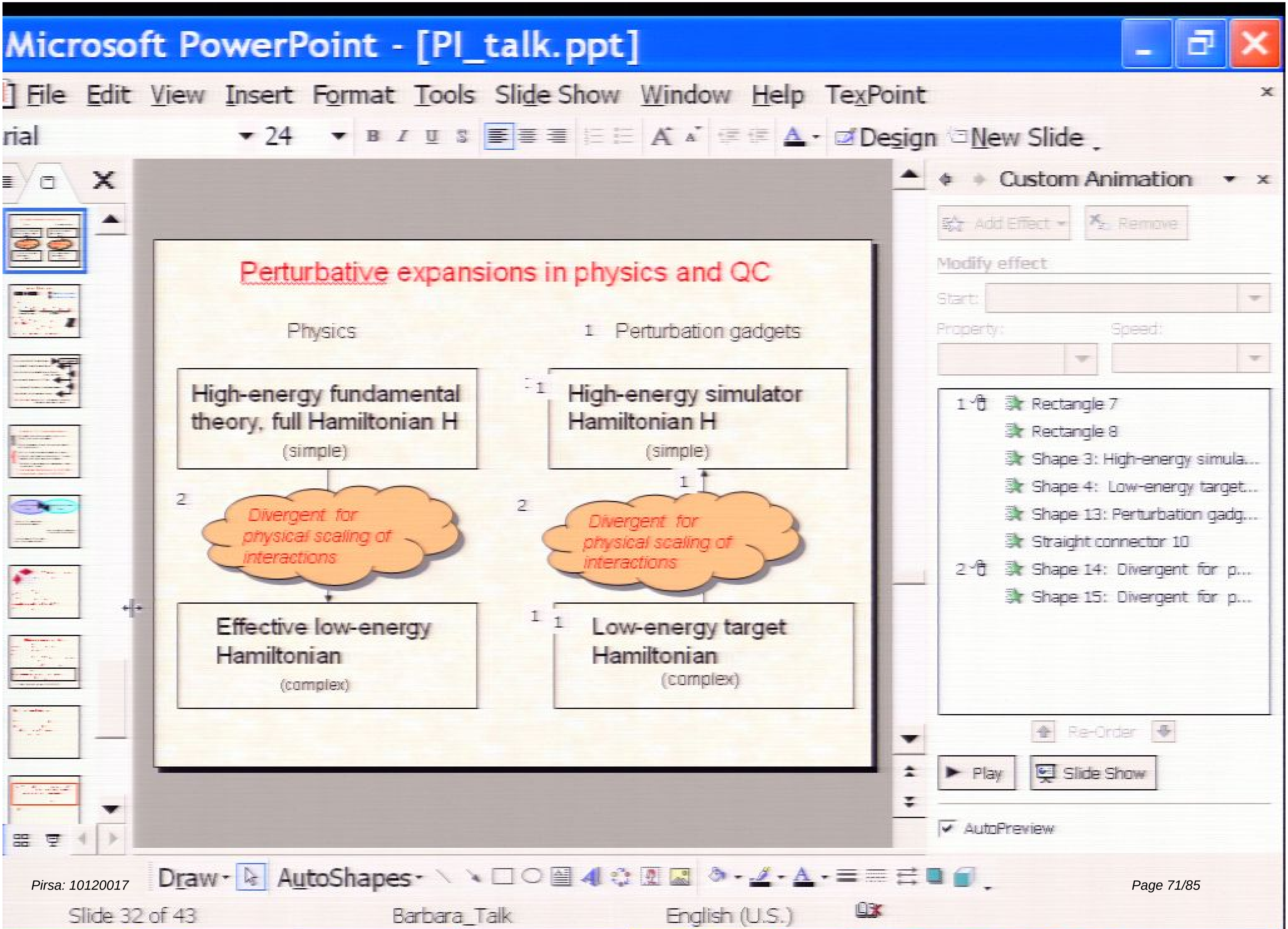
↓

Effective low-energy Hamiltonian

(complex)

