Title: Counter-factual Processes in Quantum Mechanics

Date: Nov 26, 2010 02:00 PM

URL: http://pirsa.org/10110078

Abstract: The counter-intuitive phenomena in quantum mechanics are often based on the counter-factual (or virtual) processes. The famous example is the Hardy paradox, which has been recently solved in two independent experiments. Also, the delayed choice experiment and one of quantum descriptions of the closed time like curves can be also examples of the counter-intuitive phenomena. The counter-factual processes can be characterized by the weak value initiated by Yakir Aharonov and his colleagues. In this talk, I will introduce the weak value from the probability theory and the connection to the counter-factual processes in these examples.

Counter-factual Processes in Quantum Mechanics

Yutaka Shikano

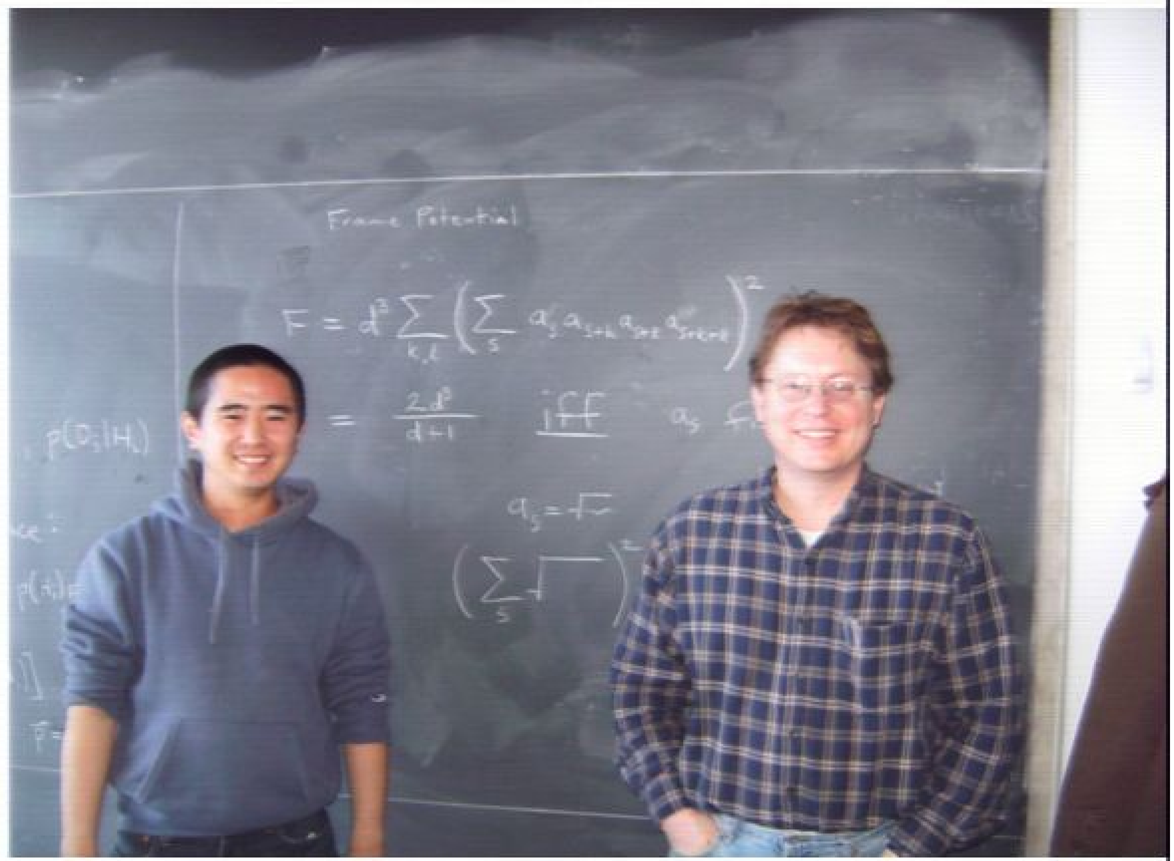
Tokyo Institute of Technology

Massachusetts Institute of Technology

Quantum Information Seminar at Perimeter Institute

November 26th, 2010.

My research interest: “What is Time?”



pictured by Michael Nielsen

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

My previous visit to PI

Feb. – Mar., 2008

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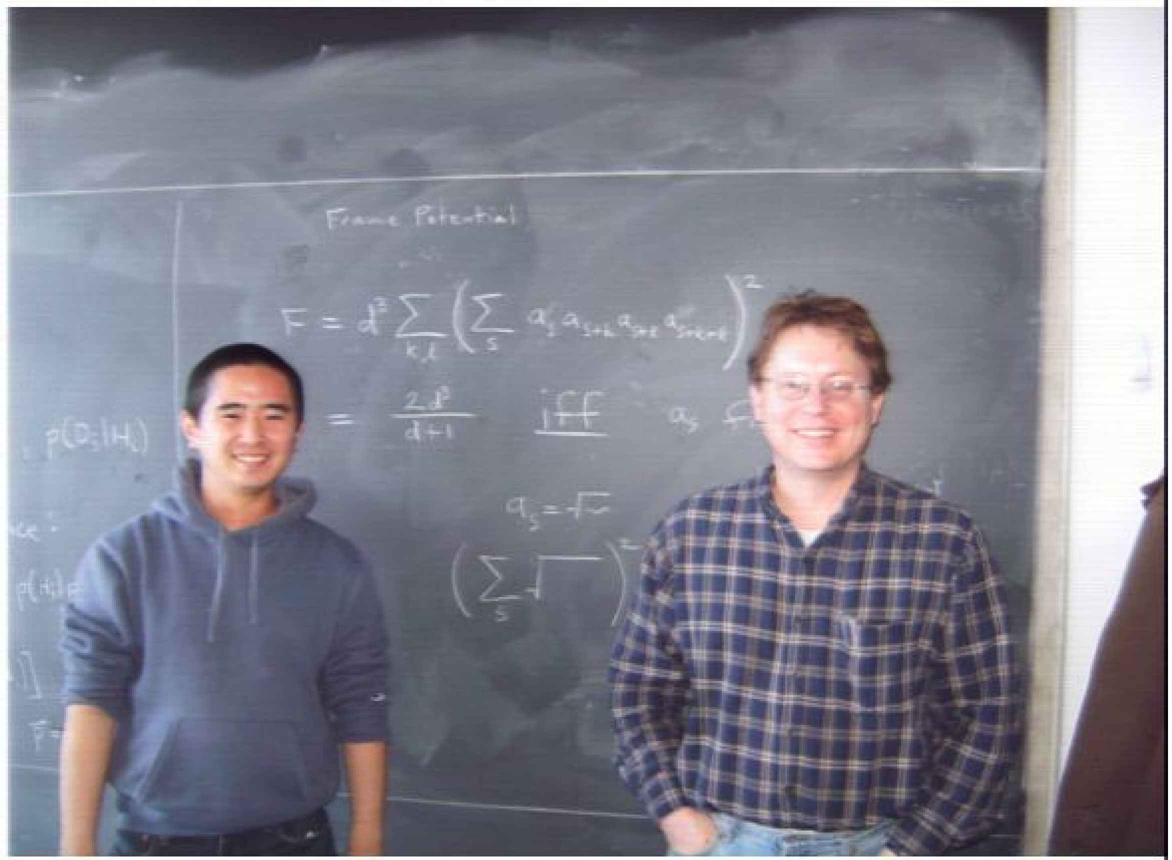
My current goal is

How to understand the time in quantum mechanics?



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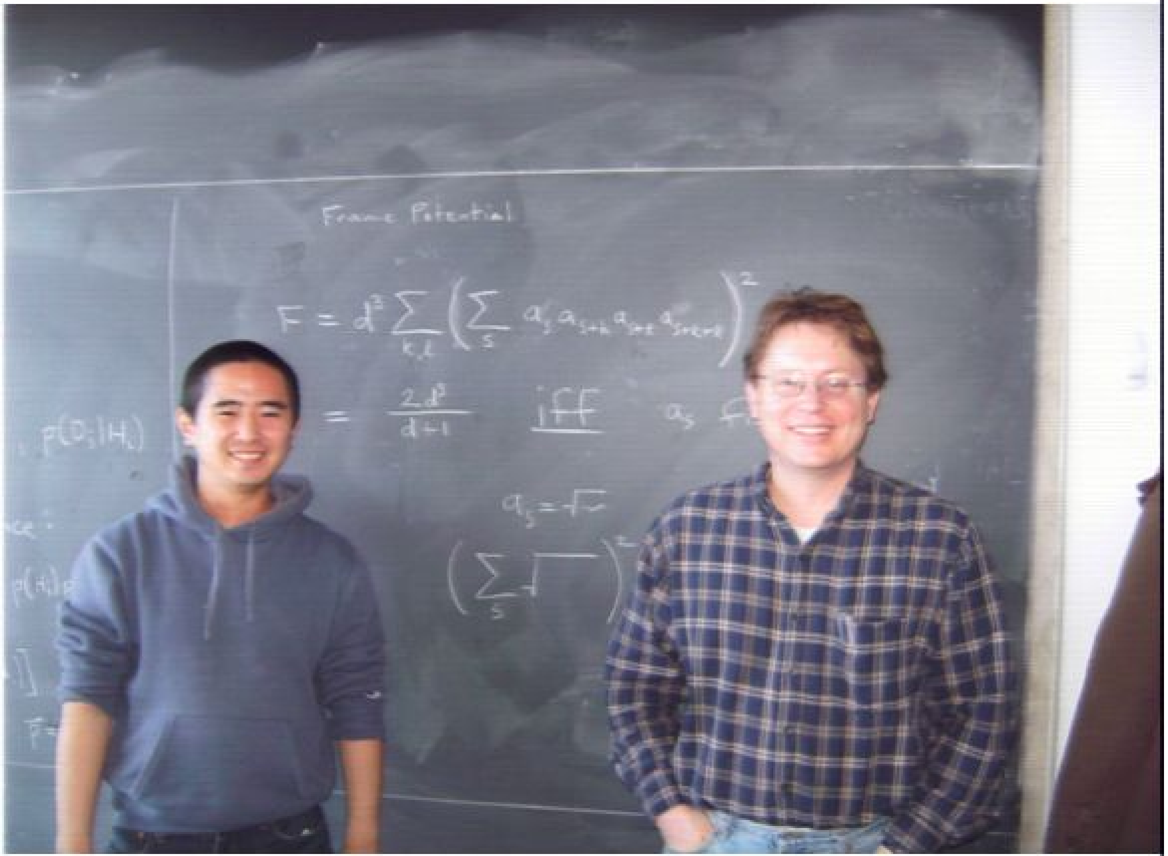
1. How to construct the time operator as the observable?
2. Is there a connection between the parameter time "t" and the measured time (clock time) "t"?



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1. Change the definition / interpretation of the observable
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International Workshop on Mathematical and Physical Foundations of Discrete Time Quantum Walk

Date: March 29th-30th, 2011

Venue: Tokyo Institute of Technology, Japan

Deadline: Dec. 31st, 2010(oral), Feb. 28th, 2011(poster)

Invited Speakers

Yakir Aharonov (Tel-Aviv University, Israel / Chapman University, USA)

Stanley Gudder (University of Denver, USA)*

Luis Velazquez (Zaragoza University, Spain)*

Takuya Kitagawa (Harvard University, USA)*

(* to be confirmed)

Conference Scope

1. Mathematical Foundations of Discrete Time Quantum Walk
   - 1-1. Stochastic Process in Quantum Probability Theory
   - 1-2. Weak Limit Theorem
   - 1-3. Classification between Localization and Delocalization
2. Physical Foundations of Discrete Time Quantum Walk
   - 2-1. Mapped to Schroedinger Equation and Dirac Equation
   - 2-2. Non-local Effect, Entanglement, and Super-oscillation
   - 2-3. Application to Quantum Information Science

Organizers

Norio Konno (Yokohama National University)
Etsuo Segawa (Tokyo Institute of Technology)
Yutaka Shikano (Tokyo Institute of Technology / Massachusetts Institute of Technology, Chair)

Contact

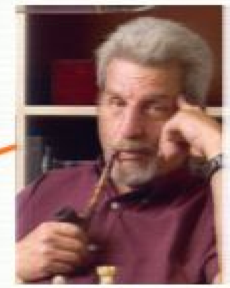
Yutaka Shikano (shikano@th.phys.titech.ac.jp)

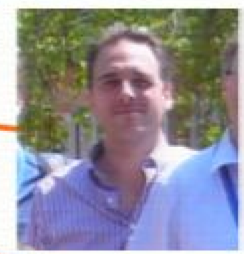
http://www.th.phys.titech.ac.jp/~shikano/dtqw/

@Tokyo Tech

Tokyo, Japan

3/29 - 30/2011





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   - Topos Approach (C. Isham, A. Doering, ...)

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Today's Outline

1. Why do we need the weak value?
    - Motivation of the "theory of weak value" – related to the probability theory
    - Definition and applications of the weak values
    - How to obtain the weak values – weak measurement
2. Counter-factual Processes
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When is the probability space defined?

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Hilbert space H

↓

Observable A

↓

Probability space

Case 1

When is the probability space defined?

Hilbert space H

↓

Observable A

↓

Probability space

Case 1

Hilbert space H

↓

Probability space

↓

Observable A

Case 2

When is the probability space defined?

Case 1	Case 2
Hilbert space H	Hilbert space H
↓	↓
Observable A	Probability space
↓	↓
Probability space	Observable A

Are they equivalent??

Definition of Probability Space

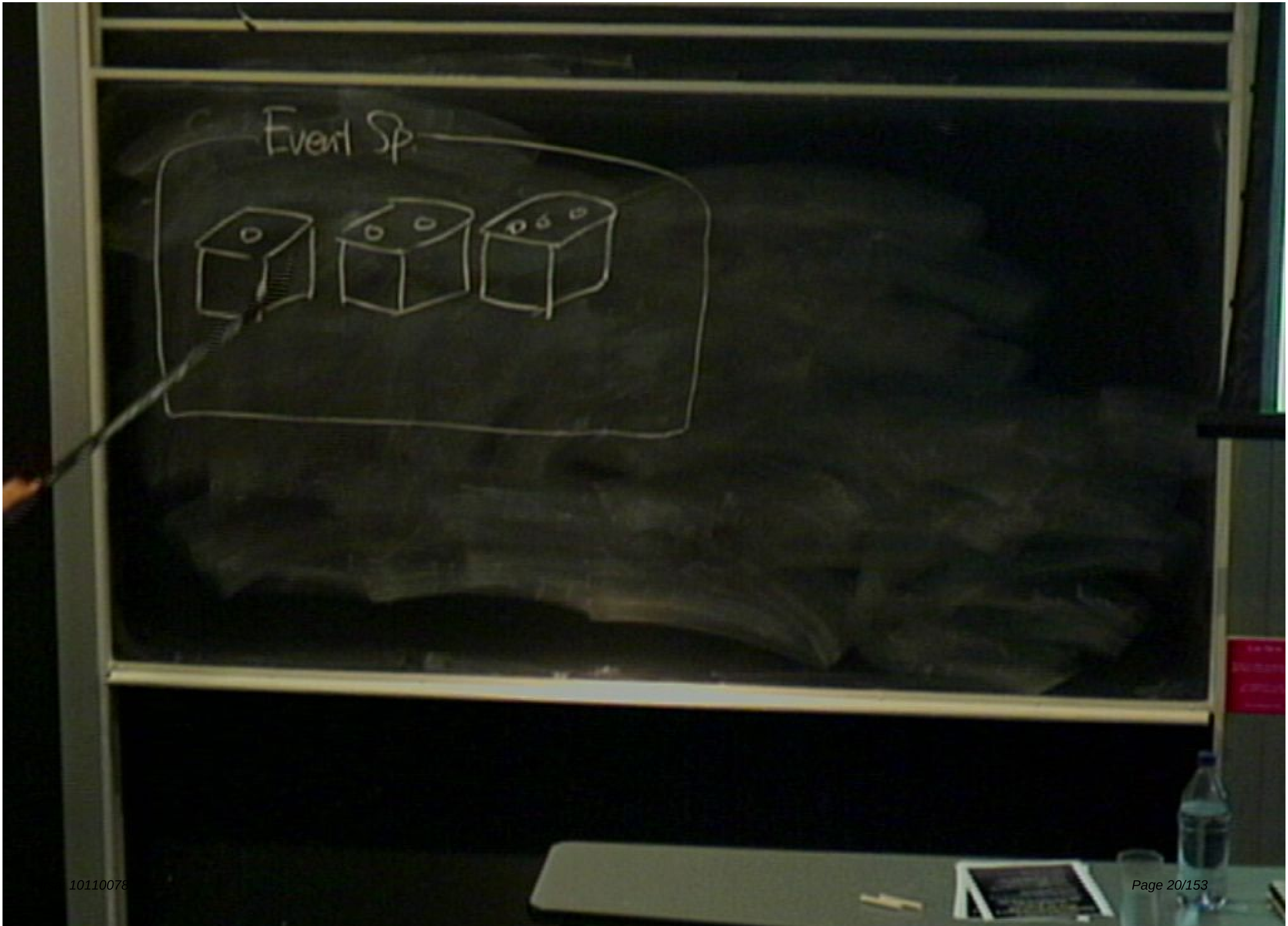
Event Space Ω

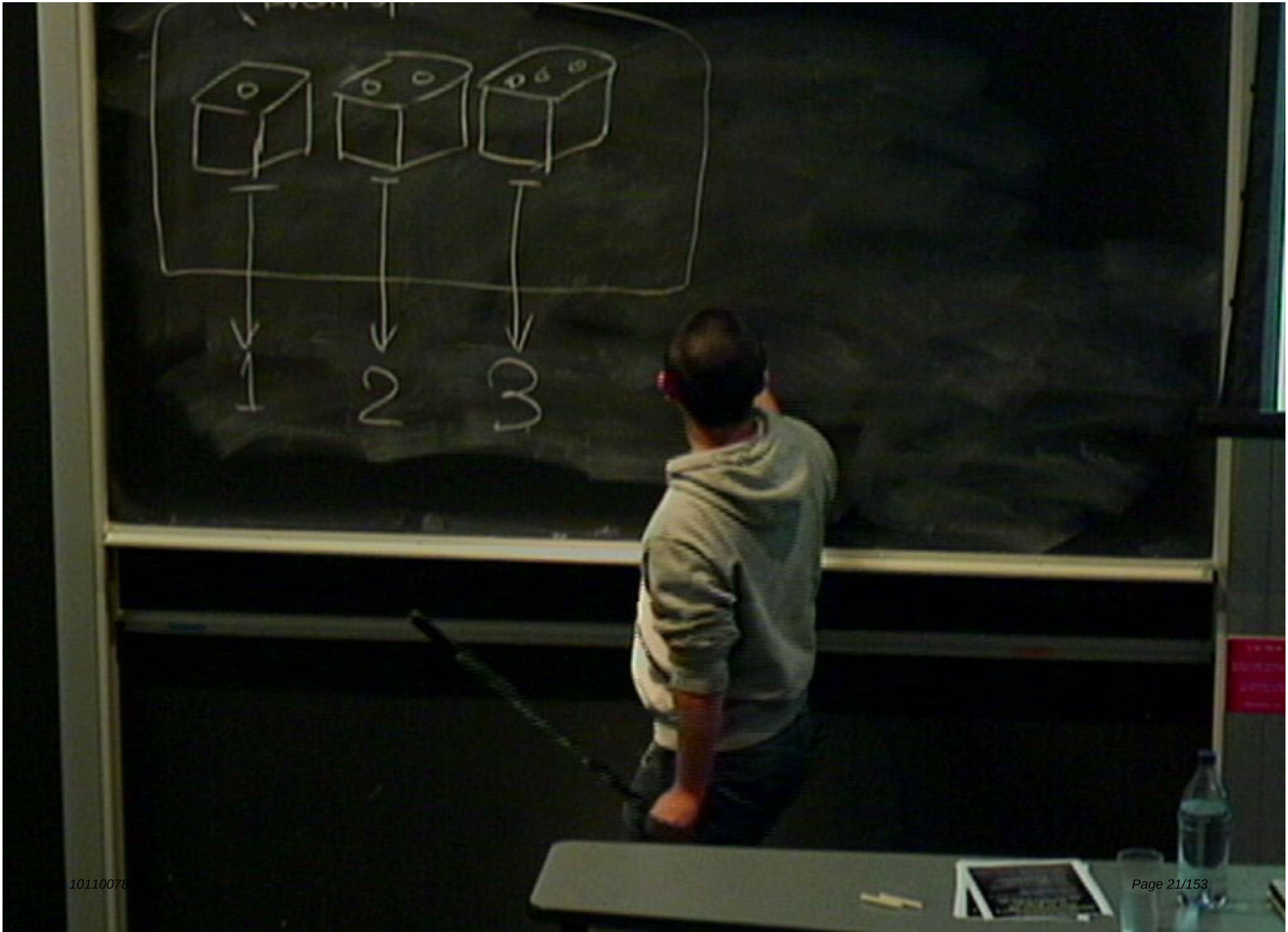
Probability Measure dP

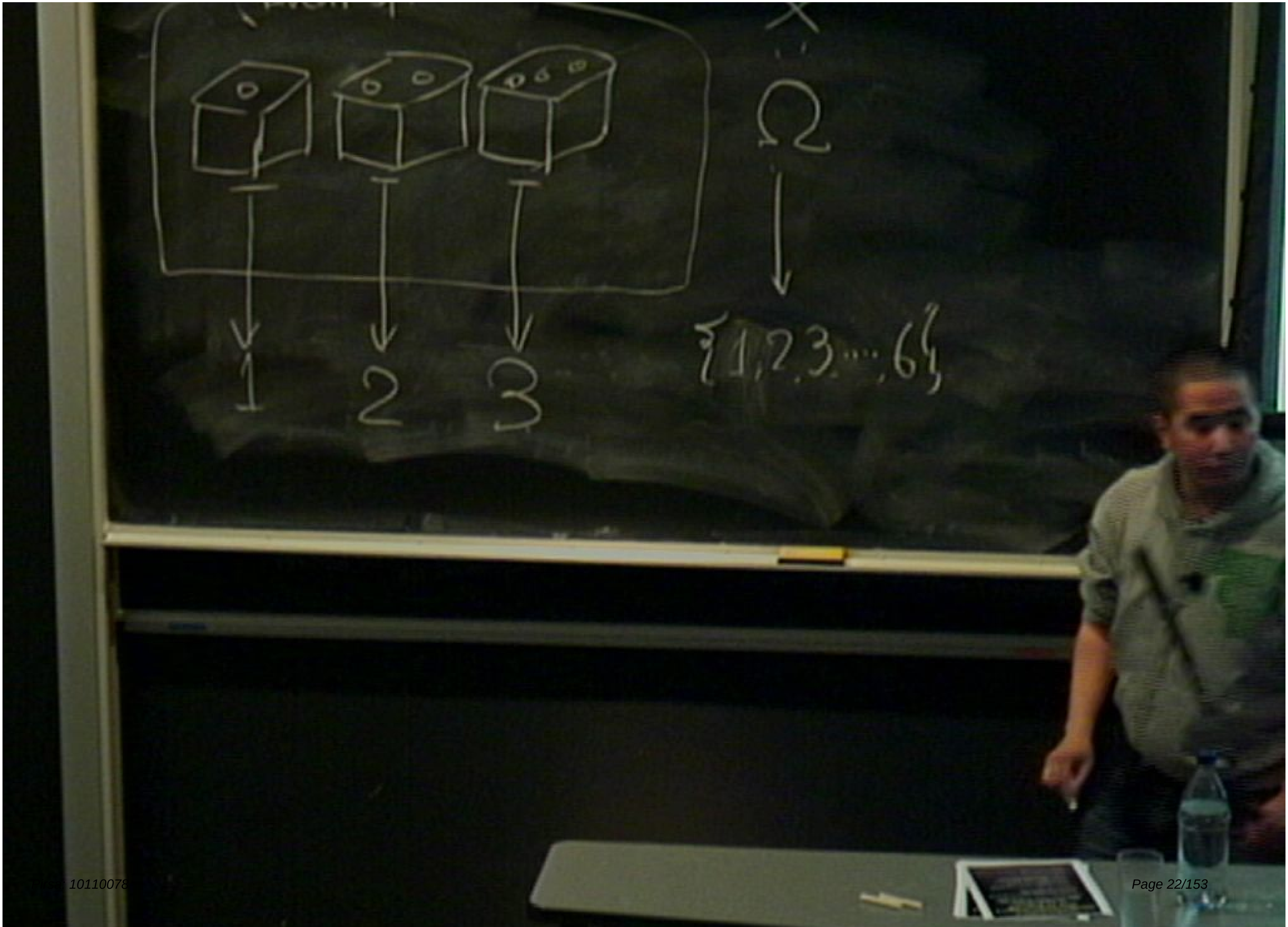
Random Variable X: Ω -> K

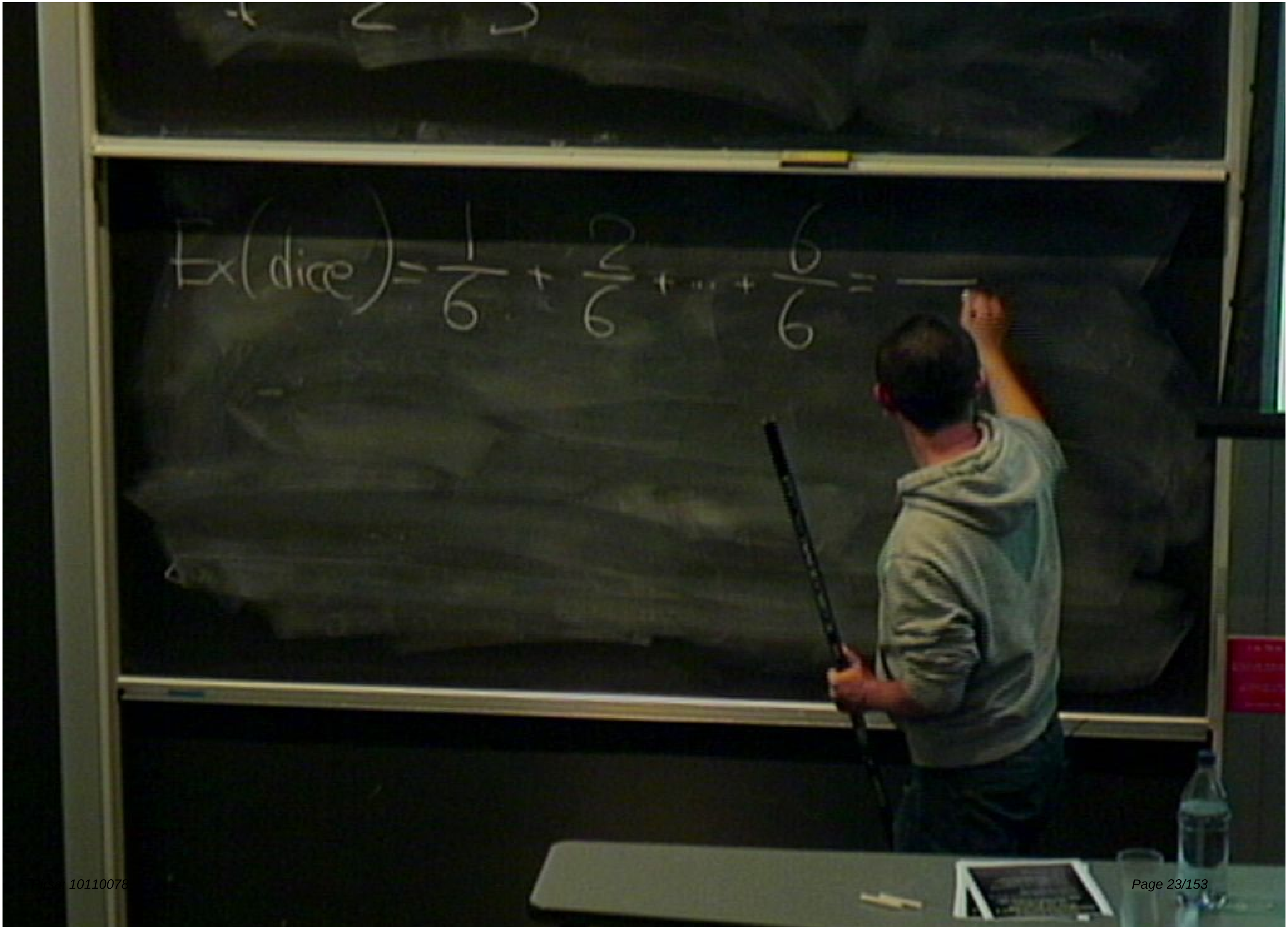
The expectation value is

$$\mathrm{Ex}[X] = \int X(\omega) dP.$$

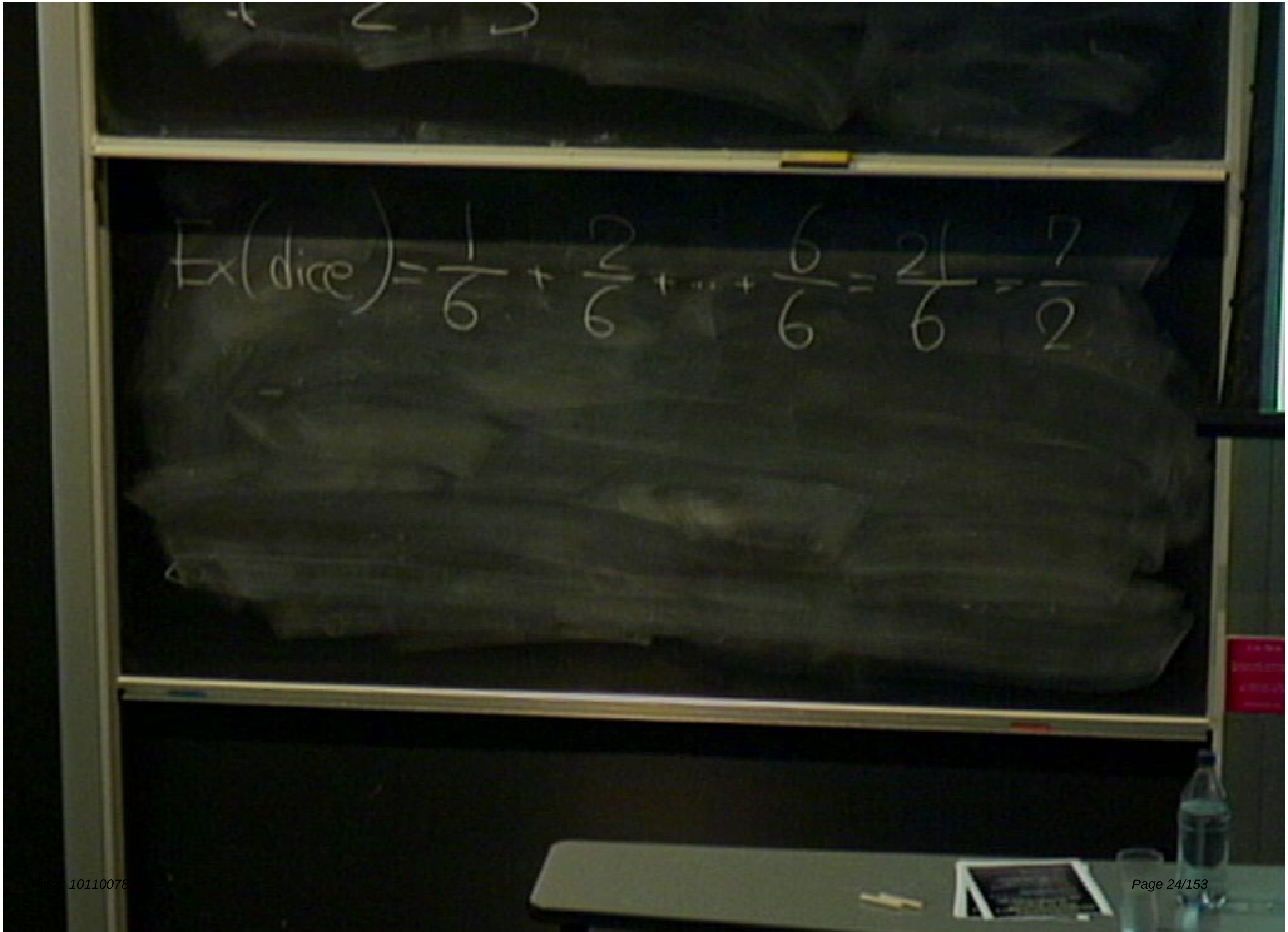

Event Sp.

