Title: 1-loop diagram in AdS space and the random disorder problem

Date: Nov 26, 2010 11:00 AM

URL: http://pirsa.org/10110079

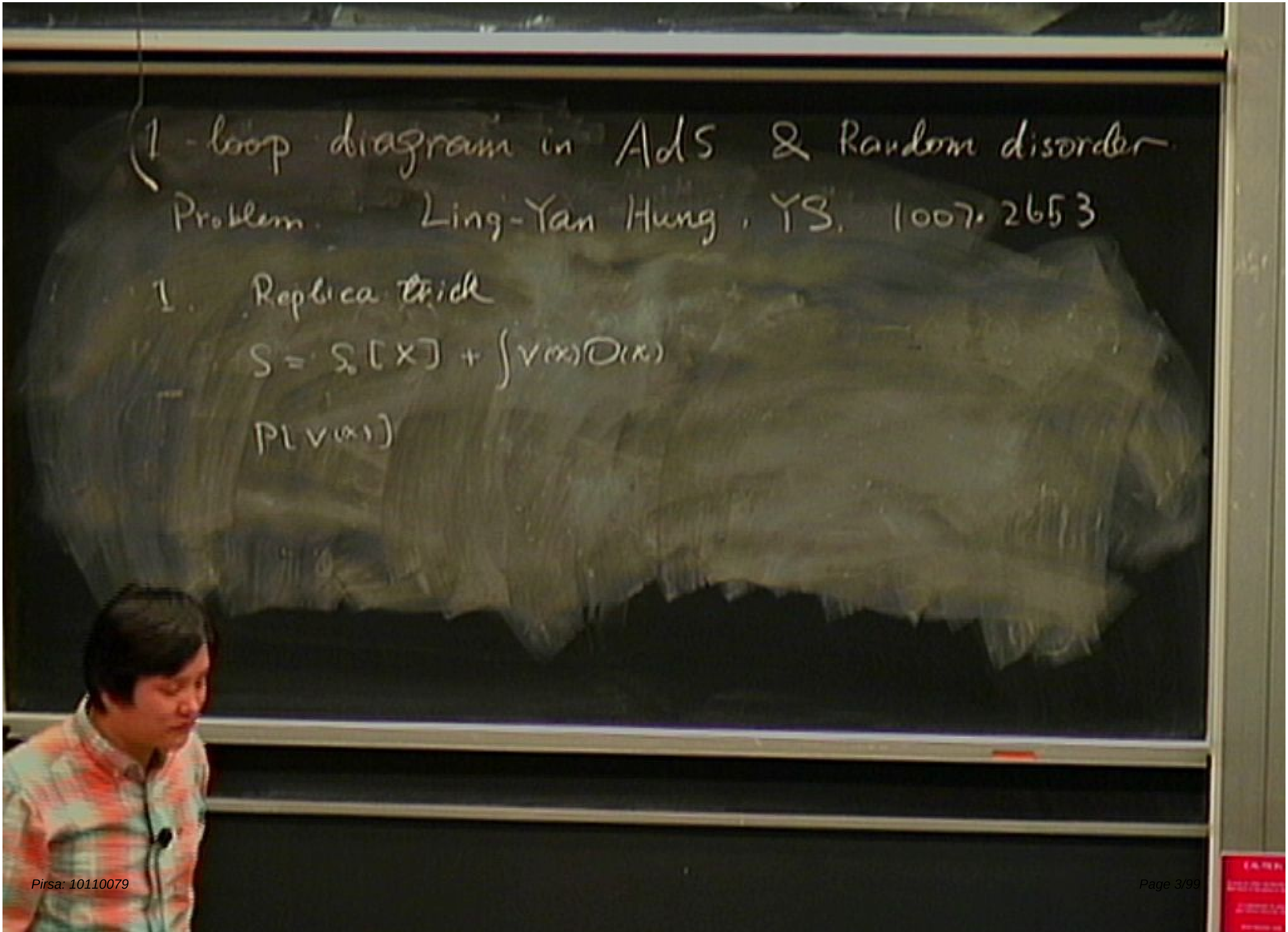
Abstract: AdS/CFT has proven itself a powerful tool in extending our understanding of strongly coupled quantum theories. While studies of AdS/CFT have predominantly focused on tree level calculations, there has been growing interest in the loop effect recently. We studied the 1-loop correction to the gauge boundary-to-boundary correlator due to its coupling to a complex scalar field. In this talk, I would outline our main results, explain the Cutkosky rule in AdS space, and discuss an extra divergence we found in both real and imaginary part of the loop integral. I would then combine our analysis with the replica trick to demonstrate a possible application where one attempts to calculate the DC conductivity in a condensed matter system with random disorder and discuss the limitation and difficulties of our method in its current form.

1-loop diagram in AdS & Random disorder

Problem    Ling-Yan Hung, YS. 1007.2653

I. Replica trick

$$S = S_0[X] + \int V(x) \mathcal{O}(x)$$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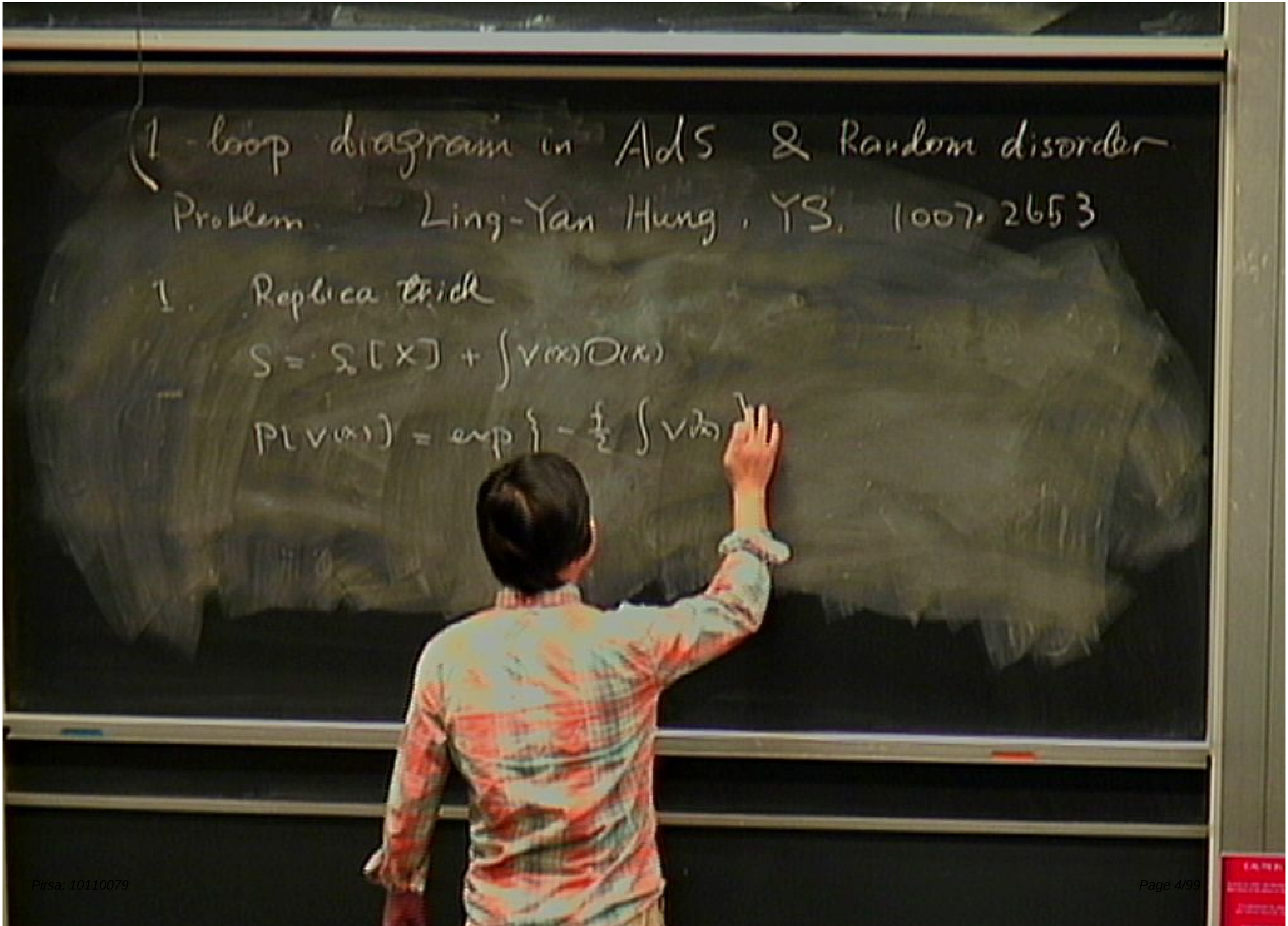
P[V(x)]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1. Replica trick
S = S₀[X] + ∫V(x)O(x)
P[V(x)] = exp{ -½ ∫V(x)

1-loop diagram in AdS & Random disorder

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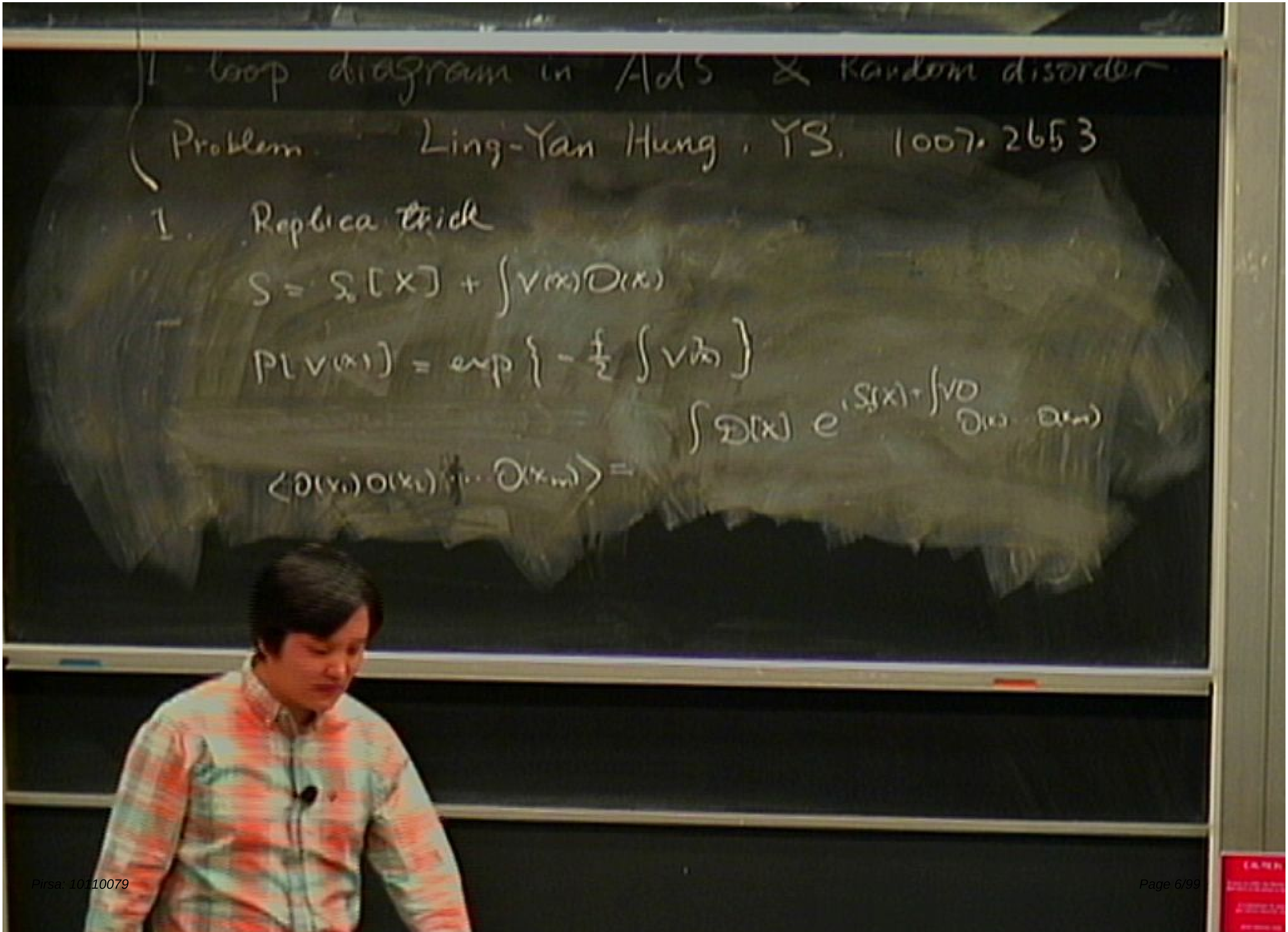
I. Replica trick

$$S = S_0[X] + \int V(x)\mathcal{O}(x)$$

$$P[V(x)] = \exp\left\{ -\frac{f}{2} \int V^2(x) \right\}$$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_m)\rangle = \int$$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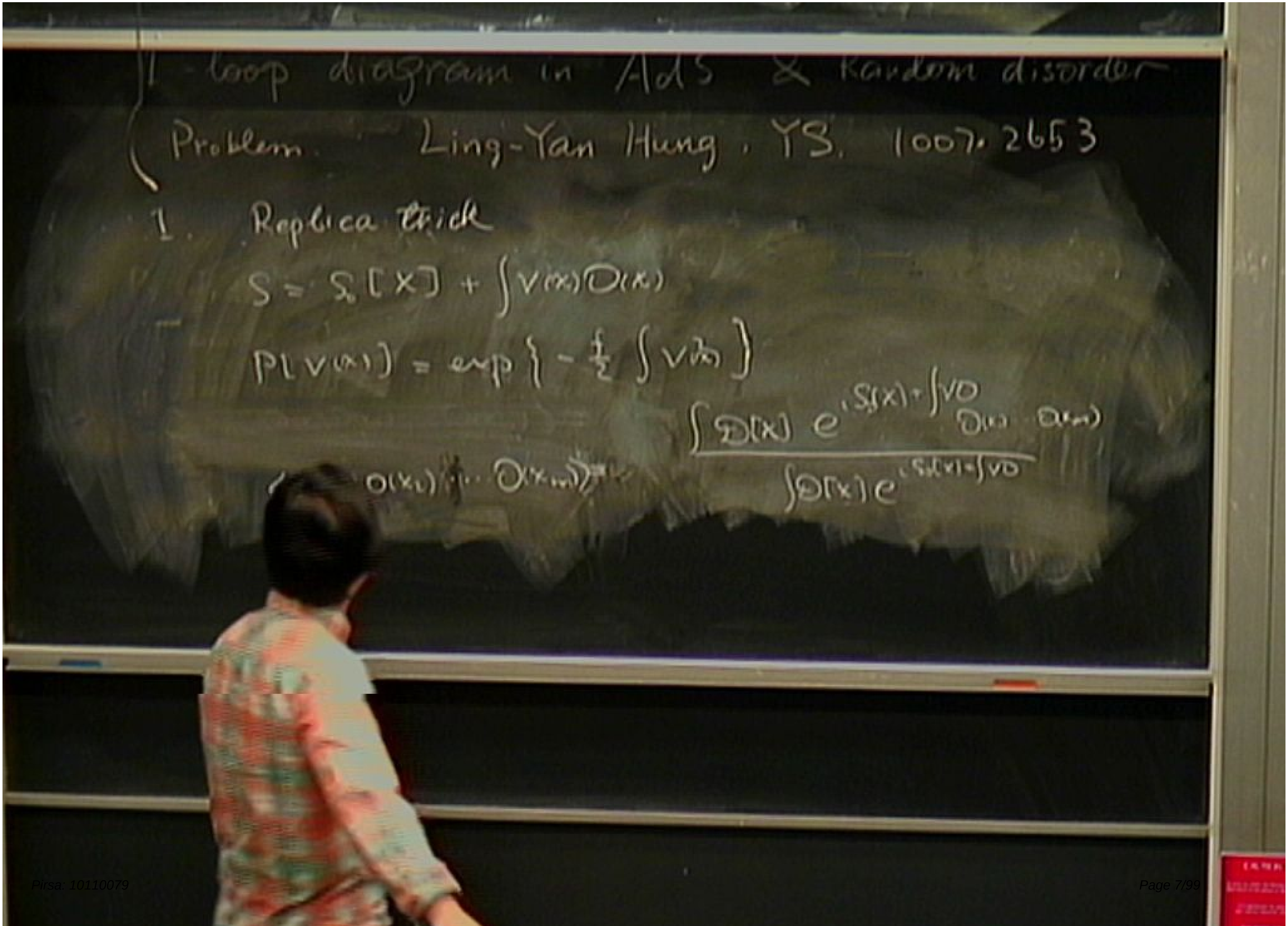
$\langle O(x_1) O(x_2) \cdots O(x_m) \rangle = \int D[X]\, e^{iS_0[X] + \int VO} O(x_1) \cdots O(x_m)$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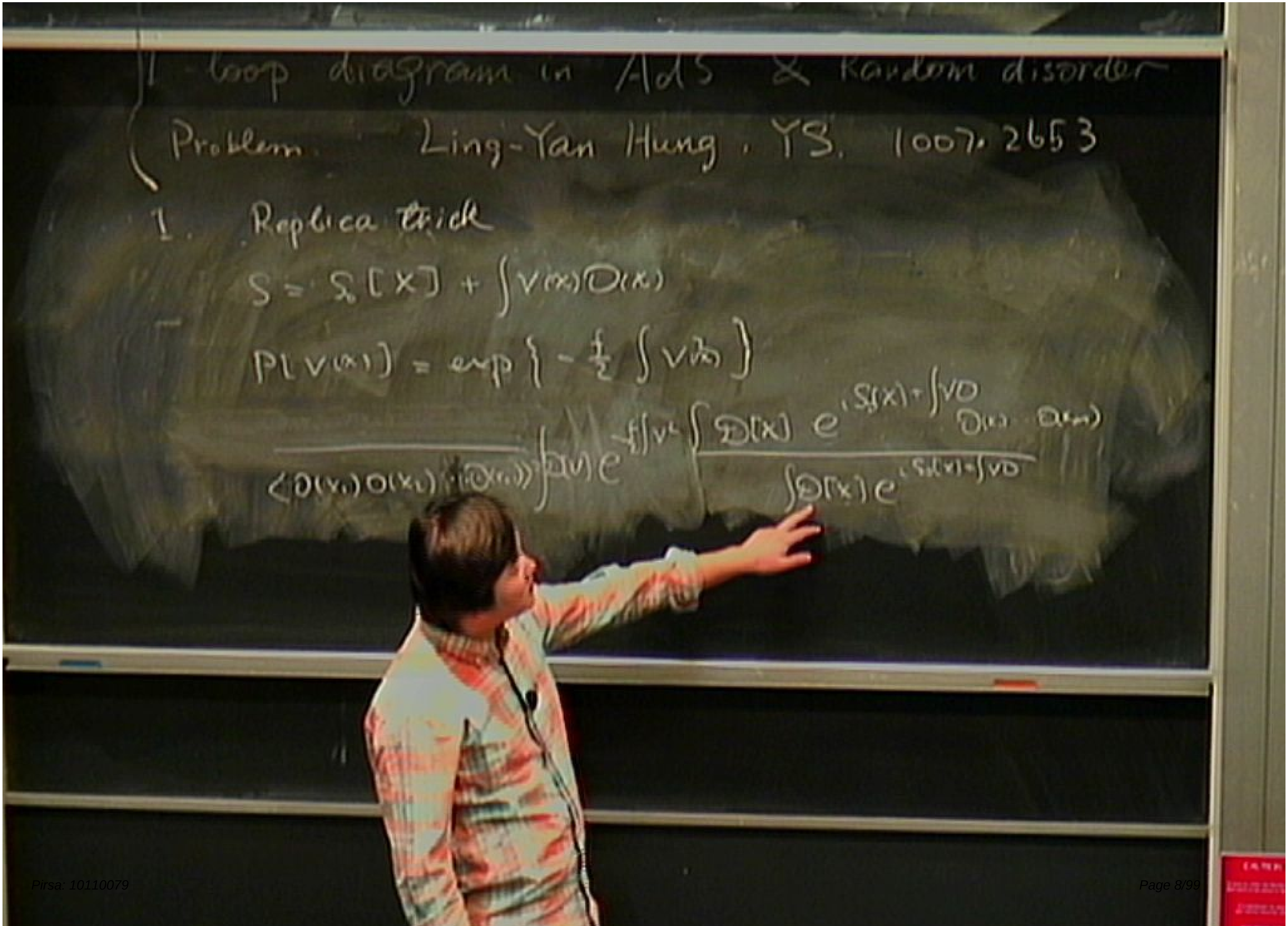
P[V(x)] = exp{ − f/2 ∫V²(x) }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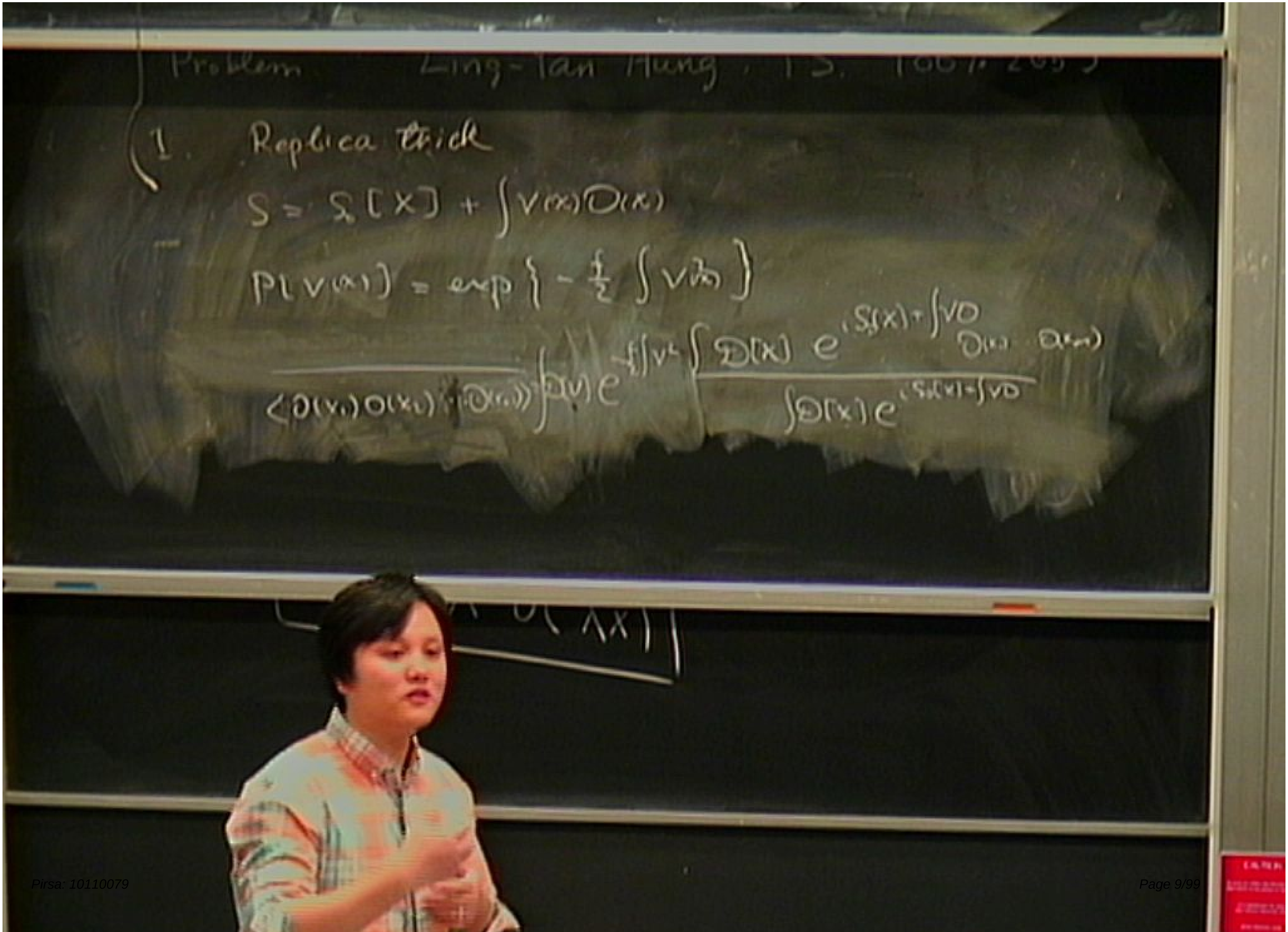
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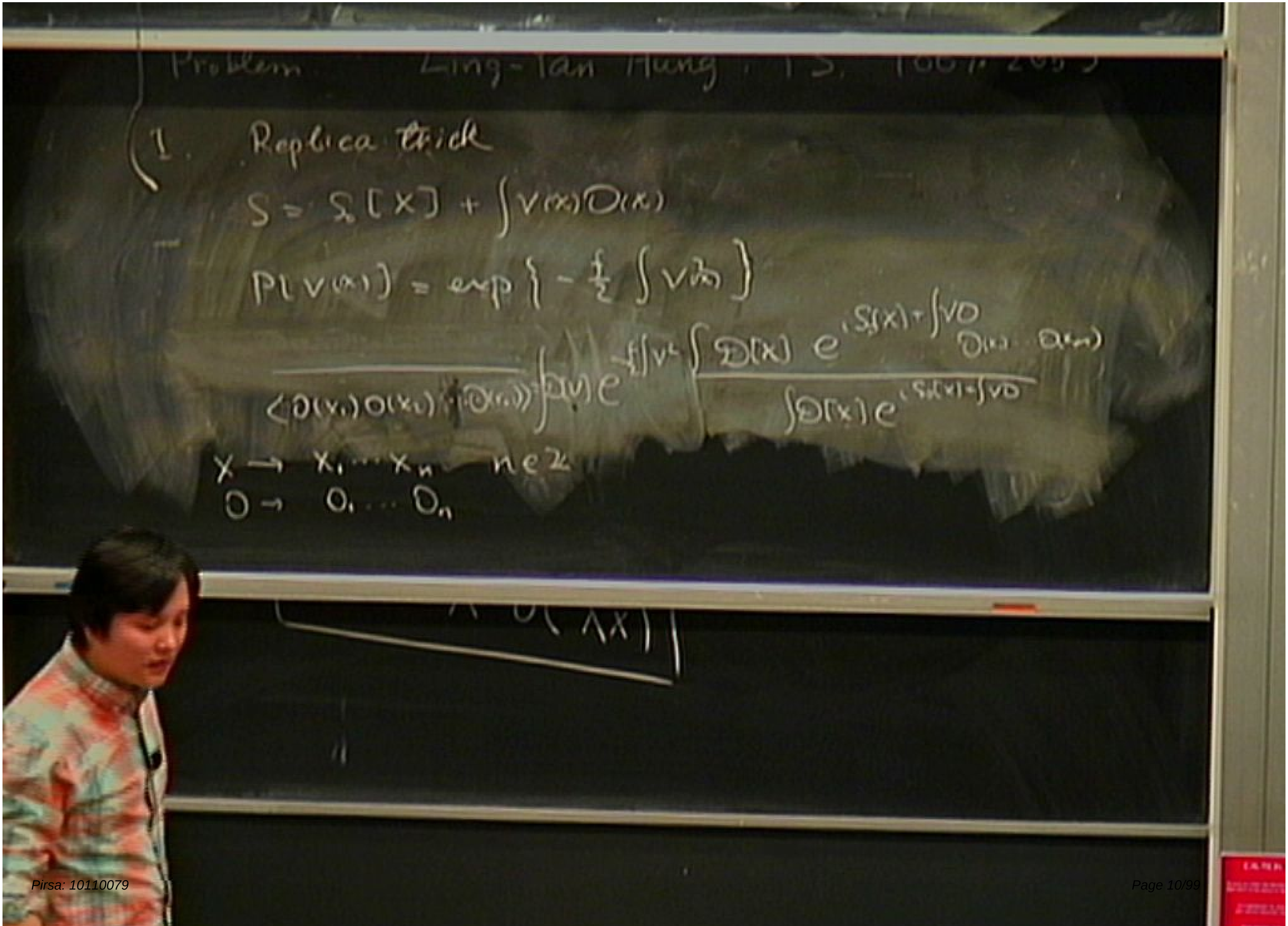
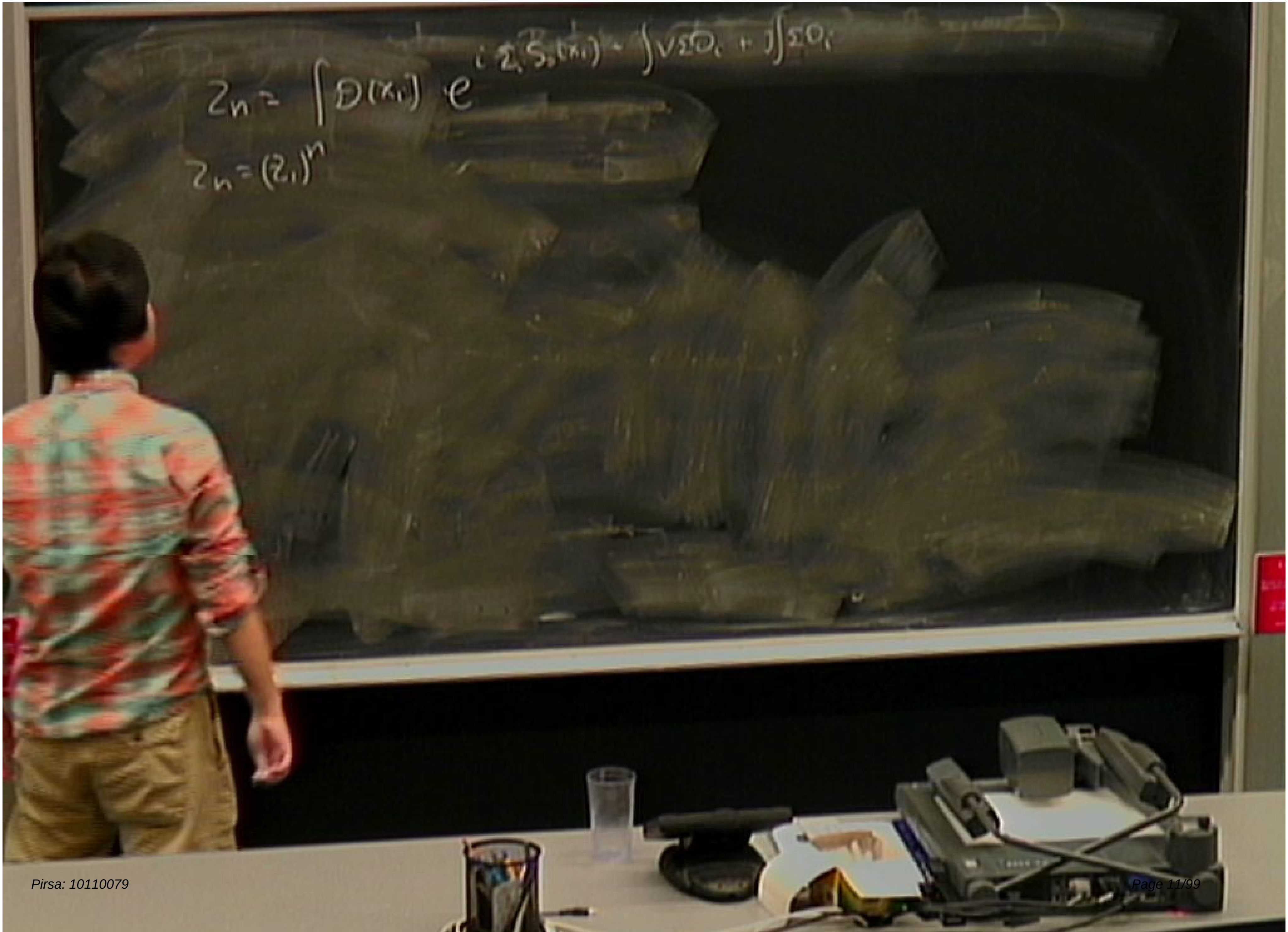
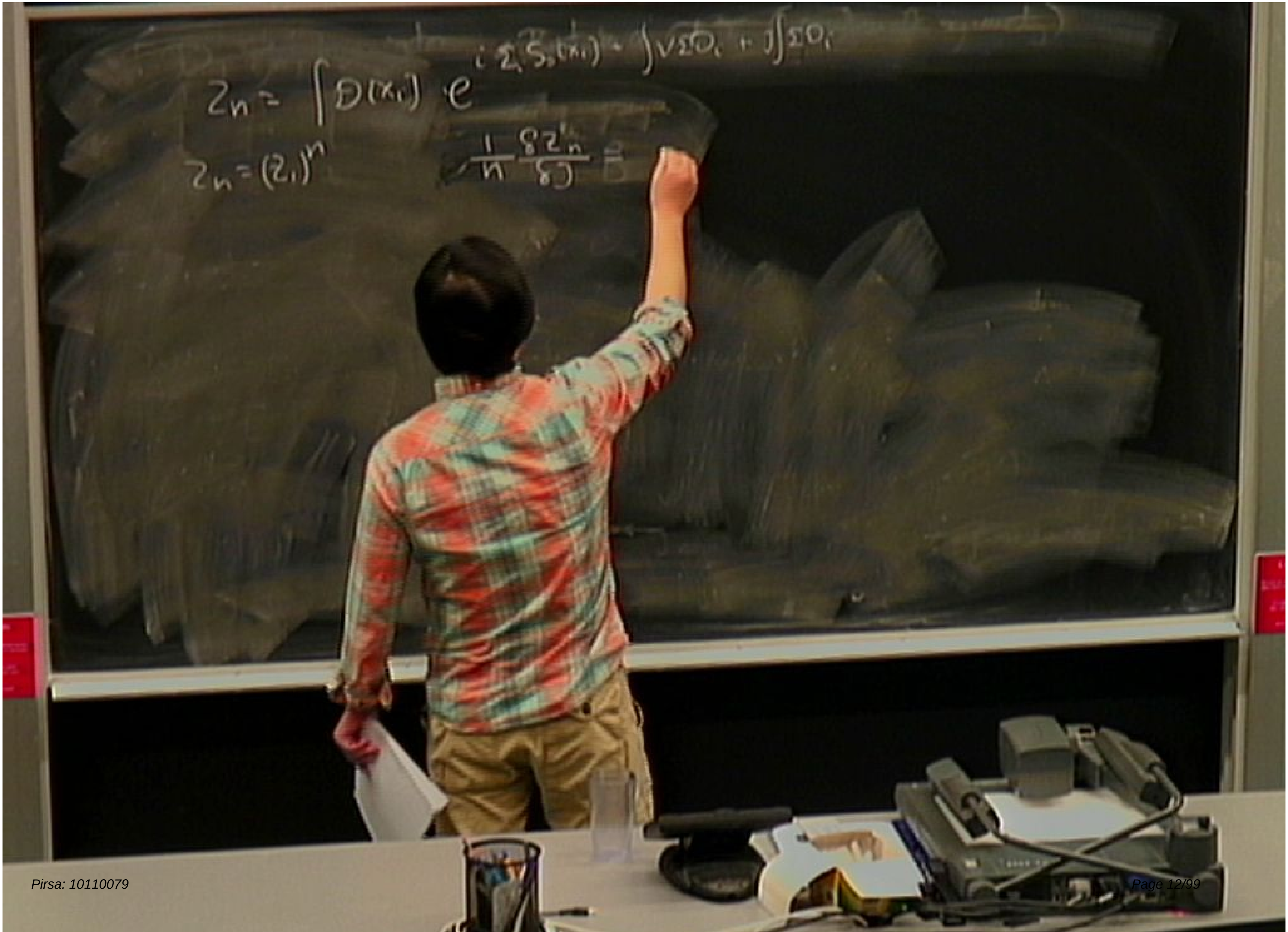
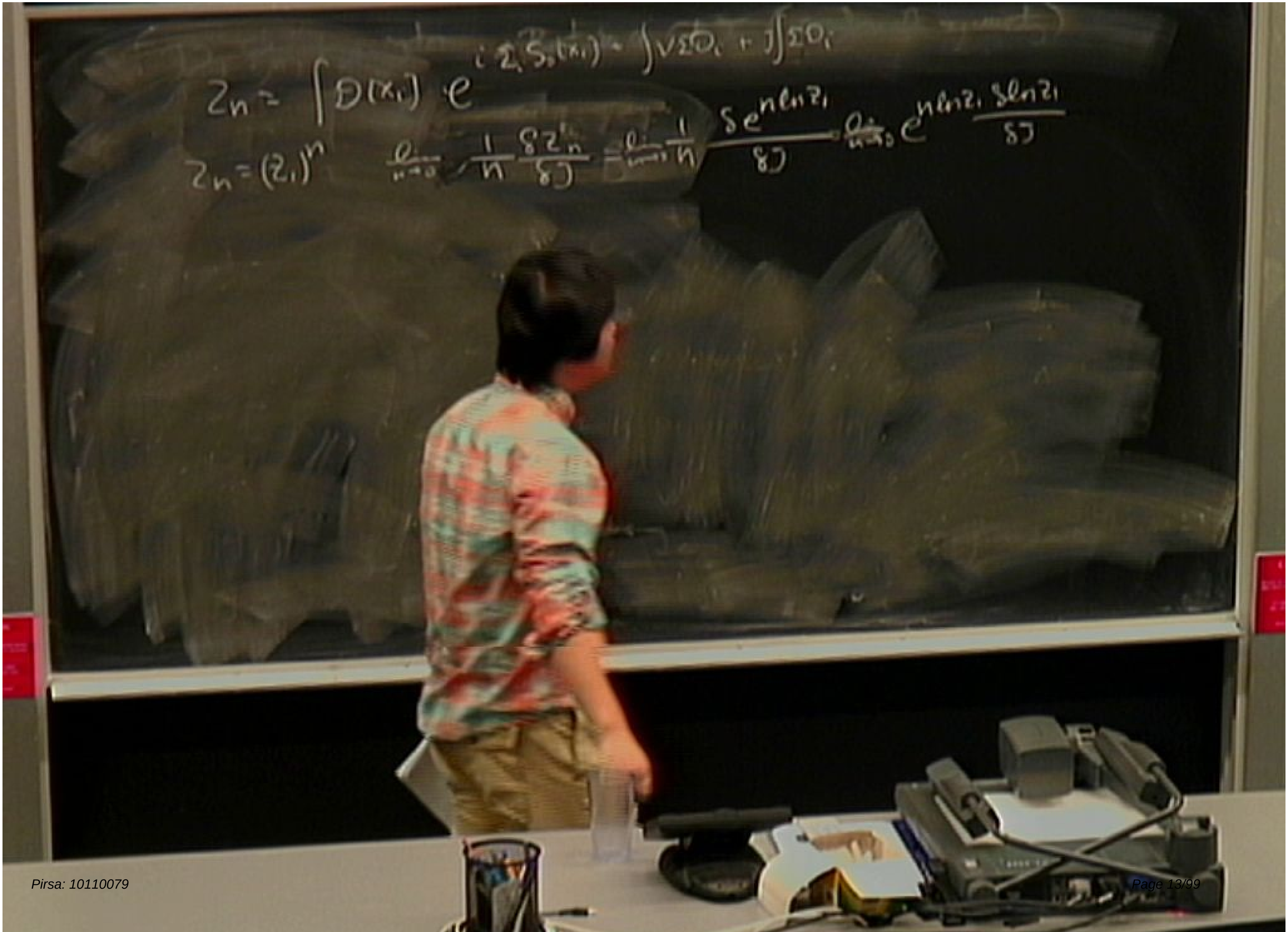
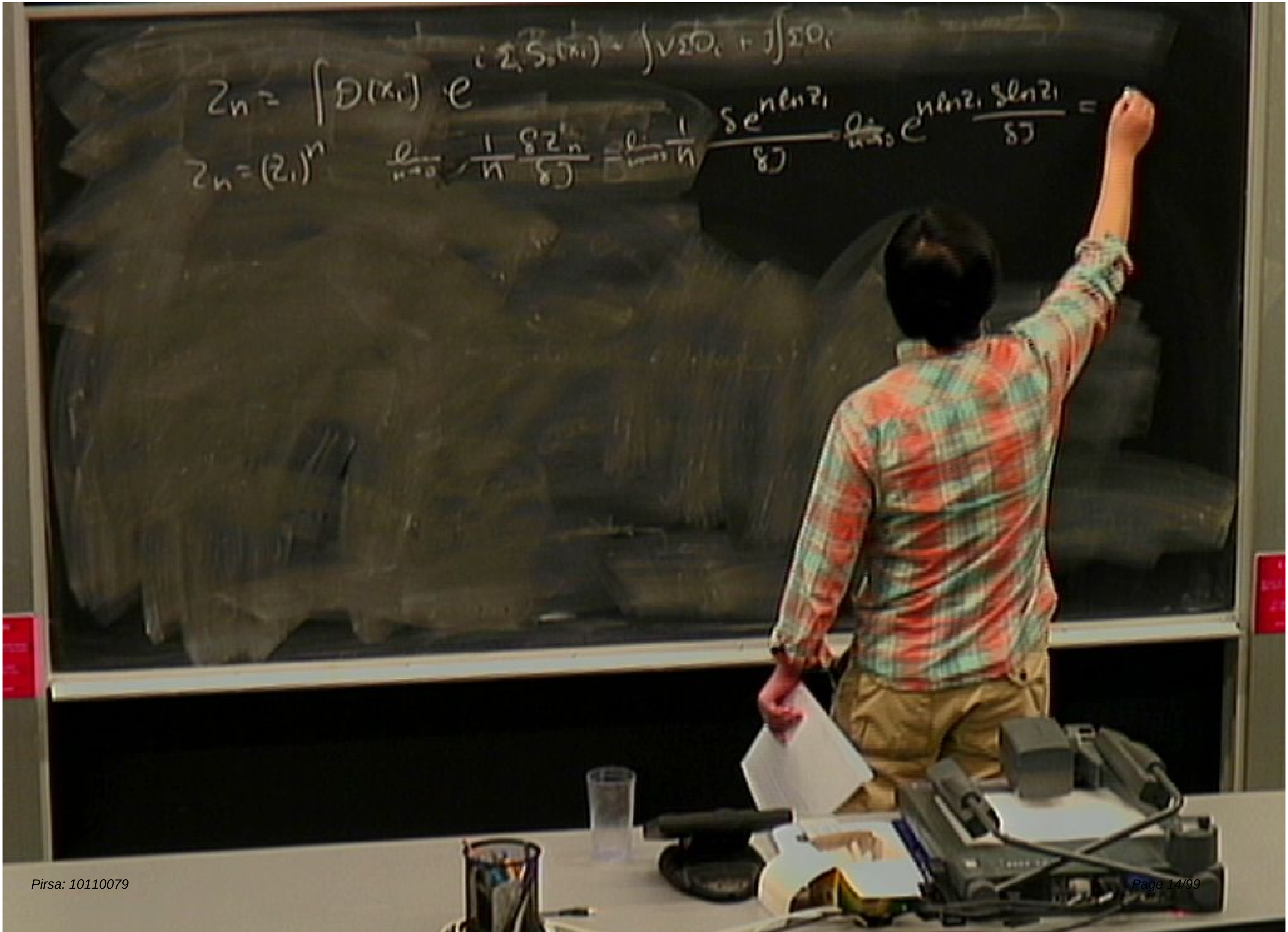
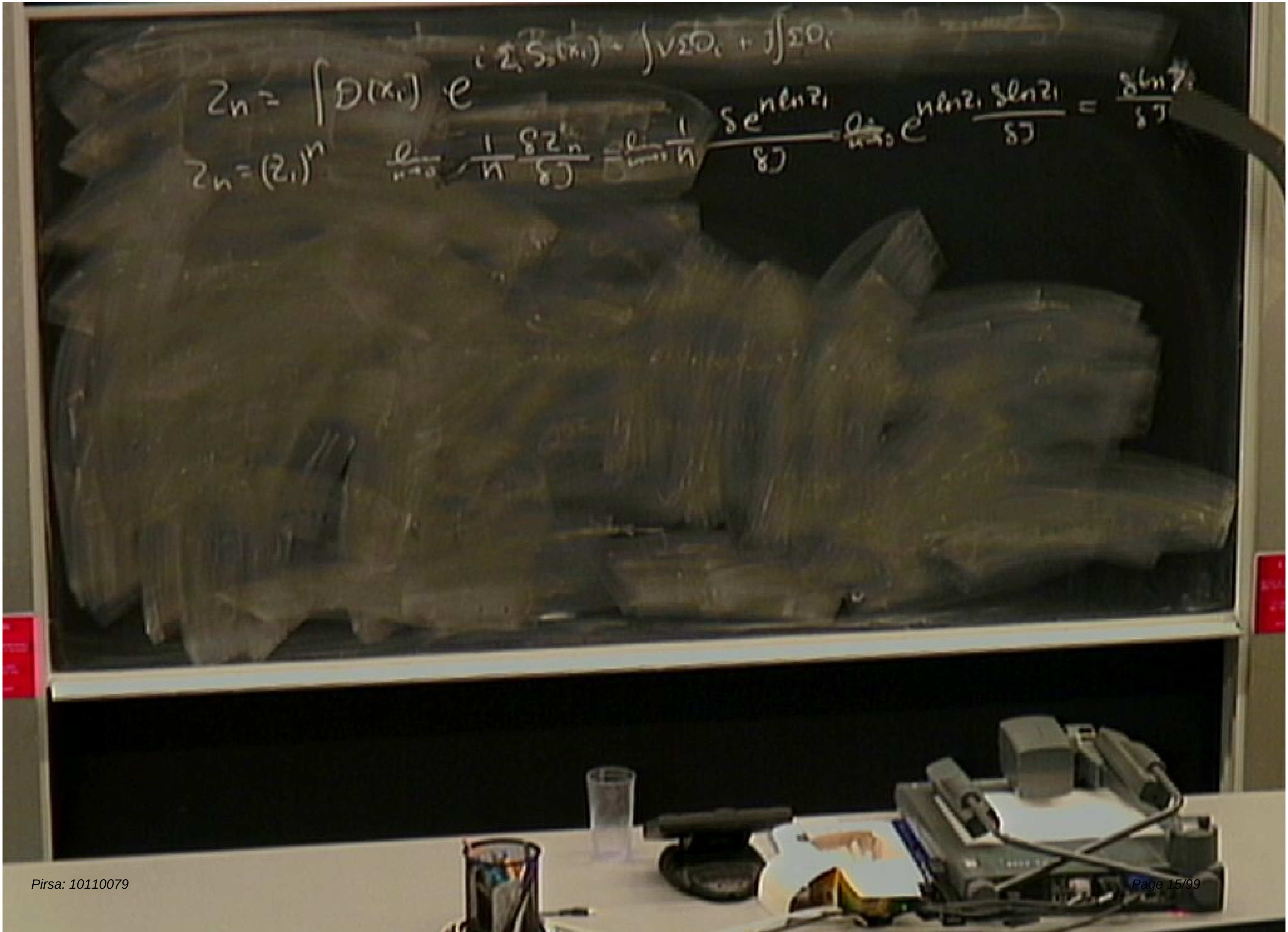