Perturbative expansions in physics and QC

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Design New Slide
Custom Animation
Perturbative expansions in physics and QC
Physics
Perturbation gadgets
High-energy fundamental theory, full Hamiltonian H
(simple)
High-energy simulator Hamiltonian H
(simple)
Divergent for physical scaling of interactions
Divergent for physical scaling of interactions
Effective low-energy Hamiltonian
(complex)
Low-energy target Hamiltonian
(complex)
Rectangle 7
Rectangle 8
Shape 3: High-energy simula...
Shape 4: Low-energy target...
Shape 13: Perturbation gadg...
Straight connector 10
Shape 14: Divergent for p...
Shape 15: Divergent for p...
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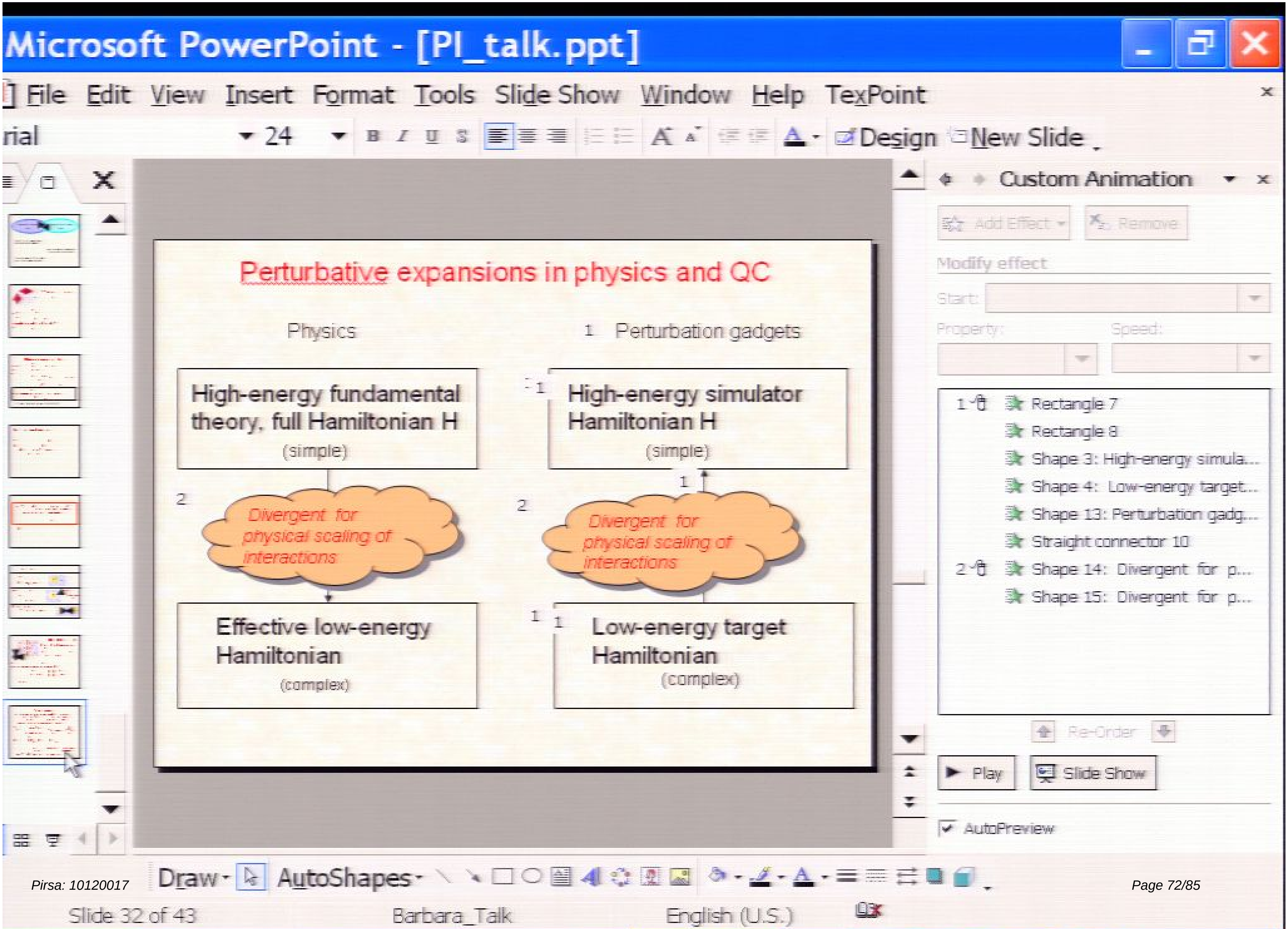
Slide Show
AutoPreview
Draw
AutoShapes
Slide 32 of 43
Barbara_Talk
English (U.S.)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Design New Slide
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Perturbative expansions in physics and QC
Physics
Perturbation gadgets
High-energy fundamental theory, full Hamiltonian H
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Rectangle 7
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Shape 14: Divergent for p...
Shape 15: Divergent for p...
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Slide 32 of 43
Barbara_Talk
English (U.S.)