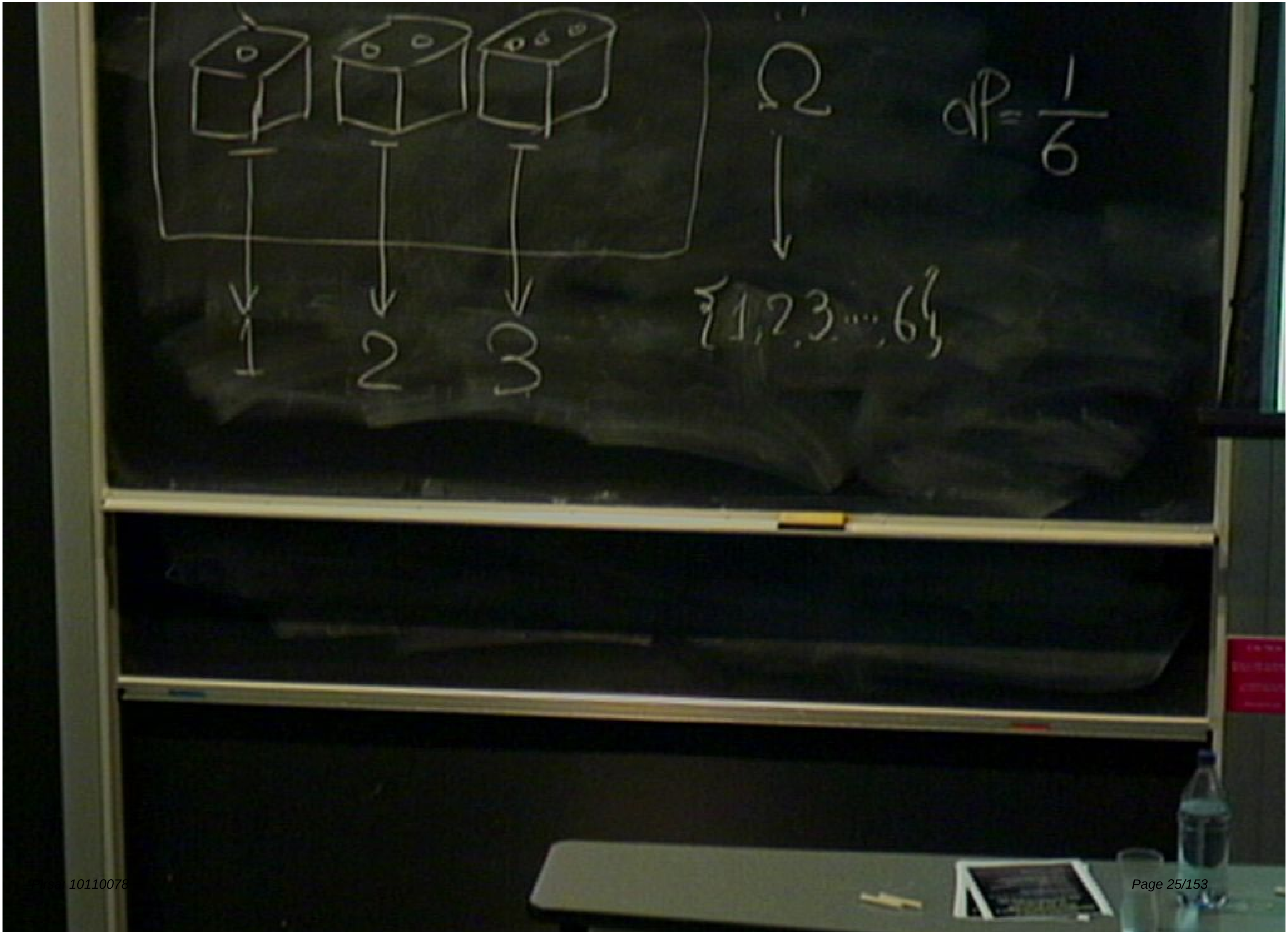

1
2
3

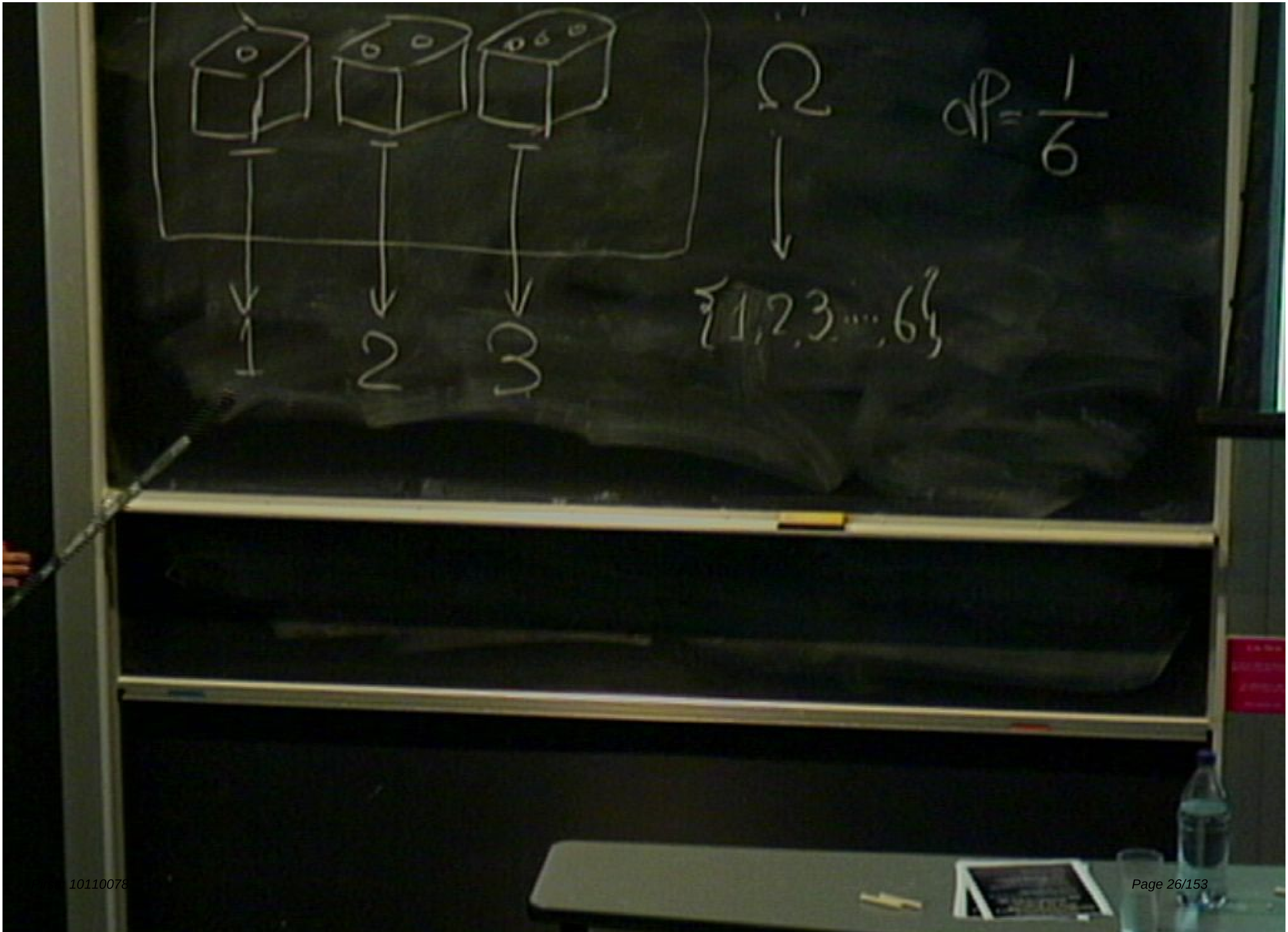

1
2
3
Ω
{1, 2, 3, ..., 6}



Ex(dice) = 1/6 + 2/6 + ... + 6/6 =


Ex(dice) = 1/6 + 2/6 + ... + 6/6 = 21/6 = 7/2


Ω
dP = 1/6
{1, 2, 3, ..., 6}
1
2
3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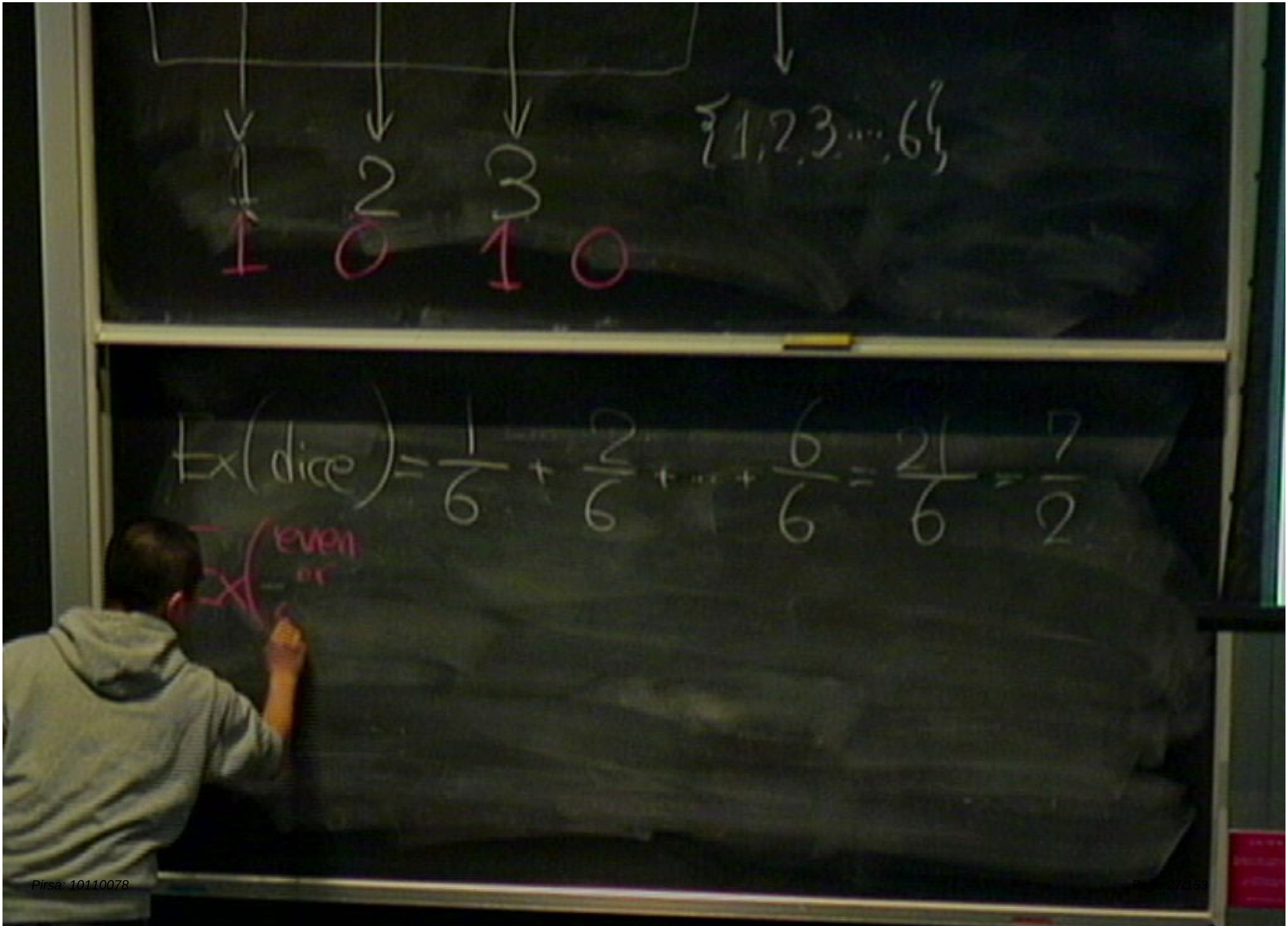
{1, 2, 3, ..., 6}
1 2 3
1 0 1 0
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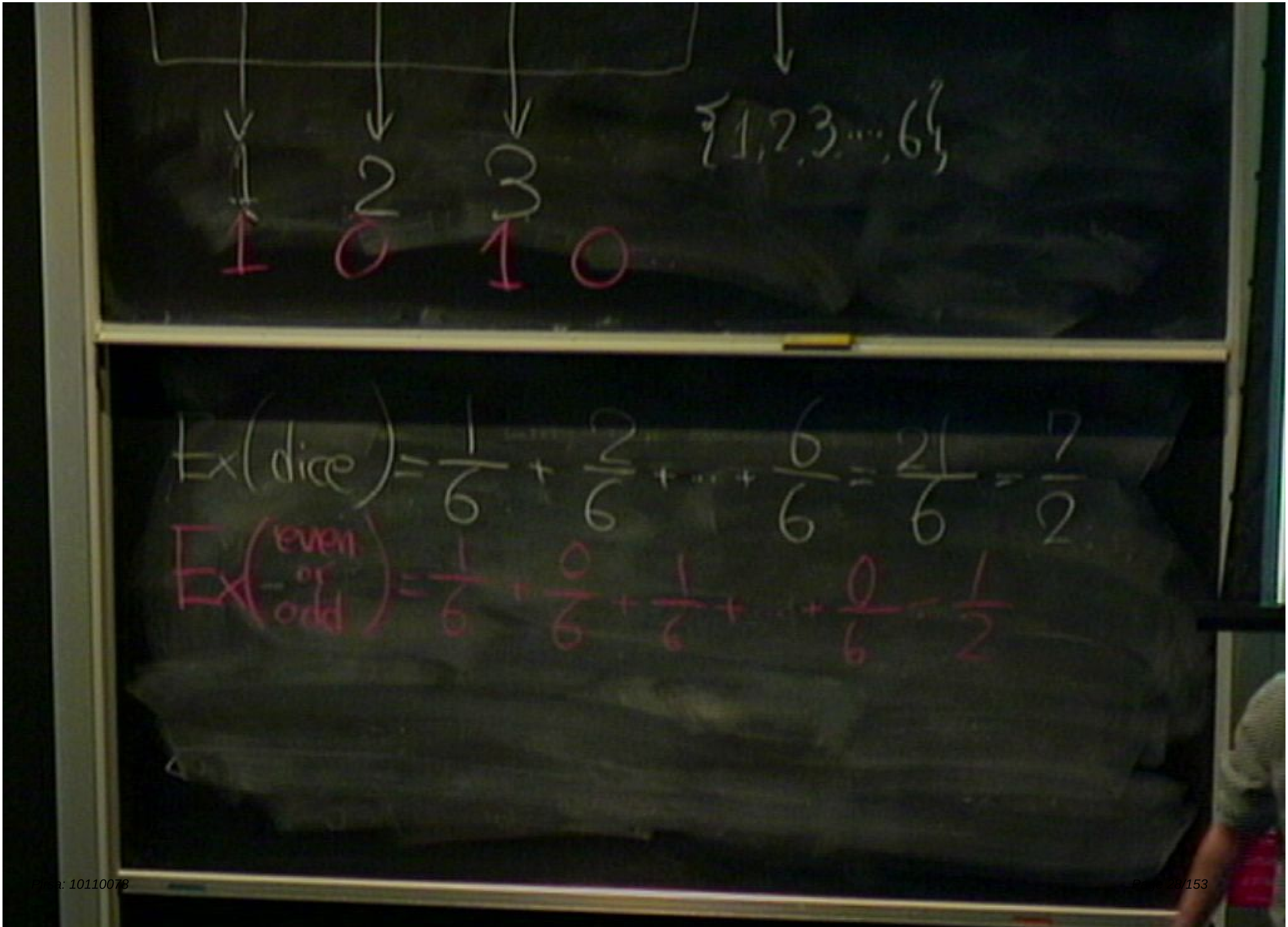
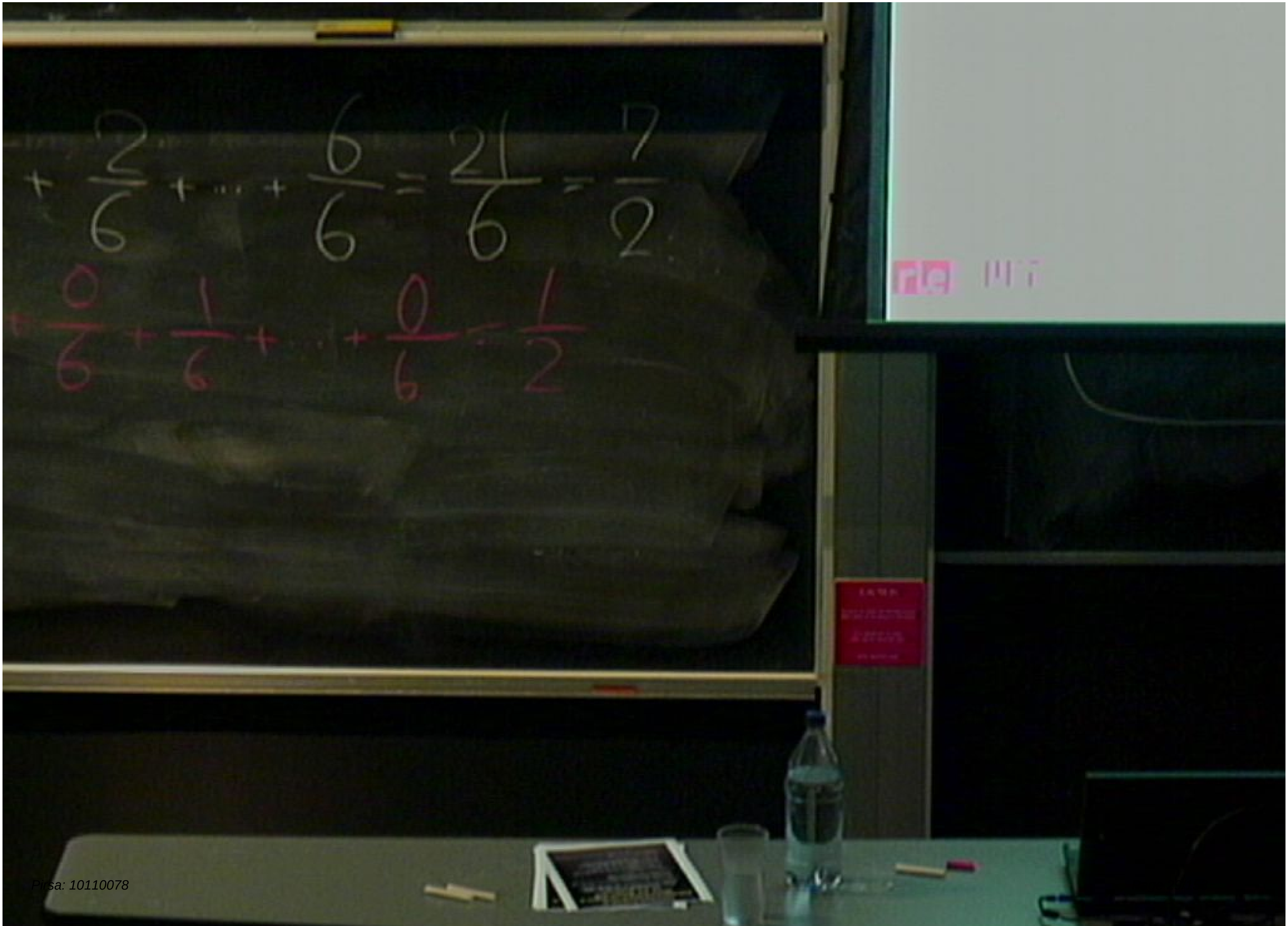
even


1 2 3
{1, 2, 3, ..., 6}
1 0 1 0
Ex(dice) = 1/6 + 2/6 + ... + 6/6 = 21/6 = 7/2
Ex(even or odd) = 1/6 + 0/6 + 1/6 + ... + 0/6 = 1/2



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Example

$$\mathcal{H} = \mathcal{L}^2(\mathbb{R})$$

Position Operator

$$\mathrm{Ex}(x,\psi) := \langle\psi|x|\psi\rangle = \int x|\psi(x)|^2 dx$$

Momentum Operator

$$\mathrm{Ex}(p,\psi) := \langle\psi|p|\psi\rangle = \int p|\psi(p)|^2 dp$$

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Not Correspondence!!

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Observable-dependent Probability Space

Observable-independent Probability Space??

Observable-independent Probability Space??

- We can construct the probability space independently on the observable by the weak values.

Def: Weak values of observable A

$$\langle A\rangle_w = \frac{\langle f|U(t_f,t)AU(t,t_i)|i\rangle}{\langle f|U(t_f,t_i)|i\rangle} \in \mathbb{C}$$

$|i\rangle$ pre-selected state $\langle f|$ post-selected state

Expectation Value?

(A. Hosoya and YS, J. Phys. A **43**, 385307 (2010))

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$dP = |\langle\phi|\psi\rangle|^2 d\phi$ is defined as the probability measure.

Born Formula ⇒ Random Variable = Weak Value

Variance?

$$Var(A) = \langle\psi|(A - \langle\psi|A|\psi\rangle)^2|\psi\rangle$$

$$= \langle\psi|A^2|\psi\rangle - (\langle\psi|A|\psi\rangle)^2$$

$$= \int d\phi \langle\psi|A|\phi\rangle\langle\phi|A|\psi\rangle - (\mathrm{Ex}(A))^2$$

$$= \int d\phi |\langle\phi|\psi\rangle|^2 \cdot \frac{\langle\psi|A|\phi\rangle}{\langle\psi|\phi\rangle} \cdot \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle} - (\mathrm{Ex}(A))^2$$

$$= \int |{}_\phi\langle A\rangle_\psi^w|^2 dP - \left(\int {}_\phi\langle A\rangle_\psi^w dP\right)^2$$

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Definition of Weak Values

Def: Weak values of observable A

$$\langle A \rangle_w = \frac{\langle f | U(t_f, t) A U(t, t_i) | i \rangle}{\langle f | U(t_f, t_i) | i \rangle} \in \mathbb{C}$$

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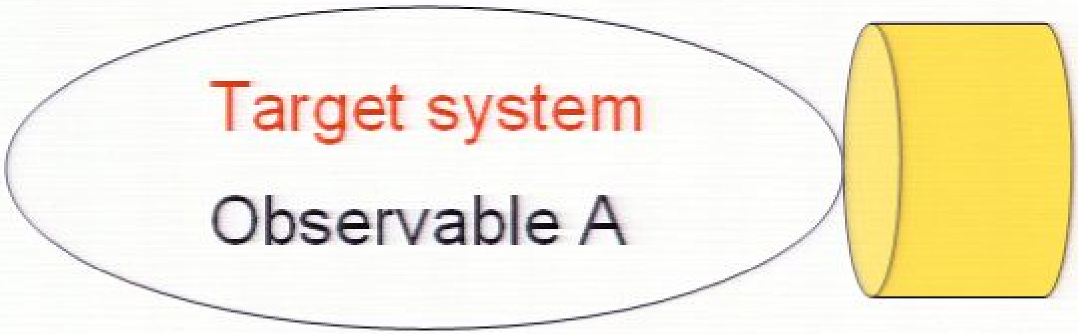
To measure the weak value...

Def: Weak measurement is called if a coupling constant with a probe interaction is very small.

(Y. Aharonov, D. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988))

Weak Measurement



Probe system

the pointer operator (position of the pointer) is q and its conjugate operator is p.

$$H_{\text{int}} = g\hat{A}\hat{p}\delta(t)$$

$$|\alpha\rangle_p = \langle f|e^{-ig\hat{A}\hat{p}}|i\rangle|\phi\rangle_p$$ State of the probe after measurement

$$\simeq \langle f|(I - ig\hat{A}\hat{p})|i\rangle|\phi\rangle_p \qquad g \ll 1$$

$$= \langle f|i\rangle(I - ig\langle A\rangle_w\hat{p})|\phi\rangle_p$$

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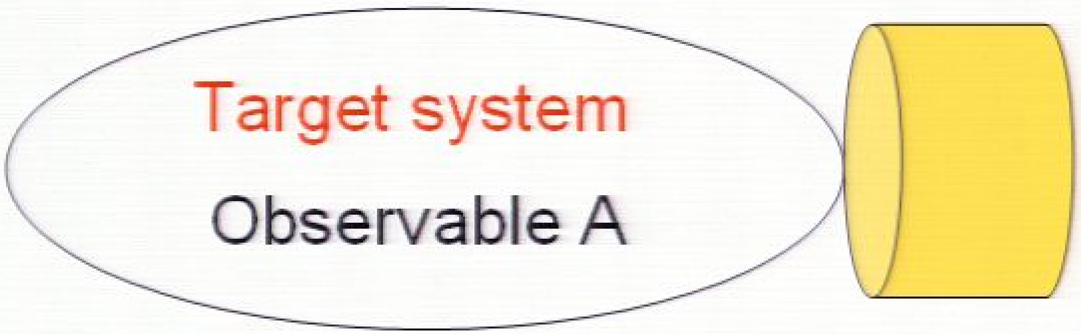

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Since the weak value of A is complex in general,

$$\begin{cases} \delta q := \langle q \rangle_f - \langle q \rangle_i = g\mathrm{Re}\langle A \rangle_w \\ \delta p := \langle p \rangle_f - \langle p \rangle_i = 2g\mathrm{Var}(p)\mathrm{Im}\langle A \rangle_w \end{cases}$$

Ve assume the probe wave function or the position be real-valued.

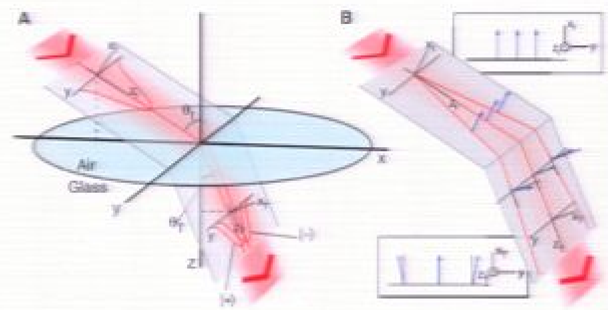
$\mathrm{Var}(p)$: Initial probe variance for the momentum

Weak values are experimentally accessible by the shifts of expectation values for the probe observables.

(R. Jozsa, Phys. Rev. A **76**, 044103 (2007))

Applications of Weak Value

- Amplification (Magnify the tiny effect)



  - Spin Hall Effect of Light

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  - Stability of Sagnac Interferometer

  (P. B. Dixon, D. J. Starling, A. N. Jordan, and J. C. Howell, Phys. Rev. Lett. **102**, 173601 (2009))

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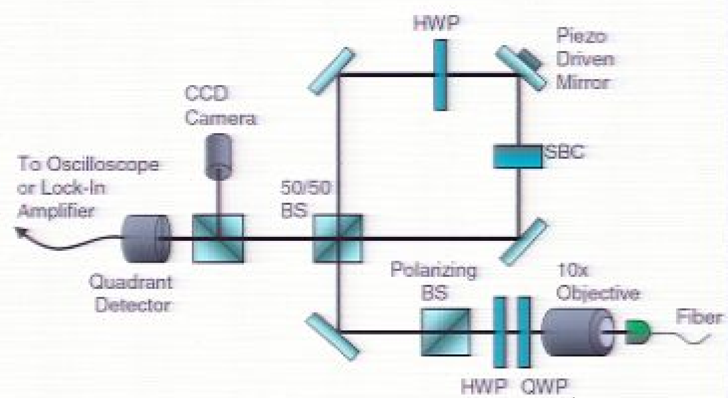

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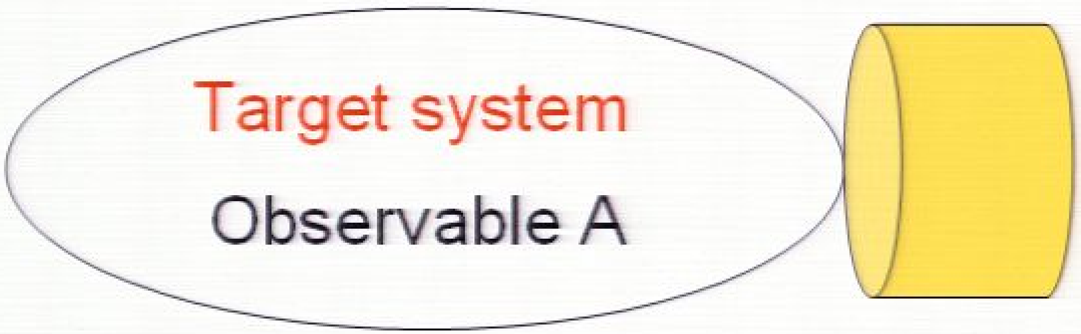

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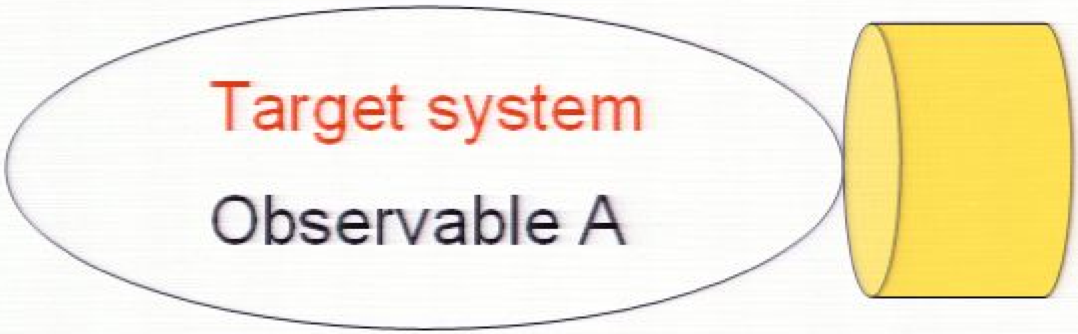

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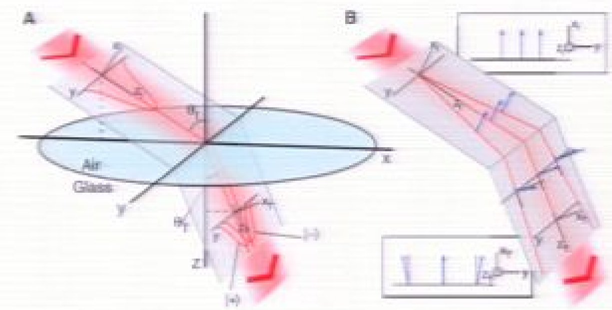

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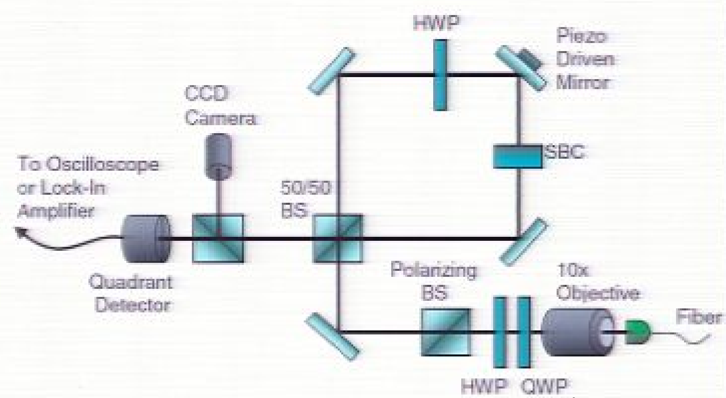
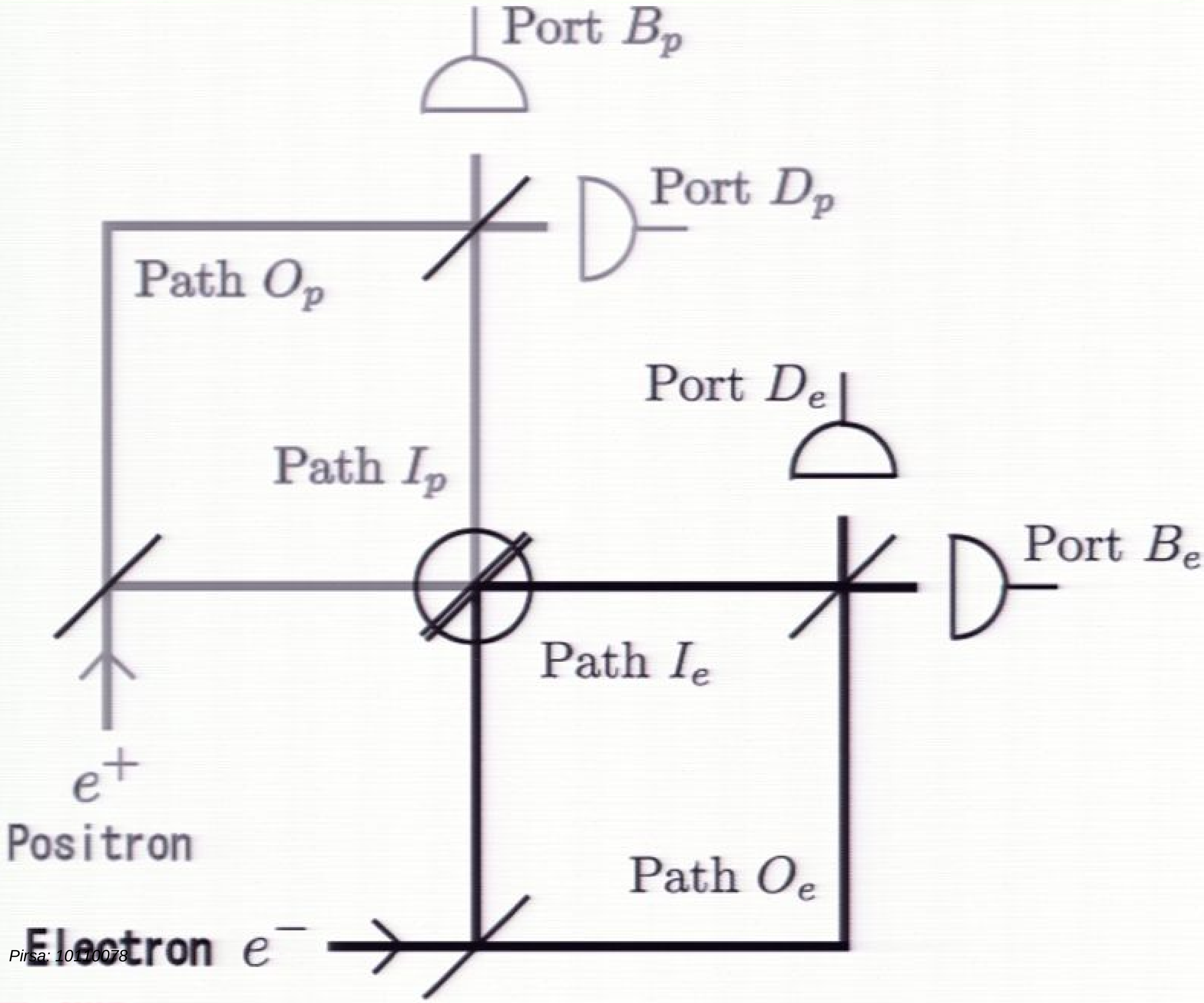


Today's Outline

1. Why do we need the weak value?
   - Motivation of the "weak value theory" – related to the probability theory
   - Definition and applications of the weak values
   - How to obtain the weak values – weak measurement
2. Counter-factual Processes
   - Hardy's paradox
   - Quantum description of the closed time-like curves
3. Conclusion

Hardy's Paradox



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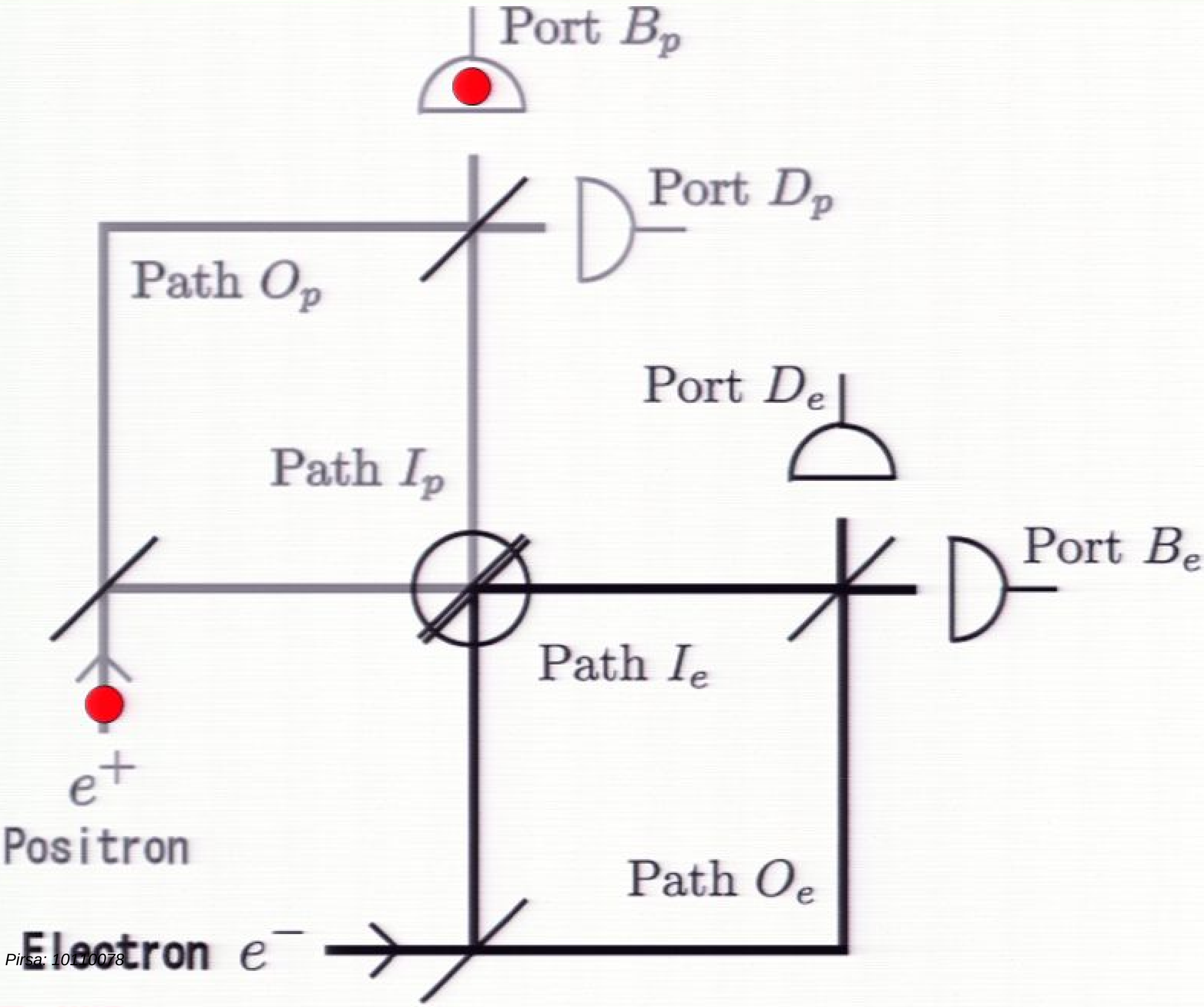

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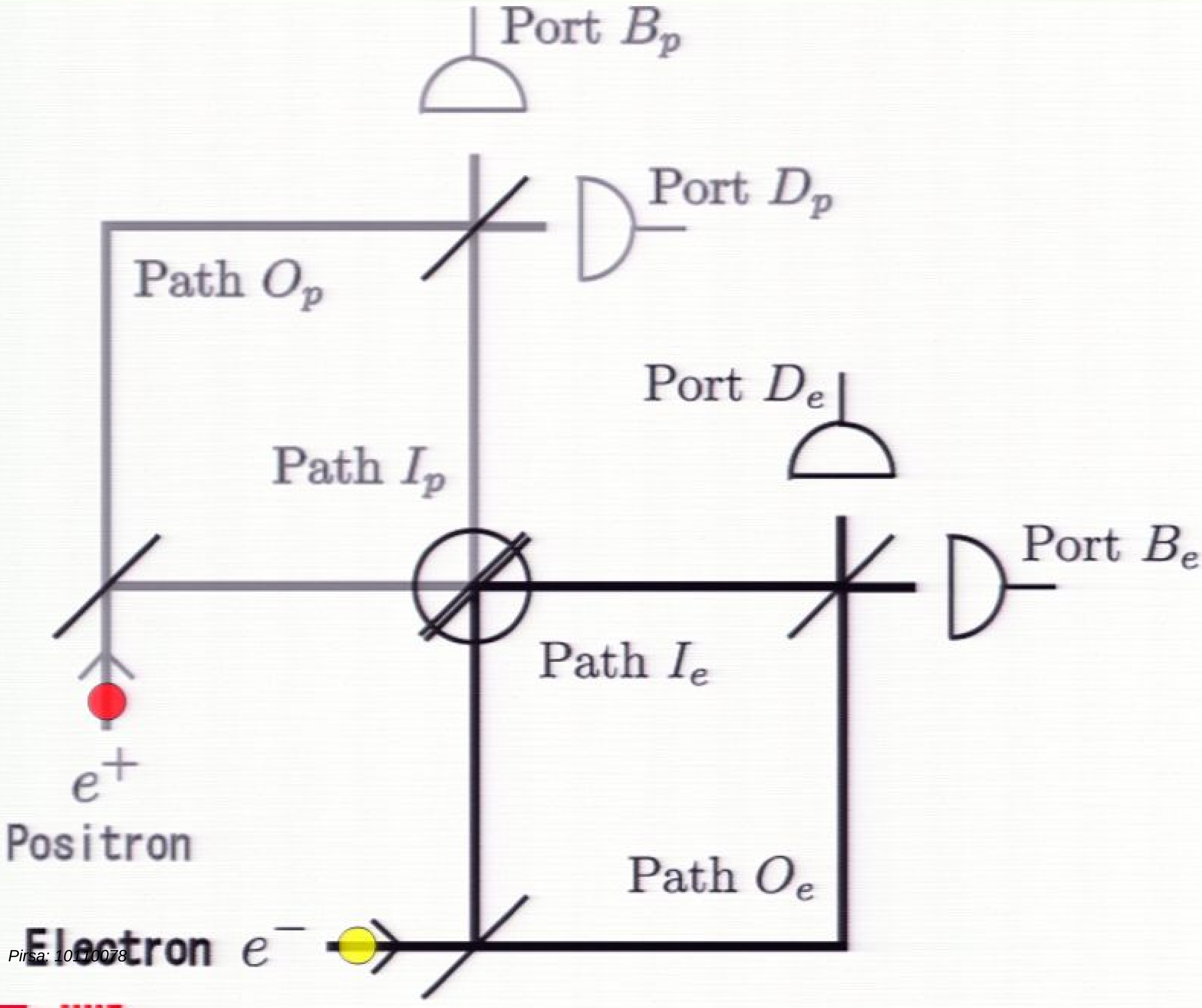