I. Replica trick
S = S₀[X] + ∫V(x)O(x)
P[V(x)] = exp{ - f/2 ∫V²(x) }
X → X₁ … Xₙ   n ∈ ℤ
O → O₁ … Oₙ

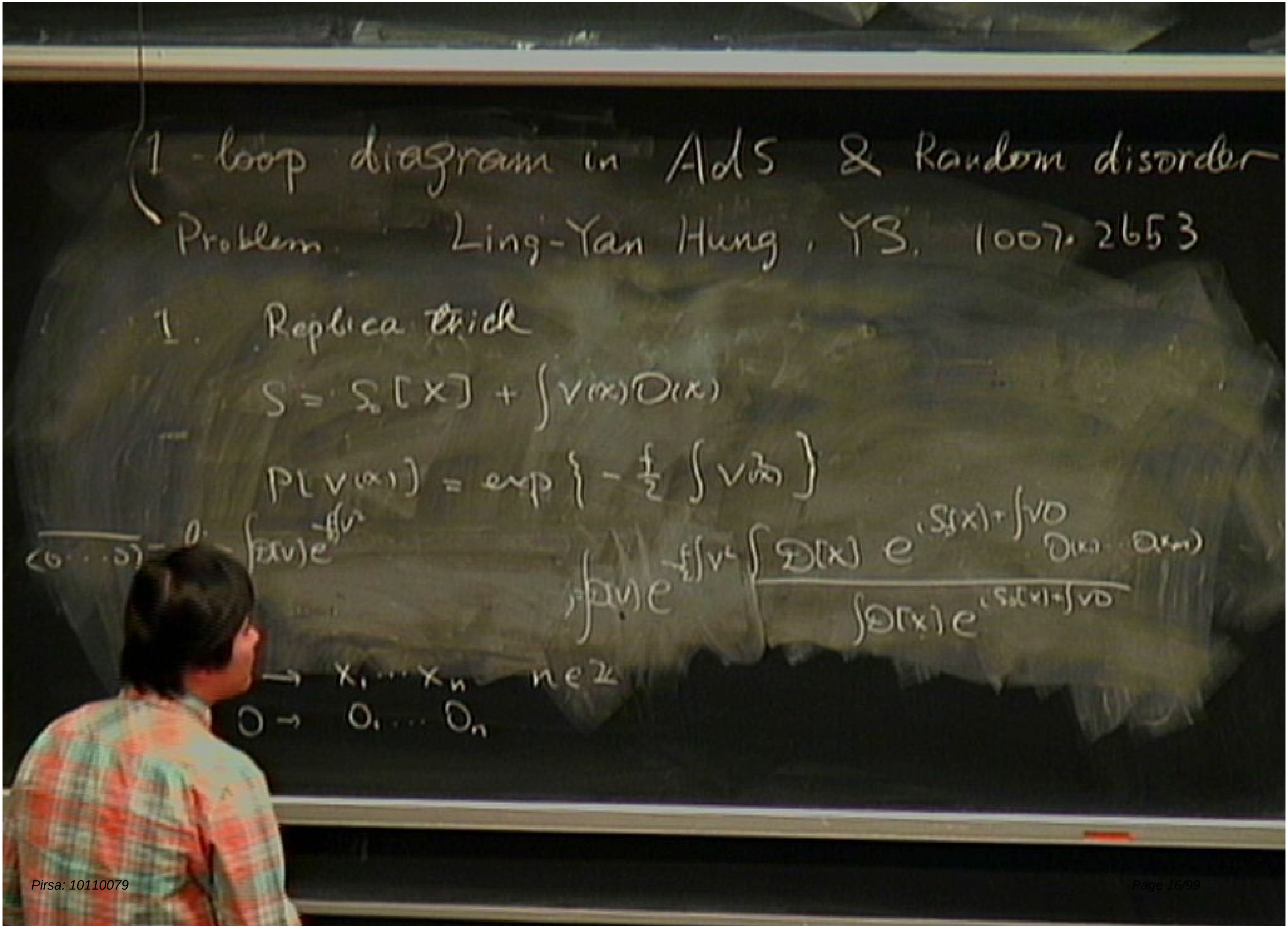

$Z_n = (Z_1)^n$




Zn = ∫D(xi) e
Zn = (Z1)^n




Zn = (Z1)n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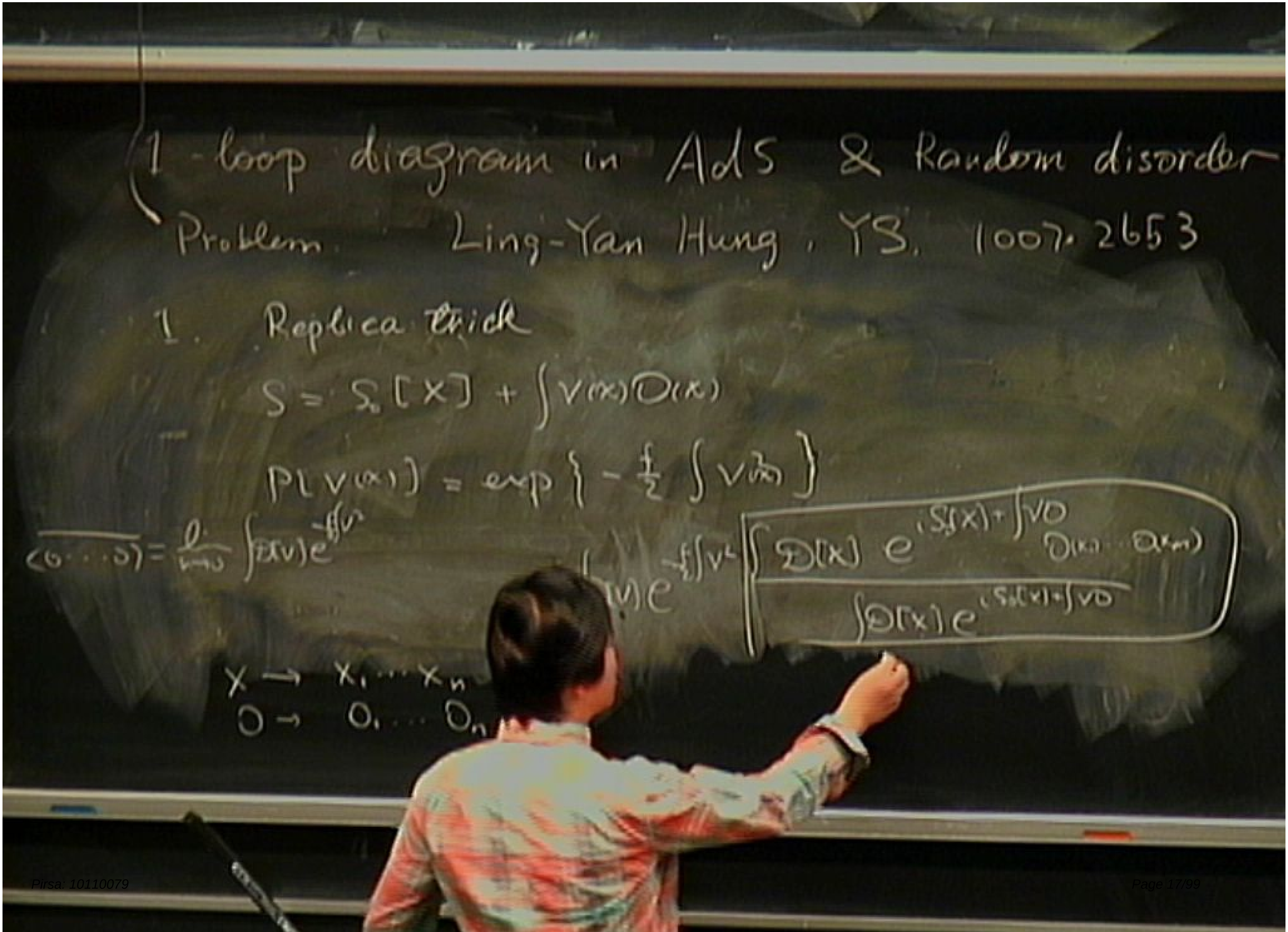
P[V(x)] = exp{ - f/2 ∫V²(x) }
X → X1 ... Xn n∈Z
O → O1 ... On


1-loop diagram in AdS & Random disorder
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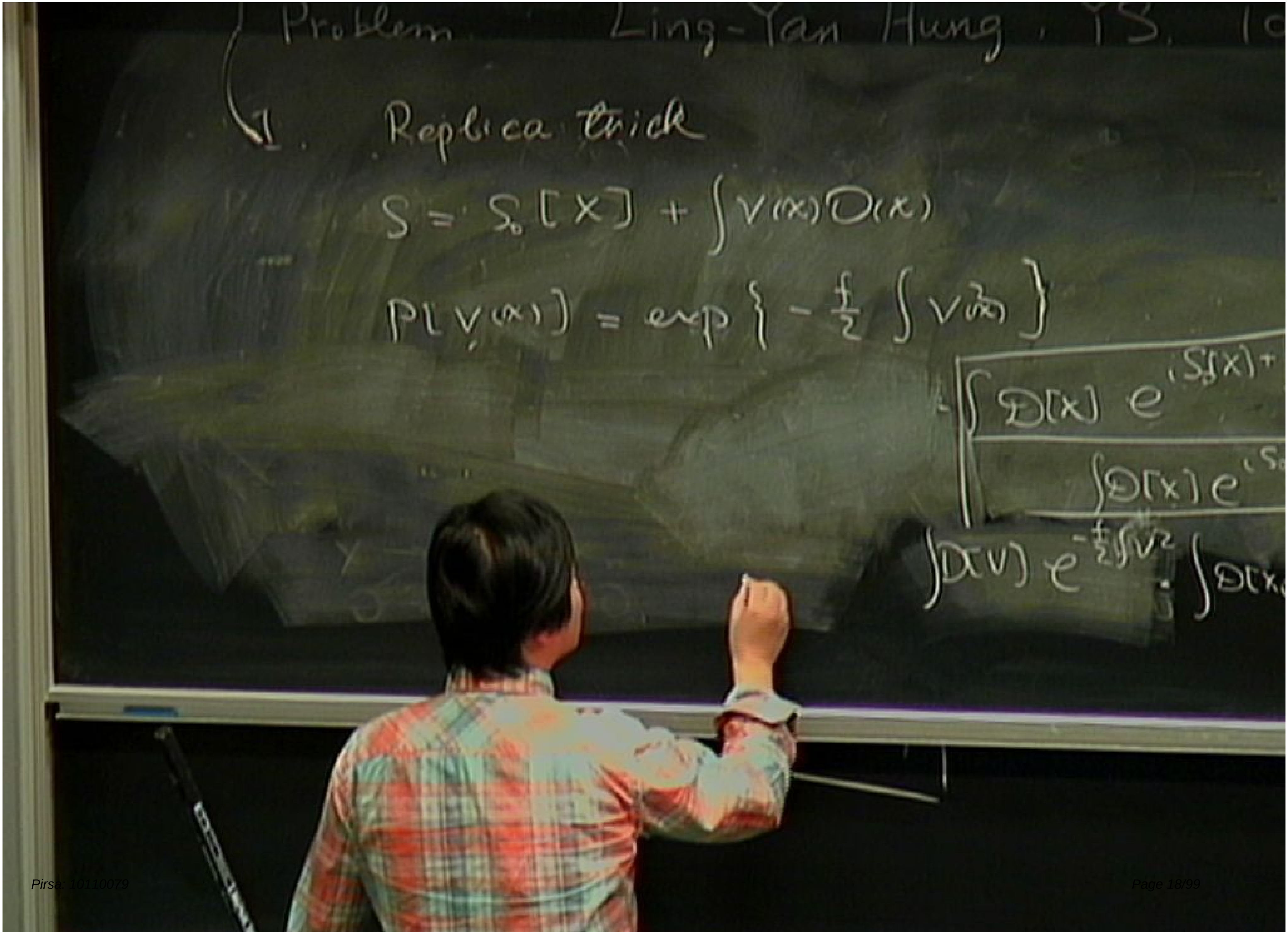
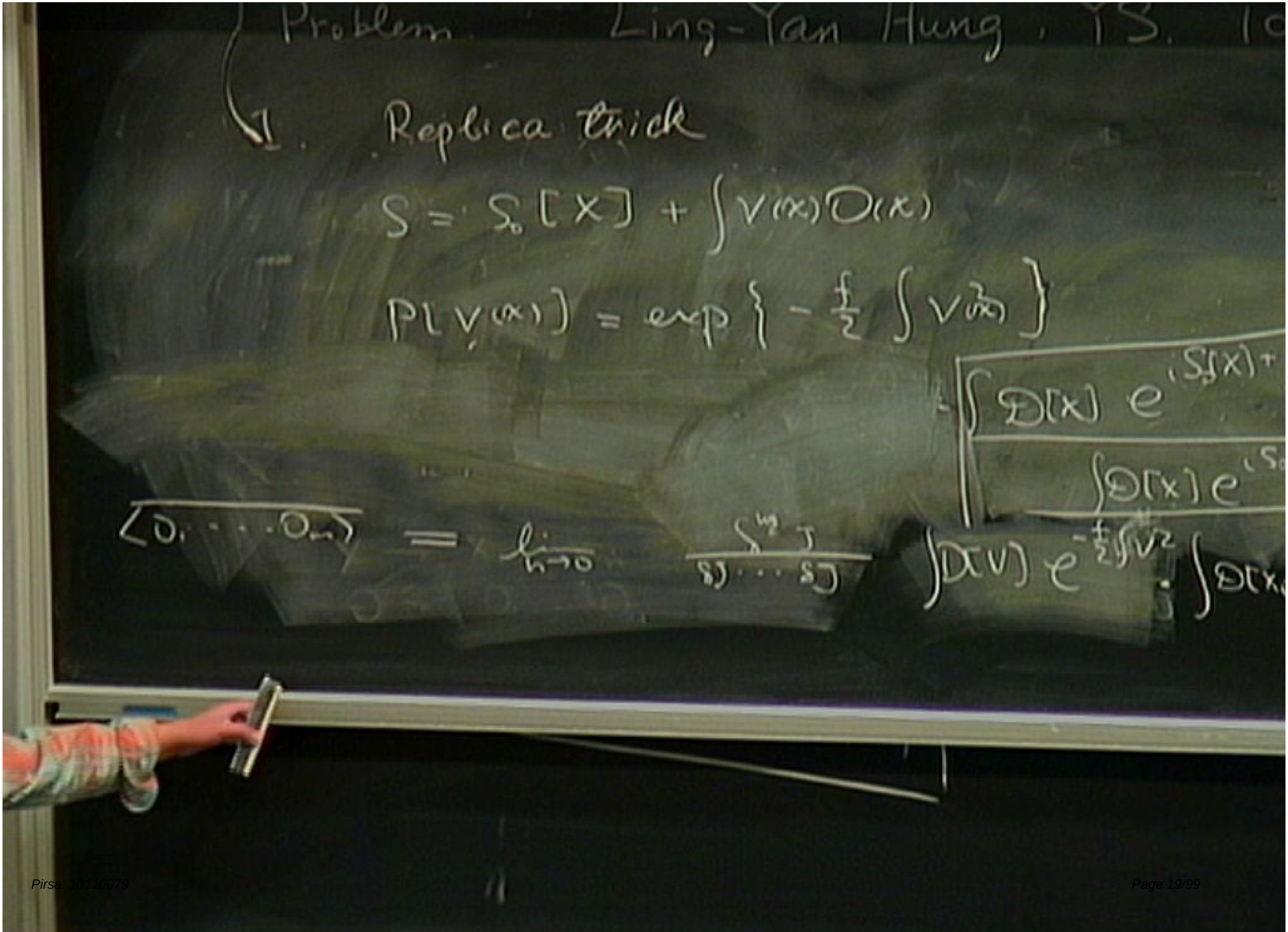
1. Replica trick
S = S₀[X] + ∫V(x)O(x)
P[V(x)] = exp{ - f/2 ∫V(x)² }
X → X₁ ⋯ Xₙ
O → O₁ ⋯ Oₙ



Problem. Ling-Yan Hung, YS,
1. Replica trick
$S = S_0[X] + \int V(x) O(x)$
$P[V(x)] = \exp\{-\frac{f}{2}\int V^2(x)\}$
$\int D[X]\, e^{iS_0[X]+}$
$\int D[X]\, e^{iS_0}$
$\int D[V]\, e^{-\frac{f}{2}\int V^2}$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I. Replica trick
S = S0[X] + ∫ V(x) O(x)
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∫ D[x] e^{iS0[x]+
∫ D[x] e^{iS0
⟨O1 . . . On⟩ = lim n→0
∫ D[V] e^{-f/2 ∫ V²} ∫ D[x

Problem. Ling-Yan Hung, T.S. ...

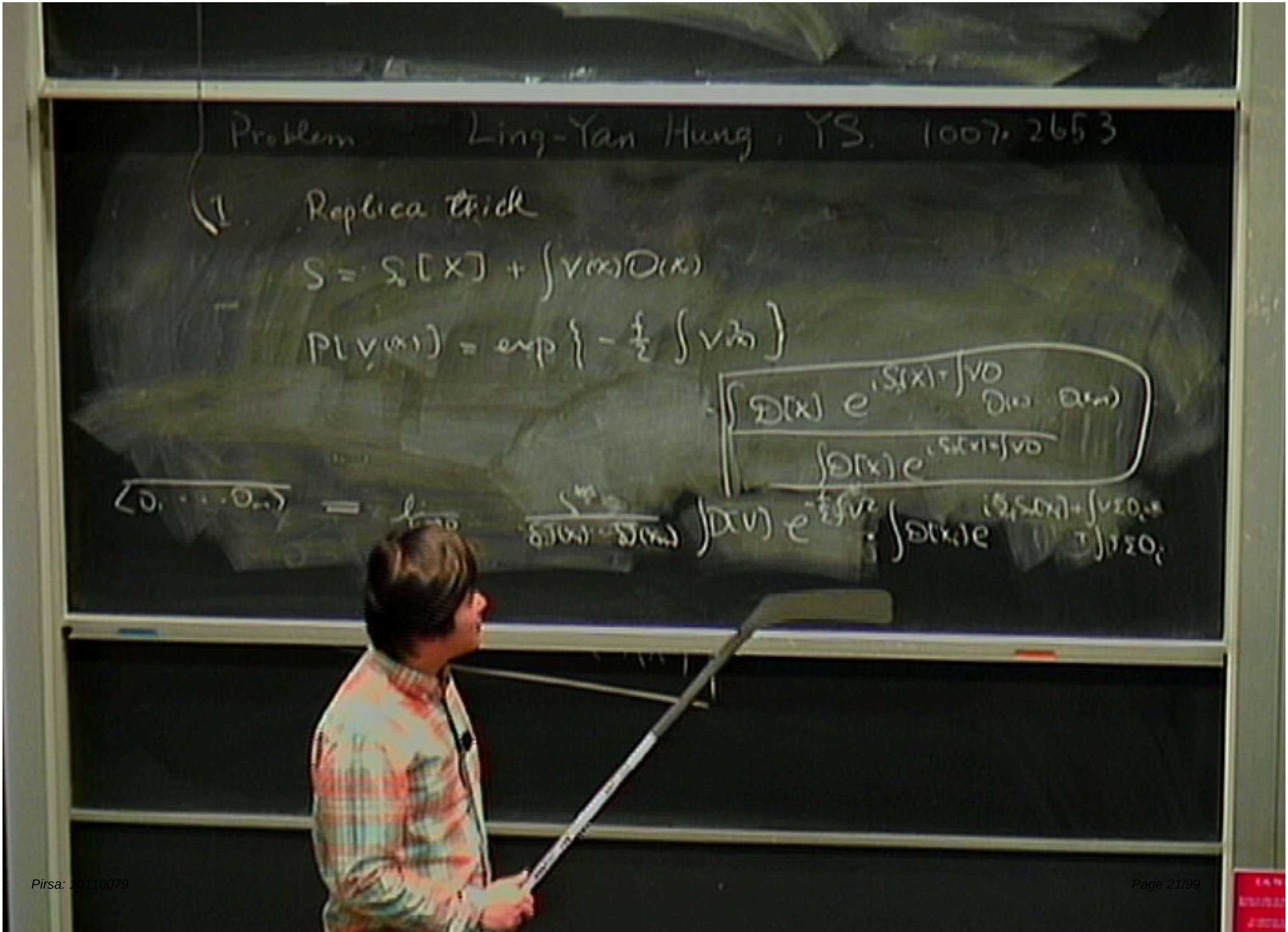
I. Replica trick

$$S = S_0[X] + \int V(x) O(x)$$

$$P[V(x)] = \exp\left\{ -\frac{f}{2} \int V^2(x) \right\}$$

$$\frac{\int D[X]\, e^{iS_0[X] + \dots}}{\int D[X]\, e^{iS_0 \dots}}$$

$$\overline{\langle O_1 \cdots O_m \rangle} = \lim_{n \to 0} \frac{\delta^m}{\delta J(x_1) \cdots \delta J(x_m)} \int D[V]\, e^{-\frac{f}{2}\int V^2} \int D[X_a \dots$$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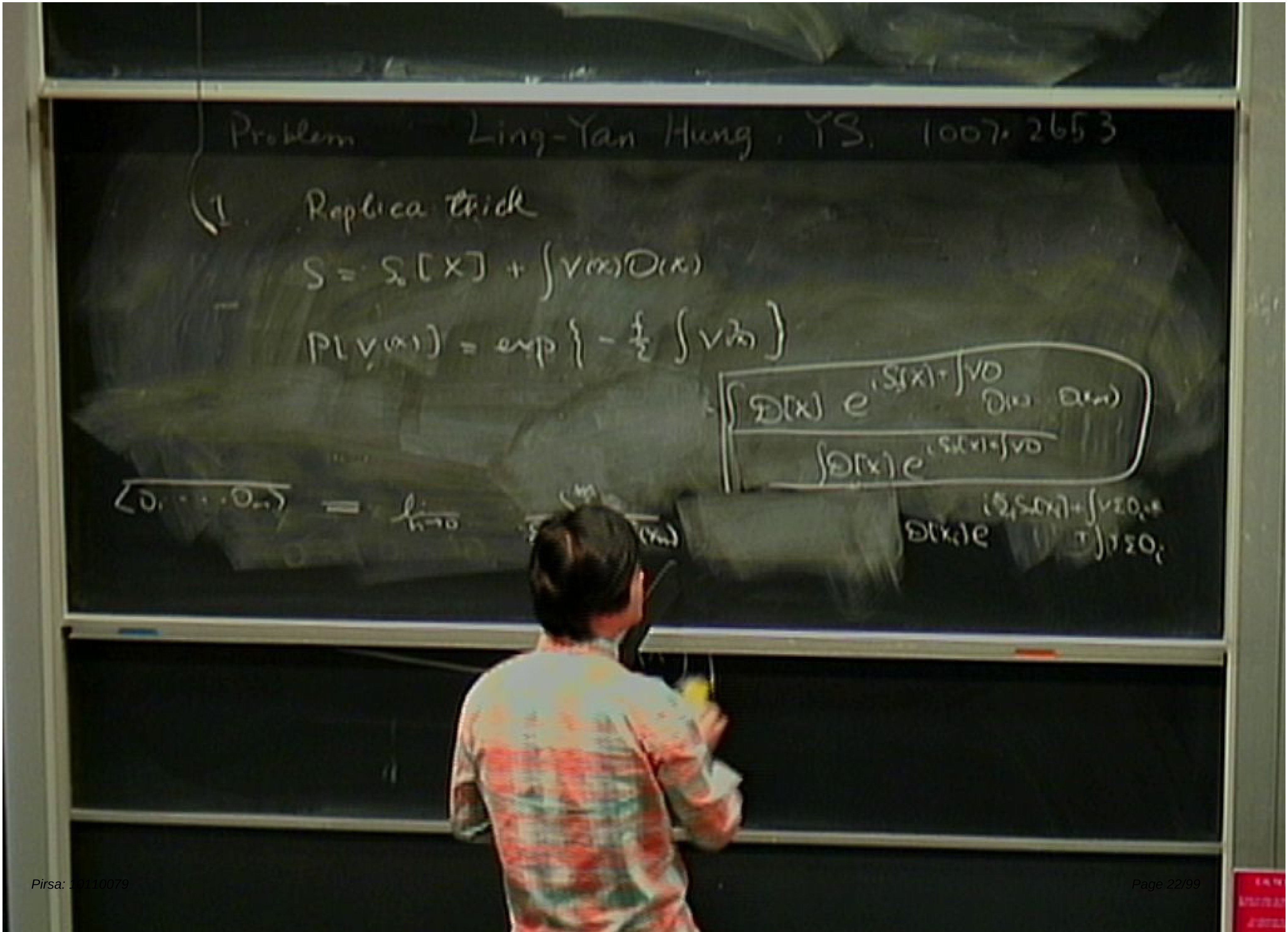
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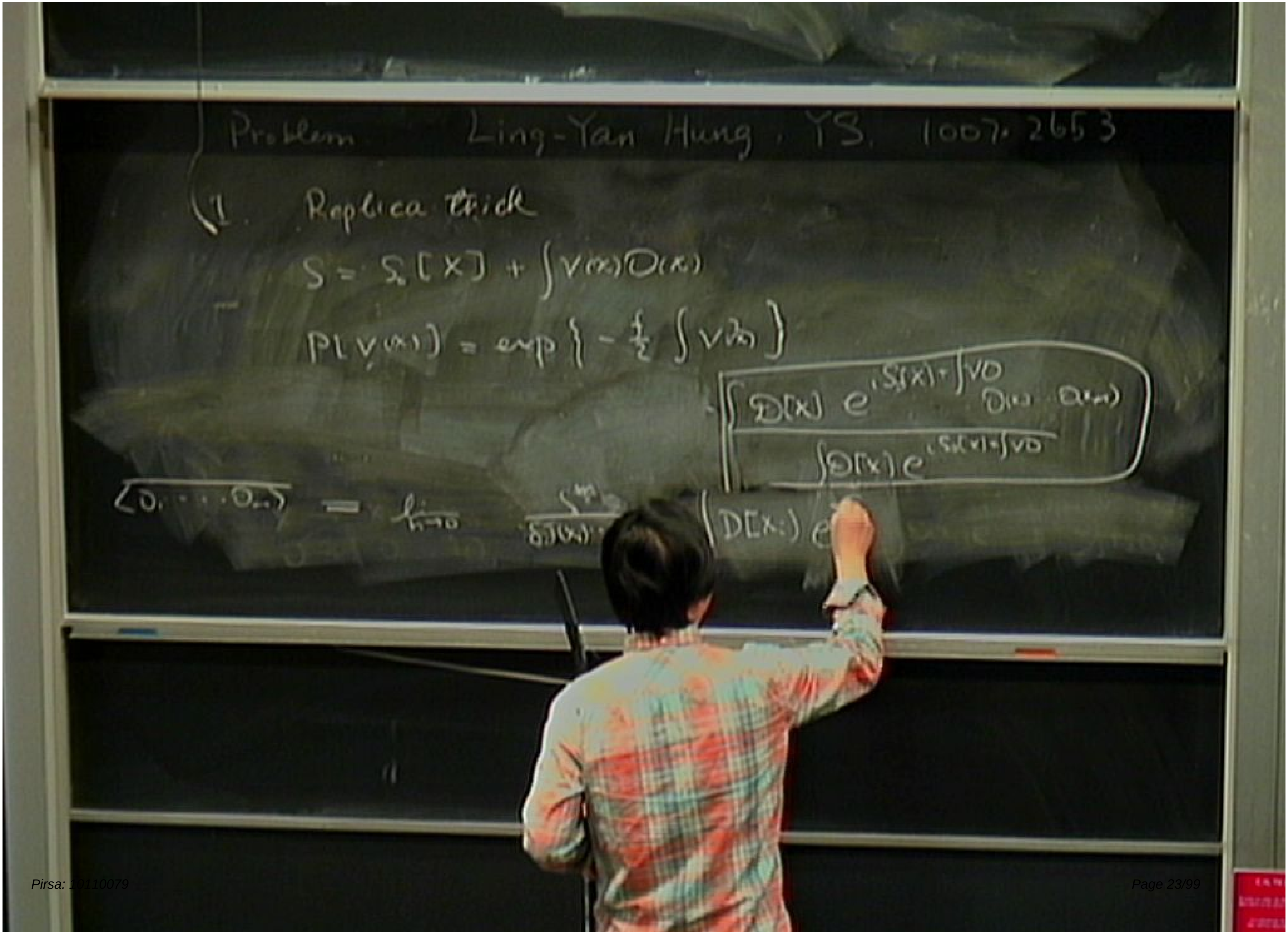
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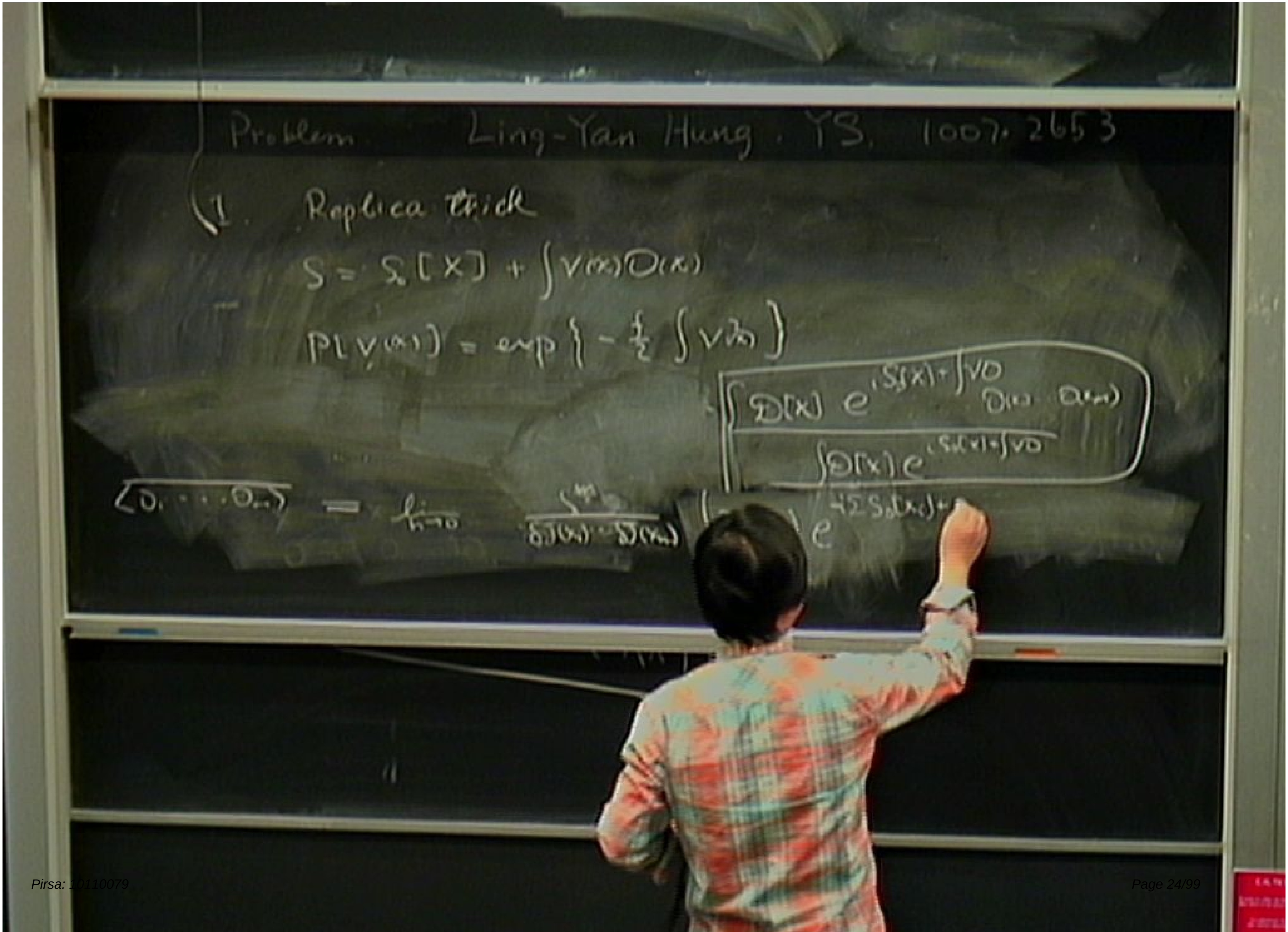
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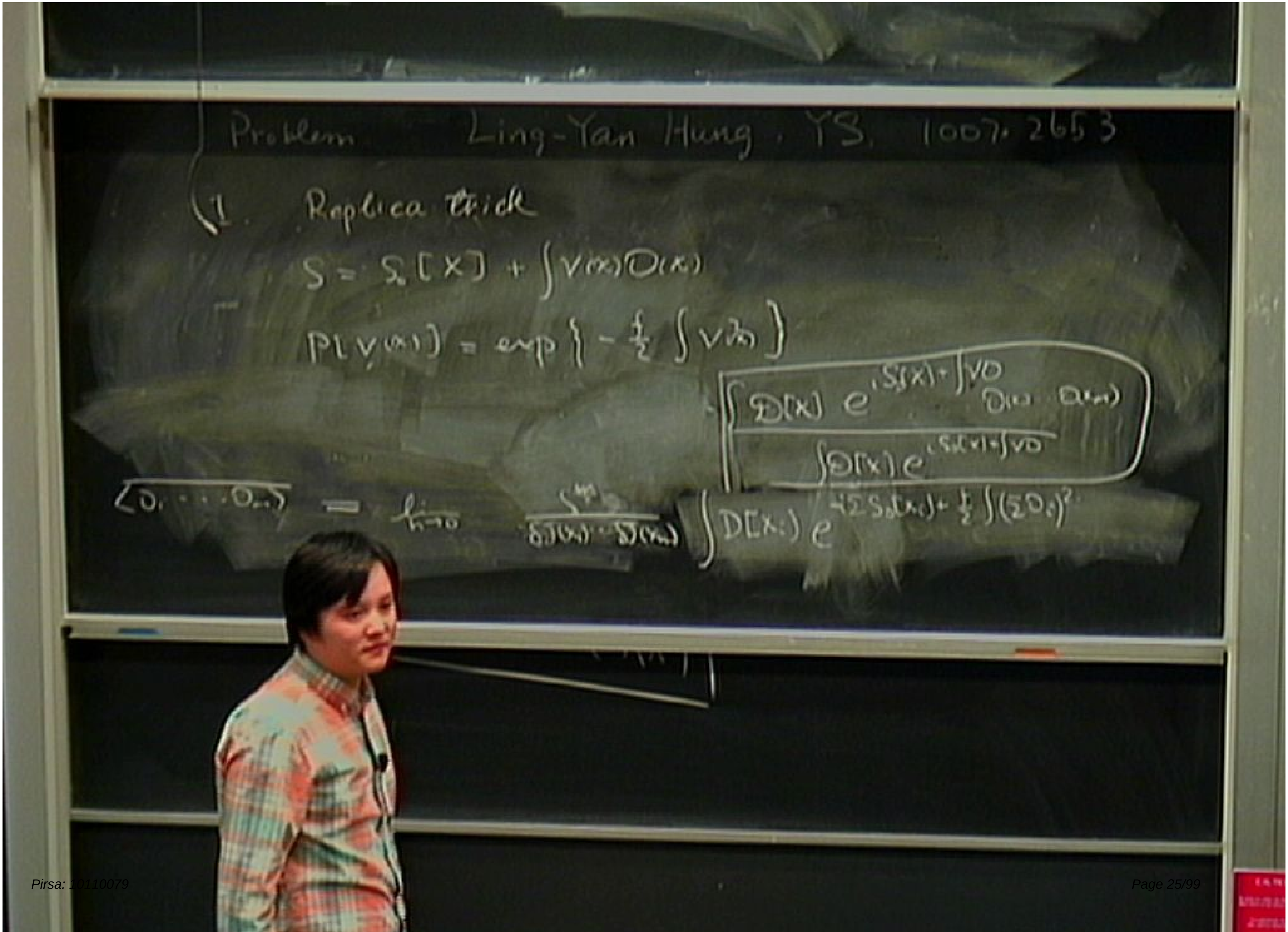
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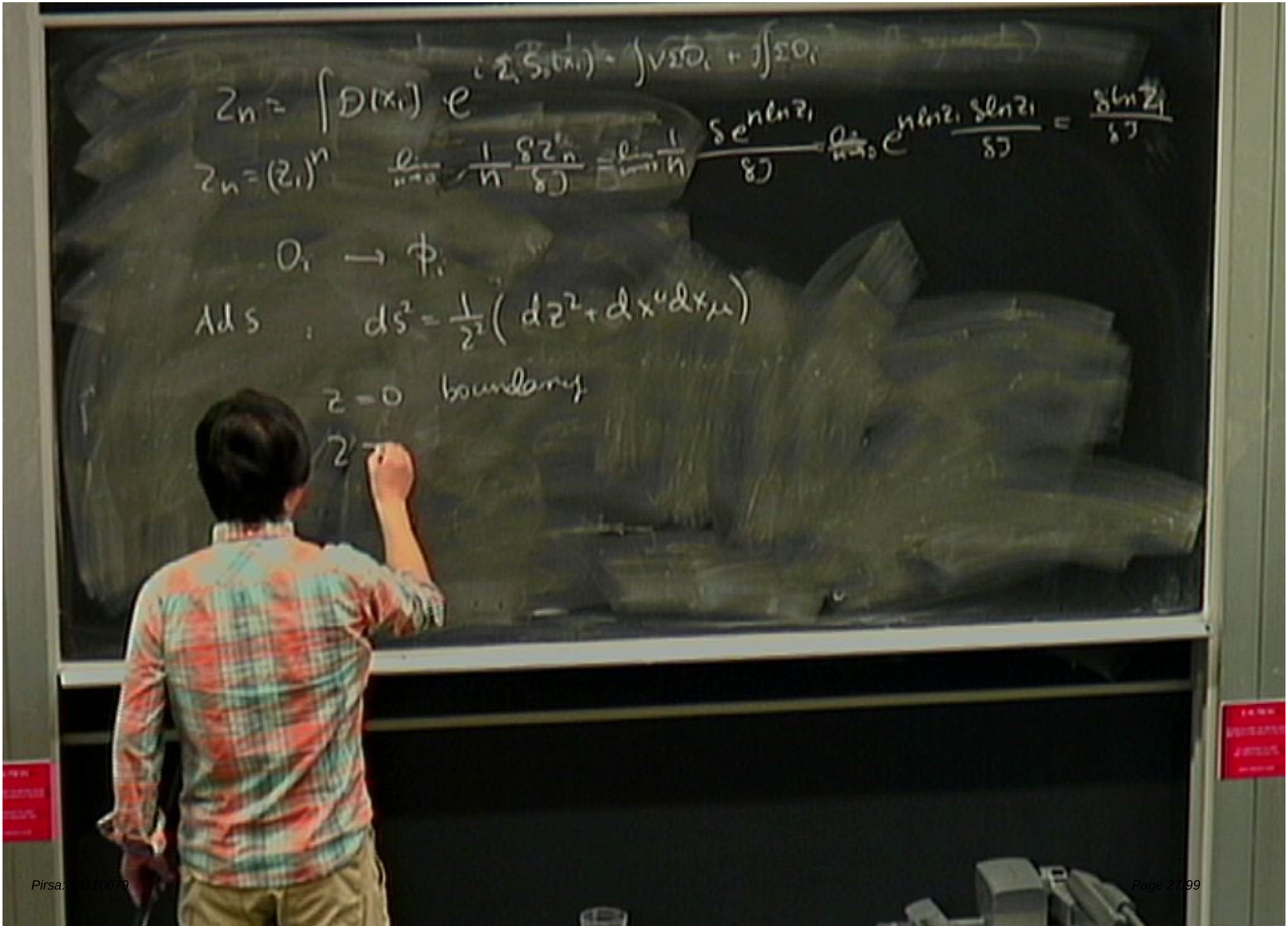
1. Replica trick