Open problems

(1) Generalize the gap stability analysis to spatially local, frustation-free, non-commuting Hamiltonians. Our result $\Rightarrow$ it suffices to prove the stability for perturbations preserving the ground state.

(2) Upper bounds for the distance of 3D quantum codes. Best known code $d \sim L^{3/2}$. Best upper bound $d = O(L^2)$. Stability of TQO phases in 3D (beyond the toric codes).

(3) Stability of TQO phases in 3D (beyond the toric codes).

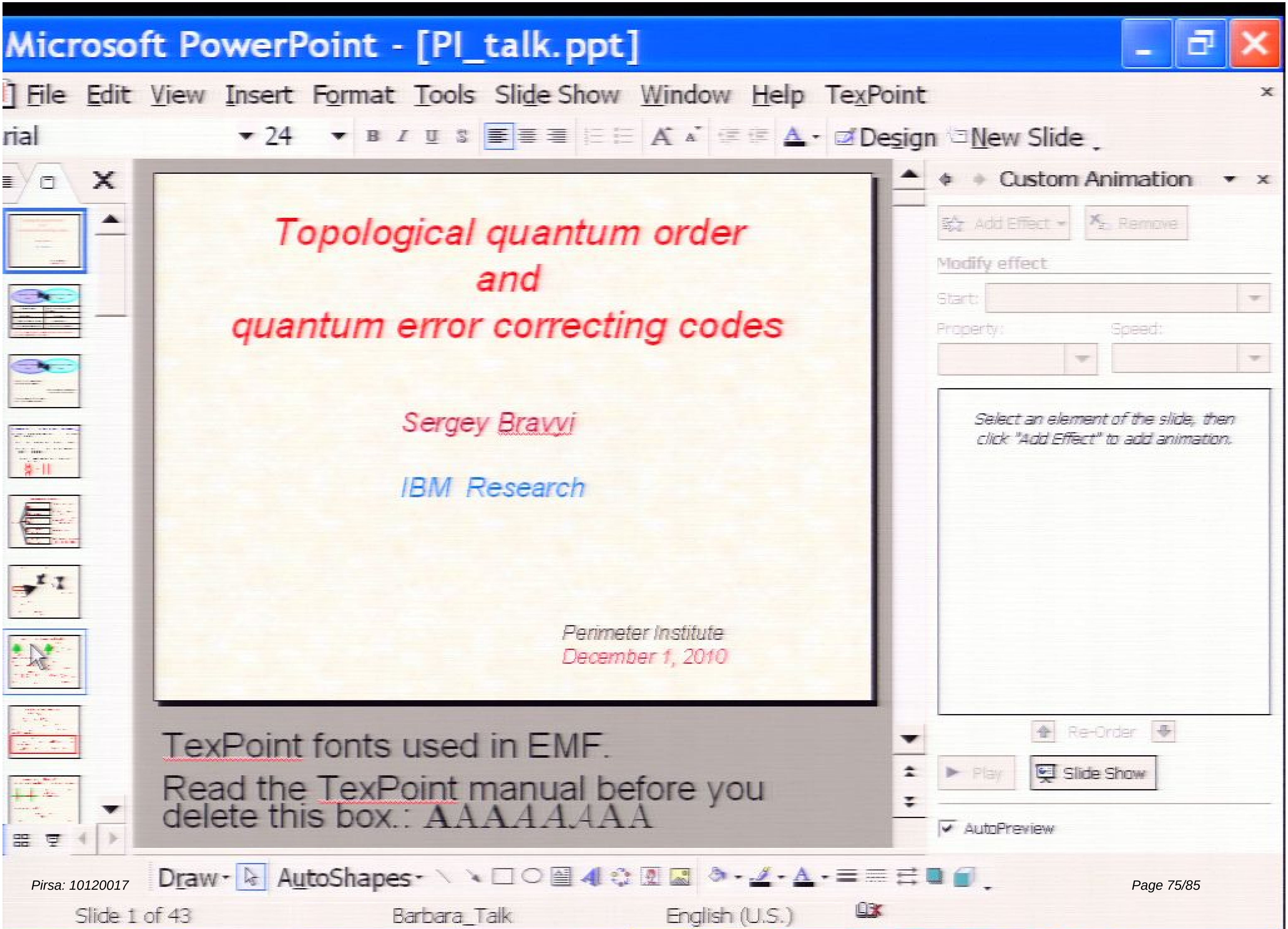
(4) Stability of TQO phases under perturbations for non-zero temperature. Does quantum entanglement help to build a more stable classical memory?

