

Title: 1-loop diagram in AdS space and the random disorder problem

Date: Nov 26, 2010 11:00 AM

URL: <http://pirsa.org/10110079>

Abstract: AdS/CFT has proven itself a powerful tool in extending our understanding of strongly coupled quantum theories. While studies of AdS/CFT have predominantly focused on tree level calculations, there has been growing interest in the loop effect recently. We studied the 1-loop correction to the gauge boundary-to-boundary correlator due to its coupling to a complex scalar field. In this talk, I would outline our main results, explain the Cutkosky rule in AdS space, and discuss an extra divergence we found in both real and imaginary part of the loop integral. I would then combine our analysis with the replica trick to demonstrate a possible application where one attempts to calculate the DC conductivity in a condensed matter system with random disorder and discuss the limitation and difficulties of our method in its current form.

1-loop diagram in ADS & Random disorder

Problem Ling-Yan Hung, YS, 1007.2653

1. Replica trick

$$S = S[x] + \int V(x) \mathcal{O}(x)$$

1-loop diagram in ADS & Random disorder
Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)]$$

1-loop diagram in ADS & Random disorder
Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S[x] + \int v(x) O(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

1-loop diagram in ADS & Random disorder

Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S_0[X] + \int v(x) O(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle O(x_1) O(x_2) \dots O(x_n) \rangle = \int$$

1 loop diagram in ADS & Random disorder

Problem Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{D}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \mathcal{D}(x_1) \mathcal{D}(x_2) \dots \mathcal{D}(x_m) \rangle = \frac{\int \mathcal{D}(x) e^{S[x] + \int v \mathcal{D}}}{\int \mathcal{D}(x) e^{S[x]}}$$

1-loop diagram in ADS & Random disorder

Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}(x)}}{\int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}(x)}}$$



1-loop diagram in ADS & Random disorder

Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \dots \mathcal{O}(x_n) \rangle = \frac{\int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}(x)} \mathcal{O}(x_1) \dots \mathcal{O}(x_n)} \int v \dots \int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}(x)}}{\int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}(x)}}$$



Problem

Ling-Ian Hung, IS, 1007-205

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \dots \mathcal{O}(x_n) \rangle = \frac{\int \mathcal{O}(x_1) \mathcal{O}(x_2) \dots \mathcal{O}(x_n) e^{-\frac{1}{2} \int v(x)} e^{S[x] + \int v(x) \mathcal{O}(x)} \mathcal{D}[x]}{\int \mathcal{O}(x) e^{-\frac{1}{2} \int v(x)} e^{S[x] + \int v(x) \mathcal{O}(x)} \mathcal{D}[x]}$$



Problem Ling-Ian Hung, IS, (00/200)

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \dots \mathcal{O}(x_n) \rangle = \frac{\int \mathcal{O}(x) e^{\int v(x) \mathcal{O}(x)} e^{S[x] + \int v \mathcal{O}}}{\int \mathcal{O}(x) e^{S[x] + \int v \mathcal{O}}}$$

$x \rightarrow x_1 \dots x_n \quad n \in \mathbb{Z}$
 $\mathcal{O} \rightarrow \mathcal{O}_1 \dots \mathcal{O}_n$

$$\frac{1}{N} \ln Z(\lambda x)$$



$$z_n = \int \mathcal{D}(x_i) e^{i \sum_i S_0(x_i) - \int \sqrt{\sigma_0} + \int \sigma_0}$$
$$z_n = (z_1)^n$$

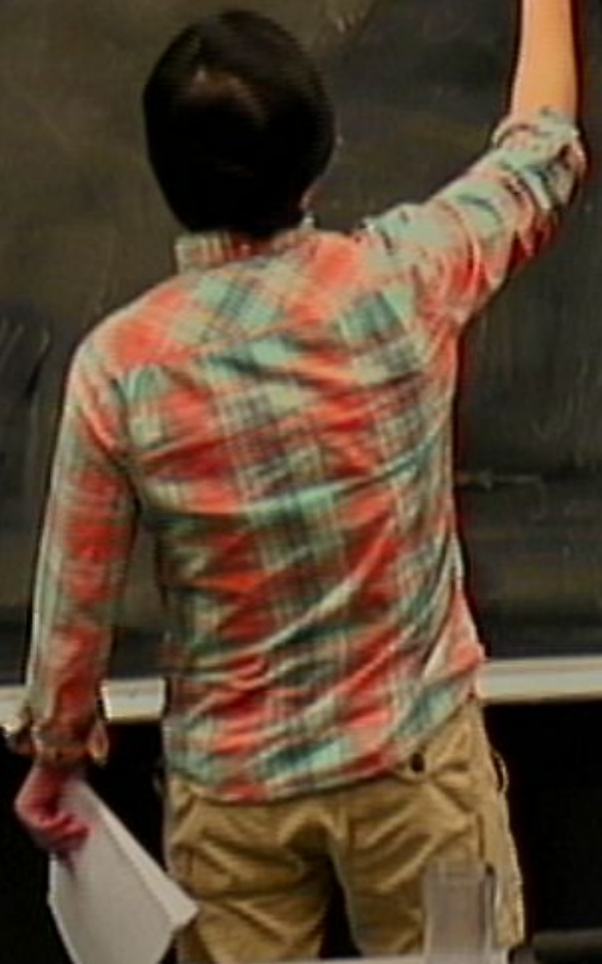


$$Z_n = \int \mathcal{D}(x_i) e$$

$$Z_n = (Z_1)^n$$

$$i \sum_i S_i(x_i) = \int \sqrt{\sigma_0} + \int \sigma_0$$

$$\frac{1}{n} \frac{\delta Z_n}{\delta J} = 0$$



$$Z_n = \int \mathcal{D}(x_i) e^{i \sum_{i=1}^n S_0(x_i) - \int \sqrt{2} \sigma_0 + \int \sigma_0}$$

$$Z_n = (Z_1)^n \xrightarrow{n \rightarrow \infty} \frac{1}{n} \frac{\delta Z_1}{\delta J} = \frac{1}{n} \frac{1}{n} \xrightarrow{\delta J} \frac{1}{n} e^{n \ln Z_1} \xrightarrow{n \rightarrow \infty} e^{n \ln Z_1} \frac{\delta \ln Z_1}{\delta J}$$



$$Z_n = \int \mathcal{D}(x_i) e^{i \sum_{i=1}^n S_0(x_i) - \int \sqrt{2} \sigma_0 + \int \sigma_0}$$

$$Z_n = (Z_1)^n \quad \xrightarrow{n \rightarrow \infty} \frac{1}{n} \frac{\delta Z_n}{\delta J} = \frac{1}{n} \frac{\delta}{\delta J} \xrightarrow{n \rightarrow \infty} \frac{1}{n} \frac{\delta}{\delta J} \xrightarrow{n \rightarrow \infty} \frac{1}{n} \frac{\delta}{\delta J} \xrightarrow{n \rightarrow \infty} \frac{1}{n} \frac{\delta}{\delta J} =$$

$$Z_n = \int \mathcal{D}(x_i) e^{i \sum_{i=1}^n S_0(x_i) - \int \sqrt{2} \sigma_0 + \int \sigma_0}$$

$$Z_n = (Z_1)^n \quad \frac{e^{-\frac{1}{n} \frac{\delta Z_1}{\delta J}}}{\frac{1}{n} \frac{\delta Z_1}{\delta J}} \xrightarrow{\frac{e^{-\frac{1}{n} \frac{\delta Z_1}{\delta J}}}{\frac{1}{n} \frac{\delta Z_1}{\delta J}}} \frac{e^{-\frac{1}{n} \frac{\delta Z_1}{\delta J}}}{\frac{1}{n} \frac{\delta Z_1}{\delta J}} \xrightarrow{\frac{e^{-\frac{1}{n} \frac{\delta Z_1}{\delta J}}}{\frac{1}{n} \frac{\delta Z_1}{\delta J}}} \frac{e^{-\frac{1}{n} \frac{\delta Z_1}{\delta J}}}{\frac{1}{n} \frac{\delta Z_1}{\delta J}} = \frac{\delta \ln Z_1}{\delta J}$$