$$S = S_0[X] + \int V(x)\mathcal{O}(x)$$

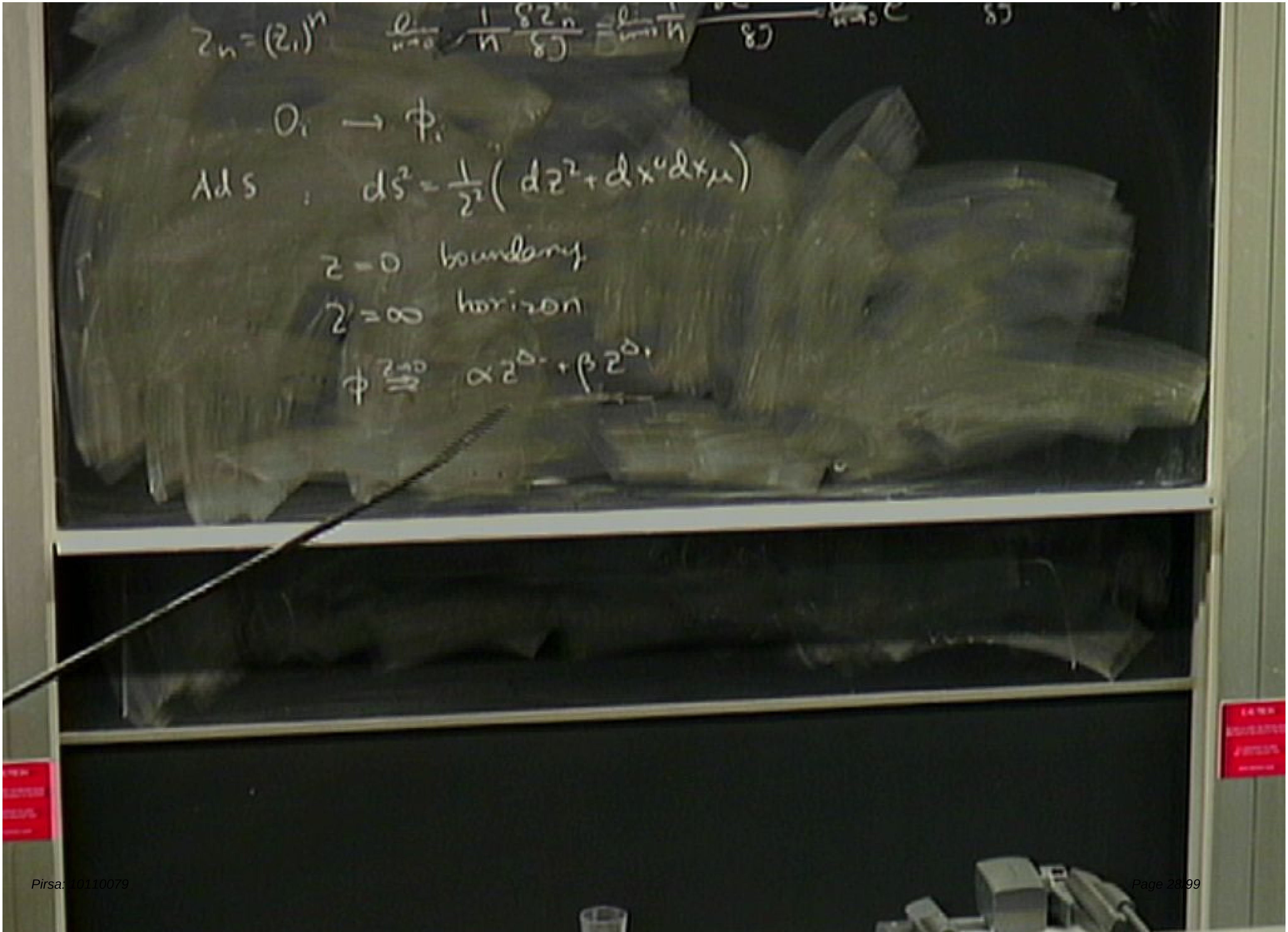
$$P[V(x)] = \exp\left\{-\frac{1}{2}\int V^2(x)\right\}$$

$$\frac{\int \mathcal{D}[X]\, e^{iS_0[X]+\int V\mathcal{O}}\, \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)}{\int \mathcal{D}[X]\, e^{iS_0[X]+\int V\mathcal{O}}}$$

$$\overline{\langle \mathcal{O}_1 \cdots \mathcal{O}_n\rangle} = \lim_{n\to 0} \frac{\delta^n}{\delta J(x_1)\cdots\delta J(x_n)} \int \mathcal{D}[X_i]\, e^{i\sum S_0[X_i] + \frac{1}{2}\int\left(\sum \mathcal{O}_i\right)^2}$$



Z_n = ∫ D(x_i) e^{i Σ_i S_i(x_i) - ∫ V Σ O_i + J ∫ Σ O_i}
Z_n = (Z_1)^n
lim_{n→0} (1/n) δZ_n/δJ = lim_{n→0} (1/n) δe^{n ln Z_1}/δJ → lim_{n→0} e^{n ln Z_1} δ ln Z_1/δJ = δ ln Z_1/δJ
O_i → φ_i
AdS : ds^2 = (1/z^2)(dz^2 + dx^μ dx_μ)
z = 0 boundary


$z_n = (z_1)^n$
$O_i \rightarrow \phi_i$
AdS : $ds^2 = \frac{1}{z^2}(dz^2 + dx^\mu dx_\mu)$
$z = 0$ boundary
$z = \infty$ horizon
$\phi \xrightarrow{z \to 0} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$

$O_i \rightarrow \phi_i$

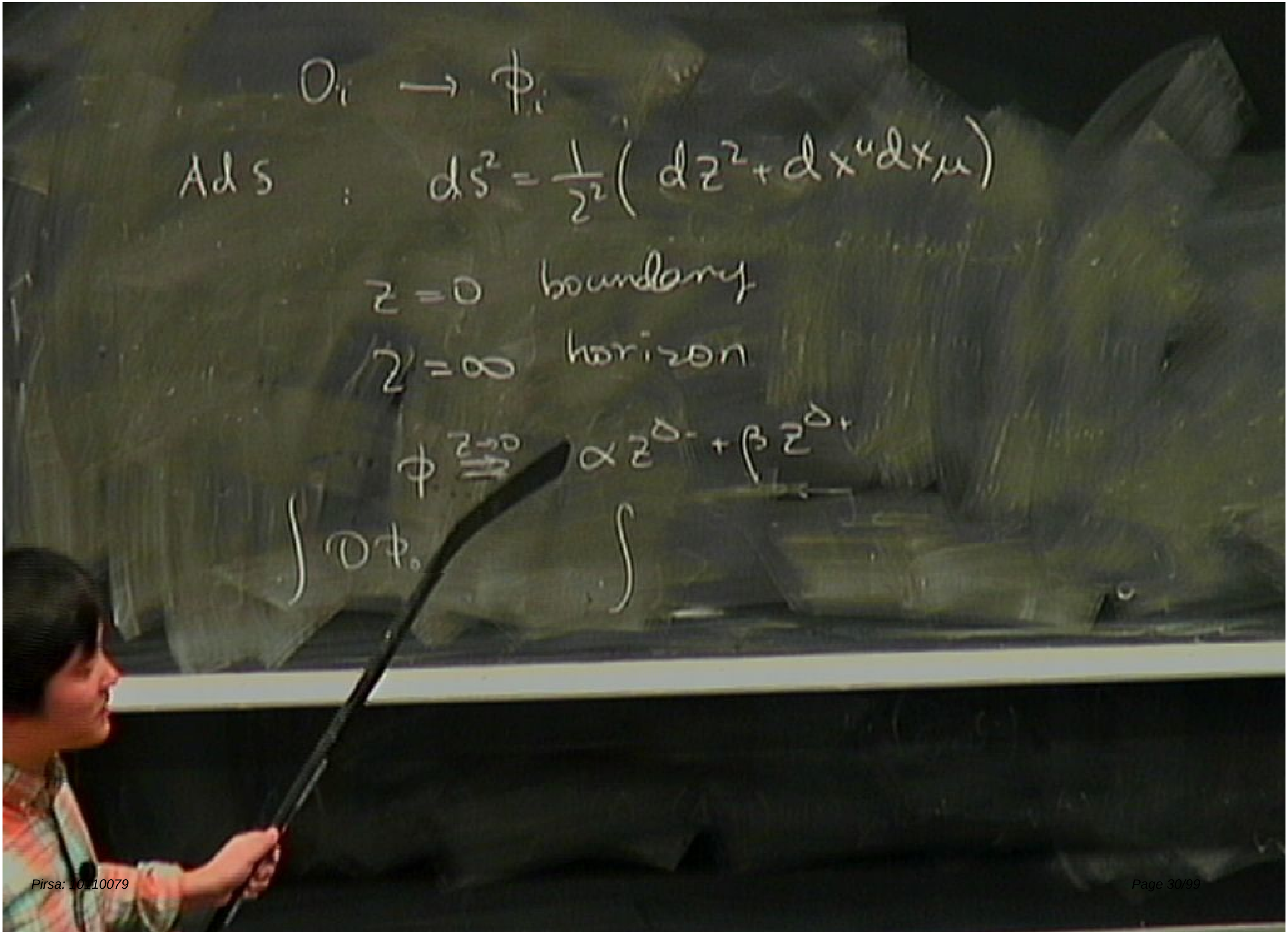
AdS : $ds^2 = \frac{1}{z^2}\left( dz^2 + dx^\mu dx_\mu \right)$

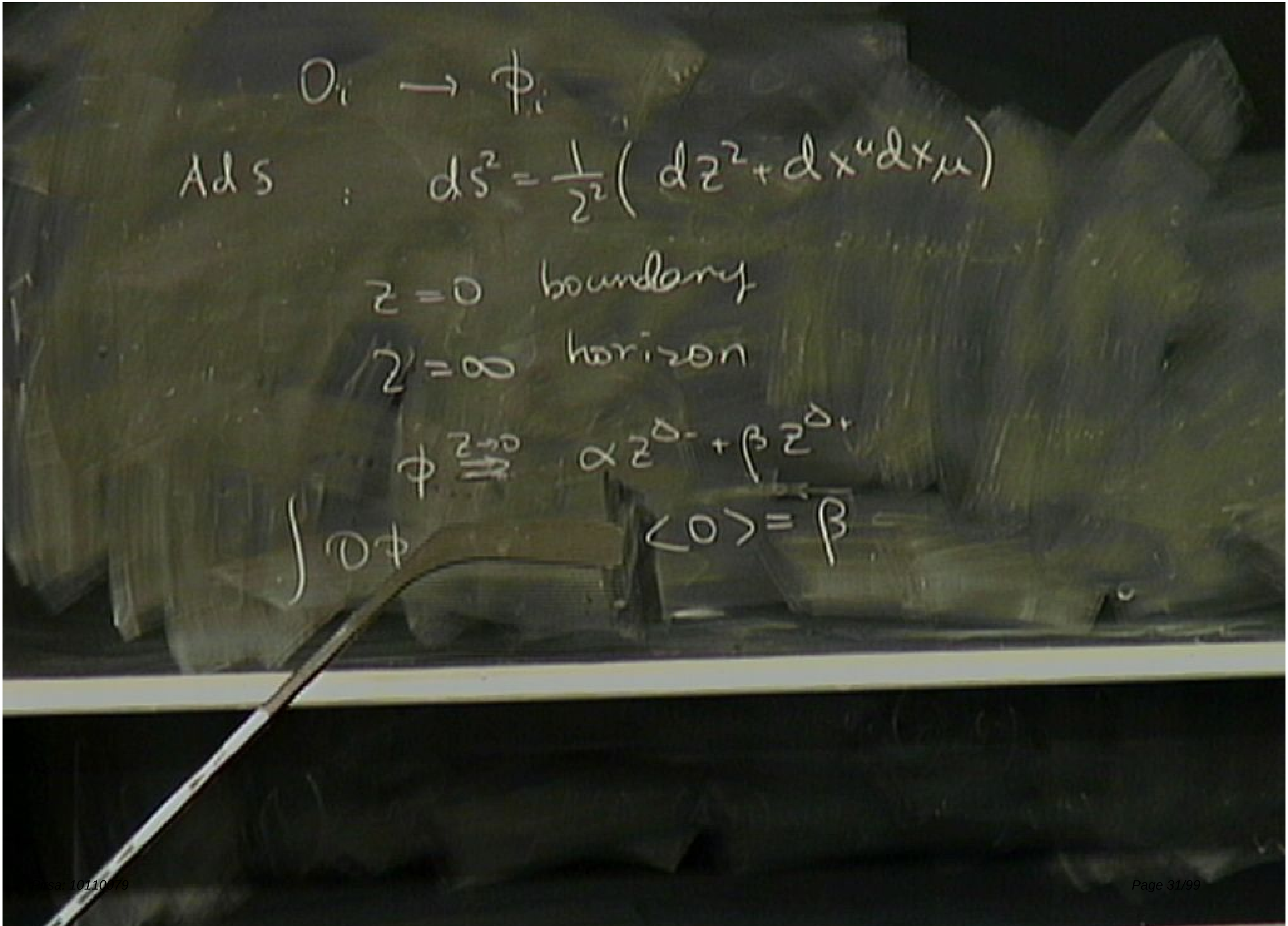
$z = 0$ boundary

$z = \infty$ horizon

$\phi \overset{z \to 0}{\Longrightarrow} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$

$\int \mathcal{D}\phi_0$


O_i → φ_i
AdS : ds² = 1/z² ( dz² + dx^μ dx_μ )
z = 0 boundary
z = ∞ horizon
φ z→0 α z^Δ- + β z^Δ+
∫ Dφ_0
∫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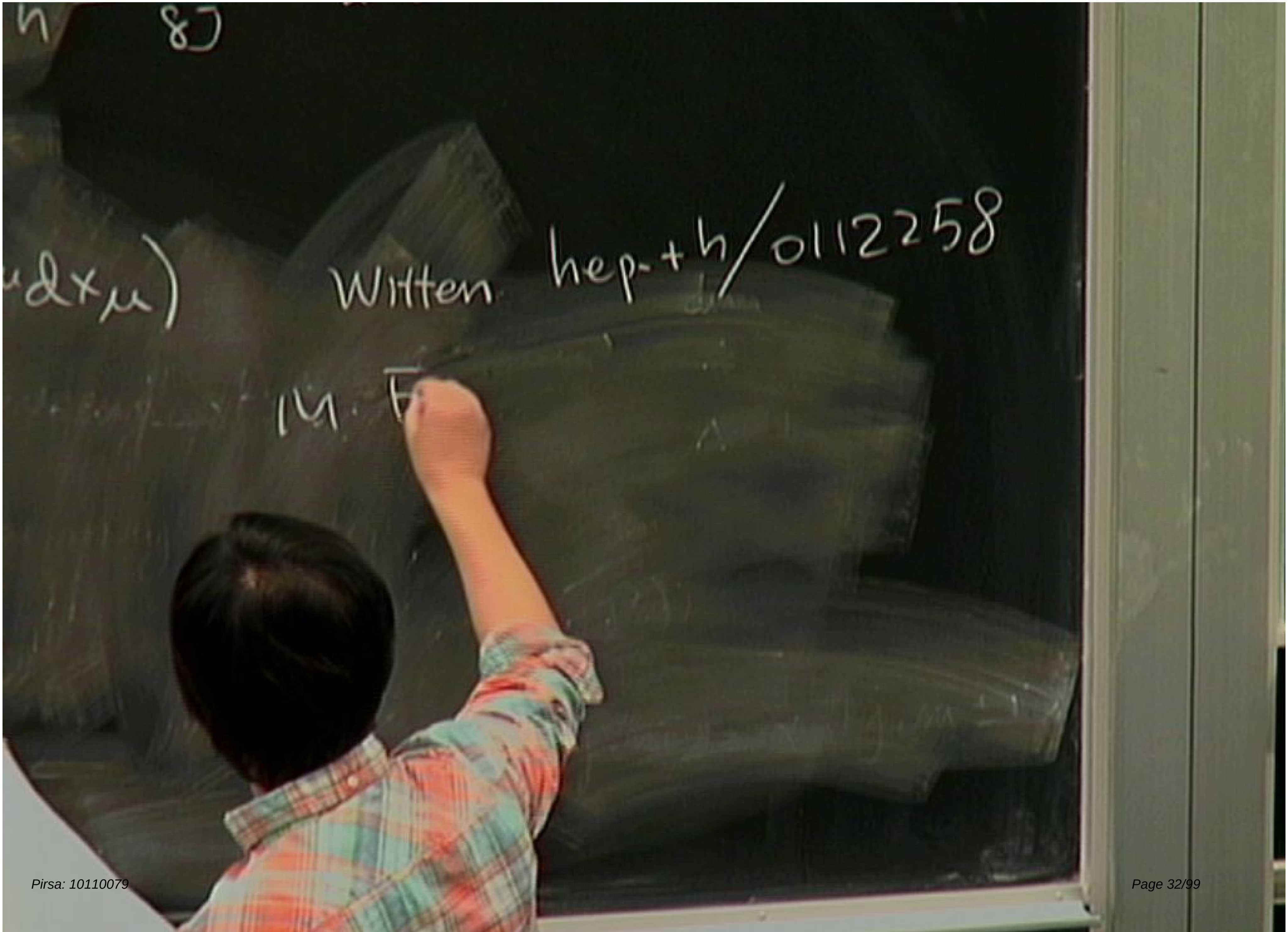
$\phi \xrightarrow{z \to 0} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$
$\int \mathcal{D}\phi$
$\langle O \rangle = \beta$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$$\frac{1}{n}\frac{\delta e^{n\ln z_1}}{\delta J} = \lim_{n\to 0} e^{n\ln z_1}\frac{\delta \ln z_1}{\delta J} = \frac{\delta \ln z_1}{\delta J}$$

$d x_\mu)$

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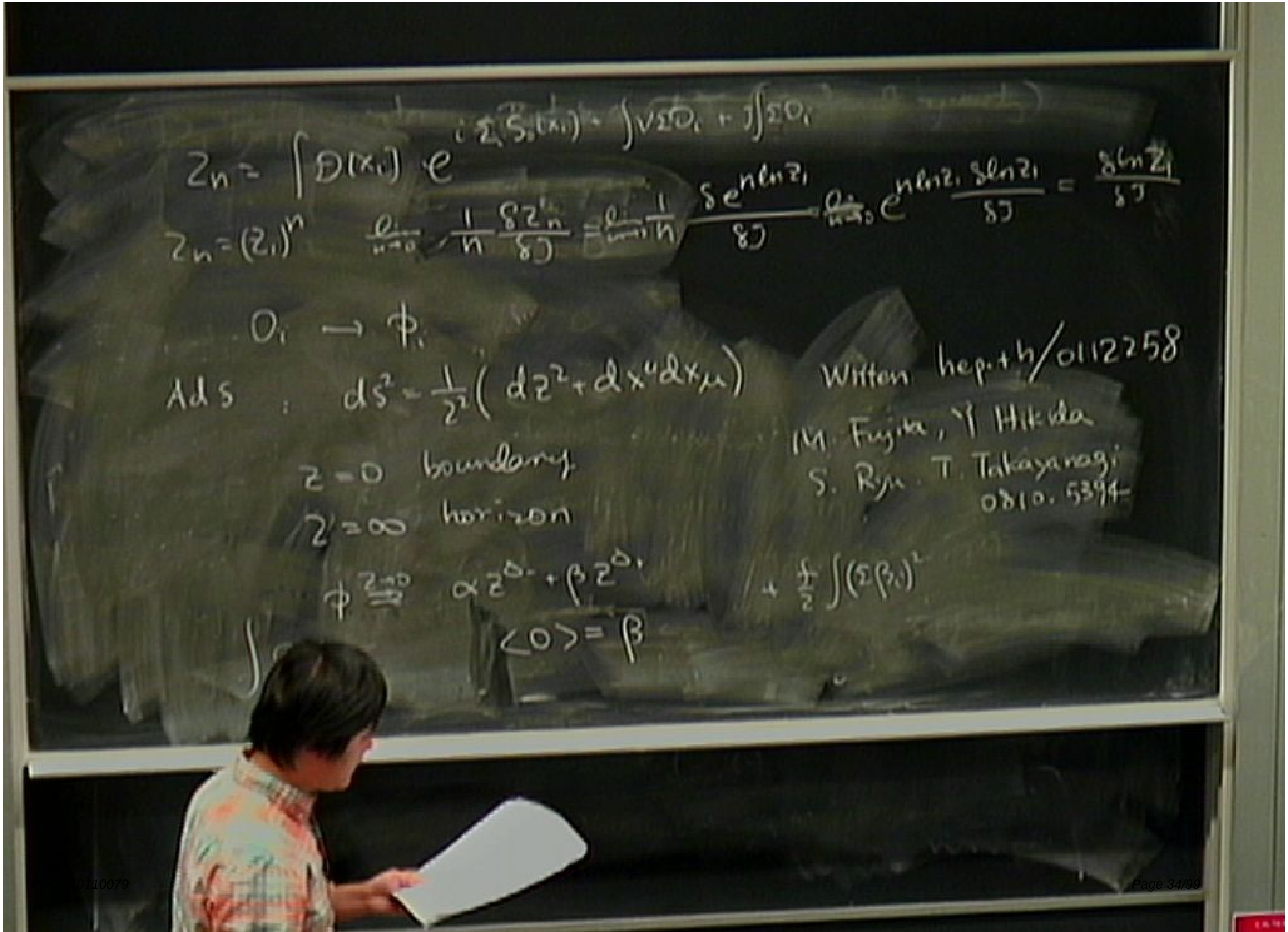
M. Fujita, Y. Hikida
S. Ryu, T. Takayanagi
0810.5394


O_i → φ_i
AdS : ds² = 1/z² ( dz² + dx^μ dx_μ )
z = 0 boundary
z = ∞ horizon
⟨O⟩ = β
Witten hep-th/0112258
M. Fujita, Y. Hikida
S. Ryu, T. Takayanagi
0810.5394

$Z_n = (Z_1)^n$ $\lim_{n\to 0} \frac{1}{n}\frac{\delta Z_n}{\delta J}$ $\lim \frac{1}{n}$ $\frac{\delta e}{\delta J}$ $\lim_{n\to 0} e$

$O_i \to \phi_i$

AdS : $ds^2 = \frac{1}{z^2}(dz^2 + dx^\mu dx_\mu)$

$z = 0$ boundary

$z = \infty$ horizon

$\phi \xrightarrow{z\to 0} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$

$\int D\phi_i$ $\begin{cases} \langle O \rangle = \beta \\ \alpha = 0 \end{cases}$

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S. Ryu T. Takayanagi

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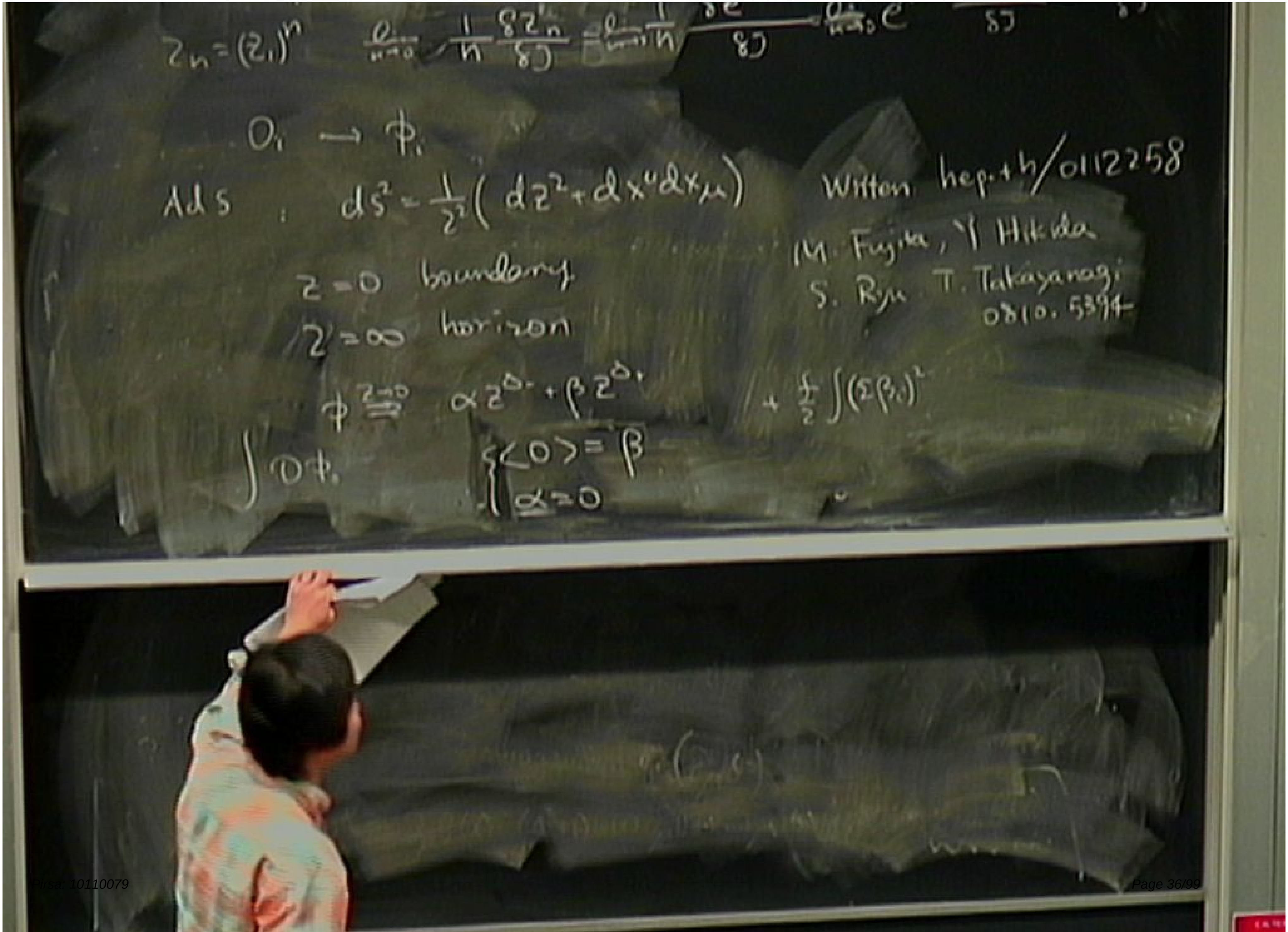
$+ \frac{f}{2}\int (\Sigma \beta_i)^2$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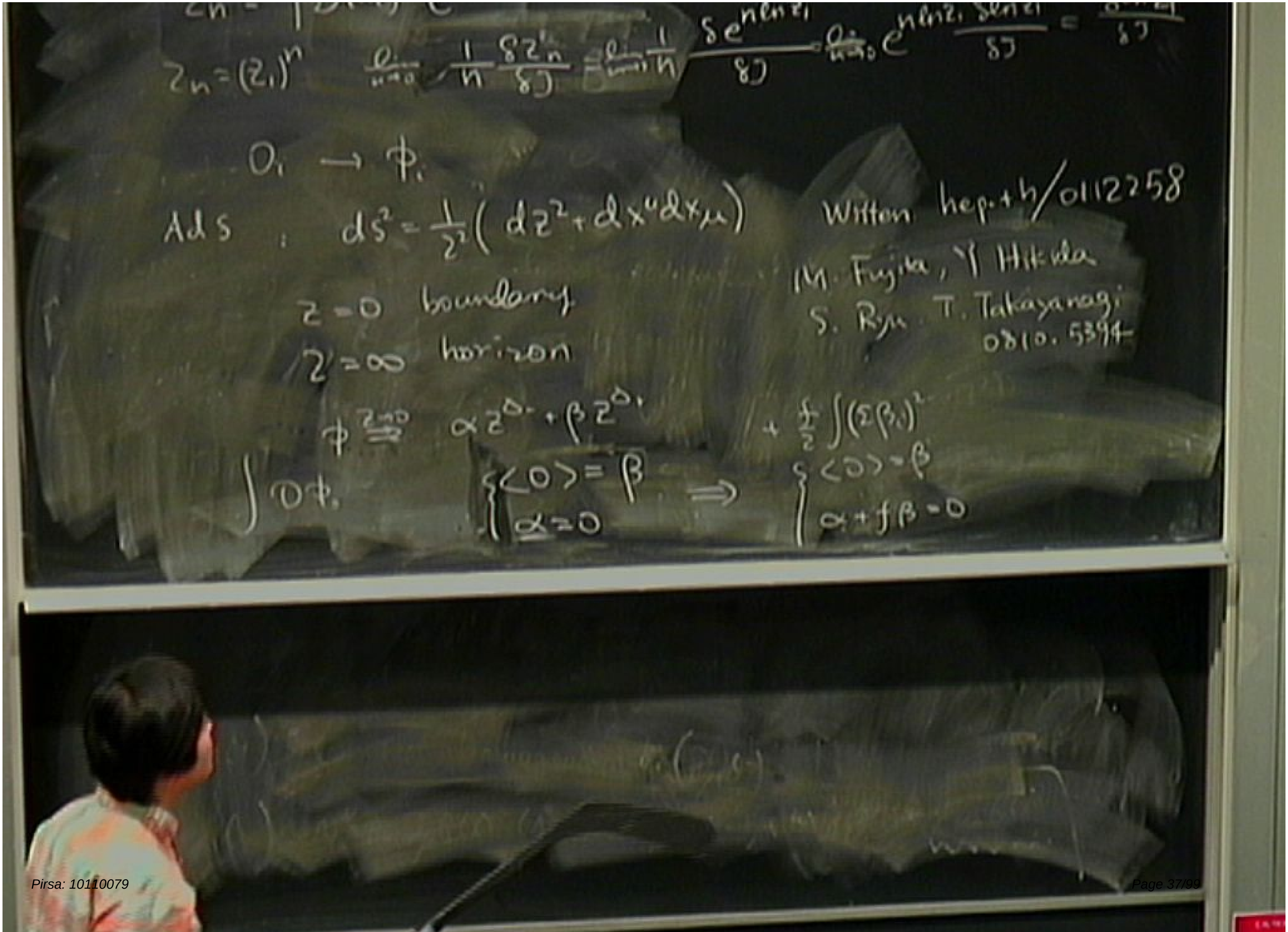
⟨O⟩ = β
α = 0


