Title: Holographic Branching and Entanglement Renormalization

Date: Nov 19, 2010 02:30 PM

URL: http://pirsa.org/10110076

Abstract: Entanglement renormalization is a coarse-graining transformation for quantum lattice systems. It produces the multi-scale entanglement renormalization ansatz, a tensor network state used to represent ground states of strongly correlated systems in one and two spatial dimensions. In 1D, the MERA is known to reproduce the logarithmic violation of the boundary law for entanglement entropy, S(L)~log L, characteristic of critical ground states. In contrast, in 2D the MERA strictly obeys the entropic boundary law, S(L)~L, characteristic of gapped systems and a class of critical systems. Therefore a number of highly entangled 2D systems, such as free fermions with a 1D Fermi surface, Fermi liquids and spin Bose metals, which display a logarithmic violation of the boundary law, S(L)~L log L, cannot be described by a regular 2D MERA. It is well-known that at low energies, a many-body system may decouple into two or more independent degrees of freedom (e.g. spin-charge separation in 1D systems of electrons). In this talk I will explain how, in systems where low energy decoupling occurs, entanglement renormalization can be used to obtain an explicit decoupled description. The resulting tensor network state, the branching MERA, can reproduce a logarithmic violation of the boundary law in 2D and, as additional numeric evidence also suggests, might be a good ansatz for the highly entangled systems with a 1D Fermi (or Bose) surface mentioned above. In addition, after recalling that the MERA can be regarded as a specific (discrete) realization of the holographic principle, we will see that the branching MERA leads to exotic holographic geometries.

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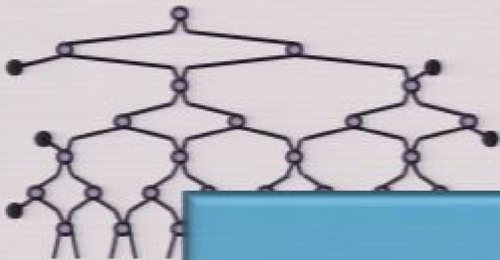
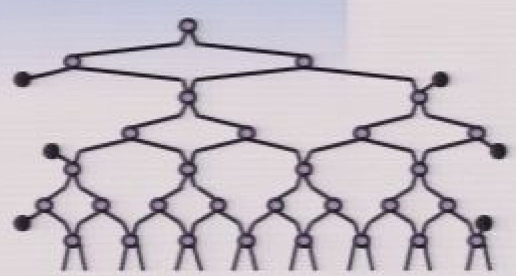


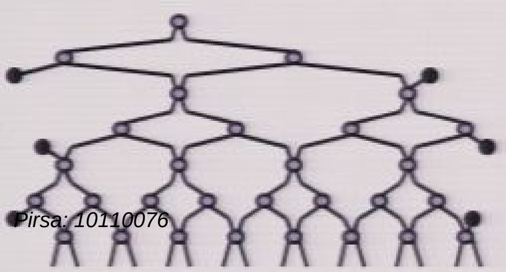
Holographic Branching and Entanglement Renormalization

Glen Evenbly

Guifre Vidal

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA





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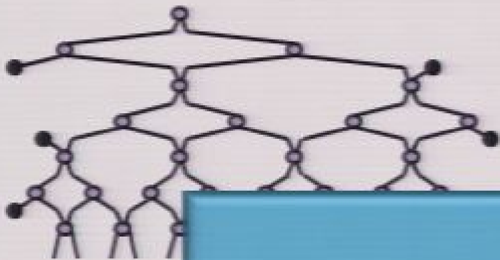

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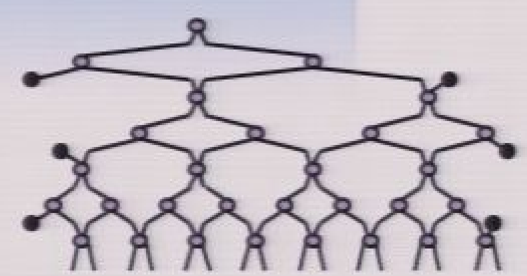
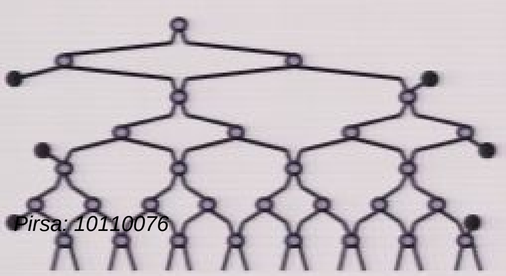
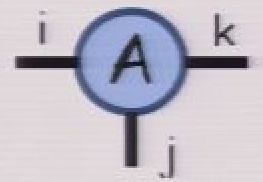




Tensor Network Methods

tensor: A_{ijk} $\longleftrightarrow$ 

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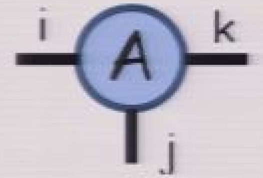
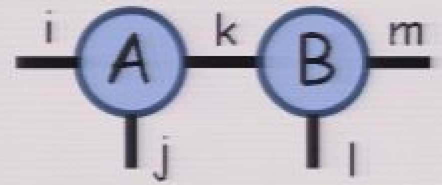
tensor: A_{ijk} $\longleftrightarrow$ 

contraction: $\sum_{k} A_{ijk} B_{klm}$ $\longleftrightarrow$ 

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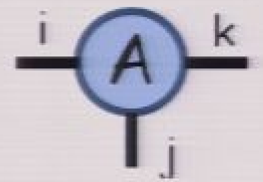

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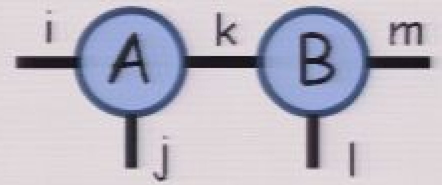

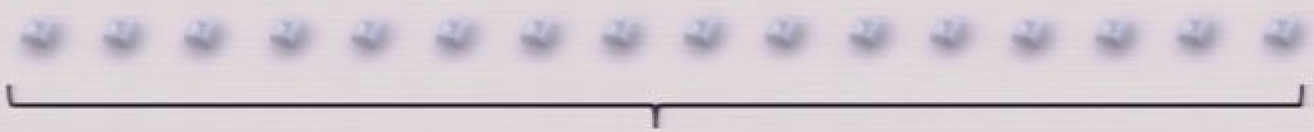

N sites

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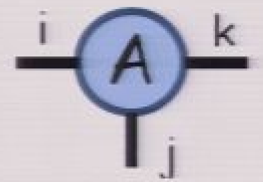

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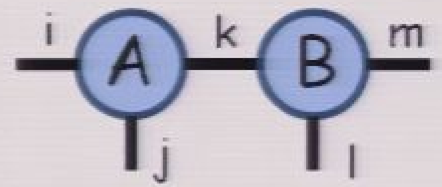
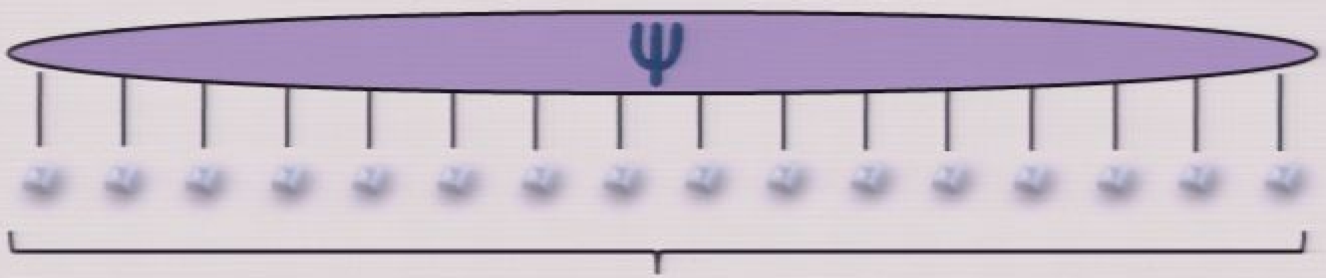


Wavefunction: $\Psi_{ijkl\ldots}$

~2^N parameters



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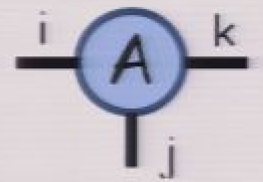

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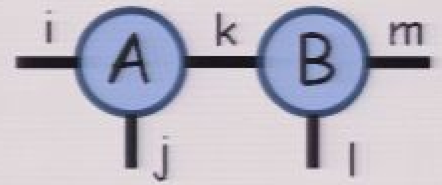

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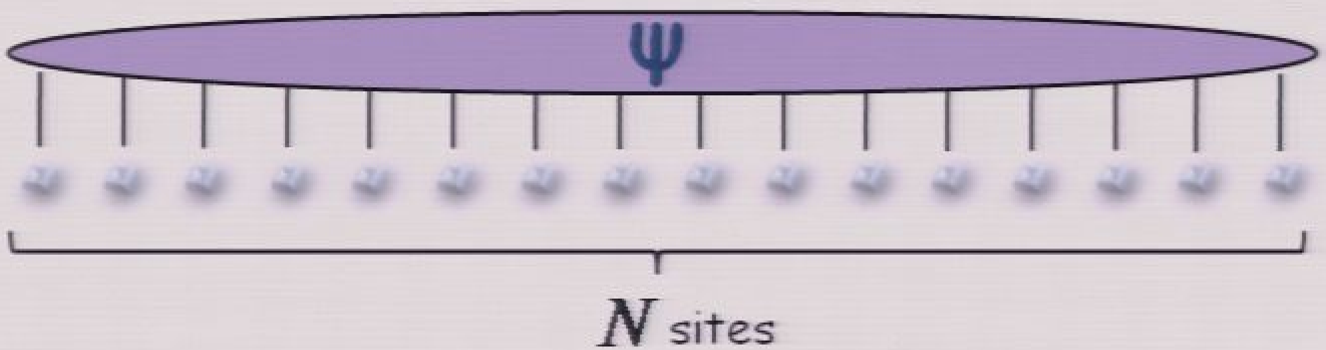
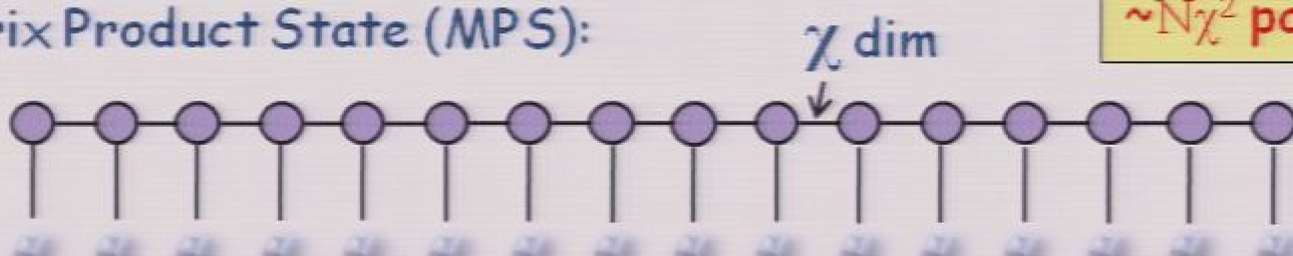


Matrix Product State (MPS):

~$N\chi^2$ parameters



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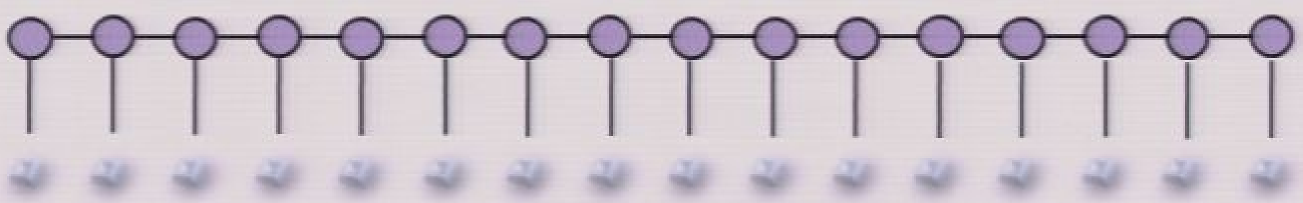

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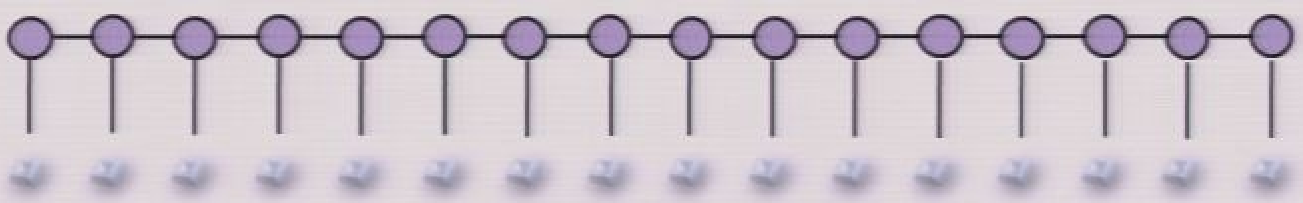

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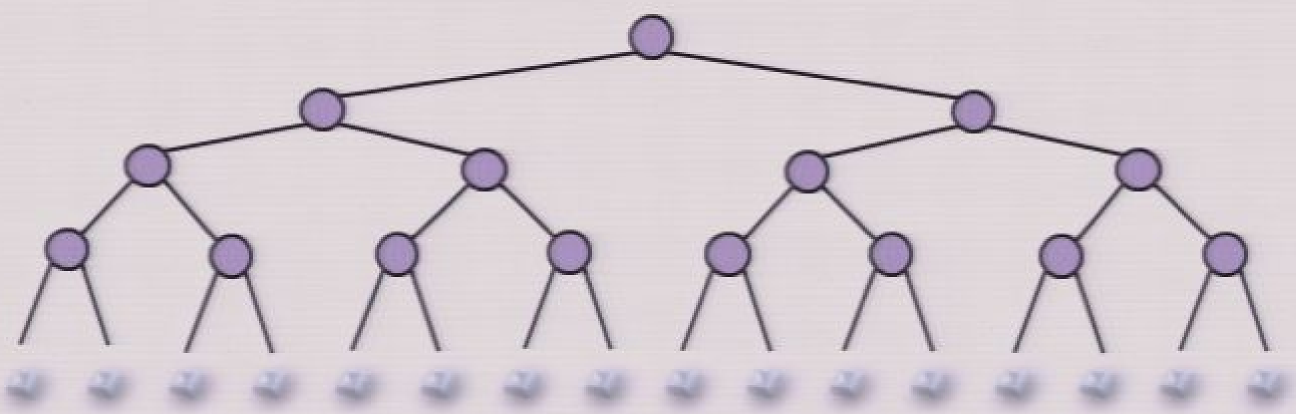

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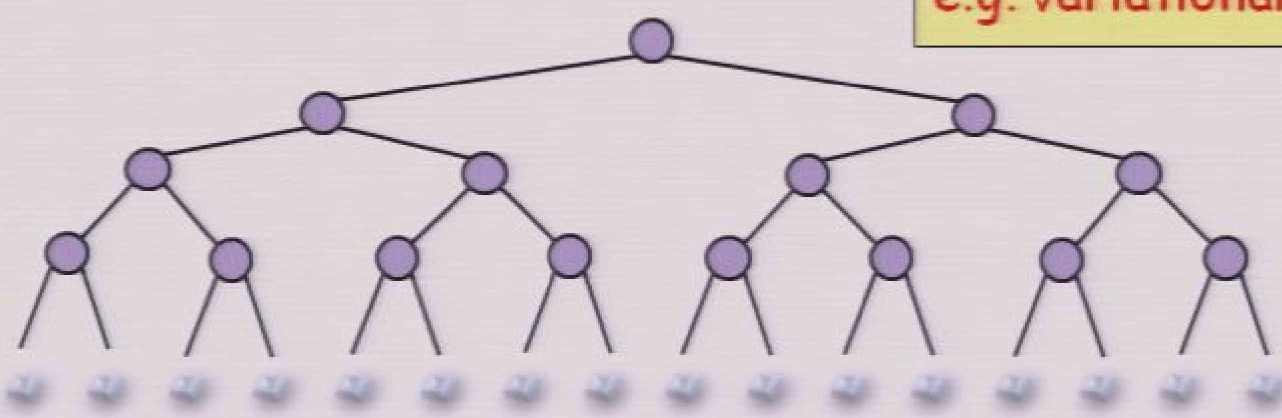

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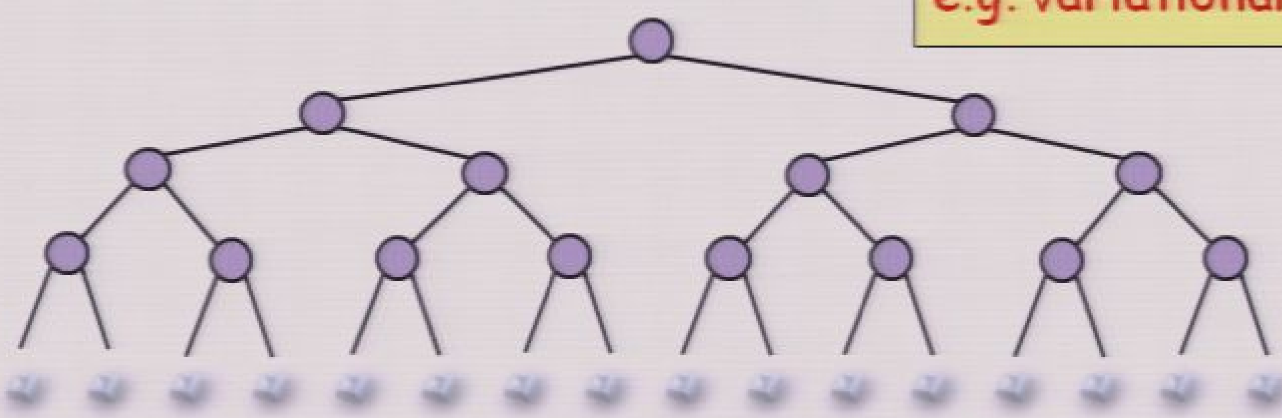

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(DMRG, PEPS, TERG, MERA)

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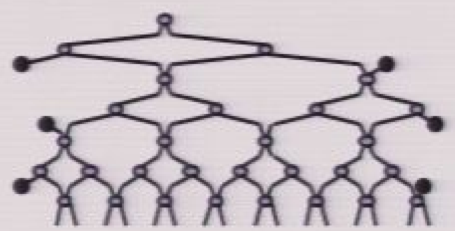
Conceptual Aspects:

- Framework for describing many-body systems - entanglement structure!

Outline

- Entanglement and tensor network methods
    - Scaling of entanglement entropy in ground states
    - Scaling of entanglement entropy in tensor network ansatz
      - physical geometry vs holographic geometry
    - Comparison of entropy scaling:
      - ground states vs tensor network ansatz

- Introduction to branching MERA



    - Decoupling a many-body theory
    - Holographic trees
    - Scaling of entropy in the branching MERA
    - Example: $S_L = L \log L$ entropy scaling in 2D fermions
    - Example: $S_L = (\log L)^2$ entropy scaling in 1D fermions

Entanglement entropy scaling in 1D systems

1D Gapped

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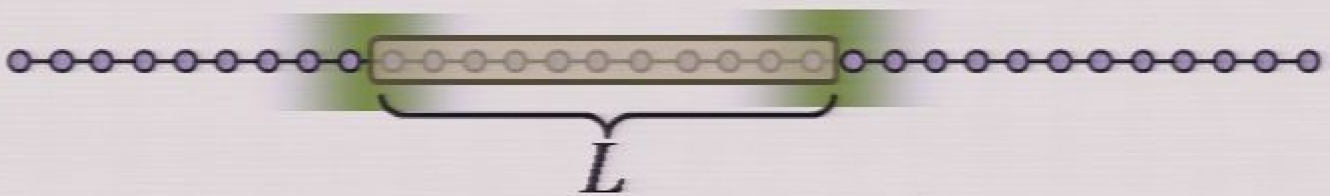
entanglement is localised near boundary



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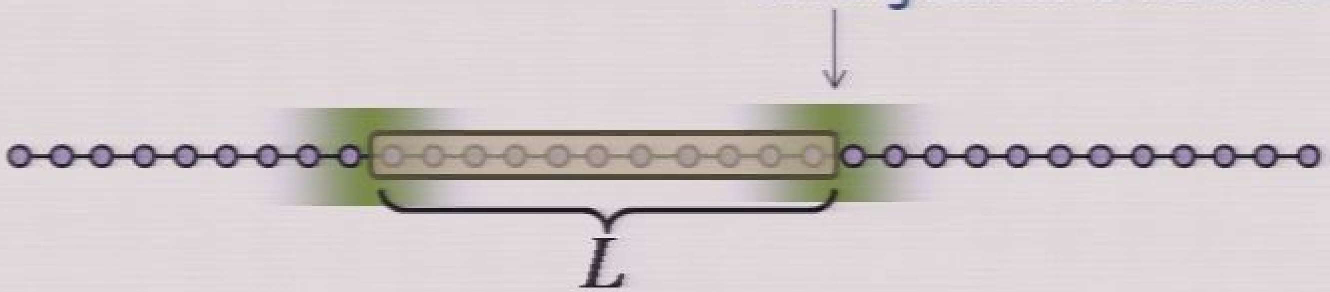

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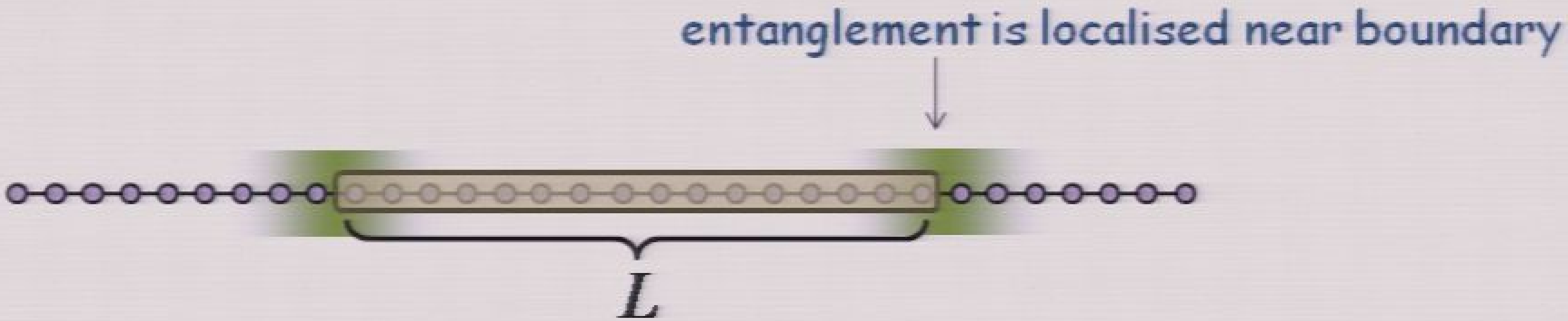

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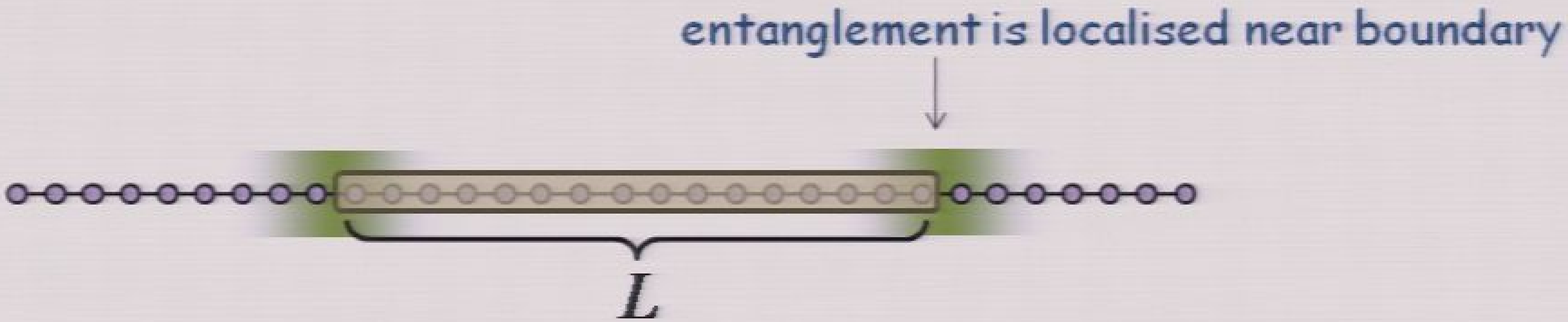

- Boundary law: $S_L = \text{const.}$ as opposed to bulk: $S_L = L$

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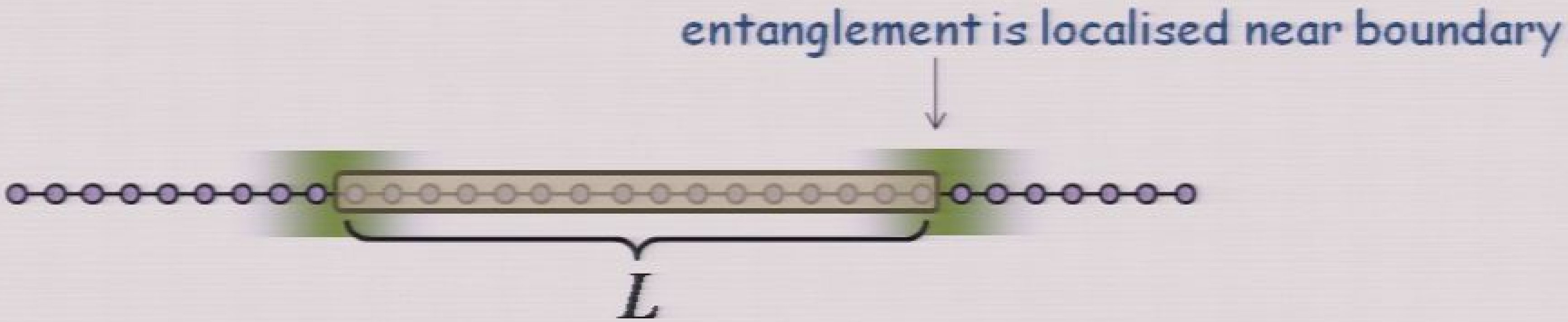

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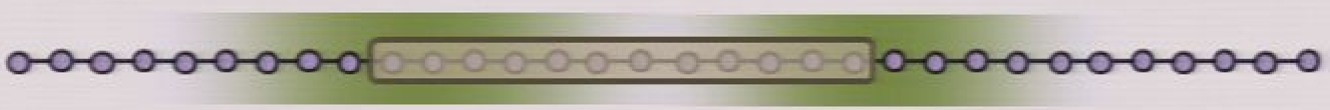
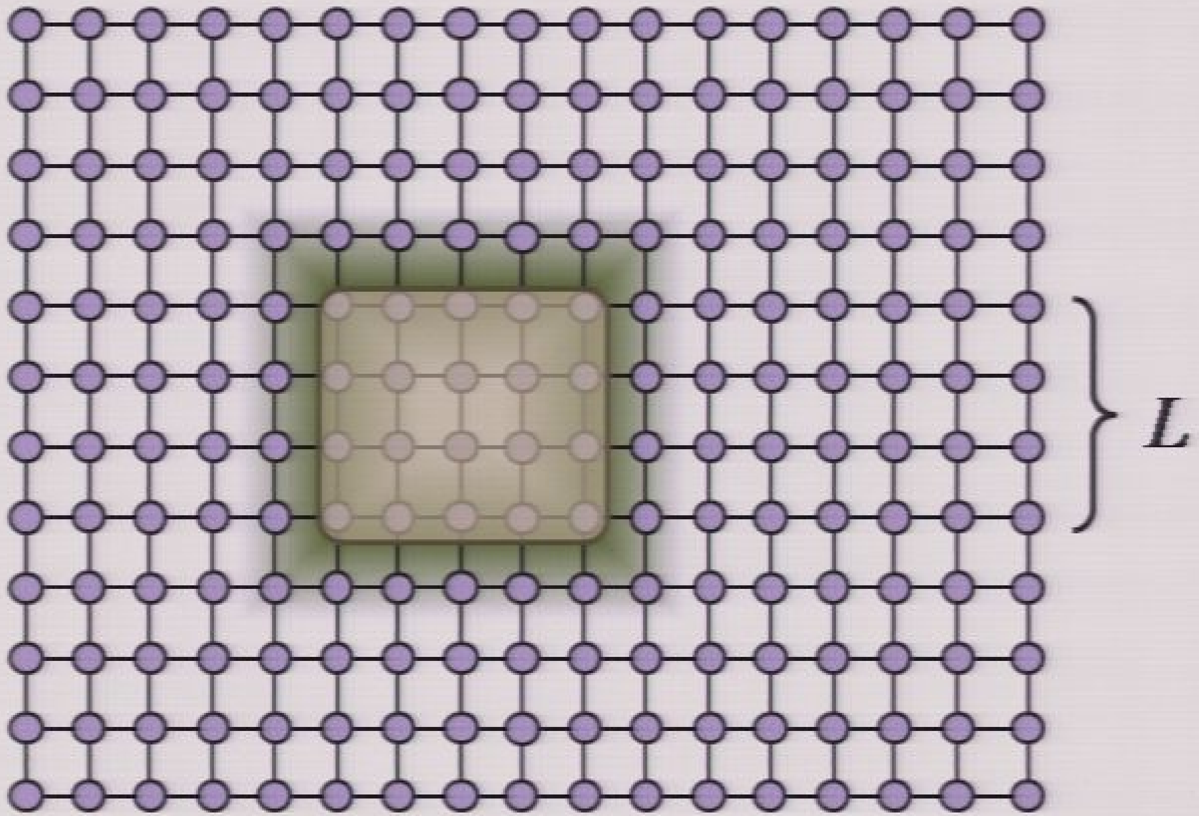
1D Critical



- All sites of the block contribute to entanglement entropy!
- Logarithmic Correction: $S_L \propto \log(L)$

Entanglement entropy scaling in 2D systems



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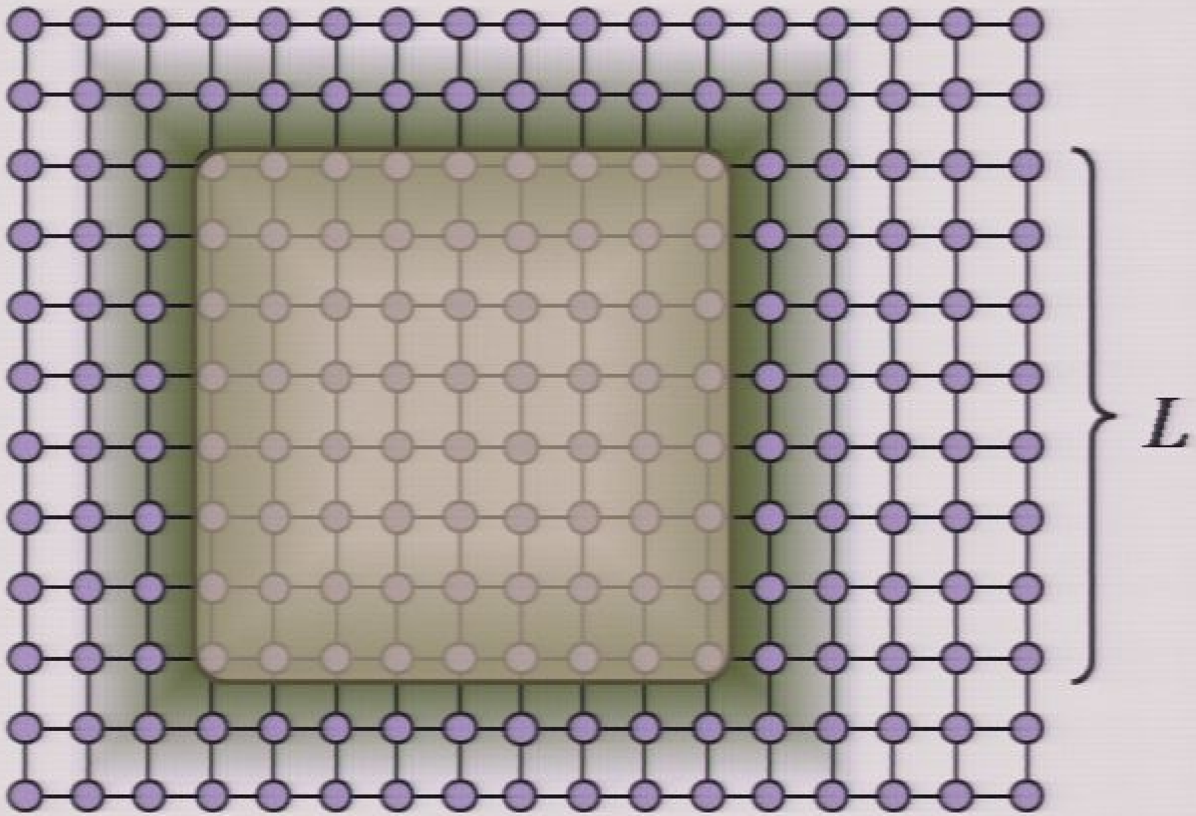

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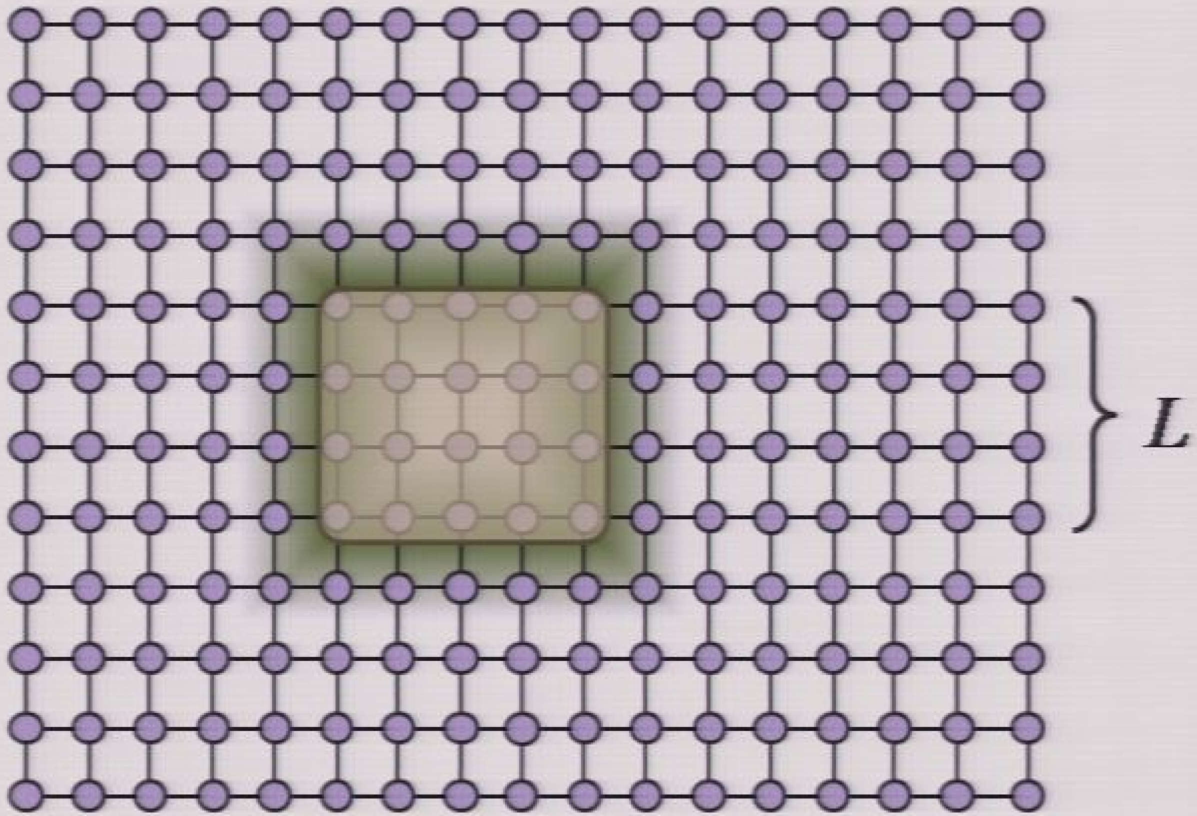