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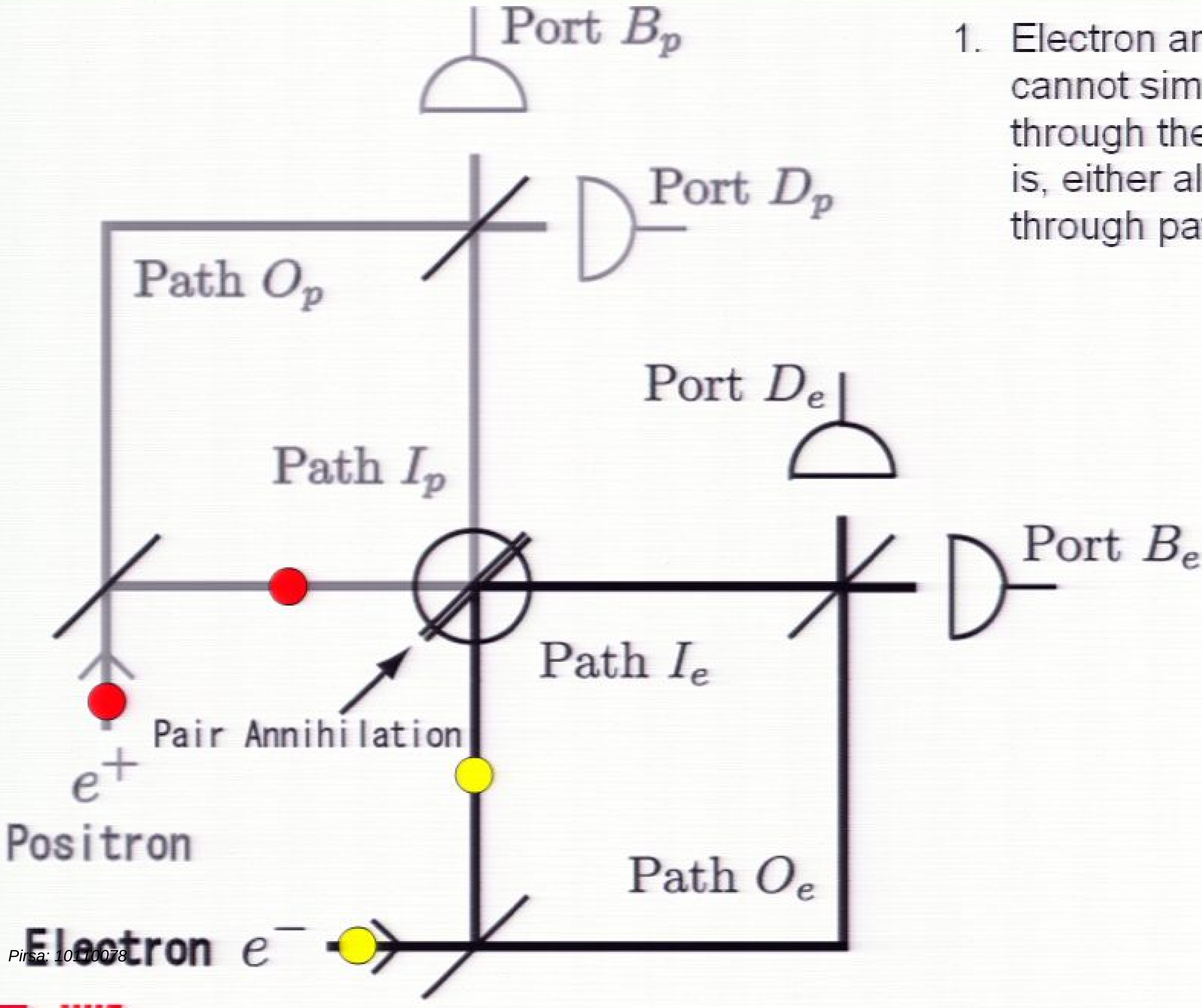

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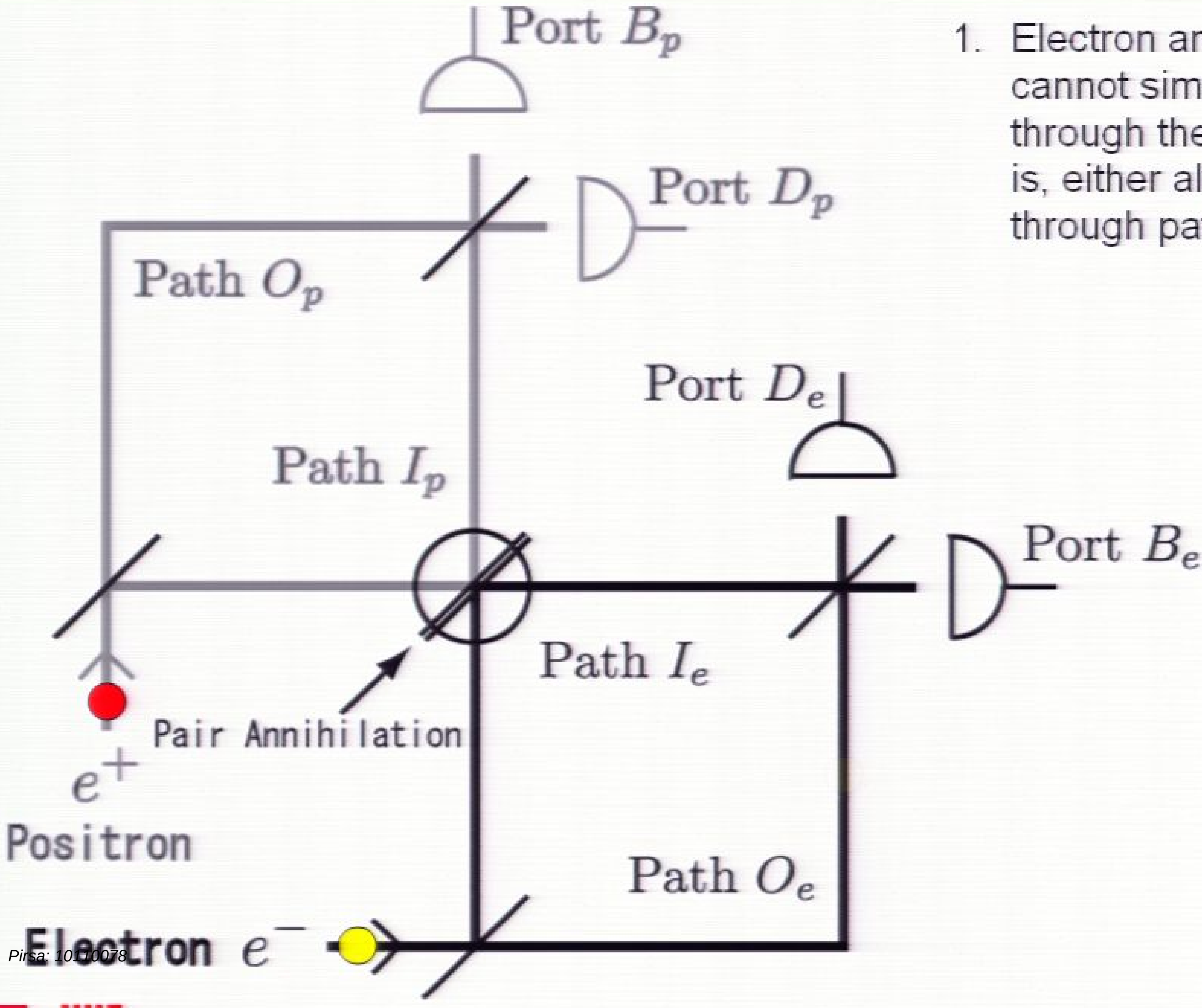

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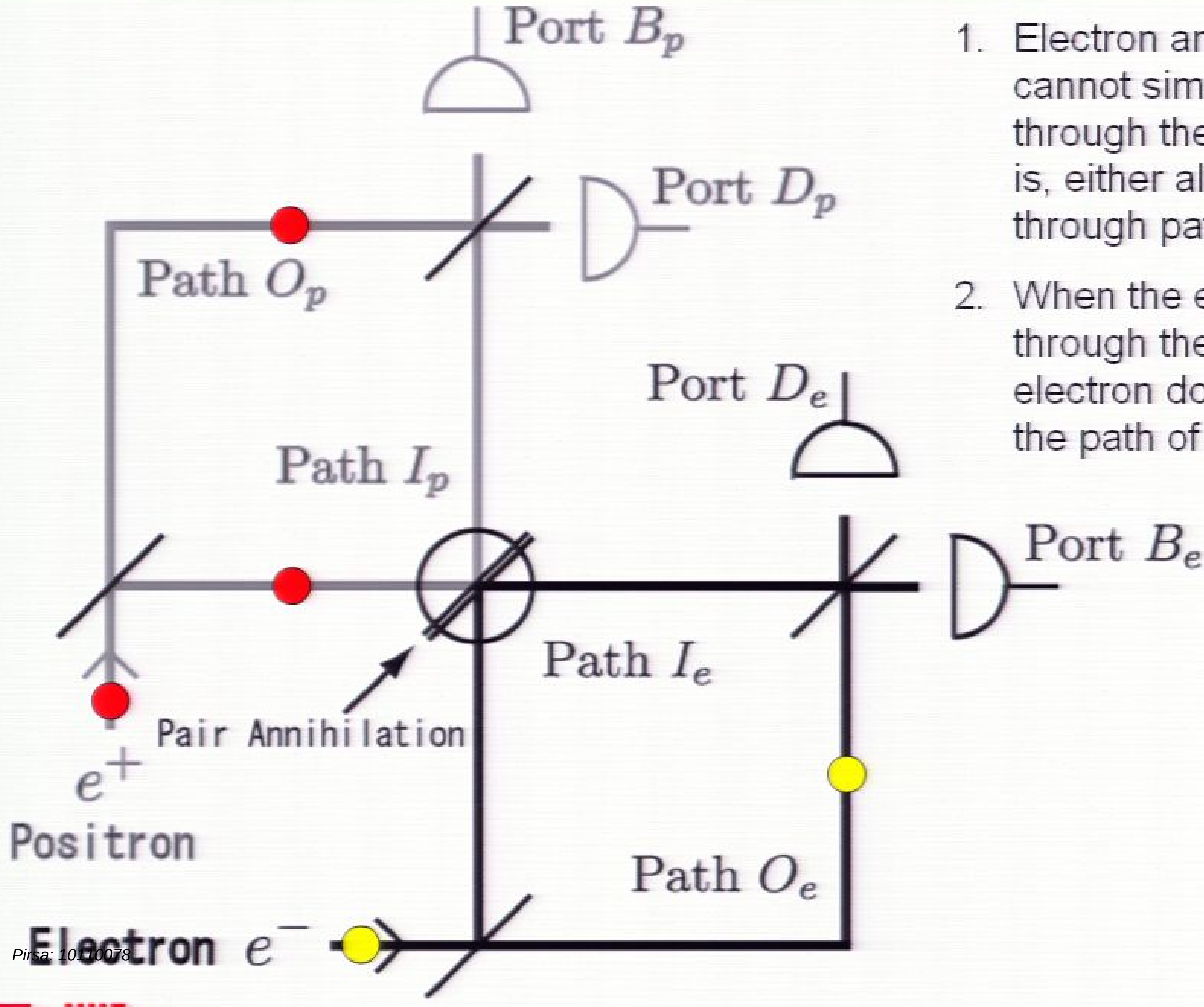

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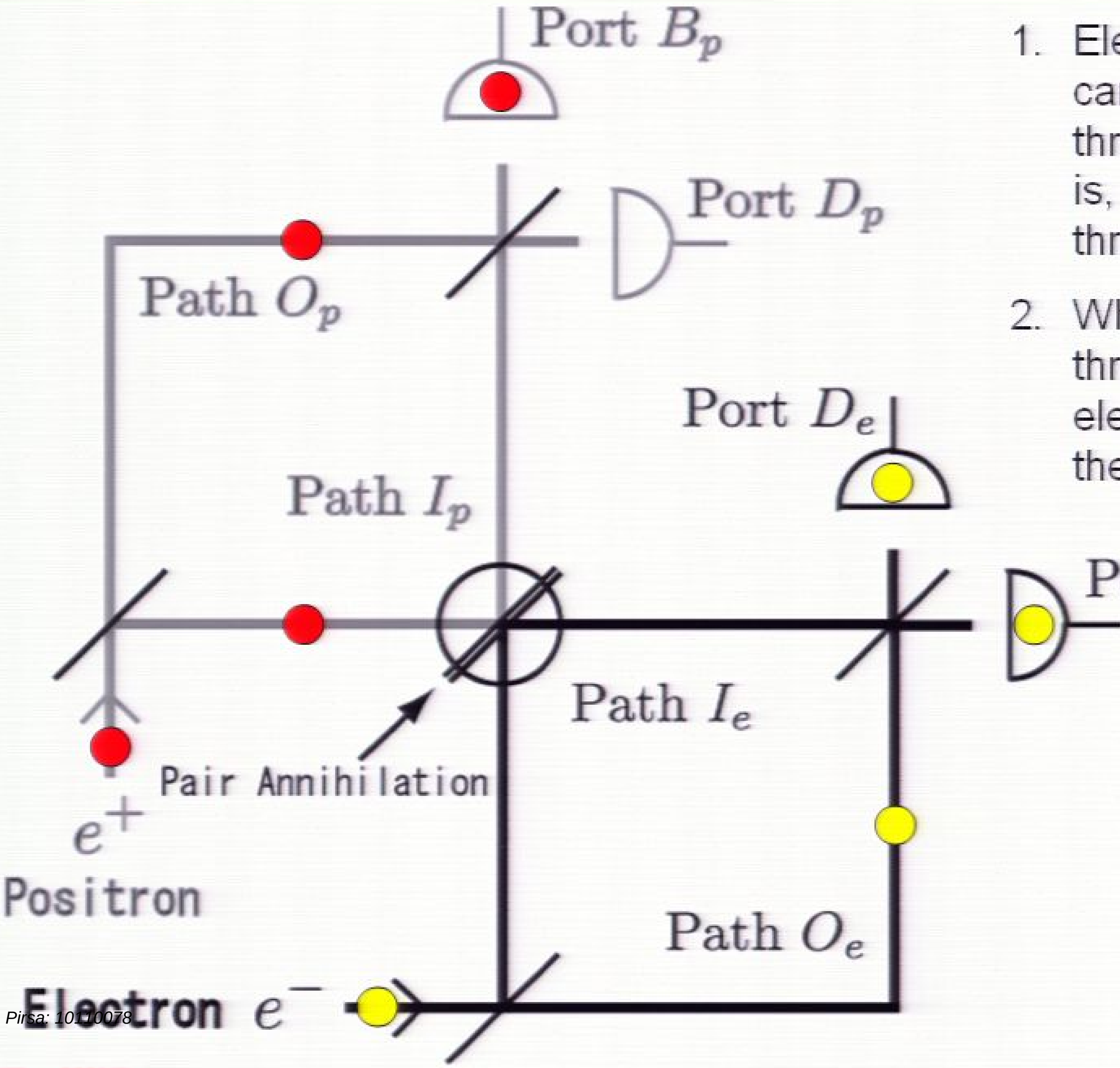

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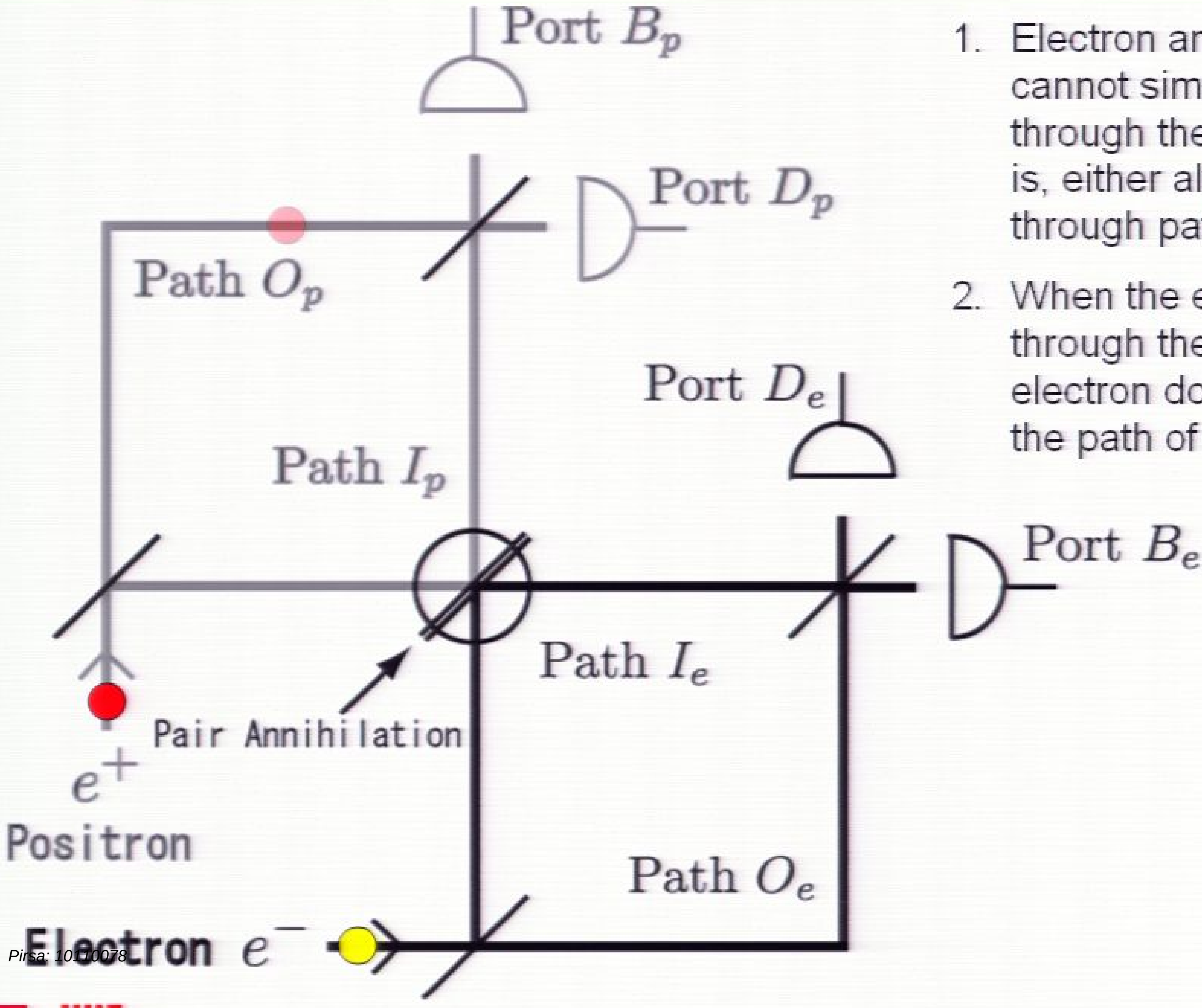

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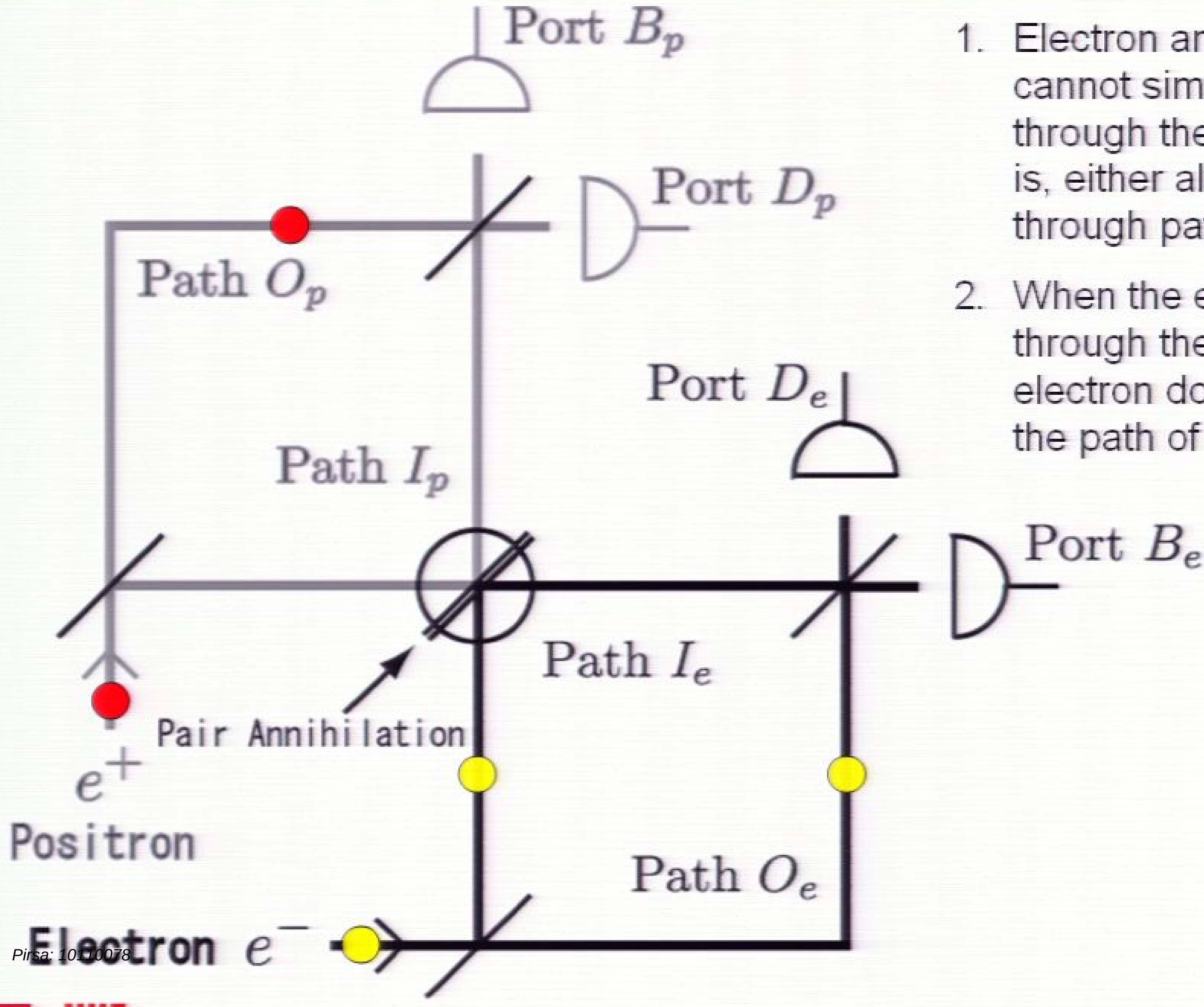

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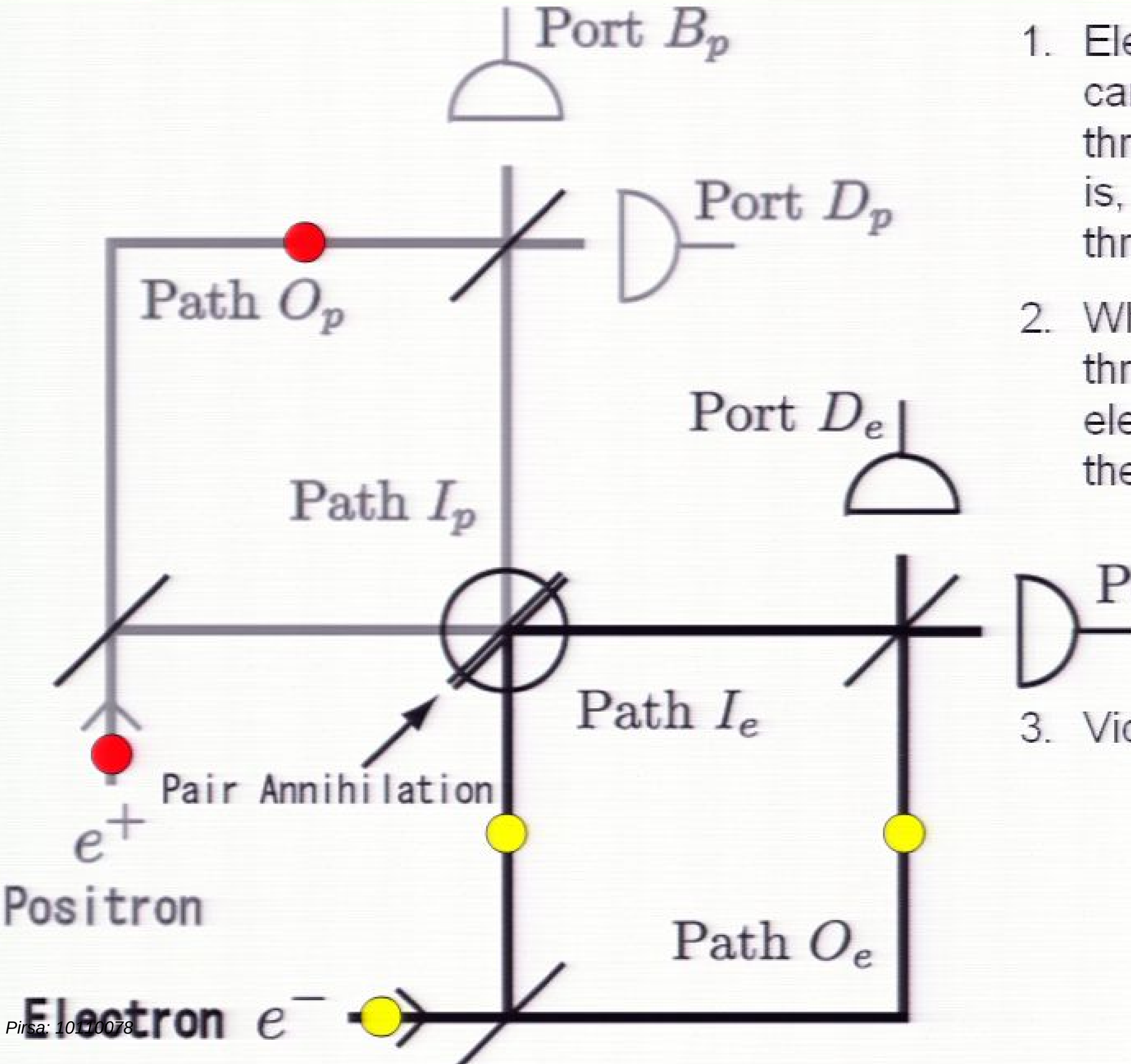

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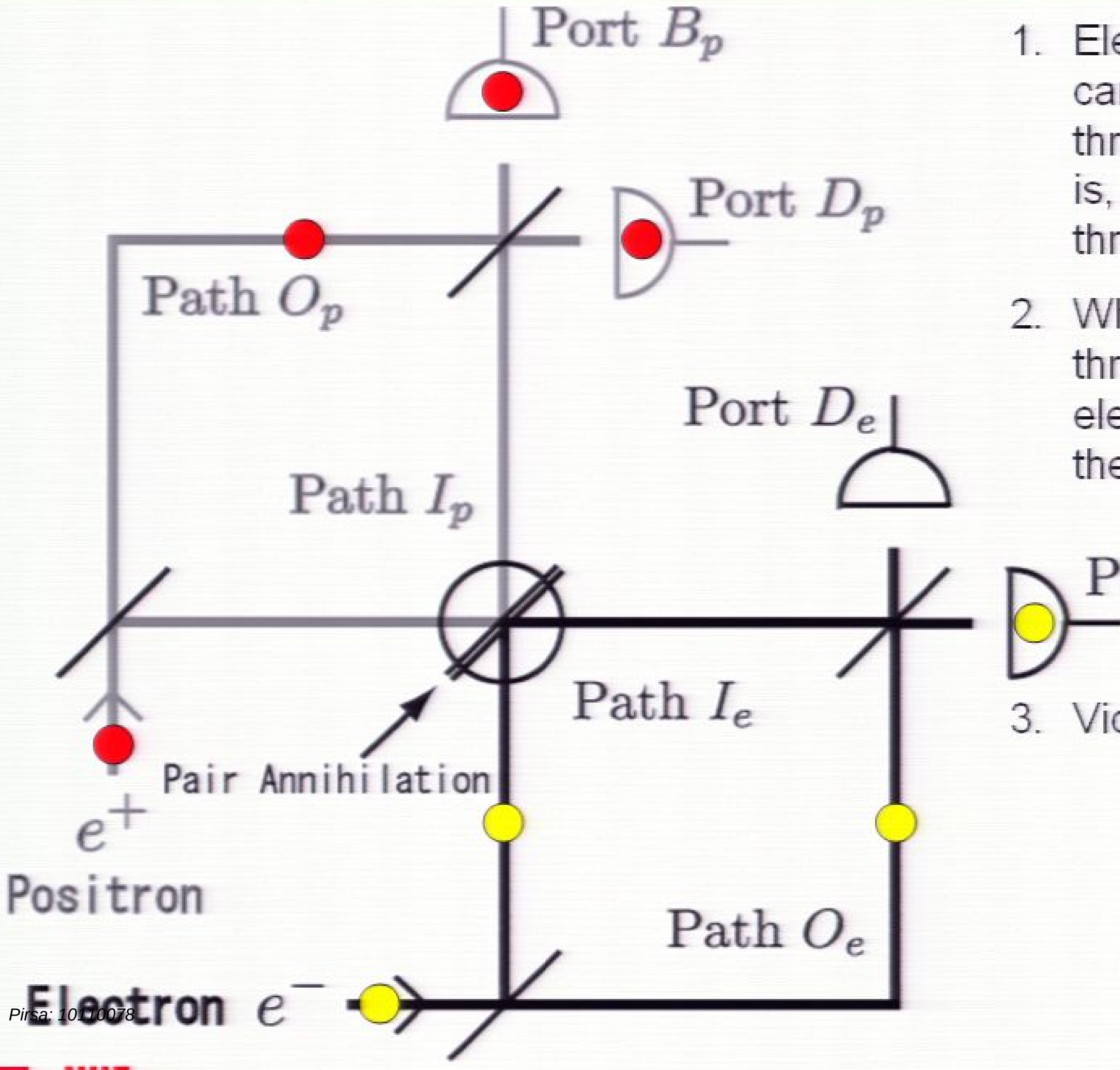

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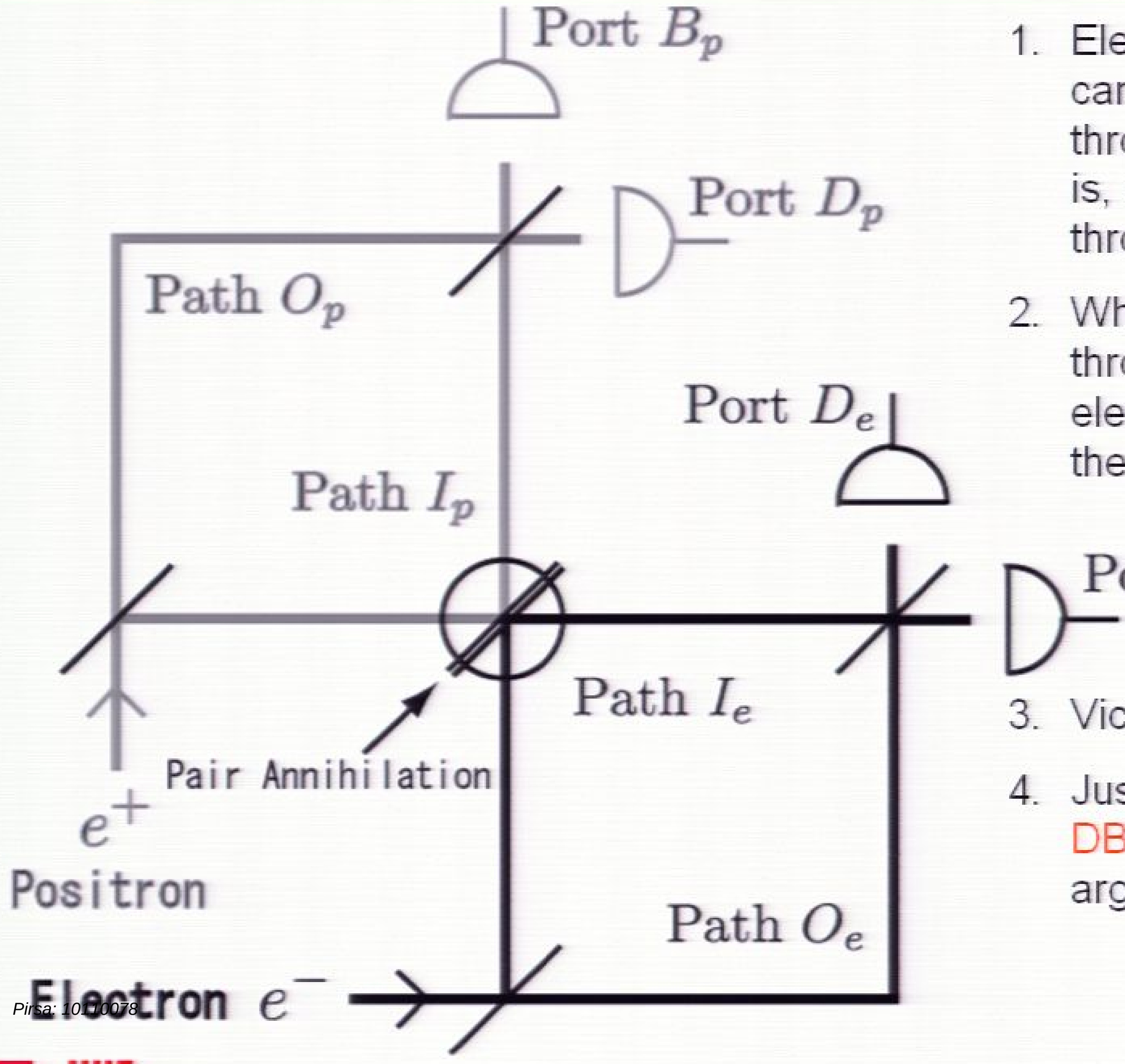

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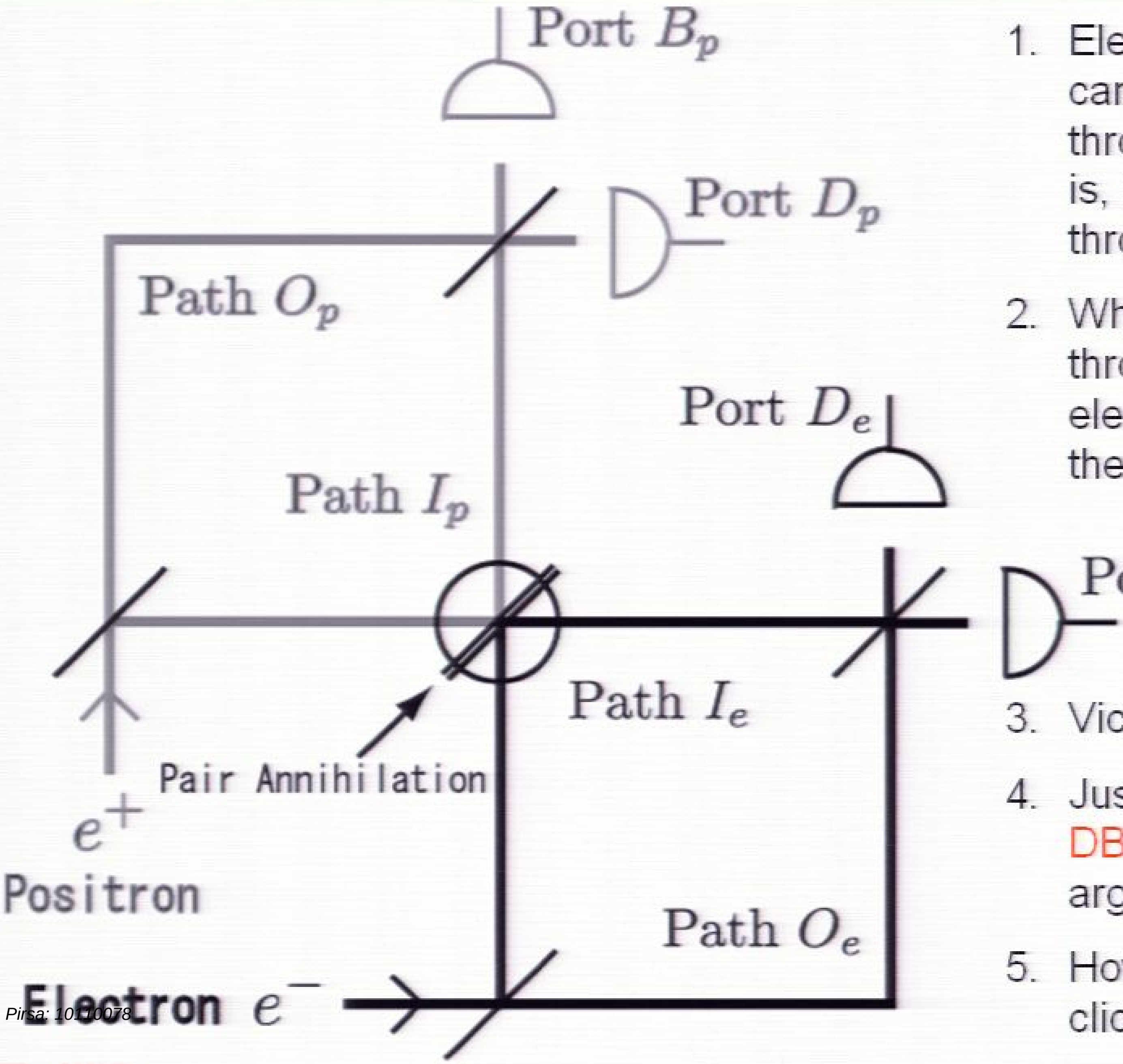

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5. However, QM tells us the click of DD. (prob. 1/12)

Important Remarks: Previous Studies

- I have not talked about the resolution of the Hardy paradox using the weak value. Please see
  - Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2002).
- Recently, this situation was experimentally realized.
  - J. S. Lundeen and A. M. Steinberg, Phys. Rev. Lett. **102**, 020404 (2009).
  - K. Yokota, T. Yamamoto, M. Koashi, and N. Imoto, New J. Phys. **11**, 033011 (2009).
- These results seemed to be very attractive for everyone.
  - Economist Mar. 5th, 2009.
  - The Wall Street Journal May 5th, 2009.

Why does the paradox be occurred?

(A. Hosoya and YS, J. Phys. A **43**, 385307 (2010))

Before the annihilation point:

$$\frac{1}{2}\left(|II\rangle + |IO\rangle + |OI\rangle + |OO\rangle\right)$$

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$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|IO\rangle + |OI\rangle + |OO\rangle\right)$$

- By this state, the probability to click DD can be calculated as 1/12.
- By the weak value analysis, this state can be used as the pre-selected state.