$Z_n = (Z_1)^n$
$\lim_{n\to 0} \frac{1}{n} \frac{\delta Z_n}{\delta J} = \lim_{n\to 0} \frac{1}{n} \frac{\delta e^{n \ln Z_1}}{\delta J} = \lim_{n\to 0} e^{n \ln Z_1} \frac{\delta \ln Z_1}{\delta J}$
$O_i \to \phi_i$
AdS : $ds^2 = \frac{1}{z^2}(dz^2 + dx^\mu dx_\mu)$
$z = 0$ boundary
$z = \infty$ horizon
$\phi \overset{z\to 0}{\Longrightarrow} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$
$\int D\phi_i$
$\langle O \rangle = \beta$, $\alpha = 0$ $\Longrightarrow$ $\langle O \rangle = \beta$, $\alpha + f\beta = 0$
$+ \frac{f}{2} \int (\Sigma \beta_i)^2$
Witten hep-th/0112258
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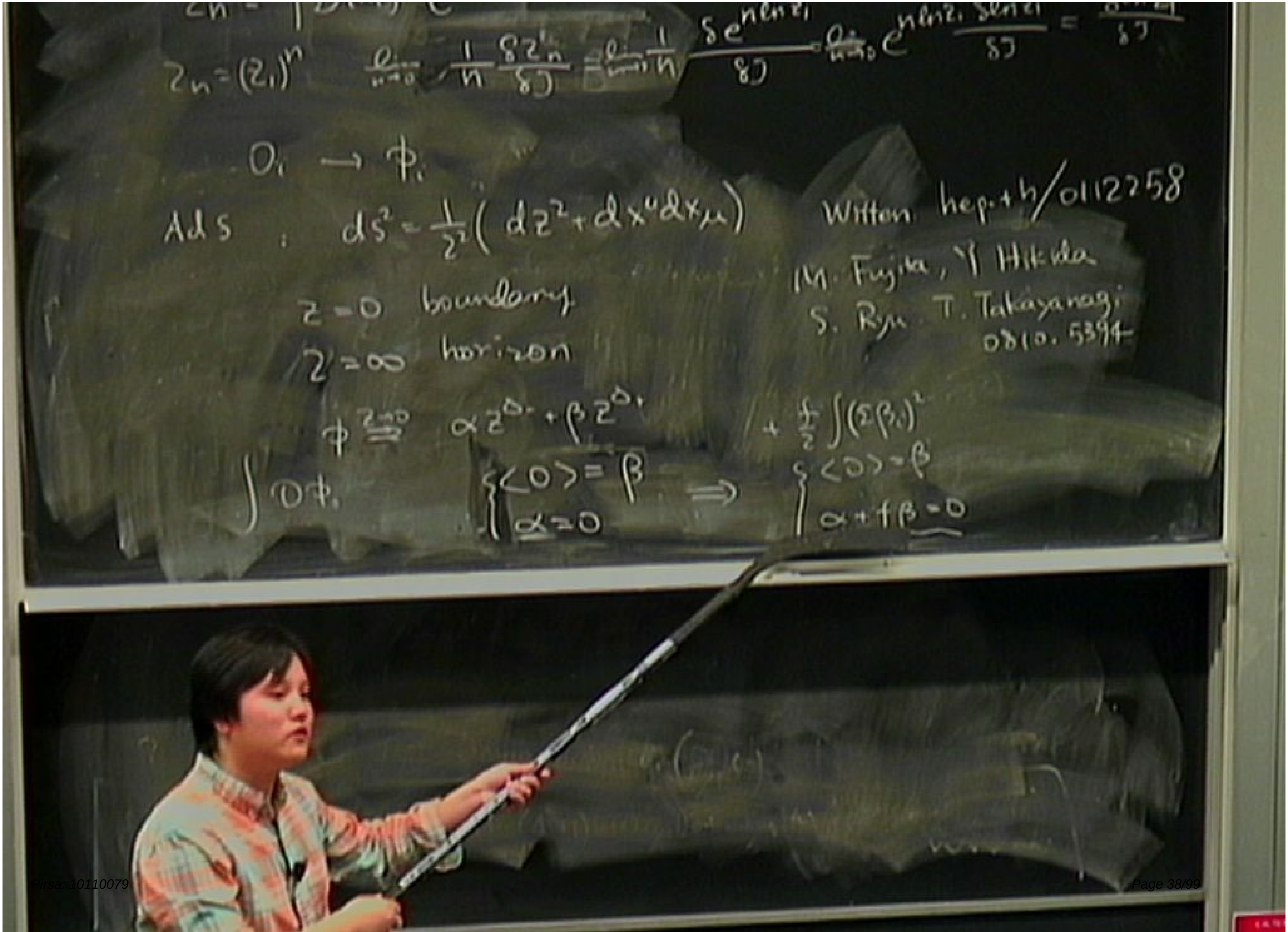
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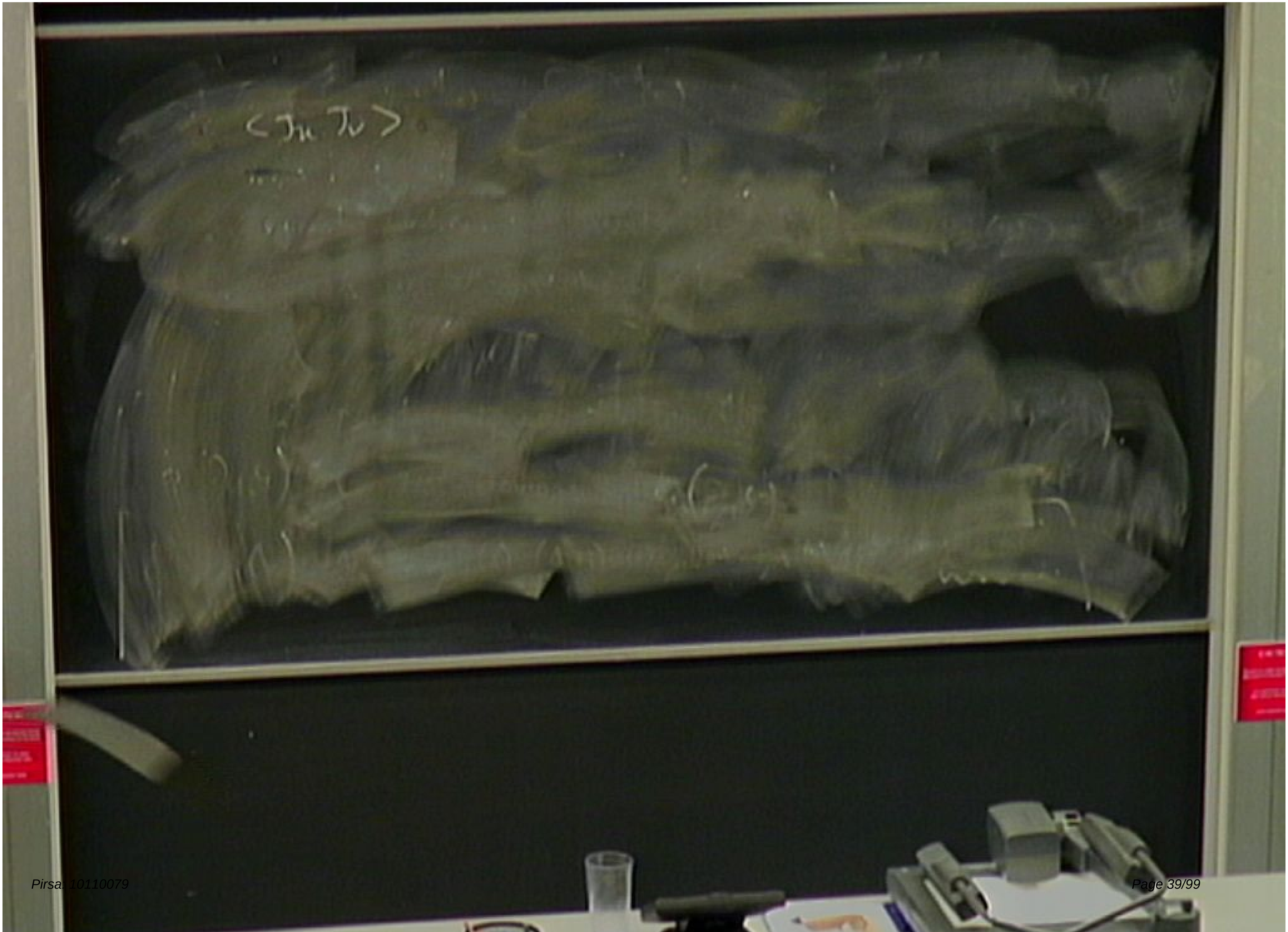
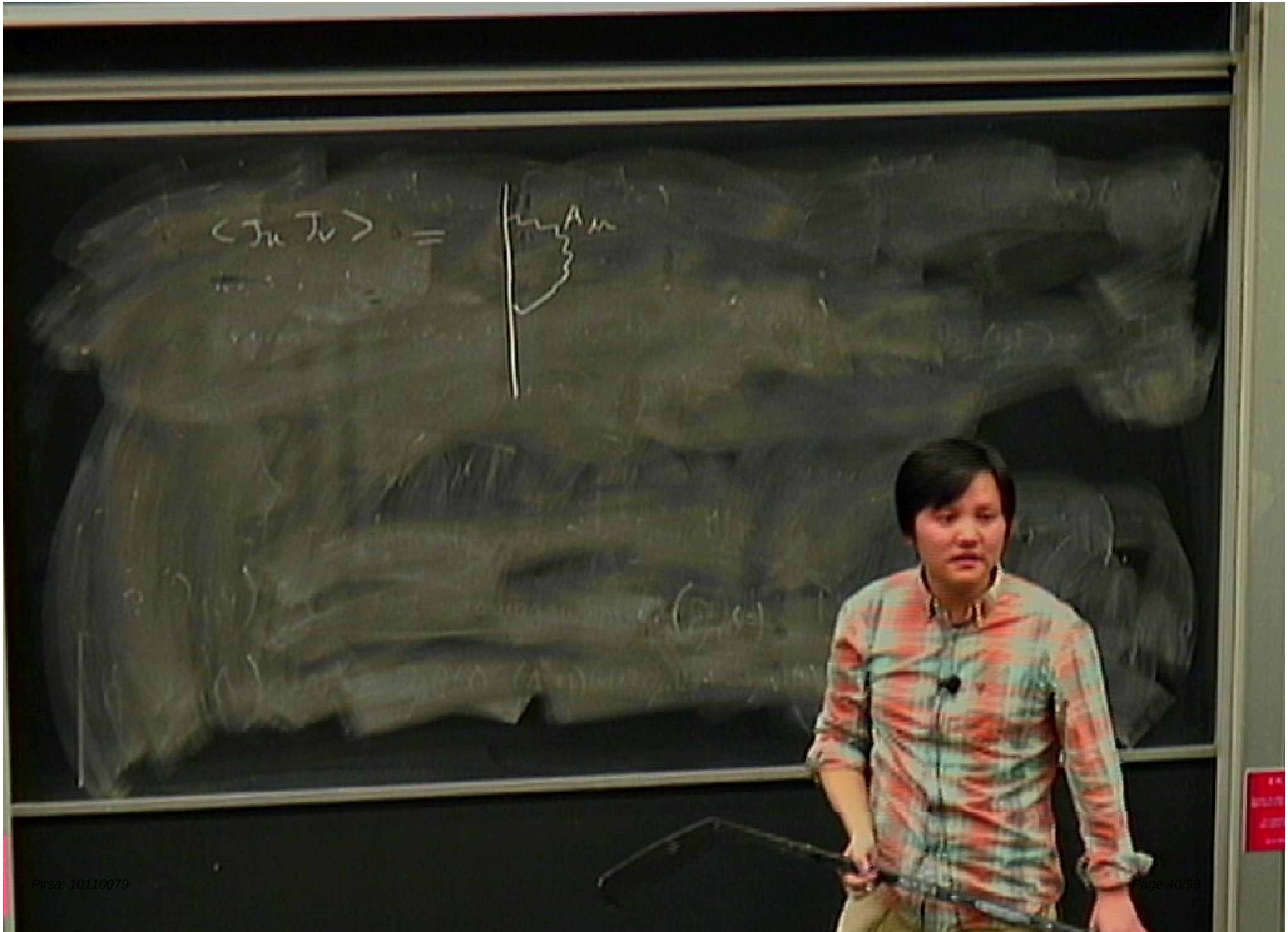
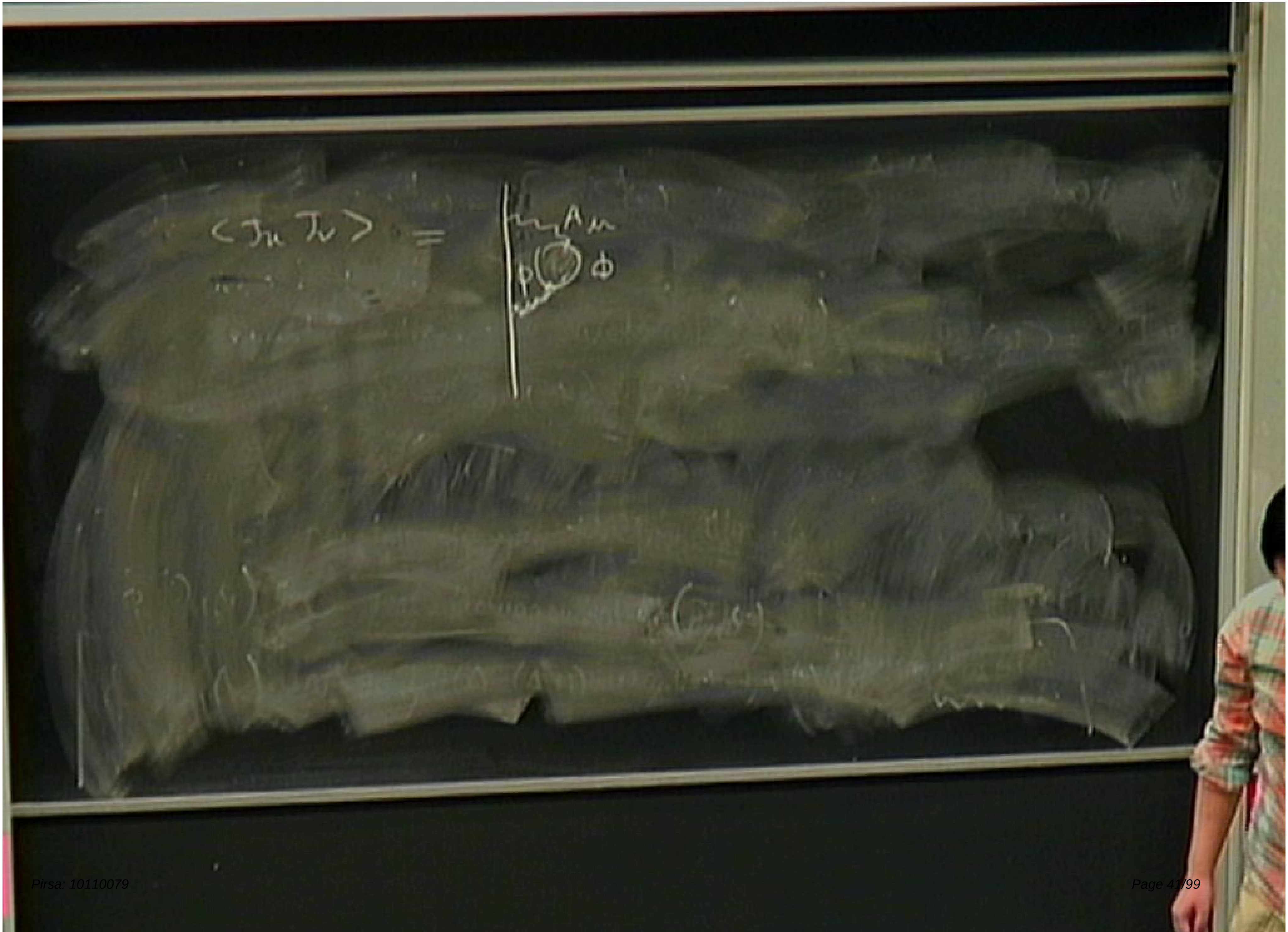
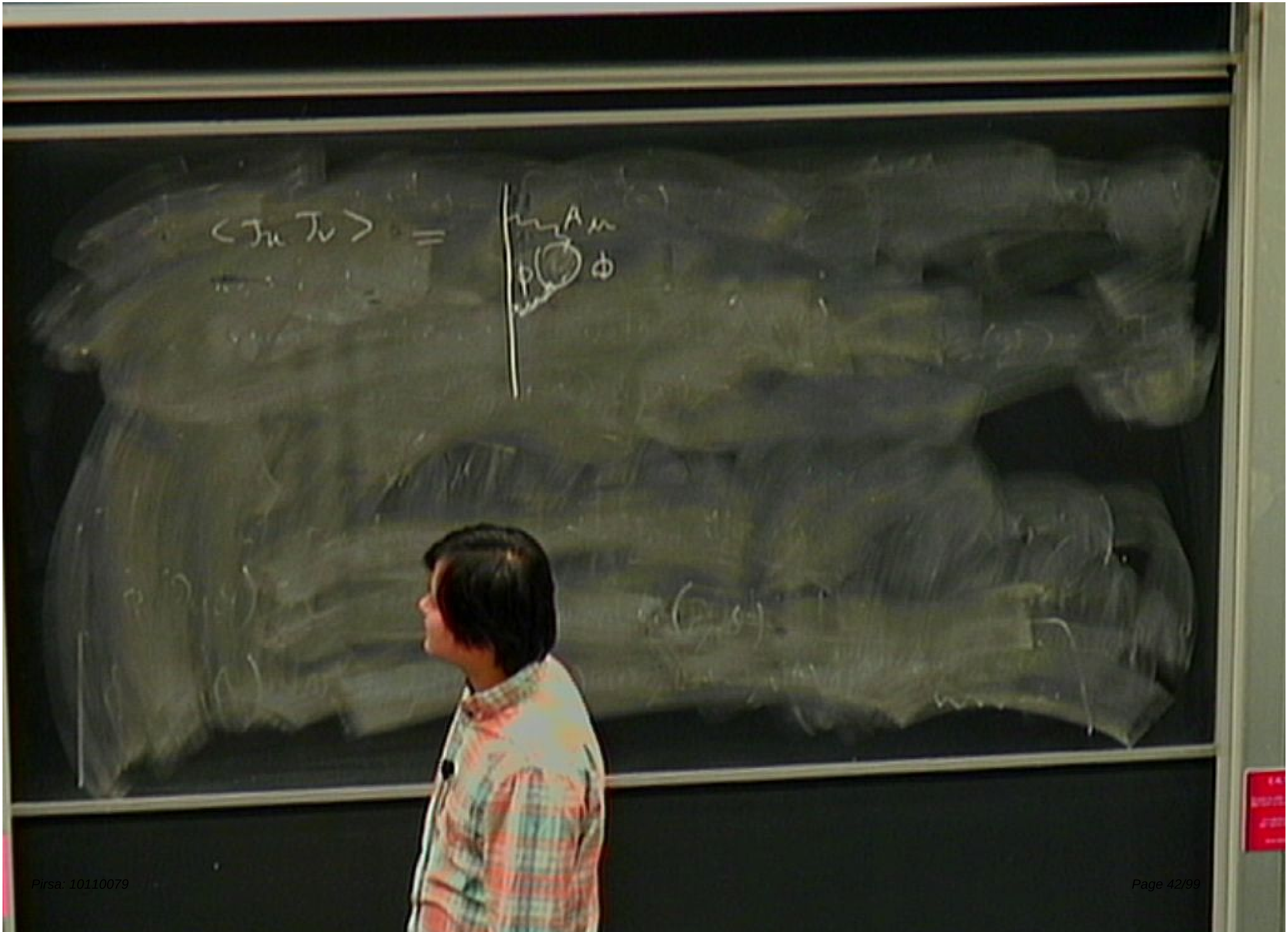
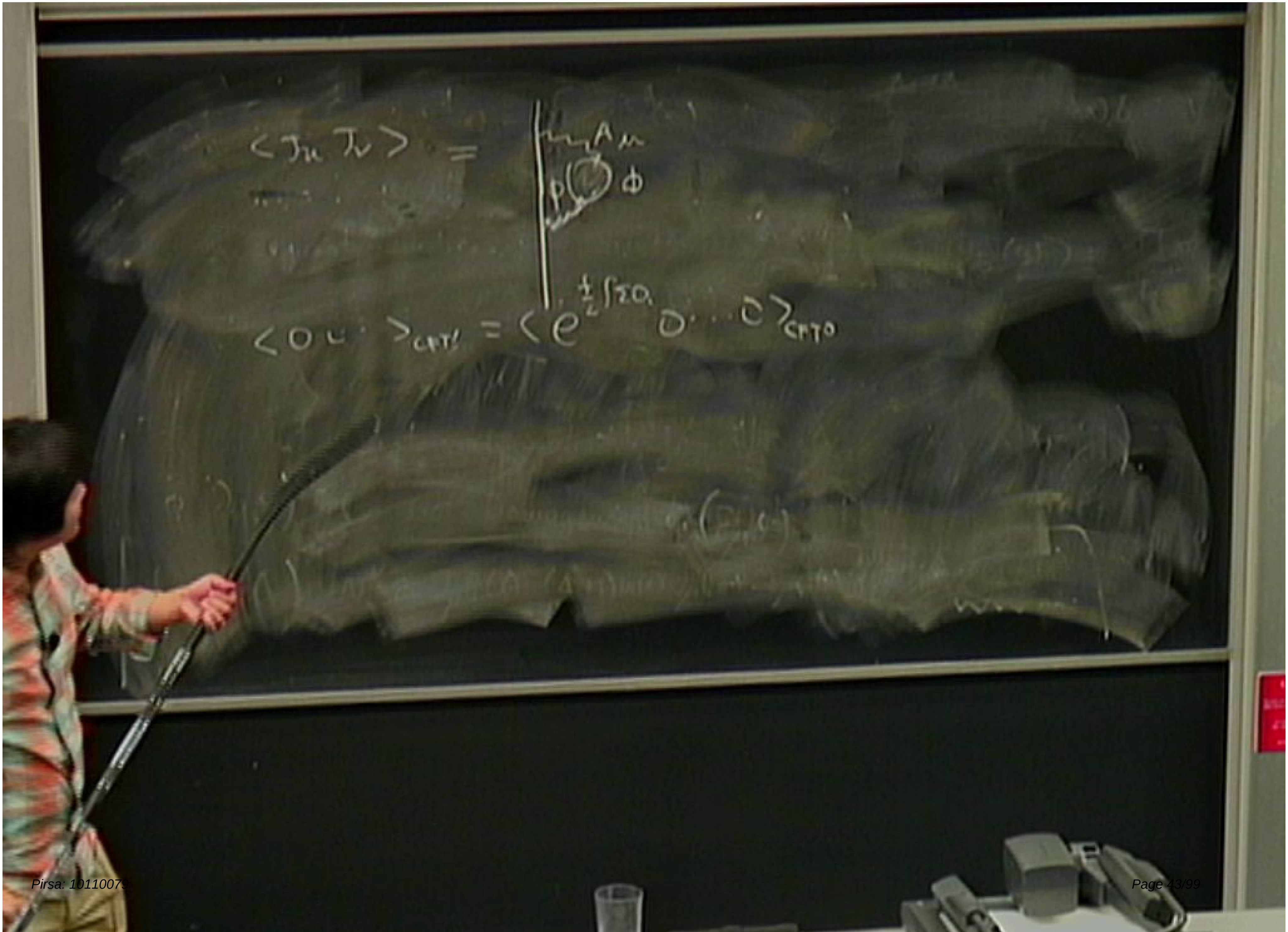
z = 0 boundary
z = ∞ horizon
∫ Dφ_i
<O> = β
α = 0
⟹
<O> = β
α + f β = 0

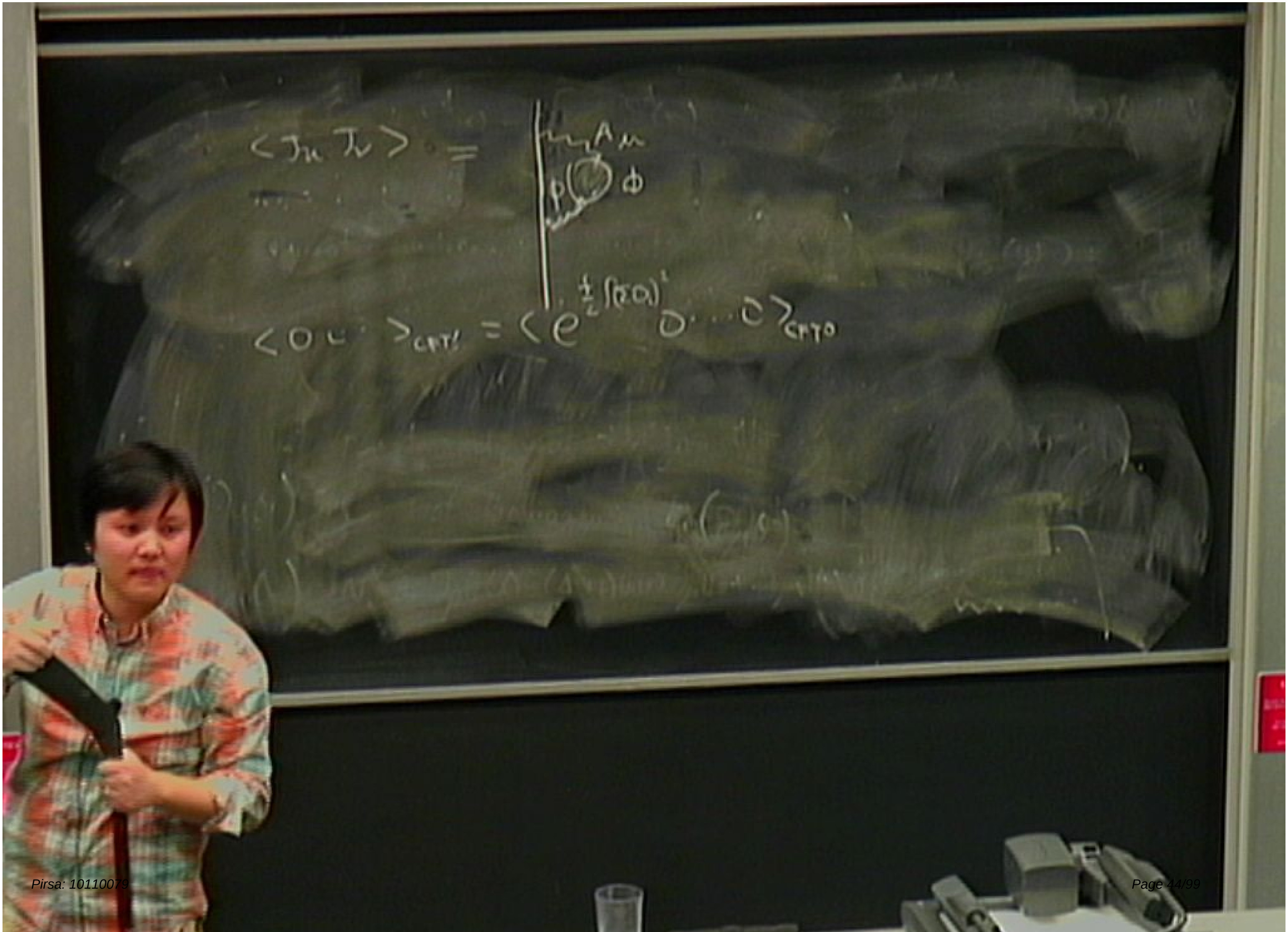


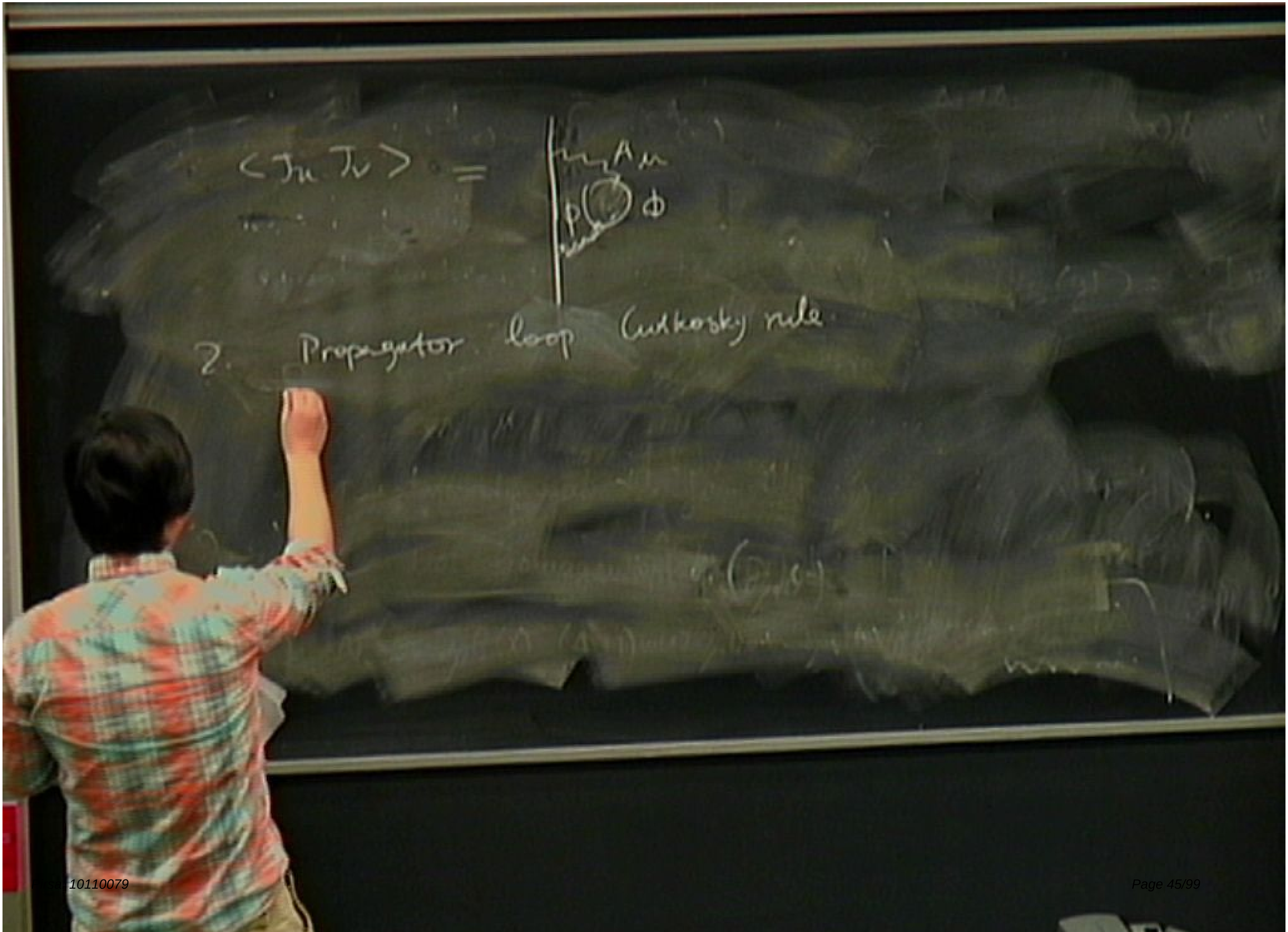

$\langle J_\mu J_\nu \rangle =$
$A_\mu$


<Jμ Jν> =
Aμ
φ

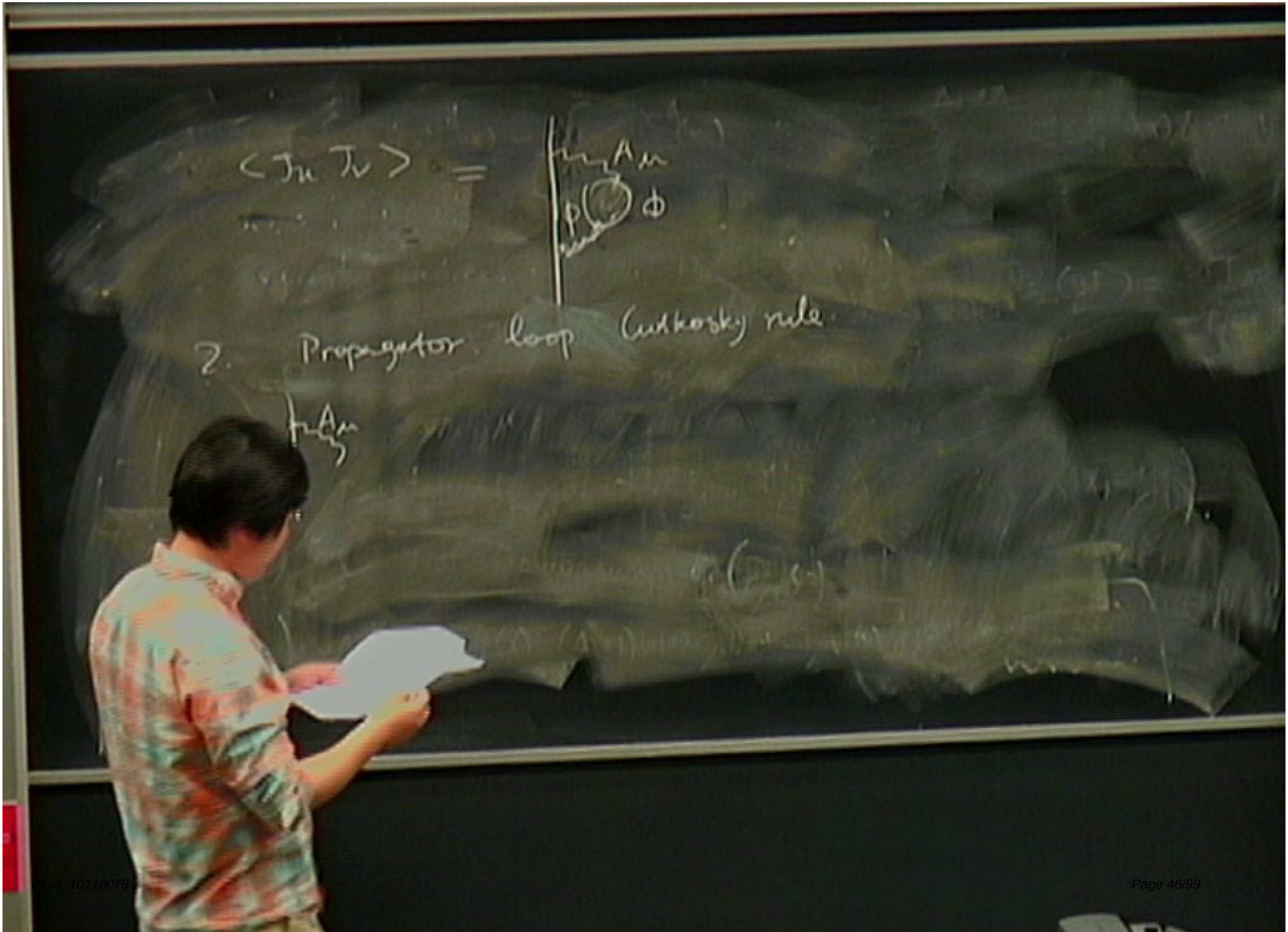

<Jμ Jν> =
Aμ


<J_μ J_ν> =
A_μ
Φ
<O … >_CFT' = <e^{½∫εO_i} O … O>_CFT0





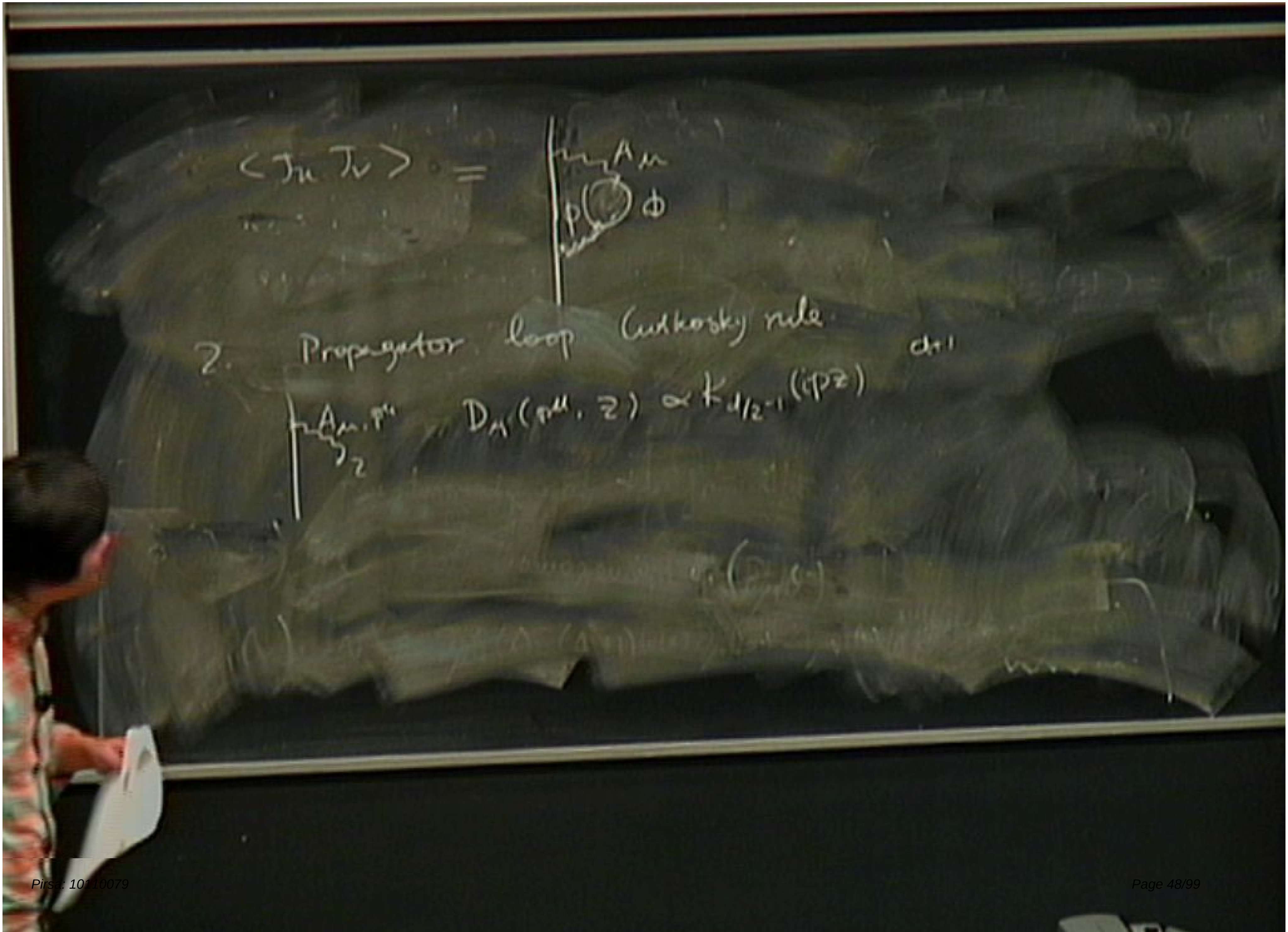
< Jμ Jν > =
Aμ
ρ
Φ
2. Propagator loop (Cutkosky rule

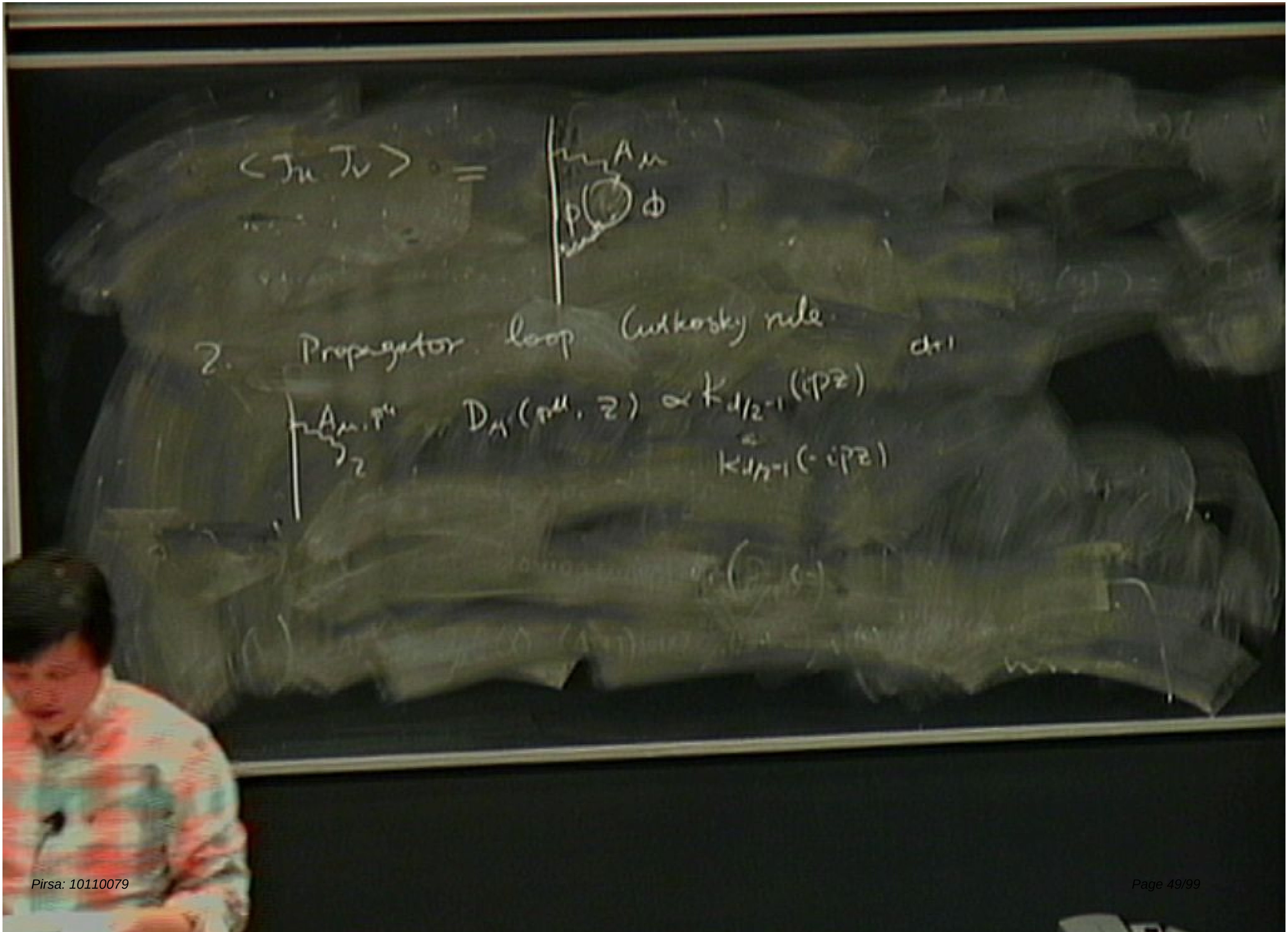

⟨J_μ J_ν⟩ =
A_μ
ρ
Φ
2. Propagator loop Cutkosky rule
A_μ

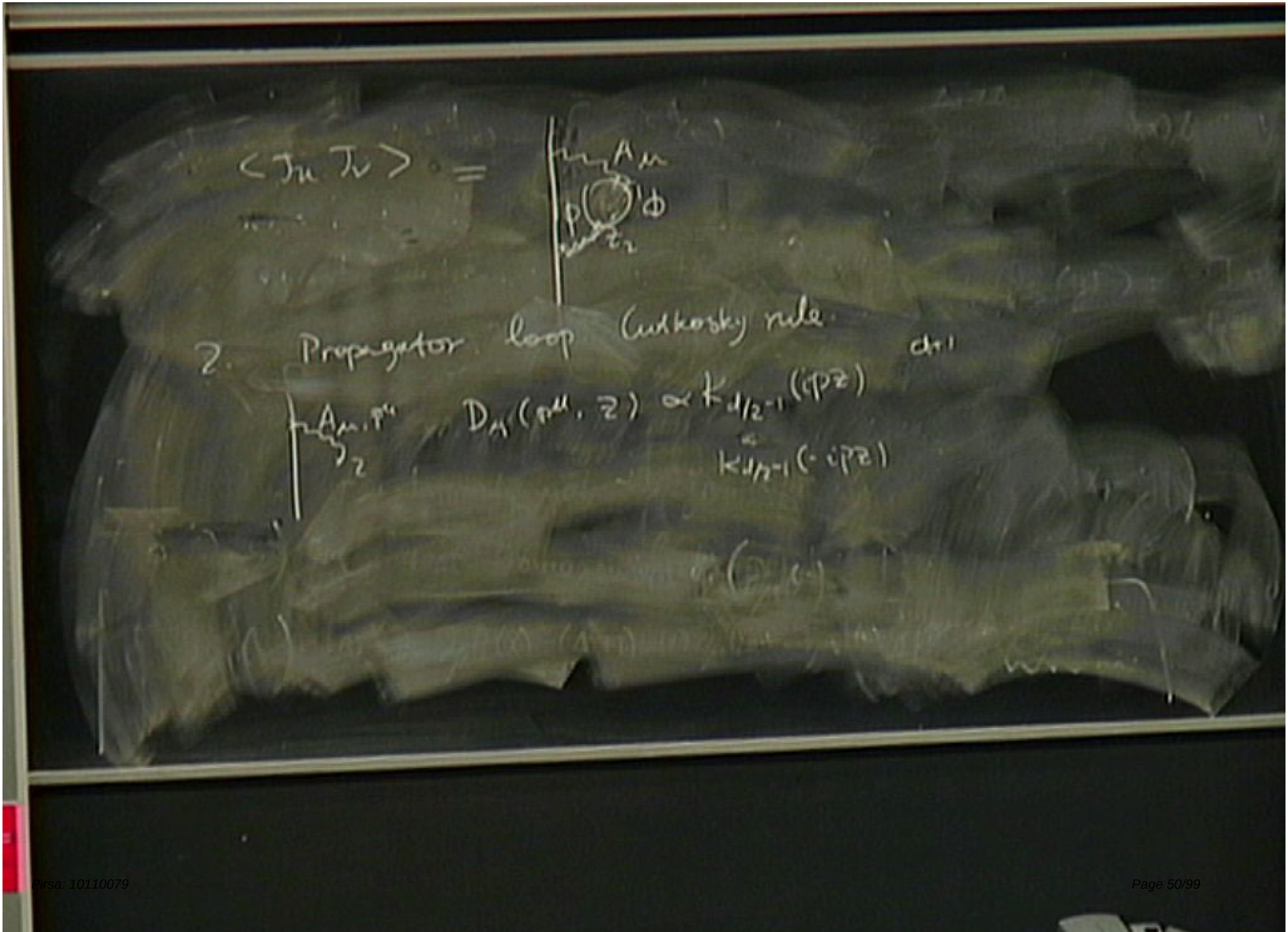
$\langle J_\mu J_\nu \rangle =$ $A_\mu$ $p$ $\Phi$