1-loop diagram in ADS & Random disorder
 Problem. Ling-Yan Hung, YS. 1007.2653

1. Replica trick

$$S = S_0[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \dots \rangle = \frac{\int \mathcal{D}(x) e^{-\frac{1}{2} \int v(x)} \int \mathcal{D}(x) e^{S_0(x) + \int v \mathcal{O}(x)}}}{\int \mathcal{D}(x) e^{-\frac{1}{2} \int v(x) + \int v \mathcal{O}(x)}}$$

$\rightarrow x_1, \dots, x_n \quad n \in \mathbb{Z}$
 $\mathcal{O} \rightarrow \mathcal{O}_1, \dots, \mathcal{O}_n$

1-loop diagram in ADS & Random disorder
 Problem Ling-Yan Hung, YS, 1007.2653

1. Replica trick

$$S = S_0[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \dots \rangle = \frac{1}{Z} \int \mathcal{D}[x] e^{-S[x]}$$

$$\langle \dots \rangle = \frac{1}{Z} \int \mathcal{D}[x] e^{-\frac{1}{2} \int v(x)} \frac{\int \mathcal{D}[x] e^{S[x] + \int v \mathcal{O}}}{\int \mathcal{D}[x] e^{S[x] + \int v \mathcal{O}}}$$

$$x \rightarrow x_1, \dots, x_n$$

$$\mathcal{O} \rightarrow \mathcal{O}_1, \dots, \mathcal{O}_n$$

Problem

Ling-Yan Hung, IS, 10

1. Replica trick

$$S = S_0[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\int \mathcal{D}(x) e^{iS(x) + \int v(x) \mathcal{O}(x)}$$
$$\int \mathcal{D}(x) e^{iS(x)}$$
$$\int \mathcal{D}(v) e^{-\frac{1}{2} \int v^2} \int \mathcal{D}(x)$$

Problem Ling-Yan Hung, IS. 10

1. Replica trick

$$S = S_0[X] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x)^2 \right\}$$

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \lim_{n \rightarrow 0} \frac{\int \mathcal{O}_1 \dots \mathcal{O}_n e^{S_0[X] + \int v(x) \mathcal{O}(x)} \mathcal{D}[X]}{\int e^{S_0[X] + \int v(x) \mathcal{O}(x)} \mathcal{D}[X]}$$

$$\int \mathcal{D}[X] e^{S_0[X] + \int v(x) \mathcal{O}(x)}$$
$$\int \mathcal{D}[X] e^{S_0[X]}$$
$$\int \mathcal{D}[v] e^{-\frac{1}{2} \int v^2} \int \mathcal{D}[X]$$

1. Replica trick

$$S = S_0[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \lim_{n \rightarrow 0} \frac{\int \mathcal{O}(x) e^{iS(x) + \int v(x) \mathcal{O}(x)} \mathcal{D}[x]}{\int \mathcal{O}(x) e^{iS(x)} \mathcal{D}[x]}$$

Problem Ling-Yan Hung, YS. 1007, 2653

1. Replica trick

$$S = S_0[X] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{O}(x) e^{iS(x) + \int v \mathcal{O}(x)}}{\int \mathcal{O}(x) e^{iS(x) + \int v \mathcal{O}(x)}}$$

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \lim_{n \rightarrow 0} \frac{\int \mathcal{O}_1 \dots \mathcal{O}_n e^{iS(x) + \int v \mathcal{O}(x)}}{\int e^{iS(x) + \int v \mathcal{O}(x)}}$$

Problem Ling-Yan Hung, YS, 1007, 2653

1. Replica trick

$$S = S_0[X] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{D}[X] e^{i(S[X] + \int v \mathcal{O})}}{\int \mathcal{D}[X] e^{i(S[X] + \int v \mathcal{O})}}$$

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \lim_{n \rightarrow 0} \frac{1}{n} \frac{\partial}{\partial v_i} \dots$$

$$\frac{\partial}{\partial v_i} \left(\int \mathcal{D}[X] e^{i(S[X] + \int v \mathcal{O})} \right)$$

Problem Ling-Yan Hung, YS, 1007, 2653

1. Replica trick

$$S = S[X] + \int v(x) \phi(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \phi(x) e^{S(x) + \int v \phi} \phi(x) \phi(x)}{\int \phi(x) e^{S(x) + \int v \phi}}$$

$$\langle \phi \dots \phi \rangle = \lim_{n \rightarrow 0} \frac{\int \phi(x) \dots \phi(x) e^{S(x) + \int v \phi}}{\int \phi(x) e^{S(x) + \int v \phi}}$$

Problem Ling-Yan Hung, YS, 1007, 2653

1. Replica trick

$$S = S[X] + \int v(x) \mathcal{O}(x)$$

$$P(v(x)) = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{O}(x) e^{S[X] + \int v \mathcal{O}}}{\int \mathcal{O}(x) e^{S[X] + \int v \mathcal{O}}}$$

$$\langle \mathcal{O} \dots \mathcal{O} \rangle = \lim_{n \rightarrow 0} \frac{\int \mathcal{O} \dots \mathcal{O} e^{S[X] + \int v \mathcal{O}}}{\int e^{S[X] + \int v \mathcal{O}}}$$

Problem Ling-Yan Hung, YS, 1007, 2653

1. Replica trick

$$S = S_0[X] + \int v(x) \mathcal{O}(x)$$

$$P[V(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{D}[X] e^{S_0[X] + \int v \mathcal{O}}}{\int \mathcal{D}[X] e^{S_0[X] + \int v}}$$

$$\langle \mathcal{O} \dots \mathcal{O} \rangle = \lim_{n \rightarrow 0} \frac{\int \mathcal{D}[X] e^{+i2S_0[X] + \frac{1}{2} \int (\sum \mathcal{O}_i)^2}}$$

Problem Ling-Yan Hung, YS, 1007, 2653

1. Replica trick

$$S = S[x] + \int v(x) \mathcal{O}(x)$$

$$P[v(x)] = \exp \left\{ -\frac{1}{2} \int v(x) \right\}$$

$$\frac{\int \mathcal{D}[x] e^{S[x] + \int v \mathcal{O}(x)}}{\int \mathcal{D}[x] e^{S[x] + \int v \mathcal{O}(x)}}$$

$$\langle \mathcal{O} \dots \mathcal{O} \rangle = \lim_{n \rightarrow 0} \frac{\sum_{\mathcal{O}}}{\delta S(x) - \delta S(x)} \int \mathcal{D}[x] e^{-i \sum S_n(x) + \frac{1}{2} \int (\sum \mathcal{O}_i)^2}$$

$$z_n = \int \mathcal{D}(x_i) e^{i \int \mathcal{L}(x_i) dt} = \int \mathcal{D}z_0 + i \int \mathcal{L}(z_0) dt$$

$$z_n = (z_1)^n \quad \frac{e^{-\frac{1}{n} \frac{\delta z_n}{\delta J}}}{n} = \frac{e^{-\frac{1}{n} \frac{1}{\delta J}}}{n} \xrightarrow{n \rightarrow \infty} e^{-n \ln z_1} \frac{\delta \ln z_1}{\delta J} = \frac{\delta \ln z_1}{\delta J}$$

$$0_i \rightarrow \phi_i$$

$$\text{AdS} \quad ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$$

$z=0$ boundary.