Topological quantum order and quantum error correcting codes

Sergey Bravyi

IBM Research

Perimeter Institute
December 1, 2010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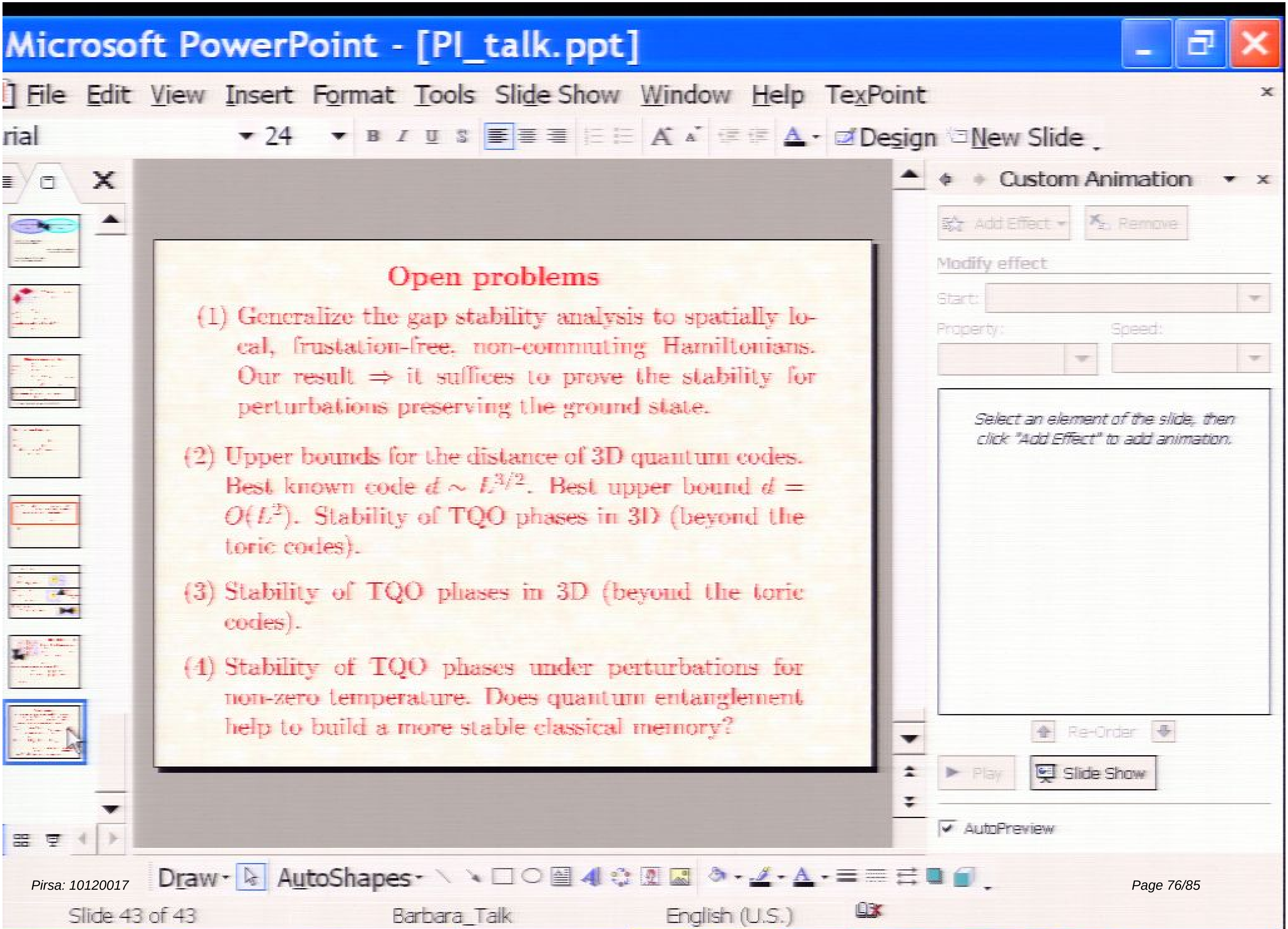