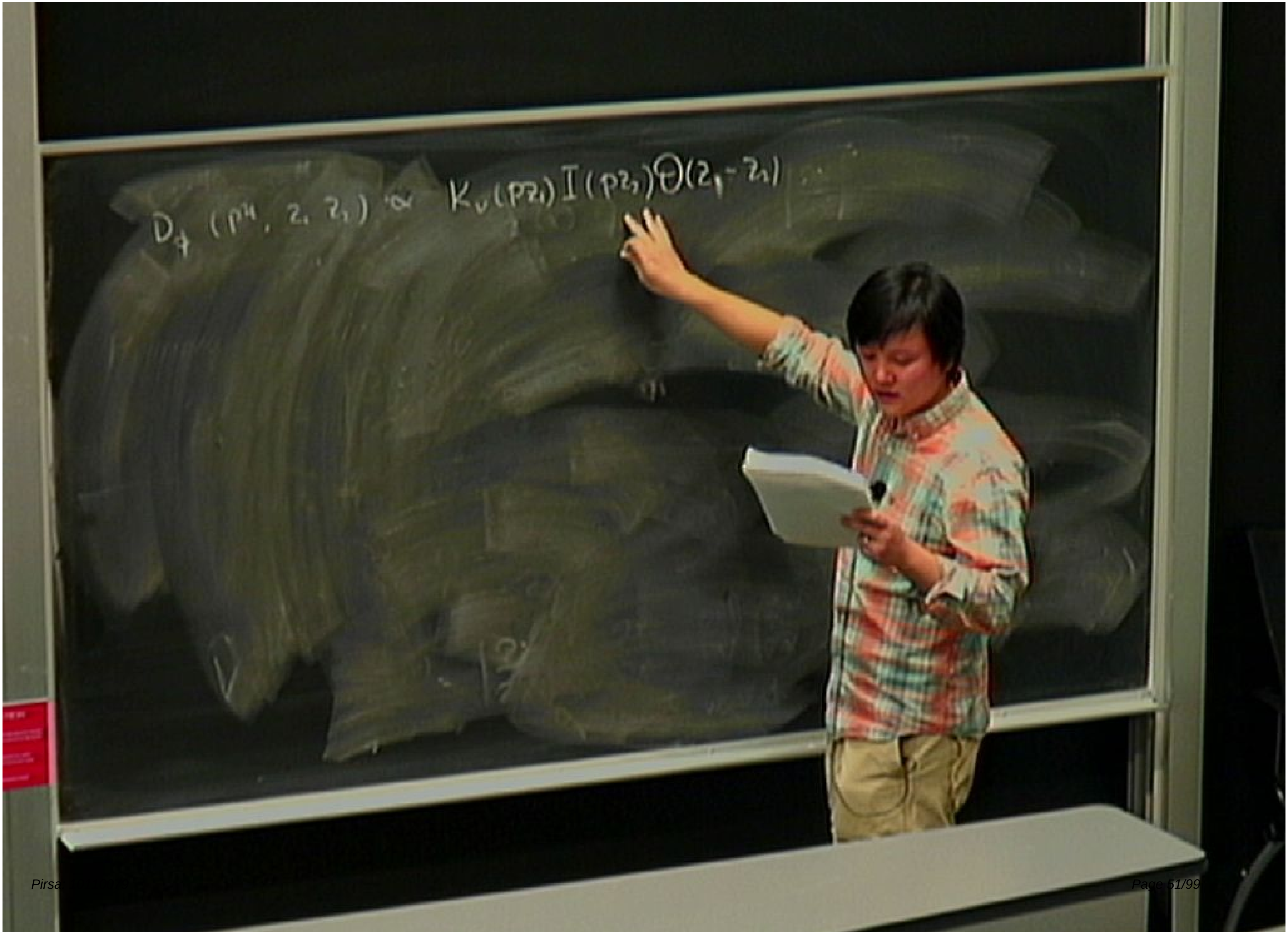
2. Propagator loop Cutkosky rule

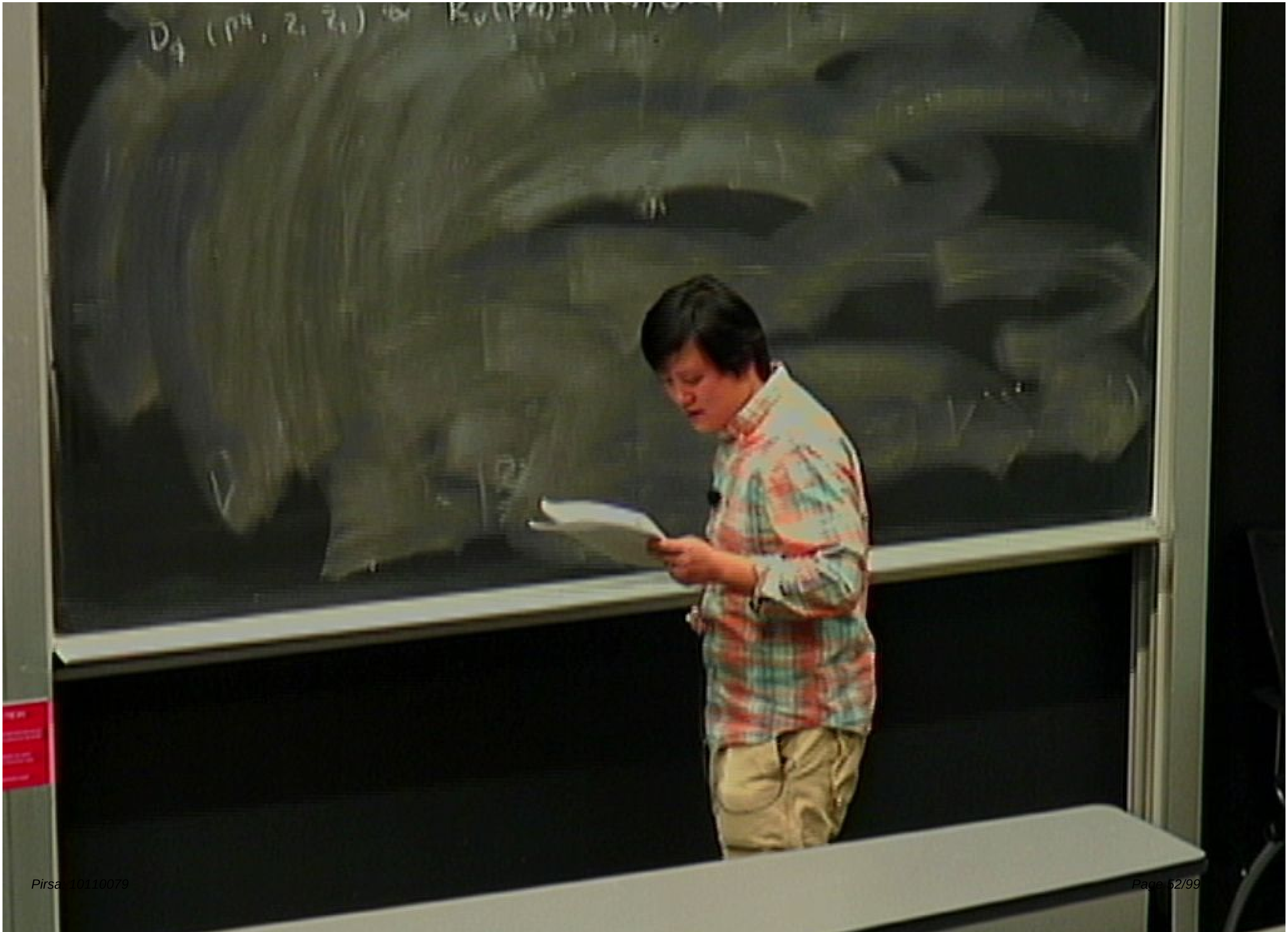
$A_\mu, p^\mu$ $D_\mu($

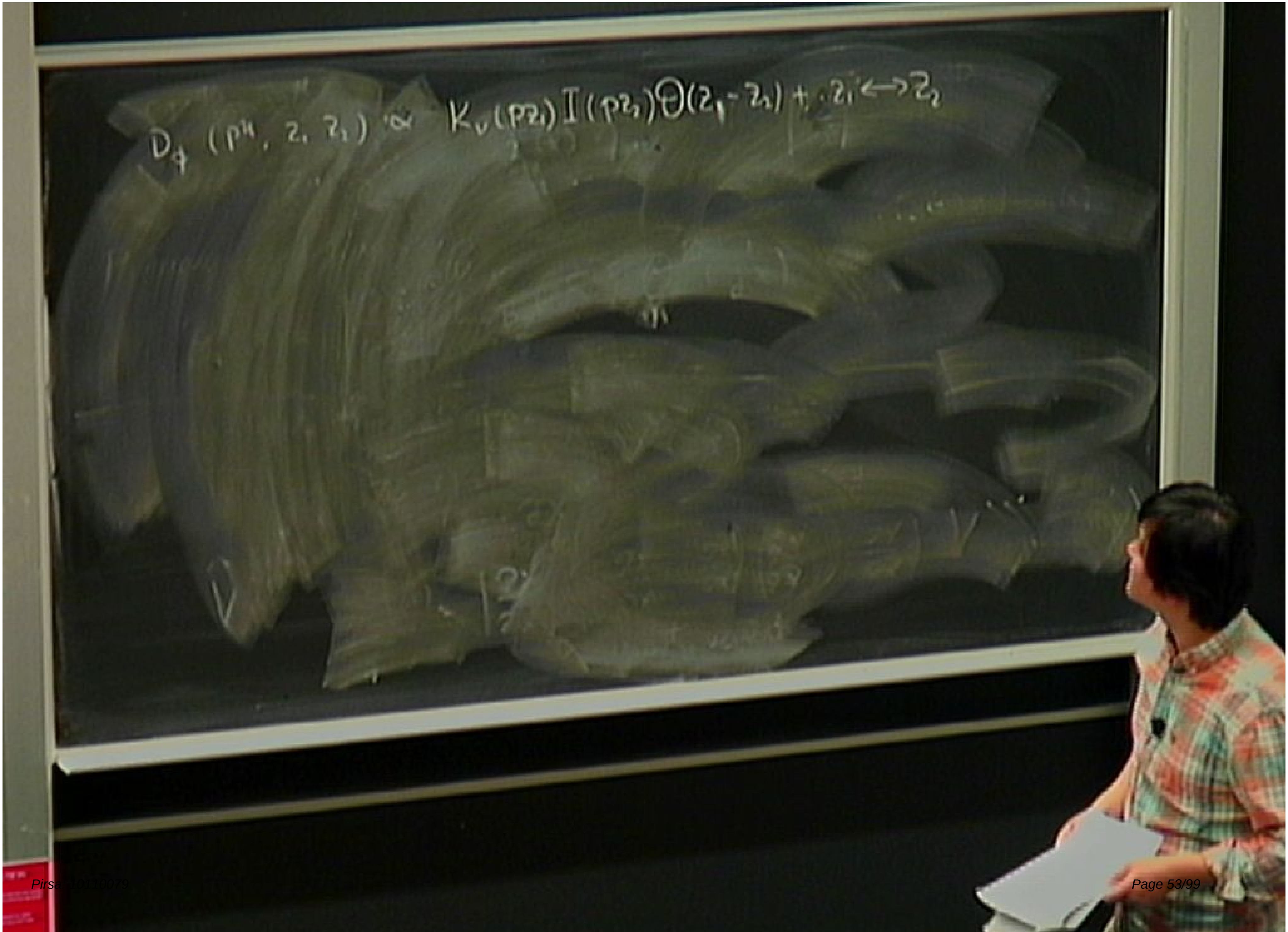

⟨J_μ J_ν⟩ =
A_μ
p
Φ
2. Propagator loop Cutkosky rule
d+1
A_μ, p^μ
z
D_μν(p^μ, z) ∝ K_d/2-1(ipz)


<J_μ J_ν> =
A_μ
p
Φ
2. Propagator loop Cutkosky rule
d+1
A_μ, p^μ
z
D_A(p^μ, z) ∝ K_{d/2-1}(ipz)
K_{d/2-1}(-ipz)

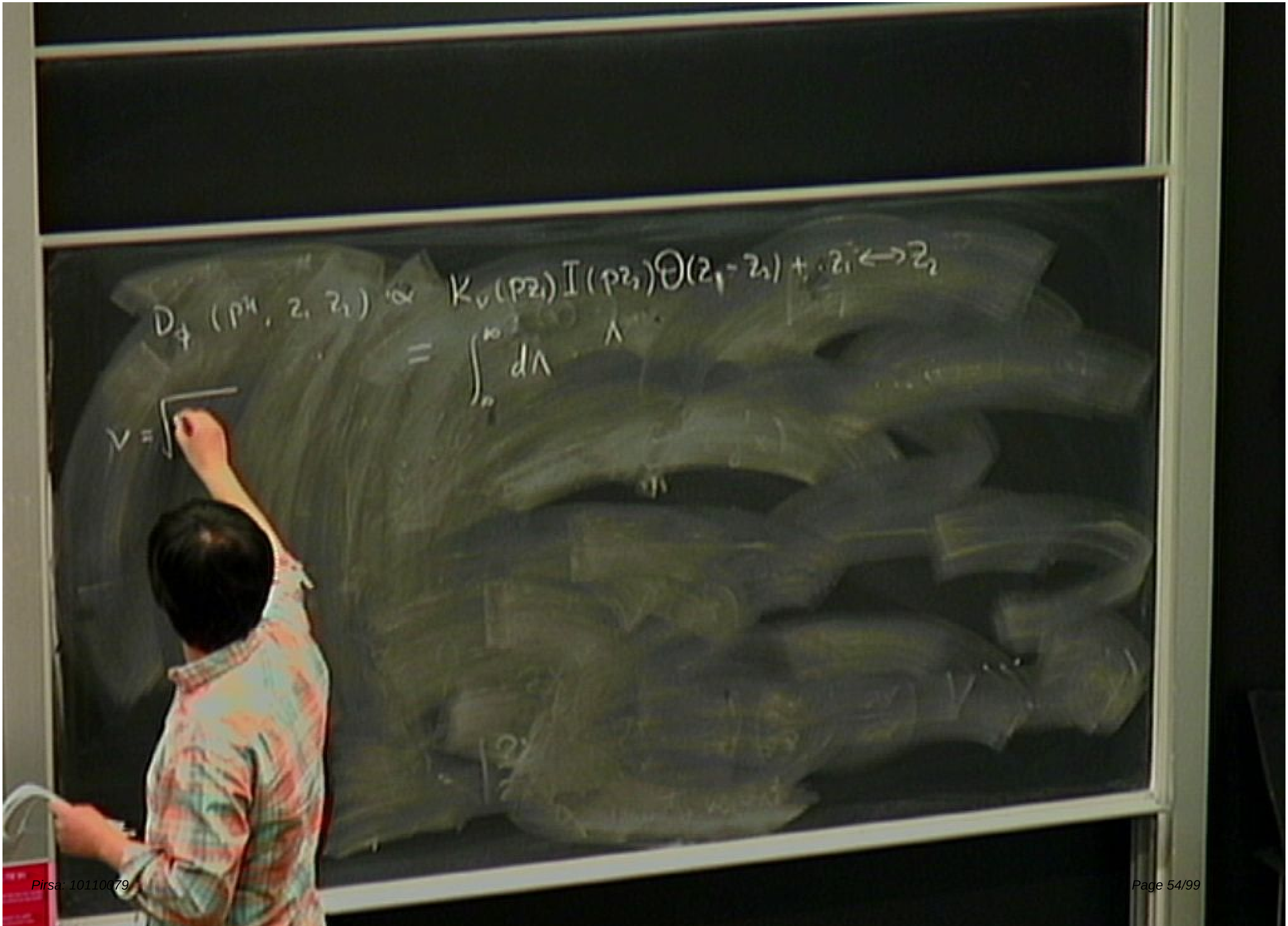

<J_μ J_ν> =
A_μ
ϕ
z_2
2. Propagator loop Cutkosky rule
d=1
A_μ, p^μ
z
D_A(p^μ, z) ∝ K_{d/2-1}(ipz)
∝ K_{d/2-1}(-ipz)

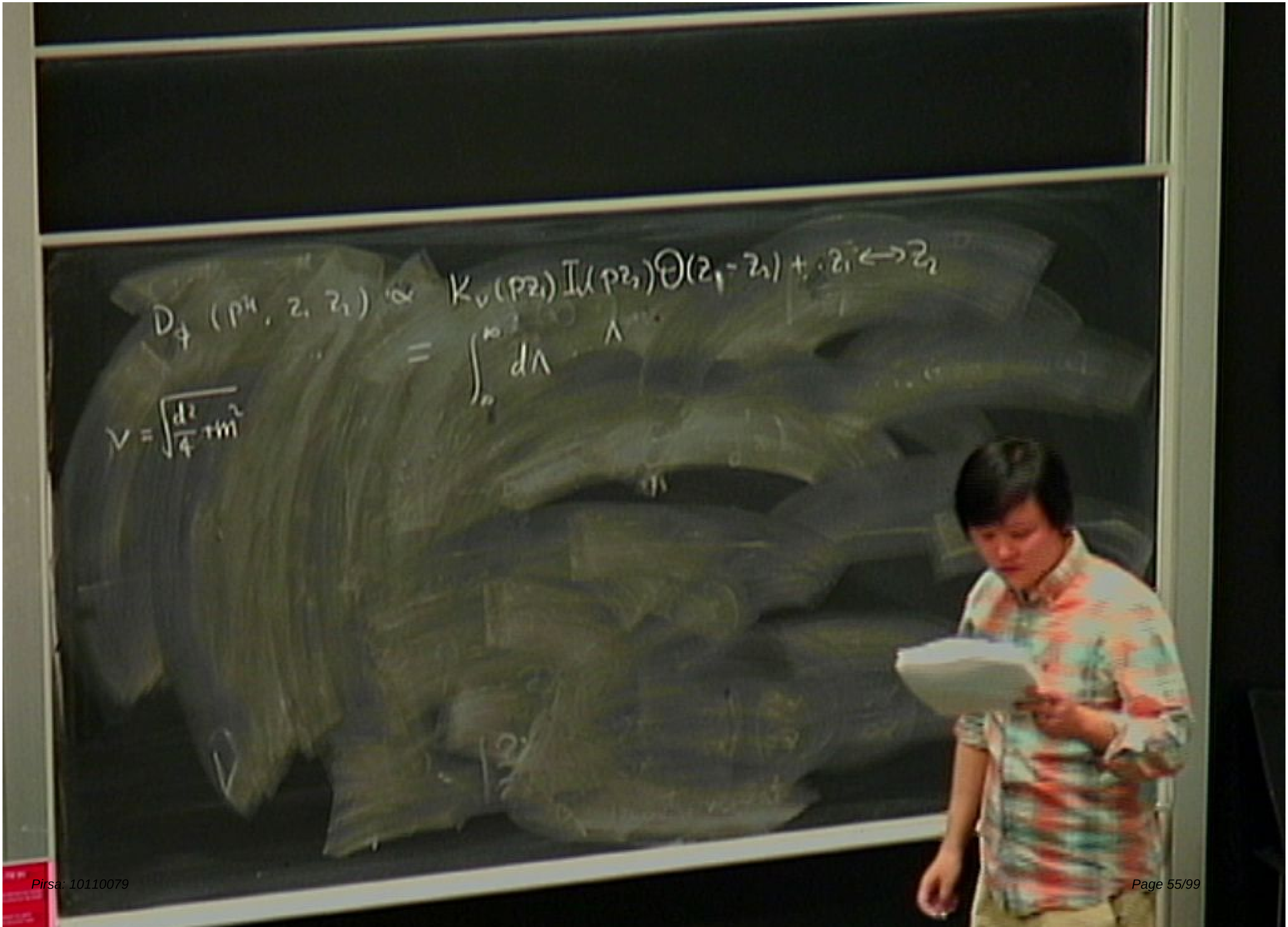

D_φ (p^4, z, z_1) ∝ K_ν(pz_1) I(pz_1) Θ(z_1 - z_1)

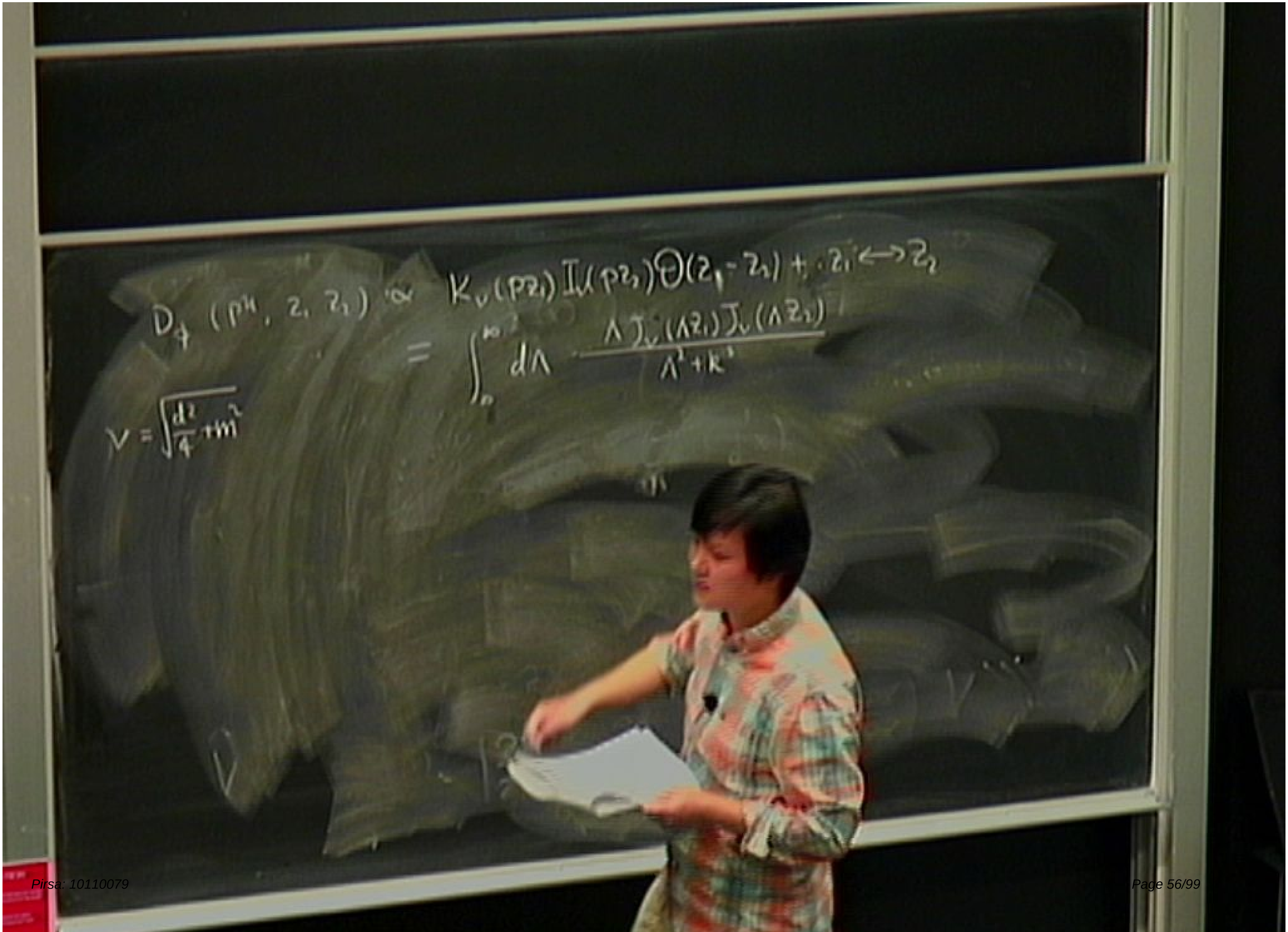


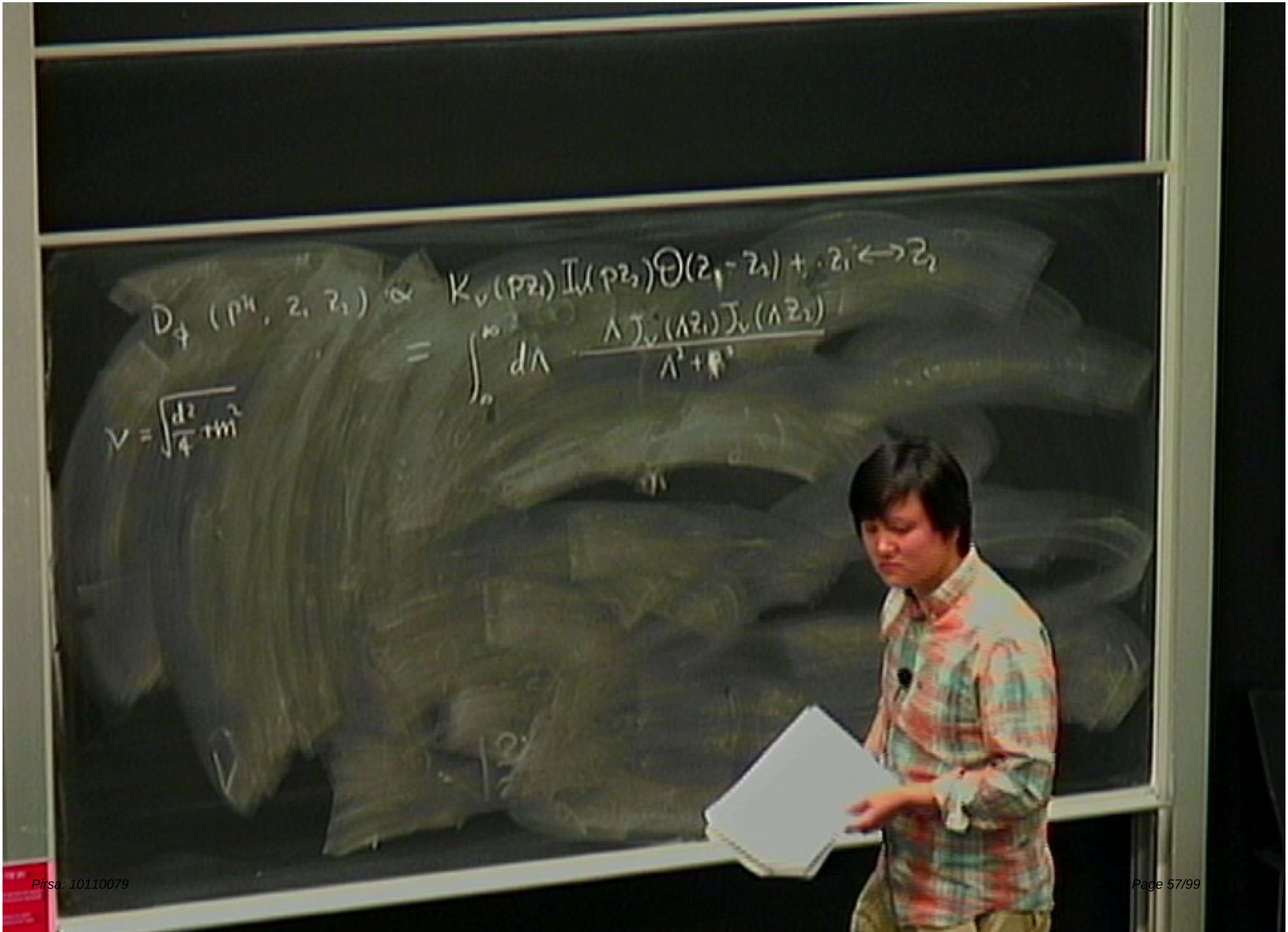


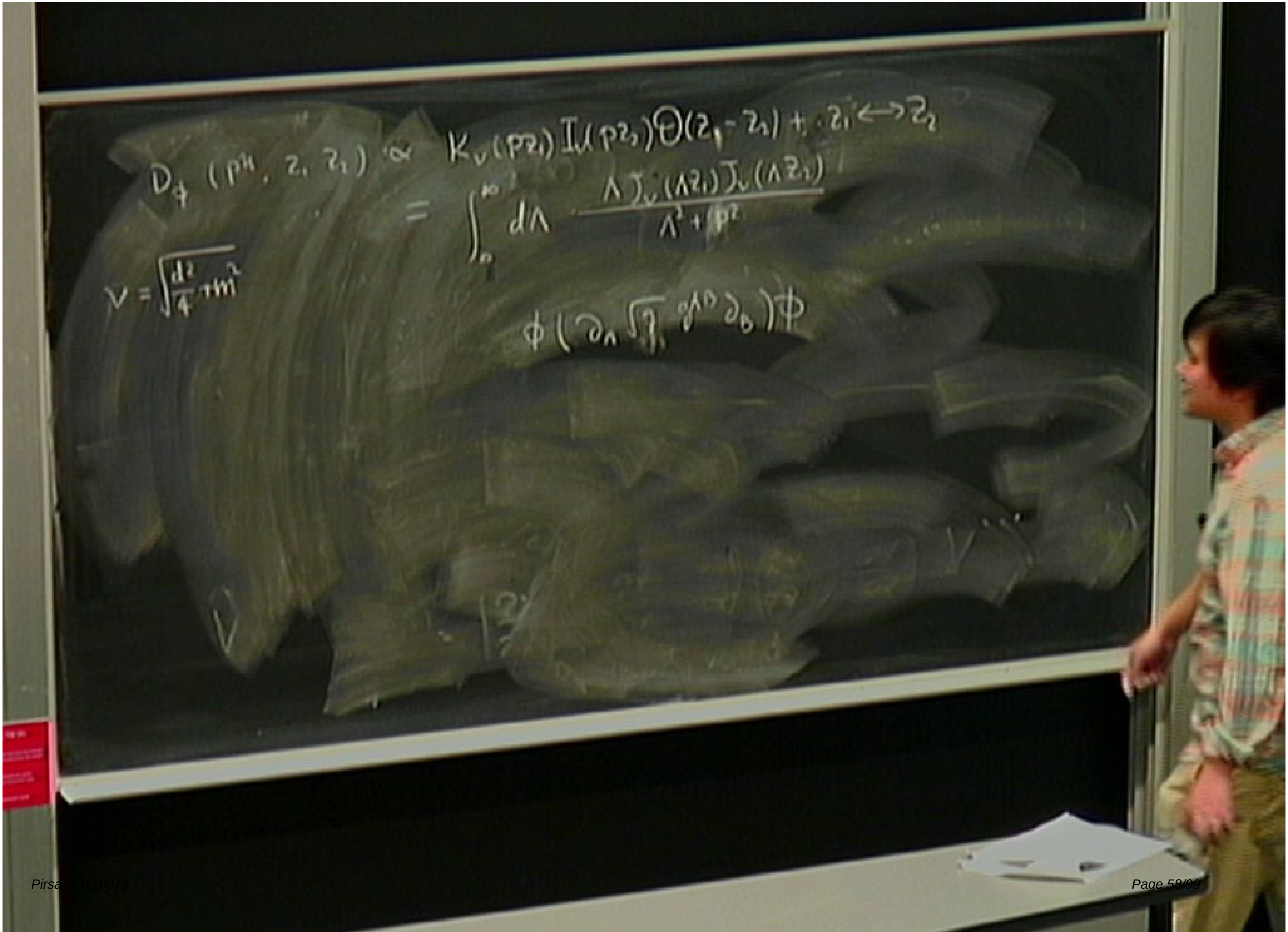
$D_{\psi}(p^4, z, z_1) \propto K_\nu(pz_1) I(pz_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$

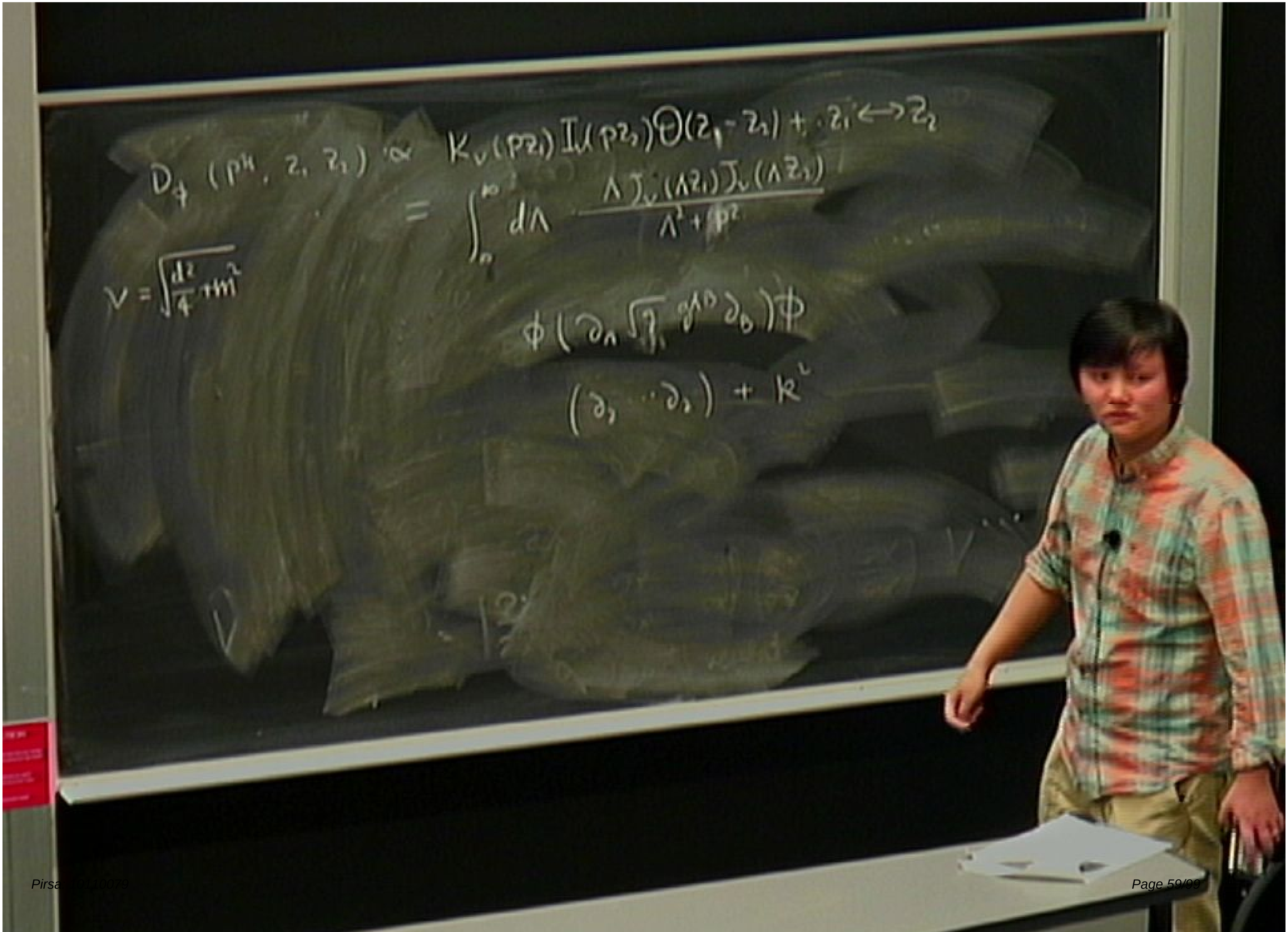

D_φ (p^μ, z, z_1) ∝ K_ν(pz_1) I(pz_2) Θ(z_1 - z_2) + z_1 ⟷ z_2
= ∫_0^∞ dΛ
ν = √


D_φ (p^μ, z, z_1) ∝ K_ν(pz_1) I_ν(pz_2) Θ(z_1 - z_2) + z_1 ⟷ z_2
= ∫_0^∞ dΛ
ν = √(d²/4 + m²)

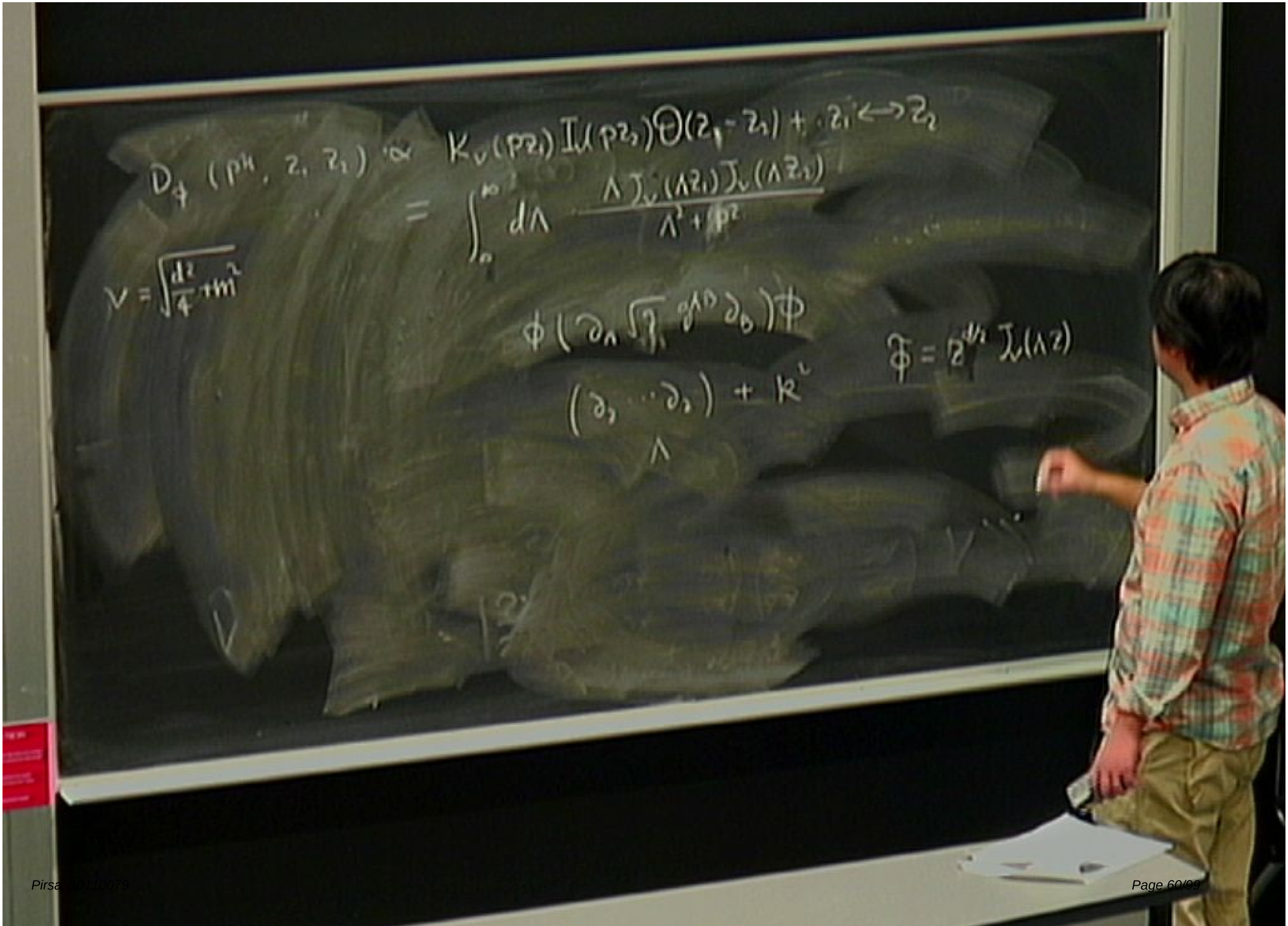

D_φ (p^μ, z, z_1) ∝ K_ν(pz_1) I_ν(pz_2) Θ(z_1 - z_2) + z_1 ⟷ z_2
= ∫_0^∞ dΛ Λ J_ν(Λz_1) J_ν(Λz_2) / (Λ^2 + k^2)
ν = √(d^2/4 + m^2)


$D_\phi(p^4, z, z_1) \propto K_\nu(pz_1)\, I_\nu(pz_2)\, \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$
$= \int_0^\infty d\Lambda \, \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$
$\nu = \sqrt{\frac{d^2}{4} + m^2}$

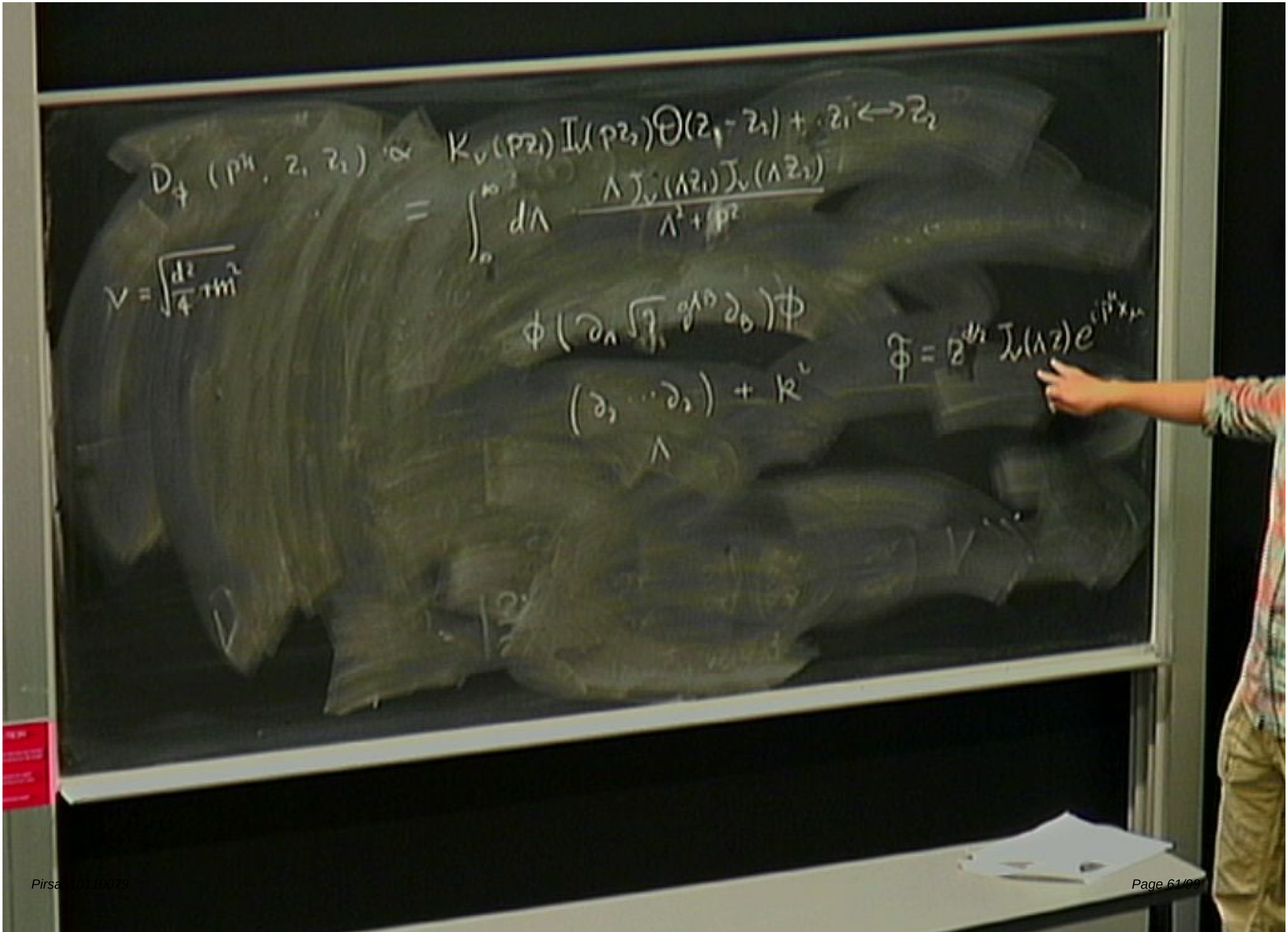

D_ϕ (p^μ, z, z_1) ∝ K_ν(pz_1) I_ν(pz_2) Θ(z_1 - z_1) + z_1 ⟷ z_2
= ∫_0^∞ dΛ Λ J_ν(Λz_1) J_ν(Λz_1) / (Λ^2 + p^2)
ν = √(d^2/4 + m^2)
ϕ (∂_A √g g^{AB} ∂_B) ϕ