$z=1$

$$z_n = (z_1)^n \quad \frac{d}{dz} \frac{1}{z} = -\frac{1}{z^2} \quad \frac{d}{dz} \frac{1}{z^n} = -\frac{n}{z^{n+1}}$$

$$O_i \rightarrow \phi_i$$

$$\text{AdS} : ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$$

$z=0$ boundary

$z=\infty$ horizon

$$\phi \stackrel{z \rightarrow \infty}{\sim} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$$

$$O_i \rightarrow \phi_i$$

$$\text{AdS} : ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$$

$z=0$ boundary

$z=\infty$ horizon

$$\phi \Big|_{z \rightarrow 0} \propto z^{\Delta_-} + \beta z^{\Delta_+}$$

$\int \mathcal{O} \phi$

$$O_i \rightarrow \phi_i$$

$$\text{AdS} : ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$$

$z=0$ boundary

$z=\infty$ horizon

$$\phi \stackrel{z \rightarrow 0}{\sim} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$$

$$\int \partial \phi \quad \int$$

$$O_i \rightarrow \phi_i$$

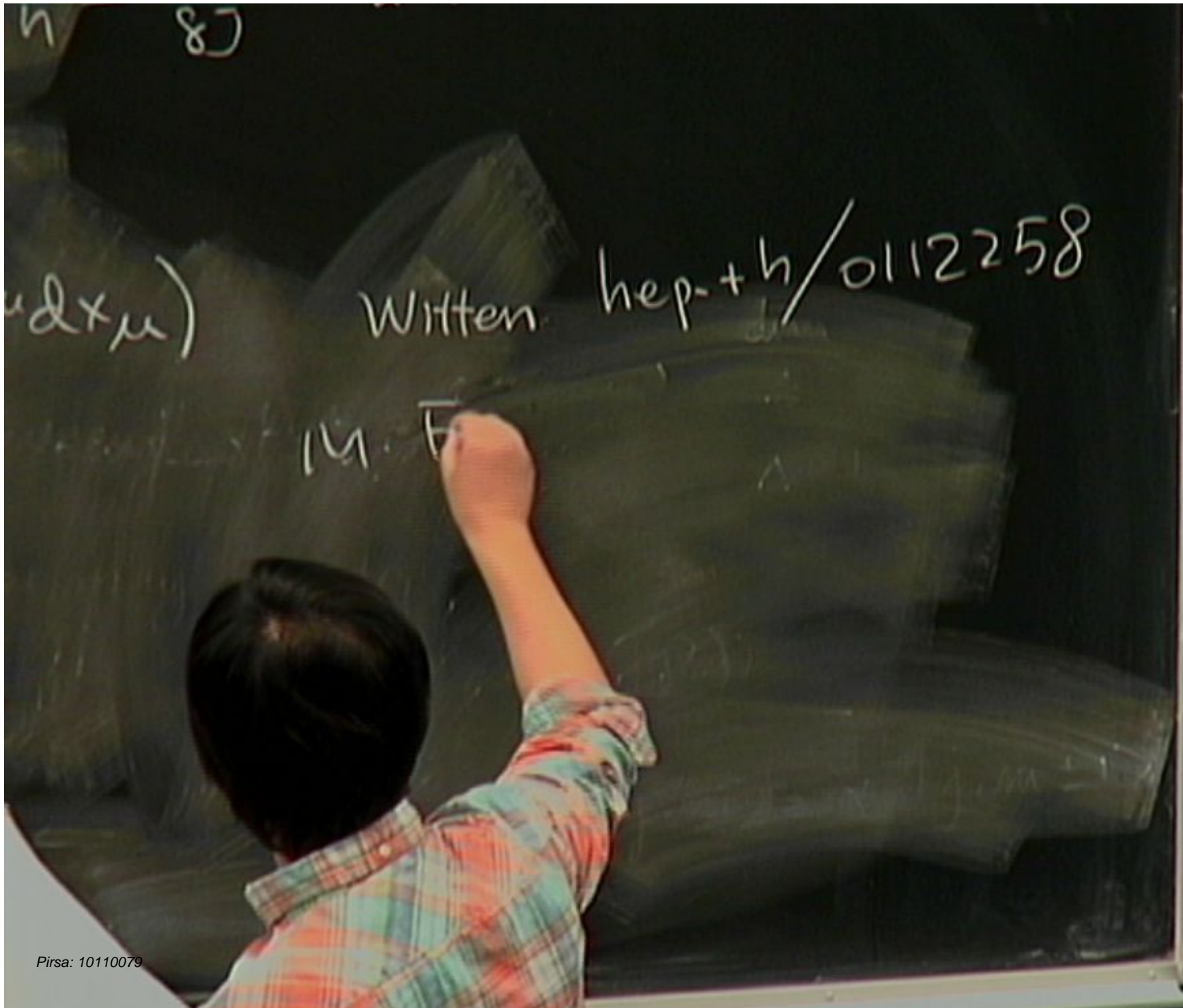
$$\text{AdS} : ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$$

$z=0$ boundary

$z=\infty$ horizon

$$\phi \stackrel{z \rightarrow 0}{\sim} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$$

$$\int \mathcal{O} \phi \quad \langle \mathcal{O} \rangle = \beta$$



(dx^μ)

Witten hep-th/0112258

14. F

$$\frac{1}{n} \text{Senenzi} \rightarrow \frac{0}{n \rightarrow 0} e^{n \ln 2} \frac{\text{Senenzi}}{8J} = \frac{0}{8J}$$

(dx, μ)

Witten hep-th/0112258

M. Fujita, Y. Hikida

S. Ryu, T. Takayanagi
0810.5394

$$Z_n = \int \mathcal{D}(x_i) e^{i \int \mathcal{L}(x_i) dx} = \int \mathcal{D}z_0 + \int \mathcal{D}z_1$$

$$Z_n = (z_1)^n \quad \frac{d}{dz_1} \frac{1}{n} \frac{\delta Z_n}{\delta z_1} = \frac{d}{dz_1} \frac{1}{n} \frac{\delta e^{n \ln z_1}}{\delta z_1} = \frac{d}{dz_1} e^{n \ln z_1} \frac{\delta \ln z_1}{\delta z_1} = \frac{\delta \ln z_1}{\delta z_1}$$

$0_i \rightarrow \phi_i$

AdS : $ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$

$z=0$ boundary
 $z=\infty$ horizon
 $\phi \xrightarrow{z \rightarrow \infty} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$
 $\langle O \rangle = \beta$

Witten hep-th/0112258
 M. Fujita, Y. Hikida
 S. Ryu, T. Takayanagi
 0810.5394

$$+ \frac{f}{2} \int (\Sigma \beta_i)^2$$



$$z_n = (z_1)^n \quad \frac{e}{\hbar} \rightarrow \frac{1}{\hbar} \frac{\delta z_n}{\delta \sigma} = \frac{e}{\hbar} \frac{1}{\hbar} \frac{\delta e}{\delta \sigma} \rightarrow \frac{e^2}{\hbar^2} e$$

$$O_i \rightarrow \phi_i$$

AdS : $ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$

Written hep-th/0112258

$z=0$ boundary

$z=\infty$ horizon

$$\phi \stackrel{z \rightarrow 0}{\sim} \alpha z^\alpha + \beta z^\beta$$

$$+ \frac{f}{z} \int (\Sigma \beta_i)^2$$

$$\int \partial \phi$$

$$\begin{cases} \langle O \rangle = \beta \\ \alpha = 0 \end{cases}$$

M. Fujita, Y. Hikida

S. Ryu, T. Takayanagi
0810.5394

$$z_n = (z_1)^n \quad \frac{e}{\hbar} \frac{1}{n} \frac{\delta z_n}{\delta \sigma} = \frac{e}{\hbar} \frac{1}{n} \frac{\delta e}{\delta \sigma} \rightarrow \frac{e}{\hbar} e \quad \delta \sigma$$

$$O_i \rightarrow \phi_i$$

Ads : $ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$

Written hep-th/0112258

$z=0$ boundary

$z=\infty$ horizon

$$\phi \Big|_{z=0} \propto \alpha z^0 + \beta z^0$$

$$+ \frac{f}{2} \int (\Sigma \beta_i)^2$$

$$\int \partial \phi$$

$$\begin{cases} \langle O \rangle = \beta \\ \alpha = 0 \end{cases}$$

M. Fujita, Y. Hikida

S. Ryu, T. Takayanagi
0810.5394

$$z_n = (z_1)^n \quad \frac{d}{dz} \frac{1}{n} \frac{\delta z'_n}{\delta z} = \frac{d}{dz} \frac{1}{n} \frac{\delta e^{n \ln z}}{\delta z} = \frac{d}{dz} \frac{e^{n \ln z} \cdot n \ln z \cdot \delta e^{n \ln z}}{\delta z} = \frac{d}{dz} \frac{e^{2n \ln z}}{\delta z}$$