Barbara_Talk
English (U.S.)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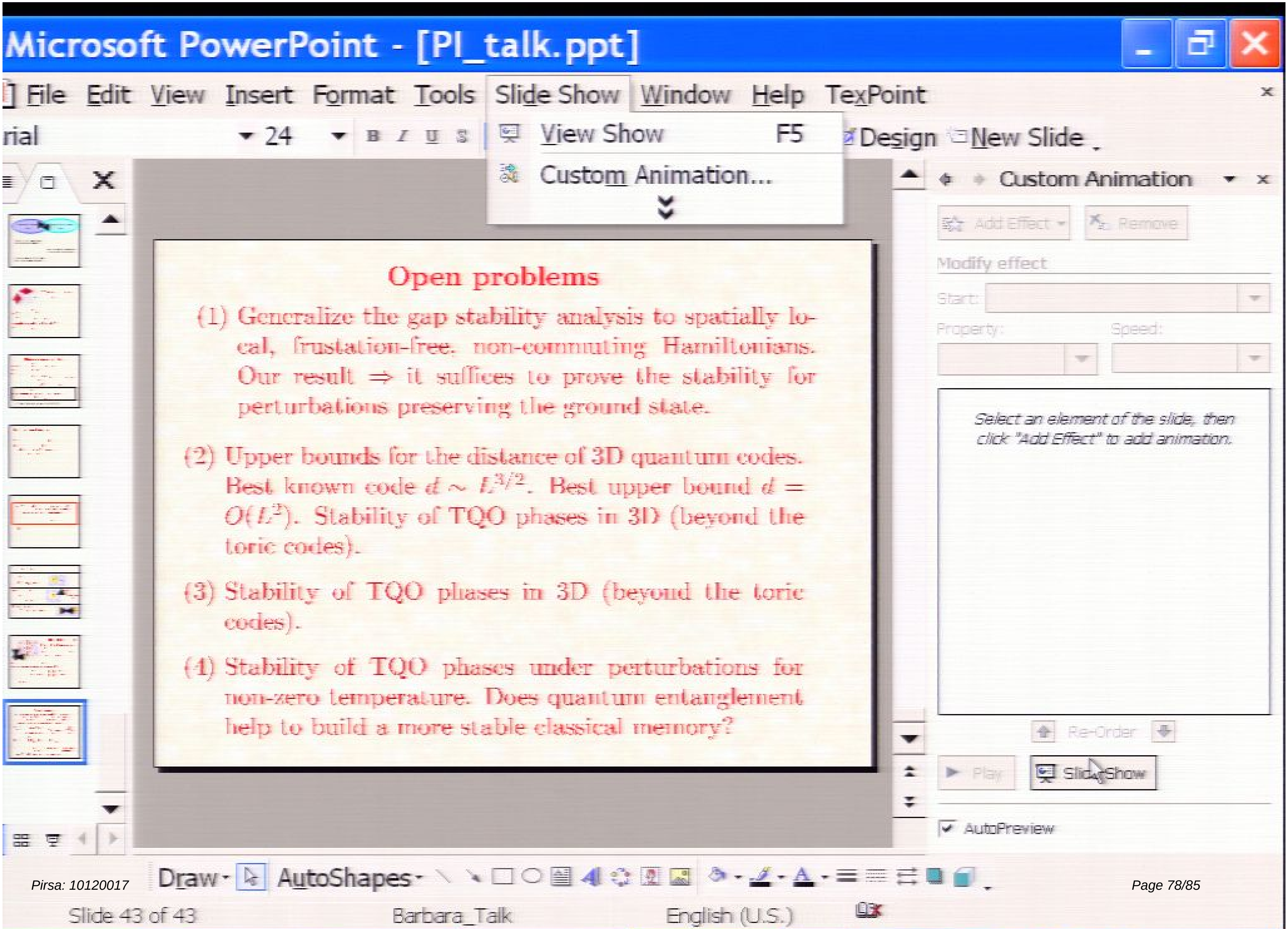
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Draw AutoShapes
Slide 43 of 43
Barbara_Talk
English (U.S.)