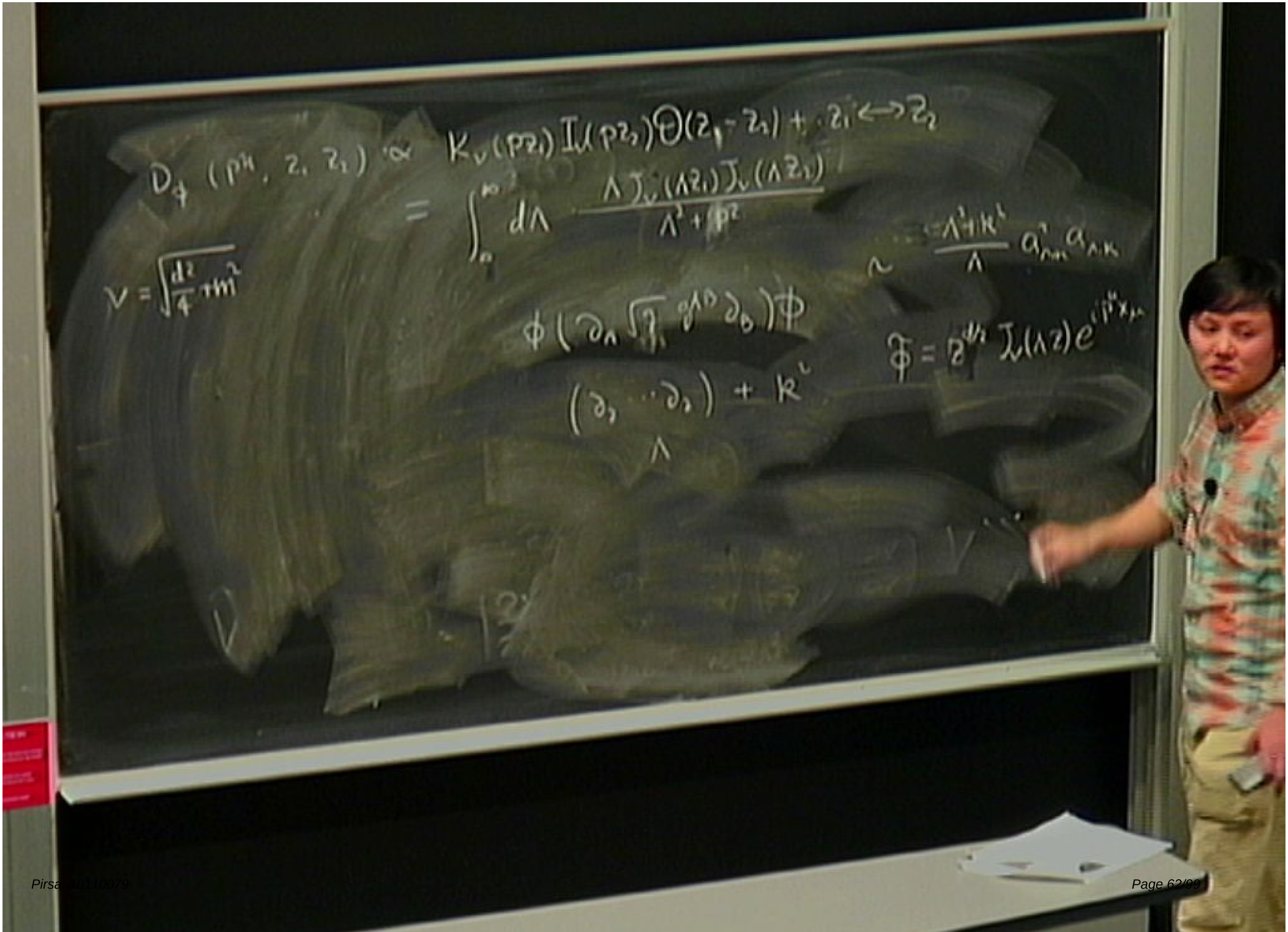


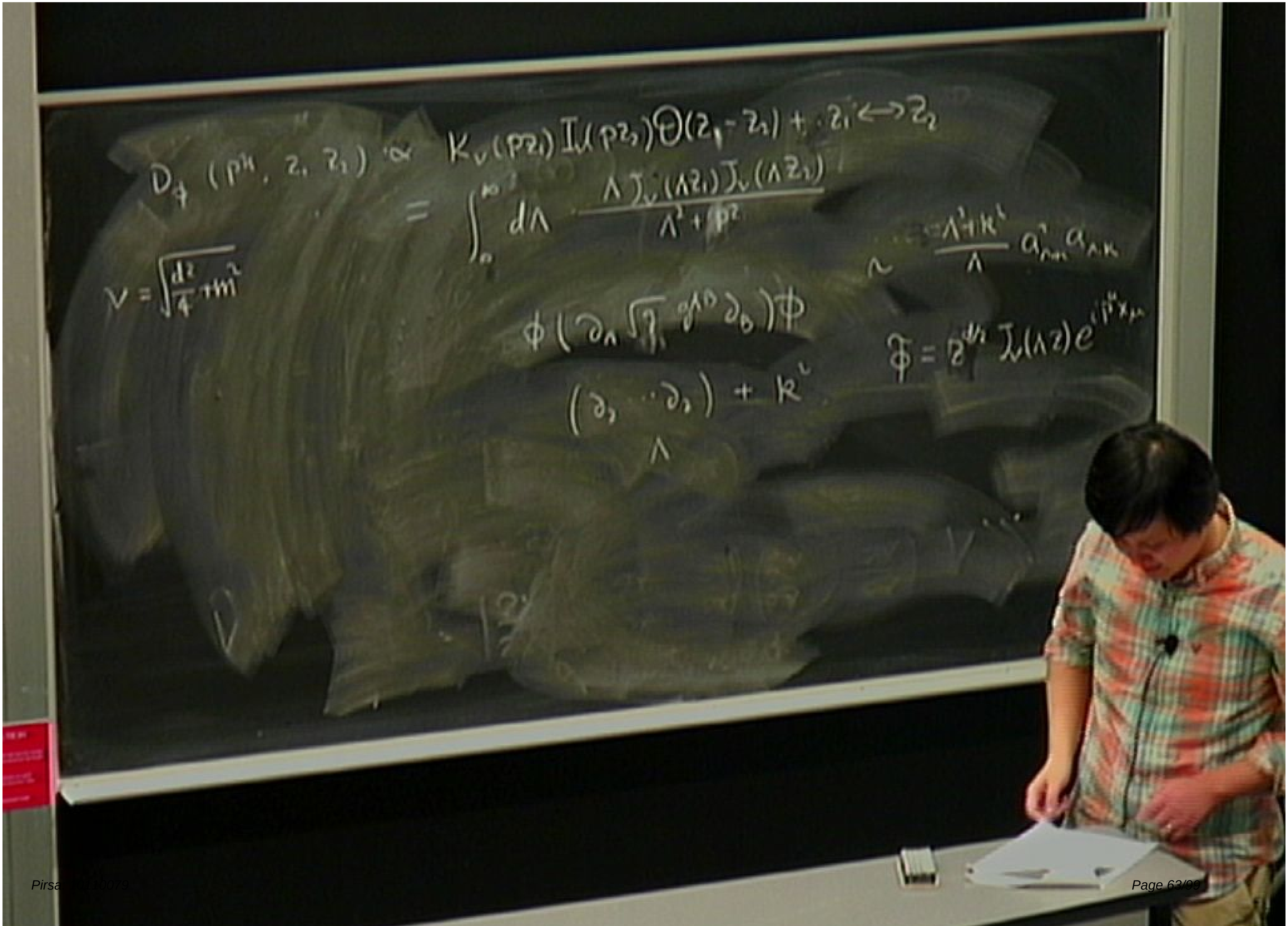
D_φ (p^μ, z, z_1) ∝ K_ν(pz_1) I_ν(pz_2) Θ(z_1 - z_2) + z_1 ⟷ z_2
= ∫_0^∞ dΛ Λ J_ν(Λz_1) J_ν(Λz_2) / (Λ² + p²)
ν = √(d²/4 + m²)
φ (∂_A √g g^{AB} ∂_B) φ
(∂_z ∂_z) + k²



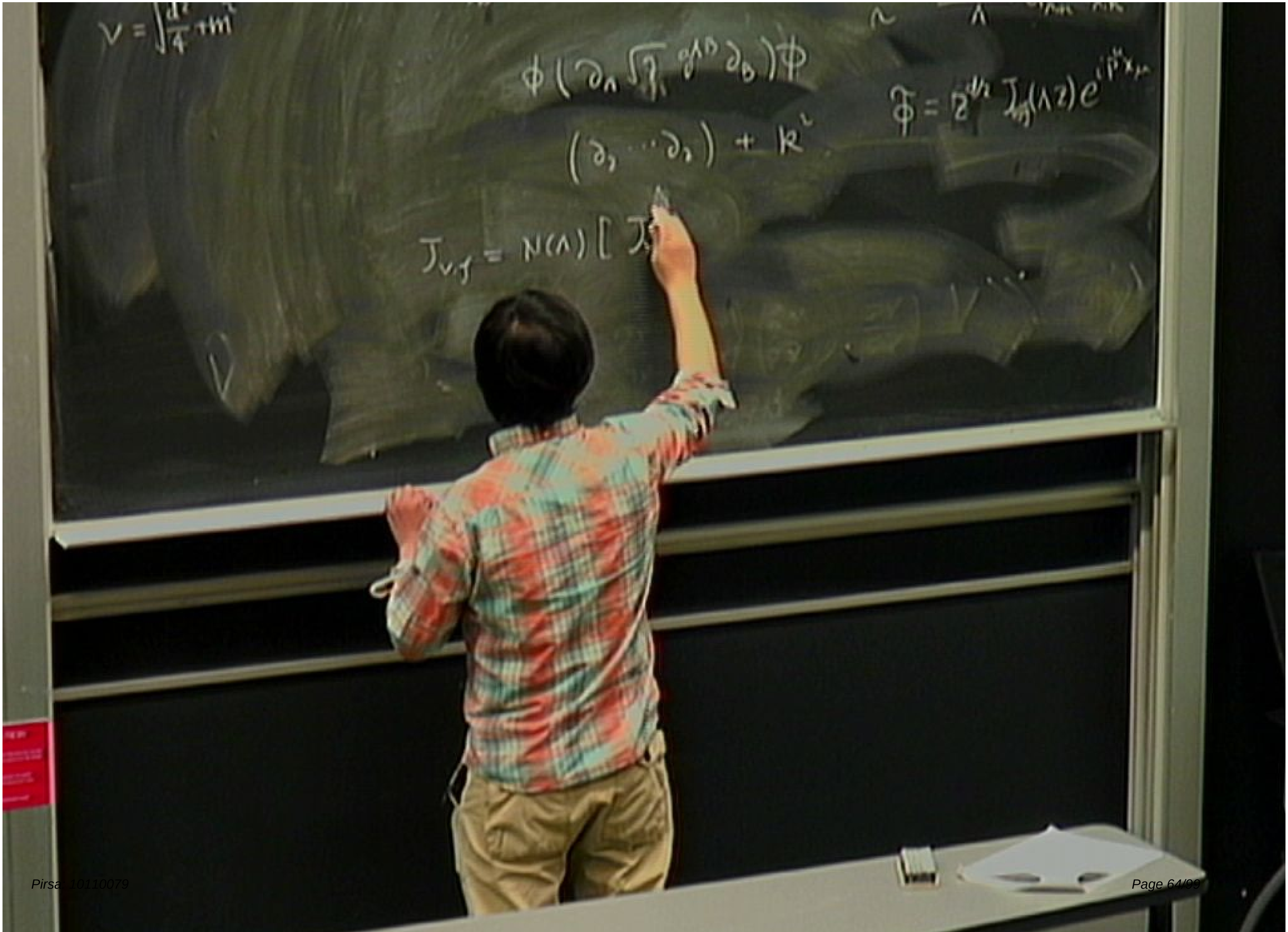
$D_\phi(p^\mu, z, z_1) \propto K_\nu(pz_1) I_\nu(pz_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$
$= \int_0^\infty d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$
$\nu = \sqrt{\frac{d^2}{4} + m^2}$
$\phi(\partial_A \sqrt{g}\, g^{AB} \partial_B)\phi$
$\Phi = z^{d/2} J_\nu(\Lambda z)$

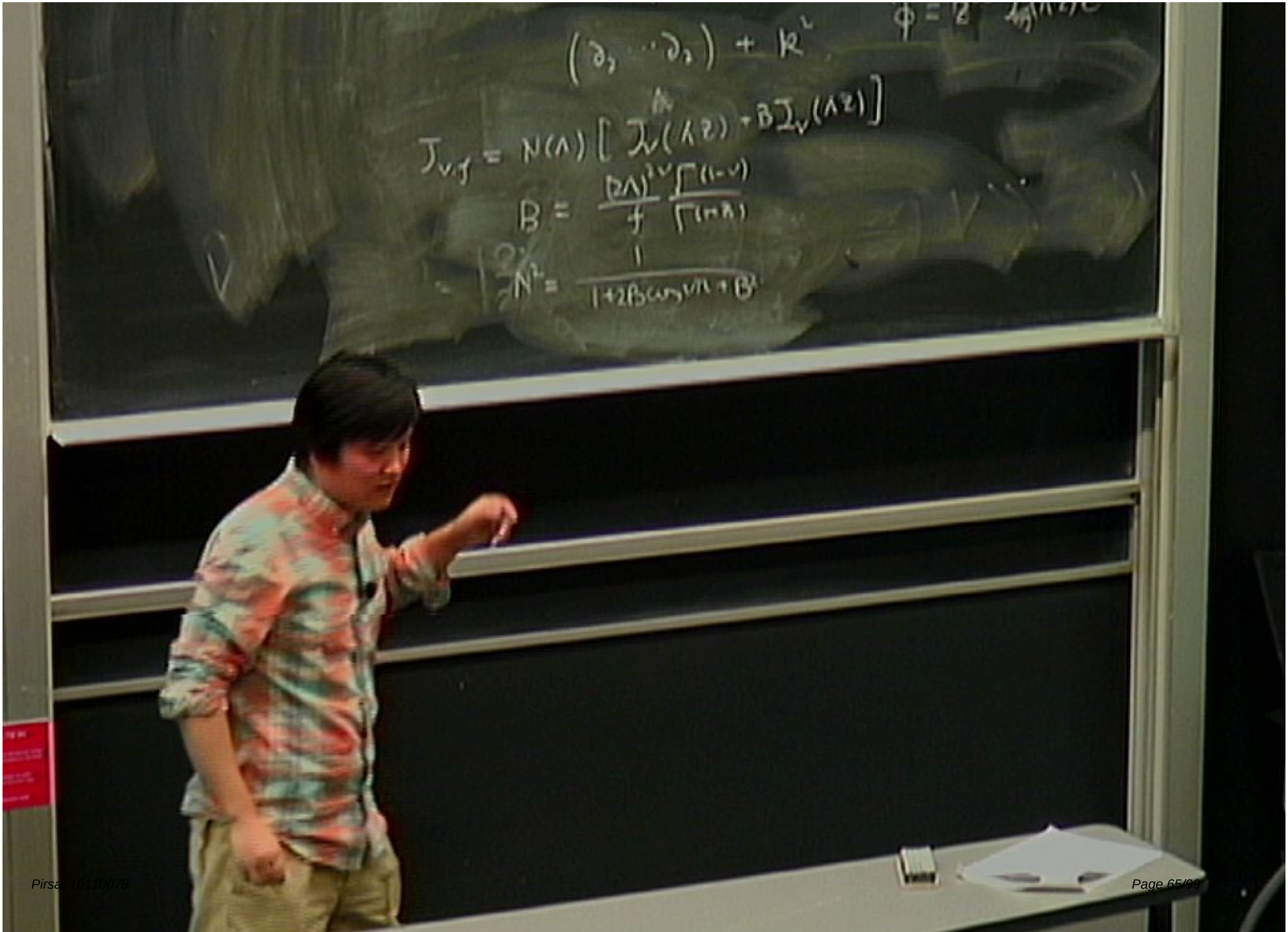

$D_\phi(p^\mu, z, z_2) \propto K_\nu(pz_1) I_\nu(pz_2)\Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$
$= \int_0^\infty d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$
$\nu = \sqrt{\frac{d^2}{4} + m^2}$
$\phi(\partial_A \sqrt{g}\, g^{AB} \partial_B)\phi$
$(\partial_z \cdots \partial_z) + k^2$
$\Lambda$
$\phi = z^{d/2} J_\nu(\Lambda z) e^{ip^\mu x_\mu}$

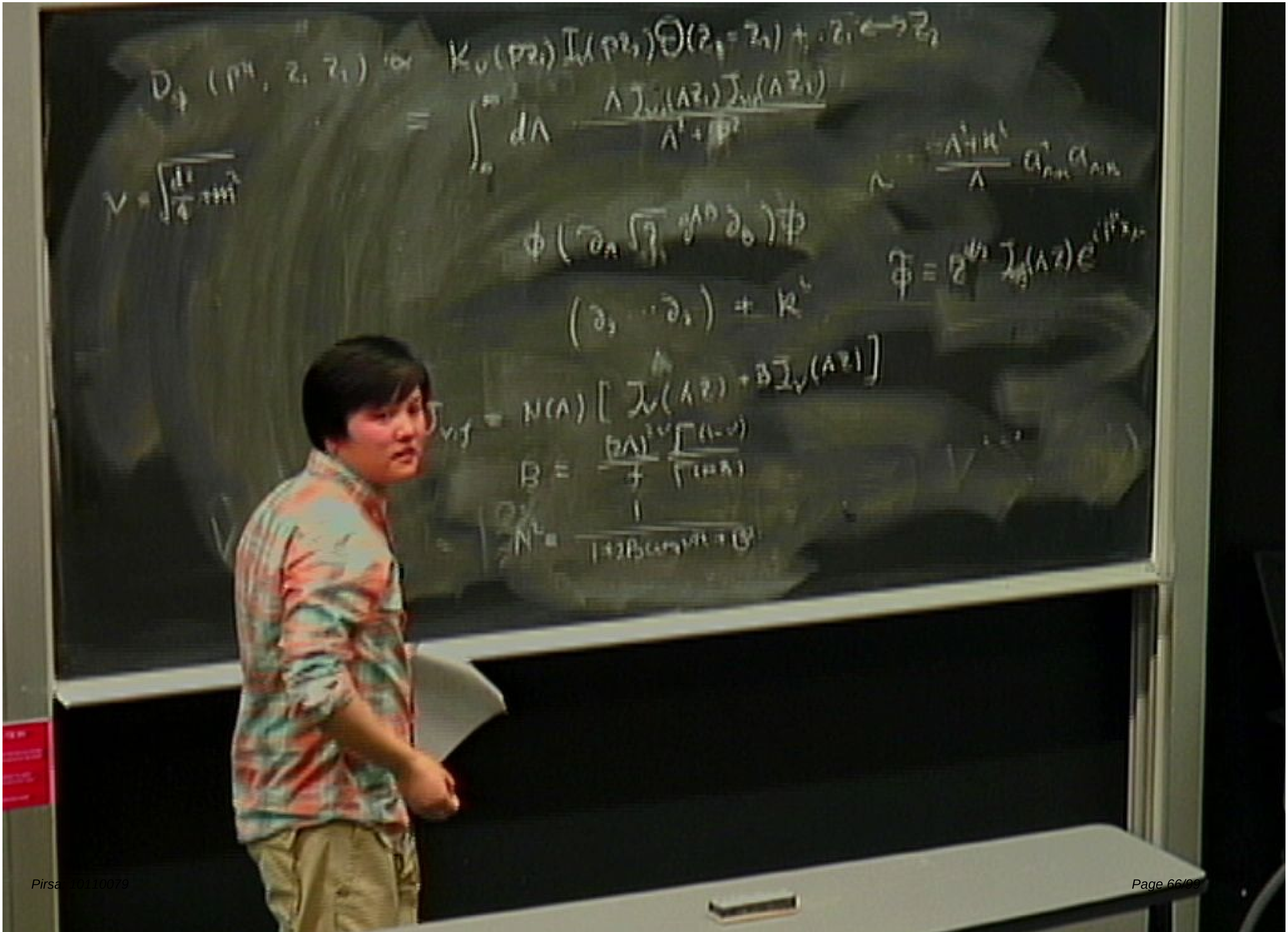

$D_\phi(p^\mu, z, z_2) \propto K_\nu(pz_1) I_\nu(pz_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$
$= \int_0^\infty d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$
$\nu = \sqrt{\frac{d^2}{4} + m^2}$
$\sim \frac{\Lambda^2 + k^2}{\Lambda} a^\dagger_{\Lambda,k} a_{\Lambda,k}$
$\phi(\partial_A \sqrt{g}\, g^{AB} \partial_B)\phi$
$\Phi = z^{d/2} J_\nu(\Lambda z) e^{ip^\mu x_\mu}$
$(\partial_z \cdot \partial_z) + k^2$
$\Lambda$

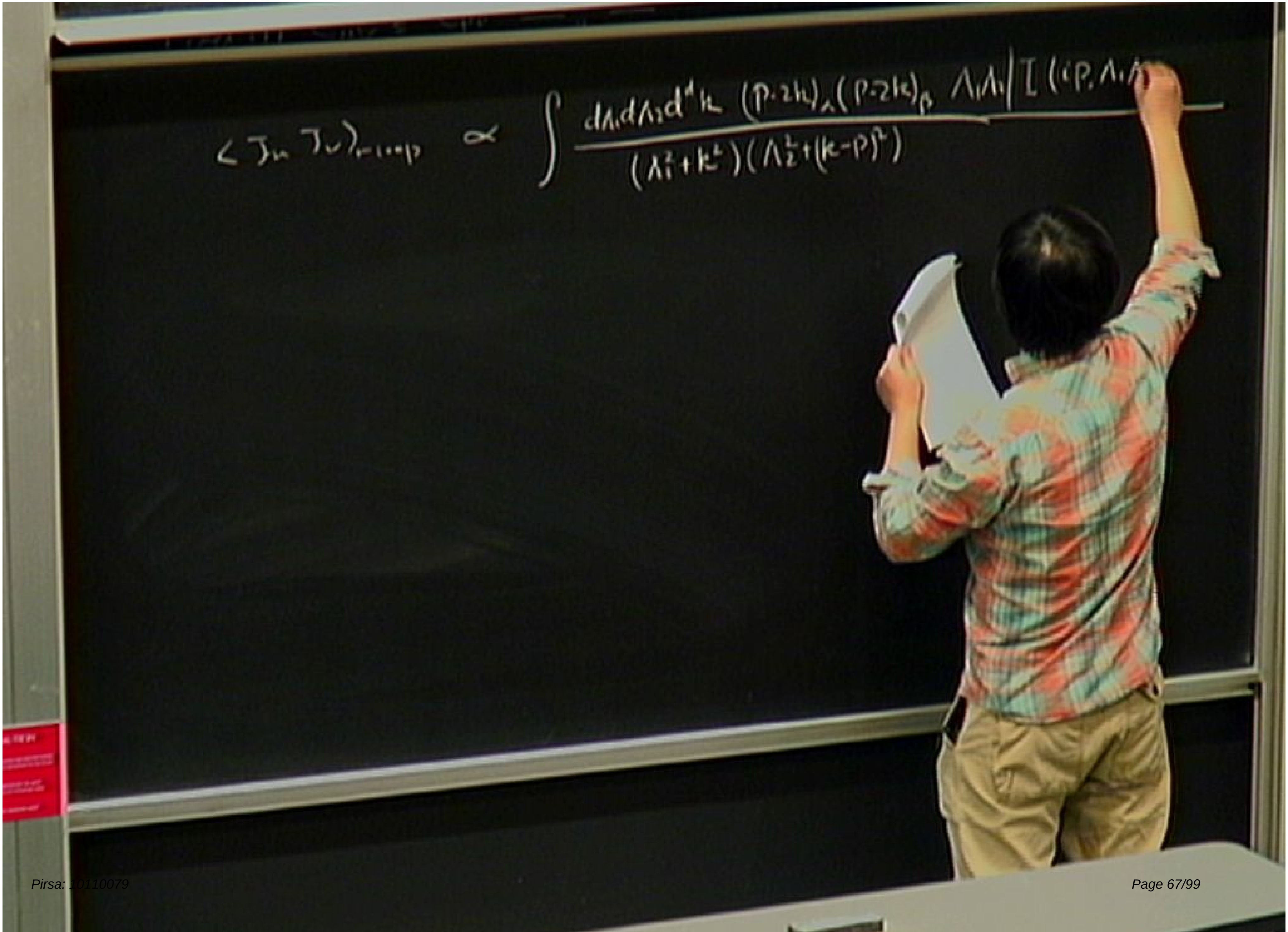


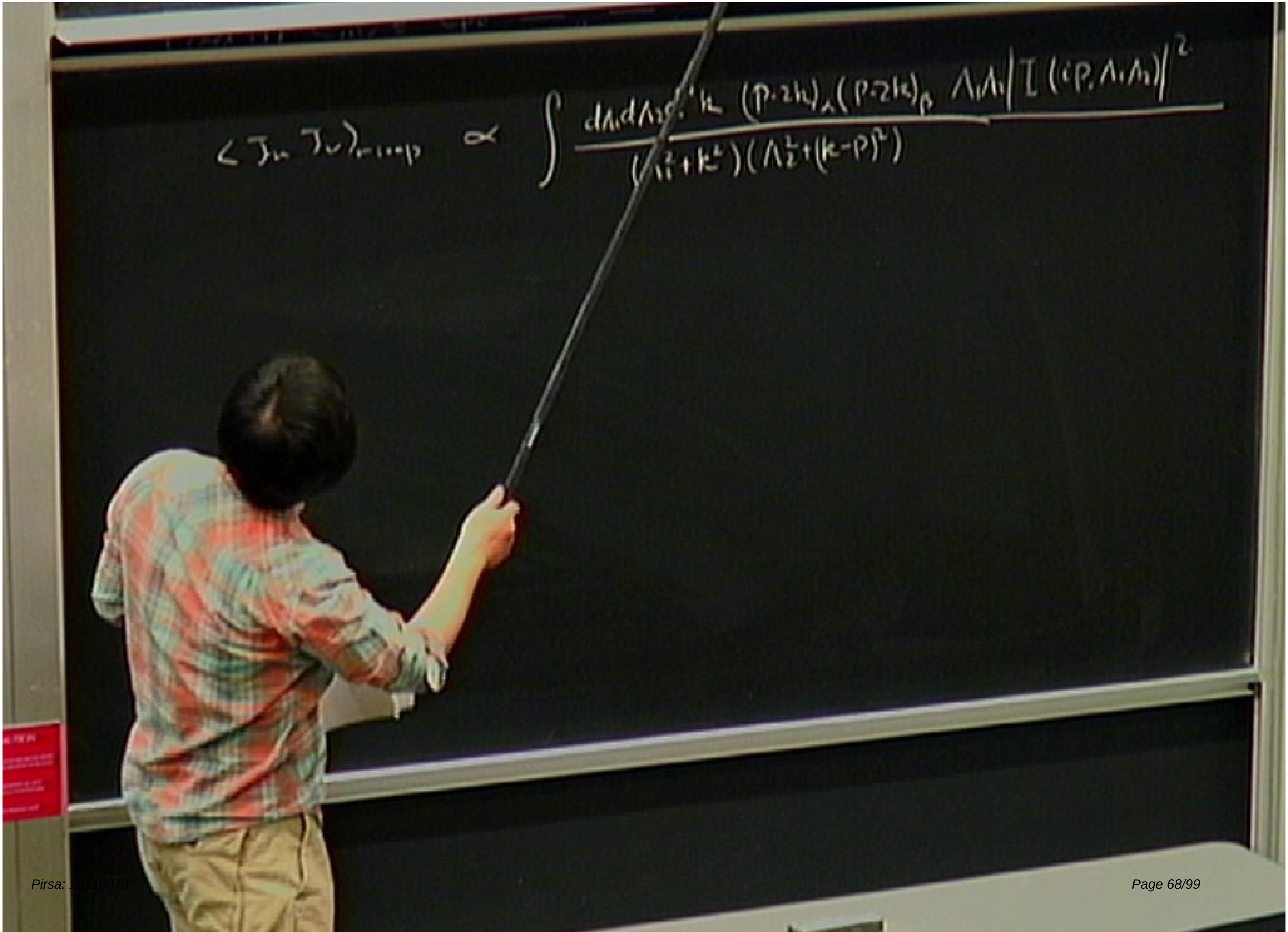
$D_\phi(p^\mu, z_1, z_2) \propto K_\nu(pz_1) I_\nu(pz_2)\Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$
$= \int_0^\infty d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$
$\nu = \sqrt{\frac{d^2}{4} + m^2}$
$\sim \frac{\Lambda^2 + k^2}{\Lambda} a^\dagger_{\Lambda k} a_{\Lambda k}$
$\phi(\partial_A \sqrt{g} g^{AB} \partial_B)\phi$
$\Phi = z^{d/2} J_\nu(\Lambda z) e^{ip^\mu x_\mu}$
$(\partial_z \cdot \partial_z) + k^2$
$\Lambda$

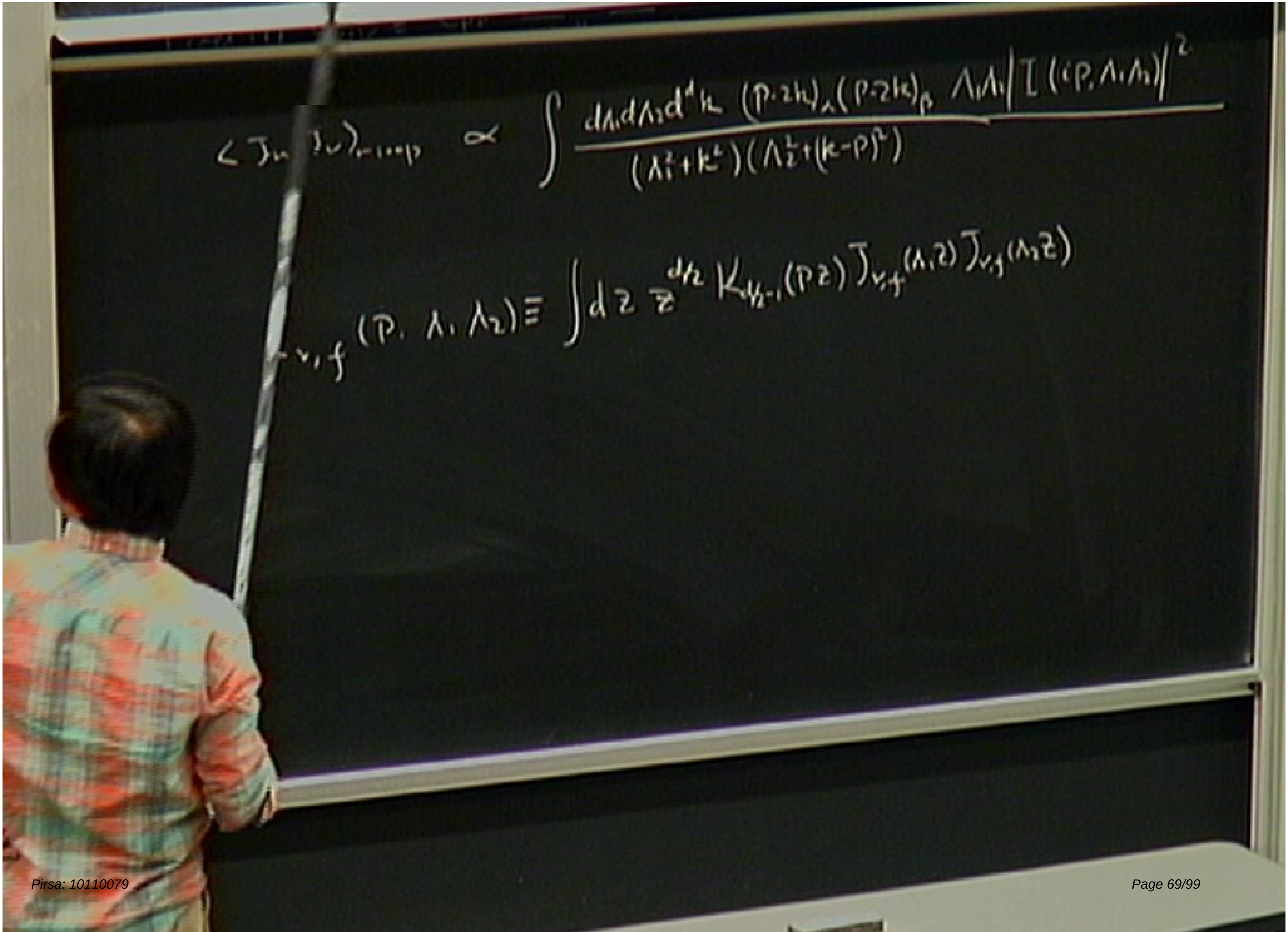

φ ( ∂_A √g g^{AB} ∂_B ) φ
Φ = z^{d/2} J_ν(Λz) e^{ip^μ x_μ}
( ∂_z ... ∂_z ) + k²
J_{ν,γ} = N(Λ) [ J






$\langle J_\mu J_\nu \rangle_{1\text{-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \; (p+2k)_\lambda (p+2k)_\rho \; \Lambda_1 \Lambda_1}{(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)}$


$\langle J_\mu J_\nu \rangle_{1\text{-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \, (p+2k)_\lambda (p+2k)_\rho \, \Lambda_1 \Lambda_1 \left| I(ip, \Lambda_1, \Lambda_2) \right|^2}{(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)}$

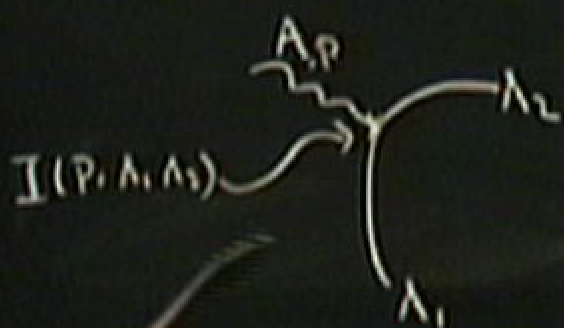

$\langle J_\mu J_\nu \rangle_{1-loop} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \, (p+2k)_\alpha (p+2k)_\beta \, \Lambda_1 \Lambda_2 \left| I(\epsilon p, \Lambda_1, \Lambda_2) \right|^2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)}$
$I_{\nu,f}(p, \Lambda_1, \Lambda_2) \equiv \int dz \, z^{d/2} K_{d/2-1}(pz) J_{\nu,f}(\Lambda_1 z) J_{\nu,f}(\Lambda_2 z)$

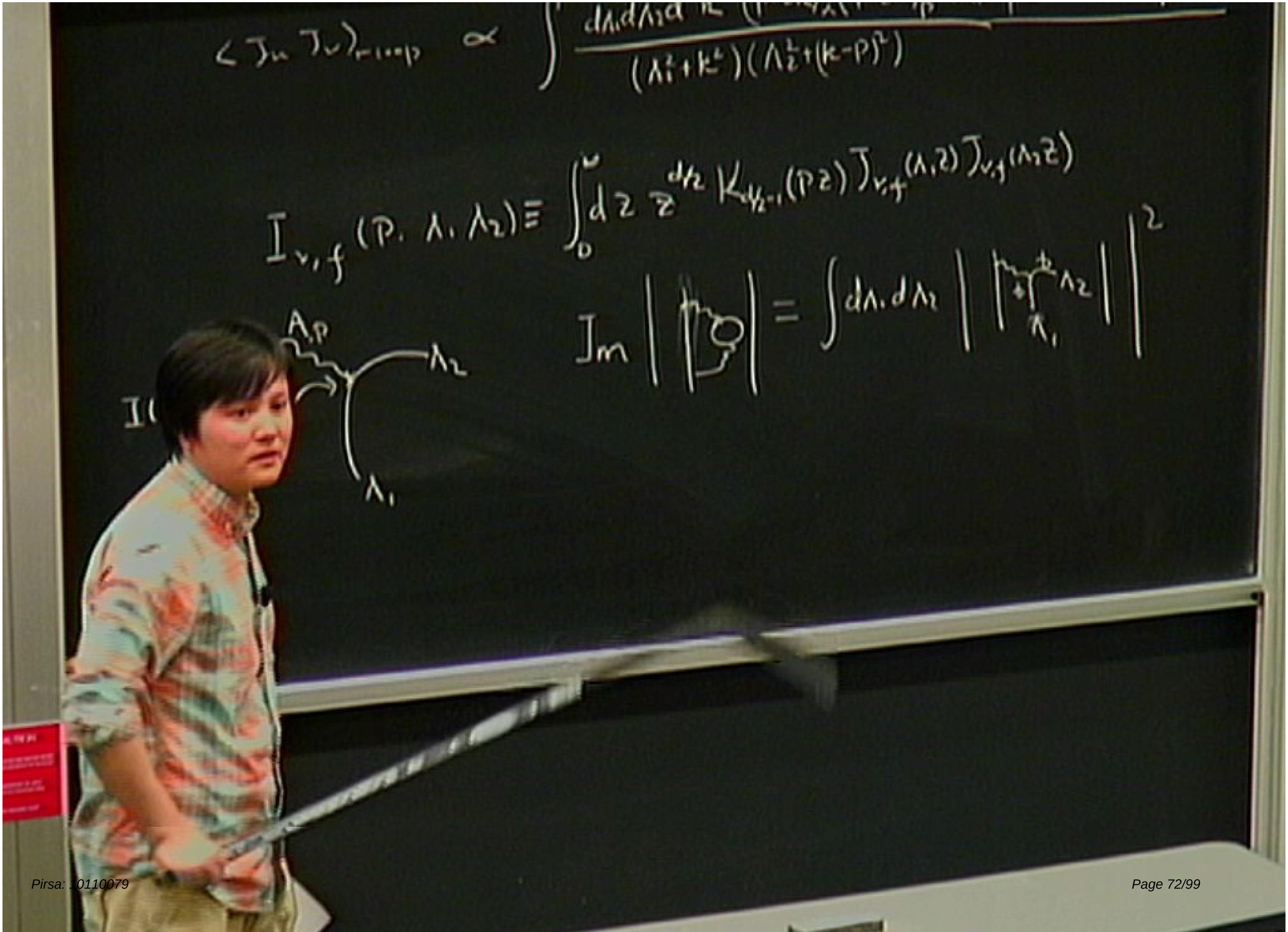
$$\langle J_\mu J_\nu \rangle_{\text{1-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \, \ldots}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)}$$

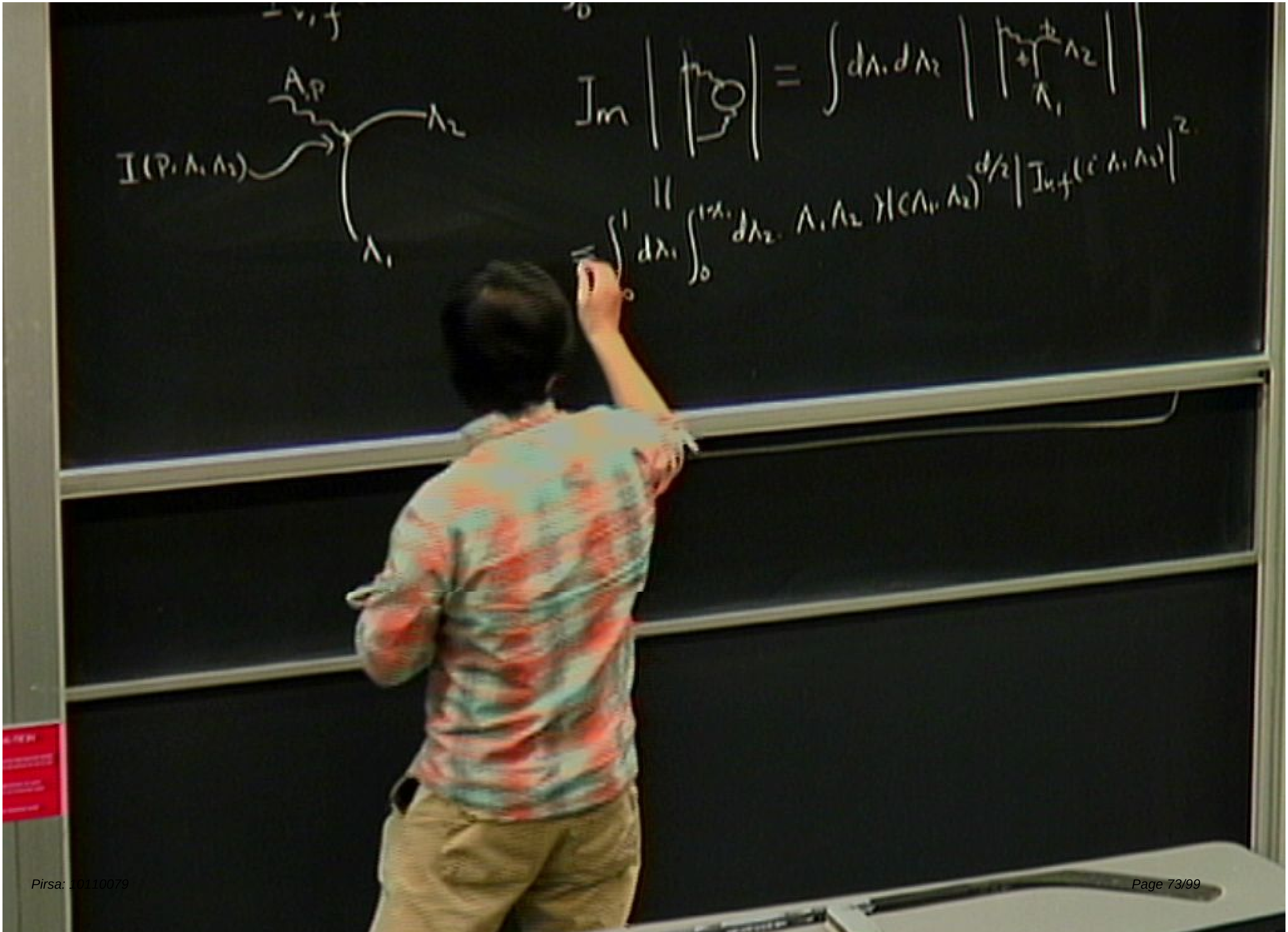
$$I_{\nu,f}(p, \Lambda_1, \Lambda_2) \equiv \int_0^\infty dz\, z^{d/2} K_{d/2-1}(pz)\, J_{\nu,f}(\Lambda_1 z)\, J_{\nu,f}(\Lambda_2 z)$$

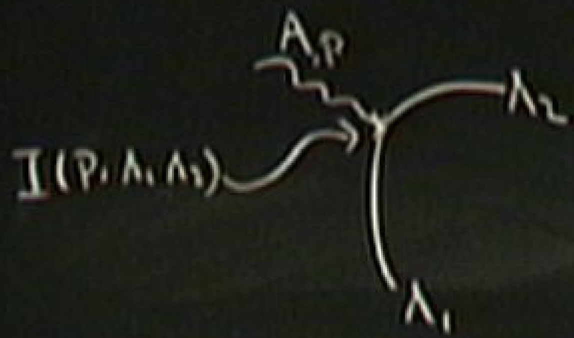
$$\langle J_\mu J_\nu \rangle_{1\text{-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \; \cdots}{(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)}$$

$$I_{\nu,f}(p,\Lambda_1,\Lambda_2) \equiv \int_0^\infty dz\, z^{d/2} K_{d/2-1}(pz)\, J_{\nu,f}(\Lambda_1 z)\, J_{\nu,f}(\Lambda_2 z)$$




⟨J_μ J_ν⟩_{1-loop} ∝ ∫ dΛ1 dΛ2 d k / (Λ1²+k²)(Λ2²+(k−p)²)
I_{ν,f}(p, Λ1, Λ2) ≡ ∫_0^∞ dz z^{d/2} K_{d/2−1}(pz) J_{ν,f}(Λ1 z) J_{ν,f}(Λ2 z)
Im = ∫ dΛ1 dΛ2
A, p
Λ2
Λ1
I(