$$0_i \rightarrow \phi_i$$

AdS $ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$

Written hep-th/0112258

$z=0$ boundary

$z=\infty$ horizon

M. Fujita, Y. Hikida

S. Ryu, T. Takayanagi
0810.5394

$$\int_0^\infty \phi \frac{dz}{z^3} \sim \alpha z^0 + \beta z^0 \Rightarrow \begin{cases} \langle O \rangle = \beta \\ \alpha = 0 \end{cases} \Rightarrow \begin{cases} \langle O \rangle = \beta \\ \alpha + f\beta = 0 \end{cases} + \frac{f}{2} \int (\Sigma \beta_i)^2$$

$$z_n = (z_1)^n \quad \frac{e^{-\frac{1}{n} \frac{\delta z'_n}{\delta z}}}{\delta z} = \frac{e^{-\frac{1}{n+1} \frac{\delta z'_n}{\delta z}}}{\delta z} \xrightarrow{\delta z} \frac{e^{-n \ln z_1}}{\delta z} = \frac{z_1^{-n}}{\delta z}$$

$$O_i \rightarrow \phi_i$$

AdS : $ds^2 = \frac{1}{z^2} (dz^2 + dx^\mu dx_\mu)$

Written hep-th/0112258

$z=0$ boundary

$z=\infty$ horizon

$$\phi \stackrel{z \rightarrow 0}{\sim} \alpha z^{\Delta_-} + \beta z^{\Delta_+}$$

$$\int_0^\infty \phi$$

$$\begin{cases} \langle O \rangle = \beta \\ \alpha = 0 \end{cases} \Rightarrow$$

$$+ \frac{f}{2} \int (\Delta \beta)^2$$

$$\begin{cases} \langle O \rangle = \beta \\ \alpha + f \beta = 0 \end{cases}$$

M. Fujita, Y. Hikida
S. Ryu, T. Takayanagi
0810.5394



$\langle J_{ii} J_{jj} \rangle$

$$\langle J_u J_v \rangle = \frac{1}{2} \frac{A_m}{\omega}$$



$$\langle T_u T_v \rangle =$$

$$\frac{\int \mathcal{A}_m}{\int \mathcal{D}\phi}$$

$$\langle T_u T_v \rangle =$$

$$\frac{\int \mathcal{A}_m}{\int \mathcal{D}\phi}$$



$$\langle T_u T_v \rangle =$$



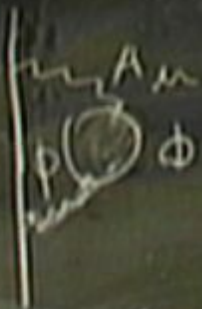
$$\langle 0 | \dots \rangle_{\text{CFT}} = \langle e^{\frac{1}{2} \int \sigma_0} \dots \rangle_{\text{CFT}}$$

$$\langle J_n J_n \rangle = \frac{\hbar^2 A_m}{p(\phi)}$$

$$\langle 0 | \dots \rangle_{\text{CFT}} = \langle e^{\frac{i}{2} \int (\partial \phi)^2} \dots \rangle_{\text{CFT}}$$

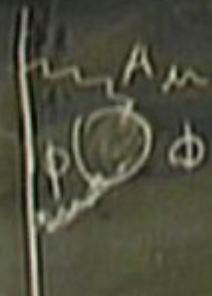


$$\langle T_{\mu\nu} T_{\alpha\beta} \rangle =$$

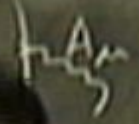


2. Propagator loop (Cutkosky rule)

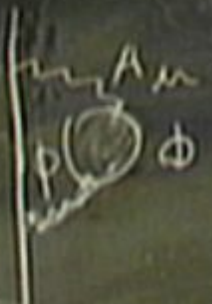
$$\langle T_{\mu\nu} T_{\alpha\beta} \rangle =$$



2. Propagator loop (Cutkosky rule)



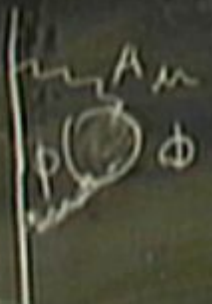
$$\langle T_{\mu\nu} \rangle =$$



2. Propagator loop (Cutkosky rule)



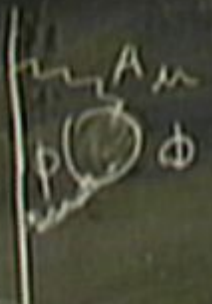
$$\langle T_{\mu\nu} T_{\nu\mu} \rangle =$$



2. Propagator loop (Cutkosky rule)

$$D_{\mu\nu}(p, z) \propto K_{d/2-1}(ipz) z^{d-1}$$

$$\langle T_{\mu\nu} T_{\mu\nu} \rangle =$$

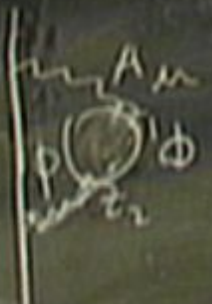


2. Propagator loop (Cutkosky rule)

$$D_{\mu\nu}(m, z) \propto k_{d/2-1}(ipz) \quad d+1$$

$$= k_{d/2-1}(-ipz)$$

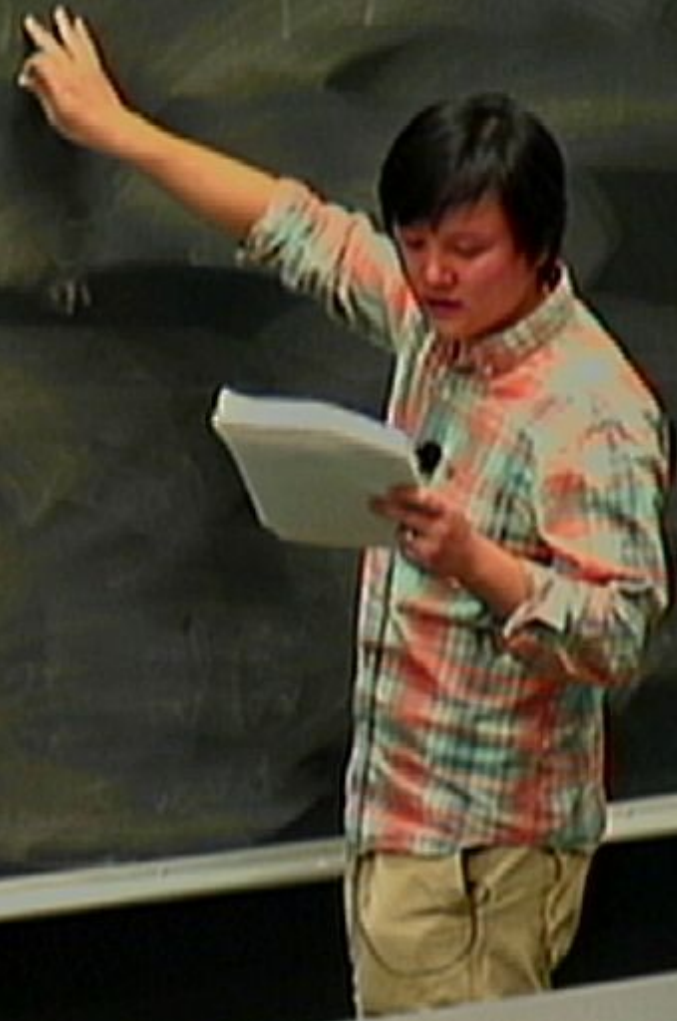
$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle =$$