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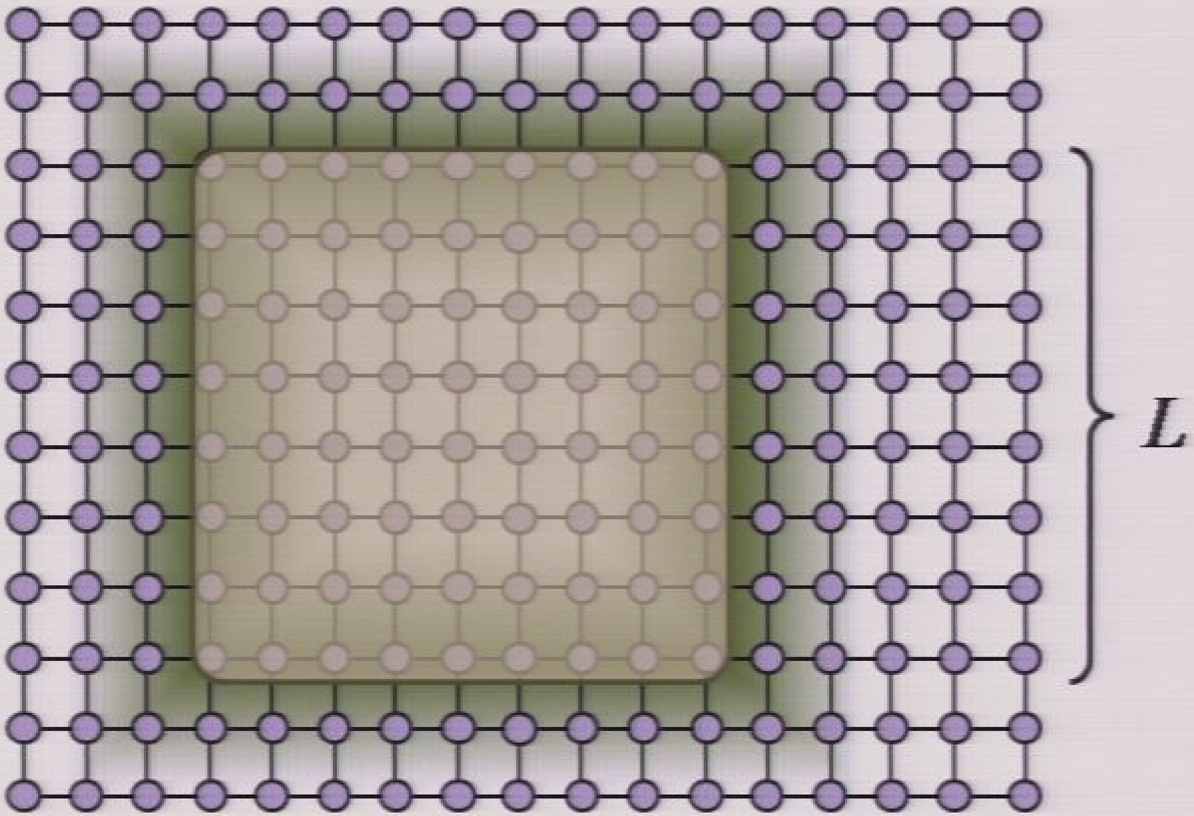

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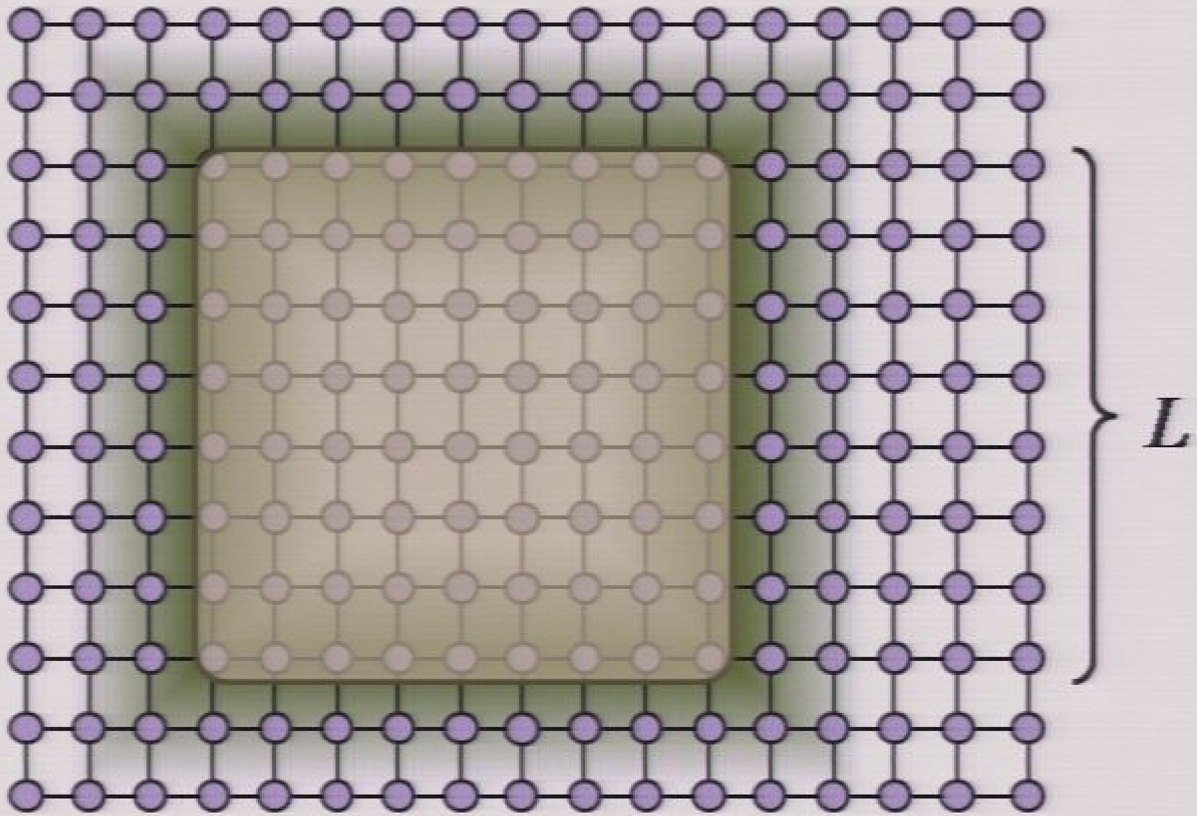
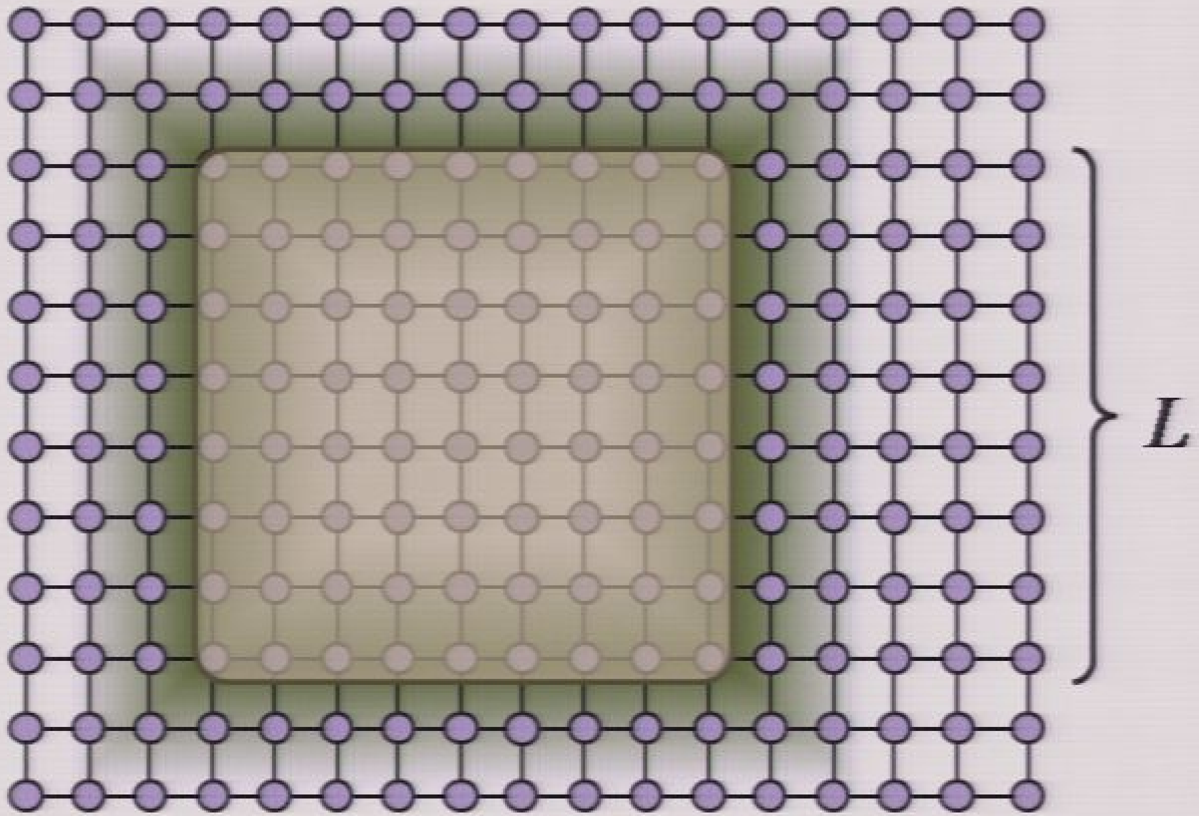


2D Gapped

- Boundary law: $S_L = L$ (boundary as opposed to bulk ~~$S_L \approx L^2$~~)

Entanglement entropy scaling in 2D systems



2D Gapped

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2D Critical

- Boundary law: $S_L = L$

or

- Logarithmic violation: $S_L = L \log L$

Scaling of entanglement entropy for free fermions

1D

	Gap.	Crit.
S_L	const.	$\log(L)$

2D

	Gap.	Crit.I	Crit.II
S_L	L	L	$L\log(L)$

1D

Vidal, Latorre, Rico, Kitaev, PRL 2003
Srednicki, PRL 1993
Callan, Wilczek, Phys Lett B 1994.
Fiola, Preskill, Strominger, Trivedi, PRD 1994.
Holzhey, Larsen, Wilczek, Nucl.Phys.B 1994.
Jin, Korepin, J. Stat. Phys. 2004
Calabrese, Cardy, J. Stat. Mech. 2004

2D

Wolf, PRL 2006.
Gioev, Klich, PRL 2006.
Barthel, Chung, Schollwock, PRA 2006.
Li, Ding, Yu, Haas, PRB 2006.
Ding, Bray-Ali, Yu, Haas, PRL 2008.
Helling, Leschke, Spitzer 2009, arXiv:0906.4946.
Swingle 2009, arXiv:0908.1724.

- Can Tensor Network methods reproduce the appropriate entanglement entropy?

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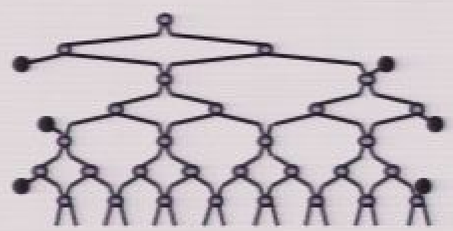

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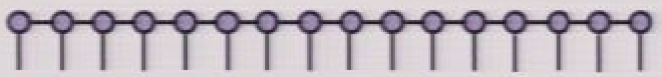
Tensor Networks in Physical Geometry

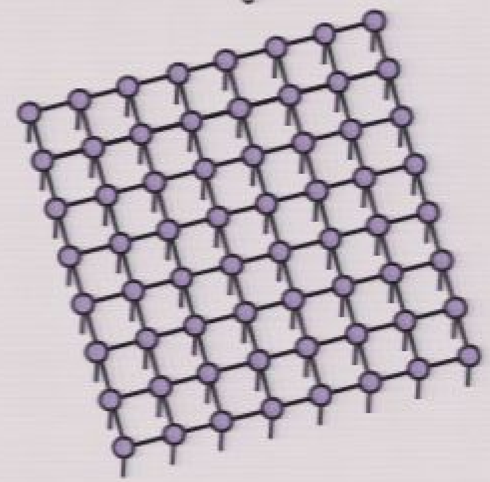
1D: MPS

Matrix product state



2D: PEPS

Projected entangled pair states



Tensor Networks in Physical Geometry

- Entanglement entropy scaling?

2D: PEPS

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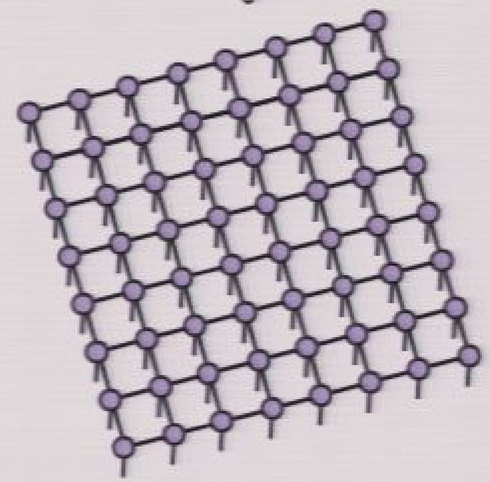

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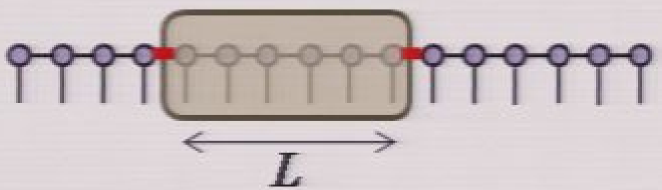

$S_L = \text{const.}$

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1D: MPS

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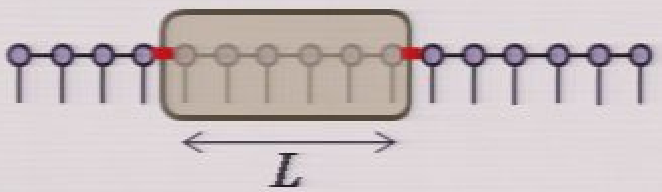

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2D: PEPS

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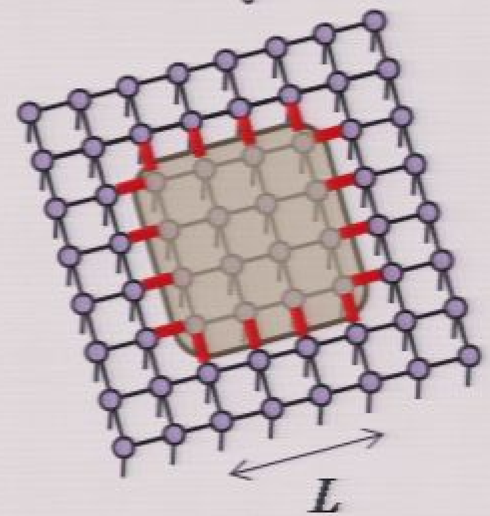

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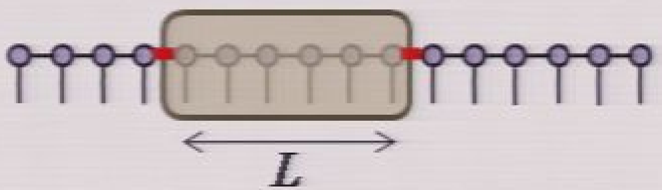

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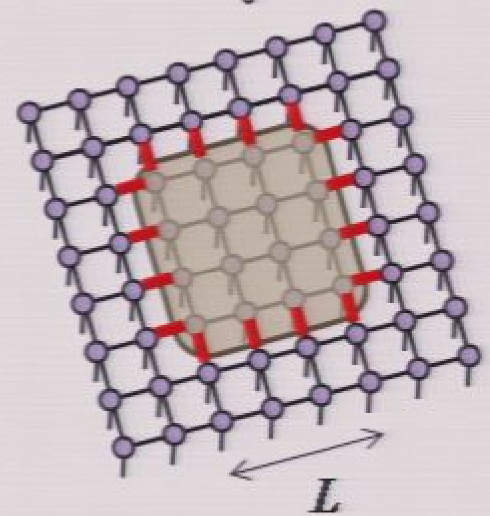

$S_L = L$

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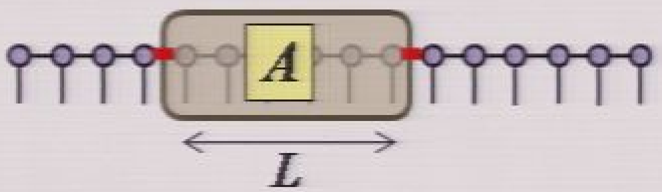

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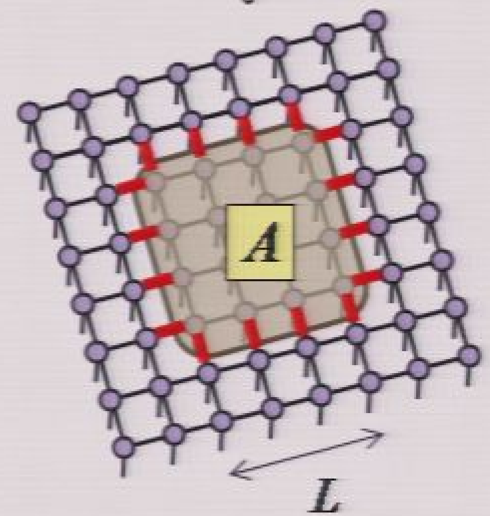

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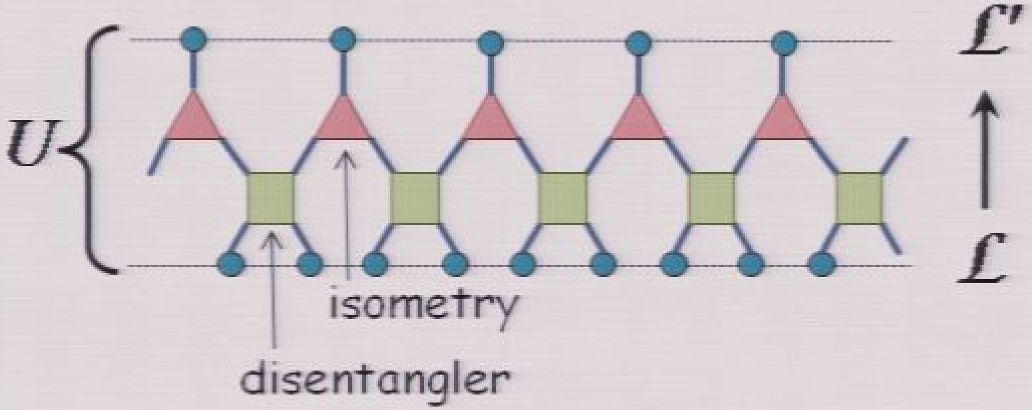
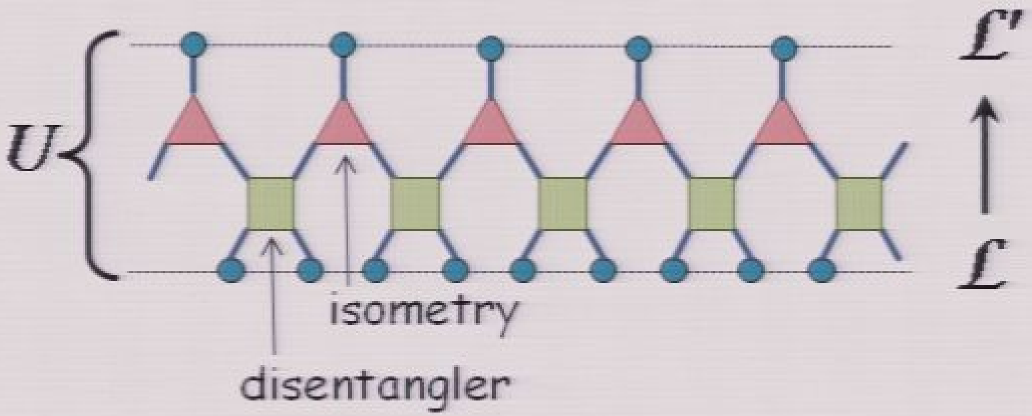
- Entropy scales as boundary in physical geometry: $S(A) \sim |\partial A|$

Entanglement Renormalization and the MERA



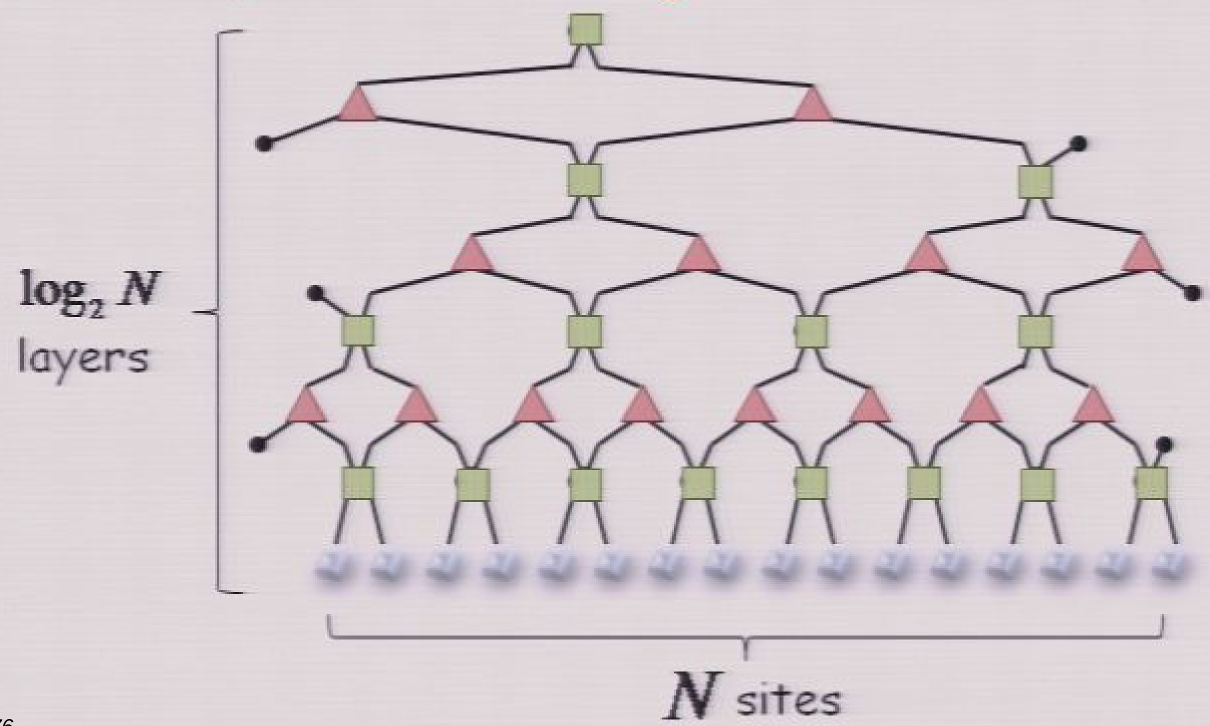
Coarse-graining transformation:
Entanglement Renormalization

Entanglement Renormalization and the MERA

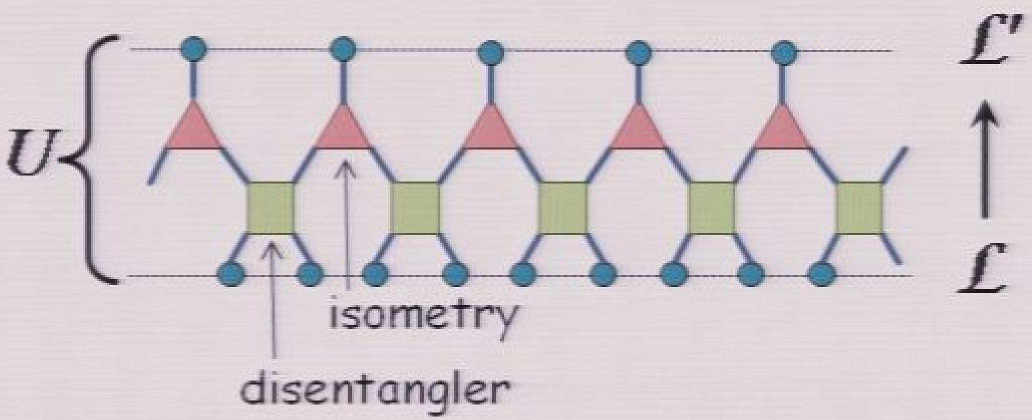


Coarse-graining transformation:
Entanglement Renormalization

MERA (multi-scale entanglement renormalization ansatz)



Entanglement Renormalization and the MERA



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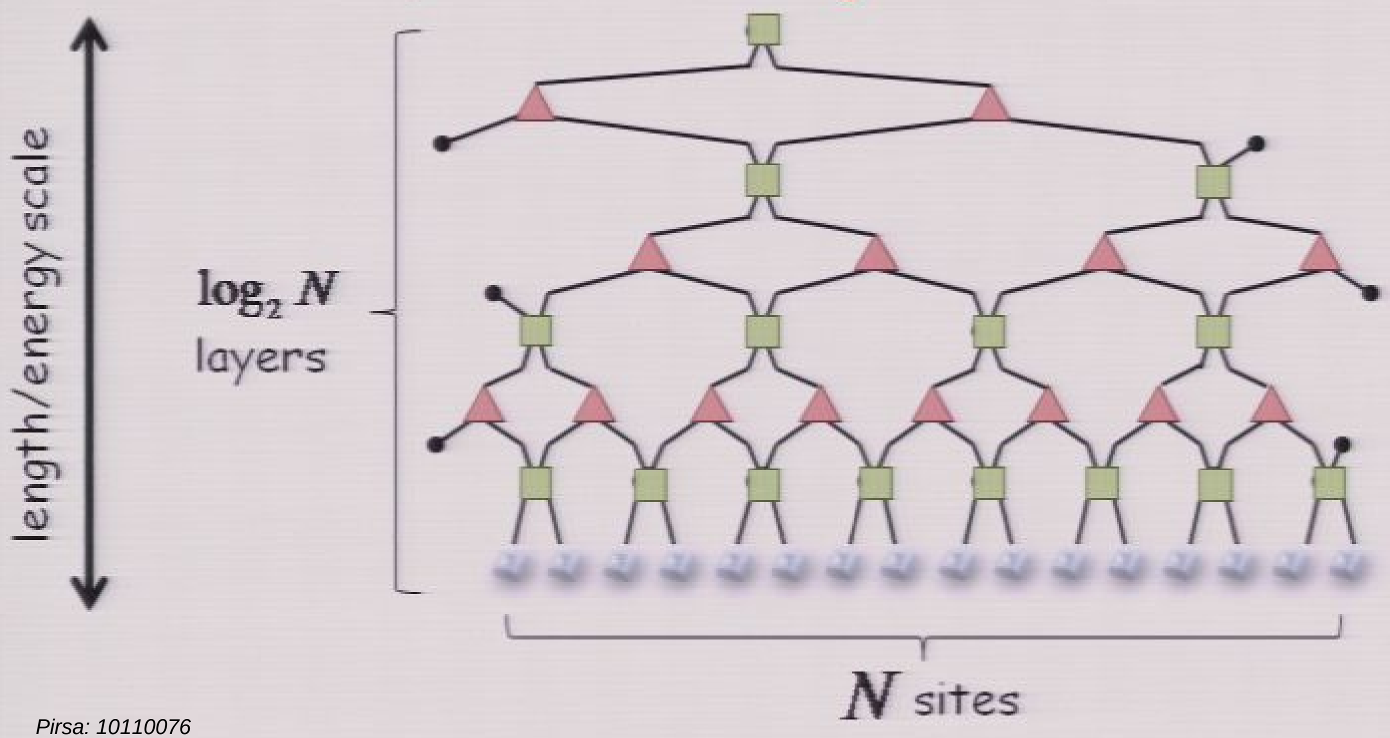
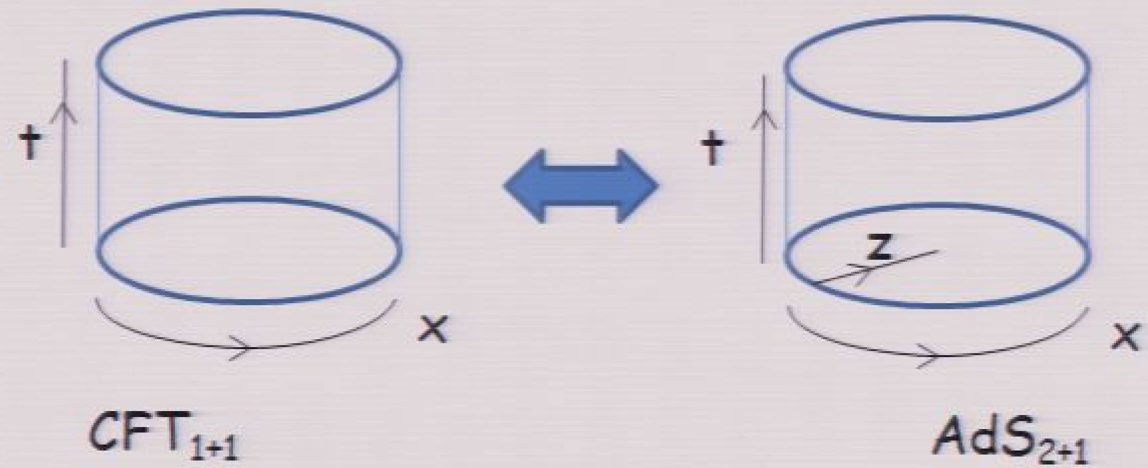


MERA ⟷ Holography
Brian Swingle
arXiv:0905.1317

Holographic geometry

• Reproduce the pattern of entanglement in the ground state

AdS/CFT Correspondance



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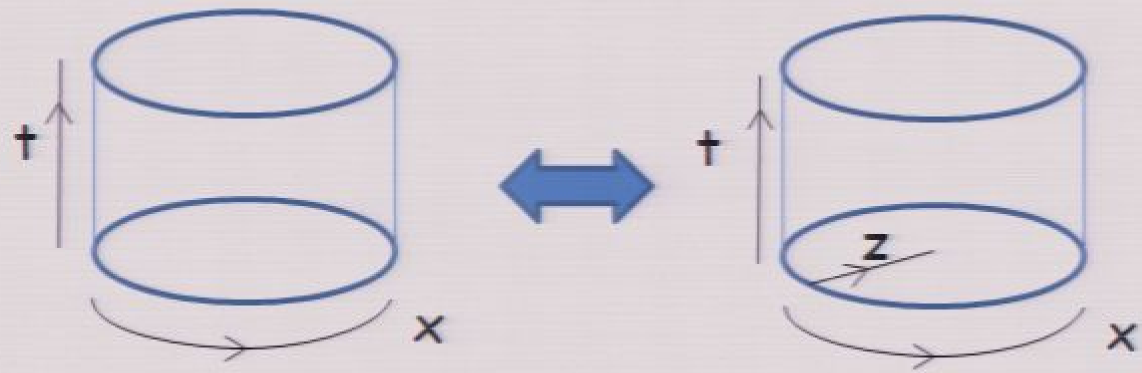
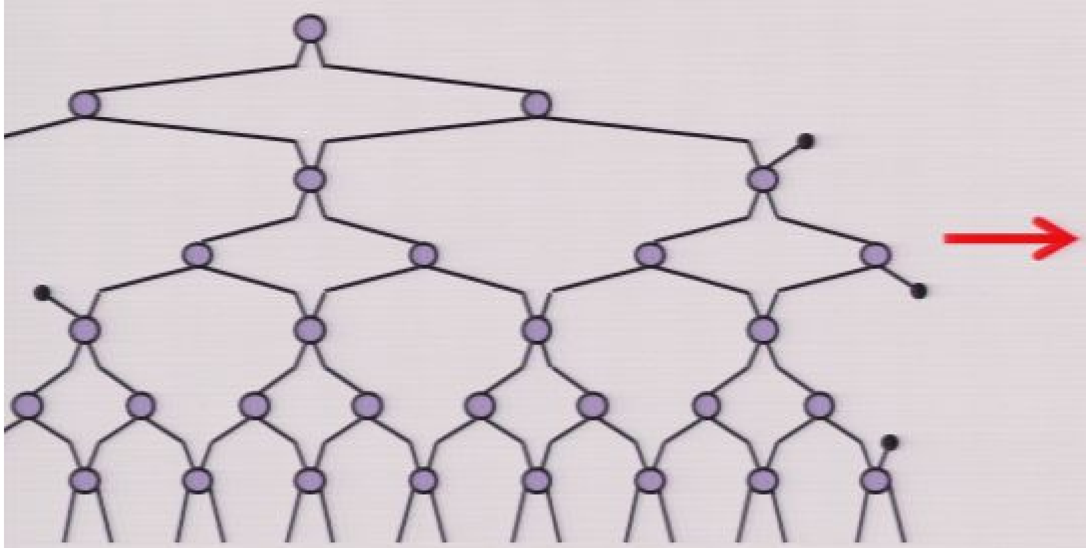
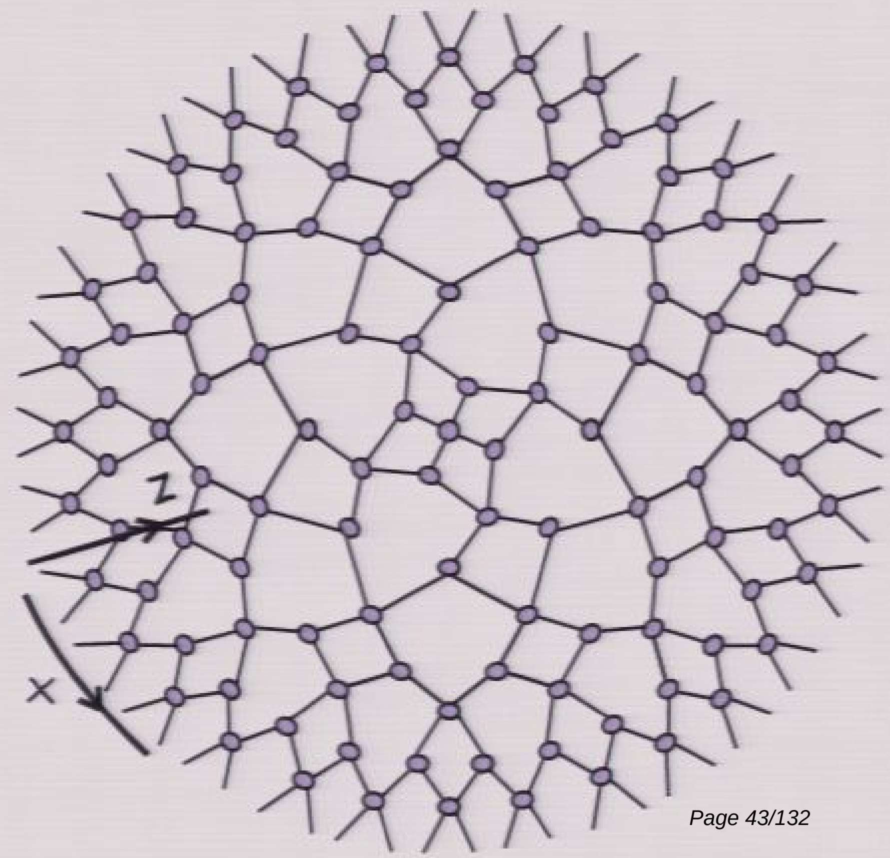