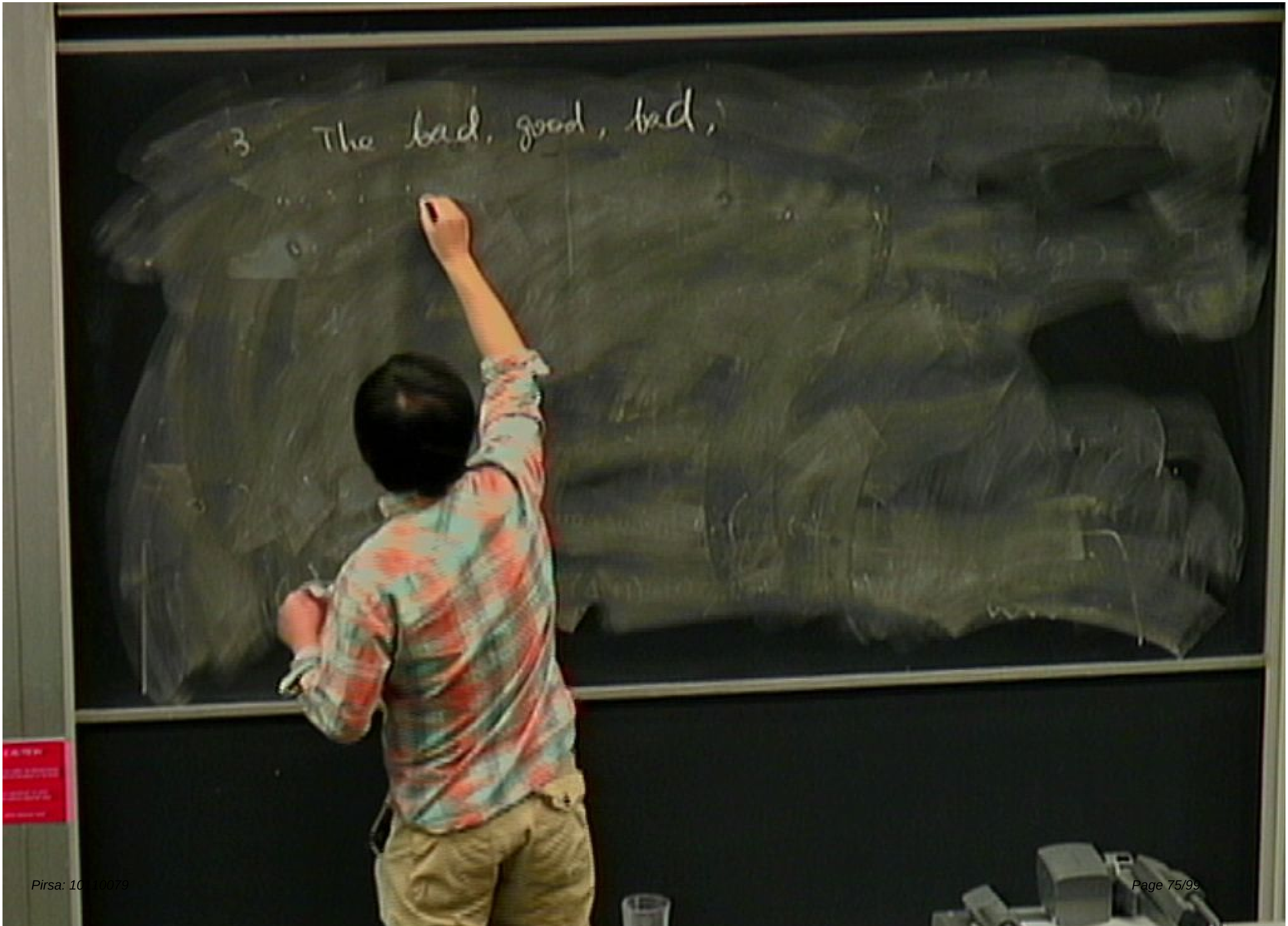

Im
I(p, λ1, λ2)

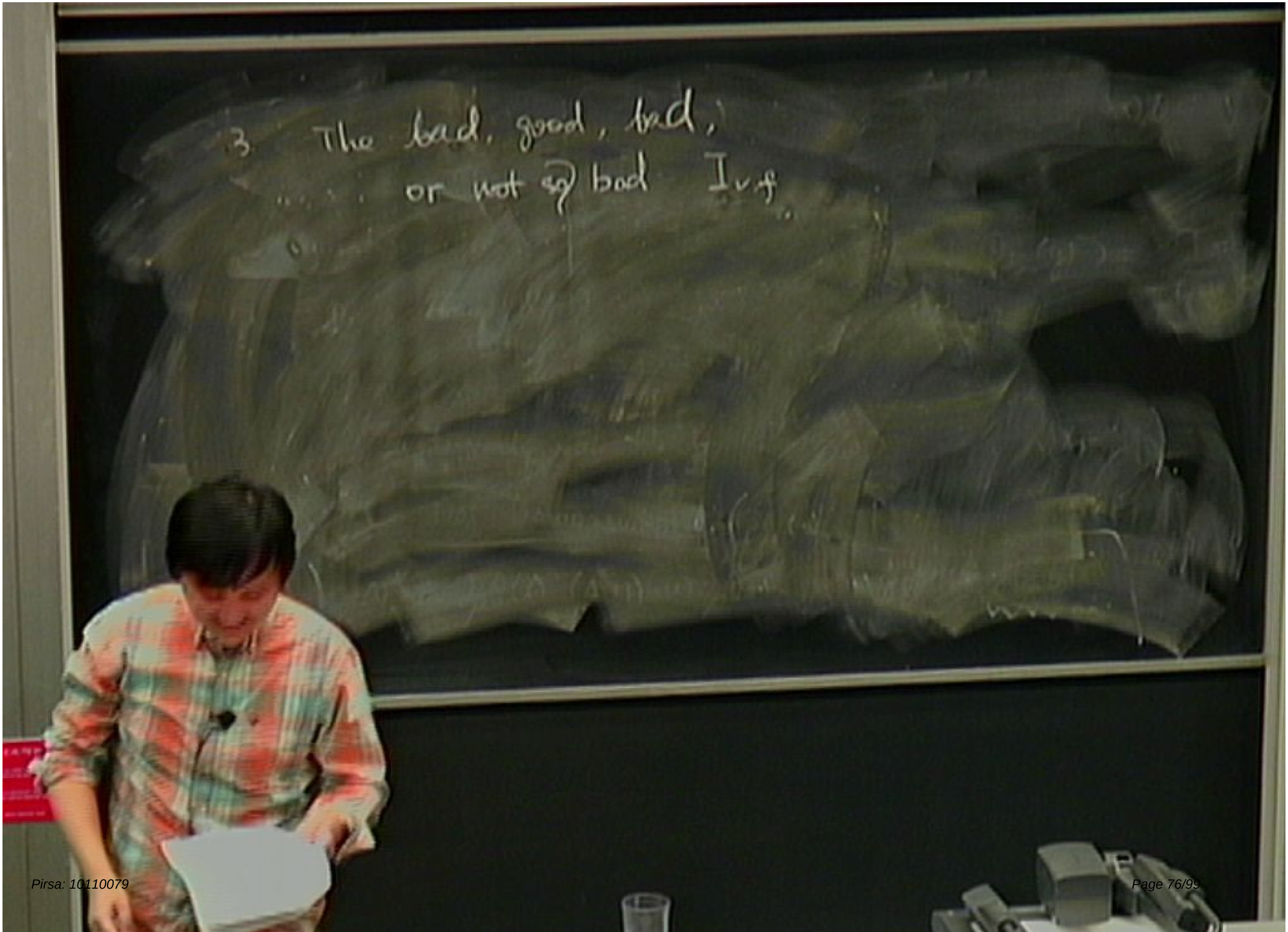


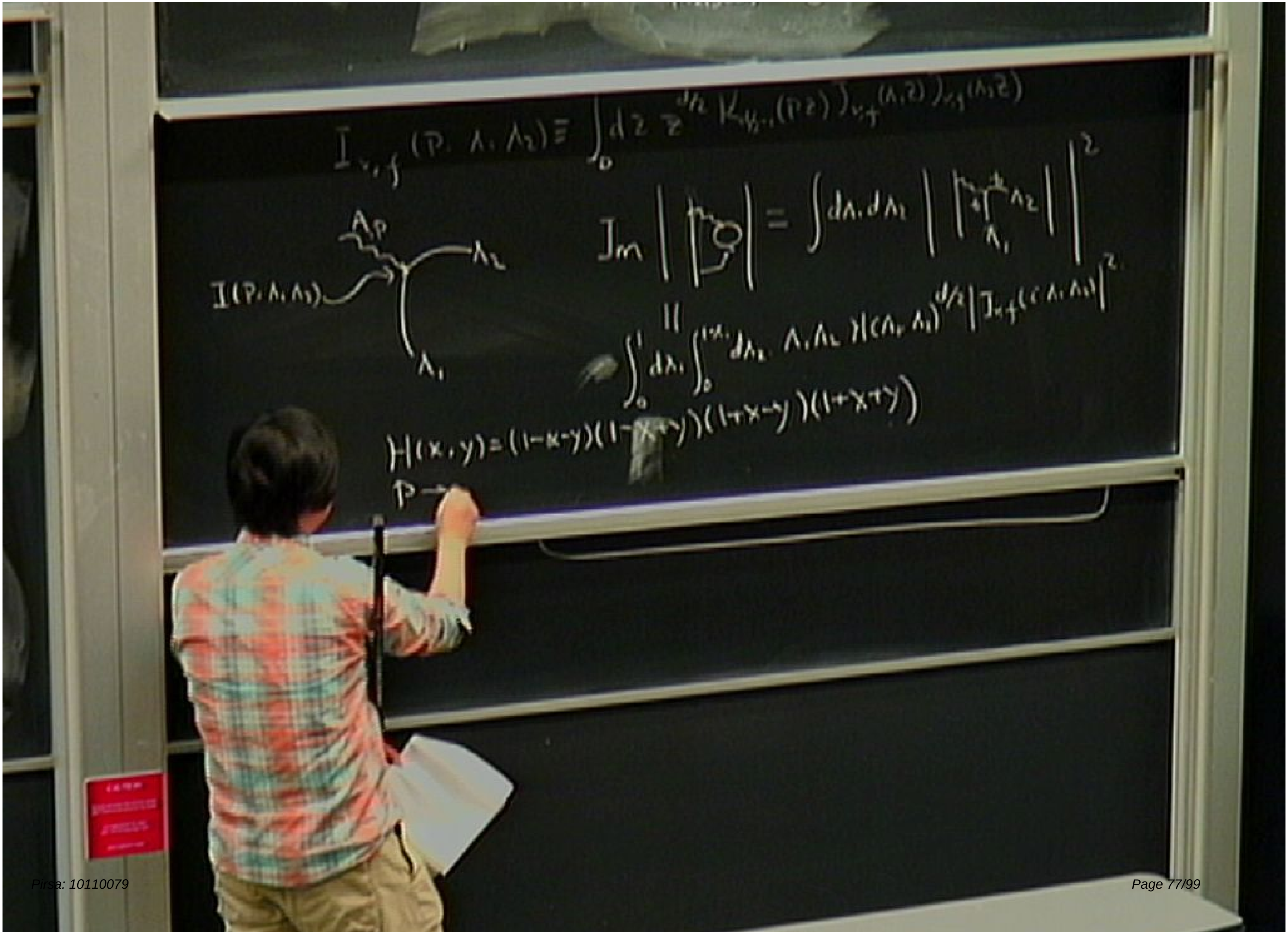

$$\mathrm{Im}\left|\left| \cdots \right|\right| = \int d\Lambda_1\, d\Lambda_2 \left|\left| \cdots \right|\right|^2$$

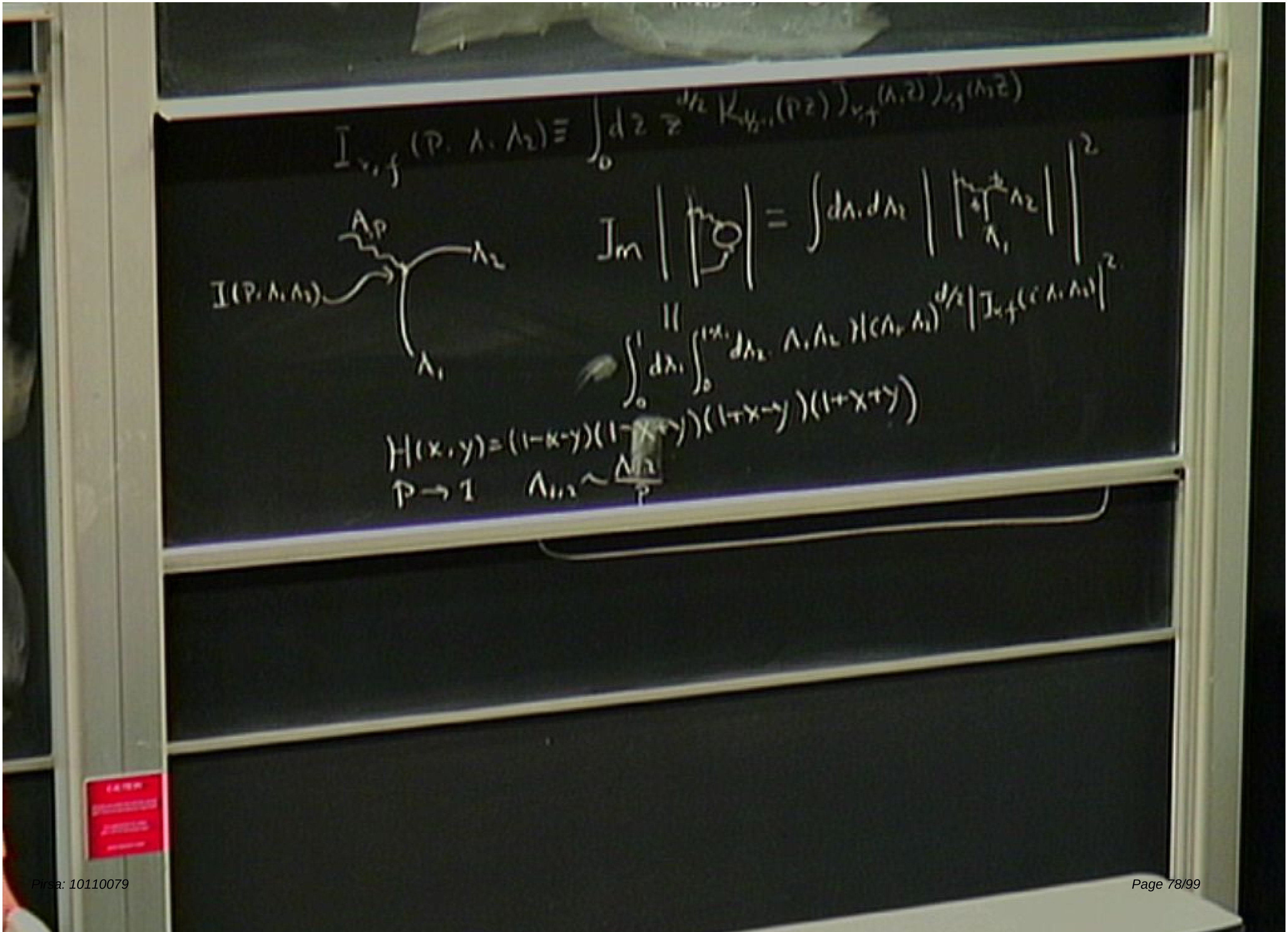
$$= \int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \;\Lambda_1 \Lambda_2\, H(\Lambda_1,\Lambda_2)^{d/2} \left| I_{i,f}(i\,\Lambda_1,\Lambda_2) \right|^2$$

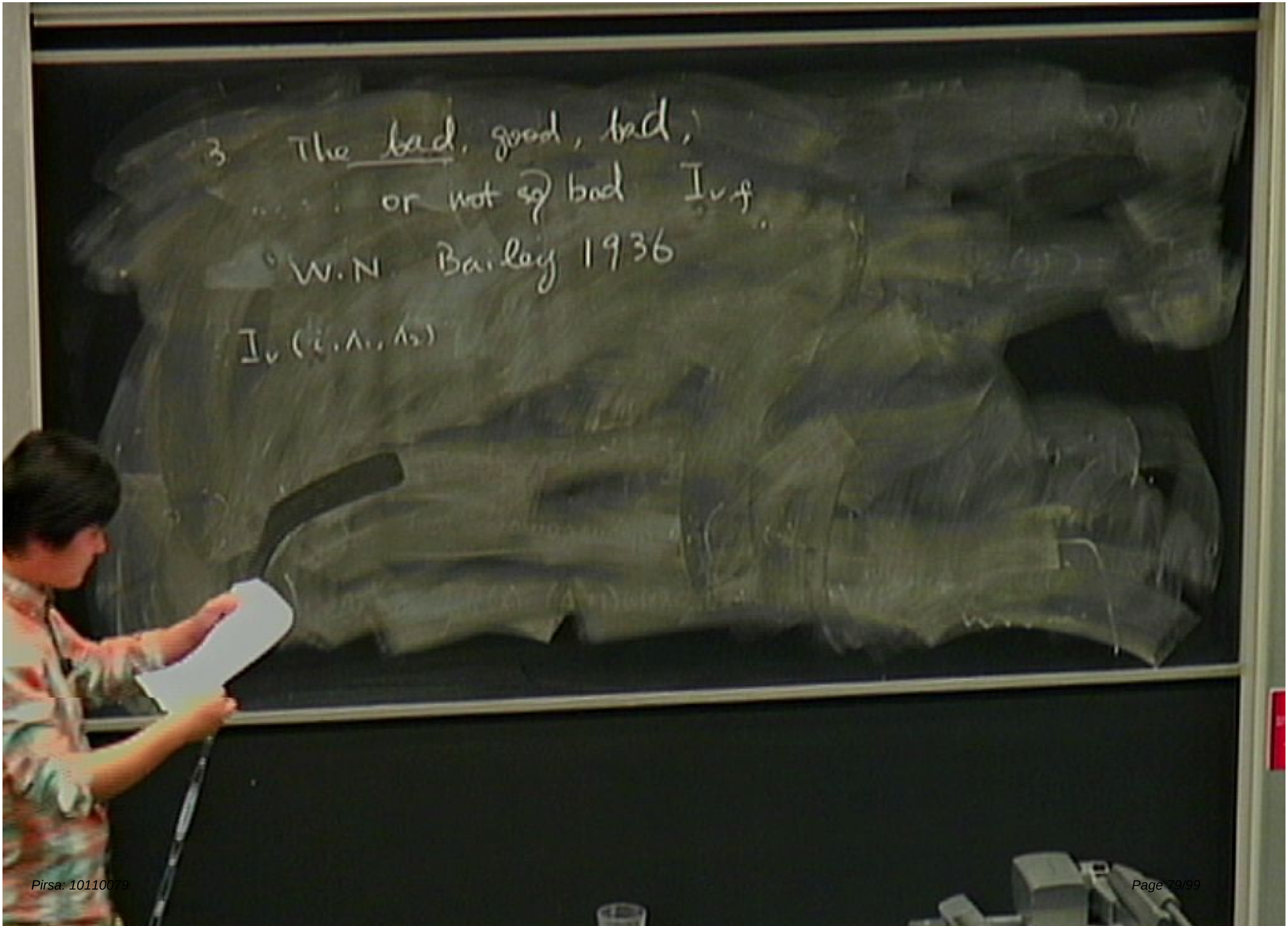
$$H(x,y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$$

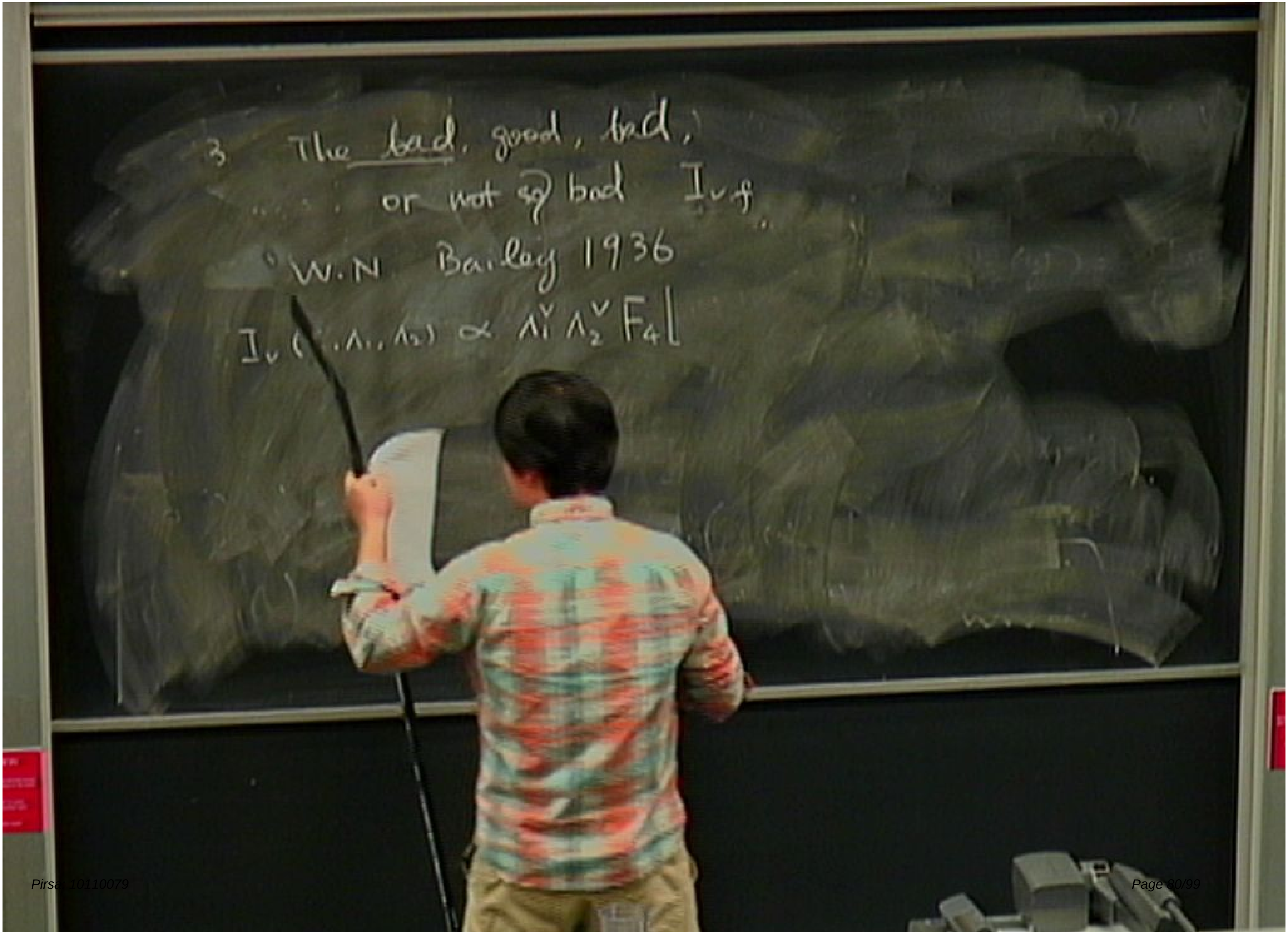

3 The bad, good, bad,

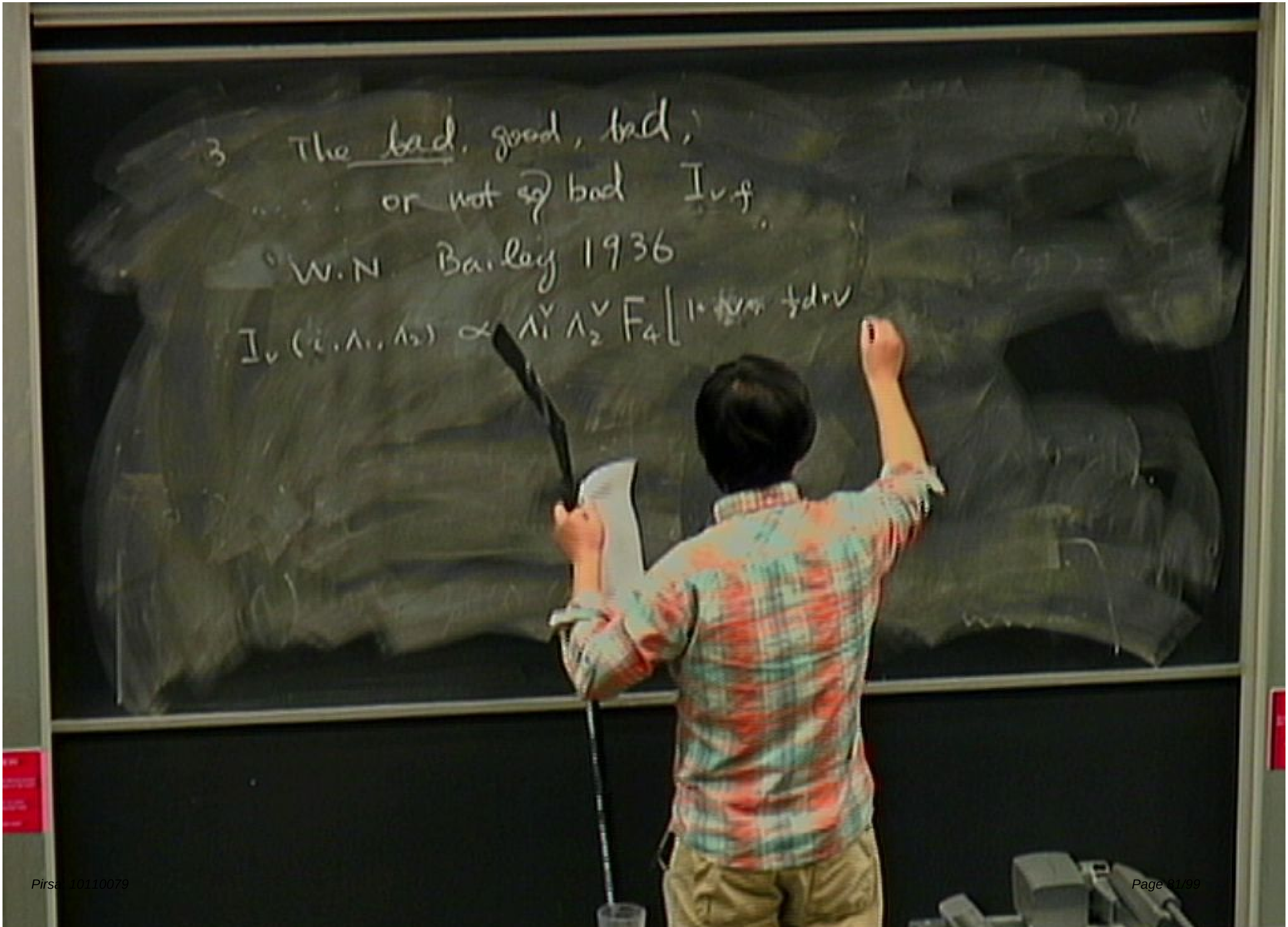

3. The bad, good, bad,
or not so bad

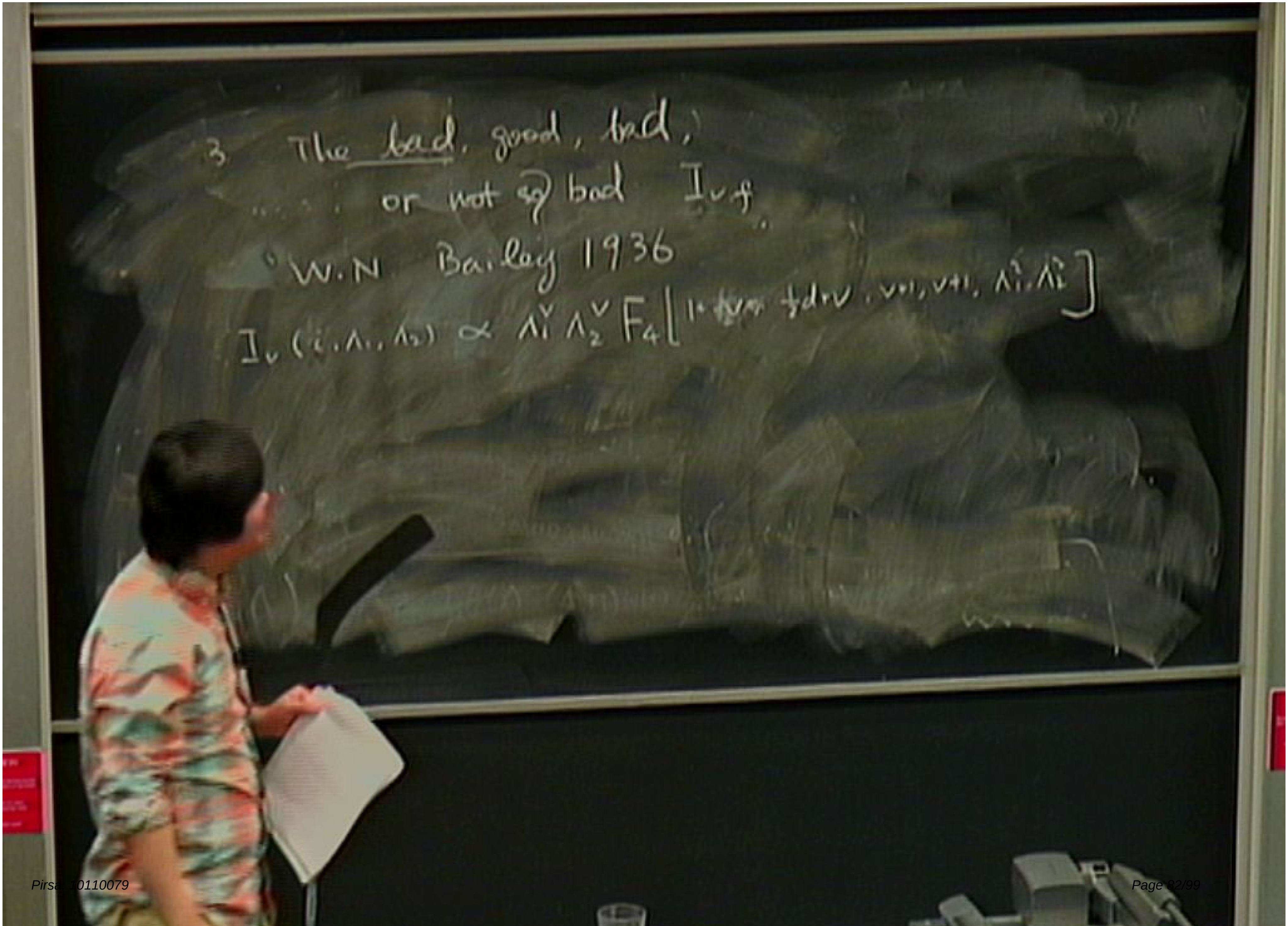

H(x, y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)


H(x,y)=(1−x−y)(1−x+y)(1+x−y)(1+x+y)
p → 1


3. The bad, good, bad,
or not so bad I_v f.
W.N Bailey 1936
I_v (i, λ1, λ2)


3 The bad, good, bad,
or not so bad Iνf
W.N Bailey 1936
Iν ( , Λ1, Λ2) ∝ Λ1ν Λ2ν F4


3 The bad, good, bad;
or not so bad
W.N Bailey 1936
I_v (i, Λ₁, Λ₂) ∝ Λ₁^v Λ₂^v F₄


3. The bad, good, bad,
or not so bad $I_\nu f$
W.N Bailey 1936
$I_\nu(i, \Lambda_1, \Lambda_2) \propto \Lambda_1^\nu \Lambda_2^\nu F_4[\ldots, \nu+1, \nu+1, \Lambda_1^2, \Lambda_2^2]$

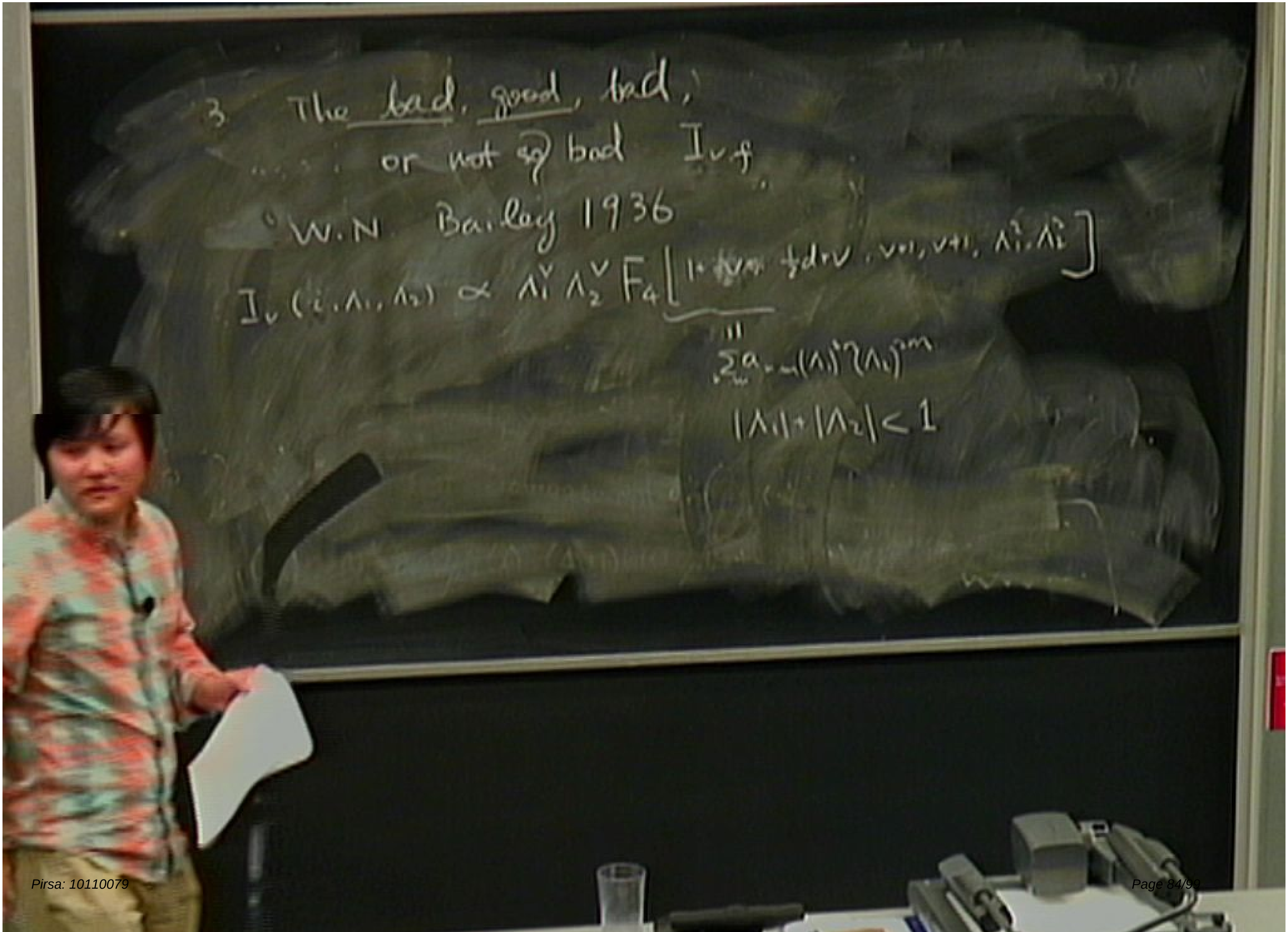
3. The bad, good, bad,

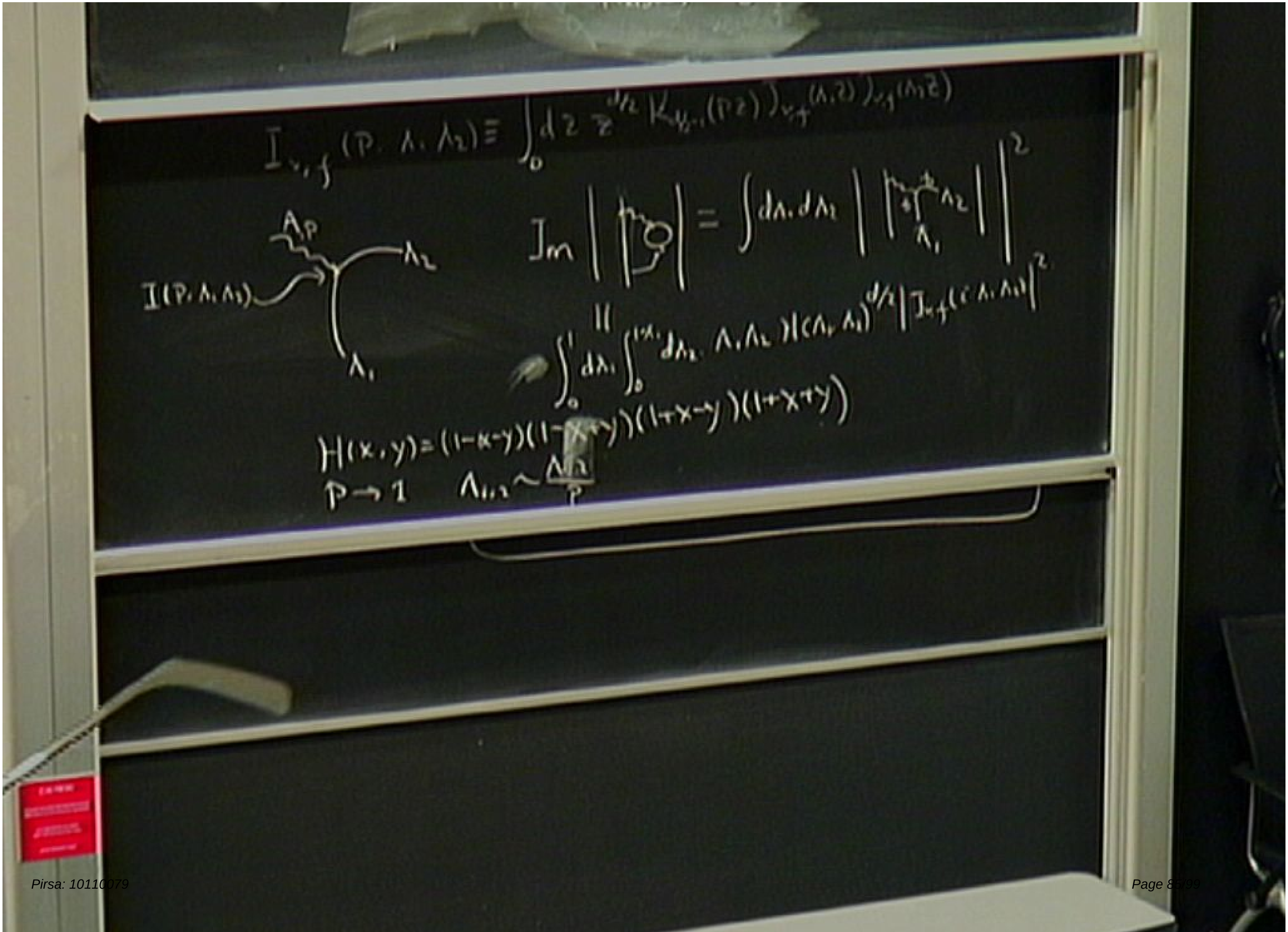
... or not so bad $I_{\nu,f}$

W.N Bailey 1936

$$I_\nu(i,\Lambda_1,\Lambda_2) \propto \Lambda_1^\nu \Lambda_2^\nu F_4\left[1+\tfrac{1}{2}d+\nu,\ \tfrac{1}{2}d+\nu,\ \nu+1,\ \nu+1,\ \Lambda_1^2,\ \Lambda_2^2\right]$$

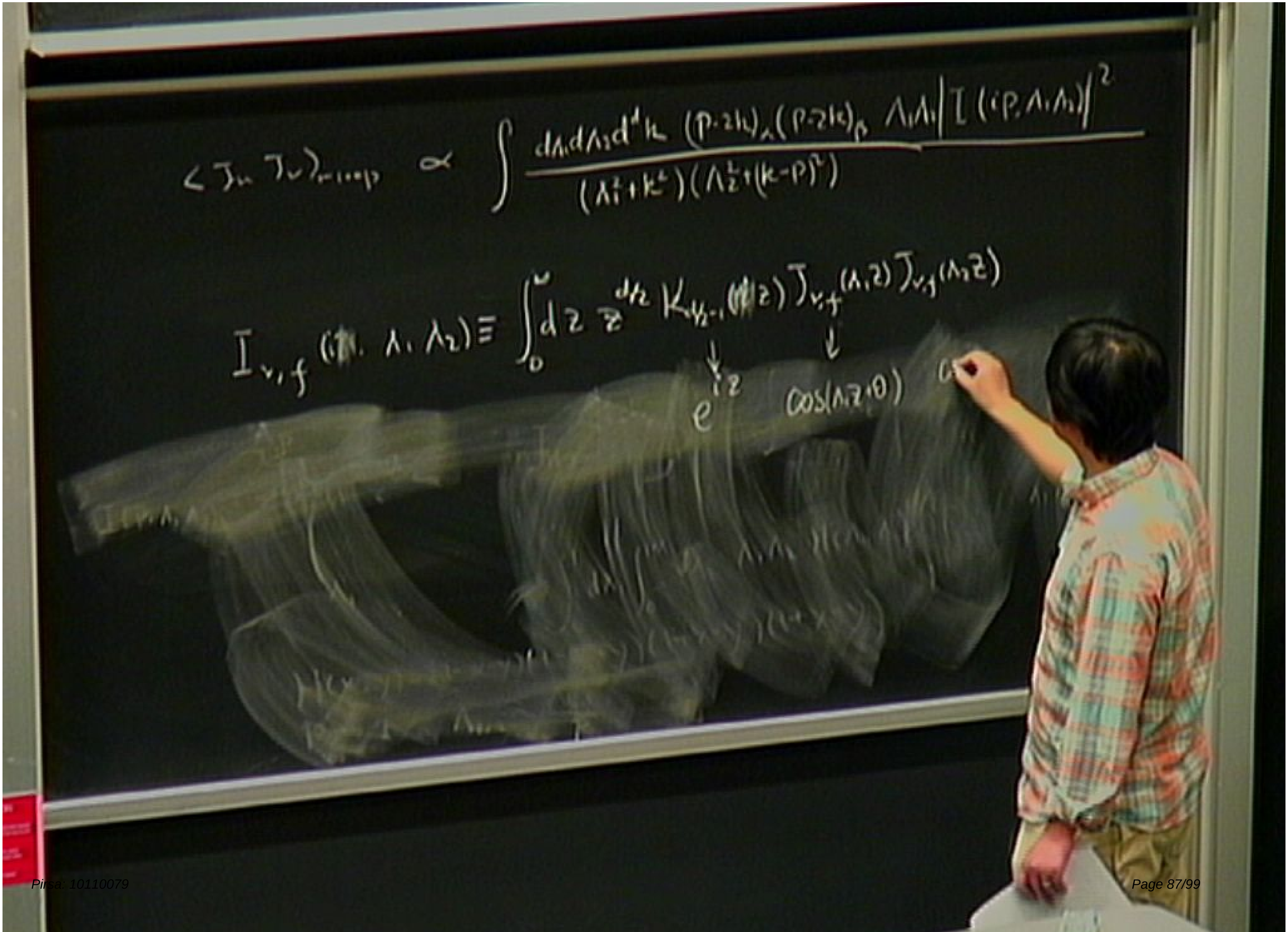
$$= \sum_{n,m} a_{n,m} (\Lambda_1)^{2n} (\Lambda_2)^{2m}$$