2. Propagator loop (Cutkosky rule)

$$D_{\mu\nu}(p, z) \propto \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - z} \frac{1}{(k-p)^2 - z}$$

$$D_4(p_4, z, z_1) \sim K_\nu(pz_1) I(pz_2) \Theta(z_1 - z_2)$$



$D_3 (m^4, z_1, z_2)$ or $K_0(P^2) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$



$$D_f(p^H, z, z_1) \propto K_\nu(pz_1) I_\nu(pz_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$D_4(p_1, z_1, z_2) \propto K_\nu(p_2) I_\nu(p_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_a^{b^2} d\Lambda \quad \Lambda$$

$$v = \sqrt{\quad}$$

$$D_4(p_1, z, z_1) \propto K_\nu(p_2) I_\nu(p_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_a^b d\Lambda$$

$$v = \sqrt{\frac{dz}{4 + m^2}}$$



$$D_4(p_1, z_1, z_2) \propto K_\nu(p_2) I_\nu(p_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{p_2} d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + k^2}$$

$$V = \sqrt{\frac{d^2}{4} + m^2}$$



$$D_{\frac{1}{2}}(p^{\mu}, z_1, z_2) \propto K_{\nu}(p z_1) I_{\nu}(p z_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\Lambda \frac{\Lambda J_{\nu}(\Lambda z_1) J_{\nu}(\Lambda z_2)}{\Lambda^2 + p^2}$$

$$V = \sqrt{\frac{d^2}{4} + m^2}$$



$$D_{\frac{1}{2}}(p^{\mu}, z_1, z_2) \propto K_{\nu}(p z_1) I_{\nu}(p z_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\Lambda \frac{\Lambda J_{\nu}(\Lambda z_1) J_{\nu}(\Lambda z_2)}{\Lambda^2 + p^2}$$

$$v = \sqrt{\frac{d^2}{4} + m^2}$$

$$\phi(\partial_n \sqrt{\frac{d^2}{4} + m^2} \partial_0) \phi$$

$$D_{\frac{1}{2}}(p, z_1, z_2) \propto K_{\nu}(p z_2) I_{\nu}(p z_1) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\Lambda \frac{\Lambda J_{\nu}(\Lambda z_1) J_{\nu}(\Lambda z_2)}{\Lambda^2 + p^2}$$

$$v = \sqrt{\frac{dz}{4 + m^2}}$$

$$\phi(\partial_n \sqrt{g} g^{\mu\nu} \partial_\mu) \phi$$

$$(\partial_1 \cdot \partial_2) + k^2$$



$$D_{\frac{1}{2}}(p, z_1, z_2) \propto K_{\nu}(p z_1) I_{\nu}(p z_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\lambda \frac{\lambda J_{\nu}(\lambda z_1) J_{\nu}(\lambda z_2)}{\lambda^2 + p^2}$$

$$v = \sqrt{\frac{dz}{4 + m^2}}$$

$$\phi(\partial_n \sqrt{g} g^{AB} \partial_B) \phi$$

$$(\partial_1 \dots \partial_n) + k^{\nu}$$

$$\Phi = z^{1/2} J_{\nu}(\lambda z)$$

$$D_{\frac{1}{2}}(p_1, z_1, z_2) \propto K_{\nu}(p_2) I_{\nu}(p_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\lambda \frac{\lambda J_{\nu}(\lambda z_1) J_{\nu}(\lambda z_2)}{\lambda^2 + p^2}$$

$$V = \sqrt{\frac{d^2}{4 + m^2}}$$

$$\phi(\partial_n \sqrt{g} g^{AB} \partial_B) \phi$$

$$(\partial_1 \dots \partial_n) + K^{\nu}$$

$$\phi = z^{\nu} J_{\nu}(\lambda z) e^{i p^{\mu} x_{\mu}}$$



$$D_{\frac{1}{2}}(p, z_1, z_2) \propto K_{\nu}(p z_1) I_{\nu}(p z_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\lambda \frac{\lambda J_{\nu}(\lambda z_1) J_{\nu}(\lambda z_2)}{\lambda^2 + p^2}$$

$$v = \sqrt{\frac{d^2}{4} + m^2}$$

$$\sim \frac{-\lambda^2 + k^2}{\lambda} a_{\lambda m}^{\dagger} a_{\lambda k}$$

$$\phi(\partial_n \sqrt{g} g^{\mu\nu} \partial_{\mu}) \phi$$

$$(\partial_1 \cdot \partial_2) + k^2$$

$$\phi = z^{\nu/2} J_{\nu}(\lambda z) e^{i p x_{\mu}}$$

$$D_{\mathbb{R}^4}(p_1, z_1, z_2) \propto K_\nu(p_2) I_\nu(p_2) \Theta(z_1 - z_2) + z_1 \leftrightarrow z_2$$

$$= \int_0^\infty d\Lambda \frac{\Lambda J_\nu(\Lambda z_1) J_\nu(\Lambda z_2)}{\Lambda^2 + p^2}$$

$$V = \sqrt{\frac{d^2}{4 + m^2}}$$

$$\sim \frac{-\Lambda^2 + \kappa^2}{\Lambda} a_{\mu\alpha}^+ a_{\mu\kappa}$$

$$\phi(\partial_n \sqrt{\eta} \partial^{\mu\nu} \partial_0) \phi$$

$$(\partial_1 \cdot \partial_2) + \kappa^2$$

$$\phi = z^{\mu\nu} J_\nu(\Lambda z) e^{i p^\mu x_\mu}$$

$$v = \sqrt{\frac{d^2}{4} + m^2}$$

$$\phi (\partial_n \sqrt{g} g^{\mu\nu} \partial_\mu) \phi$$
$$(\partial_1 \dots \partial_n) + k^2$$

$$\phi = z^{\nu/2} J_\nu(\lambda z) e^{i p^\mu x_\mu}$$

$$J_{\nu, \lambda} = N(\lambda) [J_\nu]$$

$$(\partial_1 \dots \partial_n) + R^L \quad \phi = z \quad \text{for } n \geq 2$$

$$J_{v,f} = N(\lambda) \left[J_{\nu}^{\lambda}(\lambda z) + B J_{\nu}^{\lambda}(\lambda z) \right]$$

$$B = \frac{(2\lambda)^{\nu} \Gamma(1-\nu)}{f \Gamma(1+\lambda)}$$

$$N^2 = \frac{1}{1 + 2B \cos \nu \pi + B^2}$$



$$D_{\psi}(p_1, z_1, z_2) \sim K_{\nu}(p_2) I_{\nu}(p_2) \Theta(z_1 = z_2) + \dots z_1 \leftrightarrow z_2$$

$$= \int_0^{\infty} d\Lambda \frac{\Lambda J_{\nu}(\Lambda z_1) J_{\nu}(\Lambda z_2)}{\Lambda^2 + p^2}$$

$$V = \sqrt{\frac{d^4}{4} + m^2}$$

$$\sim \frac{-\Lambda^2 + k^2}{\Lambda} \alpha_{\Lambda n}^* \alpha_{\Lambda n}$$

$$\phi(\partial_n \sqrt{\Lambda} \psi^{\Lambda n} \partial_0) \psi$$

$$(\partial_3 \dots \partial_3) + K^2$$

$$\psi = z^{\nu_3} J_{\nu_3}(\Lambda z) e^{i p_3 z_3}$$

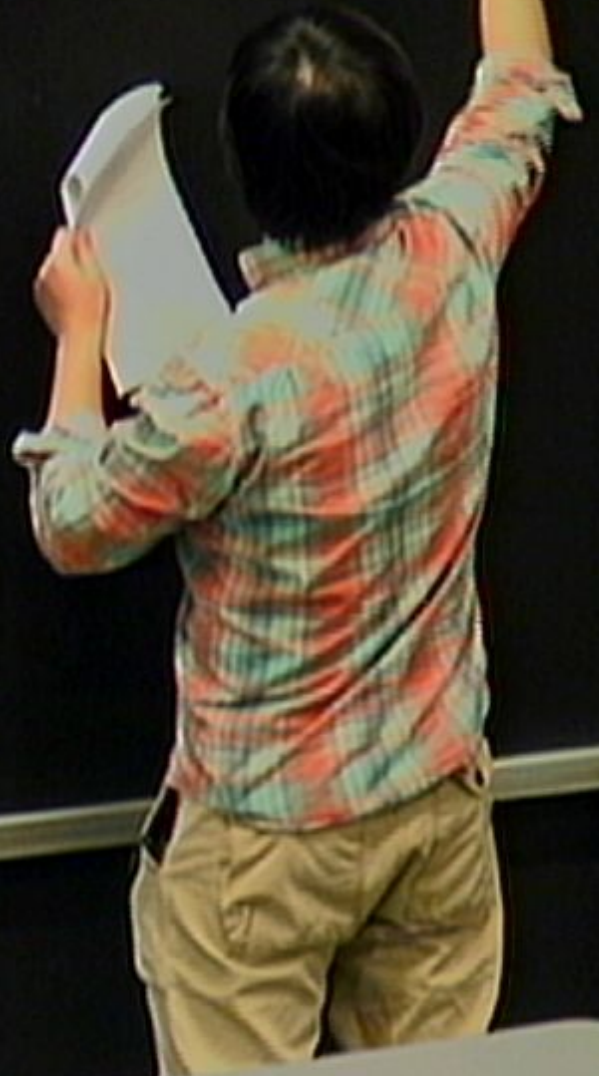
$$\psi_{\nu_3} = N(\Lambda) [J_{\nu_3}(\Lambda z) + B J_{\nu_3}(\Lambda z)]$$