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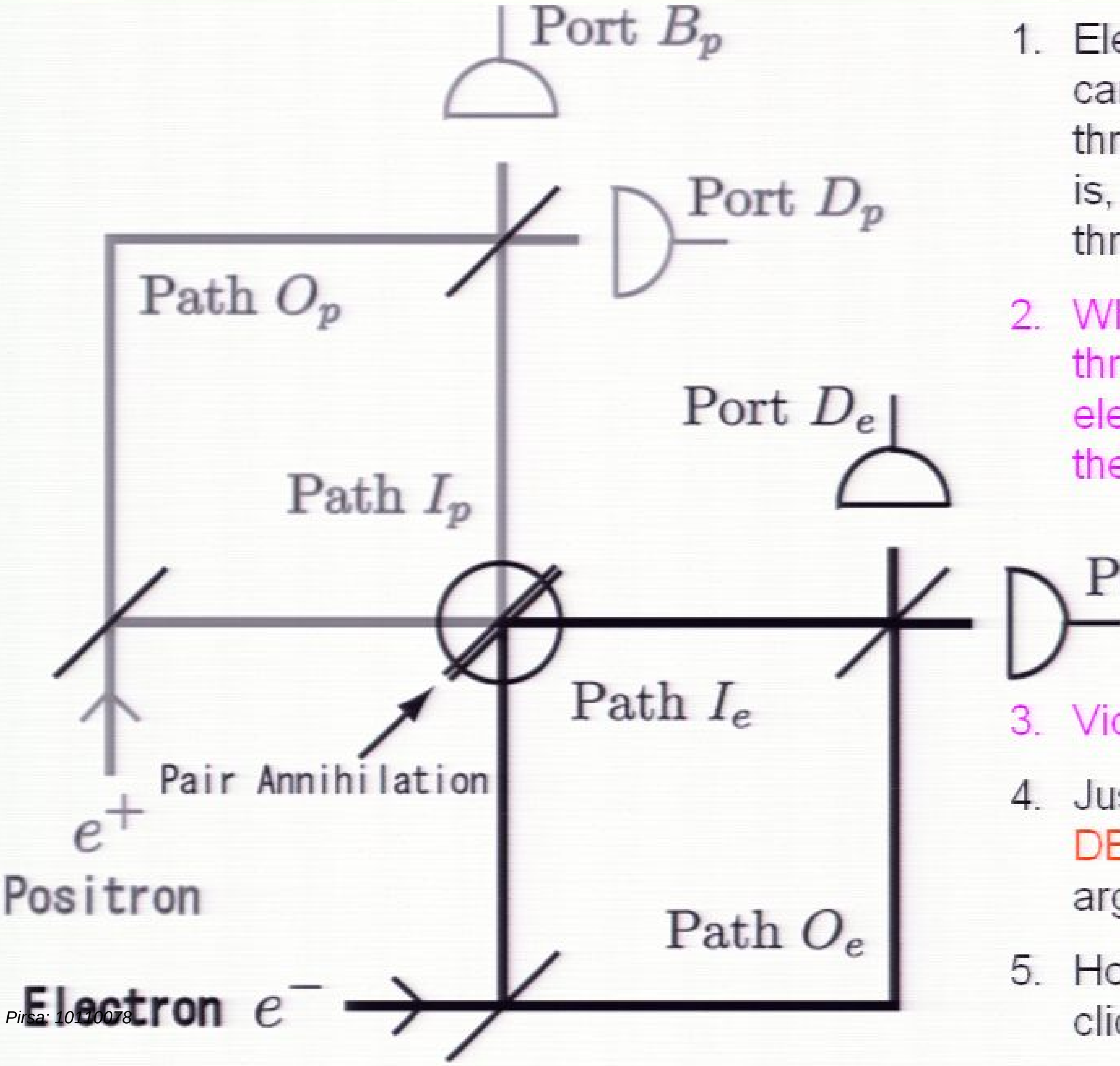
How to experimentally confirm this state?

- By the weak value analysis, this state can be used as the pre-selected state.

Only Information about...



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Counter-factual argument

- For the pre-selected state, the following operators are equivalent:

$$O(I+O) \sim_\psi O \otimes id.$$

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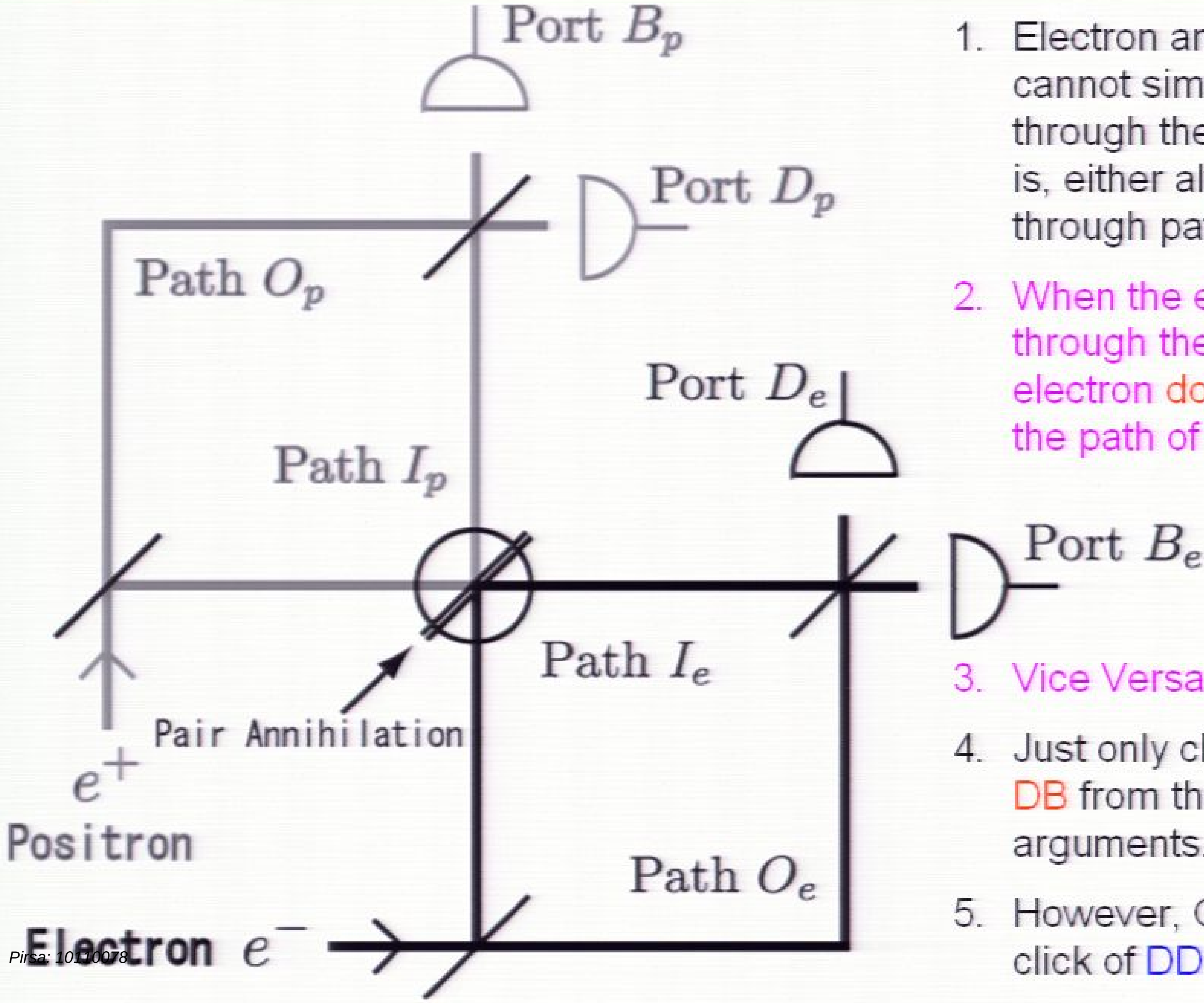

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Pre-Selected State and Weak Value

- We take the post-selected state for DD.

$$
\begin{aligned}
InP(DD) &:= {}_{DD}\langle IO\rangle_{\psi}^{w} - {}_{DD}\langle I \otimes id.\rangle_{\psi}^{w} = 0, \\
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Experimentally realizable!!

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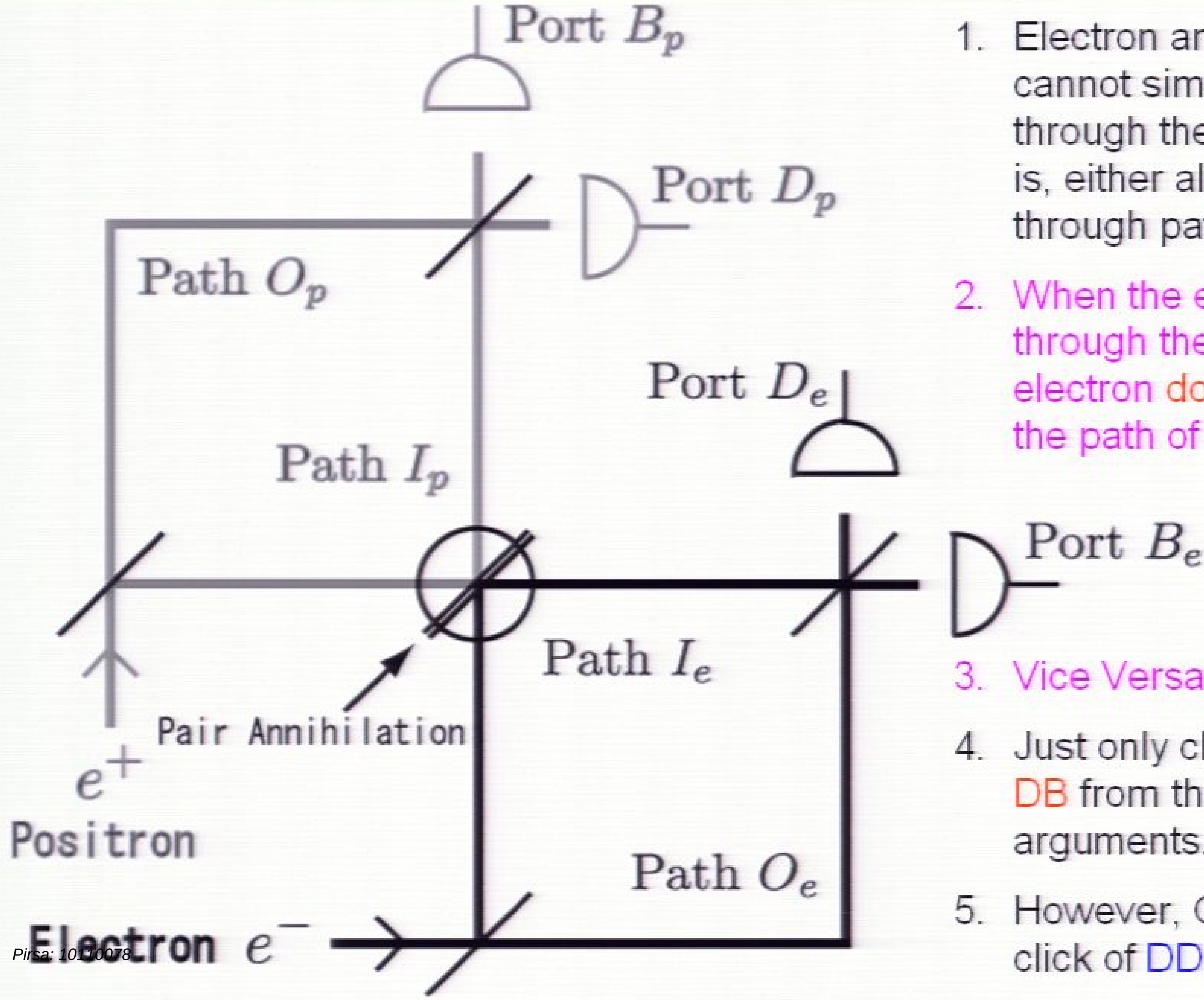

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Short Remarks

- This idea was first proposed by **Benjamin Schumacher** inspired by quantum teleportation (unpublished). This idea was talked by **Charlie Bennett** in 2002.

- This idea is related to the quantum knot theory and the spin network representation in quantum gravity.

- This work was published in
  - Along with experimental demonstration of the grandfather paradox

  S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, YS, S. Pirandola, L.A. Rozema, A. Darabi, Y. Soudagar, L.K. Shalm, and A.M. Steinberg, arXiv:1005.2219.
  - Path integral analysis (only theoretical work)

  S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, and YS, arXiv:1007.2615.

- The above analysis is generally independent to the weak value.

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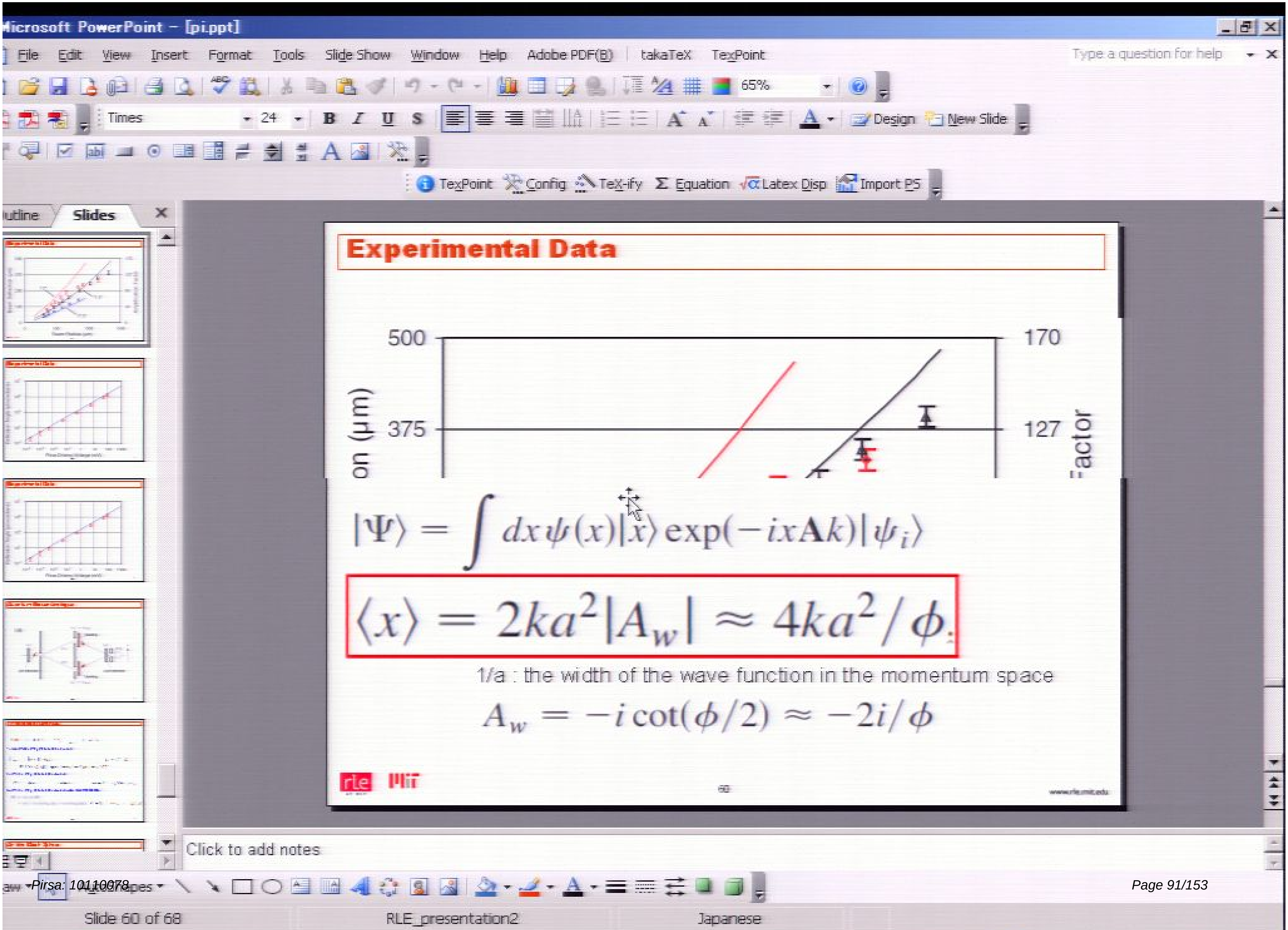
Microsoft PowerPoint - [pi.ppt]
Experimental Data
500
375
170
127
on (μm)
Factor
$|\Psi\rangle = \int dx\, \psi(x) |x\rangle \exp(-ix\mathbf{A}k) |\psi_i\rangle$
$\langle x \rangle = 2ka^2 |A_w| \approx 4ka^2/\phi$
1/a : the width of the wave function in the momentum space
$A_w = -i\cot(\phi/2) \approx -2i/\phi$
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RLE_presentation2
Japanese

Weak Value and Hardy's Paradox

$$\text{initial state} \ : |\psi\rangle = \frac{1}{\sqrt{3}}(|NO_1, O_2\rangle + |O_1, NO_2\rangle + |NO_1, NO_2\rangle)$$

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⇩

How can we observe these joint weak values experimentally ?

Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2002).

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⇩

How can we observe these joint weak values experimentally ?

Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2002).

Weak Value and Hardy's Paradox

This slide is created by Kazuhiro Yokota (Osaka Univ.)

$$\text{initial state} \ : |\psi\rangle = \frac{1}{\sqrt{3}}(|NO_1, O_2\rangle + |O_1, NO_2\rangle + |NO_1, NO_2\rangle)$$

$$\text{final state} \ : |\phi\rangle = \frac{1}{2}(|NO_1\rangle - |O_1\rangle)(|NO_2\rangle - |O_2\rangle)$$

$$\left\{\begin{array}{ll} |O_1\rangle\langle O_1|_w = 1, & |O_2\rangle\langle O_2|_w = 1 \\ |NO_1\rangle\langle NO_1|_w = 0, & |NO_2\rangle\langle NO_2|_w = 0 \\ & \\ |O_1, O_2\rangle\langle O_1, O_2|_w = 0, & |NO_1, NO_2\rangle\langle NO_1, NO_2|_w = -1 \\ |O_1, NO_2\rangle\langle O_1, NO_2|_w = 1, & |NO_1, O_2\rangle\langle NO_1, O_2|_w = 1 \end{array}\right.$$

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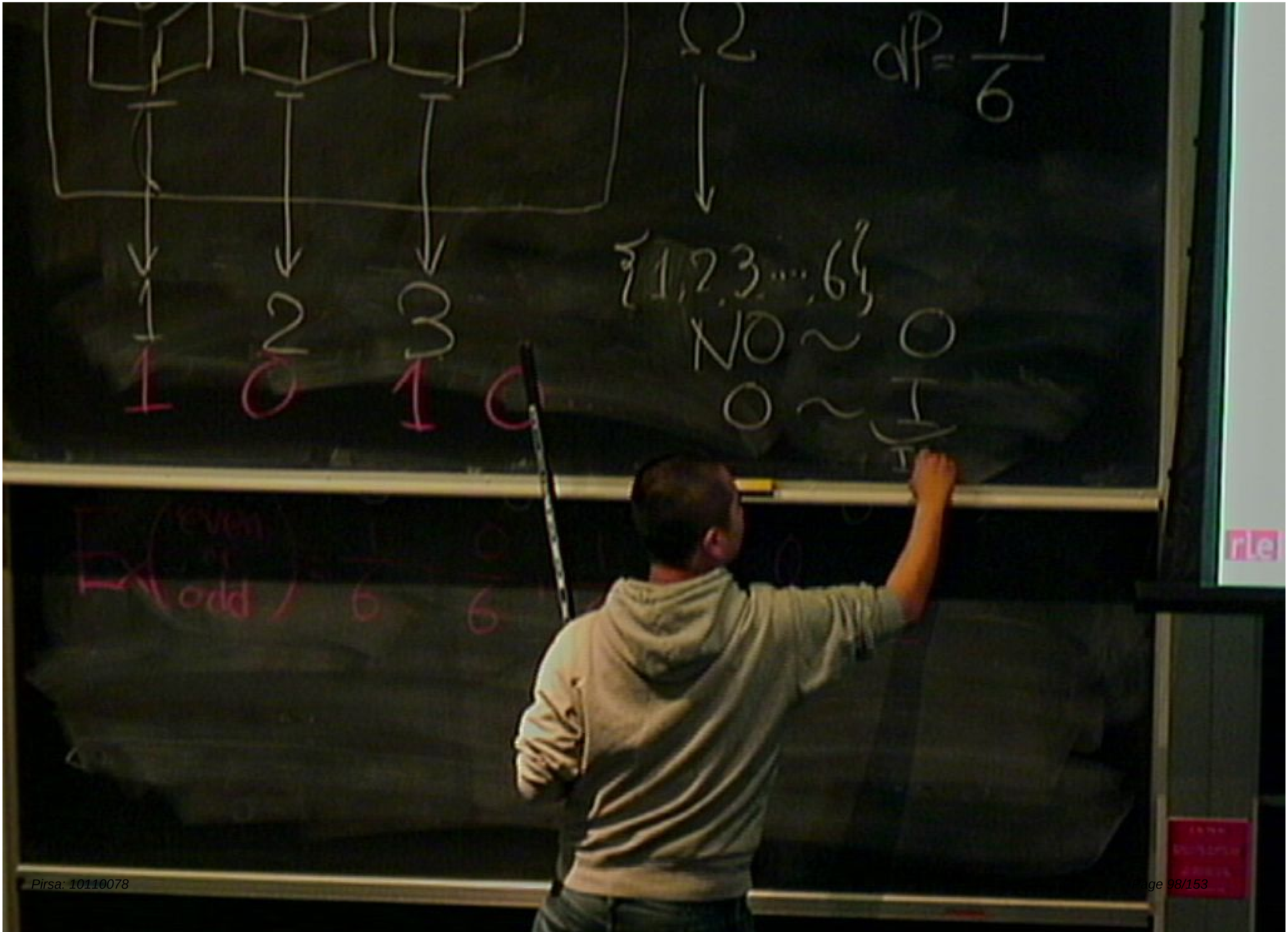
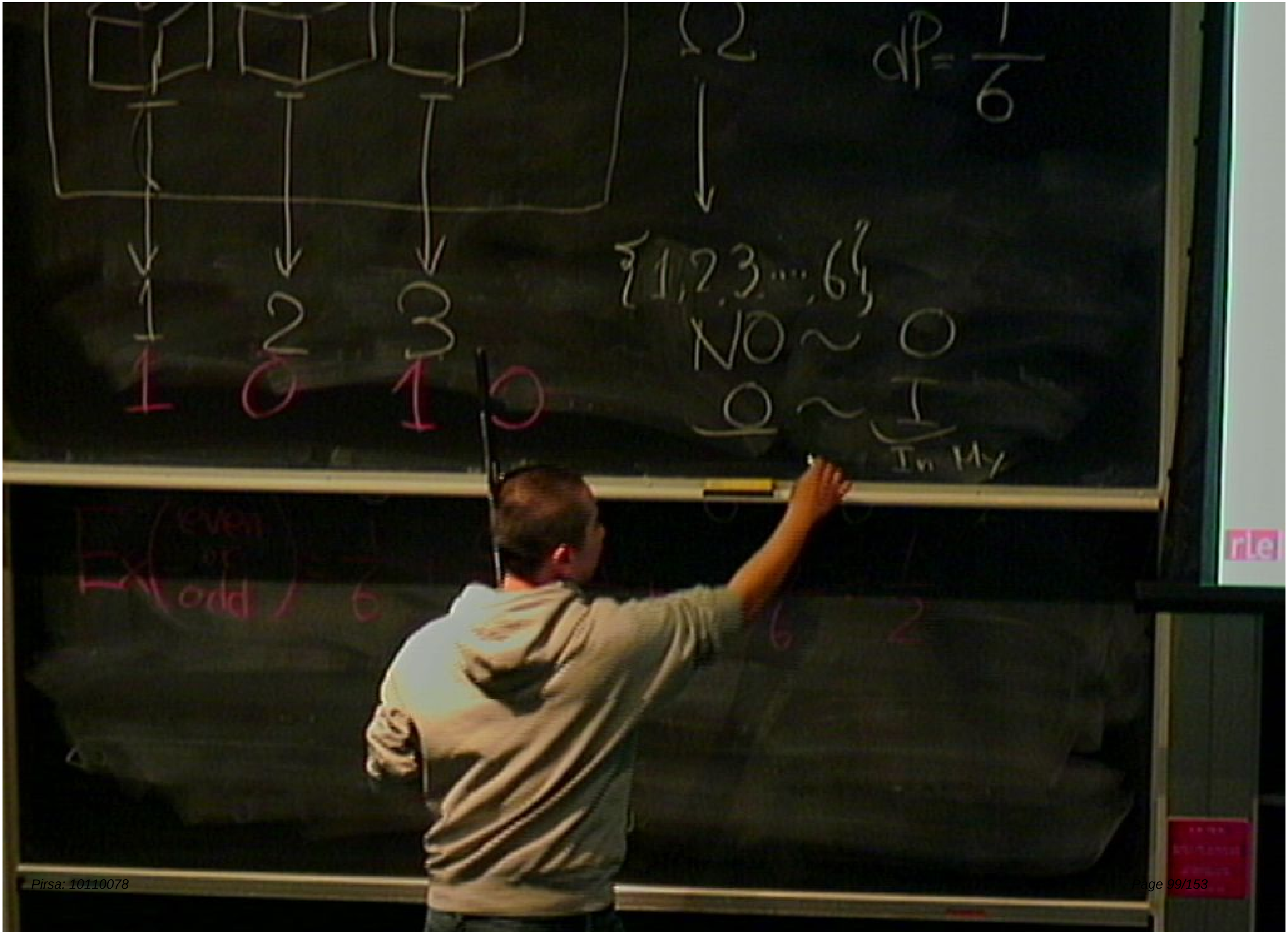

Ω
dP = 1/6
{1, 2, 3, ..., 6}
NO ~ 0
0 ~ 1
1 2 3
1 0 1 0


Ω
dP = 1/6
{1,2,3,...,6}
NO ~ 0
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In My
1 2 3
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Weak Value and Hardy's Paradox

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$$
\left\{
\begin{array}{ll}
|O_1\rangle\langle O_1|_w = 1, & |O_2\rangle\langle O_2|_w = 1 \\
|NO_1\rangle\langle NO_1|_w = 0, & |NO_2\rangle\langle NO_2|_w = 0 \\
 & \\
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\end{array}
\right.
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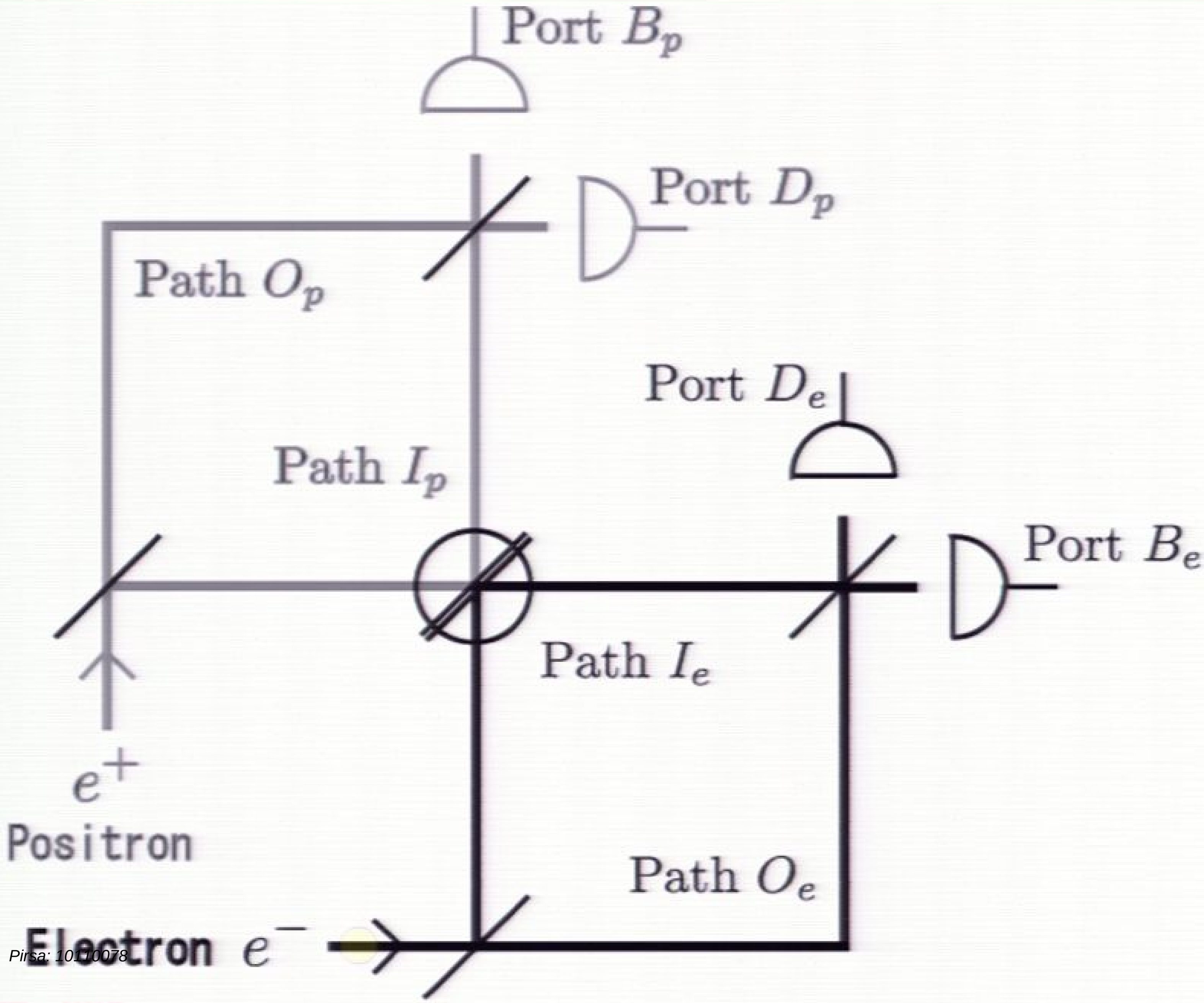
Hardy's Paradox



Important Remarks: Previous Studies

- I have not talked about the resolution of the Hardy paradox using the weak value. Please see
  - Y. Aharonov, A. Botero, S. Popescu, B. Reznik, and J. Tollaksen, Phys. Lett. A **301**, 130 (2002).
- Recently, this situation was experimentally realized.
  - J. S. Lundeen and A. M. Steinberg, Phys. Rev. Lett. **102**, 020404 (2009).
  - K. Yokota, T. Yamamoto, M. Koashi, and N. Imoto, New J. Phys. **11**, 033011 (2009).
- These results seemed to be very attractive for everyone.
  - Economist Mar. 5th, 2009.
  - The Wall Street Journal May 5th, 2009.

What is the state-dependent equivalence?

$$A = B$$

$$\Leftrightarrow \quad A|\psi\rangle = B|\psi\rangle \ \forall|\psi\rangle$$

$\Downarrow$ State-dependent equivalence

$$A \sim_\psi B$$

$$\Leftrightarrow \quad \langle\psi|(A-B)^2|\psi\rangle = 0$$

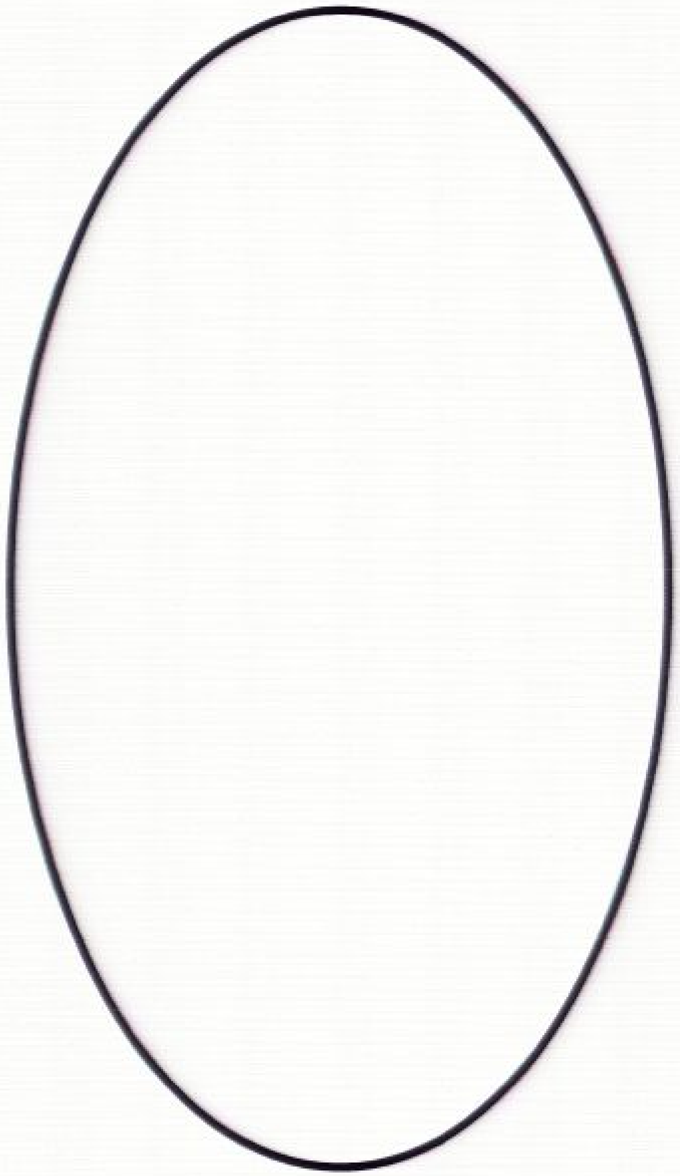
Today's Outline

1. Why do we need the weak value?
   - Motivation of the "weak value theory" – related to the probability theory
   - Definition and applications of the weak values
   - How to obtain the weak values – weak measurement

2. Counter-factual Processes
   - Hardy's paradox
   - Quantum description of the closed time-like curves

3. Conclusion

Closed Time-like Curve (CTC)



In general relativity, the solution of the Einstein equation allows to exist the closed time-like curve.

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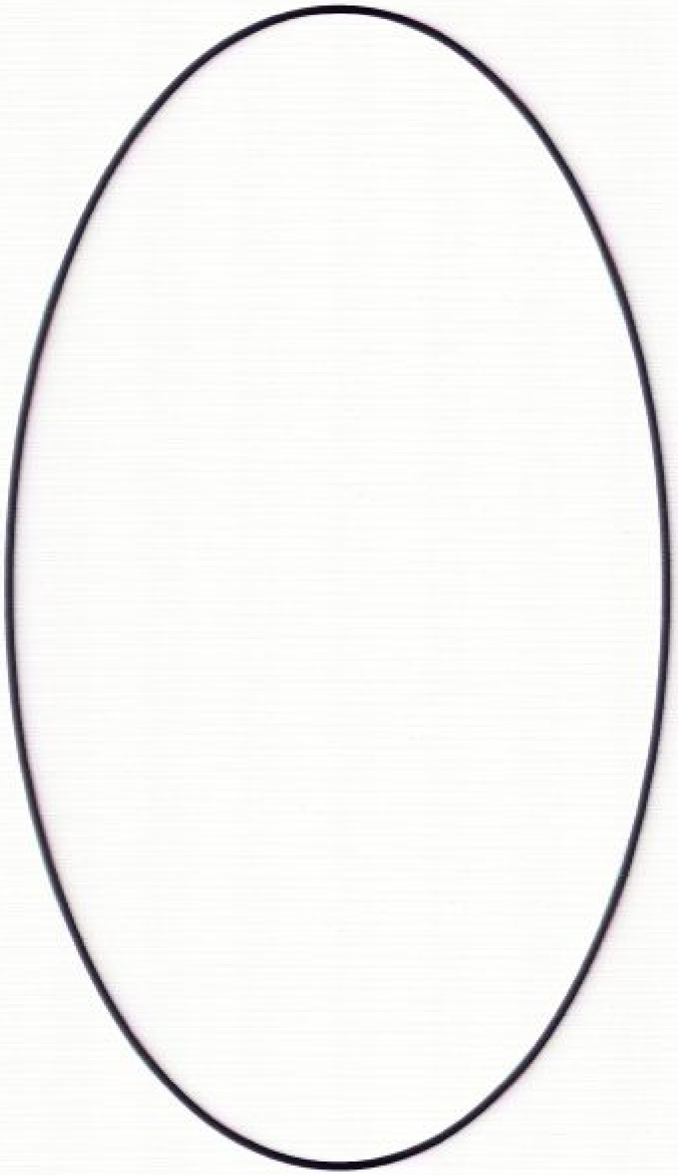

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But....

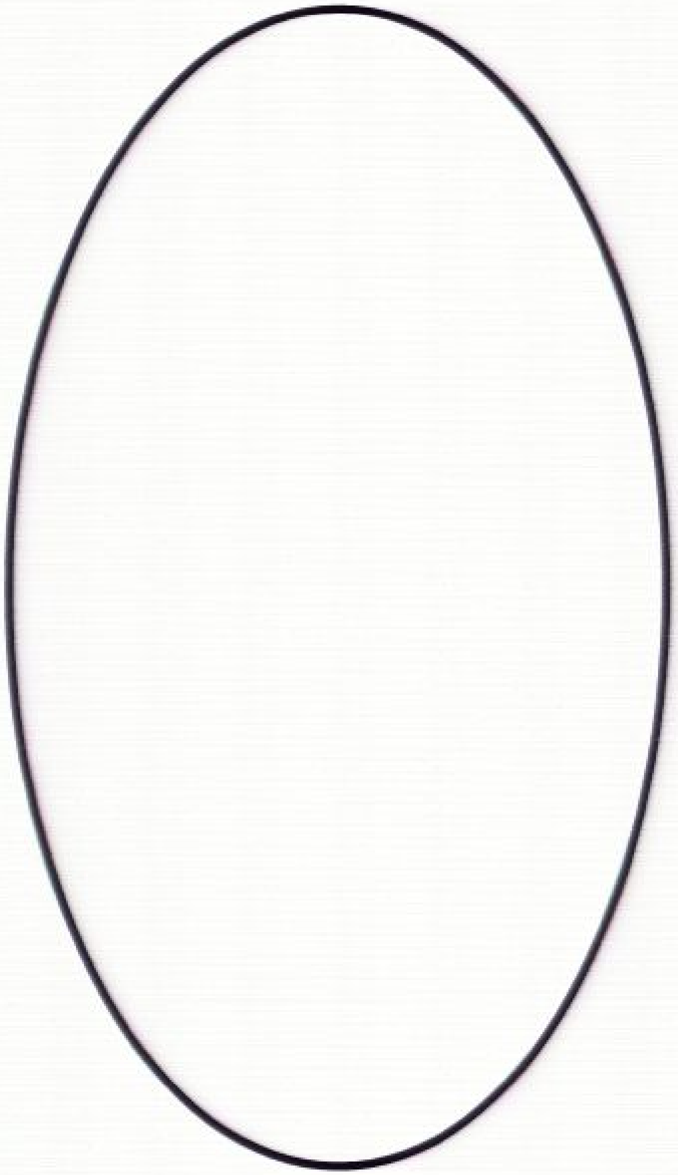
There exists the paradoxical situation.