3. The bad, good, bad,
... or not so bad I_ν f
W.N Bailey 1936
|Λ1| + |Λ2| < 1


$I_{\nu,f}(p,\Lambda_1,\Lambda_2) \equiv \int_0 dz\, z^{d/2} K_{\nu_{2}-1}(pz) J_{\nu,f}(\Lambda_1 z) J_{\nu,f}(\Lambda_2 z)$
$A_p$
$\Lambda_2$
$I(p,\Lambda_1,\Lambda_2)$
$\Lambda_1$
$\mathrm{Im} \left| \; \right| = \int d\Lambda_1 d\Lambda_2 \left| \; \right|^2$
$\int_0^1 d\Lambda_1 \int_0^{1-\Lambda_1} d\Lambda_2\, \Lambda_1 \Lambda_2\, H(\Lambda_1, \Lambda_2)^{d/2} \left| I_{\nu,f}(\ldots) \right|^2$
$H(x,y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$
$p \to 1 \quad \Lambda_{1,2} \sim \frac{\Lambda}{p}$

$$\langle J_\mu J_\nu \rangle_{1\text{-loop}} \propto \int \frac{d\lambda_1 d\lambda_2 d^d k \; (p-2k)_\alpha (p-2k)_\beta \; \lambda_1 \lambda_2 \left| I(p, \lambda_1, \lambda_2) \right|^2}{(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)}$$

$$I_{\nu,f}(p, \lambda_1, \lambda_2) \equiv \int_0^\infty dz\, z^{d/2} K_{d/2-1}(pz) J_{\nu,f}(\lambda_1 z) J_{\nu,f}(\lambda_2 z)$$

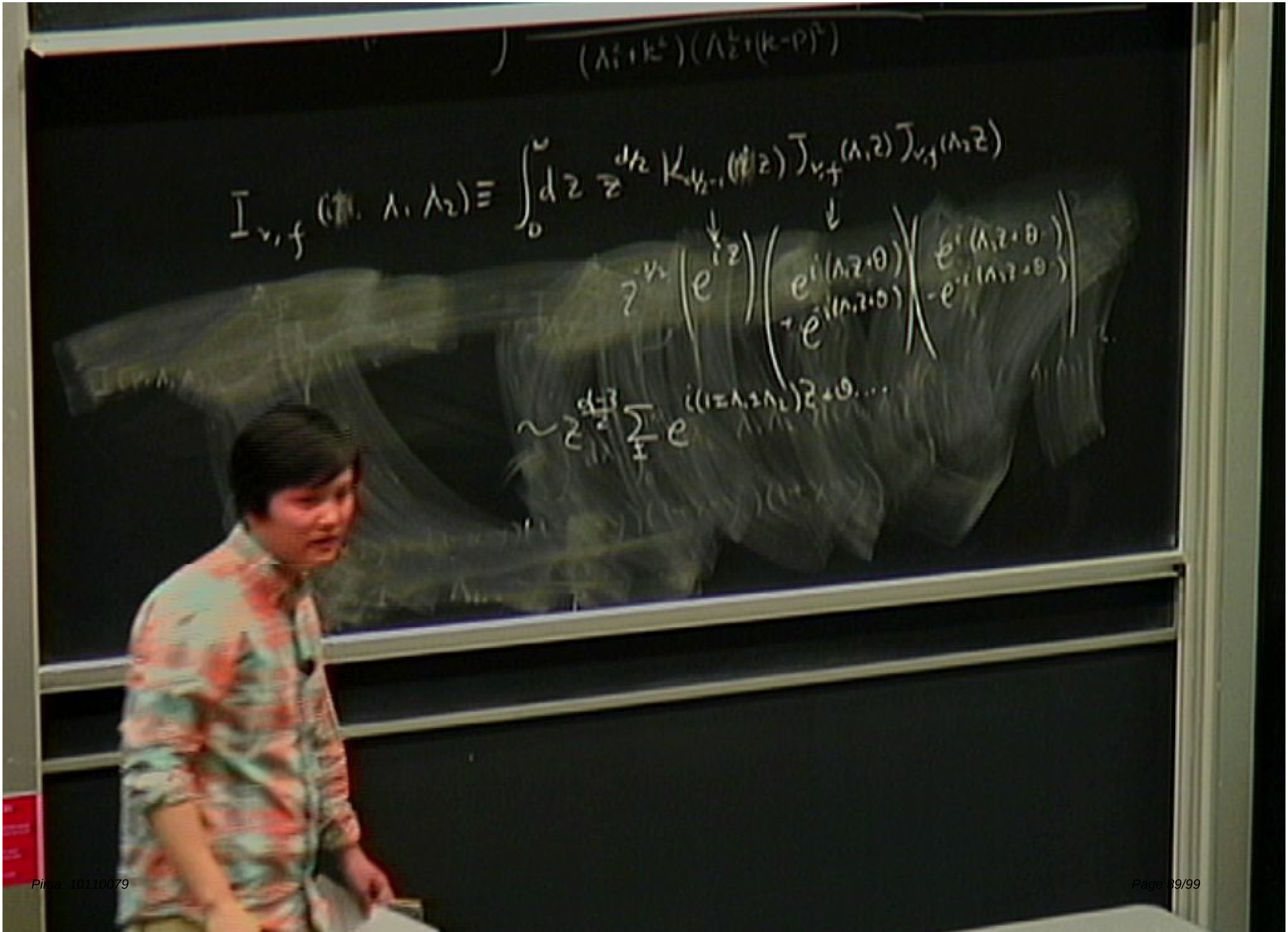

$\langle J_\mu J_\nu \rangle_{\text{1-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \, (p-2k)_\alpha (p-2k)_\beta \, \Lambda_1 \Lambda_2 \left| I(ip, \Lambda_1, \Lambda_2) \right|^2}{(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)}$
$I_{\nu,f}(ip, \Lambda_1, \Lambda_2) \equiv \int_0^\infty dz \, z^{d/2} K_{d/2-1}(pz) \, J_{\nu,f}(\Lambda_1 z) \, J_{\nu,f}(\Lambda_2 z)$
$e^{iz}$
$\cos(\Lambda_1 z + \theta)$

$$\langle J_\mu J_\nu \rangle_{\text{1-loop}} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k \,(p-2k)_\alpha (p-2k)_\beta \,\Lambda_1 \Lambda_2 \left| I(ip,\Lambda_1,\Lambda_2)\right|^2}{(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)}$$

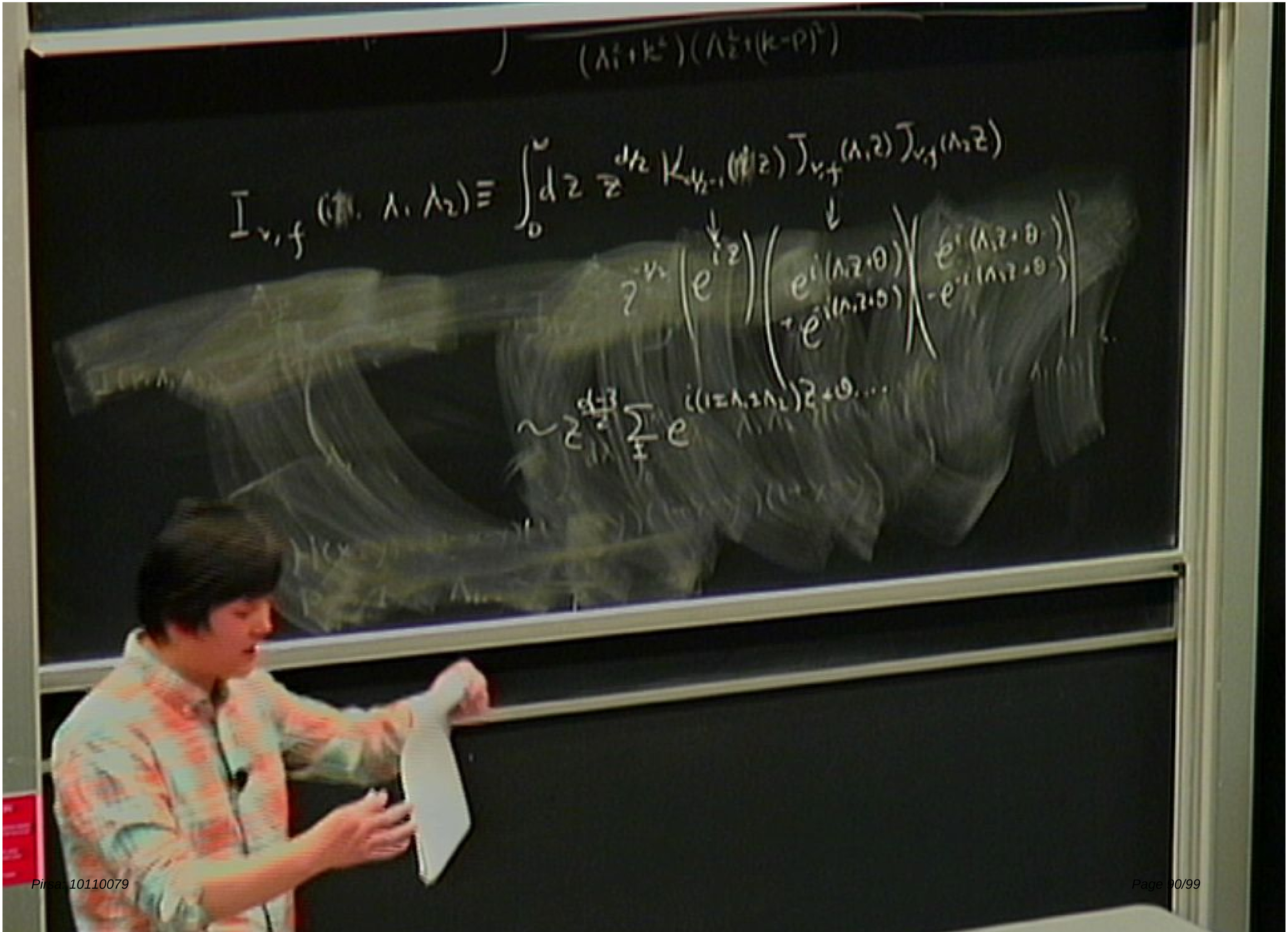
$$I_{\nu,f}(ip,\Lambda_1,\Lambda_2) \equiv \int_0^\infty dz\, z^{d/2} K_{d/2-1}(pz)\, J_{\nu,f}(\Lambda_1 z)\, J_{\nu,f}(\Lambda_2 z)$$

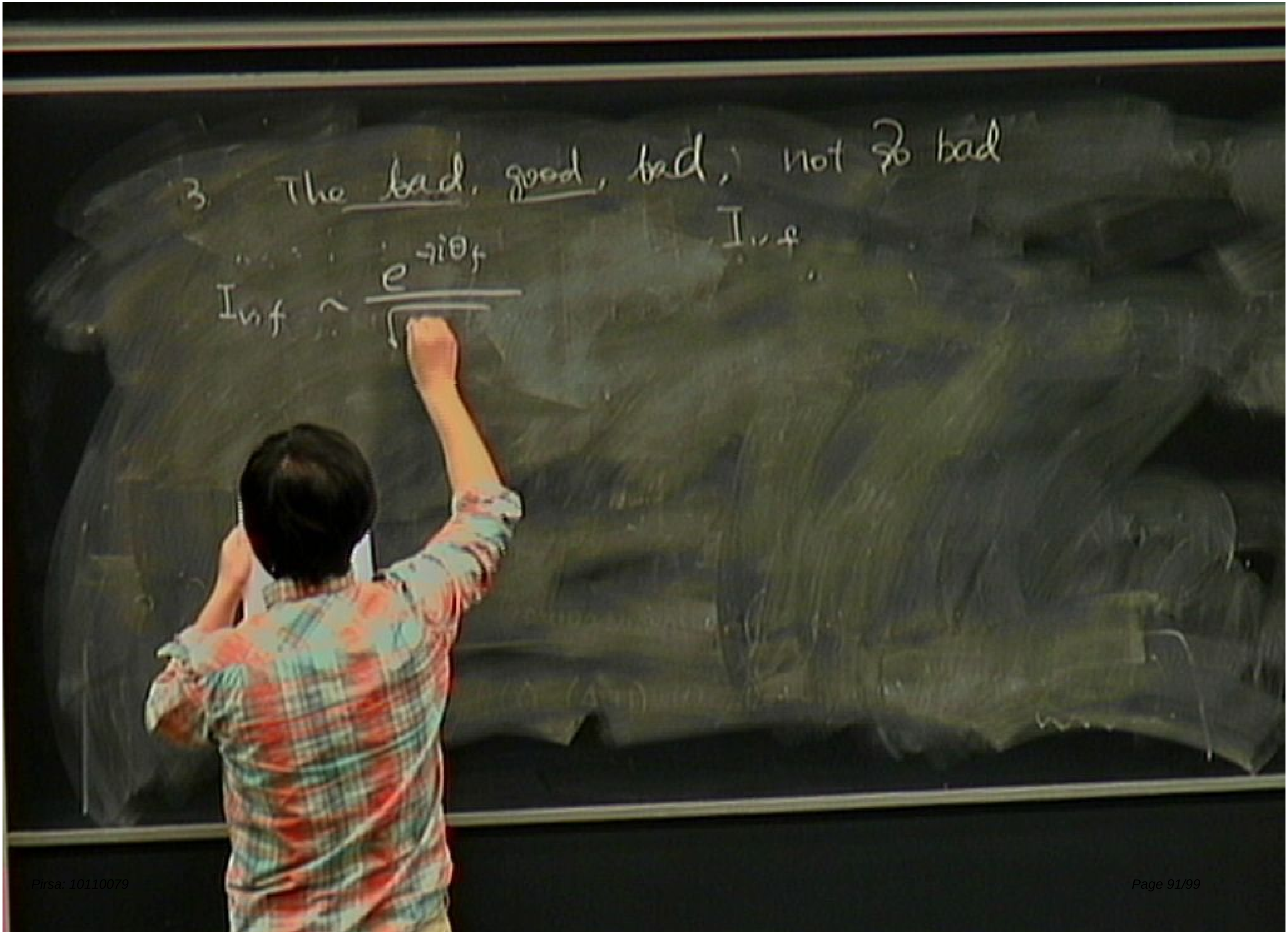
$$\downarrow \qquad\qquad \downarrow$$

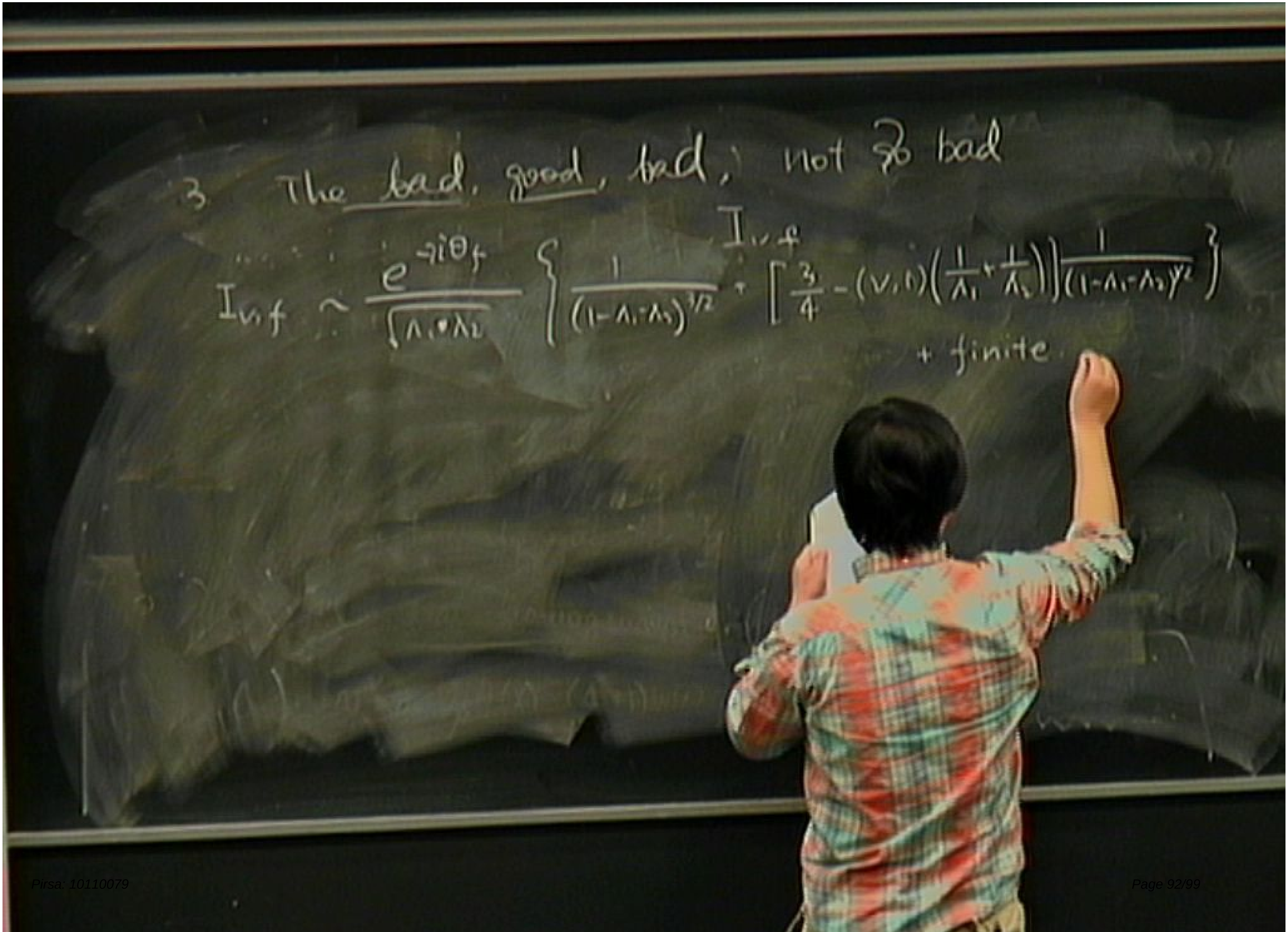
$$\left(e^{iz}\right)\left(e^{i(\Lambda_1 z+\theta)} + e^{-i(\Lambda_1 z+\theta)}\right)\left(e^{i(\Lambda_2 z+\theta_-)} - e^{-i(\Lambda_2 z+\theta_-)}\right)$$



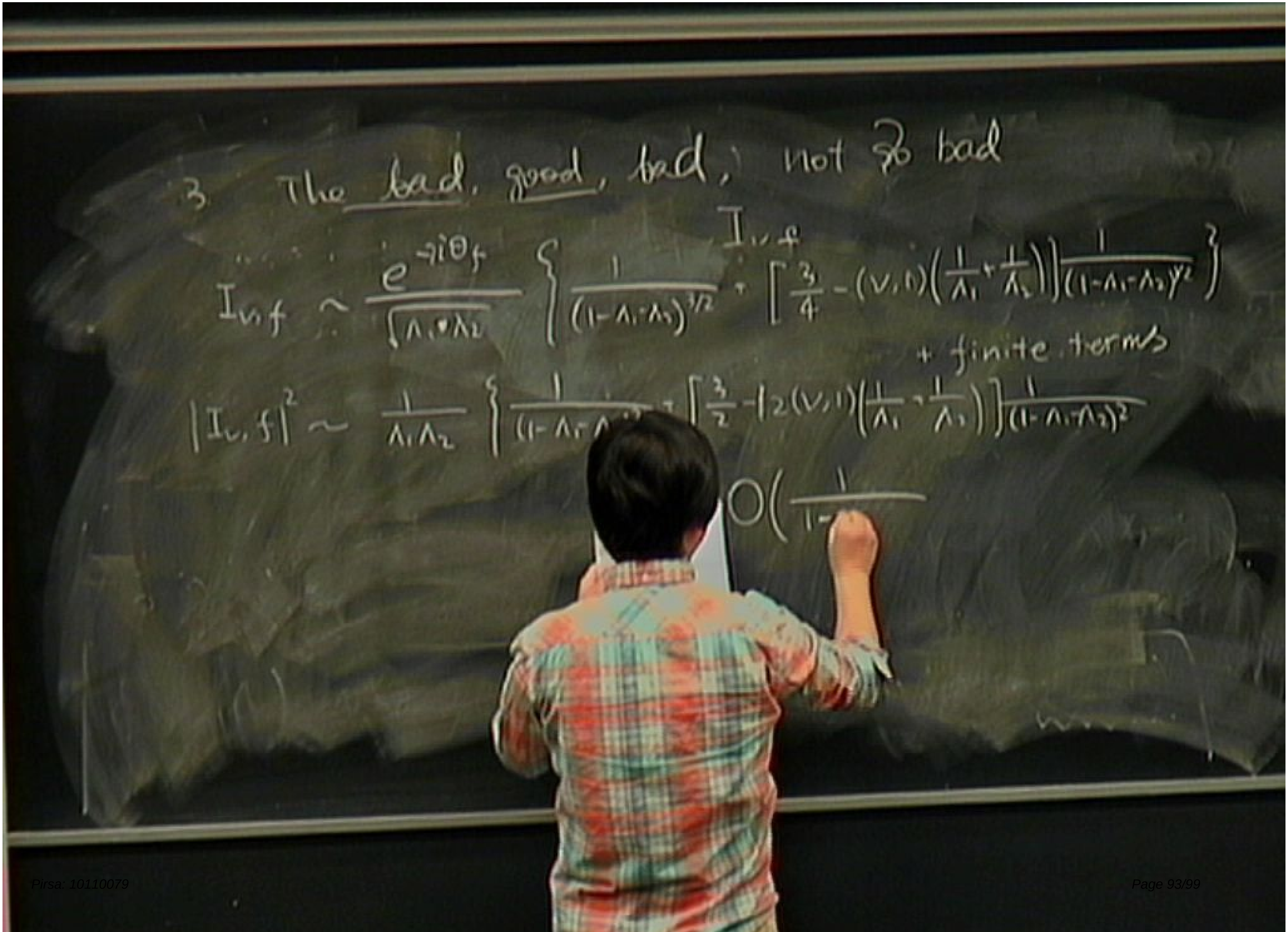
$(\Lambda_1^2+k^2)(\Lambda_2^2+(k-p)^2)$
$I_{\nu,f}(\ \ ,\Lambda_1,\Lambda_2) \equiv \int_0^\infty dz\, z^{d/2} K_{d/2-1}(\ \ z)\, J_{\nu,f}(\Lambda_1 z)\, J_{\nu,f}(\Lambda_2 z)$
$z^{-1/2}\left(e^{iz}\right)\left(e^{i(\Lambda_1 z+\theta)} + e^{-i(\Lambda_1 z+\theta)}\right)\left(e^{i(\Lambda_2 z+\theta)} - e^{-i(\Lambda_2 z+\theta)}\right)$
$\sim z^{\frac{d-3}{2}} \sum e^{i(1 \pm \Lambda_1 \pm \Lambda_2)z+\theta \ldots}$

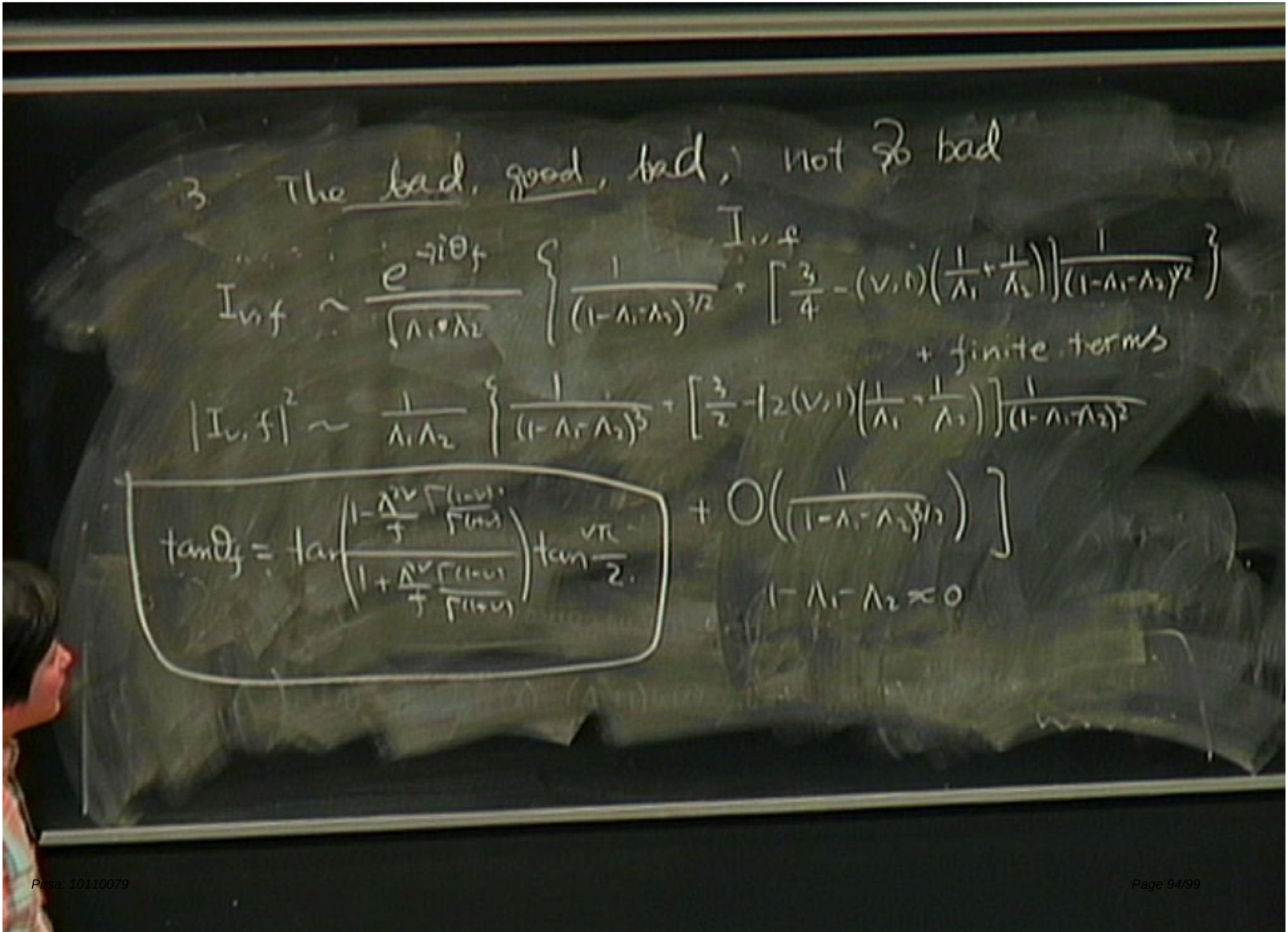

(Λ₁²+k²)(Λ₂²+(k−p)²)
I_{ν,f}(m, Λ₁, Λ₂) ≡ ∫₀^∞ dz z^{d/2} K_{d/2−1}(mz) J_{ν,f}(Λ₁z) J_{ν,f}(Λ₂z)

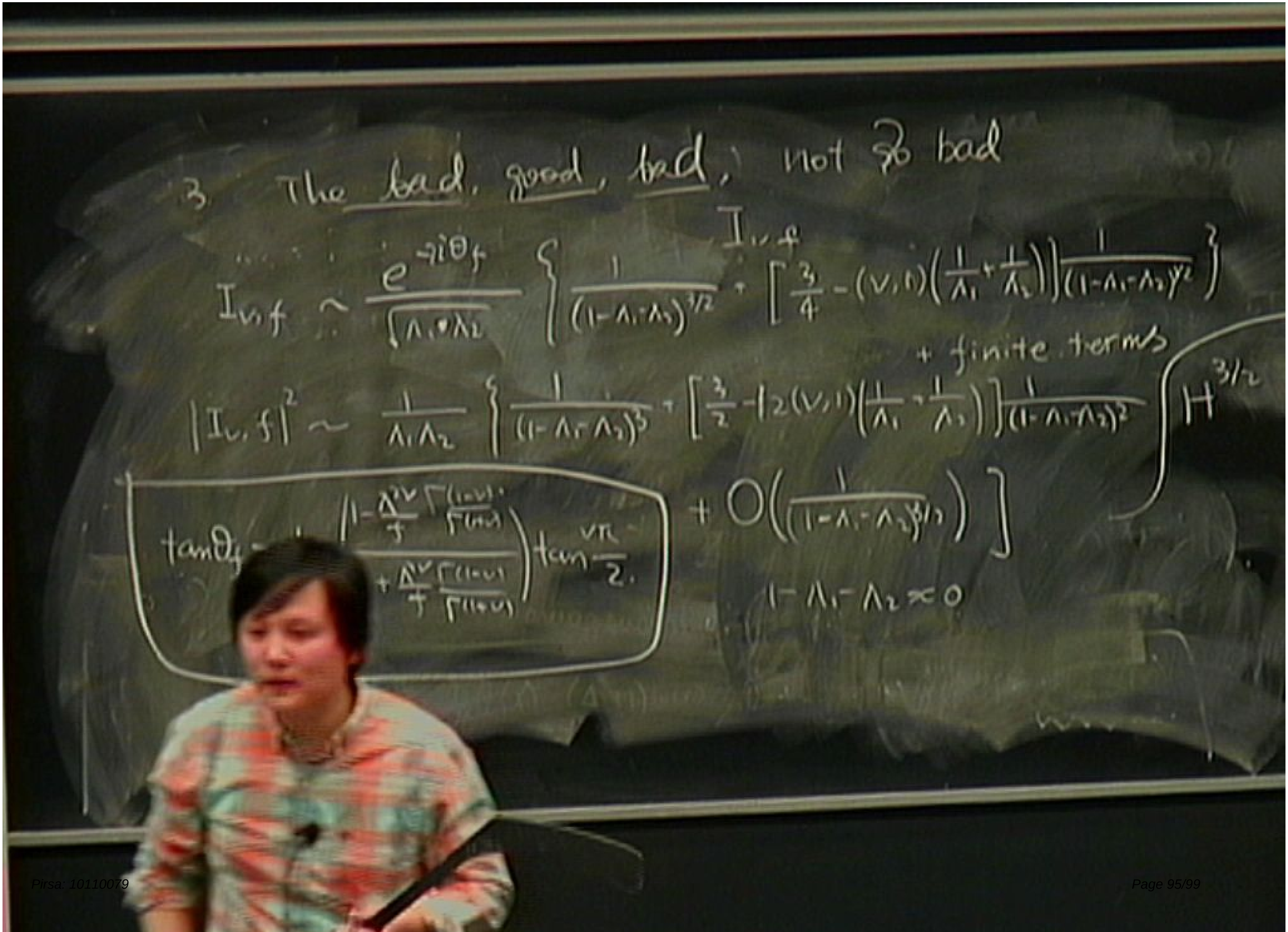

3 The bad, good, bad, not so bad
$I_{inf}$
$I_{inf} \sim \frac{e^{-i\theta_f}}{}$

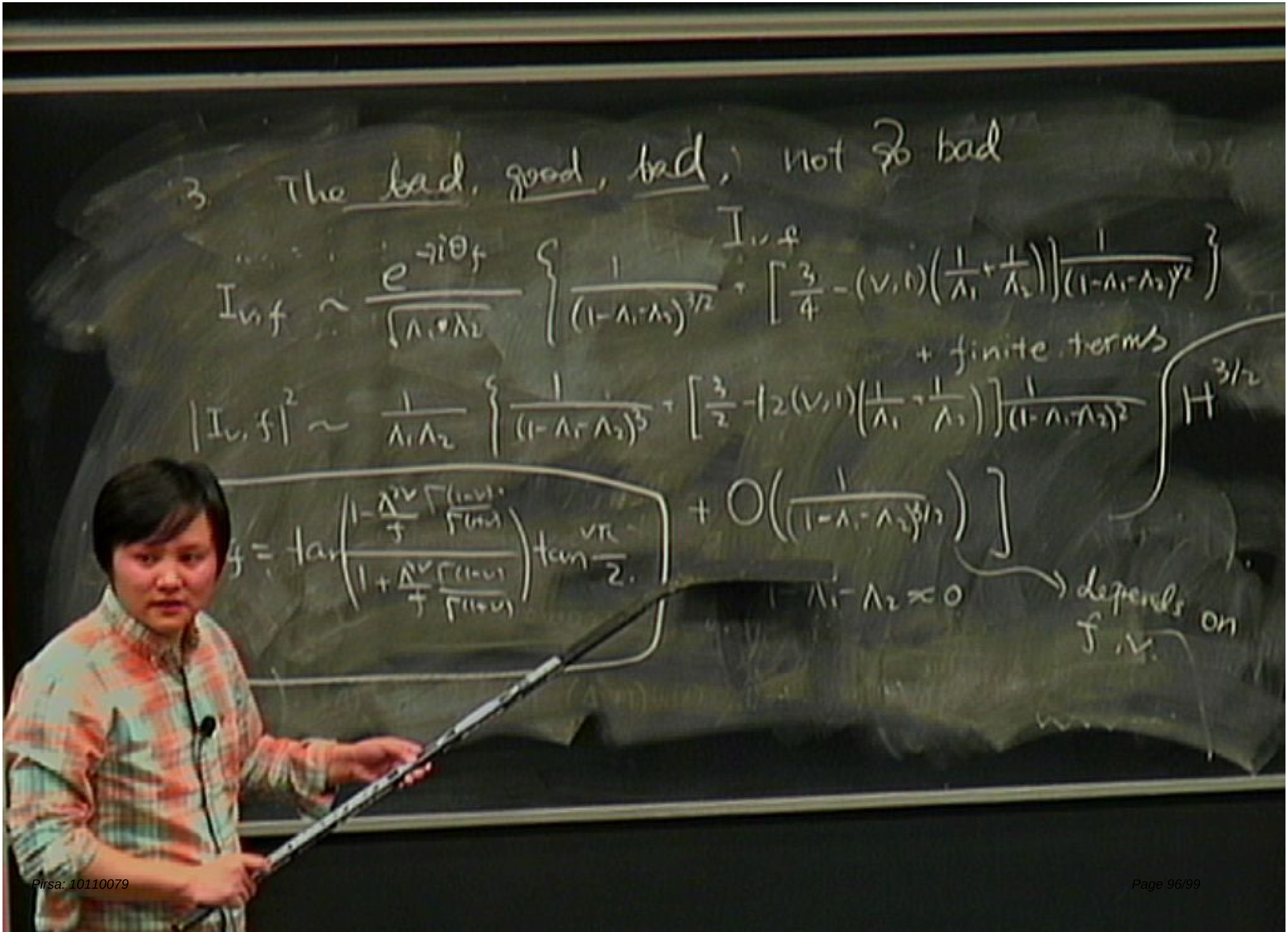


3 The bad, good, bad, not so bad
$I_{v,f}$
$I_{v,f} \sim \frac{e^{-?i\theta_f}}{\sqrt{\Lambda_1 \Lambda_2}} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}} + \left[\frac{3}{4} - (v,1)\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\right]\frac{1}{(1-\Lambda_1-\Lambda_2)^{1/2}} \right\}$
+ finite


3 The bad, good, bad, not so bad
+ finite terms


3 The bad, good, bad, not so bad
I_{ν,f} ~ e^{-2iθ_f} / √(Λ1 Λ2) { 1/(1-Λ1-Λ2)^{3/2} + [3/4 - (ν,1)(1/Λ1 + 1/Λ2)] 1/(1-Λ1-Λ2)^{1/2} }
+ finite terms
|I_{ν,f}|^2 ~ 1/(Λ1 Λ2) { 1/(1-Λ1-Λ2)^3 + [3/2 - 2(ν,1)(1/Λ1 + 1/Λ2)] 1/(1-Λ1-Λ2)^2
+ O(1/(1-Λ1-Λ2)^{3/2}) ]
tan θ_f = tan( (1 - Δ^{2ν}/f · Γ(1-ν)/Γ(1+ν)) / (1 + Δ^{2ν}/f · Γ(1-ν)/Γ(1+ν)) ) tan νπ/2
1 - Λ1 - Λ2 ≈ 0


3. The bad, good, bad, not so bad
+ finite terms
1 − Λ1 − Λ2 ≈ 0


3 The bad, good, bad, not so bad
+ finite terms
depends on

3. The bad, good, bad, not so bad

$$I_{\nu,f} \sim \frac{e^{-2i\theta_f}}{\sqrt{\Lambda_1 \Lambda_2}} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}} + \left[\frac{3}{4} - (\nu,1)\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\right]\frac{1}{(1-\Lambda_1-\Lambda_2)^{1/2}} \right\}$$

+ finite terms

$$|I_{\nu,f}|^2 \sim \frac{1}{\Lambda_1\Lambda_2}\left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^3} + \left[\frac{3}{2} - 2(\nu,1)\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\right]\frac{1}{(1-\Lambda_1-\Lambda_2)^2} + O\left(\frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}}\right)\right]$$

$H^{3/2}$

$$\tan\theta_f = \tan\left(\frac{1-\frac{\Delta^{2\nu}}{f}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}}{1+\frac{\Delta^{\nu}}{f}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}}\right)\tan\frac{\nu\pi}{2}$$

$1-\Lambda_1-\Lambda_2 \approx 0$

→ depends on $f, \nu$

3. The bad, good, bad, not so bad

$$I_{\nu,f} \sim \frac{e^{-2i\theta_f}}{\sqrt{\Lambda_1 \Lambda_2}} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}} + \left[\frac{3}{4} - (\nu,1)\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\right]\frac{1}{(1-\Lambda_1-\Lambda_2)^{1/2}} \right\}$$

+ finite terms

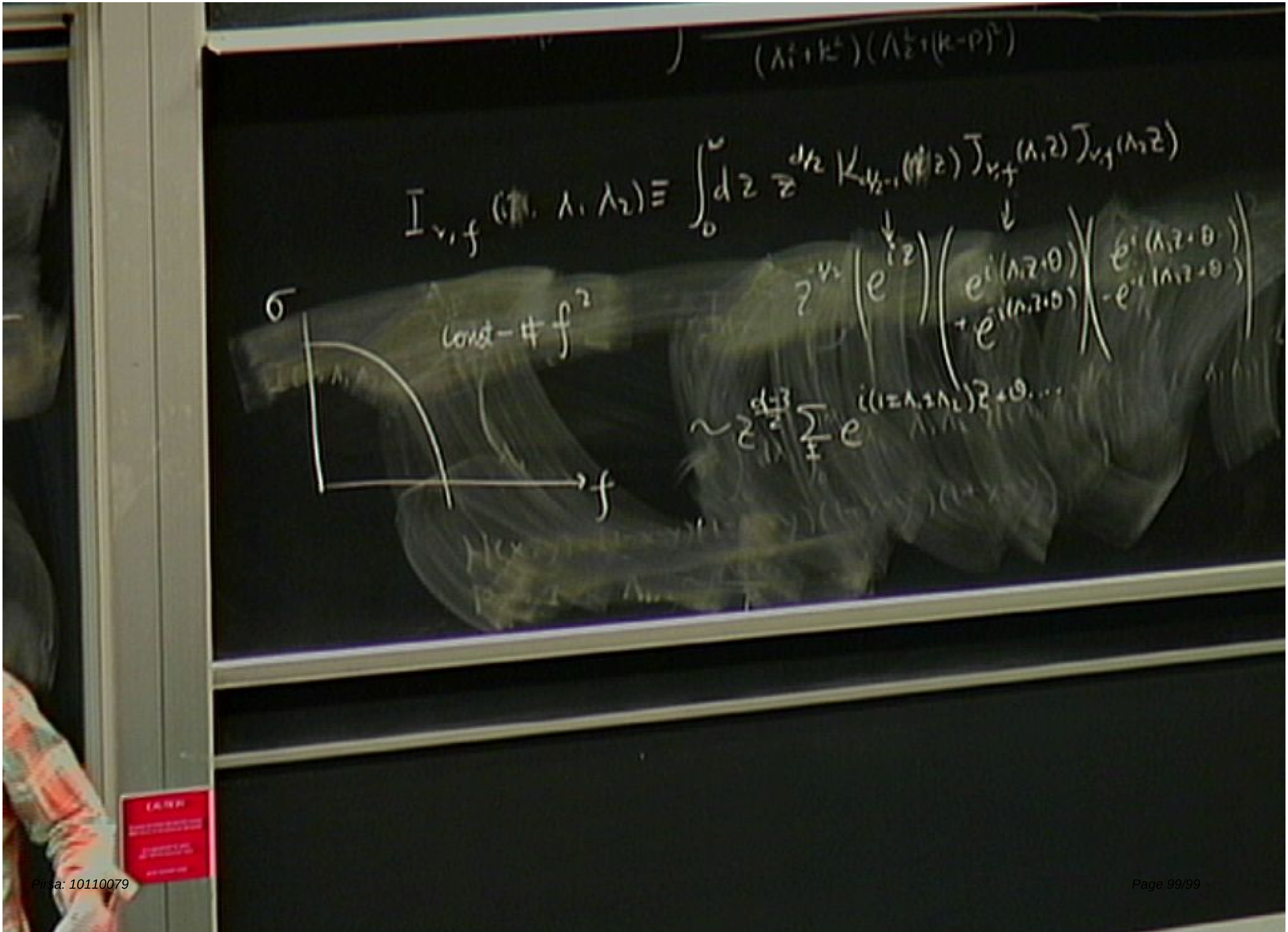
$$|I_{\nu,f}|^2 \sim \frac{1}{\Lambda_1 \Lambda_2} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^3} + \left[\frac{3}{2} - 2(\nu,1)\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\right]\frac{1}{(1-\Lambda_1-\Lambda_2)^2} + O\left(\frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}}\right) \right]$$

$\int H^{3/2}$

$$\tan\theta_f = \tan\left(\frac{1 - \frac{\Lambda^{2\nu}}{f}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}}{1 + \frac{\Lambda^{\nu}}{f}\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}}\right)\tan\frac{\nu\pi}{2}$$

$1-\Lambda_1-\Lambda_2 \approx 0$

depends on $f, \nu$


σ
const − # f²
f