$$B = \frac{(\nu_3 \Lambda)^{\nu_3} \Gamma(1-\nu_3)}{\Gamma(1+\nu_3)}$$

$$N^2 = \frac{1}{1 + 2B \cos(\nu_3 \pi) + B^2}$$



$$\langle T_{\mu\nu} \rangle_{r=100p} \propto \int \frac{d\lambda_1 d\lambda_2 d^4 k}{(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)} (p \cdot 2k)_\alpha (p \cdot 2k)_\beta \lambda_1 \lambda_2 / [(i p \cdot \lambda_1 \lambda_2)$$



$$\langle T_{\mu\nu} \rangle_{loop} \propto \int \frac{d\Lambda d\Lambda_2 d^4k (p \cdot 2k)_\mu (p \cdot 2k)_\nu \Lambda_1 \Lambda_2 |I(iP, \Lambda_1, \Lambda_2)|^2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)}$$

$$\langle \mathcal{J}_\mu \mathcal{J}_\nu \rangle_{1-loop} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^4 k (p \cdot z)_\alpha (p \cdot z)_\beta \Lambda_1 \Lambda_2 |I(iP, \Lambda_1, \Lambda_2)|^2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)}$$

$$\mathcal{J}_{\nu, f}(P, \Lambda_1, \Lambda_2) \equiv \int d^2 z \bar{z}^{d/2} K_{\frac{d}{2}-1}(Pz) \mathcal{J}_{\nu, f}(\Lambda_1, z) \mathcal{J}_{\nu, f}(\Lambda_2, z)$$

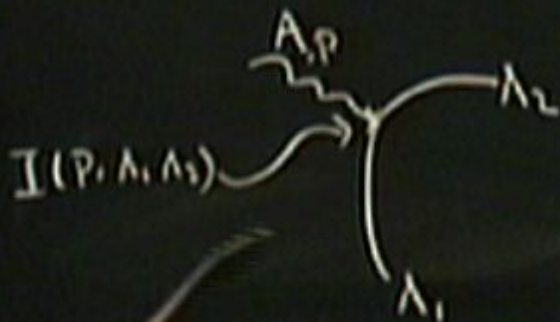
$$\langle \mathcal{T}_\mu \mathcal{T}_\nu \rangle_{1-loop} \propto \int \frac{d\lambda_1 d\lambda_2 d\lambda_3}{(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)}$$

$$\mathcal{I}_{\nu, f}(p, \lambda_1, \lambda_2) \equiv \int_0^1 dz z^{d/2} K_{\nu-1/2}(pz) \mathcal{T}_{\nu, f}(\lambda_1, z) \mathcal{T}_{\nu, f}(\lambda_2, z)$$



$$\langle \mathcal{T}_u \mathcal{T}_v \rangle_{r=1, \dots, p} \propto \int \frac{d\lambda_1 d\lambda_2 d\lambda_3 \dots}{(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)}$$

$$I_{\nu, f}(P, \lambda_1, \lambda_2) \equiv \int_0^{\infty} dz z^{d/2} K_{\nu/2-1}(Pz) \mathcal{T}_{\nu, f}(\lambda_1, z) \mathcal{T}_{\nu, f}(\lambda_2, z)$$

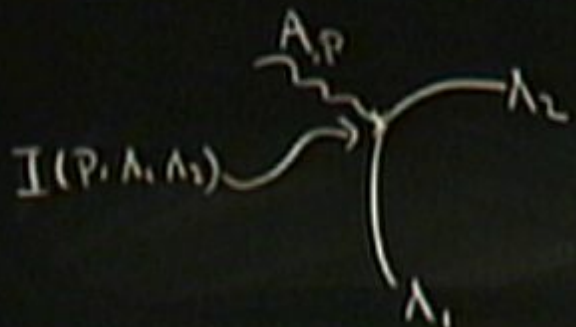


$$\langle \mathcal{T}_u \mathcal{T}_v \rangle_{100p} \propto \int \frac{d\lambda_1 d\lambda_2 d\lambda_3 \dots}{(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)}$$

$$I_{v,f}(p, \lambda_1, \lambda_2) \equiv \int_0^{\infty} dz z^{d-2} K_{\frac{d-1}{2}}(pz) \mathcal{T}_{v,f}(\lambda_1 z) \mathcal{T}_{v,f}(\lambda_2 z)$$

$$I_{\text{Im}} \left| \left| \begin{array}{c} \lambda_2 \\ \lambda_1 \end{array} \right| \right| = \int d\lambda_1 d\lambda_2 \left| \left| \begin{array}{c} \lambda_2 \\ \lambda_1 \end{array} \right| \right|^2$$

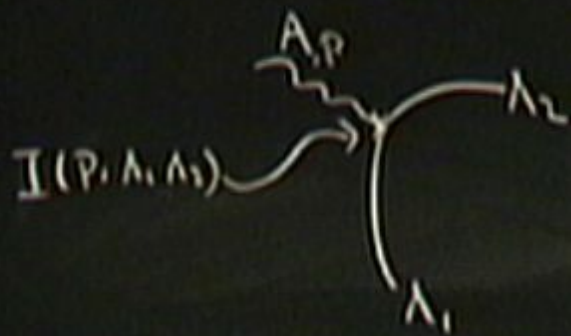




$I(P, \lambda_1, \lambda_2)$

$$I_m \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right| = \int d\lambda_1 d\lambda_2 \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|$$

$$= \int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \lambda_1 \lambda_2 \mathcal{H}(\lambda_1, \lambda_2)^{d/2} \left| I_{\mathcal{H}}(i, \lambda_1, \lambda_2) \right|^2$$



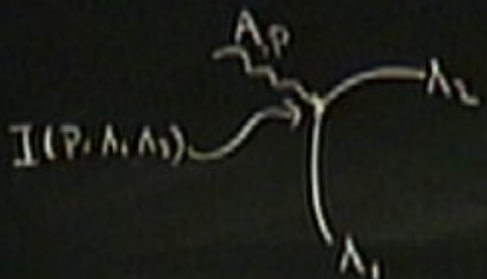
$$\text{Im} \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right| = \int d\lambda_1 d\lambda_2 \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|$$

$$\int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \lambda_1 \lambda_2 H(\lambda_1, \lambda_2)^{d/2} |I_{\text{Inf}}(i\lambda_1, \lambda_2)|^2$$

$$H(x, y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$$

3 The bad, good, bad,

3. The bad, good, bad,
or not so bad I.v.f.

$$I_{v,f}(P, \lambda_1, \lambda_2) \equiv \int_0^1 dz z^{1/2} K_{\nu, f}(Pz) J_{\nu, f}(\lambda_1 z) J_{\nu, f}(\lambda_2 z)$$