CFT_{1+1} AdS_{2+1}

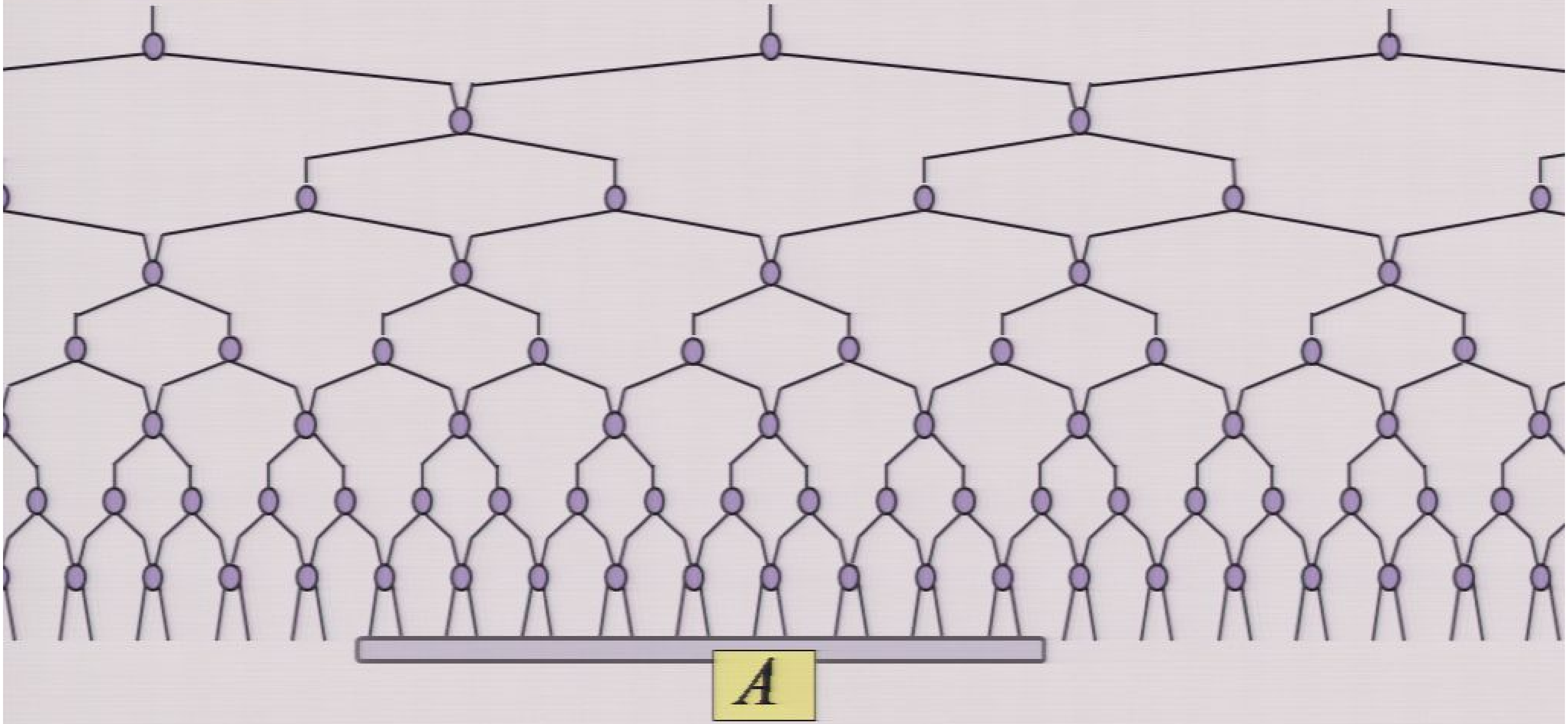
1D MERA





Computation of entanglement entropy

- D=1, MERA



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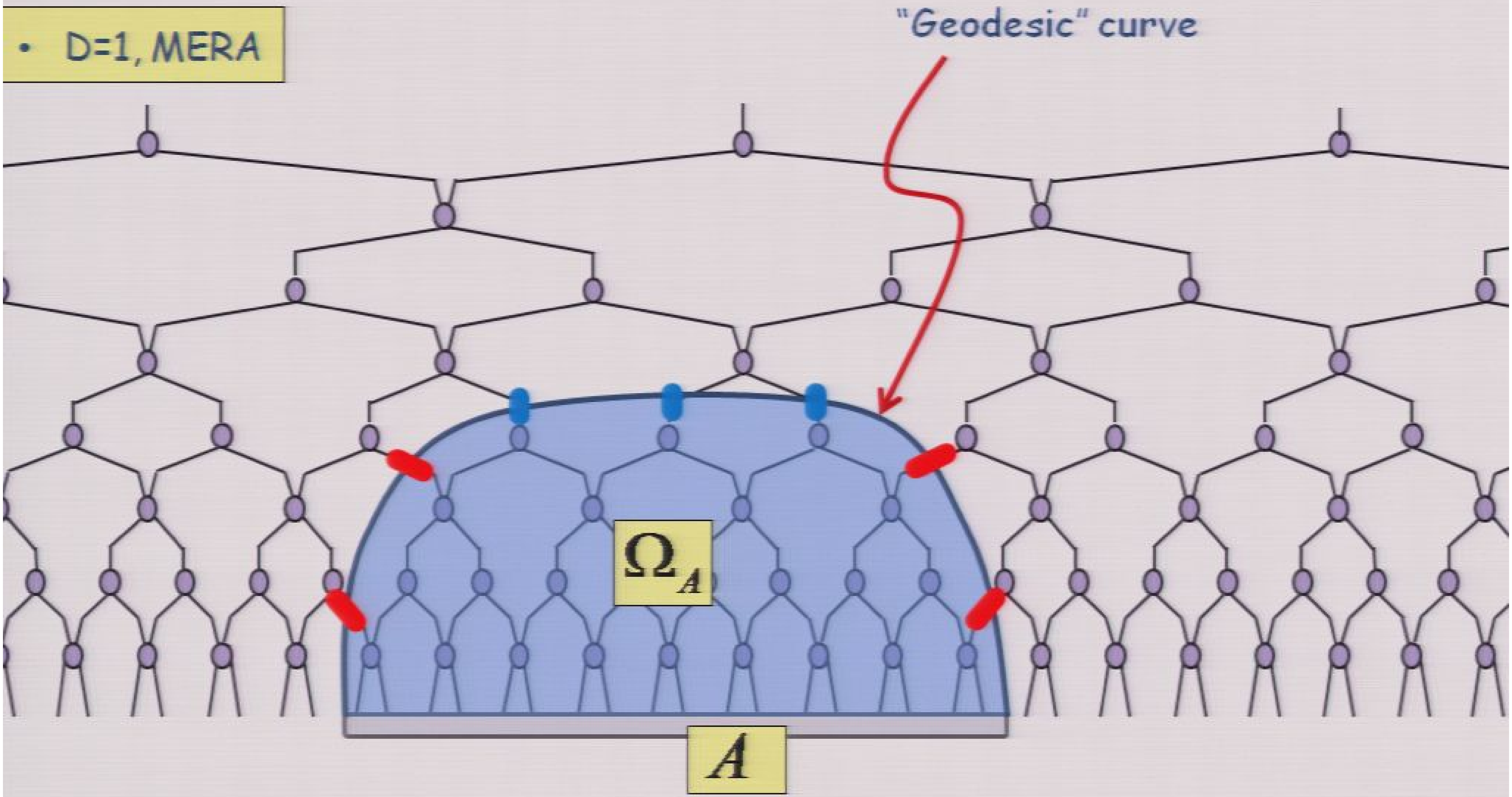

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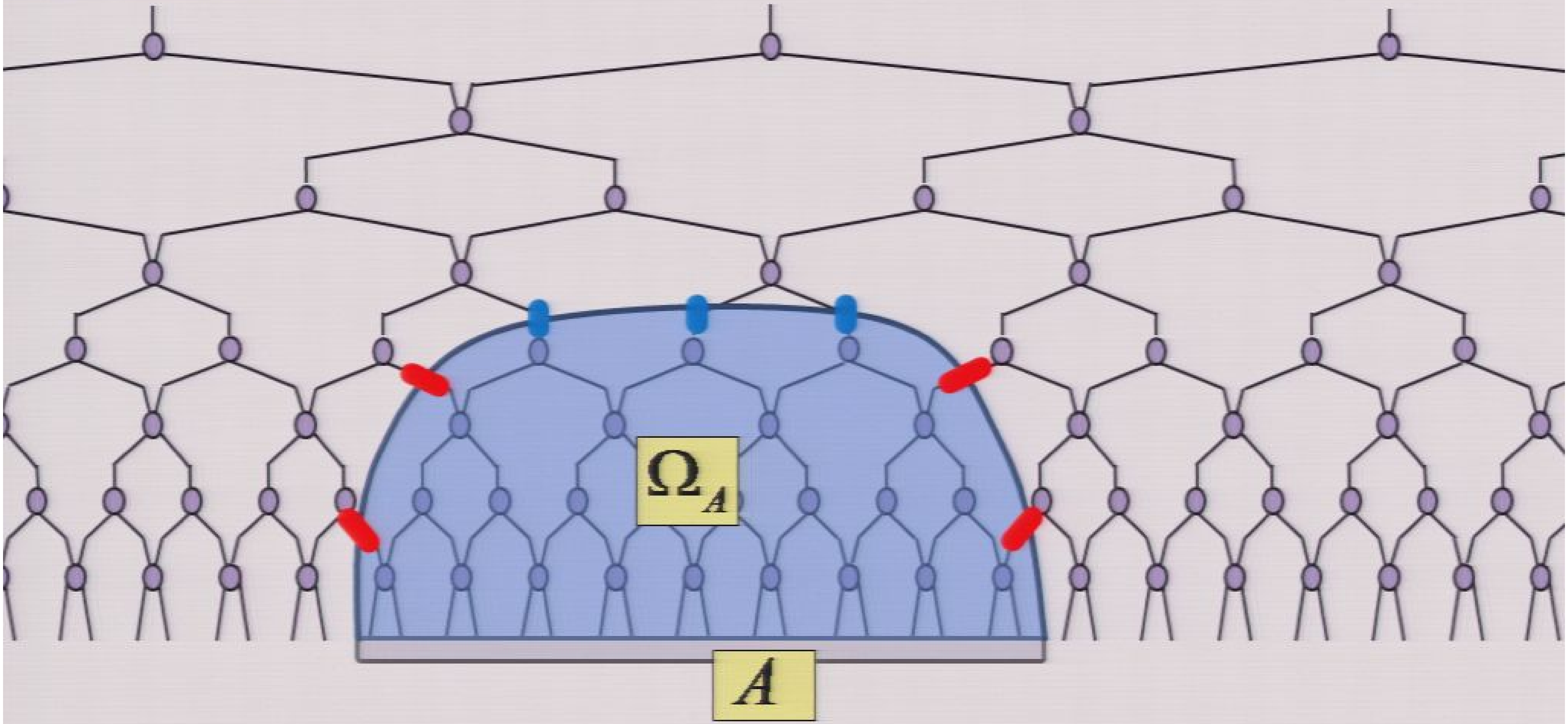

Entanglement entropy as boundary in holographic geometry: $S(A) \sim |\partial\Omega_A|$

Holographic Entanglement Entropy, e.g. Nishioka, Ryu, Takayanagi, J. Phys. A 2009

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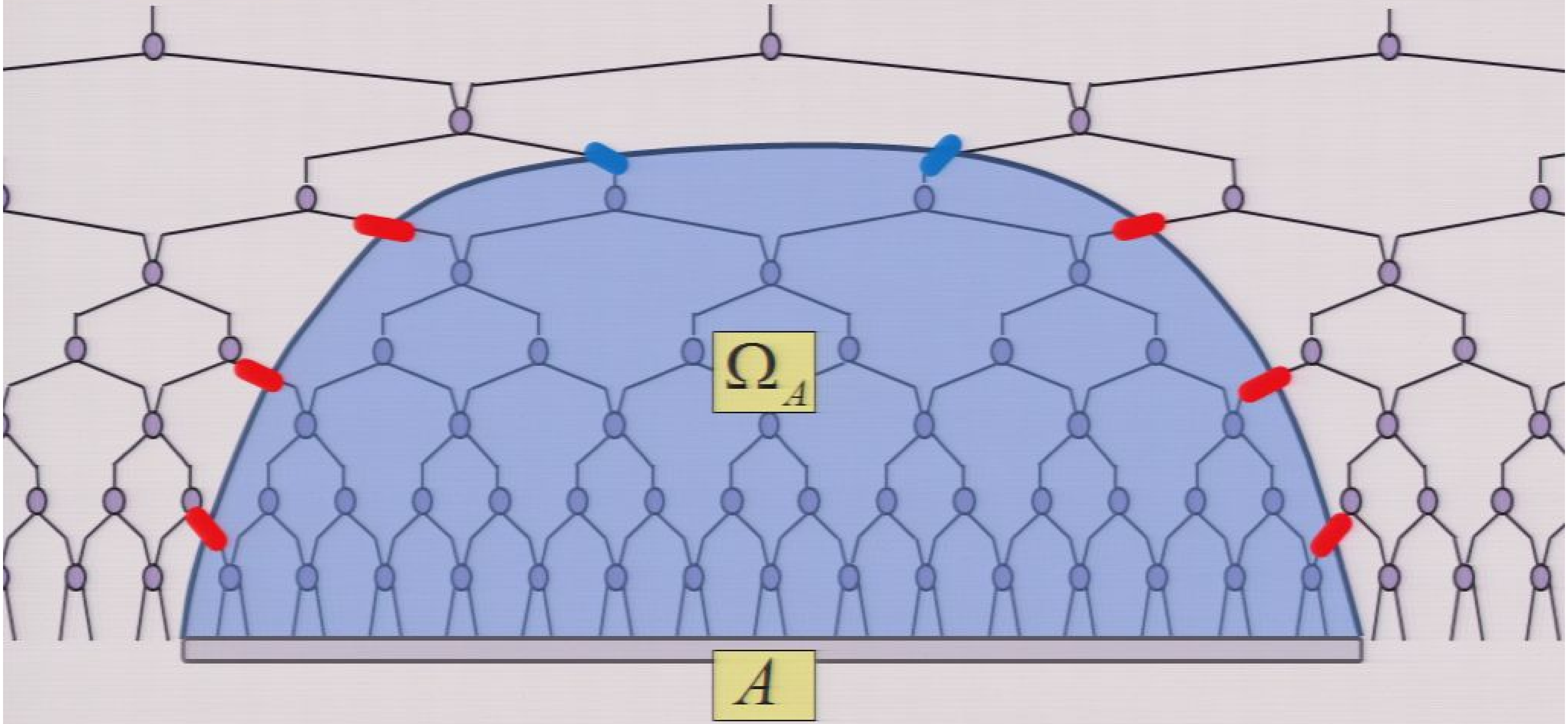

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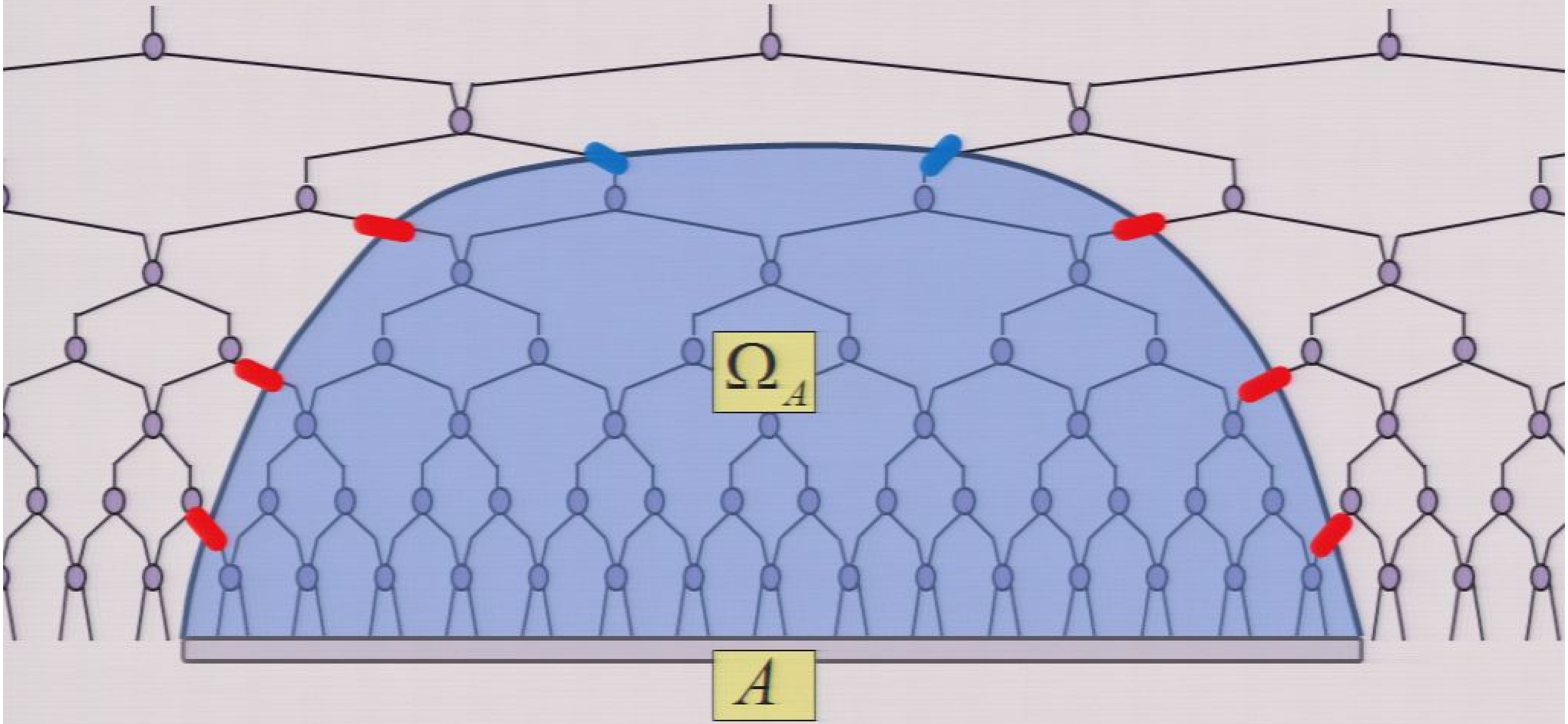

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MERA for D=1 spatial dimensions

Entanglement entropy as boundary in holographic geometry:

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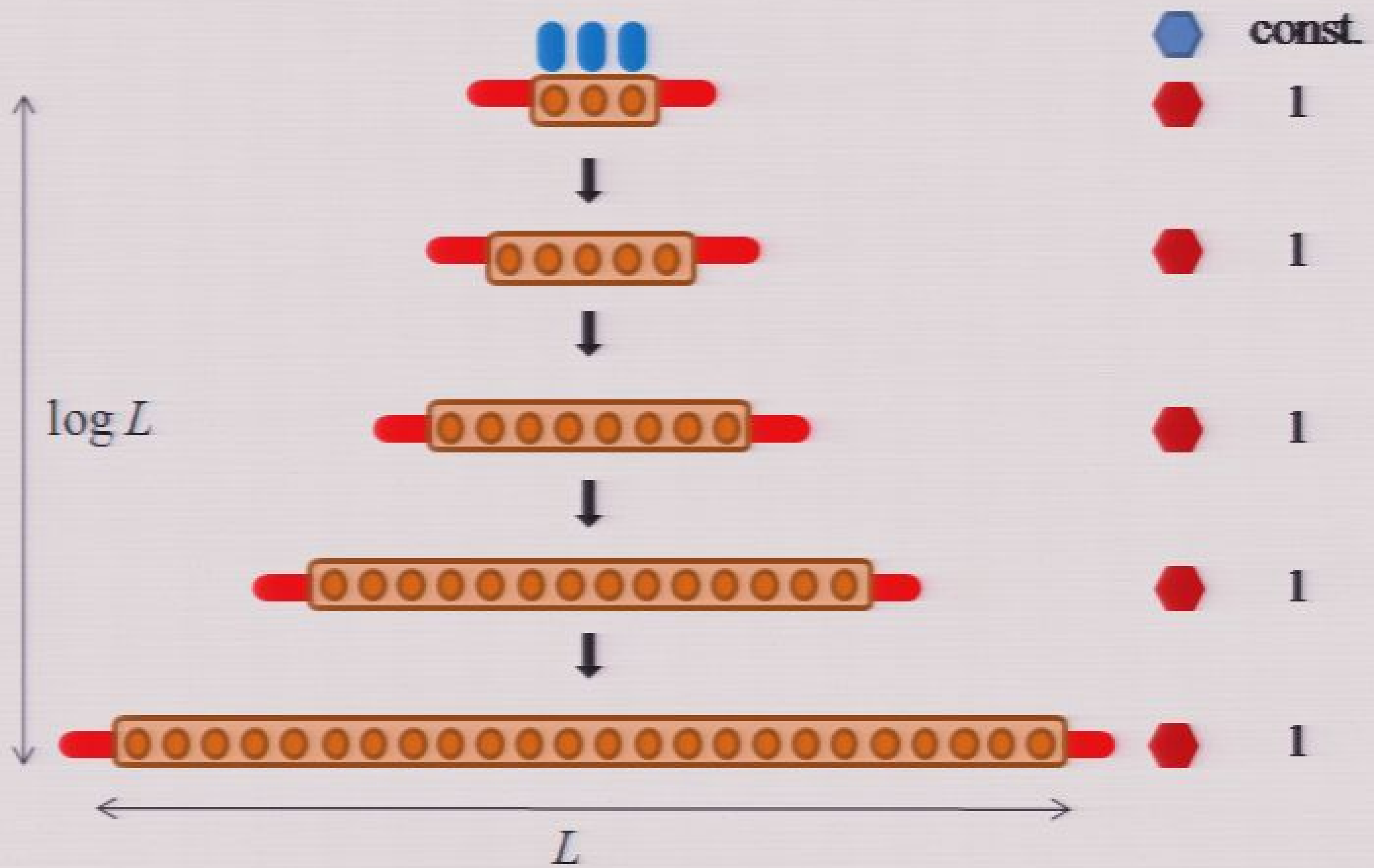
contributions to entropy



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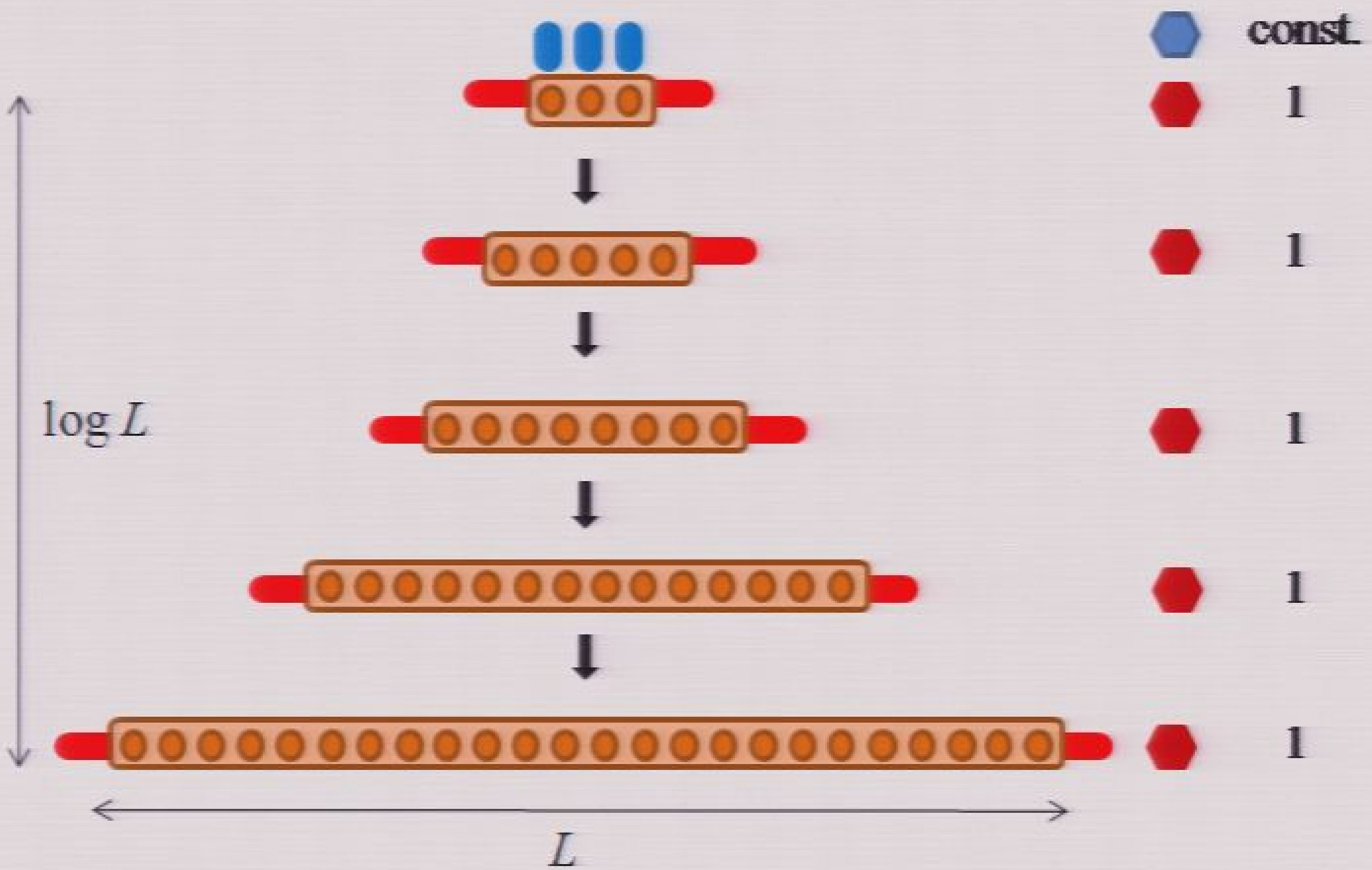
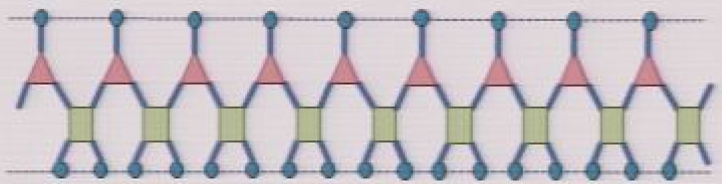


- Tensor network (MERA) based on holographic geometry can produce violations of boundary law, logarithmic violation: $S(L) \approx \log L$

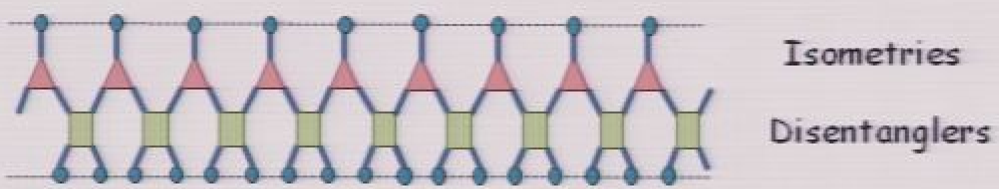
MERA for D=2 spatial dimensions

D=1 spatial dimensions

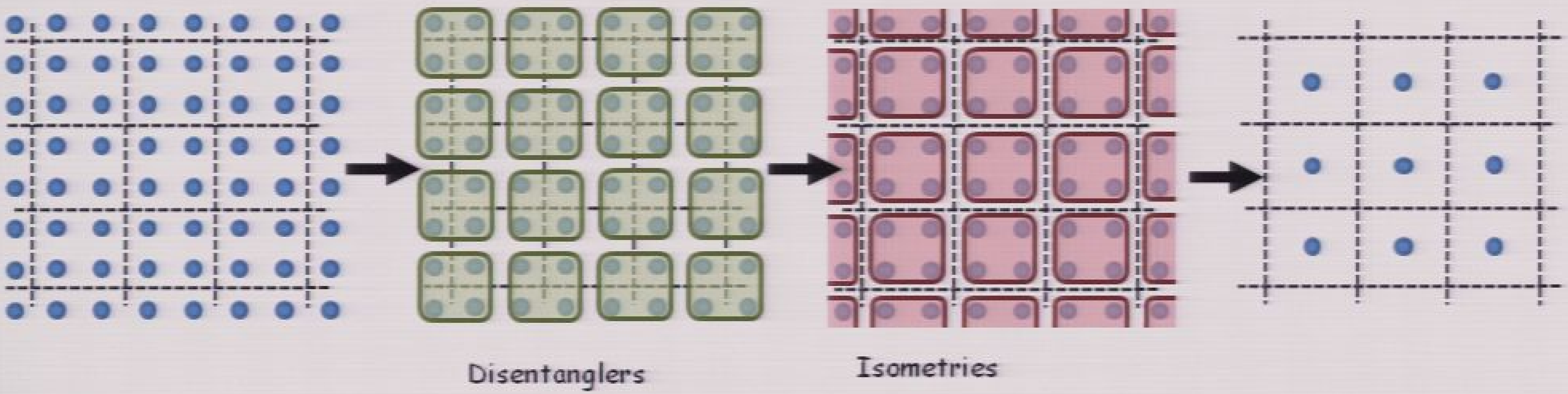


MERA for D=2 spatial dimensions

D=1 spatial dimensions



D=2 spatial dimensions



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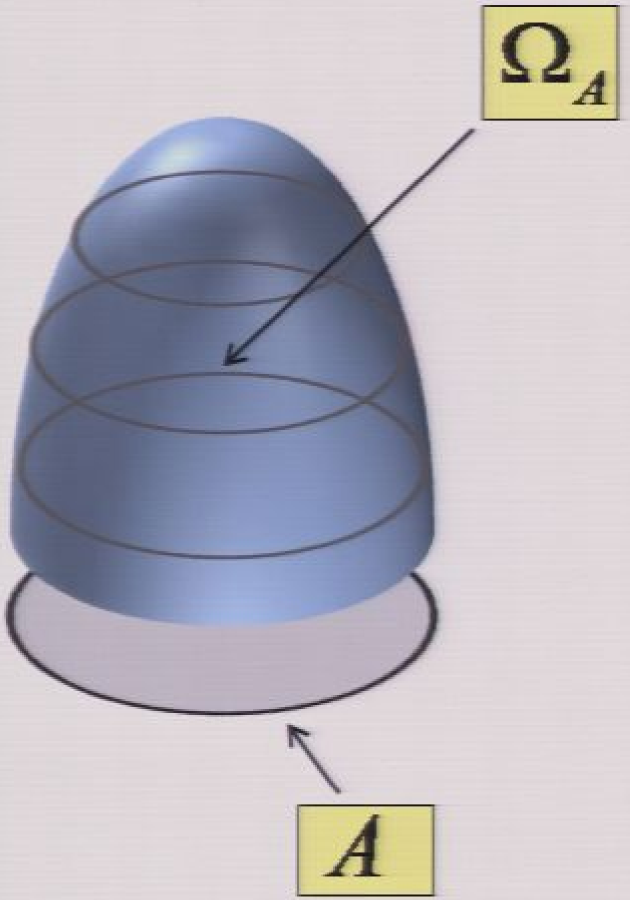

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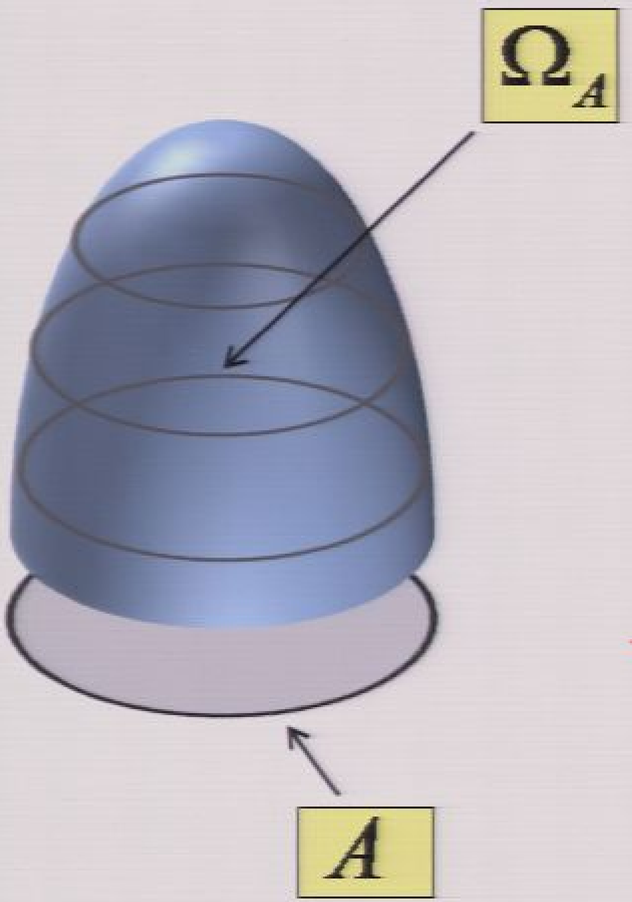

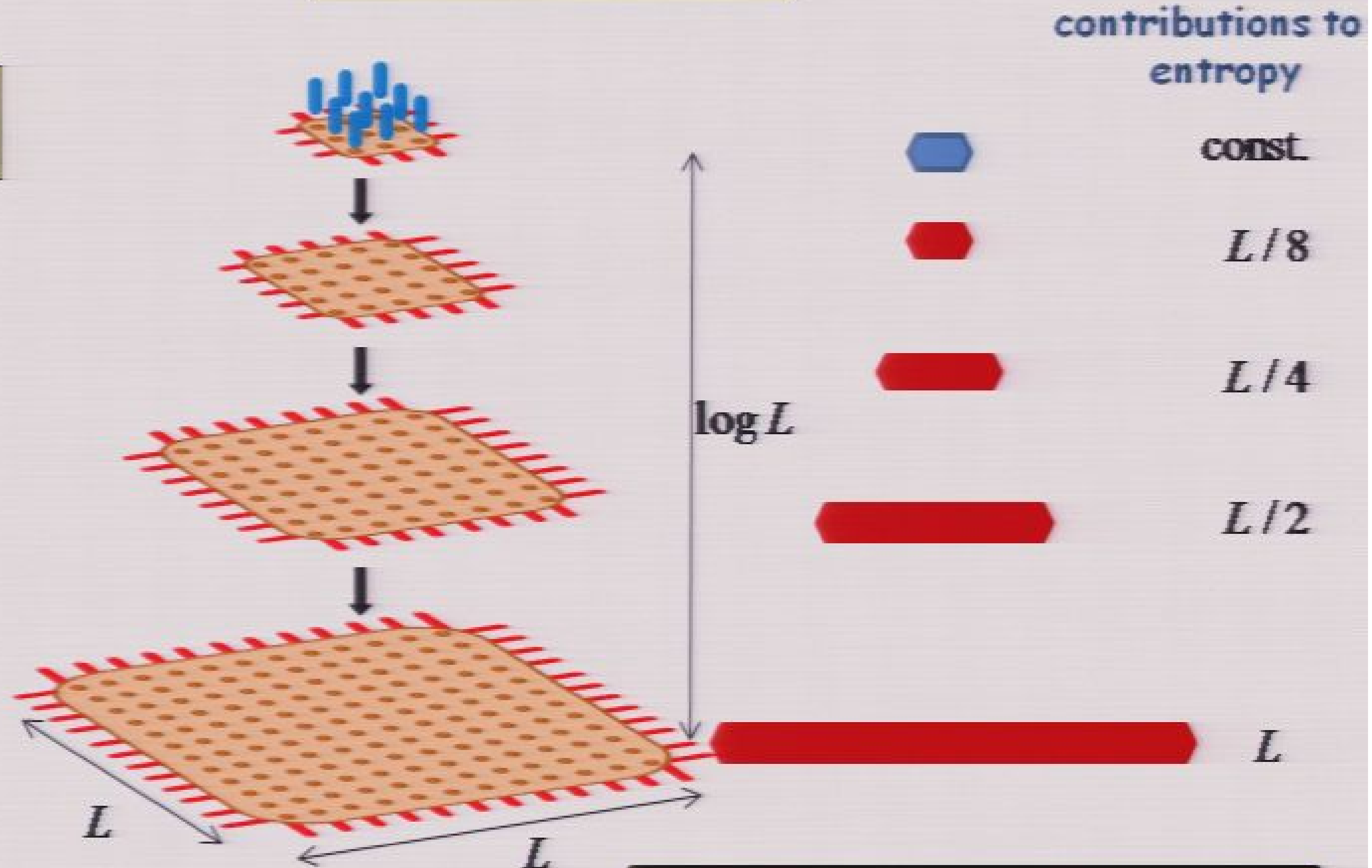

$$S_L \approx L(1+\frac{1}{2}+\frac{1}{4}+\cdots) \approx L$$
$\log L$

boundary law for entropy scaling!