Ex: Grandfather paradox

1: Alive

0: Dead

Closed Time-like Curve (CTC)



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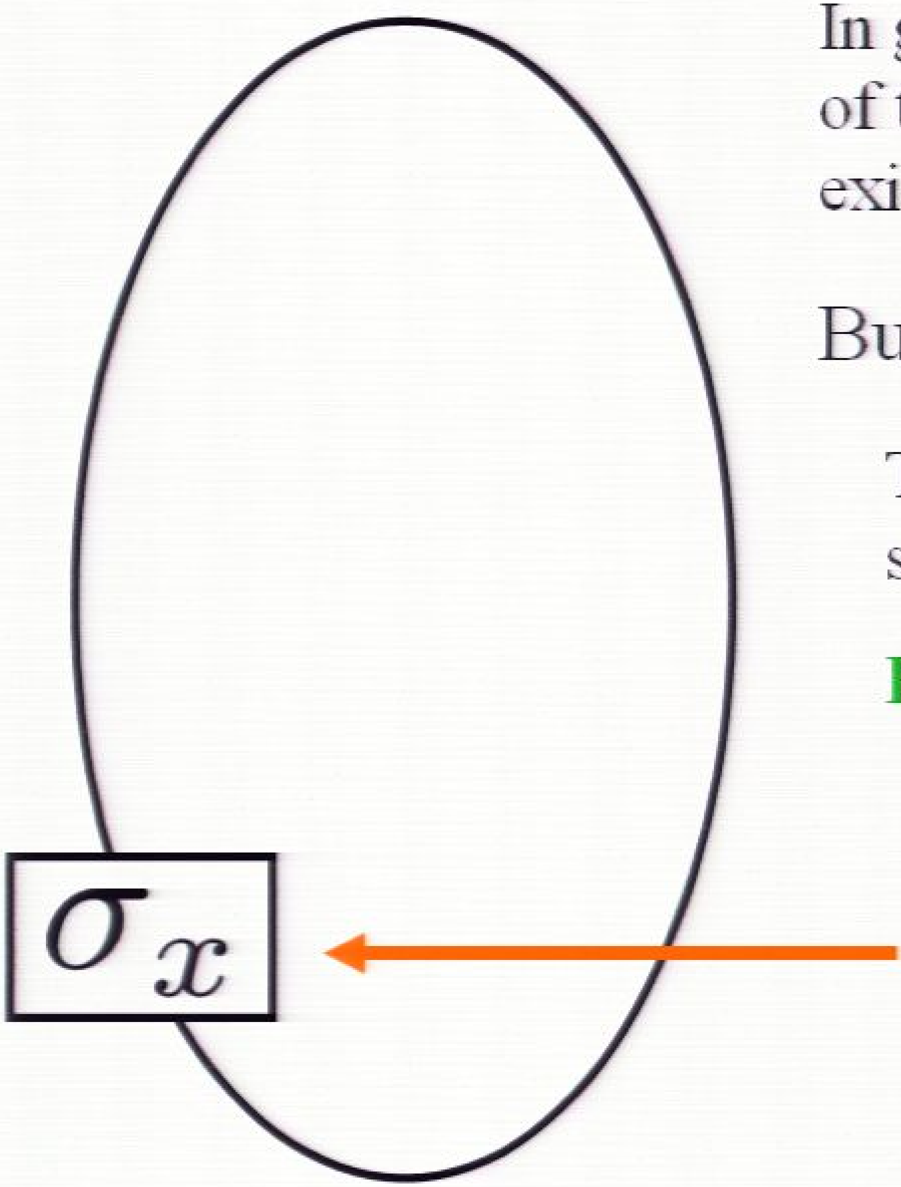

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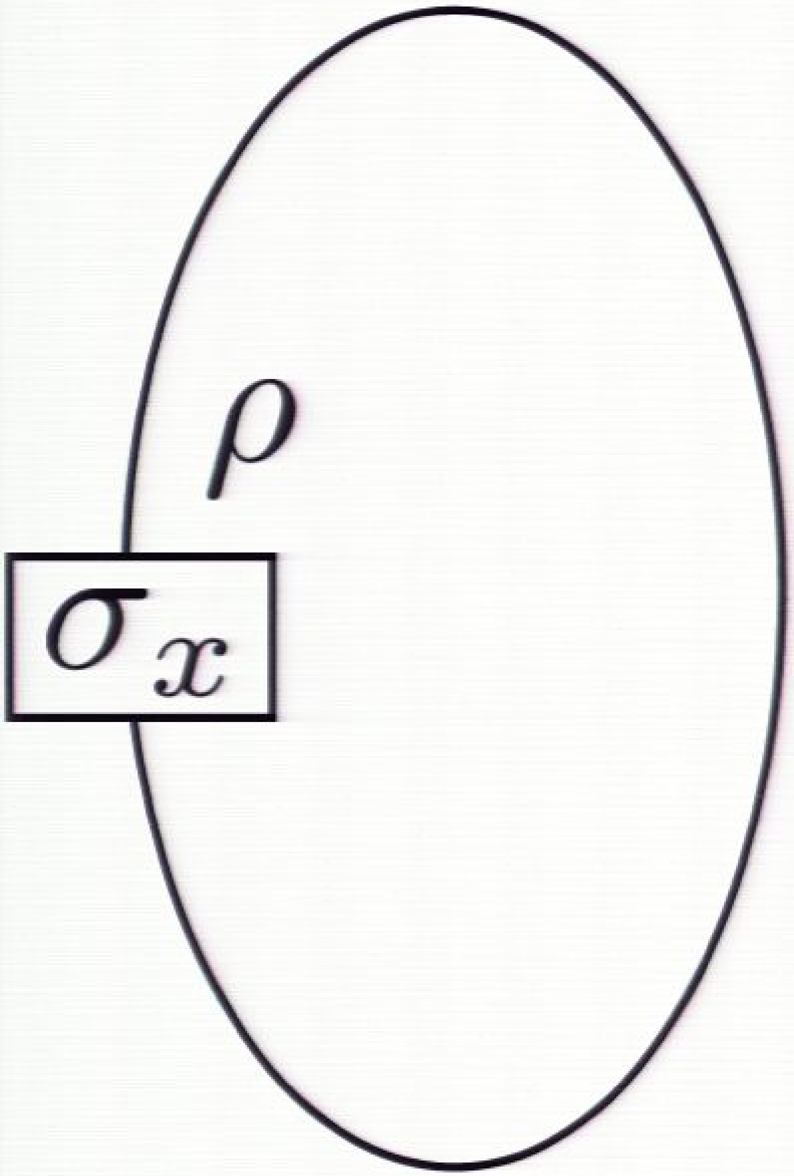
Quantum Description of CTC

(D. Deutsch, Phys. Rev. D **44**, 3197 (1991))



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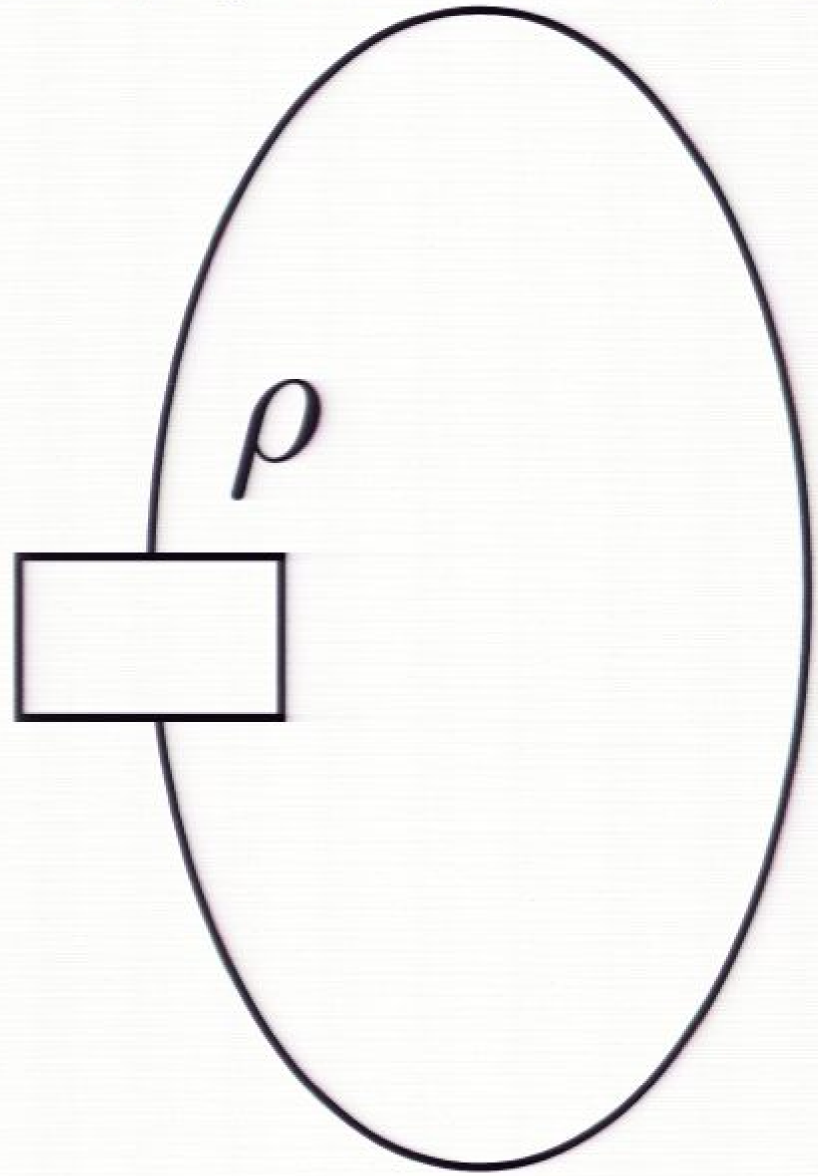

Not killing the grandfather

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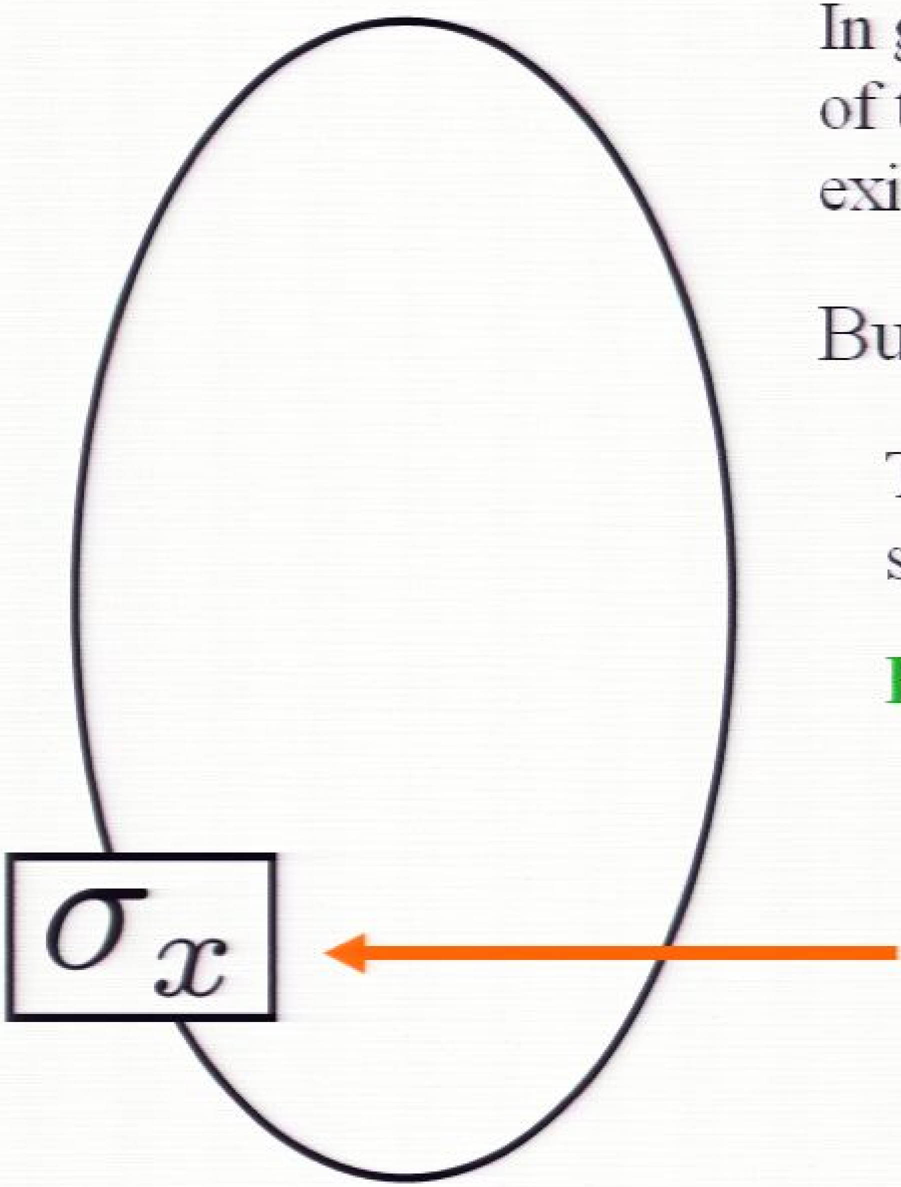

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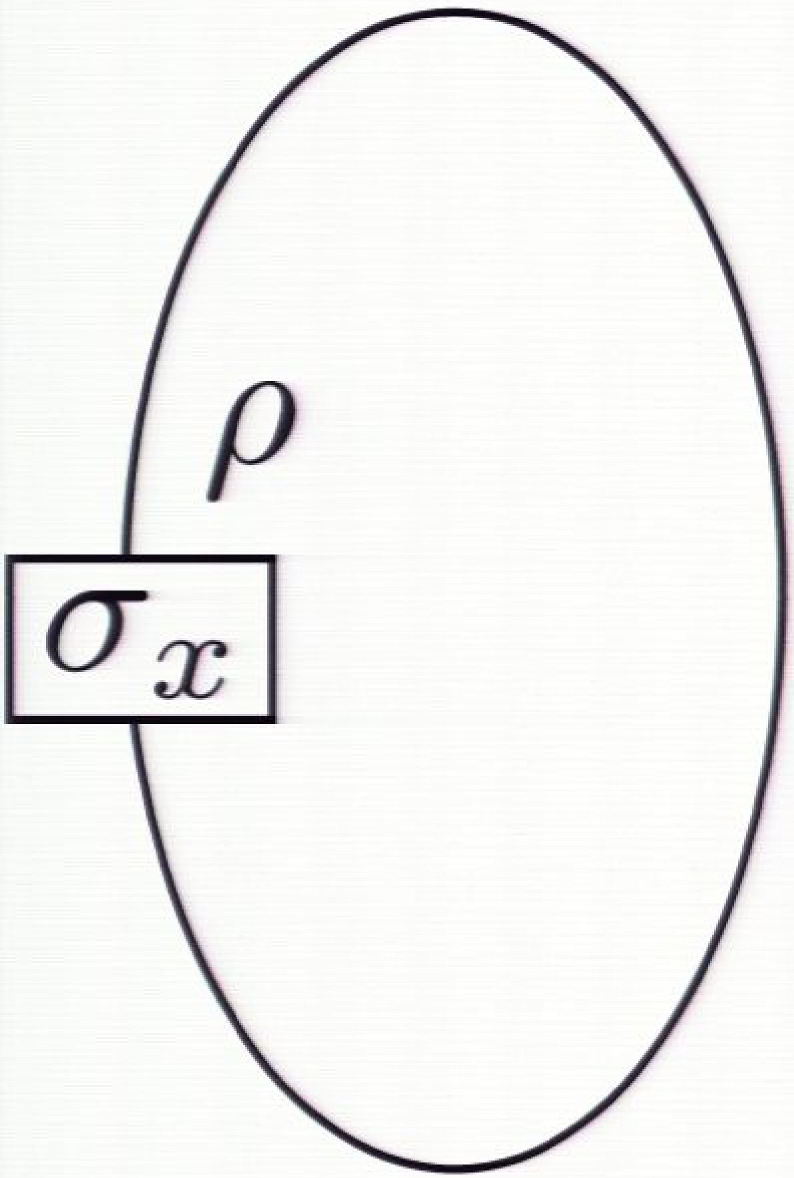

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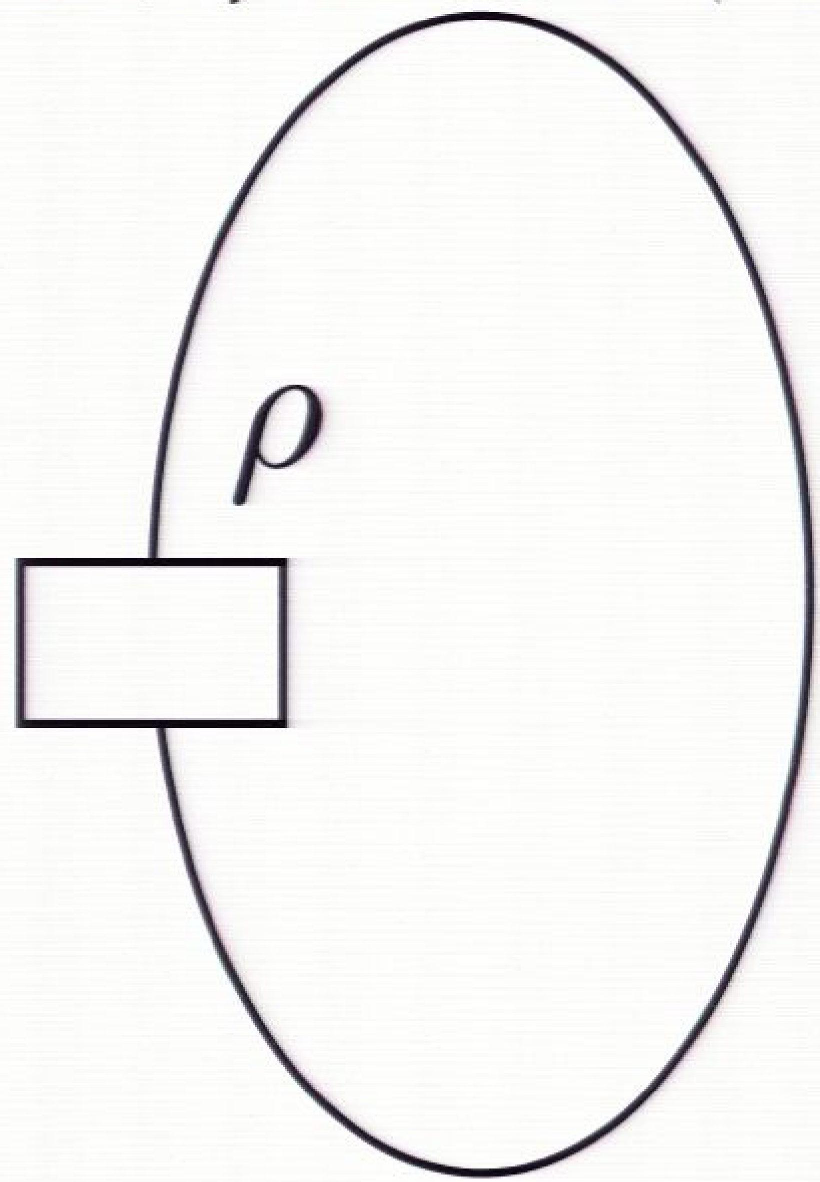

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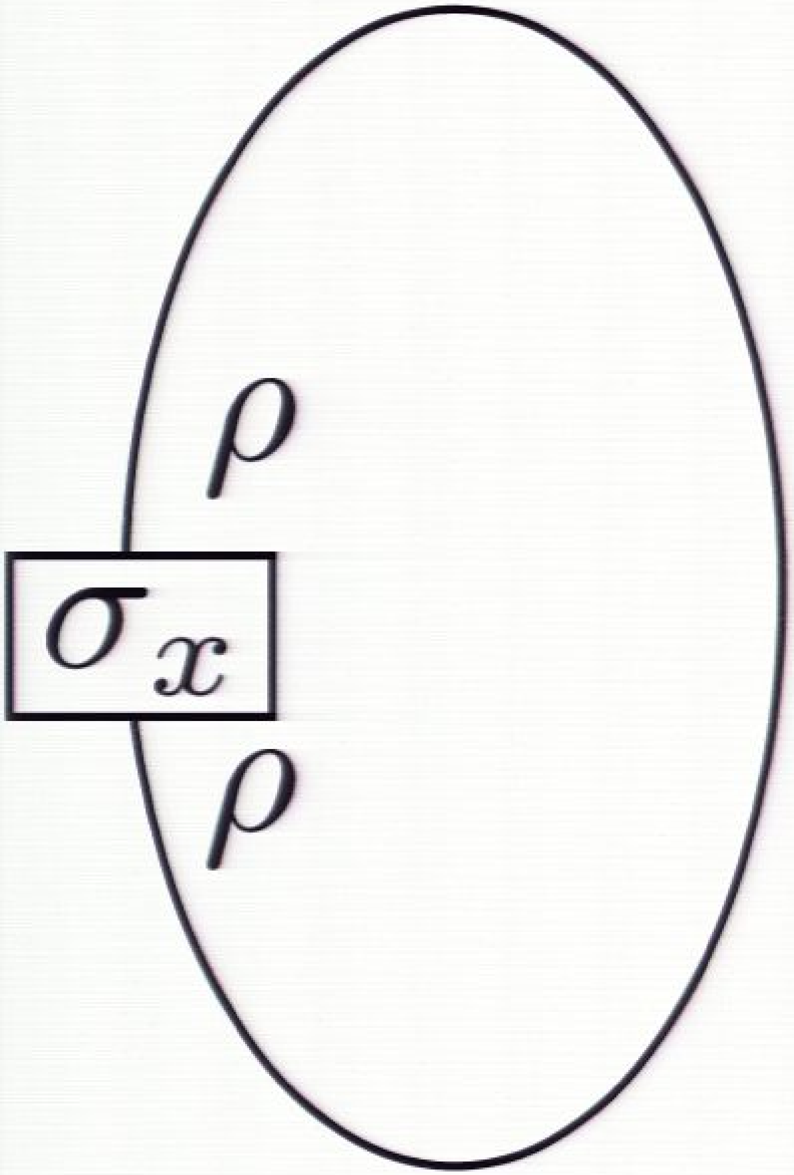

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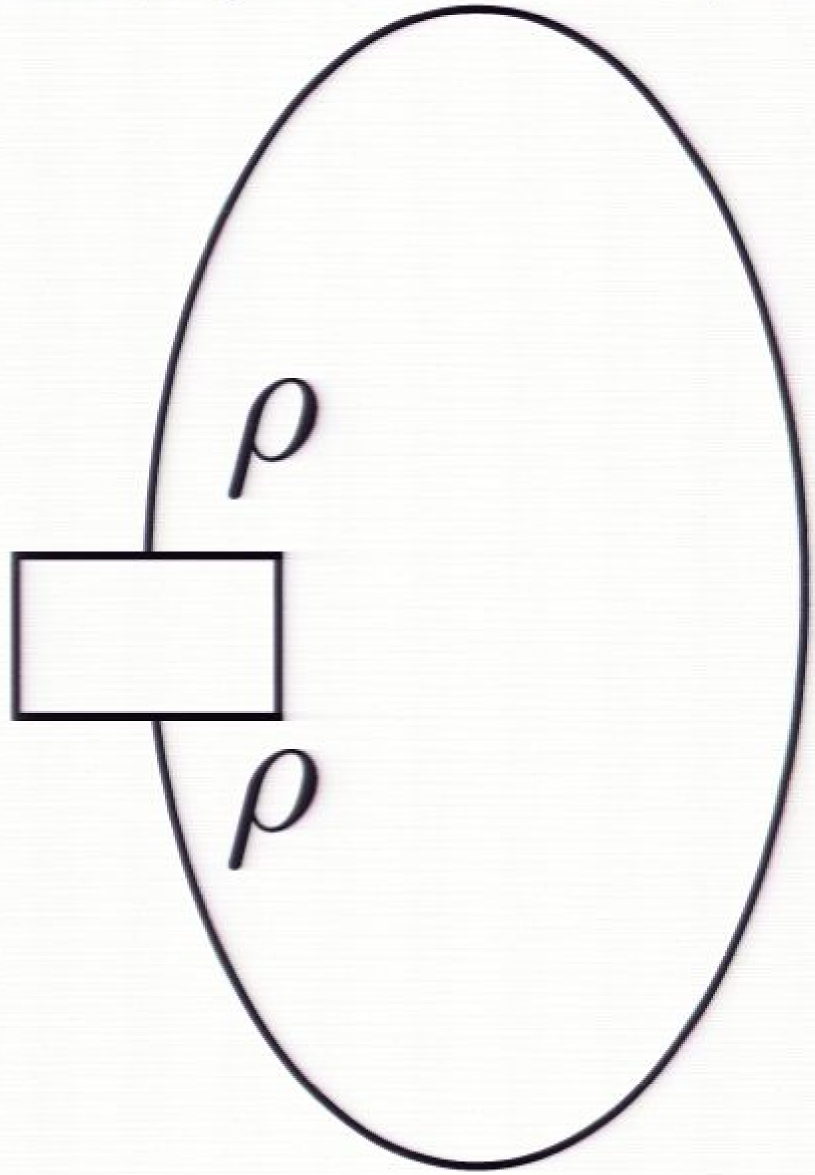

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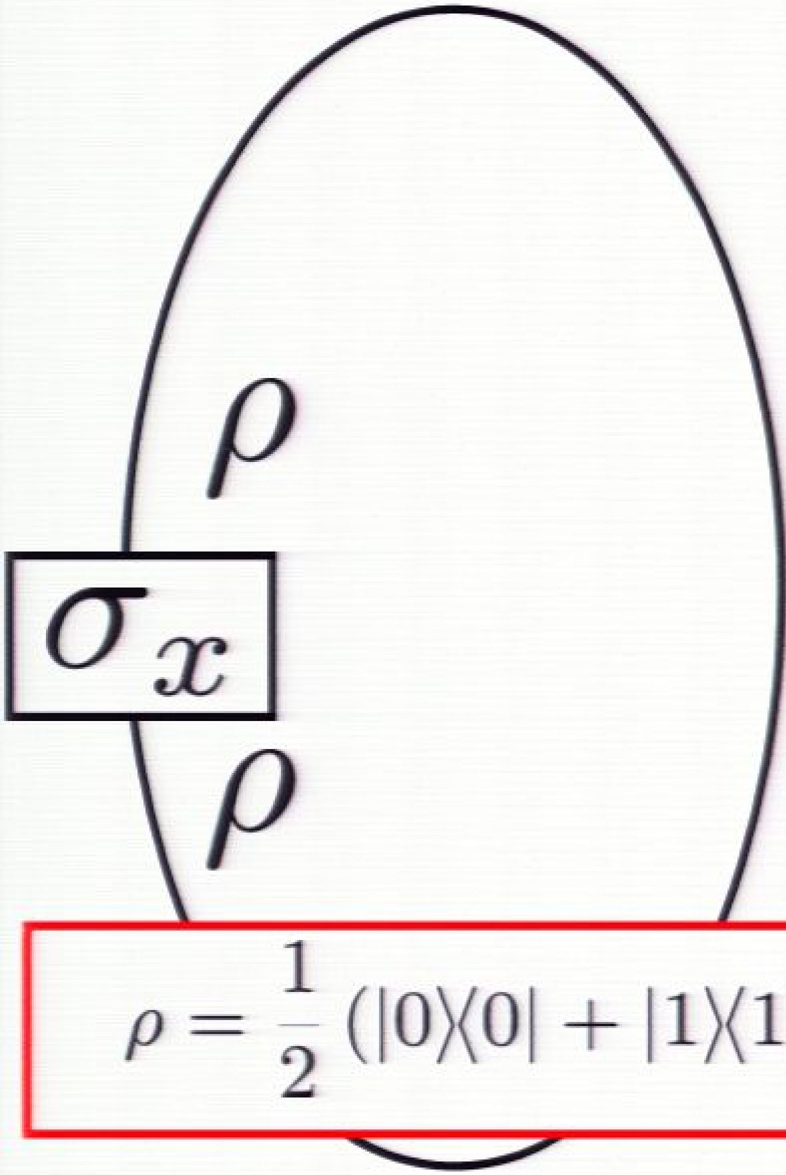
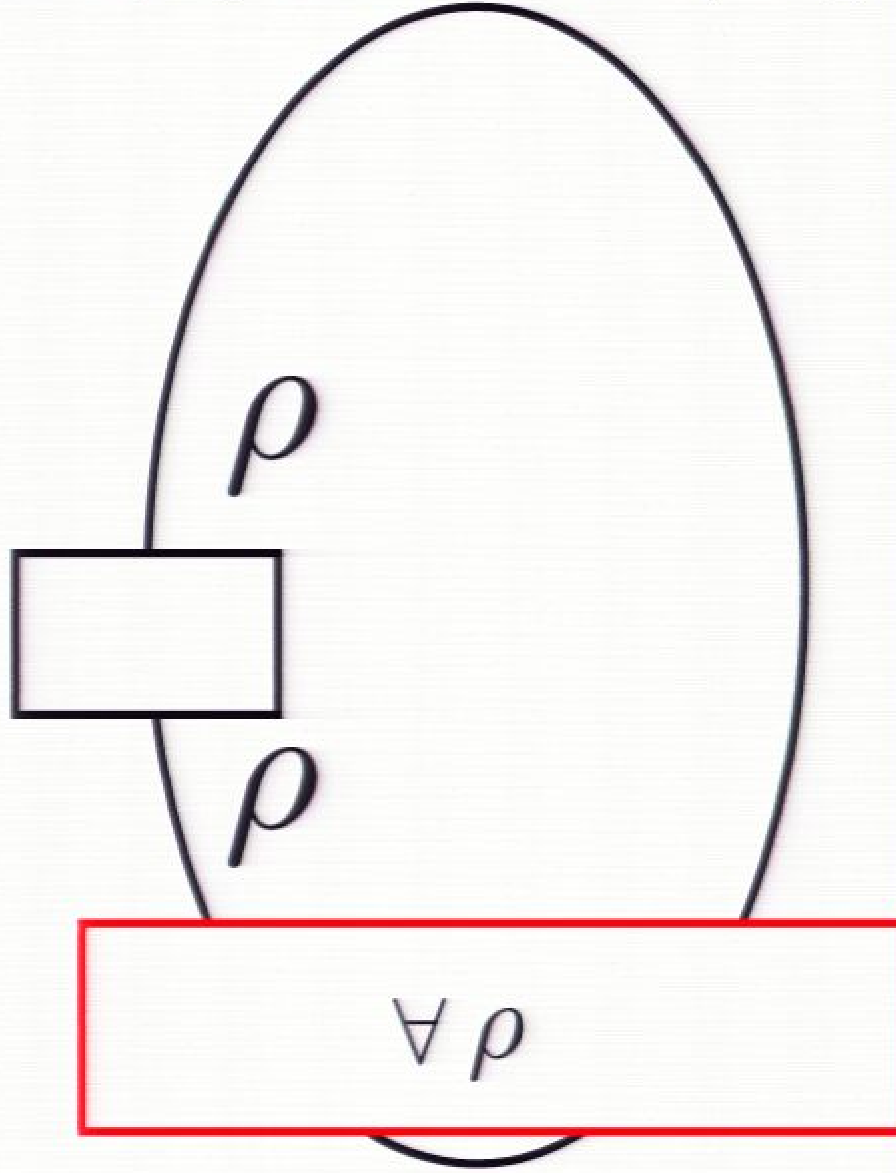


$$\rho = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

Killing the grandfather



$$\forall\, \rho$$

Not killing the grandfather

Quantum Description of CTC

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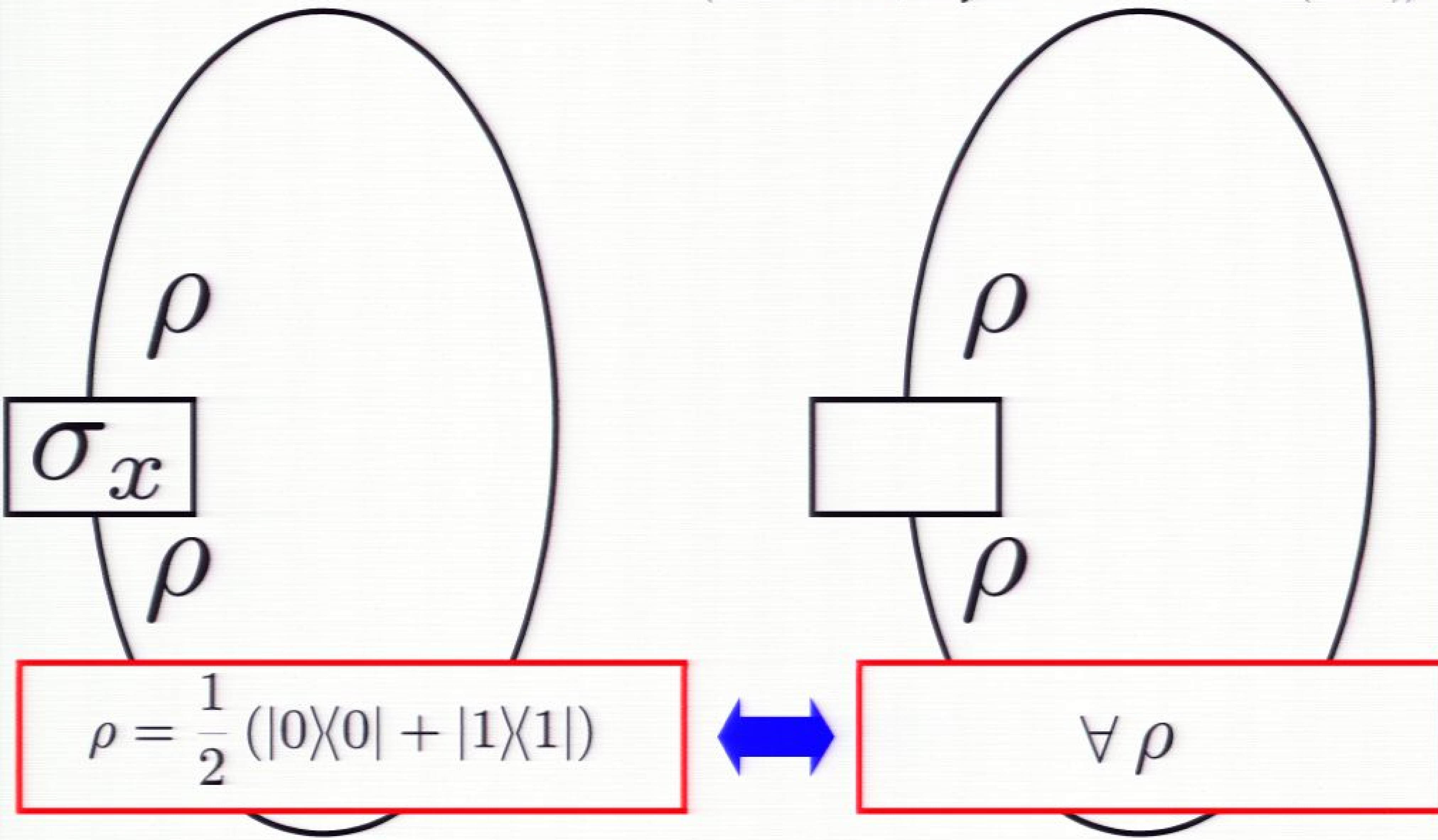
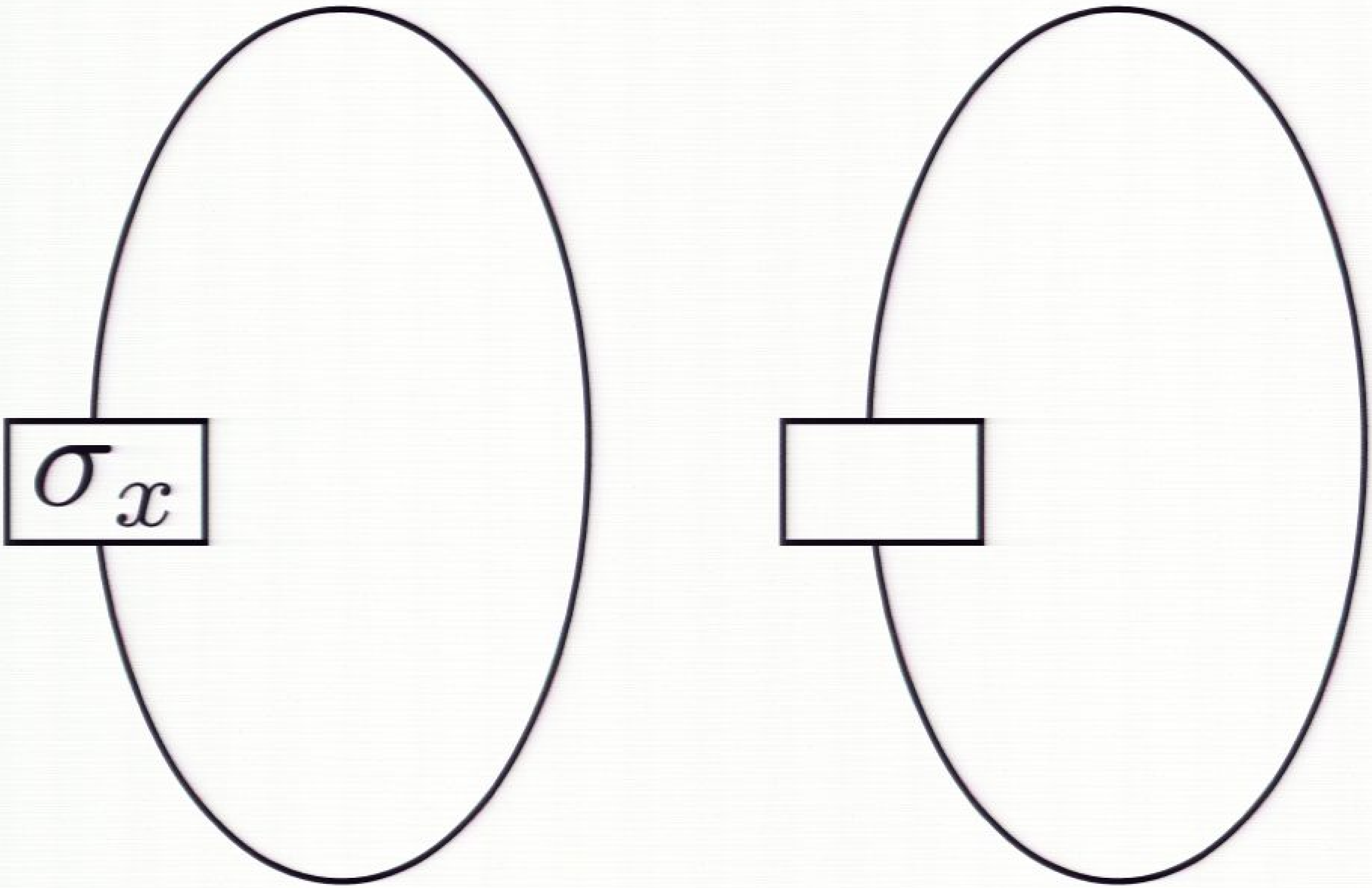


$$\rho = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \quad \Leftrightarrow \quad \forall\, \rho$$