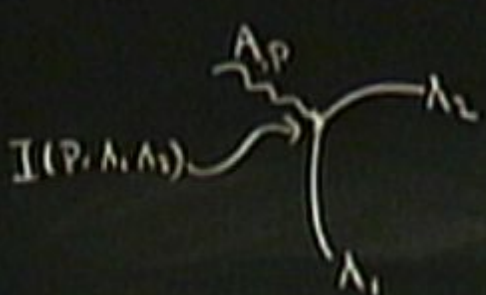
$$I_m \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|^2 = \int d\lambda_1 d\lambda_2 \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|^2$$

$$\int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \lambda_1 \lambda_2 \chi(\lambda_1, \lambda_2)^{d/2} |I_{\nu, f}(c, \lambda_1, \lambda_2)|^2$$

$$H(x, y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$$

$P \rightarrow$

$$I_{v,f}(p, \lambda, \lambda_2) \equiv \int_0^1 dz z^{d/2} K_{\nu/2}(pz) J_{\nu/2}(\lambda z) J_{\nu/2}(\lambda_2 z)$$



$$I_m \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|^2 = \int d\lambda_1 d\lambda_2 \left| \left| \begin{matrix} \lambda_2 \\ \lambda_1 \end{matrix} \right| \right|^2$$

$$\int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \lambda_1 \lambda_2 (c\lambda_1 \lambda_2)^{d/2} |I_{v,f}(c\lambda_1, \lambda_2)|^2$$

$$H(x, y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$$

$$p \rightarrow 1 \quad \lambda_{1,2} \sim \frac{\lambda_2}{p}$$



3 The bad, good, bad,
... or not so bad Inf.

W.N. Bailey 1936

$I_v(i, A, A_2)$

3 The bad, good, bad,
or not so bad $I_{\nu} f$.

W.N. Barlow 1936

$$I_{\nu}(\lambda_1, \lambda_2) \propto \lambda_1^{\nu} \lambda_2^{\nu} F_4$$

3 The bad, good, bad,
... or not so bad $I_{\nu, f}$

W.N. Bailey 1936

$$I_{\nu}(i, \lambda_1, \lambda_2) \propto \lambda_1^{\nu} \lambda_2^{\nu} F_4 \left[\begin{matrix} 1 + \nu \\ 1 + \nu, 1 + \nu \end{matrix} \right]$$

3 The bad, good, bad,

or not so bad $I_{\nu f}$.

W.N. Bailey 1936

$$I_{\nu}(i, \Lambda_1, \Lambda_2) \propto \Lambda_1^{\nu} \Lambda_2^{\nu} F_4 \left[\begin{matrix} 1 + \frac{\nu}{2} \\ \nu + 1, \nu + 1, \Lambda_1^2, \Lambda_2^2 \end{matrix} \right]$$

3 The bad, good, bad,
or not so bad $I_{\nu} f$

W.N. Bailey 1936

$$I_{\nu}(i, \lambda_1, \lambda_2) \propto \lambda_1^{\nu} \lambda_2^{\nu} F_4 \left[\begin{matrix} 1 + \frac{\nu}{2} + \frac{\nu}{2} + \nu + 1, \nu + 1, \lambda_1^2, \lambda_2^2 \\ \sum_w a_w - (\lambda_1)^2 (\lambda_2)^{2m} \end{matrix} \right]$$

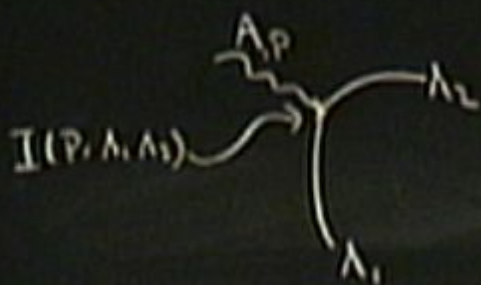
3 The bad, good, bad,
... or not so bad $I_{\nu} f$

W.N. Bailey 1936

$$I_{\nu}(i, \Lambda_1, \Lambda_2) \propto \Lambda_1^{\nu} \Lambda_2^{\nu} F_4 \left[\begin{matrix} 1 + \nu, \nu + 1, \Lambda_1^2, \Lambda_2^2 \\ \sum_{m=0}^{\infty} a_m (\Lambda_1)^m (\Lambda_2)^{2m} \end{matrix} \right]$$

$$|\Lambda_1| + |\Lambda_2| < 1$$

$$I_{v,f}(P, \lambda_1, \lambda_2) \equiv \int_0^1 dz z^{d/2} K_{\nu/2}(Pz) I_{v,f}(\lambda_1 z) I_{v,f}(\lambda_2 z)$$



$$I_m \left| \left| \begin{array}{c} \text{circle with dot} \\ \lambda_1 \end{array} \right. \right| = \int d\lambda_1 d\lambda_2 \left| \left| \begin{array}{c} \text{circle with dot} \\ \lambda_1 \end{array} \right. \right|^2$$

$$\int_0^1 d\lambda_1 \int_0^{1-\lambda_1} d\lambda_2 \lambda_1 \lambda_2 (c(\lambda_1, \lambda_2))^{d/2} |I_{v,f}(c(\lambda_1, \lambda_2))|^2$$

$$H(x, y) = (1-x-y)(1-x+y)(1+x-y)(1+x+y)$$

$P \rightarrow 1 \quad \lambda_{1,2} \sim \frac{\lambda_{1,2}}{P}$

$$\langle T_{\mu\nu} T_{\nu\mu} \rangle_{-loop} \propto \int \frac{d\lambda_1 d\lambda_2 d^d k (p-z)_\mu (p-z)_\nu \Lambda_1 \Lambda_2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)} \frac{1}{[i(p, \Lambda_1, \Lambda_2)]^2}$$

$$I_{\nu, f}(\lambda_1, \lambda_2) \equiv \int_0^1 dz z^{d/2-1} K_{d/2-1}(\lambda_1 z) J_{\nu, f}(\lambda_1, z) J_{\nu, f}(\lambda_2, z)$$

$$\langle T_{\mu\nu} T_{\mu\nu} \rangle_{-10010} \propto \int \frac{d\Lambda_1 d\Lambda_2 d^d k (p-z)_\mu (p-z)_\nu \Lambda_1 \Lambda_2 / [(i p \cdot \Lambda_1 \Lambda_2)]^2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)}$$

$$I_{\nu, f}(\Lambda_1, \Lambda_2) \equiv \int_0^\infty dz z^{d/2} K_{\nu-1/2}(\Lambda_1 z) J_{\nu, f}(\Lambda_2 z) J_{\nu, f}(\Lambda_2 z)$$

$e^{i z}$
 $\cos(\Lambda_2 z, \theta)$



$$\langle T_{\mu\nu} T_{\mu\nu} \rangle_{-10010} \propto \int \frac{d\lambda_1 d\lambda_2 d^d k (p \cdot z)_{\lambda_1} (p \cdot z)_{\lambda_2} \Lambda_1 \Lambda_2}{(\Lambda_1^2 + k^2)(\Lambda_2^2 + (k-p)^2)} \frac{1}{[i(p \cdot \Lambda_1 \Lambda_2)]^2}$$

$$I_{\nu, f}(\lambda_1, \lambda_2) \equiv \int_0^{\infty} dz z^{d-2} K_{\frac{d-2}{2}}(\lambda_1 z) J_{\nu, f}(\lambda_1 z) J_{\nu, f}(\lambda_2 z)$$

$$\left(e^{i z} \right) \left(\begin{matrix} e^{i(\lambda_1 z \cdot \theta)} \\ + e^{i(\lambda_2 z \cdot \theta)} \end{matrix} \right) \left(\begin{matrix} e^{i(\lambda_1 z \cdot \theta)} \\ - e^{i(\lambda_2 z \cdot \theta)} \end{matrix} \right)$$

$$(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)$$

$$I_{\nu, f}(\theta, \lambda_1, \lambda_2) \equiv \int_0^\infty dz z^{\nu-1} K_{\nu-1}(kz) J_{\nu, f}(\lambda_1 z) J_{\nu, f}(\lambda_2 z)$$

$$z^{-\nu} \begin{pmatrix} e^{i z} \\ e^{-i z} \end{pmatrix} \begin{pmatrix} e^{i(\lambda_1 z + \theta)} \\ + e^{i(\lambda_2 z + \theta)} \end{pmatrix} \begin{pmatrix} e^{i(\lambda_1 z + \theta)} \\ - e^{i(\lambda_2 z + \theta)} \end{pmatrix}$$

$$\sim z^{\frac{\nu-1}{2}} \sum_{\pm} e^{i(\pm \lambda_1 \lambda_2) z + \theta} \dots$$



$$(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)$$

$$I_{\nu, f}(\theta, \lambda_1, \lambda_2) \equiv \int_0^\nu dz z^{d-2} K_{\nu-1}(kz) J_{\nu, f}(\lambda_1 z) J_{\nu, f}(\lambda_2 z)$$

$$z^{-\nu} \begin{pmatrix} e^{i\lambda_1 z \cdot \theta} \\ + e^{i(\lambda_2 z \cdot \theta)} \end{pmatrix} \begin{pmatrix} e^{i(\lambda_1 z \cdot \theta)} \\ - e^{i(\lambda_2 z \cdot \theta)} \end{pmatrix}$$

$$\sim z^{\frac{d-3}{2}} \sum_{\pm} e^{i(\pm \lambda_1 \lambda_2) z \cdot \theta \dots}$$