Entanglement entropy in the MERA

Vidal, quant-ph/0610099

left out of PRL 101, 110501 (2008) !!!

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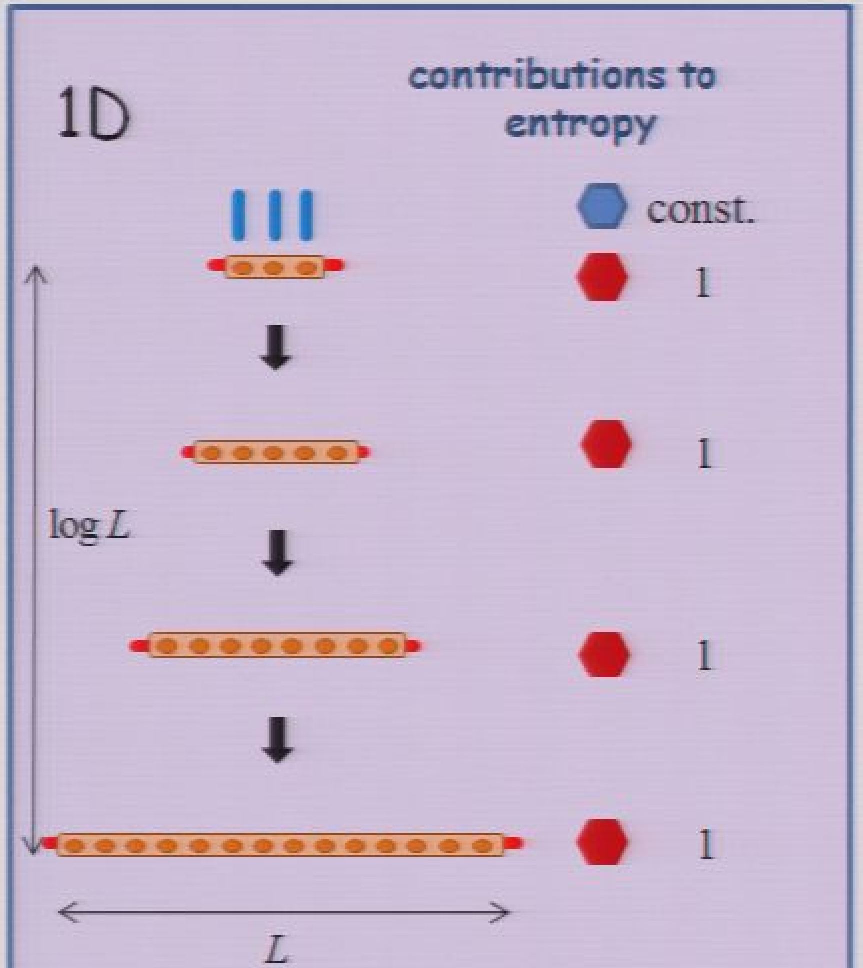
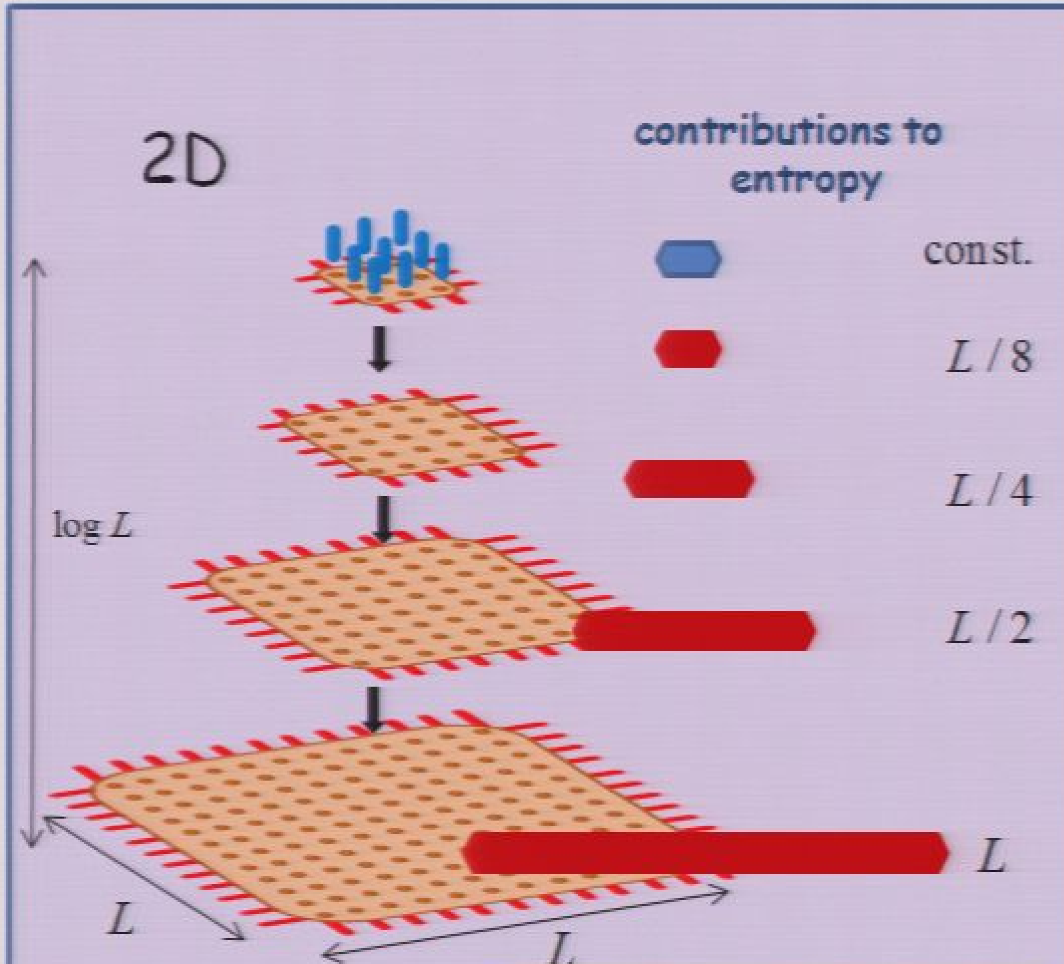


$$S_L \approx \underbrace{(1+1+\cdots+1)}_{\log L} \approx \log L$$



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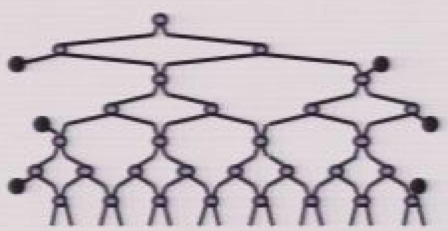

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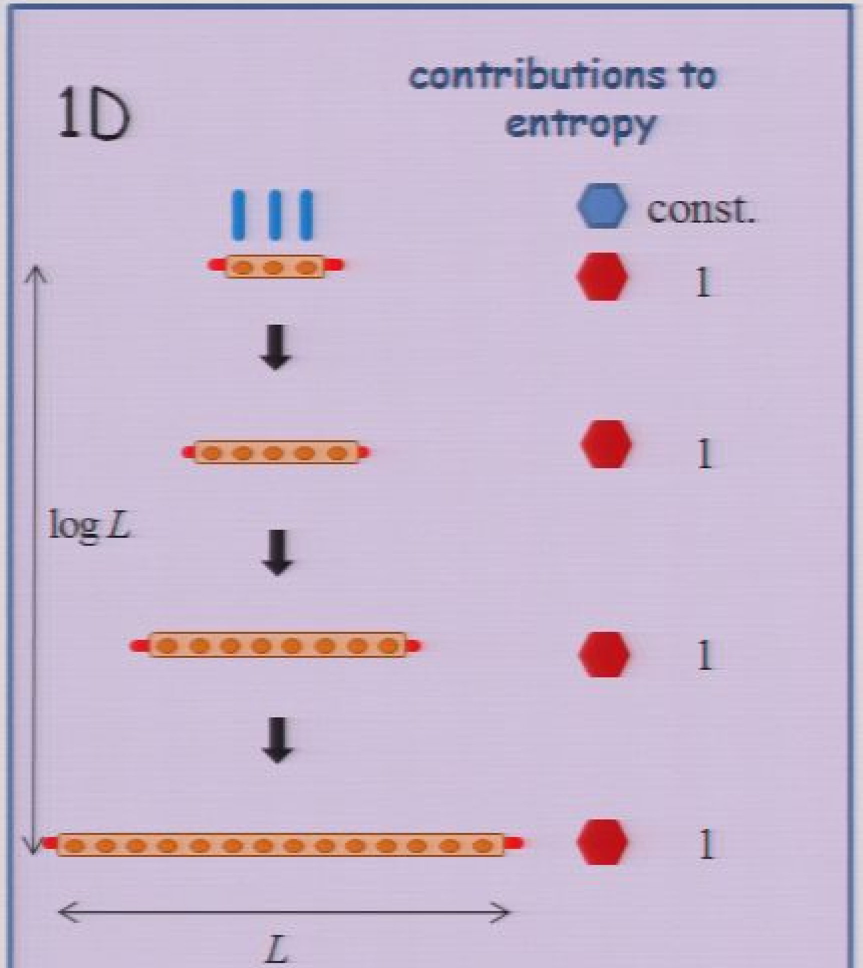

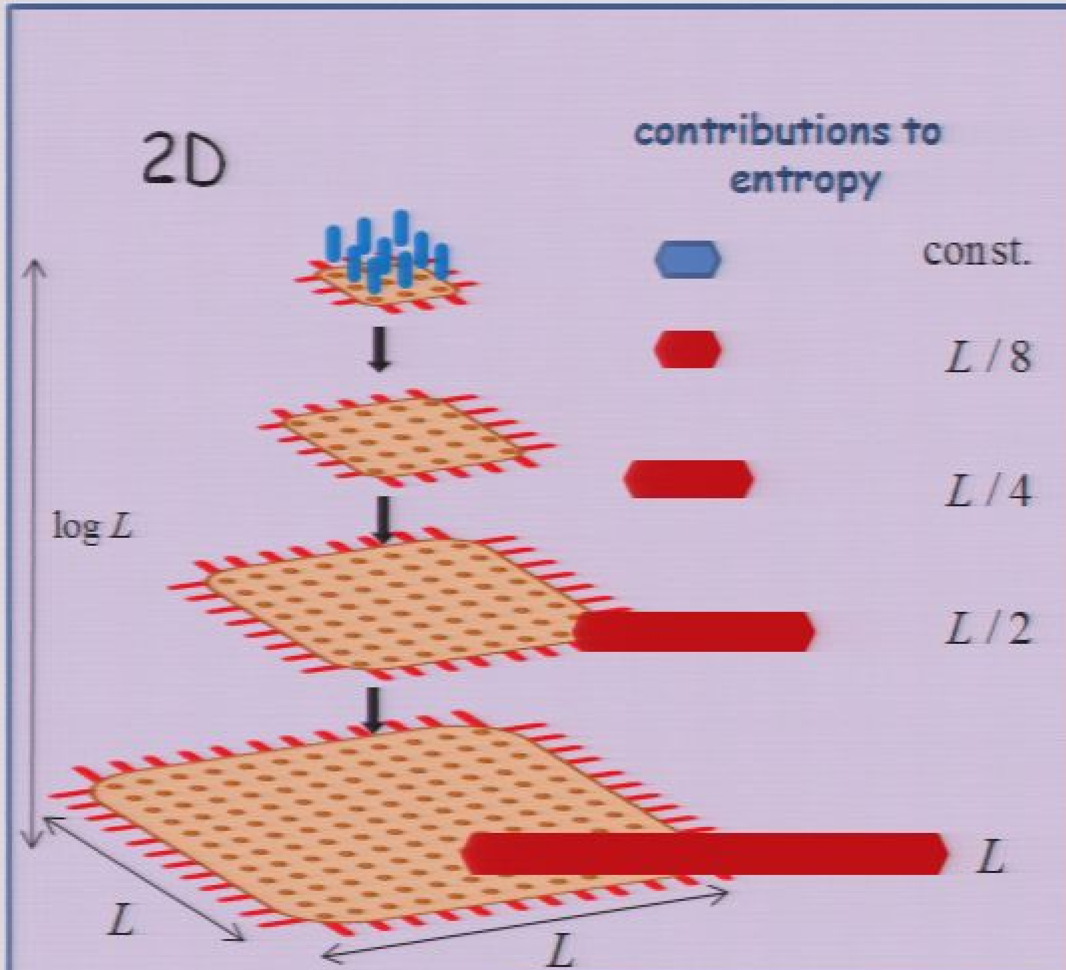

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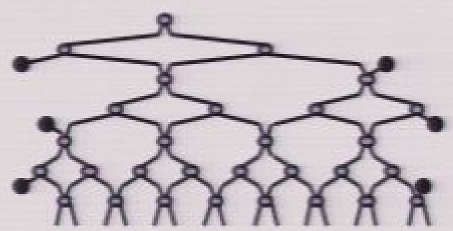

Entanglement entropy and Tensor Networks

- Can Tensor Network methods reproduce the proper entanglement entropy?

Tensor networks in physical geometry:

1D	MPS:	$S_L = \text{const.}$
2D	PEPS:	$S_L = L$

	Gap.	Crit.
1D S_L	const.	$\log(L)$

	Gap.	Crit.I	Crit.II
2D S_L	L	L	$L\log(L)$

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- Frustrated Magnets
- Interacting Fermions

All on large (or infinite) 2D lattices!

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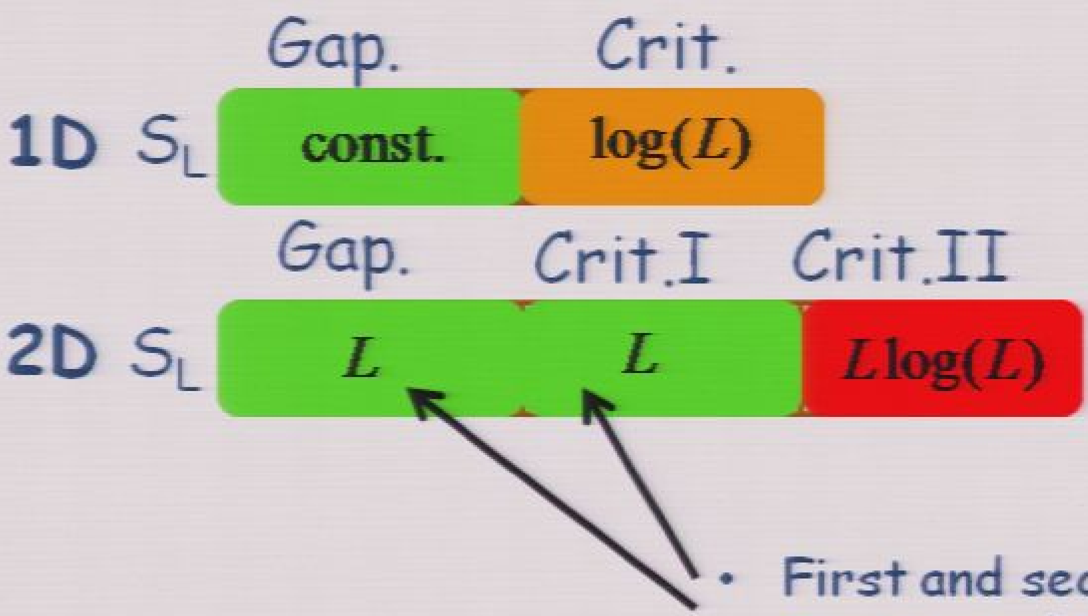

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- Natural description of scale-invariant 1D systems (arrow pointing to the $\log(L)$ entry)

	Gap.	Crit.I	Crit.II
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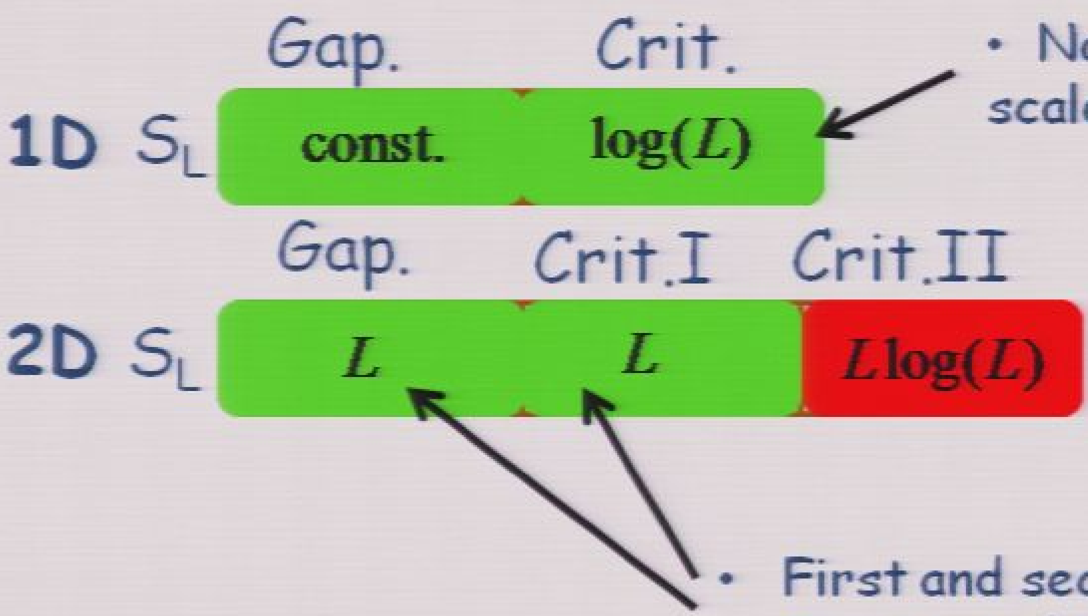

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Entanglement entropy and Tensor Networks

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		Gap.	Crit.I	Crit.II
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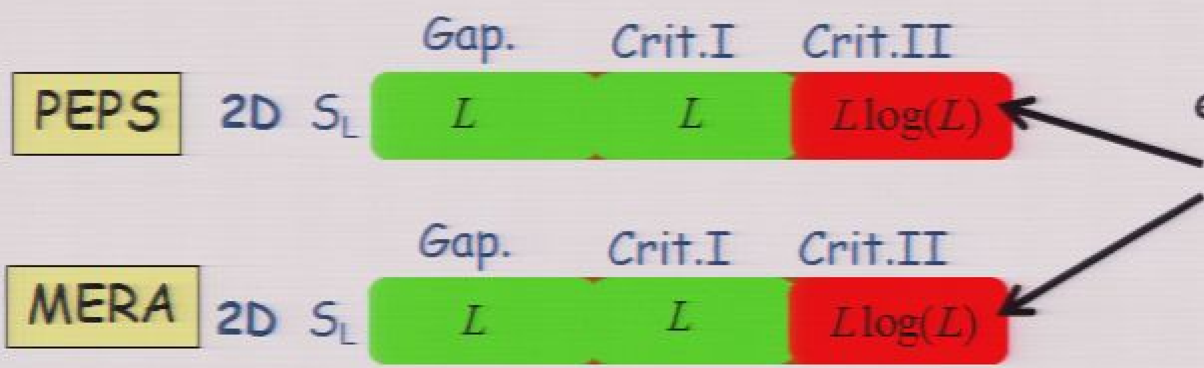
Entanglement entropy and Tensor Networks

- Certain types of critical 2D phases cannot be properly addressed (large N simulations) with current tensor-network techniques

Entanglement entropy and Tensor Networks



e.g.

- Fermi liquids
- Spin Bose Metals

- Certain types of critical 2D phases cannot be properly addressed (large N simulations) with current tensor-network techniques

- The entanglement structure of these systems is not properly understood

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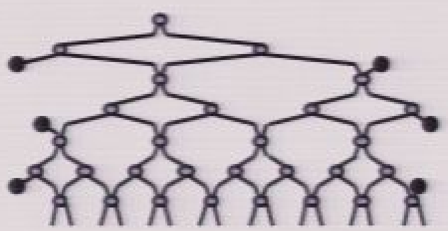

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By considering exotic holographic geometries we obtain a more general class of MERA that reproduces more entanglement entropy

→ **Branching MERA** Evenbly, Vidal, in preparation

Guifre Vidal

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Guifre Vidal

	Gap.	Crit.I	Crit.II
2D S_L	L	L	$L\log(L)$

Potentially a good ansatz for 2D systems with a 1D Fermi surface

Can reproduce other violations to boundary law for entropy scaling

- e.g. 1D quantum system with entropy: $S_L \propto (\log L)^2$

Provides a framework for understanding entanglement structure

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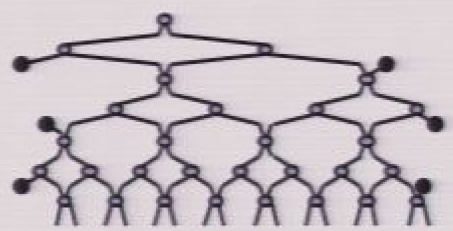


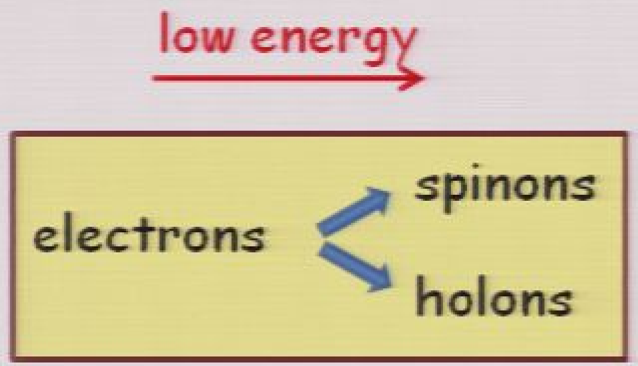
Low energy decoupling and holographic branching

MOTIVATION:

at low energies, sometimes sets of degrees of freedom decouple

Examples:

- 1D system: spin-charge separation



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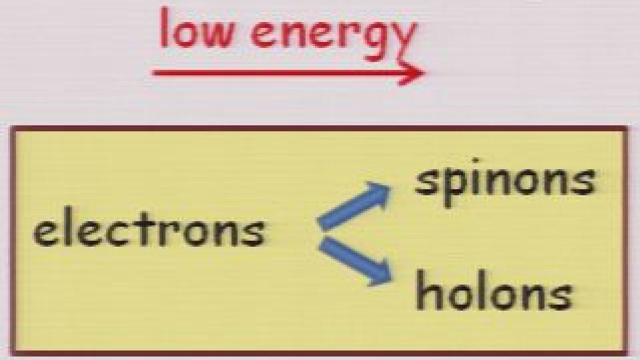


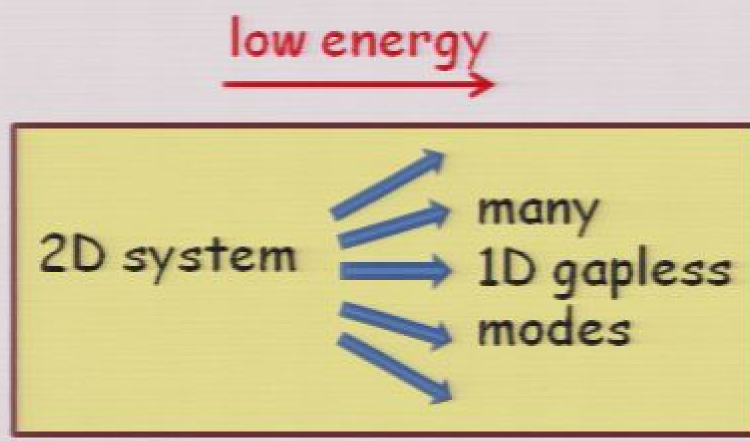
- 2D systems with 1D Fermi surface (or 1D Bose surface)

  - free fermions
  - Fermi liquids

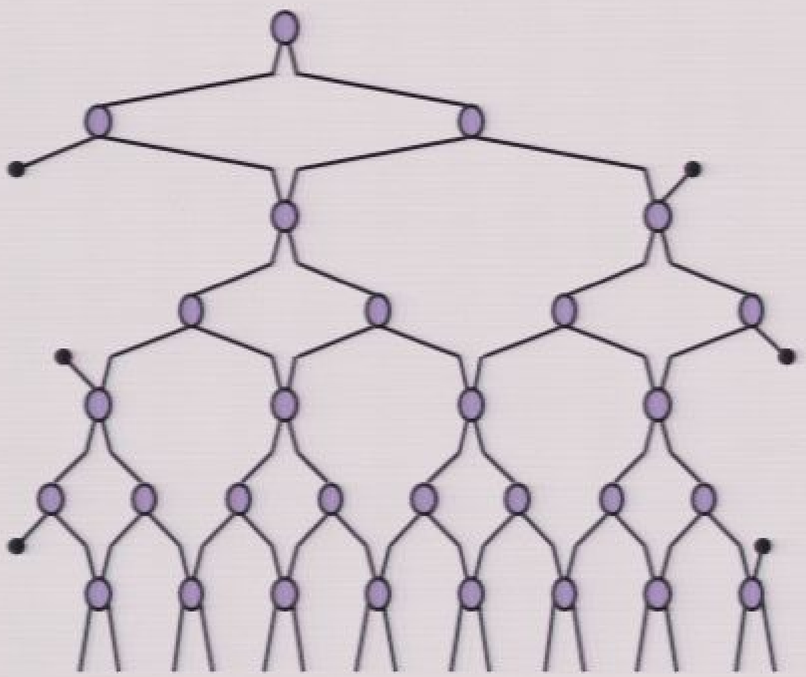
    e.g. Swingle, arXiv:0908.1724 and arXiv:1002.4635.
    Senthil, Phys. Rev. B 2008.
    Faulkner, Liu, McGreevy, Vegh, arXiv:0907.2694.

  - spin Bose metal

    e.g. Block, Sheng, Motrunich, Fisher, arXiv:1009.1179.
    Sheng, Motrunich, Fisher, arXiv:0902.4210.



- simplified diagrammatic representation for the MERA



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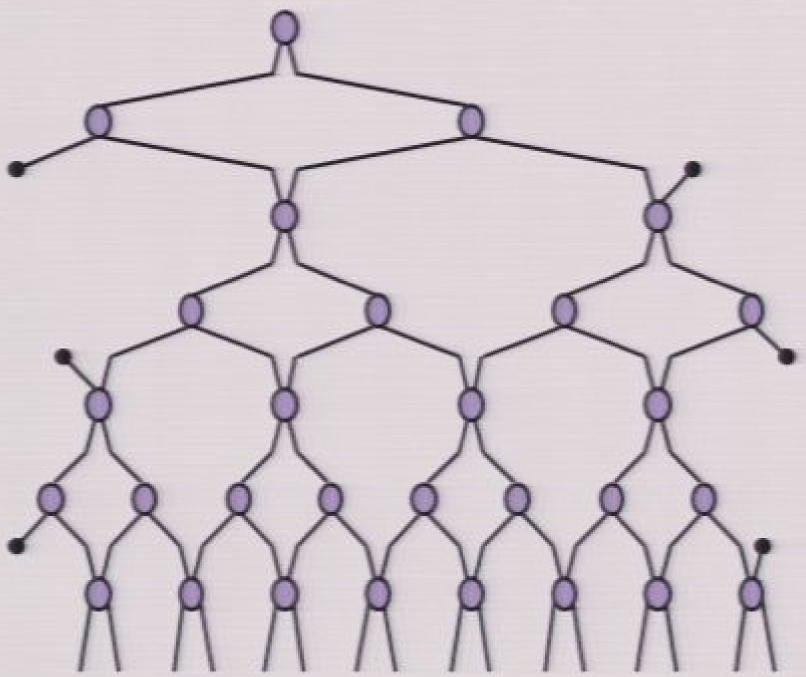

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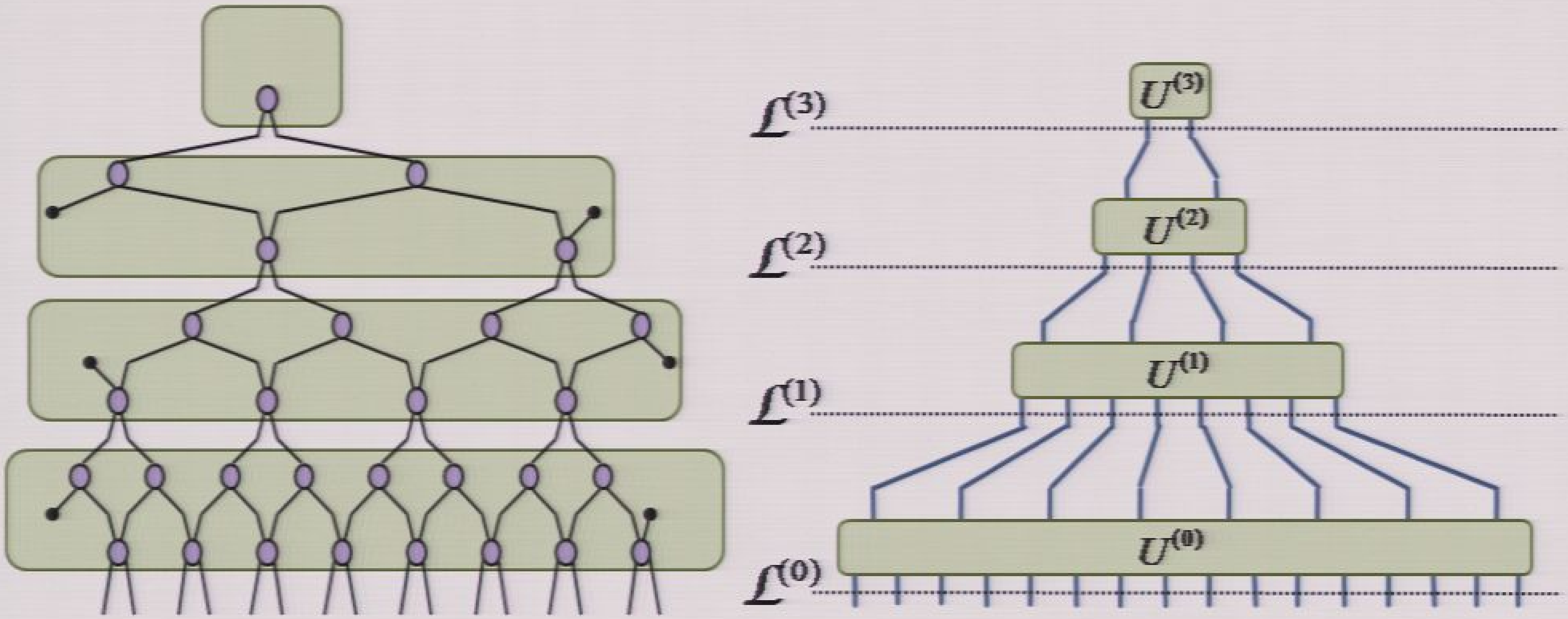