Killing the grandfather

Not killing the grandfather

Post-selected CTC



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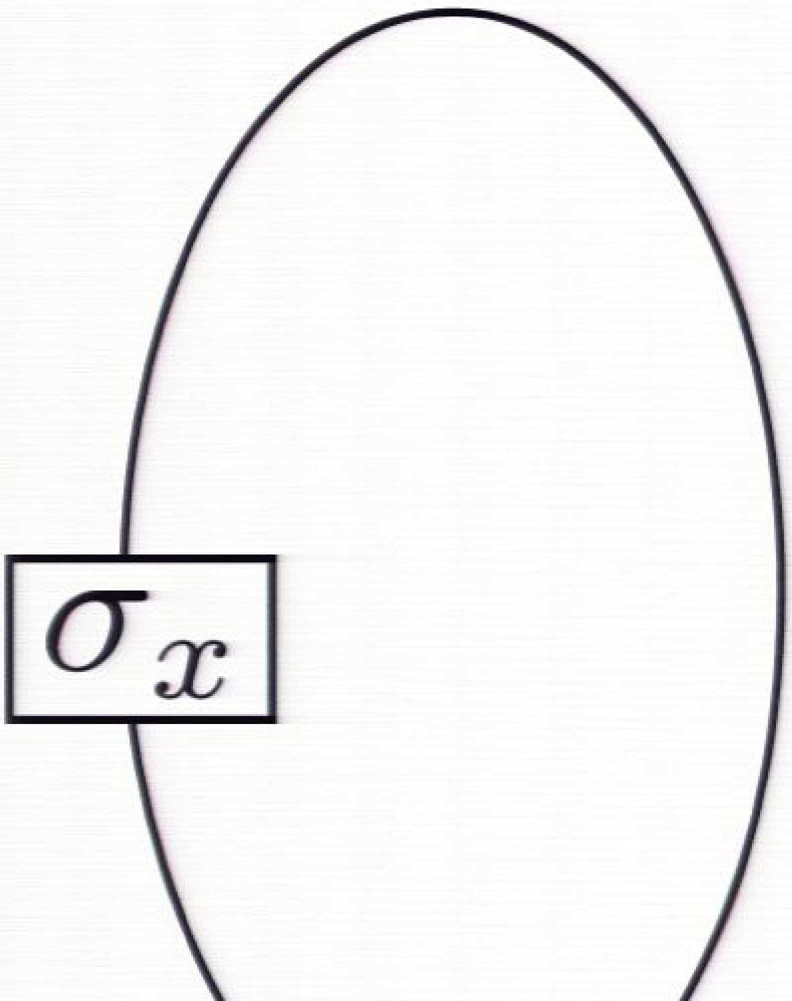
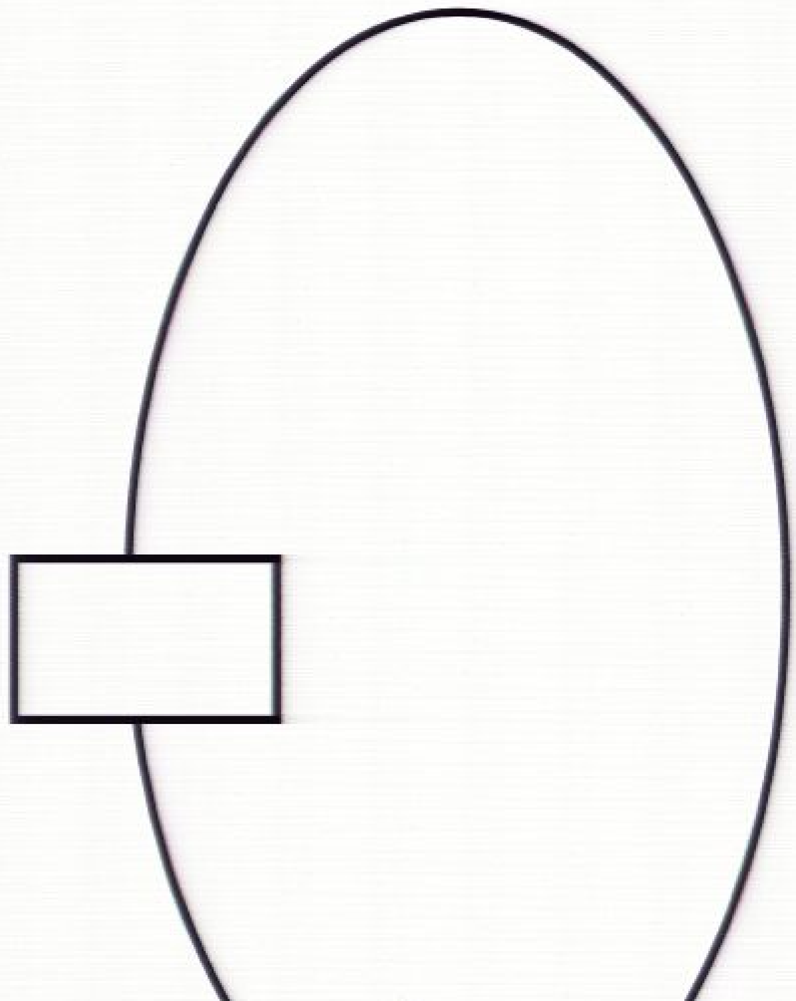


$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



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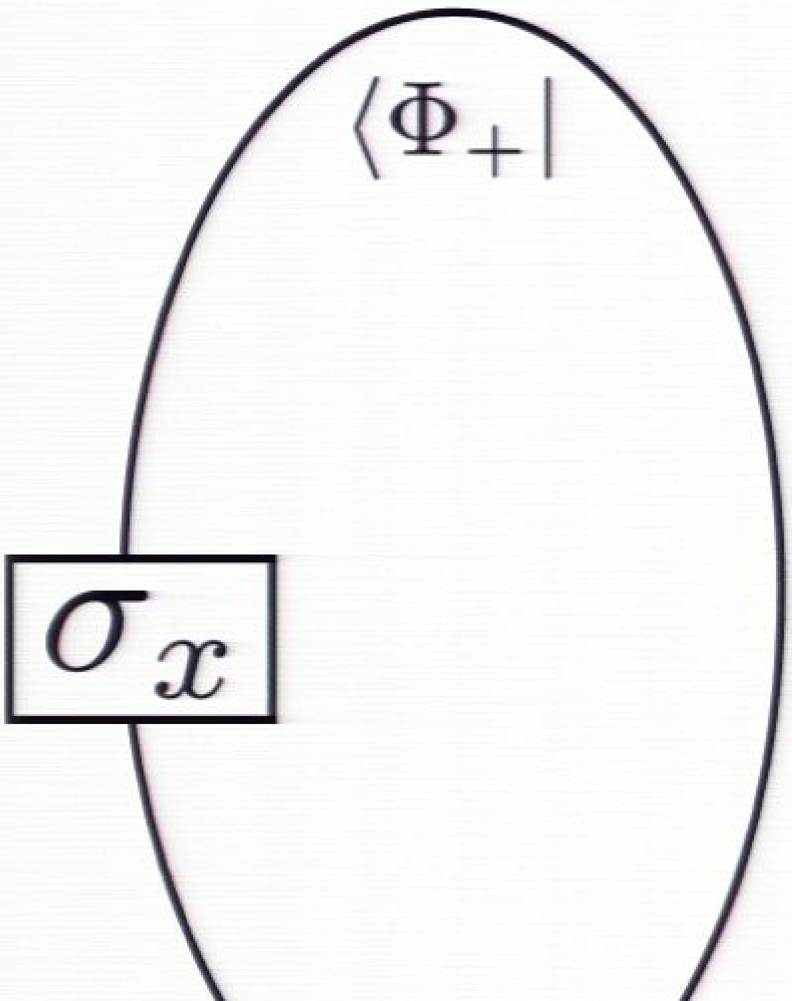

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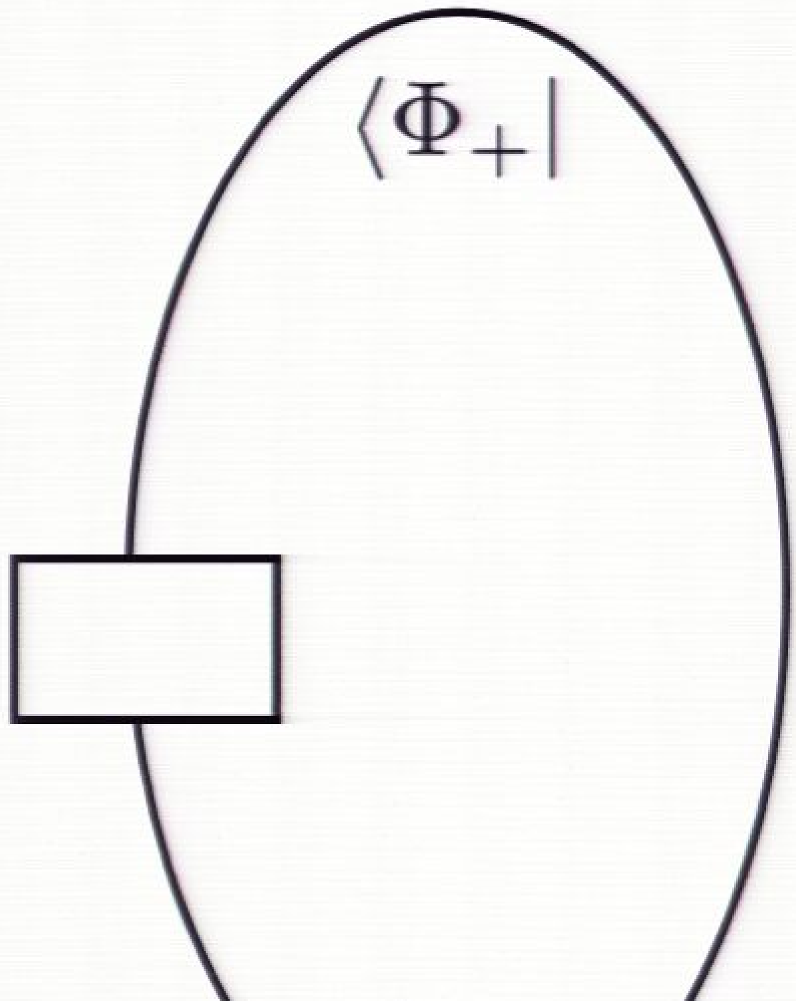

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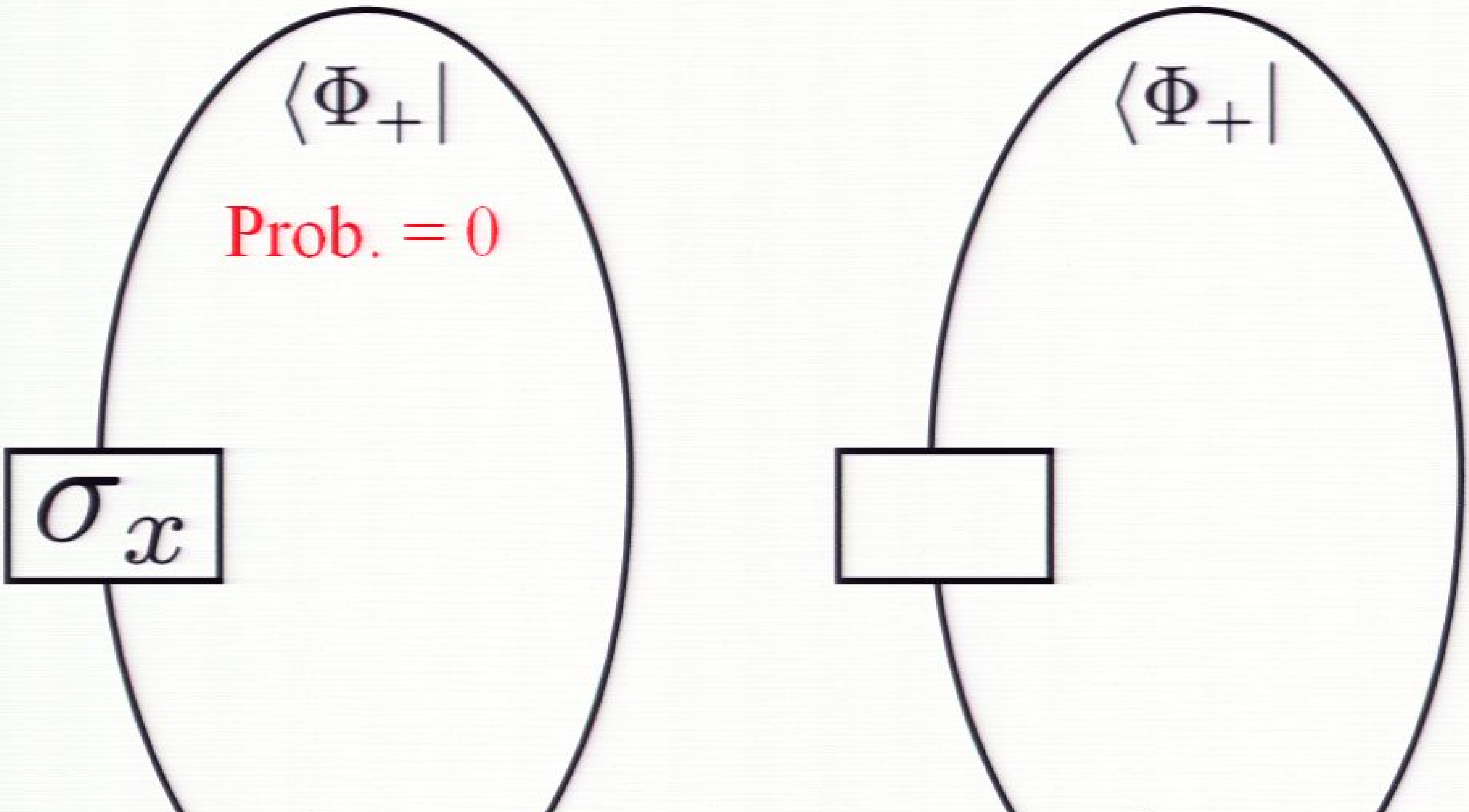

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

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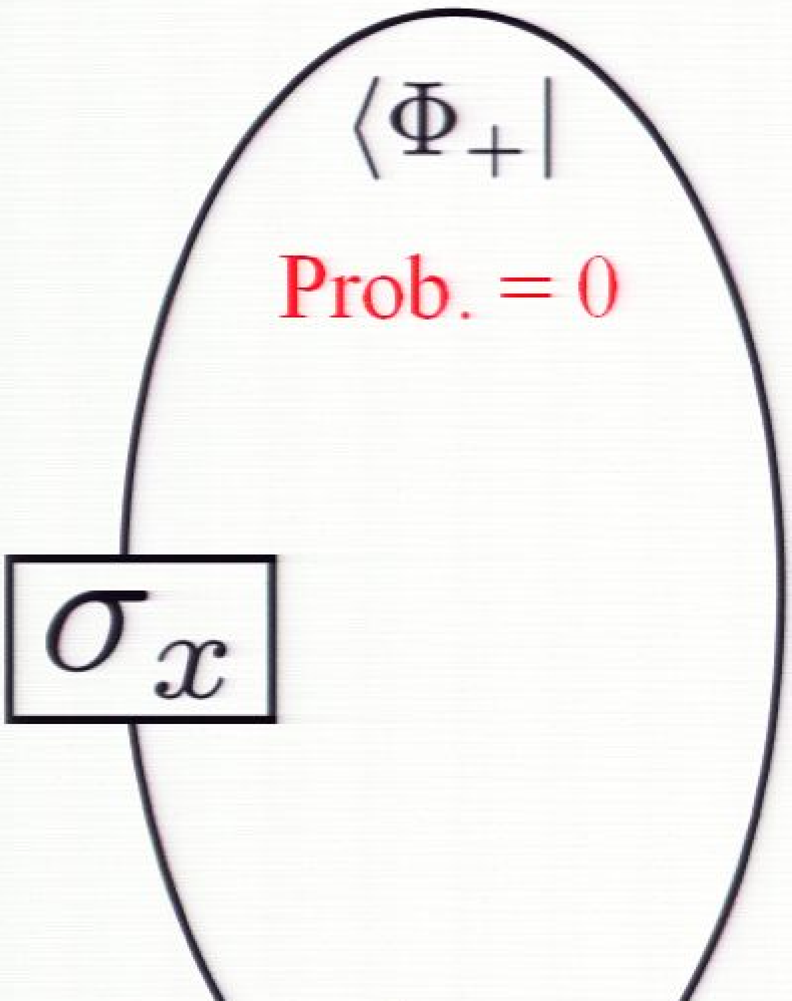

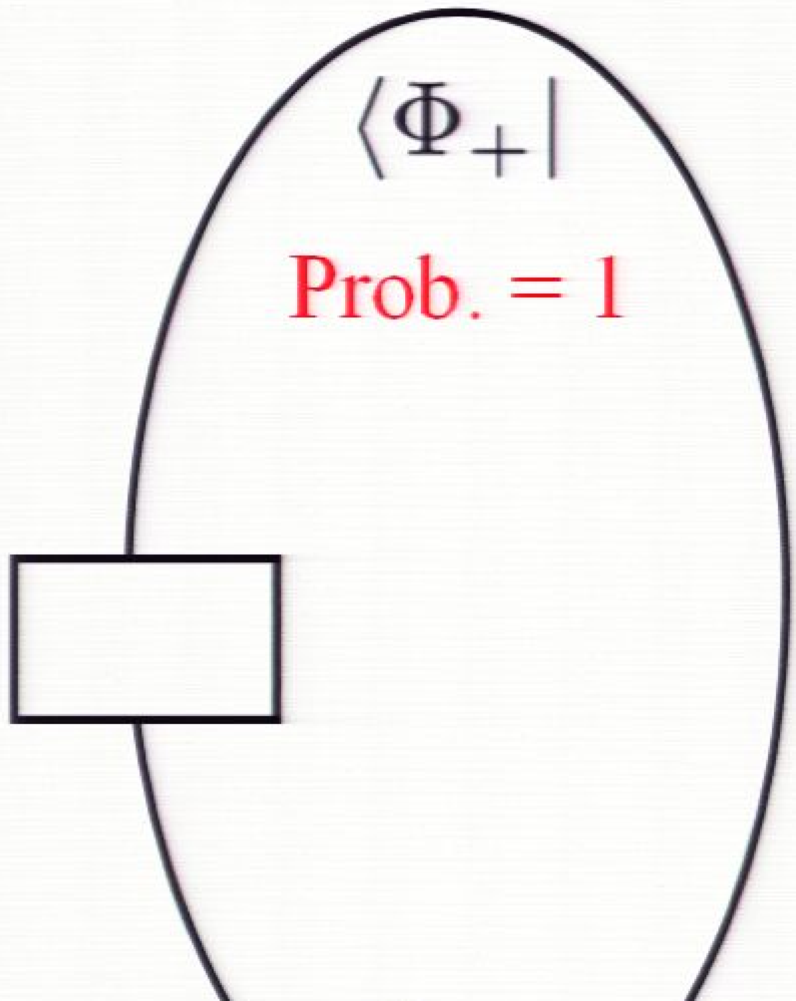

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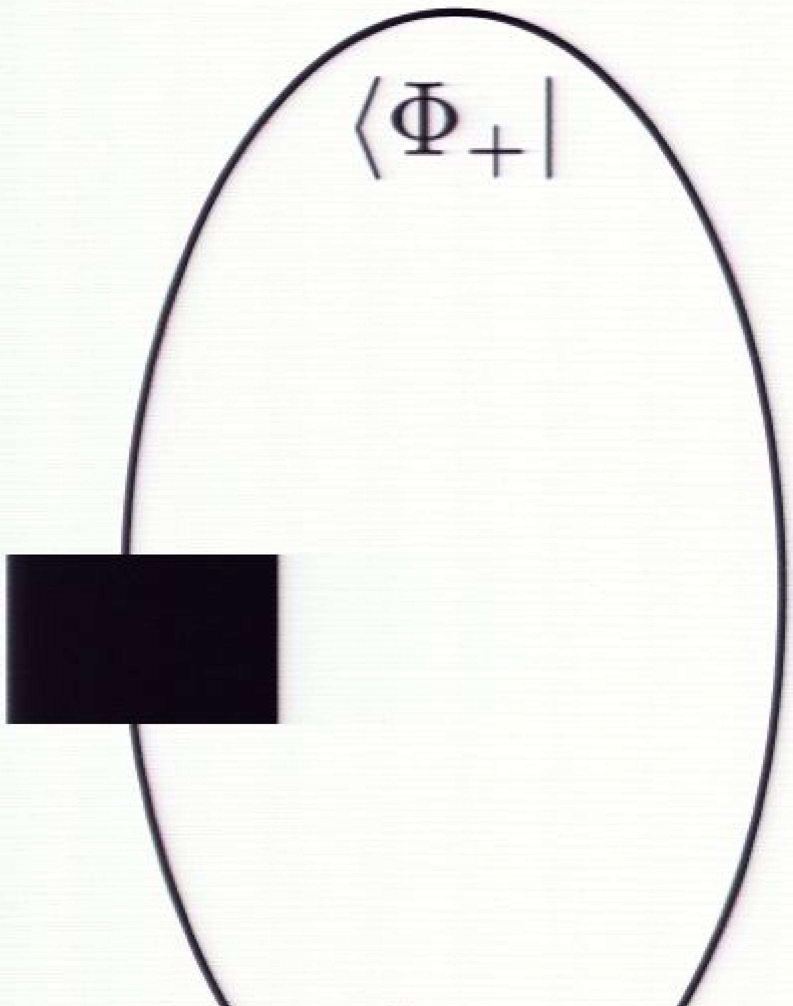
Short Remarks

- This idea was first proposed by **Benjamin Schumacher** inspired by quantum teleportation (unpublished). This idea was talked by **Charlie Bennett** in 2002.
- This idea is related to the quantum knot theory and the spin network representation in quantum gravity.
- This work was published in
  - Along with experimental demonstration of the grandfather paradox

  S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, YS, S. Pirandola, L.A. Rozema, A. Darabi, Y. Soudagar, L.K. Shalm, and A.M. Steinberg, arXiv:1005.2219.
  - Path integral analysis (only theoretical work)

  S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, and YS, arXiv:1007.2615.
- The above analysis is generally independent to the weak value.

When do we know the post-CTC is successful?



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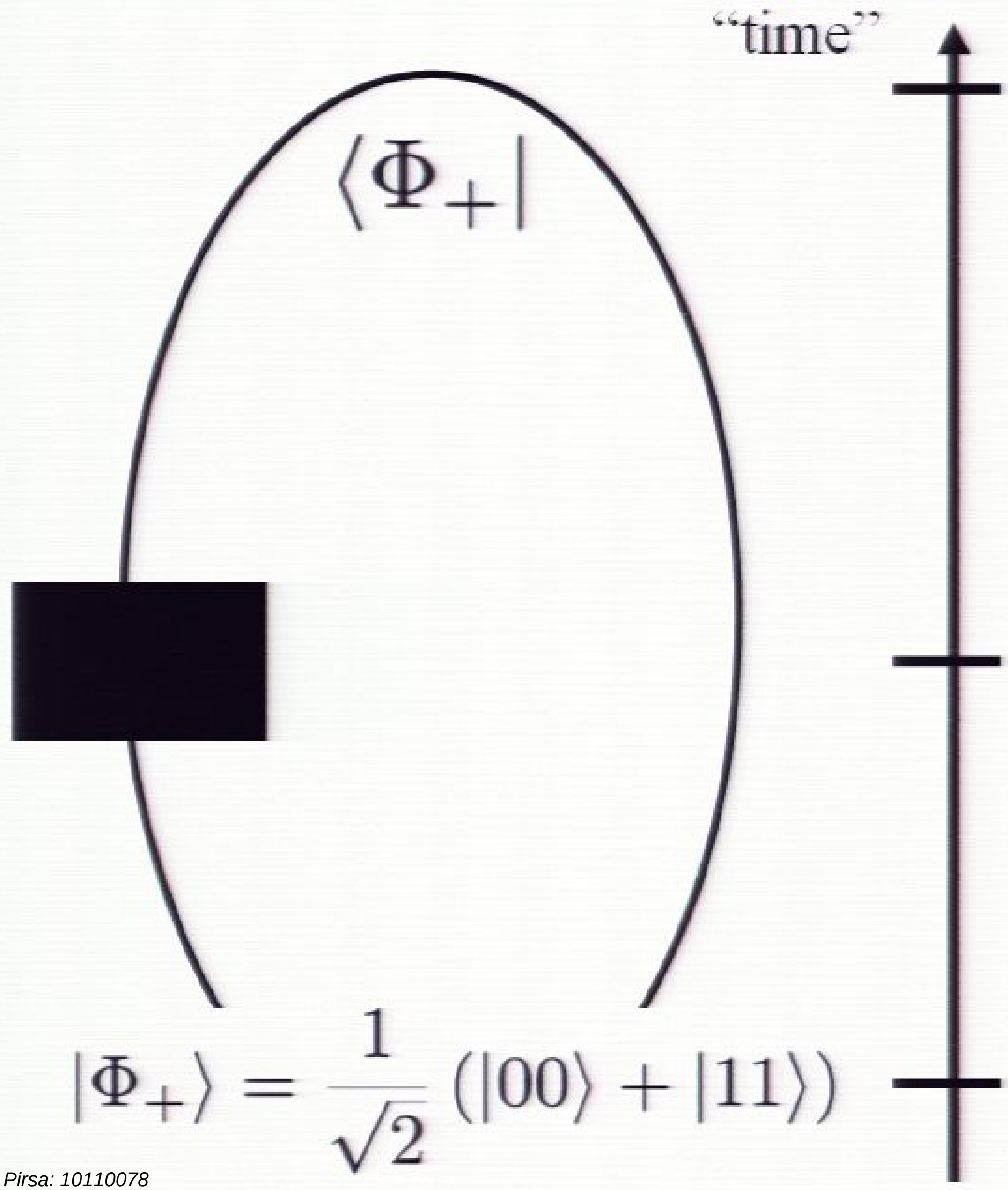

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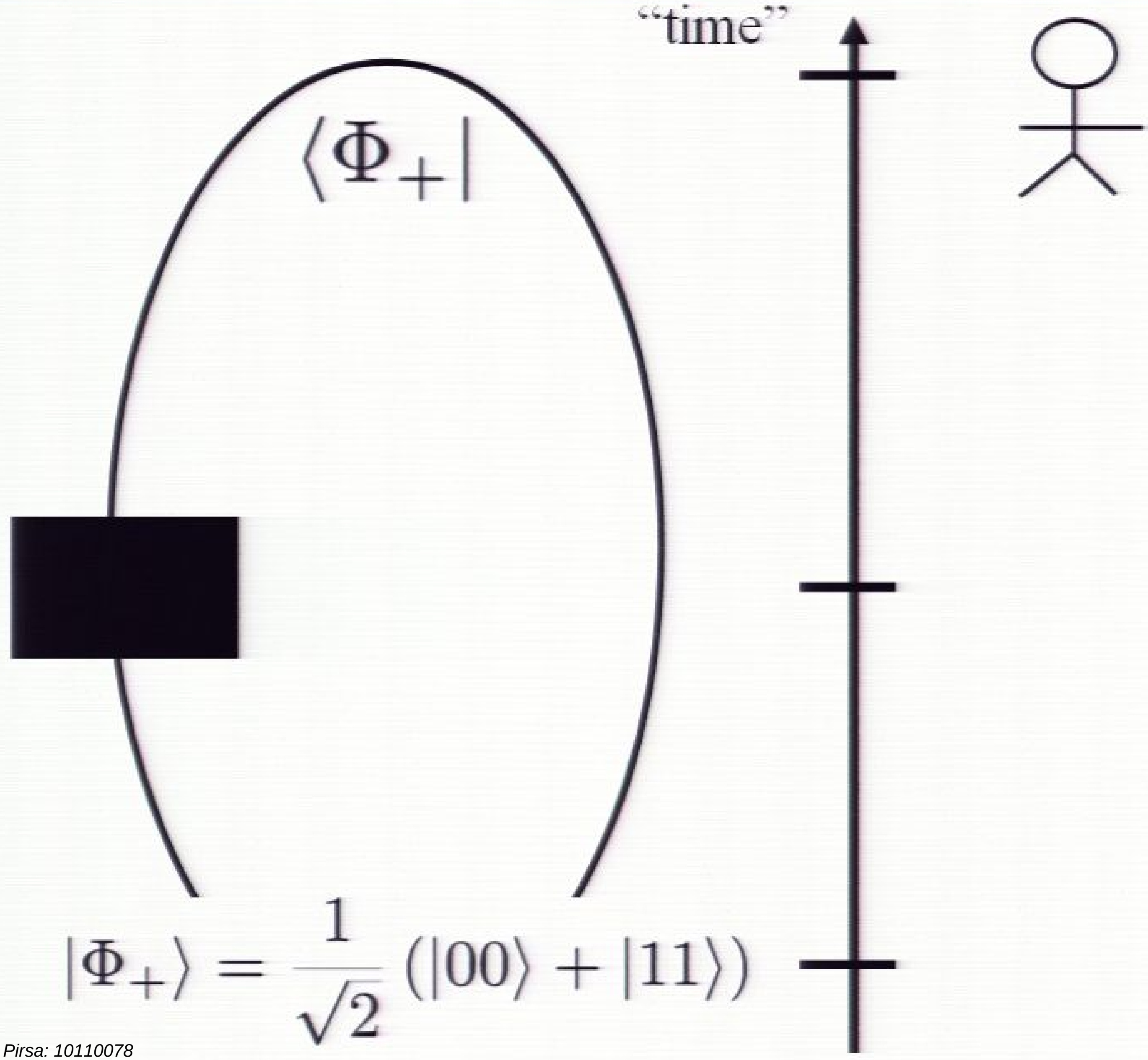

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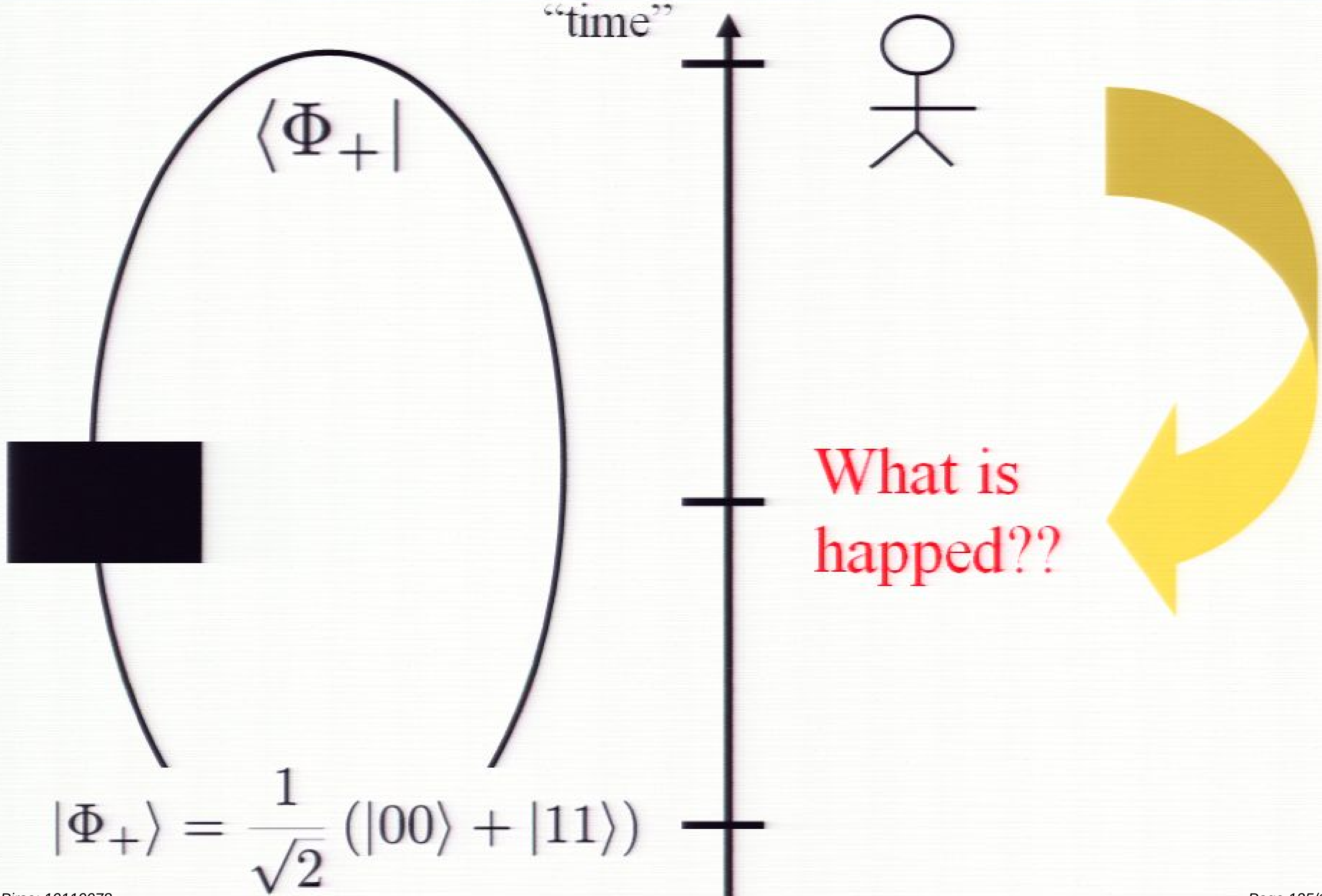

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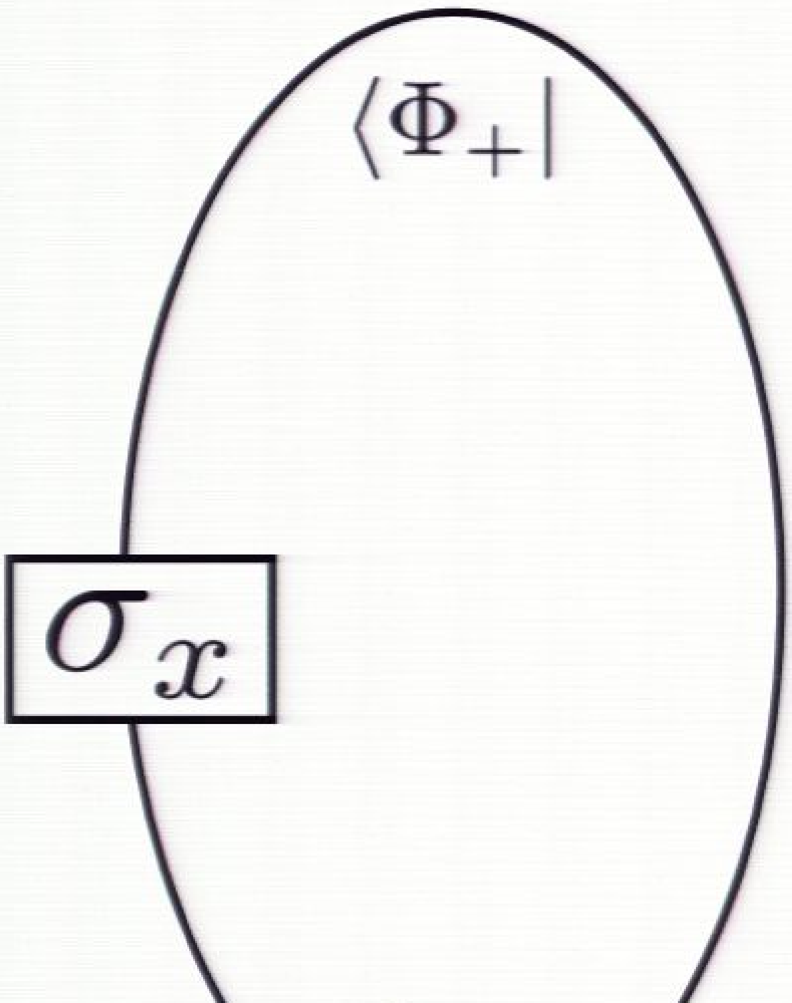
Weak Value Analysis