3 The bad, good, bad; not so bad

I, f

$$I_{mf} \sim \frac{e^{-\gamma i \theta_f}}{\sqrt{\quad}}$$



3 The bad, good, bad; not \approx bad

$$I_{inf} \sim \frac{e^{-\gamma i \theta_f}}{\sqrt{\lambda_1 \lambda_2}} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{1/2}} + \left[\frac{3}{4} - (v, n) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} \right\} + \text{finite}$$

3 The bad, good, bad, not so bad

$$I_{\text{inf}} \sim \frac{e^{-\gamma i \theta_f}}{\sqrt{\lambda_1 \lambda_2}} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{1/2}} + \left[\frac{3}{4} - (v, n) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} + \text{finite terms}$$

$$|I_{v, f}|^2 \sim \frac{1}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^2} + \left[\frac{3}{2} - 12(v, n) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^3} \right\}$$

$$O\left(\frac{1}{1-\lambda_1-\lambda_2}\right)$$

3 The bad, good, bad, not \approx bad

$$I_{\nu, f} \sim \frac{e^{-\gamma i \theta_f}}{\sqrt{\lambda_1 \lambda_2}} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} + \left[\frac{3}{4} - (\nu, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^{5/2}} \right\} + \text{finite terms}$$

$$|I_{\nu, f}|^2 \sim \frac{1}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^3} + \left[\frac{3}{2} - 2(\nu, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\}$$

$$\tan \delta_f = \tan \left(\frac{1 - \frac{\Delta^2 \nu}{f} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}}{1 + \frac{\Delta^2 \nu}{f} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}} \right) \tan \frac{\nu \pi}{2} + O \left(\frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} \right)$$

$1 - \lambda_1 - \lambda_2 \neq 0$

3 The bad, good, bad, not so bad

$$I_{\text{inf}} \sim \frac{e^{-2i\theta_f}}{\sqrt{\lambda_1 \lambda_2}} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} + \left[\frac{3}{4} - (v, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} + \text{finite terms}$$

$$|I_{\text{inf}}|^2 \sim \frac{1}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^3} + \left[\frac{3}{2} - 2(v, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} H^{3/2}$$

$$\tan \delta_f = \frac{1 - \frac{\Delta^2 v}{f} \frac{\Gamma(i+1)}{\Gamma(i+2)}}{+ \frac{\Delta^2 v}{f} \frac{\Gamma(i+1)}{\Gamma(i+2)}} \tan \frac{\nu \pi}{2}$$

$$+ O\left(\frac{1}{(1-\lambda_1-\lambda_2)^{3/2}}\right)$$

$$1 - \lambda_1 - \lambda_2 \neq 0$$

3 The bad, good, bad, not so bad

$$I_{\nu, f} \sim \frac{e^{-\gamma i \theta_f}}{\sqrt{\lambda_1 \lambda_2}} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} + \left[\frac{3}{4} - (\nu, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} + \text{finite terms}$$

$$|I_{\nu, f}|^2 \sim \frac{1}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^3} + \left[\frac{3}{2} - 2(\nu, 1) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} H^{3/2}$$

$$f = \tan \left(\frac{1 - \frac{\Delta^2 \nu}{f} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}}{1 + \frac{\Delta^2 \nu}{f} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}} \right) \tan \frac{\nu \pi}{2} + O \left(\frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} \right)$$

$$\lambda_1 - \lambda_2 \neq 0$$

depends on f, ν

3 The bad, good, bad, not so bad

$$I_{\text{inf}} \sim \frac{e^{-2i\theta_f}}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^{3/2}} + \left[\frac{3}{4} - (v, n) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} + \text{finite terms}$$

$$|I_{\text{inf}}|^2 \sim \frac{1}{\lambda_1 \lambda_2} \left\{ \frac{1}{(1-\lambda_1-\lambda_2)^3} + \left[\frac{3}{2} - 2(v, n) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right] \frac{1}{(1-\lambda_1-\lambda_2)^2} \right\} H^{3/2}$$

$$\tan \delta_f = \tan \left(\frac{1 - \frac{\lambda_1 v}{f} \frac{\Gamma(1+v)}{\Gamma(1-v)}}{1 + \frac{\lambda_1 v}{f} \frac{\Gamma(1+v)}{\Gamma(1-v)}} \right) \tan \frac{v\pi}{2}$$

$$+ O\left(\frac{1}{(1-\lambda_1-\lambda_2)^{3/2}}\right)$$

$$1 - \lambda_1 - \lambda_2 \neq 0$$

depends on f, v

3 The bad, good, bad, not so bad

$$I_{\text{inf}} \sim \frac{e^{-2i\theta_f}}{\sqrt{\Lambda_1 \Lambda_2}} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}} + \left[\frac{3}{4} - (v, n) \left(\frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} \right) \right] \frac{1}{(1-\Lambda_1-\Lambda_2)^2} \right\} + \text{finite terms}$$

$$|I_{\text{inf}}|^2 \sim \frac{1}{\Lambda_1 \Lambda_2} \left\{ \frac{1}{(1-\Lambda_1-\Lambda_2)^3} + \left[\frac{3}{2} - 2(v, n) \left(\frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} \right) \right] \frac{1}{(1-\Lambda_1-\Lambda_2)^2} \right\} H^{3/2}$$

$$\tan \delta_f = \tan \left(\frac{1 - \frac{\Lambda_1 v}{f} \frac{\Gamma(1+iv)}{\Gamma(1-iv)}}{1 + \frac{\Lambda_2 v}{f} \frac{\Gamma(1+iv)}{\Gamma(1-iv)}} \right) \tan \frac{v\pi}{2}$$

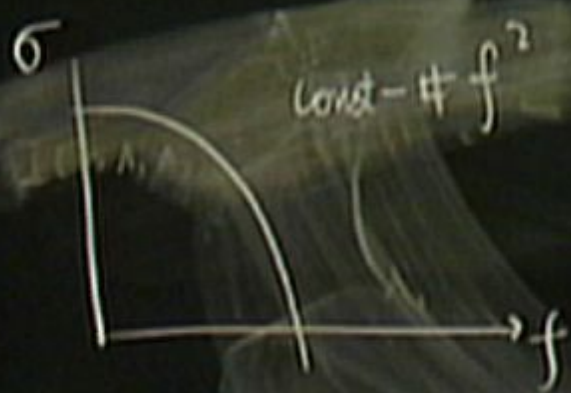
$$+ O\left(\frac{1}{(1-\Lambda_1-\Lambda_2)^{3/2}}\right)$$

$$1 - \Lambda_1 - \Lambda_2 \neq 0$$

depends on f, v

$$(\lambda_1^2 + k^2)(\lambda_2^2 + (k-p)^2)$$

$$I_{\nu, f}(\lambda_1, \lambda_2) \equiv \int_0^\infty dz z^{\nu-1} K_{\nu-1}(fz) J_{\nu, f}(\lambda_1, z) J_{\nu, f}(\lambda_2, z)$$



$$z^{-\nu} \begin{pmatrix} e^{i z} \\ e^{-i z} \end{pmatrix} \begin{pmatrix} e^{i(\lambda_1 z + \theta)} \\ + e^{i(\lambda_2 z + \theta)} \end{pmatrix} \begin{pmatrix} e^{i(\lambda_1 z + \theta)} \\ - e^{i(\lambda_2 z + \theta)} \end{pmatrix}$$

$$\sim z^{\frac{\nu-3}{2}} \sum_{\pm} e^{i(\pm \lambda_1 \pm \lambda_2) z + \theta \dots}$$

CAUTION