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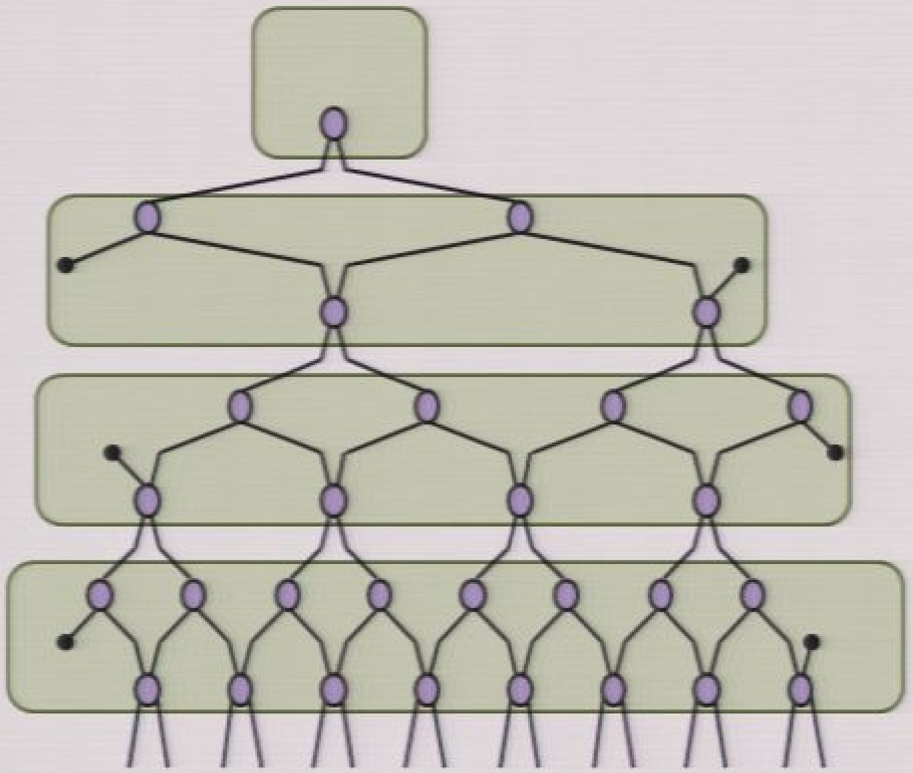

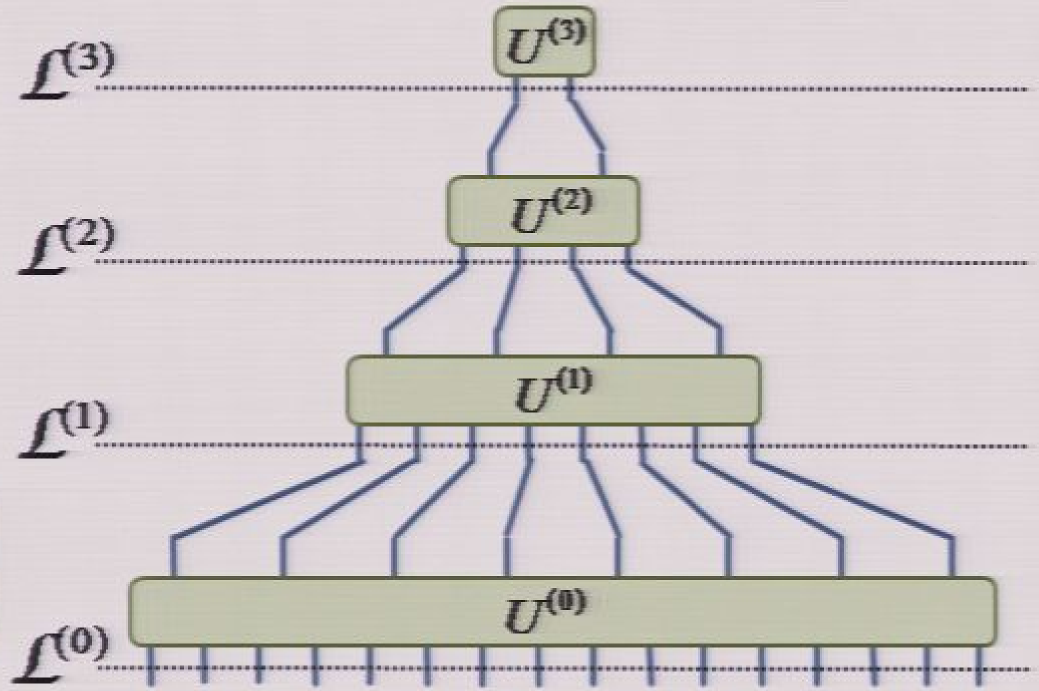

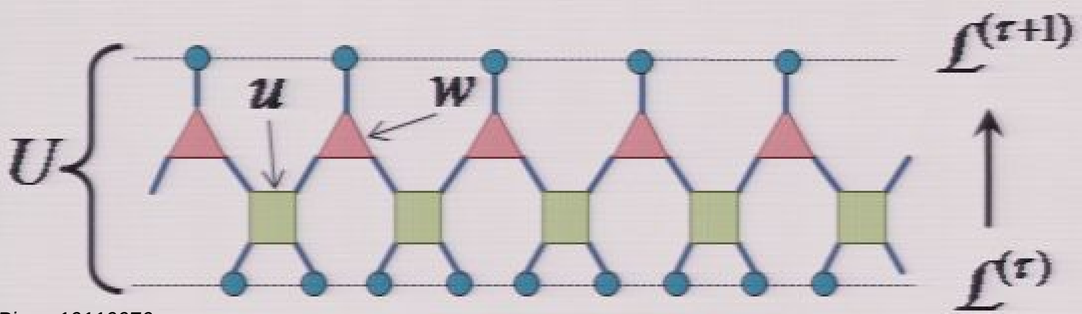

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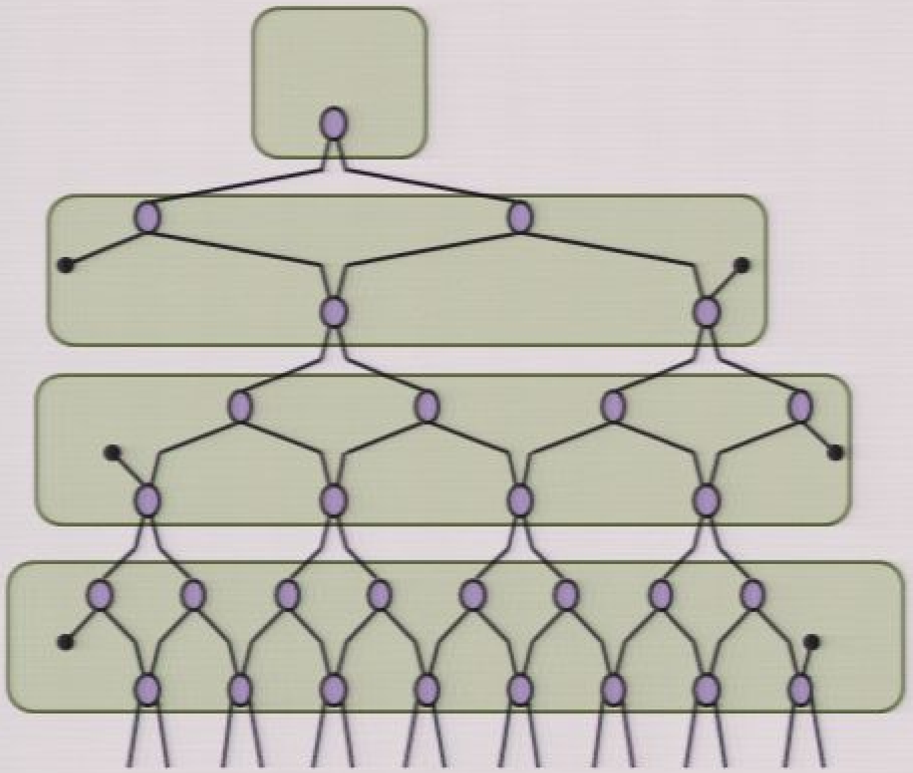
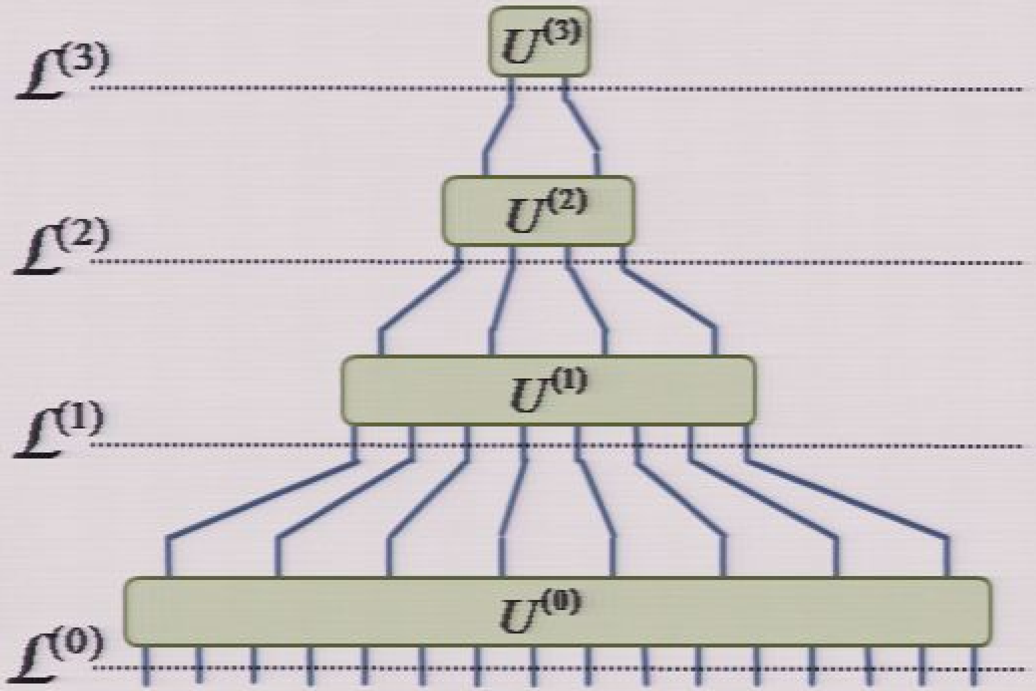
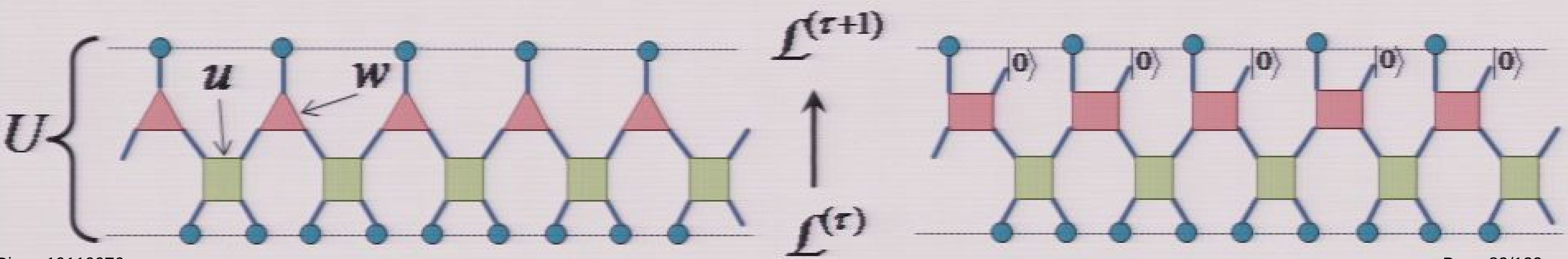
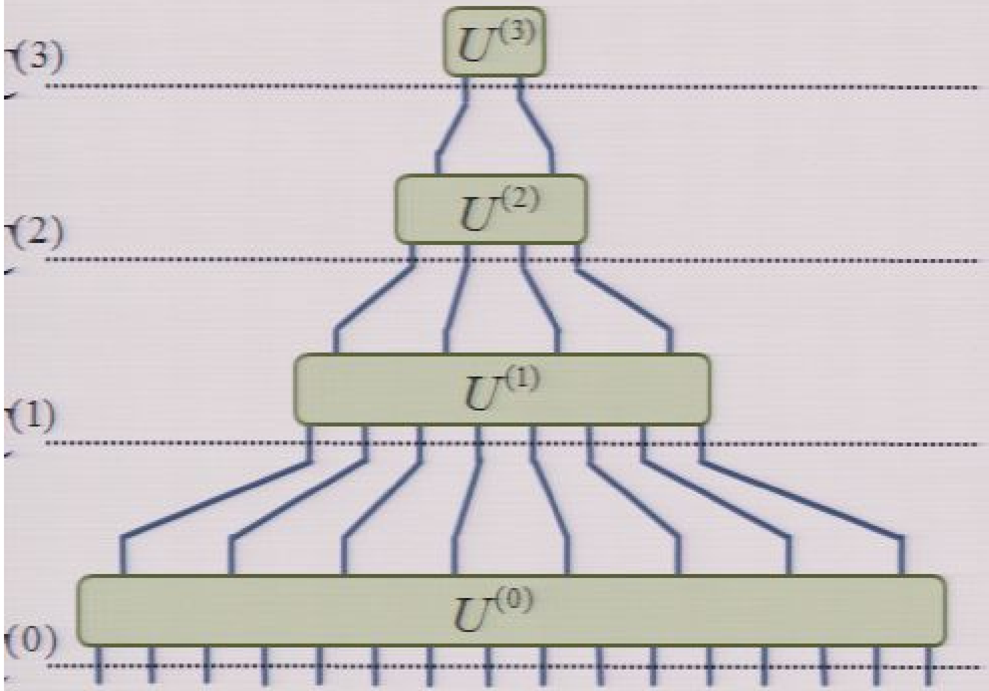
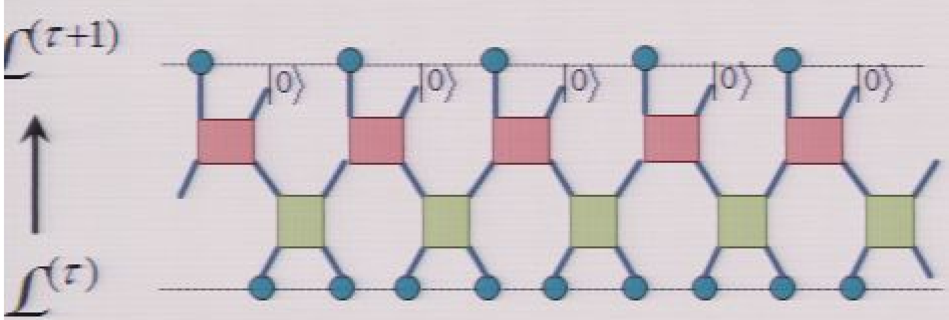






- Entanglement renormalization in the presence of decoupling





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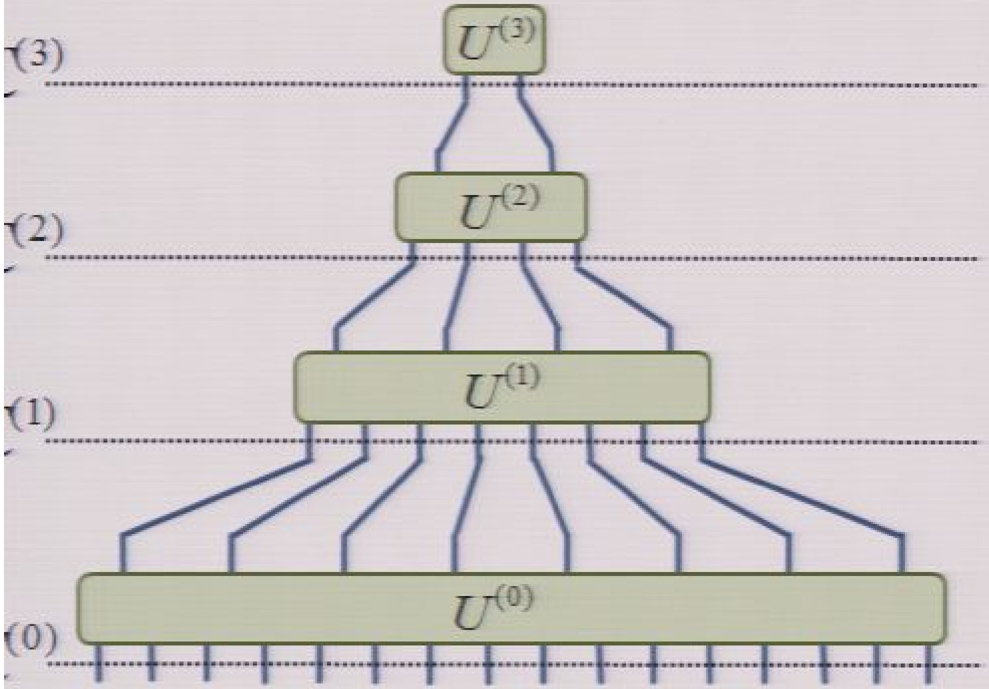
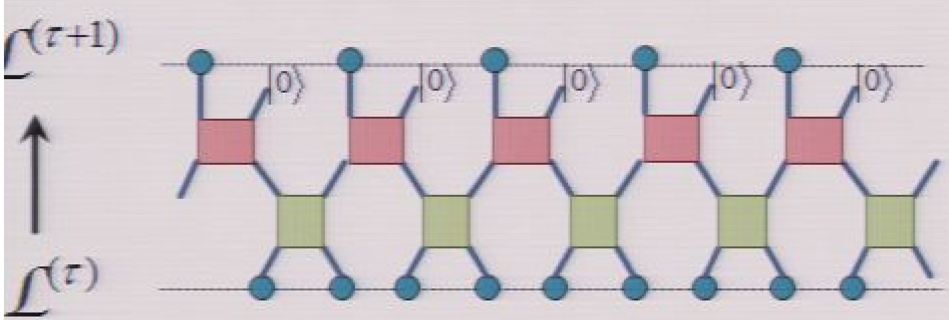
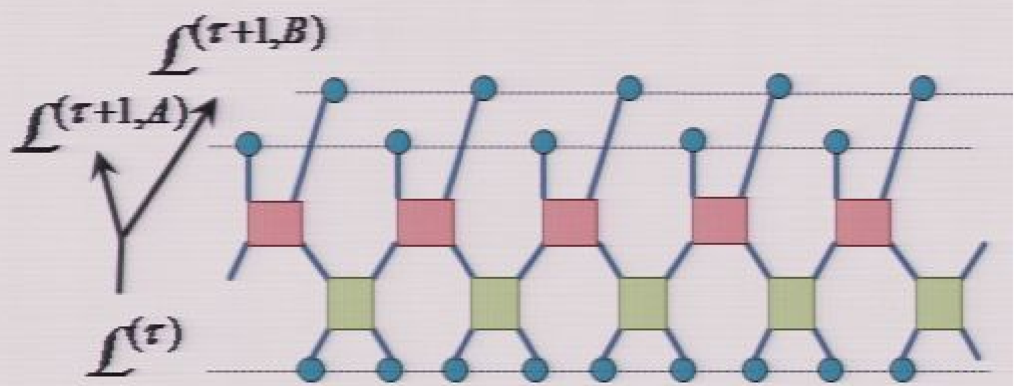
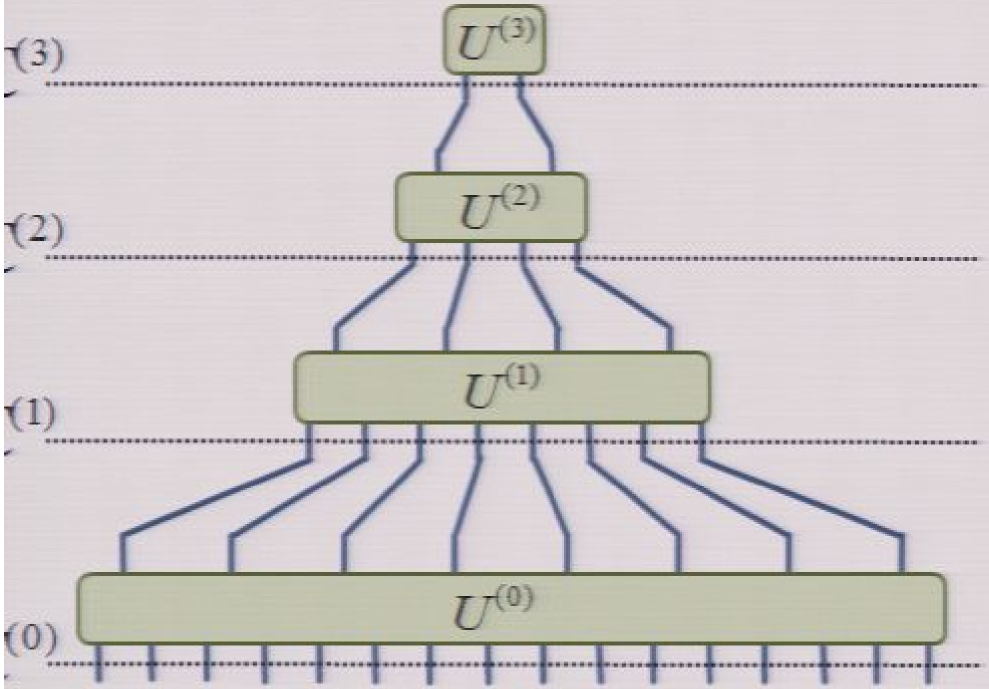


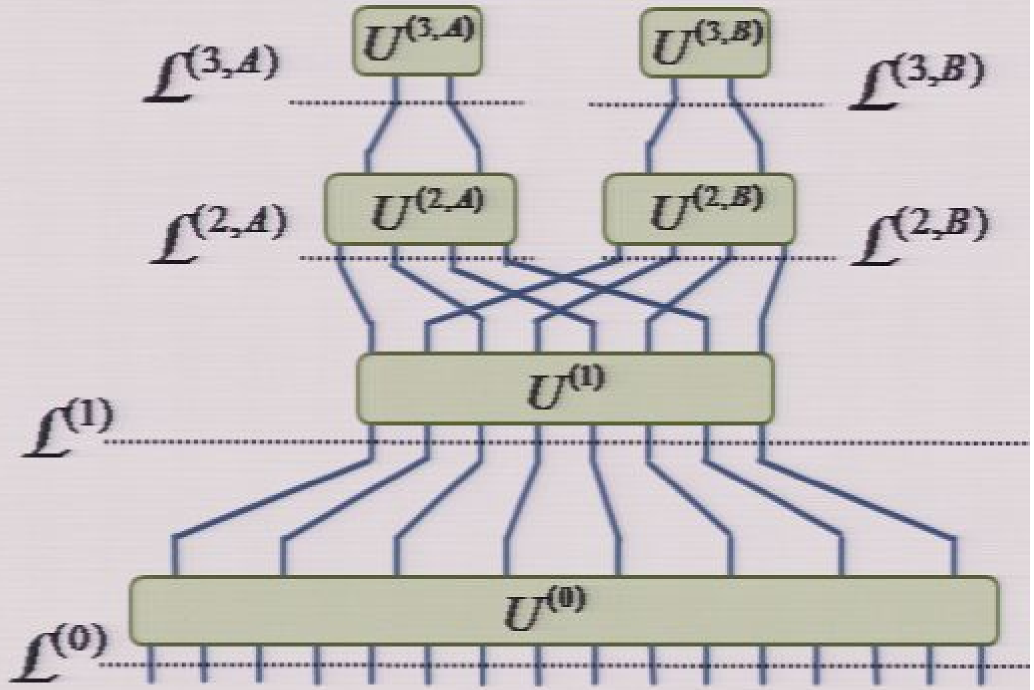


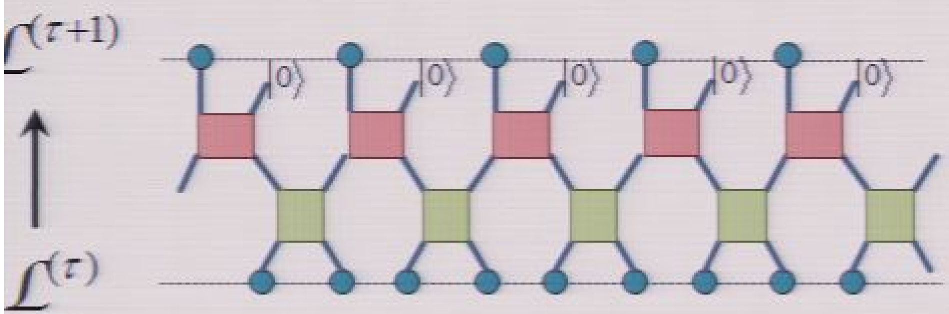


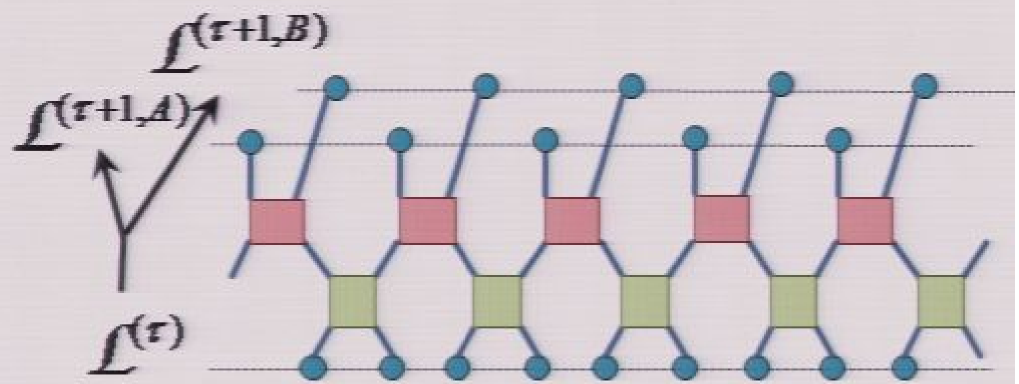
- Entanglement renormalization in the presence of decoupling

branching MERA





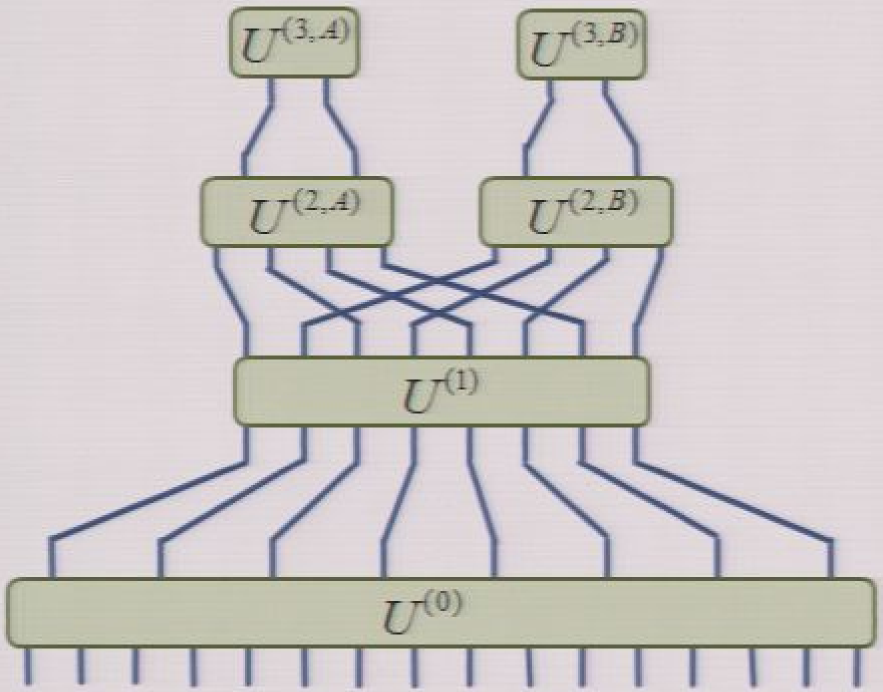


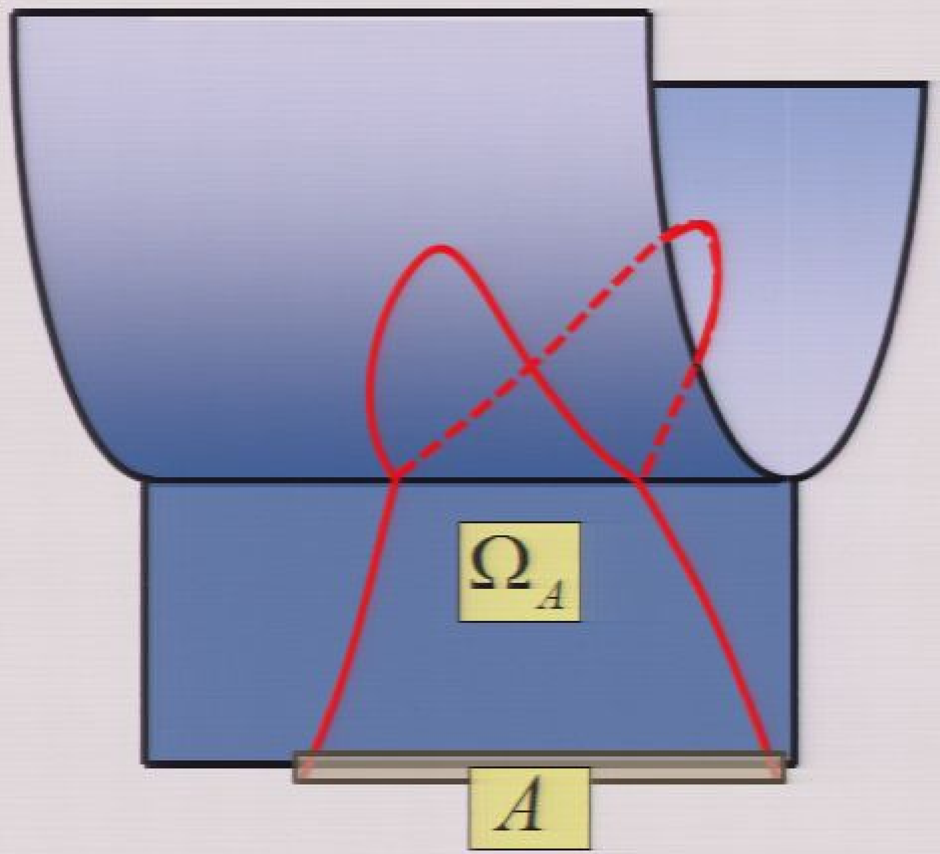


Entanglement entropy?

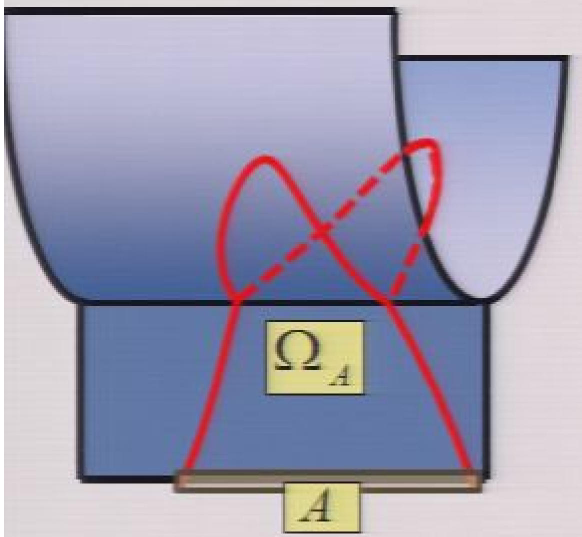
- given by boundary in exotic holographic geometry: $S(A) \sim |\partial\Omega_A|$

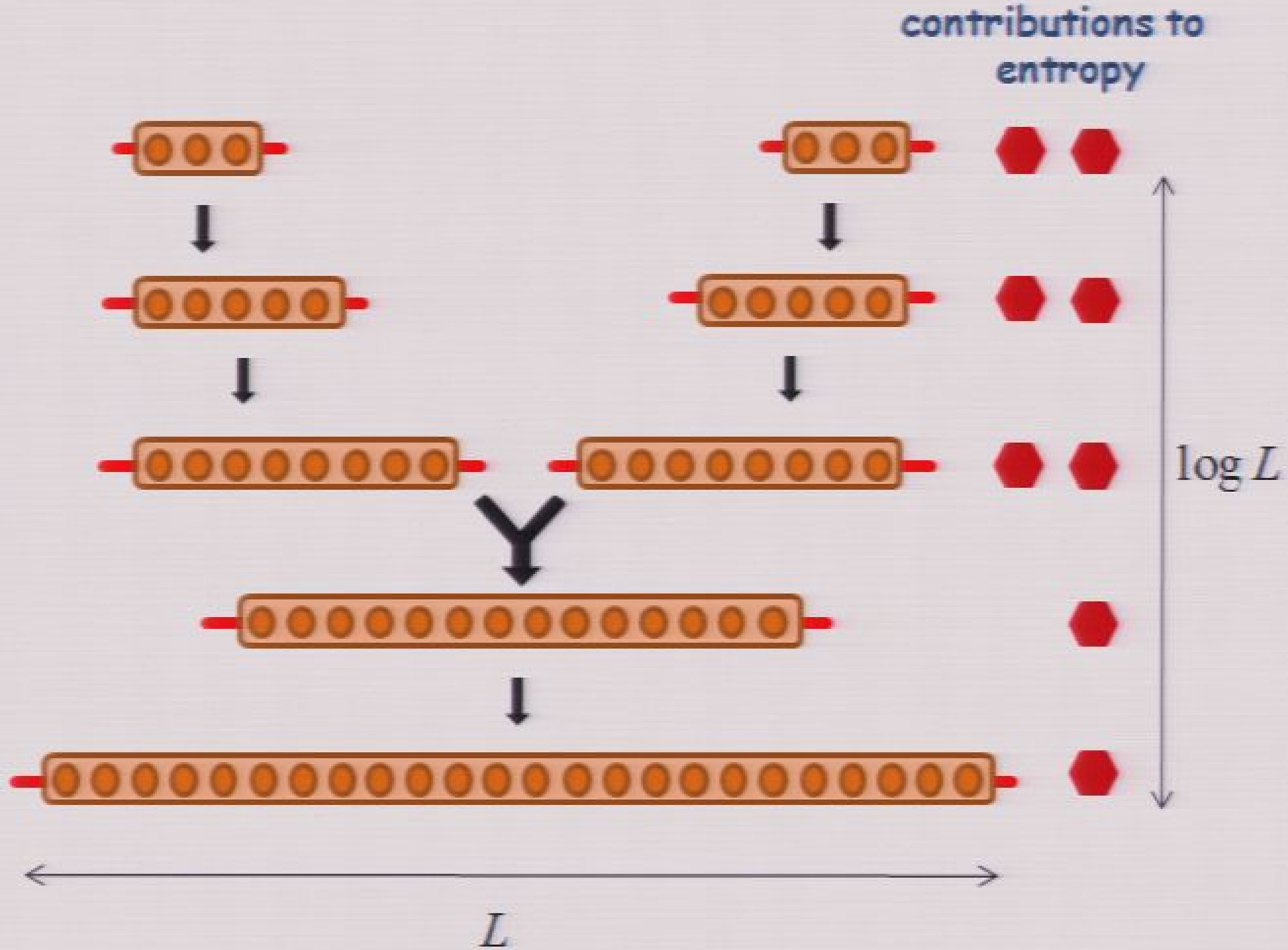
branching MERA





Entanglement entropy?





- Holographic branching can increase size of Ω_A

branching MERA can reproduce more entanglement entropy!