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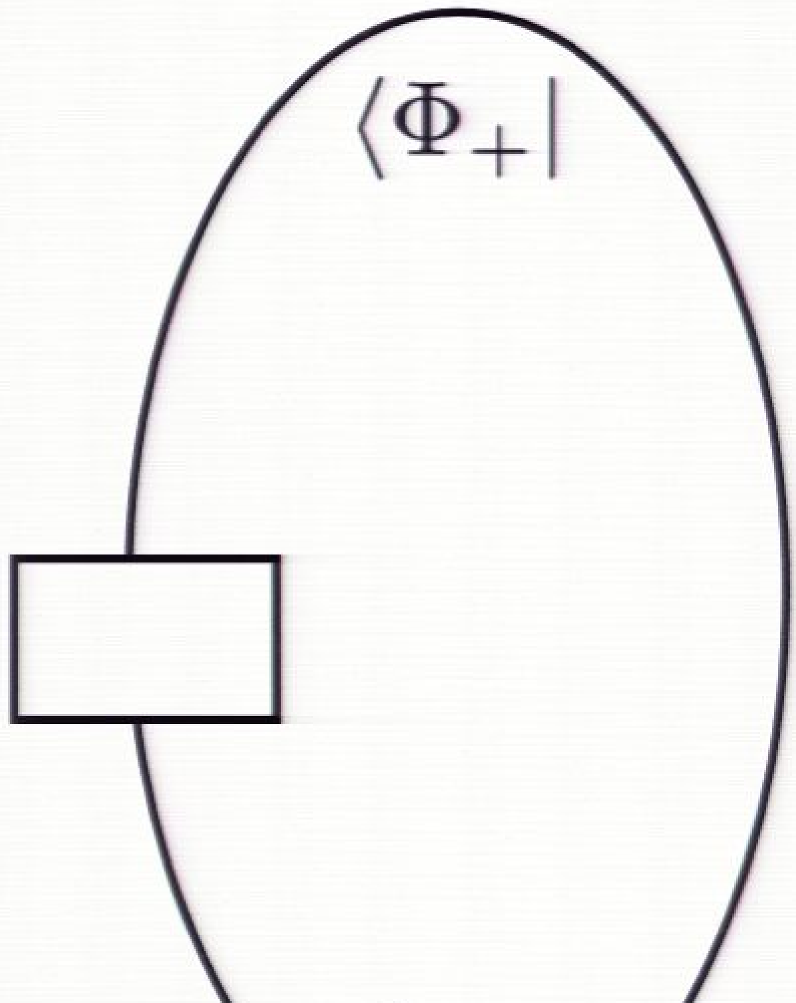

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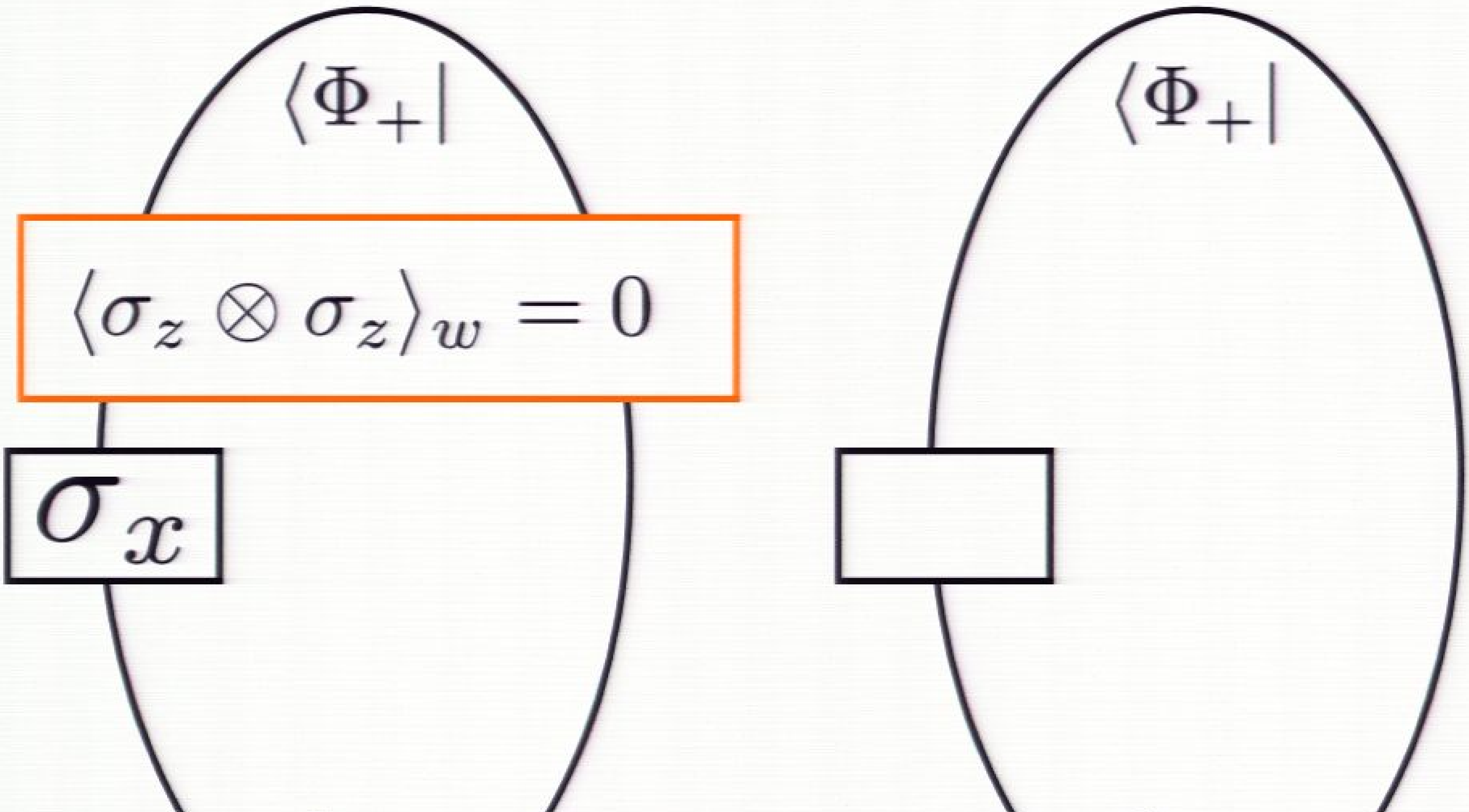

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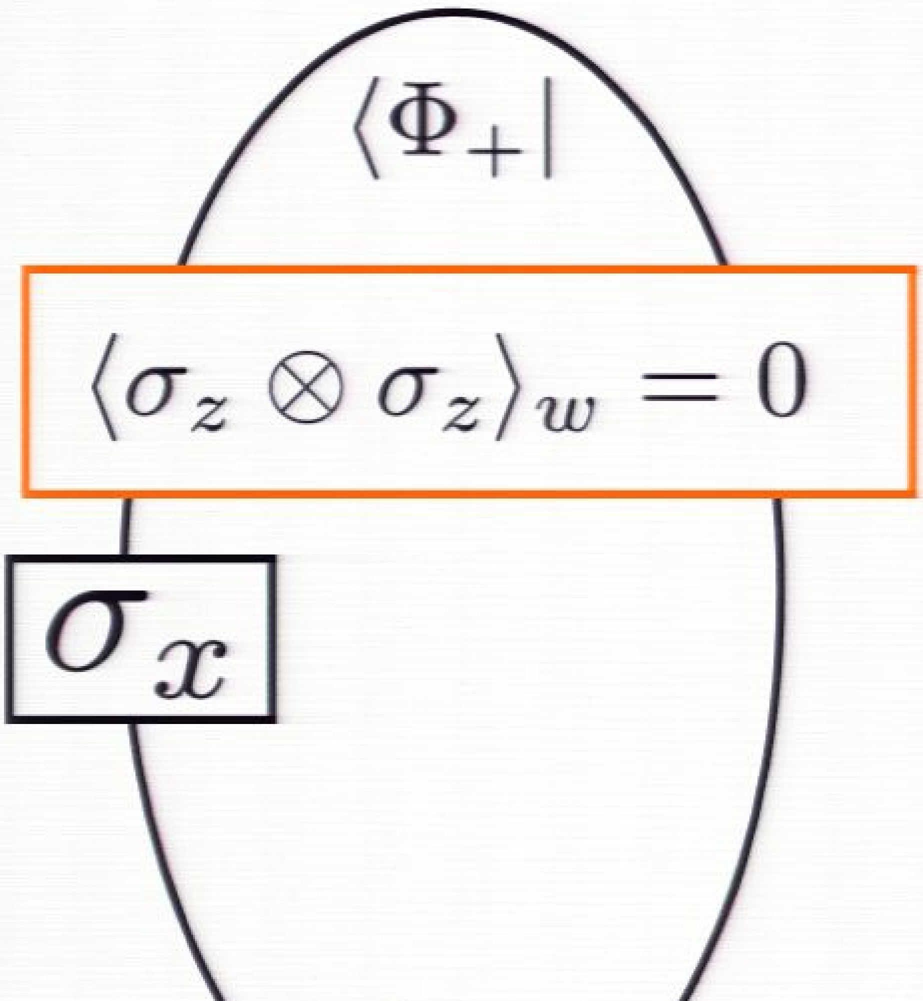

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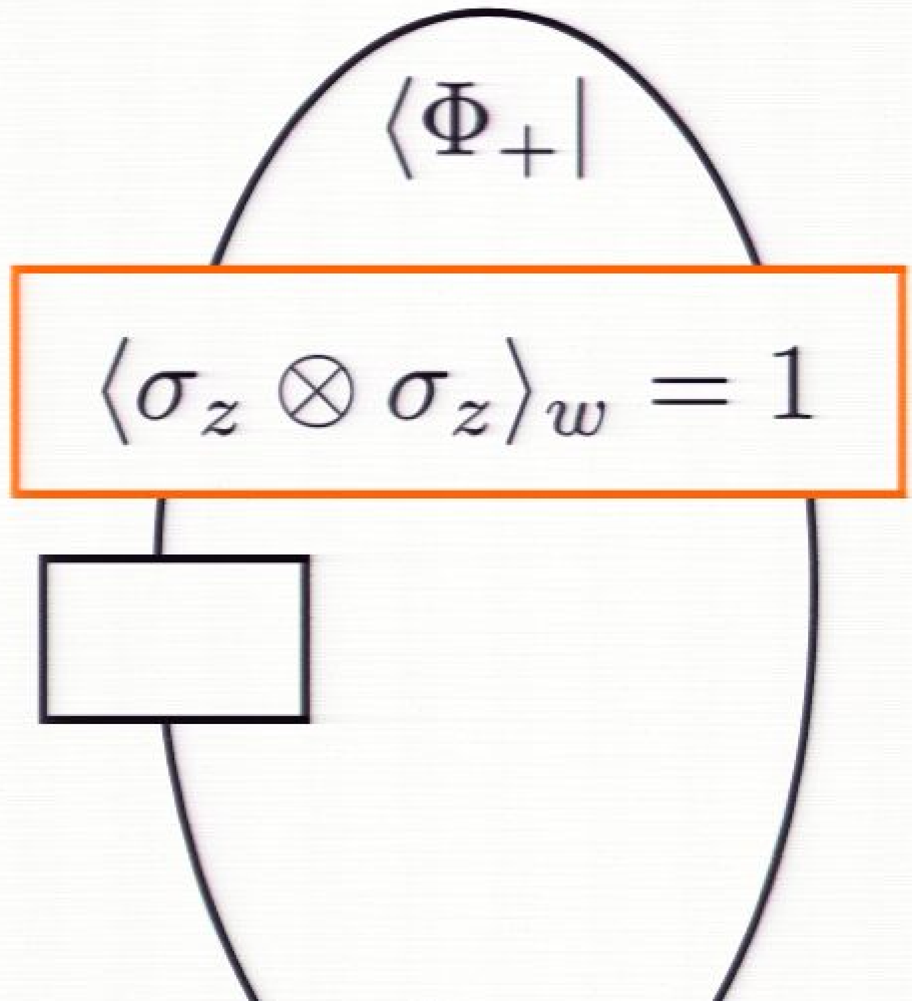

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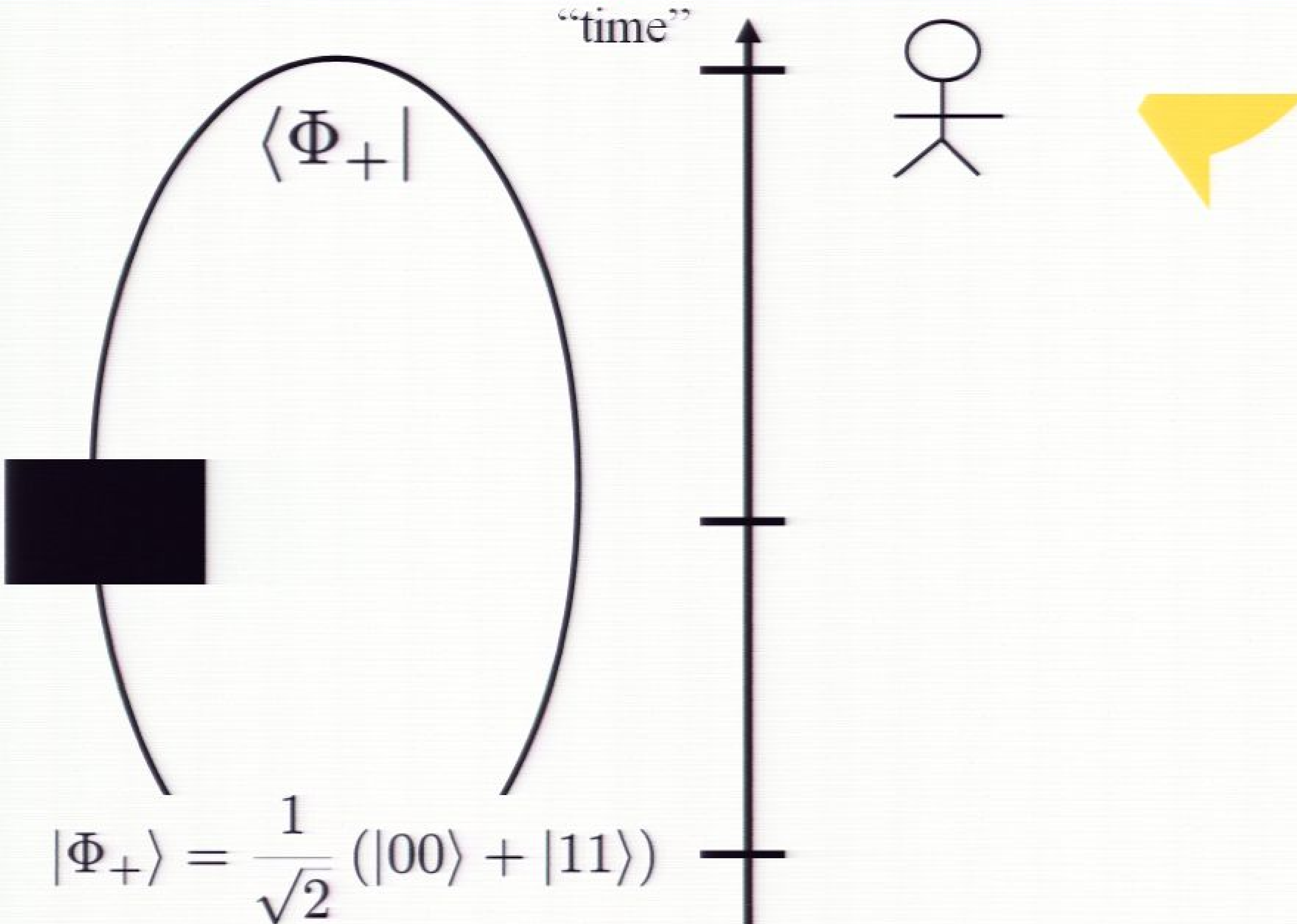

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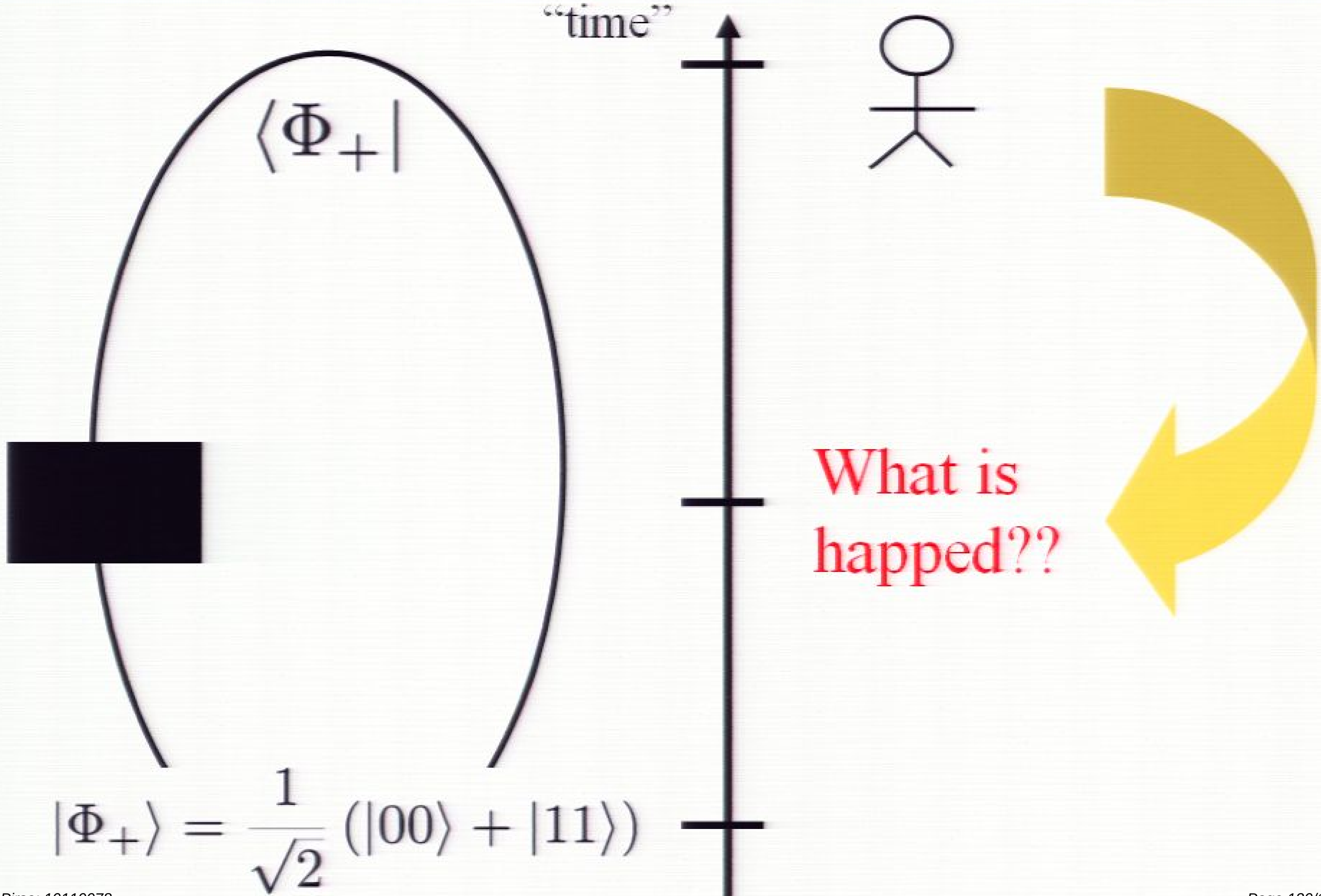

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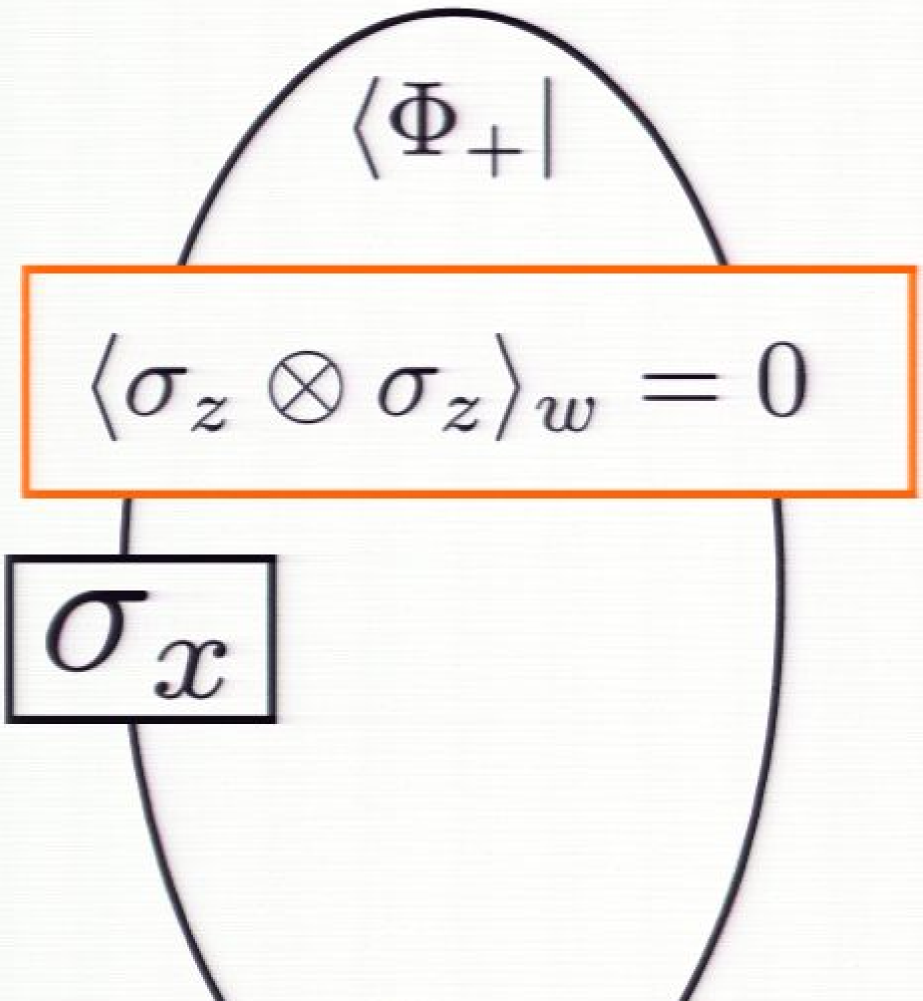

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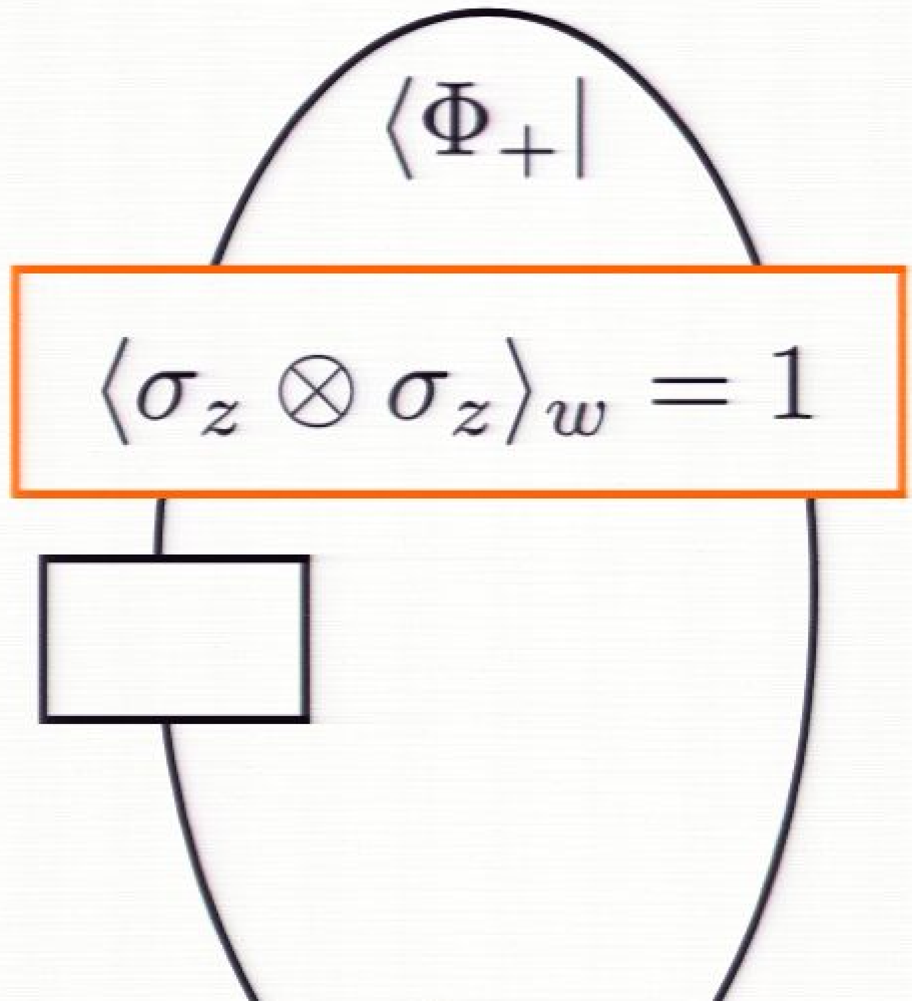

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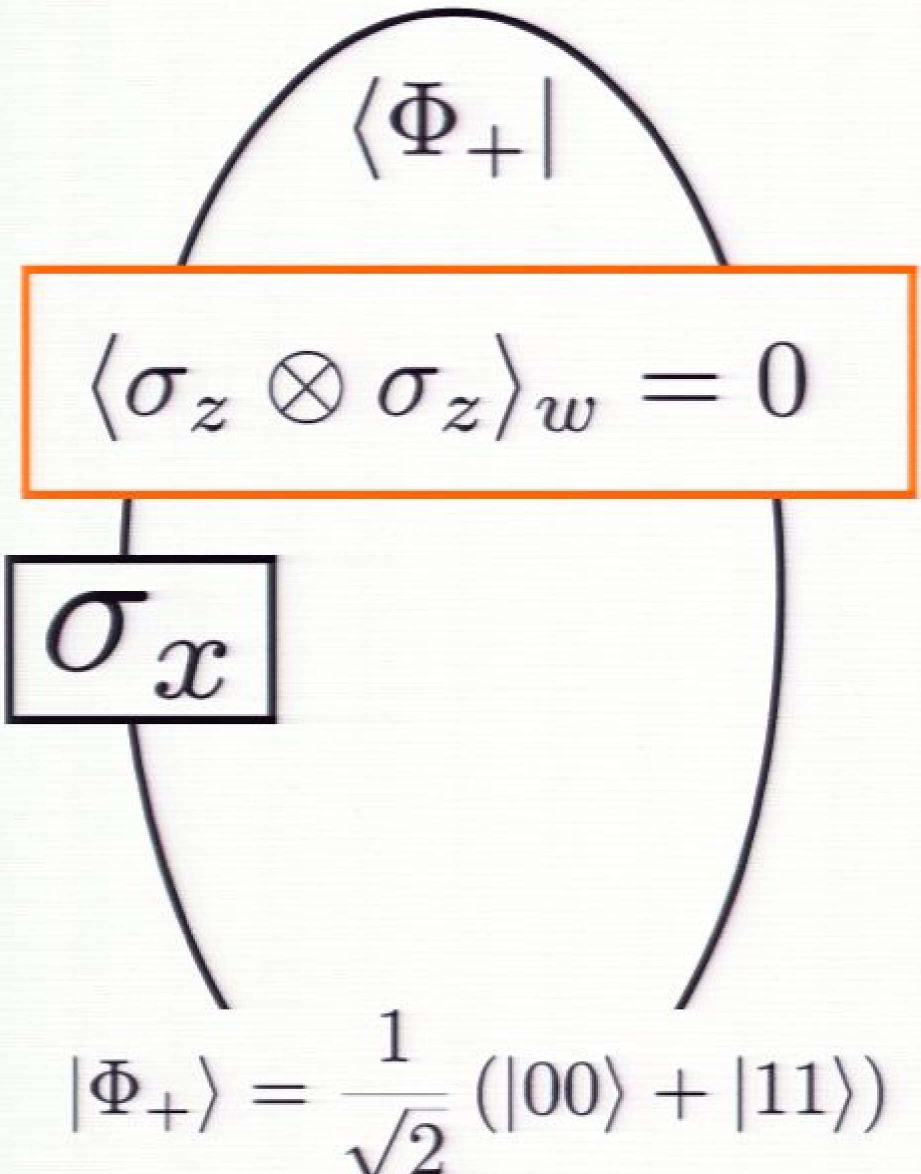
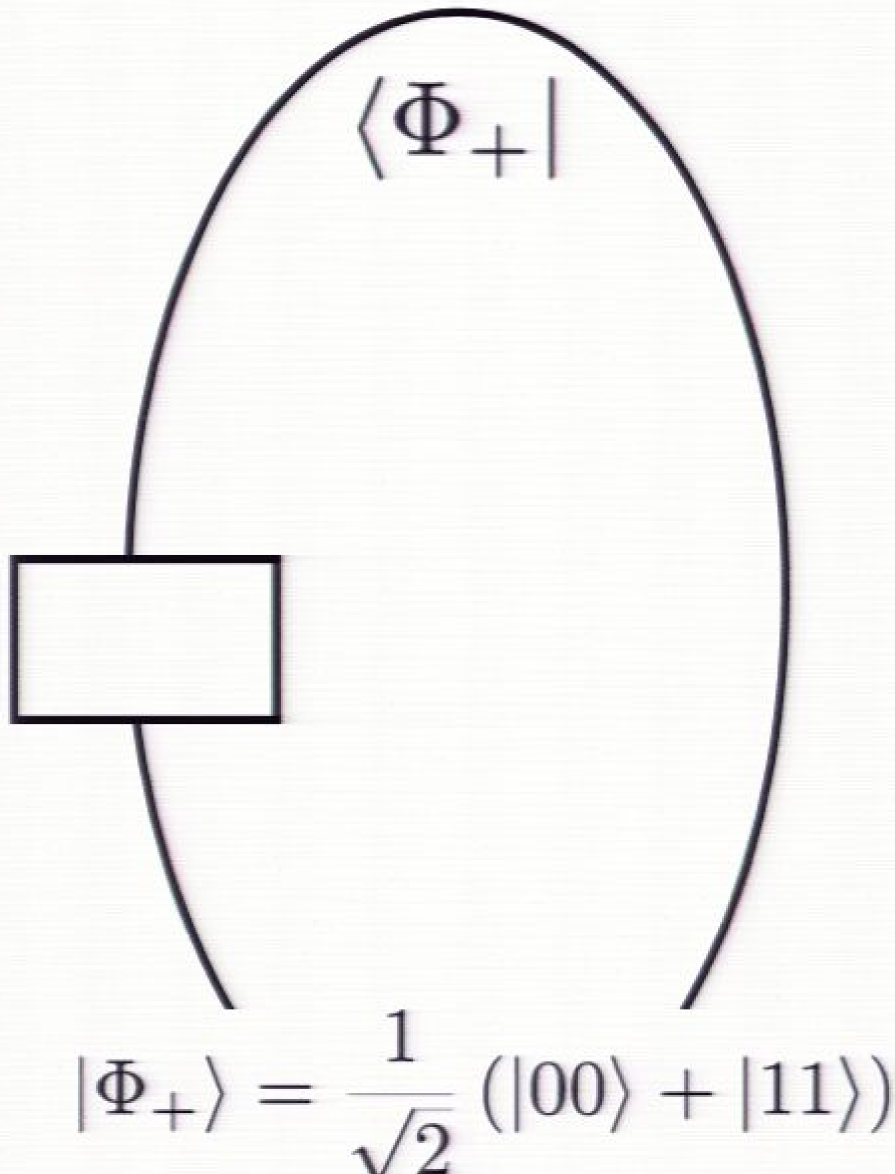
Post-CTC = Counter-factual CTC

- From the "time" of the post-selected measurement, we can construct the consistent framework.
- However, we have never measured the stuff in the black box.
- Therefore, **this consistency framework can be taken as the behavior of the closed time like curve.**
- This framework can be also characterized by the weak value.