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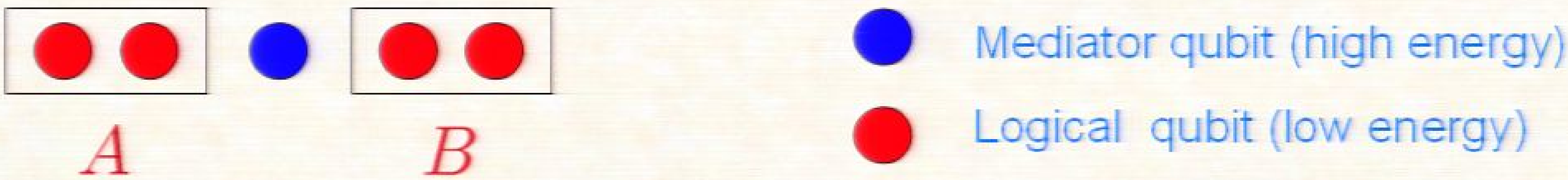
Simulation of TQO by perturbation gadgets

1. Add ancillary high-energy "mediator" qubits to the "logical" qubits acted on by H_{target}
2. Choose appropriate couplings between the mediator and the logical qubits
3. Construct a unitary operator generating an effective low-energy Hamiltonian on the logical qubits. Creates byproduct higher-order terms (exp. decaying).
4. Treat higher-order corrections as a perturbation. Apply the gap stability theorem.

S.B., DiVincenzo, Loss, Terhal, Phys. Rev. Lett. **101**, *070503 (2008)*

S.B., Hastings, Michalakis, J. Math. Phys. **51**, *093512 (2010)*

Toy model: 2-nd order gadget



$H_{\text{target}} = (1/\Delta)AB$ is a four-body interaction

Perturbative expansions in physics and QC

Physics

High-energy fundamental theory, full Hamiltonian H

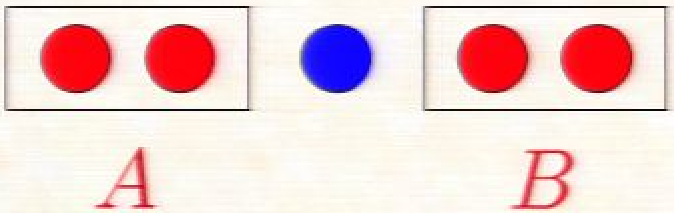
(simple)

↓

Effective low-energy Hamiltonian

(complex)

Toy model: 2-nd order gadget



Mediator qubit (high energy)

Logical qubit (low energy)

$H_{\text{target}} = (1/\Delta) AB$ is a four-body interaction

$$H = \underbrace{\Delta|1\rangle\langle 1|_m \otimes I_l}_{\text{unpertubed Hamiltonian}} + \underbrace{X_m \otimes (A - B) + I_m \otimes H_{\text{else}}}_{\text{perturbation}}$$

Using 2nd order degenerate perturbation theory one gets an effective Hamiltonian H_{eff} acting on the logical qubits:

$$H_{eff} = H_{else} - \frac{1}{\Delta}(A - B)^2$$

Idea of perturbation gadget

[Kempe, Kitaev, Regev 05] 3-local to 2-local

[Oliveira, Terhal 05] k-local to 2-local on 2D lattice

[Bravyi, DiVincenzo, Oliveira, Terhal 06] k-local to 2-local for stoquastic Hamiltonians

[Biamonte, Love 07] simulator with XZ,X,Z only

[Schuch, Verstraete 07] simulator with Heisenberg interactions

[Jordan, Farhi 08] k-th order perturbative gadgets

Main shortcoming: rigorous only if the interaction strength of the simulator Hamiltonian is poly(n).

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