Outline

- Entanglement and tensor network methods
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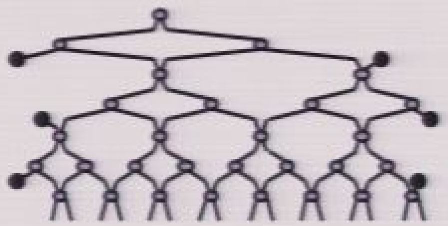
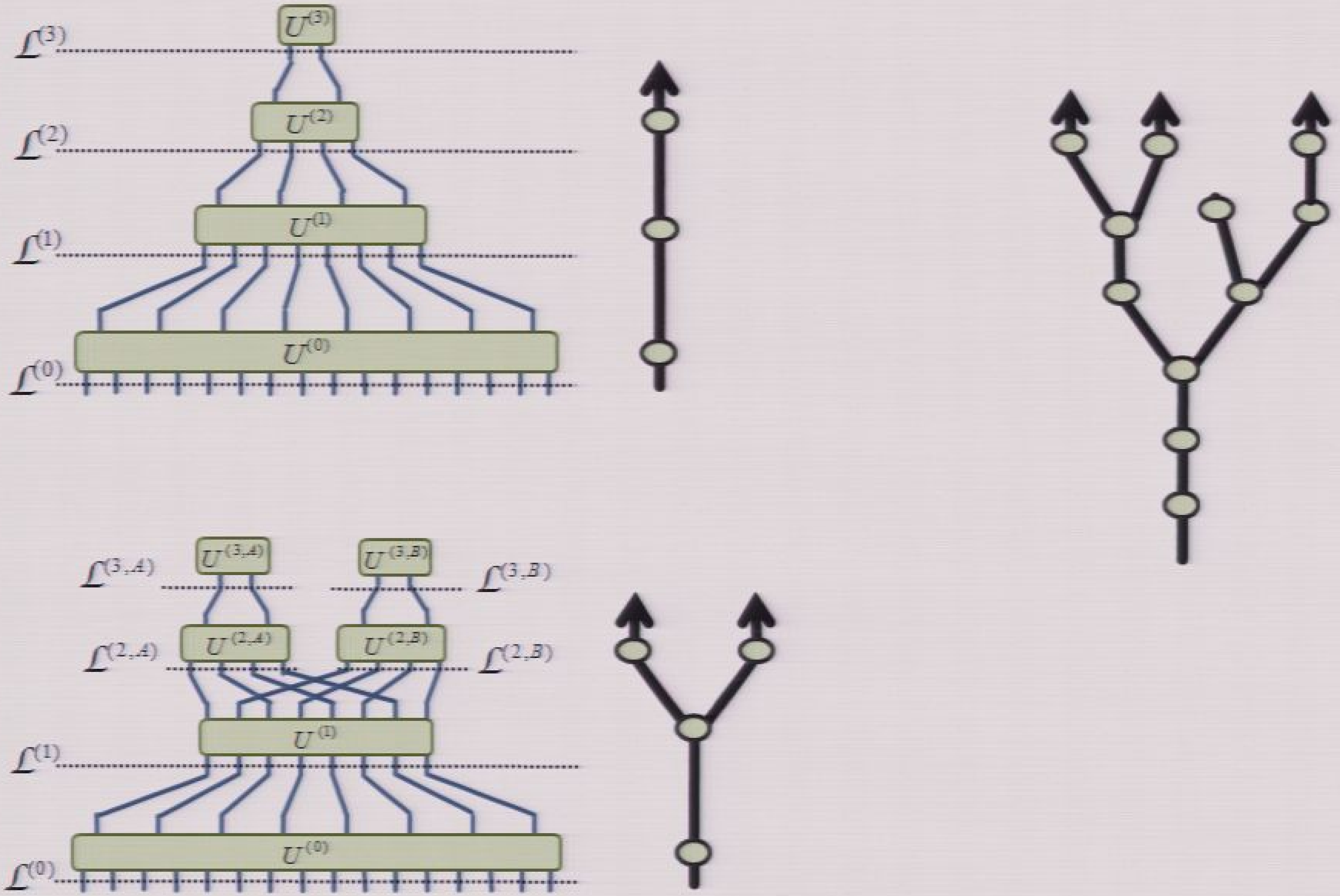
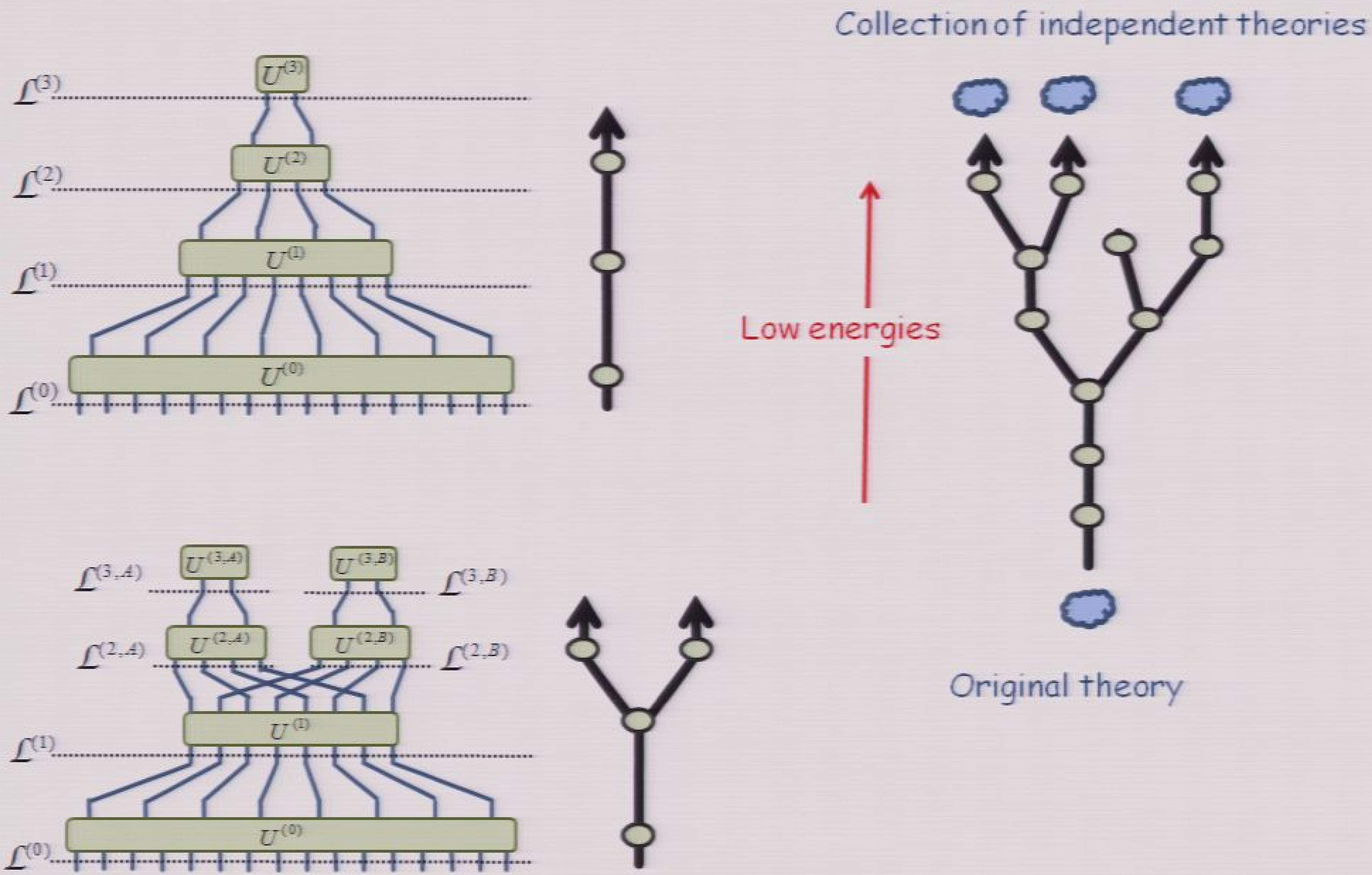
- Introduction of branching MERA
  - Decoupling a many-body theory
  - Holographic trees
  - Scaling of entropy in the branching MERA
  - Example: $S_L = L \log L$ entropy scaling in 2D fermions
  - Example: $S_L = (\log L)^2$ entropy scaling in 1D fermions



Holographic tree

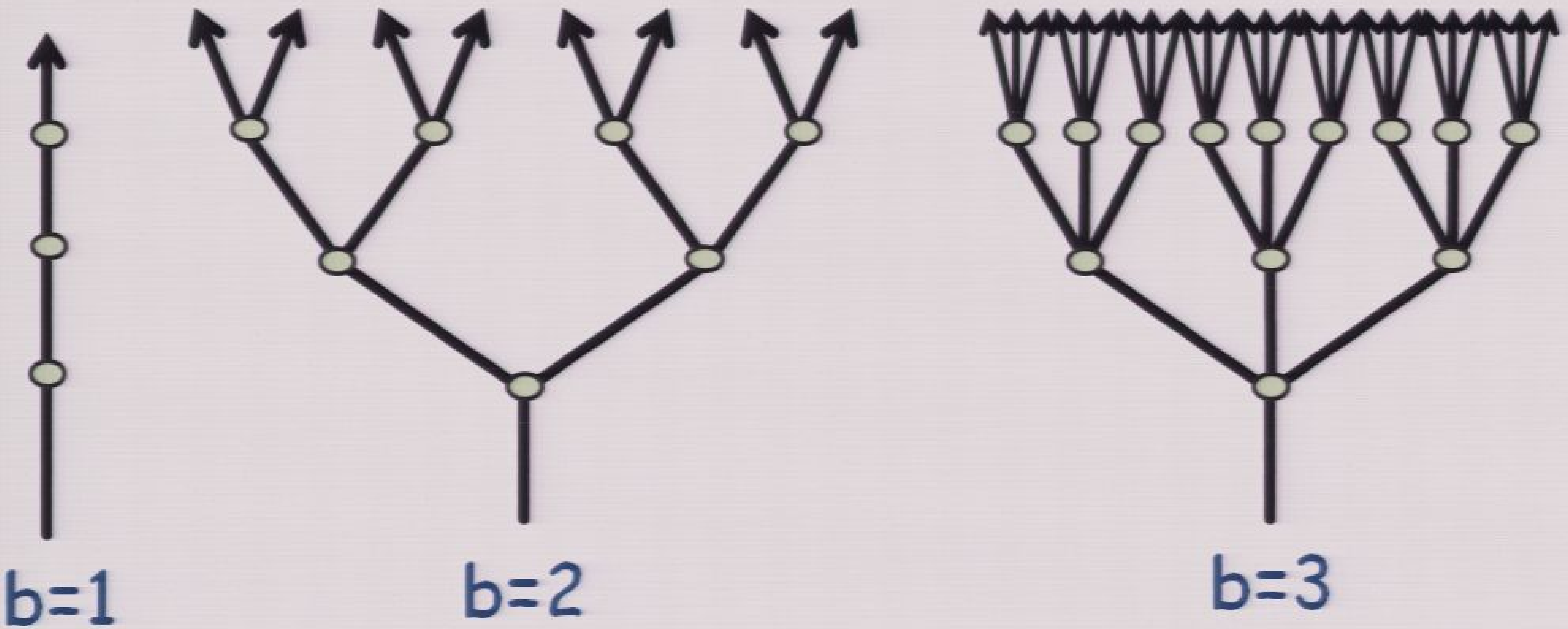


Holographic tree



Holographic Trees

- regular b-ary holographic trees:



Branching Parameter, b

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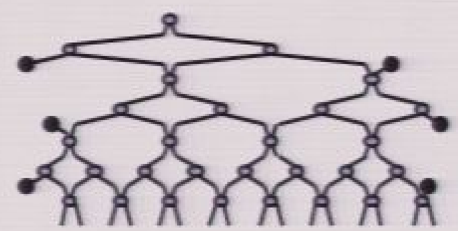
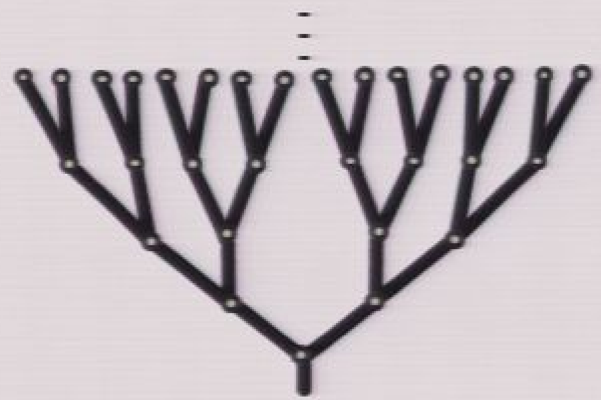
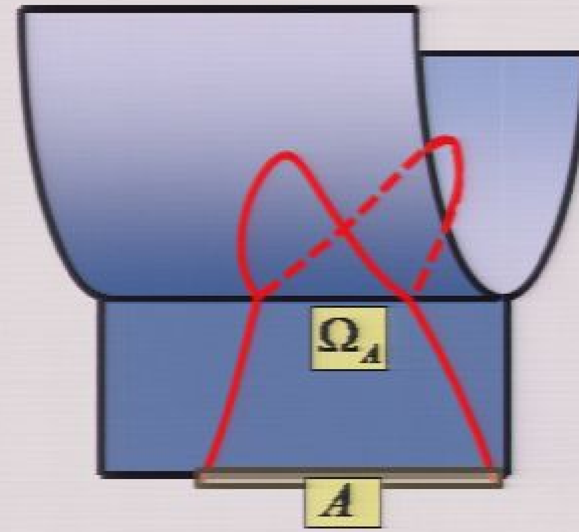
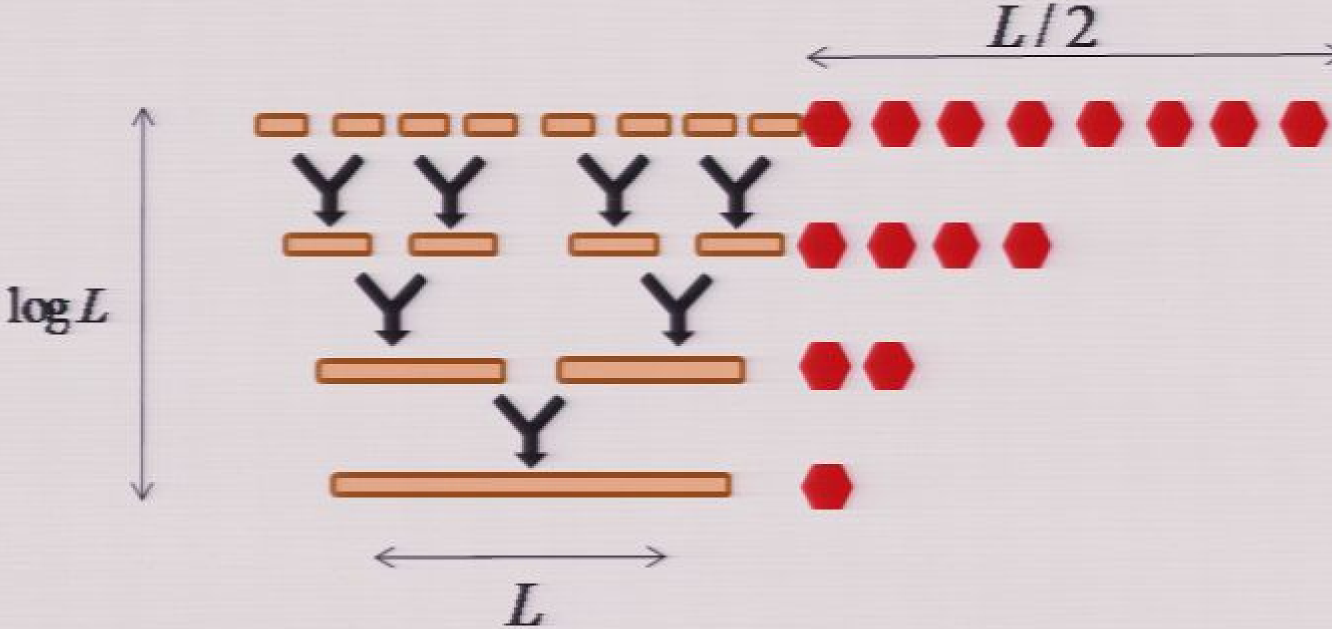


Holographic trees and entanglement entropy

b=2 branching MERA in
D=1 spatial dimensions







contributions to entropy

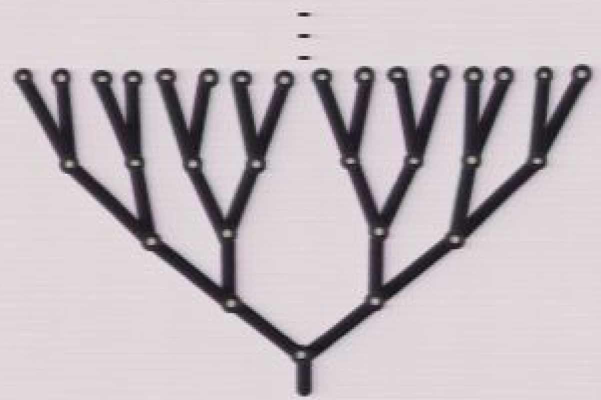
$8 = 2^{\log_2 L - 1} = L/2$

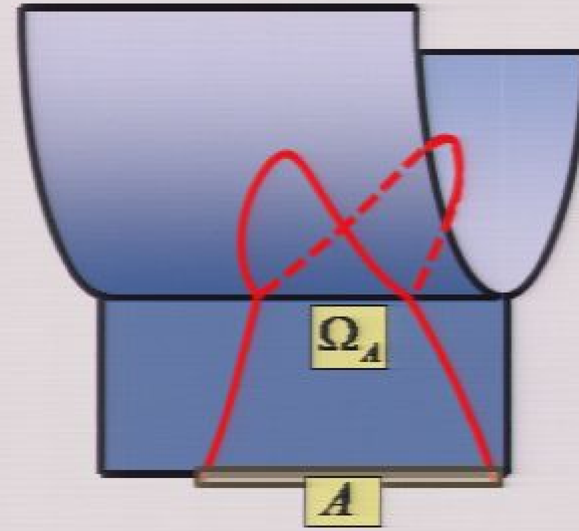
$4 = 2^2 = L/4$

$2 = 2^1 = L/8$

$1 = 2^0 = L/16$

Holographic trees and entanglement entropy





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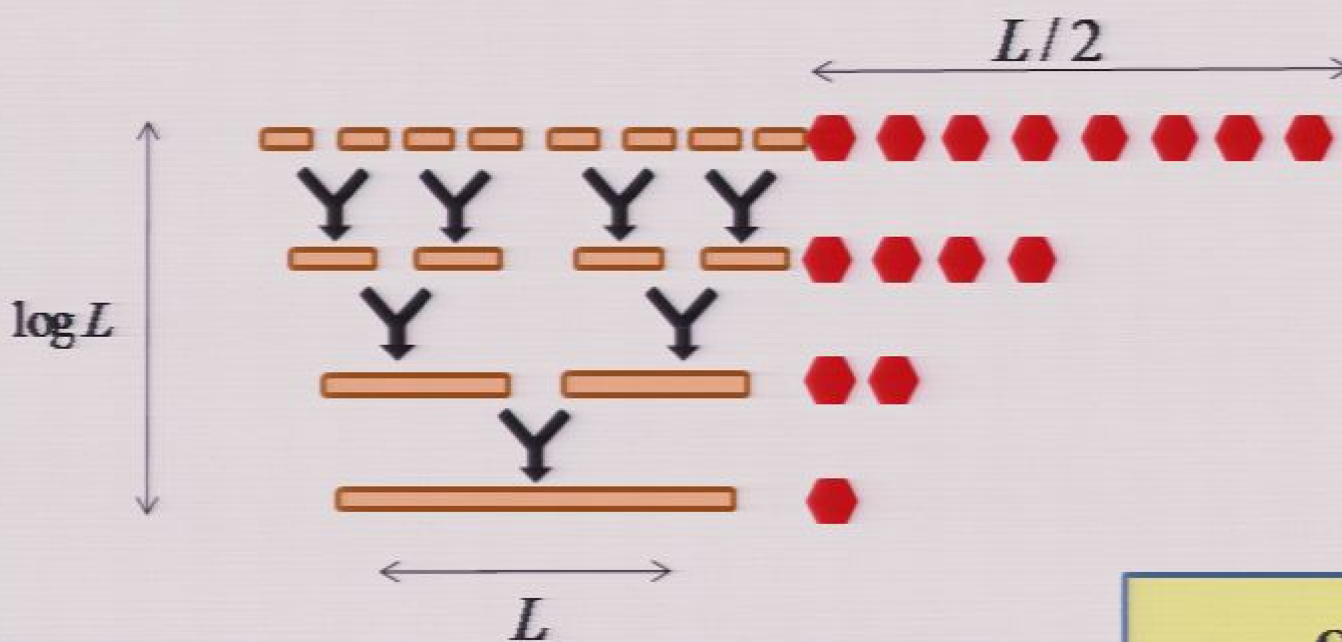

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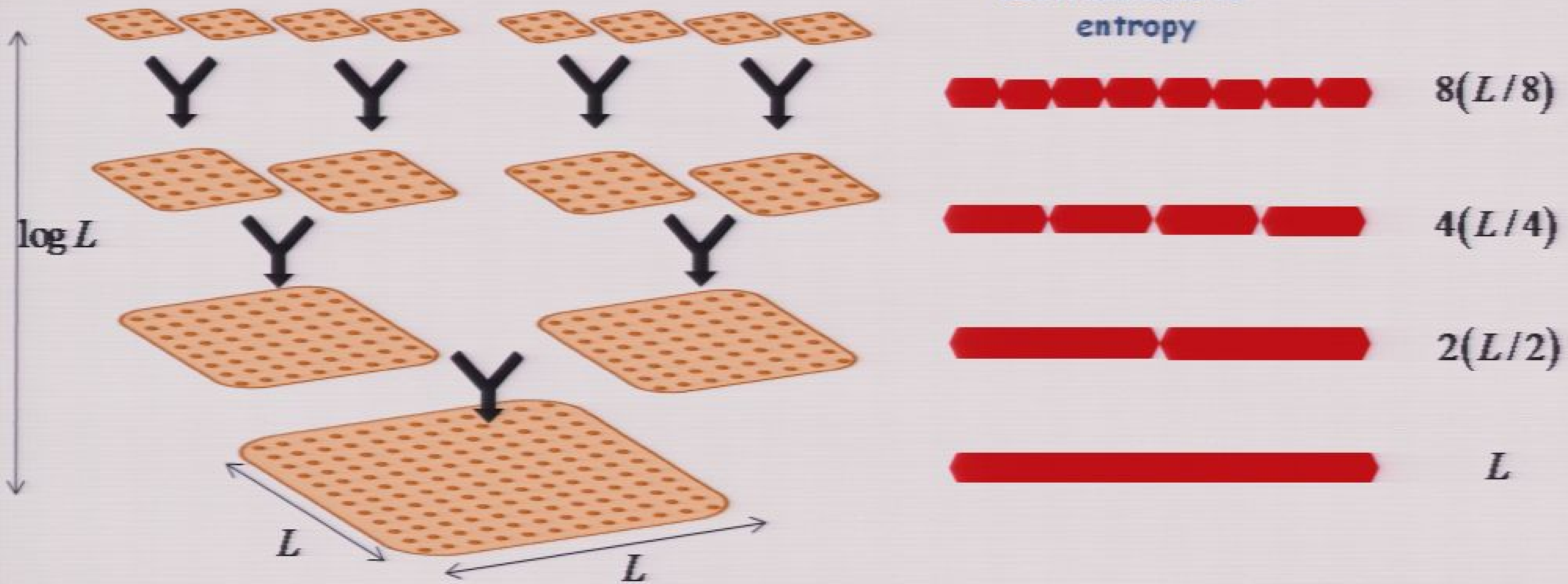
$$S_L \approx L$$

entropic "bulk" law (!)

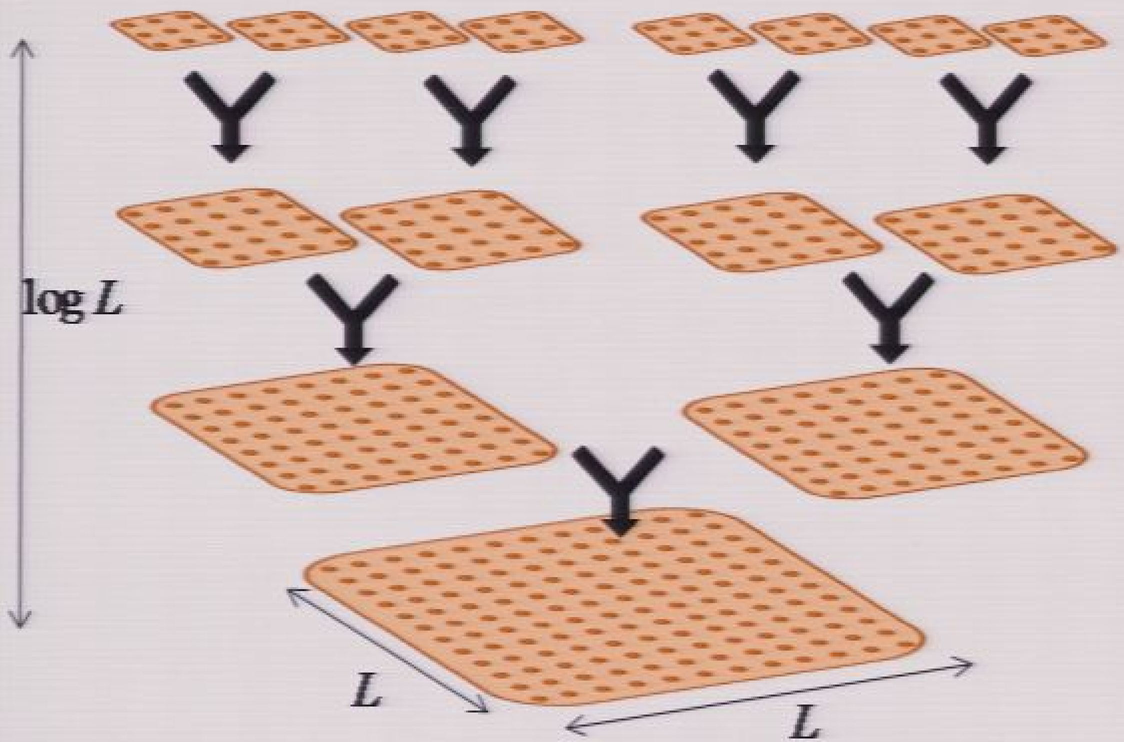
Holographic tree and entanglement entropy

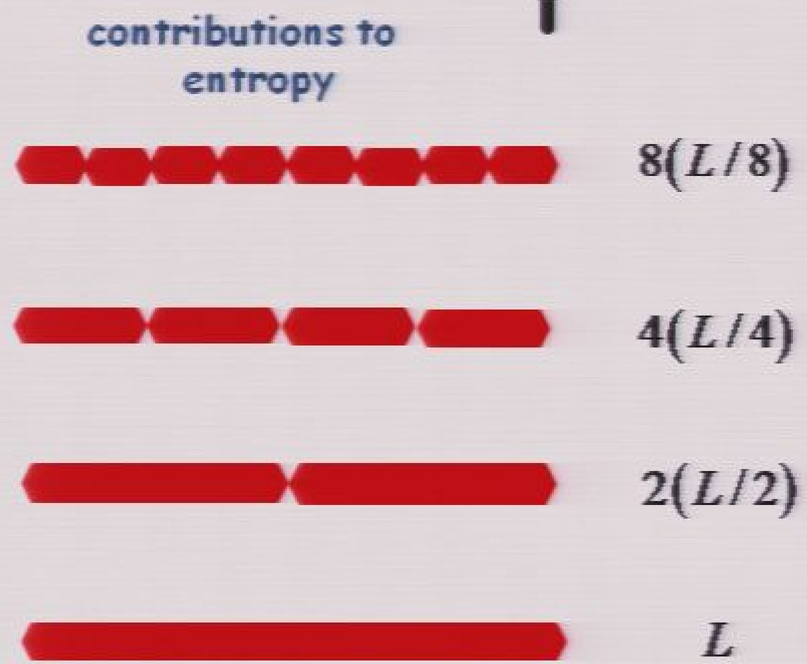
b=2 branching MERA in D=2 spatial dimensions



Holographic tree and entanglement entropy

b=2 branching MERA in D=2 spatial dimensions

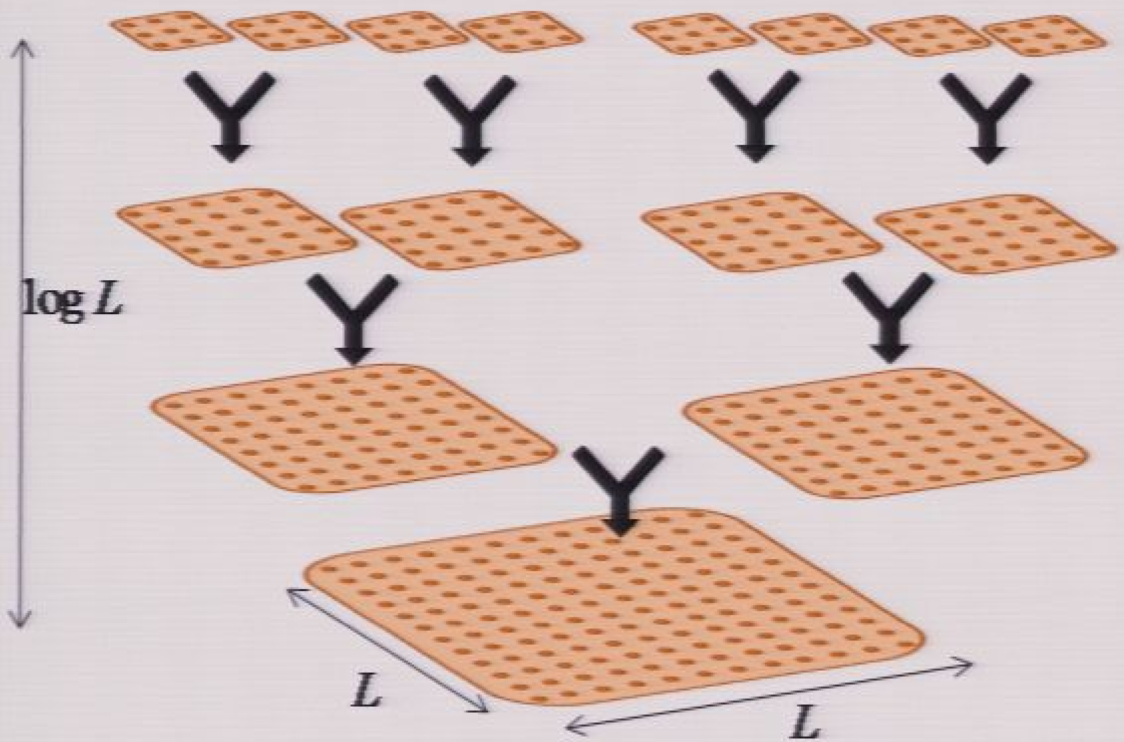


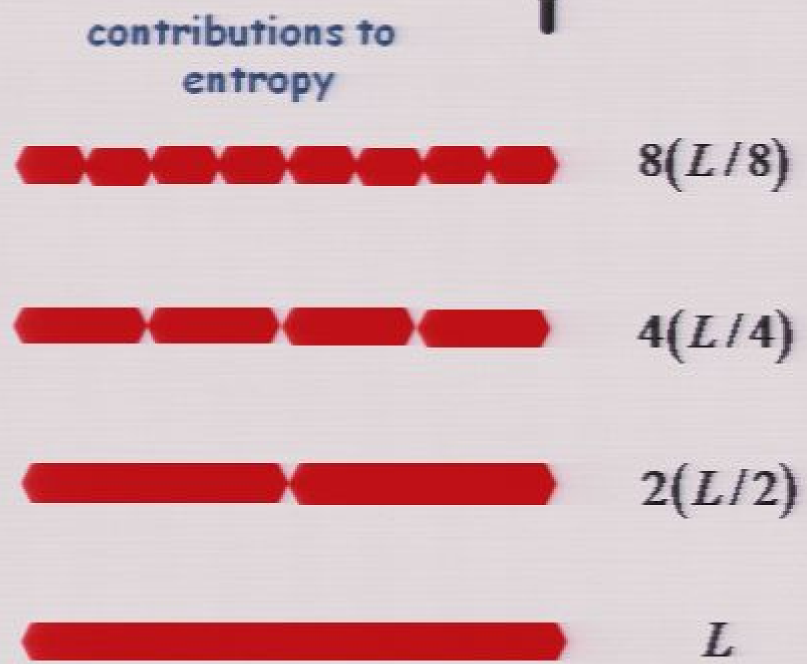


$$S_L \approx \underbrace{L + L + \cdots + L}_{\log L}$$

Holographic tree and entanglement entropy

b=2 branching MERA in D=2 spatial dimensions





$$S_L \approx \underbrace{L + L + \cdots + L}_{\log L}$$

$$S_L \approx L \log L$$

logarithmic violation (!)

Is the (b=2) branching MERA a good ansatz for $S_L = L\log L$ phase??