Weak Value Analysis





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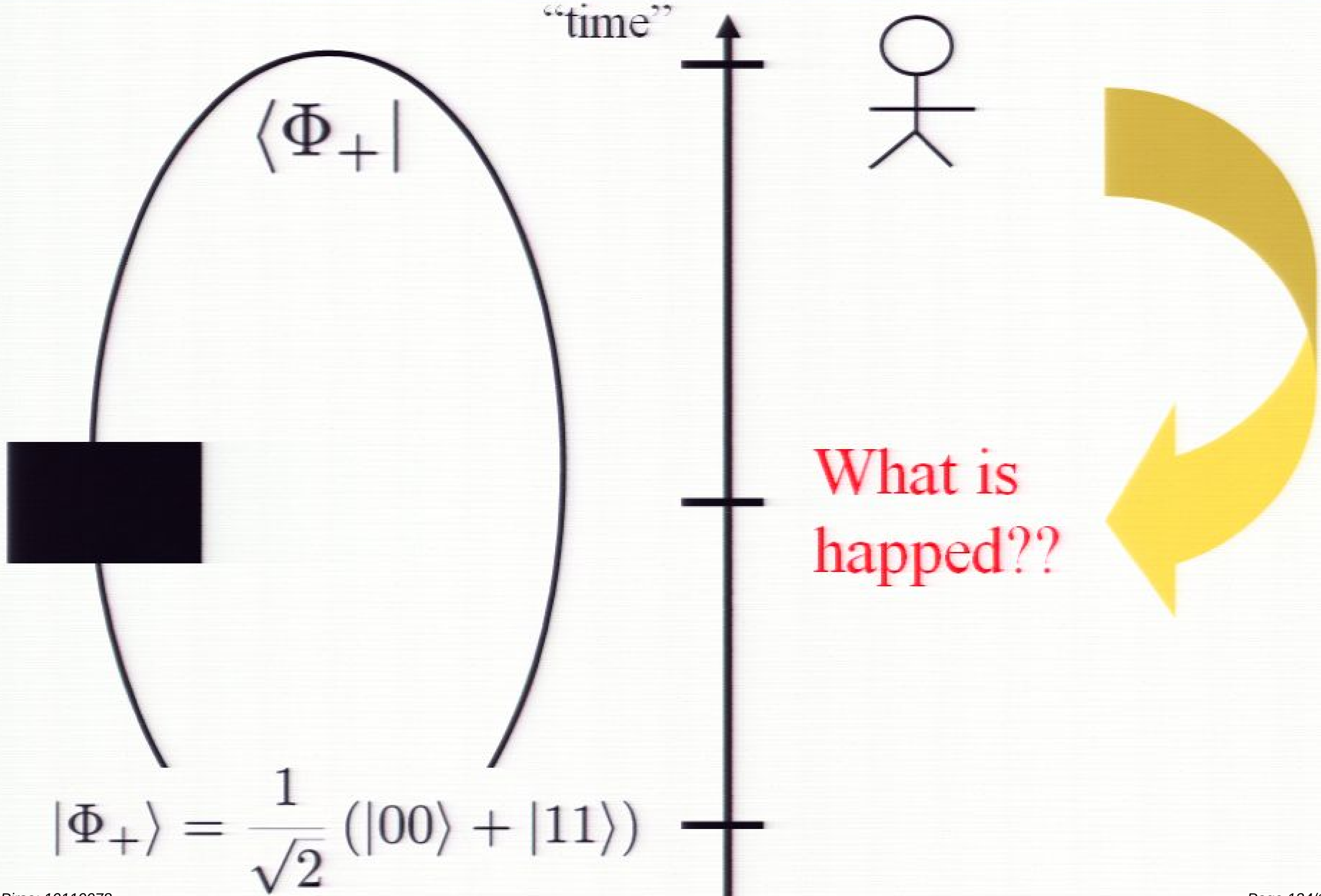

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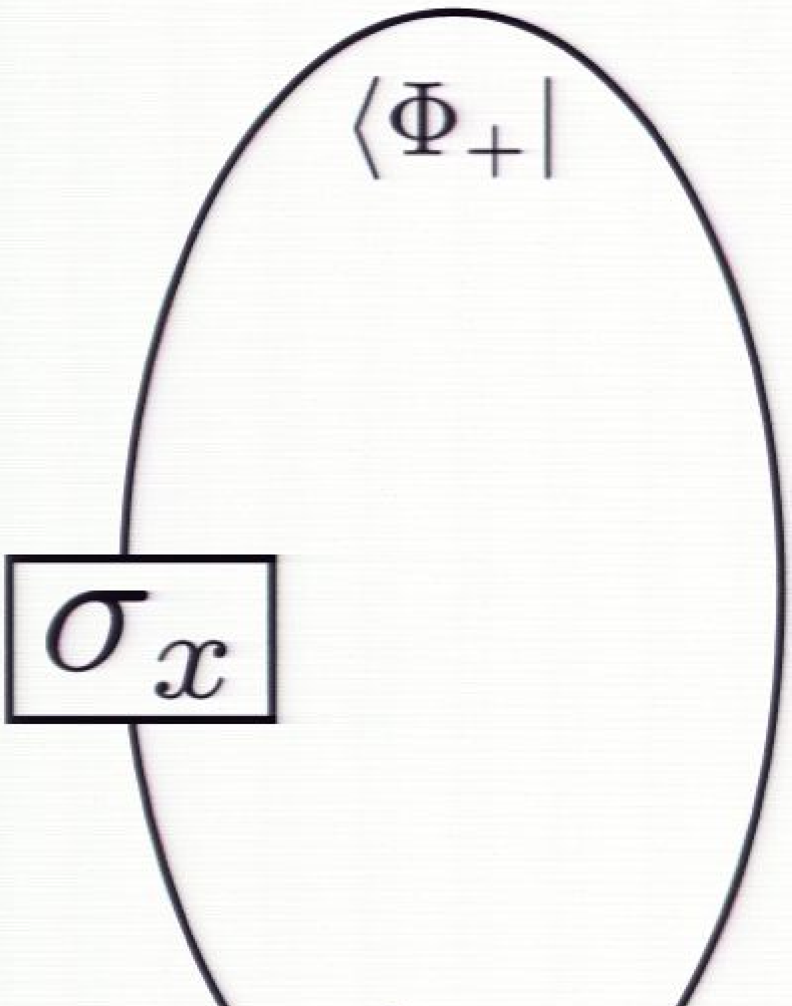

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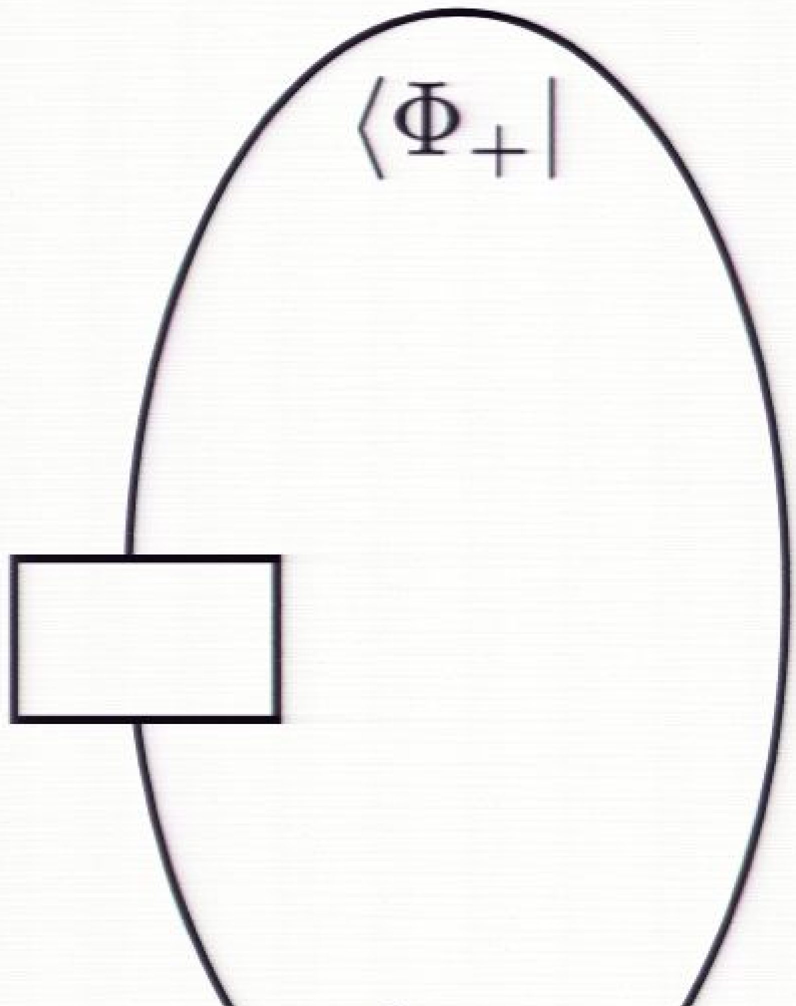

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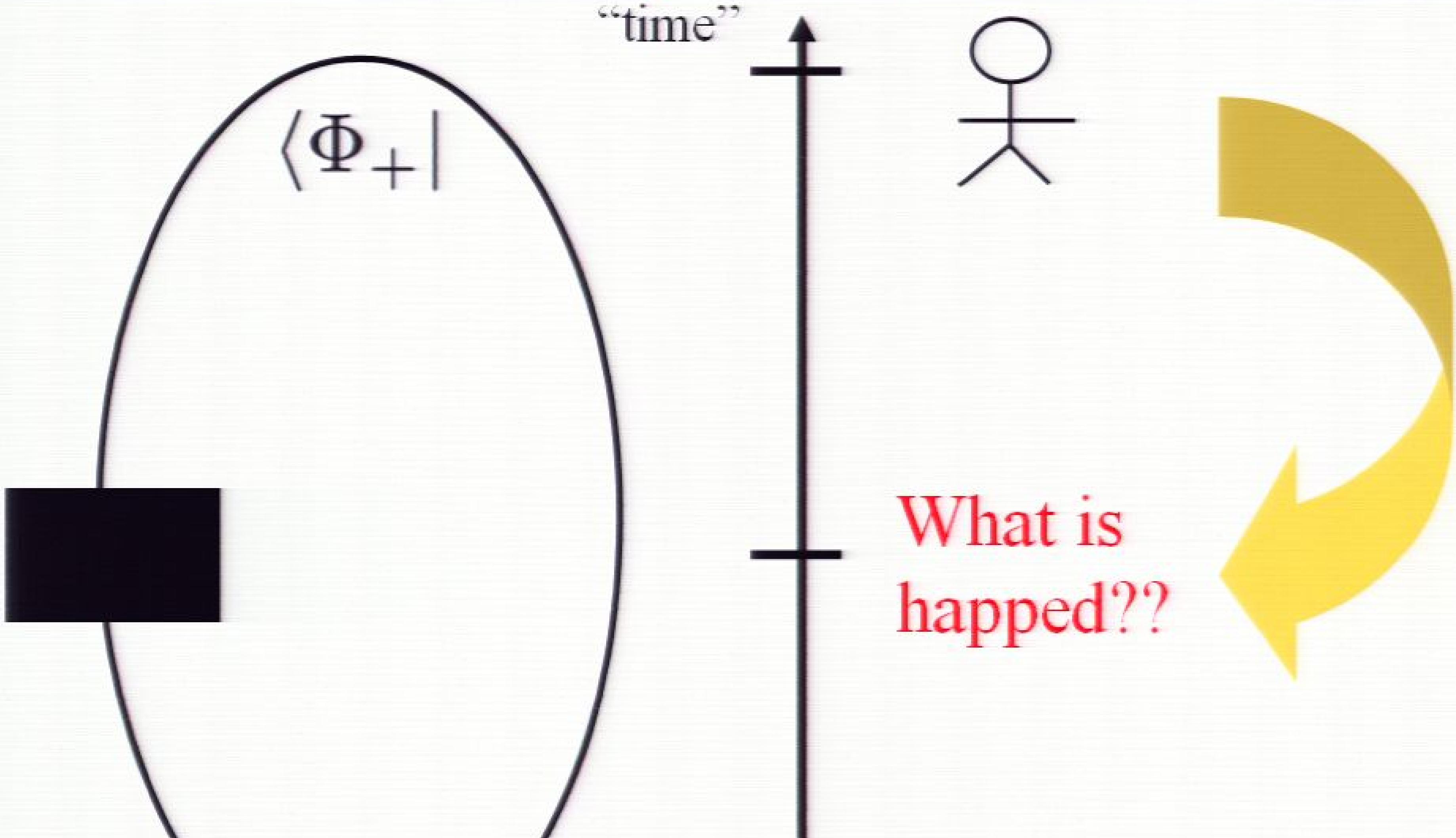

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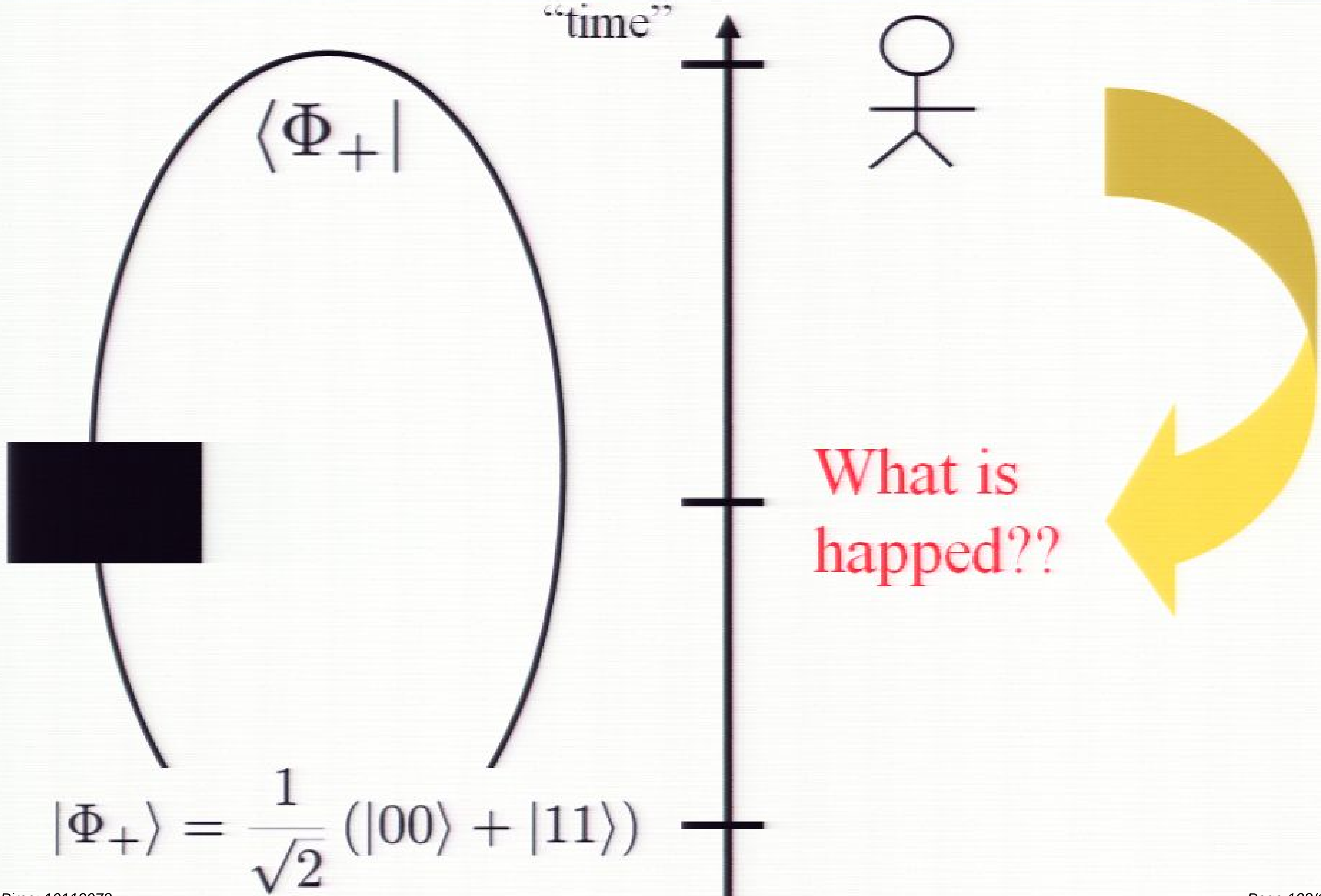

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   - Motivation of the "weak value theory" – related to the probability theory
   - Definition and applications of the weak values
   - How to obtain the weak values – weak measurement
2. Counter-factual Processes
   - Hardy's paradox
   - Quantum description of the closed time-like curves
3. Conclusion

Conclusion

- I introduced the weak value to be motivate by the observable-independent probability space.
- The weak value is a useful tool.
  - Amplification of the tiny effect
  - Geometric Phase
- The weak value can characterize the counter-factual argument.
  - Hardy's paradox
  - Quantum description of the closed time-like curve

Open Questions?

- Is it possible to construct the "theory of the weak value" as alternative approach of the standard quantum mechanics.
  - Is it possible to construct the consistent theory only using the weak value?
  - Work in progress with Richard Jozsa, Graeme Mitchison, and Akio Hosoya.
- Is the weak value represented as the "reality" (Sein / Daseinsation (in A. Doering's quote))?
  - How to understand the Kochen-Specker Theorem?
- Is the "theory of the weak value" useful?
  - How to understand mechanism of quantum speedup in quantum computation?
  - How to give a new aspect to the quantum field theory (work in progress with Izumi Ojima)?

Time is open.

written by Lee Smolin to me

Thank you so much for your attention.

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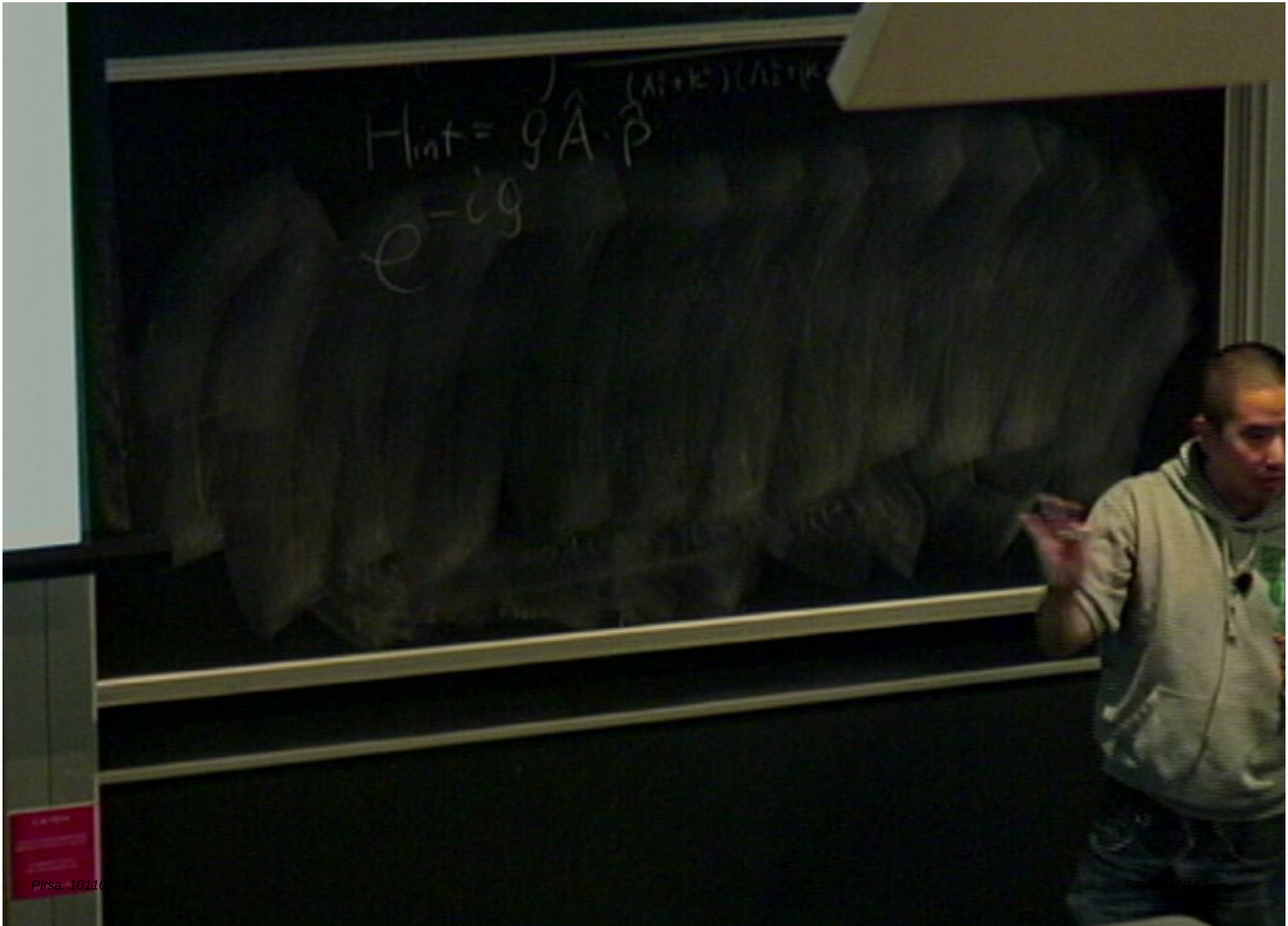
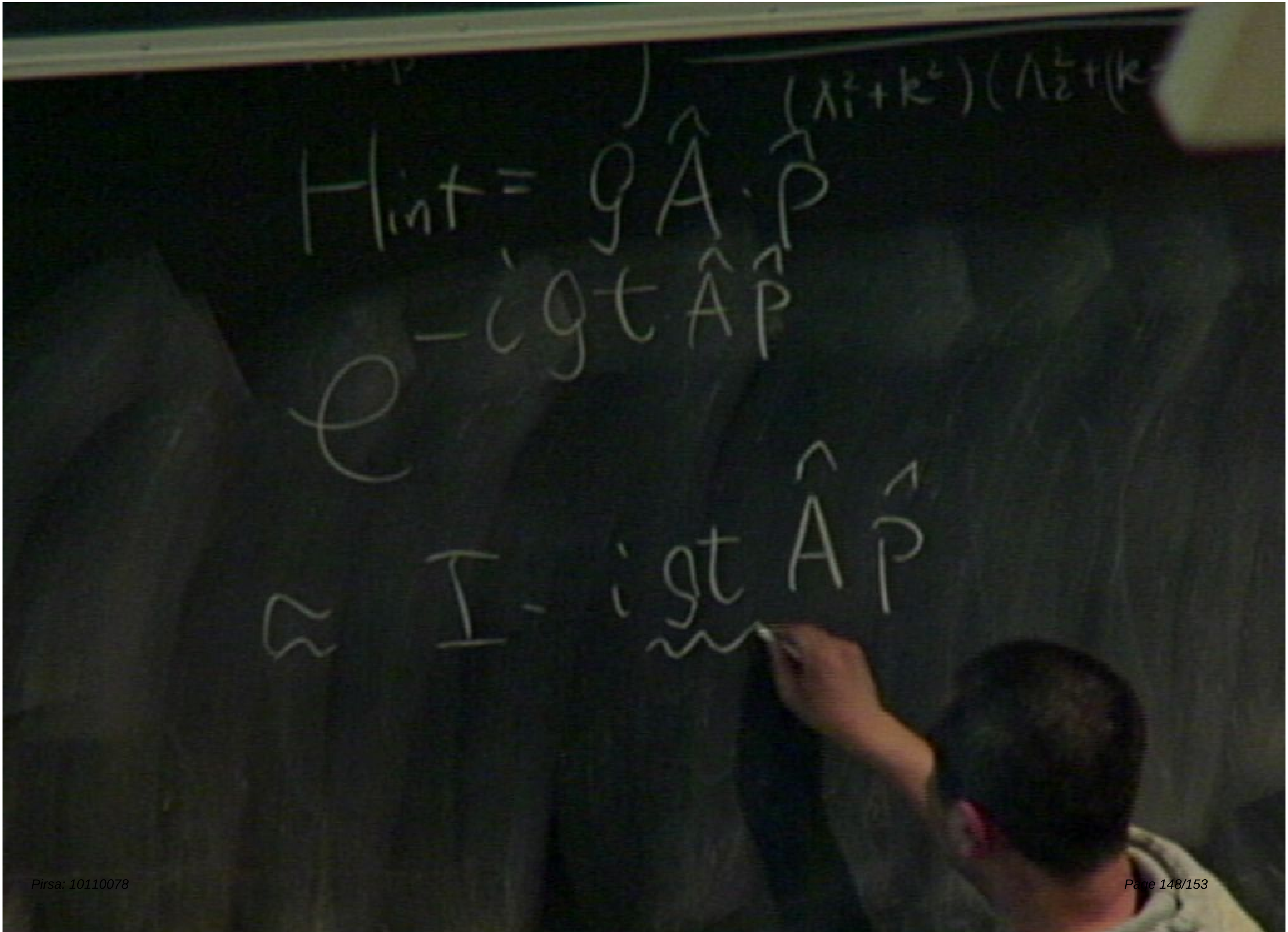

Hint = g Â · p̂
e^-ig


$(\Lambda_1^2+k^2)(\Lambda_2^2+(k$
$H_{int} = g\hat{A}\cdot\hat{p}$
$e^{-igt\hat{A}\hat{p}}$
$\approx I - igt\hat{A}\hat{p}$

$$(\Lambda_1^2+k^2)(\Lambda_2^2+(k \ldots$$

$$H_{int} = g\hat{A}\cdot\hat{P}$$

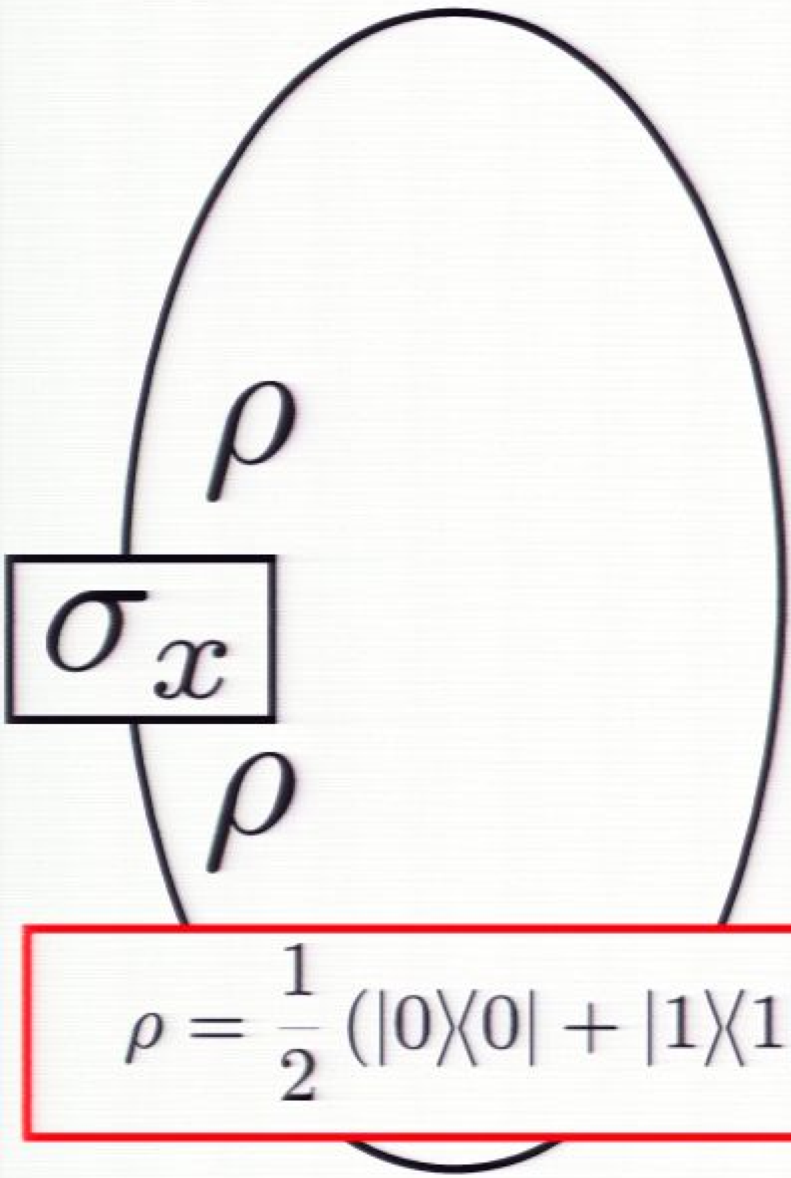
$$e^{-igt\hat{A}\hat{P}}$$

$$\simeq I - igt\hat{A}\hat{P}$$

$$gt \ll 1$$

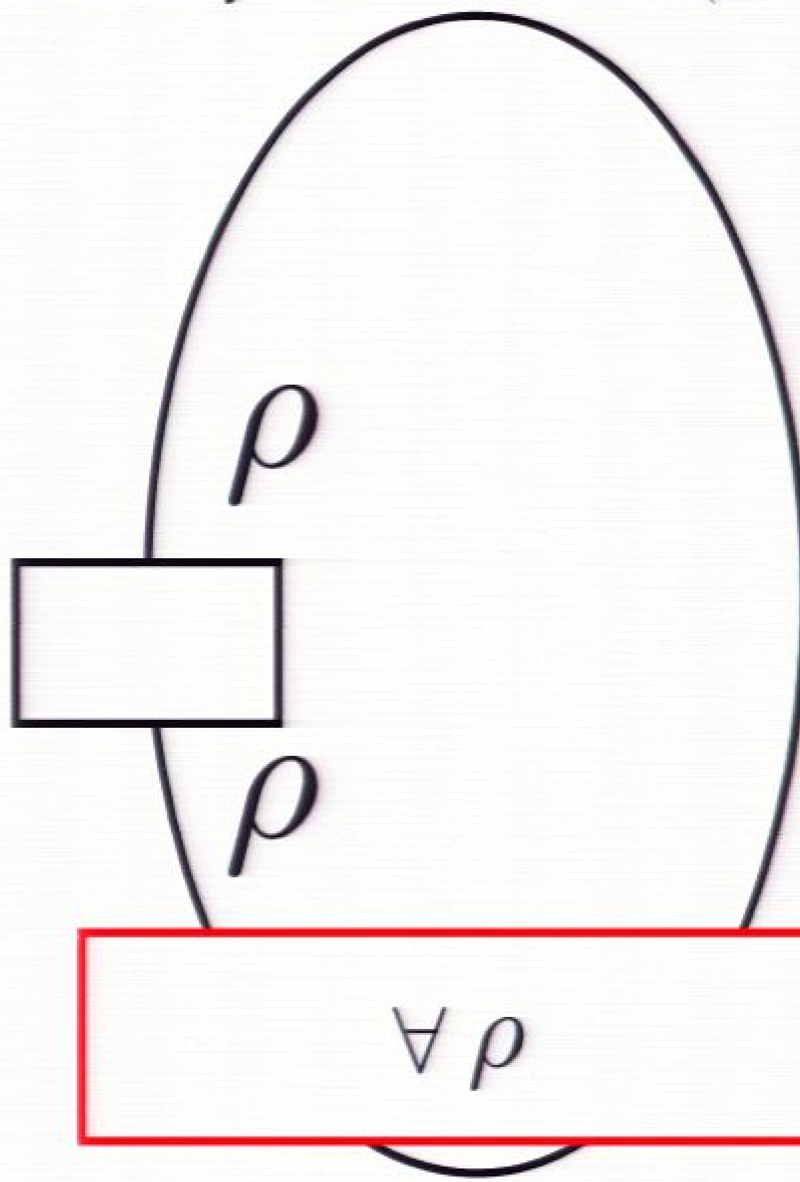
Quantum Description of CTC

(D. Deutsch, Phys. Rev. D **44**, 3197 (1991))



$$\rho = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

Killing the grandfather



$$\forall\, \rho$$

Not killing the grandfather

Quantum Description of CTC

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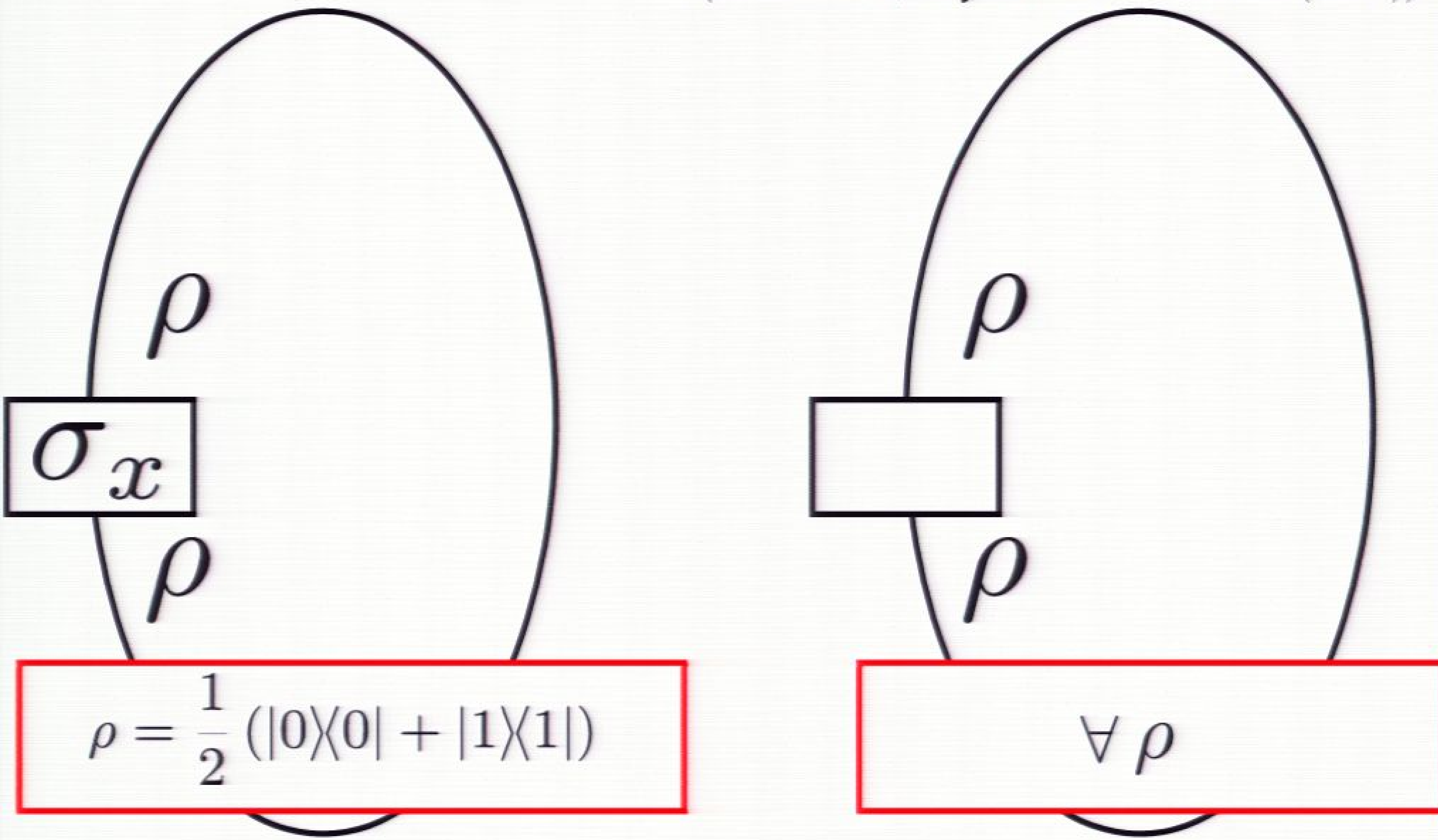
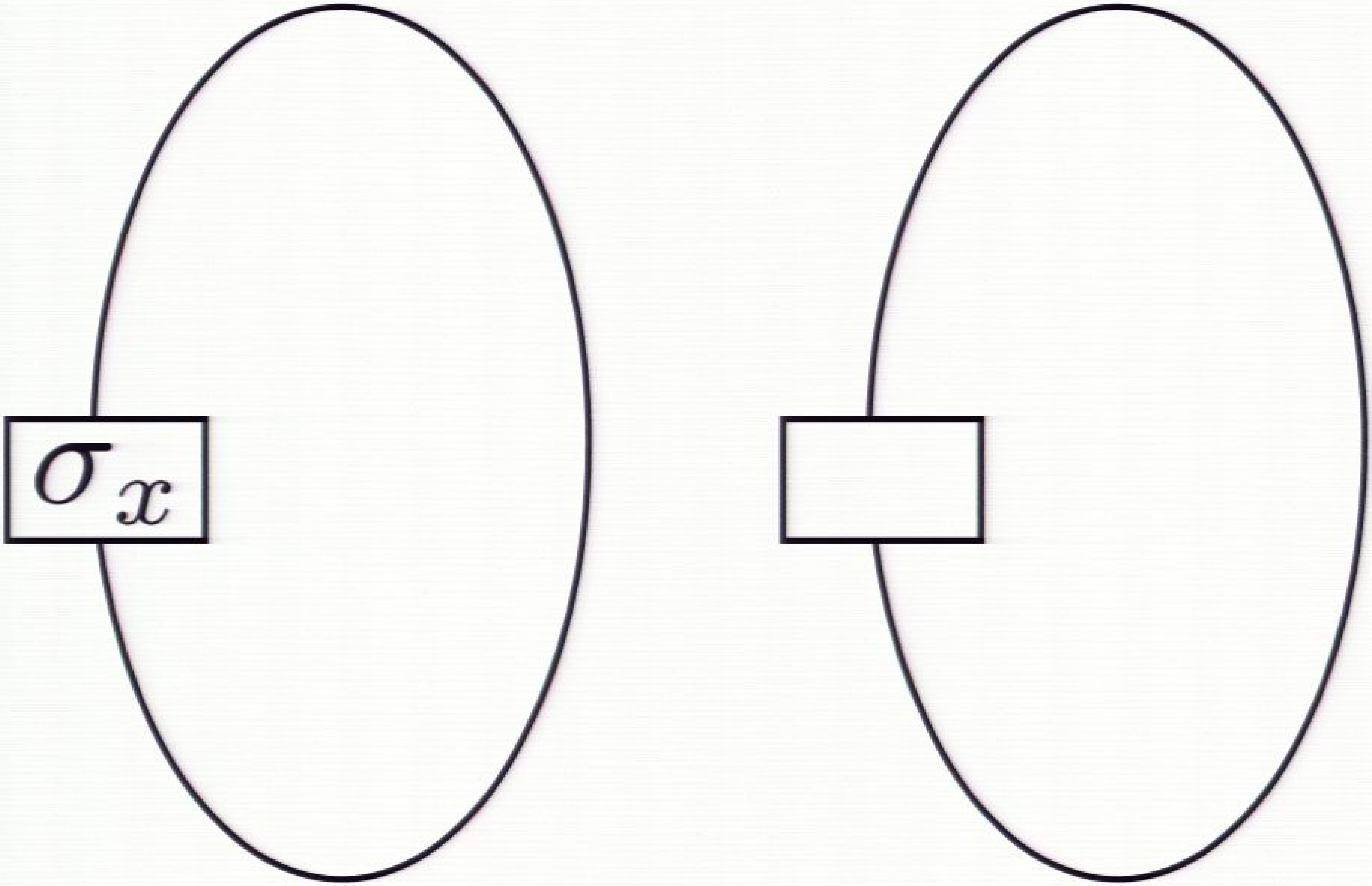


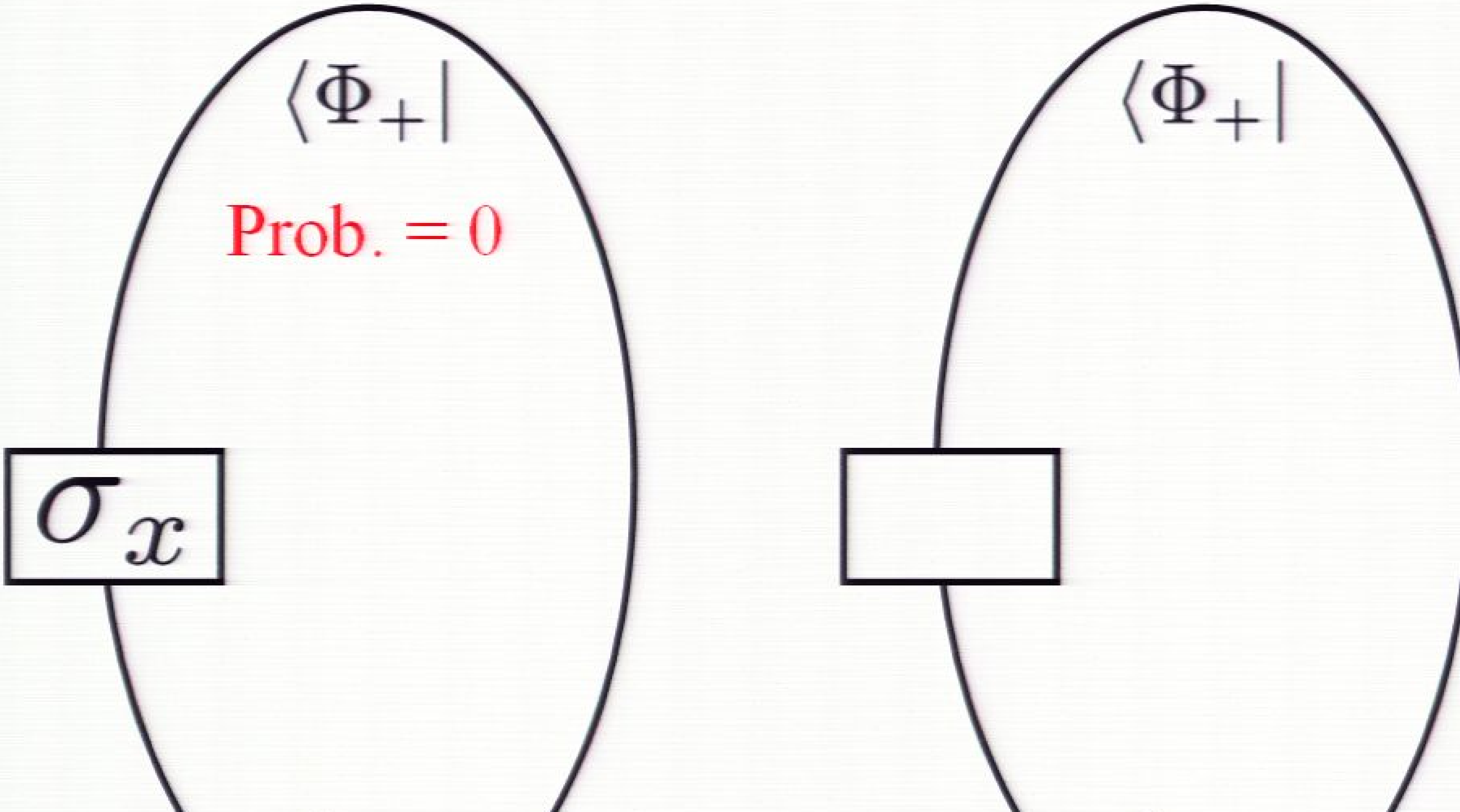
Killing the grandfather

Not killing the grandfather

Post-selected CTC



Post-selected CTC



$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \qquad |\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$