Example: free fermions in 2D

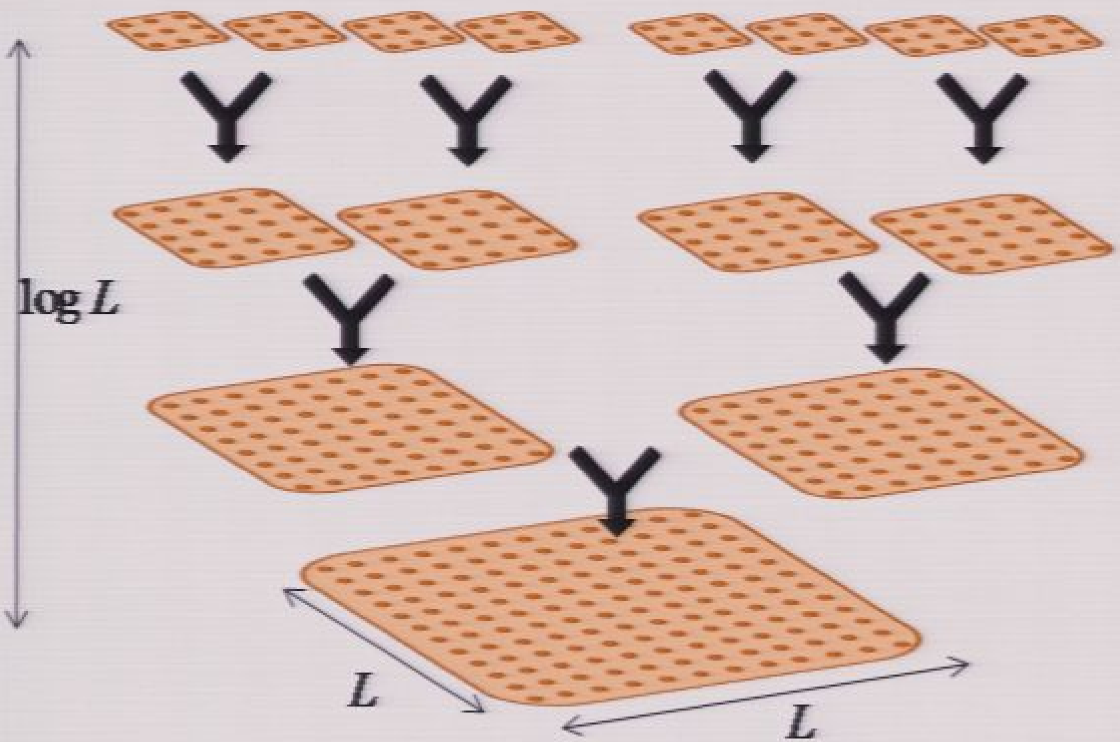
$$H = \sum_{\langle x,y \rangle} (a_x^\dagger a_y + h.c.)$$

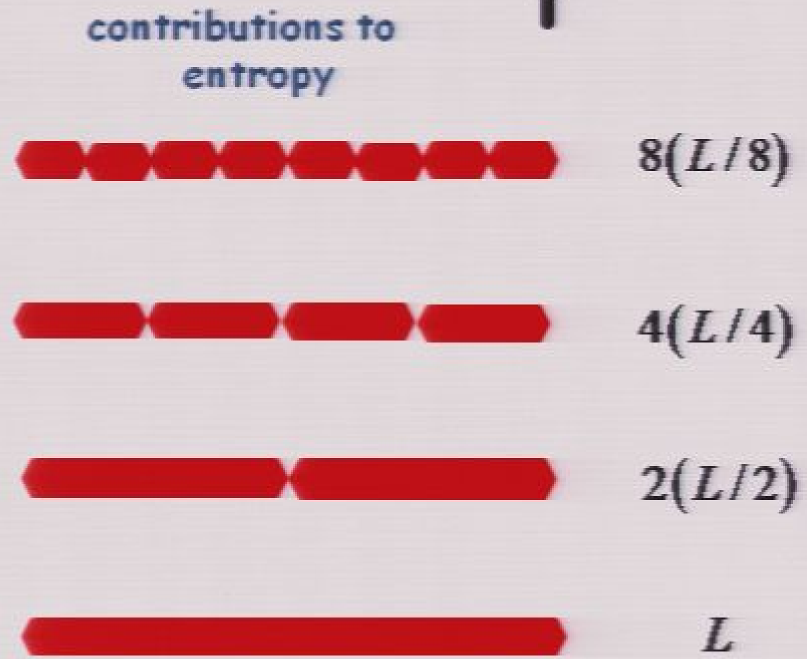
critical model type II
(1D Fermi surface)

$$S_L \approx L \log L$$

Holographic tree and entanglement entropy

b=2 branching MERA in D=2 spatial dimensions





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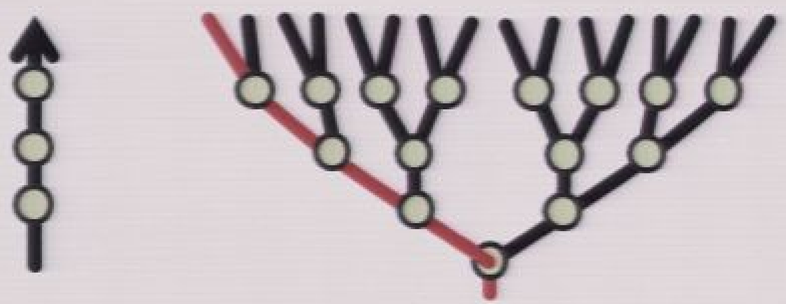

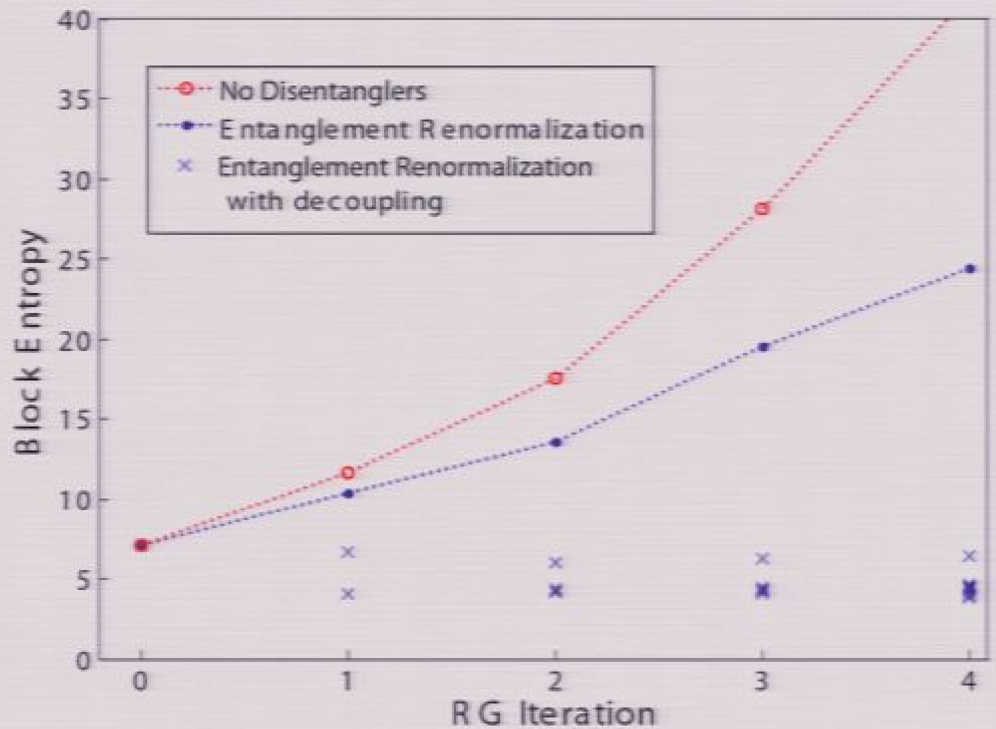

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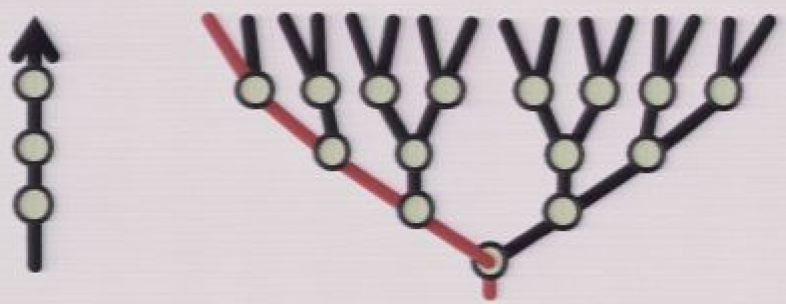
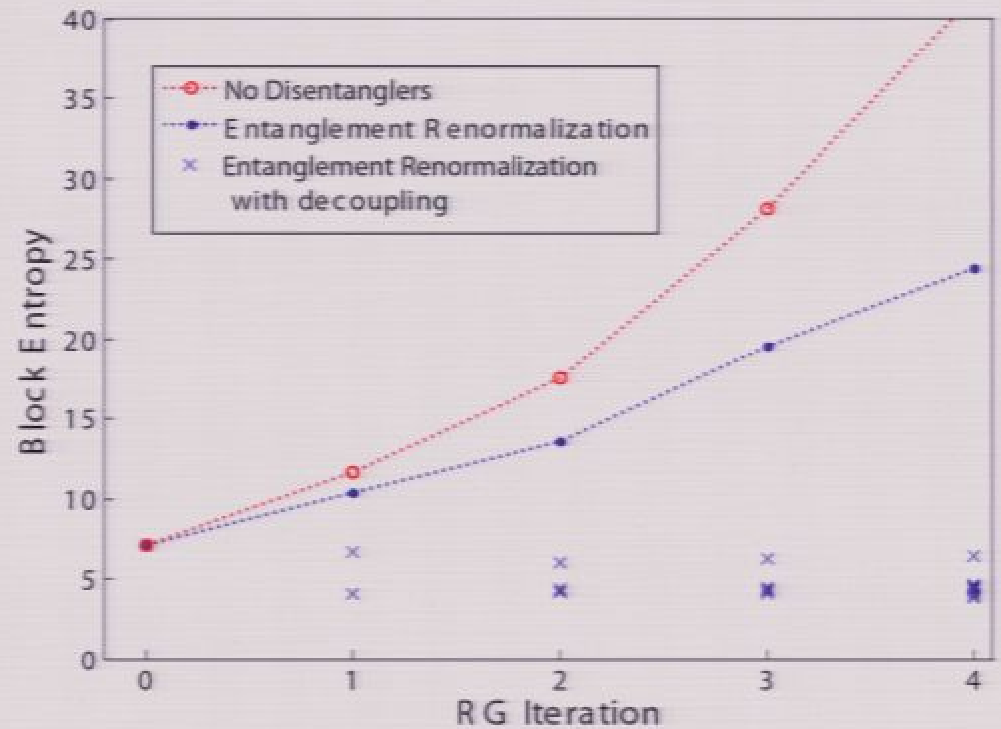




→ **Branching MERA** Evenbly, Vidal, in preparation

	Gap.	Crit.I	Crit.II
2D S_L	L	L	$L\log(L)$

Guifre Vidal

Scaling of entanglement: free fermions vs branching MERA

- Free Fermions:

Dimension of Fermi Surface, Γ

Spatial dimension

	$\Gamma=0$	$\Gamma=1$	$\Gamma=2$	
1D	$\log(L)$			
2D	L	$L\log(L)$		
3D	L^2	L^2	$L^2\log(L)$	

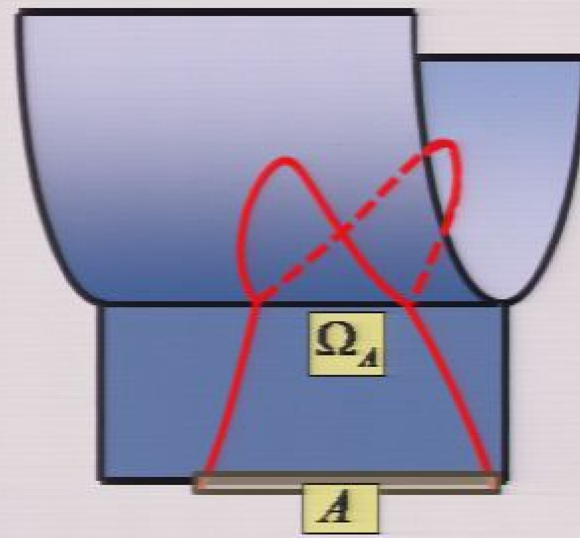
Scaling of entanglement: free fermions vs branching MERA

• Free Fermions:

Dimension of Fermi Surface, Γ

Spatial dimension

	Γ=0	Γ=1	Γ=2	
1D	log(*L*)			
2D	*L*	*L*log(*L*)		
3D	L^2	L^2	L^2log(*L*)	

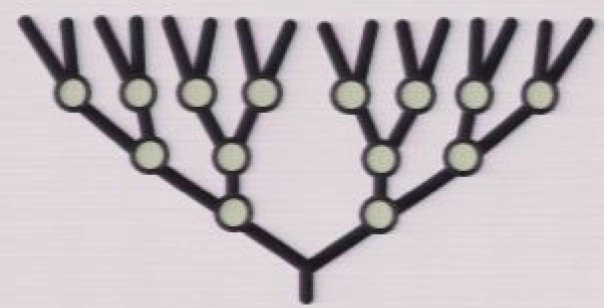


• Regular branching MERA:

Branching Parameter, b

Spatial dimension

	b=1	b=2	b=4	b=8
1D	log(*L*)	*L*		
2D	*L*	*L*log(*L*)	L^2	
3D	L^2	L^2	L^2log(*L*)	L^3



• Proposed relation between dimensionality of Fermi surface and branching parameter:

$$b = 2^{\Gamma}$$

Corrections to the Boundary Law for Entanglement Entropy

$$S_L = L^{D-1} f(L)$$

Boundary Law — Correction

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$$S_L = L^{D-1} f(L)$$

Boundary Law — Correction

$f(L) = 1$

Boundary Law

$b < 2^{D-1}$

$f(L) = \log L$

Logarithmic Violation

$b = 2^{D-1}$

$f(L) = L$

Bulk Law

$b = 2^{D}$

Corrections to the Boundary Law for Entanglement Entropy

$$S_L = L^{D-1} f(L)$$

Boundary Law — Correction

Arbitrary Corrections!

$f(L) = 1$
Boundary Law
b<2^(D-1)

$f(L) = \log L$
Logarithmic Violation
b=2^(D-1)

$f(L) = L$
Bulk Law
b=2^D

Sub-logarithmic
e.g. $f(L) = \log(\log L)$

Super-logarithmic
e.g. $f(L) = (\log L)^{\alpha}$
$f(L) = L^{\alpha}$

Example: 1D branching MERA with $S_L \approx (\log L)^2$

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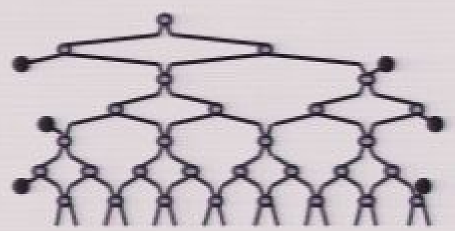

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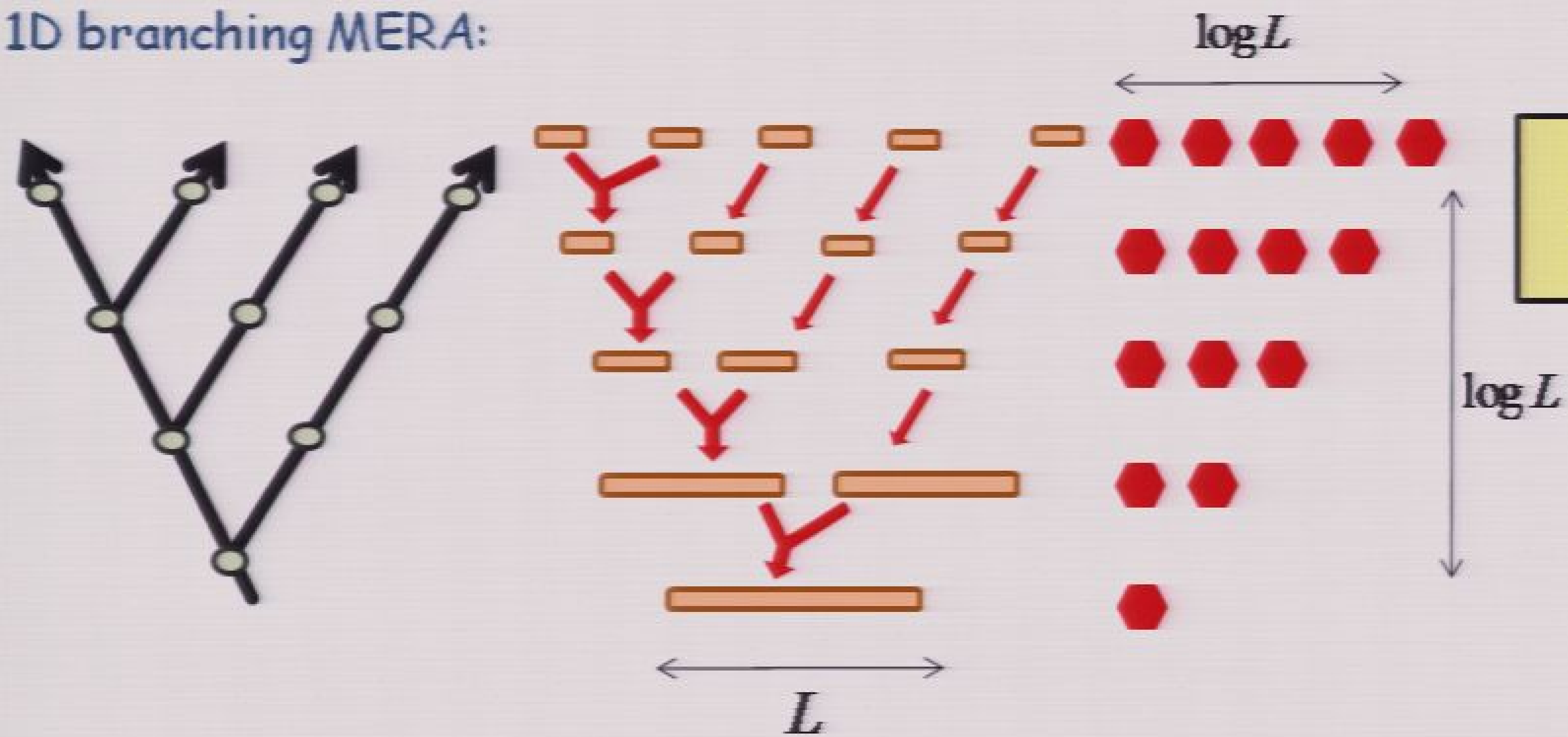
Branching MERA beyond Regular Holographic Trees

1D branching MERA:

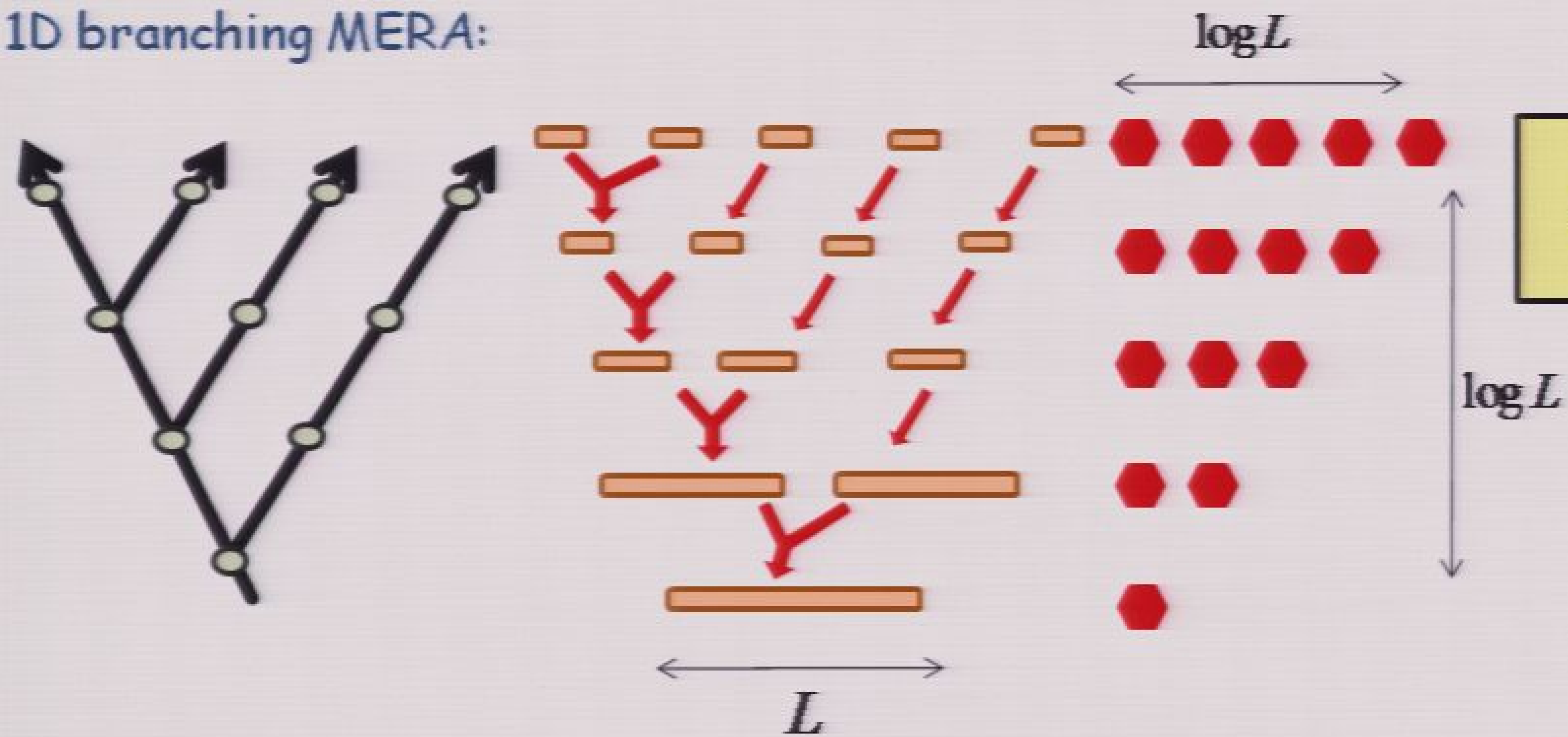


$$S_L \approx (\log L)^2$$

- Can we find a Hamiltonian that has this ground state entropy scaling?

Branching MERA beyond Regular Holographic Trees

1D branching MERA:



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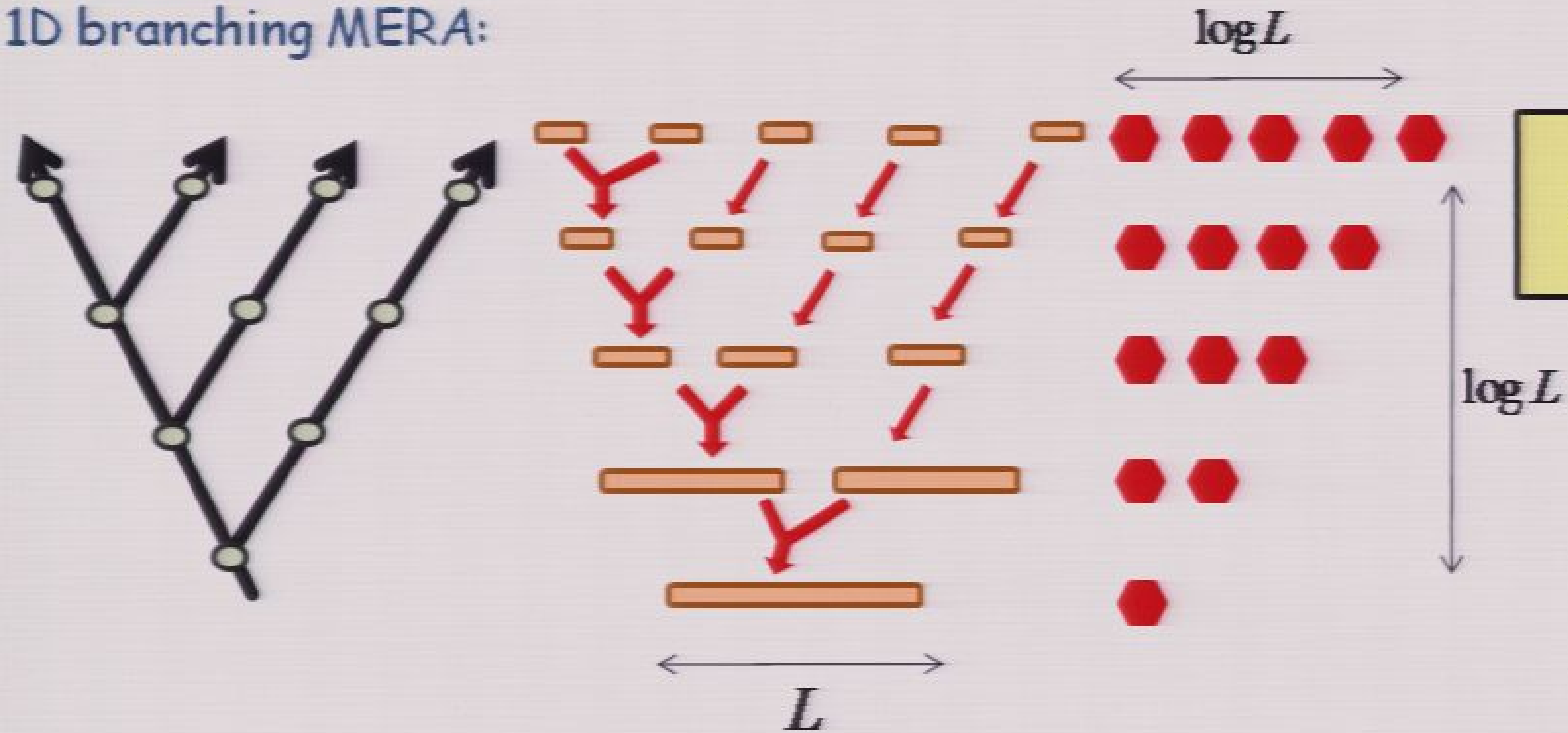


$$S_L \approx (\log L)^2$$

- Can we find a Hamiltonian that has this ground state entropy scaling?

Yes!

$$H = \sum_{r=-\infty}^{\infty}\left(\sum_{\substack{d=-\infty \\ d\neq 0}}^{\infty}\frac{\phi(d)}{d^2}\left(\hat{a}_{r+d}^{\dagger}\hat{a}_r + \hat{a}_r^{\dagger}\hat{a}_{r+d}\right)\right) - \mu\sum_r \hat{a}_r^{\dagger}\hat{a}_r$$

$$\phi(d) \approx \cos\left(\log_2 |d|\right)$$

Branching MERA beyond Regular Holographic Trees

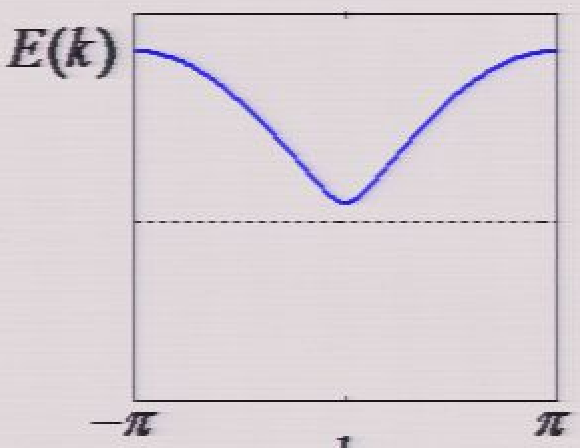
Holographic Tree:

Gapped Ising

Hamiltonian:

$$H = \frac{1}{2}\sum_r \left(\hat{a}^\dagger_{r+1}\hat{a}_r + \hat{a}^\dagger_r\hat{a}^\dagger_{r+1} + h.c.\right) - \lambda\sum_r \hat{a}^\dagger_r\hat{a}_r$$

Dispersion:



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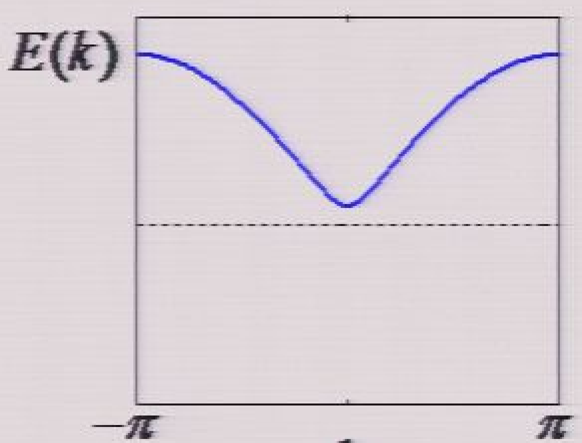
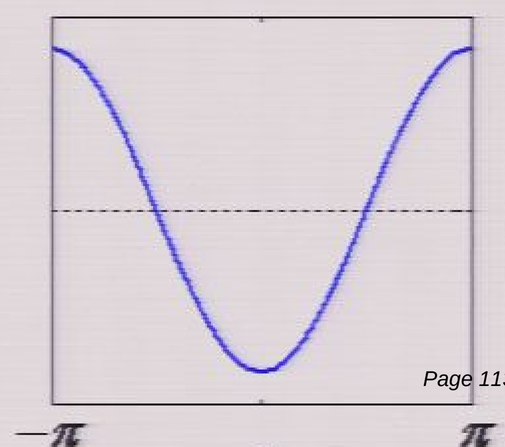
Critical XX

$$H = \frac{1}{2}\sum_r \left(\hat{a}^\dagger_{r+1}\hat{a}_r + \hat{a}^\dagger_r \hat{a}_{r+1}\right)$$

Dispersion:





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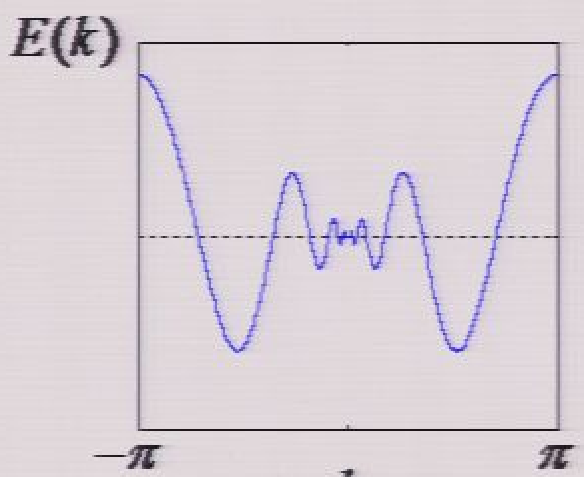
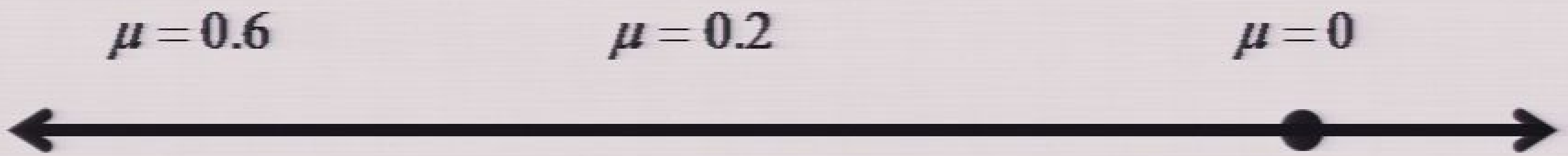
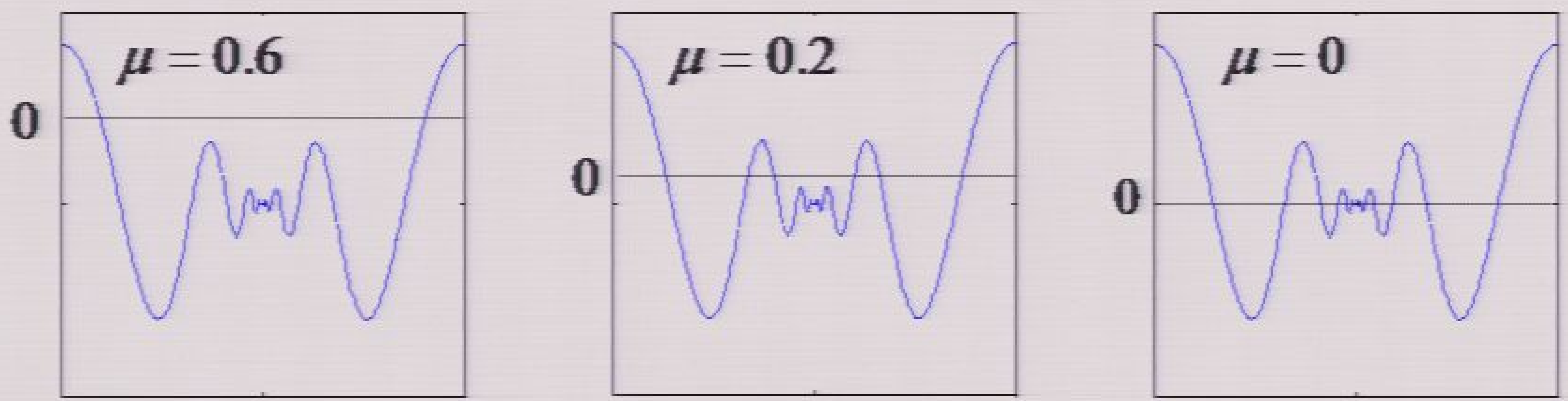


$$E(k)=\left|\sin\left(\frac{k}{2}\right)\cos\left(\pi\log_2\left|\frac{\pi}{k}\right|\right)\right|$$

hemical Potential:





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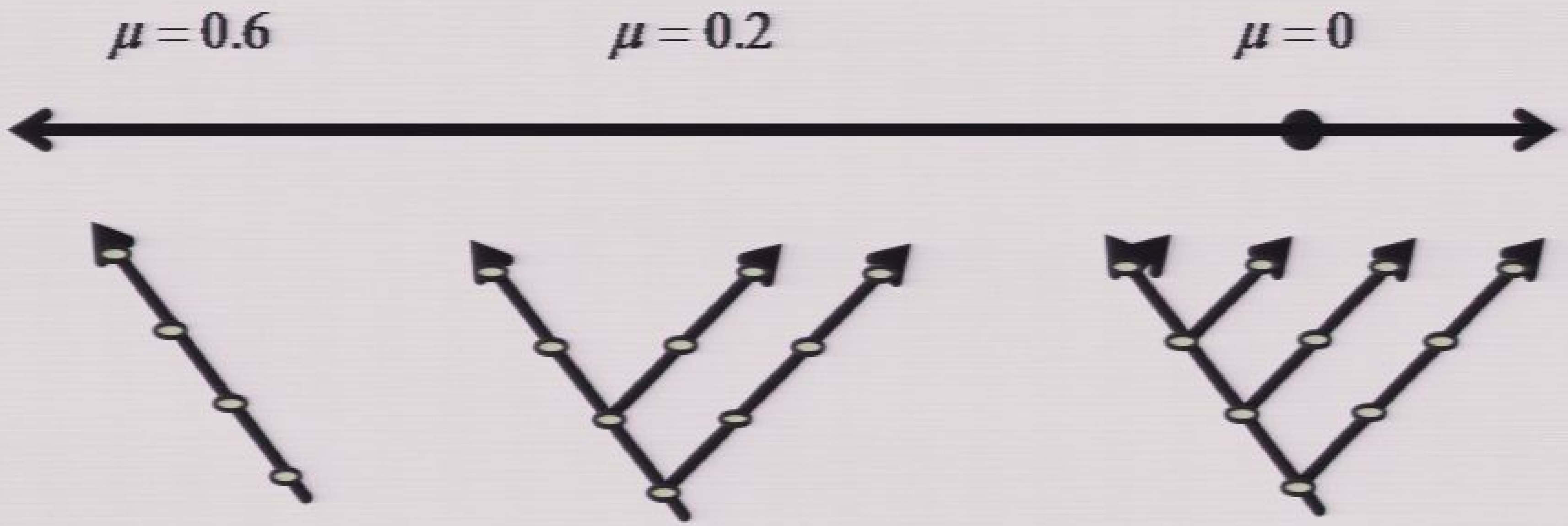

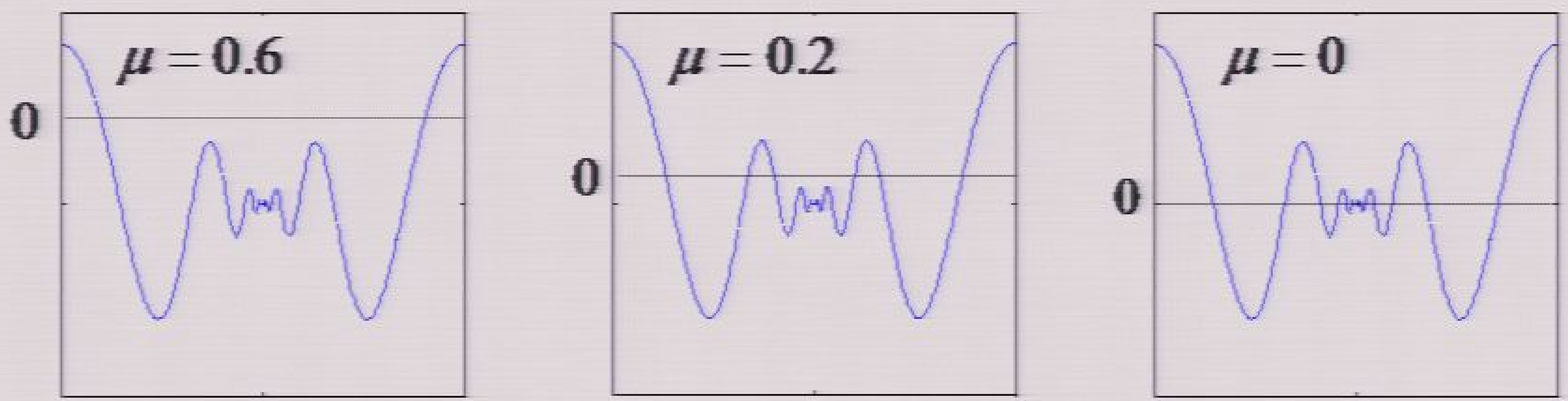

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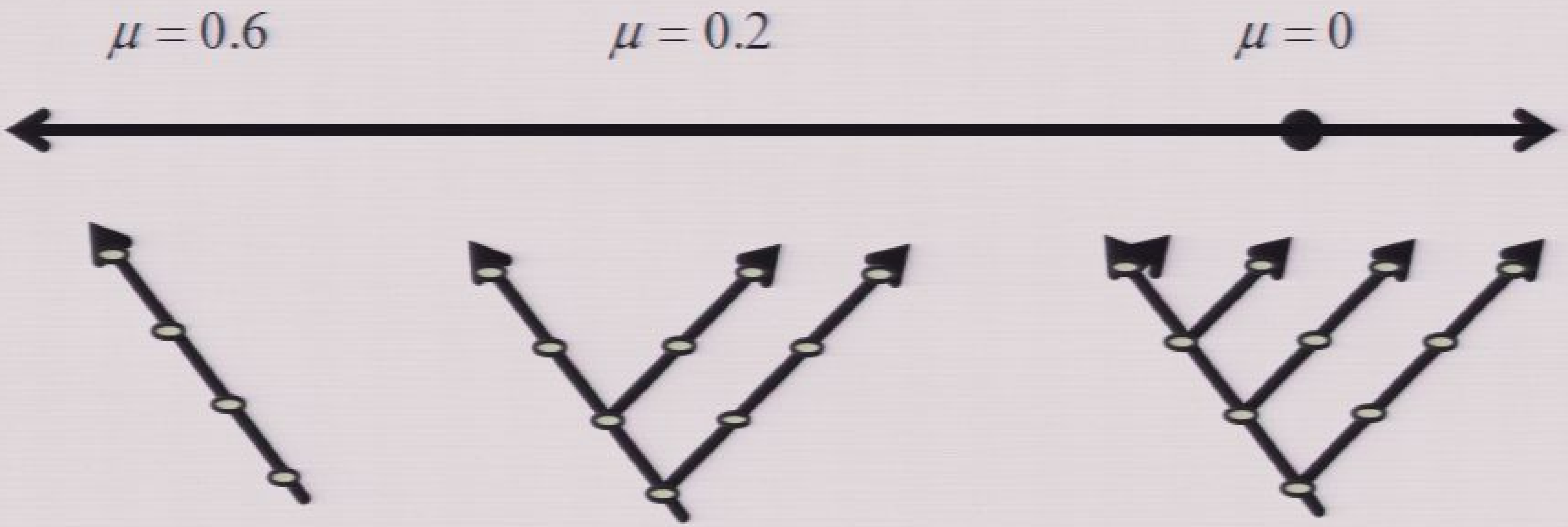

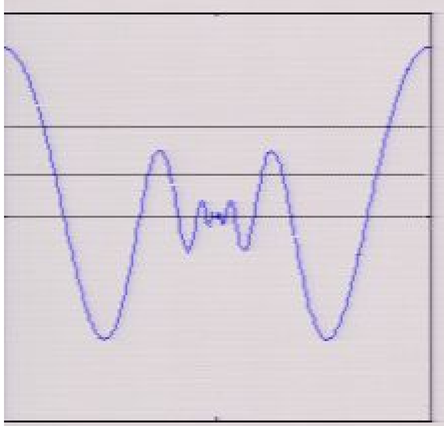

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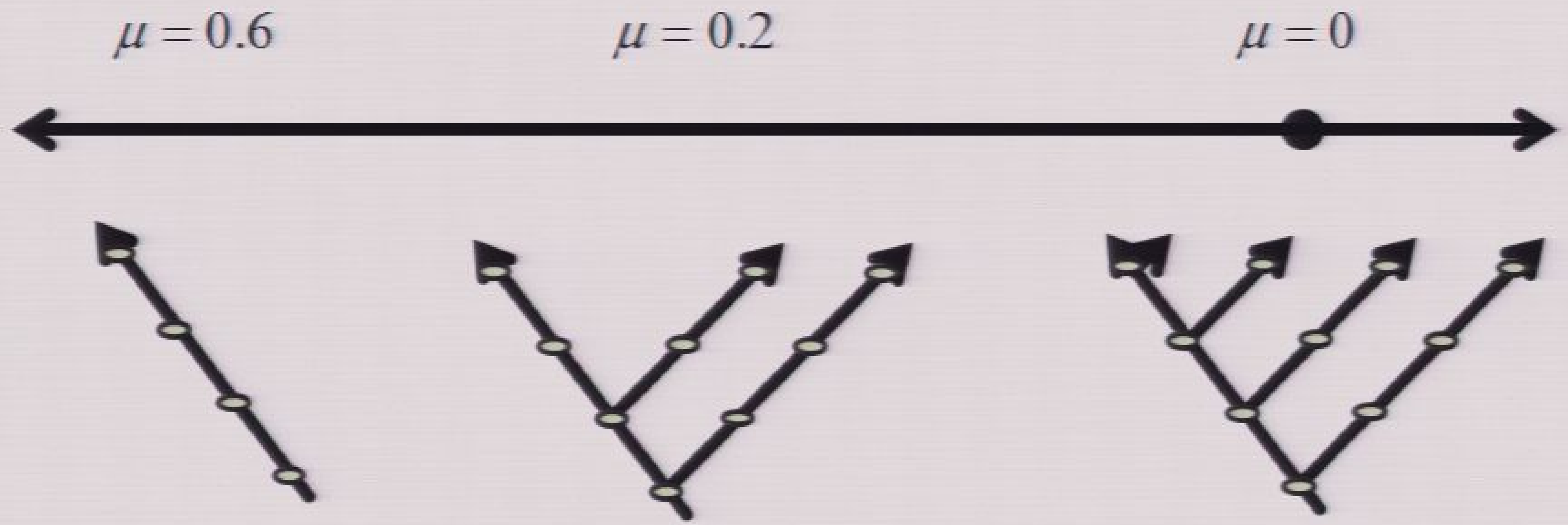

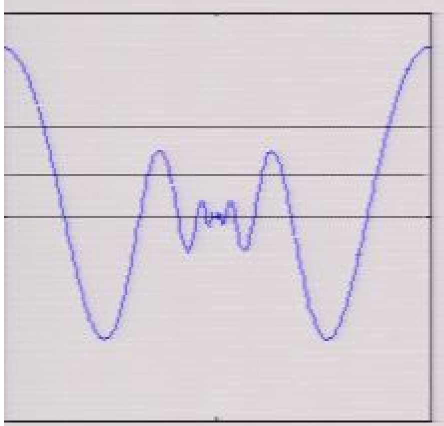

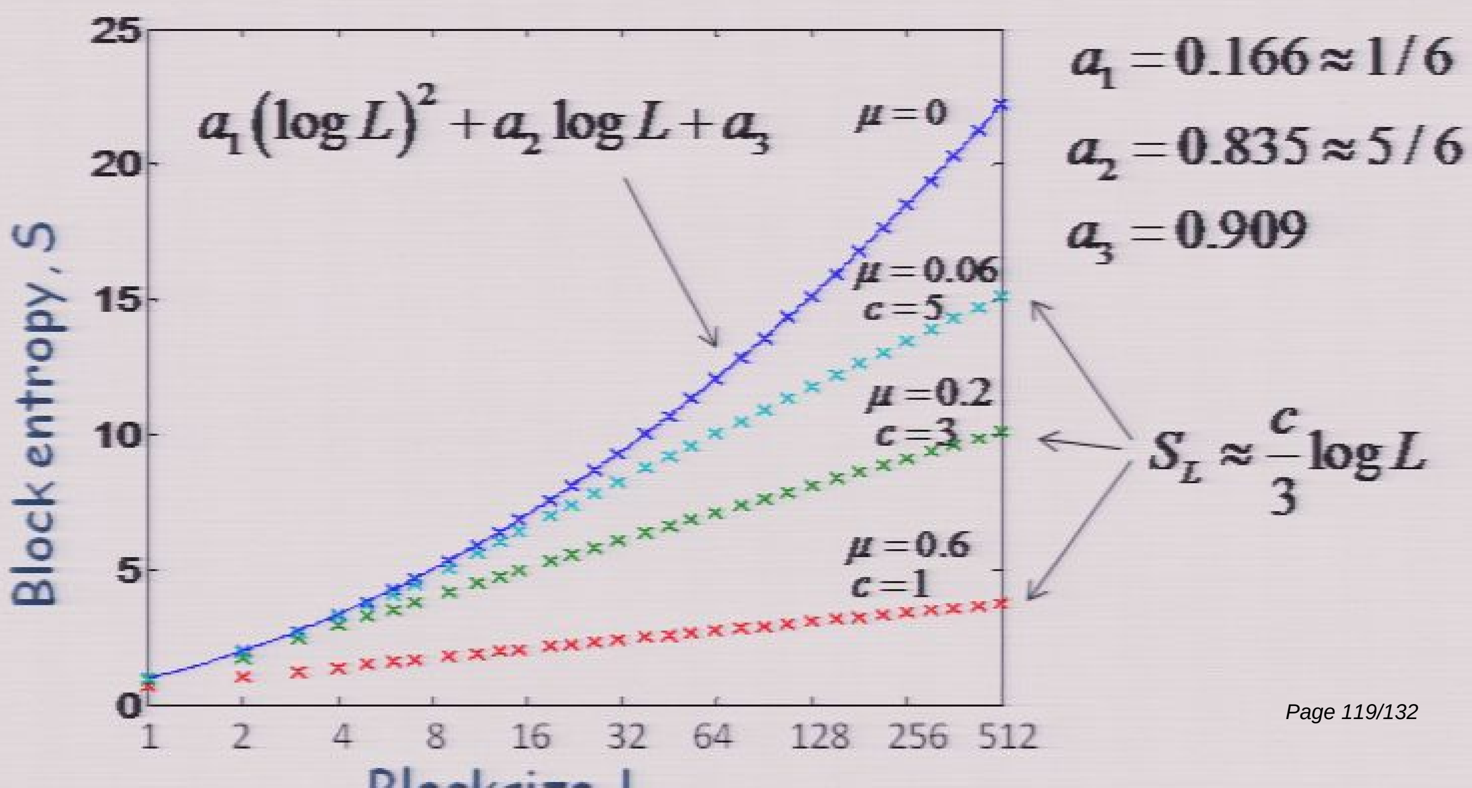

Phase Transition in D=1 Free Fermions

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$\mu < 0$	$\mu = 0$	$\mu > 0$
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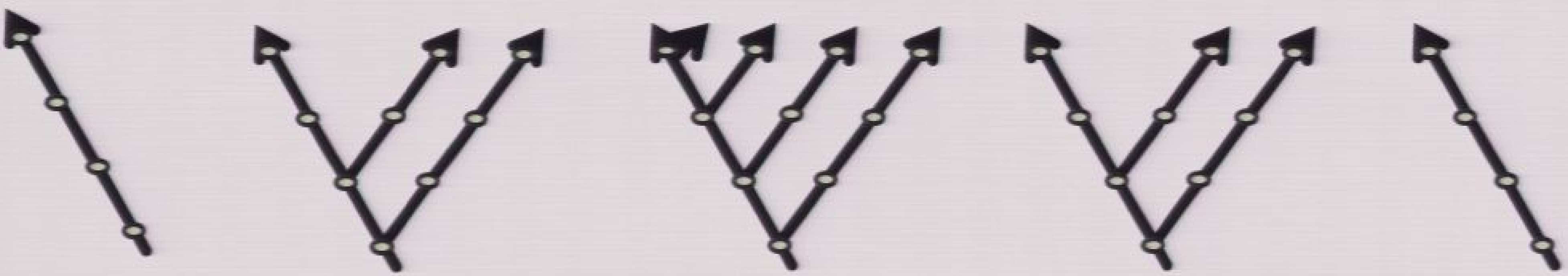

Summary/conclusions

entanglement renormalization → **MERA** ⇔ **holographic geometry**

(coarse graining transform)

(=spatial pattern of entanglement)

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new! ↓

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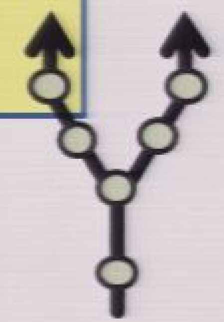

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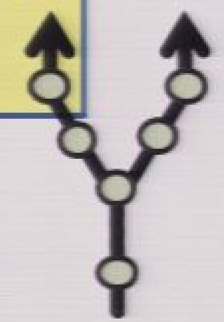

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- a formalism to explicitly factorise the theory into several theories
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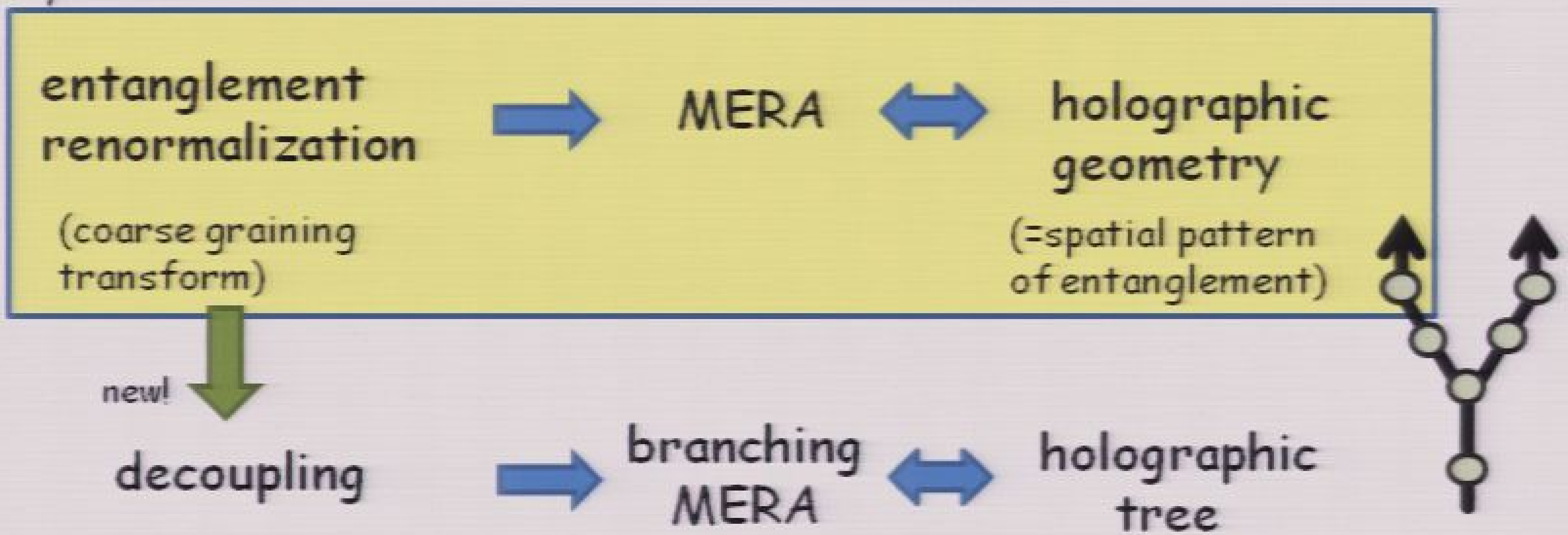
Summary/conclusions



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- an efficient ansatz for critical phases beyond reach of MPS/PEPS
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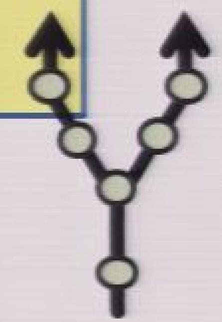

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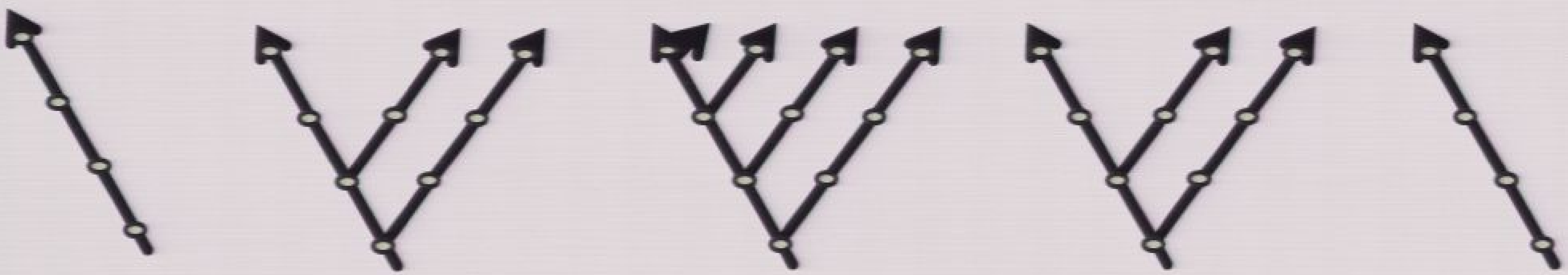

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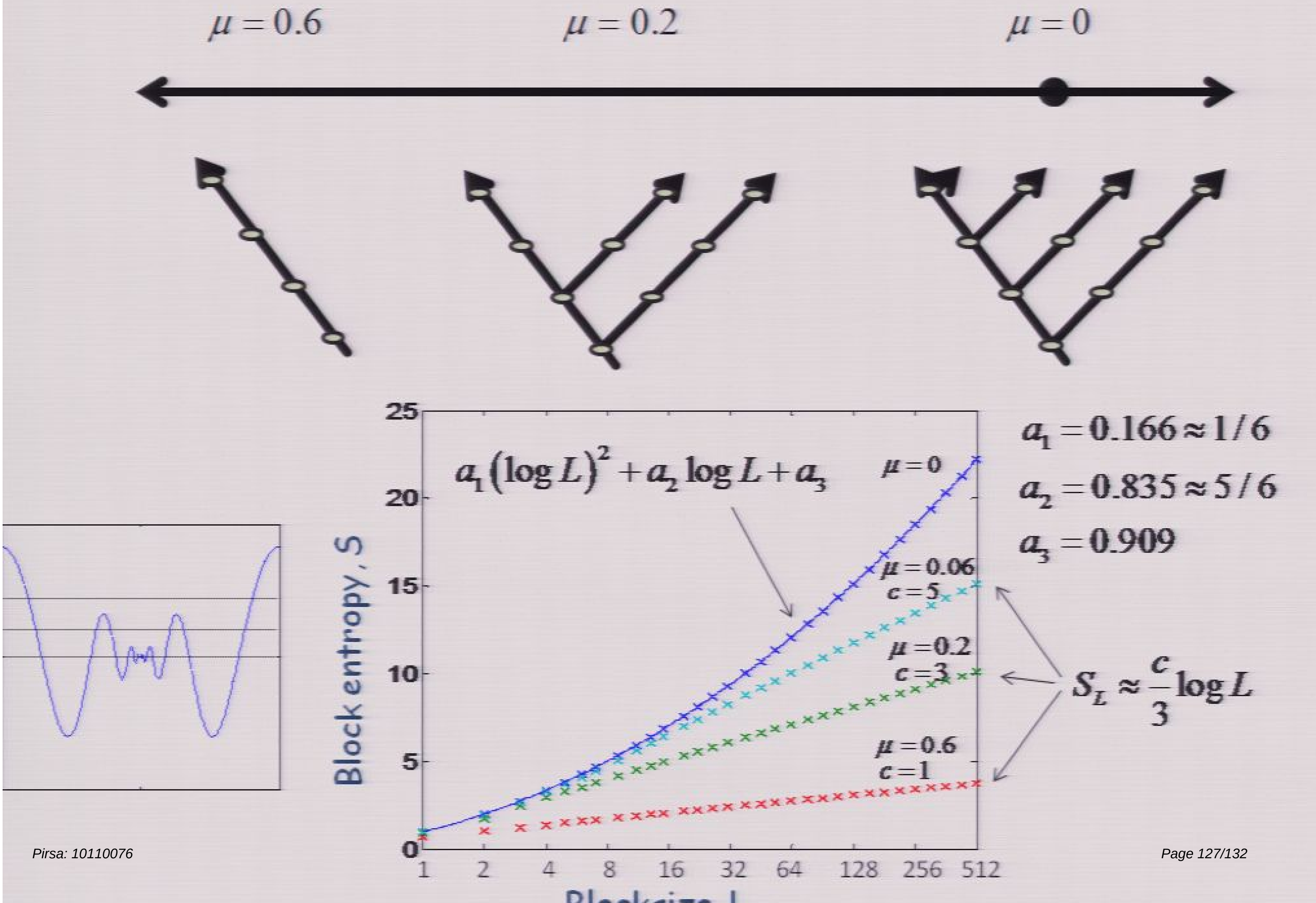


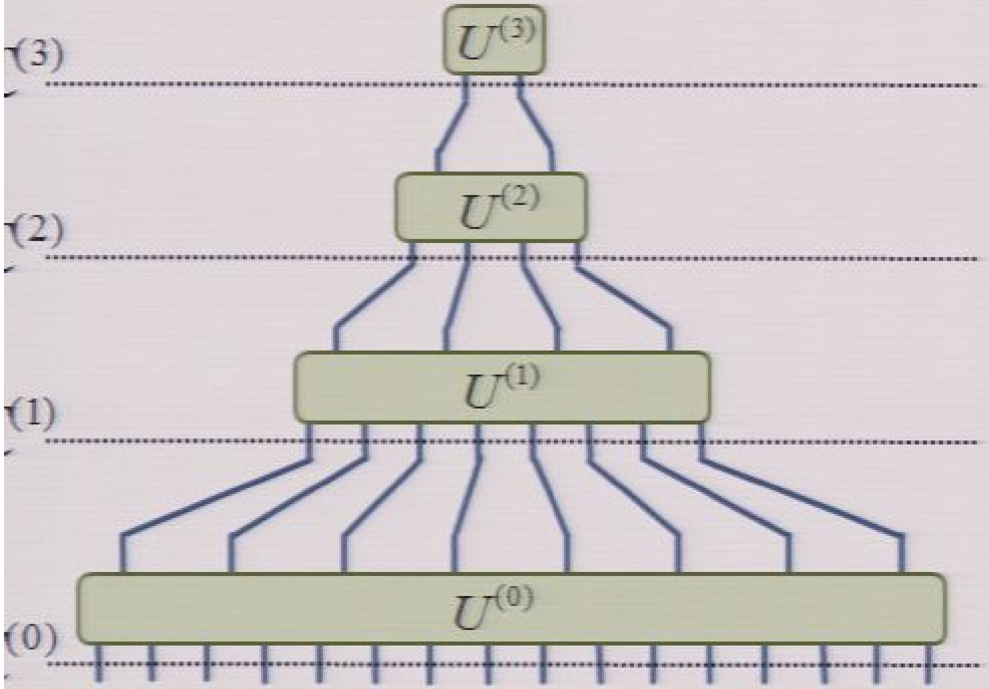
Corrections to the Boundary Law for Entanglement Entropy

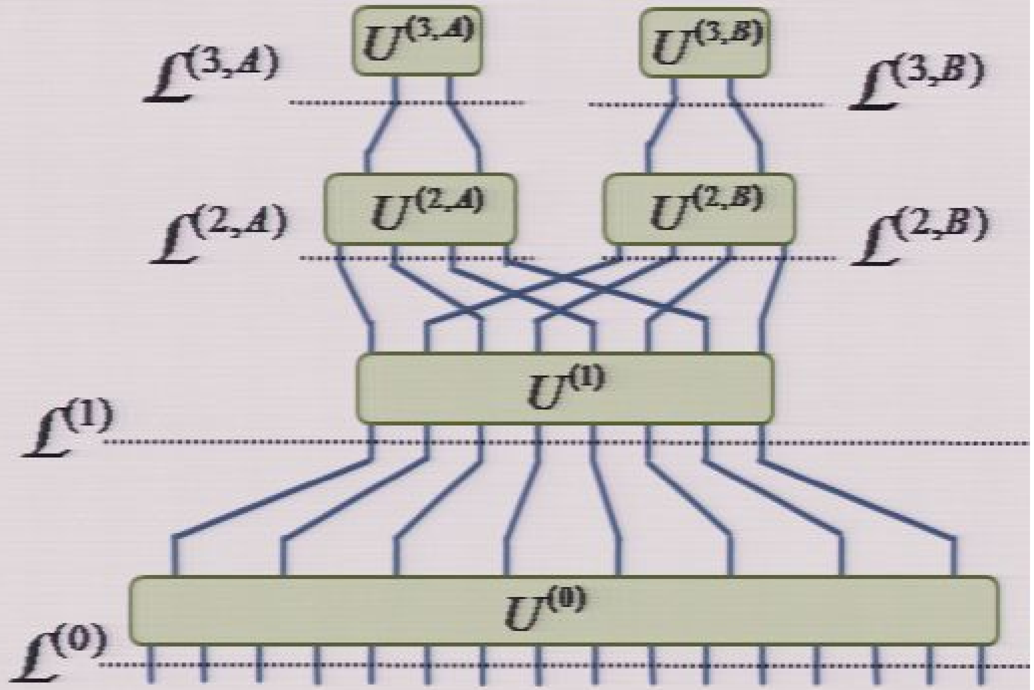
$$S_L = L^{D-1} f(L)$$

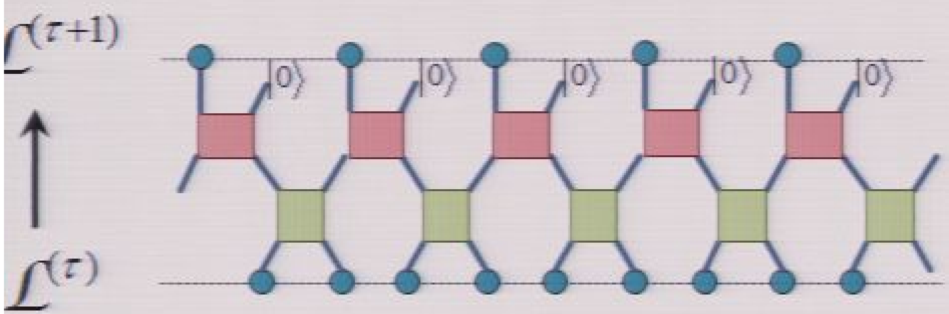
Boundary Law — Correction

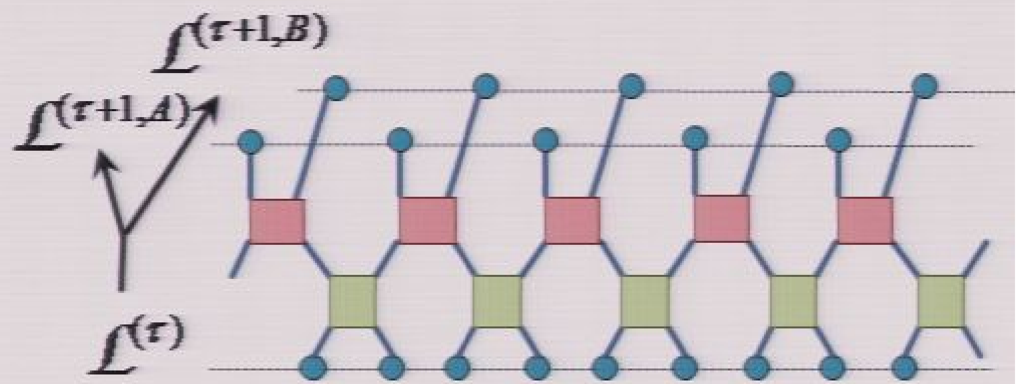
- Entanglement renormalization in the presence of decoupling

branching MERA









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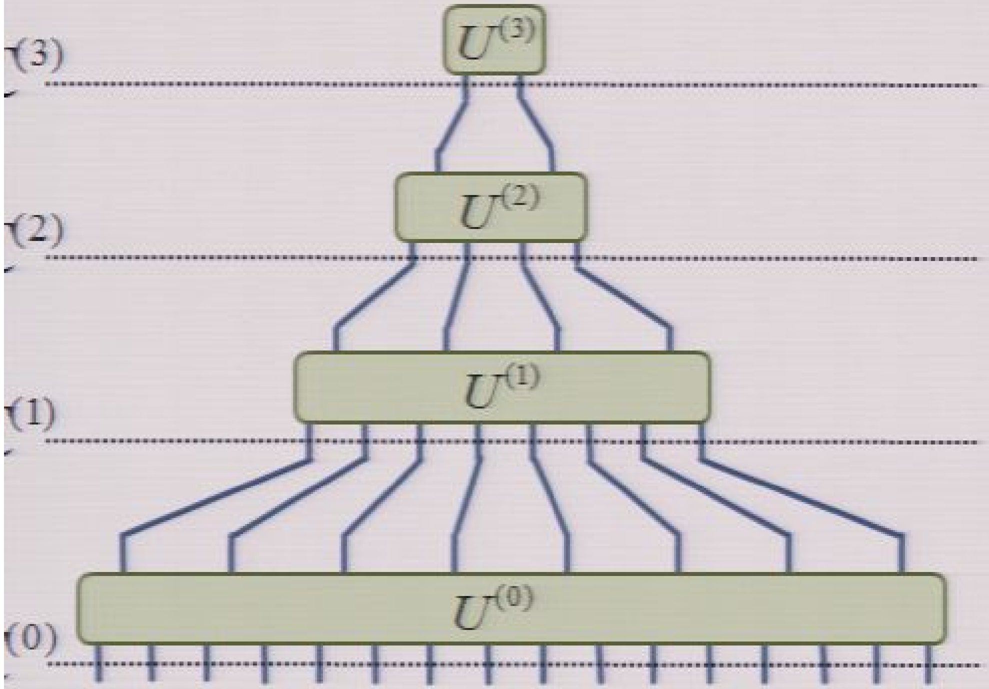

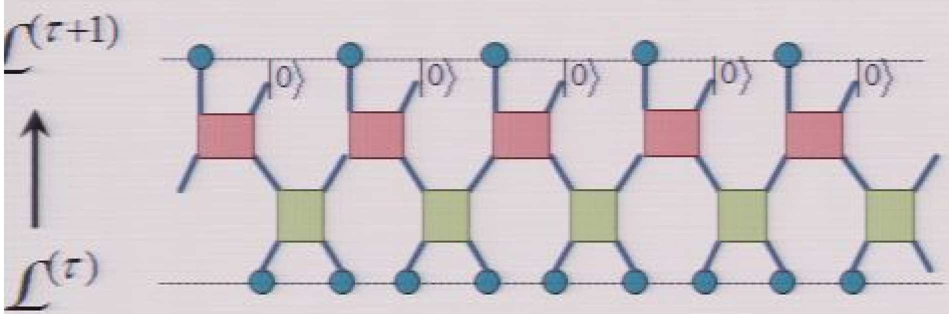

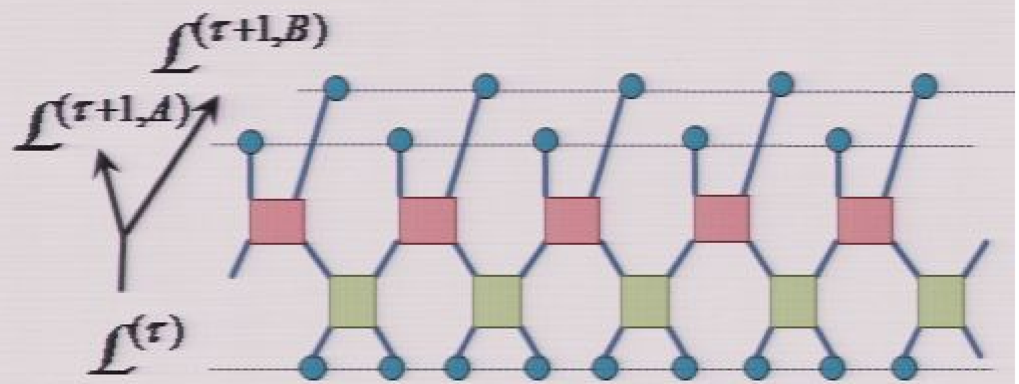

Scaling of entanglement: free fermions vs branching MERA

• Free Fermions:

Dimension of Fermi Surface, Γ

Spatial dimension

	Γ=0	Γ=1	Γ=2	
1D	$\log(L)$			
2D	L	$L\log(L)$		
3D	L^2	L^2	$L^2\log(L)$	

Scaling of entanglement: free fermions vs branching MERA

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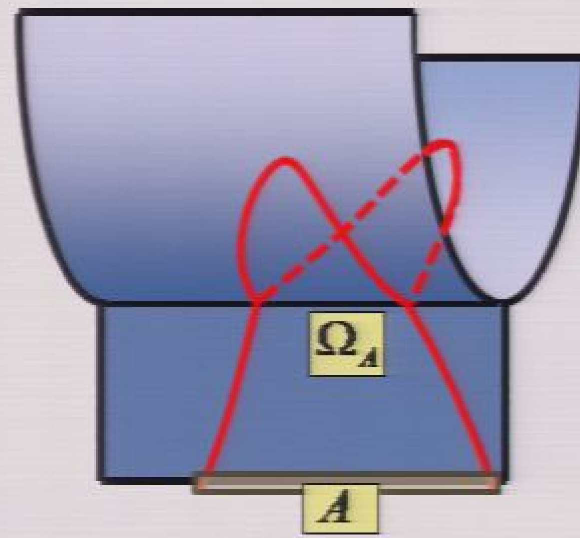
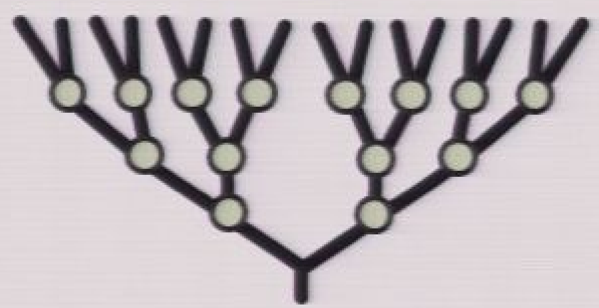


• Regular branching MERA:

Branching Parameter, b

Spatial dimension

	b=1	b=2	b=4	b=8
1D	log(L)	L		
2D	L	Llog(L)	L^2	
3D	L^2	L^2	L^2log(L)	L^3



• Proposed relation between dimensionality of Fermi surface and branching parameter:

$$b = 2^{\Gamma}$$